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MODELING DIAMETER PROTOCOL IN AN LTE ENVIRONMENT: A METHOD

by

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AN ABSTRACT OF THE THESIS OF

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The Diameter protocol, successor to RADIUS, is the de-facto AAA protocol in LTE networks. However, it acquired and is still acquiring more roles and functionalities, because the protocol was designed with the ability to expand. This expansion in the protocol functionalities led in some cases to signaling storms in the EPC. Our study thus focuses on the effect of Diameter in LTE networks.

A mathematical model that estimates Diameter traffic on the S6, Gx, Ro, and Sy interfaces, along with a set of Diameter message pairs exchanged on the interfaces in question, is derived. In addition, the model can be extended in order to study any other Diameter interfaces of interest.

Simulations were performed in MATLAB for two cases: fixed UEs and mobile ones. The mathematical results derived from the model and the simulation results were compared, and showed good agreement, with an error range from 3 to 14%. Furthermore, we conclude from our study that when it comes to UE populations with low average mobility speed, the effects of mobility on signaling are minimal.

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CHAPTER I

INTRODUCTION

Considered a privilege by select individuals in the past, Mobile Communications has evolved to become an everyday commodity to the majority of the world's population. So, since the majority of the world is concerned, achieving connectivity and interoperability between different vendors, network operators, service providers, devices, nations and continents is crucial [1]. Such a goal requires a body of standards that guides and governs the evolution of Mobile Communications. This daunting task was undertaken by Third Generation Partnership Project (3GPP). And after three generations of mobile communication technologies, the first being analog, the second being the first digital mobile systems also known as Global System for Mobile Communications (GSM) and the third and current Universal Mobile Telecommunications System (UMTS), the Mobile Communication World is finally ready to enter its fourth generation: Long Term Evolution (LTE) [2]. The mobile industry forecasts that by the end of 2013, 30% of the world's mobile operators will be committed to deploy LTE and 90% will have their infrastructure upgraded to accommodate LTE technology within by 2018 [11]. LTE comes with many promises:

- Reduced delay for connection establishment.
- Reduced transmission latency for user plane data.
- Higher data rates (download/upload) per cell.
- Better coverage at cell edges.

- Reduced cost per bit for radio transmission.
- Greater flexibility of spectrum usage.
- Seamless mobility even between different radio technologies.
- Reasonable power consumption for mobile terminals.
- More internet quota per monetary unit.
- A higher Quality of Service (QoS) that is tailored to the subscriber's needs.
- An overall higher Quality of Experience (QoE) than the current 3G networks.

This makes the lure of LTE great. However, migrating to LTE is not as easy as it seems as it introduces new challenges that were never faced in the current 3G networks [12], [13]. And such challenges rise on both the data planes that are at play, the User Plane, where user data (e-mails, pictures, voice, video, web surfing,…) propagate and the Control Plane, where different layers and protocol work intertwined to govern the body of LTE, from the Evolved Universal Terrestrial Radio Access Network (E-UTRAN) to the Evolved Packet Core (EPC).

"Diameter" is one such protocol. Its main purpose is to succeed RADIUS as the de-facto Authentication Authorization and Accounting (AAA) protocol. However, during the course of the evolution of LTE (that would be the "E" in LTE), "Diameter" acquired more and more roles and functionalities, and still is, and that is because the protocol was designed with the ability to expand. Those roles and functionalities are too many to list and discuss through the course of a single study, thus the following chapters will focus only on the original role of the protocol in question, its AAA role. And even then, it will focus on a set of message pairs and interface relative to Diameter's AAA duties. The reason of narrowing the focus to only a set on interfaces and messages pairs is because this study aims to describe Diameter's AAA traffic mathematically, i.e. to find a mathematical and logical approach that can be used for any combination of [Interfaces, Messages Pairs].

CHAPTER II

SAE AND DIAMETER PROTOCOL OVERVIEW

2.1 System Architecture Evolution

In order to better understand the challenges that LTE faces, one must first understand the LTE network's infrastructure. The architecture of LTE networks is referred to as: System Architecture Evolution (SAE) [2]. SAE is the evolution of the Global Packet Radio Service (GPRs) core network. The most notable and important difference between SAE and its predecessor is that SAE follows an All-IP structure, making it an All-IP Network (AIPN). Figure 1. The SAE: Showing E-UTRA & EPC divided into its two major parts: The Evolved Universal Terrestrial Radio Access (E-UTRA) and the Evolved Packet Core (EPC). The E-UTRA comprises of the e-NodeBs that provide the Air Interfaces over witch the User Equipment (UE) communicates with the EPC. On the other hand, the EPC is made off multiple elements, most notably the:

- **Mobility Management Entity (MME):** The MME is involved in the bearer activation/deactivation process and is also responsible for choosing the SGW for a UE at the initial attach and at time of intra-LTE handover involving Core Network (CN) node relocation. It is responsible for authenticating the user (by interacting with the HSS).
- **Serving Gateway (S-GW):** The S-GW routes user plane data packets and acts as the mobility anchor for UEs during inter-eNodeB handovers and between LTE and other 3GPP technologies.

- **Packet Data Network Gateway (P-GW/PDN-GW):** The PDN Gateway provides connectivity from the UE to external packet data networks The PGW interacts with the PCRF in order to perform policy enforcement, packet filtering for each user, charging support, lawful interception and packet screening.
- **Home Subscriber Server (HSS):** The HSS is a central database that contains user-related and subscription-related information. The functions of the HSS include, but is not limited to, mobility management, call and session establishment support, user authentication and access authorization.
- **Policy and Charging Rules Function (PCRF):** The PCRF is the software node designated in real-time to determine policy rules in a multimedia network.
- **Online Charging System (OCS):** The OCS is oriented to all subscriber types (prepaid/postpaid/other) and service types, offers unified online charging and online control capabilities and can be used as a unified charging engine for all network services, making it a core basis for convergent billing in the network [1], [2].

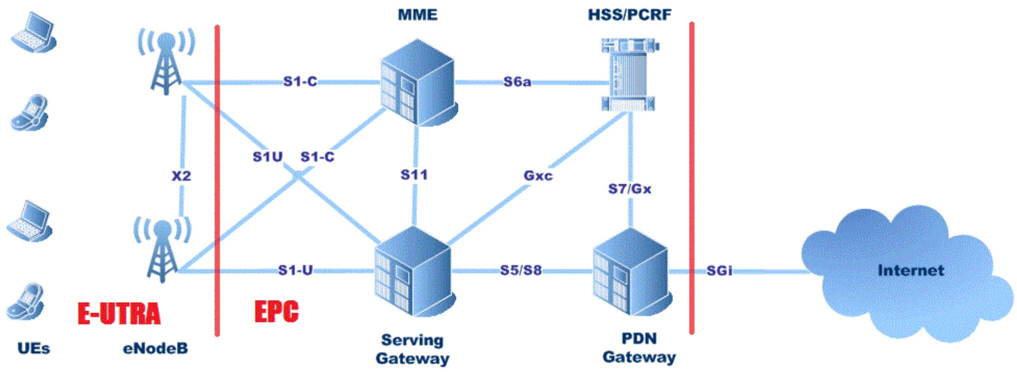


Figure 1. The SAE: Showing E-UTRA & EPC

2.2 Diameter Protocol Overview

As the Mobile Network evolved so did the signaling method. On the Control plane, the EPC is governed by the Diameter Protocol. The Diameter base protocol specification can be found in RFC 6733 [3]. Diameter is Authentication, Authorization and Accounting (AAA) protocol, however the base protocol does not cover all functionalities, so Diameter was crafted with the ability to support additional applications such as Diameter Mobile IPv4 Application [4] which allows a Mobile Node (MN) to change its point of attachment to the Internet while maintaining its fixed home address, or the Diameter Network Access Server Application [5] used for AAA in the Network Access Server (NAS) environment. Another application would be Diameter Credit-Control [6] that can be used to implement real-time credit-control for a variety of end user services such as network access, Session Initiation Protocol (SIP) services, messaging services, and download services. Another important application is the Diameter EAP application [7] that carries EAP packets between a Network Access Server (NAS) working as an EAP Authenticator and a back-end authentication server. The Diameter EAP application is based on the Diameter Network Access Server Application [NASREQ] and is intended for environments similar to NASREQ.

The basic communication unit of Diameter is referred as “Message” [23]. Each message contains a code that refers to a certain action that should be performed by the receiving end. It is important to note that Diameter Messages are synchronous. This means that each command code refers to a pair of messages whose acronyms are in the form of three letters: “XXR” and “XXA”. The last letter refers to “Request (R)” or “Answer (A)” respectively. In addition, the messages contain a set of Attribute Value Pairs (AVPs) which contain the actual data to be exchanged between Diameter nodes.

As stated previously, the Diameter base protocol supports the addition of different applications; this is generally done by adding new commands and/or AVPs to the already exiting command/AVP pool.

Furthermore, Diameter is based on a Peer-to-Peer client/server architecture as seen in **Error! Reference source not found.** [23], [16]. A client node is usually placed at the edge of the network and handles access control. The server node performs the actual authentication or authorization of remote users based on profiles. However, a Diameter node can act as a server for some requests and as a client in other situations. Diameter messages are control message; they carry vital and sensitive information. So, above all, these message need to reach their intended destinations reliably. The Diameter sits above the transport layer in the OSI's seven layer stack as seen in **Error! Reference source not found.**.

Figure 2. Client/Server Architecture

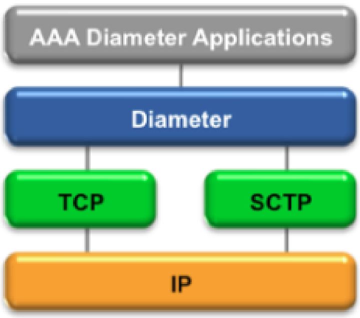


Figure 3. Diameter Protocol Position in Stack

Diameter uses Transport Control Protocol (TCP) or Stream Control Transport Protocol (SCTP) and not User Datagram Protocol (UDP) to send its messages [16], [17], [20], [22]. While TCP is a well-known protocol, a few words should be said about SCTP. The basics of SCTP are introduced in RFC 3268 [14] and its detailed specification can be found in RFC 4960 [15]. First, SCTP is a unicast protocol, and supports data exchange between exactly two endpoints. Second, SCTP provides reliable transmission in full duplex mode. Third, SCTP is rate adaptive similar to TCP. Fourth, and unlike TCP, SCTP is message oriented and supports framing of individual message boundaries whereas TCP is byte oriented [14], [15]. Thus, using TCP or SCTP as transport protocols, a Diameter node can establish multiple connections and open multiple session between entities within the EPC. This can be seen in **Error! Reference source not found.** and **Error! Reference source not found.** [22]. These connections are referred to as interfaces such as the "S6a" interface linking the MME to the HSS or the "Gx" interface between P-GW and the PCRF. Detailed information concerning these interfaces and others not shown in Figure 5 can be found in the 3GPP series of standards numbers 29 and 32 [25] - [34].

Figure 4. Connection Mesh established by the Diameter Protocol

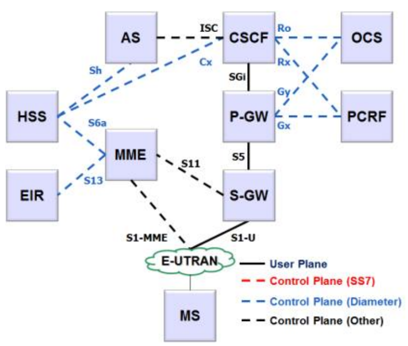


Figure 5. EPC Data Planes and Some Related Interfaces

CHAPTER III

PROBLEM DEFINITION

The problem in question is that Diameter ended up causing signaling storms [19] in the core and bottlenecking many interfaces and links. Tekelec (a provider of network signaling, policy control, and subscriber data management solutions for communications networks [35], which was acquired by Oracle [36]) estimates that by 2016, Diameter signaling would exceed 40 million messages per second on a global scale [37]. One of the causes of such large amount of traffic is that Diameter possesses a large array of control message pairs, sent over a large number of interfaces (S6a, Gx, …) between multiple LTE elements (HSS, MME, …), in order to effectively drive the multi-functionalities of the EPC.

However, the EPC's infrastructure is not the only reason for such trouble. Human life style is rapidly shifting to an "all digital" life style which makes the mobile operators' revenue mostly based on Packet Oriented Technology (Smart Phones, Tablets…) as seen in Figure 6. Subscriber Penetration vs. Operator Net Revenue [21].

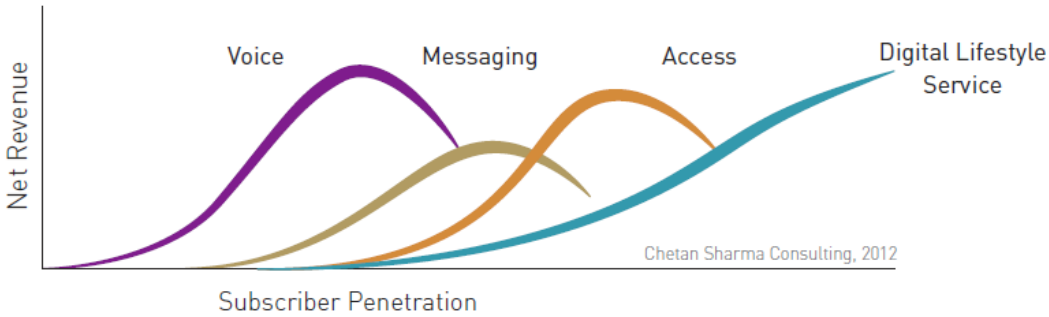


Figure 6. Subscriber Penetration vs. Operator Net Revenue

The increase in subscribers and their possession of multiple Mobile Devices plays an important role in causing signaling storms in the core [8]. The smart devices (phones, tablets …) are exploding. The issue is that smart devices rely heavily Internet services. For example, instant messaging and social networking applications are always-on services that cause regular message exchange between the client and server in order to keep the status of the subscriber up to date. This continuous and sometimes periodic usage cycle of data services increases the load on the core. In addition, policy management at the PCRF is expected to generate a large amount of traffic since mobile operators are aiming for the implementation of different “policy based usage” per subscriber [38]. A simple example can be seen in **Error! Reference source not found.**. Another extension to the implementation of multiple policies is the Personalized Service Plans, shown in **Error! Reference source not found.** [39]. The latter would require metering multiple usage limits per customer, tracking usage across multiple devices… And the more complex a subscriber’s plan is, the more signaling is generated between the charging systems, databases, applications and gateways involved in each data session.

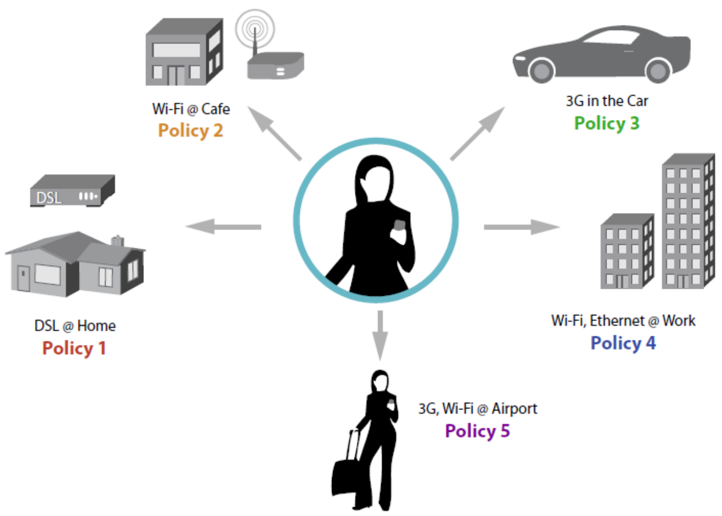


Figure 7. Multiple Policies per Subscriber

Figure 8. A Personalized Service Plan

Hence comes the big question: Is it possible to model Diameter traffic in the core? If so, how can such a model be used in order to help better understand how Diameter traffic flows in the network and figure out which interfaces and links suffer the most risk of congestion?

Modeling a protocol can be predictive at best and raises many questions [9], [10]; what parameters should be accounted for? What limitations should be considered? What reasonable assumptions should be made? Can certain real-world parameter be mapped directly to the model? The following is an example of the list of parameters that can be considered:

- Number of LTE subscribers.
- Percentage of prepaid subscribers.
- Percentage of subscribers with policy enabled.
- Average number of devices per subscriber.
- Number of simultaneous apps per device.
- Frequency of tracking area updates.
- Subscribers' activity (Call/SMS/Internet/…).

CHAPTER IV

LITERATURE REVIEW

There are many techniques that can be used in order to analyze and predict the traffic generated by the operation of a protocol within the domains of a network. The Protocol in question is the Diameter protocol and network in question is the All-IP LTE network. The LTE network mainly concerns Mobile Operators and Mobile subscribers and Wireless users. Having stated that, it is essential no to omit subscriber mobility out of the equation. Thus, this section will discuss some modeling techniques that can be used to:

1. Describe a mobile user’s movements within a cellular network.
2. Describe and estimate the traffic generated by protocols in IP networks.

4.1 Mobility

From the Network’s point of view, the mobility of a user is seen as a transition from one cell to a neighboring one that comes along with an exchange of control messages in the core marking the event. This means that choosing the correct shape of cells in a model is essential [40]. K. B. Baltzis points out that, in reality, cells are complex shaped, and heavily affected by the terrain’s features and man-made structures. Thus, for conceptual and computational simplicity, cells are often approximated to hexagonal or circular shaped objects. The hexagonal approximation is frequently employed in the planning and analysis of wireless networks since the conceptual model would not suffer the complexity of overlapping cells as it would have if using circular shaped cells.

Having determined that the best approximation of a shape of a cellular network's cell is the hexagonal one, the next step would be choosing a mobility model that satisfies the needs of modeling a particular protocol in an LTE environment. The mobility model is important when examining different events that occur to a mobile device that is able to attach and use LTE technology such as handover procedures, location updates, paging… all of which trigger a flow of control or signaling messages within the EPC. M. M. Zonoozi and P. Dassanayake argue in [41] that the mobility modeling should include changes in the direction and speed of the mobile. Moreover, it is unrealistic to assume that the speed is uniformly distributed and remains constant. It is virtually impossible to extend the analysis of the simplified case to cover the general case of mobility, and simulation appears to be the only way out. The authors propose a mobility model that considers all of the possible mobility related parameters such as initial position, direction and speed of the mobile device, changes to these attributes that occur along the path and the final destination of the mobile device. The authors conclude that that the generalized gamma distributions function is a good approximation for the cell residence time distribution in opposition to the general assumption taken by many others that the cell residence time is an exponentially distributed random variable.

The mobility model described previously maps to the random waypoint mobility model, which essentially uses speed, position and direction to estimate a mobile user's movements. A derivation of the random waypoint model is the Random Walk model. The random walk model simplifies the analysis greatly and concentrates on describing a mobile device's transition from one cell to the other or from one Location Area to the other, where, in this case, Location Area stands for a number of cells that are usually governed by the same entities in the EPC. And it is the occurrence of theses transitions that matter since each occurrence would trigger signaling within the core. Thus a simpler model for mobility that is based on Random Walk probability was considered.

I. F. Akyildiz et al. introduced a location tracking mechanism for PCN that combines a movement-based location update scheme and a selective paging scheme [42]. The authors assume that the cell residence time follows the Gamma distribution since this type of distribution does not have a specific shape. And depending on the parameters used, the gamma distribution can be used to model the Exponential, the Erlang and the Chi-Square distributions making the model more versatile. The authors than proceed to formulate and calculate the total cost per call arrival including the update cost, the polling cost, the location update movement threshold, the maximum paging delay, the call arrival rate, as well as the mean and variance of the cell residence time.

From their work in [42], I. F. Akyildiz et al. then derive a new two dimensional random walk mobility model in order to track and estimate the movements of mobile users [43]. First, the Authors consider a hexagonal plane as seen in **Error! Reference source not found.**. Then they split it into six symmetrical sectors in order to simplify the analysis. Second they give each hexagonal cell a state of the form (x,y) where “x” represents the hexagonal ring the cell belongs to and “y” represents the type the cell be longs. In this case, the “type” groups together cells that are symmetrical in terms of location and one hope neighbors. An Example of this would be the dotted cell marked (4,1) that can also be seen in **Error! Reference source not found.**. Third, and based on random walk probability, the authors derive the transition diagram seen in Figure 11 with the one-step transition from a cell to any of its six one-hop neighboring cells being equally likely with a value of $\frac{1}{6}$ which is shown in **Error! Reference source not found.**. Finally, the authors formulate a way to estimate the number of expected steps or transition that a mobile station would go through till it leaves the defined hexagonal grid seen in **Error! Reference source not found.**. The authors claim that their simulations results differ from the analytical model are by a maximum of 1%.

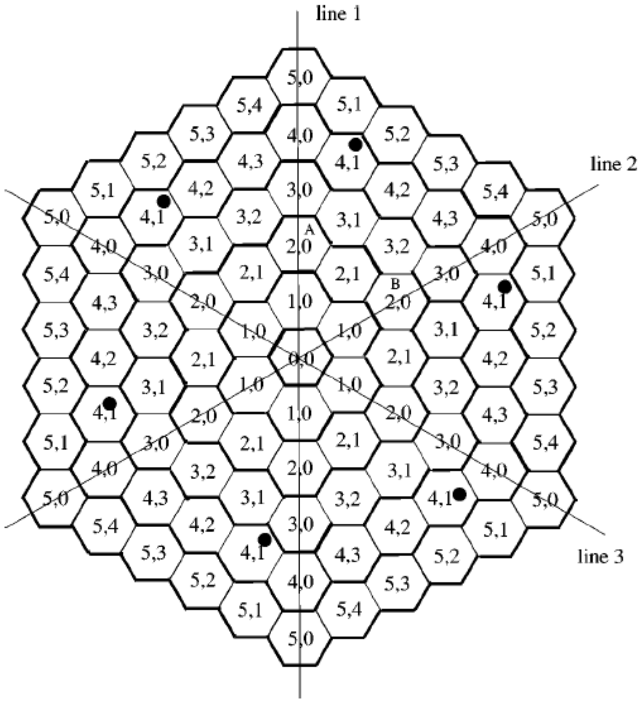


Figure 9. Hexagonal Plane/Grid

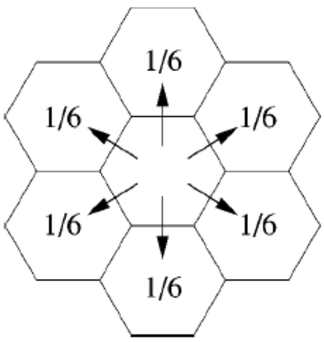


Figure 10. A one-step Transition

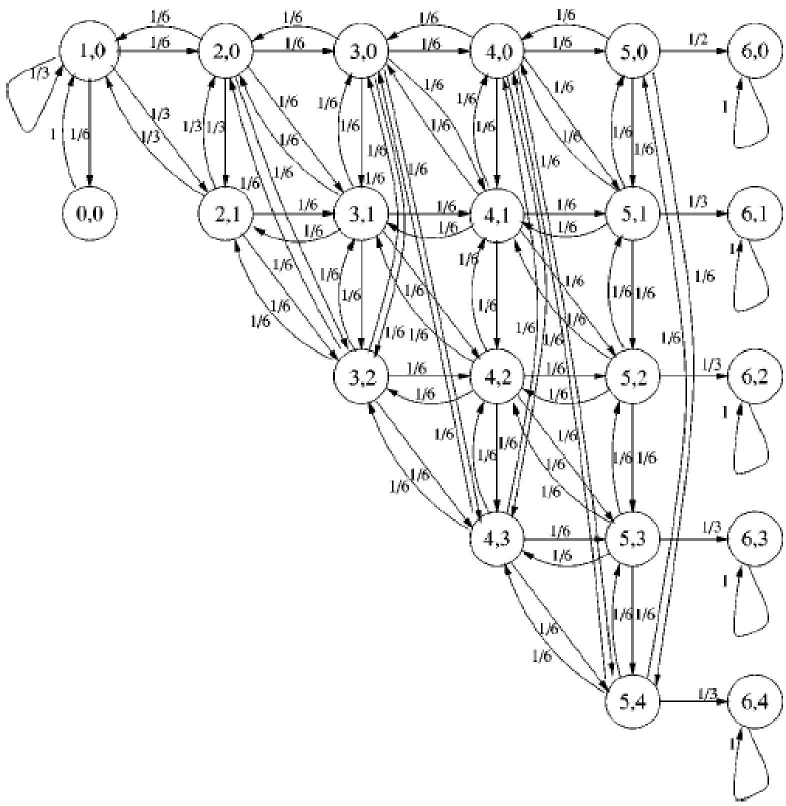


Figure 11. Markov based state transition Diagram

The work of I. F. Akyildiz et al. [42], [43] is taken a step forward by N. Shenoy and K. H. Chiang in [44] and [45]. The authors of [44] and [45] work on hexagonal and square shaped cells. However, only the hexagonal shaped cells model will be reviewed since it is the one that is most relevant to the study conducted in this report.

N. Shenoy and K. H. Chiang modified the model in [42] and [43] making it simpler, and adapted it to support both location-crossing rate and dwell-time studies. The authors also applied the “lumped-process” property of Markov chains to aggregate states and further reduce the number of computational states. They also added a wrap-around feature to reduce constraints on mobile movements. As seen in Figure 12, this wrap-around feature would be the special set of states marked by an asterisk “*” that mark the boundaries of a Location Area. This would allow the mobile station to enter the original states again from the special states, once it starts moving within the new

Location Area. Figure 13 shows the derived Markov state diagram and transition probability for this model in three parts:

- The transition diagram for states or cells within a Locations Area.
- Transitions across cells in the adjacent Location Areas.
- Transitions across cells in the adjacent Location Areas.

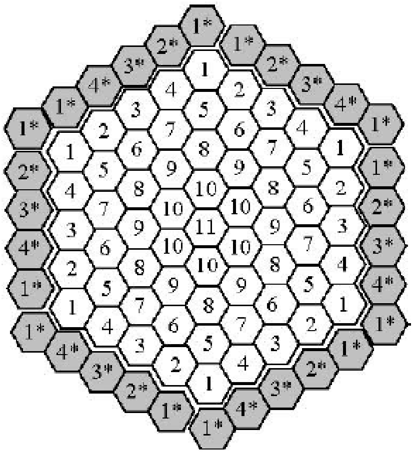


Figure 12. Hexagonal Grid with Wrap-around Feature

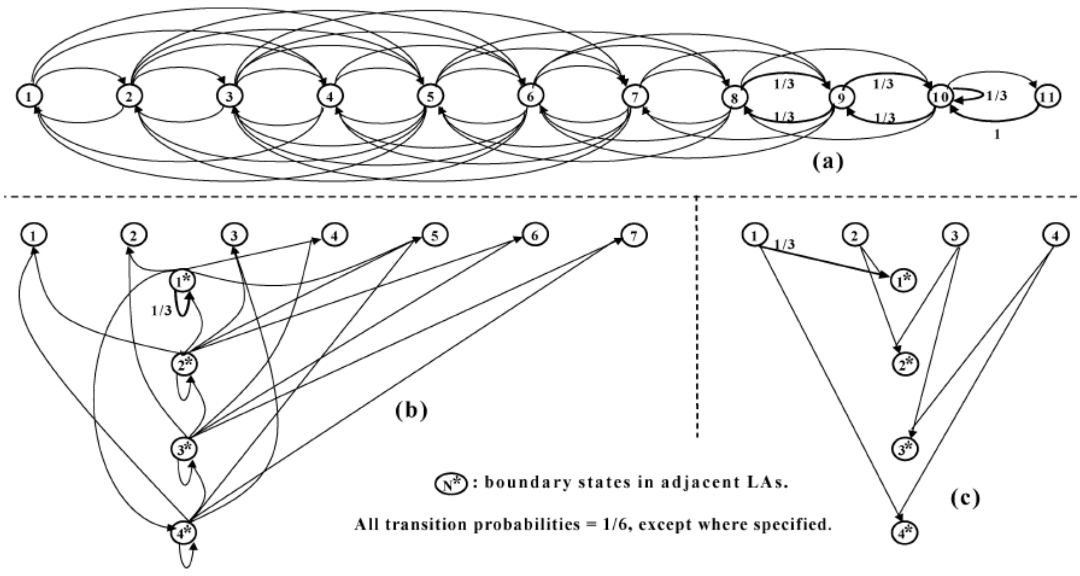


Figure 13. Markov Chain State Transition Diagram

In order to highlight the importance of the model described in [42] - [45], the following two publications, “the evaluation of Mobility Management Schemes for different Core Network Architectural Arrangements in 3GPP LTE/SAE” [46] and “The Evaluation of Location Management in UMTS” [47] will be reviewed respectively.

The authors of [46] evaluate the performance of different mobility management schemes for different SAE core network architectural options proposed in 3GPP according to signaling cost by using analytical modeling based on the random walk model. The authors consider four different architectures using three LTE-EPC entities:

1. Fully Split MME, S-GW and P-GW.
2. P-GW separate and MME and S-GW merged.
3. MME separate and S-GW and P-GW merged.
4. Fully merged MME S-GW and P-GW.

The authors then use the two dimensional random walk mobility models with the hexagonal grid architecture in order to complete their evaluations and calculate the sum of the average signaling cost of Handovers and Session Activation time. Their results show that option 1 has the least stress in terms of load per node but the highest amount of signaling and option 4 shows the least amount of signaling but introduces a heavy load on the merged node.

As for [47], its authors propose analytical and simulation models to investigate the performance of the inactivity counter mechanism. Specifically, given any mobility and traffic patterns, they determine the net costs of location update and paging. The most important thing to take from this is the “Recursive” property of the Mobility Model proposed in [43] - [46] and how it was applied in [48]. Figure 14 makes this property clearer, by showing how cells would fit in to a UTRAN Registration Area (URA) and how Multiple URAs would fit into a Routing Area (RA).

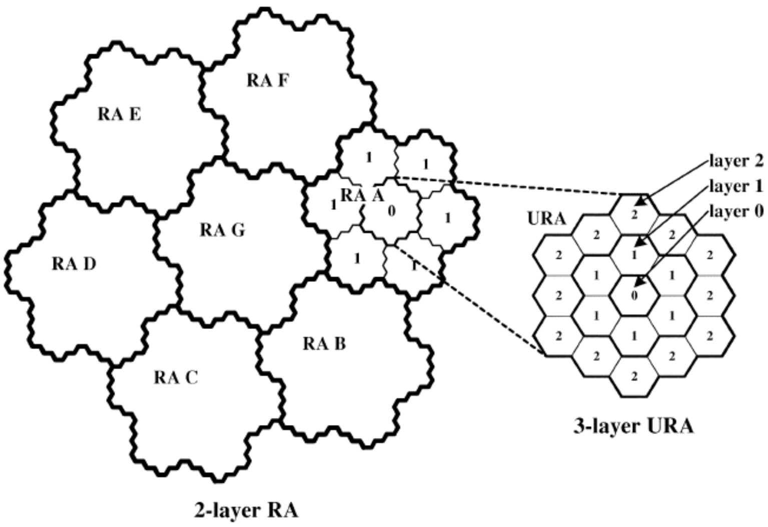


Figure 14. UMTS RA/URA/Cell

The importance and versatile use of the random walk mobility model in [43][42] has been highlighted from [44] to [47], hence, the same model that was derived in [43] by I. F. Akyildiz et al. will be adapted to the needs of this study and used accordingly to estimate the average residence time of a mobile user in a cell. This adaptation will be discussed in details in the following chapter.

4.2 Traffic Analysis

After choosing a mobility model for mobile users in cellular networks, the next step would be choosing, and adapting if necessary, a probabilistic approach to model and predict traffic flows within a network. Multiple techniques have been proposed in order to study and analyze network traffic.

