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SUPERSYMMETRIC MASSIVE GRAVITY AND DARK MATTER

by
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AN ABSTRACT OF THE DISSERTATION OF

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This dissertation is composed of two parts. The first is constructing the supersymmetric form of the Higgs Massive Gravity. The second part is forming the Hamiltonian formulation of the recently proposed Mimetic Dark Matter.

When four scalar fields with global Lorentz symmetry take a vacuum expectation value, diffeomorphism invariance is broken spontaneously and then the graviton acquires mass. To supersymmetrize this model, four $N = 1$ chiral superfields with global Lorentz symmetry are considered. The matter action is formed out of these chiral superfields and it is composed of three D-type and two F-type terms. Then, using the rules of tensor calculus, $N = 1$ supergravity Lagrangian is coupled to the four chiral multiplets. This will promote the global supersymmetry to a local one. Similar to the bosonic case, when the scalar components of the chiral multiplets acquire a vacuum expectation value, both diffeomorphism invariance and local supersymmetry are broken spontaneously. This will make the scalar fields, χ^A, vectors and the chiral spinors, ψ^A, spin-3/2 Rarita-Schwinger fields since the global Lorentz index A is then identified with the space-time Lorentz index. At the end, we show that in the broken phase the spectrum of the model consists of a massive spin-2 field, two massive spin-3/2 fields and a massive vector. It is similar to what we have for $N = 2$ supergravity, but the two gravitinos obtained have different masses.

For the second part, we construct the Hamiltonian of Mimetic Gravity. The equations of motion in this formalism are those of general relativity plus two more equations. However, these two equations are proved to be the constraint equation and the conservation of the energy-momentum tensor. Poisson brackets are computed and closure is proved. At the end, comparison with the Hamiltonian dust is done.

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Chapter 1

Introduction

Dating back to 1973, the possibility of supersymmetry, a theory unifying matter and radiation, was discovered by Bruno Zumino and Julius Wess in four dimensional spacetime [79]. At that time, it was more of a purely theoretical tool than a serious possibility for the realistic theory of nature. Shortly after, it was realized that supersymmetry could be relevant to elementary particle physics. Since then it has been an attractive subject to many physicists.

Supersymmetry implies that each boson/fermion possesses a supersymmetric particle fermion/boson. It extends the Poincare symmetry algebra by introducing anticommuting symmetry generators (spinors). It is the only possible extension of the spacetime symmetry.

A supersymmetry multiplet consists of particles having the same mass but differing by spin 1/2. Therefore, if supersymmetry exists, it should be broken at an energy which should be soon accessible by the accelerators searching for the supersymmetric partners. Nevertheless, supersymmetry is still an interesting field to many physicists for several reasons.

Supersymmetry solves the hierarchy problem which is to understand the big gap between the GUT scale and the scale of electroweak symmetry breaking. The unification scale in a grand unified theory is around 10^{15}, but below this scale the masses of the electroweak scalars are not protected by any symmetry against quadratic divergences in perturbation theory. Supersymmetric theories solve this problem by cancellations between fermion divergencies and boson divergencies at every loop order provided there is cancellation at one loop. This is an attractive feature of supersymmetry which has better renormalizability properties than of non-supersymmetric theories.

Supergravity is an extension of general relativity including supersymmetry; therefore, the fermionic superpartner of the graviton, the spin-3/2 gravitino, is included [28]. Is is the only consistent field theory for interacting spin-3/2 fields. Supergravity is a locally supersymmetric gauge theory where gravitation is implied in a natural way once the supersymmetry transformation parameters become local.
For $N = 1$ supergravity, there is one real massless gravitino, while in the extended ($N = 2, 3, ..., 8$) theories there are N gravitinos. For $N = 2$, the theory contains two Majorana (one complex) gravitinos rather than a Majorana (real) gravitino.
Chapter 2.1 is mainly an introduction to supersymmetry and its algebra. Chiral and Vector superfields are presented and then it is discussed how to form supersymmetric Lagrangians. This chapter also includes a section on supergravity and the Lagrangian is stated.

In the first part of this dissertation, we are going to construct a theory of Massive Supergravity. First, what is massive gravity? Massive gravity has been studied for around 70 years. It has gained interest because it can be an answer to open questions such as the cosmological constant naturalness problem. It started with Fierz and Pauli in 1939 when they first wrote an action describing a free massive graviton [46]. Through these years, curiosities were raised and studied, such as the vDVZ discontinuity and the Boulware-Deser ghost. Chapter 3 gives a historical review of massive gravity and discusses how to form massive gravity using the Higgs mechanism.

To construct the theory of massive Supergravity, we are going to generalize the Higgs mechanism used in the formulation of massive gravity. The validity of this setting is studied in chapter 4. Upon symmetrizing massive gravity, the graviton and the gravitino both get mass due to the breakdown of diffeomorphism invariance. Therefore, we end up with a massive spin-3/2 particle plus a massive gravitino. It is an interesting theory since it is similar to $N = 2$ Supergravity where two spin-3/2 particles exist.

To construct our globally supersymmetric action, superfields are used and the action is written in superspace using D-terms and F-terms. Then, the rules of tensor calculus are used promoting global invariance to a local one. The Supergravity Lagrangian is then coupled to the chiral and vector multiplets using these rules, where certain conditions are forced on the final action.

Chapters 5 and 6 form the second part of the work. Constrained Hamiltonian dynamics dates back to Dirac. It is used for canonical quantization and for counting degrees of freedom. Chapter 5 gives an introduction of the Hamiltonian formulation and discusses the canonical formalism constructed by Arnowitt, Deser and Misner in 1962. The equations of motion and Poisson brackets of general relativity are also presented.

Recently, Chamseddine and Mukhanov proposed a mimetic dark matter theory ([24]) where they isolated the conformal degree of freedom in a covariant way. They found that the conformal degree of freedom can mimic the contribution of an extra pressureless fluid such as Dark Matter.

Chapter 6 deals with constructing the canonical formulation of mimetic dark matter. Equations of motion are analyzed and compared to those of GR. Furthermore, we compute the Poisson brackets checking if the algebra is closed.

Chapter 7 is the conclusion. Notation and conventions used are presented in appendix (A).

Chapter 2

Supersymmetry and Supergravity

Supersymmetry is a symmetry unifying matter with radiation. It proposes to each known particle a superpartner. Up to the writing of this thesis, no supersymmetric particles have been observed in nature. Therefore, it is still a theoretical invention. Nevertheless, it remains attractive to many scientists for several reasons, mainly because it is a solution to the hierarchy problem.

2.1 Supersymmetry and its Algebra

Particles in nature are splitted into two kinds, those with integral (bosons) and half-integral (fermions) spins. Since symmetries play an important role in the description of physical phenomena, it is natural to ask if there is a symmetry that places different spin particles in the same multiplet. Several attempts to unify the spacetime symmetry of the Poincare group with the symmetry of some internal group have been done. However, until 1974 all internal symmetries were studied but the no-go theorem of Coleman and Mandula [26] stating that, under certain physical assumptions, the largest space-time symmetry possible is the Poincare symmetry shows that the invariance group can at best be the direct product of the Poincare group and an internal group.

The most general Lie algebra of symmetries of the S-matrix, as demonstrated by the theorem of Coleman and Mandula, contains P_μ, $M_{\mu\nu}$, and B_l with the following commutating relations

$$[P_\mu, B_l] = 0, \quad [M_{\mu\nu}, B_l] = 0$$
$$[B_l, B_m] = ic_{lm}^{\ \ k} B_k. \tag{2.1}$$

P_μ is the energy momentum operator, $M_{\mu\nu}$ is the Lorentz rotation generator, and B_l are Lorentz scalar operators constituting a Lie algebra where the $c_{lm}^{\ \ k}$ are the structure constants of the Lie algebra of the internal symmetry group.

The restrictions of the Coleman-Mandula theorem were avoided by relaxing one condition. Haag, Lopuszanski and Sohnius [68] generalized the notion of a Lie algebra by involving not only the usual commutators but also anticommutators. The simple extension was considering $\mathbb{Z}_2$ graded algebras (superalgebras) whose generators are classified into two classes, even (bosonic) and odd (fermionic) and they obey

$$[even, even] = even, \quad [even, odd] = odd, \quad \{odd, odd\} = even. \tag{2.2}$$

For every generator A in the graded Lie algebra, there is an associated number a such that

$$a = \begin{cases} 0 & \text{if} A \text{ is even} \\ 1 & \text{if} A \text{ is odd} \end{cases} \tag{2.3}$$

The graded commutator is defined by

$$[A, B\} = AB - (-1)^{ab} BA \tag{2.4}$$

and it satisfies the graded Jacobi identity

$$[A, [B, C\}\} + (-1)^{a(b+c)} [B, [C, A\}\} + (-1)^{c(a+b)} [C, \{A, B\}\} = 0. \tag{2.5}$$

Therefore, the simplest version to generalize the Poincare algebra to a superalgebra is enlarging it by including a spinor generator Q. This symmetry generator is fermionic and it turns a bosonic state into a fermionic one and vice versa. A supersymmetry transformation can be written as

$$Q|boson> = |fermion>, \quad Q|fermion> = |boson> . \tag{2.6}$$

Since this added generator Q to the Poincare algebra has a fermionic character, then a left handed Weyl spinor is introduced, Q_α $(\alpha = 1, 2)$. Its Hermitian adjoint is a right handed Weyl spinor, $\bar{Q}_{\dot{\beta}}$. Their algebraic relations are given by [78]

$$
\begin{aligned}
&\{Q_\alpha, Q_\beta\} = 0 \\
&\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P_\mu \\
&\left\{\bar{Q}^{\dot{\alpha}}, Q^{\beta}\right\} = 2\bar{\sigma}^{\mu\dot{\alpha}\beta} P_\mu \\
&\left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0 \\
&[M_{\mu\nu}, Q_\alpha] = -\left(\sigma_{\mu\nu}\right)_\alpha^{\beta} Q_\beta \\
&\left[M_{\mu\nu}, \bar{Q}^{\dot{\alpha}}\right] = -\left(\bar{\sigma}_{\mu\nu}\right)^{\dot{\alpha}}_{\dot{\beta}} \bar{Q}^{\dot{\beta}} \\
&[P^\mu, Q_\alpha] = 0.
\end{aligned}
\tag{2.7}
$$

Supersymmetry thus arises as an extension of Poincare symmetry mixing bosons with fermions. It combines superpartners, both bosons and fermions states, in an irreducible representation called supermultiplet which encompasses both the transformations of the Poincare group and the appropriate supersymmetry transformations.

2.2 Chiral and Vector Superfield

A superfield $S(x, \theta, \bar{\theta})$ differs from an ordinary field by being a function of both the spacetime coordinates x^μ, and also of anticommuting Grassmann variables θ_α and $\bar{\theta}_{\dot{\alpha}}$. The latter transforms as two-component Weyl spinors with

$$\{\theta_\alpha, \theta_\alpha\} = \left\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\right\} = \left\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\right\} = 0. \tag{2.8}$$

The mass dimension of the Grassmann variables is $[\theta^{\alpha}] = \left[\bar{\theta}^{\dot{\alpha}}\right] = -1/2$, whereas the dimension of the coordinate $[x] = 1$. The coefficients in an expansion of a superfield, $S(x, \theta, \bar{\theta})$, in powers of θ and $\bar{\theta}$, are the fields of the supermultiplets.

2.2.1 Chiral Superfields

If the superfield satisfies the constraint $\bar{D}_{\dot{\alpha}}\Phi(x, \theta, \bar{\theta}) = 0$, where $\bar{D}$ is the covariant derivative given by

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu} \tag{2.9}$$

then it is called a left handed chiral superfield ($LH\chi SF$). Such a superfield is given by

$$\Phi(x^{\mu}, \theta, \bar{\theta}) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\partial_{\mu}\varphi\theta\sigma^{\mu}\bar{\theta} - \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi\sigma^{\mu}\bar{\theta} - \frac{1}{4}\partial_{\mu}\partial^{\mu}\varphi\theta\theta\bar{\theta}\bar{\theta}. \tag{2.10}$$

It has the same number of fermionic and bosonic degrees of freedom where its component fields are two scalar fields, φ and F, and a Weyl spinor ψ.

Under an infinitesimal supersymmetry transformation, the behaviour of the superfield Φ is given by [78]

$$\Phi \to \Phi + \delta\Phi \tag{2.11}$$

with

$$\delta\Phi = \left(\zeta Q + \bar{\zeta}\bar{Q}\right)\Phi \tag{2.12}$$

where Q and $\bar{Q}$ are differential operators given by

$$
\begin{aligned}
Q &= \frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \\
\bar{Q} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}.
\end{aligned} \tag{2.13}
$$

From this we can derive the supersymmetry transformations of the component fields in the expansion (2.10). It is found that

$$
\begin{aligned}
&\delta\varphi = \sqrt{2}\zeta\psi \\
&\delta\psi = \sqrt{2}\zeta F - \sqrt{2}\ \partial_{\mu}\varphi\sigma^{\mu}\bar{\zeta} \\
&\delta F = i\sqrt{2}\ \partial_{\mu}\psi\sigma^{\mu}\bar{\zeta}
\end{aligned} \tag{2.14}
$$

It should be noted that the change in F is a total derivative. This will be used in constructing Lagrangians.

The conjugate superfield $\Phi^{\dagger}$ satisfies the constraint

$$D_{\alpha}\Phi^{\dagger} = 0, \tag{2.15}$$

where

$$D_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} + i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \tag{2.16}$$

It is called the right chiral superfield. Therefore, we can say that a chiral multiplet is the combination of a two-component Weyl fermion and a complex scalar field, related by supersymmetric transformations.

2.2.2 Vector Superfield

The chiral superfields introduced above do not have a vector field as component field. Thus, vector superfields are introduced to involve gauge vector bosons (with helicity eigenstates ± 1) and their fermionic supersymmetric partners, gauginos (having helicity eigenstates $\pm 1/2$) [9]. Therefore, the vector superfield should involve a real gauge field $V_\mu(x)$ and its fermionic partner $\lambda_\alpha(x)$. Starting with a general superfield given as an expansion in θ and $\bar{\theta}$, we have

$$
\begin{aligned}
S\left(x,\theta,\bar{\theta}\right) &= f\left(x\right) + \theta\varphi\left(x\right) + \bar{\theta}\bar{\chi}\left(x\right) + \theta\theta m\left(x\right) + \bar{\theta}\bar{\theta}n\left(x\right) + \theta\sigma^\mu\bar{\theta}V_\mu\left(x\right) + \theta\theta\bar{\theta}\bar{\lambda}\left(x\right) \\
&\quad + \bar{\theta}\bar{\theta}\theta\psi\left(x\right) + \theta\theta\bar{\theta}\bar{\theta}d\left(x\right)
\end{aligned}
\tag{2.17}
$$

where all higher powers of θ, $\bar{\theta}$ vanish. f, m, n, d are scalar fields, V_μ is a vector field, and $\varphi, \psi, \bar{\chi}, \bar{\lambda}$ are Weyl spinor fields. The vector superfield, V, is defined by the constraint $V(x,\theta,\bar{\theta}) = V^\dagger(x,\theta,\bar{\theta})$. This requirement gives

$$
\begin{aligned}
&f = f^* \qquad V_\mu = V_\mu^* \qquad d = d^* \\
&m^* = n \qquad \varphi = \chi \qquad \lambda = \psi.
\end{aligned}
$$

Therefore, it contains altogether sixteen degrees of freedom, eight bosonic degrees of freedom and eight fermionic degrees of freedom. It has two Weyl spinors with complex components; thus, each has four real fermionic degrees. These are matched by eight real bosonic degrees, $d(1), f(1), m(2), V_\mu(4)$. It is convenient to rewrite the vector superfield using particular field combinations (dictated by $\Phi + \Phi^\dagger$) for the coefficients of the $\theta\theta\bar{\theta}$, $\bar{\theta}\theta\theta$ and $\theta\theta\bar{\theta}\bar{\theta}$ components of V. Then we have

$$V\left(x,\theta,\bar{\theta}\right) = C\left(x\right) + i\theta\chi\left(x\right) - i\bar{\theta}\bar{\chi}\left(x\right) + \frac{1}{2}i\theta\theta\left[M(x) + iN(x)\right] \\ - \frac{1}{2}i\bar{\theta}\bar{\theta}\left[M(x) - iN(x)\right] + \theta\sigma^{\mu}\bar{\theta}V_{\mu}(x) + i\theta\theta\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^{\mu}\partial_{\mu}\chi(x)\right] \\ - i\bar{\theta}\bar{\theta}\theta\left[\lambda(x) + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\bar{\chi}(x)\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left[D - \frac{1}{2}\partial_{\mu}\partial^{\mu}C\right] \tag{2.18}$$

where C, M, N, D are real scalar fields, V_{μ} is a (real) vector field, and χ, λ are Weyl spinor fields. It is the vector V_{μ} that lends its name to the multiplet.

Under a supersymmetry transformation

$$\delta_{\zeta}V = i\left(\zeta Q + \bar{\zeta}\bar{Q}\right)V, \tag{2.19}$$

where Q and $\bar{Q}$ are defined in (2.13), we get for the transformations of the component fields

$$\delta C = i\left(\zeta\chi - \bar{\zeta}\bar{\chi}\right) \\ \delta\lambda_{\alpha} = -iD\zeta_{\alpha} - \frac{1}{2}\left(\sigma^{\mu}\bar{\sigma}^{\nu}\right)_{\alpha}^{\ \beta}\zeta_{\beta}V_{\mu\nu} \\ \delta V^{\mu} = i\left(\zeta\sigma^{\mu}\bar{\lambda} - \lambda\sigma^{\mu}\bar{\zeta}\right) - \partial^{\mu}\left(\zeta\chi - \bar{\zeta}\bar{\chi}\right) \\ \delta D = \partial_{\mu}\left(-\zeta\sigma^{\mu}\bar{\lambda} + \lambda\sigma^{\mu}\bar{\zeta}\right) \tag{2.20}$$

where $V_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$ and its transformation is given by

$$\delta V^{\mu\nu} = i\partial^{\mu}\left(\zeta\sigma^{\nu}\bar{\lambda} - \lambda\sigma^{\nu}\bar{\zeta}\right) - i\partial^{\nu}\left(\zeta\sigma^{\mu}\bar{\lambda} - \lambda\sigma^{\mu}\bar{\zeta}\right) \tag{2.21}$$

Similar to the variation of the F-field, that of the D-field is also a total divergence. Also, $\partial_\mu \partial^\mu C$ transforms as a total derivative; therefore, so does the full coefficient of $\theta\theta\bar{\theta}\bar{\theta}$ in the vector supermultiplet.

