AZARQUIEL'S RESCENSION
OF THE "ALMANAC OF AMMONIUS"

by

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PREFACE

This is a study of the tables of geocentric planetary longitudes found in the so-called "Almanac of Ammonius," in the redaction of Azarquiel. The work here is based on the tables as published in the Estudios sobre Azarquiel of José Millàs Vallicrosa.

A general discussion of the problem of planetary longitude prediction in Chapter I introduces concepts referred to later and the standard notations adopted here. Some familiarity with the modern conception of the solar system and with Ptolemaic astronomy is however assumed of the reader. Information on the historical background of the "Almanac" and a description of the tables is presented in Chapter II.

Chapters III to V contain the exposition of the work done in attempting to discover the abstract mathematical model underlying the tables. In Chapter III some general considerations applying to any model are presented. Chapter IV displays the methods used to determine from the tables the essential constants that would permit their reconstitution assuming the tables were calculated using some form of an epicyclic or

Ptolemaic model. The results obtained by comparing the subsequent recomputed longitudes with those given in the text tables are discussed in Chapter V.

The essential role played by the electronic computer (IBM 1620) should be stressed. The complicated calculations required here on such a large quantity of data would be unthinkable without it.

No footnotes are used in the following; instead, for example, the indication [Name, p. 40], in the body of the text, will refer to page 40 of the book or article listed alphabetically as Name in the bibliography. The figures referred to in the various chapters are found after Chapter V. Appendix I contains photocopies from the Estudios of some pages of the tables of planetary longitudes of the "Almanac." In Appendix II, the most important IBM 1620 programs used are given; and Appendix III contains a sample of output results.

ABSTRACT

It has been shown by this study, and to a reasonable degree of certitude, that the model used to construct the tables of planetary longitudes of the four planets, Saturn, Jupiter, Mars and Venus, found in the "Almanac of Ammonius" is essentially Ptolemaic. The model contains an equant and incorporates the <u>Almagest</u> values for the epicycle radius and eccentricity parameters. The longitude of the apogee in the tables is about ten degrees larger for the three outer planet than Ptolemy's.

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CHAPTER I

PLANETARY LONGITUDE PREDICTION

The stars appear to circle westward daily due to the earth's rotation. Seen from the earth it is as though the celestial sphere with the stars attached on the inside rotates about the earth. Within the sphere of stars, certain objects, known as the "planets," i.e. wanderers, by the ancients, share its daily motion westward, but lag and move slowly in a general eastward direction among the constellations, while the stars retain their relative positions day after day, year after year. The "planets" included the sun, moon and the five nearest of the bodies called planets today, those then known, for they appear bright in the night sky. It is the eastward paths of these planets, Saturn, Jupiter, Mars, Venus and Mercury, among the fixed stars that are of concern here.

The sun's apparent annual path around the heavens is called the ecliptic (Fig. 1). It is a great circle inclined about $23\frac{1}{2}^{\circ}$ to the celestial equator; the equinoxes are the two points where the ecliptic crosses the celestial equator. The zodiac is a band on the celestial sphere extending about 8° on each side of the ecliptic. As well as the sun this band contains the moon and the five bright planets at all times (with the occasional exception of Venus). Thus the geocentric longitude, λ, measured eastward along the ecliptic from the vernal equinox is a good indication of their positions on the celestial sphere without reference to a second coordinate. Twelve constellations are fixed at fairly regular intervals along the zodiac. The ancients marked off twelve equal divisions, the signs of the zodiac, eastward from the vernal equinox. Each 30° block took the name of the constellation it contained. (However in the intervening several thousand years the vernal equinox has moved slightly westward among the stars due to the earth's precessional motion, so that each sign has shifted past its constellation--the zodical sign of Aries now contains the constellation Pisces.) The longitudes of the sun, moon and planets were reported by the signs in which they could be seen and fractions of 30°, a method still used in some almanacs today.

The apparent paths of the planets in the zodiac are looped (Fig. 2a), the loops being somewhat similar but not identical. As already mentioned they move generally eastward among the constellations (direct motion), but at intervals they turn and move backward (retrograde motion) for a while before resuming eastward motion. At the beginning and at the end of retrogradation, there is a so-called stationary point. Fig. 2b is thus the typical graph of a planet's longitude plotted against time.

A general analysis of heliocentric and geocentric motions will clarify the phenomema described above. The planets, the earth being one of them, revolve around the sun on elliptical orbits. Suppose for a moment that the orbits are circles and that they lie in the plane of the ecliptic. To obtain the planetary motions as seen from the earth, the earth may be regarded as fixed. When the earth is stopped, it looks as though the sun moves around the earth once per year (Fig. 3); the plane of its orbit intersects the celestial sphere in the ecliptic. An "inner" planet, Mercury or Venus, moves around the sun on an orbit nearer the sun than the earth's. Subtracting the motion of the earth, Fig. 3 gives the motion of the sun and the orbit of the planet remains a circle with the sun as center. The geocentric description of the motion of an inner planet is thus given by a planet which moves on a little circle, the "epicycle," whose center is carried on a larger circle, the "deferent," whose center is the earth (Fig.4). Similarily, an "outer" planet, Mars, Jupiter, or Saturn, whose heliocentric orbit encloses the orbit of the earth, appears from the earth to be moving on a large epicycle, the center of which moves around the earth (Fig. 5b). An epicycle smaller than the deferent may be preferred. Then a point C is introduced such that, by the commutativity of vector sums, ES + SP is always equal to EC + EP. Note in particular that CP is parallel to the line from the earth to the sun. The geocentric motion of the outer planets can then be described by the planet P moving on an epicycle whose center C travels on a larger deferent of center E (Fig. 5c). [_Exact Sciences,_ p. 122-124].

Thus under the assumption of circular coplanar orbits, Fig. 6 represents the geocentric motion of any one of the planets, the earth at E, the planet at P, with varying ratio of the epicycle radius to the deferent radius according to the planet, of course. C, for the inner planets, has the same longitude as the sun, and it circles the earth E once per year; for the outer planets, C moves around E in the same period in which the planet travels around the sun, its sidereal period. The period of an inner planet's epicyclic motion is its sidereal period, the period of an outer planet's, a year, when angles are measured from a radius moving so as to remain parallel to its original direction. When angles are measured from the radius CH, the epicyclic period for any planet is called its synodic period.

For greater precision some refinements may be introduced; in particular a slightly eccentric position of the earth with respect to the center of the deferent tends to provide for second-order deviations. The geocentric longitudes of the planets, which are measured in the plane of the ecliptic, could thus be calculated from these epicyclic models.

The epicyclic motion of the planets with respect to the earth accounts for the looped paths observed. The synodic period is the mean time from loop to loop; there is one loop for each trip around the epicycle from H to H. When the planet is farthest removed from the earth the motion on the epicycle is added to the motion of C. But between A and B (Fig. 6), the planet P moves backward faster than the epicycle is carried forward and the planet retrogrades.

Before the fourth century B.C. the Babylonians realized that the apparent looped paths of the planets were not haphazard and had determined their periods accurately. They were able to predict planetary positions referring them to the zodiac. Theirs were arithmetic methods without any known use of a geometric model. [Exact Sciences, p. 97, p. 156; Dreyer, p. 1].

Babylonian period relations were adopted by Greek astronomers, but to be used in geometric models, preferably using combinations of uniform circular motions. The first of these, Eudoxus' system of concentric sphere, accounted for the looped motion of the planets, but had many shortcomings as a model designed to predict their longitudes accurately.

An epicyclic model was introduced by Apollonius in the third century B.C., and was later also used, perhaps elaborated by Hipparchus. Ptolemy at Alexandria explained and perfected these models in the Almagest (about 150 A.D.). In his system, the deferent is no longer concentric to the earth but excentric to it and for further precision the "equant" is introduced. Ptolemy found that the center of the epicycle appears to move with constant angular velocity with respect to a point later called the equant, located symmetrically to the earth with respect to the center of the deferent, G (Fig. 7). The equant E is the center of equal motion; G is the center of equal distances. In the case of Mercury an even more complicated model was necessary. The line through OEG intersects the deferent in A, the apogee. λ_a is the longitude of the apogee A, characteristic of each planet. A revolution of the epicycle is reckoned from the point H back to H, so that the epicyclic period for all planets is their synodic period, Motion in the epicycle is measured by the "anomaly," δ. The true longitude is given by

$$\lambda = \lambda_a + \alpha - C'_3 + \delta$$

where α, C'_3, and δ are shown in Fig. 7. The following rules are formulated: for Saturn, Jupiter and Mars, the radius from the center of the epicycle to the planet is parallel to the line from the earth to the sun, while for Venus and Mercury, the center of the epicycle lies on this line.

In accordance with the geocentric description of heliocentric motion presented earlier, it is clear that this theory would be a valid theory of planetary motions if the proper scale were used. However, Ptolemy was not in a position to obtain reliable information about planetary distances. In any case he was essentially interested only in the angular motions as observed from the earth, i.e. in predicting the planets' geocentric longitudes. The deferent radius for all planets was taken as 60 units or 1,0 units (sexagesimal system), while the ratio between the radii of the deferent and of the epicycle, resulting from the observed length of the retrograde arc, gave the epicycle radius characteristic of each planet. In this form the epicyclic model is a correct representation of appearances so far as angular motion is concerned. Ptolemy, though, did not pretend to explain but only to describe appearances; he probably did not claim that his models corresponded to a physical truth but considered them useful devices. It is seen that he could have been more pretentious.

Ptolemy's deviation from purely uniform circular motions was considered heretical by philosophers of the time. However, the Ptolemaic system far outlived their objections. Moreover it had the preponderant influence on all astronomy up to Kepler and Newton.

The so-called "goal-year period" of a planet is a number of years after which any point of the loop, say the stationary point at the beginning of retrogradation, occurs again at approximately the same point of the ecliptic, i.e. a number of years in which a nearly integer number of loops or epicyclic revolutions and a nearly integer number of revolutions of the planet occur. Ideally if geocentric longitudes were tabulated at regular intervals for the length of each planet's period, they would be tabulated for all time. It would be sufficient to know how many years separate the present year from the first year for which entries are made in such tables and to subtract multiples of the planets period until the number, n, is less than the planet's period (present year - base year $\equiv$ n (modulo the planet's period)); geocentric longitudes for the present year would be the same as those for the nth year of the tables. However, no planetary periods of this type are exact, and corrections of varying importance must be introduced.

Whatever periods are chosen, for the inner planets, Mercury and Venus, the number of revolutions of the planet in the ecliptic during the period is equal to the number of years of the period. For the outer planets, the number of revolutions plus the number of complete loops during the period is equal to the number of years of the period. [ACT II, p. 281-283]. These relations are mentioned in a few known texts from Babylonian to medieval times as being used in the computation of planetary longitudes. The tables under study here are based on period relations of this type.

CHAPTER II

THE TEXT

The "Almanac of Ammonius" in its present form is the work of the Arab astronomer known in the Latin West as Azarquiel. Two manuscripts of the work, one in Arabic (Munich MS 853), the second, a Spanish translation (Arsenal, Segovia MS 8.322), have been edited by José Millàs Vallicrosa in the *Estudios sobre Azarquiel*. The data for this study come from the tables for planetary longitudes of the "Almanac" as they appear in this publication, the result of a parallel study of the two manuscripts.

Azarquiel (Abū Isḥāq al-Naqqāsh al-Zarqālla, also known as Al-Zarqālī, Zarkālī, Al-Zarqellu, Azarcall, etc.) lived in eleventh century Spain. It seems that he was born in Toledo, where he began his observations and studies. Sometime before Toledo fell into Christian hands in 1085, he moved south to Cordoba, where during a number of years he continued his studies and edited his last works. [*Estudios*, p. 3-17]. In addition to the "Almanac, Azarquiel is known for the invention of an astronomical instrument, the *ṣafīḥa*, and the edition of the so-called Toledan planetary tables, which in the Latin translation influenced the development of trigonometry in Renaissance Europe. [*Struik*, p. 95]. Two extensive surveys of his works, the *Ètudes sur Zarkali* of M. Steinschneider and the more recent *Estudios*, already mentioned, can be consulted.

Other than the Arabic copy (c. 1257) and the Spanish translation of the "Almanac" edited in the *Estudios*, Millàs Vallicrosa [*Estudios*, p. 151] and Steinschneider [*Ètudes*, p. 2] both cite Latin translations of Azarquiel's work.

The original author of the "Almanac" is unknown, but he is variously referred to as Aumatius, Armeniut or Humeniz. In particular the colophon of the Arabic copy is quite clear in attributing the original material to a certain اوماتيوس. No reference to an Aūmātīūs has been found in the literature, but such almanacs were known in the Alexandrian world, and the disposition of the tables for planetary longitudes is similar to that of the tables under study here. [*Estudios*, p. 235-237]. A certain Ammonius is known to have made astronomical observations at Alexandria in the fifth century. Aūmātīūs may be a corruption of the Greek name, Ammonius, since deletion of a single dot converts the Arabic *ta* into a *nun* when these letters are medial as here, hence the name given the text, the "Almanac of Ammonius." Another tradition associates the "Almanac" with a pupil of Ptolemy.

Azarquiel would have had an Arabic translation, now lost, of the original Greek at his disposal. He updated the use of the tables to his epoch and may have introduced the Julian month names in the tables alongside the Egyptian ones. [Estudios, p. 237]. Just what other changes and additions he made is not clear. The work of Azarquiel is a bridge between the work of Ammonius and the large number of medieval almanacs, of which it is one of the first examples.

The "Almanac" consists of an introductory section [Estudios, p. 75-148], presented in the Arabic and in the Spanish, explaining the use of each group of tables, followed by the tables themselves [Estudios, p. 153-234; Tables 1-83].

