AMERICAN UNIVERSITY OF BEIRUT

EFFECTIVENESS OF DISCOUNT VOUCHERS: A THEORETICAL MODEL

by

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AN ABSTRACT OF THE PROJECT OF

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Title: Effectiveness of Discount Vouchers: A Theoretical Model

The purpose of this project is to examine the effectiveness of online discount vouchers, a marketing tool that has been gaining a lot of popularity throughout the past few years. Customers who purchase these vouchers are effectively subscribing to buy a certain product or service at a large discount. However, the discount will not be effective unless a certain threshold of subscribers is reached. The behavior of the customers is modelled taking into account the quality perception that those customers have towards the product, in addition to the distance that separates them from the seller. The model also incorporates the human interaction factor, whereby subscribers will act as sales persons and try to convince other people to subscribe as well, in order to make sure that threshold of subscribers is met, thus benefitting from the discount. Despite room for enhancement in the model, preliminary results suggest that online discount vouchers become more effective as the quality of the product increases, as well as the importance of quality in the eyes of the customers increases.

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CHAPTER I

INTRODUCTION

In the past few years, the use of discount vouchers has gained a lot of popularity in the world of e-commerce. There are several websites that sell discount vouchers. Consumers who buy these vouchers are buying the option to purchase a certain product or service at a large discount, typically around 50%. Vendors benefit from the popularity of the websites gain social exposure on their promotions. Groupon is one of the most popular companies offering such a service.

Vendors usually offer such promotions to increase sales through the combination of two main factors. The first one is that sellers that decide to use discount voucher benefit from the expertise of the voucher service companies, who already have a lot of experience in this domain. The second factor is that the decrease in price should increase the sales at the expense of lower margins, hoping that there are enough customers who would come back in the future and buy the product at full price.

In this research we shed some light on the effectiveness of discount vouchers by analyzing a two-period model that incorporates the following features:

1. The customer values the product based on the quality perception and the distance. The distance can be the physical location of the customer (e.g. a customer living far away from a coffee shop will value less the coffee less than someone living next door of the coffee shop, even if they perceive the same quality.

2. The customer would consider putting efforts seeking additional information about the product and improve the perception of the product's quality. They would possibly spend efforts convincing others with lower quality perception to buy the product as well.

The remaining chapters in this work are distributed as follows. In chapter 2, we list previous research that is relevant to vouchers and group buying, in also point out the contributions of this work and compare it to the literature. In chapter 3, we introduce the basic model, then we incorporate the social aspect of the group buying mechanism in chapter 4. In chapter 5, we discuss the results of the simulations we performed based on the model. Moreover, we present few concluding recommendations. We present the limitations of the model in chapter 6 where we also point out possible extensions of this work.

CHAPTER II

LITERATURE REVIEW

The popularity of discount vouchers has encouraged many scholars and business people to try to find the winning formula (number of coupons issued, percentage of original price to be discounted, and the period through which the discount is available) in using this kind of promotion. Extensive research has already been done to determine the relationships between the characteristics of the promotion and the reaction of the consumers. Based on previous research, several recommendations have been proposed for vendors who wish to use discount vouchers for their promotions, and also to the websites who offer such service.

The empirical research has studied the impact of several factors that influence the demand for the discount vouchers. For instance, Buyers et al. (2011) concluded that the demand for coupons is inelastic and that it could be impacted by other factors such as deal durations and limited inventory. Dhalokia (2010) in his research, found correlation between the profitability of using discount vouchers from the perspective of the merchant and a list of factors. These factors included the amount of money spent by the coupon buyers in the store, the probability of their return for a second purchase at full price, in addition to the employee satisfaction concerned with management opting to use discount vouchers. Further research supported the same findings (Dhalokia, 2011).

Other research indicated that the use of coupons by the seller is effective in bringing in new customers. It also indicated that users of coupons are likely to recommend options to others (Kimes & Dhalokia, 2011). The same research also indicated that coupon buyers are not necessarily “deal seekers” and that the non users mainly did not buy coupons for the reasons of unawareness about the availability of these vouchers (Kimes & Dhalokia, 2011).

Although the empirical research has been almost exhaustive, the theoretical work has been very limited (Edelman, Jaffe & Kominers, 2011). Up to this point we were able to find three theoretical works that are related to this research. The first one was done by Arabshahi (2011), however his research has taken the perspective of the voucher service and not that of the seller as is the case in our research.

Edelman et al. (2011) suggested a model that studies the profitability of discount vouchers for the seller. Their repeat experience model focuses on two mechanism that could influence the profitability, which are advertising and the ability to price discriminate among customers. Although their model assumes minimal formalism in capturing the functionality of the factors intended to be studied, some of the settings of our model were inspired from this work and build upon. The main difference between our model and theirs is the social impact component.

Another theoretical research from which we benefitted in building our model, was the work done by Jing and Xie (2011). In their work they modelled social interaction between customers, through which information is transferred from informed customers to the uninformed one. Our model is different from theirs, yet our social impact component was inspired from their work, further details are presented within the text.

We suggest a model that incorporates quality and distance. We introduce the possibility of customers learning and improving their perception of the product. Finally, we model the possibility of social interaction whereby convinced customers can spend some efforts convincing others with lower quality perception of the product promoted.

CHAPTER III

BASIC MODEL

The main objective of this research is to determine whether the use of discount vouchers is effective or not. In the cases where it turns out to be effective, the research also provides recommendations on some of the characteristics of discount vouchers.

A. The Settings

To simplify the analysis, we consider a two-period model, during which the customers are buying a certain product or service from the seller at a certain price. During the first period, the seller has the possibility to issue discount vouchers. In the case where the seller decides to use the vouchers, she must first reveal the characteristics of these vouchers, that will be discussed in details in the next chapter. One of these characteristics is the value of the discounted price. However, in the case where vouchers are not used, the seller will only have to optimally set the price of the product.

In the second phase, there will be no discount vouchers, no matter what the seller's decision was in the first period. In this phase the seller will have to adjust the price in a way that is complementary to the pricing policy that was started in the first period. The process is illustrated in Figure 1.

B. Customer Characteristics

In the model we take into account two aspects that influence customers' valuation towards a certain product. The first aspect is the quality perception that a

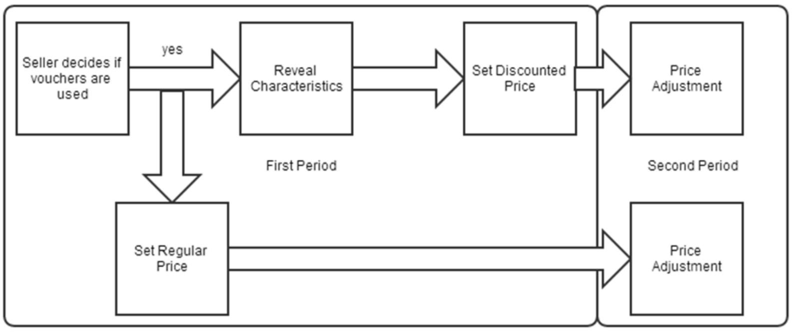

Figure 1: Key decisions in each period

certain customer has towards the product, while the second aspect is the distance that separates the buyer from the seller.

Naturally, the higher the quality perception that a customer has towards a product the more that customer values the product. On the other hand, the more the distance a customer has to travel to obtain the product, every thing else being equal, the lower the valuation that this customer has for this product.

