AMERICAN UNIVERSITY OF BEIRUT

PRICING AND ASSORTMENT DECISIONS WITH SUPPLY CHAIN INTEGRATION

by
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AN ABSTRACT OF THE THESIS OF

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Recent research has demonstrated the benefits of “horizontal integration” in retailing by *jointly* optimizing critical retail decisions on aspects such as assortment planning, pricing, and inventory levels. Another stream of research also demonstrates that “vertical integration”, by accounting for contractual and logistical considerations on the supply side (such as quantity and volume discounts, delay in payment, truck load capacity, etc.), is equally beneficial. However, very limited research has been done on optimizing retail decisions (such as assortment and pricing) while accounting for supply chain considerations. The research in this thesis is along these lines of extended horizontal and vertical integration in retailing.

Specifically, we study the effect of quantity discount contracts and truckload shipping costs on a retailer’s joint pricing and assortment decisions for a product line (category) of substitutable retail products. The study is done with a demand model aggregated from consumer preferences, based on a deterministic utility function, and in a one retailer-multiple suppliers setting. In order to gain clear insights, we propose to develop models of different flavors accounting for (i) quantity discount and (ii) truckload capacity.

With the deterministic utility model, based on a market with several customer segments having known valuations for the different products in the category, our models are based on mathematical programming, specifically, nonlinear integer programs. These models are typically hard to solve. However, by developing effective linear reformulation schemes, we reduce the computational burden. These schemes reduce the problem to an integer linear program, which can be solved efficiently with many available commercial solvers. The linearized models provide useful managerial insights and practical decision support tools.

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CHAPTER 1

INTRODUCTION AND MOTIVATION

The ever growing size of the retail industry and especially that of the supplier competition, urges these suppliers to offer more appealing contracts to retailers. Faced with many different contracts, retailers need reliable methods for their decision making processes, critical on aspects such as pricing and assortment, that account for supply chain considerations.

However, very little work in the literature considers supply chain integration with retail decision such as pricing and assortment. For example, Monahan (1984) studies the effect of quantity discount on the ordering quantity of the retailer. In another paper, Kim and Hwang (1989) study the effect of quantity discount on the ordering size of the retailer and the ordering cost. However, both of these works do not consider assortment and pricing decisions. Moreover, Glickman and White (2008) use an optimization model with truckload capacity for supplier selection, product acquisition and shipment distribution problem for known prices and demand.

In our work, we intend to study the integration of both quantity discount contracts and truckload consideration in the retailer's assortment planning, and pricing decisions for a product line of substitutable retail products. For this purpose, we use the model described in Ghoniem and Maddah (2015) as our basis model with interesting adjustments.

The model we use is a maximum utility demand model; the customers choose to buy the product with a price that maximizes their utilities. We define the customer's utility as the difference between his reservation price (maximum price the customer is willing to pay for an item) and the retailer's price of the product. In addition we consider a segmented consumer market where customers are aggregated into market segments defined by their reservation prices. The maximum utility demand model is widely used in the literature; for example Shioda et al. (2009) use this model while also considering customers as a collection of segments each characterized by the same purchasing behavior that maximizes its utility. Many other papers in the literature use the maximum utility model with a segmented market like Mussa and Rosen (1978), Dobson and Kalish (1988) and Hanson and Martin (1990).

In this thesis we apply a quantity discount contract to the model of Ghoniem and Maddah (2015). We also investigate the same model under truckload shipping costs which are a realistic aspect of retailing logistics. The shipping costs depend on the shipment size and whether it is a full truckload or less than a truckload. For the less than truckload shipments, the cost in dollars per kilogram of product is higher than that of a full truckload shipment. This affects the retailer's decision on how much product to ship and from which supplier since the shipment costs depend on the supplier involved.

The remainder of this thesis is organized as follows. In Chapter 2, we review the related work in the literature and compare it with our work. In Chapter 3, we present the quantity discount model with the related mathematical formulation, illustrative examples, and a computational study. In Chapter 4, we present the truckload shipping costs in a multi-supplier setting with the related mathematical formulation, illustrative examples, and a computational study. Finally in Chapter 5, we conclude our work and suggest directions for future research.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

Our work is an integration of marketing and operations decisions in retailing. This integration is of great importance to the retailing industry especially today with the fierce competition. High value performance is required to face the competition. This performance is enhanced by cooperation between the various departments of the firm, especially the integration of marketing and operations. The importance of this integration is evident in the literature where one can find many articles describing effective examples of linking operations and marketing. For example Bregman (1995) demonstrates a specific approach for improving the performance of retail firms by integrating the decision process. His approach provides tangible evidence of the benefits of integrating marketing, operations and purchasing decisions. Karmakar (1996) presents in his paper a series of examples of research topics that require the integration of operations and marketing. In this thesis, we develop pricing and assortment optimization models that fit into this paradigm of joint marketing and operations perspective, which we refer to as “horizontal integration”. In addition, we also consider “vertical integration” aspects by accounting for the effect of quantity discounts and truckload capacity. In this chapter, we briefly review related works on horizontal integration, in section 2.1 and on vertical integration, in section 2.2.

2.1. Horizontal Integration Literature

We focus in our work on the integration of two critical decisions made by the retailer for a product line of substitutable items; assortment size and pricing. Typically, papers in the literature consider integrating two of the following three decisions:

assortment size, pricing and inventory. For example, some papers investigating the integration of inventory decisions and assortment size decisions are Yücel et al. (2009), Gaur and Honhon (2006), Urban (1998), Hariga et al. (2007) and Mayorga et al. (2013). Other papers like Dobson and Kalish (1993), Mcintyre and Miller (1999) and Draganska et al. (2009) study the joint optimization of designing (assortment planning) and pricing a product line. Pricing and inventory integrated decision are also present in the literature. Such papers are Aydin and Porteus (2008), Dong et al. (2009), Hall et al. (2010), Huang et al. (2011), and Maddah et al. (2014).

Few papers in the literature do investigate the integration of all three critical decisions; assortment size, pricing and inventory, such as Maddah and Bish (2007), Kök and Xu (2011), Rodriguez and Aydin (2011), and Ghoniem and Maddah (2015). These papers are divided into two types, stylized (focusing on insights from simple models) and optimization driven (focusing on deriving practical decision aid tools). For a review of the stylized type, we refer to Maddah et al. (2014), and for a review of the optimization-driven papers, we refer to Ghoniem and Maddah (2015).

Ghoniem and Maddah (2015) develop a mixed integer linear program where demand is driven by exogenous consumer reservation prices and endogeneous assortment and pricing decisions in a multi-period selling horizon. They analyze the effect of seasonality of demand and costs on assortment and inventory decisions and find that these effects lead to wider assortments and higher inventory levels. In this thesis, we extend the work of Ghoniem and Maddah (2015) by considering vertical integration aspects via quantity discount models and truckload capacity.

2.2. Vertical Integration Literature

The quantity discount contract as a type of coordination between supplier and retailer was studied by Cachon and Kök (2010). This paper considers the case of multiple manufacturers selling through a single retailer. The manufacturers are competing using one of three types of contracts; a wholesale-price contract, a quantity-discount contract, or a two-part tariff. In our model, we only consider the quantity discount contract. In addition in our model the contracts offered by each supplier are independent; suppliers are blind to the competitors' contracts.

Wee (1999) develops a deterministic inventory model with quantity discount, pricing and partial backordering when the product in stock deteriorates with time. The demand rate is assumed to decrease as price for the product increases. In our study we do not include backordering and the demand depends both on the price and the reservation price of each customer segment. The retailer receives the contracts and decides on the assortment design and the pricing.

In their paper Li and Liu (2006) develop a model for illustrating how to use quantity discount policy to achieve supply chain coordination. A supplier–buyer system selling one type of product with multi-period and probabilistic customer demand is considered. In contrast, our study considers multiple substitutable products offered by multiple suppliers with a deterministic demand function.

In a review paper by Sarmah et al. (2006) the authors review literature on buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. These coordination models lead to savings in the system and improvement in the overall performance of the supply chain. In the model discussed by Monahan (1984), a vendor could encourage his customer to increase the order quantities from EOQ by offering a price discount. The amount of discount offered by the vendor compensates the buyer's increased inventory costs. Our model in this thesis does not capture inventory costs. As such, higher order quantities due to these effects are not expected.

Other papers also consider quantity discount like Yang (2004) where the author develops an optimal pricing and ordering policy for a deteriorating item with price sensitive demand with a quantity discount pricing strategy. In addition, Viswanathan and Wang (2003) evaluate the effectiveness of quantity discounts and volume discounts as coordination mechanisms with price sensitive deterministic demand. Finally, Rosenblatt and Lee (2007) study the quantity discount contracts from a supplier's point of view.