2.3 Wess Zumino Model

A vector superfield can be constructed simply from a chiral superfield Φ and an anti-chiral superfield $\Phi^\dagger$. For instance

$$\begin{aligned} i(\Phi - \Phi^\dagger) = i(\varphi - \varphi^\dagger) &+ i\sqrt{2}(\theta\psi - \bar{\theta}\bar{\psi}\psi) + i\theta\theta F - i\bar{\theta}\bar{\theta}F^\dagger \\ &- \theta\sigma^\mu\bar{\theta}\partial_\mu(\varphi + \varphi^\dagger) - \frac{1}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{1}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi} \\ &- \frac{1}{4}i\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu(\varphi - \varphi^\dagger) \end{aligned} \tag{2.22}$$

satisfies the reality requirement. The coefficient of $\theta\sigma^\mu\bar{\theta}$, the vector potential V_μ, is a pure U(1) gauge transformation. This led Wess and Zumino to suggest the following supersymmetric generalization of gauge invariance [79]

$$V(x,\theta,\bar{\theta}) \to V(x,\theta,\bar{\theta}) + i\left[\Phi(x,\theta,\bar{\theta}) - \Phi^\dagger(x,\theta,\bar{\theta})\right] \tag{2.23}$$

Under this transformation, we find by comparing with equation (2.18) that

$$\begin{aligned} &C \to C + i(\varphi - \varphi^\dagger) \\ &\chi \to \chi + \sqrt{2}\psi \\ &\frac{1}{2}(M + iN) \to \frac{1}{2}(M + iN) + F \end{aligned}$$

$$
\begin{aligned}
&V_\mu \to V_\mu - \partial_\mu(\varphi + \varphi^\dagger) \\
&\lambda \to \lambda \\
&D \to D
\end{aligned}
\tag{2.24}
$$

From (2.24) we see that in a gauge theory the fields C, χ, M and N are all gauge artifacts since they can be eliminated by adjusting $\varphi - \varphi^\dagger$, ψ, F while still leaving $\varphi + \varphi^\dagger$ arbitrary. The fields $\lambda, \bar{\lambda}, D$ are gauge invariant while V_μ transforms as $V_\mu \to V_\mu + \partial_\mu \Lambda$. Then in the 'Wess-Zumino' gauge the multiplet reduces to V_μ, λ and D, and the vector superfield reduces to

$$
V_{WZ}\left(x, \theta, \bar{\theta}\right) = \theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{2.25}
$$

The field D, which is the coefficient of $\theta\theta\bar{\theta}\bar{\theta}$, transforms as a total derivative. The advantage of this gauge is that all powers of V_{WZ}^n with $n > 2$ vanish since they will involve at least θ^3. The only non-zero power is

$$
\begin{aligned}
V_{WZ}^2\left(x, \theta, \bar{\theta}\right) &= \left(\theta\sigma^\mu\bar{\theta}\right)\left(\theta\sigma^\nu\bar{\theta}\right)V_\mu V_\nu = \theta^\alpha\left(\sigma^\mu\right)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\theta^\beta\left(\sigma^\nu\right)_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}V_\mu V_\nu \\
&= \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}V_\mu V^\mu
\end{aligned}
\tag{2.26}
$$

2.4 Supersymmetric Lagrangians

When constructing supersymmetric theories, matter fields (for example quarks) and their supersymmetric partners (squarks) are assigned to the chiral superfields discussed above. While vector superfields provide the gauge fields. In the same multiplet, their fermionic supersymmetric partners, the gauginos, are associated with them.

To construct a supersymmetric Lagrangian, we must use terms invariant under a supersymmetry transformation up to a total derivative. As discussed before, for a chiral superfield, the F-term transforms as a total derivative. While for a vector superfield, it is the D-term that is invariant up to a total derivative. Therefore, the Lagrangian of a supersymmetric theory is given by the sum of the F-term of a chiral superfield (i.e. the $\theta\theta$ component of a $LH\chi SF$ or the $\bar{\theta}\bar{\theta}$ component of a $RH\chi SF$) and the D-term of a vector superfield (i.e. the $\theta\theta\bar{\theta}\bar{\theta}$ component). This guarantees that the theory is invariant under supersymmetric transformations.

2.4.1 Lagrangians for Chiral Multiplets

Supersymmetric Lagrangians of chiral multiplets can be constructed from products of chiral superfields. The product of two left chiral superfields ($\Phi_i\Phi_j$) is again a left chiral superfield (consequently for right chiral superfields). However, the product of a left chiral superfield and a right chiral superfield ($\Phi_i^\dagger\Phi_j$) gives a vector superfield. Therefore, in general, the Lagrangian can be written as [78]

$$\mathcal{L} = \int d\theta^4 \ \Sigma_i \Phi_i^\dagger \Phi_i + \left(\int d^2\theta W(\Phi) + h.c. \right) \tag{2.27}$$

where D- and F-terms are projected out by the superspace integration. The superpotential, $W(\Phi)$, involves up to the third power of the superfield Φ_i as required by renormalizability. It is given by

$$W(\Phi) = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k, \tag{2.28}$$

where the coefficients, m_{ij} and λ_{ijk}, are real and symmetric in their indices. The mass terms and the couplings of the component fields are provided by the superpotential; however, it does not provide kinetic terms. The latter are provided by $(\Phi_i^\dagger\Phi_j)$.

The above Lagrangian expressed in terms of component fields, apart from surface terms, is given by

$$\begin{aligned}\mathcal{L} = \partial_\mu\varphi_i\partial^\mu\varphi_i^\dagger + i\bar{\psi}_i\bar{\sigma}^\mu\partial_\mu\psi_i + F_i^\dagger F_i + \Big(&m_{ij}\varphi_i F_j - \frac{1}{2}m_{ij}\psi_i\psi_j + \lambda_{ijk}\varphi_i\varphi_j F_k \\ &- \lambda_{ijk}\psi_i\psi_j\varphi_k + h.c.\Big)\end{aligned} \tag{2.29}$$

where the auxiliary field F and its complex conjugate $F^\dagger$ can be removed by their equations of motions.

2.4.2 Lagrangians Constructed out of Vector Multiplets

The Lagrangian written above does not contain spin-1 component fields since vector superfields are not included. Out of these vector superfields, a supersymmetric gauge field theory can be formed by combining gauge symmetry with supersymmetry. After all, our vector bosons are supposed to be gauge bosons.

In the non-supersymmetric case, we know that the vector potential transforms as

$$V_\mu(x) \to V_\mu^{'}(x) = V_\mu(x) + \partial_\mu \Lambda(x) \tag{2.30}$$

under the $U(1)$ gauge transformation. As discussed in section (2.3), Wess and Zumino supersymmetrized this transformation by noting that V_μ is a component of the vector superfield, and from $i(\Phi - \Phi^\dagger)$ (equation 2.22), it is clear that $\partial_\mu \Lambda \equiv \partial_\mu(\phi + \phi^\dagger)$. Therefore, they suggested that the superfield transforms as in equation (2.23), which is repeated here for convenience,

$$V\left(x,\theta,\bar{\theta}\right) \to V^{'}\left(x,\theta,\bar{\theta}\right) = V\left(x,\theta,\bar{\theta}\right) + i\left[\Phi\left(x,\theta,\bar{\theta}\right) - \Phi^\dagger\left(x,\theta,\bar{\theta}\right)\right] \tag{2.31}$$

under a $U(1)$ gauge transformation.

Noticing that [78]

$$\begin{aligned} D_\alpha V &\to D_\alpha V + i\left(D_\alpha \Phi - D_\alpha \Phi^\dagger\right) \\ &\to D_\alpha V + i D_\alpha \Phi \end{aligned}$$

since $D_\alpha \Phi^\dagger = 0$ from the chirality condition, we can obtain an invariant under the transformation (2.31) by eliminating the term $D_\alpha \Phi$. Therefore, multiplying by $\bar{D}_{\dot{\alpha}}$ we get

$$\begin{aligned} \bar{D}_{\dot{\alpha}} D_\alpha V &\to \bar{D}_{\dot{\alpha}} D_\alpha V + i \bar{D}_{\dot{\alpha}} D_\alpha \Phi \\ &\to \bar{D}_{\dot{\alpha}} D_\alpha V + i\left[\left\{\bar{D}_{\dot{\alpha}}, D_\alpha\right\} - D_\alpha \bar{D}_{\dot{\alpha}}\right]\Phi \end{aligned}$$

$$\rightarrow \bar{D}_{\dot{\alpha}} D_\alpha V - 2\sigma^{\mu}_{\alpha\dot{\alpha}} \partial_\mu \Phi$$

again using the chirality condition and also using the anticommutator relation between D_α and $\bar{D}_{\dot{\alpha}}$

$$\left\{ D_\alpha, \bar{D}_{\dot{\alpha}} \right\} = -2i\sigma^{\mu}_{\alpha\dot{\alpha}} \partial_\mu. \tag{2.32}$$

From this it is deduced that

$$\bar{D}^2 D_\alpha V \rightarrow \bar{D}^2 D_\alpha V.$$

Defining

$$W_\alpha = \bar{D}^2 D_\alpha V; \qquad \bar{W}_{\dot{\alpha}} = D^2 \bar{D}_{\dot{\alpha}} V, \tag{2.33}$$

it follows that $\bar{D}_{\dot{\beta}} W_\alpha = 0$ because $\bar{D}_{\dot{\alpha}} \bar{D} \bar{D} = 0$.

W_α and $\bar{W}_{\dot{\alpha}}$ are a left-handed chiral superfield and a right-handed chiral superfield respectively. Therefore, these superfields are chiral and gauge covariant. Due to this chirality, the supersymmetric pure gauge theory is constructed out of the F-component of $W^\alpha W_\alpha$. Then

$$\int d^2\theta W^\alpha W_\alpha \tag{2.34}$$

is a susy invariant Lagrangian. Calculation yields

$$\frac{1}{32} \left(W^\alpha W_\alpha \right)_F = -\frac{1}{4} V^{\mu\nu} V_{\mu\nu} + i\lambda\sigma^\mu \partial_\mu \bar{\lambda} - \frac{i}{4} \epsilon_{\mu\nu\rho\sigma} V^{\mu\nu} V^{\rho\sigma} + \frac{1}{2} D^2. \tag{2.35}$$

This is the supersymmetric generalization of the kinetic terms $-\frac{1}{4} V^{\mu\nu} V_{\mu\nu}$ of the U(1) gauge field. The auxiliary field D can be eliminated using its equations of motion and λ represents the gaugino contribution.

Similar to the above supersymmetric extension of the $U(1)$ abelian gauge invariance, we have that of the non-abelian gauge invariance that occurs in electroweak theory, quantum chromodynamics and grand unified theories.

The non-abelian analog of the field strength superfield W_α is the chiral superfield given by

$$W_\alpha = (2ig)^{-1}\bar{D}^2 e^{-V}\left(D_\alpha e^V\right) \tag{2.36}$$

where $V = 2gV^a t^a$. V^a are the vector superfields containing the non-abelian vector bosons and t^a are the Hermitian matrices constituting the representation of a non-abelian group G satisfying

$$\left[t^a, t^b\right] = if^{abc}t^c \tag{2.37}$$

where the f^{abc} are the totally antisymmetric structure constant of G.

In this case, the pure gauge-invariant and supersymmetric contribution to the Lagrangian is given by [9]

$$\mathcal{L} = \frac{1}{64}\left[tr\left(W^\alpha W_\alpha\right) + tr\left(W_\alpha^\dagger W^{\alpha\dagger}\right)\right]_F. \tag{2.38}$$

2.5 Supergravity

The oldest known force is the gravitational force, but it is still the least understood. This is mainly due to the fact that gravitational experiments are difficult to perform because the gravitational constant κ is small. Moreover, this gravitational constant is dimensionful and this prevented the formulation of a renormalizable quantum theory of gravity. The marriage of special relativity and quantum mechanics has been successfully done forming quantum field theory. This provided renormalizable field theories for QCD and QFT. However, up to now Einstein's gravity has proved difficult to treat with any quantum theory. Supergravity comes in as a theory that, though nonrenormalizable, still has predictive power. It is a field theory, based upon the principles of special and general relativity and of quantum mechanics, describing gravity and the other interactions [77].

Supergravity is an extension of general relativity combining the principles of supersymmetry and general relativity. Its Lagrangian field theory was formulated in 1976 [27]. Due to the symmetry between bosons and fermions, a pure supergravity theory includes the spin-2 graviton plus its supersysmmetric particle, the gravitino. The gravitino is a hypothetical spin 3/2 particle described by the Rarita-Schwinger field.

2.5.1 Rarita-Schwinger Field

The kinetic term for the gravitino, ϕ_μ, is provided by the massless Rarita-Schwinger action which is given by

$$\frac{1}{2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu \gamma_5 \gamma_\nu \partial_\rho \phi_\sigma \tag{2.39}$$

where our conventions are given in appendix (A). This action is invariant under the transformation

$$\phi_\mu \rightarrow \phi_\mu + \partial_\mu \eta \tag{2.40}$$

where η is a Majorana spinor parameter. The Rarita-Schwinger field equation is given by

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\phi_\sigma = 0 \tag{2.41}$$

which can be written in an alternative form

$$\gamma^\mu \left(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu\right) = 0. \tag{2.42}$$

2.5.2 Supergravity as Local Supersymmetry

Supersymmetry is a global symmetry of the Lagrangian; however, it may be promoted to a local symmetry. Local supersymmetry is a theory of general coordinate transformations of spacetime. To see this we consider supersymmetry transformations. In section (2.2) we saw that a supersymmetry transformation changes fermionic into bosonic particles and vice versa which can be written schematically as [1]

$$\delta B = \bar{\epsilon} F, \quad \delta F = \epsilon \partial B \tag{2.43}$$

where B and F represent bosonic and fermionic particles respectively and ϵ is the infinitesimal supersymmetry parameter. By allowing this parameter to depend on space-time coordinates, $\epsilon = \epsilon(x)$, the symmetry will become local and then the commutator of two infinitesimal supersymmetry transformations will yield

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\, B = (\bar{\epsilon}_1 \gamma^\mu \epsilon_2)\,(x)\, \partial_\mu B. \tag{2.44}$$

The space-time dependent vector field $(\bar{\epsilon}_1 \gamma^\mu \epsilon_2)\,(x)$ shows that a locally supersymmetric theory will necessarily be diffeomorphism invariant and this is achieved by including Einstein's general relativity.

2.5.3 Local Symmetry from a Global One

One way to construct a theory with local symmetry from a global one is the Noether method. Consider the Dirac Lagrangian

$$L = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \tag{2.45}$$

which is invariant under the global symmetry

$$\psi(x) \to e^{\frac{i}{4}\Lambda^{ab}\gamma_{ab}}\psi(x). \tag{2.46}$$

However, promoting this global symmetry to a local one, the phase will then depend on the space-time coordinate, $\Lambda = \Lambda(x)$. Since this is no longer a symmetry for the Lagrangian, the invariance of the Dirac equation is restored under local Lorentz transformations by introducing the covariant derivative

$$D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}. \tag{2.47}$$

This is analogous to the electromagnetic case. The Lagrangian $\bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi$ becomes invariant provided that

$$\omega'^{ab}_\mu = \partial_\mu \Lambda^{ab} + \omega_\mu{}^{ac}\Lambda_c{}^b - \omega_\mu{}^{bc}\Lambda_c{}^a. \tag{2.48}$$

The commutator of two covariant derivatives gives the curvature tensor which is defined by

$$[D_\mu, D_\nu] = \frac{1}{4}R^{ab}_{\mu\nu}\gamma_{ab}, \tag{2.49}$$

where it can be easily found that

$$R^{ab}_{\mu\nu} = \partial_\mu w^{ab}_\nu - \partial_\nu w^{ab}_\mu + w^{ac}_\mu w^{\ b}_{\nu c} - w^{ac}_\nu w^{\ b}_{\mu c} \tag{2.50}$$

and

$$R^{ab}_{\mu\nu}(w)e_{\rho a}e_{\sigma b} = R_{\mu\nu\rho\sigma}(g) \tag{2.51}$$

which is identical to the Riemann tensor as a function of the metric g.

The field e^a_μ and $\omega^b_{\nu a}$ introduced are the vierbein and the spin-connection field usually used in describing general relativity. The vierbein defines a local set of tangent frames of the spacetime manifold, while the spin-connection field is associated with (local) Lorentz transformations of these frames. $a, b, ...$ are tangent space indices, lowered and raised by the Minkowski metric, while the world indices are $\mu, \nu, ...$ The inverse of the vierbein is denoted by e^μ_a,

$$e^\mu_a = g^{\mu\nu}e_{\nu a} = g^{\mu\nu}\eta_{ab}e^b_\nu, \tag{2.52}$$

and it is related to the spin connection by

$$\partial_\nu e_a^\mu = -\omega_{\nu a}{}^b e_b^\mu - \Gamma^\mu_{\gamma\nu} e_a^\gamma \tag{2.53}$$

2.6 Supergravity Lagrangian

In a similar way, global supersymmetry can be turned into a local one. In the supersymmetric case, the transformation parameter is itself a spinor, hence a spin-3/2 field ψ_μ^α is expected since it carries both a spinor and a vector index. This field is the supersymmetric partner of the graviton identified with the gravitino's Rarita-Schwinger field. It naturally arises in the supergravity multiplets [1]. The supermultiplet of the spin-2 graviton contains the spin-3/2 gravitino. Therefore, there are two bosonic degrees of freedom (graviton) and two fermionic degrees of freedom (massless Weyl vector spinor).

There are mainly two techniques for constructing supergravity models. The first one, which was used in constructing the original supergravity lagrangians ([27], [34]), starts from a globally invariant theory and then changes the action and transformation laws by an iterative procedure so as to obtain an invariant action successively at each order of the gravitational constant. The second one ([23], [57]) is simpler where it is to construct gauge theories of the supersymmetric groups. The gauge theory of local supersymmetry introduces spin-3/2 fields identified with the gravitino's Rarita-Schwinger field. Therefore, supergravity is obtained by gauging locally supersymmetric transformation where the gauge group is the Poincare supergroup and the gauge field associated with the local supersymmetry transformation is the gravitino. Shortly after, supergravity was reformulated in superspace [78], [56] (see also [55]).

Here we will sketch how the Supergravity Lagrangian can be obtained by gauging the supersymmetry algebra. Let [23]

$$D_\mu = \partial_\mu + \omega_\mu{}^{ab} J_{ab} + e_\mu^a P_a + \psi_\mu^\alpha Q_\alpha \tag{2.54}$$

be the connection associated with the supersymmetry algebra. The curvature tensor is computed to be

$$[D_\mu, D_\nu] = R_{\mu\nu}^{ab} J_{ab} + T_{\mu\nu}^a P_a + \psi_{\mu\nu}^\alpha Q_\alpha, \tag{2.55}$$

where

$$\begin{aligned}
R_{\mu\nu}^{ab} &= \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} + \omega_\mu{}^{ac} \omega_{\nu c}{}^{b} - \omega_\nu{}^{ac} \omega_{\mu c}{}^{b}, \\
T_{\mu\nu}^a &= \partial_\mu e_\nu^a - \partial_\nu \omega_\mu^a + \omega_\mu{}^{ab} e_{\nu b} - \omega_\nu{}^{ab} e_{\mu b} + \bar{\psi}_\mu \gamma^a \psi_\nu, \\
\psi_{\mu\nu} &= \left(\partial_\mu + \frac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} \right) \psi_\nu - \left(\partial_\nu + \frac{1}{4} \omega_\nu{}^{ab} \gamma_{ab} \right) \psi_\mu.
\end{aligned} \tag{2.56}$$

By imposing the torsion free constraint

$$T_{\mu\nu} = 0, \tag{2.57}$$

${\omega_\mu}^{ab}$ are expressed in terms of e^a_μ and ψ_μ by

$$
{\omega_\mu}^{ab} = {\omega_\mu}^{ab}(e) + \frac{1}{4}\left(\bar{\psi}_\mu \gamma^a \psi^b - \bar{\psi}_\mu \gamma^b \psi^a + \bar{\psi}_a \gamma_\mu \psi^b\right) \tag{2.58}
$$

where ${\omega_\mu}^{ab}(e)$ is the usual expression in the non-supersymmetric case.