We denote the distance with d. Note that although throughout this paper we will treat d as if it is the physical distance between the buyer or the seller, it is in fact a measure of the effort that a customer needs to put in order to physically obtain the product. This could mean long geographical distance, but it could also mean that obtaining the product is more cumbersome (e.g. low value of d means a store around the corner, while a large value of d means product is not available except online). The important thing to point out is that this difficulty is there and it is beyond the control of the buyer and the seller to change that.

The quality perception that a customer has for the product will be denoted by $\hat{\mathbf{q}}$. In this model we assume that the quality perception of all the customers is uniformly distributed between 0 and q_m that is $\hat{\mathbf{q}} = \mathscr{U}(0, q_m)$. Note that the product has an intrinsic quality denoted by q_0. Initially, all customers are equally uninformed and rely on their perception to valuate the product. However, the customers can become better informed about the product, either by trying it or by putting in some effort, at a cost, to gather more information about the product. As a customer becomes more informed, the quality perception becomes closer to q_0.

We consider the following linear relationship between the (perceived) quality, the distance and the valuation of the product:

$$\hat{\mathbf{v}} = a - md + b\hat{\mathbf{q}}$$

Where a, m and b are constants. Moreover, we made the following assumptions:

- Customers are uniformly dispersed over the positive side of the distance axis, with the seller being located at $d = 0$.
- All entities (buyers and sellers) share the same time value of money that we will be accounting for using a discount factor δ.
- A customer can make only one purchase in a given period.

C. Pre-purchase Behavior

The behavior described in this section is applicable to any period once the price is revealed by the seller. We begin the discussion by defining an interested customer to be a customer who has a valuation that is higher than the price ($\hat{v} \geq p$). For a given price p, we can find the minimum quality perception that a customer must have in order to be an interested customer. To do so we use the equation $\hat{v} \geq p$, then we substitute for $\hat{v}$ the value that was suggested in the previous section. Solving for $\hat{\mathbf{q}}$, we arrive at the equation below:

$$\hat{\mathbf{q}} \geqslant q_L(d) := \frac{md}{b} + \frac{p-a}{b}$$

Where $\hat{\mathbf{q}}_{\mathbf{L}}$ is the minimum quality perception for a customer to have at a given distance and price in order to be interested in buying the product. Graphically, the proportion of interested buyers are highlighted by the shaded region in Figure 2 below.

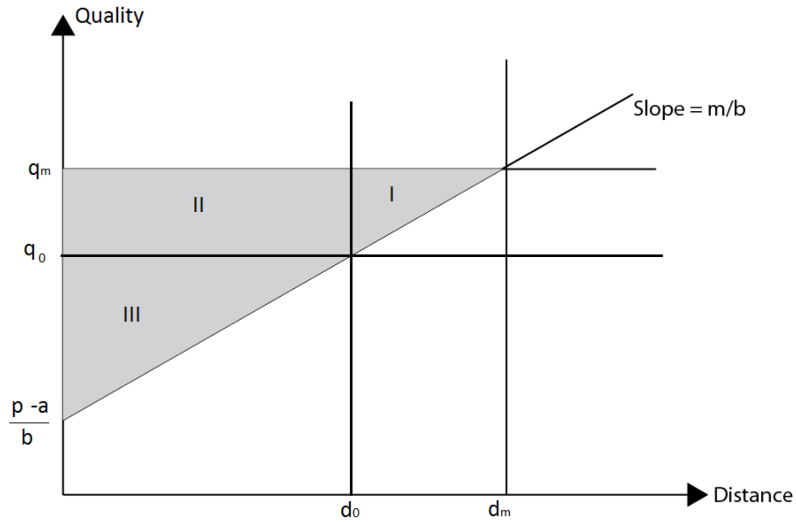

Figure 2: Graphical representation of interested customers

Since there is an upper limit for the quality perception that a consumer might have, then there exists a certain distance (d_m), beyond which no customer would be interested buyer. This distance can be calculated using the equation below:

$$d_m = \frac{a}{m} - \frac{p}{m} + \frac{b}{m} q_m$$

We can also define the distance d_0 to be the distance at which $q_0 = q_L$, and it can be calculated using the equation below:

$$d_0 = \frac{a}{m} - \frac{p}{m} + \frac{b}{m} q_0$$

Note that as indicated in Figure 2, the interested customers are classified into three categories (I, II and III). This is the result of the different dynamics that take place as these customers gain information about the product. The characteristics of the three groups are as follows:

I Buyers that have distance more than d_0. All buyers at this distance will have perception higher than q_0.

II Buyers that have distance less than d_0 and quality perception higher than q_0.

III Buyers that have distance less than d_0 and quality perception lower than q_0.

1. Seeking Information

Prior to making the purchase, the interested customer will try to gain more information about the product in order to decide whether to purchase it or not. The additional information acquired will a cost that will reduce the overall surplus if the product is purchased. In this subsection we describe the different steps that take place as the interested buyers acquire additional information about the product.

To model this phenomenon, we assume that interested buyers (i.e. $\hat{\mathbf{v}} > p$) invest some of their surplus to gain information about the product. In other words, the consumer pays part of the surplus in exchange for information about the product. Each unit of information is exchanged for c units of the surplus. The customer will stop trying to gain additional knowledge either when that customer runs out of surplus, or when the real quality of the product (q_0) is reached. The initial surplus can be calculated as:

$$S_{initial} = \hat{\mathbf{v}} - p = b(\hat{\mathbf{q}} - q_L)$$

Which is clearly uniformly distributed as it is the difference of two uniformly distributed random variables.

As the interested buyer acquires information about the product, the quality perception of that customer moves towards q_0, incurring a unit cost of c. The resulting surplus can be calculated with the equation below:

$$S = a - md + b(q + \Delta q) - c|\Delta q| - p$$

The equation indicates that for every unit change in the quality perception, the surplus is decreased by c, however, the change in the quality perception also impacts the client's valuation of the product. If the initial quality perception ($\hat{\mathbf{q}} > q_0$) then the surplus decreases by a quantity of $(b+c)\Delta q$. In the case when ($\hat{\mathbf{q}} \leqslant q_0$), the surplus increases by $(b-c)\Delta q$. We assume $b - c <= 0$.

2. Behavior of Interested Groups

Starting with the group I ($d > d_0$ *and* $\hat{\mathbf{q}} > q_L > q_0$), as the customer gains more information, the quality perception moves towards q_0, in other words it starts declining. The surplus will reach 0 while the quality perception is still more than q_L. As a result, the customer will reach a surplus of 0, while knowing that the real quality is in fact lower than the reached quality perception, which means that it is at a level lower than what would result in a surplus of 0. Thus the consumers of this category will decide not to make a purchase.