The tables of planetary longitudes [Estudios, p. 177-214; Tables 26-63] are the most lengthy. They are set up planet by planet to give geocentric longitudes at ten day intervals for the slower planets, Saturn and Jupiter, and five day intervals for Mars, Venus and Mercury and run for the following periods: Saturn, 59 years; Jupiter, 83 years; Mars, 79 years; Venus, 8 years; Mercury, 46 years. The longitudes are expressed in zodiacal signs and integer degrees and arranged in columns: for the ten day planets, the 37 entries for each year occupy one column of a table; for the five day planets, the first 36 entries for each year occupy one column of a table and the other 37 entries for that year occupy the corresponding column of the following table. Each table contains all or half of the entries for ten years of the period of the particular planet (eleven or twelve years in some cases and eight years for Venus). There are six tables for Saturn, eight for Jupiter, fourteen for Mars, two for Venus, and eight for Mercury. The complete tables for Venus and two from those of Jupiter are found in Appendix I.

The Egyptian year is composed of twelve thirty day months and five epagomenal days at the end of each year. In the tables of planetary longitudes, entries for the five day planets correspond to the 1st, 6th, 11th, 16th, 21st, and 26th of each Egyptian month, with the last entry of each year corresponding to the first epagomenal day or the 31st of the last month. For the ten day planets, there are entries for the 1st, 11th, and 21st of each month and also at the end of each year for the first epagomenal day; thus between the thirty-seventh and last entry of one year and the first entry of a following year there is a five day interval only. Julian month names are entered to the left of the Egyptian ones on the same level, but in the introductory section [Estudios, p. 80, p. 120] Azarquiel explains that one must subtract one day from the Egyptian day for each Julian month of thirty-one days that has passed and add two if February has already passed to find the corresponding day of the Julian month. Thus the longitude entry for the first of the Egyptian month of Hatur is not correct for the 1st of November but for the 31st of October, since October has thirty-one days, etc.

Also in the introductory paragraphs [*Estudios*, p. 79-85, p. 120-123] referring to the use of these tables, it is explained what corrections must be made for errors, accumulated due to the naturally somewhat inexact period relations. Azarquiel takes his epoch as 1400 of the Seleucid era (1 October 1088 - 1 October 1089 A.D.), and he indicates in what column one should enter the tables for each planet for that year and what correction should be made in the text entry. However, since each year column begins on the first of September, it is not clear whether the first entry in the column he refers to is for the first of September 1088 (first of last month of the Seleucid year 1399)--while the entries for all other months in the column are in 1400--, or whether the first entry of the said column is for the first of September 1089 (first day of last month of Seleucid year 1400)-- in which case the entries from October on are for the year 1401. After comparing the true longitudes of the planets on September first of 1088 and September first of 1089 with those obtained from the text, it may be concluded that the first hypothesis is the correct one. These longitudes are shown below.

Planet	Mercury	Venus	Mars	Jupiter	Saturn
column to enter	21	7	78	59	6
text	Virgo 17 = 167°	Cancer 25 = 115°	Virgo 1 = 151°	Gemini 24 = 84°	Taurus 24 = 54°
correction	0°	30'	30'	7°30'	3°30'
1 Sept.	167°	115°30'	151°30'	91°30'	57°30'

Planet	Mercury	Venus	Mars	Jupiter	Saturn
1 Sept. 1088 (1399)	179°	123°	159°	100°	65°
1 Sept. 1089 (1400)	151°	210°	19°	127°	78°

[Longitudes, p. 439-440]

Although it is quite clear that the initial entries in the columns indicated give the planetary positions on September 1, 1088 (rather than on September 1, 1089), there is a nearly constant difference of 8° between Azarquiel's longitudes and the actual longitudes on that date. This must mean that Azarquiel's location of the vernal equinox was too far east by about 8°.

It would have seemed reasonable to assume that year one of the tables for each planet was, at the original author's epoch, the same year. However, it proved impossible to find a horoscope with the planets' positions as indicated by the initial entries of each set of tables: Saturn, Pisces 12; Jupiter, Cancer, 28; Mars, Virgo 12; Venus, Leo 29; Mercury, Virgo 22. Also if such a year, x, existed it would be possible to find it using the information displayed above. Thus at Azarquièl's epoch, September first of year one is September 1, 1083 for Saturn, September 1, 1030 for Jupiter, September 1, 1011 for Mars, September 1, 1082 for Venus, and September 1, 1068 for Mercury; the hypothetical original year one is given by

$$\begin{array}{ll} x = 1083 - a\ 59 & x = 1082 - d\ \ 8 \\ x = 1030 - b\ 83 & x = 1068 - e\ 46 \\ x = 1011 - c\ 79 & \end{array}$$

where x, a, b, c, d, and e are integers. But this set of equations has no plausible solution. Thus apparently the tables did not have a common beginning at the epoch of the original author.

Text variants between the Arabic and the Spanish manuscripts as well as restorations are noted at the bottom of each table (P for the Arsenal manuscript; M for the Munich one). There is agreement between the two texts to a large extent. Slight variations though are frequent for the slower moving planets, Saturn and Jupiter, probably due in most cases to a copyist's or translator's confusion in deciphering similar Arabic letters [*Estudios*, p. 151].

CHAPTER III

MEAN MOTIONS

Whatever method was used in calculating the tables of planetary longitudes in the "Almanac of Ammonius", one is led to the consideration of certain underlying relations.

The periods of each planet, for which their longitudes are tabulated, were already mentioned. Inspection of the tables to see how many times during the years of its period a planet returns to its initial position in the zodiac and how many times it retrogrades in this period gives the number of revolutions of the planet in the ecliptic and the number of complete loops or complete synodic periods in the period. These satisfy the relations of Chapter I as can be seen below.

Planet	Period (years)	Revolutions of the planet	Number of synodic periods
Saturn	59	2	57
Jupiter	83	7	76
Mars	79	42	37
Venus	8	8	5
Mercury	46	46	145

These are the original Babylonian periods, still extant in the "Goal-Year-Texts" [Sachs, p. 283]. They are also quoted with required corrections for Saturn, Mars, Venus and Mercury in the Almagest [Almagest, p. 121-122] (the Jupiter period given differs--it is 71 years during which there are 6 revolutions of the planet and 65 synodic periods). Similar period relations were also used to construct longitude tables in India by Mahādeva in the early thirteenth century.

For further work it was found convenient to convert the longitudes expressed in signs of the zodiac and degrees to degrees, and also to let the longitudes build up multiples of 360°, each revolution in the ecliptic adding 360°. This, for lack of a better method at the time, was done by hand for each planet and later punched on 999 IBM cards in all to be used as the base data for the study. The punched card layout is as follows: each data card has 80 numbered punch positions; five positions are reserved for each longitude value, the last digit occupying a position divisible by five, at the rate of fifteen longitudes in order of occurence in the tables per card; the last five punch positions are for card identification, position 76 for the planet (1 for Saturn, 2 for Jupiter, 3 for Mars, 4 for Venus, and 5 for Mercury), positions 77, 78, 79 and 80 contain the serial number of the card, the data cards for each planet being numbered from 1. Thus, for instance the first fifteen longitude values for year one of Mars are punched in order on the card bearing the identification 30001. A simple program eliminating multiples of 30 brought the data back into the original form, less the signs of the zodiac of course, making it possible to check for certain calculation or punching errors.

As explained in Chapter II, Egyptian month names figure in the "Almanac" tables. But the period relations upon which the tables are based are correct for actual solar years, not for the Egyptian 365 day years. There is no evidence of a leap year of 366 days every fourth year; the fastest moving planet, Mercury, during its direct motion can reach the speed of two degrees per day, and yet no jump is visible in a graph of the longitudes. It is thus unlikely that the tables include an intercalary scheme. However, whatever the case, the average year must be a solar year. For purposes of recalculation and comparison with the text, the year has been divided into 365 equal "days", each slightly longer than the twenty-four hour mean solar day. The difference between the 1st or 300th "day" or any other "day" of a solar year composed of these hypothetical "days" and respectively the 1st or 300th or other mean solar day of a Julian year, can never be more than twelve hours, assuming the extra day of a leap year to be inserted when the Julian year is lagging the solar year by half a day. This asynchronism induces a difference of less than one degree in longitude for

Mercury, which is not significant here since the longitudes are given only to the nearest integer degree. What holds for Mercury is true a fortiori for the slower moving planets. Henceforth, the term day will refer to the artificial day here defined.

The mean motion of a planet, $\dot{\lambda}$, i.e. its average speed as it moves around the zodiac, is

$$\dot{\lambda} = \frac{\text{number of revolutions of planet in period X } 360^\circ}{\text{number of years of period X 365 days}}$$

$$= \text{degrees traveled/day}$$

The mean motion of each planet per five or ten day interval can be obtained using the equation above and the period relations.

Mean Motions	
Saturn	.33433947° / 10 days
Jupiter	.831820432° / 10 days
Mars	2.621813768° / 5 days
Venus	4.931506849° / 5 days
Mercury	4.931506849° / 5 days

Suppose one plots true longitudes, as obtained from the tables, against time, joining sucessive points by the best possible curve. Then the graph of the "mean longitude", $\bar{\lambda}$, will be the straight line that best approximates this curve. The slope of the line has been found; it is the planet's mean motion. Thus the only unknown is the initial mean longitude, $\overline{\lambda}_0$. The area between the curve showing the true longitudes and the line consists of a part below the graph of the mean longitude and a part above; the area of these two parts should be equal to one another. The difference at any given moment, $\lambda - \overline{\lambda}$, is called the "equation."

To find the initial mean longitude, $\overline{\lambda}_0$, the area, between the graph of the true longitude and that of the mean longitude, was approximated by a sum of rectangles of unit base (the unit being five or ten days depending on the intervals at which the longitudes for the planet are tabulated) and of height equal to the equation, $\lambda - \overline{\lambda}$, (at the left-hand end of the interval), which may be positive or negative. Thus the area of each rectangle will be signed: those situated above the graph of the mean longitude, positive, those below, negative. The sum of the signed areas should be zero when the initial mean longitude has been correctly located, which is equivalent to the condition that the area above be equal in absolute value to the area below.

With a dummy value for the initial mean longitude, suppose the value of the first longitude entry in the tables for the planet, and the known mean motion, the IBM 1620 Computer was programmed to read the true longitude and compare it with the corresponding dummy mean longitude it calculated to give the equation, and repeat for each true longitude value of the data for that planet. Each equation is multiplied by the length of the base, which is always one unit, except at the end of each year for the planets Saturn and Jupiter, when it is of length one-half, since although in general sucessive entries in the tables for these planets are separated by ten day intervals, the last entry of each year is followed by a five day interval. These products are summed as the operation proceeds to give a final value for the sum of signed rectangle areas, presumedly different from zero. Finally this sum is divided by the number of five or ten day intervals, and the signed quotient is added to the dummy value to give the correct initial mean longitude, $\bar{\lambda}_o$, i.e.

$$\bar{\lambda}_o = \text{dummy value} + \frac{\sum(\lambda - \bar{\lambda}[\text{dummy}]) \text{ X length of base}}{\text{number of five or ten day intervals}}$$

A parameter card, which causes all calculations in the machine to be carried to any desirable number of significant places up to twenty-eight, beyohd the usual eight, is required for Mars, Mercury, and Jupiter to avoid errors accumulated due to repeated additions in obtaining the mean longitude. For all practical purposes carrying calculations to eight significant places is enough for Saturn and Venus.

Thus by raising or lowering the graph of the mean longitude with respect to the curve of the true longitudes as obtained from the tables, the areas above and below the graph of the mean longitude were equalized to give a determination of the initial mean longitudes for each planet.

After substituting the new $\overline{\lambda}_0$ for the dummy value, the same program with a slight modification will provide a complete listing of true longitude data, mean longitudes and equations. Careful plots of these equations against time, made by hand for each planet, proved instructive. The curve of the equation is a periodic function with, like the true longitude curve (Fig. 2b), one relative maximum and one relative minimum in each synodic period, but its median line is the horizontal axis, not the graph of the mean longitude.

First, large deviations of isolated entries from the general trend of the longitude curve became evident. A few of the worst displayed in the table below, were restored to give points lying along the curve of the equation.

Planet	Year	Date	Text Entry	Restored Value
Saturn	45	Toth 11	10	20
Jupiter	18	Quiach 11	16	26
	30	Mocre 1	12	19
	74	Mocre 31	0	5
Mars	10	Anxir 26	9	29

It is a question open to discussion whether it would be wise to attempt restoration of the numerous other entries that appear to be obvious errors or inconsistencies in the text, i.e. whether the comparison later between recomputations of the tables and the tables themselves will be more meaningful if the latter are left nearly unchanged or if they are corrected in light of the equation curve as a whole. The attitude is taken here that since it is difficult to decide at this point what changes are legitimate, no more corrections should be made.

Secondly, it is seen from visual inspection of the plots that successive maximum and minimum equations are not equal, but oscillate in a repeating pattern similar to that shown in Fig. 8. There are two motions, one superposed on the other. This phenomenon is clearest in the case of Saturn, but it can be discovered in each plot. The underlying model is therefore fairly sophisticated, approximating quite accurately the true characteristics of geocentric longitudes.

Since some corrections have been made in the data, the initial mean longitude may be redetermined. The final results for $\bar{\lambda}_0$ for each planet are given below.

Planet	$\bar{\lambda}_0$
Saturn	347.415615°
Jupiter	108.380878°
Mars	174.187619°
Venus	157.640410°
Mercury	158.345144°

From the discussion of planetary longitudes in Chapter II, it is to be expected that the initial mean longitudes of Venus and Mercury should be the same, since the first entry of year one of each planet's period is for the first of September of some year, they should be equal to the longitude of the mean sun. In fact the values obtained are equal when rounded off to the nearest integer degree.

The Greeks adopted Babylonian period relations. They are also known to have revised some of the Babylonian ephemerides and worked with their linear methods based on zigzag functions. It would thus be possible that the tables are Babylonian, either in origin or in method.

There is an Indian model (which may be of Greek origin) with two epicycles used to predict planetary longitudes, and it has already been mentioned that period relations such as those found in the text figure in some Indian work. Therefore the tables under study could also be Indian.