Accounting for the truckload costs of shipping the product from supplier to retailer is a realistic approach. These shipping costs do affect the overall cost of the products selected and therefore affects the assortment, pricing and supplier selection decisions. Truckload costs are investigated in a paper by Glickman and White (2008). The authors study the retailer's problem to decide what to order from each supplier and where to send it when products are sold by multiple suppliers in various locations. To solve this decision problem the authors develop an optimization model that they apply to a wholesale distribution of grocery products. Comparing the model's solution with the actual record of shipments reveals instances in which the model selected higher-priced vendors in order to capitalize on truckload cost savings, which are seen to be an important factor in vendor selection. We aim in our work to see the effects of truck load costs on vendor selection and pricing and assortment decisions made by the retailer. Our work differs from Glickman and White's work by endogenizing pricing and assortment decisions via a deterministic utility model.

Truckload shipping costs are usually investigated in the literature as a part of supplier selection mechanism, e.g., Aguezzoul and Ladet (2004), Ghodsypour and O'Brien (2001), Glickman and White (2008), and Smytka and Clemens (1993), or for choosing the best transportation method for goods like Chu (2005) where the author presents heuristic algorithms for the truckload and less than truckload costs.

CHAPTER 3

QUANTITY DISCOUNT MODEL

In this chapter, we introduce a model similar to the model developed earlier by Ghoniem and Maddah (2015) with important alterations. Our model includes a quantity discount contract available to the retailer. In section 3.1., we formulate the problem as a non-linear model over a one-period selling horizon. In section 3.2., we linearize our model. In section 3.3., we introduce illustrative examples and in section 3.4., we perform a computational study of the model.

3.1. Formulation of the non-linear model

The problem examines a single period selling horizon where the retailer jointly optimizes assortment planning and the pricing decisions for a product line of substitutable products within a market of multiple consumer segments with quantity discount contracts.

Let $J = \{1, 2, ..., n\}$ be the set of substitutable products from which the retailer composes her product line, and let $I = \{1, 2, ..., m\}$ be the set of consumer segments present in the market that make purchasing decisions over the selling period. Each consumer segment $i \in I$, purchases at most one product from J in order to maximize its utility. The utility of consumer segment i from product $j \in J$ is defined as the difference between its reservation price, α_{ij}, and the retail price p_j. The utility of the no purchase option denoted by $j = 0$ is scaled to be zero. The quantity discount contracts allow the retailer to buy the products at lower unit costs if the quantities ordered are above certain thresholds; U_j. Specifically, the unit cost of product *j* is given by $c_j = \begin{cases} c_{j1} & if \;\; Q_j < U_j \\ c_{j2} & if \;\; Q_j \geq U_j \end{cases}$, where Q_j is the amount ordered for product *j*.

The following are the parameters used in our model.

- α_{ij} is the reservation price of customer segment *i* for product *j*.
- k_j is the fixed cost incurred by ordering product *j*.
- c_{j1} and c_{j2} are the unit cost of product *j* without and with quantity discount respectively.
- s_i is the size of consumer segment *i*.
- d_j is the demand of product *j*.
- U_j is the quantity order threshold for product *j*.

The model decision variables are as follows.

- Q_{j1} and Q_{j2} are the quantities ordered of product *j* without and with quantity discounts respectively.
- Q_j is the quantity ordered of product *j*; $Q_j = Q_{j1} + Q_{j2}$.
- z_j is a binary variable such that $z_j = 1$ if product *j* is included in the assortment, and $z_j = 0$ otherwise.
- x_{ij} is a binary variable such that $x_{ij} = 1$ if consumer segment *i* purchases product *j*, and $x_{ij} = 0$ otherwise.
- p_j is the retail price of product *j*.

- v_{j1} is a binary variable such that $v_{j1} = 1$ if product *j* is purchased without discount (at a cost of c_{j1}), and $v_{j1} = 0$ otherwise.
- v_{j2} is a binary variable such that $v_{j2} = 1$ if product *j* is purchased with discount (at a cost of c_{j2}), and $v_{j2} = 0$ otherwise.

The model formulation with quantity discount contract is as follows:

$$\text{Maximize} \quad \sum_{j=1}^{n} \left[p_j d_j - k_j z_j - c_{j1} Q_{j1} - c_{j2} Q_{j2} \right] \qquad (1a)$$

Subject to

$$\sum_{k=1}^{n} (\alpha_{ik} - p_k) x_{ik} \geq (\alpha_{ij} - p_j) z_j, \forall i, j \qquad (1b)$$

$$\sum_{k=1}^{n} (\alpha_{ik} - p_k) x_{ik} \geq 0, \forall i \qquad (1c)$$

$$\sum_{j=0}^{n} x_{ij} = 1, \forall i \qquad (1d)$$

$$x_{ij} \leq z_j, \forall i, j \qquad (1e)$$

$$d_j = \sum_{i=1}^{m} x_{ij} s_i, \forall j \qquad (1f)$$

$$Q_j = Q_{j1} + Q_{j2}, \forall j \qquad (1g)$$

$$Q_{j1} \leq v_{j1}(U_j - 1), \forall j \qquad (1h)$$

$$Q_{j2} \geq v_{j2} U_j, \forall j \qquad (1i)$$

$$v_{j1} + v_{j2} = z_j, \forall j \qquad (1j)$$

$$Q_{j2} \leq v_{j2} \sum_{i=1}^{m} s_i, \forall j \qquad (1k)$$

$$p_j \leq z_j \max_i \{\alpha_{ij}\}, \forall j \qquad (1l)$$

$$Q_j = d_j \qquad (1m)$$

x, z, v are binary variables (1n)

$$Q, Q_{j1}, Q_{j2}, d_j, p \geq 0 \qquad (1o)$$

Q_{j1}, Q_{j2}, Q_j, d_j are integers *(1p)*

The objective function in (1a) maximizes the retailer's profit composed of sales revenues minus the fixed costs, and the variable cost which depends on the quantity discount scheme. Constraints (1b)-(1c) reflect the customers' behavior of maximizing their utility under the deterministic choice model. Constraints (1d) ensure that each customer buys at most one variant from the product line. Constraints (1e) guarantee that a customer will select a product from the assortment offered by the retailer, or buy nothing. Constraints (1f) aggregate the demand for each product from the customer preferences, and constraints (1m) set the order quantity for each product equal to its aggregated demand. Constraints (1f)-(1j) enforce the all-unit quantity discount scheme that we adopt for all products. Constraints (1k) and (1l) impose valid upper bounds on the order quantity and the price of each product. These constraints serve to tighten the formulation, and, eventually, reduce the computational effort for solving the model. Constraints (1n)-(1p) define the required types of our decision variables.

3.2. Linearization of the model

We now linearize the model in section 3.1. For this purpose, we introduce the following two sets of variables, g_{ij} and w_j, such that:

$$g_{ij} = p_j x_{ij}, \forall i, j, \qquad (2a)$$

$$w_j = p_j z_j, \forall j \tag{2b}$$

This linearization is similar to the one in Ghoniem and Maddah (2015), where further details and discussions are presented along with related linearization constraints.

The model formulation over one period selling horizon is as follows:

$$\text{Maximize} \qquad \sum_{j=1}^{n}\left[\sum_{i=1}^{m} s_i g_{ij} - k_j z_j - c_{j1}Q_{j1} - c_{j2}Q_{j2}\right] \tag{3a}$$

Subject to

$$\sum_{k=1}^{n}\left(\alpha_{ik}x_{ik} - g_{ik}\right) \geq \alpha_{ij}z_j - w_j, \forall i, j \tag{3b}$$

$$\sum_{k=1}^{n}\left(\alpha_{ik}x_{ik} - g_{ik}\right) \geq 0, \forall i \tag{3c}$$

$$\sum_{j=0}^{n} x_{ij} = 1, \forall i \tag{3d}$$

$$g_{ij} \leq \max_k\{\alpha_{kj}\}x_{ij}, \forall i, j \tag{3e}$$

$$g_{ij} \geq p_j - \max_k\{\alpha_{kj}\}(1 - x_{ij}), \forall i, j \tag{3f}$$

$$g_{ij} \leq p_j, \forall i, j \tag{3g}$$

$$w_j \leq z_j \max_i\{\alpha_{ij}\}, \forall j \tag{3h}$$

$$w_j \geq p_j - \max_i\{\alpha_{ij}\}(1 - z_j), \forall j \tag{3i}$$

$$w_j \leq p_j, \forall j \tag{3j}$$

$$x_{ij} \leq z_j, \forall i, j \tag{3k}$$

$$p_j \leq z_j \max_i\{\alpha_{ij}\}, \forall i, j \tag{3l}$$

$$Q_{j1} \leq v_{j1}(U_j - 1), \forall j \tag{3m}$$

$$Q_{j2} \geq v_{j2}U_j, \forall j \tag{3n}$$

$$Q_{j2} \leq v_{j2} \sum_{i=1}^{m} s_i, \forall j \qquad (3o)$$

$$Q_{j1} + Q_{j2} = \sum_{i=1}^{m} x_{ij} s_i, \forall j \qquad (3p)$$

$$v_{j1} + v_{j2} = y_j, \forall j \qquad (3q)$$

$x_{ij}, v_{j1}, v_{j2}, z_j$ are binary variables and Q_{j1}, Q_{j2} are integers *(3r)*

$$g_{ij}, p_j, Q_{j1}, Q_{j2}, w_j \geq 0 \qquad (3s)$$

The objective function and constraints in the above model are similar to those in section 3.1, except for constraints (3e)-(3j) which are linearization constraints ensuring that (2a) and (2b) hold.