As for group II ($d < d_0$ *and* $\hat{\mathbf{q}} > q_0$), $\hat{\mathbf{q}}$ will decrease to get closer to q_0. There exists a certain quality level q_{max}, such that if the initial quality perception is greater than q_{max}, then the customer will reach a surplus of 0 before $\hat{\mathbf{q}}$ reaches q_0. Thus that customer will run out of surplus knowing that the real quality is below the reached one, the customer decides not to make the purchase. However, if the initial quality perception is less than q_{max}, then the customer reaches q_0, while maintaining a positive surplus. The customer decides to purchase the product. The value of q_{max} can be calculated using the equation below:

$$q_{max} = \frac{1}{c}(bq_0 + cq_0 - bq_L)$$

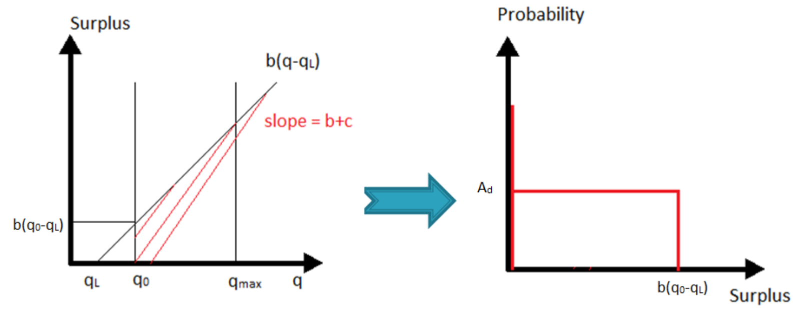

Figure 3: Surplus of Group II customers as they seek information

A_d is the area of the uniformly distributed portion of the surplus. The calculation of A_d is shown in Appendix A, the calculated expression is defined by the equation below:

$$A_d = \frac{2b}{Cq_m}(q_0 - q_L)$$

The above explanation is illustrated graphically in Figure 3. In the figure, the surplus is plotted against the quality perception. Note that as calculated earlier in this section, the initial surplus is uniformly distributed over the straight line of equation $b(\hat{\mathbf{q}} - q_L)$. When customers seek knowledge, the surplus decreases at a rate of $b + c$. Figure 3 shows that all initial quality perceptions that are greater than q_{max} will result in reaching a surplus of 0 while $\hat{\mathbf{q}}$ is still larger than q_0.

Figure 3 also shows that initially the surplus is uniformly distributed along the line $b(\hat{\mathbf{q}} - q_L)$. However, for a fixed distance d, the distribution of the surplus will change and it will be made up of two components. The first one is a spike at 0 that is the result from all customers who had an initial quality perception greater than q_{max}. The remaining part of the group who had an initial quality perception less than q_{max} will result in a uniform surplus distribution between 0 and the original surplus at $\hat{\mathbf{q}} = q_0$, which can be calculated to be $b(q_0 - q_L)$.

In the case of group III ($d < d_0$ *and* $\hat{\mathbf{q}} < q_0$), $\hat{\mathbf{q}}$ increases to get closer to q_0.

There exists q_{min}, such that if the initial quality perception is less than q_{min}, then the customer will reach a surplus of 0 before $\hat{\mathbf{q}}$ reaches q_0, so they will be aware that the real quality is greater than the one that would result for them in a surplus of 0. The customer decides to purchase the product. In effect all customers belonging to this group end up buying, but the ones with an original quality perception greater than q_{min} end up with a positive surplus, while the rest will end up with a surplus of 0. The value of q_{min} can be calculated using the equation below:

$$q_{min} = \frac{1}{c}(bq_L + cq_0 - bq_0)$$

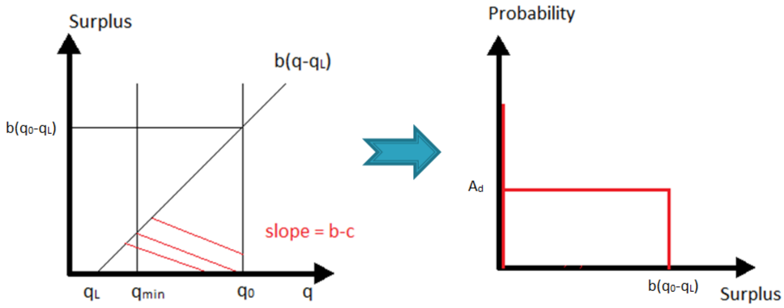

Figure 4: Surplus of Group III customers as they seek information

Graphically, the process is explained in Figure 4. The analysis is similar to that of group II, the only difference is that as a customer gains knowledge, $\hat{\mathbf{q}}$ increases, which pushes the resulting surplus upwards at a rate of b. However, the cost of gaining information decreases the resulting surplus at a rate of c. Thus the net effect would be a change at a rate of $b-c$, which is an increase in the case of our assumption where $b-c>0$ and a decrease in surplus otherwise.

Note that at a given distance d, the resulting shape of the distribution of the surplus is similar to that in the case of group II. The aggregated sum of the customers who had an initial quality perception more than q_{min} will result in a surplus distribution that is uniform between 0 and $b(q_0 - q_L)$. The remaining part of the group will result in a spike at zero surplus. Notice that the limits of the uniformly distributed parts are the same for the two groups at a given distance d.

If we add the two distributions of the surplus, that are resulting from Groups II and III at the same distance d, we will end up with a spike at $S = 0$ in addition to a uniform distribution between 0 and $b(q_0 - q_L)$. At this stage, we will mention that the upper limit of the rectangle is dependant on d. Hence the area will be a function of d that we will be denoting with A_d. This means that the spike at $S = 0$ will have a height of $1 - A_d$.

CHAPTER IV

A MODEL FOR SOCIAL IMPACT

In this chapter we focus on the events that take place only in the case when discount vouchers are introduced. Before explaining the social interactions that will be occurring, we first explain some of the details about the characteristics of the voucher, these would help in understanding why customers are willing to put effort in convincing each other.

A. Discount Vouchers Characteristics

In chapter 3, we have limited the use of the discount vouchers to the first period. There are two other important characteristics that the seller needs to decide in order to end up with optimal profits.

The first characteristic, as discussed in chapter 3, is the discounted price. When a seller decides to offer coupons, the seller must set the optimal pricing policy. The policy is comprised of selecting a price for each period knowing that the first one is the discounted price p_d. The second characteristic, is the number of buyers threshold that is denoted by M. If the number of subscribers does not reach M, the discount voucher is cancelled. Otherwise, all subscribers get to buy the product at a discount. This minimum threshold is set by the seller at the beginning of the period.

The procedure of events that take place when vouchers are introduced start with the seller revealing p_d and M. Following that, interested buyers subscribe to purchasing the product. If the number of subscribers is greater or equal to M, then all subscribers will pay p_d in exchange for the product. In the other case, when the number of subscribers is less than M, then the discount is put on hold, thus the interested customers will attempt to convince the uninterested ones to subscribe as well. If enough uninterested customers are convinced to make the number of subscribers greater or equal to M, then all subscribers will buy the product atp_d. Otherwise the subscribers will have the option to buy the product at its regular price. This summary is illustrated graphically in Figure 5.

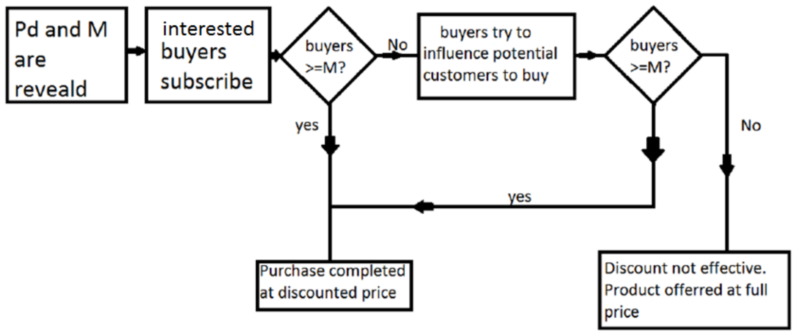

Figure 5: Summary of the events taking once discount vouchers are issued

B. Information Sharing

From the discussion in the previous section, the discount will not take effect unless there are at least M subscribers. Hence, for the interested buyers, wanting to benefit from the discount, try to convince other customers of the community to subscribe as well.