Otherwise it is highly probable that the "Almanac" tables of planetary longitudes were calculated using some form of the epicyclic theory described in Chapter I. Such models may have been of three types: a simple epicyclic model; the same with the earth occupying an excentric position with respect to the center of the deferent; and the Ptolemaic type with an eccentricity and an equant.

The possibility of a simple epicyclic model can be immediately eliminated for such a model always produces identical loops, all maximum and minimum equations are equal in absolute value. This is not the case here, as seen above.

In the following, the possibility that one of the other two types of epicyclic models mentioned is at the base of the tables, will be tested.

CHAPTER IV

PRELIMINARY DETERMINATION OF PARAMETERS

The basic problem of discovering what system was used to calculate the tables of planetary longitudes in the "Almanac of Ammonius" has been limited to the examination of two principal alternatives, an excentric epicyclic model or a model of the Ptolemaic type with equant. Both incorporate certain parameters that must now be extracted from the data.

The number of epicyclic revolutions measured from H (Fig. 7) is equal to the number of synodic periods in the number of years of the planet's period, for once in each epicyclic revolution the planet retrogrades. The anomalistic motion of the epicycle is

$$\dot{\gamma} = \frac{\text{number of revolutions of the epicycle X } 360^{\circ}}{\text{number of years of period X 365 days}}$$

$$\simeq \text{ degrees traveled / day.}$$

The motion in the anomaly per five or ten day interval for each planet, found using this equation and the period relations, are given below.

Anomalistic Motion	
Saturn	9.52867425° / 10 days
Jupiter	9.03119327° / 10 days
Mars	2.30969308° / 5 days
Venus	3.08219178° / 5 days
Mercury	15.54496724° / 5 days

In the simple epicyclic model any maximum equation is equal to any other maximum or any minimum in absolute value. The entire equation is due to the movement of the epicycle, and a maximum or minimum occurs when the line joining the earth to the planet is tangent to the epicycle (Fig. 9). If $a = \max\,(\lambda - \bar{\lambda})$, then

$$r = R \sin a$$

where r is the radius of the epicycle and R is the radius of the deferent. One has chosen here to assume R equals 60 units which is used by Ptolemy. Thus

$$r = 60 \sin a$$

When the model has an eccentricity with or without equant embedded in it, then the equation is not due to the anomalistic motion alone. However, one can attempt to eliminate the effect of the eccentricity or equant by replacing a by the quantity

$$b = \frac{\sum \text{relative maximum equations} - \sum \text{relative minimum equations}}{\text{number of maximum and minimum}}$$

where the number of relative maxima and minima is equal to twice the number of epicyclic revolutions. Then

$$r = 60 \sin b$$

is a fair estimate of the epicycle radius. This has been done here. For each planet the relative maxima and minima were found from the complete listings of equations previously obtained and summed on a desk calculator to give the numerator of the expression for b; this is divided by the number of maxima and minima to yield b. The values then obtained for the epicycle radius, $r = 60 \sin b$, are shown below alongside the corresponding <u>Almagest</u> parameters for each planet.

Epicycle Radius r		
Planet	Value obtained from data	<u>Almagest</u> value
Saturn	6.740	6.500
Jupiter	11.622	11.500
Mars	41.074	39.500
Venus	43.240	43.167
Mercury	22.969	22.500

The calculated estimates of r for Saturn, Jupiter, and Venus are nearly Ptolemaic, though for Mars there is a significant difference. However, it should be pointed out that the method used in arriving at these estimates of the epicycle radius is not equally good for all planets. In the case of Venus the maxima and minima do not vary much in size, i.e. the eccentricity or equant is small, and one would expect the estimate to be good. It should also be good for Jupiter and Saturn because there are many revolutions of the epicycle, occuring at very random distances from the apsidal line OEG (Fig. 7). On the other hand, the value for Mars is dubious since a smaller number of epicyclic revolutions occur in a large number of planetary revolutions,and the randomness of distances of maximum and minimum equations from the apsidal line is not assured; moreover the variations in size of the maxima and minima is largest for Mars, and it may be expected that eliminating the effect of the eccentricity or equant will be difficult.

Studying the motions of Mercury has always presented particular difficulties. In this study, certain problems met in working with the tables for this planet have led to the decision not to deal further with parameters and recomputations for Mercury here.

Considering again the epicyclic model of Fig. 9, it can be seen that when the planet is at H or H' the equation is zero degrees. At H the anomaly is $0^{\circ} + n\ 360^{\circ}$, and the equation passes from negative to positive; at H' the anomaly is $180^\circ + n\ 360^\circ$, and the equation passes from positive to negative. If the anomalistic motion is known, the anomaly of any point H or H' can be used to find the anomaly γ of any other point desired. If some point H is used, then for the corresponding n

$$\gamma = n\ 360^\circ - \dot{\gamma}\text{ per interval X number of intervals between H and the point}$$

If a point H' is used

$$\gamma = 180^\circ + 360^\circ - \dot{\gamma}\text{ per interval X number of intervals between H' and the point}$$

In a model with eccentricity or with eccentricity and equant, a zero degree equation no longer corresponds to zero or 180° anomalies since the eccentricity and equant also contribute to the equation. But somewhere between each minimum equation and the following maximum, the anomaly is $0^\circ + n\ 360^\circ$, and somewhere between a maximum and a minimum equation, the anomaly is $180^\circ + n\ 360^\circ$. Approximations to such places are the longitude values midway between the beginnings of the phenomena of retrogradation and their ends (anomaly near $180^\circ + n\ 360^\circ$), and midway between their ends and beginnings (anomaly near $0^\circ + n\ 360^\circ$). Each revolution of the epicycle in the period could thus give a tentative determination of γ_0, the initial anomaly or for each planet the anomaly of the first entry in its longitude tables.

$[\gamma_o] = n\,360^\circ - \dot{\gamma}$ X number of intervals between midpoint longitude and initial longitude

or $[\gamma_o] = 180^\circ + n\,360^\circ - \dot{\gamma}$ X number of intervals

The correct value of n must of course be used for each determination. Averaging all these first estimates, the parameter γ_o can be approximated.

$$\gamma_o = \frac{\sum [\gamma_o]}{\text{number of estimates}}$$

where the number of estimates equals twice the number of epicyclic revolutions. The values for the initial anomaly obtained in this manner, but for Jupiter and Saturn using only part of the entire period, are: Saturn, 167°; Jupiter, 49°; Mars, 326°; and Venus, 343°.

Another method was also used. Figures were drawn showing the position of the planet on day one of the tables. in which various apogees and the Ptolemaic eccentricity and equant and the parameters already determined here were used. The initial anomaly could then be read from the figure. In this way the following estimates were obtained: Saturn, 160° - 170°; Jupiter, 45° - 50°; Mars, 340° - 345°; Venus, 340° - 345°.

But there remains an important fact to be used in the determination of the initial anomaly of Mars, Jupiter, and Saturn, the outer planets. In Chapter I, it has been seen that on the same day or on the same date of different years (this being a yearly phenomenon depending on the sun) the radius from the center of the epicycle to an outer planet is in the same direction as the line from the earth to the sun, while for an inner planet the center of the epicycle lies on this line. This is equivalent to stating that for an outer planet the mean longitude $\bar{\lambda}$ plus the anomaly γ at any given moment of the year is always equal to the mean longitude of Venus or Mercury or of the sun (Fig. 5, Fig. 7). Since the determination of $\bar{\lambda}_0$ is based on every entry in the tables, it is a reliable parameter; also it is for the first of September of some year for each planet. Therefore the various values above of γ_0 for the outer planets will be corrected to that given by

$$\bar{\lambda}_0 \text{ Venus } - \bar{\lambda}_0 \text{ outer planet} = \gamma_0 \text{ outer planet (modulo } 360^\circ)$$

where $\bar{\lambda}_0$ Venus $= 157.6^\circ$. The new values, Saturn 170°, Jupiter 49° and Mars 343°, do not differ greatly from those obtained earlier.

Two other attempts were made to obtain a trustworthy value of γ_0 for Venus as well, one arithmetic, the other graphical. The arithmetic method is a variation of one described earlier. Since the departure from a purely epicyclic model is small in the case of Venus, the so-called points H and H' having zero equation were used as they would be to obtain γ_0 in an epicyclic model. All points did not give the same value for γ_0, as is to be expected, but the average namely 342.4°, of the ten values thus obtained was considered another fair estimate of the initial anomaly. Lastly a graphical method, using the plot of the Venus equation, was tried. A transparent strip of paper with ten equal divisions of the eight year period, to the same scale as the plot, was fitted over the graph of equations such that in general all the end points of the divisions were as close to the points H and H' as possible. Then the point at which the beginning of the first equal division fell, was marked on the graph; the anomaly of this point, which differs slightly from the point H, was considered to be 360°. The initial anomaly was then found using the number of days that separate this point from beginning of the tables; this gave γ_0 equals 342.8°.

The final values arrived at for the initial anomaly of the various planets are displayed below.

Planet	γ_0
Saturn	170°
Jupiter	49°
Mars	343°
Venus	342°-343°

The difference between the longitude of the center of the epicycle C and the mean longitude is a function having for period one revolution of C and due to the eccentricity or to the eccentricity and the equant. This difference is zero when the longitude of C is equal to the longitude of the deferent apogee λ_a plus or minus a multiple of 180°. When the difference passes from small positive to small negative values, C passes through the apogee; when the difference passes from small negative to small positive, C passes through the perigee, and its longitude is equal to $\lambda_a + 180^\circ + n\ 360^\circ$, where n is an integer.

To arrive at the longitude of the apogee λ_a, an attempt to obtain the curve of these differences between the longitude of the center of the epicycle C and the mean longitude, by minimizing the effect of the anomalistic motion on the equation, has been made. The equations of the longitude values midway between the beginnings of the phenomena of retrogradation and their ends, and midway between their ends and beginnings were recorded. In the discussion concerning γ_0 it has been seen that the anomaly at such places, is near zero or 180°, i.e. the equation is due to the eccentricity and equant alone, for the effect of the epicycle is minimum in the equation of longitude values located as specified. Thus all recorded equations which occur in one revolution of C lie on one period length of a curve which approximates the curve of the differences between the longitudes of C and the corresponding mean longitudes. Each revolution of the planet should give another copy of the desired curve. For greater accuracy in drawing this curve equations from every revolution of the planet are superposed on the same plot of one period length (of 360° or of the number of years C requires to make one revolution for the particular planet.) Also in drawing the curve, one can make use of the fact that the part of the curve which lies above the horizontal axis is the mirror image of the part below, i.e. the maximum of curve equals the minimum, etc. The longitude of the point where this curve crosses the horizontal axis going from positive to negative provides a value for the longitude of the apogee λ_a.

The usefulness of this method is limited to Saturn and Jupiter and, to a lesser degree, Mars. It is quite good for Saturn and Jupiter; their epicycle radii are small, and for each trip of the planet around the zodiac a good curve can be obtained using the equations of longitude values located midway with respect to the phenomena of retrogradation as described above; if the curves are superposed as in the plot made here, a perhaps even better idea can be gained of the position of the apogee and of the maximum and minimum differences between the longitude of the center of the epicycle and the mean longitude caused by the eccentricity and the equant. For Mars and Venus it cannot be expected that one could draw the curve in question from midway equations occuring in a single revolution of the planet; the points obtained are too few, for the epicycles of these planets are large, and their anomalistic motion is slower than that of Saturn or Jupiter, their mean motion, faster. However, if the points from all revolutions of the planet are superposed on the same plot as done here, a fair curve can be obtained for Mars. Not very surprisingly this proved impossible for Venus, the eccentric effect being very small and the number of revolutions of the epicycle, few; a cluster of points resulted through which no reasonable curve could be sketched.

The values found for the longitude of the apogee of the three outer planets are shown below with the Ptolemaic parameters for the four planets.

Longitude of apogee λ_a		
Planet	Value obtained from data	Almagest value
Saturn	232°	224°
Jupiter	167°	152°
Mars	130°	107°
Venus		46°

In a model without equant, the curve of the differences of the longitude of the center of the epicycle C and the mean longitude is due solely to the eccentricity. Consideration of Fig. 10 will show that if this is the case, the eccentricity is given by

$$e = 60 \sin g$$

where g is the maximum difference between the longitude of C and the mean longitude or the maximum of the curve discussed above.

In a model of the Ptolemaic type, the eccentricity is

$$e = 60 \tan \tfrac{1}{2} g$$

as can be seen in Fig. 11, where again g is the maximim of the curve of longitude differences between C and the mean.

An approximation to this curve has already been plotted for the study of the longitude of the apogee. In order to have an idea of the importance of the effect of the eccentricity or the combined effect of the eccentricity and the equant, it will be assumed first that the model incorporates an eccentricity only. Then by taking $e = 60 \sin g'$, where g' is the maximum of the curve plotted, the eccentricity e can be found approximately to be Saturn, 6.8 parts, Jupiter, 5.8, Mars, 11.5. Secondly, assuming that the model contains an equant, $e = 60 \tan \frac{1}{2} g'$, where g' is the same as above. Values of e thus obtained are given below with the Ptolemaic eccentricity, from a model in which, it should be remembered, an equant figures.

Eccentricity e		
Planet	Value obtained from data	<u>Almagest</u> value
Saturn	3.4	3.417
Jupiter	2.9	2.750
Mars	5.8	6.000
Venus		1.250

The same comments apply to the usefulness of this method for the various planets as those already made concerning the determination of the longitude of the apogee, in particular no estimate of e can be reached for Venus.

CHAPTER V

RECOMPUTATION OF THE TABLES

Having obtained values for the parameters that figure in the models under consideration, one is now in a position to attempt a reconstitution of the tables.

The mean motion, $\dot{\bar{\lambda}}$ per five or ten day intervals, and the anomalistic motion, $\dot{\gamma}$ per five or ten day interval, can be considered fixed at the values given in the preceding chapters. The initial mean longitude $\bar{\lambda}_0$ has also been determined accurately for each planet, and as seen above, this determination dictates the initial anomaly γ_0 for the three outer planets. The other parameters found in Chapter IV are at best approximations to the true values of these same parameters embedded in any such model at the base of the tables, but they certainly give plausible material with which to begin recomputations.