S. Sun et al. presented a method called Sampling Markov Chain in order to carry out short term prediction of traffic flow [48]. The goal of this method is to accurately forecast future traffic for durations ranging from 5 to 30 minutes. They take on the challenge of accomplishing this with incomplete data. They apply the Gaussian Mixture Model and Competitive EM algorithm to estimate the state transition probability density function for the Markov Chain. They also resort to Monte Carlo integration in order to approximate missing data by replacing it by its historical average. They test their theory on vehicle traffic flows. The authors approach shows better accuracy than their predecessors.

According to D. Dudek, first order Markov Chains can be used in order to detect DoS-related traffic anomalies [49]. The author applies his theory to Wireless Sensor Networks (WSNs) since DoS attacks can be very harmful to WSN nodes in terms of operation, memory flooding and power consumption. The author's method focuses on detecting the attacks by exploiting the fact that WSN nodes show distinct traffic patterns. The traffic in the network is regarded as a sequence of states and time is divided into equal-length observation intervals. During the first phase – the learning phase – nodes collect information about the expected normal traffic. At the end of the learning phase, each node possesses a "time series" of traffic observation points. From those points, a traffic profile is built that describes the "time series" in question. The traffic profile would be the Markov Chain itself. It is used to detect low probability events, i.e. a sequence of low probability transitions in the chain. If unlikely transitions amass over some period of time, the traffic is identified as anomalous. The author's simulations show good detection results with only 3% false positive detections for 85% of the WSN nodes.

X. Liu and T. N. Saadawi investigate the greedy behavior of in IEEE 802.11 wireless networks, and provide information on theperformances of the MAC layer using a two dimensional Markov chain analytical model [50]. The authors sum up the greedy behaviors of nodes to three categories: nodes who randomly select a small value for the contention window but do not double it after a collision is detected, nodes who select a smaller value for the contention window instead of increasing it and nodes that always select a fixed back off counter. The authors investigate the cases when there are no packets in the nodes buffer and when there is at least one packet in the nodes buffer under the assumption that the packet arrival rate follows a Poisson distribution and that the packet service queue can be modeled using the M/M/1/∞ queue. The authors then use Opnet 15.0 for their simulation, and use it to validate their derived analytical model.

L. Quigfu et al. integrate the GM (1,1) model and the Markov model and derive the Grey-Markov model in order to predict traffic volumes. Combinations of the GM and Markov models are used to predict the annual traffic accident, freight volume and traffic volume of China [51]. The GM (1,1) model is used to forecast trends in the data sequences while the Markov Model can be used to detect the vibration regulation of their development since data sequences retrieved from the real word are rarely stable. Thus the authors provide a new method to predict these stochastic data sequences. However, they clearly point out that their derivation still requires further study.

A. Dainotti et al. proposes a packet-level traffic classification approach based on Hidden Markov Model (HMM) by relying on Packet Size (PS) and Inter Packet Delay Time characteristics (IPDT) [52]. The authors derived a mathematical model which they validate by applying their model to real traffic traces collected from the usage of different applications such as Age of Mythology and Counter Strike (two Multi-Player Network Games), HTTP, SMTP, Edonkey, PPlive (a peer-to-peer IPTV application), and MSN Messenger. One motivation for their study is to offer proper Quality of Service (QoS) depending on the category of traffic carried by flows, and to perform a billing not only based on bandwidth usage but also on the traffic category. The second motivation would be enforcing security policies for different applications and the identification of malicious traffic flows. The authors achieve 90% correct classification, though they point out that the tests were done only on user outgoing traffic and that applying their technique to incoming traffic as well can increase classification accuracy by studying the bonds of in/out traffic of the same application.

The HMM is also used by M. K. Yasu et al. in [53]. The authors design a tool that takes into account the IPDT and PS metrics in order to measure wireless Internet traffic and build an analytical model that can predicate such traffic.

In [54], Y. Xie et al. use Hierarchical HMM to model and synthesize stationary and non-stationary oscillatory processes of network traffic. The structure includes two nested hidden Markov chains and one observable process. The first-layer hidden Markov chain with variable state-duration controls the time-varying oscillatory process. Thus, using the first-layer Markov chain as a condition, they model the local fluctuation process by using the second-layer hidden Markov chain and derive a model that can reproduce the input trace's structured oscillation.

In [55] A. Jukan and S. Zaghloul derive a mathematical model for Remote Authentication Dial In User Service (RADIUS) traffic based on the assumption that the number of user sessions at a given period of time follows Poisson's distribution, the user session length is exponentially distributed and the session length is independent of the session rate. This model sums the AAA traffic and the accounting traffic thus estimating the total traffic generated by RADIUS in a network. The authors conduct their study for a single AAA server and for multiple AAA servers that are connected as a proxy chain, where the intermediate AAA servers simply forward the messages to the intended destination. The authors extend their study again in [56] to include the corresponding AAA traffic rate generated by the Network Access Server (NAS) in response to the RAN requests. They validate their model using event-driven simulations (MATLAB and C++). The Authors continue their work in [57] to include the mean total signaling rate at the Policy Control Function (PCF) for IP Multimedia Subsystem (IMS) signaling. They resort to Markov chains in order to include the effects of handoffs on mobile user sessions use event-driven simulations for validation. In [58] A. Jukan and S. Zaghloul finally tackle Diameter signaling issues. They use the stochastic and renewal theoretic techniques to develop an analytical model for the AAA signaling rate as a function of protocol parameters, users' access rates, session durations, and mobility. They conclude that the AAA signaling rate is a monotonic non-linear function of the mobility rate and asymptotically converges to the AAA signaling rate in fixed networks as the residence time increases. The results also include the effects of the session dropping during handoffs, roaming users (preliminary), the mean time between accounting updates. Mobility was again accounted for using Markov techniques and validation was done using event-driven simulations.

The previous work concerning IP-Network traffic modeling is extensive and the work done on AAA signaling by A. Jukan and S. Zaghloul from [55] to [58] is commendable. Hence, the Diameter protocol traffic model that will be discussed and derived in the following chapter differentiates itself from the previous work by:

- Targeting specific Diameter related Interfaces and the corresponding message pairs exchanged on them though limiting the study to an LTE environment, i.e.

LTE specific Diameter Interfaces while attempting to categorize LTE Diameter interfaces into two categories:

1. Interfaces that are heavily affected by a UE's mobility.
2. Interfaces where the effects of a UE's mobility, even at high speeds, is minimal.

- Targeting the UE's behavior and its effects on the signaling in the core instead of the aggregated traffic for all UEs:
    - Targets the cumulative number per session per UE instead of the average traffic, though, given the cumulative, the average can be easily calculated by dividing the cumulative over the average session duration.
- Making no assumptions on the distribution on the UE's session duration and residence time in a cell except that they are independent and using the exponential distribution as a test scenario instead of restricting the model to it, allowing the use of any desired distribution.
- Considering residence time per cell instead of per Location Area (LA):
    - That was done by using the Markov based mobility model in [43] to estimate the number of handoffs per UE by adapting it from a model that estimates the residence time per LA to one that estimates the residence time per cell in function of the UE's average speed and cell radius.
    - Since the model is restricted to a single LA, it links the number of re-authentication and re-authorization during an active UE session not only to the auth/author lifetime but also to the number of handoffs from one cell to the next and thus brings light to the case were a change to a UE's session occurs because of the "newly moved into" cell's status, thus

forcing the UE to re-authenticate and re-authorize, and if it is necessary, drop its current active session.

- Making no assumptions on the success rate of re-authentication and re-authorization.
- Considering the effects of the grace period during re-authentication and re-authorization.
- Being more of a logical and mathematical way for calculating the traffic on Diameter specific interfaces than being a fixed model for specific diameter functionalities, hence it can be easily adapted and re-derived for other interfaces and the corresponding messages pairs exchanged on them.

CHAPTER V

MATHEMATICAL DERIVATION

5.1 Objectives

The primary objective of the following traffic model is to relate Diameter traffic to a user's activity. However, modeling the entire protocol over all the interfaces while considering all the message pairs is a daunting task, thus four different interfaces will be considered along with eight different message pairs because the combination of [Interface, Message Pairs] scene in Figure can be used to:

- Embody the simplest AAA spirit of the protocol.
- Represent a UE's session from start (authentication and authorization) to finish (session termination).
- Develop a general method, that would allow deriving the needed model equation for any [Interface, Message Pair] combination of interest.
- Show that the interfaces are indeed characterized into two groups:
    1. Interfaces that are heavily affected by a UE's mobility.
    2. Interfaces where the effects of a UE's mobility, even at high speeds, is minimal.

5.2 Derivation – The Number of Message Pairs per Session per UE

First, The derivation and calculation of the number of messages is per session per UE. Since Diameter operates via client / server architecture, each request warrants an answer even if the answer is to deny a UE certain privileges. Thus, in ideal conditions, the number of requests is equal to the number of answers. So, a single expression will derived for each interface.

Second, the LTE Network's architecture that will be used for the model along with the Diameter interfaces in question is presented in Figure 15. In addition, the different LTE elements are considered to be physically separate except the MME and the PGW since no interface is being considered between them. They can be though off as a single element communicating on two separate Diameter interfaces. It is also important to note, that for this model, the LTE elements are considered to be physically separate and each Diameter interface is located on a separate physical interface than the others.

Third, the LTE elements are the Diameter interfaces between them are defined:

- LTE elements:
  - The User Equipment (UE), the UE's data session to be more precise.
  - The Mobility Management Entity (MME).
  - The Home Subscriber Server (HSS).
  - The Packet-data-network Gateway (PGW).
  - The Policy and Charging Rules Function (PCRF).
  - The Online Charging System (OCS).
- Diameter interfaces:
  - The **S6** interface: between the MME and the HSS.
  - The **Gx** interface: between the PGW and the PCRF.
  - The **Ro** interface: between the PGW and the OCS.

– The **Sy** interface: between the PCRF and the OCS.

Fourth, the interaction between the elements is specified. Figure 16 [60] shows the message pairs that are exchanged on the different Diameter interfaces between the separate LTE elements.

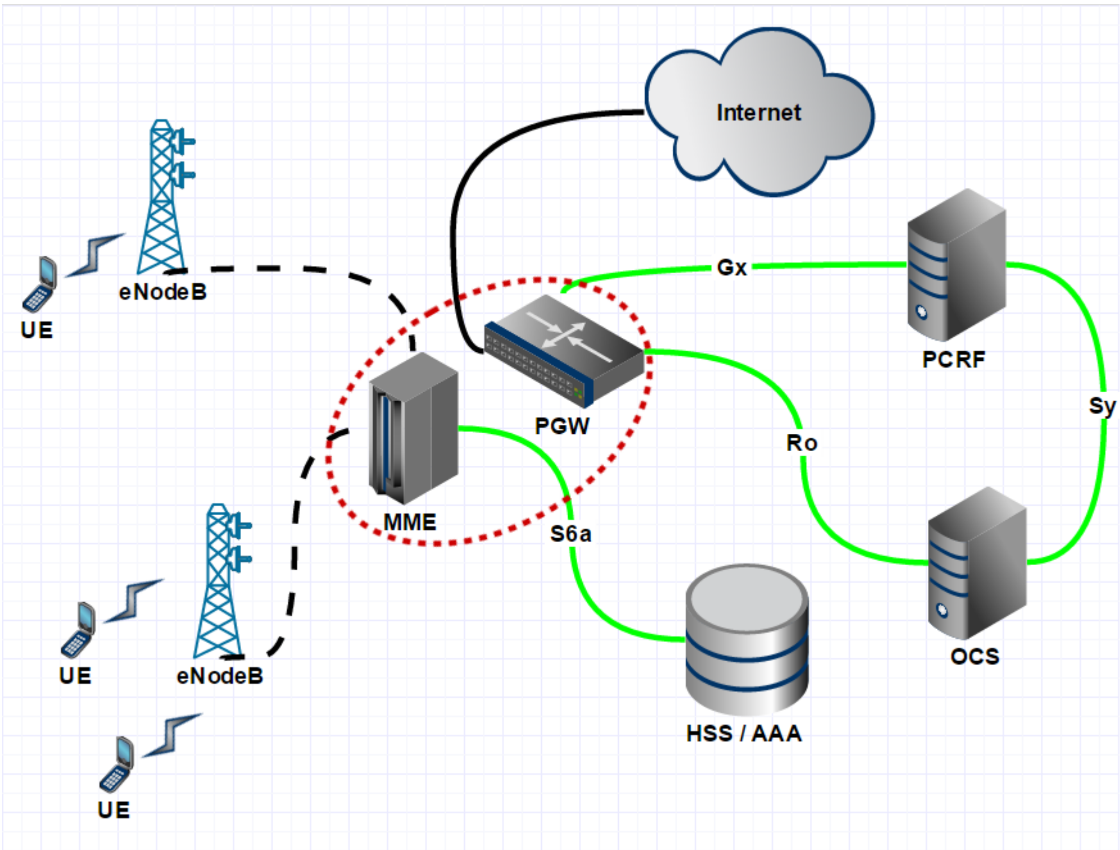


Figure 15. LTE Network Architecture and Diameter Interfaces

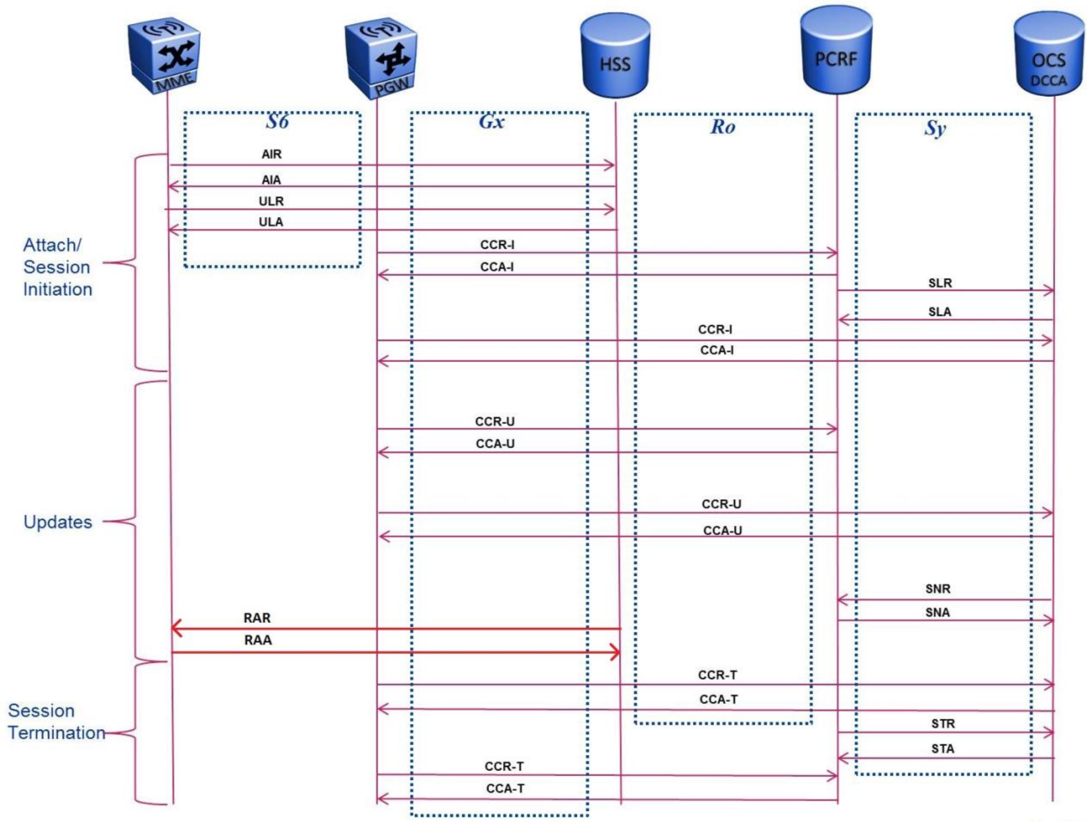


Figure 16. Diameter Example Message Flow

Fifth, the messages exchanged over the Diameter LTE interfaces in question are defined:

- On the S6 interface:
  - Authentication Information Request (AIR) / Authentication Information Answer (AIA).
  - Update Location Request (ULR) / Update Location Answer (ULA).
  - Re Auth Request (RAR) / Re Auth Answer (RAA) (the assumption made here is that the AAA server is the same as the HSS, but it can be a separate entity).
- On the Gx interface:
  - Credit Control Request (CCR) / Credit Control Answer (CCA) (Initiate, Update, terminate).

- On the Ro interface:
  - Credit Control Request (CCR) / Credit Control Answer (CCA) (Initiate, Update, terminate).
- On the Sy interface:
  - Spending Limit Request (SLR) / Spending Limit Request (SLA).
  - Spending-Status Notification Request (SNR) / Spending-Status Notification Request (SNA).
  - Session Termination Request (STR) / Session Termination Request (STA).

Sixth, the UE's data session related parameters must be defined:

- $\Delta t_{session}$: The UE's data session lifetime. The scenario considered here is that every time a session is established authentication/authorization occurs.
- $\Delta t_{auth/author}$: The authentication/authorization lifetime, a parameter that is fixed by the Mobile Operator. Once it expires, the UE will have to re-authenticate/re-authorize, else the UE's session is terminated. It important to note that the re-authentication/re-authorization procedure is initiated by the AAA server, in the case considered her, that would be the HSS.
- $\Delta t_{garce}^{\max}$: The authentication/authorization grace period, a parameter that is also set by the Mobile Operator. The grace period is a tiny window of time that starts each time $\Delta t_{auth/author}$ expires. The UE must complete his re-authentication/re-authorization procedure before the grace period ends else his data session is terminated.

- Δt_U : The Interim Update Duration is the time interval between two consecutive CCR-U messages.
- $\Delta t_{residence}$: The UE's residence time in an LTE cell. This parameter will be further described later on in function of the LTE's cell's radius and the UE's average movement velocity.

5.2.1 *Authentication and Authorization – The S6a Interface*

The total number of auth/author messages during a UE's data session on the S6 interface (MME - HSS) can be expressed as:

$$N_{auth/author}^{session/UE} = N_{AIR/AIA} + N_{ULR/ULA} + N_{RAR/RAA}$$

$$E\left[N_{auth/author}^{session/UE}\right] = E\left[N_{AIR/AIA}\right] + E\left[N_{ULR/ULA}\right] + E\left[N_{RAR/RAA}\right]$$

where:

- $N_{AIR/AIA}$: is the number of AIR / AIA message pairs exchanged during a UE's data session.
- $N_{ULR/ULA}$: is the number of ULR / ULA message pairs exchanged during a UE's data session.
- $N_{RAR/RAA}$: is the number of RAR / RAA message pairs exchanged during a UE's data session.
- $E\left[random\ variable\right]$: is the expected value of $\{random\ variable\}$.

The AIR/AIA message pair are exchanged only once at the beginning of a UE's session. Given that the AIA returns with a positive answer (i.e. the success of the AIR/AIA message pair exchange), the ULR/ULA message pair occurs and the UE's data session starts. Then:

$$N_{AIR/AIA} = 1$$

$$E\left[N_{AIR/AIA}\right] = 1$$

$$N_{ULR/ULA} = 1$$

$$E\left[N_{ULR/ULA}\right] = 1 \times p_a = p_a$$

where p_a is the success rate of the AIR/AIA message pair exchange. Furthermore, the number of RAR/RAA message pairs exchanged during a UE's data session can be expressed as:

$$N_{RAR/RAA} = \left[\left(N_{RAR/RAA}^{timeout}\right) + \left(N_{RAR/RAA}^{crossing|timeout}\right)\right]$$

$$E\left[N_{RAR/RAA}\right] = \left[\left(E\left[N_{RAR/RAA}^{timeout}\right]\right) \times p_{fixed-UE} + \left(E\left[N_{RAR/RAA}^{crossing|timeout}\right]\right) \times p_{cell-cross}\right] \times p_a$$

where:

- $N_{RAR/RAA}^{timeout}$: is the total number of possible RAR / RAA message pairs that occur via $\Delta t_{auth/author}$ expiry (fixed UE).
- $N_{RAR/RAA}^{crossing|timeout}$: is the total number of possible RAR/RAA message pairs that could occur via $\Delta t_{auth/author}$ expiry or cell boundary crossing (mobile UE).
- $p_{fixed-UE}$: is the probability that the next possible RAR/RAA message pair occurs via $\Delta t_{auth/author}$ expiry.
- $p_{cell-cross}$: is the probability that the next possible RAR/RAA message pair occurs via cell boundary crossing.
- ***Condition:*** $\Delta t_{auth/author} \leq \Delta t_{session}$

So, under the assumption that all re-auth/re-author are successful (***Note:*** this assumption will be removed later on), for the case of a fixed user:

$$N_{RAR/RAA}^{timeout} = \left(\frac{\Delta t_{session}}{\Delta t_{auth/author}} - 1 \right)$$

$$E\left[N_{RAR/RAA}^{timeout} \right] = \left(\frac{E\left[\Delta t_{session}\right]}{\Delta t_{auth/author}} - 1 \right)$$

where the (-1) in the expression removes the first auth/author made via the AIR/AIA message pair. And, for the case of a mobile user:

$$E\left[N_{RAR/RAA}^{crossing|timeout} \right] = \left(E\left[N_{RAR/RAA}^{timeout} \right] \times p_{Fixed-UE} \right) + E\left[N_{RAR/RAA}^{crossing} \right]; \; given\, that\, the\, UE\, is\, not\, fixed$$

$$N_{RAR/RAA}^{crossing} = \left(\frac{\Delta t_{session}}{\Delta t_{residence}} - 1 \right)$$

$$E\left[N_{RAR/RAA}^{crossing} \right] = \left(\frac{E\left[\Delta t_{session}\right]}{E\left[\Delta t_{residence}\right]} - 1 \right)$$

under the assumption that $\Delta t_{session}$ and $\Delta t_{residence}$ are independent. Finally, if the UE moves fast enough to always cross a cell's boundary before the current $\Delta t_{auth/author}$ expires, then $p_{fixed-UE} = 0$ and $N_{RAR/RAA}^{crossing|timeout}$ defaults to:

$$N_{RAR/RAA}^{crossing|timeout} = N_{RAR/RAA}^{crossing}$$

$$E\left[N_{RAR/RAA}^{crossing|timeout} \right] = E\left[N_{RAR/RAA}^{crossing} \right]$$

The probabilities $p_{fixed-UE}$ and $p_{cell-cross}$ can be determined by examining Figure 17.

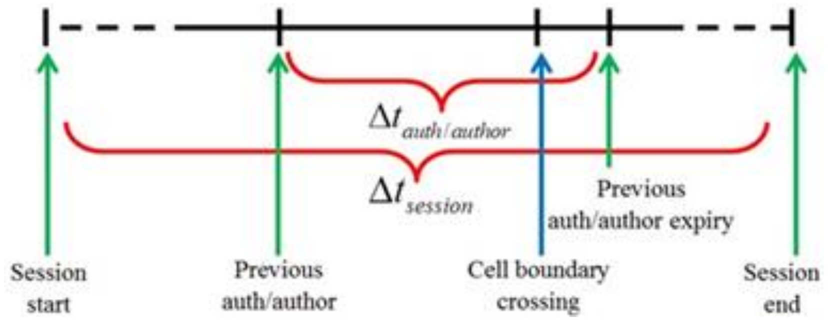


Figure 17. UE data Session vs. Mobility

Given a successful previous auth/author (re-auth/re-author), i.e. an RAA with a positive answer, for the next re-auth/re-author to occur via a cell boundary crossing, the crossing event must occur while the UE is still in session and before $\Delta t_{auth/author}$ of the previous auth/author (re-auth/re-author) expires, then:

$$p_{cell-cross} = \left(\int_{\Delta t_{residence}}^{+\infty} f_{session}(t)dt \right) \times \left(\int_{0}^{\Delta t_{auth/author}} f_{residence}(t)dt \right)$$

where $f_{session}(t)$ and $f_{residence}(t)$ are the Probability Density Functions (PDF) for $\Delta t_{session}$ and $\Delta t_{residence}$ respectively. And for case where the $E[\Delta t_{residence}]$ and $E[\Delta t_{residence}]$ are used:

$$p_{cell-cross} = \left(\int_{E[\Delta t_{residence}]}^{+\infty} f_{session}(t)dt \right) \times \left(\int_{0}^{\Delta t_{auth/author}} f_{residence}(t)dt \right)$$

Thus for a re-auth/re-author that occurs via $\Delta t_{auth/author}$ expiry:

$$p_{fixed-UE} = 1 - p_{cell-cross}$$

With Diameter, an answer is warranted for every request regardless of whether the answer itself is positive or negative with regard to the request. This means that the events of “an answer returning” and of “the answer is positive” are independent. Thus, given that the auth/author via AIR/AIA was successful, the probability of the first RAA occurring $(p_{occurence})$ and returning with positive answer can be expressed as

$p_{occurence} = p_{fixed-UE}$ or $p_{occurence} = p_{cell-cross}$, depending on whether the re-auth/re-author occurred via the UE crossing from one cell to another or via $\Delta t_{auth/author}$ expiry. So, if a second re-auth/re-author is to possibly occur, then the first RAA must have returned carrying a positive answer, thus the probability of the RAA occurring and returning with a positive answer can be expressed as $p = p_{occurence} \times p_{success}$, where $p_{success}$ is the probability of the previous RAA returning with a positive answer. Hence, for the third RAA message, $p = p_{occurrence} \times p_{success}^2$ and so on up to "m" RAA messages. Thus, for the case of a fixed UE, i.e. $p_{cell-cross} = 0$ and $p_{fixed-UE} = 1$, $E\left[N_{RAR/RAA}^{timeout}\right]$ can be expressed as:

$$E\left[N_{RAR/RAA}^{timeout}\right] = 1 + p_{success} + p_{success}^2 + p_{success}^3 + \ldots + p_{success}^m$$

$$E\left[N_{RAR/RAA}^{timeout}\right] = \sum_{k=0}^{m_{fixed-UE}} p_{success}^k$$

Similarly, for the case of a mobile UE where $p_{cell-cross} = 1$ and $p_{fixed-UE} = 0$, $E\left[N_{RAR/RAA}^{crossing|timeout}\right]$ can be expressed as:

$$E\left[N_{RAR/RAA}^{crossing|timeout}\right] = \sum_{k=0}^{m_{cell-cross}} p_{success}^k$$

Both extreme cases are shown in Figure 18 with the green and orange bars representing re-auth/re-author via timeout (case of fixed UE) and via cell boundary crossing respectively.

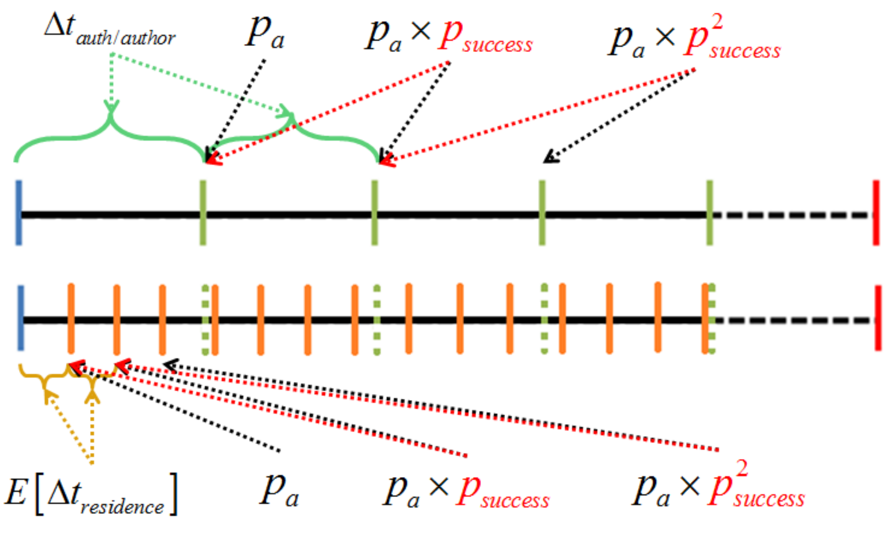


Figure 18. Extreme Cases of Re-authentication and Re-authorization

The term $\sum_{k=0}^{m} p_{success}^{k}$ is the sum of a geometric series with $a=1$ and $r=p_{success}$.

Thus, by removing the assumption from before that all re-auth/re-author are successful

$\Leftrightarrow p_{success} \neq 1$, $\sum_{k=0}^{m} p_{success}^{k}$ can be reduced to:

$$\sum_{k=0}^{n-1} a \times r^{k} = a \times \frac{1-r^{n}}{1-r} \text{ with } m=n-1$$

$$\sum_{k=0}^{m} p_{success}^{k} = \frac{1-p_{success}^{m+1}}{1-p_{success}}$$

The success of a returning RAA depends on two things:

- The RAA carrying a positive answer.
- The re-auth/re-author procedure started and finished within the grace period $\Delta t_{grace}^{\max}$.

Thus, $p_{success} = p_{ra} \times p_{\Delta t_{grace}}$ where:

- p_{ra}: is the probability that he RAA returns carrying a positive answer.
- $p_{\Delta t_{grace}}$: is the probability that the re-auth/re-author procedure started and finished within the grace period $\Delta t_{grace}^{\max}$. By examining Figure 19, $p_{\Delta t_{grace}}$ can be expressed as:

$$p_{\Delta t_{grace}} = \int_{0}^{\Delta t_{grace}^{\max}} f_{\Delta t_{grace}}(t)dt$$

where $f_{\Delta t_{grace}}(t)$ is the PDF of the UE's duration from "Grace period start" till "Re-auth/re-author procedure" end.