Our model is inspired from the work of Jing et al. (2011). In their model, the informed customers transfer some of their knowledge to the uninformed customers at a cost that is deducted from their surplus. In our model, the informed ones also transfer information to the uninformed. However, instead of paying for the transferred information, we put a restriction on it that is equal to βS, where $\beta \leqslant 1$. This way the amount of information transferred will have an upper bound that is equal to the surplus.

For a customer to be a “convincer”, that customer must have a positive surplus after additional information has been sought. Therefore, the convincer's quality perception will be equal to q_0. Since the customer has gained all the needed knowledge needed, we know that this customer is informed. So from chapter 3, we know that members of group II who had an initial quality perception less than q_{max} and members of group III who have an initial quality perception greater than q_{min}, will end up informed and with a positive surplus, hence they are able to convince.

In the model, we suggest to depict that the social impact relies on the following. The knowledge that a customer has is influenced through the quality perception. When an informed customer is trying to convince an uninformed one, in effect the informed customer will end up increasing the quality perception of the uninformed. The increase in the quality perception is equal to the surplus of the informed customer multiplied by a factor (β), which is a measure of the efficiency of the communication in the channels selected by the convincers.

In summary, the informed customer with quality perception and distance (q,d) and a positive surplus S, and tries to convince an uninformed customer that has a quality perception q' at a distance d'. The uninformed customer's quality perception will increase to become q_N. q_N is assumed to increase proportionally to the effort, which in turn is proportional to the surplus of the person who is convincing. The value of q_N can be calculated according to the equation below:

$$q_N = q' + \beta S$$

It is worth mentioning that as the price increases, $q_L(d)$ also increases, which in turn increases the probability of having a surplus of zero. This means that as the price increases, q_N approaches q'. Note that we showed earlier that interested buyers located at a distance that is greater than d_0, decide not to purchase the product after gaining enough information. This means that all convincers are located within the limits of $d = 0$ and $d = d_0$, coupled with the fact that customers at a distance greater than d_0 are not going to buy, provides enough motivation for the convincers to target uninterested customers in the same region in which they are located.

Practically the distribution of q_N is obtained randomly a customer from the interested buyers, and another one from the uninterested buyers. The quality perception of the uninterested buyer is increased by βS where $0 \leqslant \beta \leqslant 1$ and S is the surplus of the interested buyer. The problem is an addition of two uniformly distributed random variables.

Result: Following the social impact model we obtain a closed form solution of the distribution of q_N, which is the distribution of the quality perception of the customers who were initially (prior to the social impact effect) not interested in purchasing the product. The calculations are shown in Appendix A, the results are presented directly here:

$$P(uninterested\ customer\ buys) = P(q_N \leqslant q_m) - P(q_N \leqslant q_L(d'))$$

$$= 1 - P(q_N \leqslant q_L(d'))$$

$$= \frac{q_m}{q_L(d')} - 1$$

To get the expected fraction of the convinced customers, we integrate the result over the region where the customers who were exposed to convincing are located, which is between $d = 0$ and $d = d_0$. The result follows:

$$EX = \frac{1}{d_m} \int_0^{d_0} P(q_N < q_m) - P(q_N < q_L) dd$$

$$= \frac{bq_m}{md_m} \ln(md_0 + p - a) - \frac{d_0}{d_m} - \frac{bq_m}{md_m} \ln(p - a)$$

CHAPTER V

RESULTS AND RECOMMENDATIONS

To calculate the expected profit a Monte Carlo simulation was used (Risk Solver on MS Excel). In Appendix C we show how to calculate the profits for the first periods (with and without vouchers). We did not do the calculation analytically for the second periods, this problem will be added to the future work related to this project. The model on excel portrays the behavior that was described in the previous chapters, the flowchart that describes the used algorithm on excel is shown in Appendix B.

We studied the effect of varying the real quality of the product (q_0), the sensitivity (b), that the customers have to the perceived quality, in addition to the cost (c), of the gained information. Moreover, in the case of vouchers, we studied the impact of the channel efficiency factor (β).

A. Profits Without Using Vouchers

1. Impact of Real Quality

We varied q_0 between 0 and q_m, which was taken to be equal to 5 in our analysis. The results show that if the real quality of the product is very low, the price in the first period should be set high enough to drive the demand in the first period to 0. Then the problem is reduced to a one period problem where the optimal price is to be calculated. The reasoning behind these results is that it is not beneficial to have many customers knowing the real quality of the product early on. It is better to work in only one period where most customers over estimate the quality of the product. The variation between the price in the first period (p_1) and the price in the second period (p_2) and the profits is shown in figure 6.

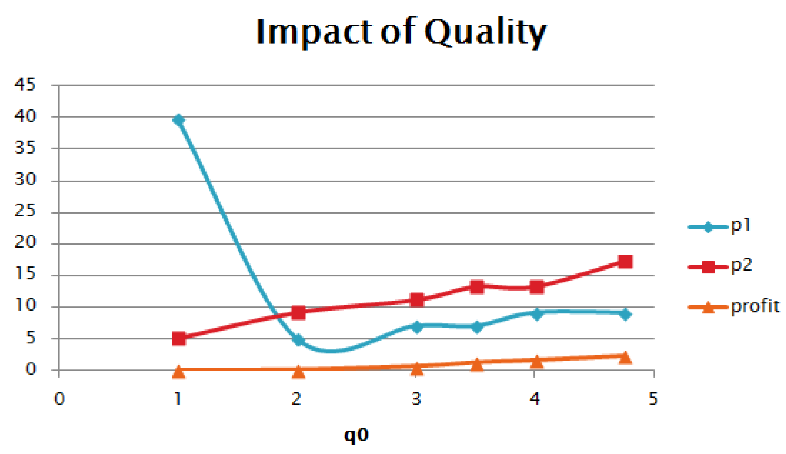

Figure 6: Impact of varying q_0 when vouchers are not used

Notice that as the quality of the product gets better, all the other variables (p_1, p_2, *and profits*) will increase as well. In the case shown in the graph, p_2 happens to be more than p_1, however, that is not always the case. The main conclusion is that for low values of q_0, p_1 is high enough to drive demand to 0. Then for moderate values of q_0, p_1 drops. From that point on, all the variables increase at the same rate with q_0.

2. *Impact of Sensitivity to Quality*

When the customers do not care about the quality of the product, that is when $b = 0$, the only factor that will end up affecting the customer's valuation will be the distance. Knowing that d is uniformly distributed, the price in the first and the second period should be very close.

Consider a moderate value of q_0, as b, the sensitivity to the quality increases, p_1, p_2, *and profits* also increase. However, when the real quality of the product is low, the impact of b on the prices and the profits tends to diminish. Figure 7 shows the variation for a relatively high q_0.

