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A TRANSMISSION NETWORK EXPANSION GAME

by
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AN ABSTRACT OF THE THESIS OF

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The transmission network plays a critical role in the electricity market. Nowadays, making decisions concerning network investments has become harder due to the new patterns of electricity consumption and the competition among electricity producers. In fact, electric power systems are now operated horizontally instead of vertically.

In this work an expansion plan is assessed using a performance function that is a trade-off between the cost of new lines and the social welfare resulting from the expansion. To estimate such welfare, the correlated equilibrium (CE) of the electricity market is studied, and no regret algorithm is applied to the competition in the power pool.

The methodology is illustrated using graver six bus system and IEEE 24 bus system. Simulation results show that CE can be used to account for the strategic behavior of the generators and that it has low computational requirements when compared to Nash equilibrium (NE) as well as it always exits even when NE doesn't.

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CHAPTER I

INTRODUCTION

Decision about electrical transmission network expansion has become harder in the last two decades. As a matter of fact, the power sector has been decentralized and the electricity market is now a competitive market. Thus a transmission expansion plan cannot follow the traditional planning criteria anymore. As a result, new techniques are developed in order to solve the transmission network expansion problem (TNEP).
According to [1] the traditional planning models can be divided into two categories. The first is the mathematical optimization models that incude linear and nonlinear programming, dynamic programming, mixed integer programming benders decomposition and others. The second category is the heuristic models which usually starts with a certain plan and improves on it using heuristic algorithms such as genetic algorithms (GA), Tabu search (TS) and several heuristic and metaheuristic techniques. The planning models can be divided depending if they consider static or dynamic scenarios. The previously mentioned models are called static scenarios. Dynamic scenarios are the one based on game theory which represents the behavior of different participants in the electricity market. Different games and models like cournot,Stackelberg, bargaining,etc.. are recently used to obtain an optimal decision concerning TNEP.
Competition in power markets was adopted to increase the efficiency and reduce prices. The aim of a competitive market is to exchange a greater quantity of electricity with the least possible prices. On the other side, the network operation and power flows are

limitied to different constraints. Consequently, when modeling the electricity pool, the strategic actions of the market agents under the market constraints should be taken into account. The tool to such a modeling can be provided by game theory. In addition, by taking into consideration the benefit of the consumer while allocating prices, the game model prevents market power exertion.

In this thesis, we propose a methodology to search for the best expansion alternatives in a transmission system using non-cooperative game theory. The focus is on how the expansion will influence the behavior of the power producers and consequently the social benefit. An expansion plan is then assessed using a performance function that is a trade-off between the cost of new lines and the social welfare resulting from the expansion. Thus, as a first step, a certain set of expansion plans is considered. This set may contain plans suggested by the system operator or other authorities. In reality, not all right of ways can be subject to expansion. There are conditions, requirements, and regulations that should be met. Thus, considering a specific set of possible plans is not far from being realistic. In the second step, the power generators interact in the market by submitting their own bids. When choosing his action, a producer should take into consideration the cost and bids of his opponents in addition to the influence of his bid on the market clearance performed by the system operator.

Usually Nash equilibrium (NE) is adopted as the solution for the bidding game of power producers. However, it can be shown from the literature, that multiple market do not have pure NE or have multiple market equilibrium. This is due to the transmission capacity constraints. To overcome such issue, we apply in this work a generalization of NE, introduced by Aumann [2]. It’s the correlated equilibrium (CE).

The CE is a NE where the players choose their strategies based on a signal form a trusted party. A strategy tuple (signal) is chosen by the party based on a certain distribution. This distribution is a CE if and only if, the players will have no incentive to choose unitarily an action different from the recommended one. CE can be more efficient than the NE and it always exist for finite games.

To obtain the correlated equilibria of the power market, and in order to model the interaction between different power producers, we consider that every player will play regret matching. At each stage T, a player i already played action j, will choose an action k for the next stage (T+1) based on a certain probability. In the regret matching algorithm, this probability is proportional to its regret corresponding to action k at time T. That is the extra benefit player i would have gained if he played k instead of j in all the stages in the past that he actually played j.

At the end the joint distribution of play will converge to the set of correlated equilibria of the game. The joint distribution of play is a probability distribution where the probability assigned for each action tuple is the relative frequency of this tuple being played.

The expansion game in our work consists of three decision making levels. We start by the network planner that will take the decision on the expansion. He specifies first the set of expansion plans under study. This set may contain plans suggested by the system operator, power producers or other authorities. In the second level come the players that interact with each other’s and bid. When bidding, the power producers aim to maximize their payoff subject to the market clearance performed by the system operator. The lower level is that of the Independent System Operator (ISO) who performs a market clearance that results in generating the nodal injections, nodal withdrawals and nodal prices, by maximizing the social welfare subject to electrical constraints.

Finally, a performance function that is a trade-off between welfare resulting from the expansion and the expansion cost is computed for every plan in the set. The plan with the highest performance function is considered to be the best solution.

As already mentioned, considering only a limited set of expansion plans is not far from being realistic. In large networks, where huge number of right of ways exist, it is very difficult to assess every possible expansion and compute its performance function. For that purpose, we use a linear regression model built using a certain set of proposed plans to predict the social welfare or performance function of any new plan. The proposed expansion set with the computed social welfare of each plan is used as training data for the model. However, due to the high dimensionality of the problem in case of wide-area networks, and its nature (sparsity), principal component analysis (PCA) is performed on the data for dimensionality reduction. The new components or features are the independent variables while the social welfare will constitute the dependent variable.

In addition two methods, support vector machine (SVM) and logistic regression, used for classification, were applied in order to reduce the search space of the plans. A threshold is set and resulted in two classes of plans. The space is reduced almost by half. The class with the higher PF can be examined using another classification or ranking prediction, or even by calculating the corresponding performance function of the plans as described previously.

The 6-bus Garver system and IEEE 24-bus system are used for numerical validation. A numerical comparison between the use of CE and the use of NE is presented.

The contribution of this work lies in the application of the no regret algorithm to a two sided cost based electricity pool to obtain the correlated equilibrium of the electricity market and the use of this correlated equilibria to compute the social welfare. Then investigating the ability of the correlated equilibrium as the competition's output to serve the planner as a decision criteria for the expansion.

As limitations, the results of the competition in the electricity market are only an estimation of the possible output since we did not consider different states of the market that accounts for peak and off-peak hours. In addition, the variation in generation and demand over the expansion time horizon was not taken into account.

The organization of this thesis is as follows. Chapter II presents a survey of previous work on traditional and game theoratical methods that were used to solve the TNEP. In Chapter III the expansion problem is formulated starting by modeling the interaction between producers together with the market clearence mechanism. The correlated equilibrium and No Regret algorithm are then described in details. In addition, liner regression and PCA application along with SVM and logistic regression application to the TNEP is presented. Chapter IV presents the experimental results generated by applying the methodology to the classical Garver six-bus system and the IEEE 24-bus system. Finally conclusions are drawn and future work is stated in Chapter V.

CHAPTER II

LITERATURE REVIEW

Due to the decentralization of the power market, game theory was established as an efficient way to come up with solutions to the electricity market issue. Giving the numerous generations, the ability to bid and take strategic action had a great influence in eliminating market power exertion and providing electricity at best prices and managing electricity demands.

Previously, when the market was operated vertically, traditional ways were used to solve the TNEP. These traditional methods were either heuristics methods or optimization models and in many cases a combination of both was used. Optimization models are like linear programming [3], [4], dynamic programming [5], mixed integer programming etc... In [6], the nonlinear and nonconvex mixed integer problem is transformed into a linear mixed integer problem taking into account losses with guaranteed convergence. [7] presents a heuristic method to reduce the search space, combined with mixed-binary linear programing to find the optimal solution of multistage transmission expansion planning problem. At every stage, the expansion cost was minimized subject to the network constraints to obtain an optimal plan. The expansion plans obtained at each stage constituted the reduced search space. Decomposition techniques like Benders' were also used [8]. GA is combined with Benders' decomposition in [9].

GA and simulated annealing (SA) are typical examples of heuristic methods used to solve the TNEP. In [10], the multistage and coordinated TNEP modeled as a mixed integer linear programming is solved using an efficient GA. [11] models the TNEP as a mixed integer nonlinear program and solves it using an improved SA approach. In [12] the static TNEP is solved by hybrid SA. The algorithm perform a low-cost local search at each temperature.

All the above approaches used the DC model flow used in this work. However AC power flow model was used in different approaches in the literature. In [13] the AC network model is presented as a mixed integer program, and its convex relaxation is solved by the branch and cut algorithm. [14] presents a constructive heuristic algorithm that includes an interior point method to solve the TNEP using the AC model.

In these mentioned traditional methods the criteria for expansion the most used were network security, congestion, operational and investment cost. Nowadays, with the market decentralization and renewable energy integration, new criteria rises that should be considered. There isn't one single entity that sells electricity anymore, and interaction between power generators is a fact. Here comes the total benefit of all participants as a new criteria to consider when expanding the network. Thus, game theory is an efficient tool to model such interaction in the electricity market.

Game theory was used in two ways: cooperative game theory where players form coalitions and non-cooperative game theory where each player is at its own.

As said previously, cooperative game theory is based on coalitions. A kernel approach was used in [15] and [16] where the goal was to distribute the transmission cost in a decentralized way. The agents of the game can be the generators, the consumers and/or the transmission line owners. At each iteration, coalitions are formed by a certain number of players as to increase their payoffs. Thus costs are distributed based on the kernel stability concept. At the end of the process, the cost allocation is the last calculated one.

Also in the cooperative game, private field planning was considered in [17], where the players pay for the plan they support. The only generation companies participating in the expansion are those that increase their generation output due to transmission expansion. The optimal plan is the expansion plan (satisfying all the required constraints) that is the most suitable for the greater coalition.

In addition, to give the electricity market participants incentives to support the transmission network expansion, the transmission investment problem in [18] was formulated in a way that aims to maximize the social welfare. The network expansion is decided by the result of a poll in which each consumer and each producer is assigned a certain weight (which is a measure of the influence of each firm on the expansion decision). The sum of the total weights of a coalition should be greater than 0.5 for the coalition to gain the right of deciding about the expansion.