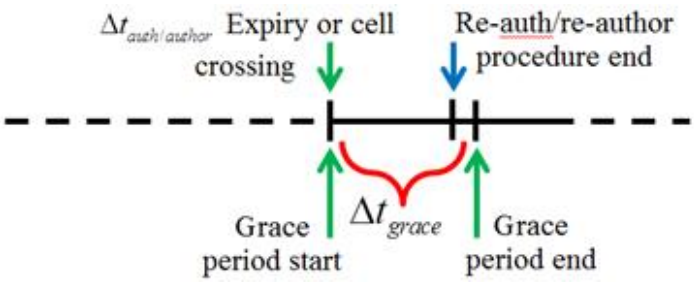


Figure 19. Re-auth/re-author within the Grace period

Finally, the parameter "m" can be determined by comparing the expressions of $N_{RAR/RAA}^{timeout}$ and $N_{RAR/RAA}^{crossing}$ without the assumption on the success of the re-auth/re-author to their counterpart expressions with the assumption that all the re-auth/re-author are successful (i.e. $p_{success} = 1$):

- $$\sum_{k=0}^{m_{fixed-UE}} p_{success}^{k} = \left(\frac{\Delta t_{session}}{\Delta t_{auth/author}} - 1 \right) \ with \ \sum_{k=0}^{m_{fixed-UE}} 1^{k} = m_{fixed-UE} + 1$$

$$\rightarrow m_{fixed-UE} = \frac{\Delta t_{session}}{\Delta t_{auth/author}} - 2$$

$$\rightarrow m_{fixed-UE} = \frac{E\left[\Delta t_{session}\right]}{\Delta t_{auth/author}} - 2; \ when \ using \ E\left[N_{RAR/RAA}^{timeout}\right]$$

- $$\sum_{k=0}^{m_{cell-cross}} p_{success}^{k} = \left(\frac{\Delta t_{session}}{\Delta t_{residence}} - 1 \right) \ with \ \sum_{k=0}^{m_{cell-cross}} 1^{k} = m_{cell-cross} + 1$$

$$\rightarrow m_{cell-cross} = \frac{\Delta t_{session}}{\Delta t_{residence}} - 2$$

$$\rightarrow m_{cell-cross} = \frac{E\left[\Delta t_{session}\right]}{E\left[\Delta t_{residence}\right]} - 2; \ when \ using \ E\left[N_{RAR/RAA}^{crossing}\right]$$

Thus, for the following message pair exchange seen in Figure 16, the total number of message pairs on the S6 interface between the MME and HSS can be calculate using Table 1:

Table 1. Parametrized Equation - the Number of Message Pairs - S6 Interface

Term	**Expression**
$E\left[N_{auth/author}^{session/UE}\right]$	$E\left[N_{AIR/AIA}\right]+E\left[N_{ULR/ULA}\right]+E\left[N_{RAR/RAA}\right]$
$E\left[N_{AIR/AIA}\right]$	1
$E\left[N_{ULR/ULA}\right]$	$1\times p_a$
$E\left[N_{RAR/RAA}\right]$	$\left[\left(E\left[N_{RAR/RAA}^{timeout}\right]\right)\times p_{fixed-UE}+\left(E\left[N_{RAR/RAA}^{crossing\|timeout}\right]\right)\times p_{cell-cross}\right]\times p_a$
$E\left[N_{RAR/RAA}^{timeout}\right]$	$\frac{1-p_{success}^{m_{fixed-UE}+1}}{1-p_{success}}$
$E\left[N_{RAR/RAA}^{crossing\|timeout}\right]$	$\left(E\left[N_{RAR/RAA}^{timeout}\right]\times p_{Fixed-UE}\right)+E\left[N_{RAR/RAA}^{crossing}\right]$
$E\left[N_{RAR/RAA}^{crossing}\right]$	$\frac{1-p_{success}^{m_{cell-cross}+1}}{1-p_{success}}$
$m_{fixed-UE}$	$\frac{E\left[\Delta t_{session}\right]}{\Delta t_{auth/author}}-2$
$m_{cell-cross}$	$\frac{E\left[\Delta t_{session}\right]}{E\left[\Delta t_{residence}\right]}-2$
$p_{cell-cross}$	$\left(\int_{E\left[\Delta t_{residence}\right]}^{+\infty} f_{session}(t)dt\right)\times\left(\int_{0}^{\Delta t_{auth/author}} f_{residence}(t)dt\right)$
$f_{session}(t)$	PDF of $\Delta t_{session}$
$f_{residence}(t)$	PDF of $\Delta t_{residence}$
$p_{fixed-UE}$	$1-p_{cell-cross}$
$p_{success}$	$p_{ra}\times p_{\Delta t_{grace}}$
$p_{\Delta t_{grace}}$	$\int_{0}^{\Delta t_{grace}} f_{\Delta t_{grace}}(t)dt$

$f_{\Delta t_{grace}}(t)$	PDF of the UE's duration from "Grace period start" till "Re-auth/re-author procedure" end
p_a	Probability that AIA returns with (+) answer
p_{ra}	Probability that RAA returns with (+) answer
$E[\Delta t_{session}]$	Average Session Lifetime
$E[\Delta t_{residence}]$	Average Residence Time (addressed later)
$\Delta t_{auth/author}$	Auth/Author lifetime
$\Delta t_{grace}^{\max}$	Grace Period

5.2.2 Credit Control – The Gx, Ro and Sy Interfaces

The Credit Control traffic on the Gx (PGW - PCRF) and Ro (PGW - OCS) can be grouped and described using the same mathematical expression since the CCR / CCA message pairs exchange is the same on both interfaces. As for the SLR / SLA, SNR / SNA and STR / STA message pairs exchanged on the Sy (PCRF - OCS) interface, they are related to the CCR / CCA message exchange and can be grouped with the Gx and Ro interfaces because:

- The SLR / SLA message pair is exchanged when CCR-I / CCA-I message pair (Initiate) is exchanged.
- The SNR / SNA message pair is exchanged when CCR-U / CCA-U message pair (Update) is exchanged.
- The STR / STA message pair, which is a generic Diameter message pair used to signal the termination of a session, is exchanged when CCR-T / CCA-T message pair (Terminate) is exchanged.

The total number of credit control message pairs for each of the Gx, Ro and Sy interfaces can expressed as:

$$N_{CC-Total}^{Gx,Ro,Sy} = N_I^{session/UE} + N_U^{session/UE} + N_T^{session/UE}$$

$$E\left[N_{CC-Total}^{Gx,Ro,Sy}\right] = E\left[N_I^{session/UE}\right] + E\left[N_U^{session/UE}\right] + E\left[N_T^{session/UE}\right]$$

where:

- $N_I^{session/UE}$: is the number of CCR-I / CCA-I or SLR / SLA message pairs exchanged during a UE's data session.
- $+N_U^{session/UE}$: is the number of CCR-U / CCA-U or SNR / SNA message pairs exchanged during a UE's data session.
- $N_T^{session/UE}$: is the number of CCR-T / CCA-T or STR / STA message pairs exchanged during a UE's data session.
- $E[random\ variable]$: is the expected value of $\{random\ variable\}$.

The number of CCR-I / CCA-I, CCR-T / CCA-T, SLR / SLA and STR/STA message pairs are exchanged once per session, and depend on p_a (the probability that the AIA message from the AIR / AIA message pair returned carrying a positive answer, i.e. the UE's session was opened, or has started). Thus:

$$N_I^{session/UE} = N_T^{session/UE} = N_{CCR-I/CCA-I}^{session/UE} = N_{CCR-T/CCA-T}^{session/UE} = N_{SLR/SLA}^{session/UE} = N_{STR/STA}^{session/UE} = 1$$

$$E\left[N_I^{session/UE}\right] = E\left[N_T^{session/UE}\right] = 1 \times p_a$$

The update messages that are sent periodically during a session depend on the length of the session, which depends on it being opened (positive AIA) and the RAA messages returning with a positive answer, else a user's session is terminated. Thus, given that the SNR / SNA message pair are exchanged when CCR-U / CCA-U message pair are exchanged then:

$$N_U^{session/UE} = N_{CCR-U/CCA-U}^{session/UE} = N_{SNR/SNA}^{session/UE}$$

$$E\left[N_U^{session/UE}\right] = E\left[N_{CCR-U/CCA-U}^{session/UE}\right] = E\left[N_{SNR/SNA}^{session/UE}\right]$$

$$E\left[N_U^{session/UE}\right] = N_1 \times p_1 + N_2 \times p_2 + N_3 \times p_3 + ... + N_k \times p_k$$

where:

- N_i : is the number of update message pairs exchanged between two consecutive auth/author (re-auth/re-author) procedures.
- p_i : is some probability to be determined later on.

N_i can be determined by following the same procedure done for the number of auth/author during a session. Thus, for the case where the UE is fixed, $E\left[N_U^{session/UE}\right]$ can be expressed as:

$$E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] = \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_1 + \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_2 + \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_3 + ... + \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_k$$

$$E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] = \frac{\Delta t_{auth/author}}{\Delta t_U} \times \left(p_1 + p_2 + p_3 + ... + p_k\right)$$

where:

- Δt_U : is Interim Update Duration, the interval of time between two consecutive updated message pairs. Similarly to $\Delta t_{auth/author}$, Δt_U is fixed by the Mobile Operator.
- N_i : is the number of update message between two successive auth/author instances (where between the AIR / AIA pair and the first RAR / RAA pair or two successive RAR / RAA pairs) as scene in Figure 20 it is important to note that the auth/author instances (red bars) and the update messages (blue bars) are occurring on separate interfaces, thus can occur simultaneously.

- p_i : is the probability that the AIA message from the AIR / AIA message pair returned carrying a positive answer and the previous RAR / RAA was also successful, with p_1 depending only on the AIR / AIA message pair since it is relative to the first RAR / RAA pair. Thus:
    - $p_1 = p_a$
    - $p_2 = p_a \times p_{success}$
    - $p_3 = p_a \times p_{success}^2$
    - $\ldots \rightarrow p_{m_{fixed-UE}} = p_a \times p_{success}^{m_{fixed-UE}+1}$

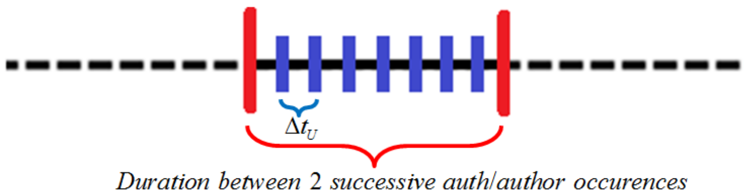


Figure 20. Credit Control Updates

Thus $E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right]$ can be expressed as:

$$E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] = \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_a \times \left(1 + p_{success} + p_{success}^2 + \ldots + p_{success}^{m_{fixed-UE}+1}\right)$$

$$E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] = \frac{\Delta t_{auth/author}}{\Delta t_U} \times p_a \times \left(\sum_{k=0}^{m_{fixed-UE}+1} p_{success}^k\right)$$

The final term in the series is raised to the power "$m_{fixed-UE}+1$" and not "$m_{fixed-UE}$" because "$m_{fixed-UE}$" was first derived by taking $\left(\frac{E\left[\Delta t_{session}\right]}{\Delta t_{auth/author}} - 1\right)$ where the (-1) removes the AIA / AIR message pair and considers only the number of RAA / RAR message pairs. Thus taking "$m_{fixed-UE}+1$", includes the messages of the first interval, i.e. the messages between the AIA / AIR and the first RAA / RAR pair. Furthermore, for the case where the UE moves fast enough such that the auth/author (re-auth/re-author) always occur via cell boundary crossing and never via $\Delta t_{auth/author}$ expiry, the expression of $E\left[\left(N_U^{session/UE}\right)_{cell-cross}\right]$ can be derived by analogy:

$$E\left[\left(N_U^{session/UE}\right)_{cell-cross}\right] = \frac{E\left[\Delta t_{residence}\right]}{\Delta t_U} \times p_a \times \left(\sum_{k=0}^{m_{cell-cross}+1} p_{success}^k\right)$$

Finally, and in contracts with the $E\left[N_{RAR/RAA}\right]$, the $E\left[N_U^{session/UE}\right]$ does not contain a hybrid term like $E\left[N_{RAR/RAA}^{crossing|timeout}\right]$ that considers both cases (fixed and mobile UE) since, unlike $N_{RAR/RAA}$, $N_U^{session/UE}$ does not increase by some constant number of message pairs "k" each time cell crossing occurs ($k_{RAR/RAA} = 1$), but relies more on the $\Delta t_{session}$. Then, the $E\left[N_U^{session/UE}\right]$ can be expressed as:

$$E\left[N_U^{session/UE}\right] = \left(E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] \times p_{fixed-UE} + E\left[\left(N_U^{session/UE}\right)_{cell-cross}\right] \times p_{cell-cross}\right)$$

Thus, for the following message pair exchange seen in Figure 16, the total number of message pairs on the Gx, Ro and Sy interfaces can be calculate using Table 2:

Table 2. Parametrized Equation - Number of Message Pairs - Gx, Ro & Sy Interfaces

Term	Expression
$E\left[N_{CC-Total}^{Gx,Ro,Sy}\right]$	$E\left[N_{I/T}^{session/UE}\right] + E\left[N_U^{session/UE}\right] + E\left[N_{I/T}^{session/UE}\right]$
$E\left[N_I^{session/UE}\right]$	$1 \times p_a$
$E\left[N_T^{session/UE}\right]$	$1 \times p_a$
$E\left[N_U^{session/UE}\right]$	$\left(E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right] \times p_{fixed-UE} + E\left[\left(N_U^{session/UE}\right)_{cell-cross}\right] \times p_{cell-cross}\right)$

$E\left[\left(N_U^{session/UE}\right)_{fixed-UE}\right]$	$\frac{\Delta t_{auth/author}}{\Delta t_U}\times p_a\times\left(\sum_{k=0}^{m_{fixed-UE}+1} p_{success}^k\right)$
$E\left[\left(N_U^{session/UE}\right)_{cell-cross}\right]$	$\frac{E\left[\Delta t_{residence}\right]}{\Delta t_U}\times p_a\times\left(\sum_{k=0}^{m_{cell-cross}+1} p_{success}^k\right)$
$\sum_{k=0}^{m+1} p_{success}^k$	$\frac{1-p_{success}^{m+2}}{1-p_{success}}$
$m_{fixed-UE}$	$\frac{E\left[\Delta t_{session}\right]}{\Delta t_{auth/author}}-2$
$m_{cell-cross}$	$\frac{E\left[\Delta t_{session}\right]}{E\left[\Delta t_{residence}\right]}-2$
$p_{cell-cross}$	$\left(\int_{E\left[\Delta t_{residence}\right]}^{+\infty} f_{session}(t)dt\right)\times\left(\int_{0}^{\Delta t_{auth/author}} f_{residence}(t)dt\right)$
$f_{session}(t)$	PDF of $\Delta t_{session}$
$f_{residence}(t)$	PDF of $\Delta t_{residence}$
$p_{fixed-UE}$	$1-p_{cell-cross}$
$p_{success}$	$p_{ra}\times p_{\Delta t_{grace}}$
$p_{\Delta t_{grace}}$	$\int_{0}^{\Delta t_{grace}} f_{\Delta t_{grace}}(t)dt$
$f_{\Delta t_{grace}}(t)$	PDF of the UE’s duration from “Grace period start” till “Re-auth/re-author procedure” end
p_a	Probability that AIA returns with (+) answer
p_{ra}	Probability that RAA returns with (+) answer
$E\left[\Delta t_{session}\right]$	Average Session Lifetime
$E\left[\Delta t_{residence}\right]$	Average Residence Time (addressed later)

$\Delta t_{auth/author}$	Auth/Author lifetime
$\Delta t_{grace}^{\max}$	Grace Period
Δt_U	Duration between two successive Interim Updates

5.2.3 Session Lifetime and Average Residence Time in a Cell – Considerations

5.2.3.1 The Session Lifetime

$\Delta t_{session}$ can be generalized for a population by replacing it in the equation by $E\left[\Delta t_{session}\right]$. $E\left[\Delta t_{session}\right]$ can also be profiled per application, i.e. $E\left[\Delta t_{session}\right]_{VoIP}$, $E\left[\Delta t_{session}\right]_{Web}$, $E\left[\Delta t_{session}\right]_{VideoCall}$ … and be used in order to study the effects of an application of interest on the EPC. Profiling $E\left[\Delta t_{session}\right]$ can be done by collecting statistics from real networks, or simply mapping them to the session lifetimes that are collected/inputted from/into a simulator.

5.2.3.2 The Residence Time in a Cell

$\Delta t_{residence}$ can also be generalized for a population by replacing it in the equation by $E\left[\Delta t_{residence}\right]$. $E\left[\Delta t_{residence}\right]$ can be determined through various methods such as (but not limited to):

- Collecting statistics, either from the real network data or from a simulator.
- Using a pre-derived expression for $E\left[\Delta t_{residence}\right]$ from previous literature.

- Derive an expression for $E[\Delta t_{residence}]$ by using information from previous literature as a basis.

K. L. Yeung and S. Nanda required an expression of the average sojourn time or residence in a cell in order to complete their analysis for “optimal Mobile Determined Micro-Macro Cell Selection” [59]. They based their expression of the sojourn time in an arbitrary shape cell based on the following assumptions:

- Cells are circular with radius “R”.
- Mobiles are uniformly distributed in the system.
- Mobiles in microcells move in a straight line with direction uniformly distributed between $[0, 2\pi)$.
- The mean mobile speed is: $E[V]$.

Thus, $E[\Delta t_{residence}]$ can be expressed as: $E[\Delta t_{residence}] = \frac{\pi \times R}{2 \times E[V]}$.

In addition, the random walk Markov Chain based model derived by I. F. Akyildiz et al. in [43] for modeling mobility in hexagonal grids formed by hexagonal shaped cells can be adapted to derive an expression for $E[\Delta t_{residence}]$ by looking at the hexagonal grid as a hexagonal shaped cell that is formed by small identical hexagonal shaped areas, as shown in Figure 21. Furthermore, deriving another expression for $E[\Delta t_{residence}]$ can help cross-validate both expressions, the one from [59] and the derived one using the method in [43], against each other.

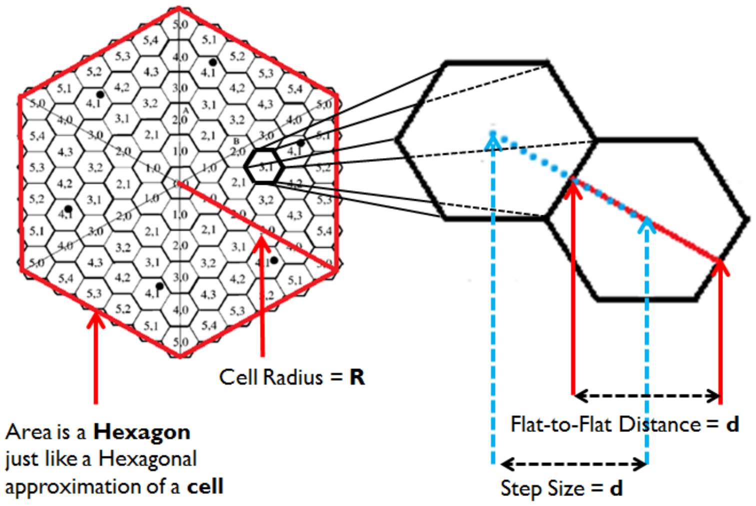


Figure 21. Hexagons, A Cell and the Step Size

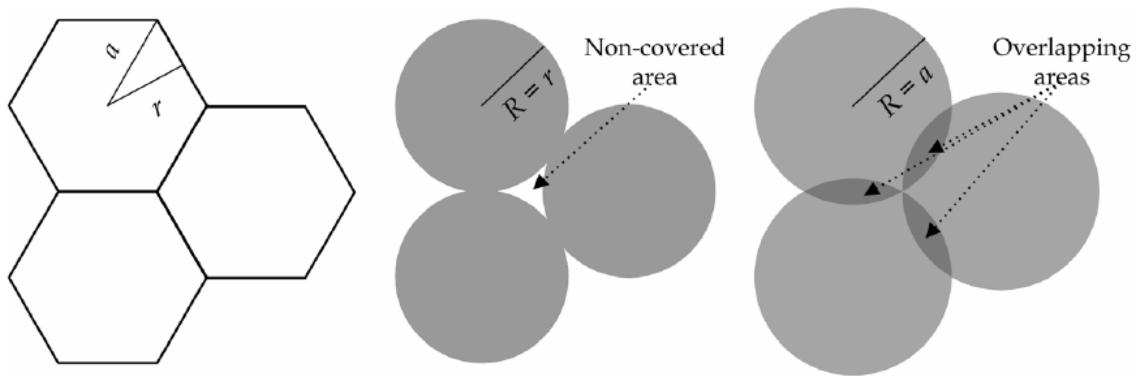


Figure 22. Circular Cells vs. Hexagonal Cells

Thus, the expected residence time of a mobile UE in such a cell would be the expected number of steps the UE needs to make, from one small hexagon to the next, in order to get out of the cell. But first, the step size " d " must be related to the cell radius " R " and the average UE velocity $E[V]$. Hence, let:

- d : be the flat-to-flat distance, i.e. the distance between two parallel sides. This distance is the same between the centers of two identical adjacent hexagons. " d " would then be the step size.

- a: be the distance from the center of a hexagon to one of its vertices. Based on the book chapter by K. B. Baltzis [40], the radius of the cell "R" can be taken to be approximately equal to "a" as shown in Figure 22.
- $A = \frac{\sqrt{3}}{2} \times d^2$: be the area of any hexagon, with d^2 being the flat-to-flat distance squared.
- $\frac{d}{2} = \frac{\sqrt{3}}{2} \times a = \frac{\sqrt{3}}{2} \times R$, with "$\frac{d}{2}$" being the height of the equilateral triangle formed by the center of the hexagon and two consecutive vertices.

Second, a relationship between the step size "d", the cell radius "R" and the number of hexagons inside the cell, as well as a relationship between the number of hexagons and the number of layers following which the cells were arrayed is found. Using Figure 21 as a reference, take:

- n: The number of tiny hexagons in the big hexagonal shaped cell.
- n_L: The number of layers in the big hexagon.
- $n_L = 6$

$$n = 1 + 6 + 12 + 18 + 24 + 30$$

$$n = 1 + 6 \times (1 + 2 + 3 + 4 + 5)$$

The sum $(1 + 2 + 3 + 4 + 5)$ is the finite sum of natural numbers and is equal to $\frac{5 \times (5+1)}{2}$, thus:

$$n = 1 + 6 \times \frac{5 \times (5+1)}{2}$$

$$n = 1 + 6 \times \frac{(6-1) \times ((6-1)+1)}{2} = 1 + 6 \times \frac{(6-1) \times ((6))}{2}$$

This equation can be generalized to:

$$n = 1 + 6 \times \frac{(n_L - 1) \times n_L}{2}$$

The relation between the cell (big hexagon) and the little ones forming its inside it can be expressed as:

$$A_{cell} \simeq n \times A_{tinyHexa}$$

$$\frac{\sqrt{3}}{2} \times d_{cell}^2 \simeq n \times \frac{\sqrt{3}}{2} \times d_{tinyHexa}^2$$

(with d_{item}^2 being the flat-to-flat distance squared of "item")

$$\frac{\sqrt{3}}{2} \times \left(R \times \sqrt{3}\right)^2 \simeq n \times \frac{\sqrt{3}}{2} \times d_{tinyHexa}^2$$

$$\left(R \times \sqrt{3}\right)^2 \simeq n \times d_{tinyHexa}^2$$

$$d_{tinyHexa} = R \times \sqrt{\frac{3}{n}}$$

$$d_{tinyHexa} = R \times \sqrt{\frac{3}{1 + 6 \times \frac{(n_L - 1) \times n_L}{2}}}$$

Thus the average number distance traveled by a UE inside a cell, starting from any random location within the cell until leavung leaves it can be expressed as:

$$avgDistance = avgNumberOfSteps \times StepSize$$

$$avgDistance = avgNumberOfSteps \times d$$

And given $E[V]$ to be the average velocity of the UE, then $E[\Delta t_{residence}]$ can be expressed as:

$$E[\Delta t_{residence}] = \frac{avgDistance}{E[V]} = \frac{avgNumberOfSteps \times d}{E[V]}$$

$$E\left[\Delta t_{residence}\right] = \frac{avgNumberOfSteps \times R \times \sqrt{\dfrac{3}{1+6\times\dfrac{(n_L - 1)\times n_L}{2}}}}{E\left[V\right]}$$

Finally, for each value of "n_L", the average number of steps $L(n_L - 1, z)$ must be recalculated based on the following formula developed by I. F. Akyildiz et al. [43] and then divided by 6 since each boundary state has 5 other symmetrical states:

$$L(n_L - 1, z) = \sum_{k=1}^{\infty} \sum_{x=0}^{n_L - 1} \sum_{y=0}^{n_L - 2} k \times p_{k,(x,y),(n_L, z)}$$

$$p_{k,(x,y),(i,j)} = \begin{cases} p_{(x,y),(i,j)} & for\ k = 1 \\ p^{k}_{(x,y),(i,j)} - p^{k-1}_{(x,y),(i,,j)} & for\ k > 1 \end{cases}$$

with "$p_{(x,y),(i,j)}$" being the 1-step Markovian probability matrix derived from Figure 9**Error! Reference source not found.** with each tiny hexagon being treated as a Markovian state. The calculation of $L(n_L - 1, z)$ can be easily done using a Matlab script, and k can be taken from 1 to 200, since for $k > 200$, the values of $L(n_L - 1, z)$ converges to the same value.

CHAPTER VI

SIMULATION AND VALIDATION

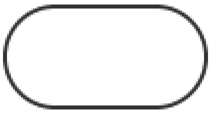
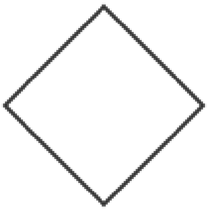
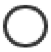
The validity of any model is best solidified by comparing it to real life data. However, in the case of a AAA protocol, getting traces and real time data is extremely difficult because the information handled by such protocols is of sensitive nature, and Diameter is no exception. Hence, validating the mathematical model discussed in the previous chapter is to be done via simulation. After much investigation, no third party simulator was found sufficient enough for the needs of this study and both of the strongest candidates, Riverbed Modeler 18.0 (previously Opnet) and NS3 lack the modules of an LTE core (EPC) - which is the main target of this study - and most importantly, an implementation of Diameter protocol. Thus, the only solution was to come up with a MATLAB simulator, albeit simple, which possesses the needed functionalities to study the effects of Diameter signaling in an EPC.

6.1 Experimental Setup: Simulating on MATLAB

Multiple simulations were run on MATLAB. Two separate scripts were written. One for Fixed UEs and another that incorporates the effects of a UE’s mobility to the amount of signaling in the EPC that is caused by the UE’s active data session and the UE’s movements. Figure 23 and Figure 25 show the flowcharts for the MATLAB simulation scripts respectively. The flowcharts blocks hold the keyword “step №” given that the operations done within some of the steps can be many. Each step is explained in detail in the respective sections for each of the MATLAB scripts. Furthermore, Figure

24 and Figure 26 show an example of the Authentication / Authorization (AIR / AIA) and Reauthentication / Reauthorization (RAR / RAA) instances on the S6 interface during a fixed and mobile UE's active data session respectively. Finally, Table 3 shows the flowchart blocks and entities used along with a description of each.

Table 3. Flowchart Blocks and Entities

Flowchart Symbol	**Symbol Description**
	Start / End of a MATALAB script.
	Block of successive operations: calculation, assignment, …
	Condition which could be: • An If / Else statement. • Checking a condition at the start of a loop (for, while, …) and subsequently rechecking the condition at loopback.
	Used for drawing purposes only as a point where multiple arrows cross.
	Represents the "direction of flow" of the step execution sequence within a MATLAB script.

6.1.1 Fixed UE

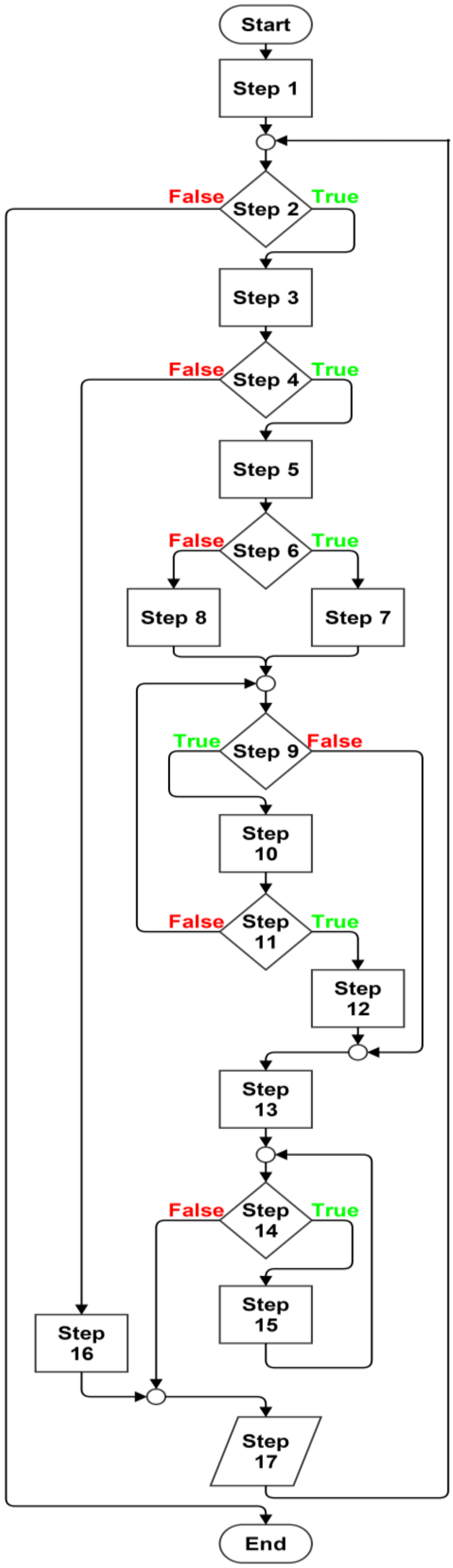


Figure 23. MATLAB Simulation Flowchart - Fixed UE

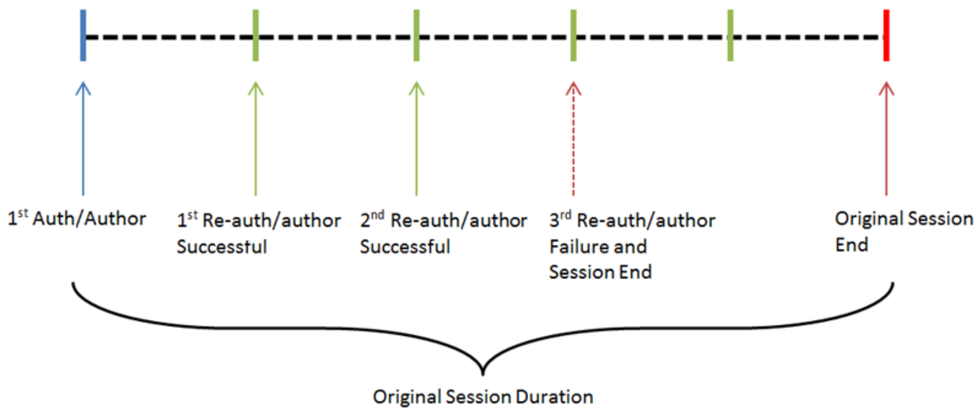


Figure 24. Example Auth/Author Sequence for Fixed UE

<u>Step 1: Variable Initialization and Folder / File creation</u>

1. At the start of the MATLAB scripts multiple variables are initialized:

    1. p_a : The AIR / AIA success rate is set to the desired value from $[0\ ;\ 1]$. This value will be used for the entire UE population during a script run.

    2. p_{ra} : The RAR / RAA success rate is set to the desired value from $[0\ ;\ 1]$. This value will be used for the entire UE population during a script run.

    3. $\Delta t_{session}$: The session lifetime for each UE is set to 0. This value will be reinitialized randomly for each UE. The session lifetime follows the desired distribution set by the tester. (Unit: seconds).

    4. $E[\Delta t_{session}]$: The average session lifetime is the value around which the random $\Delta t_{session}$ are created. (Unit: seconds).

    5. $\Delta t_{auth/author}$: The authentication / authorization lifetime is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).

6. Δt_U : The interim duration interval is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).
7. $\Delta t_{grace}^{\max}$: The grace period is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).
8. $E\left[\Delta t_{grace}\right]$: This term is the average time around which a UE starts and finishes its reauthentication and reauthorization procedure. This value will be used for the entire UE population during a "script run". (Unit: seconds).
9. $p_{\Delta t_{grace}}$: The probability that a UE starts and finishes its reauthentication and reauthorization procedure during $\Delta t_{grace}^{\max}$ is calculated using the desired Probability Density function chosen by the tester. This term will be combined with p_{ra} and used later.
10. Num_of_UE : The UE population size is set to desired value.
11. noa : The total number of message pairs per UE on the S6 interface is initialized to 0. This value is reset to 0 for each UE.
12. nou : The total number of message pairs per UE on the Gx, Ro and Sy interfaces is initialized to 0. This value is reset to 0 for each UE.
13. $temp$: A temporary variable used as needed.