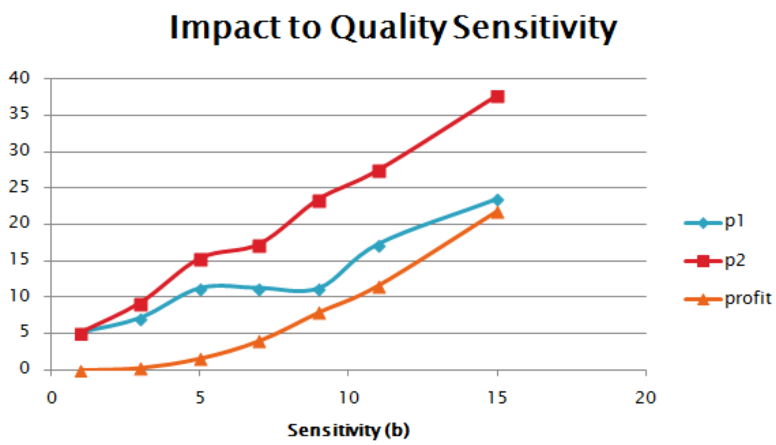

Figure 7: Impact of varying b when vouchers are not used

3. Impact of Cost of Information

The Monte Carlo simulation showed that for lower values of c, $p1$, $p2$ and the profits are not sensitive to the cost of the gained information. However, after a certain limit, $p1$ and the profits start to be influenced negatively as c increases. This is explained by the fact that as c increases q_{max} decreases, thus decreasing the number of buyers and convincers in the first period. The variation of p_1, p_2, and the profits are shown in Figure 8.

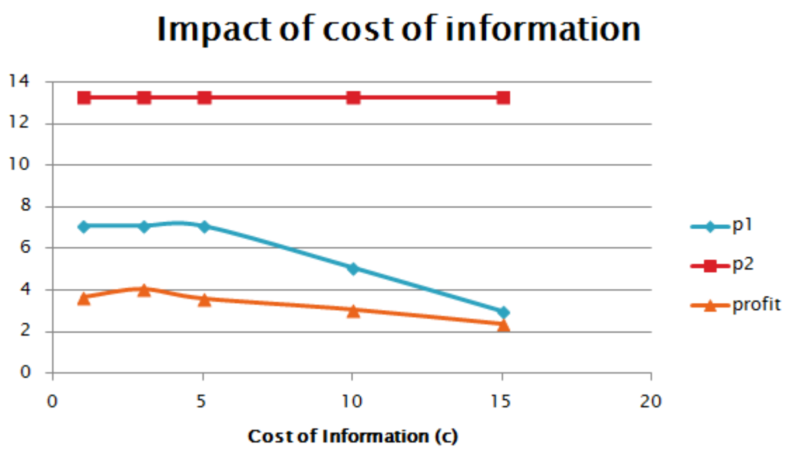

Figure 8: Impact of varying c when vouchers are not used

Notice that p_2 is not affected by c, but p_1 as well as the profit start to decrease as the cost of acquiring information increases.

B. Profits With Vouchers

In this section we will show the results of the same exercise that we did in the previous section, but this time we will be taking into consideration that the seller decided to use discount vouchers.

1. Impact of Real Quality

The behavior of p_1, p_2 and the profits are similar to the results in the previous section when discount vouchers were not used. That is for a low real quality of the product (q_0) the price in the first period will be high to remove the demand in that period, thus reducing the problem to a one period problem. Once q_0 becomes large enough, p_1 drops below p_2. From that point on, all of the three variables that we are studying will increase linearly with q_0, but the rate of increase of p_2 will be higher than that of p_1 and the profits. The variation of these variables with respect to q_0 is shown in Figure 9.

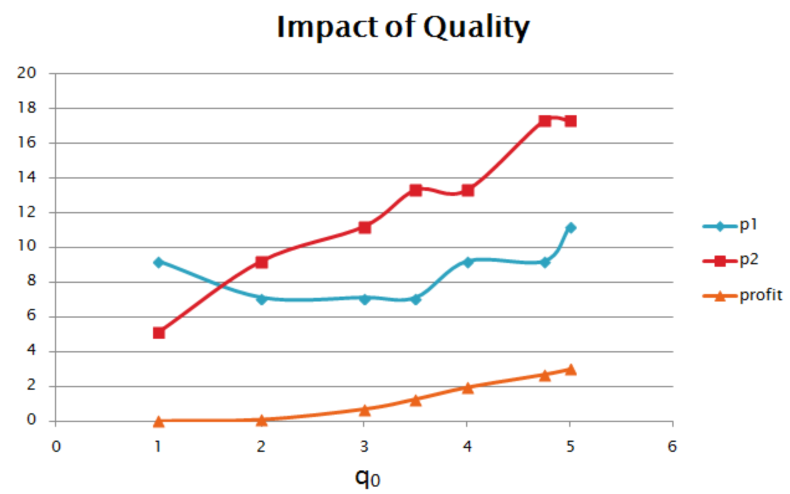

Figure 9: Impact of varying q_0 when vouchers are used

Note that the difference in the case of using vouchers is that p_1 will always be less than p_2 (without setting this as a constraint), unlike the case of no discount vouchers where that would depend on the configuration of the inputs.

2. *Impact of Sensitivity to Quality*

The results were a little bit similar to the case of no discount vouchers. In the case when b is very low, the two prices will be almost equal. As the customers' sensitivity to quality increases, the optimal prices in of the two periods as well as the profits also increase. The change is plotted in Figure 10.

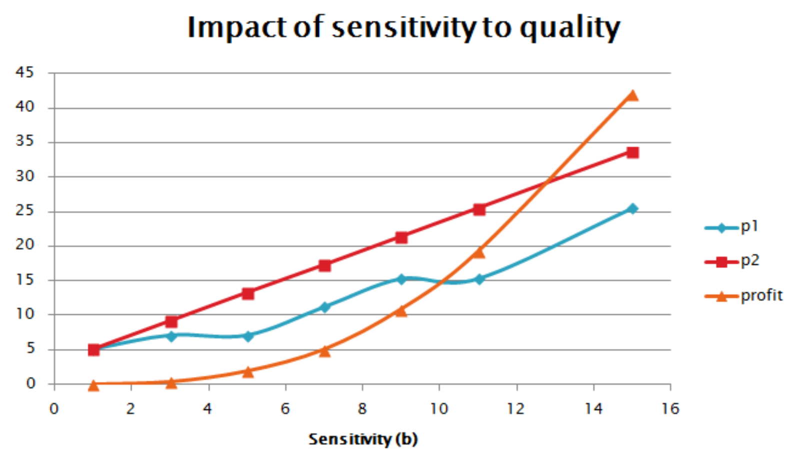

Figure 10: Impact of varying b when vouchers are used

Note that p_2 and p_1 vary at a linear rate with b, but the rate of p_2 is greater than that of p_1. As for the profit, they vary exponentially with b, this suggests that vouchers will be relevant in the case where customers are very sensitive with respect quality.

3. *Impact of Cost of Information*

The behavior $p1$, $p2$ and the profits is very similar to the case when no discount vouchers are used when it comes to varying the cost of acquiring information. That is p_2 will not be affected, but p_1 and the profits will start to decline linearly once the cost of acquiring information reaches a certain limit. The reasoning is the same, if it is costly to acquire information, many customers will decide not to seek information and will end up deciding not to buy. The variation of p_1, p_2 and the profits, with respect to c are shown in Figure 11.