Concerning non-cooperative game theory, different criteria were taken into account. In some cases, the cost of the expansion was considered and the game was established to guarantee the best expansion cost. In other cases, the social welfare resulting from the expansion is studied and the game was established to maximize it. In addition, there are cases were the trade-off between the two criteria is used.

As an example of the first case, [19] and [20] present a methodology to search for an optimal transmission expansion using multiple optimization techniques. Once the optimal TEP is obtained, the costs of the included projects are evaluated considering the bargaining solution of the right-of-ways between investor and land owner, the asset cost and the level of effort of the corresponding agent (investor). A principal-agent game where the principal is the central planner and the agent is the investor is used to model the expected utility of the game participants. Finally the bidding competition occur in order to win the project contract. The optimal bid is concluded depending on the number of bidders participating and assuming a risk preference.

Considering the Social welfare as a criteria, the model presented in [21] studies the interaction between generation and transmission enterprises. Based on the nodal price and the decision variables of generation operators (generation capacity) and transmission enterprises (line power flow) the profit maximization problem of each enterprise is formulated. Thus a cournot equilibrium solution is derived. An optimal model of market clearing is presented balancing between demand and supply. To test the model a three-bus system is used .

In [22] the interrelationship between generation expansion planning and transmission expansion planning concerning optimal development of the network is modeled in a competitive electricity market. Two stackelberg games are formulated, where the transmission operator act as leader and the generation firms as the follower. In the first, a central organization manage both generation and transmission system development at the same time. In the second, a transmission operator manages the development of the transmission system while predicting the decisions of generation firms. The two methodologies were illustrated by a simplified three node network in order to compare results, and a 21 nodes network to test the applicability of the methods.

The Stackelberg-Worst NE is introduced in [23] in three steps. The first step consists of modeling the strategic behavior of generation companies, considering the potential need of generation capacity expansion. Second step is the formulation of the NE Concept as an optimization Problem. Step 3 is the formulation of the Planner Problem, in which the Stackelberg-Worst NE concept is used to solve the multiple equilibrium problem (given the transmission plan). To test the developed model a three-node example system is studied.

The concept of reference transmission network (RTN) is introduced in [24]. It is considered to be the sub game perfect NE of the dynamic game. The game agents in this paper are the planner and all the power producers. Transmission expansion plans are proposed by the investors and the planner chooses one possible feasible plan. For each plan a multi-stage game is performed and every stage is formed by a number of spot market sessions. At every spot market an auction is organized and both power producers and consumers participate and bid.

In [25], an algorithm that maximize the individual welfare of a player in the electricity market is presented. This algorithm is used then to compute the NE by assuming that every player, when bidding, aim to maximize its individual welfare given the bids of the other players in the market. The payoff maximazation is subject to welfare maximization that is done subject to electrical constraints. The algorithm uses the newton step method to update bids. To illustrate the algorithm a 2 bus system and a 9 bus system were used. Cases with and without constraints were studied. It is showed that in the case where transmission constraints exists no pure equilibrium was found nor a continuum of nash equilibria was detected.

As in [25] the strategic game presented in [26] consider a number of dominant firms, each submitting a bid also in a way to maximize its own profit. The penalty interior point algorithm is used to solve the firm problem here. To illustrate the proposed algorithm, it is applied to a 30 bus network. The work states that a convergence to a NE is not guarantee.

In [27] the impact of the transmission constraint is showed. A two level optimization problem is considered where an ISO solves an OPF to get generated quantities and nodal prices at the lower level and producers aim to maximize their payoffs at the upper level. The optimal response curve is generated when transmission constraints exist and in the case when no constraints. It can be shown that in the presence of constraints, the optimal response curves are parallel thus no equilibrium exists. A three bus-system is used for testing.

The bilevel programming method in [28] uses an active-set method to solve the economic dispatch performed by ISO which is the lower level problem. The upper level problem is a payoff maximization problem done by the producers. The active-set method is used here to overcome the influence of inequality constraints. The constraints are separated into active and inactive. Inactive constraints are then eliminated. Applying the KKT conditions, the generation levels and the prices are given in function of the producers' bids. A single level problem is obtained that maximizes the producer payoff subject to the inactive constraints. The trust-region Newton method is used to solve the aforementioned problem. All possible combinations of active-inactive constraints were considered. The global NE is obtained by comparing all NEs resulting from different combinations. The IEEE 9-bus system and the IEEE 30-bus system were used for testing.

A 3 stage model is presented in [29] to generate an investment decision. The model is based on a trade-off between the expansion cost and the social welfare after the expansion. In the third stage, wich is the lower level, the ISO takes the strategic level of generation of each producer and perform a market clearance in a way to maximize the social welfare (producers profit minus production costs). In the second stage, producers choose their strategic decisions in order to maximize their payoffs, taking into consideration the effect of their decisions on the ISO's market clearence. In the first stage the planner decides on the expansion based on a cost-welfare trade-off while expecting how the expansion could influence the equilibria in the second stage. It is tested using a three-bus system.

As it can be seen from the above, the NE was always the equilibrium used to model the outcome of the competition in the power market, However A generalization of the Nash equlibrium, the Correlated equilibrium, was used in different fields. This notion was developed by Auman [2] were he breaks the separation between the “ "Bayesian" and the "game-theoretic" view of the world ”. Hart in [30] shows that taking actions based on natural regret measures will lead after a sufficiently high number of stages, to the correlated equilibria of the game. In [31] the no regret is applied to obtain the correlated equilibrium between distributive users accessing the spectrum for cognitive radio.

On another side, pattern recognition tools were used in the transmission expansion problems. In [32], linear regression and neural network models were build, using a certain set of plans, to predict the cost of the other plans.

This work applies the no regret learning algorithm to the competition in the power market to estimate the resulting social welfare of the expansion. A set of proposed plans is considered and each plan is associated with its expected social welfare. This set is used to build a linear regression model that aim at predicting the ranking of plans. The quadratic model of the market clearance and the players' payoffs in [24] is used.

CHAPTER III

THE EXPANSION GAME

In this work, we apply the concept of correlated equilibrium to the electricity market using the no-regret algorithm. The goal is to study the social welfare resulting from the expansion and to choose afterwards, the best plan based on a performance function that is a trade-off between the cost of building new lines and the benefit that will result from the expansion.

We model the electricity market as a bidding game between power producers. These producers will choose their price function in a way to maximize their payoff while taking into consideration the cost and bids of their opponents in addition to the influence of their own bids on the market clearance performed by the system operator.

A. The Pool Model

A power pool consists of generators offering bids that contain price-quantity pairs. Two types of pairs exist; the first one is a function of the variable costs of generation, this is the case of cost-based pools. In the case of price-based pools, the generators can offer any price they want. Another classification of pools can be done based on the demand side. If the demand is forecasted and the market operator dispatch units to meet this demand, then we are in a one-sided pool. The second type is the two-sided pools where buyers also submit price-quantity bids so we obtain a demand curve.

In this work, we consider a cost-based two-sided pool where the market clearance is done on an hourly basis.

One way of pricing is to consider a single price for the whole pool without considering network constraints. The cheapest generators are given priority and the chosen price is the one of the last unit used (the most expensive) to cover the demand. After that the feasibility of the solution is studied to check if there is any congestion. In this case out-of-merit generators may be used to replace in-merit generators. The congestion price is added to the energy price.

The second way which is the one we used in our work is using the locational marginal price (LMP) that considers both the generation marginal price and the transmission constraints. Thus we get a price for each node. These LMP's are generated by market clearance, and are defined as the change in cost needed to supply the increment of demand at the nodes. In our case, the KKT conditions of the optimization maximizing the social welfare subject to electrical constraints, will yield prices. Details of such optimization will be discussed throughout the coming sections

B. Bidding in the power market

The generators marginal cost function and demand function (price-quantity pairs) are considered continuous as follows:

$$p(G_k) = a_k + b_k G_k \qquad (3.1)$$

$$p(L_k) = d_k - s_k L_k \qquad (3.2)$$

Where:

p represent the price,

G_k is the generated power by the generator at node k,

L_k is the power withdrawn by the load at node k ,

a_k , b_k , d_k and s_k are all positive coefficients.

The slope b_k is considered to be the strategic variable of the producer at node k. Thus the price function becomes : $p(G_k) = a_k + \beta_k G_k$, with β_k is the bid of the generator at node k. For simplicity, we consider that at node k all generators correspond to the same producer and all demands to the same consumer.

The spot market is organized as an auction in which both power producers and consumers are allowed to participate. After the biding process is done, the independent system operator perform a market clearance that maximizes the social welfare, where as mentioned earlier, locational marginal prices λ_k of each node are generated together with generation and demand level.

In fact, the marginal cost/price is defined as the change in cost/price that will occur when supplying an increment of one unit of demand or producing an increment of one unit:

$$\frac{dCost}{dG_k} = a_k + \beta_k G_k \qquad (3.3)$$

Then the cost of generator k for a level of generration G_k is

$$Cost = \int_0^{G_k} a_k + \beta_k G_k = a_k G_k + \frac{1}{2}\beta_k (G_k)^2 \qquad (3.4)$$

Same for the consumers, what a consumer is willing to pay can be expressed by the following:

$$\text{Payout} = \int_0^{L_k} d_k - s_k L_k = d_k L_k - \frac{1}{2} s_k (L_k)^2 \qquad (3.5)$$

The payoff U_k of a power producer at node k is the difference between the revenues from the sale and the cost of generating power:

$$U_k = (\lambda_k G_k - (a_k G_k + \frac{1}{2} b_k (G_k)^2)) \qquad (3.6)$$

Where: λ_k are the nodal prices.

Note that when producers are acting strategically they submit to the market operator a cost function with β_k as variable costs and we call it declared cost function. However the real cost function of the producers is the one including b_k as variable costs. Thus consideration related to the producer marginal cost function is different between the market operator's point of view and the producer's point of view.

The load benefit is the difference between what the consumer is willing to pay and the actual payment:

$$LB_k = ((d_k L_k - \frac{1}{2} s_k (L_k)^2) - \lambda_k L_k) \qquad (3.7)$$

C. *Market clearance*

The independent system operator's goal is to maximize the hourly declared social welfare subject to the network's electrical constraints.