2. Once the above variables are initialized, the current script run files and folder are created in order to save the MATLAB simulation's information and results.
   1. c : The scripts gets the current clock time and saves it in the variable c . This is used to name the created folders and files automatically.
   2. The "current run" folder is created and named using c .
   3. The following files are created within the "current run" folder:

- Info.txt: This file contains information on the current run, i.e. the original values of the initialized variables above.
- Num_of_auth.txt: This file contains the total number of message pairs per UE on the S6 interface for the "current run".
- Num_of_up.txt: This file contains the total number of message pairs per UE on the Gx, Ro and Sy interfaces for the "current run".

3. The script proceeds to **Step 2**.

Step 2: For Loop for the UE Population

This is the start and loopback point for the "for loop" that goes through the activity of each UE from UE "1" to UE "Num_of_UE". As long as the number of UEs is less than the population size, the script creates a new UE and proceeds to **Step 3**. Else, the script proceeds to the state "**end**".

Step 3: Initializing the ith UE's Relative Variables and the AIR / AIA Message Pair

1. $temp = 0.0$: The variable temp is set to 0.
2. $\Delta t_{session}$: A new session lifetime is randomly created of the ith UE based on the desired distribution set by the tester and $E\left[\Delta t_{session}\right]$. (Unit: seconds).
3. $t\_UE\_sim = 0.0$: This variable denotes to track the simulation time for the ith UE. (Unit: seconds).
4. $t\_UE\_start = 0.0$: This variable denotes the data session start instance for the ith. (Unit: seconds).
5. $t\_UE\_end = t\_UE\_start + \Delta t_{session}$: This variable denotes the data session end instance for the ith UE. This variable is update accordingly later on in case of (re)authentication and (re)authorization failure. (Unit: seconds).

6. $noa = 0$: noa is reset to 0 for the i^{th} UE.

7. $nou = 0$: nou is reset to 0 for the i^{th} UE.

8. Num_of_Reauth: The total number of possible RAR / RAA message pairs is reset to 0 for the i^{th} UE. This particular variable appears only in the MATLAB script for Fixed UEs because Fixed UEs always reauthenticate and reauthorize via $\Delta t_{auth/author}$ expiry, hence at fixed intervals. So the total number of RAR / RAA message pairs for the i^{th} UE that has a $\Delta t_{session}$ can be calculated. However, this is not the number reported at the end since each RAR / RAA message pair is checked for it procedure's success. This will be further elaborated from **Step 9** to **Step 12**.

9. $Num\_of\_Updates$: The total number of possible CCR-U / CCA-U message pairs and SNR / SNA message pairs is reset to 0 for the i^{th} UE.

10. $t\_AIR = t\_UE\_start$: Set the time instance of the AIR message to the start time of the i^{th} UE's session start time instance. (Unit: seconds).

11. $t\_UE\_sim = t\_AIR$: Update the time tracking variable. (Unit: seconds).

12. $\Delta t_{AIR/AIA}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the AIR message and receiving the AIA message for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

13. $t\_AIA = t\_AIR + \Delta t_{AIR/AIA}$: Calculate the time instance of sending the AIA message. (Unit: seconds).

14. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.

15. $temp$: create a uniformly distributed random variable from $[0 : 1]$ to represent the ith UE probability of its AIA message returning with a positive answer. This variable and its respective value will be used in **Step 4**.

Step 4: If statement – The ith UE first Authentication and Authorization

1. If $(temp \leq p_a)$: if this condition is **True**, it means the ith UE passed the first authentication and authorization procedure, i.e. AIA returned with a positive value. The script proceeds to **Step 5**.
2. Else: if the condition is **False**, it means that the ith UE failed its first authentication and authorization procedure. The script proceeds to **Step 16**.

Step 5: The ULR / ULA Message Pair

1. $\Delta t_{AIA/ULR}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the AIA message and sending the ULR message for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).
2. $t\_ULR = t\_AIA + \Delta t_{AIA/ULR}$: Calculate the time instance of sending the ULR message. (Unit: seconds).
3. $\Delta t_{ULR/ULA}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the ULR message and receiving the ULA message for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

4. $t\_ULA = t\_ULR + \Delta t_{ULR/ULA}$: Calculate the time instance of receiving the ULA message. (Unit: seconds).

5. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.

Step 6: If Statement - Determine the Total Possible Number of RAR / RAA Message Pairs for the ith UE

If $\Delta t_{session} \left(\bmod \Delta t_{auth/author} \right) = 0$: This statement checks if $\Delta t_{session}$ is divisible by $\Delta t_{auth/author}$. If it is, the script proceeds to **Step 7**, else the script proceeds to **Step 8**.

Step 7: The ith UE's Session Lifetime is Divisible by the Authentication and Authorization Lifetime

If the statement in **Step 6** is **True,** then $Num\_of\_Reauth = \left\lfloor \frac{\Delta t_{session}}{\Delta t_{auth/author}} \right\rfloor - 1$.

The (-1) is used to remove the RAR / RAA message pair whose time instance would coincide with the ith UE's session's end time instance t_UE_end. The script proceeds to **Step 9**.

Step 8: The ith UE's Session Lifetime is Not Divisible by the Authentication and Authorization Lifetime

Else: if the statement in **Step 6** was **False**, then

$Num\_of\_Reauth = \left\lfloor \frac{\Delta t_{session}}{\Delta t_{auth/author}} \right\rfloor$ since the last RAR / RAA message pair's time instance would not coincide with the ith UE's session's end time instance t_UE_end.

The script proceeds to **Step 9.**

Step 9: For Loop for the RAR / RAA Message Pairs Relative to the ith UE

This is the start and "loop back" point for the "for loop" that goes through the ith UE's RAR / RAA message pairs sequentially, from the 1st one till the last $(Num\_of\_Reauth)^{th}$. However, the script could exit this loop if the UE fails its current reauthentication and reauthorization process. This is further elaborated at **Step 11**. But, as long as the previous reauthentication and reauthorization process is successful, the loop would proceed to Step 10. Else, the script proceeds to **Step 13**.

Step 10: The ith UE's kth RAR / RAA Message Pair

1. $t\_RAR = t\_UE\_sim + \Delta t_{auth/author}$: Calculate the time instance of sending the kth RAR message. (Unit: seconds).
2. $\Delta t_{RAR/RAA}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the RAR message and receiving the RAA message for the ith UE's kth RAR / RAA message pair. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).
3. $t\_RAA = t\_RAR + \Delta t_{RAR/RAA}$: Calculate the time instance of receiving the RAA message. (Unit: seconds).
4. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.
5. $t\_UE\_sim = t\_RAR$: Update the simulation time tracker to the time instance of the kth RAR message. (Unit: seconds).
6. $temp$: create a uniformly distributed random variable from $[0\ :\ 1]$ to represent the ith UE's kth RAR / RAA message pair's probability of its RAA message returning with a positive answer and that the kth RAR / RAA procedure finished with the

limits of the grace period. This variable and its respective value will be used in **Step 11**.

Step 11: If Statement – The i^th UE's K^th RAR / RAA Message Pair's RAA

1. If $\left(temp > \left(p_{ra} \times p_{\Delta t_{grace}}\right)\right)$: if this condition is **False**, it means the i^th UE's k^th RAR / RAA message pair's RAA passed the reauthentication and reauthorization procedure, i.e. the RAA returned with a positive value and the i^th UE was able to complete the procedure within the limits of the grace period $\left(\Delta t_{grace}^{\max}\right)$. The script loops back to **Step 9**.
2. Else: if the condition is **True**, it means the i^th UE's k^th RAR / RAA message pair's RAA failed the reauthentication and reauthorization procedure, i.e. either the RAA returned with a negative value or the i^th UE was not able to complete the procedure within the limits of the grace period $\left(\Delta t_{grace}^{\max}\right)$. The script proceeds to **Step 12**.

Step 12: The i^th UE's K^th RAR / RAA Message Pair's RAA Failure

Given that the k^th reauthentication and reauthorization procedure failed:

1. $t\_UE\_end = t\_RAA$: Update the i^th UE's session's end time instance to the time instance of the last RAA message received.
2. Break from "for loop" (**Step 9**) and proceed to **Step 13**.

Step 13: The Credit Control Message Pair Initialization and Termination on the Gx, Ro and Sy Interfaces

1. The Credit Control Initialzation Messages are determined first:
    1. $\Delta t_{ULA/CCR-I}^{Gx}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the ULA message and sending the CCR-I message for the i^th UE on the Gx

interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).

2. $t\_CCR\_I\_Gx = t\_ULA + \Delta t_{ULA/CCR-I}^{Gx}$: Calculate the time instance of sending the CCR-I message on the Gx interface. (Unit: seconds).
3. $t\_UE\_sim = t\_CCR\_I\_Gx$: Update the time tracking variable. This will be used later on in **Step 15**. (Unit: seconds).
4. $\Delta t_{CCR-I/CCA-I}^{Gx}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-I message and receiving the CCA-I message for the i^{th} UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).
5. $t\_CCA\_I\_Gx = t\_CCR\_I\_Gx + \Delta t_{CCR-I/CCA-I}^{Gx}$: Calculate the time instance of receiving the CCA-I message on the Gx interface. (Unit: seconds).
6. $\Delta t_{CCA-I/SLR}^{Gx/Sy}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-I message on the Gx interface and sending the SLR message on the Sy interface for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).
7. $t\_SLR\_Sy = t\_CCA\_I\_Gx + \Delta t_{CCA-I/SLR}^{Gx/Sy}$: Calculate the time instance of sending the SLR message on the Sy interface. (Unit: seconds).

8. $\Delta t_{SLR/SLA}^{Sy}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the SLR message and receiving the SLA message for the ith UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

9. $t\_SLR\_Sy = t\_SLA\_Sy + \Delta t_{SLR/SLA}^{Sy}$: Calculate the time instance of receiving the SLA message on the Sy interface. (Unit: seconds).

10. $\Delta t_{SLA/CCR-I}^{Sy/Ro}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the SLA message on the Sy interface and sending the CCR-I message on the Ro interface for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

11. $t\_CCR\_I\_Ro = t\_SLA\_Sy + \Delta t_{SLA/CCR-I}^{Sy/Ro}$: Calculate the time instance of sending the CCR-I message on the Ro interface. (Unit: seconds).

12. $\Delta t_{CCR-I/CCA-I}^{Ro}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-I message and receiving the CCA-I message for the i^{th} UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

13. $t\_CCA\_I\_Ro = t\_CCR\_I\_Ro + \Delta t^{Ro}_{CCR-I/CCA-I}$: Calculate the time instance of receiving the CCA-I message on the Ro interface. (Unit: seconds).
14. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

2. Given that the i[th] UE's session lifetime, and hence t_UE_end was determined after **Steps 11** and **12**, the sequence of termination messages can be determined while referring to Figure 16 as a reference. Doing that would help pinpoint the interval during the i[th] UE's activity where the interim credit control update messages (CCR-U / CCA-U, SNR / SNA) were exchanged and would make mapping their occurrence time instances easier on their respective interfaces. ***The sequence of message is determined in reverse order*** with t_UE_end as the reference time instance:
    1. $t\_CCA\_T\_Gx = t\_UE\_end$: Calculate the time instance of receiving the CCA-T message on the Gx interface. The CCA-T on the Gx interface is the last message exchanged. (Unit: seconds).
    2. $\Delta t^{Gx}_{CCA-T/CCR-T}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-T message and receiving the CCA-T message for the i[th] UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).
    3. $t\_CCR\_T\_Gx = t\_CCA\_T\_Gx - \Delta t^{Gx}_{CCA-T/CCR-T}$: Calculate the time instance of sending the CCA-T message on the Gx interface. (Unit: seconds).

4. $\Delta t^{Sy/Gx}_{STA/CCR-T}$: Create a uniformly distributed random number from $[1 \,:\, 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the STA message on the Sy interface and sending the CCR-T message on the Gx interface for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \,:\, 9]$. (Unit: seconds).

5. $t\_STA\_Sy = t\_CCR\_T\_Gx + \Delta t^{Sy/Gx}_{STA/CCR-T}$: Calculate the time instance of receiving the STA message on the Sy interface. (Unit: seconds).

6. $\Delta t^{Sy}_{STA/STR}$: Create a uniformly distributed random number from $[1 \,:\, 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the STR message and receiving the STA message for the ith UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \,:\, 9]$. (Unit: seconds).

7. $t\_STR\_Sy = t\_STA - \Delta t^{Sy}_{SLA/SLR}$: Calculate the time instance of sending the STR message on the Sy interface. (Unit: seconds).

8. $\Delta t^{Ro/Sy}_{CCA-T/STR}$: Create a uniformly distributed random number from $[1 \,:\, 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-T message on the Ro interface and sending the STR message on the Sy interface for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \,:\, 9]$. (Unit: seconds).

9. $t\_CCA\_T\_Ro = t\_STR\_Sy - \Delta t^{Ro/Sy}_{STR/CCA-T}$: Calculate the time instance of receiving the CCA-T message on the Ro interface. (Unit: seconds).

10. $\Delta t^{Ro}_{CCA-T/CCR-T}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-T message and receiving the CCA-T message for the i^th UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).

11. $t\_CCR\_T\_Ro = t\_CCA\_T\_Ro - \Delta t^{Ro}_{CCA-T/CCR-T}$: Calculate the time instance of sending the CCR-T message on the Ro interface. (Unit: seconds).

12. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

3. $Num\_of\_Updates = \left\lfloor \dfrac{t\_CCR\_T\_Ro - (t\_UE\_sim + \Delta t_U)}{\Delta t_U} \right\rfloor$: Calculate the total number of CCR-U / CCA-U and SNR / SNA message pairs within the time interval $[t\_CCR\_U\_Gx\ :\ t\_CCR\_T\_Ro]$, where $t\_CCR\_U\_Gx = (t\_UE\_sim + \Delta t_U)$. In this case, the time instance $t\_CCR\_U\_Gx$ stands for the time instance of the 1^st CCR-U message on the Gx interface.

4. The script proceeds to **Step 14**.

Step 14: For Loop for the Credit Control Message Pairs Relative to the ith UE on the Gx, Ro and Sy Interfaces

This is the start and "loop back" point for the "for loop" that goes through the ith UE's CCR-U / CCA-U and SNR / SNA message pairs sequentially, from the 1st one till the last $(Num\_of\_Updates)^{th}$. Once all the message pairs's time instances are mapped on their respective interfaces, the script proceeds to **Step 17**.

Step 15: Credit Control Interim Updates on the Gx, Ro Aand Sy Interfaces

1. $t\_CCR\_U\_Gx = t\_UE\_sim + \Delta t_U$: Calculate the time instance of sending the jth CCR-U message on the Gx interface. (Unit: seconds).
2. $\Delta t_{CCR-U/CCA-U}^{Gx}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-U message and receiving the CCA-U message for the ith UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).
3. $t\_CCA\_U\_Gx = t\_CCR\_U\_Gx + \Delta t_{CCR-U/CCA-U}^{Gx}$: Calculate the time instance of receiving the CCA-I message on the Gx interface. (Unit: seconds).
4. $\Delta t_{CCA-U/CCR-U}^{Gx/Ro}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-U message on the Gx interface and sending the CCR-U message on the Ro interface for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

5. $t\_CCR\_U\_Ro = t\_CCA\_U\_Gx + \Delta t^{Gx/Ro}_{CCA-U/CCR-U}$: Calculate the time instance of sending the CCR-U message on the Ro interface. (Unit: seconds).

6. $\Delta t^{Ro}_{CCR-U/CCA-U}$: Create a uniformly distributed random number from $[1 \;:\; 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-U message and receiving the CCA-U message for the i^th UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \;:\; 9]$. (Unit: seconds).

7. $t\_CCA\_U\_Ro = t\_CCR\_U\_Ro + \Delta t^{Ro}_{CCR-U/CCA-U}$: Calculate the time instance of receiving the CCA-U message on the Ro interface. (Unit: seconds).

8. $\Delta t^{Ro/Sy}_{CCA-U/SNR}$: Create a uniformly distributed random number from $[1 \;:\; 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-U message on the Ro interface and sending the SNR message on the Sy interface for the i^th UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \;:\; 9]$. (Unit: seconds).

9. $t\_SNR\_Sy = t\_CCA\_U\_Ro + \Delta t^{Ro/Sy}_{CCA-U/SNR}$: Calculate the time instance of sending the SNR message on the Sy interface. (Unit: seconds).

10. $\Delta t^{Sy}_{SNR/SNA}$: Create a uniformly distributed random number from $[1 \;:\; 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the SNR message and receiving the SNA message for the i^th UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \;:\; 9]$. (Unit: seconds).

11. $t\_SNA\_Sy = t\_SNR\_Sy + \Delta t^{Sy}_{SNR/SNA}$: Calculate the time instance of receiving the SNA message on the Sy interface. (Unit: seconds).

12. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

13. $t\_UE\_sim = t\_CCR\_U\_Gx$: Update the simulation time tracker to the time instance of the jth CCR-U message exchanged on the Gx interface. (Unit: seconds).

14. The script loops back to **Step 14**.

Step 16: If Statement – ith UE's First Authentication and Authorization Failure

$t\_UE\_end = t\_AIA$: Set the ith UE's session's end time instance to t_AIA given that the condition from **Step 4** is **False**, i.e the ith UE failed its first authentication and authorization procedure and its session is terminated. The script then proceeds to **Step 17**.

Step 17: Output to Files of the ith UE's Results

1. Export noa and nou of the ith UE to their respective output files:
   - $noa \rightarrow$ Num_of_auth.txt
   - $nou \rightarrow$ Num_of_up.txt
2. The script loops back to **Step 2**.

6.1.2 Mobile UE

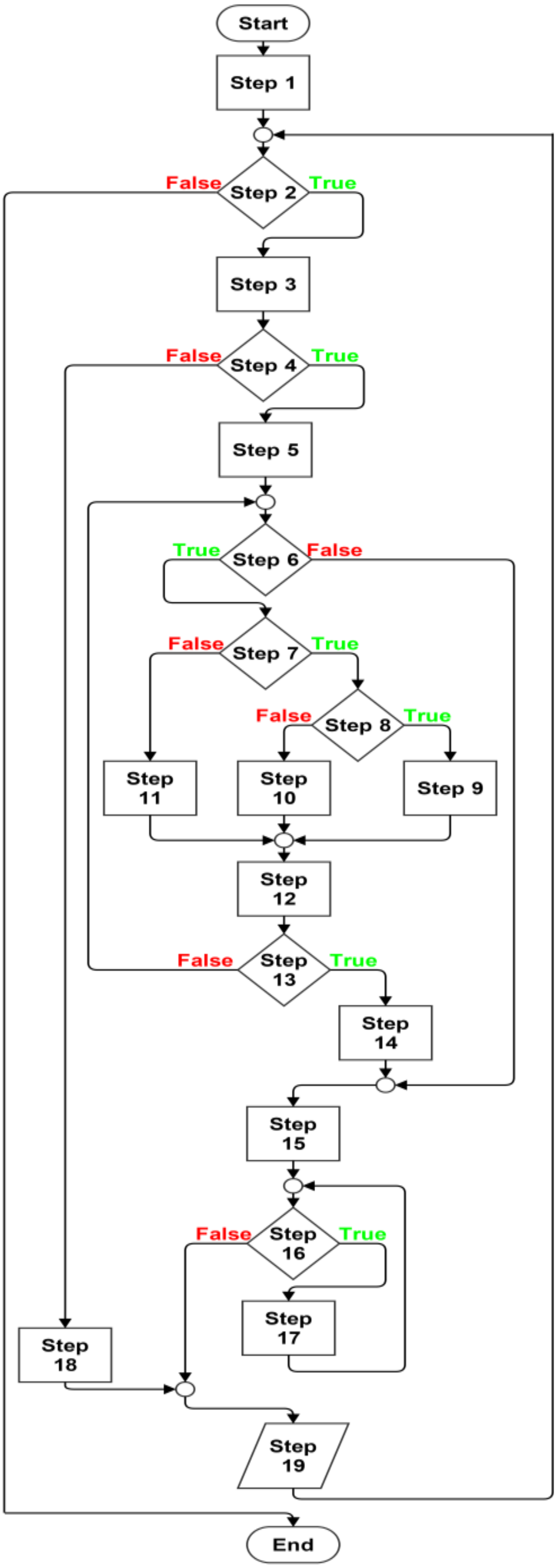


Figure 25. MATLAB Simulation Flowchart - Mobile UE

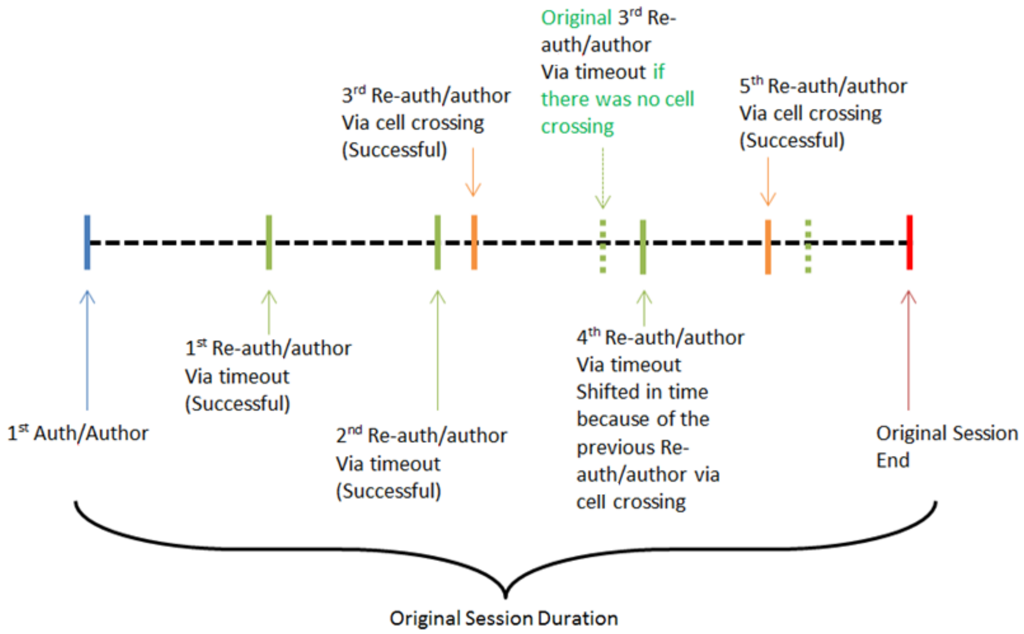


Figure 26. Example Auth/Author Sequence of Mobile UE

Step 1: Variable Initialization and Folder / File creation

1. At the start of the MATLAB scripts multiple variables are initialized:

    1. p_a: The AIR / AIA success rate is set to the desired value from $[0\ ;\ 1]$. This value will be used for the entire UE population during a script run.

    2. p_{ra}: The RAR / RAA success rate is set to the desired value from $[0\ ;\ 1]$. This value will be used for the entire UE population during a script run.

    3. $\Delta t_{session}$: The session lifetime for each UE is set to 0. This value will be reinitialized randomly for each UE. The session lifetime follows the desired distribution set by the tester. (Unit: seconds).

    4. $E[\Delta t_{session}]$: The average session lifetime is the value around which the random $\Delta t_{session}$ are created. (Unit: seconds).

5. $\Delta t_{auth/author}$: The authentication / authorization lifetime is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).

6. Δt_{U} : The interim duration interval is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).

7. $\Delta t_{grace}^{\max}$: The grace period is set to the desired value. This value will be used for the entire UE population during a script run. (Unit: seconds).

8. $E\left[\Delta t_{grace}\right]$: This term actually is the average time around which a UE starts and finishes its reauthentication and reauthorization procedure. This value will be used for the entire UE population during a script run. (Unit: seconds).

9. $p_{\Delta t_{grace}}$: The probability that a UE starts and finishes its reauthentication and reauthorization procedure during $\Delta t_{grace}^{\max}$ is calculated using the desired Probability Density function chosen by the tester. This term will be combined with p_{ra} and used later.

10. Num_of_UE : The UE population size is set to desired value.

11. noa : The total number of message pairs per UE on the S6 interface is initialized to 0. This value is reset to 0 for each UE.

12. nou : The total number of message pairs per UE on the Gx, Ro and Sy interfaces is initialized to 0. This value is reset to 0 for each UE.

13. $temp$: A temporary variable used as needed.

14. NoS : Number of steps needed to exit a hexagonal cell. This number is generated using the algorithm from Chapter V, section 2.3.2.

15. n_L : The number of layers within the hexagonal shaped cell. This parameter is used when generating the NoS .
16. R : The radius of the LTE cell. It is set to the desired value and assumed to be the same for all the cells a UE moves into during the MATLAB simulation. (Unit: meters)
17. $E[V]$: The average UE speed of the UE population. This parameter is set to the desired value and used in calculating $E[\Delta t_{residence}]$. (Unit: meters/second).
18. $E[\Delta t_{residence}]$: The UE's average residence time in an LTE cell. This term is calculated using the formula derived in Chapter V, section 2.3.2. $E[\Delta t_{residence}]$ is the value around which the random $\Delta t_{residence}$ are created. (Unit: seconds).
19. $\Delta t_{residence}$: The residence time per cell for each UE is set to 0. This value will be reinitialized randomly for each UE, and each and every time a UE moves from a cell to another. The residence time follows the desired distribution set by the tester. (Unit: seconds).

2. Once the above variables are initialized, the current script run files and folder are created in order to save the MATLAB simulation's information and results.
    1. c : The scripts gets the current clock time and saves it in the variable c . This is used to name the created folders and files automatically.
    2. The "current run" folder is created and named using c .
    3. The following files are created within the "current run" folder:
        - Info.txt: This file contains information on the current run, i.e. the original values of the initialized variables above.

- Num_of_auth.txt: This file contains the total number of message pairs per UE on the S6 interface for the “current run”.
- Num_of_up.txt: This file contains the total number of message pairs per UE on the Gx, Ro and Sy interfaces for the “current run”.

3. The script proceeds to **Step 2**.

Step 2: For Loop for the UE Population

This is the start and loopback point for the “for loop” that goes through the activity of each UE from UE “1” to UE “ Num_of_UE ”. As long as the number of UEs is less than the population size, the script creates a new UE and proceeds to **Step 3**. Else, the script proceeds to the state “**end**”.

Step 3: Initializing the ith UE’s Relative Variables and the AIR / AIA Message Pair

1. $temp = 0.0$: The variable temp is set to 0.
2. $\Delta t_{session}$: A new session lifetime is randomly created of the ith UE based on the desired distribution set by the tester and $E\left[\Delta t_{session}\right]$. (Unit: seconds).
3. $t\_UE\_sim = 0.0$: This variable denotes to track the simulation time for the ith UE. (Unit: seconds).
4. $t\_UE\_start = 0.0$: This variable denotes the data session start instance for the ith. (Unit: seconds).
5. $t\_UE\_end = t\_UE\_start + \Delta t_{session}$: This variable denotes the data session end instance for the ith UE. This variable is update accordingly later on in case of (re)authentication and (re)authorization failure. Unit: seconds).
6. $noa = 0$: noa is reset to 0 for the ith UE.
7. $nou = 0$: nou is reset to 0 for the ith UE.

8. $Num\_of\_Updates$: The total number of possible CCR-U / CCA-U message pairs and SNR / SNA message pairs is reset to 0 for the ith UE.

9. $\Delta t_{residence}$: A new residence time is randomly created of the ith UE based on the desired distribution set by the tester and $E\left[\Delta t_{residence}\right]$. (Unit: seconds).

10. $\theta_{residence}$: Create a uniformly distributed random number from $\left[0\ :\ \Delta t_{residence}\right]$ to represent the duration the ith UE has already spent in its current cell before starting its data session. This is used only for the residence time in the 1st cell given that, when the UE moves into a new cell, his data session is already active. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $\left[0\ :\ \Delta \mathrm{t}_{residence}\right]$. (Unit: seconds).

11. $t\_previous = t\_UE\_start$: This variable marks the time instance of the previous (re)authentication and (re)authorization procedure. At the begging of the MATLAB simulation. It is set the start time instance of the ith UE's session. (Unit: seconds).

12. $t\_current\_cross$: This variable marks the time instance on which the ith UE will cross from it's the current cell it is residing in to a new one. This variable will be updated later on. (Unit: seconds).

13. $t\_next\_auth = t\_UE\_start + \Delta t_{auth/author}$: This variable marks the time instance of the next reauthentication and reauthorization procedure via $\Delta t_{auth/author}$ expiry. (Unit: seconds).

14. $t\_AIR = t\_UE\_start$: Set the time instance of the AIR message to the start time of the ith UE's session start time instance. (Unit: seconds).

15. $t\_UE\_sim = t\_AIR$: Update the time tracking variable. (Unit: seconds).

16. $\Delta t_{AIR/AIA}$: Create a uniformly distributed random number from $[1 \; : \; 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the AIR message and receiving the AIA message for the i^th^ UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \; : \; 9]$. (Unit: seconds).

17. $t\_AIA = t\_AIR + \Delta t_{AIR/AIA}$: Calculate the time instance of sending the AIA message. (Unit: seconds).

18. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.

19. $temp$: create a uniformly distributed random variable from $[0 \; : \; 1]$ to represent the i^{th} UE probability of its AIA message returning with a positive answer. This variable and its respective value will be used in **Step 4**.

Step 4: If Statement – The i^{th} UE first Authentication and Authorization

1. If $(temp \leq p_a)$: if this condition is **True**, it means the i^{th} UE passed the first authentication and authorization procedure, i.e. AIA returned with a positive value. The script proceeds to **Step 5**.
2. Else: if the condition is **False**, it means that the i^{th} UE failed its first authentication and authorization procedure. The script proceeds to **Step 18**.

Step 5: The ULR / ULA Message Pair

1. $\Delta t_{AIA/ULR}$: Create a uniformly distributed random number from $[1 \; : \; 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the AIA message and sending the ULR message for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \; : \; 9]$. (Unit: seconds).

2. $t\_ULR = t\_AIA + \Delta t_{AIA/ULR}$: Calculate the time instance of sending the ULR message. (Unit: seconds).

3. $\Delta t_{ULR/ULA}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the ULR message and receiving the ULA message for the i[th] UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).

4. $t\_ULA = t\_ULR + \Delta t_{ULR/ULA}$: Calculate the time instance of receiving the ULA message. (Unit: seconds).

5. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.

6. $t\_current\_cross = t\_ULA + \Delta t_{residence} - \theta_{residence}$: The time instance of the i[th] UE cell boundary crossing is calculated. $\theta_{residence}$ is subtracted only for the i[th] UE's 1[st] cell boundary crossing. All additional $t\_current\_cross$ will not be in function of $\theta_{residence}$. (Unit: seconds).

7. The script proceeds to **Step 6**.

Step 6: While Loop to Check if the i[th] UE's Next Auth / Author Even Is Within its Session Lifetime

The while loop check if the time instance of the i[th] UE's next reauthentication and reauthorization event is smaller than this UE's session end time instance. As long as the condition remains true, the script proceeds to **Step 7**, i.e. the ith UE's session is stil active and ongoing, else the Script moves to **Step 15** while singnaling that this UE reached the end of its session lifetime $\Delta t_{session}$ and that t_UE_end remains unchanged from **Step 3**.

Step 7: If Statement – Checking if the Next Cell Boundary Crossing Event Is Within the i^th Session Lifetime

1. $if\ (t\_current\_cross < t\_UE\_end)$: This statement compares the i^th UE's next cell boundary crossing's time instance to the UE's session end time instance. If the condition it **True**, the script proceeds to **Step 8**.
2. Else: if the condition is **False**, the script proceeds to **Step 11**.