For this purpose an epicyclic model with eccentricity only, and also a model of the Ptolemaic type with equant were programmed for the IBM 1620; the programs can be found in Appendix II. A measure of the correctness of a set of parameters was gained by squaring the differences between corresponding entries in the resulting tables of true longitudes and those of the text, and summing. This operation was built into the program. Dividing the sum of squared differences by the number of the values and taking the square root gives the average deviation of the computed values from the text values. If this falls below one degree, the agreement between the text and the recomputation can be considered very good, for longitudes in the text are given in integer degrees. The program calls for each "Almanac" true longitude to be printed as it occurs in the text (i.e. residue modulo 30°) minus the signs of the zodiac, and also as it is on the data card, on a single horizontal line together with the calculated longitude, the mean longitude, the corresponding equations (both from the data longitude and from the calculated value), their difference, the sum of the squared differences up to that point, the anomaly, and other variable angles in the model. The next line begins with the next entry in the tables followed by the same longitude value as it appears in the data and the corresponding calculated longitude, the mean, etc. (Appendix III).

Since the period of Venus is short, reconstitution of the tables for this planet was begun first, supposing a model with equant, and using the Ptolemaic eccentricity and apsidal position, along with the values for the epicycle radius and the initial anomaly determined from the data in Chapter IV. As explained, the deviation from a simple epicyclic theory is small for Venus; the equant and eccentricity are relatively unimportant. However, after several runs with some variations in the apogee, it became clear that the longitude of the apogee in the tables was larger than the Ptolemaic one. By methodically increasing it by five degrees, a better value for λ_a was found to be near 85°. Increasing and decreasing the epicycle radius r slightly in the model revealed that the Ptolemaic value of 43.1667° (43°10') gives the best results. Likewise the Ptolemaic eccentricity proved best, any variation in it increasing the sum of squared differences. A small change in the initial anomaly does not affect the computed values radically, but taking γ_o equals 342.75° was a little better than 342° or 343°. Returning to the longitude of the apogee again now that other parameters have been corrected, and varying it degree by degree, it was found that the optimum value for λ_a is 87.4°, a rather large value compared to the one in the Almagest, namely 46°. Any further changes in any of the parameters worsened instead of improving the results.

The trial and error factor was allowed full play in the case of Venus, for with thirty-nine data cards the time required for one computation of the tables on the 1620 was only about twenty minutes. One was obliged to rely on it completely for the determination of λ_a and to a lesser extent of e. Moreover, in the case of the other planets as well, since computations were begun during the determination of the parameters, on occasion by consistent varying of a parameter, its value was found without reference to the independent study made of it in Chapter IV.

Again for Saturn and Jupiter, programming the equant model with different sets of parameters disclosed that the Ptolemaic values for the epicycle radius and the eccentricity are the best. The initial anomaly is already fixed. The longitudes of the apogees of Saturn and Jupiter obtained in the preceeding chapter (only approximations) must be modified by a few degrees to induce greatest agreement as variations of this parameter showed; λ_a is then 234.9° for Saturn and 162° for Jupiter.

In the case of Jupiter, and in spite of the tremendous speed of the computer compared to manual calculation, the 205 cards of data (83 years) and the use of a parameter card, made attempted reconstitutions very time consuming. Printing on the console typewriter further increased the time required, and until a high speed printer became available, the sum of squared differences alone printed at the end of all calculations had to serve as the indication of how well the computed longitudes agreed with the data. In spite of this disadvantage, enough information was gleaned to permit rapid changes to give optimum results as soon as it was possible to make complete listings. What has just been stated applies _a fortiori_ to Mars; a complete run through the 385 data cards with parameter card, to recalculate the tables, requires over six hours of 1620 computer time using the high speed printer. The time element precluded the variations of parameters found useful for the other planets. However, at this point the results obtained for Saturn, Jupiter, and Venus furnish significant indices as to what parameter values could be meaningfully tested.

Thus, again for Mars in an equant model the _Almagest_ values for the epicycle radius r and the eccentricity e prove the best. Neither the Ptolemaic value for the longitude apogee nor the value obtained experimentally provide good results. But since the values of λ_a for Jupiter and Saturn are approximately ten degrees larger than the Ptolemaic ones, it was reasonable to suppose that the same would hold for the third outer planet. And in fact a longitude of 117° for the apogee permits the calculation of longitudes very close to the text longitudes. The largest deviations occur during retrogradations of the planet near perigee (Fig. 12).

If a simple eccentric model is run taking the eccentricity equal to approximately double that used in an equant model, but with the other parameters unchanged, sensibly the same results can be obtained for Venus and Saturn. In fact in the recomputations made the best results for each planet are obtained using a model with equant. However, conceivably that with further variation of the eccentricity a model without equant could theoretically be made nearly as satisfactory with respect to the sum of squared differences as the equant model. But historically the eccentricity was obtained from a study of the oppositions and stationary points, and as far as is known was thus very nearly the same in tables before and after the introduction of Ptolemy's equant. The unnaturally large eccentricity values that would have been used in these tables, had they been calculated using a model without an equant, argue strongly for the rejection of such a hypothesis.

Programs were written for the 1620 that would have the printer plot the equation of the text longitudes and the equation of the calculated longitudes on the same axis (Appendix II). This permitted a further evaluation of the final results by making irregularities in the text evident. It can be seen that a considerable number of the larger deviations occur in such situations. For example, in Appendix III, the sample output from the Jupiter plot shows an instance where a sum of squared differences of 300 is run up for twelve entries, or while less than one card of the 205 data cards is passing, caused by a jump in the text curve. This type of discrepancy may be due to an error on the part of the original calculator of the tables, since it is not a question of an isolated variation from the general trend easily introduced by some copyist. Thus visual inspection demonstrates that correspondance between the text and the reconstitution is, over most of the curve, better than the tabulated deviation indicates. In all cases the results displayed below are good enough to permit the rejection of the Babylonian and Indian hypotheses formulated in Chapter III.

Optimum Sets of Parameters

Planet	r	e	$\hat{\gamma}_0$	λ_a		Sum of squared differences	Average deviation
				text	<u>Almagest</u>		
Saturn	6.5	3.4167	170°	234.9°	224.17°	740	.6
Jupiter	11.5	2.75	49°	162°	152.15°	4100	1.2
Mars	39.5	6.0	343°	117°	106.67°	16000	1.6
Venus	43.1667	1.25	342.75°	87.4°	46.17°	980	1.2

For each planet the mean motion $\dot{\bar{\lambda}}$, anomalistic motion $\dot{\gamma}$, and initial mean longitude found in the tables of Chapter III and IV are used.

The relatively better agreement of the recomputed values with the text for Saturn is to be expected, since the variation of the longitude curve about the graph of the mean is small; the equation is never larger than 13°. Therefore with the correct mean and anomalistic motions and initial mean longitude the results should be good.

The fact that the apsidal longitudes of Saturn, Jupiter, and Mars are all about ten degrees larger than Ptolemy's values suggests the possibility of dating the original composition of the "Almanac." However, since the rate of precession used by the author is not known it is difficult to draw any conclusions; a date anywhere between the seventh and eleventh century could be indicated.

The leading conclusion of the study, that the tables seem to have been based on the Ptolemaic model and contain Almagest parameters need occasion little surprise. The Almagest, as remarked earlier, was the handbook of astronomers in the Greek and Arab worlds for many centuries.

FIGURES

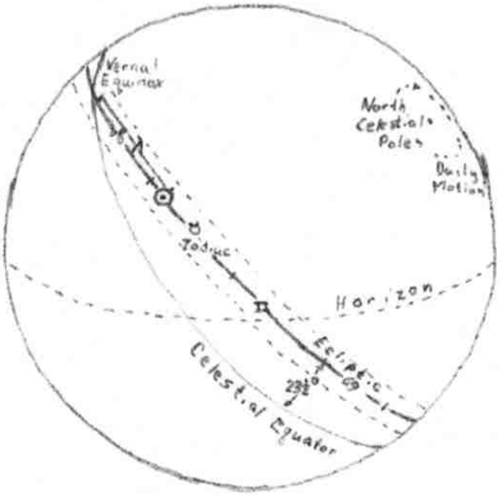


Fig. 1 Celestial sphere and zodiac, showing longitude of sun in Taurus and Gemini rising

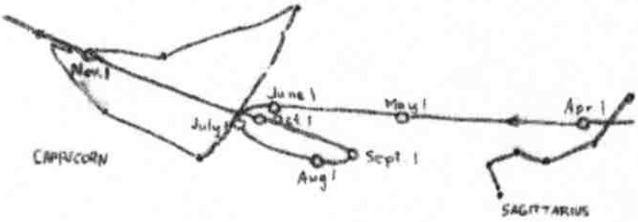


Fig. 2a The path of Mars in 1939 [Baker, p. 140]

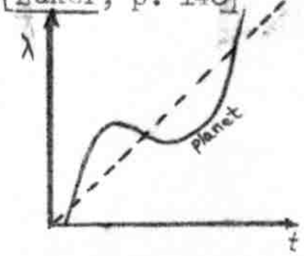


Fig. 2b Graph of planet's geocentric longitude

Heliocentric and Geocentric Planetary Motions [Exact Sciences, p. 123-124]

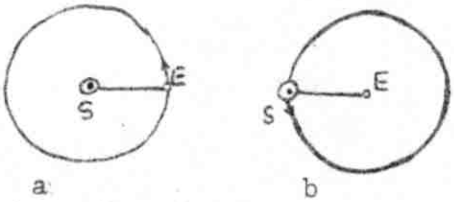


Fig. 3 Sun's motion

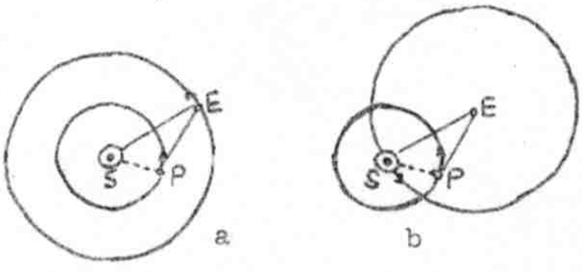


Fig. 4 Inner planet's motion

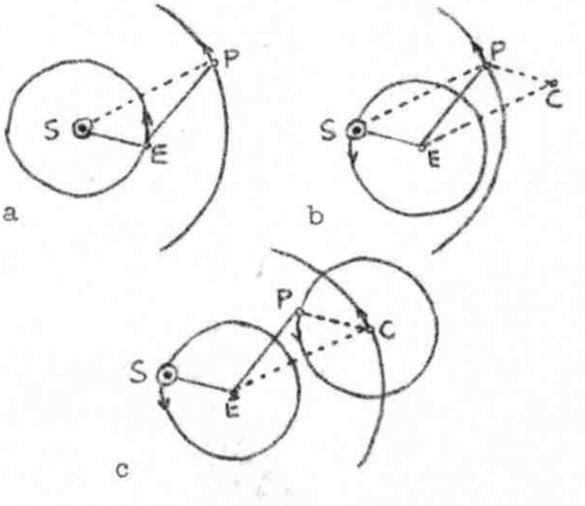


Fig. 5 Outer planet's motion

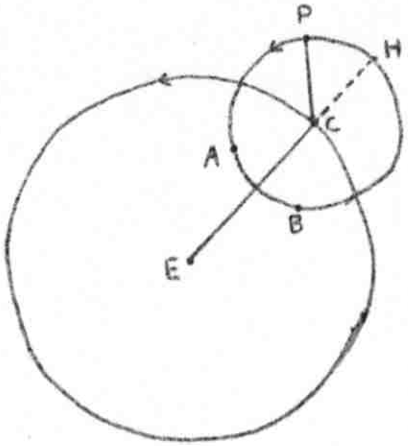


Fig. 6 Epicyclic model

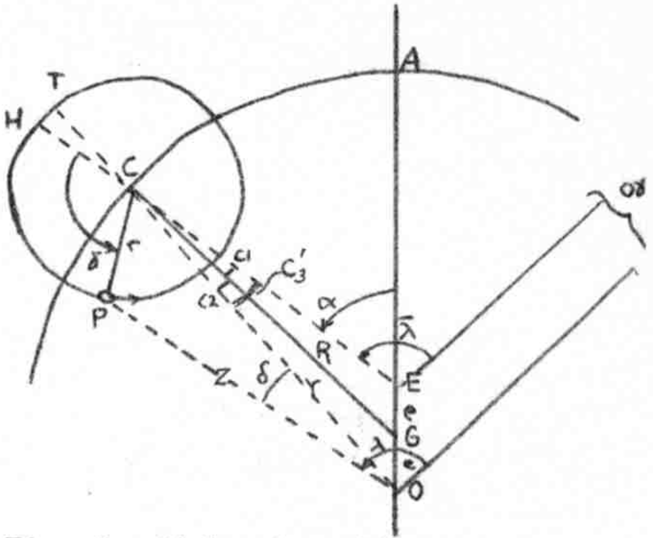


Fig. 7 Ptolemaic model
[Exact Sciences, p. 199]