Definition 1: the concept of social welfare in electricity markets is defined as the difference between what the consumers are willing to pay and the declared cost of the producers.

$$SW = (\sum_{L_k}(d_k L_k - \frac{1}{2} s_k (L_k)^2) - \sum_{G_k}(a_k G_k + \frac{1}{2} \beta_k (G_k)^2)) \qquad (3.8)$$

Thus the ISO is then performing the following optimization problem:

$$\max_{G_k, L_k, f_{km}, \delta_k} \sum_{L_k} (d_k L_k - \frac{1}{2} s_k \left(L_k\right)^2 \Big) - \sum_{G_k} (a_k G_k + \frac{1}{2} \beta_k (G_k)^2) \qquad (3.9)$$

subject to

$$G_k - L_k + \sum_{mk} (f^0_{mk} + \sum_y f_{mk,y}) - \sum_{km} (f^0_{km} + \sum_y f_{km,y}) = 0 \quad \forall\, k \in B \qquad (3.10)$$

$$f^0_{km} = - b_{km} n^0_{km} \, (\delta_k - \delta_m) \quad \forall\, km \in \Omega \qquad (3.11)$$

$$-2\delta_{max}\left(1 - x_{km,y}\right) \leq \frac{f_{km}}{b_{km}} + (\delta_k - \delta_m) \leq 2\delta_{max}\left(1 - x_{km,y}\right) \quad \forall\, km \in \Omega, \forall\, y \in Y$$

(3.12)

$$-n^0_{km} f_{km,max} \leq f^0_{km} \leq n^0_{km} f_{km,max} \quad \forall\, km \in \Omega \qquad (3.13)$$

$$0 \leq G_k \leq G_{max} \quad \forall\, k \in B \qquad (3.14)$$

$$0 \leq L_k \leq L_{max} \quad \forall\, k \in B \qquad (3.15)$$

$$- \delta_{max} \leq \delta_k \leq \delta_{max} \quad \forall\, k \in B \qquad (3.16)$$

Where

- $x_{km,y}$ is the binary decision variable for line y (expand or no)
- f^0_{mk} and $f_{km,y}$ are the power flow in the initial case and in line y
- $f_{km,max}$ is the capacity of the transmission line
- δ_k is the phase angle at bus k
- δ_{max} is the maximum phase angle
- b_{km} is the susceptance for the right of way km
- n^0_{km} and $n_{km,\ max}$ are the number of lines in the initial case and the maximum number of lines that can be added
- G_{max} and L_{max} are the maximum generation and demand at node k
- B, Ω, and Y are respectively the set of buses, right of ways and lines.

Kirchhoff's first law is represented in Equation (3.10). Equations (3.11) and (3.12) guarantee the feasibility of power flows, while equations (3.13) to (3.16) set the bounds of generation, demand, power flows, and angles.

D. Generators' Bi-level problem

As said previously, every power producer will choose his bid in a way to maximize his payoff while taking into consideration the cost and bids of his opponents in addition to the influence of his own bid on the market clearance performed by the system operator. As a result, every power producer will be facing the following bi-level optimization problem:

$$\max_{\beta_k} \lambda_k G_k - (a_k G_k + \frac{1}{2} b_k (G_k)^2 \quad (3.17)$$

s.t.:

$$\max_{G_k, L_k, f_{km}, \delta_k} \sum_{L_k} (d_k L_k - \frac{1}{2} s_k \left(L_k)^2\right) - \sum_{G_k} (a_k G_k + \frac{1}{2} \beta_k (G_k)^2)$$

s.t.:

(3.10) to (3.16)

E. Market Equilibrium

Usually Nash equilibrium (NE) is adopted as the solution for the bidding game of power producers. The meaning of a Nash equilibrium is that,no player has the incentive to uniterally change his action.

As it can be seen from the literature, multiple market do not have pure NE or have multiple market equilibrium. This is due to the transmission capacity constraints. In fact, finding a NE is finding the intersection of the optimal responses of different players. According to [27], when limits on transmission capacities are introduced, these optimal responses do not intersect anymore, or do intersect in multiple points.

Due to the aformentioned disadvantages of NE, we apply in this work a generalization of NE that is the correlated equilibrium (CE). Before proceeding with the correlated equilibrium, a brief explanation about game theory and how to compute Nash equilibrium is presented.

1. *Game Theory Notations*

The branch of mathematics studying games is called game theory. In this work we consider only finite games where players have a finite set of possible choices.

Let n be the number of players, and S_i the set of possible actions for player i. A tuple of actions **s** is denoted by $(s_1, ..., s_i, ..., s_n)$ with $s_i \in S_i$ is the action of player i.

$(s_1, ..., s_i, ..., s_n) \in S$ where $S = \prod_i S_i$.

s_{-i} is a notation for the tuple $(s_1, ..., s_{i-1}, s_{i+1} ..., s_n)$, in other words the actions of all players other than i.

Each player has its utility function $U_i(s_i, s_{-i})$ that is used to calculate his/her payoff or the benefit s/he gets from choosing action s_i knowing that the action set of others is s_{-i}

2. *Nash equilibrium*

An action tuple is a NE, if no player will gain higher payoff in case of changing unilaterally his/her action while fixing the others' actions.

Definition 1: A strategy tuple $(s_1^*, ..., s_i^*, ..., s_n^*)$ is a Nash equilibrium of the game if, for each player i ($\forall s_i \in S_i$):

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \qquad (3.18)$$

We define the best response of player i to s_{-i} , the action s_i that will give player i the highest payoff. A way to compute the nash equilibrium is to find $s_i^* = \max_{s_i} U(s_i , s_{-i}^*)$ for each player i. In other words to find best responses of all players and compute the intersection. Below is NE corresponding algorithm.

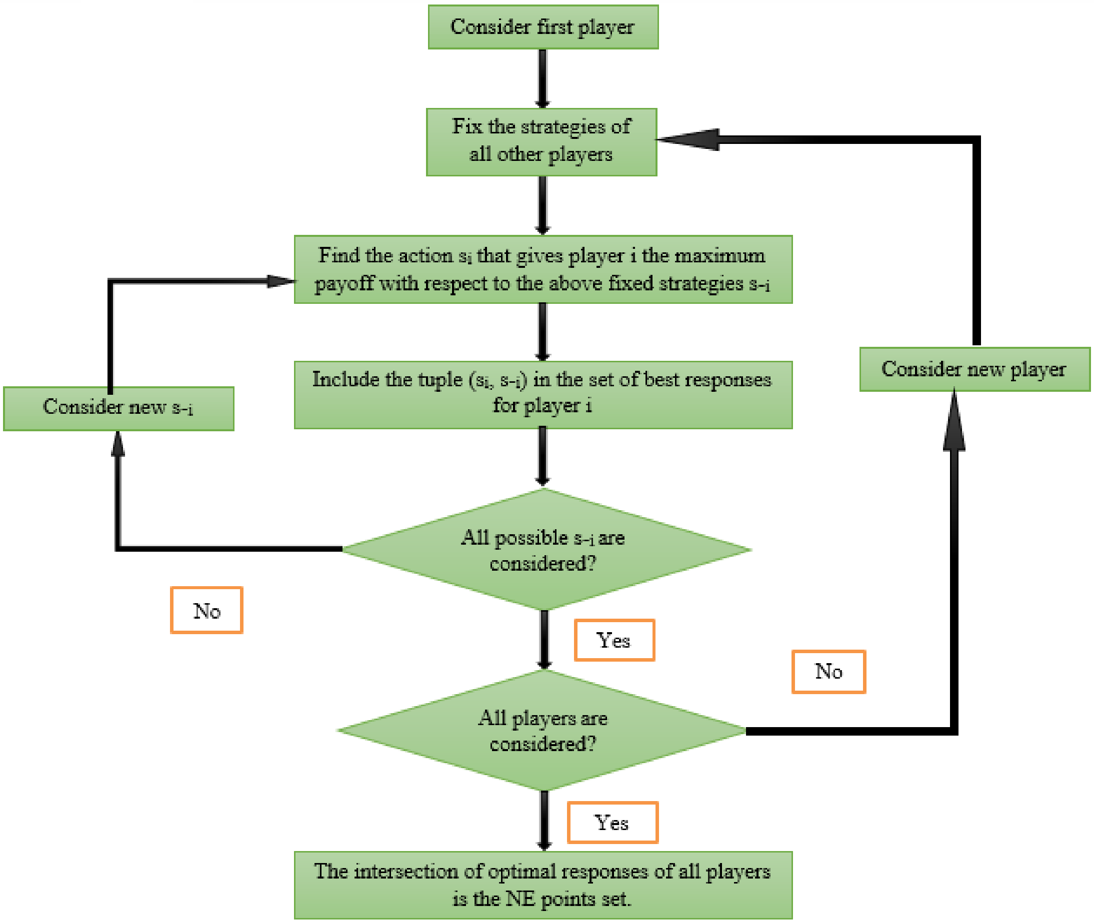

Figure 1: Algorithm for computing Nash Equilibrium

Below is a small example on how to find a NE:

Example 3.1: Consider two drivers speeding from different streets towards an intersection. Each driver has two actions: STOP and GO. The payoffs are showed in Table 3.1

Table 3.1: Drivers Game

	Stop	Go
Stop	4,4	1.5
Go	5,1	0,0

The strategies of player 1 are the rows and player 2 strategies are shown in the columns.

Considering player 1

- The best response of player 1 knowing that player 2 has chosen GO is STOP since it will give player 1 higher payoff. So (STOP, GO) is included into the set of best responses of 1 and (GO, GO) is dropped.
- The best response of player 1 knowing that player 2 has chosen STOP is GO. So (GO, STOP) is included into the set of best responses of 1 and (STOP, STOP) is dropped.

Considering player 2

- The best response of player 2 knowing that player 1 has chosen STOP is GO. So (STOP, GO) is included into the set of best responses of 2 and (STOP, STOP) is dropped.
- The best response of player 2 knowing that player 1 has chosen GO is STOP. So (GO, STOP) is included into the set of best responses of 1 and (GO, GO) is dropped.

There are two pure Nash equilibria (Stop, Go) and (Go, Stop) that are the intersection of the two best responses set.

In our application β_k is the action of the generator at node k and U_k is its utility function based on which the payoff is calculated as mentioned before.

Impact of the transmission constraints on the existence of Nash equilibrium

We say that there is congestion in the network, in case one or more of the transmission capacity constraints is binding. In other words, the power flow on a transmission line is at its maximum.