Step 8: If Statement – Cheking if the Next Re-Auth / Re – Author Event Occurs via Cell Boundary Crossing or via Auth / Author Lifetime Expiry

1. $if\ (t\_current\_cross < t\_next\_auth)$: This statement checks if the ith UE's next cell boundary crossing time instance occurs before the current $\Delta t_{auth/autor}$ expiry (given that t_next_auth is used to point to the time instance when the current $\Delta t_{auth/autor}$ expires). If this statement is **True**, the script proceeds to **Step 9**.
2. Else: If this statement is **False**, the script proceeds to **Step 10**.

Step 9: Cell Boundary Crossing

1. $t\_RAR = t\_current\_cross$: Calculate the time instance of sending the k^th RAR message and set it to the time instance of the current cell boundary crossing. (Unit: seconds).
2. $\Delta t_{RAR/RAA}$: Create a uniformly distributed random number from $[1\ :\ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the RAR message and receiving the RAA message for the i^th UE's k^th RAR / RAA message pair. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1\ :\ 9]$. (Unit: seconds).

3. $t\_RAA = t\_RAR + \Delta t_{RAR/RAA}$: Calculate the time instance of receiving the RAA message. (Unit: seconds).
4. $t\_previous = t\_current\_cross$: Update the time instance of the previous reauthentication and reauthorization to the time instance of the kth one, i.e. the one that just occurred. (Unit: seconds).
5. $t\_next\_auth = t\_previous + \Delta t_{auth/author}$: Update the time instance of the next reauthentication and reauthorization via $\Delta t_{auth/author}$ expiry. (Unit: seconds).
6. $\Delta t_{residence}$: Given that the ith UE has moved to a new cell, its residence time is refreshed. A new residence time is randomly created for it based on the desired distribution set by the tester and $E\left[\Delta t_{residence}\right]$. (Unit: seconds).
7. $t\_current\_cross = t\_previous + \Delta t_{residence}$: Update the time instance of the next cell boundary crossing. (Unit: seconds).
8. The script proceeds to **Step 12**.

Step 10: The ith UE's kth RAR / RAA Message Pair via Auth / Author Lifetime Expiry

1. $t\_RAR = t\_next\_auth$: Calculate the time instance of sending the kth RAR message. (Unit: seconds).
2. $\Delta t_{RAR/RAA}$: Create a uniformly distributed random number from $\left[1 : 9\right]$ with a 10^{-6} order of magnitude to represent the duration between sending the RAR message and receiving the RAA message for the ith UE's kth RAR / RAA message pair. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $\left[1 : 9\right]$. (Unit: seconds).

3. $t\_RAA = t\_RAR + \Delta t_{RAR/RAA}$: Calculate the time instance of receiving the RAA message. (Unit: seconds).
4. $t\_previous = t\_next\_auth$: Update the time instance of the previous reauthentication and reauthorization to the time instance of the kth one, i.e. the one that just occurred. (Unit: seconds).
5. $t\_next\_auth = t\_previous + \Delta t_{auth/author}$: Update the time instance of the next reauthentication and reauthorization via $\Delta t_{auth/author}$ expiry to the next one (also via $\Delta t_{auth/author}$ expiry). (Unit: seconds).
6. The script proceeds to **Step 12**.

<u>Step 11: (Replica of Step 10 – Occurs in The Flow of Operation Execution) The ith UE's kth RAR / RAA Message Pair via Auth / Author Lifetime Expiry</u>

1. $t\_RAR = t\_next\_auth$: Calculate the time instance of sending the kth RAR message. (Unit: seconds).
2. $\Delta t_{RAR/RAA}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the RAR message and receiving the RAA message for the ith UE's kth RAR / RAA message pair. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).
3. $t\_RAA = t\_RAR + \Delta t_{RAR/RAA}$: Calculate the time instance of receiving the RAA message. (Unit: seconds).
4. $t\_previous = t\_next\_auth$: Update the time instance of the previous reauthentication and reauthorization to the time instance of the kth one, i.e. the one that just occurred. (Unit: seconds).

5. $t\_next\_auth = t\_previous + \Delta t_{auth/author}$: Update the time instance of the next reauthentication and reauthorization via $\Delta t_{auth/author}$ expiry to the next one (also via $\Delta t_{auth/author}$ expiry). (Unit: seconds).
6. The script proceeds to **Step 12**.

Step 12: Increment "noa", Update "temp" and Proceed to Step 13

1. $noa = noa + 1$: Increment the number of messages on the S6 interface by one.
2. $temp$: Create a uniformly distributed random variable from $[0 : 1]$ to represent the i^{th} UE's k^{th} RAR / RAA message pair's probability of its RAA message returning with a positive answer and that the k^{th} RAR / RAA procedure finished with the limits of the grace period. This variable and its respective value will be used in **Step 13**.

Step 13: If statement – The i^{th} UE's K^{th} RAR / RAA Message Pair's RAA

1. If $\left(temp > \left(p_{ra} \times p_{\Delta t_{grace}}\right)\right)$: if this condition is **Flase**, it means the i^{th} UE's k^{th} RAR / RAA message pair's RAA passed the reauthentication and reauthorization procedure, i.e. the RAA returned with a positive value and the i^{th} UE was able to complete the procedure within the limits of the grace period $\left(\Delta t_{grace}^{max}\right)$. The script loops back to **Step 6**.
2. Else: if the condition is **True**, it means the i^{th} UE's k^{th} RAR / RAA message pair's RAA failed the reauthentication and reauthorization procedure, i.e. either the RAA returned with a negative value or the i^{th} UE was not able to complete the procedure within the limits of the grace period $\left(\Delta t_{grace}^{max}\right)$. The script proceeds to **Step 14**.

Step 14: The i^{th} UE's K^{th} RAR / RAA Message Pair's RAA Failure

Given that the k^{th} reauthentication and reauthorization procedure failed:

1. $t\_UE\_end = t\_RAA$: Update the i^th UE's session's end time instance to the time instance of the last RAA message received.
2. Break from "for loop" (**Step 6**) and proceed to **Step 15**.

Step 15: The Credit Control Message Pair Initialization and Termination on the Gx, Ro and Sy Interfaces

1. The Credit Control Initialzation Messages are determined first:
    1. $\Delta t^{Gx}_{ULA/CCR-I}$: Create a uniformly distributed random number from $[1 \;:\; 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the ULA message and sending the CCR-I message for the i^th UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \;:\; 9]$. (Unit: seconds).
    2. $t\_CCR\_I\_Gx = t\_ULA + \Delta t^{Gx}_{ULA/CCR-I}$: Calculate the time instance of sending the CCR-I message on the Gx interface. (Unit: seconds).
    3. $t\_UE\_sim = t\_CCR\_I\_Gx$: Update the time tracking variable. This will be used later on in **Step 17**. (Unit: seconds).
    4. $\Delta t^{Gx}_{CCR-I/CCA-I}$: Create a uniformly distributed random number from $[1 \;:\; 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-I message and receiving the CCA-I message for the i^th UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \;:\; 9]$. (Unit: seconds).

5. $t\_CCA\_I\_Gx = t\_CCR\_I\_Gx + \Delta t_{CCR-I/CCA-I}^{Gx}$: Calculate the time instance of receiving the CCA-I message on the Gx interface. (Unit: seconds).

6. $\Delta t_{CCA-I/SLR}^{Gx/Sy}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-I message on the Gx interface and sending the SLR message on the Sy interface for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

7. $t\_SLR\_Sy = t\_CCA\_I\_Gx + \Delta t_{CCA-I/SLR}^{Gx/Sy}$: Calculate the time instance of sending the SLR message on the Sy interface. (Unit: seconds).

8. $\Delta t_{SLR/SLA}^{Sy}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the SLR message and receiving the SLA message for the i^{th} UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

9. $t\_SLR\_Sy = t\_SLA\_Sy + \Delta t_{SLR/SLA}^{Sy}$: Calculate the time instance of receiving the SLA message on the Sy interface. (Unit: seconds).

10. $\Delta t_{SLA/CCR-I}^{Sy/Ro}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the SLA message on the Sy interface and sending the CCR-I message on the Ro interface for the i^{th} UE. The uniform distribution is used in this case and

similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).

11. $t\_CCR\_I\_Ro = t\_SLA\_Sy + \Delta t_{SLA/CCR-I}^{Sy/Ro}$: Calculate the time instance of sending the CCR-I message on the Ro interface. (Unit: seconds).

12. $\Delta t_{CCR-I/CCA-I}^{Ro}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-I message and receiving the CCA-I message for the i^{th} UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).

13. $t\_CCA\_I\_Ro = t\_CCR\_I\_Ro + \Delta t_{CCR-I/CCA-I}^{Ro}$: Calculate the time instance of receiving the CCA-I message on the Ro interface. (Unit: seconds).

14. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

2. Given that the i^{th} UE's session lifetime, and hence t_UE_end was determined after **Steps 13** and **14**, the sequence of termination messages can be determined while referring to Figure 16 as a reference. Doing that would help pinpoint the interval during the i^{th} UE's activity where the interim credit control update messages (CCR-U / CCA-U, SNR / SNA) were exchanged and would make mapping their occurrence time instances easier on their respective interfaces. ***The sequence of message is determined in reverse order*** with t_UE_end as the reference time instance:

1. $t\_CCA\_T\_Gx = t\_UE\_end$: Calculate the time instance of receiving the CCA-T message on the Gx interface. The CCA-T on the Gx interface is the last message exchanged. (Unit: seconds).

2. $\Delta t^{Gx}_{CCA-T/CCR-T}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-T message and receiving the CCA-T message for the i^th^ UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).

3. $t\_CCR\_T\_Gx = t\_CCA\_T\_Gx - \Delta t^{Gx}_{CCA-T/CCR-T}$: Calculate the time instance of sending the CCA-T message on the Gx interface. (Unit: seconds).

4. $\Delta t^{Sy/Gx}_{STA/CCR-T}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the STA message on the Sy interface and sending the CCR-T message on the Gx interface for the i^th^ UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).

5. $t\_STA\_Sy = t\_CCR\_T\_Gx + \Delta t^{Sy/Gx}_{STA/CCR-T}$: Calculate the time instance of receiving the STA message on the Sy interface. (Unit: seconds).

6. $\Delta t^{Sy}_{STA/STR}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the STR message and receiving the STA message for the i^th^ UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any

kind of biasing towards particular values within the interval $[1 \; : \; 9]$. (Unit: seconds).

7. $t\_STR\_Sy = t\_STA - \Delta t^{Sy}_{SLA/SLR}$: Calculate the time instance of sending the STR message on the Sy interface. (Unit: seconds).

8. $\Delta t^{Ro/Sy}_{CCA-T/STR}$: Create a uniformly distributed random number from $[1 \; : \; 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-T message on the Ro interface and sending the STR message on the Sy interface for the ith UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \; : \; 9]$. (Unit: seconds).

9. $t\_CCA\_T\_Ro = t\_STR\_Sy - \Delta t^{Ro/Sy}_{STR/CCA-T}$: Calculate the time instance of receiving the CCA-T message on the Ro interface. (Unit: seconds).

10. $\Delta t^{Ro}_{CCA-T/CCR-T}$: Create a uniformly distributed random number from $[1 \; : \; 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-T message and receiving the CCA-T message for the ith UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \; : \; 9]$. (Unit: seconds).

11. $t\_CCR\_T\_Ro = t\_CCA\_T\_Ro - \Delta t^{Ro}_{CCA-T/CCR-T}$: Calculate the time instance of sending the CCR-T message on the Ro interface. (Unit: seconds).

12. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

3. $Num\_of\_Updates = \left\lfloor \frac{t\_CCR\_T\_Ro - \left(t\_UE\_sim + \Delta t_U\right)}{\Delta t_U} \right\rfloor$: Calculate the total number of CCR-U / CCA-U and SNR / SNA message pairs within the time interval $\left[t\_CCR\_U\_Gx \ : \ t\_CCR\_T\_Ro\right]$, where $t\_CCR\_U\_Gx = \left(t\_UE\_sim + \Delta t_U\right)$. In this case, the time instance $t\_CCR\_U\_Gx$ stands for the time instance of the 1st CCR-U message on the Gx interface.
4. The script proceeds to **Step 18**.

Step 16: For Loop for the Credit Control Message Pairs Relative to the ith UE on the Gx, Ro and Sy Interfaces

This is the start and "loop back" point for the "for loop" that goes through the ith UE's CCR-U / CCA-U and SNR / SNA message pairs sequentially, from the 1st one till the last $\left(Num\_of\_Updates\right)^{th}$. Once all the message pairs's time instances are mapped on their respective interfaces, the script proceeds to **Step 19**.

Step 17: Credit Control Interim Updates on the Gx, Ro Aand Sy Interfaces

1. $t\_CCR\_U\_Gx = t\_UE\_sim + \Delta t_U$: Calculate the time instance of sending the jth CCR-U message on the Gx interface. (Unit: seconds).
2. $\Delta t_{CCR-U/CCA-U}^{Gx}$: Create a uniformly distributed random number from $\left[1 \ : \ 9\right]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-U message and receiving the CCA-U message for the ith UE on the Gx interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $\left[1 \ : \ 9\right]$. (Unit: seconds).

3. $t\_CCA\_U\_Gx = t\_CCR\_U\_Gx + \Delta t^{Gx}_{CCR-U/CCA-U}$: Calculate the time instance of receiving the CCA-I message on the Gx interface. (Unit: seconds).

4. $\Delta t^{Gx/Ro}_{CCA-U/CCR-U}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-U message on the Gx interface and sending the CCR-U message on the Ro interface for the i^th^ UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

5. $t\_CCR\_U\_Ro = t\_CCA\_U\_Gx + \Delta t^{Gx/Ro}_{CCA-U/CCR-U}$: Calculate the time instance of sending the CCR-U message on the Ro interface. (Unit: seconds).

6. $\Delta t^{Ro}_{CCR-U/CCA-U}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the CCR-U message and receiving the CCA-U message for the i^{th} UE on the Ro interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

7. $t\_CCA\_U\_Ro = t\_CCR\_U\_Ro + \Delta t^{Ro}_{CCR-U/CCA-U}$: Calculate the time instance of receiving the CCA-U message on the Ro interface. (Unit: seconds).

8. $\Delta t^{Ro/Sy}_{CCA-U/SNR}$: Create a uniformly distributed random number from $[1 : 9]$ with a 10^{-7} order of magnitude to represent the duration between receiving the CCA-U message on the Ro interface and sending the SNR message on the Sy interface for the i^{th} UE. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 : 9]$. (Unit: seconds).

9. $t\_SNR\_Sy = t\_CCA\_U\_Ro + \Delta t^{Ro/Sy}_{CCA-U/SNR}$: Calculate the time instance of sending the SNR message on the Sy interface. (Unit: seconds).

10. $\Delta t^{Sy}_{SNR/SNA}$: Create a uniformly distributed random number from $[1 \ : \ 9]$ with a 10^{-6} order of magnitude to represent the duration between sending the SNR message and receiving the SNA message for the i^th UE on the Sy interface. The uniform distribution is used in this case and similar ones to remove any kind of biasing towards particular values within the interval $[1 \ : \ 9]$. (Unit: seconds).

11. $t\_SNA\_Sy = t\_SNR\_Sy + \Delta t^{Sy}_{SNR/SNA}$: Calculate the time instance of receiving the SNA message on the Sy interface. (Unit: seconds).

12. $nou = nou + 1$: Increment the number of messages on the Gx, Ro and Sy interfaces by one.

13. $t\_UE\_sim = t\_CCR\_U\_Gx$: Update the simulation time tracker to the time instance of the j^th CCR-U message exchanged on the Gx interface. (Unit: seconds).

14. The script loops back to **Step 16**.

Step 18: If Statement – i^th UE's First Authentication and Authorization Failure

$t\_UE\_end = t\_AIA$: Set the i^th UE's session's end time instance to t_AIA given that the condition from **Step 4** is **False**, i.e the i^th UE failed its first authentication and authorization procedure and its session is terminated. The script then proceeds to **Step 19**.

Step 19: Output to Files of the i^th UE's Results

1. Export noa and nou of the i^th UE to their respective output files:

   - $noa \rightarrow$ Num_of_auth.txt
   - $nou \rightarrow$ Num_of_up.txt

2. The script loops back to **Step 2**.

6.1.3 Request / Answer Delay and Inter-Message Pair Delay

The following concers the use of the 10^{-6} seconds delay between each request and its respective answer along with the use of the 10^{-7} seconds delay between two successive message pairs. Both delay values were retieved after observing a trace for a test-UE from a local operator. Howerver a data from a single UE, especialy a test-UE, is not enough to study the probabilistic distribution of Request / Answer Delay and Inter-Message Pair Delay. Thus, the uniform distribution is chosen to avoid any biasing towards any particular interval of numbers.

6.2 Case Study - 1

6.2.1 Reference Set - 1

Reference Set - 1 comprises of the following Variable / Value pairs and is shown in Table 4. It is the set from which all simulations start, i.e. when studying one of the variables included in the mathematical model, the rest of the variables will be fixed to the values of the reference set.

Table 4. Reference Set - 1

Variable	**Value**
p_a	0.99
p_{ra}	0.99
$E\left[\Delta t_{session}\right]$	20 min

$E\left[\Delta t_{residence}\right]$ || for mobile UEs MatLab scipt only || $\frac{E[V]}{R}$ ratio used for analysis	Depends on R and $E[V]$
R || for mobile UEs MatLab scipt only	1000 m
$E[V]$ || for mobile UEs MatLab scipt only	1 m/s
$\Delta t_{auth/author}$	1 min
$\Delta t_{grace}^{\max}$	9 sec
$E\left[\Delta t_{grace}\right] = \frac{1}{\lambda_{grace}}$ || $\frac{\Delta_{grace}^{\max}}{\lambda_{grace}}$ ratio used for analysis	1 sec
Δt_U	1 sec

6.2.2 *Mobility*

The mobility model used to estimate the UE's average residence time is the one derived in Chapter V, section 2.3.2. The number of layers used is 6, with a corresponding avgNumberOfSteps = 9.2863 because using 6 layers gave the closest answer to the model derived in [59],with a difference of 7.14 %.

6.2.3 *Variable Distributions*

For Case Study - 1, the distributions of $\Delta t_{session}$, $\Delta t_{residence}$ and Δt_{grace} are assumed to be exponential:

- Let $\Delta t_{session}$ be exponentially distributed with $E\left[\Delta t_{session}\right] = \frac{1}{\lambda_{session}}$. i.e.

  $f_{session}(t) = \lambda_{session} \times e^{-\lambda_{session} \times t}$ for $t > 0$.

- Let $\Delta t_{residence}$ be exponentially distributed with $E\left[\Delta t_{residence}\right] = \frac{1}{\lambda_{residence}}$. i.e.

  $f_{residence}(t) = \lambda_{residence} \times e^{-\lambda_{residence} \times t}$ for $t > 0$.

- Let Δt_{grace} be exponentially distributed with $E\left[\Delta t_{grace}\right] = \frac{1}{\lambda_{\Delta t_{grace}}}$. i.e.

  $f_{\Delta t_{grace}}(t) = \lambda_{\Delta t_{grace}} \times e^{-\lambda_{\Delta t_{grace}} \times t}$ for $t > 0$.

- $\int_{\theta}^{+\infty} \lambda \times e^{-\lambda \times t} dt = e^{-\lambda \times \theta}$.

- $\int_{\theta}^{\theta+T} \lambda \times e^{-\lambda \times t} dt = e^{-\lambda \times \theta} \times \left(1 - e^{-\lambda \times T}\right)$.

6.2.4 *Experiment Description*

A total of eight experiments were done for Study Case - 1. Each experiment consists of multiple MATLAB Simulations (or multiple script runs) depending on the variable whose effects were being studied.

1. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The Populations Size

   $Num\_of\_UE = \left\{10,\ 100,\ 1000,\ 10^4,\ 10^5,\ 10^6,\ 10^7\right\}$.

2. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The AIR / AIA Message Pair Success Rate

   $p_a = \left\{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 0.99\right\}$.

3. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The RAR / RAA Message Pair Success Rate

   $p_{ra} = \{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 0.99\}$.

4. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. [The Grace Period / The Average Duration to Start and Finish the Re-Auth / Re-Autho Process] (Unitless)

   $\lambda_{grace} \times \Delta t_{grace}^{\max} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$.

5. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The Average Session Duration (min)

   $E\left[\Delta t_{session}\right] = \{5,\ 10,\ 15,\ 20,\ 25,\ 30\}$.

6. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The Authentication and Authorization Lifetime (min)

   $\Delta t_{auth/author} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$.

7. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The Interim Update Interval (sec)

   $\Delta t_U = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$.

8. The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces vs. The Ratio [Average UE Speed / Cell Radius] (10^{-3}/second)

   $\frac{E\left[V\right]}{R} = \{1:30\ //\ Step\ Size = 1\}$.

6.2.5 Result Collection and Data Processing

The MATLAB Simulation results were processed on Microsoft Excel. Figure 27 shows an example output for each of the Diameter Interface groups, the {S6} and the {Gx, Ro, Sy} groups. The data is transferred to Excel and the average for each of the groups is calculated and plotted.

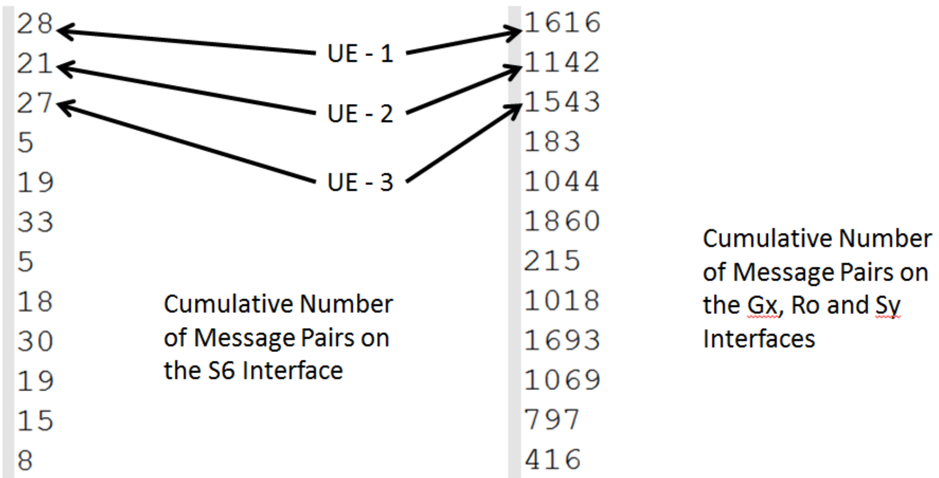


Figure 27. MATLAB Script Example Output

6.3 Case Study - 2

6.3.1 Reference Set - 2

Reference Set - 2 comprises of the following Variable / Value pairs and is shown in Table 5Table **4**. It is the set from which all simulations start, i.e. when studying one of the variables included in the mathematical model, the rest of the variables will be fixed to the values of the reference set.

Table 5. Reference Set - 2

Variable	**Value**
p_a	0.99
p_{ra}	0.99
$E\left[\Delta t_{session}\right]$	10 min
$E\left[\Delta t_{residence}\right]$ || for mobile UEs MatLab scipt only || $\frac{E[V]}{R}$ ratio used for analysis	Depends on R and $E[V]$
R || for mobile UEs MatLab scipt only	1000 m
$E[V]$ || for mobile UEs MatLab scipt only	1 m/s
$\Delta t_{auth/author}$	1 min
$\Delta t_{grace}^{\max}$	2 sec
$E\left[\Delta t_{grace}\right]$	1.015 sec
Δt_U	5 sec

6.3.2 Mobility

The mobility model used to estimate the UE’s average residence time is the one derived in Chapter V, section 2.3.2. The number of layers used is 6, with a corresponding avgNumberOfSteps = 9.2863 because using 6 layers gave the closest answer to the model derived in [59],with a difference of 7.14 %.

6.3.3 Variable Distributions

For Case Study - 2, the distributions of $\Delta t_{session}$, $\Delta t_{residence}$ and Δt_{grace} are assumed to be:

- Let $\Delta t_{session}$ be lognormally distributed with parameters μ and σ such that

  $E\left[\Delta t_{session}\right] = e^{\mu+\frac{\sigma^2}{2}}$ and $Var\left[\Delta t_{session}\right] = \left(e^{\sigma^2} - 1\right)e^{2\times\mu+\sigma^2}$ i.e.

  $f_{session}(t) = \frac{1}{t\times\sigma\times\sqrt{2\times\pi}} e^{-\frac{(\ln(t)-\mu)^2}{2\times\sigma^2}}$ for $t > 0$.

  - $\mu = \ln\left(\frac{\left(E\left[\Delta t_{session}\right]\right)^2}{\sqrt{Var\left[\Delta t_{session}\right] + \left(E\left[\Delta t_{session}\right]\right)^2}}\right)$

  - $\sigma = \sqrt{\ln\left(1 + \frac{Var\left[\Delta t_{session}\right]}{\left(E\left[\Delta t_{session}\right]\right)^2}\right)}$

  - $\int_0^t f_{session}(t) = \frac{1}{2} + \frac{1}{2}\times erf\left[\frac{\ln(t)-\mu}{\sqrt{2\times\sigma}}\right]$

  - $\int_t^{+\infty} f_{session}(t) = 1 - \int_0^t f_{session}(t)$

- Let $\Delta t_{residence}$ be exponentially distributed with $E\left[\Delta t_{residence}\right] = \frac{1}{\lambda_{residence}}$. i.e.

  $f_{residence}(t) = \lambda_{residence} \times e^{-\lambda_{residence}\times t}$ for $t > 0$.

  - $\int_\theta^{+\infty} \lambda \times e^{-\lambda\times t} dt = e^{-\lambda\times\theta}$.

  - $\int_\theta^{\theta+T} \lambda \times e^{-\lambda\times t} dt = e^{-\lambda\times\theta} \times \left(1 - e^{-\lambda\times T}\right)$.

- Let Δt_{grace} be uniformly distributed with $a \le t \le b$ and $E\left[\Delta t_{grace}\right] = \frac{1}{2} \times \left(a+b\right)$.

  i.e. $f_{\Delta t_{grace}}(t) = \frac{1}{b-a}$ with $t \in \left[0\ ,\ b\right]$.

  – $\int_{0}^{t} f_{\Delta t_{grace}}(t) = \frac{t}{b}$

6.3.4 *Experiment Description*

An experiment was done for Study Case - 2. It consists of multiple MATLAB Simulations (or multiple script runs). The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces was plotted vs. The Ratio [Average UE Speed / Cell Radius] (10^{-3}/second) $\frac{E\left[V\right]}{R} = \left\{1:30\ //\ \mathrm{S}tep\ \mathrm{S}ize = 1\right\}$. Furthermore, the variance for the lognormally distributed variables are taken to be $Var\left[X\right] = 5\% E\left[X\right]$.

6.3.5 *Result Collection and Data Processing*

The MATLAB Simulation results were processed on Microsoft Excel. Figure 27 shows an example output for each of the Diameter Interface groups, the {S6} and the {Gx, Ro, Sy} groups. The data is transferred to Excel and the average for each of the groups is calculated and plotted.

6.4 Case Study – 3

6.4.1 Reference Set - 3

Reference Set - 3 comprises of the following Variable / Value pairs and is shown in Table 6Table **4**. It is the set from which all simulations start, i.e. when studying one of the variables included in the mathematical model, the rest of the variables will be fixed to the values of the reference set.

Table 6. Reference Set - 3

Variable	**Value**
p_a	0.99
p_{ra}	0.99
$E\left[\Delta t_{session}\right]$	10 min
$E\left[\Delta t_{residence}\right]$ || for mobile UEs MatLab scipt only || $\frac{E\left[V\right]}{R}$ ratio used for analysis	Depends on R and $E\left[V\right]$
R || for mobile UEs MatLab scipt only	1000 m
$E\left[V\right]$ || for mobile UEs MatLab scipt only	1 m/s
$\Delta t_{auth/author}$	2 min
$\Delta t_{grace}^{\max}$	9 sec
$E\left[\Delta t_{grace}\right]=\frac{1}{\lambda_{grace}}$ || $\frac{\Delta_{grace}^{\max}}{\lambda_{grace}}$ ratio used for analysis	1 sec
Δt_U	5 sec

6.4.2 *Mobility*

The mobility model used to estimate the UE's average residence time is the one derived in Chapter V, section 2.3.2. The number of layers used is 6, with a corresponding avgNumberOfSteps = 9.2863 because using 6 layers gave the closest answer to the model derived in [59],with a difference of 7.14 %.

6.4.3 *Variable Distributions*

For Case Study – 1, the distributions of $\Delta t_{session}$, $\Delta t_{residence}$ and Δt_{grace} are assumed to be:

- Let $\Delta t_{session}$ be lognormally distributed with parameters μ and σ such that

  $E\left[\Delta t_{session}\right] = e^{\mu + \frac{\sigma^2}{2}}$ and $Var\left[\Delta t_{session}\right] = \left(e^{\sigma^2} - 1\right) e^{2\times\mu+\sigma^2}$ i.e.

  $$f_{session}(t) = \frac{1}{t\times\sigma\times\sqrt{2\times\pi}} e^{-\frac{\left(\ln(t)-\mu\right)^2}{2\times\sigma^2}} \quad \text{for } t > 0.$$

  – $$\mu = \ln\left(\frac{\left(E\left[\Delta t_{session}\right]\right)^2}{\sqrt{Var\left[\Delta t_{session}\right] + \left(E\left[\Delta t_{session}\right]\right)^2}}\right)$$

  – $$\sigma = \sqrt{\ln\left(1 + \frac{Var\left[\Delta t_{session}\right]}{\left(E\left[\Delta t_{session}\right]\right)^2}\right)}$$

  – $$\int_0^t f_{session}(t) = \frac{1}{2} + \frac{1}{2} \times erf\left[\frac{\ln(t) - \mu}{\sqrt{2\times\sigma}}\right]$$

  – $$\int_t^{+\infty} f_{session}(t) = 1 - \int_0^t f_{session}(t)$$

- Let $\Delta t_{residence}$ be lognormally distributed with parameters μ and σ such that

  $E\left[\Delta t_{residence}\right] = e^{\mu+\frac{\sigma^2}{2}}$ and $Var\left[\Delta t_{residence}\right] = \left(e^{\sigma^2} - 1\right) e^{2\times\mu+\sigma^2}$ i.e.

  $f_{residence}(t) = \frac{1}{t\times\sigma\times\sqrt{2\times\pi}} e^{-\frac{(\ln(t)-\mu)^2}{2\times\sigma^2}}$ for $t > 0$.

  - $\mu = \ln\left(\frac{\left(E\left[\Delta t_{residence}\right]\right)^2}{\sqrt{Var\left[\Delta t_{residence}\right] + \left(E\left[\Delta t_{residence}\right]\right)^2}}\right)$

  - $\sigma = \sqrt{\ln\left(1 + \frac{Var\left[\Delta t_{residence}\right]}{\left(E\left[\Delta t_{residence}\right]\right)^2}\right)}$

  - $\int_0^t f_{residence}(t) = \frac{1}{2} + \frac{1}{2}\times erf\left[\frac{\ln(t)-\mu}{\sqrt{2\times\sigma}}\right]$

- Let Δt_{grace} be exponentially distributed with $E\left[\Delta t_{grace}\right] = \frac{1}{\lambda_{\Delta t_{grace}}}$. i.e.

  $f_{\Delta t_{grace}}(t) = \lambda_{\Delta t_{grace}} \times e^{-\lambda_{\Delta t_{grace}}\times t}$.

  - $\int_{\theta}^{\theta+T} \lambda\times e^{-\lambda\times t} dt = e^{-\lambda\times\theta} \times \left(1 - e^{-\lambda\times T}\right)$.