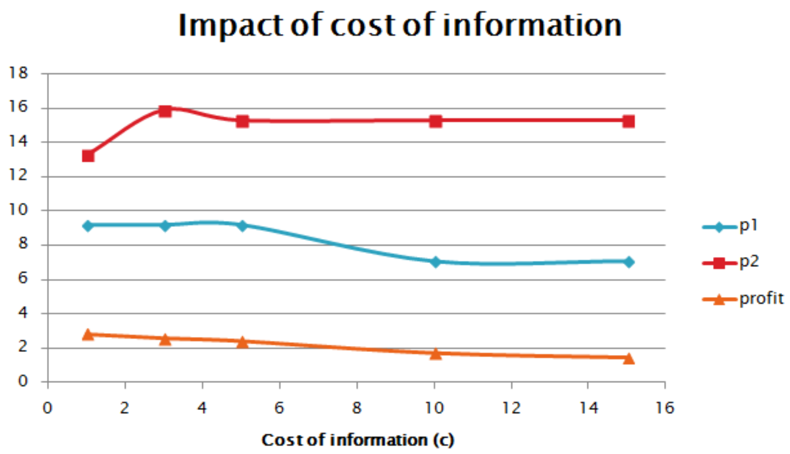

Figure 11: Impact of varying c when vouchers are used

4. Impact of Beta

The factor β can be thought of as a measure of how efficient the communication channel is. The higher the value of β, the bigger the impact every unit of surplus has on the implied quality. A lower value of β will make communication inefficient, where a large amount of surplus will be required to transfer a small amount of information.

We varied the results of beta between 0 and 1. We noticed that for lower values of β, the profit is very sensitive to any change in β. However, after a certain limit, the profits will become insensitive to β. This can be explained by the fact that for a certain value of β all the uninterested customers will be convinced to buy, thus increasing the value of β will be like "beating on a dead horse". The variation of the profits versus β is plotted in Figure 12.

C. Are Vouchers Effective?

The ultimate aim of the project was to determine whether discount vouchers are effective or not. In the previous two sections, we saw that there are similarities in the variation of the prices and the profits when we studied there sensitivity against several parameters. The question remains whether the profit of the seller is higher with or

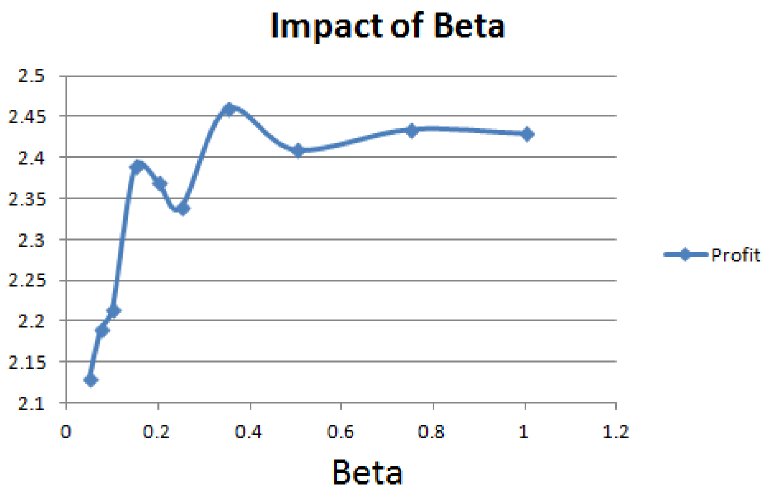

Figure 12: Impact of varying c vouchers are used

without vouchers.

To answer the question we performed a sensitivity analysis by varying the difference in the profits of the two cases (with and without vouchers) against q_0, b and c in order to find out how the profitability of discount vouchers is affected. In the first case, we performed the sensitivity analysis against q_0. The result is shown in Figure 13.

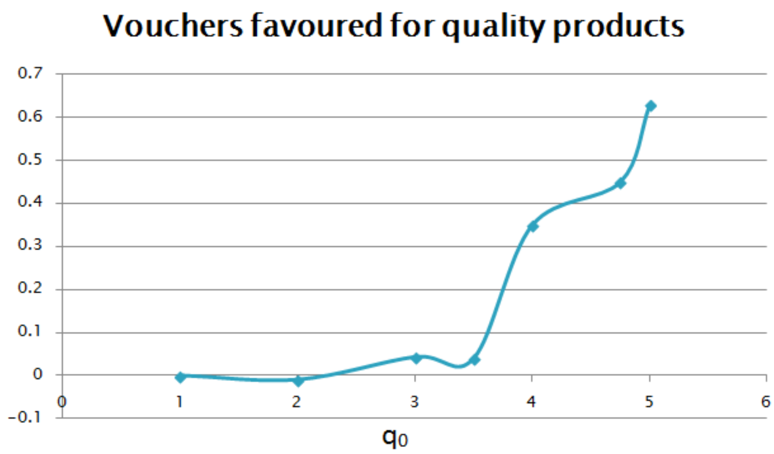

Figure 13: Impact of q_0 on effectiveness of vouchers

The figure illustrates that as q_0 increases, vouchers become more profitable. According to our model, increasing q_0 would increase q_{max} and also decreases q_{min}. This means that:

- There will be more buyers from groups II and III.
- There will be more convincers from groups II and III
- The convinced customers will probably return to buy the product in the second period to buy at full price.

This result is inline with previous research in the sense that the profitability of vouchers increases as the number of customers who come back to buy at full price increases.

We then did the same analysis but this time we varied the customers' sensitivity to quality (*b*) on the x-axis. The results are shown in Figure 14.

Figure 14: Impact of varying *b* on effectiveness of vouchers

The results are inline in our conclusion in the previous sections where we showed that as the customer's sensitivity to the quality increases, the use of vouchers will become more and more effective. Finally, we did not find any relation between the effectiveness of discount vouchers and the cost of acquiring information. Figure 15 shows the difference in profits in the case of vouchers and the case without, plotted against the change in the cost of acquiring information.

In summary, as the real quality of the product increases, and as customers tend to care more about the quality of the product, the more and more discount vouchers

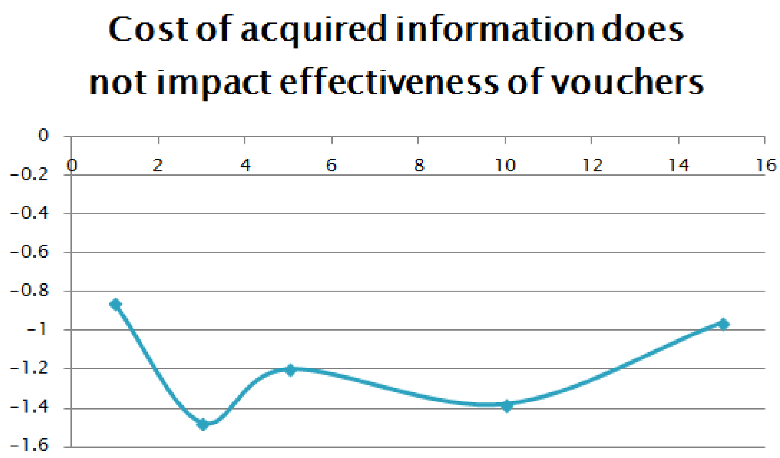

Figure 15: Impact of varying c on the effectiveness of vouchers

become effective. This is not necessarily intuitive as one might first assume that the reason why the seller is trying promotion is the bad quality of the product. This conclusion enforces the fact that vouchers are mainly used for exposure and not for price discrimination, unlike what other research have suggested.