However, this torsion constraint is not preserved under the supersymmetry transformations which are given by

$$
\begin{aligned}
\delta e^a_\mu &= \bar{\epsilon}\gamma^a \psi_\mu, \\
\delta {\omega_\mu}^{ab} &= 0, \\
\delta \psi_\mu &= \left(\partial_\mu + \frac{1}{4}{\omega_\mu}^{ab}\gamma_{ab}\right)\epsilon.
\end{aligned} \tag{2.59}
$$

To preserve it, the $\delta {w_\mu}^{ab}$ should be modified to become

$$
\delta' \omega_{\mu ab} = -\frac{1}{2} e^a_\nu e^\rho_b \left(\bar{\epsilon}\gamma_\rho \psi_{\mu\nu} - \bar{\epsilon}\gamma_\mu \psi_{\nu\rho} + \bar{\epsilon}\gamma_\nu \psi_{\rho\mu}\right). \tag{2.60}
$$

It is found that the Lagrangian invariant under these supersymmetry transformations is the Supergravity Lagrangian given by

$$
e^{-1} L_{SG} = -\frac{1}{4}R + \bar{\psi}_\mu \gamma^{\mu\nu\rho}\left(\partial_\nu + \frac{1}{4}{\omega_\nu}^{ab}\gamma_{ab}\right)\psi_\rho. \tag{2.61}
$$

However, soon after the construction of the gauge action of supergravity, the non-closure of the algebra on-shell was shown. For this, auxiliary fields were added to the action and the transformation laws, similar to the situation in global supersymmetry.

In [45] and [5] the problem was solved and a minimal set of auxiliary fields, closing the gauge algebra and leaving the action invariant, were derived. It was found out that only a scalar, a pseudoscalar and an axial world vector field are needed.

Therefore, the Lagrangian of supergravity contains the Einstein-Hilbert Lagrangian of general relativity and the Rarita-Schwinger Lagrangian for the gravitino field and auxiliary fields. The field content of $N = 1$ Supergravity with the minimal set of auxiliary fields consists of the spin-2 field, $e_{a\mu}$, the spin-3/2 field, ϕ_μ, and the auxiliary fields S, P and A_μ. It is given by [45], [63]

$$L_{S.G} = -\frac{e}{2\kappa^2} R(e,\omega) - \frac{e}{3}|u|^2 + \frac{e}{3} A_\mu A^\mu - \frac{1}{2}\bar{\phi}_\mu R^\mu \tag{2.62}$$

where

$$u = S - iP \tag{2.63}$$

$$R_{\mu\nu}{}^{rs} = \partial_\mu \omega_\nu{}^{rs} + \omega_\mu{}^{rp} \omega_{\nu p}{}^{s} - \mu \leftrightarrow \nu \tag{2.64}$$

$$R^\mu = \epsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma_5 D_\rho(\omega) \phi_\sigma \tag{2.65}$$

$$R = e_r{}^\mu e_s{}^\nu R_{\mu\nu}{}^{rs} \tag{2.66}$$

$$D_\mu = \partial_\mu + (1/2) \omega_{\mu rs} \sigma^{rs} \tag{2.67}$$

$$\omega_{\mu rs} = \omega_{\mu rs}(e) + K_{\mu rs}(e, \phi_\mu) \tag{2.68}$$

$$K_{\mu rs}(e, \phi_\mu) = (\kappa^2/4)(\bar{\phi}_\mu \gamma_r \phi_s - \bar{\phi}_\mu \gamma_s \phi_r + \bar{\phi}_r \gamma_\mu \phi_s) \tag{2.69}$$

and e is the determinant of the vierbein. This Supergravity Lagrangian is invariant under local supersymmetry transformations up to a total divergence.

Chapter 3

Massive Gravity

General relativity (GR) [44], formulated in 1915 by Einstein, is the theory of gravity for a four dimenional spacetime. It is the theory of a massless spin-2 particle. Einstein added to his theory in 1917 a cosmological constant to make the universe static, which what was believed at that time. However, Hubble's 1929 discovery showed that the universe is expanding. Physicists then believed that the expansion would be slowed down as time goes on by the effect of gravity. However, in 1998, supernova data revealed that the universe has not been slowing down due to gravity, it is expanding in an accelerated motion. To explain this, theorists included dark energy, a form of energy density with negative pressure. It is poorly understood, but we know that it makes around 68% of the energy density. The simplest interpretation of this dark energy is represented by the cosmological constant, a constant in space and time. However, still more is unknown than what is known.

3.1 History of Massive Gravity

When we discuss massive gravity, it is normal to ask why massive gravity. One of the main motivations for considering massive theories of gravity is that they provide a new point of view of the cosmic acceleration. A long-distance modification of general relativity can be obtained by the graviton having a small mass while keeping the physics at small scales equivalent to general relativity.

There are two main idea of how massive gravity could be useful in interpreting the cosmological constant. The first idea is that mass of the graviton weakens gravity in the infrared and this may weaken the sensitivity of the dynamics to an already existing large cosmological constant. This is what referred to as screening or degravitating solutions [47, 48]. The second is that gravitons can condense to form a condensate whose energy density sources self-acceleration which explains that acceleration of the universe without the need of a cosmological constant [32, 17]. Simple cosmological solutions with flat, open, and closed spatial geometries were found in [73]. Their solutions exhibited self-acceleration, while being free from ghost instabilities. Also, in [25], cosmological solutions were found within the tetrad formulation of massive gravity [22]. It was found that the effect of a graviton mas is equivalent a matter source introduced to the Einstein equations that can consist of several different matter types; a cosmological term, quintessence, gas of cosmic strings, and non-relativistic cold matter.

Massive gravity is an interesting topic for many authors. It dates back for more than 70 years. It is a theory which propagates a massive spin-2 particle. To construct such a theory, the graviton should be given mass. This can be done by adding to the Einstein-Hilbert action a mass term such that as the mass goes to zero, general relativity should be restored. The first action written for a free massive gravity was in 1939 by Fierz and Pauli [46] in the perturbative limit. This is given by

$$S = \int d^4x \left(-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. - \frac{1}{2}m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2 \right) \right). \tag{3.1}$$

where $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$. For $m = 0$, the action is the linearized Einstein-Hilbert action having the gauge symmetry

$$\delta h_{\mu\nu} = \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu. \tag{3.2}$$

However, the mass term breaks this general coordinate invariance. The Fierz-Pauli tuning (the -1 coefficient between $h_{\mu\nu}h^{\mu\nu}$ and h^2) is crucial where any deviation will not be description of a single massive spin-2 particle. This action propagates 5 degrees of freedom, the right number for a massive spin-2 particle.

In 1970, van Dam, Veltman and, independently, Zakharov [76], [80] coupled the linear theory to a source and discovered that the limit as the graviton's mass goes to zero is not continuous. There is a difference between what the theory predicts for a small non-zero value and general relativity ($m = 0$). For example, it gives a 25 percent off for the bending of light by the sun. It was then concluded that the mass of the graviton is exactly zero and not some extremely small value. This discontinuity of the physics in the parameters is known as the van Dam-Veltman-Zakharov discontinuity (vDVZ) [80]. Due to this discontinuity, massive gravity was considered not physically possible since it was experimentally ruled out.

However, Vainshtein then argued that the discontinuity is due to the fact that not all degrees of freedom decouple as the graviton's mass goes to zero. As the mass goes to zero, a massive graviton becomes a massless graviton plus a scalar coupled to the trace of the stress tensor. He studied the non-linear theory presuming that the linear theory does not give the whole picture [75]. He showed that as the mass approaches zero, the perturbation theory fails because higher order contributions becomes much stronger due to singularities in the graviton mass. Then he resolved this problem by considering a distance scale known as Vainshtein radius around any mass. He found that below this radius, massive graviton acts like a massless particle. The linear approximation is not suitable in the massless limit since Vainshtein radius goes to infinity as the mass goes to zero. This saved massive gravity by having no contradiction with experiments even if the graviton has a small mass. In [18] (see also [42]), further developments of this scale were considered.

Unfortunately, in the same year, Deser and Boulware [13] showed that the theory of massive gravity is ill-behaved at the non-linear level because it has a ghost scalar that remains coupled. In the non-linear theory, there is an extra degree of freedom that appears as a scalar field with a negative kinetic energy. This scalar is known as the Boulware-Deser ghost. This led them to conclude that Einstein's gravity is an isolated theory since massive gravity is theoretically sick.

A Lagrangian theory mixing a graviton with a massive $2^+ f$ meson was formulated by Isham, Salam and Strathdee [19]. This was generalized by Chamseddine, Salam and Strathdee [2]. They introduced the mixing terms through a spontaneous symmetry breaking mechanism. Extra dimensions theories were considered by Dvali, Gabadadze, and Porrati [49]. They built a five dimensional model which seems, in the decoupling limit, to be ghost-free when considered around a true background.

Since there was no Higgs mechanism that is ghost-free and returns a massive graviton, it was believed that we cannot obtain a ghost-free massive Einstein's gravity in four dimensions. Siegel [71] restored diffeomorphism invariance by using four scalars in open-string field theories. However, his theory is not free of ghosts when studied around a trivial background. Then, in 2003, Arkani-Hamed, Georgi and Schwartz [62] by applying this method to massive gravity restored general coordinate invariance, but their model is not ghost free. They also studied massive gravity as an effective field theory where they found a maximum UV cutoff.

't Hooft [72] (see also [52]) broke diffeomorphism invariance by using four scalar fields having vacuum expectation values. These scalars make the graviton massive where we end up with a massive spin-2 boson and a massive scalar. However, in the unbroken phase, his theory includes a ghost (negative kinetic energy of the scalar field). In the broken symmetry phase, there is no Fierz-Pauli term for the massive graviton, and the ghost state does not decouple.

Not long ago, Chamseddine and Mukhanov [21], using Higgs mechanism, were able to form massive gravity. They considered four scalars with global Lorentz symmetry. The action considered is made up of two parts, the action of the four scalar particles added to the Einstein action. As a consequence of spontaneous symmetry breaking, the four scalar fields will have non-zero expectation values, that breaks diffeomorphism invariance. The graviton absorbs three of the scalar degrees of freedom, while the fourth degree will remain coupled. Then the graviton will have a Fierz-Pauli mass term making it massive and having five degrees of freedom. It was found that their theory is free of ghosts below scales related to Vainshtein scales. In [54], the massless limit of Higgs gravity was considered and the Vainshtein scale was determined. It was found that this scale depends on the interactions of scalar fields. It was also proved that below this scale, massive gravity goes smoothly to Einstein's gravity. A simplified reformulation of massive gravity was given in [22]. In this new formulation, the action depends quadratically on the scalar fields.

3.2 Higgs Massive Gravity

To make the graviton massive via Higgs mechanism, four scalar fields ϕ^A are considered with $A = 0, 1, 2, 3$. The index A is a global Lorentz index; it mixes the scalar fields and preserves the metric $\eta_{AB} = (1, -1, -1, -1)$. From these scalar fields, a field space tensor is constructed

$$H^{AB} = g^{\mu\nu}\partial_\mu\varphi^A\partial_\nu\varphi^B, \tag{3.3}$$

An action providing the graviton mass is then constructed using the space tensor.

This action is diffeomorphism and Lorentz invariant and is given by

$$S = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R + \frac{m^2}{8}\int d^4x\sqrt{-g}\left(H^2 - H^A_B H^B_A\right) + 3\left(\frac{1}{16}H^2 - 1\right)^2 .$$

where $\kappa^2 = 8\pi G$. This is not a unique action. There exist many other actions agreeing at the second level but differing at higher orders.

Expanding around the vacuum solution, which breaks diffeomorphism invariance [21]

$$\varphi^A = \left(x^A + \chi^A\right) \ , \quad g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} \ . \tag{3.4}$$

and introducing

$$\bar{h}^{AB} = H^{AB} - \eta^{AB}, \tag{3.5}$$

this gives

$$\bar{h}^{AB} = h^{AB} + \left(\partial^A\chi^B + \partial^B\chi^A\right) + \cdots . \tag{3.6}$$

Then the action is rewritten as

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}R + \frac{m^2}{8}\int d^4x\sqrt{-g}\left[\ \left(\bar{h}^2 - \bar{h}^A_B\bar{h}^B_A\right) + \cdots\right] . \tag{3.7}$$

We can write the Einstein action also in terms of $\bar{h}^A_B$ since it is invariant under infinitesimal transformations $\tilde{x}^A = x^A + \xi^A$ and the metric perturbations around Minkowski space-time transform similarly, with χ^A instead of ξ^A. Therefore, the full action up to second order terms in $\bar{h}^A_B$ is given by

$$S = \frac{1}{2}\int d^4x \left[\bar{h}^{A,C}_B \bar{h}^B_{A,C} - 2\bar{h}^{A,C}_C \bar{h}^{\ D}_{A,D} + 2\bar{h}^{A,C}_C \bar{h}_{,A} \right.$$
$$\left. -\bar{h}_{,A}\bar{h}^{,A} - m^2\left(\bar{h}^A_B\bar{h}^B_A - \bar{h}^2\right)\right].$$

This theory is free of ghosts since no linear term in H was included in the action. The constructed action is at least quadratic in the fields φ^A since the field H^{AB} is quadratic in φ^A.

3.3 vDVZ Discontinuity

The vDVZ discontinuity is easily seen by comparing the propagators of the massless and massive graviton. The graviton propagator in flat spacetime in the momentum space is given by [80]

$$D_{\mu\nu,\lambda\sigma}(k) = \frac{1}{2}\frac{\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}}{k^2 + i\epsilon}. \tag{3.8}$$

While if we propose gravity to be due to the exchange of a massive spin-2 particle with mass m_g, then the propagator takes the form

$$D^{(m)}_{\mu\nu,\lambda\sigma}(k) = \frac{1}{2}\frac{G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda} - \frac{2}{3}G_{\mu\nu}G_{\lambda\sigma}}{k^2 - m_g^2 + i\epsilon} \tag{3.9}$$

with $G_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu/m_g^2$. We can replace $G_{\mu\nu}$ by $\eta_{\mu\nu}$ since the spin-2 particle is coupled to a conserved source with $k_\mu T^{\mu\nu} = 0$. This propagator becomes in the limit $m_g \to 0$

$$D^{(m)}_{\mu\nu,\lambda\sigma}(k) = \frac{1}{2}\frac{\eta_{mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \frac{2}{3}\eta_{\mu\nu}\eta_{\lambda\sigma}}{k^2 + i\epsilon}. \tag{3.10}$$

This differs from the propagator of the massless graviton by a factor of 2/3 instead of 1 appearing in front of the last term. This discontinuity is traced back to the fact that the scalar mode of the massive graviton does not decouple. A massless graviton has only two degrees of freedom, while a massive one has five degrees of freedom. These five degrees can be thought of as coming from a massless spin-2, spin-1 and spin-0 fields. The coupling of the helicity ± 1 vanishes because of the condition $k_\mu T^{\mu\nu} = 0$. However, the extra coupling of the helicity zero to the trace of the energy-momentum tensor survives in addition to the two tensor degrees of freedom.

This discontinuity was shown at the classical level without going to quantum theory in [54]. The method usually applied in cosmological perturbation theory was used. They first start by classifying the metric perturbations according to the irreducible representations of the spatial rotation group [61]. In the Newtonian gauge, the metric then takes the form

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\psi)\delta_{ik}dx^i dx^k. \tag{3.11}$$

The static interaction between two massive bodies is entirely due to the two potentials ϕ and ψ. Solving the equations of motion, it follows immediately that $\psi = \phi/2$

and the gravitational potential ϕ for a central source of mass M_0 is given by

$$\phi = -\frac{4}{3}\frac{M_0}{r}e^{-m_g r} = \frac{4}{3}\phi_N e^{-m_g r} \tag{3.12}$$

where ϕ_N is the Newtonian gravitational potential. m_g is the graviton's mass which is of the order of the present Hubble constant, about $10^{-33}eV$.

For distance scales much smaller than $1/m_g$, the static potentials will be related to the Newtonian potential by $\phi = \frac{4}{3}\phi_N$ and $\psi = \frac{4}{3}(2)\phi_N$. This however won't affect the bending of light which is determined by the sum $\phi + \psi$. In massive gravity, this combination is equal to

$$\phi + \psi = \frac{4}{3}\phi_N + \frac{2}{3}\phi_N = 2\phi_N \tag{3.13}$$

which is what we have in General Relativity with $\psi = \phi_N$. Therefore, we obtain the same prediction for the bending of light. However, the gravitational potential is modified by a 4/3 factor even in the limit of a zero mass. This will modify the motion of planets as predicted from general relativity. Fixing the gravitational potential by redefining the gravitational constant will give a wrong bending of light. This is the classical version of the van Dam, Veltman and Zakharov discontinuity.