The existence of congestion highly depends on the producers' bids. [27] divided the actions space of the generators ($S = \prod_i S_i$) into two region. Region 1 or S^1 where the system is not congested and Region 2 or S^2 where congestion exists. The best response function of player i, is as discussed earlier a function of s_{-i}. It can be denoted then by $\Theta_i(s_{-i})$. The best response curves were studied in the two regions. In region 1 the transmission constraints were dropped since these constraints are slack. The curves admitted an intersection point while in region two (when these constraints were included) the curves are parallel which means that no Nash equilibrium exists since the NE is defined as the intersection of optimal responses of the players.

3. *Mixed strategy Nash equilibrium*

A Mixed strategy is when a player chooses his possible actions with a certain probability. A mixed strategy is a Nash equilibrium if no player has the incentive to change its probability distribution over its possible actions. In the above example, one mixed equilibrium exists where each player mixes the pure strategies with probability 1/2 each.

4. *Correlated equilibrium*

CE is introduced by Aumann [19]. The CE is a NE where the players choose their strategies based on a signal from a trusted party. A strategy tuple (signal) is chosen by the party based on a certain distribution and the player will have no incentive to choose an action different from the recommended one. It can be more efficient than the NE and it always exist for finite games.

Definition 2: A probability distribution *p* is a correlated strategy of game G, if and only if, for each players i, $s_i \in S_i$, and $s_{-i} \in S_{-i}$,

$$\sum_{s_{-i} \in \Omega_{-i}} p(s_i, s_{-i}) U_i(s_i, s_{-i}) \geq \sum_{s_{-i} \in \Omega_{-i}} p(s_i, s_{-i}) U_i(s'_i, s_{-i}) \quad \forall s_i \in S_i \quad (3.19)$$

This means that when the signal is to choose action s_i, then by choosing action s'_i instead of s_i player i cannot obtain a higher expected payoff.

Now suppose that in the example 3.1, prior to playing the game the players received a signal and agree to follow the strategy suggested. No player will have the incentive to change his action. It can be interpreted as a traffic light that chooses the following strategy for the players based on the probability distribution shown in Table 3.2.

Table 3.2: Drivers Game with received signal

	Stop	Go
Stop	0	1/3
Go	1/3	1/3

It can be seen that, in terms of total benefits (16/3 in this case), CE does better than the NE. We can notice then from the above that the best correlated equilibrium will yield higher profit than the NE.

In our work, we use the no regret algorithm to obtain the correlated equilibrium. A detailed description of the algorithm is included in the following section.

F. No Regret Algorithm

Regret matching is defined in [30] as follows: "Switch next period to a different action with a probability that is proportional to the regret for that action, where regret is defined as the increase in payoff had such a change always been made in the past".

It is proved in [30] that if each player play regret matching, then the joint distribution of play converges to the set of correlated equilibria of the game. An explanation on the meaning of the joint distribution of play is presented in Section G.

Consider at time T the current action tuple (s_i^T, s_{-i}^T). And let $\mathrm{Avpayoff}_i$ be the average payoff that player i has obtained up to current time T:

$$\mathrm{Avpayoff}_i = \frac{1}{T}\sum_{t=1}^{T} U_i(s_i^t, s_{-i}^t) \qquad (3.20)$$

The regret at time T for player i of playing action s_i^T instead of any other possible action k, is the difference between the current average payoff and the average payoff that player i would have obtained if he played action k in all periods that he actually played s_i^T.

So we consider all possible actions $k \neq s_i^T$ and let $V_i(k)$ be the average payoff i would have obtained had he played k instead of s_i^T every time in the past that he actually played s_i^T:

$$V_i(k) = \frac{1}{T}\sum_{t=1}^{T} v_t \qquad (3.21)$$

Where:

$$v_t = U_i(k, s_{-i}^t) \quad \text{if} \quad s_i^t = s_i^T \qquad (3.22)$$

$$v_t = U_i(s_i^t, s_{-i}^t) \quad \text{if} \quad s_i^t \neq s_i^T$$

The regret is then defined by:

$$R_i (k) = [V_i (k) – Avpayoff_i]_+ \qquad (3.23)$$

Player i will then take, in the next period, the action with the highest probability. This probability of playing the action k at time T+1 is calculated proportionally to the regret as follows:

$$Prob_i^{T+1}(k) = cR_i(k) \qquad (3.24)$$

Player i decides whether to play the same action as in the previous period or to switch to another action depending on the probabilities (proportional to the regret) obtained. The action k at time T+1 is chosen with a probability $Prob_i^{T+1}(k)$.

G. Joint Distribution of Play

The joint distribution of play calculates the relative frequency at which a certain action tuple occurs and expressed by:

$$f_T(s) = \frac{1}{T} \# \{t \leq T : s_t = s\} \qquad (3.25)$$

The sequence of these frequencies for large T will converge to the set of correlated equilibrium.

It is very important to mention that the joint distribution of play differs from the product of its marginal distribution: it is fully determined by the history of play. For example, consider the game of matching pennies. The players will play half the time HH and half the time TT. This will be noticed quickly and at least one player will change his behavior. However the marginal distribution of play is (1/2, 1/2) for each player, thus the player have no incentive to change. So to be reasonable, the joint distribution of play should be taken into consideration.

It can be seen here that the history of play act as the trusted party that gives signals to the players.

H. Performance Function

After computing the optimal correlated equilibria and the social welfare for each considered plan, the corresponding value of the performance function is calculated based on the following:

- The expansion is made considering a time horizon of 10 years. That is 87600 hours knowing that the above computed social welfare is the hourly declared social welfare.
- The meaning of declared social welfare is that the power producer submits to the system operator his strategic price function $p(G_k) = a_k + \beta_k G_k$, that this operator will account for in the producer's benefit when maximizing the social welfare. However, in the final performance function we consider the true social welfare thus accounting for the real cost function $p(G_k) = a_k + b_k G_k$.

The resulting performance function expressing a trade-off between total true social welfare and expansion cost, is the following:

$$87600 \sum_{L_k} \left(d_k L_k - \frac{1}{2} s_k (L_k)^2 \right) - 87600 \sum_{G_k} \left(a_k G_k + \frac{1}{2} b_k (G_k)^2 \right) - \sum_{km,y} \left(C_{km} x_{km,y} \right) \quad (3.26)$$

Where C_{km} is the cost of building a line between node k and node m.

I. Linear Regression and Classification

In an electrical transmission network, lines cannot be built in all right of ways. There are restrictions and conditions. In addition there is a maximum number of lines that can be built in a right of way. Therefore, considering a set of possible plans that could be proposed by market agents or any other entity is not far from being realistic.

However, in wide-area systems with today's interconnected transmission network, thousands of possible investments are there. These possible plans cannot be examined in a reasonable time frame. Linear regression is used here to be able to predict the performance function or social welfare of any new plan using a certain number of expansion plans as observations to build the regression model.

Multiple timescales exists for expansion plans. The planning horizon may vary between near term (0-5 years), mid-term (5-10 years), long term (10-20 years) and very long term (20-30 years). The importance of the time needed to generate the expansion decision differs from a timescale to another. However even in the cases when the time frame of the expansion decision is not that important (case of long term strategic expansion), a regression model could serve as a first estimation on the desired plan or few plans, in case other decisions related to the transmission network right of ways and the lines surrounding need to be taken.

1. Dimensionality reduction using principal component analysis PCA

Due to high dimensionality and the nature (sparsity) of the TNEP data especially in large systems, it would be hard to build a regression model without reducing the dimensionality. As in [32], PCA is used in this work for that purpose.

PCA is “defined as an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by any projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on.”

The steps of such transformation are the following:

- Create an m x n matrix data with m is the number of observations and n the number of features: In our case the features are the number of lines built in each right of ways, so the number of features is equal to the number of existing right of ways with the number of lines as the corresponding value.
- Compute the covariance matrix of the data matrix
- Compute the eigenvectors and eigenvalues of the covariance matrix
- Choosing the principal components (PC) (the eigenvectors) that corresponds to the highest eigenvalues, and forming the new feature vector.

  NB: Every PC accounts for a variance in the data. Thus the more components used the more accurate the data can be reconstructed. In large transmission networks, a trade-off should be made between number of features and accuracy of representation.
- Deriving the new data matrix:

$$\text{NewData} = \text{FeatureVector}^T \times \text{InitialData}^T \qquad (3.25)$$

2. Regression

Regression studies the relationship between an independent variable X (explanatory or predictor variable) and a dependent variable Y (response or outcome). A regression model is build using a number of observation. Its aim is to find the best fit line to predict the response of a given independent variable:

$$y_p = b_1 x_i + b_0 \qquad (3.26)$$

We denote by:

x_i: the predictor of the observation i.

y_i: the real response of the observation i

y_p: the predicted response of the observation i

b_1: the slope of the line.

b_0: the intercept of the line

The prediction error is defined as: $e = y_p - y_i$

The NewData matrix that is the output of the PCA as described in the previous section is used as X, it is an m x n' matrix where n' is the new size of features equal to the number of principal components. The performance function values corresponding to each plan form the dependent variables vector Y. Thus every new plan is transformed into the new system of principal components chosen previously and its performance function is predicted by the regression model.

3. *Classification*

Two methods, support vector machine and logistic regression, used for classification, were applied in order to reduce the search space of the plans. We consider divided the plans space into two classes. The class with the high performance function and the class with the low performance function separated by a threshold. Afterwards, the class with the higher PF can be examined using another classification or ranking prediction, or even by calculating the corresponding performance function of the plans as described previously.

Support Vector Machine (SVM)

SVM is a classifier that generates, based on the observed labeled data, an optimal hyperplane that will categorize new data. A hyperplane is optimal if it is as far as possible from all points. This distance between the hyperplane and the nearest point is called margin.

Logistic Regression

The possible dependent variables of the logistic regression are two categories. It generates the probability that a certain variable x belongs to this category or that. It measures the relationship between these categories and the independent variables. We used this method due to the nature of the problem.

The basic idea used by the logistic regression model is to find w_0 and w_1 where:

Y = category 1 if $w_0 + w_1x + e > 0$

Y = category 2 otherwise

Where e is the error term.

CHAPTER IV

EXPERIMENTAL RESULTS AND ANALYSIS

The no regret algorithm detailed in Chapter III is applied to the classical Garver six-bus system and the IEEE 24 bus system (IEEE reliability test system).