3.3. Illustrative examples

The model is coded in AMPL and solved using CPLEX solver. The results are shown below. It is important to note that for comparison purposes, the model was first implemented in AMPL without the quantity discount contract and the results were used to study the effect of the quantity discount contract.

We consider a product line with two variants and two customer segments.

The base parameter values are as follows.

For product 1,

$\alpha_{11} = 9$, $\alpha_{21} = 8.5$, $k_1 = 40$, $c_{11} = 8$, $c_{12} = 6$, and $U_1 = 110$.

For product 2,

$\alpha_{12} = 10.5$, $\alpha_{22} = 9.5$, $k_2 = 40$, $c_{21} = 8$, $c_{22} = 7$, and $U_2 = 110$.

The customer segments volumes are $s_1 = 1000$ and $s_2 = 100$.

The results of the base case are as follows:

Assortment chosen: Product {2}

Prices: p_2 = \$10.5

Quantity ordered: $Q_{22} = 1000$

Consumer segments choices: Consumer segment 1 chose product 2 and consumer segment 2 chose not to buy.

$v_{22} = 1$; meaning that product 2 was bought at the discounted price.

Profit: \$3460

Offering product 2 at a price of \$10.5, appeals to consumer segment 1. Although lowering the price of product 2 to \$9.5 would lead to consumer segment 2 buying the product, the size of consumer segment 2 is not large enough to compensate the profit provided by consumer segment 1 at a price of \$10.5 with discount.

In Table 1 we show results for different variations of the base case. Each case in Table 1 involves the change(s) from the base case shown in the second column of the table.

Table 1: Results on the illustrative quantity discount example. Base Case: $\alpha_{11} = 9$, $\alpha_{21} = 8.5$, $\alpha_{12} = 10.5$, $\alpha_{22} = 9.5$, $k_1 = 40$, $k_2 = 40$, $U_1 = 110$, $U_2 = 110$, $c_{11} = 8$, $c_{12} = 6$, $c_{21} = 8$, $c_{22} = 7$, $s_1 = 1000$, $s_2 = 100$

Case #	Change	v	Assortment	Prices \$	Quantity	x	Profit \$
0	No change	(0,0,0,1)	{2}	10.5	1000	(0,0,1) (1,0,0)	3460
1	$s_1 = 100$	(0,0,0,1)	{2}	9.5	200	(0,0,1) (0,0,1)	460
2	$s_2 = 680$	(0,1,0,1)	{1, 2}	{8.5 10}	{0,680 0,1000}	(0,0,1)(0,1,0)	4620
3	$\alpha_{12} = 9$	(0,1,1,0)	{1, 2}	{9 9.5}	{0,1000 100,0}	(0,1,0) (0,0,1)	3070
4	$\alpha_{22} = 11$	(0,0,0,1)	{2}	10.5	{0,0 0,1100}	(0,0,1) (0,0,1)	3810
5	$c_{11} = 6$	(0,0,0,1)	{2}	10.5	{0,0 0,1000}	(0,0,1) (1,0,0)	3460
6	$\alpha_{12} = 9$,	(0,1,0,0)	{1}	9	{0,1000	(0,1,0)	2000

	$K_1 = 1000$ $K_2 = 1800$				0,0}	(1,0,0)	
7	$\alpha_{21} = 10$, $\alpha_{12} = 9$, $s_1 = 200$, $s_2 = 1000$	(0,1,0,0)	{1}	10	{0,1000 0,0}	(1,0,0) (0,1,0)	3960
8	$\alpha_{11} = 8.5$, $\alpha_{21} = 9$, $\alpha_{12} = 9$, $s_1 = 100$, $s_2 = 1000$	(0,1,0,0)	{1}	9	{0,1000 0,0}	(1,0,0) (0,1,0)	$2960
9	$\alpha_{11} = 10.5$, $\alpha_{21} = 10.5$	(0,1,0,0)	{1}	10.5	{0,1100 0,0}	(0,1,0) (0,1,0)	$4910
10	$\alpha_{21} = 10$	(1,0,0,1)	{1, 2}	{10 10.5}	{100,0 0,1000}	(0,0,1) (0,1,0)	$3620
11	$s_2 = 680$, $U_1 = 1000$, $U_2 = 1000$	(0,1,0,0)	{1}	8.5	{0,1680 0,0}	(0,1,0) (0,1,0)	$4160
12	$\alpha_{21} = 9.5$, $\alpha_{12} = 9$, $\alpha_{22} = 11$, $s_1 = 100$, $s_2 = 1000$	(0, 0,0,1)	{2}	11	{0,0 0,1000}	(1,0,0) (0,0,1)	$3960
13	$\alpha_{12} = 9, U_1 = 1050$	(0,1,0,0)	{1}	8.5	{0,1100 0,0}	(0,1,0) (0,1,0)	$2710
14	$\alpha_{21} = 9.5$	(1,0,0,1)	{1,2}	{9.5 10.5}	{100,0 0,1000}	(0,0,1) (0,1,0)	$3570
15	$U_1 = U_2 = 1050, c_{22} = 6$	(0,0,0,1)	{2}	9.5	{0,0 0,1100}	(0,0,1) (0,0,1)	$3810

Table 2: Results on the illustrative example without quantity discounts.

Case #	Change	Assortment	Prices $	Quantity	x	Profit $
0	No change	{2}	10.5	1000	(0,0,1) (1,0,0)	2460
1	$s_1 = 100$	{2}	9.5	200	(0,0,1) (0,0,1)	260
2	$s_2 = 680$	{2}	9.5	1680	(0,0,1) (0,1,0)	2480
3	$\alpha_{12} = 9$	{1, 2}	{9,9.5}	{1000,100}	(0,1,0),(0,0,1)	1070

4	$\alpha_{22} = 11$	{2}	10.5	{1100}	(0,0,1) (0,0,1)	2710
5	$c_{11} = 6$	{1}	9	{1000}	(0,1,0) (1,0,0)	2960
6	$\alpha_{12} = 9$, $K_1 = 1000$ $K_2 = 1800$	–	–	–	(1,0,0) (1,0,0)	–
7	$\alpha_{21} = 10$, $\alpha_{12} = 9$, $s_1 = 200$, $s_2 = 1000$	{1}	10	{1000}	(1,0,0) (0,1,0)	1960
8	$\alpha_{11} = 8.5$, $\alpha_{21} = 9$, $\alpha_{12} = 9$, $s_1 = 100$, $s_2 = 1000$	{2}	9.5	{1000}	(1,0,0) (0,0,1)	$1460
9	$\alpha_{11} = 10.5$, $\alpha_{21} = 10.5$	{1}	10.5	{1100}	(0,1,0) (0,1,0)	$2710
10	$\alpha_{21} = 10$	{1, 2}	{10 10.5}	{100,1000}	(0,0,1) (0,1,0)	$2620
11	$s_2 = 680$	{2}	9.5	1680	(0,0,1) (0,1,0)	$2480
12	$\alpha_{21} = 9.5$, $\alpha_{12} = 9$, $\alpha_{22} = 11$, $s_1 = 100$, $s_2 = 1000$	{2}	11	{1000}	(1,0,0) (0,0,1)	$2960
13	$\alpha_{12} = 9$	{1,2}	9,9.5	{1000,100}	(0,1,0) (0,0,1)	$1070
14	$\alpha_{21} = 9.5$	{1,2}	{9.5 10.5}	{100,1000}	(0,0,1) (0,1,0)	$2570
15	No change to no discount parameters	{2}	10.5	{1000}	(0,0,1) (1,0,0)	$2460

We make the following comments on some cases of Tables 1 and 2. These comments serve to validate the model results.

case #2: In this case both products are offered in the assortment at prices of p_1 = \$8.5 and p_2 = \$10. Consumer segment 1 buys product 2 and consumer segment 2 buys product 1.