6.4.4 Experiment Description

An experiment was done for Study Case - 2. It consists of multiple MATLAB Simulations (or multiple script runs). The Cumulative Number of Message Pairs on the {S6} and {Gx, Ro, Sy} interfaces was plotted vs. The Ratio [Average UE Speed / Cell

Radius] (10^{-3}/second) $\frac{E[V]}{R} = \{1:30 \text{ // } Step\ Size = 1\}$. Furthermore, the variance for the lognormally distributed variables are taken to be $Var[X] = 5\%E[X]$.

6.4.5 Result Collection and Data Processing

The MATLAB Simulation results were processed on Microsoft Excel. Figure 27 shows an example output for each of the Diameter Interface groups, the {S6} and the {Gx, Ro, Sy} groups. The data is transferred to Excel and the average for each of the groups is calculated and plotted.

6.3 Results and Interpretation

6.3.1 Case Study - 1

<u>Case Study - 1 - Experiment - 1</u>

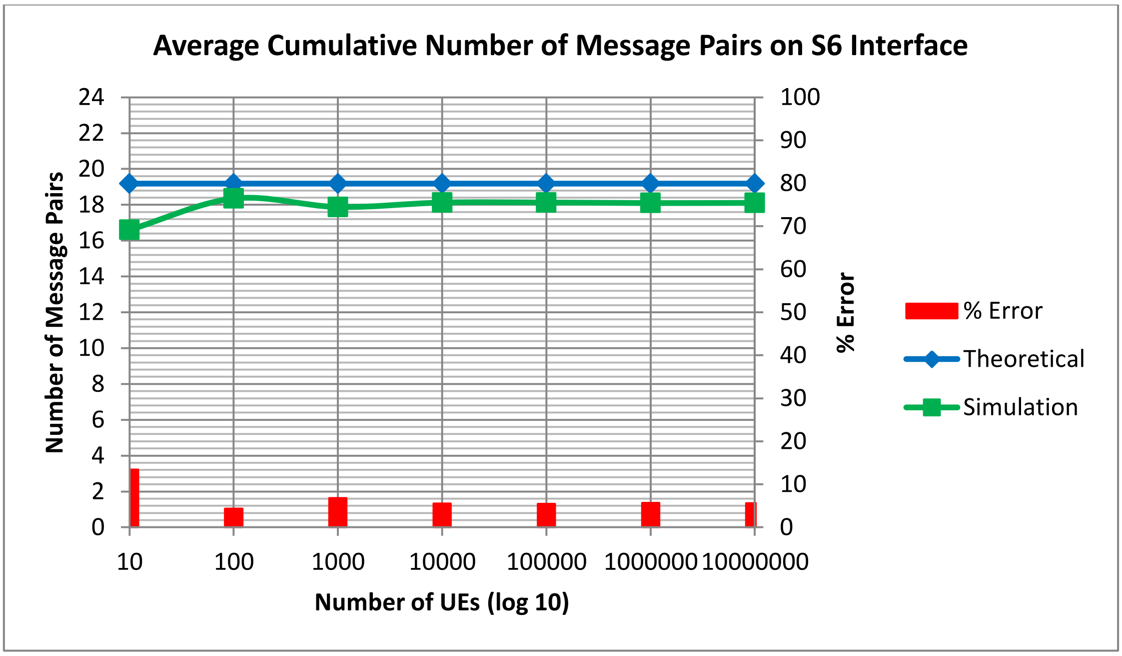


Figure 28. Number of Message Pairs vs. UE Population Size - S6

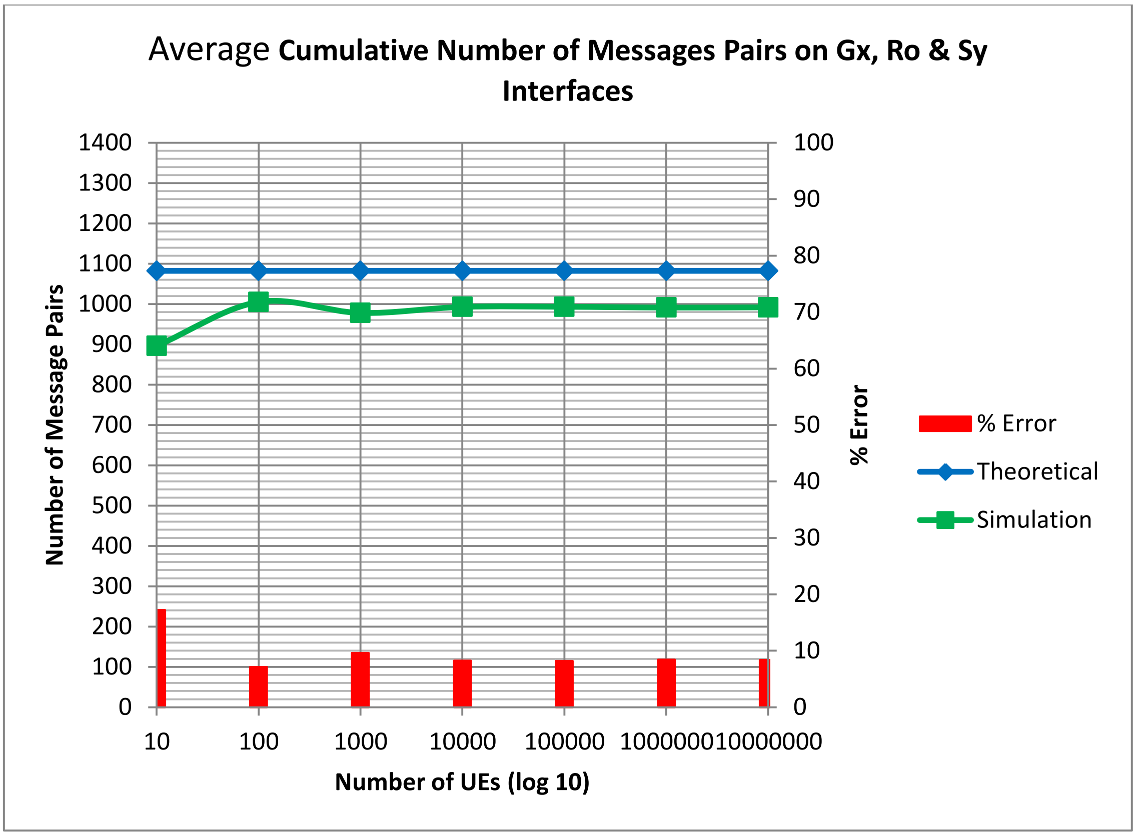


Figure 29. Number of Message Pairs vs. UE Population Size - Gx, Ro & Sy

Figure 28 and Figure 29 show the average number of cumulative message pairs on the S6, Gx, Ro and Sy interfaces. The main purpose of these two figures is to the compare the accuracy of the derived mathematical model vs. the UE Population Size. Thus, these two figures show that the model is more accurate when it comes to large populations of UEs with the absolute percent relative error stabilizing for 100,000 UEs and more (5.5% for the S6 interfaces and 8.5% for the Gx, Ro and Sy interfaces). The following results were obtained using the Reference Set defined in Table 4. Observing the mathematical model become more accurate vs. a large number of UEs is not surprising since this it is based on statistical variables ($E\left[\Delta t_{session}\right], E\left[\Delta t_{residence}\right]$, …), i.e. the larger the sample (number of UEs) the better the statistical data. So, based on the results shown in Figure 28 and Figure 29, all the simulations that follow were run for a

Population Size of 100,000 UEs. Furthermore, the mathematical and simulation results were compared using the Percent Mean Absolute Relative Error (PMARE) given by

$PMARE = \frac{1}{n} \times \sum_{i=1}^{n} \left| \frac{(f_i - y_i)}{f_i} \right|$ where n is the number of simulated values per tested variable, f_i the theoretical (mathematical) value calculated using the model and y_i the simulation's result.

Case Study - 1 - Experiment - 2

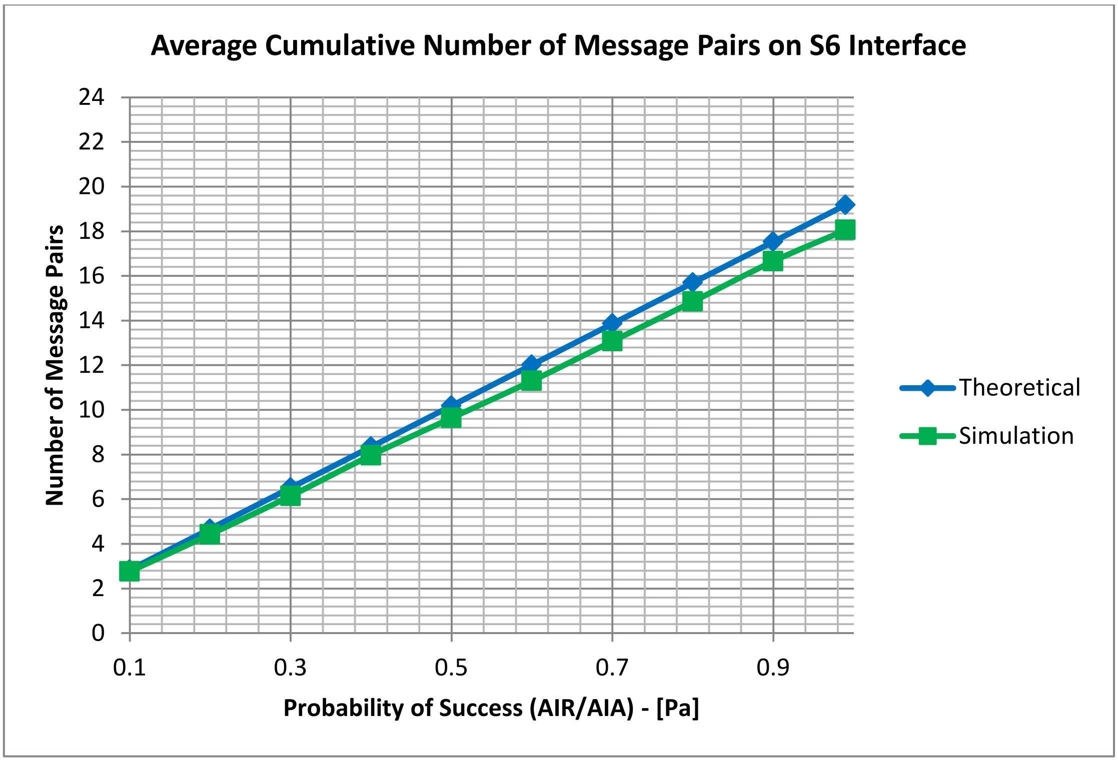


Figure 30. Number of Message Pairs vs. p_a - S6

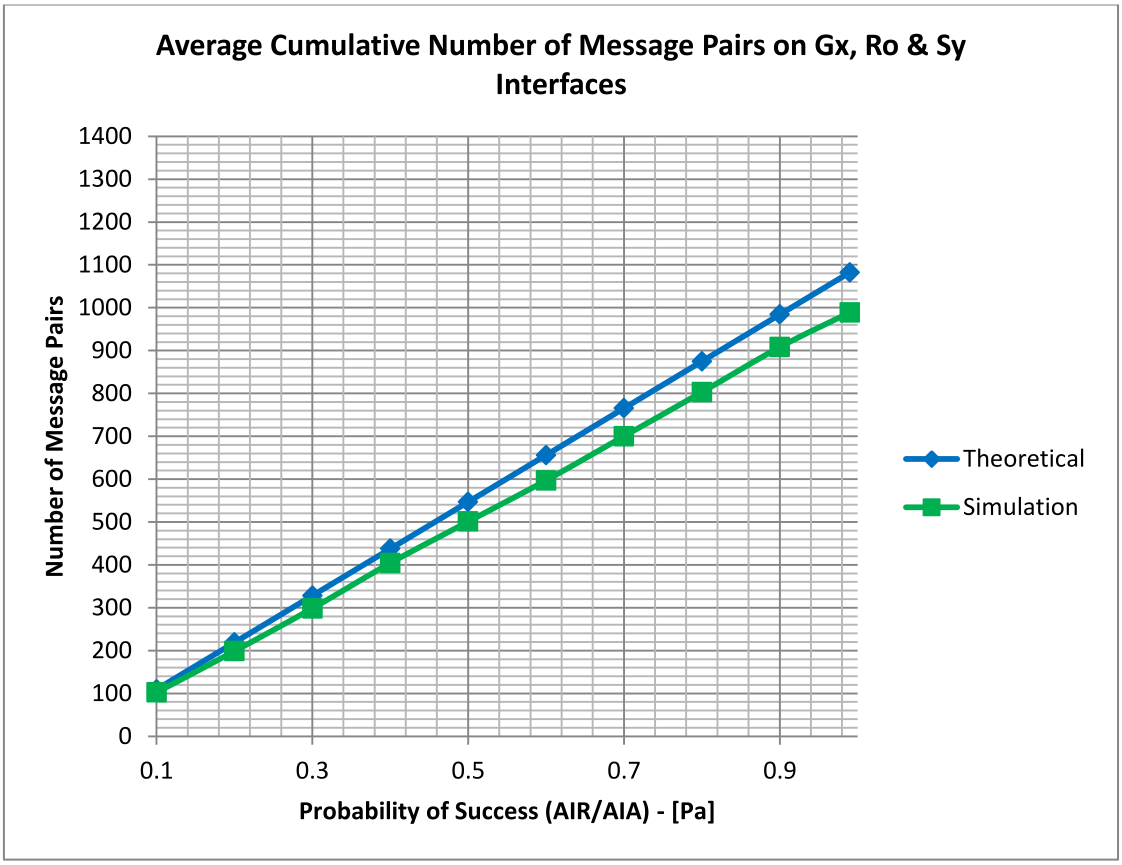


Figure 31. Number of Message Pairs vs. p_a - Gx, Ro, Sy

Figure 30 and Figure 31 show that the average cumulative number of message pairs on all the interfaces in question varies linearly as p_a increases with a PMARE of 5.13% on the S6 interface and 8.62% on the Gx, Ro and Sy interfaces. This of course is to be expected because p_a refers to the probability that a UE was the granted the ability to use network resources, and thus, the larger the number of UEs with active sessions, the larger the average number of messages.

Case Study - 1 - Experiment - 3

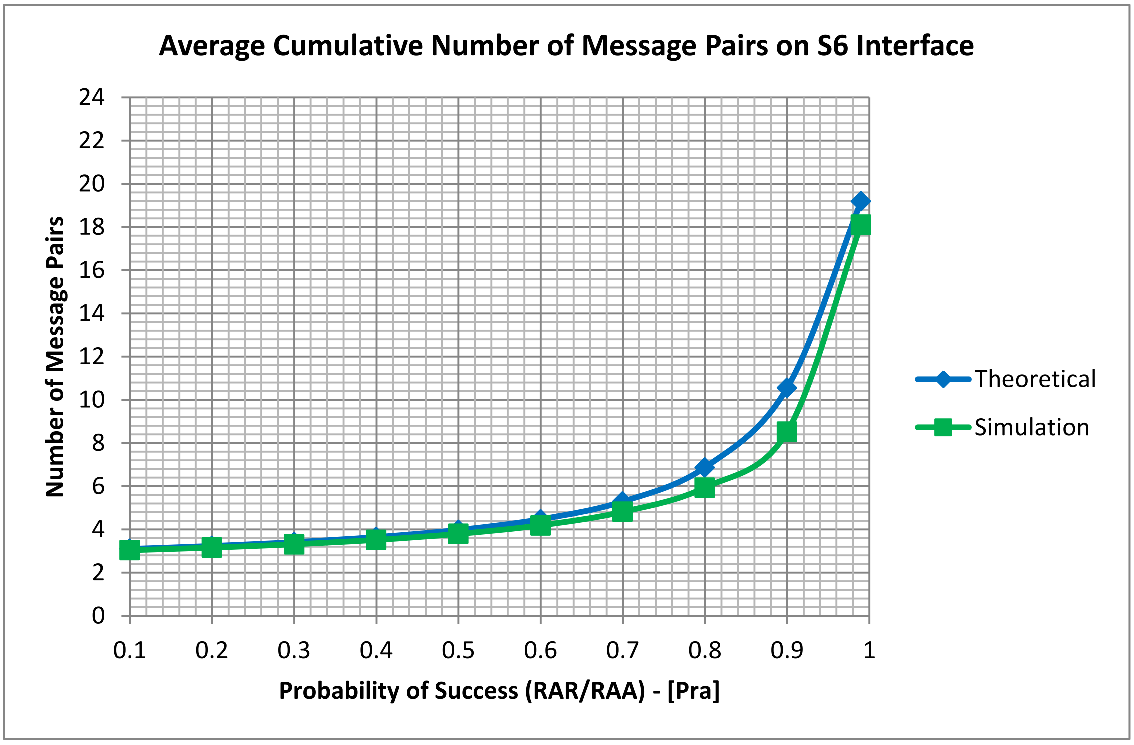


Figure 32. Number of Message Pairs vs. p_{ra} - S6

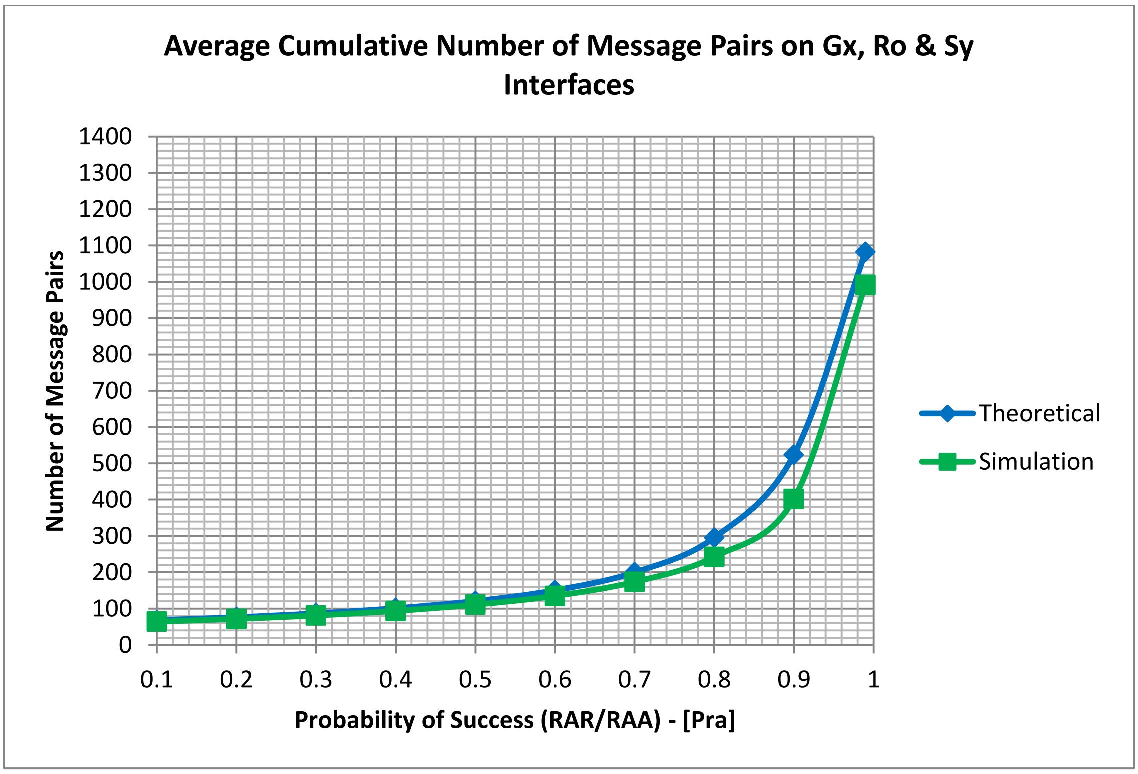


Figure 33. Number of Message Pairs vs. p_{ra} - Gx, Ro, Sy

Figure 32 and Figure 33 show the variation of the average cumulative number of message pairs on the S6, Gx, Ro and Sy interfaces with an PMARE of 6.89% on the S6 interface and 10.71% on the Gx, Ro and Sy interfaces. The most interesting feature here is that the number of messages on all the interfaces is relatively low and changes slowly $0.1 \le p_{ra} \le 0.8$ then it increases dramatically by a factor of 5 for $p_{ra} > 0.8$. Furthermore, the overall trend vs an increasing p_{ra} is an increasing exponential. This shape is mostly due to removing the assumption that all subsequent reauthentication and reauthorization procedures are always successful. The removal of the latter assumption caused the inclusion of the term $\frac{1-p_{success}^{m+1}}{1-p_{success}}$ with $p_{success} = p_{ra} \times p_{\Delta t_{grace}}$ which is the value the geometric sum that estimates the cumulative number of sequentially successful RAR / RAA message pair exchange of a UE.

Case Study - 1 - Experiment - 4

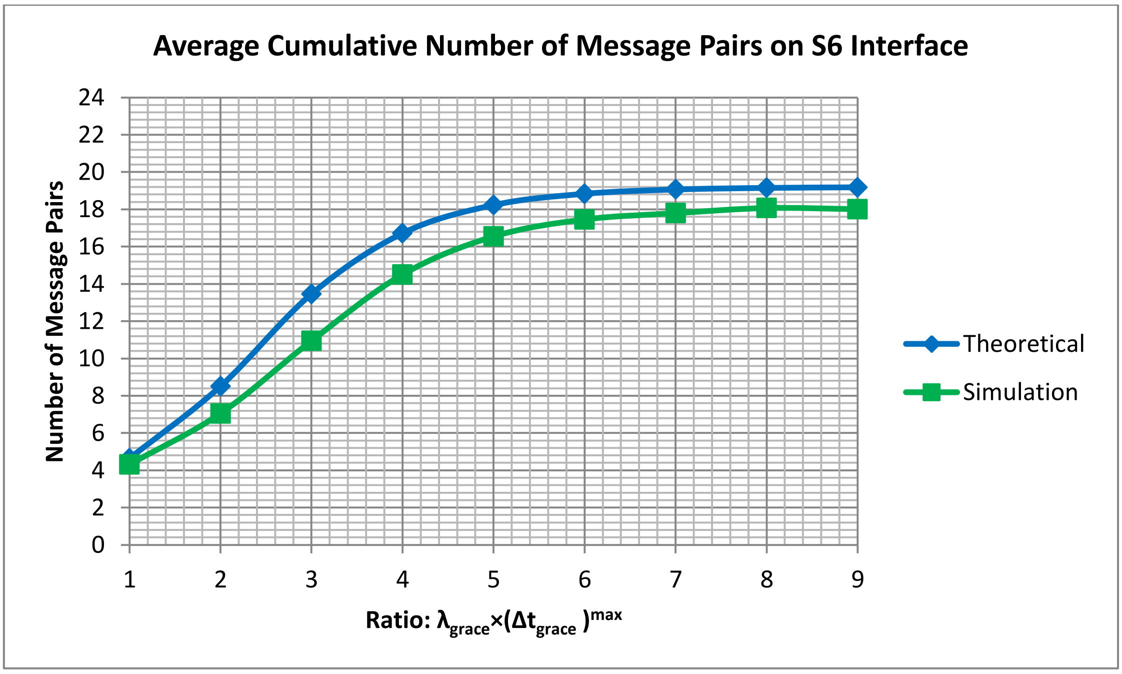


Figure 34. Number of Message Pairs vs. [$\lambda_{grace}\times(\Delta t_{grace})^{max}$] - S6

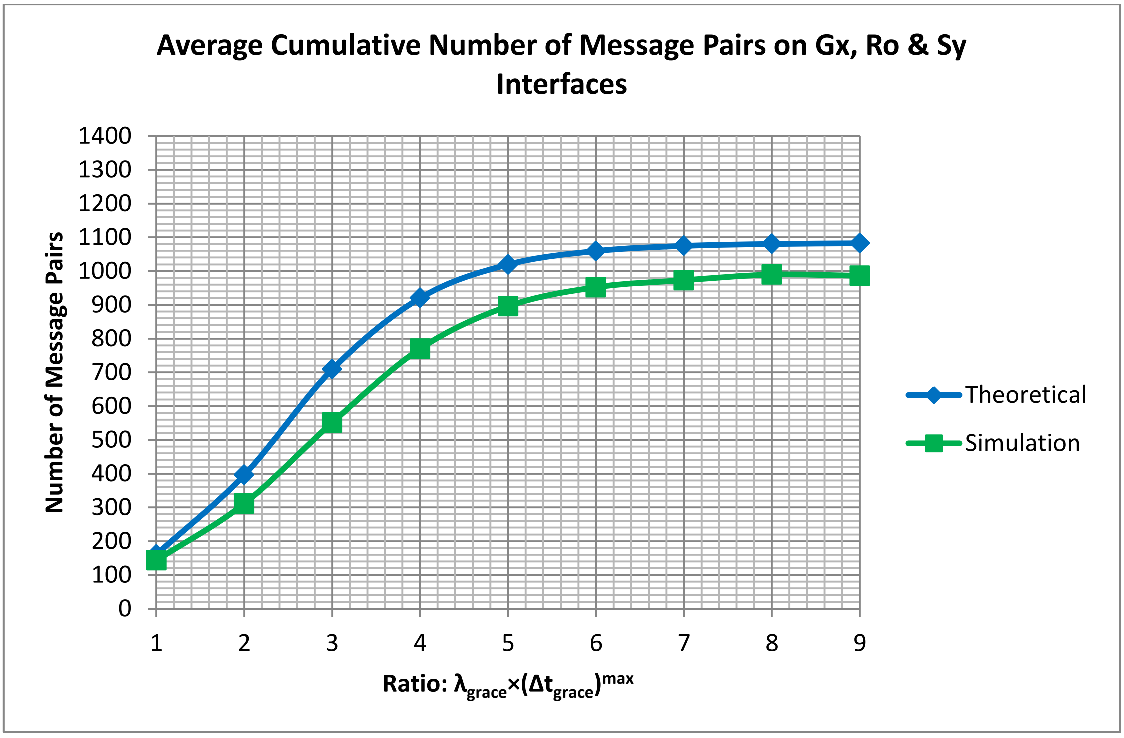


Figure 35. Number of message Pairs vs. [$\lambda_{grace}\times(\Delta t_{grace})^{max}$] - Gx, Ro, Sy

Figure 34 and Figure 35 show the variation of the average cumulative number of message pairs on the S6, Gx, Ro and Sy interfaces with an PMARE of 10.1% on the S6 interface and 13.32% on the Gx, Ro and Sy interfaces. What the ratio $\left[\lambda_{grace}\times\Delta t_{grace}^{max}\right]$ represents is the average time a UE needs to finish its re-Auth/re-Author procedure within the limited duration Δt_{grace}^{max}. Thus, the bigger the ratio, the more likely a UE, which is not planning on terminating its data session, would be able to have longer session durations without facing the need to re-establish a new one. Furthermore, the overall trend vs an increasing $\lambda_{grace}\times\Delta t_{grace}^{max}$ ratio is an increasing truncated logarithmic function. And given that logarithms and exponential are mathematically related, this logarithmic curvature is also due to removing the assumption that all subsequent reauthentication and reauthorization procedures are always successful, but in this case, it is because the procedure did not finish it the specified grace period. The removal of the latter assumption caused the inclusion of the term $\frac{1-p_{success}^{m+1}}{1-p_{success}}$ with $p_{success} = p_{ra} \times p_{\Delta t_{grace}}$

which is the value the geometric sum that estimates the cumulative number of sequentially successful RAR / RAA message pair exchange of a UE.

Case Study - 1 - Experiment - 5

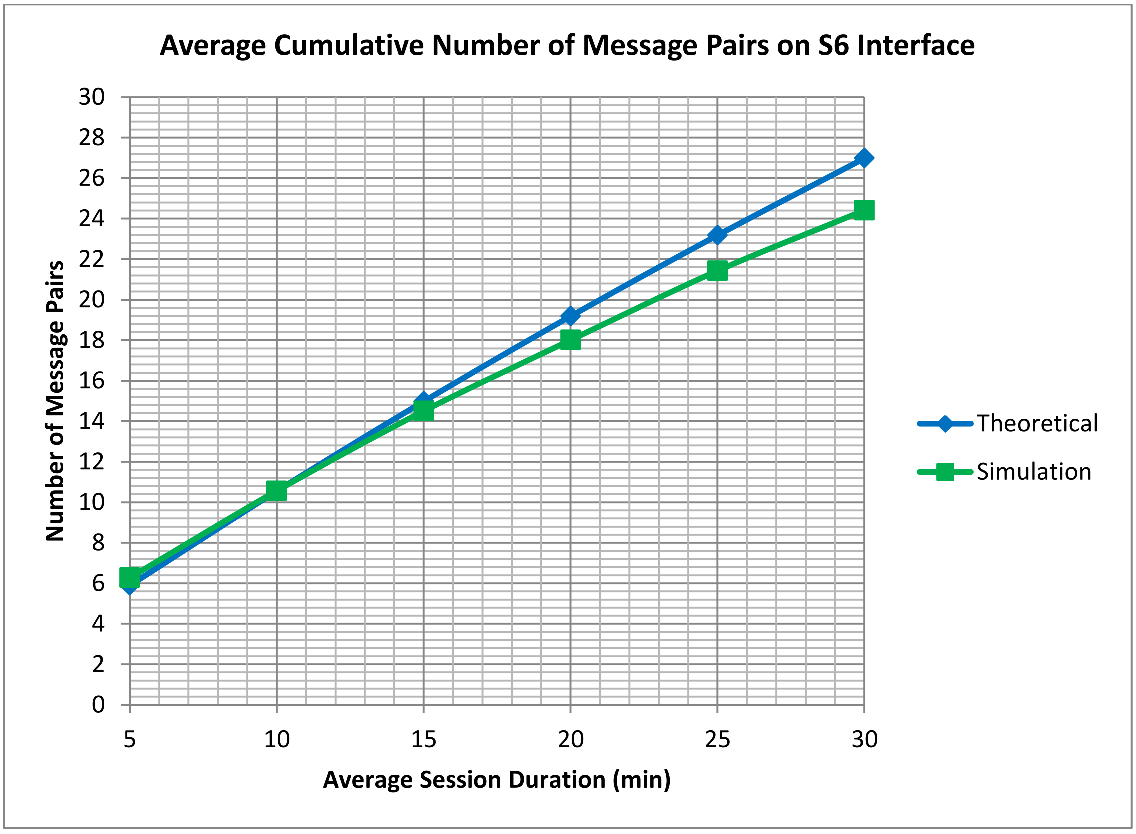


Figure 36. Number of Message Pairs vs. Average Session Duration - S6

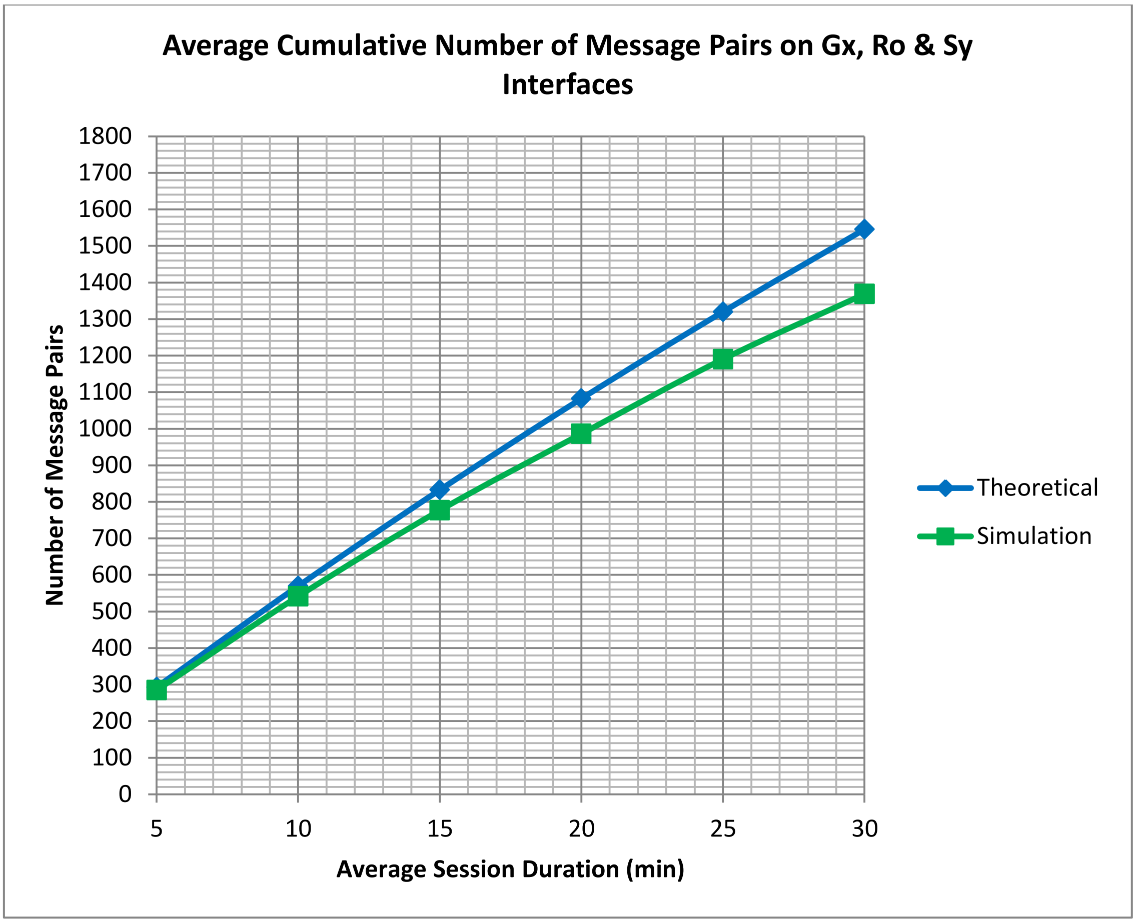


Figure 37. Number of Message Pairs vs. Average Session Duration - Gx, Ro & Sy

Figure 36 and Figure 37 show the variation of the average cumulative number of message pairs on the S6, Gx, Ro and Sy interfaces with an PMARE of 5.49% on the S6 interface and 7.44% on the Gx, Ro and Sy interfaces. The rate of change of the average cumulative number of message pairs on all the interfaces in question is constant with the average cumulative number increasing linearly vs a linearly increasing UE session lifetime $\left(\Delta t_{session}\right)$. This is by far the simplest and most expected result since it is very logical that, given that the message pairs are occurring periodically, then the longer a session lasts, the more requests and answers are exchanged.