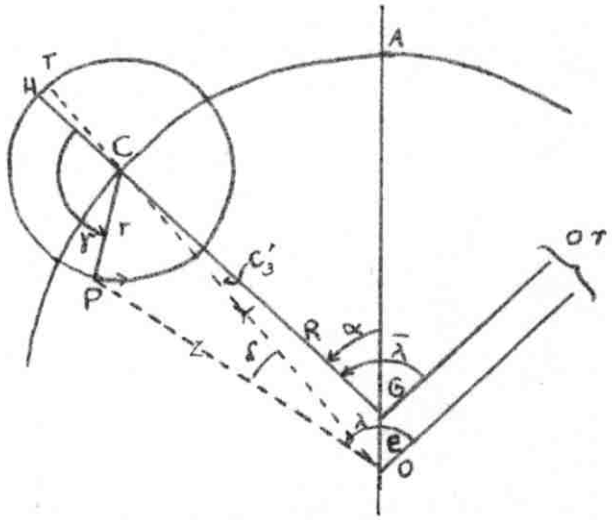


Fig. 7' Model without equant

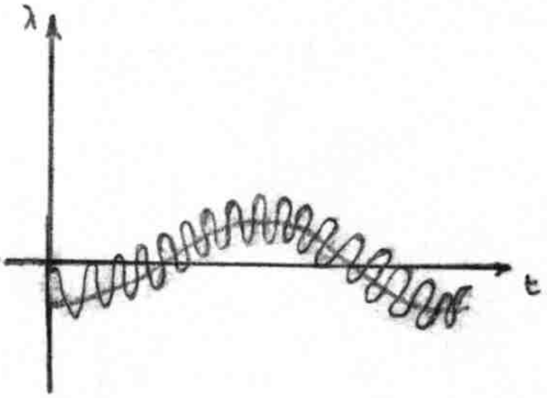
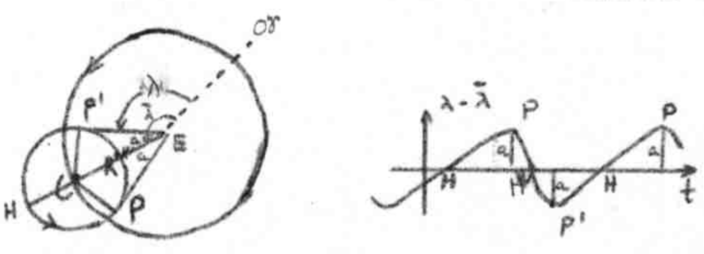


Fig. 8 Curve of equations similar to Saturn plot



a

b

Fig. 9 Epicyclic model

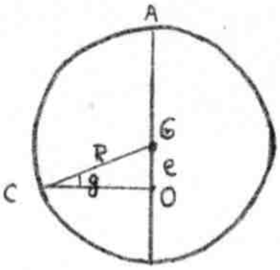


Fig. 10 Model without equant

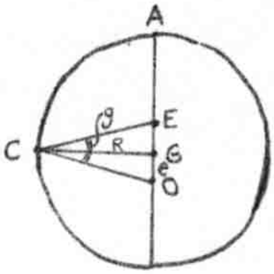


Fig. 11 Model with equant

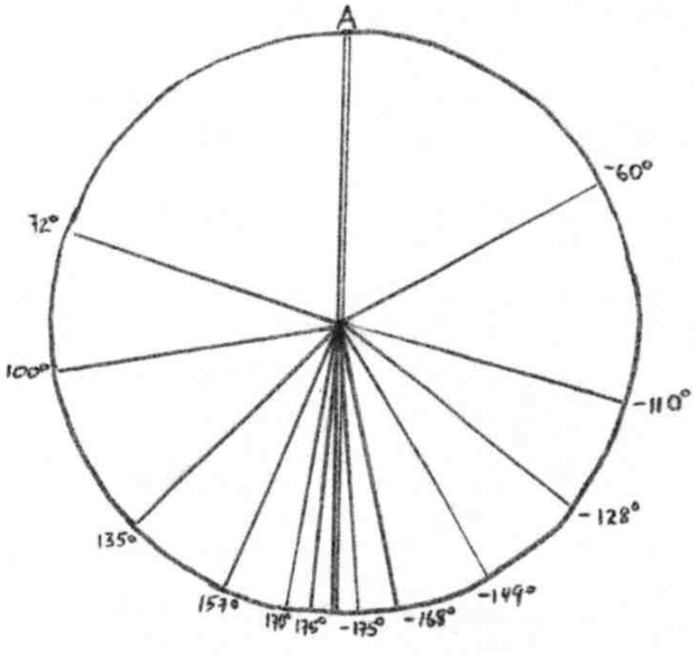


Fig. 12 Distances of the planet Mars from apogee when the worst deviations of recomputed values occur

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APPENDIX I

The "Almanac" tables of planetary longitudes for Venus, whose position is given at five day intervals, follow. Two tables from the set of those for Jupiter, a ten day planet, are also included. The exceptional five day intervals between the last entry of each year and the first of the next year may be noted, as well as the column fifty-nine Azarquiel indicates for his epoch. The tables shown are reproduced from the Estudios.

Mon., f° 36 v.
Ars., f° 120 v.

Tabla 54

مواضع الزهرة المقومة

TABLA DE LOS LOGARES ENDEREÇADOS DE VENUS

Los meses romanos	Los meses egipcianos	Los annos	1	2	3	4	5	6	7	8
		Los dias	Leo	Cancer	Libra	Leo	Libra	Virgo	Cancer	Libra
Setiemb.	Toth	1	29	28	6	11	5	19	25	20
.	.	6	Virgo 9	Leo 1	12	17	3	25	Leo 1	25
.	.	11	11	5	18	23	1	Libra 1	7	Scorpio 1
.	.	16	17	9	24	29	Virgo 29	7	13	6
.	.	21	24	13	Scorpio 1	Virgo 5	26	13	19	12
.	.	26	Libra 0	18	7	11	25	20	25	17
Ochubre	Beba	1	6	22	13	17	22	26	Virgo 1	22
.	.	6	12	27	19	24	20	Scorpio 2	7	27
.	.	11	19	Virgo 2	25	Libra 0	19	8	13	Sagitar. 1
.	.	16	25	7	Sagitar. 1	6	19	14	19	6
.	.	21	Scorpio 1	12	7	12	21	20	25	10
.	.	26	7	17	12	18	23	27	Libra 2	14
Noviemb.	Hatur	1	14	22	18	25	26	Sagitar. 3	8	17
.	.	6	20	28	24	Scorpio 1	29	9	14	20
.	.	11	26	Libra 4	Capricor. 0	7	Libra 3	15	20	22
.	.	16	Sagitar. 3	9	5	13	7	21	26	24
.	.	21	9	15	11	20	11	27	Scorpio 3	26
.	.	26	15	21	16	26	15	Capricor. 4	9	25
Dezemb.	Quiach	1	22	27	22	Sagitar. 2	20	10	15	24
.	.	6	28	Scorpio 3	27	8	25	16	21	21
.	.	11	Capricor. 4	9	Aquarius 3	15	Scorpio 0	23	28	19
.	.	16	11	15	8	22	5	29	Sagitar. 4	17
.	.	21	17	22	13	28	10	Aquarius 5	10	13
.	.	26	23	28	18	Capricor. 4	15	11	16	11
Enero	Tobih	1	Aquarius 0	Sagitar. 4	23	11	21	16	23	9
.	.	6	6	10	27	16	26	23	29	9
.	.	11	12	16	Piscis 1	23	Sagitar. 2	29	Capric. 5	10
.	.	16	18	22	4	29	8	Piscis 5	11	12
.	.	21	25	28	7	Aquarius 6	13	11	17	15
.	.	26	Piscis 1	Capricor. 5	10	12	19	16	26	18
Febreru	Amxir	1	7	11	11	18	25	20	Aquar. 1	22
.	.	6	13	17	12	26	Capricor. 1	27	6	26
.	.	11	19	23	12	Piscis 1	7	Aries 2	12	Capric. 0
.	.	16	26	Aquarius 0	9	6	13	7	19	5
.	.	21	Aries 2	6	9	12	19	13	25	10
.	.	26	8	12	8	18	25	18	Piscis 1	14

9 : 6

Mon., fº 37
Ars., fº 121

Tabla 55

مواضع الزهرة المقومة

TABLA DE LOS LOGARES ENDEREÇADOS DE VENUS

Los meses romanos	Los meses egipcianos	Los annos	1	2	3	4	5	6	7	8
		Los dias	Aries	Aquarius	Piscis	Piscis	Aquarius	Aries	Piscis	Capric
Março	Baramhat	1	14	19	4	25	2	23	8	20
»	»	6	20	25	1	Aries 1	8	28	14	29
»	»	11	26	Piscis 1	Aquarius 29	7	14	Taurus 3	20	Aquar. 1
»	»	16	Taurus 2	7	28	13	20	8	26	6
»	»	21	7	13	27	20	26	12	Aries 2	12
»	»	26	13	20	27	26	Piscis 2	16	9	17
Abril	Barmodi	1	18	26	28	Taurus 2	8	19	15	23
»	»	6	23	Aries 2	Piscis 0	8	14	21	21	29
»	»	11	29	8	3	14	20	22	27	Piscis 5
»	»	16	Gemini 4	14	6	20	26	22	Taurus 3	11
»	»	21	9	20	10	27	Aries 2	21	10	16
»	»	26	15	26	14	Gemini 3	8	19	16	22
Mayo	Baxanz	1	20	Taurus 3	19	9	15	17	22	28
»	»	6	26	9	24	15	21	14	28	Aries 4
»	»	11	Cancer 0	15	Aries 0	21	27	11	Gemini 4	10
»	»	16	5	21	5	27	Taurus 3	9	10	16
»	»	21	9	27	11	Cancer 3	9	8	16	22
»	»	26	13	Gemini 3	16	9	15	7	22	28
Junio	Baroni	1	17	10	22	15	22	7	28	Taurus 4
»	»	6	21	16	27	21	28	8	Cancer 4	10
»	»	11	24	22	Taurus 2	26	Gemini 4	10	10	16
»	»	16	27	28	8	Leo 1	10	12	16	22
»	»	21	29	Cancer 4	13	6	16	15	22	28
»	»	26	0	10	19	12	22	18	28	Gemini 4
Julio	Abib	1	0	17	25	18	29	21	Leo 4	10
»	»	6	29	23	Gemini 1	23	Cancer 5	25	10	16
»	»	11	27	29	7	28	11	29	16	22
»	»	16	24	Leo 5	12	Virgo 4	17	Gemini 3	22	28
»	»	21	21	11	18	9	23	8	28	Cancer 4
»	»	26	19	17	24	14	29	13	Virgo 4	11
Agosto	Moçre	1	16	23	29	18	Leo 6	18	10	17
»	»	6	16	Virgo 0	Cancer 5	22	12	23	16	23
»	»	11	16	6	11	26	18	28	22	29
»	»	16	17	12	17	29	24	Cancer 3	27	Leo 5
»	»	21	19	18	23	Libra 2	Virgo 0	9	Libra 4	11
»	»	26	21	24	29	4	7	14	9	17
			25	Libra 0	Leo 5	5	13	20	15	23

Shoud
(25)

Tabla 37

Mon., fº 27 y 28.
Ars., fº 112.

مواضع المشتري المقومة

TABLA DE LOS LOGARES ENDEREÇADOS DE JUPITER

Los meses romanos	Los meses egipcianos	Los annos	51	52	53	54	55	56	57	58	59	60	61
		Los días	Libra	Libra	Scorpio	Sagita.	Capric	Piscis	Aries	Taurus	Gemini	Cancer	Leo
Setiemb.	Toth	1	1	25	22	22	27 i	8	19	25	24	21	16
·	·	11	3	27	23	23	27 i	6	18	25	25	23	18
·	·	21	5	29	25	25	27 i	5	17	25	26	24	20
Ochubre	Beha	1	7	Scorpio 1	26	25	27	4	16	24	26	26	22
·	·	11	9	3	28	26	28 j	4	15 o	23	27	27	23
·	·	21	11	5	Sagita. 0	27	29	3	13	23	27	28	24
Noviemb	Hatur	1	13	7	2	28	Aquar 1	3	12	22	27	29,	26
·	·	11	15	10	4	Capric. 0 e	2	3	11	20	26	29	28
·	·	21	17	12	6	3	4	4	11	30	25	29	29
Deziemb	Quiach	1	19	14	9	5	6	4	10	18	25	29	29
·	·	11	21	16	11	7	8	6	10	17	24	29	0
·	·	21	23	18	13	10	10	7	10	16	22	28	0
Enero	Tobih	1	24	20	16	12	13	8	10	15	21	27	0
·	·	11	25	22	18	15	15	10	11	15	20	27	29
·	·	21	27	23	20	17	17	12	12	15	19	25	29
Febrero	Anxir	1	27	25	22	19	20	14	14	16	18	23	28
·	·	11	27	26	24	20 f	22	16	15	16	18	22	27
·	·	21	27 a	27	26	23 g	24	19	17	17	18	21	26
Març	Baramhat	1	26	28	27	26	25	21	19	18	18	20	24
·	·	11	26	29	29	28	28	23	21 p	20	18	20	23
·	·	21	26	29	Capric. 0	Aquar. 0	29	26	23	21	19	20	22
Abril	Barmudi	1	25	29	1	2	Piscis 1	28 k	26	23	20	20	21
·	·	11	23	27	1	3	2	Aries 1 l	29	25	21	20	20
·	·	21	22	27	1	4	4	2 m	Taurus 0	27	23	21	20
Mayo	Baxanz	1	21	26	1	5	6	6 n	3	29	24	22	20
·	·	11	20	25	1	6	8	7	5	Gemini 2 q	27	23	21
·	·	21	19	24	1	7	9	9	7	5	29	24	21
Junio	Baruni	1	18	22	0 d	7	10	11	10	7 r	Cancer 1	26	22
·	·	11	18	21	Sagit c 28 d	7	12	13	12	10	3	27	23
·	·	21	18	20	27	6	12	14	14	11	5	Leo 0	25
Julio	Abib	1	18	19	26	5	13	15	16	13	7	2	26
·	·	11	18	19	25	4	13	16	18	15	11	4	28
·	·	21	19	19	23	3 h	12	18	20	17	12	6	Vir. 0
Agosto	Moçre	1	20	19	22	2	12	18	21	19	14	8	3
·	·	11	21	19	22	0	11	19	23	21	17	10	4
·	·	21	22	20	22	Capric. 29	9	19	24	23	18	12	6
·	·	31	23	21	22	28	8	19	24	24	20	15	8

a P, 26. — *c* falta en los dos manuscritos. — *d* P, 29. — *e* P, 1. — *f* P, 21. — *g* P, 22. — *h* M, 2. — *i* P, 26. — *j* M, 29. — *k* P, 26. — *l* P, 28. — *m* P, 29. — *n*: el copista de P, se equivocó de columna al copiar el manuscrito de M. Sólo damos los valores de M. — *o* P, 14. — *p* M, 20. — *q* P, 3. — *r* M, 6.