3.4 Boulware-Deser Ghost

In this section we show that the nonlinear ghost propagates around broken symmetry background following what was done in [4]. To trace the appearance of the nonlinear ghost it is convenient to work in the Newtonian gauge where the scalar field perturbations are not equal to zero. In this case, the metric takes the form

$$ds^2 = (1 + 2\phi)\, dt^2 + 2S_i dt dx^i - \left[(1 - 2\psi)\, \delta_{ik} + \tilde{h}_{ik}\right] dx^i dx^k \tag{3.14}$$

where $S_{i,i} = 0$ and $\tilde{h}_{ij,i} = \tilde{h}_{ii} = 0$. $\tilde{h}_{ij}$ are irreducible tensor perturbations having two independent components and describing the graviton with two degrees of freedom in a diffeomorphism invariant way. Also, the scalar fields perturbations transforming as scalars under the three-dimensional rotation group, χ^0 and $\chi^i = \pi_{,i}$, are only considered. In this choice, the ghost is traced as a dynamical degree of freedom of the scalar field χ^0. It was shown in [54] that the linear perturbations of the scalar fields can be expressed in terms of the metric potential ψ as

$$\begin{aligned} \pi &= \frac{2\Delta - 3m_g^2}{m_g^2 \Delta}\psi \\ \chi^0 &= -\frac{2\Delta + 3m_g^2}{m_g^2 \Delta}\dot{\psi} \end{aligned} \tag{3.15}$$

and then the action up to second order in perturbations simplifies to

$${}^{(S)}\delta_2 S = -3 \int d^4x \left[\psi \left(\partial_t^2 - \Delta + m_g^2\right) \psi\right]. \tag{3.16}$$

Due to the accidental $U(1)$ symmetry of the scalar fields χ_A, the field χ^0 enters as a Lagrange multiplier around Minkowski background [21]. However, this symmetry is not preserved on a background deviating from Minkowski space. Therefore, the nonlinear ghost will reappear in the cubic order in the metric and scalar field perturbations since the χ^0 starts to propagate. To see the ghost, third order terms involving the powers of $\dot{\chi}^0$ are considered

$$\delta_3 S = \frac{m_g^2}{2}\int d^4x \left\{ \left[\left(g^{00} - 1 + \sqrt{-g} \right) \bar{h}_i^i + \left(g^{0i} + \dot{\chi}^i - \chi^0_{,i} \right) \left(g^{0i} + \dot{\chi}^i \right) \right] \dot{\chi}^0 + \frac{1}{2} \bar{h}_i^i \left(\dot{\chi}^0 \right)^2 \right.$$
$$\left. + \cdots \right\}. \tag{3.17}$$

The term linear in the time derivative of χ^0 does not induce dynamics for the mode χ^0, while the term proportional to $\left(\dot{\chi}^0\right)^2$ induces the propagation of χ^0 on the background deviating from Minkowski space for which $\bar{h}_i^i \neq 0$.

This action can be expressed entirely in terms of ψ. Considering small perturbations around a background field ψ_b, $\psi = \psi_b + \delta\psi$, the action up to second order in $\delta\psi$ is given by

$$\delta S = -3\int d^4x \left\{ \delta\psi \left(\partial_t^2 - \Delta + m_g^2 \right) \delta\psi + \frac{1}{m_{GH}^2} \left[\left(\partial_t^2 \delta\psi \right)^2 + 2\frac{\ddot{\psi}_b}{\Delta\psi_b} \left(\Delta\delta\psi \right) \left(\partial_t^2 \delta\psi \right) \right] \right.$$
$$\left. + \cdots \right\}, \tag{3.18}$$

where

$$m_{GH}^2 = -\frac{3m_g^4}{4\Delta\psi_b}, \tag{3.19}$$

Taking for the background field the scalar mode of gravitational wave with the wavenumber $k \sim m_g$, for which $\ddot{\psi}_b \sim \Delta\psi_b \sim m_g^2\psi_b$ and $m_{GH}^2 \sim m_g^2/\psi_b$. By considering perturbations $\delta\psi$ with wave-numbers $m_{GH}^2 \gg k^2 \gg m_g^2$ and skipping subdominant terms we can rewrite the action above as

$$\delta S \approx -\frac{3}{m_{GH}^2} \int d^4x \delta\psi \left(\partial_t^2 + \cdots\right) \left(\partial_t^2 + m_{GH}^2 \cdots\right) \delta\psi. \tag{3.20}$$

The perturbation propagator is then given by

$$\frac{1}{\partial^2 \left(\partial^2 + m_{GH}^2\right)} \simeq \frac{1}{m_{GH}^2} \left(\frac{1}{\partial^2} - \frac{1}{\partial^2 + m_{GH}^2}\right). \tag{3.21}$$

This obviously describes the scalar mode of the graviton together with non perturbative Boulware-Deser ghost of mass $m_{GH} \sim m_g/\sqrt{\psi_b}$. As ψ_b vanishes, m_{GH} becomes infinite and the ghost disappears. This ghost would not be essential if m_{GH} would be larger than the Vainstein scale $\Lambda_5 = m_g^{4/5}$ since the linearized consideration above breaks down at energies above Λ_5 and the scalar fields enter the strong coupling regime. However, in strong enough background we have $m_g < m_{GH} < \Lambda_5$ and then the nonlinear ghost appears below the Vainshtein scale, where the perturbative expansion is trustable.

In [31] and [30] it was claimed that there is a unique ghost free nonlinear extension of massive gravity related with $\Lambda = m_g^{2/3}$ Vainshtein scale. Their claim was proved in the decoupling limit neglecting the vector modes of the graviton. However, it was shown in [4] that away from the decoupling limit the nonlinear ghost reappears in the fourth order of perturbation theory which cannot be removed by adding fifth and higher order terms.

To summarize, Higgs gravity uses four scalar fields and the Higgs mechanism to resolve the problems that faced finding a consistent theory for massive gravitons. This model reproduces at the leading order the Fierz-Pauli mass term, but the Boulware-Deser ghost arises at higher non-linear order of perturbation theory. However, it was shown in [4] that this nonlinear ghost exists only at the scales below the Vainshtein energy scale which is extremely low, about $10^{-20}eV$, considering the mass of the graviton to be of the order of the present Hubble scale. Therefore, the ghost is in the strong coupling regime where it is completely harmless and thus irrelevant.

Chapter 4

Supersymmetrizing Massive Gravity

In this chapter, we are going to construct a theory of massive supergravity by generalizing the Higgs Gravity formulation [58], [59]. It is interesting to explore what are we going to get upon taking massive gravity into supersymmetry.

4.1 Going into Supersymmetry

To supersymmetrize Higgs Gravity, we start by using four chiral superfields, $\Phi^A\left(x,\theta,\overline{\theta}\right)$, instead of the four scalars used in the bosonic case (section 3.2). These superfields are subject to the condition

$$\overline{D}_{\dot{\alpha}}\Phi^A\left(x,\theta,\overline{\theta}\right)=0, \tag{4.1}$$

where $A=0,1,2,3$ is a global Lorentz index and $\bar{D}_{\dot{\alpha}}$ is defined in equation (2.9). From section 2.2, we have

$$\Phi_A=\varphi_A+i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi_A-\frac{1}{4}\,\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi_A+\sqrt{2}\,\theta\psi_A-\frac{i}{\sqrt{2}}\,\theta\theta\left(\partial_\mu\psi_A\sigma^\mu\bar{\theta}\right)+\theta\theta F_A. \tag{4.2}$$

To form our generalized induced metric, we start by writing a quartic interaction. The most general term is given by

$$D_\alpha \Phi_A D_\beta \Phi_B \overline{D}^{\dot{\alpha}} \Phi^{*C} \overline{D}^{\dot{\beta}} \Phi^{*D} M^{\alpha\beta AB}_{\dot{\alpha}\dot{\beta}CD} \tag{4.3}$$

where $M^{\alpha\beta AB}_{\dot{\alpha}\dot{\beta}CD}$ is a multispinor constructed in such a way as to make the action invariant under Lorentz transformations. There are two possible strategies to adopt: to symmetrize and antisymmetrize with respect to the fermionic indices $\alpha\beta$ and $\dot{\alpha}\dot{\beta}$, or to use the equivalence of $\alpha\dot{\alpha}$ to a vector index

$$V_{\alpha\dot{\alpha}} = \sigma^\mu_{\alpha\dot{\alpha}} V_\mu. \tag{4.4}$$

We thus define H_{ABC} as the basic field

$$H_{ABC} = D^\alpha \Phi_A (\sigma_B)_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \Phi^*_C = D\Phi_A \sigma_B \bar{D} \Phi^*_C \tag{4.5}$$

Its Hermitian conjugate is

$$H^*_{ABC} = D\Phi_C \sigma_B \overline{D} \Phi^*_A = H_{CBA} \tag{4.6}$$

We also denote $H_{ABC}\eta^{AB}$ by H_{AAC} and we define the contracted field

$$H_C = H_{AAC}, \qquad H^*_A = H_{ACC} \tag{4.7}$$

to simplify our expressions. The products that could be formed from this H field are given in appendix B. These represent all the possible D-type terms that can be included in our action. Therefore, the action to start with is the linear combination of all these D-type terms constructed. This is given by

$$
\begin{aligned}
& c_1 H_{ABC} H_{ABC} + c_2 H_{ABC} H_{ACB} + c_2^* H_{ABC} H_{BAC} + c_3 H_{ABC} H_{BCA} \\
& + c_4 H_{ABC} H_{CAB} + c_5 H_{ABC} H_{CBA} + c_6 H_A H_A^* + c_7 H_A H_A + c_7^* H_A^* H_A^* \\
& + \epsilon^{ABCD} H_{ABC} \left(c_8 H_D - c_8^* H_D^* \right) + \epsilon^{ABCD} \left(c_9 H_{ABE} H_{CDE} + c_9^* H_{EAB} H_{ECD} \right) \\
& + \epsilon^{ABCD} \left(c_{10} H_{AEB} H_{CED} + c_{11} H_{AEB} H_{ECD} + c_{11}^* H_{AEB} H_{CDE} + c_{12} H_{EAB} H_{CDE} \right) .
\end{aligned}
\tag{4.8}
$$

where all the constants c_i are real except for those whose conjugate appear (i.e. c_2, c_7, c_8, c_9, c_{11}, are complex). All these terms are computed term by term up to all orders in θ.

4.2 Coupling to Supergravity

To couple our supersymmetric action to supergravity, we use the rules of tensor calculus. These rules provide us the method of coupling Supergravity to the components of vector and chiral multiplets [63] (see also [43]). This will promote the global supersymmetry to a local one. Let us first present a review of these rules.

4.2.1 Rules of Tensor Calculus

The vector multiplet discussed in section (2.2) has as component fields two scalars (C and a complex M), two Majorana spinors (ξ and λ), a vector (V_μ), and one auxiliary scalar field (D). It is represented by

$$V = (C, \xi, M, V_\mu, \lambda, D)\,. \tag{4.9}$$

While the component fields of a left-handed chiral multiplet (F-type) are a complex scalar field z, left-handed Weyl spinors χ_L, and a complex auxiliary field h. It is given by

$$F = (z, \chi_L, h)\,. \tag{4.10}$$

To couple the F-type multiplet to supergravity, we use the action formula given by

$$e^{-1}L_F = h + \kappa u z + \kappa\bar{\phi}_\mu\gamma^\mu\chi + i\kappa^2\bar{\phi}_\mu\gamma^{\mu\nu}\phi_{vR}z + h.c. \tag{4.11}$$

That of the D-type multiplet is given by

$$\begin{aligned} e^{-1}L_D = D &+ \frac{i\kappa}{2}\bar{\phi}_\mu\gamma^5\gamma^\mu\lambda - \frac{\kappa}{3}\left(uM^* + u^*M\right) + \frac{i\kappa^2}{8}\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_\nu\phi_\rho\bar{\xi}\phi_\sigma \\ &+ \frac{2}{3}\kappa V_\mu\left(A^\mu + \frac{3}{8}ie^{-1}\epsilon^{\mu\rho\sigma\tau}\bar{\phi}_\rho\gamma_\tau\phi_\sigma\right) - i\frac{\kappa}{3}e^{-1}\bar{\xi}\gamma_5\gamma_\mu R^\mu \\ &- \frac{2}{3}\kappa^2 Ce^{-1}L_{S.G.} + e^{-1}L_{S.G.} \end{aligned} \tag{4.12}$$

where $L_{S.G.}$ is the supergravity Lagrangian given in section (2.5). These two equations contain the auxiliary fields u and A^μ. To eliminate them, we use their equations of motion which are given by respectively

$$\kappa z - \frac{\kappa}{3}M^* - \frac{1}{3}u^* = 0$$

$$\frac{2}{3}\kappa V_\mu + \frac{1}{3}A_\mu = 0 \tag{4.13}$$

Plugging back for A_μ, u and u^* in $e^{-1}L_F + e^{-1}L_D$, we get

$$\begin{aligned} e^{-1}L_F + e^{-1}L_D = D &- \frac{1}{2\kappa^2}R(e,w) - \frac{1}{2}e^{-1}\bar{\phi}_\mu R^\mu + (h+h^*) - \kappa^2\left(Mz + M^*z^*\right) \\ &+ 3\kappa^2 zz^* + \left(\left(\kappa\bar{\phi}_\mu\gamma^\mu\chi + i\kappa^2\bar{\phi}_\mu\gamma^{\mu\nu}\phi_{vR}z\right) + h.c.\right) - \frac{2}{3}\kappa^2 Ce^{-1}L_{S.G.} \\ &+ \frac{i\kappa}{2}\bar{\phi}_\mu\gamma^5\gamma^\mu\lambda - i\frac{\kappa}{3}e^{-1}\bar{\xi}\gamma_5\gamma_\mu R^\mu + \frac{k^2}{3}MM^* + \frac{i\kappa^2}{8}\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_\nu\phi_\rho\bar{\xi}\phi_\sigma \\ &+ \frac{i}{4}e^{-1}\kappa\epsilon^{\mu\rho\sigma\tau}V_\mu\bar{\phi}_\rho\sigma_\tau\phi_\sigma. \end{aligned} \tag{4.14}$$

4.2.2 Constructing the Full Action

Using the above equations we can construct our full action. We require an action with the following conditions

- It has a Fierz-Pauli term for the vierbeins $(\bar{e}^\mu_A\bar{e}^A_\mu - \bar{e}^2)$. This choice decouples the sixth degree of freedom of the massive graviton.
- It has no constant.
- It contains no linear vierbein term
- It gives Maxwell form for the χ_A fields

$$l\left(\partial_\mu\chi_A\partial^\mu\chi^{A*} - \partial_A\chi^A\partial_B\chi^{B*}\right) \tag{4.15}$$

where l is a constant

- It is ghost free where there should be no terms like

$$\partial_\mu \chi_A \partial^\mu \chi^A, \quad \text{or} \quad \partial_A \chi^A \partial_B \chi^B \tag{4.16}$$

- The gravitino should be massive.

Calculations show that working instead with

$$\overline{H}_{ABC} = H_{ABC} - Dx_A \sigma_B \overline{D} x_C^*, \tag{4.17}$$

where x_A are the coordinates, will cancel the linear vierbein term without including higher order terms in H_{ABC}. This is similar to what was done in the bosonic case to give an action with the correct behaviour (as discussed in section 3.2). In that case $\overline{h}_{AB}$ was defined by

$$\overline{h}_{AB} = H_{AB} - \eta_{AB} \equiv H_{AB} - \partial_\mu x_A \partial^\mu x_B \tag{4.18}$$

so that there is no need to include higher order terms in H_{AB} and only

$$\left(\bar{h}^A_B \bar{h}^B_A - \bar{h}^2\right) \tag{4.19}$$

were considered.

It was also found that if we consider only D-type terms, all of the above required conditions are well satisfied except making the gravitino massive. To solve this issue, F-type terms are included because only such terms will return a mass term for the gravitino.

Several F-type terms can be written. Two such terms are given below. These will be added to the action formed of D-type terms composed above (equation 4.8)

$$
\begin{aligned}
&c_{13}\overline{D}^2\left(D\Phi_A\sigma^{AB}D\Phi_B\right)+c_{13}^*D^2\left(\overline{D}\Phi_A^*\overline{\sigma}^{AB}\overline{D}\Phi_B^*\right)\\
&+c_{14}\overline{D}^2\left(D\Phi_A D\Phi^A\bar{D}\Phi_B^*\bar{D}\Phi^{B*}\right)+c_{14}^*D^2\left(\bar{D}\Phi_A^*\bar{D}\Phi^{A*}D\Phi_B D\Phi^B\right)
\end{aligned}
\tag{4.20}
$$

To find the full Lagrangian that satisfies the required conditions, we have first to express the supermultiplets in terms of their component fields. Expressing the metric in terms of vierbeins, $g^{\mu\nu} = e_a^{\mu}e^{\nu a}$, and expanding the fields around the vacuum solution

$$
\varphi^A = x^A + \chi^A, \qquad e_a^{\mu} = \delta_a^{\mu} + \bar{e}_a^{\mu}, \tag{4.21}
$$

then up to quadratic orders, the components of our superfields are found. Coupling the components of the vector and chiral multiplets to supergravity, it is found that the required matter action is formed of three D-type terms and two F-type terms. It is given by

$$
\begin{aligned}
&m^4\int\left(c_1\bar{H}_{ABC}\bar{H}_{BCA}+c_2\bar{H}_{ABB}\bar{H}_{CCA}+c_3\bar{H}_{AB}\bar{H}_{AB}^*\right)d\theta^2 d\bar{\theta}^2 d^4x\\
&+\frac{m^2}{\kappa}\int\left(c_4\bar{D}^2\left(D\Phi_A\sigma^{AB}D\Phi_B\right)+c_4^*D^2\left(\bar{D}\Phi_A^*\bar{\sigma}^{AB}\bar{D}\Phi_B^*\right)\right)d\theta^2 d^4x\\
&+m^4\int c_5\bar{D}^2\left(D\Phi_A D\Phi^A\bar{D}\Phi_B^*\bar{D}\Phi^{B*}\right)d\theta^2 d^4x\\
&+m^4\int c_5^*D^2\left(\bar{D}\Phi_A^*\bar{D}\Phi^{A*}D\Phi_B D\Phi^B\right)d\theta^2 d^4x
\end{aligned}
\tag{4.22}
$$

where $H_{AB} = D\Phi_A D\Phi_B$ and m and κ are used to fix the dimensions where $[\Phi_A] = -1$, $[D^\alpha] = 1/2$, $[d\theta] = 1/2$ and $[d^4x] = -4$.