Case Study - 1 - Experiment - 6 And Experiment - 7 (Respectively)

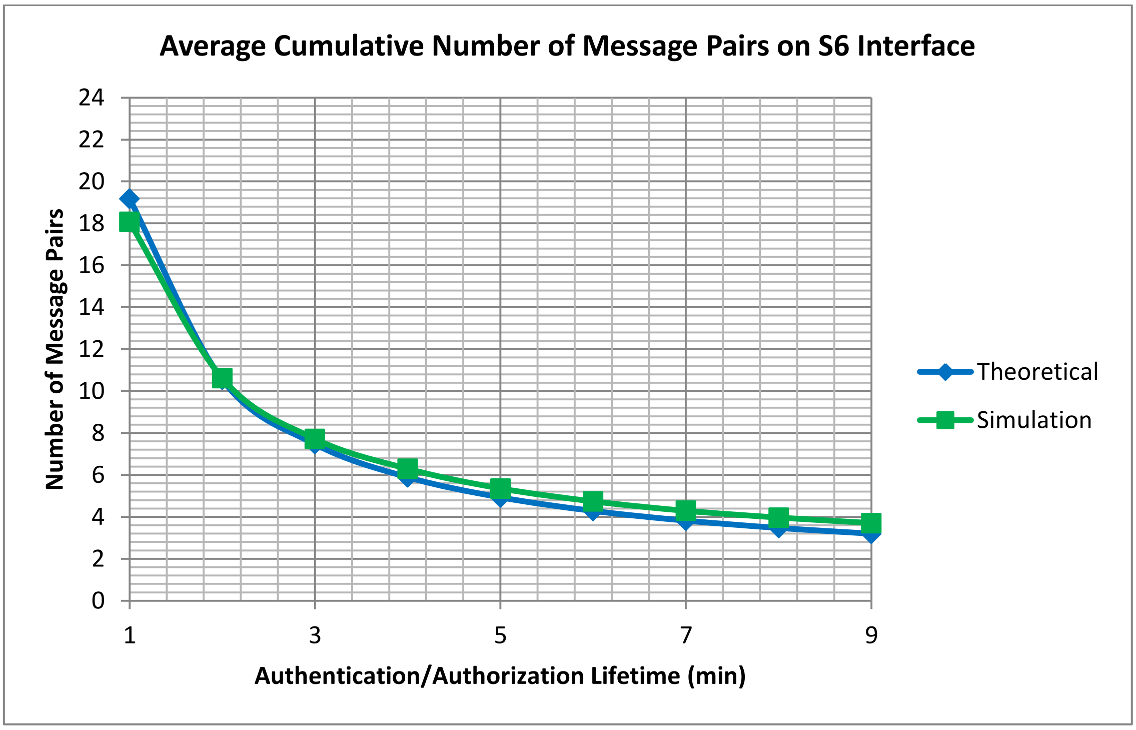


Figure 38. Number of Message Pairs vs. Auth/Author Lifetime - S6

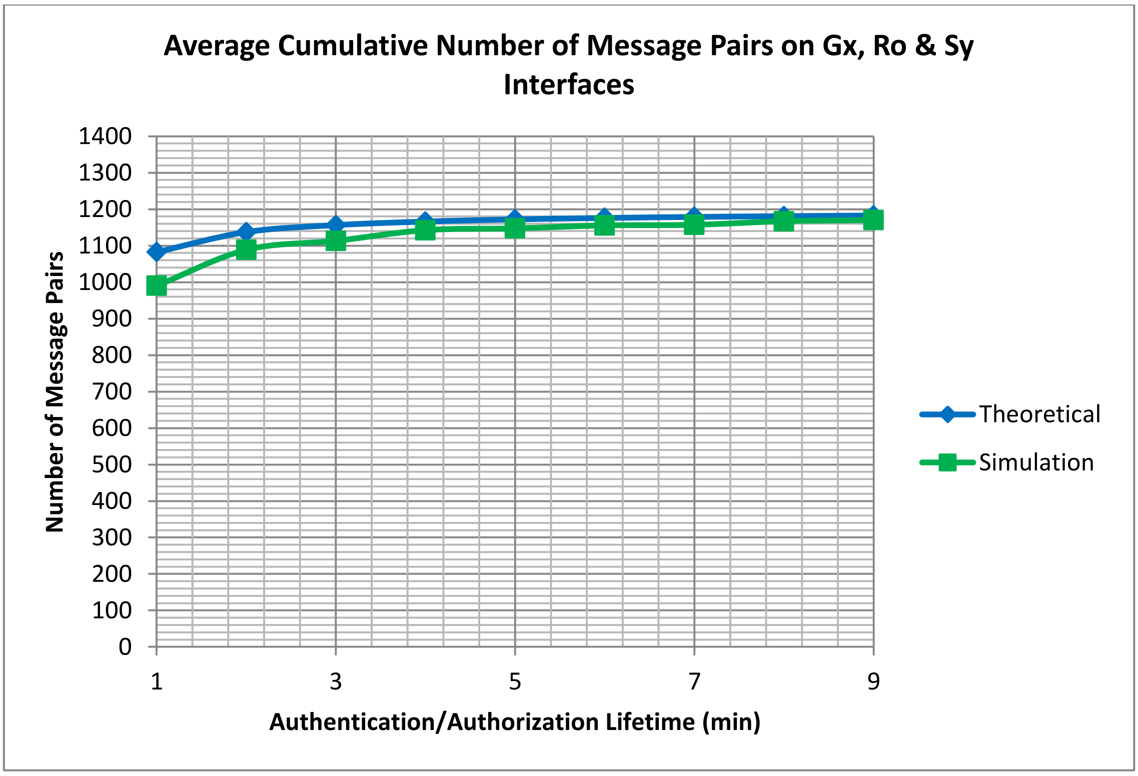


Figure 39. Number of Message Pairs vs. Auth/Author Lifetime - Gx, Ro & Sy

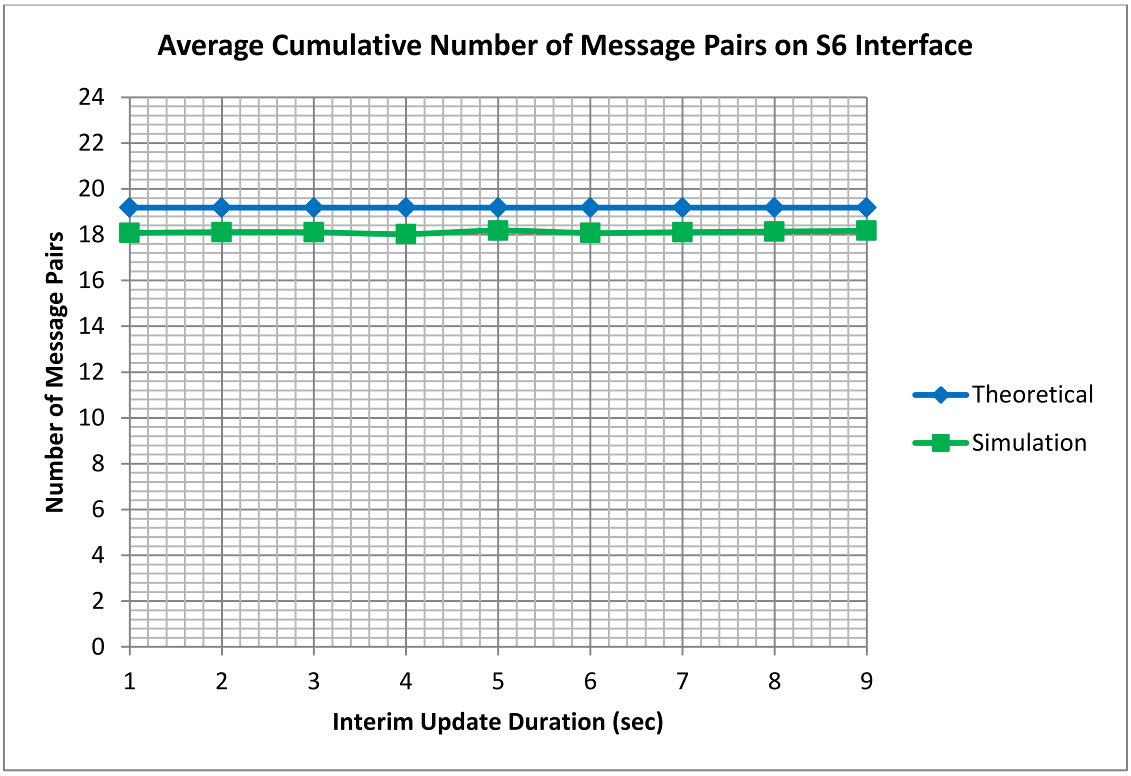


Figure 40. Number of Message Pairs vs. Interim Update Duration - S6

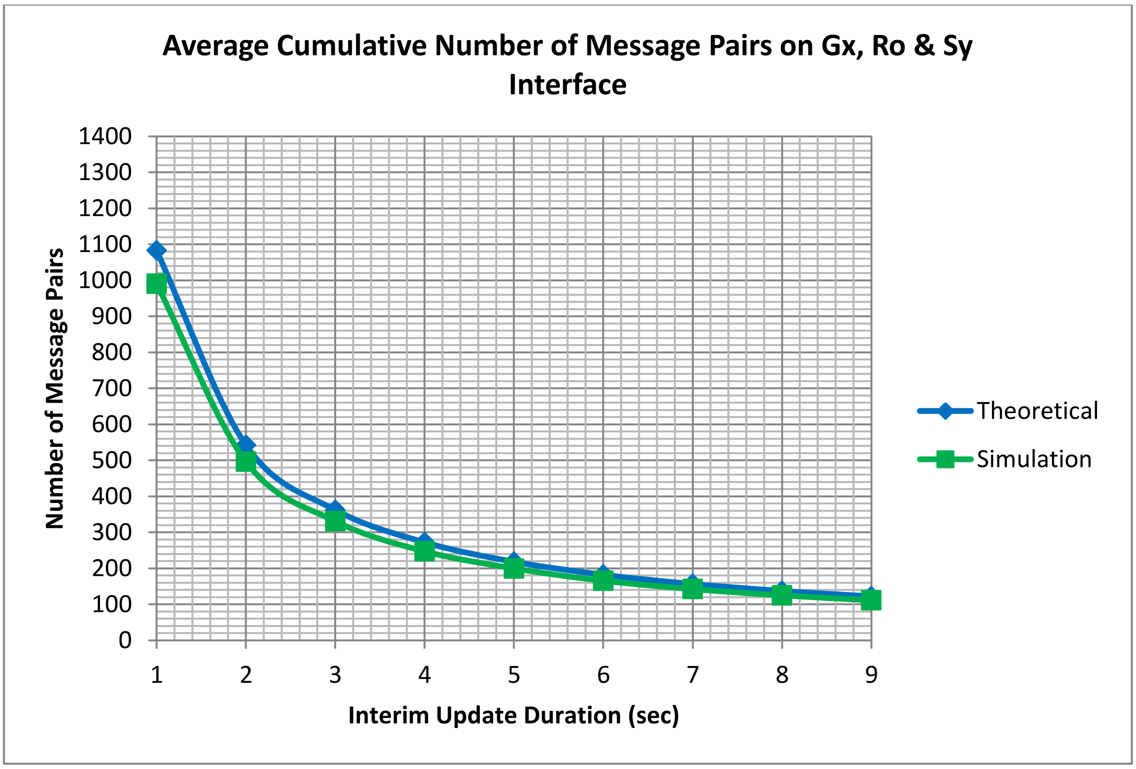


Figure 41. Number of Message Pairs vs. Interim Update Duration - Gx, Ro & Sy

Figure 38 and Figure 39 show the variation in the average cumulative number of message pairs vs. the Auth/Author lifetime, while Figure 40 and Figure 41 show the variation in the average cumulative number of message pairs vs. the Interim Update Duration. The results behave as expected, with the number of message pairs on the S6 interface decreasing as the Auth/Author lifetime $\left(\Delta t_{auth/author}\right)$ increases while the number of messages on the Gx, Ro and Sy interfaces remains practically the same (experiment - 6). This is because the number of message pairs on the later interfaces depends more on the average session duration $E\left[\Delta t_{session}\right]$ and the Interim Update Duration $\left(\Delta t_U\right)$ than the Auth/Author lifetime $\left(\Delta t_{auth/author}\right)$. While in Figure 40 and Figure 41 (experiment - 7), it is the number of message pairs on the Gx, Ro and Sy interfaces that decreases while the number of message pairs on the S6 interface is unaffected. This is also expected because the S6 interface is independent of the Interim Update Duration $\left(\Delta t_U\right)$. For experiment - 6, the PMARE was 8.69% and 2.94% for the S6 and the Gx, Ro and Sy interfaces respectively, while the PMARE for experiment - 7 was 5.58% and 8.83% for the S6 and the Gx, Ro and Sy interfaces respectively.

Case Study - 1 - Experiment - 8

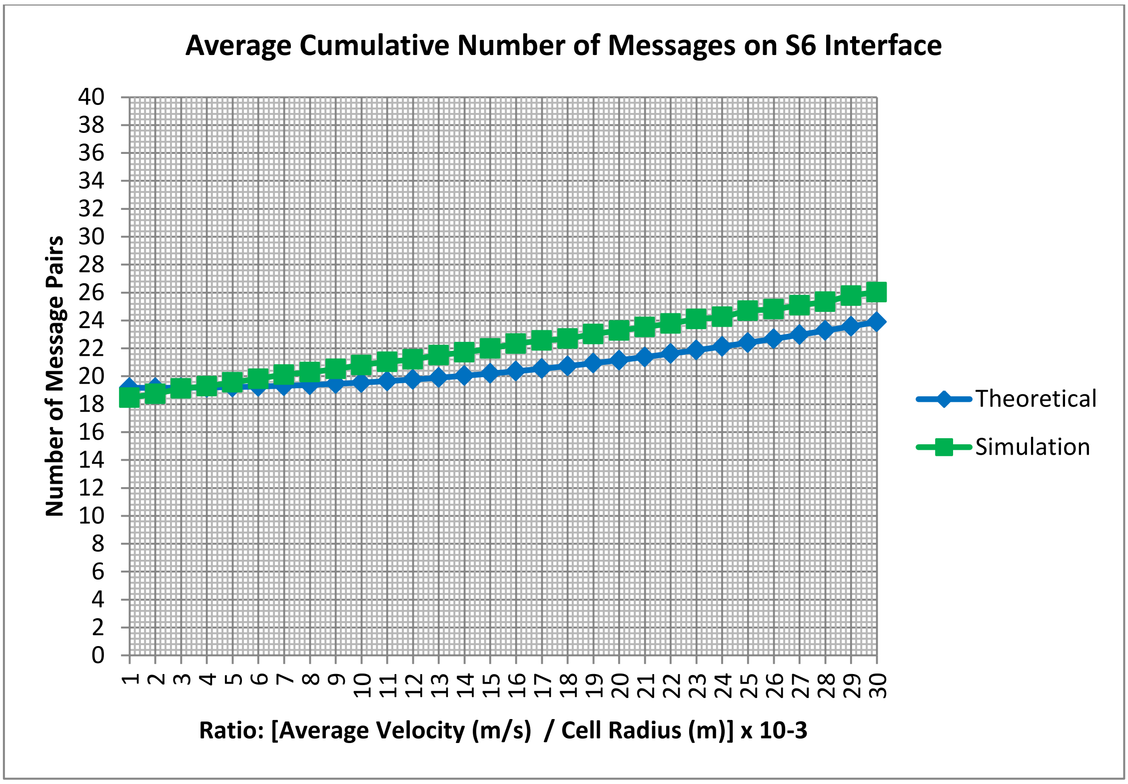


Figure 42. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - S6

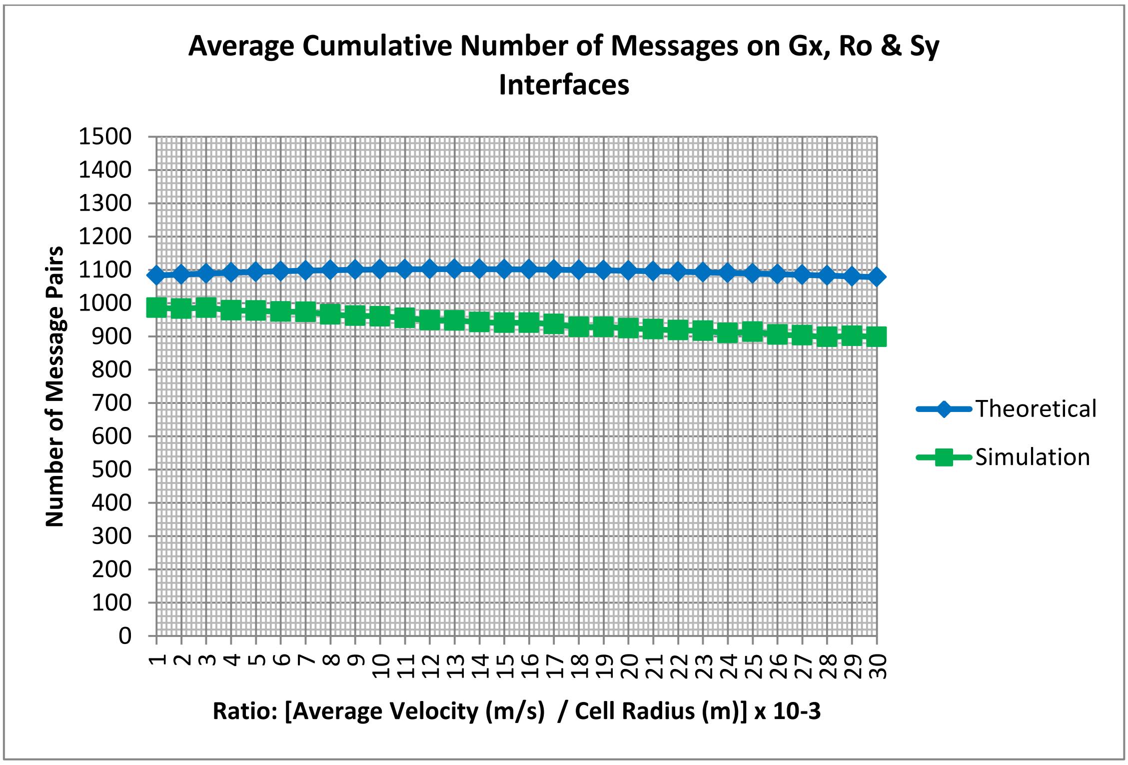


Figure 43. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - Gx, Ro & Sy

First, no study on cellphone networks can be said to be complete without taking in to consideration the effects of mobility, i.e. the movement of a UE from one cell to the next and so on. Hence, using the mobility model discussed in Chapter V section 2.3.2, simulations were run for a fixed cell radius of 1 Km, and the UE's average speed varied from 1 m/s to 30 m/s (3.6 Km/h to 108 Km/h). Figure 42 and Figure 43 show the average cumulative number of message pairs on the S6 interface and the Gx, Ro and Sy interfaces vs. the ratio $\frac{E[V]}{R}$ respectively. Given that, from the point of view of the EPC, a UE's mobility is seen as the transition from one cell to another, the $\frac{E[V]}{R}$ ration is used since it expresses the amount of time spent by a UE in a cell better than using the average UE speed on its own $\left(E[V]\right)$. Using the ratio also helps generalize the results in Figure 42 and Figure 43, because, for example, a UE moving at an average speed of 4m/s in a 1Km cell is expected to reside in that cell the same amount of time as a UE moving at 12m/s in a 3Km cell.

Second, Figure 42 shows that the increase of the average cumulative number of message pairs on the S6 interface vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall linear trend. This result is expected because the chances of a UE crossing from a cell to another along with the number of cell crossings that this UE would do during its active session lifetime $\left(\Delta t_{session}\right)$ increase as the UE's relative $\frac{E[V]}{R}$ ratio increases, i.e. as the UE's expected residence time in a cell $\left(E\left[\Delta t_{residence}\right]\right)$ decreases. Furthermore, it is noticeable that, for relatively low $\frac{E[V]}{R}$ ratios, the rate of change in the average cumulative number of message pairs on the S6 interface is minimal. Thus, it can be concluded that for low $\frac{E[V]}{R}$ ratios, the effects of mobility on a UE's activity is minimal. The importance of such a conclusion is that it helps simplify the analysis along with the mathematical expression of the Model derived in Chapter V section 2.1 by approximating the case of a UE with a low $\frac{E[V]}{R}$ to that of a fixed UE. This is easily done by taking $p_{cell-cross} = 0$ and $p_{fixed-UE} = 1$. The PMARE for the S6 interface was 7.21%.

Third, Figure 43 that the average cumulative number of message pairs on the Gx, Ro and Sy interfaces vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall linearly horizontal trend. This result is somewhat similar to that for experiment - 6, and more specifically Figure 39 where it was shows that the average cumulative number of message pairs on the Gx, Ro and Sy interfaces is minimally affected by the activity seen on the S6 interface. Although it must be noted that the theoretical and simulation result show a small deviation at high $\frac{E[V]}{R}$ ratios. The reason for slight decrease is because a UE expected session lifetime decreases slightly as the number of the reauthentications and reauthorizations increases. This makes sense because, for a very high number of reauthentication and reauthorization events, the chances of a UE failing one of the procedures increase. However, even with a high number reauthentication and reauthorization events, the rate of change for the average cumulative number of message pairs on the Gx, Ro and Sy interfaces is small as seen in Figure 43. Thus it can be concluded that overall, a UE's mobility, which is represented by the $\frac{E[V]}{R}$ ratio, has a minimal effect on the interfaces whose activity depends more on the length of a UE data session $(\Delta t_{session})$ than the UE's mobility. The PMARE for the Gx, Ro and Sy interfaces was 13.9%.

6.3.1 Case Study – 2

Case Study - 2 - Experiment

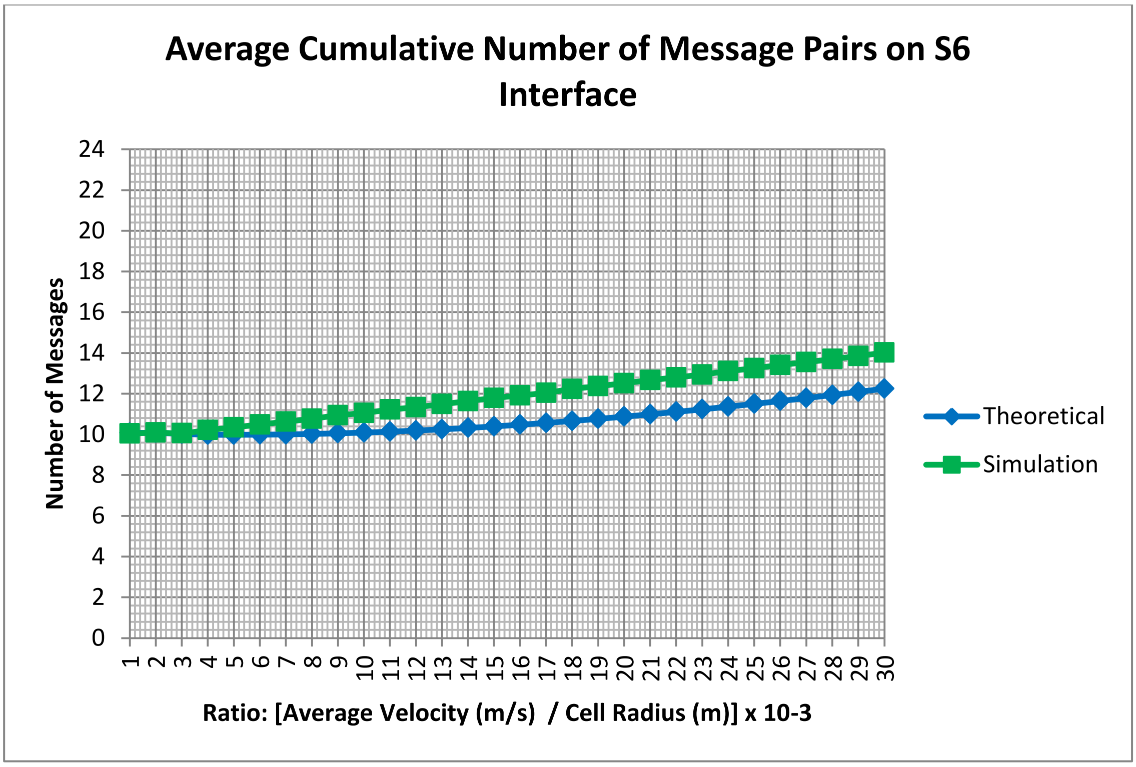


Figure 44. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - S6

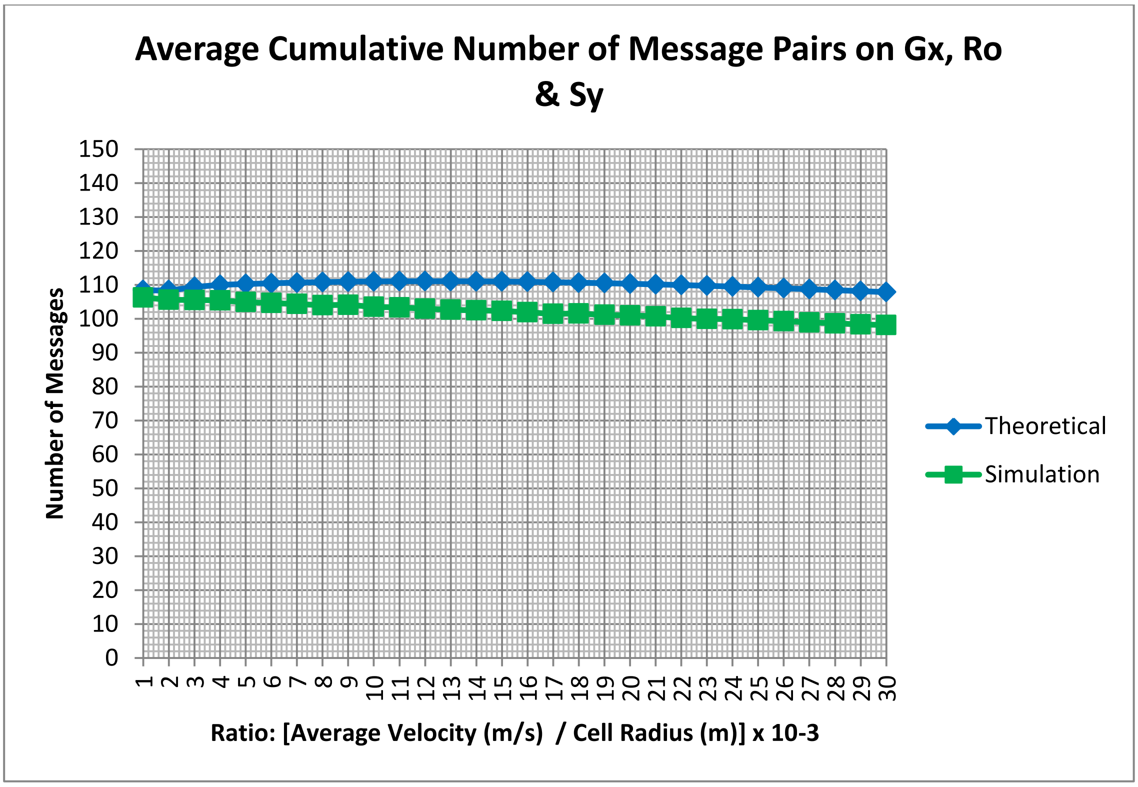


Figure 45. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - Gx, Ro & Sy

First, Figure 44 shows that the average cumulative number of message pairs on the S6 interface vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall linear trend. Similar to the results seen in Case Study - 1 - Experiment - 8, the average cumulative number of message pairs increase as the UE's relative $\frac{E[V]}{R}$ ratio increases, i.e. as the UE's expected residence time in a cell $\left(E[\Delta t_{residence}]\right)$ decreases. Furthermore, it is noticeable that, for relatively low $\frac{E[V]}{R}$ ratios, the rate of change in the average cumulative number of message pairs on the S6 interface is minimal. The results show a similar behavior to the results in Figure 43 that for low $\frac{E[V]}{R}$ ratios, i.e. the effects of mobility on a UE's activity is minimal and that analysis along with the mathematical expression of the Model derived in Chapter V section 2.1 can be simplified by approximating the case of a UE with a low $\frac{E[V]}{R}$ to that of a fixed UE. The PMARE for the S6 interface was 10.88%.

Second, Figure 45 shows that the average cumulative number of message pairs on the Gx, Ro and Sy interfaces vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall linearly horizontal trend which is also similar to the results seen in Case Study - 1 - Experiment - 8 Figure 43. The theoretical and simulation result show a similar small deviation at high $\frac{E[V]}{R}$ ratios like Figure 43, although it is less pronounced. This is due to chosing Δt_u equal to 5 seconds instead of 1 second. Thus, a bigger Interim Update Interval gives a smaller number of message pairs on the Gx, Ro and Sy interfaces along with a less pronounced rate of change than the one seen in Figure 43. The same conclusion as Case Study - 1 - Experiment - 8 can be made, i.e.the UE's mobility, which is represented by the $\frac{E[V]}{R}$ ratio, has a minimal effect on the interfaces whose activity depends more on the length of a UE data session $(\Delta t_{session})$ than the UE's mobility. The PMARE for the Gx, Ro and Sy interfaces was 7.15%.