Tabla 38

Mon., fº 28 y 28 v.
Ars., fº 112 v.

مواضع المشتري المقومة

TABLA DE LOS LOGARES ENDEREÇADOS DE JUPITER

Los meses romanos	Los meses egipcianos	Los annos	62	63	64	65	66	67	68	69	70	71	72
		Los dias	Virgo	Libra	Libra	Scorpio	Sagit	Aquar.	Piscis	Aries	Taurus	Gemini	Can
Setiemb	Toth	1	9	1	28	25	24	3	14	23	25	28	25
.	.	11	11 a	5	29	27	27	2	12	22	28 q	29	25
.	.	21	14	7	Scorpio 1	28	28	2	11	21	29	Canc. r 1	28
Ochubre	Beba	1	16	9	3	Sagit. 0 e	29	2	10	20	28	2	Leo 0
.	.	11	18	11	5	2	Capric 0	2	9	19	28	2	1
.	.	21	20	14	8	4	1	2	9 n	18	27	2	2
Noviemb	Hatur	1	22	16	10	6	3	3	8	17	26	2	3
.	.	11	23	18	12	8	5	4	8	16	25	1	4
.	.	21	25	20	14	10	6	6	9	15	24	1	4
Deziemb	Quiach	1	26	22	16	12	9	7	9	14	23	Gemini 29	4
.	.	11	28	23	18	15	11	9	10	14	21	28	3
.	.	21	29	25	21	17	14	11	11	14	20	26	3
Enero	Tubih	1	29	27	23	19	16	13	13	15	19	25	2
.	.	11	0	28	24 s	22	18	15	14	15	19	24	1
.	.	21	0	29	26	24	21	18	16	16	19	23	0
Febrero	Amxir	1	0	Scorpio 0	28	26	23	20	18	17	19	23	C. u 28
.	.	11	29	0	29	28	25	22	20	19	20	22	27
.	.	21	29	0	Sagit. 0	Capric 0	27	25	23	22	20	22	26
Marco	Baramhat	1	28	0	1	1	Aqua. 0	27	25	25	22	22	25
.	.	11	26	0	2	3	2	Piscis 0	27	27	23	23	24
.	.	21	25	Libra 29	2	5	4 i	2	Aries 0	Taurus 0	24	23	24
Abril	Barmoda	1	24	28	2	5	5	4	2	2	26	25	24
.	.	11	23	27	2	6	7 k	6	4	4	28	26	24
.	.	21	22	26	1	6	9 l	8	7	7	Gemin. 2	27	25
Mayo	Basanz	1	21	24	0	6 g	10	10	9	9	4	29	26
.	.	11	20	23	Scorpio 29	6 h	11	12	11	11	7	Cancer 1	27
.	.	21	20	22	28	6	11	14	16	17	9	2	28
Junio	Baruni	1	20	22 d	26	6	12	15	17	17	11	5	L. v 0
.	.	11	20	21	25	4	12	16	18 o	17	13	7	1
.	.	21	21	21	24	3	11	17	19	19	16	8	2
Julio	Abib	1	22	21	23	1	11	18	21	21	18	11	5
.	.	11	23	21	23	0	10	18	21	22	20	13	7
.	.	21	24 b	22	22	Sagit. n 29	9	18	22	23	22	15	9
Agosto	Mucre	1	26	23	22	28	7	18	28	24	24	17	11
.	.	11	28	24	22	27	6	16 m	23	25	25	19	14
.	.	21	Libra c 0	25	23	27 s	5	16 m	23	26 p	26	21	16
.	.	31	2	26	24	26 t	4	15 m	23	27	27	22	18

---

a M, 10. — *b* P, 25. — *c* sólo en M. — *d* P, 21. — *e* P, 25. — *f* P, 29. — *g* P, 7. — *h* en ninguno de los dos manuscritos. — *i* M, 25. — *j* M, 3. — *k* M, 8. — *l* M, 10. — *m* P, 17. — *n* M, 8. — *o* P, 17. — *p* P, 27. — *q* P, 29. — *r* P, 26. — *s* en los dos manuscritos Taurus. — *t* M, 3. — *u* M, no consta; en P, Gemini. — *v* en P, Cancer.

APPENDIX II

In all the programs presented here the following notation is used. Reference should be made to Fig. 7 and Fig. 7'.

Parameters	
XLA	λ_a
AN	γ_0
E	e
P	r
XLM	$\bar{\lambda}_0$
XAM	$\dot{\gamma}$
XMM	$\dot{\bar{\lambda}}$
R	R
D	conversion factor degrees to radians

Variables	
XLM	$\bar{\lambda}$
ALP	α
C1	C1
Y	Y
C2	C2
C	C_3'
Z	Z
G	δ
TL	λ
AN	γ

The core of the program for a model with equant and that of the program for a model without equant are shown below. Any of the other programs, which are for the Ptolemaic model, can be modified to calculate longitudes from a model without equant by changing the core of the program only.

Ptolemaic Model

```
6 ALP=XLM-XLA
9 IF(ALP-360.)3,8,8
8 ALP=ALP-360.
  GO TO 9
3 C1=ATANF(E*SINF(ALP*D)/R/SQRTF(1.-(E*SINF(ALP*D)/R)**2))/D
  B=ALP-C1
  Y=SQRTF(R*R+E*E+2.*R*E*COSF(B*D))
  C2=ATANF(E*SINF(B*D)/Y/SQRTF(1.-(E*SINF(B*D)/Y)**2))/D
  C=C1+C2
  Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
  G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
  TL=XLM-C+G
```

Model without Equant

```
6 ALP=XLM-XLA
9 IF(ALP-360.)3,8,8
8 ALP=ALP-360.
  GO TO 9
3 Y=SQRTF(R*R+E*E+2.*R*E*COSF(ALP*D))
  C =ATANF(E*SINF(ALP*D)/Y/SQRTF(1.-(E*SINF(ALP*D)/Y)**2))/D
  Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
  G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
  TL=XLM-C+G
```

Two programs are given for the ten day planets, Saturn and Jupiter, one to obtain complete numerical listings, the other to plot the equations based on the text and on the recomputed longitudes. The parameters contained in the numerical output program, as given, are those for Saturn. The same program is used to get the numerical output for Jupiter by changing the parameters. The program to plot displays the Jupiter parameters, but the same program is good for Saturn with the proper change in parameters.

Likewise two programs are shown for Mars and Venus. The numerical output program contains the parameter values for Mars, the program to plot, those for Venus. The comments made above also apply here, i.e. the program for Mars also serves for Venus, with a change of parameters, and vice versa.

```
      DIMENSION IX(15)
      J=0
      T=0
      XLA=234.9
      AN=170.
      E=3.41667
      P=6.50
      XLM=347.415615
      XAM=9.52867425
      XMM=.33433945
      R=60
      D=3.14159265359/180.
   31 PRINT 1
    1 FORMAT(1H1,70X,16H PTOLEMAIC MODEL/)
      PRINT 2
    2 FORMAT(74X,7H SATURN)
      PRINT 30
   30 FORMAT(/5X,137H   TEXT TL(DATA) TL(CALC)  MLONG  EQ(DATA) EQ(CALC
     1)  DIFF       SSD                   AN    ALP   C1     B     Y
     2C2     C     Z     G)
      K=0
   10 READ 11,IX,N
   11 FORMAT (16I5)
      PRINT 12,N
   12 FORMAT(/80X,I5)
      DO 4 I=1,15
      J=J+1
      IF(IX(I))21,21,6
    6 ALP=XLM-XLA
    9 IF(ALP-360.)3,8,8
    8 ALP=ALP-360.
      GO TO 9
    3 C1=ATANF(E*SINF(ALP*D)/R/SQRTF(1.-(E*SINF(ALP*D)/R)**2))/D
      B=ALP-C1
      Y=SQRTF(R*R+E*E+2.*R*E*COSF(B*D))
      C2=ATANF(E*SINF(B*D)/Y/SQRTF(1.-(E*SINF(B*D)/Y)**2))/D
      C=C1+C2
      Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
      G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
      TL=XLM-C+G
      W1=TL-XLM
      IZ=IX(I)
   20 IF(IZ-30)17,18,18
   18 IZ=IZ-30
      GOTO 20
   17 X=IX(I)
      W2=X-XLM
      U=W2-W1
      T=T+U**2
      PRINT 7,IZ,IX(I),TL,XLM,W2,W1,U,T,AN,ALP,C1,B,Y,C2,C,Z,G
    7 FORMAT(5X,I6,I8,2F10.2,3F8.2,F12.2,15X,9F6.1)
      IF(37-J)40,40,50
   40 J=0
      XLM=XLM+XMM/2.
      AN=AN+XAM/2.
      GOTO 60
   50 XLM=XLM+XMM
      AN=AN+XAM
   60 IF(AN-360.)4,5,5
    5 AN=AN-360.
```

CONT'D

```
    4 CONTINUE
      K=K+1
      IF(K-3)10,31,10
   21 CONTINUE
      END
```

```
1506
      DIMENSION BLANK(142), PT(2)
  101 FORMAT(4A1)
      READ101,BLK, DOT, AST, PLUS
      DO 102 IP = 1,142
  102 BLANK(IP) = BLK
      PT(1)=DOT
      PT(2) = AST
      DIMENSION IX(15)
      J=0
      T=0
      XLA=162.
      AN=49.            JUPITER
      P=11.50
      E=2.75
      XLM=108.380878
      XAM=9.031193266
      XMM=.8318204324
      R=60
      D=3.14159265359/180.
   10 READ 11,IX
   11 FORMAT (15I5)
      DO 4 I=1,15
      J=J+1
      IF(IX(I))21,21,6
    6 ALP=XLM-XLA
    9 IF(ALP-360.)3,8,8
    8 ALP=ALP-360.
      GO TO 9
    3 C1=ATANF(E*SINF(ALP*D)/R/SQRTF(1.-(E*SINF(ALP*D)/R)**2))/D
      B=ALP-C1
      Y=SQRTF(R*R+E*E+2.*R*E*COSF(B*D))
      C2=ATANF(E*SINF(B*D)/Y/SQRTF(1.-(E*SINF(B*D)/Y)**2))/D
      C=C1+C2
      Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
      G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
      TL=XLM-C+G
      W1=TL-XLM
      X=IX(I)
      W2=X-XLM
      U=W2-W1
      T=T+U**2
      LT=3.*W2+55.+.5
      LC=3.*W1+55.+.5
      IF (LC-LT)103,104,105
  103 NP = 1
      KK = LC-1
      MP = 2
      GO TO 106
  105 NP = 2
      KK = LT-1
      MP = 1
  106 ALC = LC
      ALT = LT
      KP= ABSF(ALC-ALT)
      IF(KP-1) 104,107,108
  104 KK = LC-1
      KJ=111-KK
      PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PLUS,(BLANK(IP),IP=1,
     1KJ)
  109 FORMAT(I8,3F8.2,56A1,56A1)
      GO TO 110
  107 KJ=110-KK
```

CONT'D

```
    PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PT(NP),PT(MP),(BLANK(I
   1P),IP=1,KJ)
    GO TO 110
108 KJ=110-(KK+KP)
    PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PT(NP),(BLANK(IP),IP=1,
   1KP),PT(MP),(BLANK(IP),IP=1,KJ)
110 CONTINUE
    IF(37-J)40,40,50
 40 J=0
    XLM=XLM+XMM/2.
    AN=AN+XAM/2.
    GOTO 60
 50 XLM=XLM+XMM
    AN=AN+XAM
 60 IF(AN-360.)4,5,5
  5 AN=AN-360.
  4 CONTINUE
    GO TO 10
 21 PRINT 13,T
 13 FORMAT (F12.2)
    END
```

```
1506
      DIMENSION IX(15)
      T=0
      XLA=117.
      AN=343.
      P=39.5
      E=6.
      XLM=174.187619
      XAM=2.30969308
      XMM=2.621813768
      R=60
      D=3.14159265359/180.
   31 PRINT 1
    1 FORMAT(1H1,30X,16H PTOLEMAIC MODEL/)
      PRINT 2
    2 FORMAT(35X,5H MARS)
      PRINT 30
   30 FORMAT(/137H     TEXT TL(DATA) TL(CALC)  MLONG  EQ(DATA) EQ(CALC)
     1DIFF        SSD                          AN     ALP   C1      B      Y     C2
     2     C      Z      G)
      K=0
   10 READ 11,IX,N
   11 FORMAT (16I5)
      PRINT 12,N
   12 FORMAT(/75X,I5)
      DO 4 I=1,15
      IF(IX(I))21,21,6
    6 ALP=XLM-XLA
    9 IF(ALP-360.)3,8,8
    8 ALP=ALP-360.
      GO TO 9
    3 C1=ATANF(E*SINF(ALP*D)/R/SQRTF(1.-(E*SINF(ALP*D)/R)**2))/D
      B=ALP-C1
      Y=SQRTF(R*R+E*E+2.*R*E*COSF(B*D))
      C2=ATANF(E*SINF(B*D)/Y/SQRTF(1.-(E*SINF(B*D)/Y)**2))/D
      C=C1+C2
      Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
      G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
      TL=XLM-C+G
      W1=TL-XLM
      IZ=IX(I)
   20 IF(IZ-30)17,18,18
   18 IZ=IZ-30
      GOTO 20
   17 X=IX(I)
      W2=X-XLM
      U=W2-W1
      T=T+U**2
      PRINT 7,IZ,IX(I),TL,XLM,W2,W1,U,T,AN,ALP,C1,B,Y,C2,C,Z,G
    7 FORMAT(I6,I8,F10.2,F10.2,F8.2,F8.2,F8.2,F12.2,15X,9F6.1)
      XLM=XLM+XMM
      AN=AN+XAM
      IF(AN-360.)4,5,5
    5 AN=AN-360.
    4 CONTINUE
      K=K+1
      IF(K-3)10,31,10
   21 CONTINUE
      END
```

```
      DIMENSION BLANK(142), PT(2)
  101 FORMAT(4A1)
      READ101,BLK, DOT, AST, PLUS
      DO 102 IP = 1,142
  102 BLANK(IP) = BLK
      PT(1)=DOT
      PT(2) = AST
      DIMENSION IX(15)
      T=0
      XLA=87.4
      AN=342.75
      E=1.25          VENUS
      P=43.1667
      XLM=157.640410
      XAM=3.08219178
      XMM=4.93150685
      R=60
      D=3.14159265359/180.
   10 READ 11,IX
   11 FORMAT (15I5)
      DO 4 I=1,15
      IF(IX(I))21,21,6
    6 ALP=XLM-XLA
    9 IF(ALP-360.)3,8,8
    8 ALP=ALP-360.
      GO TO 9
    3 C1=ATANF(E*SINF(ALP*D)/R/SQRTF(1.-(E*SINF(ALP*D)/R)**2))/D
      B=ALP-C1
      Y=SQRTF(R*R+E*E+2.*R*E*COSF(B*D))
      C2=ATANF(E*SINF(B*D)/Y/SQRTF(1.-(E*SINF(B*D)/Y)**2))/D
      C=C1+C2
      Z=SQRTF(Y*Y+P*P+2.*Y*P*COSF(AN*D+C*D))
      G=ATANF(P*SINF(AN*D+C*D)/Z/SQRTF(1.-(P*SINF(AN*D+C*D)/Z)**2))/D
      TL=XLM-C+G
      W1=TL-XLM
      X=IX(I)
      W2=X-XLM
      U=W2-W1
      LT=W2+55.+.5
      LC=W1+55.+.5
      IF (LC-LT)103,104,105
  103 NP = 1
      KK = LC-1
      MP = 2
      GO TO 106
  105 NP = 2
      KK = LT-1
      MP = 1
  106 ALC = LC
      ALT = LT
      KP= ABSF(ALC-ALT)
      IF(KP-1) 104,107,108
  104 KK = LC-1
      KJ=111-KK
      PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PLUS,(BLANK(IP),IP=1,
     1KJ)
  109 FORMAT(I8,3F8.2,56A1,56A1)
      GO TO 110
  107 KJ=110-KK
      PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PT(NP),PT(MP),(BLANK(I
     1P),IP=1,KJ)
      GO TO 110
```

```
108 KJ=110-(KK+KP)
    PRINT109,IX(I),W2,W1,U,(BLANK(IP),IP=1,KK),PT(NP),(BLANK(IP),IP=1,  CONT'D
   1KP),PT(MP),(BLANK(IP),IP=1,KJ)
110 CONTINUE
    XLM=XLM+XMM
    AN=AN+XAM
    IF(AN-360.)4,5,5
  5 AN=AN-360.
  4 CONTINUE
    GOTO 10
 21 CONTINUE
    END
```