The components of the basic field, H_{ABC}, and of the superfields forming the action are given in appendix C. This action is not unique. Many other actions exist serving the same purpose. For example, the three D-type terms can be substituted by three other terms

$$\begin{aligned}
&\epsilon^{ABCD} D^\alpha \Phi_A \left(\sigma_B\right)_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \Phi^\dagger_C D^\beta \Phi_D \left(\sigma^E\right)_{\beta\dot{\beta}} \bar{D}^{\dot{\beta}} \Phi^\dagger_E \\
&\epsilon^{ABCD} D^\alpha \Phi_A \left(\sigma_B\right)_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \Phi^\dagger_E D^\beta \Phi_D \left(\sigma_C\right)_{\beta\dot{\beta}} \bar{D}^{\dot{\beta}} \Phi^{E\dagger} \\
&\epsilon^{ABCD} D^\alpha \Phi_A \left(\sigma^E\right)_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \Phi^\dagger_C D^\beta \Phi_D \left(\sigma_B\right)_{\beta\dot{\beta}} \bar{D}^{\dot{\beta}} \Phi^\dagger_E .
\end{aligned} \tag{4.23}$$

The constants, c_1, c_2, c_3, c_4 and c_5, are found by forcing the constraints on the action. This will return a system of equations for the constants given by

- No ghost $\rightarrow -16c_1 + 32c_2 - 256\left(c_5 + c_5^*\right) = 0$
- Maxwell: $l\ \left(\partial_\mu \chi_A \partial^\mu \chi^{A*} - \partial_A \chi^A \partial_B \chi^{B*}\right) \rightarrow 32c_1 + 128c_2 + 32c_3 = l$

  and $32c_1 + 32c_2 + 32c_3 + 48^2 \times 3\ c_4 c_4^* - 128\left(c_5 + c_5^*\right) = -l$
- Constant $\rightarrow -64 \times 16\left(c_5 + c_5^*\right) + 96^2 \times 3\ c_4 c_4^* = 0$

Solving this system, we obtain

$$\begin{aligned}
c_1 &= \frac{l}{24} - 432\ c_4 c_4^* \\
c_2 &= \frac{l}{48} \\
c_3 &= -\frac{3l}{32} + 432\ c_4 c_4^*
\end{aligned}$$

$$c_5 = \frac{27}{2}\ c_4 c_4^* \tag{4.24}$$

l is fixed to be $-1/2$ by normalizing the kinetic term. Moreover, c_4 is arbitrary, but we set it to be $\frac{i}{48\sqrt{6}}$ for the term $\bar{\psi}_A\gamma^A\gamma^B\psi_B$ to cancel out. The full Lagrangian $(e^{-1}L_F + e^{-1}L_D)$ is then reduced to

$$\begin{aligned}
&-\frac{1}{2}m^4\ \left(\partial_\mu\chi_A\partial^\mu\chi^{A*} - \partial^A\chi_A\partial^B\chi_B^*\right) + \frac{7}{3}m^4\ \left(\bar{e}_\mu^A\bar{e}_A^\mu - \bar{e}^2\right) - m^4\ F_A F^{A*} \\
&-\frac{7}{3}\left(\bar{e}\partial_A\chi^A + \bar{e}\partial_A\chi^{A*}\right) + \frac{7}{3}\left(\bar{e}_A^\mu\partial_\mu\chi^A + \bar{e}_A^\mu\partial_\mu\chi^{A*}\right) \\
&-\frac{5}{24}m^4\ \epsilon^{ABCD}\bar{\psi}_A\gamma_B\gamma_5\partial_C\psi_D + \frac{3i}{8}m^4\ \bar{\psi}_A\gamma_\mu\partial^\mu\psi^A - \frac{\sqrt{6}}{8}m^6\kappa\ \bar{\psi}_A\psi^A \\
&+\frac{\sqrt{6}}{18}m^6\kappa\ \bar{\psi}_A\gamma^A\gamma^B\psi_B - \frac{5\sqrt{6}}{36}m^6\kappa\ \bar{\psi}_A\gamma^B\gamma^A\psi_B + \frac{1}{2}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_5\gamma_\nu\partial_\rho\phi_\sigma \\
&+\frac{\sqrt{2}i}{4}m^4\kappa\ \bar{\phi}_\mu\gamma^\mu\gamma^A\psi_A - \frac{\sqrt{2}i}{4}m^4\kappa\ \bar{\psi}_A\gamma^A\gamma^\mu\phi_\mu + \frac{\sqrt{3}}{6}m^2\bar{\phi}_\mu\gamma^\mu\partial^A\psi_A \\
&+\frac{\sqrt{3}}{6}m^2\partial^A\bar{\psi}_A\gamma^\mu\phi_\mu + \frac{\sqrt{6}i}{3}m^2\kappa\ \bar{\phi}_\mu\gamma^{\mu\nu}\phi_v + \frac{\sqrt{3}}{12}m^2\kappa\ \bar{\phi}_\mu\gamma^\mu\gamma^A\gamma^B\partial_B\psi_A \\
&+\frac{\sqrt{3}}{12}m^2\kappa\ \partial_B\bar{\psi}_A\gamma^B\gamma^A\gamma^\mu\phi_\mu - \frac{1}{2\kappa^2}R(e,w).
\end{aligned}$$

Now it is clear how the dimensions are fixed by m and κ, where $[\chi_A] = -1, [\bar{e}] = 0, [F_A] = 0, [\psi_A] = -1/2$ and the gravitino $[\phi_\mu] = 3/2$.

4.3 Equations of Motion

The equations of motion can be found from the full Lagrangian. The Euler-Lagrange equations are given by

$$\frac{\delta L}{\delta \Phi^a} = \frac{\partial L}{\partial \Phi^a} - \partial_\mu \left[\frac{\partial L}{\partial \left(\partial_\mu \Phi^a \right)} \right] = 0 \tag{4.25}$$

From this, the equations of motion for $\bar{\psi}_A$ and $\bar{\phi}_\mu$ are found to be given respectively by

$$\begin{aligned} &\frac{-5}{24} m^4 \epsilon^{ABCD} \gamma_B \gamma_5 \partial_C \psi_D + \frac{3i}{8} m^4 \gamma_\mu \partial^\mu \psi^A - \frac{\sqrt{6}}{8} m^6 \kappa \ \psi^A - \frac{5\sqrt{6}}{36} m^6 \kappa \ \gamma^B \gamma^A \psi_B \\ &+ \frac{\sqrt{6}}{18} m^6 \kappa \ \gamma^A \gamma^B \psi_B - \frac{\sqrt{3}}{6} m^2 \ \gamma^\mu \partial^A \phi_\mu - \frac{\sqrt{2}i}{4} m^4 \kappa \ \gamma^A \gamma^\mu \phi_\mu \\ &- \frac{\sqrt{3}}{12} m^2 \kappa \ \gamma^B \gamma^A \gamma^\mu \partial_B \phi_\mu = 0 \end{aligned} \tag{4.26}$$

and

$$\begin{aligned} &\frac{\sqrt{3}}{6} m^2 \ \gamma^\mu \partial_A \psi^A + \frac{\sqrt{2}i}{4} m^4 \kappa \ \gamma^\mu \gamma^\nu \psi_\nu + \frac{\sqrt{3}}{12} m^2 \ \gamma^\mu \gamma^A \gamma^B \partial_B \psi_A + \frac{\sqrt{6}i}{3} m^2 \kappa \gamma^{\mu\nu} \phi_\nu \\ &+ \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu \partial_\rho \phi_\sigma = 0. \end{aligned} \tag{4.27}$$

The gravitino is a vector-spinor and hence it is reducible because it contains both spin 3/2 and spin 1/2 degrees of freedom. Therefore, to simplify the equations of motion and count the degrees of freedom, we decompose ψ_A into a spin-3/2 helicity,

$\hat{\psi}_A$, and a spin-1/2 helicity, λ

$$\psi_A = \hat{\psi}_A + \frac{1}{4}\gamma_A\gamma_5\lambda$$
$$\Rightarrow \bar{\psi}_A = \bar{\hat{\psi}}_A + \frac{1}{4}\bar{\lambda}\gamma_A\gamma_5, \tag{4.28}$$

where $\gamma_A\hat{\psi}^A = 0$. Similarly, we decompose ϕ_μ

$$\phi_\mu = \hat{\phi}_\mu + \frac{1}{4}\gamma_\mu\gamma_5\eta$$
$$\Rightarrow \bar{\phi}_\mu = \bar{\hat{\phi}}_\mu + \frac{1}{4}\bar{\eta}\gamma_\mu\gamma_5, \tag{4.29}$$

where $\gamma_\mu\hat{\phi}^\mu = 0$. Again $\hat{\phi}_\mu$ is a spin-3/2 helicity, and η is spin-1/2 helicity.

The equations of motion expressed using this decomposition are then given by

$$\begin{aligned}
&\frac{-5}{24}m^4\epsilon^{ABCD}\gamma_B\gamma_5\partial_C\hat{\psi}_D + \frac{5}{96}m^4\epsilon^{ABCD}\gamma_B\gamma_D\partial_C\lambda + \frac{3i}{8}m^4\gamma_\mu\partial^\mu\hat{\psi}^A \\
&+ \frac{3i}{32}m^4\gamma_\mu\gamma^A\gamma^5\partial^\mu\lambda + \frac{3\sqrt{6}}{32}m^6\kappa\ \gamma^A\gamma^5\lambda - \frac{29\sqrt{6}}{72}m^6\kappa\ \hat{\psi}^A - \frac{\sqrt{3}}{6}m^2\ \gamma^5\partial^A\eta \\
&- \frac{\sqrt{2}i}{4}m^4\kappa\ \gamma^A\gamma^5\eta - \frac{\sqrt{3}}{12}m^2\kappa\ \gamma^B\gamma^A\gamma^5\partial_B\eta = 0
\end{aligned} \tag{4.30}$$

and

$$\begin{aligned}
&\frac{\sqrt{3}}{3}m^2\ \gamma^\mu\partial_A\hat{\psi}^A + \frac{\sqrt{2}i}{4}m^4\kappa\ \gamma^\mu\gamma^5\lambda + \frac{\sqrt{6}}{6}m^2\kappa\ \hat{\phi}^\mu - \frac{\sqrt{6}}{8}m^2\kappa\gamma^\mu\gamma^5\eta \\
&+ \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\hat{\phi}_\sigma + \frac{1}{2}\gamma^5\gamma^{\mu\rho}\partial_\rho\eta = 0.
\end{aligned} \tag{4.31}$$

To simplify these field equations, we multiply equations (4.30) and (4.31) by γ_A and γ_μ respectively. This will yield

$$\frac{i}{3}\partial_A\hat{\psi}^A - \frac{i}{8}\gamma^5\gamma^A\partial_A\lambda + \frac{3\sqrt{6}}{8}m^2\kappa\ \gamma^5\lambda - \sqrt{2}i\kappa\ \gamma^5\eta = 0 \tag{4.32}$$

$$\frac{i}{3}m^2\ \partial_A\hat{\psi}^A - \frac{\sqrt{6}}{12}m^4\kappa\ \gamma^5\lambda - \frac{\sqrt{2}}{8}im^2\kappa\ \gamma^5\eta + \frac{\sqrt{3}}{12}\partial_\mu\hat{\phi}^\mu + \frac{\sqrt{3}}{16}\gamma^5\gamma^\mu\partial_\mu\eta = 0 \tag{4.33}$$

A further simplification of equations (4.30) and (4.31) can be done by tracing them with ∂_A and ∂_μ respectively. This gives

$$\begin{aligned}&\frac{3i}{8}m^4\gamma^\mu\partial_\mu\partial_A\hat{\psi}^A + \frac{3i}{32}m^4\gamma^5\partial^A\partial_A\lambda + \frac{3\sqrt{6}}{32}m^6\kappa\ \gamma^A\gamma^5\partial_A\lambda - \frac{29\sqrt{6}}{72}m^6\kappa\ \partial_A\hat{\psi}^A\\ &- \frac{\sqrt{3}}{4}m^2\ \gamma^5\partial^A\partial_A\eta - \frac{\sqrt{2}i}{4}m^4\kappa\ \gamma^A\gamma^5\partial_A\eta = 0\end{aligned} \tag{4.34}$$

and

$$\frac{\sqrt{3}}{3}\ \gamma^\mu\partial_\mu\partial_A\hat{\psi}^A + \frac{\sqrt{2}i}{4}m^2\kappa\ \gamma^\mu\gamma^5\partial_\mu\lambda + \frac{\sqrt{6}}{6}\kappa\ \partial_\mu\hat{\phi}^\mu - \frac{\sqrt{6}}{8}\kappa\ \gamma^\mu\gamma^5\partial_\mu\eta = 0. \tag{4.35}$$

We can then express $\hat{\psi}^A$ in terms of λ and η using equation (4.32),

$$\partial_A\hat{\psi}^A = \frac{3}{8}\gamma^5\gamma^A\partial_A\lambda + \frac{9\sqrt{6}i}{8}m^2\kappa\gamma^5\lambda + 3\sqrt{2}\kappa\gamma^5\eta. \tag{4.36}$$

Upon plugging this into equation (4.34), we get an equation relating λ to η

$$\frac{-3i}{8}m^2\partial_A\partial^A\lambda+\frac{17\sqrt{6}}{12}m^4\kappa\;\gamma^A\partial_A\lambda-\frac{87i}{4}m^6\kappa^2\;\lambda-7\sqrt{2}im^2\kappa\;\gamma^A\partial_A\eta$$
$$-\frac{58\sqrt{3}}{3}m^4\kappa^2\;\eta-2\sqrt{3}\;\partial_A\partial^A\;\eta=0. \tag{4.37}$$

Similarly from (4.33) and (4.36) we get an equation for $\partial_A\hat{\phi}^A$

$$\partial_A\hat{\phi}^A=\frac{-\sqrt{3}i}{2}m^2\gamma^5\gamma^A\partial_A\lambda+\frac{11\sqrt{2}}{2}m^4\kappa\;\gamma^5\lambda-\frac{3}{4}\gamma^5\gamma_A\partial^A\eta-\frac{7\sqrt{6}i}{2}m^2\kappa\gamma^5\eta \tag{4.38}$$

Plugging this into (4.35), we get another equation relating λ and η

$$\frac{-3}{8}\partial_A\partial^A\lambda-\frac{13\sqrt{6}i}{8}m^2\kappa\gamma^A\partial_A\lambda+\frac{11}{2}m^4\kappa^2\;\lambda-3\sqrt{2}\kappa\;\gamma^A\partial_A\eta-\frac{7\sqrt{3}i}{2}m^2\kappa^2\;\eta=0. \tag{4.39}$$

Combining equations (4.37) and (4.39), we get

$$-\frac{5\sqrt{6}}{24}m^4\kappa\;\gamma^A\partial_A\lambda-\frac{109i}{4}m^6\kappa^2\;\lambda-4\sqrt{2}i\;m^2\kappa\;\gamma^A\partial_A\eta-\frac{137\sqrt{3}}{6}m^4\kappa^2\;\eta$$
$$-2\sqrt{3}\;\partial_A\partial^A\eta=0. \tag{4.40}$$

Since the action is invariant under local supersymmetry transformations, we can choose the gauge $\eta=0$

$$\gamma_A\partial^A\lambda+\frac{109\sqrt{6}}{5}im^2\kappa\;\lambda=0. \tag{4.41}$$

This gives a Dirac type equation for the spin-1/2 helicities.

It should be noted that the divergence of $\hat{\phi}$ is found in terms of lambda. However, we can find a combination of $\hat{\phi}_A$ and $\hat{\psi}_A$

$$\hat{\phi}'_A = \hat{\phi}_A + \alpha\hat{\psi}_A \tag{4.42}$$

such that the divergence of $\hat{\phi}'$ equals zero ($\partial^A \hat{\phi}'_A = 0$) [29]. Then $\hat{\phi}'$ has two helicities 3/2 and $-3/2$.

4.4 Degrees of Freedom

Counting degrees of freedom, we start with a $N = 1$ supersymmetry model, and then couple it to Supergravity. We know from section (2.5) that supergravity contains a massless spin-2 graviton with two bosonic degrees of freedom and one massless spin-3/2 gravitino with two fermionic degrees of freedom. For the $N = 1$ supersymmetry model, it is similar to the Wess-Zumino model discussed in section (2.3). It contains four spin-0 particles, φ^A; however, φ^0 decouples due to the Fierz-Pauli choice. Therefore, there are only six degrees of freedom (3 times 2). For this, we have six fermionic degrees of freedom forming a multiplet. Therefore, we are starting with an overall eight bosonic degrees of freedom and eight fermionic degrees of freedom.

After coupling to Supergravity, we obtain $N = 1$ massive representation. It contains a single massive spin-2 particle, a single massive vector field and two massive spin-3/2 particles. Each massive spin-3/2 particle has four degrees of freedom, and this constitute the eight fermionic degrees of freedom. For the bosonic degrees of freedom, the massive spin-1 particle has three degrees of freedom, and the massive graviton has five degrees of freedom. Therefore, we have an overall of eight bosonic degrees of freedom which gives us the same number of degrees of freedom as before.

What is noticeable is that at the end we are left with two massive gravitinos. This is similar to the $N = 2$ supersymmetry in which we have two massive spin-3/2 particles. However, in our case, the two gravitinos have different masses. This is because supersymmetry is completely broken. It is a space-time symmetry and then it is broken exactly at the same scale as the diffeomorphism breaking. We are starting with a supergravity action and a matter action which are independent; therefore, we are left with two massive spin-3/2 particles with completely different masses. There is no $N = 2$ supersymmetry to start with since before diffeomorphism breaking we had spin 1/2 and not spin 3/2, then the two gravitinos would not have the same mass. When coordinate invariance is broken by the four scalars, the Lorentz symmetry of the tangent manifold gets identified with that of space-time, the four scalars become four-vectors and the spin 1/2 supersymmetric partners become spin 3/2 fields. Therefore, one is a genuine gravitino ϕ_μ, while the other ψ_A becomes identified with a gravitino after the breaking.

4.5 LHC and Our Superpartners

The LHC, Large Hadron Collider, with a 27 km loop built near Geneva is the world's largest and most powerful particle accelerator. It was built between 1998 and 2008 and it first started on 10 September 2008. It has many purposes where it allows to test the predictions of the Standard model and to prove (or disprove) the existence of the Higgs boson, and also to search for the superpartners predicted by supersymmetry.

The LHC is now shutdown for upgrades and it will resume early in 2015 where the beam energy will be increased to reach 6.5 TeV per beam. We hope that some of the open questions will be answered and that superpartners will be found to prove supersymmetry. If this so, then the experiments will tell us at what energies we expect to find our predicted particles in the model built in this thesis.

Chapter 5

Canonical Formulation

The Hamiltonian formulations provide insights mainly for counting degrees of freedom, for canonical quantization purposes and for numerical investigation of solutions. Canonical structures play an important role in quantum gravity and in numerical relativity.

To set up a canonical formulation, one starts by defining momenta to be the time derivatives of the fields. This is used to replace only time derivatives in the Hamiltonian by momenta; therefore, spacial derivatives do not change. In this formulation, space-time symmetry is hidden, but of course still present.

The aim of this chapter is to write Einstein's action in canonical form and find the constraints. First, we will briefly review the Hamiltonian formalism.

5.1 History

Hamiltonian formulation is not a new topic. Dirac, in [36, 37, 40] (see also [51]), set up a general theory for constrained Hamiltonian dynamics which is used for canonical quantization. Then he applied this procedure to general relativity [38], [41]. Further work was done by Bergmann ([6, 11, 15]) to quantize covariant field theories, like the Einstein's theory of gravity. Then, Arnowitt, Deser and Misner identified the ADM energy as the Hamiltonian after gauge fixing, and they constructed a canonical formulation of gravity in 1962 ([64, 65, 66, 67]). Their canonical formulation is most widely used. The tetrad form of this ADM formalism was derived in [33].

There are other canonical formulations such as the one introduced by Ashtekar where he introduced new variables which lead to simplifications in the gravitational constraints ([7, 8]). His theory led to loop quantum gravity. Later, the canonical formulations of matter couplings to gravity were considered. For example, such canonical formulations for massive scalar fields were found in [69, 70] and for spin-1/2 Dirac field in [39, 53].