6.3.1 Case Study – 3

Case Study - 3 - Experiment

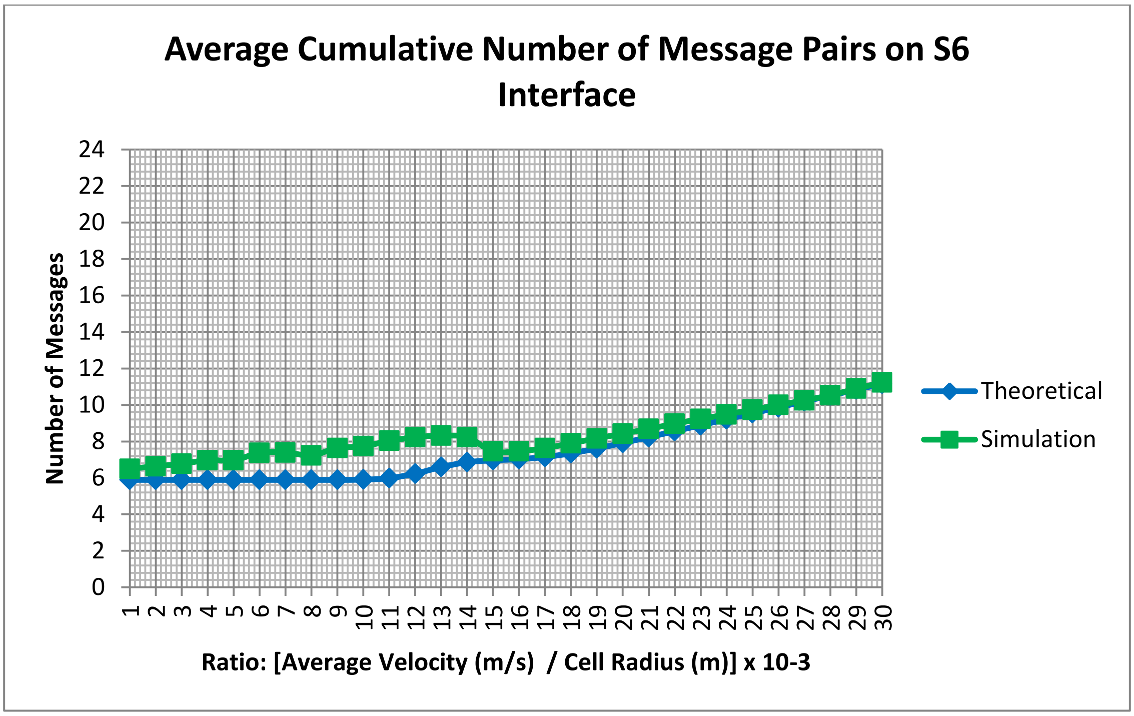


Figure 46. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - S6

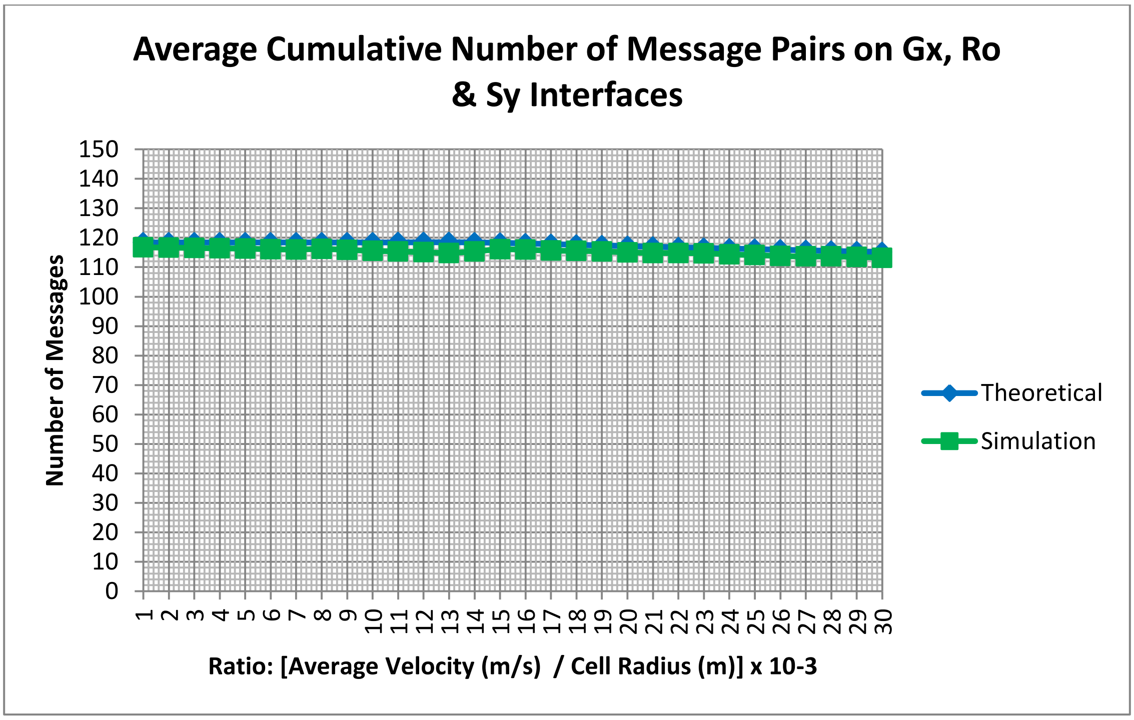


Figure 47. Number of Message Pairs vs. [Avg Velocity / Cell Radius] - Gx, Ro & Sy

First, Figure 46 shows that the average cumulative number of message pairs on the S6 interface vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall increasing trend. This is similar to the results seen in Case Study - 1 - Experiment - 8. However, for relatively medium ranged $\frac{E[V]}{R}$ ratios, a step like trend can be observed in the simulation results. This step like trend is also mirrored in the theoretical results although it is a bit less pronounced than the simulation results. This step like trend is due to the choice of variable distributions. The lognormal distribution differs from the exponential distribution by it rate of change. While the exponential distribution follows a decreasing trend in general, the lognormal distribution trend starts at a low value, increases to a maxima and then decreases. The general shape of the lognormal distribution, i.e. the location and height of the maxima, along with the rate of change on both sides is due to the distribution's parameters μ and σ (which are **not** its mean and variance). In addition, for this experiment both the $\Delta t_{session}$ and $\Delta t_{residence}$ follow a lognormal distribution. Thus the step like trend is due to both distributions reaching values close to their maxima and then shifting away from it towards a significantly lower value, hence showing a high rate of change then decreasing, all the while, their cumulative distribution functions keep on increasing as $\frac{E[V]}{R}$ increases. Thus, the overall trend recovers quickly after the step like trend for relative middle ranged $\frac{E[V]}{R}$ ratios and returns to its increasing behavior. Furthermore, and similarly to the results of both cases, the rate of change in the average cumulative number of message pairs on the S6 interface is minimal for relatively low $\frac{E[V]}{R}$ ratios. These results strengthen the conclusion made in the previous two cases that the effects of mobility on a UE's activity is minimal for relatively low $\frac{E[V]}{R}$ ratios and that analysis along with the mathematical expression of the Model derived in Chapter V section 2.1 can be simplified by approximating the case of a UE with a low $\frac{E[V]}{R}$ to that of a fixed UE. The PMARE for the S6 interface was 12.82%.

Second, Figure 47 that the average cumulative number of message pairs on the Gx, Ro and Sy interfaces vs a linearly increasing $\frac{E[V]}{R}$ ratio follows an overall linearly horizontal trend which is also similar to the results seen in Case Study - 1 - Experiment - 8 Figure 43. The theoretical and simulation result show a similar small deviation at high $\frac{E[V]}{R}$ ratios like Figure 43, although it is less pronounced. This is also due to choosing Δt_u equal to 5 seconds instead of 1 second. Thus, a bigger Interim Update Interval gives a smaller number of message pairs on the Gx, Ro and Sy interfaces along with a less pronounced rate of changed than the one seen in Figure 43. The same conclusion as Case Study - 1 - Experiment - 8 can be made that the UE's mobility, which is represented by the $\frac{E[V]}{R}$ ratio, has a minimal effect on the interfaces whose activity depends more on the length of a UE data session $(\Delta t_{session})$ than the UE's mobility. The PMARE for the Gx, Ro and Sy interfaces was 1.91%.

6.4 Sepcial Cases

The purpose of this section is to show the proximity of the MATLAB simulation scripts to the message exchange in real scenarios. Thus, three special cases were calculated manually and then compared to their respective MATLAB simulation runs:

6.4.1 Case - 1

The folowing MATLAB simulation is set up with the following parameters:

1. All UEs start their activity at the same time.
2. The UEs are fixed (not mobile).
3. All UEs have a fixed $\Delta t_{session}$ of 5min.
4. All UE's have a fixed $\Delta t_{auth/author}$ of 1min.
5. All UEs have a fixed Δt_{U} of 10sec.
6. All UEs are able to succeffuly complete all their Authentication and Authorization and Re-Authentication and Re-Authorization procedures.

Figure 48 shows the manual calculation of Special Case – 1 and the message exchange on all four interfaces can be seen. On the S6 interface, no reauthentication and reauthorization occurs after the fourth one since the UE's session reaches its natural end. The first bar on the S6 interface represents the AIA / AIR message pair with the ULR / ULA message pair as the second vertical bar. The dotted lines represent the time instance at which each pair occurs hence the sequence of message can also be observed. The CCR-I / CCA-I (Gx and Ro interfaces) and the SLR / SLA (Sy interface) message pairs are represented by the first bars on the Gx, Ro and Sy interfaces while the last bars on these interfaces represent the CCR-T / CCA-T (Gx and Ro interfaces) and STR /

STA (Sy interface) message pairs respectively. The Subsequent update messages, i.e the CCR-U / CCA-U (Gx and Ro interfaces) and SNR / SNA message pairs (Sy interface) are represent by the bars in between the first and the last bars on these interfaces. The Ro and Sy interfaces are similar to the Gx interface.

Furthermore, Figure 49 shows the MATLAB simulation results for the first 9 UEs of Special Case 1. Thus, given that all the UEs start their activit at the same time and have equivqlent sessions, the total cumulative number of message pairs for the population of 100,000 UEs is calculated by summing the cumulative number of message pairs of each UE which is equal to 600,000 message pairs on the S6 interface and 3,000,000 message pairs on the Gx, Ro and Sy interfaces. Using Figure 49 as a reference, the total number of message pairs can be estimated by multiplying the number of message pairs on each interface by the number of active UEs. Since all UEs start and stop their activity at the same time, then:

- The Cumulative Number of Message Pairs on the S6 interface is: $6 \times 100,000 = 600,000$.
- The Cumulative Number of Message Pairs on the Gx, Ro and Sy interfaces is: $30 \times 100,000 = 3,000,000$.

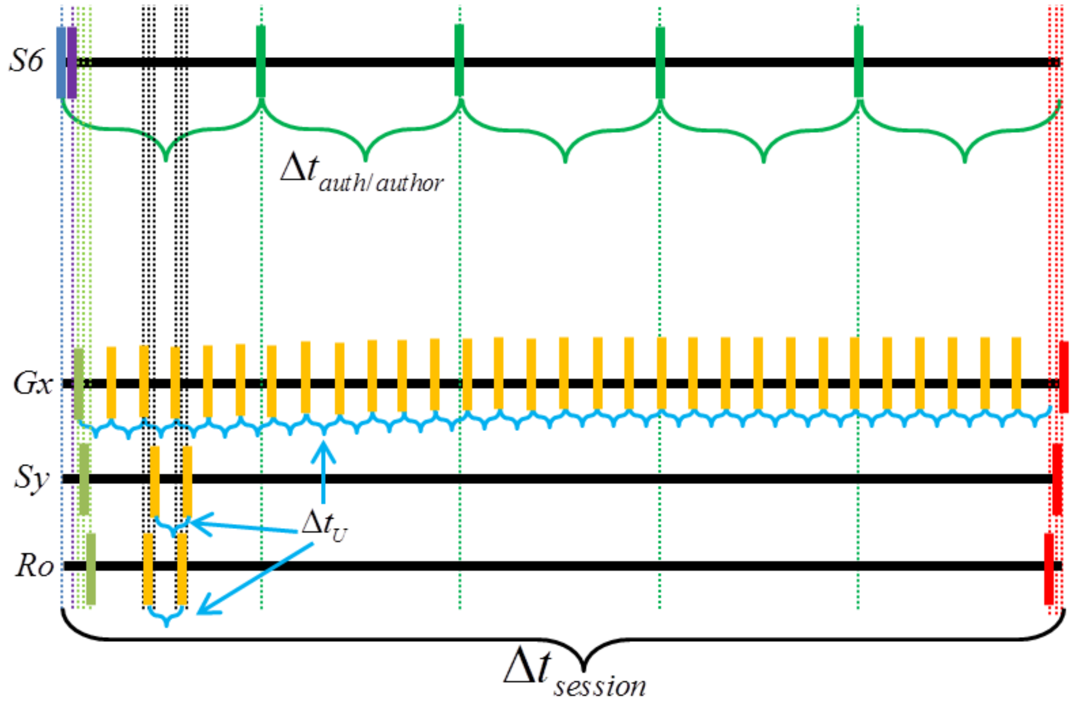


Figure 48. Special Case - 1 - A UE's Session

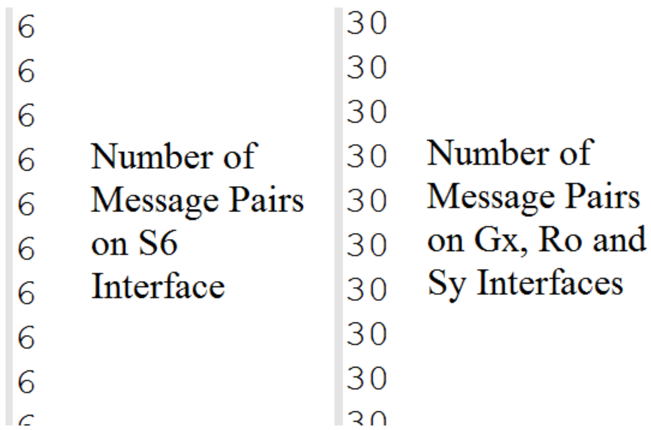


Figure 49. Special Case - 1 - MATLAB Simulation Output

6.4.1 Case - 2

The folowing MATLAB simulation is set up with the following parameters:

1. All UEs start their activity at the same time.
2. The UEs are fixed (not mobile).
3. All UEs have a fixed $\Delta t_{session}$ of 5min.
4. All UE's have a fixed $\Delta t_{auth/author}$ of 1min.
5. All UEs have Δt_U of 10sec.
6. All UEs are able to succeffuly complete their Authentication and Authorization and their 1st Re-Authentication and Re-Authorization procedure but all of them will fail the 2nd Re-Authentication and Re-Authorization procedure.

Figure 50 shows the manual calculation of Special Case - 2. The message exchange on all four interfaces can be observed. On the S6 interface, the 2nd reauthentication and reauthorization fails and the UE's session is stoped. The bars in Figure 50 hold the same meaning as those in Figure 48.

Furthermore, Figure 51 shows the MATLAB simulation results for the first 9 UEs of Special Case 2. Thus, given that all the UEs start their activit at the same time and have equivqlent sessions, the total cumulative number of message pairs for the population of 100,000 UEs is calculated by summing the cumulative number of message pairs of each UE which is equal to 400,000 message pairs on the S6 interface and 1,200,000 message pairs on the Gx, Ro and Sy interfaces. Using Figure 49 as a reference, the total number of message pairs can be estimated by multiplying the number of message pairs on each interface by the number of active UEs. Since all UEs start and stop their activity at the same time, then:

- The Cumulative Number of Message Pairs on the S6 interface is: $4 \times 100,000 = 400,000$.
- The Cumulative Number of Message Pairs on the Gx, Ro and Sy interfaces is: $12 \times 100,000 = 1,200,000$.

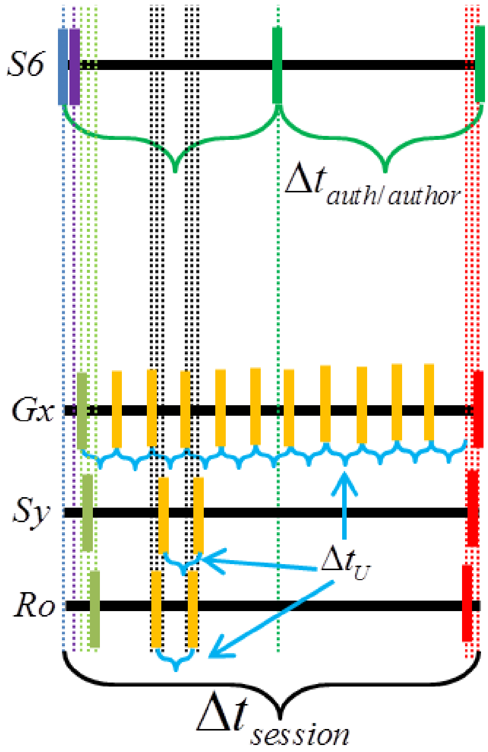


Figure 50. Special Case - 2 - A UE's Session

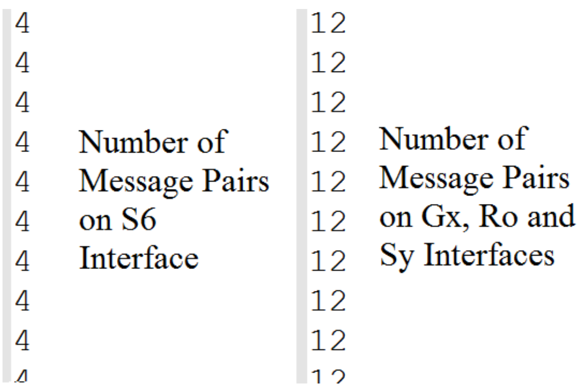


Figure 51. Special Case - 2 - MATLAB Simulation Output

6.4.1 Case - 3

The folowing MATLAB simulation is set up with the following parameters:

1. All UEs start their activity at the same time.
2. The UEs are fixed (not mobile).
3. All UEs have a fixed $\Delta t_{session}$ of 6min.
4. All UE's have a fixed $\Delta t_{auth/author}$ of 1min.
5. All UEs have Δt_U of 10sec.
6. All UEs first residence time $(\Delta t_{residence})$ is equal to 3min with all subsequent residence times equal to 2.5min.
7. All UEs start their activity after spending 0.5min $(\theta_{residence})$ of their first residence time.
8. All UEs are able to succeffuly complete all their Authentication and Authorization and Re-Authentication and Re-Authorization procedures.

Figure 52 shows the manual calculation of Special Case - 2. The message exchange on all four interfaces can be seen. It can be seen that the UE crosses the first cell boundary after 2.5min have passed from the session start along with the $\Delta t_{auth/author}$ being reset after each reauthentication and reauthorization via cell boundary crossing. The bars in Figure 52 hold the same meaning as those in Figure 48.

Furthermore, Figure 53 shows the MATLAB simulation results for the first 9 UEs of Special Case - 3. Thus, given that all the UEs start their activit at the same time and have equivqlent sessions, the total cumulative number of message pairs for the population of 100,000 UEs is calculated by summing the cumulative number of message pairs of each UE which is equal to 800,000 message pairs on the S6 interface and 3,600,000 message pairs on the Gx, Ro and Sy interfaces. Using Figure 49 as a reference, the total number of message pairs can be estimated by multiplying the number of message pairs on each interface by the number of active UEs. Since all UEs start and stop their activity at the same time, then:

- The Cumulative Number of Message Pairs on the S6 interface is: $8 \times 100,000 = 800,000$.
- The Cumulative Number of Message Pairs on the Gx, Ro and Sy interfaces is: $36 \times 100,000 = 3,600,000$.

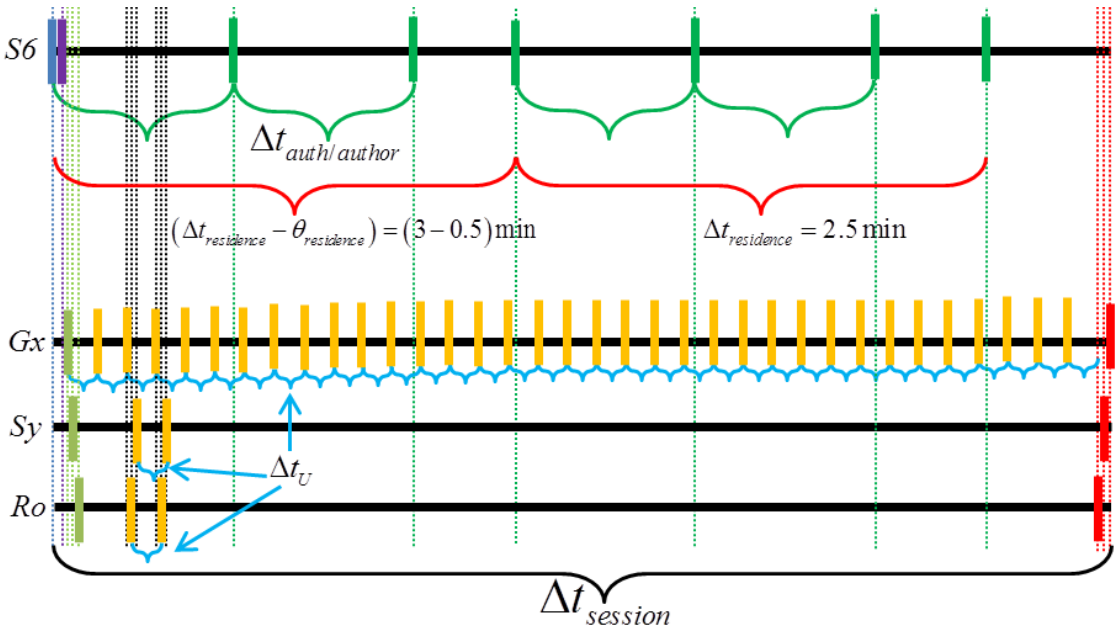


Figure 52. Special Case - 3 - A UE's Session

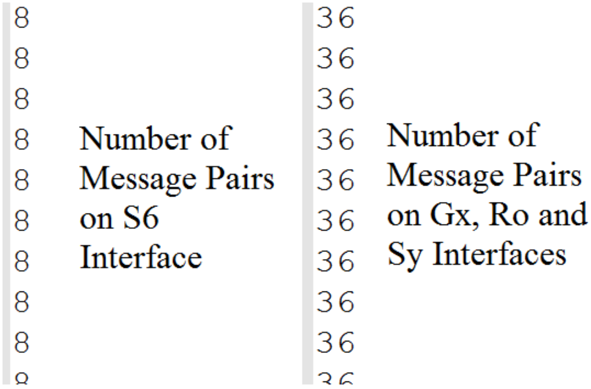


Figure 53. Special Case - 3 - MATLAB Simulation Output

Thus, using the 3 special cases shown in this section, it can be concluded that the Diameter interfaces within the MATLAB simulation scripts behave very similarly to what a Diameter interface would behave in reality.

6.5 Section Conclusion

Thus, the mathematical model derived in Chapter 5 was validated using MATLAB simulations. Two scripts where used, on for Fixed UEs and another that includes the effects of a UE's mobility to the message exchange. The MATLAB Simulations were built around two pillars:

1. The Diameter Base Protocol from RFC 6733 [3] and the Diameter Credit-Control Application from RFC 4006 [6] and uses Authentication and Authorization lifetime $\left(\Delta t_{auth/author}\right)$, the Interim Update Interval $\left(\Delta t_U\right)$ for credit control update message pairs, the Grace Period $\left(\Delta t_{grace}^{max}\right)$ and the RAR / RAA message pair exchange reasons and causes.
2. The message exchange sequence in Figure 16.

The theoretical and MATLAB simulation results were compared using the Percent Mean Average Relative Error (PMARE) and showed good agreement with the lowest PMARE of 1.91% and the highest PMARE 13.9%.

The results also showed the importance of not overlooking the success rate of the reauthentication and reauthorization procedure during a UE's data session, including the effects of the grace period, which manifests itself in the exponential and logarithmic rate of change seen in experiments 3 and 4 respectively.

Finally, the results from Case Study - 1 - Experiment - 8, Case Study - 2 and Case Study - 3 highlighted two major points. First, the effects of mobility are minimal on UEs with a relatively low $\frac{E[V]}{R}$ ratio. Second, the Diameter interfaces can indeed be grouped into two groups:

1. Interfaces that are highly affected my Mobility.
2. Interfaces where the effects of Mobility are minimal.

CHAPTER VII

MODEL APPLCATION

Extending the current model to estimate the aggregate total traffic generated by all the UEs cannot be done simply by multiplying the number or total UEs in the collection of cells in question by the numbers generate by the model. The following presents the general approach that can be followed:

First, given that the traffic per session per UE was calculated, what remains is to express the total traffic that is created by all active UEs in the LTE network. Expressing the total traffic is done by using three pieces of data parameters:

1. $N_X^{session/UE}$: This is the number of message pairs exchanged for Authentication/Authorization or Accounting on the different interfaces (where X represents an interface)
2. $\psi(t)$: This is the total number of active sessions in function of time. This parameter does not have an explicit form and is gathered by observing a network for a period of time. For example: $\psi(t = 5\,\text{min}) = 4$ active sessions, $\psi(t = 10\,\text{min}) = 27$ active sessions. Thus, the smaller the sampling interval, the more accurate the results are.
3. Hence, the total number of messages per second $\phi(t)$ seen on a particular interface "Y" at some time "t_i" can be expressed as:

$$\phi(t = t_i) = \psi(t = t_i) \times \frac{N_{X=Y}^{session/UE}}{E\left[\Delta t_{session}\right]} \ (msg/\text{sec})$$

Second, changing the values in the Reference Set of variables (Table 4) according to the time during the day (example: from 1 pm to 3 pm) during which the Diameter traffic needs to be estimated, the living area (town, city, country, ...) in which the UEs are located and the desired application (Facebook, VoIP, …) to be studied.

Third and finally, when predicting future traffic $\phi^*(t = t_i)$, previous statistical data can be used $\left\{ \psi(t = t_i) \ ; \ \frac{N_{X=Y}^{session/UE}}{E\left[\Delta t_{session}\right]} \right\}$ along with with the predicted futur total number of active sessions $\psi^*(t = t_i) = \left(1 + \frac{k}{100}\right) \times \psi(t = t_i)$, where k is the percentage of the expected increase or decrease for the future traffic. Thus the predicted future total aggregated traffic can be expressed as:

$$\phi^*(t = t_i) = \psi^*(t = t_i) \times \frac{N_{X=Y}^{session/UE}}{E\left[\Delta t_{session}\right]} \ (msg / \sec)$$

$$\phi^*(t = t_i) = \left(1 + \frac{k}{100}\right) \times \psi(t = t_i) \times \frac{N_{X=Y}^{session/UE}}{E\left[\Delta t_{session}\right]} \ (msg / \sec)$$

CHAPTER VIII

CONCLUSION

The target of this study was to develop a mathematical and logical approach that can be used to describe Diameter Control traffic in an LTE environment for any combination of [Interfaces, Messages Pairs] for a single Location Area, i.e., as defined in this study, a number of LTE cells that are governed by set of LTE elements (and single instance of each element, i.e. one MME, one HSS, one …). It used a set of interfaces (S6, Gx, Ro and Sy) and a set of corresponding Diameter message pairs that are exchanged on them, and split them into two cases:

- The case where an interface (S6) and its corresponding message pairs depend on the UE’s mobility and residence time in a cell.
- The case where the interfaces (Gx, Ro and Sy) and their corresponding message pairs depend more on the UE’s session duration.

Two different expressions were derived for the cumulative number of message pairs per session per UE based on the UE’s behavior , parametrized, presented in the form of tables (Table 1 and Table 2), and are in function of:

1. The average session duration.
2. The average residence time, which is in function of:
    1. The cell radius.
    2. The UE’s average speed.
3. The re-authentication and re-authorization lifetime.
4. The interim update duration.

5. The grace period.
6. The authentication and authorization success rate.
7. The re-authentication and re-authorization success rate.

The mathematical model included the effects of the success rate of re-authentication and re-authorization and considered the effects of the grace period during re-authentication and re-authorization.

Furthermore, the model followed a general derivation by making no assumptions on the distribution on the UE's session duration and residence time in a cell except that they are independent instead of restricting the model to it to a particular distribution, allowing the use of any desired distribution.

In addition, the Markov based mobility model from [43] was adapted from a model that estimates the residence time per Location Area, i.e. a large area formed by a group of cells, to one that estimates the residence time per cell in function of the UE's average speed and cell radius and thus sheds light on the effects of a UE's mobility on the number of re-authentication and re-authorization during a UE's active session. This in turn brings light to the case were a change to a UE's session occurs because of the "newly moved into" cell's status, thus forcing the UE to re-authenticate and re-authorize, and if it is necessary, drop its current active session.

The model was validated on MATLAB. Two spate scripts were written, one for fixed UEs and another that includes the effects of a UE's mobility to the message pairs exchanged in the EPC. The MATLAB simulation scripts where based in general on information from RFC 6733 [3] and RFC 4006 [6] and uses Authentication and Authorization lifetime, the Interim Update Interval for credit control update message pairs, the Grace Period , the Re-Authentication and Re-Authorization message pair exchange reasons and causes along with the message exchange sequence in Figure 16 as a scaffold.

Mutiple experiements were conducted. The results showed the importance of not overlooking the success rate of the reauthentication and reauthorization procedure during a UE’s data session and including the effects of the grace period. Furthermore, the result showed that, when it comes to mobility, it is better to study the traffic in the EPC vs the $\frac{E[V]}{R}$ ratio and that the effects of mobility are minimal on UEs with a relatively low $\frac{E[V]}{R}$ ratio.

The simulation and theoretical results were compared using the Percent Mean Absolute Relative Error (PMARE). The highest and lowest PMARE were 13.9% and 1.91% respectively.

Finally, a method was presented in order to estimate the aggregate total traffic generated in the EPC by a population of UEs. The Method is based on studying the population’s past statistics and combining it with the mathematical model to predict the future aggregate total traffic.

CHAPTER IX

FUTURE WORK

The future work of this study revolves around two main points: first solidifying the validity of the model by using additional simulators with a good representation of an EPC once they are available and second, extending the work and covering additional variables.

When it comes to the second point, the possible extensions are many and are not limited to the following possibilities:

9.1 Delay Analysis

Delay Analysis is mainly a queuing problem. Thus in order to proceed with it, the idea discussed in the Chapter VII must be implemented first. Now, given that the number of messages per second for the desired interface is estimated, queuing techniques such as the M/M/1/K queue can be applied, with "K" being the buffer size, in order to estimate the service time or the delay between a request and its appropriate response. Another queuing model, the M/M/C/K queue can be used in case of load balancing between multiple entities with similar functions (example: load balancing between multiple MMEs, multiple HSSs, …).

9.2 Adding the effects of the lower layers

Including the effects of lower layers, such as the transport layer, the network layer and so on, would show the "bigger picture", or better yet, the complete picture of the traffic on a particular interface, whether a logical one or physical one. However, there is no real contribution there since the lower OSI layers such as the transport, network and MAC layers have been the subject of extensive studies and modeling in the past. Hence one can simply pick up the desired model for these layers and use the number of messages per second discussed in Chapter VII as an input to the desired model.

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