It is possible to offer only product 2 at p_2 = \$9.5 which would appeal to both consumer segments, which is what happens when no discount contracts are available, however that would lower the overall profit of the retailer when discount was available.

In this case, the utility of both products with respect to consumer segment 1 is 0.5. To break the tie, we assume the retailer prices the product with the higher profit at $p-\varepsilon$. ε has a very small value. In this example, p_2 would be set at \$(10 - ε).

case #5: In this case the original unit cost for product 1 is set equal to that of the discounted unit cost. In the no discount model it led to only having product 1 in the assortment at p_1 = \$9. Segment 1 bought the product while segment 2 bought nothing. However in this model, decreasing c_{11} had no effect on the result; it is identical to that of the base case. Product 2 is chosen at p_2 = \$10.5 and only bought by segment1.

The retailer in the case did not choose product 1 simply because the profit from product 2 at the discounted unit cost (10.5 – 7 = \$3.5) is higher than that of product 1 (9 – 6 = \$3).

case #6: The changes in this case led to having no product in the assortment with the no discount model since the fixed costs were too high to be compensated. However with the quantity discount contract product 1 is chosen by the retailer and bought by segment 1 at a price of p_1 = \$9.

case #13: If we compare this case to case #3, we find that increasing the quantity threshold when applying a quantity discount contract leads to a smaller assortment.

case #14: This case is also similar to case #3. Only this time the reservation price of product 1 to segment 2 is increased to \$9.5. In both cases we have a similar assortment. However the prices of the products change.

From Table 1 and Table 2, we notice the occurrence of the following four situations:

1. Quantity discount led to a wider assortment, e.g. case# 13.
2. Quantity discount led to a smaller assortment, e.g. case# 2.
3. Quantity discount led to higher prices, e.g. case# 8.
4. Quantity discount led to lower prices, e.g. case# 2.

3.4. Computational study

In this section, we perform a computational study to observe the effects of the quantity discount model on a larger scale. All mathematical programs were coded in AMPL and solved using CPLEX. The following data was generated for the study:

- Five problem instances were generated for each of the following problem instance sizes/characteristics: (*n,m*) = (30,6), (*n,m*) = (50,7), and (*n,m*) = (75,8).
- The size of the customer segments, s_i, was randomly set using $floor\lfloor U(100{,}1000) \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer $\le x$ and $U(a,b)$ denotes a random variable which is uniformly distributed over (*a*, *b*).
- The fixed cost, k_j, was set using *U*(30, 100).
- The quantity order threshold, U_j, was set as $U_j = \min_i\{s_i\} \times U(0.99, 5)$.

- The variable ordering cost with no quantity discount was set as $c_{1j} = U(7,12)$.
- The variable ordering cost with quantity discount was set according to these ranges.

  i) For the high discount range: $c_{2j} = c_{1j} \times U(0.8, 0.9)$

  ii) For the medium discount range: $c_{2j} = c_{1j} \times U(0.85, 0.95)$

  iii) For the low discount range: $c_{2j} = c_{1j} \times U(0.9,1)$
- The customer reservation price was set as $\alpha_{ij} = c_{1j} \times U(0.9,1.05)$.

Tables 3-14 report our computational results. The results are reported as follows:

- Columns 1 and 2 report the instance number and the CPU time in seconds for solving the model to optimality using CPLEX.
- Column 3 reports the total profit, the total revenue, and the total profit as a percentage of the total revenue.
- Column 4 reports the optimal assortment, the optimal selling prices, and the profit margin for each product. The profit margin for product *j* was calculated as follows: $\frac{p_j - c_j}{c_j} \times 100$. In cases where the quantity discount contract is available, column 4 also reports if the quantity discount cost is used.

Analyzing these tables, we found that, even for larger scale problems, the model behaves in a similar manner to the simple scale problems. Applying the quantity discount contract led to cases with smaller, larger, and even identical assortments. In addition, the products' selling prices increased, decreased and in some cases remained the same.

The following are the reported results.

Table 3: Results for (n, m) = (30, 6) with no quantity discount

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin%
I.1	0.093	1620.2 44195 3.67	(6, 7, 10, 18, 26, 28) (12.06, 9.69, 12.34, 11.62, 11.66, 10.00) (4.26, 4.74, 4.46, 4.78, 4.70, 4.78)
I.2	0.122	1081.5 34863.8 3.10	(3, 11, 13, 15, 29) (9.23, 9.88, 9.73, 10.67, 8.70) (3.64, 4.53, 4.97, 4.19, 3.58)
I.3	0.128	1168.6 31892.9 3.66	(5, 11, 12, 13, 26) (10.33, 12.23, 12.55, 10.46, 11.68) (4.69, 4.34, 4.88, 4.54, 4.79)
I.4	0.117	1077.2 32952.5 3.27	(1, 15, 28) (11.50, 12.70, 9.75) (4.40, 3.27, 4.96)
I.5	0.098	1554.9 41637.9 3.73	(4, 15, 19, 27) (10.57, 10.53, 9.62, 12.22) (4.56, 4.44, 4.85, 4.10)

Table 4: Results for high discount range for (n, m) = (30, 6)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
I.1	0.217	8135.9 43340.4 18.77	(4, 6, 12, 23, 26) (10.85, 12.06, 11.65, 10.02, 11.65) (24.89, 21.65, 25.42, 25.92) (1, 1, 1, 1, 1)
I.2	0.109	7644.1 41839.9 18.27	(15, 18, 28, 30) (10.67,8.95,10.96,11.81) (23.74, 28.91, 23.10, 20.57) (1, 1, 1, 1)
I.3	0.093	7041.8 35146.5 20.04	(9, 11, 12, 14) (11.51,12.11,12.30,12.29) (24.63, 27.60, 21.22, 26.80) (1, 1, 1, 1)
I.4	0.184	6165.2 34328.2 17.96	(1, 8, 15, 22) (11.41,11.78,12.40,11.50) (26.10, 20.25, 17.19, 28.96) (1, 1, 1, 1)
I.5	0.179	8110.1 39570.5 20.50	(3, 18, 30) (8.46,10.98,11.49) (26.33, 27.10, 26.14) (1, 1, 1)

Table 5: Results for medium discount range for (n, m) = (30, 6)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
I.1	0.183	6689.0 43701.5 15.30	(11, 20, 23, 26) (12.18,11.36,10.02,11.66) (19.42, 15.23, 24.48, 21.59) (1, 1, 1, 1)
I.2	0.188	5780.1 43285.7 13.35	(1, 10, 15, 17) (11.92,11.37,10.61,10.65) (14.10, 14.50, 21.91, 15.69) (1, 1, 1, 1)
I.3	0.113	5262.5 34769.2 15.13	(12, 13, 14, 24) (12.30,10.46,12.21,12.30) (13.40, 22.63, 18.82, 18.29) (1, 1, 1, 1)
I.4	0.147	4534.0 33300.3 13.61	(15, 25, 27) (12.31,10.65,10.17) (15.10, 16.71, 19.65) (1, 1, 1)
I.5	0.198	5857.8 41407.4 14.15	(2, 22, 27) (11.67,9.45,12.14) (12.69, 18.15, 20.55) (1, 1, 1)

Table 6: Results for low discount range for (n, m) = (30, 6)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
I.1	0.1	4444.5 45564.5 9.75	(10, 11, 20, 27) (12.01,12.18,11.43,11.85) (12.73, 12.02, 10.97, 10.32) (1, 1, 1, 1)
I.2	0.087	4404.0 45338.1 9.71	(1, 10, 30) (11.92,11.37,11.81) (11.20, 11.82, 10.41) (1, 1, 1)
I.3	0.149	3525.4 33593.4 10.49	(7, 13, 14, 24) (8.37,10.41,12.21,12.30) (13.03, 12.37, 13.35, 11.82) (1, 1, 1, 1)
I.4	0.114	3272.7 32468.5 10.08	(1, 3, 11) (11.50,10.57,10.93) (12.64, 11.14, 12.62) (1, 1, 1)
I.5	0.125	3884.2 35900.7 10.82	(3, 9, 12) (8.47,11.93,8.27) (10.14, 12.74, 15.00) (1, 1, 1)

Table 7: Results for case for (n, m) = (50, 7) with no quantity discounts

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin%
II.1	0.198	2166.5 62864.5 3.45	(29, 32, 37, 39, 43, 47) (11.30,8.67,10.78,10.96,11.13,12.33) (4.75, 4.62, 4.40, 4.53, 4.77, 2.91)
II.2	0.193	2357.9 59862.6 394	(13, 40, 46) (10.19,11.77,11.97) (4.88, 4.37, 4.00)
II.3	0.199	1452.3 43903.2 3.31	(3, 6, 8, 9, 15, 17, 45) (11.43,10.21,12.46,8.49,11.76,12.02,11.76) (4.62, 4.99, 5.00, 3.71, 3.99, 4.70, 3.53)
II.4	0.302	1101.7 30986.2 3.56	(15, 16, 28, 46, 49) (11.17,11.60,9.59,8.22,12.28) (4.16, 3.65, 4.12, 4.79, 4.98)
II.5	0.213	969.7 31615 3.07	(10, 12, 15, 39, 41) (12.04,12.18,9.48,8.39,12.35) (3.93, 4.39, 4.90, 3.44, 3.87)