5.2 Hamiltonian Formalism

Starting with the action

$$S = \int L(q^i, \dot{q}^i) dt \tag{5.1}$$

for a system of n degrees of freedom, $i = 1, ..., n$, the conjugated momentum is defined to be

$$p_i = \frac{\partial L}{\partial \dot{q}^i}, \tag{5.2}$$

and the Hamiltonian is given by

$$H = \dot{q}^i p_i(q, \dot{q}) - L(q, \dot{q}). \tag{5.3}$$

It is a well-defined functional of q^i and p_j with the equations of motion given by

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q^i}. \tag{5.4}$$

where they are of first order in the time derivative. These field equations can be expressed in terms of the classical Poisson bracket which is defined to be

$$\{f(q,p), g(q,p)\} = \Sigma_{i=1}^{n} \left(\frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i} \right). \tag{5.5}$$

The equations of motion are then written in the compact form

$$\dot{q}^i = \{q^i, H\}, \quad \dot{p}_i = \{p_i, H\}. \tag{5.6}$$

The Hamiltonian formulation of field theories is similar to the finite-dimensional system; however, the independent variables are labeled by a continuous parameter x and not by a discrete parameter i. Mathematical care is required for the variational equations or Poisson brackets.

A basic functional derivative is

$$\frac{\delta\phi(x)}{\delta\phi(y)} = \delta(x-y) \quad \text{instead of} \quad \frac{\partial q^i}{\partial q^j} = \delta^i_j, \tag{5.7}$$

and the generalized Poisson bracket is then defined using functional derivatives

$$\{f,g\} = \int dx^n \left(\frac{\partial f}{\partial\phi(x)} \frac{\partial g}{\partial p_\phi(x)} - \frac{\partial f}{\partial p_\phi(x)} \frac{\partial g}{\partial\phi(x)} \right). \tag{5.8}$$

5.3 ADM Formalism

Arnowitt, Deser and Misner constructed a canonical formulation of gravity (ADM) in 1962 ([67]). This Hamiltonian formulation requires time to be singled out where spacetime is split into space and time. To write the theory in a $3+1$ dimensional form, we start by considering two hypersurfaces with t = constant (earlier) and $t+dt$ = constant (later) [20]. The 3-geometry of the earlier hypersurface is described by the metric

$$h_{ij}(t,x,y,z)dx^i dx^j. \tag{5.9}$$

while that of the later hypersurface is

$$h_{ij}(t+dt,x,y,z)dx^i dx^j. \tag{5.10}$$

The proper distance and proper time from the earlier to the later 3-geometry are given in terms of the lapse and shift functions, N and N_i. The proper interval between $x^\mu = (t,x^i)$ and $x^\mu + dx^\mu = (t+dt, x^i + dx^i)$ is found by using the 4-

dimensional form of the Pythagorean theorem and this yields

$$ds^2 = h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) - (Ndt)^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{5.11}$$

The full metric, $g_{\mu\nu}$, is then given by [67]

$$\begin{bmatrix} N_i N^i - N^2 & N_k \\ N_i & h_{ik} \end{bmatrix} \tag{5.12}$$

and its inverse is

$$\begin{bmatrix} -\frac{1}{N^2} & \frac{N^i}{N^2} \\ \frac{N^k}{N^2} & h^{ik} - N^i N^k / N^2 \end{bmatrix} \tag{5.13}$$

where h_{ij} is an induced metric on the 3-dimensional hypersurfaces with $h^{ij} h_{jk} = \delta^i_k$. After this splitting of spacetime, Einstein's action

$$S = -\frac{1}{2} \int \sqrt{-g} R \tag{5.14}$$

is written as

$$S_{ADM} = \int d^4x \, \left(\dot{h^{ij}} \pi_{ij} - N^\mu H_\mu \right) \quad = \int d^4x \, \left(\dot{h^{ij}} \pi_{ij} - N H_0 - N^i H_i \right) \tag{5.15}$$

where π_{ij} is the canonical momentum given by

$$\pi_{ij} = \frac{\partial}{\partial \dot{h}^{ij}} \left(\sqrt{-g} L_{ADM} \right) = \sqrt{h} \left(h_{ij} K - K_{ij} \right) \tag{5.16}$$

and K_{ij} is the extrinsic curvature

$$K_{ij} = \frac{1}{2N}(N_{i|j} + N_{j|i} - \dot{h}_{ij}). \tag{5.17}$$

H_0 and H_i are called the Hamiltonian and the diffeomorphism constraints respectively. They are given by the intrinsic curvatures

$$\begin{aligned} H_0 &= R_0 \equiv -\sqrt{h}\left[{}^3R + h^{-1}\left(\frac{1}{2}\pi^2 - \pi^{ij}\pi_{ij}\right)\right] \\ H_i &= R_i \equiv -2h_{ik}\pi^{kj}{}_{|j} \end{aligned} \tag{5.18}$$

The action is in the first order form (first order time derivatives enter) where only the time derivative of h_{ij} enter into the action. $\dot{N}$ and $\dot{N}^i$ are not involved; i.e. their momenta are not defined and they are not dynamical variables. Therefore, the Hamiltonian of general relativity is a function of h_{ij} and its conjugate momentum π^{ij}, while the lapse and shift functions enter as Lagrange multipliers. Variation with respect to N and N_i give constraint equations.

5.4 Equations of Motion of GR

Computing the Hamiltonian field equations, we first have the propagation of the h_{ij}

$$\dot{h}_{ij} = \{h_{ij}, H_g\} = 2Nh^{-1/2}\left(\pi_{ij} - \frac{1}{2}h_{ij}\pi\right) + N_{i|j} + N_{j|i}. \tag{5.19}$$

where

$$H_g = \int d^3x \left(NR_0 + N^i R_i\right). \tag{5.20}$$

This restates equation (5.16), the defining equation of the canonical momentum in terms of the metric tensor and its derivatives. Second, there is another set of six equations determining the propagation of π^{ij}

$$\dot{\pi}^{ij} = \{\pi^{ij}, H_g\} = -\frac{\delta H_g}{\delta h_{ij}}. \tag{5.21}$$

Varying the Hamiltonian with respect to h_{ij} and using

$$\frac{\delta h_{kl}(x)}{\delta h_{ij}(y)} = \delta^i{}_{(k}\delta^j{}_{l)}\delta(x,y) = \frac{1}{2}\left(\delta^i_k\delta^j_l + \delta^i_l\delta^j_k\right)\delta(x,y), \tag{5.22}$$

gives

$$\begin{aligned}\dot{\pi}^{ij} = &-N\sqrt{h}\left({}^3R^{ij} - \frac{1}{2}h^{ij}\,{}^3R\right) + \frac{1}{2}Nh^{-1/2}h^{ij}\left(\pi^{mn}\pi_{mn} - \frac{1}{2}\pi^2\right)\\ &- 2Nh^{-1/2}\left(\pi^{im}\pi_m^{\ j} - \frac{1}{2}\pi\pi^{ij}\right) + \sqrt{h}\left(N^{|ij} - h^{ij}N^{|m}{}_{|m}\right) + \left(\pi^{ij}N^m\right)_{|m}\\ &- N^i_{|m}\pi^{mj} - N^j_{|m}\pi^{mi}\end{aligned} \tag{5.23}$$

These six equations correspond to the six field equations $G^{ij} = 0$. Variation with respect to N and N^i yield four constraint equations, $H_\mu = 0$. These are equivalent to the Einstein's field equations

$$G^0_\mu = R^0_\mu - \frac{1}{2}\delta^0_\mu R = 0. \tag{5.24}$$

The existence of the constraints shows that initial values cannot be arbitrarily chosen. Their presence also means that there are underlying symmetries. These first class constraints generate gauge transformations, which are equivalent to coordinate changes [12].

Counting the degrees of freedom, the basic variables, g_{ij} and π^{ij}, are both symmetric variables thus having twelve independent components. Four are eliminated by the four independent constraints. Out of the eight remaining, another four degrees of freedom can be gauged away by gauge transformations. Then we are left with four phase space degrees of freedom which recover the two degrees of freedom of the graviton.

5.5 Poisson Brackets of GR

To construct the algebra of the first class constraints, it is more convenient to work with smeared forms. This will ensure well-defined algebraic relations since the derivatives by field will then be free of delta distributions.

Consider the smeared quantities, $H[N]$ and $D[N^i]$, given by the integration with respect to the multipliers N and N^i

$$H[N] = \int d^3xN(x)H^0(x) \quad \text{and} \quad D[N^i] = \int d^3xN^i(x)H_i(x) \tag{5.25}$$

Working out the Poisson brackets of the constraints, then we obtain [12]

$$\begin{aligned}
\{\mathbf{H}[N_1], \mathbf{H}[N_2]\} &= \mathbf{D}\left[h^{ij}\left(N_2\partial_j N_1 - N_1\partial_j N_2\right)\right] \\
\left\{\mathbf{D}[N^i{}_1], \mathbf{D}[N^i{}_2]\right\} &= \mathbf{D}\left[N_1^i\partial_i N_2^j - N_2^i\partial_i N_1^j\right] \\
\left\{\mathbf{D}[N^i{}_1], \mathbf{H}[N_2]\right\} &= \mathbf{H}[N_1^i\partial_i N_2].
\end{aligned} \tag{5.26}$$

These equations are equivalent to the Dirac algebra [40] (see also [35]).

Chapter 6

Canonical Formulation of Mimetic Dark Matter

Dark matter does not interact with electromagnetic waves; it neither emits nor absorbs electromagnetic radiation, making it hard to be observed. Its existence was proposed to give the galaxies extra mass, since they are spinning faster than we can explain with known laws of physics and the galaxies observed mass. They cannot be held together with the gravity generated by their observable matter; the extra mass generates extra gravity. This gravitational effect of dark matter on visible matter inferred its existence.
Dark matter outweighs visible matter. The universe contains 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy out of the total mass-energy. So the matter we know accounts only for approximately 5% of the content of the universe; while dark matter and dark energy constitute 95% of the total content of the universe. Out of the total matter in the universe, 84.5% is dark matter.

In this chapter, we review the recently proposed theory of mimetic dark matter [24] and we construct its Hamiltonian formulation [60].

6.1 Mimetic Dark Matter

Not long ago, Chamseddine and Mukhanov [24] modified Einstein's theory of gravity to propose a theory of mimetic dark matter. Their theory is a conformal extension of Einstein's general relativity. They considered a physical metric, $g_{\mu\nu}$, in terms of an auxiliary metric, $\tilde{g}_{\mu\nu}$, and a scalar field ϕ. It is defined by

$$g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu}, \tag{6.1}$$

where the scalar field enter only through first derivative. This physical metric is invariant with respect to the conformal transformation of the auxiliary metric, $\tilde{g}_{\mu\nu} \to \Omega^2\tilde{g}_{\mu\nu}$.

The action is constructed in terms of the physical metric which is considered as a function of the auxiliary metric and the scalar. Varying the action with respect to the auxiliary metric results in a traceless equation of motion, while the variation with respect to the scalar returns a differential equation for the trace part. These equations are given respectively by

$$\begin{aligned}
&\left(G^{\mu\nu} - T^{\mu\nu}\right) - \left(G - T\right) g^{\mu\alpha} g^{\nu\beta}\partial_\alpha\phi\partial_\beta\phi = 0 \\
&\nabla_\kappa \left(\left(G - T\right)\partial^\kappa\phi\right) = 0.
\end{aligned} \tag{6.2}$$

where $G^{\mu\nu}$ and $T^{\mu\nu}$ are Einstein's tensor and the energy momentum tensor respectively. The trace of the former equation is

$$(G-T)\left(1-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)=0 \tag{6.3}$$

which is satisfied even for $G-T\neq 0$ because the scalar field satisfies the constraint equation

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi=1. \tag{6.4}$$

It was shown that even in the absence of matter, $T_{\mu\nu}=0$, the conformal degree of freedom can mimic Dark matter. This dust fluid has a stress tensor given by

$$\tilde{T}^{\mu\nu}=(G-T)\,g^{\mu\alpha}g^{\nu\beta}\partial_\alpha\phi\partial_\beta\phi. \tag{6.5}$$

Therefore, it is a pressureless fluid with energy density $G-T$, and a four-velocity $u^\mu=g^{\mu\alpha}\partial_\alpha\phi$ satisfying the normalization condition $u^\mu u_\mu=1$.

In [50], an equivalent formulation of this model is given without introducing the auxiliary metric. Their action is given by

$$S=-\int\left(R(g)+\lambda\left(1-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)\right)\sqrt{-g}d^4x. \tag{6.6}$$

Then the ghost free models of this theory were discussed in [10]. It was found that the theory is free of ghost instabilities for a positive energy density. Recently, an arbitrary potential was added to the action to form an extension of the mimetic dark matter theory [3]. Cosmological solutions were studied and it was shown how choosing an appropriate potential can lead to various cosmological solutions.

In short, this model is predicting dark matter without introducing matter. It will be experimentally ruled out once a dark matter particle is found. One could add the proposed Lagrangian to a galaxy and then compare with the existed data to check the validity of the theory.

6.2 Hamiltonian Formulation of the Extended Theory

The action of the extended mimetic dark matter theory, where an arbitrary potential, $V(\phi)$, is added, can be written as the following

$$S = -\int d^4x(-g)^{1/2}\left(\frac{1}{2}R + \frac{1}{2}\lambda\left(1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + V(\phi) + L_m\right) \tag{6.7}$$

where L_m is the matter Lagrangian. Variation with respect to λ gives the constraint equation

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1. \tag{6.8}$$

The first step in constructing the canonical formalism is to write the action in a $3+1$ dimensional form by splitting spacetime into space and time. The part of the action depending on the scalar field is given by

$$S = -\int d^4x N h^{1/2}\left(\frac{1}{2}\lambda\left(1 - g^{00}\partial_0\phi\partial_0\phi - 2g^{0i}\partial_0\phi\partial_i\phi + h^{ij}\partial_i\phi\partial_j\phi - \frac{N^iN^j}{N^2}\partial_i\phi\partial_j\phi\right)\right)$$

$$- N\sqrt{h}V(\phi) \tag{6.9}$$

where g^{00}, g^{0i} and g^{ij} are defined with the signature $(+,-,-,-)$.

Varying the Lagrangian with respect to the time derivative of ϕ, we get the momentum conjugate to ϕ which is found to be

$$\begin{aligned} p = \frac{\partial L}{\partial \dot{\phi}} &= N\sqrt{h}\lambda \left(g^{00}\partial_0\phi + g^{0i}\partial_i\phi\right) \\ \Rightarrow \dot{\phi} &= \frac{Np}{\sqrt{h}\lambda} + N^i\partial_i\phi \end{aligned} \tag{6.10}$$

while the variation with respect to $\dot{\lambda}$ will result in a primary constraint

$$p_\lambda = \frac{\partial L}{\partial \lambda} = 0. \tag{6.11}$$

Demanding that this primary constraint is preserved under the time evolution gives a secondary constraint

$$\dot{p}_\lambda = -\{p_\lambda, H\} = -\frac{\partial H}{\partial \lambda} = 0 \tag{6.12}$$

Writing the Lagrangian as

$$L = -\frac{1}{2}Nh^{1/2}\lambda\left(1 + g^{00}\dot{\phi}^2 + h^{ij}\partial_i\phi\partial_j\phi - \frac{N^iN^j}{N^2}\partial_i\phi\partial_j\phi\right) + p\dot{\phi} - N\sqrt{h}V(\phi) \tag{6.13}$$

then the Hamiltonian is given by

$$H = p\dot{\phi} - L$$

$$= \frac{1}{2} N h^{1/2} \lambda \left(1 + g^{00} \dot{\phi}^2 + h^{ij} \partial_i \phi \partial_j \phi - \frac{N^i N^j}{N^2} \partial_i \phi \partial_j \phi \right) + N \sqrt{h} V(\phi) \quad (6.14)$$

Using equation (6.10), we plug for $\dot{\phi}$ in terms of p, then we get

$$H = \frac{N p^2}{2\sqrt{h}\lambda} + \frac{1}{2} N \sqrt{h} \lambda \left[1 + h^{ij} \partial_i \phi \partial_j \phi \right] + p N^i \partial_i \phi + N \sqrt{h} V(\phi). \quad (6.15)$$

This Hamiltonian is still a function of the lagrange multiplier λ. To find it, we solve the secondary constraint $\frac{\partial H}{\partial \lambda} = 0$. This gives

$$\lambda = \frac{p}{\sqrt{h}\sqrt{h^{ij} \partial_i \phi \partial_j \phi + 1}} \quad (6.16)$$

Plugging back we get the total action

$$\int d^4x \, \left(L_{ADM} + p\dot{\phi} - N p \sqrt{h^{ij} \partial_i \phi \partial_j \phi + 1} - N^i p \partial_i \phi - N \sqrt{h} V(\phi) \right) \quad (6.17)$$

where L_{ADM} is given in section 5.3. The total action is then by

$$S_t = \int d^4x \, \left(\dot{h^{ij}} \pi_{ij} + p\dot{\phi} - N \left(p \sqrt{h^{ij} \partial_i \phi \partial_j \phi + 1} + R^0 \right) - N^i \left(p \partial_i \phi + R_i \right) - N \sqrt{h} V(\phi) \right) \quad (6.18)$$

6.3 Equations of Motion

The field variables of the total action are (h^{ij}, π_{ij}) and (p, ϕ). The lapse and shift functions, N and N^i, still appear linearly and variation of the Lagrangian with respect to both will return constraint equations. The equations of motion are found to be

- Varying with respect to π_{ij}

$$\dot{h}^{ij} = 2Nh^{-1/2}\left(\pi^{ij} - \frac{1}{2}h^{ij}\pi\right) + N^{i|j} + N^{j|i} \tag{6.19}$$

  This equation is that of Einstein's theory of gravity. It is independent of ϕ since the part of the action depending on the scalar field is independent of π_{ij}.

- With respect to h^{ij}

$$\begin{aligned}\dot{\pi}_{ij} = &-N\sqrt{h}\left({}^3R_{ij} - \frac{1}{2}h_{ij}\,{}^3R\right) + \frac{1}{2}Nh^{-1/2}h_{ij}\left(\pi^{mn}\pi_{mn} - \frac{1}{2}\pi^2\right) \\ &- 2Nh^{-1/2}\left(\pi_{im}\pi_j^m - \frac{1}{2}\pi\pi_{ij}\right) + \sqrt{h}\left(N_{|ij} - h_{ij}N^{|m}{}_{|m}\right) + \left(\pi_{ij}N^m\right)_{|m} \\ &- N_i^{|m}\pi_{mj} - N_j^{|m}\pi_{mi} + \frac{Np\partial_i\phi\partial_j\phi}{2\sqrt{h^{kl}\partial_k\phi\partial_l\phi + 1}} - \frac{1}{2}N\sqrt{h}V(\phi)h_{ij}\end{aligned} \tag{6.20}$$

  This equation includes two more terms than those of general relativity, coming from the part of the action depending on the scalar field ϕ.

- With respect to N

$$R^0 + p\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1} + \sqrt{h}V(\phi) = H_{grav} + H_\phi = 0 \tag{6.21}$$

- With respect to N^i

$$R_i + p\partial_i\phi = H_{i\,grav} + H_{i\,\phi} = 0 \tag{6.22}$$

The above equations, (6.21) and (6.22), are the four modified constraint equations. They now include also the fields (ϕ, p).

