AMERICAN UNIVERSITY OF BEIRUT

IMPORTANCE OF LOWER-BOUND SHEAR STRENGTH IN THE ASSESSMENT OF THE PROBABILITY OF FAILURE OF SPATIALLY RANDOM CLAYEY SLOPES

by
ZEINA DAOUD FARAH

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AN ABSTRACT OF THE THESIS OF

Zeina Daoud Farah for Master of Engineering
Major: Geotechnical Engineering

Title: Importance of Lower-Bound Shear Strength in the Assessment of the Probability of Failure of Spatially Random Clayey Slopes

The stability of a soil slope is traditionally evaluated by adopting a deterministic approach that is based on a target global factor of safety that is calculated either through limit equilibrium methods or through finite element analyses.

Due to the uncertainties that affect the risk of slope failures, recent studies have attempted to solve slope stability problems using reliability theory. Recently, probabilistic geotechnical analyses in which nonlinear finite-element methods are combined with random field generation techniques have been adopted to quantify the effect of spatial variability in soil properties on the risk of failure of slopes. This approach is currently referred to in the literature as the Random Finite Element Method (RFEM).

In this study, a robust probabilistic slope stability analysis using the RFEM is conducted to investigate the effect of including a lower-bound shear strength in the probabilistic model describing the uncertainty in the undrained shear strength of clayey slopes. Another objective of this study is to investigate the sensitivity of the reliability of clayey slopes to the random field generated in the analysis.

Finally, the results obtained from the simulated analyses are used to recommend design factors of safety that would result in acceptable probabilities of failure for undrained clayey slopes.

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To my parents David and Randa Farah,

To my pretty little sister Mary-Rose

and to my loving fiancé Tony Kiwan

For their patience, love and support throughout my life…

CHAPTER 1

INTRODUCTION

A. Background

Slope stability analysis is an established discipline in geotechnical engineering that is continuously attracting interest at the levels of fundamental research and practice (Griffiths and Marquez, 2007). The stability of a soil slope is traditionally evaluated by adopting a deterministic approach that is based on a target global factor of safety (FS). FS is generally evaluated either through limit equilibrium methods (LEM) or through finite element analyses and it is calculated using nominal soil parameters’ values. Decisions regarding the stability of soil slopes are based on this nominal FS which may not reflect the actual risk level that is inherent in the slope design. The common practice adopts factors of safety that generally exceed 1.5 (Terzaghi and Peck 1948).

The uncertainties inherent in slope stability analyses are many and include the different dimensions of geometry and soil parameters. The deterministic approach relies on the global FS to account for the different sources of uncertainties and reduce the slopes’ risk of failure. Li and Lumb (1987) report that adopting the factor of safety as a measure of risk does not show consistency due to the fact that different slopes might show the same FS yet, present different risk levels depending on the variability of the soil properties.

The nominal FS may not reflect the actual stability situation presented in the real slope due to the spatial variability that could exist in the field (Jha and Ching, 2013).

Slope stability problems exhibit uncertainties on different parametrical and methodological dimensions. These uncertainties pertain to the soil heterogeneity even within the same layer but most importantly, they do exist on the level of the method itself. Deterministic approaches do not deal with the uncertainties of soil properties. On the other hand, probabilistic approaches are more amenable to a realistic assessment of the risk of failure by incorporating all sources of uncertainty which will affect the reliability of the slope.

Reliability analyses that targeted slope stability problems grabbed the attention of researchers in the past few decades. The main differences among the probabilistic methods adopted in the literature pertain to the assumptions, limitations, capability to handle complex problems, and mathematical complexity (Faour 2014). Several researchers have analyzed the effect of spatial variability in soil properties on the stability of the slopes. Li and Lumb (1987), Christian et al. (1994), Malkawi et al. (2000), El- Ramly et al. (2002), Low (2003), Babu and Mukesh (2004), Cho (2007), Cho (2010), and Wang et al. (2011) accounted for the effect of spatial variability by using different limit equilibirum methods (LEM) along with random field theory. Both the random field theory and Limit equilibrium methods were used to assess the reliability of soil slopes.