Table 8: Results for high discount range for (n, m) = (50, 7)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
II.1	0.395	11882.5 59587.3 19.94	(24, 44, 47) (9.53,8.58,12.32) (25.75, 25.22, 25.63) (1, 1, 1)
II.2	0.196	11959.4 61790.4 19.35	(9, 40, 46) (12.24,11.77,11.97) (27.73, 24.75, 22.58) (1, 1, 1)
II.3	0.235	8999.1 43937.1 20.48	(3, 4, 17, 19) (11.43,11.20,11.81,10.17) (26.42, 24.86, 28.64, 26.50) (1, 1, 1, 1)
II.4	0.243	7350.3 35604.2 20.65	(2, 13, 34, 49) (11.42,12.00,12.24,12.28) (25.26, 26.35, 23.49, 30.80) (1, 1, 1, 1)
II.5	0.217	6348.1 32167.5 19.73	(10, 12, 27) (12.04,11.82,10.53) (26.57, 24.83, 26.27) (1, 1, 1)

Table 9: Results for medium discount range for (*n*, *m*) = (50, 7)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
II.1	0.267	9497.7 66939.3 14.19	(19, 22, 29, 47) (11.86,11.73,11.15,12.33) (16.29, 14.92, 20.80, 15.51) (1, 1, 1, 1)
II.2	0.334	7523.3 58929.3 12.77	(13, 19, 40) (10.17,11.99,11.77) (17.79, 14.07, 13.99) (1, 1, 1)
II.3	0.422	6433.2 42306.4 15.21	(3, 10, 20) (11.43,11.67,7.82) (20.92, 17.12, 19.01) (1, 1, 1,)
II.4	0.262	5009.6 32178.7 15.57	(3, 16, 25, 34, 43) (9.88,11.42,10.38,12.24,12.00) (21.29, 14.41, 21.27, 21.59, 20.13) (1, 1, 1, 1, 1)
II.5	0.267	4079.8 28517.3 14.31	(10, 26, 31, 39) (12.04,11.45,10.41,8.35) (14.61, 18.87, 13.27, 20.90) (1, 1, 1, 1)

Table 10: Low discount range for (n, m) = (50, 7)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
II.1	0.352	5714.8 54483 10.49	(7, 20, 24, 44) (9.41,10.61,9.54,8.58) (14.55, 10.59, 11.50, 13.79) (1, 1, 1, 1)
II.2	0.205	6604.4 61790.4 10.69	(9, 40, 46) (12.24,11.77,11.97) (14.51, 13.46, 10.13) (1, 1, 1)
II.3	0.207	4796.7 43095.6 11.13	(10, 22, 23, 32) (11.86,11.72,7.61,12.05) (13.01, 11.45, 13.42, 14.07) (1, 1, 1, 1)
II.4	0.315	3520 33304.2 10.57	(17, 26, 40, 41, 49) (10.15,10.39,11.88,10.89,12.28) (9.99, 14.84, 10.73, 12.29, 15.35) (1, 1, 1, 1, 1)
II.5	0.222	2855.1 31259.1 9.13	(11, 26, 31, 35) (11.21,11.45,10.10,12.23) (9.69, 14.38, 10.98, 8.73) (1, 1, 1, 1)

Table 11: Results for (*n*, *m*) = (75, 8) with no quantity discounts

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin%
III.1	0.462	1725.32 47681.1 3.62	(14, 16, 17, 42, 57, 67) (12.24,12.20,10.24,11.61,11.16,10.75) (4.84, 4.61, 4.73, 4.16, 4.78, 4.35)
III.2	0.318	1837.78 49313.9 3.72	(7, 9, 25, 32, 49, 59) (11.09,11.18,11.74,10.17,11.24,10.39) (4.64, 4.10, 4.88, 4.99, 4.49, 4.57)
III.3	0.437	1686.25 45858 3.67	(12, 17, 21, 55, 65, 70) (10.24,12.37,10.74,10.07,10.66,11.57) (4.97, 4.38, 4.02, 4.20, 4.69, 4.88)
III.4	0.359	1302.32 42913.3 3.03	(20, 25, 51, 59) (11.84,12.01,10.44,11.38) (3.98, 4.51, 4.10, 3.50)
III.5	0.413	1637.37 44204.9 3.70	(3, 11, 32, 40, 48, 55) (11.76,11.72,11.03,12.15,11.08,11.24) (4.51, 4.20, 4.70, 4.61, 4.30, 4.96)

Table 12: Results for high discount range for (n, m) = (75, 8)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
III.1	0.51	10004.59 48885.5 20.4	(2, 5, 8, 14, 49, 62) (10.09,12.27,11.62,12.24,11.87,11.31) (28.69, 24.28, 30.19, 29.08, 24.41, 25.72) (1, 1, 1, 1, 1, 1)
III.2	0.371	10179.12 49341.6 20.63	(52, 55, 62) (11.16, 10.41, 11.41) (25.37, 28.91, 27.02) (1, 1, 1)
III.3	0.619	9090.44 43591.5 20.85	(10, 45, 72) (9.37, 10.85, 11.03) (27.33, 26.60, 28.96) (1, 1, 1)
III.4	0.748	8340.75 43271.3 19.28	(8, 10, 20, 25, 49, 53, 59) (11.24, 10.42, 11.80, 12.01, 8.54, 12.26, 11.54) (28.02, 27.52, 28.14, 26.10, 26.93, 22.15, 20.17) (1, 1, 1, 1, 1, 1, 1)
III.5	0.336	9303.47 43738.7 21.27	(1, 3, 11, 48) (11.60, 11.48, 11.72, 10.95) (28.30, 27.08, 27.42, 28.12) (1, 1, 1, 1)

Table 13: Results for medium discount range for (n, m) = (75, 8)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
III.1	0.402	7797.83 47082 16.56	(2, 14, 30, 42, 62) (10.00, 12.14, 12.06, 11.61, 11.31) (21.41, 21.42, 21.01, 17.59, 20.82) (1, 1, 1, 1, 1)
III.2	0.419	7567.16 49259.8 15.36	(25, 49, 64, 67) (11.74, 11.24, 11.63, 10.21) (20.79, 19.11, 16.34, 19.33) (1, 1, 1, 1)
III.3	0.366	7034.09 44695.9 15.74	(2, 11, 23, 38, 65) (10.90, 10.16, 10.78, 11.62, 10.53) (18.60, 20.98, 19.16, 18.55, 21.16) (1, 1, 1, 1, 1)
III.4	0.389	6721.19 42941 15.65	(20, 25, 51, 54) (11.84, 11.79, 10.44, 10.48) (20.01, 18.95, 20.48, 20.95) (1, 1, 1, 1)
III.5	0.469	7288.90 45444.6 16.04	(3, 11, 24, 55, 63) (11.76, 11.72, 11.95, 11.12, 12.15) (4.51, 20.85, 20.17, 21.52, 20.17) (0, 1, 1, 1, 1)

Table 14: Results for low discount range for $(n, m) = (75, 8)$

Instance	CPU (s)	Profit Revenue Profit%	Assortment Price Profit margin% Quantity discount
III.1	0.391	5314.30 47252.7 11.25	(5, 7, 30, 42, 49, 60) (12.18, 9.02, 12.06, 11.61, 12.10, 11.55) (13.91,15.67, 13.02, 12.99, 11.80, 13.83) (1, 1, 1, 1, 1, 1)
III.2	0.373	5441.65 49876.7 10.91	(23, 27, 36, 50, 59) (9.29, 12.00, 11.48, 11.70, 10.39) (16.11, 12.18, 11.58, 13.53) (1, 1, 1, 1, 1)
III.3	0.417	5385.21 46342.6 11.62	(8, 20, 37, 38, 59) (11.47, 8.14, 9.87, 11.57, 12.17) (13.61, 13.59, 12.56, 14.59, 14.85) (1, 1, 1, 1, 1)
III.4	0.315	5064.42 42913.3 11.80	(20, 25, 51, 59) (11.84, 12.01, 10.44, 11.38) (14.65, 14.37, 15.66, 13.01) (1, 1, 1, 1)
III.5	0.422	4836.33 45145.4 10.71	(3, 24, 40, 42, 68) (11.52, 11.95, 12.15, 11.71, 11.11) (11.68, 14.46, 13.03, 11.72, 12.62) (1, 1, 1, 1, 1)

CHAPTER 4

TRUCKLOAD CAPACITY MODEL

In this chapter, the problem setting consists of a one period selling horizon with multi-suppliers while considering truckload capacity costs. This contract is mainly based on the model described in the paper by Glickman and White (2008). In section 4.1 we formulate the problem. In section 4.2 we provide some illustrative examples and in section 4.3 we perform a computational study.