- Two more equations are left. The first is obtained by varying with respect to p

$$\dot{\phi} - N\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1} - N^i\partial_i\phi = 0 \tag{6.23}$$

- The second is the variation with respect to ϕ

$$\dot{p} - \partial_k\left(\frac{Np\partial^k\phi}{\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1}} + N^k p\right) + N\sqrt{h}\,\frac{dV(\phi)}{d\phi} = 0 \tag{6.24}$$

Therefore, we have two more equations than those of Einstein's theory of gravity, coming from the variation of ϕ and p. However, if we write the constraint equation $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1$ in a $3+1$ dimensional form,

$$g^{00}\dot{\phi}^2 + g^{0i}\dot{\phi}\partial_i\phi + h^{ij}\partial_i\phi\partial_j\phi - \frac{N^iN^j}{N^2}\partial_i\phi\partial_j\phi = 1, \tag{6.25}$$

then we get a quadratic equation for $\dot{\phi}$. Its solution is exactly equation (6.23). This means that the equation for the scalar field ϕ gives us no new info.

Considering the other equation, eq. (6.24), we are going to show that it is exactly the Bianchi identity. To prove this we will first start by finding the energy-momentum tensor.

Consider the action part depending on the scalar field ϕ, it is given by

$$S_\phi = \int d^4x \ \left(p\dot{\phi} - Np\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1} - N^i p\partial_i\phi - N\sqrt{h}V(\phi)\right). \tag{6.26}$$

The canonical components of the stress energy tensor, found by varying the action, are given by [12]

$$\begin{aligned}
T_{00} &= -\frac{N}{\sqrt{h}}\left(N\frac{\delta S_\phi}{\delta N} + 2N^i\frac{\delta S_\phi}{\delta N^i} + 2\frac{NiNj}{N^2}\frac{\delta S_\phi}{\delta h^{ij}}\right) \\
T_{0i} &= -\frac{N}{\sqrt{h}}\left(\frac{\delta S_\phi}{\delta N^i} + 2\frac{N^j}{N^2}\frac{\delta S_\phi}{\delta h^{ij}}\right) \\
T_{ij} &= -\frac{2}{N\sqrt{h}}\left(\frac{\delta S_\phi}{\delta h^{ij}}\right).
\end{aligned} \tag{6.27}$$

Computing the variations, we have

$$\begin{aligned}
\frac{\delta S_\phi}{\delta N} &= -p\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1} - \sqrt{h}V(\phi) \\
\frac{\delta S_\phi}{\delta N^i} &= -p\partial_i\phi \\
\frac{\delta S_\phi}{\delta h^{ij}} &= -\frac{Np}{2}\frac{\partial_i\phi\partial_j\phi}{\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1}} + \frac{N\sqrt{h}}{2}\ h_{ij}V(\phi)
\end{aligned} \tag{6.28}$$

Plugging back, we get the components of $T_{\mu\nu}$. These are given by

$$\begin{aligned}
T_{00} &= \frac{N^2 p}{\sqrt{h}}\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1} + \frac{2N}{\sqrt{h}}N^i p\partial_i\phi + \frac{N^iN^j}{\sqrt{h}}\frac{p\partial_i\phi\partial_j\phi}{\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1}} \\
&\quad + N^2V(\phi) - N^iN_iV(\phi) \\
T_{0i} &= \frac{N}{\sqrt{h}}p\partial_i\phi + \frac{N^j}{\sqrt{h}}\frac{p\partial_i\phi\partial_j\phi}{\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1}} - N_iV(\phi)
\end{aligned}$$

$$T_{ij} = \frac{p\partial_i\phi\partial_j\phi}{\sqrt{h}\sqrt{h^{ij}\partial_i\phi\partial_j\phi + 1}} - h_{ij}V(\phi). \tag{6.29}$$

This in complete agreement with the energy-momentum tensor found in the Lagrangian formalism, $T_{\mu\nu} = \lambda\partial_\mu\phi\partial_\nu\phi$, upon using equation (6.23).

Let us start from the definition of the covariant derivative

$$\nabla_\mu T^\mu_\nu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}T^\mu_\nu\right) - \Gamma^\mu_{\nu\rho}T^\rho_\mu. \tag{6.30}$$

The T^μ_ν components are obtained by raising the index appropriately. Expressing then ${}^{(4)}\Gamma$ in term ${}^{(3)}\Gamma$ and simplifying, it is easy to show that equation (6.24) is just the identity $\nabla_\mu T^\mu_i = \nabla_0 T^0_i + \nabla_j T^j_i = 0$.

This shows that the mimetic gravity has the same number of equations as that of Einstein's gravity since the two added equations are the constraint equation and the conservation of the energy-momentum tensor. However, the equations resulting from the variation with respect to h^{ij}, N and N^i are those of pure Einstein's gravity plus extra terms as a function of the scalar field ϕ. This is how the mimetic dark matter enters the picture.

6.4 Poisson Brackets

As shown in the previous section, the constraints can be decomposed into the gravitational part plus the scalar field part. These are written as

$$H = H_{grav} + H_{\phi}; \quad H_i = H_{i\,grav} + H_{i\,\phi}. \tag{6.31}$$

This is a $'$non-derivative$'$ coupling where the H_{ϕ} is independent of the gravitational momentum π_{ij} [74]. Working with the smeared functions introduced before (in section 5.5), then the Poisson bracket of two Hamiltonian constraints splits into

$$\begin{aligned}
\{\mathbf{H}[N_1], \mathbf{H}[N_2]\} &= \{\mathbf{H}_{grav}[N_1] + \mathbf{H}_{\phi}[N_1], \mathbf{H}_{grav}[N_2] + \mathbf{H}_{\phi}[N_2]\} \\
&= \{\mathbf{H}_{grav}[N_1], \mathbf{H}_{grav}[N_2]\} + \{\mathbf{H}_{grav}[N_1], \mathbf{H}_{\phi}[N_2]\} \\
&+ \{\mathbf{H}_{\phi}[N_1], \mathbf{H}_{grav}[N_2]\} + \{\mathbf{H}_{\phi}[N_1], \mathbf{H}_{\phi}[N_2]\}
\end{aligned} \tag{6.32}$$

This can be simplified because the gravitational Hamiltonian constraint is independent of the spatial derivatives of momenta, then the two Poisson brackets $\{H_{grav}, H_{\phi}\}$ and $\{H_{\phi}, H_{grav}\}$ cancel out, and we get

$$\{\mathbf{H}[N_1], \mathbf{H}[N_2]\} = \{\mathbf{H}_{grav}[N_1], \mathbf{H}_{grav}[N_2]\} + \{\mathbf{H}_{\phi}[N_1], \mathbf{H}_{\phi}[N_2]\} \tag{6.33}$$

To compute this, we first find the functional derivatives and then plug in equation

(5.8). The functional derivatives of the scalar Hamiltonian constraint are

$$\frac{\delta \mathbf{H}_\phi[N]}{\delta\varphi(x)} = -\partial_i\left(\frac{pNh^{ij}\partial_j\phi}{\sqrt{h^{kl}\partial_k\phi\partial_l\phi+1}}\right) + N\sqrt{h}\frac{dV}{d\phi};$$
$$\frac{\delta \mathbf{H}_\phi[N]}{\delta p(x)} = N\sqrt{h^{kl}\partial_k\phi\partial_l\phi+1} \tag{6.34}$$

This gives upon plugging in the integral

$$\{\mathbf{H}_\phi[N_1], \mathbf{H}_\phi[N_2]\} = \mathbf{D}_\phi\left[N_3^i\right] \tag{6.35}$$

where

$$N_3^i = h^{ij}\left(N_2\partial_j N_1 - N_1\partial_j N_2\right). \tag{6.36}$$

Similarly the Poisson bracket of two diffeomorphism constraints is computed by first finding their functional derivatives

$$\frac{\delta \mathbf{D}_\phi}{\delta\phi} = -\partial_i(pN^i)$$
$$\frac{\delta \mathbf{D}_\phi}{\delta p} = N^i\partial_i\phi \tag{6.37}$$

and then upon integration

$$\left\{\mathbf{D}_\phi[N^i{}_1], \mathbf{D}_\phi[N^i{}_2]\right\} = \mathbf{D}_\phi\left[N_1^i\partial_i N_2^j - N_2^i\partial_i N_1^j\right]. \tag{6.38}$$

In computing the third Poisson bracket, $\{\mathbf{D}[N^i], \mathbf{H}[N]\}$, the cross term $\{\mathbf{D}_{grav}[N^i], \mathbf{H}_\phi[N]\}$

survives because the scalar Hamiltonian constraint H_ϕ is a function of the metric h^{ij}. Therefore, we have

$$\begin{aligned}\left\{\mathbf{D}[N^i],\mathbf{H}[N]\right\} &= \left\{\mathbf{D}_{grav}[N^i],\mathbf{H}_{grav}[N]\right\} + \left\{\mathbf{D}_\phi[N^i],\mathbf{H}_\phi[N]\right\} \\ &\quad + \left\{\mathbf{D}_{grav}[N^i],\mathbf{H}_\phi[N]\right\}. \end{aligned} \tag{6.39}$$

The Poisson bracket for the scalar constraints is found by using equations (6.34) and (6.37). Upon integrating by parts we get

$$\left\{\mathbf{D}_\phi[N^i],\mathbf{H}_\phi[N]\right\} = \mathbf{H}_\phi[N^i\partial_i N] - \frac{N^k{}_{|i}pNh^{ij}\partial_j\phi\partial_k\phi}{\sqrt{h^{rs}\partial_r\phi\partial_s\phi+1}}. \tag{6.40}$$

To find $\{\mathbf{D}_{grav}[N^i],\mathbf{H}_\phi[N]\}$, we use equation (5.18). Then we have

$$\frac{\delta D_{grav}}{\delta\pi_{ij}} = 2N^i{}_{|k}\ h^{kj} \tag{6.41}$$

which gives upon integrating by parts

$$\left\{\mathbf{D}_{grav}[N^i],\mathbf{H}_\phi[N]\right\} = \frac{N^k{}_{|i}pNh^{ij}\partial_j\phi\partial_k\phi}{\sqrt{h^{rs}\partial_r\phi\partial_s\phi+1}}. \tag{6.42}$$

Then the combined result is the one expected,

$$\left\{\mathbf{D}[N^i],\mathbf{H}[N]\right\} = \mathbf{H}\left[N^i\partial_i N\right]. \tag{6.43}$$

Therefore the full constraint algebra is equivalent to that of general relativity (equation 5.26).

6.5 Canonical Formulation of Dust

In this section, we compare the Hamiltonian formulation of mimetic dark matter to that of dust which was done by Brown and Kuchar [14]. They wrote the four-velocity u_α of the dust as $u_\alpha = -\partial_\alpha T + W_i \partial_\alpha Z^i$ where Z_i are the comoving coordinates of the dust particles, T is the proper time and W_i are the 3-velocity components. The canonical action is then given by

$$S^D = \int d^4x \left(P\dot{T} + P_i \dot{Z}^i - NH^D - N^i H_i^D \right) \tag{6.44}$$

where Z_i and T are the canonical coordinates,

$$H^D = P\sqrt{h^{ij}u_i u_j - 1} \quad and \quad H_i^D = -Pu_i = P\partial_i T - PW_k \partial_i Z^k \tag{6.45}$$

This agrees with our results when $Z_i = 0$. The scalar field ϕ is then the proper time T. This agrees with the Lagrangian analysis when it was shown that in the synchronous gauge $\phi = T$.

Chapter 7

Conclusions and Future Work

In this work, two problems were considered, supersymmetrizing massive gravity and constructing the canonical formulation of mimetic dark matter.
Starting from four chiral superfields with global Lorentz index instead of the four scalars used in the bosonic case, the basic field was defined. Also, supermultiplets were formed and the matter action was constructed. When this was coupled to the Supergravity action using the rules of tensor calculus, global symmetry was promoted to a local one. Applying the conditions forced on the total action, it was found out that the matter action is formed of three D-type terms and two F-type terms.
Once the scalar fields of the chiral multiplets acquire a vacuum expectation value, then local supersymmetry is broken exactly at the same scale as the diffeomorphism breaking. This breaking of coordinate invariance identified the four scalars with four-vectors and the spin-1/2 with a spin-3/2 Rarita Schwinger field. Therefore, we are left with a massive vector, two massive spin-3/2 fields with different masses, and a massive spin-2 particle. The two gravitinos does not have the same mass since there is no N=2 supersymmetry to start with. Before diffeomorphism breaking we

had spin-1/2 and not spin 3/2; then, the two gravitinos would not have the same mass since one is a genuine gravitino, while the other becomes identified with a gravitino after the breaking.

Equations of motion were found and analyzed and then degrees of freedom were counted. Before coupling to supergravity, we started with eight bosonic and eight fermionic degrees of freedom. This is preserved after the coupling where we are left with the same degrees of freedom.

Further work can be done by generalizing the work in [16] to the supersymmetric case using the quadratic formulation in [22]; however, this is much more complicated since it needs a superspace formulation of supergravity [78]. Also, in this work, the structure at the non-linear level, a much more difficult task, was not touched. Analysis at higher orders may reveal ghosts. But then the Vainshtein scale could be found and we expect things to be similar to massive gravity. This means that when we go to higher orders, we expect the non-linear ghost to be found in the strong coupling regime where it is completely harmless.

Next, mimetic dark matter was studied in its canonical form. The Hamiltonian was first constructed and then the equations of motion were analyzed. It was found that the number of equations is equal to that in general relativity plus two more equations coming from the variation with respect to the two extra phase variables. However, the equation for the scalar field was reinterpreted as the conservation of the energy-momentum tensor, and the other is just the constraint equation, $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1$. Therefore, we have an extra field, ϕ, modifying Einstein's equations while keeping the same number of equations. This is how dark matter is represented.

Poisson brackets are computed and closure is proved. Also, comparison with the Hamiltonian formulation of dust is done. This shows the equivalence between the two models and proves that the modification done to Einstein's action is representing dark matter. Further work could be done on applications of the Hamiltonian quantization such as to solve the Wheeler-DeWitt equations.

Appendix A

Notation and Convention

Metric: $g_{\mu\nu} = diag\left\{1, -1, -1, -1\right\}$

Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Dirac spinor:

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix};$$

Adjoint Dirac spinor:

$$\bar{\Psi} \equiv \Psi^* \gamma_0 = \begin{pmatrix} \chi^\alpha \ \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

Majorana spinor:

$$\Psi_M = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}_{\dot{\alpha}} \end{pmatrix}; \qquad \bar{\Psi}_M = \begin{pmatrix} \psi^\alpha \ \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

Grassman spinor:

$$\theta^\alpha = \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}; \ \bar{\theta}^{\dot{\alpha}} = \begin{pmatrix} \bar{\theta}^1 \\ \bar{\theta}^2 \end{pmatrix}$$

$\bar{\psi}_{\dot{\alpha}} \equiv \left(\psi_\alpha\right)^*$ and $\chi^\alpha \equiv \left(\bar{\chi}^{\dot{\alpha}}\right)^*$

Antisymmetric ϵ-matrices:

$$\epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \; \epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

As a convention, repeated spinor indices contracted like ${}^{\alpha}{}_{\alpha}$ $\quad or \quad$ ${}_{\dot{\alpha}}{}^{\dot{\alpha}}$

$$\gamma^{\mu} \equiv \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}; \quad \gamma^{5} \equiv i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.1}$$

There are also the following useful identities:

$$\left(\sigma^{\mu}\right)_{\alpha\dot{\beta}} = \epsilon_{\dot{\beta}\dot{\alpha}}\epsilon_{\alpha\beta}\left(\bar{\sigma}^{\mu}\right)^{\dot{\alpha}\beta}; \;\; \left(\bar{\sigma}^{\mu}\right)^{\dot{\alpha}\beta} = \epsilon^{\beta\alpha}\epsilon^{\dot{\alpha}\dot{\beta}}\left(\sigma^{\mu}\right)_{\alpha\dot{\beta}}$$

$$\left(\sigma^{\mu}\right)_{\alpha\dot{\alpha}}\left(\bar{\sigma}^{\nu}\right)^{\dot{\alpha}\alpha} = Tr\left(\sigma^{\mu}\bar{\sigma}^{\nu}\right) = 2\eta^{\mu\nu}$$

$$\left(\sigma^{\mu}\right)_{\alpha\dot{\alpha}}\left(\sigma_{\mu}\right)_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}$$

$$\left(\sigma^{\mu}\right)_{\alpha\dot{\alpha}}\left(\bar{\sigma}_{\mu}\right)^{\beta\dot{\beta}} = 2\delta_{\alpha}^{\beta}\delta_{\dot{\alpha}}^{\dot{\beta}}$$

$$\left(\sigma^{\mu\nu}\right)_{\alpha}{}^{\beta} \equiv \frac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right)_{\alpha}{}^{\beta}; \quad \left(\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}} \equiv \frac{i}{4}\left(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}$$

$$\sigma^{\mu\nu} = \frac{1}{2i}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}; \quad \bar{\sigma}^{\mu\nu} = \frac{-1}{2i}\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma}$$

$$\left(\sigma^{\mu}\bar{\sigma}^{\nu} + \sigma^{\nu}\bar{\sigma}^{\mu}\right)_{\alpha}{}^{\beta} = 2\eta^{\mu\nu}\delta_{\alpha}^{\beta}; \quad \left(\bar{\sigma}^{\mu}\sigma^{\nu} + \bar{\sigma}^{\nu}\sigma^{\mu}\right)^{\dot{\alpha}}{}_{\dot{\beta}} = 2\eta^{\mu\nu}\delta_{\dot{\beta}}^{\dot{\alpha}}$$

$$\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho} + \sigma^{\rho}\bar{\sigma}^{\nu}\sigma^{\mu} = 2\left(\eta^{\mu\nu}\sigma^{\rho} + \eta^{\nu\rho}\sigma^{\mu} - \eta^{\mu\rho}\sigma^{\nu}\right)$$

$$\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\rho} + \bar{\sigma}^{\rho}\sigma^{\nu}\bar{\sigma}^{\mu} = 2\left(\eta^{\mu\nu}\bar{\sigma}^{\rho} + \eta^{\nu\rho}\bar{\sigma}^{\mu} - \eta^{\mu\rho}\bar{\sigma}^{\nu}\right)$$

$$\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho} - \sigma^{\rho}\bar{\sigma}^{\nu}\sigma^{\mu} = -2i\epsilon^{\mu\nu\rho\kappa}\sigma_{\kappa}$$

$$\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\rho} - \bar{\sigma}^{\rho}\sigma^{\nu}\bar{\sigma}^{\mu} = 2i\epsilon^{\mu\nu\rho\kappa}\bar{\sigma}_{\kappa}$$

$$Tr\left(\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho}\bar{\sigma}^{\kappa}\right) = 2\left(\eta^{\mu\nu}\eta^{\rho\kappa} + \eta^{\mu\kappa}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\kappa} - i\epsilon^{\mu\nu\rho\kappa}\right) \tag{A.2}$$

where $\epsilon_{0123} = +1$.

$$\theta^\alpha \theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}(\theta\theta); \quad \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\left(\bar{\theta}\bar{\theta}\right);$$
$$\theta_\alpha \theta_\beta = \frac{1}{2}\epsilon_{\alpha\beta}(\theta\theta); \quad \bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\left(\bar{\theta}\bar{\theta}\right)$$

The derivatives with respect to a Grassmann variable are defined as follows:

$$\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha}; \;\; \partial^\alpha \equiv -\epsilon^{\alpha\beta}\partial_\beta; \;\; \bar{\partial}_{\dot{\alpha}} \equiv \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}; \;\; \bar{\partial}^{\dot{\alpha}} \equiv -\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\beta}}.$$

This implies that

$$\partial_\alpha \theta^2 = 2\theta_\alpha; \;\; \partial^\alpha \theta^2 = -2\theta^\alpha;$$

$$\bar{\partial}_{\dot{\alpha}}\bar{\theta}^2 = -2\bar{\theta}_{\dot{\alpha}}; \;\; \bar{\partial}^{\dot{\alpha}}\bar{\theta}^2 = 2\bar{\theta}^{\dot{\alpha}}.$$

Appendix B

D-type terms

In this appendix, we list the products that can be formed as D-type terms.