APPENDIX III

The output displayed in the following few sheets is obtained using the four programs in Appendix II. Only a short stretch of the output from each, not necessarily from the beginning, is given of course. The complete numerical output format is shown for Saturn and Mars, where the parameters used in the recomputed longitudes are the same as those in the corresponding programs. A section of the plots for Jupiter and Venus are included, where again the programs and parameters are those shown for these planets in Appendix II. In the Venus plot a change of one print position indicates a change of one degree in the equation plotted; the scale is tripled in the Jupiter plot, and a difference of three print positions means a difference of one degree in the equation. The jump in the curve of the equation derived from the text longitudes will be remarked for Jupiter. An * plots a point obtained from the text longitudes, a · , a point from the recomputed longitudes, while a + indicates that the two points coincide. In the complete numerical output TL is the true longitude, MLONG, the mean longitude, EQ, the equation, and SSD, the sum of squared differences up to that point.

```
                                              PTOLEMAIC MODEL

                                                  SATURN

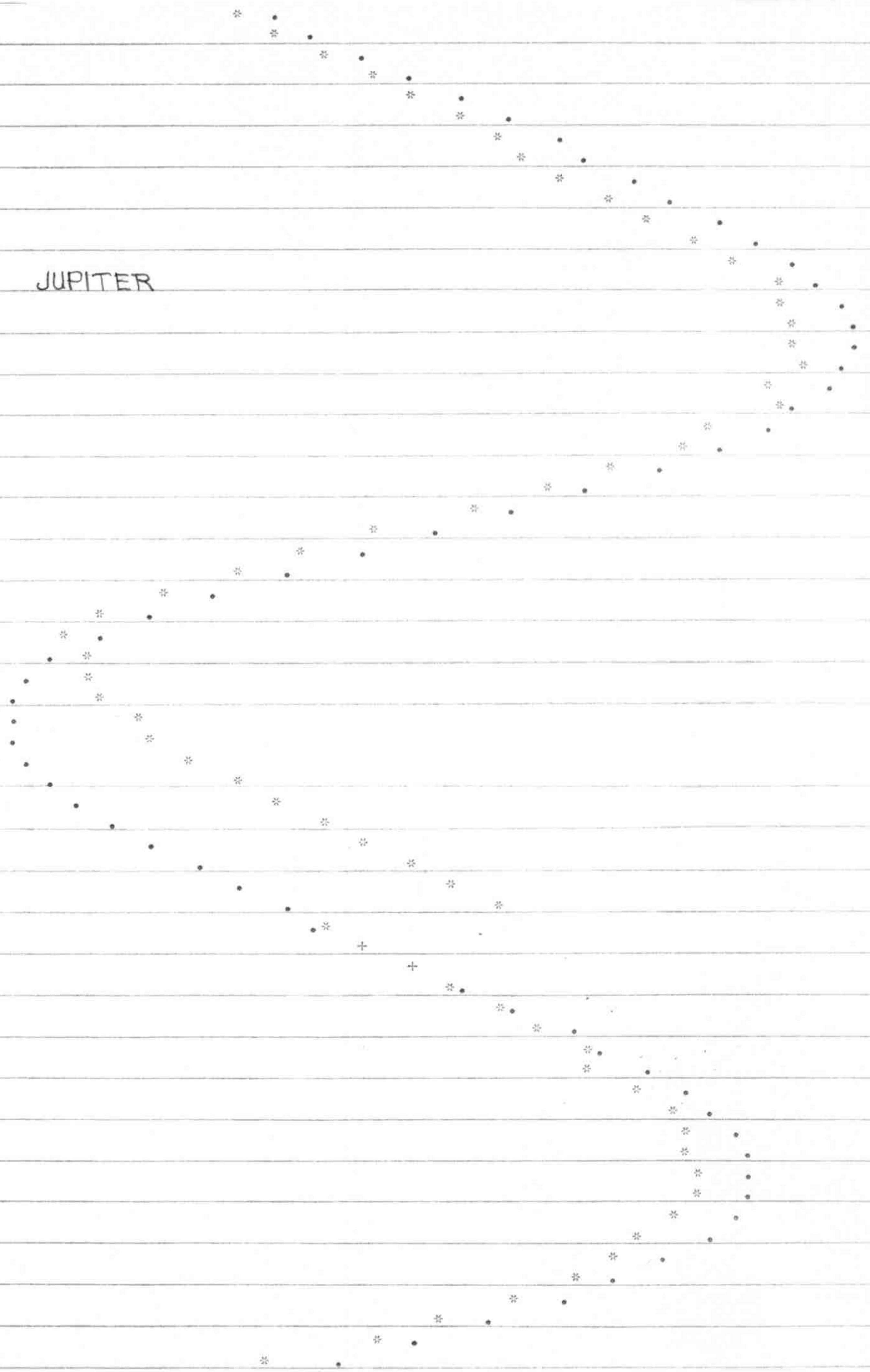
TEXT TL(DATA) TL(CALC)  MLONG  EQ(DATA) EQ(CALC)  DIFF       SSD                    AN   ALP     C1    B     Y     C2     C     Z      G

                                                                  10001
 12     342    341.74  347.41   -5.41    -5.67     .25        .06              17 .0 112.5   3.0 109.5  58.9   3.1   6.1  52.4    .4
 11     341    340.91  347.74   -6.74    -6.83     .08        .07              17 .5 112.8   3.0 109.8  58.9   3.1   6.1  52.4   -.7
 11     341    340.11  348.08   -7.08    -7.97     .88        .86              18 .0 113.1   3.0 110.1  58.9   3.1   6.1  52.6  -1.8
 10     340    339.37  348.41   -8.41    -9.04     .62       1.25              19 .5 113.5   2.9 110.5  58.8   3.1   6.1  53.0  -2.9
  9     339    338.75  348.75   -9.75   -10.00     .24       1.31              20 .1 113.8   2.9 110.8  58.8   3.1   6.0  53.6  -3.9
  9     339    338.26  349.08  -10.08   -10.82     .73       1.85              21 .6 114.1   2.9 111.2  58.8   3.1   6.0  54.3  -4.7
  8     338    337.94  349.42  -11.42   -11.48     .05       1.85              22 .1 114.5   2.9 111.5  58.8   3.0   6.0  55.1  -5.4
  8     338    337.79  349.75  -11.75   -11.96     .20       1.89              23 .7 114.8   2.9 111.8  58.8   3.0   6.0  56.1  -5.9
  8     338    337.83  350.09  -12.09   -12.25     .16       1.92              24 .2 115.1   2.9 112.2  58.7   3.0   6.0  57.1  -6.2
  8     338    338.05  350.42  -12.42   -12.36    -.05       1.92              25 .7 115.5   2.9 112.5  58.7   3.0   6.0  58.2  -6.3
  9     339    338.45  350.75  -11.75   -12.30     .54       2.22              26 .2 115.8   2.9 112.9  58.7   3.0   6.0  59.2  -6.2
  9     339    339.02  351.09  -12.09   -12.07    -.02       2.22              27 .8 116.1   2.9 113.2  58.7   3.0   5.9  60.2  -6.0
 10     340    339.74  351.42  -11.42   -11.68     .25       2.29              28 .3 116.5   2.9 113.6  58.7   3.0   5.9  61.2  -5.7
 10     340    340.59  351.76  -11.76   -11.16    -.59       2.65              29 .8 116.8   2.9 113.9  58.6   3.0   5.9  62.1  -5.2
 12     342    341.57  352.09  -10.09   -10.52     .42       2.83              30 .4 117.1   2.9 114.2  58.6   3.0   5.9  62.9  -4.5

                                                                  10002
 13     343    342.65  352.43   -9.43    -9.77     .34       2.95              31 .9 117.5   2.8 114.6  58.6   3.0   5.9  63.6  -3.8
 14     344    343.81  352.76   -8.76    -8.95     .18       2.98              32 .4 117.8   2.8 114.9  58.6   3.0   5.9  64.2  -3.0
 16     346    345.03  353.09   -7.09    -8.06     .96       3.92              33 .9 118.1   2.8 115.3  58.6   3.0   5.8  64.6  -2.1
 17     347    346.30  353.43   -6.43    -7.13     .69       4.40              34 .5 118.5   2.8 115.6  58.6   3.0   5.8  64.9  -1.2
 18     348    347.59  353.76   -5.76    -6.17     .40       4.57              35 .0 118.8   2.8 116.0  58.5   3.0   5.8  65.0   -.3
 19     349    348.89  354.10   -5.10    -5.20     .10       4.58                 .5 119.2   2.8 116.3  58.5   2.9   5.8  65.0    .6
 21     351    350.18  354.43   -3.43    -4.25     .81       5.24               1 .1 119.5   2.8 116.6  58.5   2.9   5.8  64.8   1.5
 22     352    351.44  354.77   -2.77    -3.32     .55       5.55               1 .6 119.8   2.8 117.0  58.5   2.9   5.8  64.4   2.4
 23     353    352.65  355.10   -2.10    -2.45     .34       5.67               2 .1 120.2   2.8 117.3  58.5   2.9   5.7  63.9   3.3
 24     354    353.78  355.43   -1.43    -1.64     .21       5.72               3 .6 120.5   2.8 117.7  58.4   2.9   5.7  63.2   4.1
 25     355    354.84  355.77    -.77     -.93     .15       5.74               4 .2 120.8   2.8 118.0  58.4   2.9   5.7  62.5   4.8
 26     356    355.78  356.10    -.10     -.32     .21       5.79               5 .7 121.2   2.7 118.4  58.4   2.9   5.7  61.6   5.4
 27     357    356.60  356.44     .55      .16     .39       5.95               6 .2 121.5   2.7 118.7  58.4   2.9   5.7  60.6   5.8
 28     358    357.28  356.77    1.22      .50     .71       6.46               7 .8 121.8   2.7 119.1  58.4   2.9   5.7  59.6   6.2
 28     358    357.80  357.11     .88      .69     .19       6.50               8 .3 122.2   2.7 119.4  58.3   2.9   5.6  58.5   6.3

                                                                  10003
 29     359    358.14  357.44    1.55      .70     .85       7.22               9 .8 122.5   2.7 119.7  58.3   2.9   5.6  57.4   6.3
 29     359    358.31  357.77    1.22      .53     .68       7.69              10 .3 122.8   2.7 120.1  58.3   2.9   5.6  56.3   6.1
 29     359    358.29  358.11     .88      .18     .70       8.18              11 .9 123.2   2.7 120.4  58.3   2.8   5.6  55.3   5.8
 29     359    358.09  358.44     .55     -.35     .90       9.00              12 .4 123.5   2.7 120.8  58.3   2.8   5.6  54.3   5.2
 28     358    357.71  358.78    -.78    -1.06     .28       9.08              13 .9 123.8   2.7 121.1  58.3   2.8   5.5  53.5   4.5
 28     358    357.18  359.11   -1.11    -1.93     .81       9.75              14 .5 124.2   2.6 121.5  58.2   2.8   5.5  52.8   3.6
 27     357    356.51  359.45   -2.45    -2.93     .48       9.99              15 .0 124.5   2.6 121.8  58.2   2.8   5.5  52.2   2.6
 26     356    356.13  359.61   -3.61    -3.47    -.13      10.01              15 .7 124.7   2.6 122.0  58.2   2.8   5.5  52.0   2.0
 26     356    355.33  359.95   -3.95    -4.61     .66      10.45              16 .3 125.0   2.6 122.3  58.2   2.8   5.5  51.7    .8
 25     355    354.50  360.28   -5.28    -5.78     .49      10.69              17 .8 125.3   2.6 122.7  58.2   2.8   5.4  51.7   -.2
 24     354    353.67  360.62   -6.62    -6.94     .32      10.80              18 .3 125.7   2.6 123.0  58.2   2.8   5.4  51.8  -1.4
 23     353    352.90  360.95   -7.95    -8.04     .09      10.81              19 .9 126.0   2.6 123.4  58.1   2.8   5.4  52.1  -2.5
 22     352    352.23  361.29   -9.29    -9.05    -.23      10.86              20 .4 126.3   2.6 123.7  58.1   2.7   5.4  52.6  -3.6
 22     352    351.69  361.62   -9.62    -9.92     .30      10.95              21 .9 126.7   2.6 124.1  58.1   2.7   5.4  53.3  -4.5
 21     351    351.31  361.95  -10.95   -10.64    -.31      11.05              22 .4 127.0   2.6 124.4  58.1   2.7   5.3  54.1  -5.2
```