A. Garver 6-bus system

As it can be shown in Fig.2, Garver system has 6 nodes with power producers at node 1,3, and 6, and power consumers at nodes 1 to 5. It has 15 right of way. We consider here that a maximum of 4 lines can be added to each right of way.

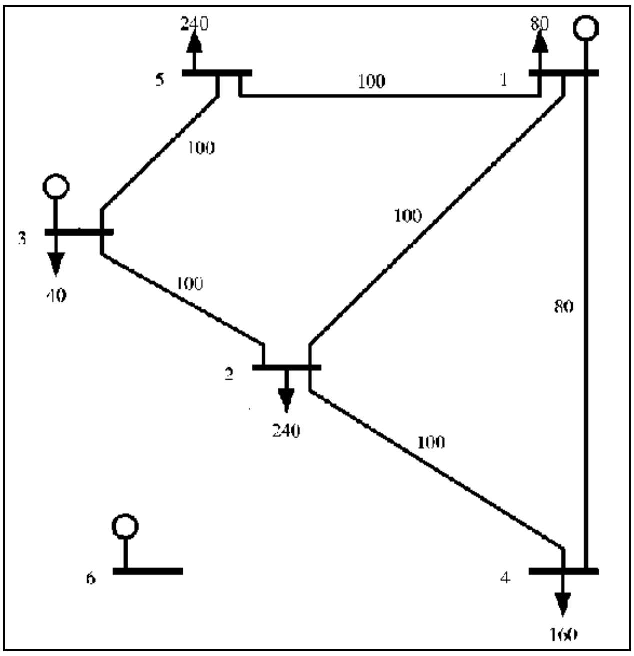

Figure 2: Garver six-bus system

Transmission Lines data of the Garver system along with its initialy built lines are given in Table 4.1. The system has intially six built lines.

Table 4.1: Transmission lines data of the Garver system.

From (node)	To (node)	Capacity (MW)	Cost (M$)	Already built
1	2	100	40	1
1	3	100	38	0
1	4	80	60	1
1	5	100	20	1
1	6	70	68	0
2	3	100	20	1
2	4	100	40	1
2	5	100	31	0
2	6	100	30	0
3	4	82	59	0
3	5	100	20	1
3	6	100	48	0
4	5	75	63	0
4	6	100	30	0

5	6	78	78	0

1. *Correlated equilibria of the Garver system*

We consider a set of 10 expansion plans. These plans were the results of different algorithm and scenarios used to solve the TNEP. The lines to be expanded in each of the ten cases are shown in Table 4.2.

Table 4.2: Proposed expansion plans for Garver system

Plan	Corridors
1	2-6=4, 3-5=1, 4-6=2
2	2-6 = 1, 3-5 = 1, 4-6 = 2
3	2-6 = 2, 3-5 = 1, 4-6 = 2
4	2-6=1, 3-5=1, 4-6=1, 5-6=1
5	2-6=1, 2-3=1, 4-6=1, 5-6=1
6	2-6=1, 1-2=1, 4-6=1, 5-6=1
7	4-6=3, 3-5 = 1, 2-3=1
8	4-6=3, 3-5 = 2, 2-3=1, 2-6 = 1
9	2-6 = 4, 3-5 = 2, 4-6 = 2, 3-6=1
10	2-6 = 4, 3-5 = 2, 4-6 = 3, 3-6=1, 1-5=1

Taking plan 1 for example, 2-6= 4 means that between node 2 and 6, 4 lines to be added, between node 3 and 5, 1 line will be added, and between node 4 and 6, 2 lines will be added. This can be seen in Fig.3.

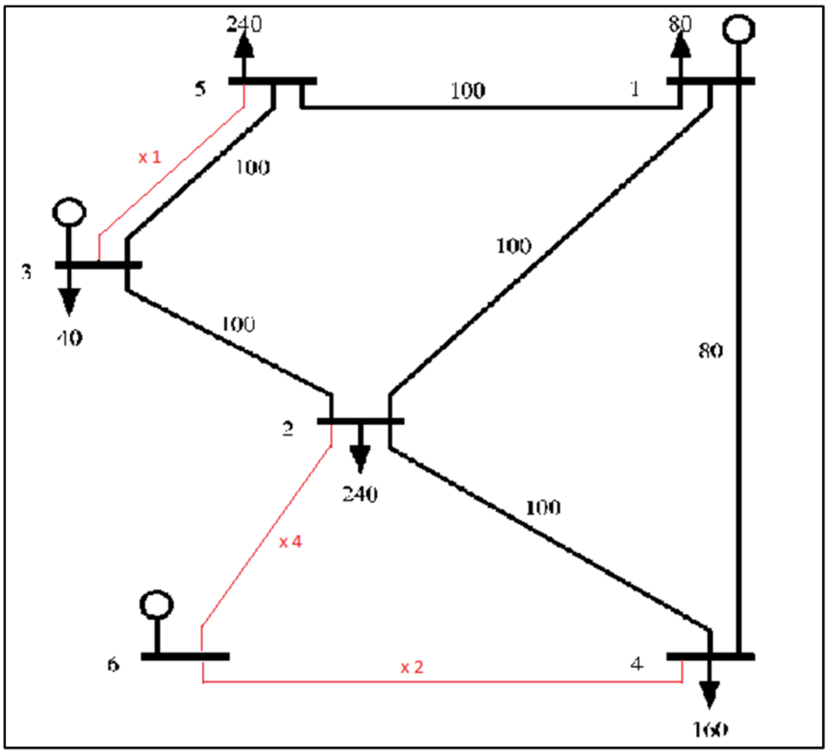

Figure 3: Expansion plan 1

Table 4.3 shows the marginal cost function's parameters (discussed in Section 3.B) for each of the three generators. And Table 4.4 shows the marginal cost function's parameters for each of the five consumers.

Table 4.3: Parameters of generator's marginal cost function

Generator	a_k ($/MWh)	b_k ($/(MW)^{2}h)

G_1	10	0.3
G_3	11	0.4
G_6	12	0.5

Table 4.4: parameters of demand function

Load	d_k (\$/MWh)	s_k (\$/(MW)^{2}h)
L_1	100	0.52
L_2	100	0.65
L_3	350	0.73
L_4	320	0.8
L_5	200	0.76

As mentioned earlier in Chapter III, the correlated equilibria is a probability distribution over the set of possible action tuples. The tuples with nonzero probabilities and their corresponding probabilities in each of the ten expansion cases are presented in Tables 4.5 to 4.14, where β_k represents the action of the generator at node k.

Table 4.5: Probability distribution on action tuples for plan 1

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple1	0.9	0.9	0.9	0.333

Action Tuple 2	0.9	0.8	0.8	0.3317
Action Tuple 3	0.8	0.8	0.9	0.3317

Instantly, taking the case of plan 1, if player 1 receives a recommendation to play 0.9 for example, it means that the chosen tuple (by the third party represented here by the history of play) could be either 1 or 2 (according to Table 4.5). These tuples are not Nash equilibrium points which means that players will have incentive to change their actions knowing the actions of their opponents. Thus 0.9 is not the best response for s_{-1} = (0.8 0.8) neither for s_{-1} = (0.9 0.9). However, the incomplete information of player 1, (not knowing which action tuple has been actually chosen) will lead him to choose the action with the highest probability of getting a high payoff. In other words player 1 here is not risking and thus following the recommendations.

Table 4.6: Probability distribution on action tuples for plan 2

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple1	1	0.7	0.8	0.163
Action Tuple 2	0.9	0.7	0.8	0.161
Action Tuple 3	0.9	1	0.7	0.079
Action Tuple 4	0.9	1.1	0.7	0.095
Action Tuple 5	0.9	0.7	0.7	0.25

Table 4.7: Probability distribution on action tuples for plan 3

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple 1	1	1	0.7	0.0880
Action Tuple 2	0.8	0.7	0.7	0.0580
Action Tuple 3	0.8	1	0.7	0.116
Action Tuple 4	0.9	1	0.7	0.184
Action Tuple 5	1	0.6	0.7	0.095
Action Tuple 6	0.8	0.6	0.7	0.147
Action Tuple 7	0.9	0.6	0.7	0.1715

We can observe from Table 4.6 and Table 4.7 that the relation is not always proportional between lines added and prices. As we see lower prices for the generator at node 3 in plan 3 knowing that the number of total lines added in plan 1 is higher. Although the additional line between plan 3 and 2 lowered the prices in the case of plan 2. This ensures the combinatorial nature of the TNEP problem.

Table 4.8: Probability distribution on action tuples for plan 4

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple1	0.8	0.9	0.9	0.034
Action Tuple 2	0.7	0.7	0.9	0.055
Action Tuple 3	0.7	0.6	0.9	0.537

Action Tuple 4	0.7	0.9	0.9	0.299
Action Tuple 5	0.8	0.6	0.9	0.058

Table 4.9: Probability distribution on action tuples for plan 5

	β_1 ($/(MW)^{2}h)	β_2 ($/(MW)^{2}h)	β_3 ($/(MW)^{2}h)	Probability
Action Tuple1	0.8	0.7	0.9	0.4915
Action Tuple 2	0.7	0.8	1	0.0239
Action Tuple 3	0.7	0.8	0.8	0.0137
Action Tuple 4	0.7	0.6	0.8	0.0648
Action Tuple 5	0.7	0.6	1	0.3857

Table 4.10: Probability distribution on action tuples for plan 6

	β_1 ($/(MW)^{2}h)	β_2 ($/(MW)^{2}h)	β_3 ($/(MW)^{2}h)	Probability
Action Tuple 1	0.6	0.6	0.9	0.5
Action Tuple 2	0.8	0.7	0.7	0.5

According to Table 4.10, the occurring frequency in case of plan 6 is approximately 0.5 for action tuple 1 and 0.5 for action tuple 2. This means that the third party represented here by the history of play will choose action tuple 2 with a probability 0.5 and action tuple 3 with probability 0.5. In this case, the correlated equilibrium is a combination of Nash equilibrium points.