CHAPTER VI

LIMITATIONS OF THE MODEL

There are several factors that the model accounts for, but in some cases these factors are modelled in a simplified way or are not included from the first place. These items that will be analyzed in future work.

Our first limitation is in the way the quality perception is modelled in the valuation of the product. The way that we modelled it was by using the term "bq", where q is the quality perception which is uniformly distributed and b is the sensitivity of the customers towards the quality, in other words the importance of quality in a product in the eyes of the customer. In reality, when customers are looking at a certain product and they are all equally uninformed, they will probably all have close perceptions of quality. However, what will vary more is how important they value quality in the product. For that, we could consider the distribution of q around q_0 and one could also consider the coefficient to be random.

The other limitation that is worth discussing is as follows: We assumed that people with positive surplus will seek information and that only people with positive surplus might put efforts in convincing others with lower quality perception. However, this might not be a necessary condition, as a customer might be willing to seek information and put effort to convince others even if that customer had a negative surplus, if that customer was expecting a high reward in the end.

A third limitation is that we assumed that the quality gained varies linearly as investment in surplus is increased. We will be expanding our research to find a more realistic way to model this phenomenon.

The forth limitation is that we did not associate any cost for improved quality.

That is the cost for a product with a high q_0 is the same as that for a product with low q_0. In reality, as the quality of a product increases, the costs associated with producing the product tend to rise as well. This factor should be taken into account in the model, especially when it comes to comparing profits across different values of q_0.

Last but not least, we mentioned that there exists a threshold of M subscribers that must be reached in order for the discount to be effective. However, we did not include this parameter in our calculation and simulations. We implicitly assumed that M will be reached. In reality, M must be higher than the demand in the no voucher case, since the optimal price in that case has been determined based on the demand during that period, using vouchers with that demand will result in a situation that is not optimal.

In conclusion, some of the obtained results were inline with previous literature. As the quality of the product increases the chance of customers returning and buying the product in the second period at full price (Dhalokia, 2011). It was also inline with previous research when the profitability of vouchers increased as the importance of quality increased in the eyes of the consumers (Dhalokia, 2011). The contributions to the literature is that now the driver behind bringing customers back are now identified as the high quality of the product. Thus through proper market research, the customer will be able to predict whether the vouchers will be profitable or not.

On the other hand the above findings contradict other research by suggesting that the main profitability of the vouchers is by increasing awareness among consumers and not through price discrimination as was suggested by Edelman et al. (2011). Since price discrimination would not encourage customers to come again and purchase at full price, but encourages "deal seekers" to take a one time advantage from the deal.

APPENDIX A

CALCULATING DISTRIBUTION CONVINCED CUSTOMERS

In this appendix, we will show the calculation of the distribution of the buyers who were convinced by the customers who were initially interested. In our scenario, the interested customer with quality perception and distance of (q, d) is picked at random and is trying to convince a non-buyer who is also selected at random with quality perception and distance of (q', d'). Note that by definition, since the first customer is interested then prior to the knowledge gaining step $q \geqslant q_L(d)$, conversely for the non-buyer $q' < q_L(d')$. In effect the quality perception of the non-buyer (q') will increase by a value equal to βS, where S is the surplus of the interested customer.

$$P(uninterested\ customer\ buys) = P(q_N \geqslant X)$$

$$= P(q_N \leqslant q_m) - P(q_N \leqslant q_L(d'))$$

To calculate $P(q_N \leqslant q_m)$ and $P(q_N \leqslant q_L(d'))$ we will calculate $P(q_N \leq X)$.

$$P(q_N \leq X) = P(q' + \beta S \leq X)$$

$$= P(q' \leqslant X - \beta S)$$

Note here that S, the surplus of the interested customer was shown to be made of two components in chapter 3. The first component is made of a spike at $S = 0$, from the customers that have a distance less than d_0 and an initial quality perception that is either greater than q_{max} or less than q_{min}. The other part of the customers that are located at a distance less than d_0 will make up a uniform distribution for the given distance d that is spread between 0 and $S_d = b(q_0 - q_L)$.

We can calculate the area of the rectangle (A_d) as follows:

$$A_d = \frac{q_{max} - q_{min}}{q_m} = \frac{2b}{Cq_m}(q_0 - q_L)$$

This means that the hight of the rectangle is $\frac{A_d}{S_d}$ and its length is $b(q_0 - q_L)$, while the length of the spike at $S = 0$ is equal to $(1 - A_d)$. Since both A_d and S_d are functions of d, we will fix d and the probability that we are calculating will become:

$$P(q_N \leq X) = \frac{1}{d_m}(\int_0^{d_0}(\int_0^{b(q_0-q_L)} \frac{A_d}{S_d}\frac{X - \beta S}{q_L(d')}ds + (1 - A_d)P(q' < X)) + \int_{d_0}^{d_m} \frac{X}{q_L(d')}dd)$$

So by fixing d we get two integrals over the distance domain. One part of the integral is between $d = d_0$ and $d = d_m$. In that region, all interested buyers will have a surplus of 0 and will end up not buying. The expression $P(q' \leqslant X - \beta S)$ is reduced to $P(q' \leqslant X)$. Note that q' is a uniform distribution between 0 and $q_L(d')$ so the solution of the integral is a function of d'.

The other part of the integral represents the region between $d = 0$ and $d = d_0$. As explained the distribution of the surplus of the interested customers in this region after the knowledge gaining process is made of the two components, which are the spike and the rectangle whose dimensions were calculated earlier in this section. The integral is thus split into two sub integrals to cover the two components of the distribution of the surplus.

The first sub integral spans the rectangle, making the limits of the sub integral from $S = 0$ to $S = b(q_0 - q_L)$. The probability of the surplus to be equal to any of the values in this range has already been calculated to be $\frac{A_d}{S_d}$, while $P(q' \leqslant X - \beta S)$ can be written as $\frac{X - \beta S}{q'(d')}$.

The second sub integral is to cover the spike at surplus =0. The interested customers of this component will have a surplus of 0 and will not be willing to do any convincing, thus the expression is reduced to become similar to the region between $d = d_0$ and $d = d_m$.