4.1. Formulation of the model

We now introduce the truckload costs incurred from shipping the products. The truckload costs are different for each supplier and they consist of the costs of shipping the products from the supplier to the retailer. These shipment costs are set in two categories; truckload shipments (TL) when the truck is at its weight limit capacity and less-than-truckload shipments (LTL) when the truckload is less than the weight limit of the truck.

Let $L = \{1, 2, ..., N\}$ be the set of suppliers available to the retailer. We introduce for this part a new binary parameter a_{jl} such that $a_{jl} = 1$ if product *j* is supplied by supplier *l* and $a_{jl} = 0$ otherwise. Minor changes affect the rest of the parameters as well as the problem's variables. These changes are as follows;

- The cost parameter becomes function of the supplier as well as the product c_{jl} .
- The fixed costs become:

1- k_{jl} the fixed cost of ordering product *j* from supplier *l*

2- F_l the fixed cost of establishing a channel with supplier *l*

- The quantity ordered becomes Q_{jl}, the quantity of product *j* ordered from supplier *l*; this quantity is divided into three parts:

a- q_{1jl} which is the quantity ordered of product *j* from supplier *l* in a full truckload shipment.

b- q_{2jl} which is the quantity ordered of product *j* from supplier *l* in a less-than-truckload shipment.

c- q_{3jl} which is the quantity ordered of product *j* from supplier *l* to fill a less-than-truckload shipment.

q_{3jl} *is needed because in some situations it is profitable to order a full truck form a certain supplier and not use all of it.*

- The binary variable y_{jl} is such that $y_{jl} = 1$ if product *j* is ordered from supplier *l* and $y_{jl} = 0$ otherwise.

- The binary variable Z_l is such that $Z_l = 1$ if any products are bought from supplier *l* and $Z_l = 0$ otherwise.

For the truckload contract we introduce the following new parameters:

- r_l the TL shipping rate from supplier *l* in $/truckload

- θ_l the LTL shipping rate from supplier *l* in $/kg

- u_j the per unit weight of product *j* in kg

- W_l the truck weight limit capacity from supplier *l* in kg

We also introduce for this model the following new variables:

- T_l the number of TL shipments from supplier *l*

- H_l the weight of the LTL shipments from supplier l

The new model with truckload capacity contracts is as follows:

Maximize $$\sum_{j\in J}\sum_{i\in I} s_i g_{ij} - \sum_{l\in L}\sum_{j\in J}\left[K_{jl} y_{jl} + c_{jl} Q_{jl}\right] - \sum_{l\in L}\left[r_l T_l + \theta_l H_l + Z_l F_l\right] \quad (4a)$$

Subject to

$$\sum_{k=1}^{n}\left(\alpha_{ik} x_{ik} - g_{ik}\right) \geq \alpha_{ij} z_j - w_j, \forall i, j \quad (4b)$$

$$\sum_{k=1}^{n}\left(\alpha_{ik} x_{ik} - g_{ik}\right) \geq 0, \forall i \quad (4c)$$

$$\sum_{j=0}^{n} x_{ij} = 1, \forall i \quad (4d)$$

$$g_{ij} \leq \max_k\{\alpha_{kj}\} x_{ij}, \forall i, j \quad (4e)$$

$$g_{ij} \geq p_j - \max_k\{\alpha_{kj}\}(1 - x_{ij}), \forall i, j \quad (4f)$$

$$g_{ij} \leq p_j, \forall i, j \quad (4g)$$

$$w_j \leq z_j \max_i\{\alpha_{ij}\}, \forall j \quad (4h)$$

$$w_j \geq p_j - \max_i\{\alpha_{ij}\}(1 - z_j), \forall j \quad (4i)$$

$$w_j \leq p_j, \forall j \quad (4j)$$

$$\sum_{l\in L} Q_{jl} \geq \sum_{i=1}^{m} x_{ij} s_i, \forall j \quad (4k)$$

$$z_j \geq y_{jl}, \forall j, l \quad (4l)$$

$$x_{ij} \leq z_j, \forall i, j \quad (4m)$$

$$p_j \leq z_j \max_i\{\alpha_{ij}\}, \forall i, j \quad (4n)$$

$$Q_{jl} \le y_{jl} \sum_{i \in I} s_i^2, \forall j,l \qquad (4o)$$

$$Z_l \ge y_{jl}, \forall j,l \qquad (4p)$$

$$T_l \ge \left(\frac{1}{W_l}\right)\left(\sum_{j \in J} u_j Q_{jl}\right) - 1, \forall l \qquad (4q)$$

$$T_l \le \left(\frac{1}{W_l}\right)\left(\sum_{j \in J} u_j Q_{jl}\right), \forall l \qquad (4r)$$

$$H_l \ge \sum_{j \in J} u_j Q_{jl} - W_l T_l, \forall l \qquad (4s)$$

$$y_{jl} \le a_{jl}, \forall j,l \qquad (4t)$$

$$Q_{jl} = q_{1jl} + q_{2jl}, \forall j,l \qquad (4u)$$

$$\sum_j q_{2jl} u_j = H_l, \forall l \qquad (4v)$$

$$q_{3jl} u_j \ge W_l - H_l, \forall j,l \qquad (4w)$$

$$H_l \theta_l \le r_l + q_{3jl} c_{jl}, \forall j,l \qquad (4x)$$

x_{ij}, y_{jl}, z_j and Z_l are binary variables *(4y)*

Q_{jl} and T_l are integers ≥ 0 *(4za)*

$$g_{ij}, p_j, H_l, w_{jl} \ge 0 \qquad (4zb)$$

Constraints (4b)-(4j) and constraints (4l)-(4n) are similar to those in Chapter 3. Constraints (4k) and (4o)-(4x) can be explained as follows. Constraints (4k) ensure that the quantity order can be greater than the demand. Constraints (4o) set an upper limit to the quantity ordered. Constraints (4p) ensure that if any product is ordered from a certain supplier, the fixed cost of establishing a channel with that retailer is paid.

Constraints (4q)-(4r) set the number of full trucks from supplier *l*. Constraints (4s) sets the LTL weight for supplier *l*. Constraints (4u) ensure that the retailer can order product *j* from supplier *l* only if that supplier provides this product. Constraints (4v) set q_{2jl} as the quantity ordered in LTL trucks. Constraints (4w) set q_{3jl} as the slack quantity needed to continue filling the LTL trucks. Constraints (4x) ensure that the retailer orders a full truck and not use all of it if it is more profitable than ordering an LTL truck.

4.2. Illustrative examples

The above model is coded in AMPL and solved using CPLEX solver and the following results were found. Again for comparison purposes, we coded the model in AMPL with no truckload costs and use the results to study the effect of adding truckload shipping costs.

In this setting, the problem consists of two suppliers offering two substitutable products with truckload costs to a single retailer with a market of two consumer segments.

It is important to note that for the LTL argument to be valid, the cost of shipping a full truck with the LTL rates must be greater than the cost of the TL shipment, i.e. $W_l\theta_l \geq r_l, \forall l$.

For product 1 we have, $\alpha_{11} = 14$, $\alpha_{21} = 12.5$, and $u_1 = 1$.

For product 2 we have, $\alpha_{12} = 10$, $\alpha_{22} = 10.5$, and $u_2 = 1$.

For supplier 1 we have, $K_{11} = 40$, $K_{21} = 45$, $c_{11} = 8.5$, $c_{21} = 9$, $F_1 = 100$, $a_{11} = 1$, $a_{21} = 1$, $W_1 = 1000$, $\theta_1 = 1.5$, and $r_1 = 300$.

For supplier 2 we have, $K_{12} = 40$, $K_{22} = 45$, $c_{12} = 9.5$, $c_{22} = 7.5$, $F_2 = 100$, $a_{12} = 1$, $a_{22} = 1$, $W_2 = 800$, $\theta_2 = 1$, and $r_2 = 280$.

The size of the consumer segments are $s_1 = 900$ and $s_2 = 100$.

Running these settings initially with no truckload costs considerations led to the following results.

Assortment chosen: Products {1,2} with product 1 ordered from supplier 1 only and product 2 ordered from supplier 2 only.

Prices: $p_1 = 14; p_2 = 10.5$

Quantity ordered: $Q_{11} = 900; Q_{22} = 100$

Consumer segments choices: Consumer segment 1 chose product 1 and consumer segment 2 chooses product 2.