Table 4.11: Probability distribution on action tuples for plan 7

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple 1	1	1.1	0.8	0.0920
Action Tuple 2	1.2	0.7	0.8	0.14
Action Tuple 3	1.1	0.7	0.8	0.136
Action Tuple 4	1.3	1.1	0.8	0.031
Action Tuple 5	1	0.7	0.8	0.061
Action Tuple 6	1.1	1.1	0.8	0.219
Action Tuple 7	1.2	1.1	0.8	0.2080

Table 4.12: Probability distribution on action tuples for plan 8

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple 1	0.7	0.7	0.6	0.0265
Action Tuple2	0.7	0.7	0.8	0.0973
Action Tuple 3	0.8	0.6	0.7	0.1003
Action Tuple 4	0.8	0.8	0.7	0.0177
Action Tuple 5	0.8	0.9	0.7	0.0619
Action Tuple 6	0.9	0.7	0.6	0.1711
Action Tuple 7	0.9	0.7	0.6	0.0118
Action Tuple 8	0.9	0.7	0.6	0.2507

Action Tuple 9	1	0.6	0.7	0.0206
Action Tuple 10	1	0.7	0.8	0.1947
Action Tuple 11	1	0.8	0.8	0.0088
Action Tuple 12	1	0.9	0.7	0.0295
Action Tuple 13	1	1	0.8	0.0118

Table 4.13: Probability distribution on action tuples for plan 9

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple 1	0.7	0.7	0.6	0.2718
Action Tuple 2	0.7	0.7	0.8	0.03123
Action Tuple 3	0.8	0.6	0.7	0.2815
Action Tuple 4	0.8	0.8	0.7	0.1488
Action Tuple 5	0.8	0.9	0.7	0.0733
Action Tuple 6	0.9	0.7	0.6	0.02153
Action Tuple 7	1	0.7	0.6	0.1715

We can see that occurring action tuples can be common between plans. Thus, what is important here is the probability distribution.

Even for the same action tuple, the dispatched generation and demand quantities (influenced by the configuration) will influence on the resulting social welfare that will be different from plan to another.

Table 4.14: Probability distribution on action tuples for plan 10

	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	Probability
Action Tuple 1	0.5	0.6	0.6	0.5
Action Tuple 2	0.6	0.7	0.7	0.5

We can see from Table 4.14 that for plan 10, prices are lower compared to those observed for the other plans.

The social welfare of each plan, its cost and its performance function are presented in Table 4.15.

Table 4.15: Social welfare, cost and performance function for each plan

Plan	SocialWelfare (10^9 $)	Cost (10^6$)	Performance function ($)
1	7.9482	200	7.7482
2	7.6820	110	7.5720
3	7.8504	140	7.7104
4	8.0390	141	7.8980
5	8.0531	141	8.0739
6	8.0176	161	8.2167
7	7.0145	130	6.8845
8	7.6421	180	7.4621

9	8.1176	298	7.8196
10	8.6018	318	8.4868

As it can be seen, plan 10 has the highest performance function.

2. *Comparison between NE and CE*

For each plan, the Nash equilibrium points of the market were computed using the algorithm described in Section 3.E.2. These points existed only in three expansion cases (plan 5, 6, and 10). Results are presented in Table 4.16.

Table 4.16: Nash equilibrium for each plan

Plan	NE			Social Welfare (10^9)
	β_1 (\$/(MW)^{2}h)	β_2 (\$/(MW)^{2}h)	β_3 (\$/(MW)^{2}h)	
1	X	X	X	X
2	X	X	X	X
3	X	X	X	X
4	X	X	X	X
5	0.7	0.6	0.9	8.0721
6	0.6	0.6	0.9	8.0060
7	X	X	X	X

8	X	X	X	X
9	X	X	X	X
10	0.5	0.6	0.7	8.4247

As it can be seen, in many cases the Nash equilibrium does not exist. This can put limitations to the social welfare estimation.

3. Linear Regression and Classification

In addition to the above 10 plans, 7 new additional plans were considered for the number of observation to be enough. Table 4.17 shows a 17 x 15 matrix representing the 17 plans with its 15 features.

Table 4.17: Matrix representation of the plans data

Plan	1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
1	0	0	0	0	0	0	0	0	4	0	1	0	0	2	0
2	0	0	0	0	0	0	0	0	1	0	1	0	0	2	0
3	0	0	0	0	0	0	0	0	2	0	1	0	0	2	0
4	0	0	0	0	0	0	0	0	1	0	1	0	0	1	1
5	0	0	0	0	0	1	0	0	1	0	0	0	0	1	1
6	1	0	0	0	0	0	0	0	1	0	0	0	0	1	1
7	0	0	0	0	0	1	0	0	0	0	1	0	0	3	0
8	0	0	0	0	0	1	0	0	1	0	2	0	0	3	0
9	0	0	0	0	0	0	0	0	4	0	2	1	0	3	0
10	0	0	0	1	0	0	0	0	4	0	2	1	0	3	0
11	1	0	0	0	1	0	0	0	0	0	0	0	0	2	0

12	1	0	0	0	0	0	0	0	0	0	2	0	0	2	0
13	0	0	0	1	0	0	0	0	0	0	2	2	0	2	0
14	2	0	0	0	0	0	2	0	0	0	0	0	0	2	0
15	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0
16	0	0	0	2	0	0	0	0	0	0	1	1	0	1	0
17	0	0	0	0	0	0	1	0	0	0	1	1	0	3	0

After performing PCA, nine eigenvectors corresponding to the highest eigenvalues are chosen as the components of the new system. Testing is done using leave-one-out method. At every iteration 16 plans are used to train the model and one plan is used for testing. The vector of dependent variables Y contains the corresponding Social Welfare of these plans.

Table 4.18 shows the real and predicted hourly declared social welfare along with the percentage of error.

Table 4.18: Predicted plans using linear regression

Plan	Predicted Social Welfare	Real Social Welfare	Error (%)
Test 1	88900	88182	0.81
Test 2	89970	88728	1.4
Test 3	91870	92955	1.17
Test 4	85080	92168	7.69
Test 5	91870	93798	2.05

Test 6	89420	80090	11.65
Test 7	82640	90595	8.78
Test 8	97820	91601	6.79
Test 9	94440	96882	2.52
Test 10	98060	88609	10.66
Test 11	90210	90160	0.05
Test 12	90660	97553	7.07
Test 13	86070	91503	5.94
Test 14	10483	94301	11.17
Test 15	88770	87195	1.81
Test 16	10095	90881	11.08
Test 17	89550	90754	1.33

The aim of obtaining the social welfare is deciding on the expansion that will result in the highest benefit for all parties. Thus what matters for our problem is the ranking of these plans in terms of social welfare or performance function more than knowing the exact expected social welfare/performance function. For that purpose we check the ability of the model to perform ranking prediction. The above plans are taken two at a time and the predicted ranking is checked. 137 possible pairs exist. 118 (86%) are predicted correctly which means a level of error of 14%. The least major error was the wrong ranking of plans with difference in social welfare equal to 100$ which is a very small difference. Four of the errors were in the range of hundreds. The largest error was a wrong ranking for two plans with a difference in social welfare of 13000$ which is a very big difference. This was the only error of this range. The other errors corresponded to a difference in social welfare in the range of 1000$.

For the classification we had to use a minimum of 40 plans to obtain a model with the ability to generalize. A threshold of 90000 was taken for the social welfare. The same way of representing the data used in linear regression is used here and PCA is also performed. In the case of classification using SVM, 85% were classified correctly and 14 plans out of 40 were used as support vectors. In the case of logistic regression, 77% were classified correctly.

4. Cost criteria vs Social welfare criteria

We emphasize in this section on the necessity to account for the resulting social welfare when choosing an expansion plan. First only the cost is considered as criteria for the expansion. An optimization problem is defined minimizing the cost subject to electrical constrained.

$$\min_{x_{km}} \sum_{km} C_{km} x_{km}$$
$$\text{s.t.}$$

Equations (3.10) to (3.16)

The problem was implemented using MOSEK and the obtained results showed that the lines to be expanded are 26 (2 lines), 46(2 lines), and 35 (1 line).

From the results of section 4.1, the performance function of the above expansion plan is 7.7104*10^9 (plan 3 in Table). The value of the highest performance function is that of plan 10, which is 8.4868*10^9. The loss resulting in choosing plan 3 instead of plan 10 is 7.764*10^8 $.

5. *Perfect competition vs Strategic behavior*

A perfect competition is when generators bids their marginal cost. In this case the social welfare will be higher since prices will be the lowest possible and thus the value of the elastic demand will be higher. However, this is not a realistic assumption since private generators will aim at maximizing their payoffs. For that target, they will choose the optimal bid higher than the marginal cost that will guarantee the best payoff. We conduct here a comparison between the decision on expansion in the case of perfect competition and in the case of strategic behavior. Results are shown in Table 4.19

Table 4.19: Performance function in case of perfect competition

Plan	SocialWelfare (10^9 $)	Cost (10^6$)	Performance function (10^9 $)
1	8.4828	200	8.2828
2	8.4540	110	8.3440
3	8.4707	140	8.3307
4	8.3936	141	8.2526
5	8.3728	141	8.2318
6	8.4911	161	8.3301
7	8.3252	130	8.1952
8	8.4085	180	8.2285

9	8.4774	298	8.1794
10	8.6675	318	8.3495

As it can be seen from the results, the best plan chosen while accounting for strategic behavior, its performance function is very close to the one when perfect competition is assumed. This shows that the best expansion is the expansion that diminish as much as possible the market power and the ability of generators to submit bids a lot higher than their marginal cost.

B. Correlated equilibria of the IEEE 24-bus system

The system has 24 nodes, 276 right of ways. It has 10 generators and 15 consumers. There are already 34 existing right of ways. Based on the literature we consider that the possible expansions are in these 34 right of ways in addition to ten new ones. A maximum of 4 lines can be added in each right of way.