129	-4.33	-3.76	-.57
131	-3.16	-2.59	-.57
133	-1.99	-1.34	-.65
135	-.83	-.04	-.78
137	.33	1.28	-.94
139	1.50	2.63	-1.12
141	2.67	3.96	-1.29
142	3.25	4.62	-1.37
144	4.42	5.91	-1.48
146	5.59	7.13	-1.54
148	6.76	8.27	-1.51
150	7.93	9.31	-1.38
152	9.09	10.21	-1.11
154	10.26	10.96	-.69
155	10.43	11.52	-1.08
156	10.60	11.87	-1.26
157	10.77	11.98	-1.20
158	10.93	11.82	-.88
158	10.10	11.38	-1.27
159	10.27	10.63	-.36
158	8.44	9.58	-1.13
158	7.61	8.22	-.61
157	5.78	6.59	-.81
156	3.94	4.73	-.78
155	2.11	2.71	-.59
153	-.71	.61	-1.32
152	-2.54	-1.48	-1.06
151	-4.37	-3.48	-.89
150	-6.21	-5.31	-.89
149	-8.04	-6.90	-1.14
149	-8.87	-8.21	-.66
150	-8.70	-9.21	.51
151	-8.53	-9.91	1.37
152	-8.36	-10.31	1.94
154	-7.20	-10.43	3.22
155	-7.03	-10.28	3.25
157	-5.86	-9.90	4.03
159	-4.69	-9.31	4.61
161	-3.52	-8.54	5.01
163	-2.36	-7.62	5.26
165	-1.19	-6.58	5.38
167	-.02	-5.43	5.40
169	1.14	-4.20	5.34
171	2.31	-2.91	5.22
167	-2.10	-2.25	.15
169	-.93	-.92	0.00
171	.23	.41	-.17
173	1.40	1.73	-.32
175	2.56	3.01	-.44
177	3.73	4.23	-.49
179	4.90	5.37	-.47
180	5.07	6.41	-1.34
182	6.24	7.32	-1.08
184	7.40	8.07	-.66
185	7.57	8.64	-1.06
186	7.74	9.00	-1.26
187	7.91	9.13	-1.21
188	8.08	8.99	-.91
188	7.25	8.56	-1.31
188	6.41	7.84	-1.42
188	5.58	6.80	-1.21
188	4.75	5.46	-.71
187	2.92	3.85	-.93
186	1.09	2.00	-.91
185	-.74	0.00	-.73
183	-3.57	-2.11	-1.46



```
              PTOLEMAIC MODEL

                MARS

TEXT TL(DATA) TL(CALC)  MLONG   EQ(DATA) EQ(CALC)  DIFF       SSD                   AN   ALP    C1     B     Y     C2     C     Z      G

                                                                        30158
 28    6298   6298.15  6309.20  -11.20  -11.04    -.15     7455.23            348.1  72.2   5.4  66.7  62.6   5.0  10.5 102.1    -.5
  1    6301   6301.56  6311.82  -10.82  -10.26    -.56     7455.54            350.4  74.8   5.5  69.2  62.3   5.1  10.7 101.8     .4
  5    6305   6304.97  6314.44   -9.44   -9.46     .02     7455.54            352.7  77.4   5.6  71.8  62.1   5.2  10.8 101.5    1.3
  8    6308   6308.41  6317.06   -9.06   -8.65    -.41     7455.72            355.0  80.0   5.6  74.4  61.8   5.3  11.0 101.2    2.3
 12    6312   6311.86  6319.68   -7.68   -7.82     .13     7455.73            357.3  82.6   5.6  76.9  61.6   5.4  11.1 100.8    3.3
 15    6315   6315.33  6322.31   -7.31   -6.97    -.33     7455.85            359.6  85.3   5.7  79.5  61.3   5.5  11.2 100.4    4.2
 19    6319   6318.82  6324.93   -5.93   -6.10     .17     7455.88              1.9  87.9   5.7  82.1  61.1   5.5  11.3  99.9    5.2
 22    6322   6322.33  6327.55   -5.55   -5.22    -.33     7455.99              4.2  90.5   5.7  84.8  60.8   5.6  11.3  99.4    6.1
 26    6326   6325.85  6330.17   -4.17   -4.31     .14     7456.01              6.5  93.1   5.7  87.4  60.5   5.6  11.4  98.8    7.0
 29    6329   6329.40  6332.79   -3.79   -3.39    -.40     7456.17              8.9  95.7   5.7  90.0  60.2   5.7  11.4  98.2    8.0
  3    6333   6332.96  6335.42   -2.42   -2.45     .03     7456.17             11.2  98.4   5.6  92.7  60.0   5.7  11.4  97.6    8.9
  6    6336   6336.54  6338.04   -2.04   -1.50    -.54     7456.47             13.5 101.0   5.6  95.4  59.7   5.7  11.3  96.9    9.8
 10    6340   6340.13  6340.66    -.66    -.52    -.13     7456.48             15.8 103.6   5.5  98.0  59.4   5.7  11.3  96.3   10.7
 14    6344   6343.75  6343.28     .71     .46     .24     7456.55             18.1 106.2   5.5 100.7  59.1   5.7  11.2  95.5   11.6
 18    6348   6347.38  6345.90    2.09    1.48     .61     7456.92             20.4 108.9   5.4 103.4  58.8   5.6  11.1  94.8   12.5

                                                                        30159
 21    6351   6351.04  6348.52    2.47    2.51    -.04     7456.92             22.7 111.5   5.3 106.1  58.6   5.6  10.9  94.0   13.4
 25    6355   6354.71  6351.15    3.84    3.55     .28     7457.00             25.0 114.1   5.2 108.9  58.3   5.5  10.8  93.2   14.3
 29    6359   6358.39  6353.77    5.22    4.62     .60     7457.37             27.3 116.7   5.1 111.6  58.0   5.5  10.6  92.4   15.2
  2    6362   6362.10  6356.39    5.60    5.70    -.10     7457.38             29.6 119.3   4.9 114.3  57.7   5.4  10.4  91.5   16.1
  6    6366   6365.82  6359.01    6.98    6.80     .17     7457.41             32.0 122.0   4.8 117.1  57.5   5.3  10.1  90.7   16.9
 10    6370   6369.56  6361.63    8.36    7.92     .43     7457.60             34.3 124.6   4.7 119.9  57.2   5.2   9.9  89.8   17.8
 14    6374   6373.31  6364.26    9.73    9.05     .68     7458.06             36.6 127.2   4.5 122.6  56.9   5.0   9.6  88.9   18.7
 17    6377   6377.08  6366.88   10.11   10.20    -.08     7458.07             38.9 129.8   4.4 125.4  56.7   4.9   9.3  88.0   19.5
 21    6381   6380.87  6369.50   11.49   11.37     .12     7458.09             41.2 132.5   4.2 128.2  56.4   4.7   9.0  87.1   20.3
 25    6385   6384.67  6372.12   12.87   12.54     .32     7458.19             43.5 135.1   4.0 131.0  56.2   4.6   8.6  86.2   21.2
 29    6389   6388.48  6374.74   14.25   13.73     .51     7458.45             45.8 137.7   3.8 133.8  56.0   4.4   8.2  85.3   22.0
  3    6393   6392.31  6377.36   15.63   14.94     .68     7458.93             48.1 140.3   3.6 136.7  55.7   4.2   7.8  84.4   22.8
  6    6396   6396.14  6379.99   16.00   16.15    -.14     7458.95             50.4 142.9   3.4 139.5  55.5   4.0   7.4  83.5   23.6
 11    6401   6399.99  6382.61   18.38   17.38    1.00     7459.96             52.7 145.6   3.2 142.3  55.3   3.7   7.0  82.6   24.4
 14    6404   6403.85  6385.23   18.76   18.61     .14     7459.98             55.0 148.2   3.0 145.2  55.1   3.5   6.5  81.6   25.1

                                                                        30160
 18    6408   6407.71  6387.85   20.14   19.85     .28     7460.06             57.4 150.8   2.7 148.0  54.9   3.3   6.0  80.7   25.9
 22    6412   6411.58  6390.47   21.52   21.10     .41     7460.23             59.7 153.4   2.5 150.9  54.8   3.0   5.6  79.8   26.7
 26    6416   6415.45  6393.10   22.89   22.35     .54     7460.53             62.0 156.1   2.3 153.7  54.6   2.7   5.1  78.9   27.4
  0    6420   6419.33  6395.72   24.27   23.61     .66     7460.97             64.3 158.7   2.0 156.6  54.5   2.4   4.5  78.0   28.1
  4    6424   6423.21  6398.34   25.65   24.87     .78     7461.58             66.6 161.3   1.8 159.5  54.4   2.2   4.0  77.0   28.9
  7    6427   6427.09  6400.96   26.03   26.13    -.09     7461.59             68.9 163.9   1.5 162.3  54.3   1.9   3.4  76.1   29.6
 11    6431   6430.97  6403.58   27.41   27.38     .02     7461.59             71.2 166.5   1.3 165.2  54.2   1.6   2.9  75.2   30.3
 15    6435   6434.84  6406.20   28.79   28.63     .15     7461.61             73.5 169.2   1.0 168.1  54.1   1.3   2.3  74.3   31.0
 19    6439   6438.71  6408.83   30.16   29.88     .28     7461.69             75.8 171.8    .8 171.0  54.0    .9   1.8  73.4   31.6
 23    6443   6442.57  6411.45   31.54   31.12     .42     7461.87             78.1 174.4    .5 173.8  54.0    .6   1.2  72.5   32.3
 26    6446   6446.43  6414.07   31.92   32.35    -.43     7462.06             80.5 177.0    .2 176.7  54.0    .3    .6  71.6   33.0
  0    6450   6450.27  6416.69   33.30   33.57    -.27     7462.13             82.8 179.6   0.0 179.6  54.0   0.0   0.0  70.7   33.6
  4    6454   6454.10  6419.31   34.68   34.78    -.10     7462.14             85.1 182.3   -.2 182.5  54.0   -.2   -.5  69.8   34.2
  8    6458   6457.91  6421.94   36.05   35.97     .08     7462.15             87.4 184.9   -.4 185.4  54.0   -.6  -1.0  68.9   34.8
 12    6462   6461.70  6424.56   37.43   37.14     .29     7462.23             89.7 187.5   -.7 188.3  54.0   -.9  -1.6  68.0   35.4
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2416	-15.06	-15.21	.15
2423	-12.99	-13.80	.81
2429	-11.92	-12.39	.47
2435	-10.85	-10.98	.13
2441	-9.78	-9.57	-.20
2447	-8.71	-8.17	-.54
2456	-4.65	-6.76	2.11
2461	-4.58	-5.36	.78
2466	-4.51	-3.97	-.53
2472	-3.44	-2.59	-.85
2479	-1.37	-1.21	-.16
2485	-.30	.15	-.45
2491	.76	1.50	-.74
2498	2.82	2.84	-.02
2504	3.89	4.18	-.28
2510	4.96	5.49	-.53
2516	6.03	6.80	-.76
2522	7.10	8.09	-.99
2529	9.17	9.36	-.19
2535	10.23	10.63	-.39
2541	11.30	11.87	-.56
2547	12.37	13.10	-.72
2553	13.44	14.32	-.87
2560	15.51	15.52	0.00
2566	16.58	16.70	-.12
2572	17.65	17.87	-.22
2578	18.71	19.02	-.30
2584	19.78	20.16	-.37
2590	20.85	21.28	-.43
2596	21.92	22.39	-.47
2602	22.99	23.49	-.49
2608	24.06	24.57	-.51
2614	25.13	25.63	-.50
2620	26.19	26.68	-.48
2626	27.26	27.72	-.45
2632	28.33	28.74	-.41
2638	29.40	29.75	-.34
2644	30.47	30.74	-.27
2650	31.54	31.72	-.17
2656	32.60	32.68	-.07
2662	33.67	33.62	.05
2668	34.74	34.54	.20
2674	35.81	35.45	.36
2680	36.88	36.33	.55
2686	37.95	37.19	.76
2692	39.02	38.02	.99
2697	39.08	38.82	.26
2704	41.15	39.59	1.56
2709	41.22	40.32	.90
2715	42.29	41.01	1.28
2720	42.36	41.64	.71
2725	42.43	42.22	.21
2731	43.50	42.72	.77
2736	43.56	43.15	.41
2742	44.63	43.47	1.15
2747	44.70	43.69	1.01
2752	44.77	43.76	1.00
2757	44.84	43.67	1.16
2761	43.91	43.39	.51
2766	43.97	42.86	1.11
2770	43.04	42.05	.99
2774	42.11	40.88	1.23
2777	40.18	39.28	.90
2780	38.25	37.15	1.09
2782	35.32	34.38	.93
2784	32.39	30.85	1.53

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