Now that the problem is broken down into a set of integral, the problem becomes more of an arithmetic problem, we will not go over the involved steps in solving it, we will directly show the solution.

$$P(q_N \leq X) = \frac{1}{q_L(d')}(\frac{2m^3 d_0^3}{3cq_m d_m} - \frac{\beta q_0^2 b^2 d_0}{cq_m d_m} - \frac{2m_0 d_0 q_0^2 b^2}{cq_m d_m} + X)$$

APPENDIX B

MODEL IMPLEMENTATION ALGORITHM

A. Algorithm for No Discount Vouchers

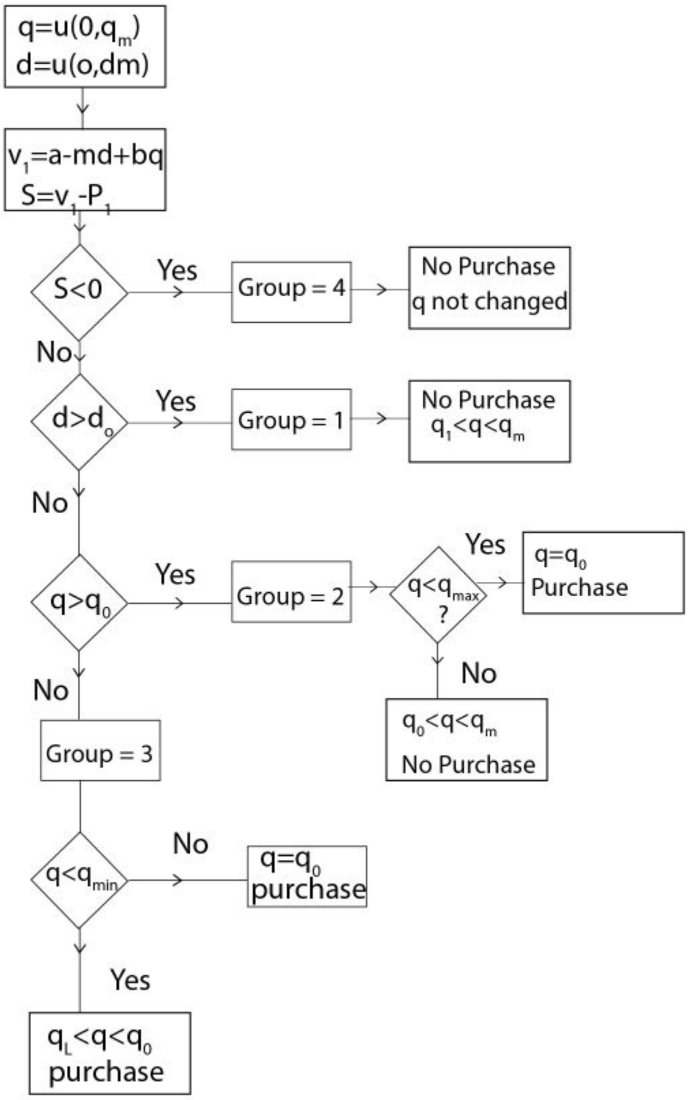

Figure 16: Algorithm implementation algorithm when vouchers are not used

B. Algorithm for Using Discount Vouchers

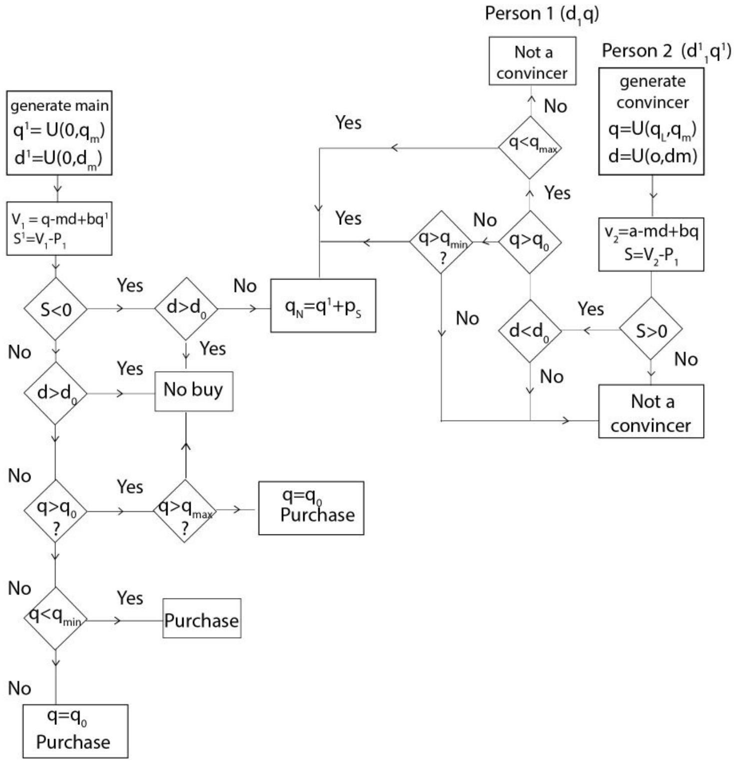

Figure 17: Algorithm implementation algorithm when discount vouchers are used

APPENDIX C

CALCULATING PROFITS ANALYTICALLY

In this section, we will derive the expected demand and the expected profits in the first period for the case when vouchers are not used and also when vouchers are used. For the expected demand and profits in the second period we relied on simulation. Coming up with the expressions analytically is part of the future work related to this project.

In chapter 3 we concluded that all the interested buyers in group I will decide not to buy, while all the interested customers in in group III will end up buying the products. As for the customers in group II, only part of them will end up buying, depending on whether their original perception (the ones who start out with a quality perception that is less than q_{max}. Since all interested buyers are located within the distance d_m, then the expected fraction of buyers can be evaluated as:

$$\frac{\int_0^{d_0} q\_max - q_L}{q_m d_m}$$

$$= \frac{d_0(\frac{1}{b} + \frac{a}{c})(bq_0 - p + a - \frac{md_0}{2})}{q_m d_m}$$

If we assume that there are N potential customers between $d = 0$ and d_m, then we can say that the expected demand can be defined in the equation below:

$$E(D(p)) = N\frac{d_0(\frac{1}{b} + \frac{a}{c})(bq_0 - p + a - \frac{md_0}{2})}{q_m d_m}$$

A. Profits Without Using Discount Vouchers

In this section, we are going to calculate the expected demand and expected profits in the case where the seller decides not to utilize the discount vouchers. The seller sets the price of p_1 in the first period. At the beginning of the second period, the seller has the chance to change the price, this new price will be denoted by p_2.

To calculate the expected profits in the case when the seller decides not to use discount vouchers, it is enough to use the equation that was derived in the previous section to calculate the demand in each of the two periods, then we multiply the demand of each period by the respective price set by the seller at that period.

$$E(\pi) = (p_1 - ci)E(D_{ND1}(p_1)) + \delta(p_2 - c)E(D_{ND2}(p_2))$$

where ci is the cost encountered by the seller to obtain and sell the product, δ is used to account for the time value of money, $E(D_{ND1}(p_1)$ is the expected demand in the first period and it can be calculated as follows:

$$E(D_{ND1}(p_1)) = N\frac{d_0(\frac{1}{b} + \frac{a}{c})(bq_0 - p_1 + a - \frac{md_0}{2})}{q_m d_m}$$

The value of $E(D_{ND2}(p_2)$ has not been calculated to this moment.

B. Profits When Using Discount Vouchers

The analysis when using the discount vouchers has some similarities with the case of no discount vouchers. The expression for the profits remain as follows:

$$E(\pi) = (p_1 - ci)E(D_{ND1}(p_1)) + \delta(p_2 - c)E(D_{ND2}(p_2))$$

The main difference is that the we need to adjust the expression for the demand to account for the uninterested customers who have been convinced into buying. The fraction of additional buyers has already been calculated in chapter 3. The expression for the expected demand becomes:

$$E(D_{D2}(p_1)) = N\frac{d_0(\frac{1}{b}+\frac{a}{c})(bq_0 - p_1 + a - \frac{md_0}{2})}{q_m d_m} + \frac{bq_m}{m}\ln(md_0 + p - a) - d_0 - \frac{bq_m}{m}\ln(p-a)$$

The expression for the demand in the second period has not been calculated to this point.

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