In the above publications, different approaches were adopted in incoporating spatial variability in soil properties in the analysis of slopes. For instance, Malkawi et al. (2000) and Low (2003) studied the effect of spatial variability by varying the coefficient of variation (COV) of the soil properties. Li and Lumb (1987), El- Ramly et al. (2002), Cho (2007), and Wang et al. (2011) adopted isotropic correlation structures to generate their random fields. On the other hand, Babu and Mukesh (2004) used random fields that are decribed by anisotropic correlation structure. They reported that the isotropic representation would induce an overestimation in the probabilities of failure of slopes. Chok et al. (2007) conducted reliability analyses by assuming that the undrained shear strength of clayey slopes (Su) is an uncertain parameter. They conducted stability analyses by varying the slope geometry (angle and height ratio of the slope), spatial correlation length (smooth and rough), and statistical parameters of the lognormal distribution characterizing Su.

From the finite element school perspective, a major drawback of the LEM is that it requires a priori assumption of the shape and location of the failure surface, which may be a misleading representation of the realistic failure case. It isn't until the late 90's that the idea of the finite element started to grab the attention of the researchers particularly in the area of slope stability. Griffiths and Lane (1999) introduced the finite element method in the slope stability analyses and showed its advantages over the traditional LEM. According to Griffiths and Lane (1999), nonlinear FE overcomes the limitations of LEM due the natural occurrence of the failure surface through the elements of the soil mass witnessing the weakest shear strength values that are incapable of sustaining the applied shear stresses. Hence, the need for preliminary assumptions about the failure surface is eliminated by FE. Moreover, the assumption about side forces in the LEM formulation is also eliminated by FE.

Given the advantages of the FE in analyzing the stability of soil slopes, it was naturally extended to what is referred to now as the Random Finite Element Method (RFEM) by which Griffiths and many other researchers pursued a more rigorous method of probabilistic geotechnical analysis. The RFEM combines nonlinear finite-element methods with random field generation techniques to quantify the effect of spatial variability in soil properties on the risk of failure of slopes. Some of these studies include the work of Griffiths and Fenton (2000), Griffiths and Fenton (2004), Griffiths et al. (2009a), Griffiths et al. (2009b), Griffiths et al. (2010a), Griffiths et al. (2010b) and Jha and Ching (2013). The approach captures the effect of soil spatial variability and fully accounts for spatial correlation and averaging. The advantages of RFEM over LEM are quite similar to those of FE in that it does not require a predetermination of the shape or the location of the failure plane.

The RFEM was incorporated by Fenton and Griffiths in a slope stability analysis software called Rslope2d. The Software models a single-layered soil system using anisotropic random fields and is capable of evaluating the probability of failure of a slope using Monte Carlo simulations within a Finite Element setting. The software divides the slope into a mesh of 8-node quadrilateral elements and computes the plane strain deformation of an elastic-plastic Von-Mises solid via the viscoplastic strain method. The elements are assigned with random soil properties based on a predetermined statistical distribution (Normal, Lognormal or Bounded). The probability of slope failure is then computed as a function of the assigned properties (Griffiths and Fenton, 2004).

The RFEM is capable of accounting for the soil spatial variability effect by taking into account the spatial correlation and averaging approaches. Thus, the Random Finite Element Method (RFEM) has been adopted to quantify the effect of spatial variability in soil properties on the risk of failure of slopes (Griffiths and Fenton 2000, Griffiths and Fenton 2004, Griffiths et al. 2007, Griffiths et al. 2009, Griffiths et al. 2010 and Jha and Ching 2013).

Griffiths (2000) studied the effect of the scale of fluctuation and the coefficient of variation of soil properties on the reliability of slopes. Griffiths and Fenton (2004) compared simple and advanced probabilistic approaches to study the effect of spatial variability and local averaging on the probability of failure of slope. Moreover, Griffiths (2009) built on the work done by Griffiths and Fenton (2004) and studied the effect of the inclination angle of the slope on the probability of failure. Additionally, Griffiths (2010) performed a comparison between limit equilibrium methods and the random finite element method to indicate the importance and superiority of the RFEM in assessing the stability of slopes.

In a recent study, Jha and Ching (2013) performed a robust and rigorous probabilistic slope stability analysis using the Random Finite Element Method to study the effect of slope geometry, mean and coefficient of variation of the soil parameters, and the scale of fluctuation on the probability of failure of undrained slopes. The authors conducted the study by collecting a database for 34 real undrained engineered slope cases. The paper aimed at quantifying the effect of spatial variability in the undrained shear strength of clays on the probability of failure of the slopes. An advanced model of spatial variability that takes into account vertical and horizontal spatial variability was adopted. The vertical scale of fluctuation in the undrained shear strength was back-calculated for each case in the database using the simplified method presented in Phoon and Kulhawy (1999). The horizontal scale of fluctuation in the undrained shear strength was assumed due to the lack of soil data (boreholes) needed to quantify the lateral spatial variability. One of the major contributions of the paper is a relationship between the mean and the coefficient of variation of the factor of safety from one hand and the slope geometry, mean and the coefficient of variation of the soil properties, and the scale of fluctuation in the undrained shear strength on the other hand.