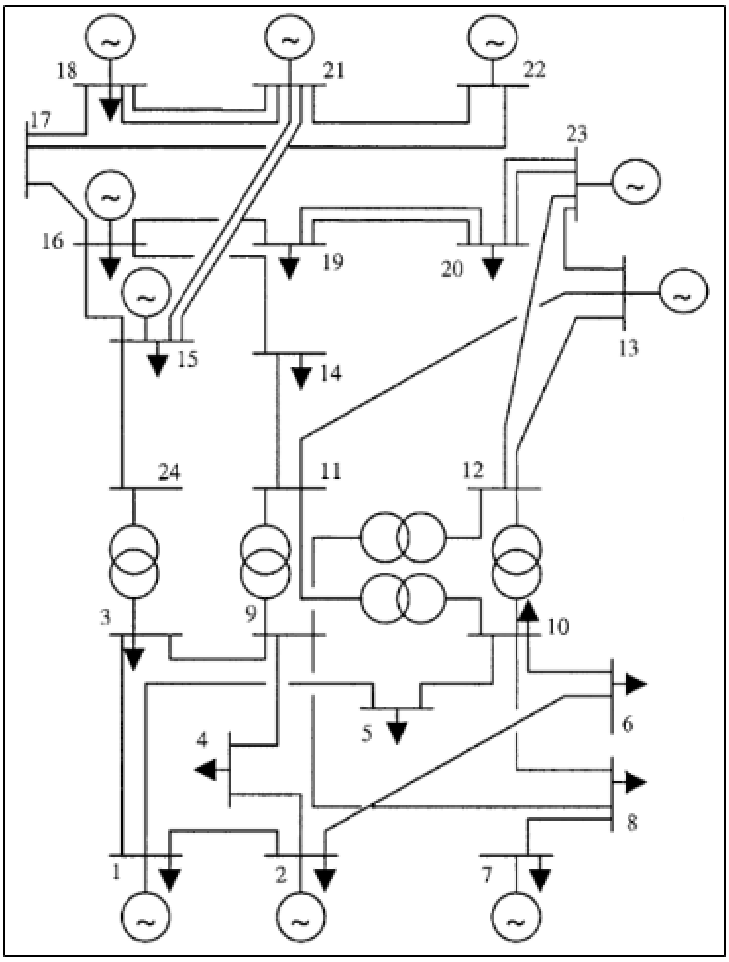

Figure 4: IEEE24 bus system

The transmission network data is given in Table 4.20.

Table 4.20: Transmission lines data of the IEEE24 bus system

Line	Reactance(p.u.)	Capacity(MW)	Line	Reactance(p.u.)	Capacity(MW)
1-2	0.0146	175	11-13	0.0488	500
1-3	0.2253	175	11-14	0.0426	500
1-5	0.0907	350	12-13	0.0488	500
2-4	0.1356	175	12-23	0.0985	500
2-6	0.205	175	13-23	0.0884	500
3-9	0.1271	175	14-16	0.0594	500
3-24	0.084	400	15-16	0.0172	500
4-9	0.111	175	15-21	0.0249	1000
5-10	0.094	350	15-24	0.0529	500
6-10	0.0642	175	16-17	0.0263	500
7-8	0.0652	350	16-19	0.0234	500
8-9	0.1762	175	17-18	0.0143	500
8-10	0.1762	175	17-22	0.1069	500
9-11	0.084	400	18-21	0.0132	1000
9-12	0.084	400	19-20	0.0203	1000
10-11	0.084	400	20-23	0.0112	1000
10-12	0.084	400	21-22	0.0692	500

Ten expansion plans are considered in Table 4.21. As said previously in the case of the Garver system these plans were the results of different algorithm and scenarios used to solve the TNEP.

Table 4.21: Proposed expansion plans for the IEEE24 bus system.

Plan	Corridors
1	1-5=1, 6-10=1, 7-8=2, 10-12=1, 11-13=1, 12-13=1, 14-16=1, 16-17=2, 17-18=1, 15-16=1
2	1-5 = 1, 6-10 = 1, 7-8 = 2
3	6-10=1, 7-8=1, 10-11=1, 11-13=1, 14-16=1, 16-17=2, 20-23=1

4	6-10=1, 7-8=1, 10-12=1, 14-16=1
5	6-10=1, 7-8=1, 10-12=1, 12-13=1, 14-16=1
6	3-24=1, 6-10=1, 8-9=1, 14-16=1
7	3-24=1, 6-10=1, 8-9=1, 14-16=1 , 16-17=1, 17-18=1
8	6-10=1, 7-8=1, 8-9=1, 14-16=1
9	6-10=1, 7-8=1, 14-16=1, 16-17=1
10	3-9=1, 6-8=1, 6-10=1, 7-8=1, 10-12=1, 14-16=1

The type of generators at each node of the system is given in Table 4.22 and in a more detailed manner in Table 4.23 where at each node the number and type of existing generators was included.

Table 4.22: Type of generators at each node

Type	Node
Nuclear	18 21
Coal/Stream	1 2 15 16 23
Oil/Stream	7 13 15
Hydro	22

Table 4.23: Detailed generators type at each node

Node	U12 oil	U20 Turbine	U50 Hydro	U76 Coal	U100 Oil	U155 Coal	U197 Oil	U350 Coal	U400 Nuclear
1		2		2					
2		2		2					
7					3				
13							3		
15	5					1			

16						1			
18									1
21									1
22			6						
23						2		1	

Table 4.24 presents the generators data by generator type. As it can be seen the coefficients of the different generators are not in the same range which represents a challenge for the generators with high production cost.

Table 4.24: Generators data by generator type

Type	Fuel	Cost coef. a ($/MWh)	Cost coef. b ($/(MW)^{2}h)
U12	#6 oil	56.5640	0.328412
U20	#2 oil	130	0
U50	Water	0.001	0
U76	Coal	16.0811	0.014142
U100	#6 oil	43.6615	0.052672
U155	Coal	12.3883	0.008342
U197	#6 oil	48.5804	0.00717
U350	Coal	11.8495	0.004895
U400	LWR/ Uranium	4.4231	0.000213

The hydro generator at node 22 is replaced by a nuclear generator to consider the ability of this generator to bid knowing that the variable costs of hydro generators are equal to zeros.

Cost function parameters of the power producers are considered to be the average of the parameters of all generators at the corresponding node, since we model each node with one generator only for simplicity. These parameters are shown in Table 4.25 and consumer's function parameters are shown in Table 4.26.

Table 4.25: Generators cost function parameters

Generator	a_k (\$/MWh)	b_k (\$/(MW)^{2}h)
G_1	73	0.003
G_2	73	0.003
G_7	44	0.020
G_{13}	48.58	0.003
G_{15}	34	0.050
G_{16}	12.39	0.003
G_{18}	4.42	0.000050
G_{21}	4.42	0.000050
G_{22}	4.42	0.000050
G_{23}	12.12	0.003

Table 4.26: Consumers cost function parameters

Load	d_k (\$/MWh)	s_k (\$/(MW)^{2}h)
L_1	110	0.52
L_2	150	0.65
L_3	120	0.73
L_4	200	0.8
L_5	140	0.76
L_6	100	0.53
L_7	130	0.64

L_8	160	0.74
L_9	155	0.81
L_{10}	135	0.75
L_{13}	125	0.77
L_{14}	125	0.54
L_{15}	110	0.62
L_{16}	150	0.76
L_{18}	125	0.79
L_{19}	140	0.65
L_{20}	145	0.52

The correlated equilibria for each of the 10 plans are presented in Table 4.27 to 4.36.

Table 4.27: Probability distribution on action tuples for plan 1

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.017	0.017	0.09	0.017	0.75	0.017	0.00075	0.00075	0.00075	0.017	0.002
2	0.016	0.016	0.085	0.016	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.002
3	0.011	0.011	0.055	0.01	0.1	0.004	0.0001	0.0001	0.0001	0.004	0.002
4	0.012	0.012	0.065	0.012	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.002
5	0.013	0.013	0.07	0.013	0.05	0.004	0.0001	0.0001	0.0001	0.004	0.495
6	0.012	0.012	0.065	0.012	0.05	0.005	0.00015	0.00015	0.00015	0.005	0.495

For plan 1 we see that two main action tuples can be chosen with approximately a probability of 0.5 each. Similar observations to that mentioned in the case of Garver system can be noticed in the case of IEEE 24 system. However we consider here more realistic size and capacities for the generating companies as we differentiate between generators type and used fuel. As a result different sets of possible actions appeared with different ranges for the actions. This is highlighted more through the results of the remaining plans.

Table 4.28: Probability distribution on action tuples for plan 2

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.017	0.017	0.09	0.017	0.75	0.017	0.00075	0.00075	0.00075	0.017	0.002
2	0.008	0.008	0.045	0.008	0.3	0.008	0.00005	0.00005	0.00005	0.003	0.002
3	0.011	0.011	0.06	0.011	0.45	0.011	0.00075	0.00075	0.00075	0.017	0.002
4	0.004	0.004	0.025	0.004	0.1	0.004	0.00005	0.00005	0.00005	0.003	0.499
5	0.013	0.013	0.07	0.013	0.55	0.013	0.00015	0.00015	0.00015	0.005	0.495

Also, plan 2 resulted in two action tuples with nonzero probability (equal to 0.5 each) according to Table 4.28. These tuples are different than these of plan 1 since every plan can give a different equilibrium point. But in fact, a similar combination of actions for some of the generators can be noticed: the combination [0.013 0.013 0.07 0.013] for generators 1 to 4 and the combination [0.00015 0.00015 0.00015 0.005] for generators 7 to 10 is repeated. A look into the results of plan 3 and 4 will show again the repetition of such combinations.

Table 4.29: Probability distribution on action tuples for plan 3

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.005	0.005	0.03	0.005	0.15	0.005	0.00005	0.00005	0.00005	0.003	0.0459
2	0.011	0.011	0.06	0.011	0.45	0.011	0.0001	0.0001	0.0001	0.004	0.0479
3	0.014	0.014	0.075	0.014	0.6	0.014	0.00005	0.00005	0.00005	0.003	0.1796
4	0.004	0.004	0.025	0.004	0.1	0.004	0.00005	0.00005	0.00005	0.003	0.1816
5	0.012	0.012	0.065	0.012	0.5	0.012	0.00015	0.00015	0.00015	0.005	0.1776
6	0.008	0.008	0.045	0.008	0.3	0.008	0.0001	0.0001	0.0001	0.004	0.0439
7	0.007	0.007	0.04	0.007	0.25	0.007	0.0001	0.0001	0.0001	0.004	0.1137

For plan 3, the 7 action tuples that occurred most frequently were included in Table 4.29. It can be seen, as mentioned in the case of the Garver system, that different number of scenarios can occur (ranging from 2 tuples for plan 1 in Table 4.27, to 8 tuples for plan 4 in Table 4.30).