Profit: $5415

When running the same settings with truckload considerations, we got the following results.

Assortment chosen: Products {1,2} from supplier {1}

Prices: $p_1 = 14; p_2 = 10.5$

Quantity ordered: $Q_{11} = 900; Q_{21} = 100$

Consumer segments choices: Consumer segment 1 chose product 1 and consumer segment 2 chose product 2.

Profit: $5065

#TL =1

Weight of LTL H_1 = 0 kg

Even though considering truckload costs did not affect the size of the assortment or the pricing in this particular case, it did change the retailer's decision. The retailer chose to consolidate his order from one supplier to save on the shipping costs, even though this entails a high unit cost for products.

In the following table we analyze more instances of the problem.

The first base case that we analyze is the same as earlier with these changes, r_1=200, θ_1=0.5, W_1=800, s_1=1000, r_2=180, θ_2=0.2, W_2=700, and s_2=100. We call this case # 0.

The second base that we analyze is the same as case# 0 with these changes, c_{11}=8, θ_1=3.5, W_1=1000, s_1=900, r_2=180, θ_2=2.5, W_2=800, $\alpha_{22}=11.5$, and $\alpha_{12}=12$. We call this case # 0'.

The results of running these cases without truck costs are shown in the following table. We then run case# 0 with truck costs have cases# 1 and 2 and run case#0' with truck costs and get cases# 3 and 4. The results are in Table 16.

Table 15: Result with no truck costs

Case	Assortment	Prices	Quantity	x	suppliers	Profit
0	{1,2}	14 10.5	Q_{11}=1000 Q_{22}= 100	(0,1,0) (0,0,1)	Product 1 from supplier 1 and product 2 from supplier 2	6015
0'	{1}	14	Q_{11}=900	(0,1,0) (1,0,0)	Supplier 1	5260

Table 16: Results with truck costs

Case	Change	Assortment	Prices	Quantity	x	#TL Weight LTL	Profit
0	_______	{1,2}	14 10.5	Q_{11}=1000 Q_{22}= 100	(0,1,0) (0,0,1)	1,0 200,100	5195

1	$\theta_1 = 50.5$ $\theta_2 = 40.5$	{1,2}	14 10.5	Q_{11}= 800 Q_{12}= 200 Q_{22}=1000	(0,1,0) (0,0,1)	1,1 0,0	2295
2	$\theta_1 = 3.5$ $\theta_2 = 2.5$	{1}	14	Q_{11}=1000	(0,1,0) (1,0,0)	1,0 200,0	4960
3	________	{1,2}	13.5 11.5	Q_{11}= 900 Q_{21}= 100	(0,1,0) (0,0,1)	1,0 0,0	4815

In *case#1*, very high LTL costs led the retailer to order more than the demand to save on the LTL costs and order 2 full trucks from each supplier.

In *case#2*, the retailer chose to only buy offer one product to the largest segment with the highest profit to cut the costs on the truck shipments.

In case# 3, the retailer ordered a bigger assortment compared to case # 0' and also decreased the price of product 1.

4.3. Computational study

In this section, we performe a computational study to observe the effects of considering truckload costs on the model on a larger scale. All mathematical programs were coded in AMPL and solved using CPLEX. The following data was generated for the study:

- Five problem instances were generated for each of the following problem instance sizes/characteristics: (n, m, L) = (30, 6, 3), (n, m, L) = (50, 7, 4), and (n, m, L) = (75, 8, 5).

- The size of the customer segments, s_i, was randomly set using *floor [U(1000, 10000)].*
- The fixed cost, k_j, was set using $k_j = U\lfloor(30,100)\rfloor$.
- The weight of the products, u_j, was set using $u_j = U\lfloor(0.5, 2.5)\rfloor$.
- The variable ordering cost was set as $c_{jl} = U\lfloor(9,12)\rfloor$.
- The full TL ordering rate was set as $r_l = \left[2 - 0.1 * \left(\frac{W_l}{5000}\right)\right] * \left[\frac{W_l}{10}\right]$.
- The LTL ordering rate was set as $\theta_l = \left[\frac{r_l}{W_l}\right] * U\lfloor(1,2)\rfloor$.
- The full TL capacity was set as $W_l = 5000 * (ceil\lfloor U(3,6)\rfloor)$.
- The supplier selection cost was set as $F_l = U\lfloor(500,1000)\rfloor$.
- The binary parameter for the availability of product *j* at supplier *l* was set as $a_{jl} = U\lfloor(0,2)\rfloor$.
- The customer reservation price was set as $\alpha_{ij} = \lfloor c_{jl}(9,12)\rfloor \times \lfloor U(0.9,1.05)\rfloor$.

The following tables report our computational results. The results are reported as follows:

- Columns 1 and 2 report the instance number and the CPU time in seconds for solving the model to optimality using CPLEX.
- Column 3 reports the total profit, the total revenue, and the total profit as a percentage of the total revenue.

- Column 4 reports the optimal assortment, the suppliers selected, the optimal selling prices, and the profit margin for each product. The profit margin for product *j* was calculated as follows: $\frac{p_j - c_j}{c_j} \times 100$.
- In cases where the truckload costs were considered, column 5 also reports the number of full TL and the weights carried LTL.

Analyzing these tables, we found that, even for larger scale problems, the model behaves in a similar manner to the simple scale problems. Truckload costs considerations leads to smaller and larger assortments, and higher and lower prices. The following are the reported results.

Table 17: Results for (*n, m, L*) = (30, 6, 3) with no TL capacity

Instance	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin%
I.1	0.134	62938.23 358814 17.54	(6, 18, 20, 21, 27) (2, 1, 2, 2, 3) (11.95, 11.68, 11.74, 11.73, 10.94) (26.99, 25.81, 20.33, 22.13, 20.17)
I.2	0.14	67016.33 333800 20.08	(9, 11, 13, 15, 20, 21) (2, 2, 2, 3, 1, 2) (11.98,11.08, 12.47, 11.50, 11.49, 11.42) (22.30,21.01, 37.90, 24.54, 20.83, 22.82)
I.3	0.129	81703.55 388701 21.02	(3, 10, 22, 25, 26, 27) (2, 2, 1, 1, 1, 1) (11.69,11.42, 12.40, 11.62, 12.12, 12.10) (25.40,22.59, 31.28, 27.36, 32.30, 25.60)
I.4	0.133	103163.88 530655 19.44	(2, 3, 7, 19, 30) (1, 1, 3, 3, 2) (11.41, 11.97, 11.69, 11.90, 11.32) (15.14, 32.07, 25.51, 25.37, 19.44)
I.5	0.144	58096.08 323067 17.98	(1, 7, 8, 23, 24) (2, 2, 1, 2, 3) (11.32, 11.88, 12.06, 12.33, 12.31) (20.57, 22.18, 16.39, 25.72, 30.55)

Table 18: Results with TL capacity for (*n*, *m*, *L*) = (30, 6, 3)

Inst.	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin%	# of TL LTL weight
I.1	0.173	69249.39 375120 18.46	(5, 6, 7, 17, 19, 25) (3, 2, 2, 1 2, 2, 1) (12.13,11.95,12.42,12.47,11.65,11.7) (31.31,26.99,25.52,36.68,28.32, 22.21,18.84)	(1, 1, 0) (3463.63, 1.87, 1378.02)
I.2	0.126	65384.87 337467 19.38	(7, 9, 13, 15, 21, 22) (3, 3, 2, 3, 2, 1) (12.13,10.95,11.99,12.22,12.14,11.2) (27.31,22.54,37.90,24.53,22.82,26.1)	(0, 0, 0) (5957.37,7878.52,17005.5)
I.3	0.222	73050.73 388701 18.79	(3, 10, 22, 25, 26, 27) (2, 2, 1, 1 2, 1 3, 1) (11.69,11.42,12.40,11.62,12.12,12.1) (25.40,22.59,31.28,27.36 12.50, 32.29 33.32, 25.60)	(1, 1, 0) (0.73, 0.36, 4071.68)
I.4	0.18	103596.1 5 542659 19.09	(8, 9, 16, 19, 25, 30) (2,1, 2, 1, 2, 2) (12.09,11.92,12.5311.90,11.98,11.32) (22.16,28.42,37.77,26.19,21.84,19.4)	(0, 1, 0) (15817.9, 3661.6, 0)
I.5	0.122	56032.96 322491 17.38	(1, 7, 8, 22, 23, 24) (1, 2, 1, 2,1, 3) (11.32,11.89,12.06,11.74,12.33,12.3) (24.87,22.31,16.39,25.78,30.30,30.5)	(1, 0, 0) (4688.02,12989.1,016.69)