1. $H_{ABC}H_{ABC}$ where $(H_{ABC}H_{ABC})^* = H_{CBA}H_{CBA} = H_{ABC}H_{ABC}$ and is self adjoint.

2. $H_{ABC}H_{ACB}$ where $(H_{ABC}H_{ACB})^* = H_{CBA}H_{BCA} = H_{ABC}H_{BAC}$

3. $H_{ABC}H_{BCA}$ where $(H_{ABC}H_{BCA})^* = H_{CBA}H_{ACB} = H_{ABC}H_{BCA}$ and is self adjoint.

4. $H_{ABC}H_{CAB}$ where $(H_{ABC}H_{CAB})^* = H_{CBA}H_{BAC} = H_{ABC}H_{CAB}$ is self adjoint

5. $H_{ABC}H_{CBA}$ where $(H_{ABC}H_{CBA})^* = H_{CBA}H_{ABC}$ and is self adjoint

6. $H_A H_A^*$ is self adjoint

7. $H_A H_A$ where $(H_A H_A)^* = H_A^* H_A^*$

8. $\epsilon^{ABCD}H_{ABC}H_D$ where $\left(\epsilon^{ABCD}H_{ABC}H_D\right)^* = -\epsilon^{ABCD}H_{ABC}H_D^*$

9. $\epsilon^{ABCD}H_{ABE}H_{CDE}$ where $\left(\epsilon^{ABCD}H_{ABE}H_{CDE}\right)^* = \epsilon^{ABCD}H_{EBA}H_{EDC}$
$= \epsilon^{ABCD}H_{EAB}H_{ECD}$

10. $\epsilon^{ABCD}H_{AEB}H_{CED}$ where $\left(\epsilon^{ABCD}H_{AEB}H_{CED}\right)^* = \epsilon^{ABCD}H_{AEB}H_{CED}$ is self adjoint

11. $\epsilon^{ABCD}H_{AEB}H_{ECD}$ where $\left(\epsilon^{ABCD}H_{AEB}H_{ECD}\right)^* = \epsilon^{ABCD}H_{AEB}H_{CDE}$

12. $\epsilon^{ABCD}H_{EAB}H_{CDE}$ where $\left(\epsilon^{ABCD}H_{EAB}H_{CDE}\right)^* = \epsilon^{ABCD}H_{BAE}H_{EDC}$
$= \epsilon^{ABCD}H_{EAB}H_{CDE}$ is self adjoint.

Appendix C

Components of the Superfields

The components of the basic field H_{ABC} to all orders in θ and $\bar{\theta}$ are given by

- No $\theta, \bar{\theta}$ terms:

$$2\psi_A \sigma_B \bar{\psi}_C$$

- θ terms:

$$2\sqrt{2}i\ \psi_A \sigma_B \bar{\sigma}_C \theta + 2\sqrt{2}i\ \psi_A \sigma_B \bar{\sigma}^\mu \theta\ \partial_\mu \chi_C^* + 2\sqrt{2}\theta \sigma_B \bar{\psi}_C\ F_A$$

- $\bar{\theta}$ terms:

$$-2\sqrt{2}i\ \bar{\theta} \bar{\sigma}_A \sigma_B \bar{\psi}_C - 2\sqrt{2}i\ \bar{\theta} \bar{\sigma}^\mu \sigma_B \bar{\psi}_C\ \partial_\mu \chi_A + 2\sqrt{2}\psi_A \sigma_B \bar{\theta}\ F_C^\dagger$$

- $\theta\theta$ terms:

$$4i\ \eta_{BC}F_A\theta\theta + 4i\ F_A\ \partial_B\chi_C^*\theta\theta$$

- $\bar{\theta}\bar{\theta}$ terms:

$$-4i\ \eta_{BA}F_C^\dagger\bar{\theta}\bar{\theta} - 4i\ F_C^\dagger\ \partial_B\chi_A\bar{\theta}\bar{\theta}$$

- $\theta\bar{\theta}$ terms:

$$\begin{aligned}
&-2i\ \left(\theta\sigma_B\bar{\psi}_C\right)\left(\partial_\mu\psi_A\sigma^\mu\bar{\theta}\right) - 2i\ \left(\theta\partial_\mu\psi_A\right)\left(\bar{\theta}\bar{\sigma}^\mu\sigma_B\bar{\psi}_C\right) + 2i\ \left(\psi_A\sigma_B\bar{\theta}\right)\left(\theta\sigma^\mu\partial_\mu\bar{\psi}_C\right)\\
&+2i\ \left(\psi_A\sigma_B\bar{\sigma}^\mu\theta\right)\left(\bar{\theta}\partial_\mu\bar{\psi}_C\right) + 4\ \bar{\theta}\bar{\sigma}_A\sigma_B\bar{\sigma}_C\theta + 4\ \bar{\theta}\bar{\sigma}_A\sigma_B\bar{\sigma}^\mu\theta\ \partial_\mu\chi_C^*\\
&+4\ \bar{\theta}\bar{\sigma}^\mu\sigma_B\bar{\sigma}_C\theta\ \partial_\mu\chi_A + 4\ \bar{\theta}\bar{\sigma}^\mu\sigma_B\bar{\sigma}^\nu\theta\ \partial_\mu\chi_A\partial_\nu\chi_C^* + 4\theta\sigma_B\bar{\theta}\ F_AF_C^\dagger
\end{aligned}$$

- $\theta\theta\bar{\theta}$ terms:

$$\begin{aligned}
&-\sqrt{2}i\ \left(\bar{\theta}\bar{\sigma}^\mu\sigma_B\bar{\psi}_C\right)\theta\theta\partial_\mu F_A - \sqrt{2}\ \left(\psi_A\sigma_B\bar{\theta}\right)\theta\theta\partial_\mu\partial^\mu\chi_C^* + \sqrt{2}i\ \left(\partial_\mu\bar{\psi}_C\bar{\sigma}^\mu\sigma_B\bar{\theta}\right)\theta\theta F_A\\
&+2\sqrt{2}i\ \left(\bar{\theta}\partial_B\bar{\psi}_C\right)\theta\theta F_A + 3\sqrt{2}\ \left(\partial_\mu\psi_A\sigma^\mu\bar{\theta}\right)\ \eta_{BC}\theta\theta + 3\sqrt{2}\ \left(\partial_\mu\psi_A\sigma^\mu\bar{\theta}\right)\ \partial_B\chi_C^*\theta\theta\\
&-\sqrt{2}i\ \bar{\theta}\bar{\sigma}^F\partial^\mu\psi_A\ \epsilon_{\mu BCF}\theta\theta - \sqrt{2}i\ \bar{\theta}\bar{\sigma}^F\partial^\mu\psi_A\partial^\nu\chi_C^*\ \epsilon_{\mu B\nu F}\theta\theta - \sqrt{2}\bar{\theta}\bar{\sigma}_C\partial_B\psi_A\theta\theta\\
&+\sqrt{2}\bar{\theta}\bar{\sigma}_B\partial_C\psi_A\theta\theta - \sqrt{2}\ \bar{\theta}\bar{\sigma}_\mu\partial_B\psi_A\ \partial^\mu\chi_C^*\ \theta\theta + \sqrt{2}\ \bar{\theta}\bar{\sigma}_B\partial_\mu\psi_A\ \partial^\mu\chi_C^*\ \theta\theta
\end{aligned}$$

- $\theta\bar{\theta}\bar{\theta}$ terms:

$$
\begin{aligned}
&\sqrt{2}i\ \left(\psi_A\sigma_B\bar{\sigma}^\mu\theta\right)\bar{\theta}\bar{\theta}\partial_\mu F_C^\dagger + 3\sqrt{2}\ \left(\theta\sigma^\mu\partial_\mu\bar{\psi}_C\right)\eta_{BA}\bar{\theta}\bar{\theta} + 3\sqrt{2}\ \left(\theta\sigma^\mu\partial_\mu\bar{\psi}_C\right)\partial_B\chi_A\bar{\theta}\bar{\theta} \\
&- \sqrt{2}i\ \left(\partial^\nu\bar{\psi}_C\bar{\sigma}^F\theta\right)\bar{\theta}\bar{\theta}\epsilon_{AB\nu F} - \sqrt{2}i\ \left(\partial^\nu\bar{\psi}_C\bar{\sigma}^F\theta\right)\bar{\theta}\bar{\theta}\ \partial^\mu\chi_A\epsilon_{\mu B\nu F} - \sqrt{2}\left(\partial_B\bar{\psi}_C\bar{\sigma}_A\theta\right)\bar{\theta}\bar{\theta} \\
&+ \sqrt{2}\left(\partial_A\bar{\psi}_C\bar{\sigma}_B\theta\right)\bar{\theta}\bar{\theta} - \sqrt{2}\left(\partial_B\bar{\psi}_C\bar{\sigma}_\mu\theta\right)\bar{\theta}\bar{\theta}\ \partial^\mu\chi_A + \sqrt{2}\left(\partial_\mu\bar{\psi}_C\bar{\sigma}_B\theta\right)\bar{\theta}\bar{\theta}\ \partial^\mu\chi_A \\
&- \sqrt{2}i\ \left(\theta\sigma_B\bar{\sigma}^\mu\partial_\mu\psi_A\right)\bar{\theta}\bar{\theta}F_C^\dagger - 2\sqrt{2}i\ \left(\theta\partial_B\psi_A\right)\bar{\theta}\bar{\theta}F_C^\dagger - \sqrt{2}\ \left(\theta\sigma_B\bar{\psi}_C\right)\bar{\theta}\bar{\theta}\ \partial_\mu\partial^\mu\chi_A
\end{aligned}
$$

- $\theta\theta\bar{\theta}\bar{\theta}$ terms:

$$
\begin{aligned}
&\frac{1}{2}\psi_A\sigma_B\partial_\mu\partial^\mu\bar{\psi}_C + \frac{1}{2}\partial_\mu\partial^\mu\psi_A\sigma_B\bar{\psi}_C + 2i\ F_A\partial_B F_C^\dagger - 2i\ F_C^\dagger\partial_B F_A \\
&+ 2i\ \eta_{AB}\partial_\mu\partial^\mu\chi_C^* - 2i\ \eta_{BC}\partial_\mu\partial^\mu\chi_A + 2i\ \partial_B\chi_A\partial_\mu\partial^\mu\chi_C^* - 2i\ \partial_B\chi_C^*\partial_\mu\partial^\mu\chi_A \\
&- \frac{3}{2}\partial_B\psi_A\sigma_\mu\partial^\mu\bar{\psi}_C - \frac{3}{2}\partial^\mu\psi_A\sigma_\mu\partial_B\bar{\psi}_C + \frac{1}{2}\partial^\mu\psi_A\sigma_B\partial_\mu\bar{\psi}_C \\
&+ \frac{1}{2}\partial^\mu\bar{\psi}_C\bar{\sigma}_\mu\partial_B\psi_A + \frac{1}{2}\partial_B\bar{\psi}_C\bar{\sigma}_\mu\partial^\mu\psi_A - \frac{1}{2}\partial_\mu\bar{\psi}_C\bar{\sigma}_B\partial^\mu\psi_A
\end{aligned}
$$

C.1 D-type terms

1. For the superfield $\bar{H}_{ABC}\bar{H}^{BCA}$:

   - $C = 0,\ \xi = 0,\ M = -8\ \left(\psi_A\sigma_B\bar{\sigma}^A\psi^B\right)$
   - $V_\mu =$ quadratic, $\lambda =$ quadratic
   - $D = -16\ \left(\partial_\mu\chi_A\partial^\mu\chi^A + \partial_\mu\chi_A^*\partial^\mu\chi^{A*}\right) + 32\ \left(\partial_\mu\chi_A\partial^\mu\chi^{A*} + \partial^A\chi_A\partial^B\chi_B^*\right)$

     $+ 80\ F_AF^{A*} - 8\ \epsilon^{ABCD}\psi_A\sigma_B\partial_C\bar{\psi}_D - 8i\ \left(\psi_A\sigma^A\partial_B\bar{\psi}^B + \psi_A\sigma^B\partial^A\bar{\psi}_B\right)$

     $- 56i\ \psi_A\sigma^\mu\partial_\mu\bar{\psi}^A + 32\ \bar{e}\partial_A\chi^A + 32\ \bar{e}\partial_A\chi^{A*} + 32\ \bar{e}^2$

where V_μ and λ don't affect our results since they will give terms with higher orders.

2. $\bar{H}_{AB}{}^{B}\bar{H}_{C}{}^{CA}$:

- $C = 0,\ \xi = 0,\ M = 16\ \left(\psi_A\sigma^A\bar{\sigma}^B\psi_B\right)$
- $V_\mu =$ quadratic, $\lambda =$ quadratic
- $D = 32\ \left(\partial_\mu\chi_A\partial^\mu\chi^A + \partial_\mu\chi_A^*\partial^\mu\chi^{A*}\right) + 32\ \partial^A\chi_A\partial^B\chi_B^* + 128\ \partial_\mu\chi_A\partial^\mu\chi^{A*}$
  $+ 272\ F_AF^{A*} + 8\ \epsilon^{ABCD}\psi_A\sigma_B\partial_C\bar{\psi}_D - 8i\ \left(\psi_A\sigma^A\partial_B\bar{\psi}^B + \psi_A\sigma^B\partial^A\bar{\psi}_B\right)$
  $- 200i\ \psi_A\sigma^\mu\partial_\mu\bar{\psi}^A + 96\ \bar{e}^2 + 128\ \bar{e}_\mu^a\bar{e}_a^\mu + 96\ \bar{e}\partial_A\chi^A + 96\ \bar{e}\partial_A\chi^{A*}$
  $+ 128\ \bar{e}_A^\mu\partial_\mu\chi^A + 128\ \bar{e}_A^\mu\partial_\mu\chi^{A*}$

and the third

3. $\left(D\Phi_AD\Phi_B\right)\left(\bar{D}\Phi^{B*}\bar{D}\Phi^{A*}\right)$:

- $C = 0,\ \xi = 0,\ M = 0$
- $V_\mu = 0,\ \lambda =$ quadratic
- $D = 32\ \left(\partial^A\chi_A\partial^B\chi_B^* + \partial_\mu\chi_A\partial^\mu\chi^{A*}\right) + 80\ F_AF^{A*} - 8\ \epsilon^{ABCD}\psi_A\sigma_B\partial_C\bar{\psi}_D$
  $- 8i\ \left(\psi_A\sigma^A\partial_B\bar{\psi}^B + \psi_A\sigma^B\partial^A\bar{\psi}_B\right) - 56i\ \psi_A\sigma^\mu\partial_\mu\bar{\psi}^A + 64\ \bar{e}_\mu^a\bar{e}_a^\mu$
  $+ 64\ \bar{e}_A^\mu\partial_\mu\chi^A + 64\ \bar{e}_A^\mu\partial_\mu\chi^{A*}$

C.2 F-type terms

The components of the F-type terms are

1. $\bar{D}^2\left(D\Phi_A\sigma^{AB}D\Phi_B\right)$:

- $z = -96i - 48i\ \partial^A\chi_A - 48i\ \bar{e} - 16i\ \bar{e}\partial^A\chi_A - 32i\ \bar{e}^\mu_A\partial_\mu\chi^A + 8i\ \bar{e}^A_\mu\bar{e}^\mu_A$

  $-8i\ \bar{e}^2$

- $X_\alpha = -4\sqrt{2}\ \left(\sigma^{AB}\psi_A\right)_\alpha\partial_\mu\partial^\mu\chi_B - 16\sqrt{2}i\ \left(\partial^A\psi_A\right)_\alpha\partial^B\chi_B$

  $-\ 48\sqrt{2}i\ \left(\partial^A\psi_A\right)_\alpha + 16\sqrt{2}i\ (\partial^A\psi_B)_\alpha\partial^B\chi_A$

- $h =$ total derivative

h is a total derivative, therefore it won't affect our calculations. The components of the other term are

2. $\bar{D}^2\left(D\Phi_A D\Phi^A\bar{D}\Phi^*_B\bar{D}\Phi^{B*}\right)$:

   - $z = -128\ \bar{\psi}_A\bar{\psi}^A$

   - $X_\alpha = 128\sqrt{2}i\ \left(\sigma^B\bar{\psi}_B\right)_\alpha\partial^A\chi_A + 64\times 4\sqrt{2}i\ \left(\sigma^\nu\bar{\psi}_B\right)_\alpha\partial_\nu\chi^*_B$

   - $h = -64\times 4\ \left(\partial_\mu\chi_A\partial^\mu\chi^A + \partial_\mu\chi^*_A\partial^\mu\chi^{A*} + \partial^A\chi_A\partial^B\chi^*_B\right) - 64\times 2\ \ F_AF^{A*}$

     $-\ 64\ \epsilon^{ABCD}\psi_A\sigma_B\partial_C\bar{\psi}_D + 64i\ \left(\psi_A\sigma^A\partial_B\bar{\psi}^B + \psi_A\sigma^B\partial^A\bar{\psi}_B + \psi_A\sigma^\mu\partial_\mu\bar{\psi}^A\right)$

     $-\ 64\times 16 - 64\times 16\ \bar{e} - 64\times 8\ \bar{e}^A_\mu\bar{e}^\mu_A - 64\times 4\ \bar{e}^2$

     $+\ 64\ \left(\bar{e}\partial_A\chi^A + \bar{e}\partial_A\chi^{A*}\right) + 64\times 4\ \left(\bar{e}^\mu_A\partial_\mu\chi^A + \bar{e}^\mu_A\partial_\mu\chi^{A*}\right).$

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