Table 4.30: Probability distribution on action tuples for plan 4

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.003	0.003	0.02	0.003	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.1078
2	0.017	0.017	0.09	0.017	0.75	0.017	0.00075	0.00075	0.00075	0.017	0.002
3	0.013	0.013	0.07	0.013	0.55	0.013	0.00005	0.00005	0.00005	0.003	0.0020
4	0.004	0.004	0.025	0.004	0.1	0.004	0.00005	0.00005	0.00005	0.003	0.0020
5	0.011	0.011	0.06	0.011	0.45	0.011	0.00005	0.00005	0.00005	0.003	0.2615
6	0.003	0.003	0.02	0.003	0.05	0.003	0.0001	0.0001	0.0001	0.004	0.2595
7	0.008	0.008	0.045	0.008	0.3	0.008	0.00015	0.00015	0.00015	0.005	0.2595
8	0.008	0.008	0.045	0.008	0.3	0.008	0.0001	0.0001	0.0001	0.004	0.1058

Examining the first four plans shows that the companies are grouped into action groups where each group has the same action for a certain action tuple. For example generators 1, 2, and 4 and generators 7, 8, and 9. This can be observed in the other expansion plans too, and it is due to the size and capacity of each company.

Table 4.31: Probability distribution on action tuples for plan 5

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.016	0.016	0.085	0.016	0.7	0.016	0.00005	0.00005	0.00005	0.003	0.1637
2	0.016	0.016	0.085	0.016	0.7	0.016	0.00075	0.00075	0.00075	0.017	0.1657
3	0.007	0.007	0.04	0.007	0.25	0.007	0.00005	0.00005	0.00005	0.003	0.1657
4	0.006	0.006	0.035	0.006	0.2	0.006	0.00075	0.00075	0.00075	0.017	0.1657
5	0.006	0.006	0.035	0.006	0.2	0.006	0.00005	0.00005	0.00005	0.003	0.1637

6	0.007	0.007	0.04	0.007	0.25	0.007	0.0001	0.0001	0.0001	0.004	0.1617

Moreover, looking to action tuples 7 and 8 in plan 4, and action tuples (1, 2), (3, 6), and (4, 5) in plan 5. It can be seen that for all these couples the only difference is the combination of actions of generators 7 to 10.

Table 4.32: Probability distribution on action tuples for plan 6

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.003	0.003	0.02	0.003	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.002
2	0.017	0.017	0.09	0.017	0.75	0.017	0.00075	0.00075	0.00075	0.017	0.002
3	0.016	0.016	0.085	0.016	0.7	0.016	0.00005	0.00005	0.00005	0.003	0.002
4	0.003	0.003	0.02	0.003	0.05	0.003	0.0001	0.0001	0.0001	0.004	0.3313
5	0.012	0.012	0.065	0.012	0.5	0.012	0.00015	0.00015	0.00015	0.005	0.3313
6	0.017	0.017	0.09	0.017	0.75	0.017	0.00005	0.00005	0.00005	0.003	0.3313

In plan 6, observing the tuples 1, 3, and 6 shows that the only difference between these tuples is in the actions of generators 1 to 6 while the actions of generators 7 to 10 is the same.

Table 4.33: Probability distribution on action tuples for plan 7

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.003	0.003	0.02	0.003	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.2689
2	0.014	0.014	0.075	0.014	0.6	0.014	0.0001	0.0001	0.0001	0.004	0.2378
3	0.005	0.005	0.03	0.005	0.15	0.005	0.0001	0.0001	0.0001	0.004	0.0422
4	0.015	0.015	0.08	0.015	0.65	0.015	0.00005	0.00005	0.00005	0.003	0.0422
5	0.009	0.009	0.05	0.009	0.35	0.009	0.00005	0.00005	0.00005	0.003	0.0422

6	0.004	0.004	0.025	0.004	0.1	0.004	0.0001	0.0001	0.0001	0.004	0.0356
7	0.016	0.016	0.085	0.016	0.7	0.016	0.00005	0.00005	0.00005	0.003	0.0378

Table 4.34: Probability distribution on action tuples for plan 8

Tuple	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	Prob. Dist.
1	0.01	0.01	0.0550	0.01	0.4	0.01	0.00005	0.00005	0.00005	0.003	0.491
2	0.012	0.012	0.065	0.012	0.5	0.012	0.0001	0.0001	0.0001	0.004	0.491

As mentioned before, having two actions with the probability of 0.5 for each means that the correlated equilibrium is a combination of Nash equilibrium points. Taking action tuple 1 for plan 8 in Table 4.34, we can see that a Nash equilibrium point will not be always fair for the players or leading to a good payoff. As all the generators are bidding with very low prices and thus having a very low payoff. Such Nash equilibrium is not realistic.

Table 4.35: Probability distribution on action tuples for plan 9

Tuple	β1	β2	β3	β4	β5	β6	β7	β8	β9	β10	Prob. Dist.
1	0.005	0.005	0.03	0.005	0.15	0.005	0.0001	0.0001	0.0001	0.004	0.0719
2	0.014	0.014	0.075	0.014	0.6	0.014	0.00005	0.00005	0.00005	0.003	0.0878
3	0.006	0.006	0.035	0.006	0.2	0.006	0.00075	0.00075	0.00075	0.017	0.1776
4	0.008	0.008	0.045	0.008	0.3	0.008	0.00005	0.00005	0.00005	0.003	0.1776
5	0.014	0.014	0.075	0.014	0.6	0.014	0.00075	0.00075	0.00075	0.017	0.0878
6	0.008	0.008	0.045	0.008	0.3	0.008	0.0001	0.0001	0.0001	0.004	0.0778
7	0.005	0.005	0.03	0.005	0.15	0.005	0.00005	0.00005	0.00005	0.003	0.0778

Table 4.36: Probability distribution on action tuples for plan 10

Tuple	β1	β2	β3	β4	β5	β6	β7	β8	β9	β10	Prob.

											Dist.
1	0.003	0.003	0.02	0.003	0.05	0.003	0.00005	0.00005	0.00005	0.003	0.2475
2	0.016	0.016	0.085	0.016	0.7	0.016	0.00005	0.00005	0.00005	0.003	0.2435
3	0.007	0.007	0.04	0.007	0.25	0.007	0.00005	0.00005	0.00005	0.003	0.2435
4	0.006	0.006	0.035	0.006	0.2	0.006	0.00075	0.00075	0.00075	0.017	0.2435

Again we can see that occurring action tuples can be common between plans. Thus what is important here is the probability distribution. In addition, it is important to mention that the generation capacity of the first six generators is almost double or triple the capacity of the last four generators. Consequently, looking to the occurring action tuples we can see that the generators are divided into two groups depending on their generation capacity. The big picture that can be seen is small generators (7, 8, 9, and 10) reacting against large generators (1, 2,3,4,5, and 6). In fact a fix combination of actions of the first group occurs with a fix combination of actions in the second group.

This in fact shows the importance of taking the companies' size when deciding on an expansion plan

The resulting social welfare and performance function of the plans are shown in Table 4.37.

Table 4.37: Social welfare of each plan for the IEEE 24 bus system

Plan	SW(10^10$)	Cost (10^6$)	PF(10^10$)
1	1.1522	203.42	1.1319
2	1.1427	87.99	1.1339
3	1.1461	124.19	1.1337
4	1.1419	100.09	1.1319

5	1.1457	124.19	1.1333
6	1.1430	176.38	1.1254
7	1.1440	196.78	1.1243
8	1.1394	164.99	1.1229
9	1.1429	94.52	1.1334
10	1.1459	215.2	1.1244

That is the best expansion plan is plan 2. As it can be seen plan 1 is better in terms of social welfare. However Plan 2 is better in terms of performance function. Here comes the importance of the trade-off to prevent losses.

Regression-Ranking prediction:

We consider 20 plans. 157 (91%) out 171 pairs are predicted correctly. The least major error was the wrong ranking of plans with the difference in social welfare of 14$ which a negligible difference. The other errors were in the range of hundreds with the largest error corresponding to a difference in social welfare of 760$.

Perfect competition vs Strategic behavior

Plan 1 had the lowest difference between its Social welfare in the case of perfect competition and its social welfare in case of strategic behavior. It is the plan with the maximum social welfare that will minimize market as much as possible.

CHAPTER IV

CONCLUSION AND FUTURE WORK

In this work, we propose the application of the notion of correlated equilibrium to the competition in the power pool to decide on the best transmission network expansion plan. The CE is used to estimate the total social welfare of the pool. In The TNEP three decision making levels were considered for the set of proposed plans. At the upper level the system planner needs to decide when and where to expand the network. The criteria used in the decision is a performance function including a trade-off between the Social welfare that the expansion will result in and the cost of the expansion. In the power pool, power producers, constituting the second level, submit their bids to the independent system operator. The later perform a market clearance which is the third level in our game. The base of market clearance is maximizing the hourly declared social welfare. Thus a power producer will choose his bid while accounting for the bids of others and the clearance performed by the system operator. The correlated equilibrium of the bidding game between producers is used as the expected outcome of the game. Simulation results showed that that CE can be used to account for the strategic behavior of the generators and that it has low computational requirements when compared to Nash equilibrium (NE) as well as it always exits even when NE doesn't. The Garver six bus system and the IEEE24 bus system were used for numerical applications.

The above method is used when choosing from a determined set of proposed plans. In the case of a very large number of possible plans, the ability of a regression model to predict the ranking of plans in terms of social welfare is studied. And the ability of two classification method to reduce correctly the search space is investigated.

In addition a comparison between the Social welfare-Cost trade-off and the Cost only criteria. Results showed that dropping the resulting social welfare when deciding on the expansion will result in big losses.

As future work, different demand levels could be considered to study the influence of expansion and the efficiency of the CE in peak hours and off peak hours, what will give a more detailed picture on the state after the expansion.

In addition, using the proposed methodology with the AC power flow model instead of the DC model will result in more accurate estimation of the social welfare. Additional criteria can also be added like network security and detailing the expansion cost into operational, investment.... Also, transmission expansion can be considered along with generation expansion using this methodology

From an economical point of view, the size of each generation company can be taken into account. As the bidding of small companies is different from big companies' bidding. Small companies tends to submit their marginal cost while large companies submit bids higher than its marginal cost.

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