Table19: Results for (n, m, L) = (50, 7, 4) with no TL capacity

Instance	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin%
II.1	0.385	98788.67 470765 20.98	(7, 11, 14, 18, 35, 42, 50) (4, 3, 2, 2, 1, 3, 4) (10.81,11.53,12.25,12.28,11.98,12.00,12.03) (18.39,27.08,35.15,28.94,27.09,30.78,33.32)
II.2	0.32	131870.52 55340 23.83	(2, 13, 17, 21, 23, 36) (3, 4, 4, 2, 3, 3) (12.46, 12.35, 12.35, 12.36, 12.28, 12.40) (34.75, 28.36, 34.51, 32.73, 29.60, 36.94)
II.3	0.372	125499.21 545808 22.99	(5, 8, 22, 27, 30, 40) (4, 1, 4, 4, 3, 4) (11.55, 12.30, 12.03, 12.30, 11.81, 12.15) (26.61, 33.65, 31.17, 36.68, 26.37, 29.73)
II.4	0.385	96474.03 505139 19.10	(1, 9, 11, 23, 30, 43, 45) (1, 3, 4, 1, 1, 1, 3) (12.04,12.10,12.09,11.44,11.82,11.48,12.21) (27.15,22.50,23.85,24.76,23.25,21.81,31.33)
II.5	0.315	106833.54 494034 21.62	(20, 35, 41, 43, 44) (4, 2, 3, 2, 3) (12.02, 12.06, 12.02, 11.46, 11.79) (32.31, 27.11, 28.78, 26.96, 26.30)

Table 20: Results with TL for ((*n, m, L*) = (50, 7, 4)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin% Quantity discount	# of TL LTLweight
II.1	0.672	94784.12 470925 20.13	(11, 14, 19, 21, 43, 49, 50) (1 2 3, 2, 1, 3, 1, 1, 1 4) (11.53,12.25,11.86,11.68,11.53,12.27,12.03) (20.18,27.40,27.08,35.15,26.69,27.49,24.57, 32.78, 9.52 33.36)	(1, 0, 0, 0) (0.20, 5981.38, 6868.17, 7270.97)
II.2	0.59	121168.23 551152 21.98	(2, 17, 21, 23, 27, 36) (3, 3 4, 2, 3, 2, 3) (12.46, 12.35, 12.36, 12.28, 12.01, 12.40) (34.75,35.5034.50,32.73,29.60,31.86, 36.94)	(0, 1, 2, 0) (0, 8524.66, 1.15, 6518.57)
II.3	0.341	129977.64 554128 23.46	(5, 8, 20, 27, 30, 39, 42) (4, 1, 4, 4, 1, 4, 4) (11.90,12.30,12.00,12.30,11.81,12.12,12.36) (30.44,33.65,30.56,36.68,29.45,34.51,36.68)	(0, 0, 0, 1) (13545.5, 0, 0, 7149.3)
II.4	0.613	99224.26 509332 19.48	(1, 9, 12, 17, 23, 45) (1, 4, 1, 4, 4, 3) (12.04, 11.91, 12.18, 11.65, 11.44, 12.39) (27.15, 31.57, 22.61, 23.71, 25.81, 33.30)	(1, 0, 0, 0) (9528.24, 0, 3637.75, 18430.3)
II.5	0.374	104180.12 503966 20.67	(19, 20, 30, 31, 41, 43, 46) (1, 4, 3, 3 4, 4, 2, 4) (11.61,12.02,11.73,12.50,12.02,11.5812.22)	(0, 0, 1, 1) (4843.72, 7486.72, 1.22, 4447.06)

Table 21: Results for (n, m, L) = (75, 8, 5) with no TL capacity

Instance	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin%
III.1	1.097	122475.22 530003 23.11	(5, 7, 16, 18, 52, 55, 72, 74) (4, 2, 1, 1, 5, 3, 2, 2) (12.04, 11.66, 12.35, 12.09, 12.06, 12.51, 12.10, 12.28) (33.26, 23.40, 32.75, 28.63, 33.86, 29.66, 31.05, 33.83)
III.2	1.095	141430.26 595123 23.76	(4, 17, 20, 22, 24, 41, 71, 72) (4, 1, 1, 2, 2, 2, 1, 3) (12.49, 12.30, 12.01, 12.41, 11.88, 12.39, 12.39, 11.95) (36.47, 29.21, 30.22, 36.71, 31.19, 32.04, 35.58, 27.92)
III.3	1.945	112055.86 506835 22.11	(1, 13, 26, 44, 64, 66) (2, 3, 2, 1, 1, 4) (11.59, 11.96, 12.14, 12.06, 11.94, 11.83) (26.61, 32.83, 31.47, 28.18, 27.01, 24.32)
III.4	1.364	132241.40 589650 22.43	(6, 7, 20, 28, 41, 49, 56, 59) (1, 1, 3, 1, 1, 3, 3, 3) (12.30,12.00,11.58,12.18, 12.05, 12.08, 11.47, 12.05) (28.85, 30.27, 27.71, 27.75, 33.33, 33.46, 22.86, 31.25)
III.5	1.454	132397.64 608445 21.76	(17, 29, 33, 38, 47, 48, 57, 70) (2, 1, 1, 5, 2, 1, 5, 1) (11.58,11.93,12.15,12.27, 11.91, 12.49, 12.40, 12.12) (25.53,31.82,27.08,33.75, 27.98, 32.71, 20.92, 28.93)

Table 22: Results with TL (n, m, L) = (75, 8, 5)

Instance	CPU (s)	Profit Revenue Profit%	Assortment Supplier Price Profit margin%	# of TL LTL weight
III.1	1.437	115626.74 528002 21.90	(5, 7, 19, 52, 55, 71, 72, 74) (4, 2, 2, 5, 5, 1, 2, 2) (12.04, 11.66, 11.66, 12.06, 12.51, 12.20, 12.10, 12.28) (33.26, 28.40, 29.29, 33.86, 33.00, 34.57, 31.05, 33.83)	(0, 1, 0, 0, 1) (1656.99, 2610.81,0, 5055.47, 5179.83)
III.2	1.804	137216.76 598842 22.91	(1, 4, 20, 22, 24, 41, 63, 71) (4, 4, 1, 2, 2, 2, 4, 1, 1 4) (12.36, 12.49, 12.01, 12.41, 11.88, 12.39, 12.20, 12.39) (37.28, 36.47, 30.22, 36.71, 31.19, 32.04, 27.81, 35.58 28.04)	(0, 1, 0, 1, 0) (9510.88, 11251.9, 0, 1.51, 0)
III.3	1.699	111990.87 510494 21.94	(1, 7, 15, 18, 23, 26, 45) (2 3, 3, 3, 3, 4, 2, 1) (11.69, 11.86, 11.98, 11.96, 12.28, 12.14, 11.98) (27.69 25.19, 29.03, 32.44, 26.45, 34.82, 31.47, 29.34)	(0, 0, 1, 0, 0) (4909.37, 12050.8, 0.11, 8180.21)
III.4	1.568	124446.85 593170 20.98	(5, 7, 17, 37, 41, 49, 56, 59) (3, 1 5, 4, 4, 1, 3, 3, 3) (12.26, 12.00, 12.00, 12.39, 12.05, 12.08, 11.47, 12.05) (29.17, 30.27 29.90, 29.96, 31.93, 33.33, 33.46, 22.09, 31.25)	(1, 0, 1, 0, 0) (0.39, 0, 16097.7, 7960.48, 225.242)
III.5	1.085	136373.89 614968 22.18	(28, 30, 38, 55, 56, 63, 70) (2, 1, 2, 2, 2, 1, 1) (12.19, 12.05, 12.27, 12.10, 12.46, 12.28, 12.12) (31.00, 29.08, 34.22, 33.66, 28.45, 32.77, 28.93)	(1, 1, 0, 0, 0) (761.11, 2729.02, 0, 0, 0)

CHAPTER 5

CONCLUSIONS
AND SUGGESTIONS FOR FUTURE WORK

In this thesis, we study the effect of vertical integration on retailing decisions. Specifically, we demonstrate that quantity discount contracts can lead to different assortments and interestingly to bigger assortment. The quantity discount contracts were shown to also affect the pricing of the products chosen, both upward and downward. W e then studied the effect of TL capacity on assortment and pricing and observed similar effects.

Future work could consider integrating other supply chain effects into assortment planning and pricing. These include effects such as delay in payments (Shinn et al (1996)), volume discount (Xia and Wu (2007)), and rebates (Saha (2013)).

Future work can also integrate two or more supply chain effects simultaneously. In particular, quantity discounts and TL capacity can be considered jointly. Several recent papers considered these two effects jointly (Massini et al (2012), Burwell et al (1997)).

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