In slope stability problems involving clayey slopes, the uncertainty in the undrained shear strength of the clay generally governs the probability of failure of the slope. In most of the published work where reliability analyses were conducted for undrained slopes, the uncertainty in the undrained shear strength was modeled using conventional lognormal distributions. It could be argued that for clays with relatively low sensitivities, the undrained shear strength will always have a minimum non-zero value that is represented by the fully remolded undrained shear strength of the same clay. For deposits of clay with typical high spatial variabilities (typical coefficients of variation between 0.3 and 0.5), the left hand tail of a conventional lognormal distribution is incapable of incorporating the presence of a non-zero lower-bound shear strength and may not constitute a realistic model of the uncertainty in the undrained shear strength.

A more realistic probability distribution with the capability of incorporating a physical lower-bound value of shear strength may result in more reliable estimates of the risk of failure of an undrained slope. Gilbert et al. (2005) and Najjar and Gilbert

(2009) adopted a lognormal distribution that is truncated at a lower-bound value to model the uncertainties in the factor of safety of a drained clayey slope and in the capacity of deep foundations in sands and clays, respectively. A truncated lognormal distribution is convenient because the parameters describing the distribution are the same as those of the nontruncated distribution with the addition of one extra parameter, the lower-bound value. In all of the above studies, it was found that the incorporation of a realistic, physical lower bound to model a geotechnical capacity or a factor of safety could result in significant reductions in the probability of failure of a geotechnical structure and would this increase the reliability of the design. The major objective of this thesis will be to investigate the effectiveness of introducing a lower-bound shear strength in slope reliability assessments involving the RFEM.

B. Objectives and approach of research

The major objective of this thesis is to investigate the incorporation of a physical lower-bound shear strength in the probability model describing the uncertainty in the undrained shear strength of a clayey slope. The lower bound will be represented by the remolded undrained shear strength of the clay, which is a well-known property that could be determined using information about the sensitivity of the clay. The basis for the existence of the lower bound shear strength is the fact that the remolded undrained shear strength of clay constitutes the minimum possible value of strength for that particular clay. The remolded strength is reflected in the sensitivity of the clay which generally varies from 1.5 to 4 in clays of low to average sensitivity.

The main goal of this thesis is to investigate the effect of including lower-bound shear strength on the design factor of safety that is required to yield a given target level of reliability (or probability of failure). The effect of the lower-bound will be showcased for different design problems, whereby the parameters that will be investigated are the COV of the undrained shear strength Su, horizontal and vertical correlation distances, and sensitivity of the clay. The results pertaining to the assumption of random fields will be compared to those of the more traditional "homogeneous" slopes cases to shed light on the importance of realistically modeling the spatial variability in the reliability-based design of slopes.

The main backbone of the thesis will be RFEM (the random finite element method); The Rslope2D software will be the tool adopted for the quantification of the probability of failure of the undrained slopes that are designed with different deterministic factors of safety with and without a lower bound undrained shear strength. The originality of this work centers on incorporating the lower bound shear strength within the random field. It is worth noting that no published work in the literature has addressed this aspect of the problem.

C. Organization of thesis

In chapter 2, a literature review of some of the recent works done in this field will be addressed. The literature review covers some of the work done on spatial variability in slope stability analyses that are based on traditional limit equilibrium methods and more advanced finite element methods. In chapter 3, a description of the tools used throughout this study is conducted; the RFEM software input parameters are discussed in addition to the assumptions made hereafter. In chapter 4, the reliability of a homogeneous vs. spatially variable clayey slope is assessed while highlighting the effect of the lower-bound undrained shear strength. In chapter 5, the sensitivity of the slope reliability to the random field of the undrained strength is investigated. Typical correlation length values were adopted in the analyses. In chapter 6, an attempt is made to recommend factors of safety that would result in target probabilities of failure for spatially random clayey slopes for the case where the COV of Su is equal to 0.5. Conclusions and contributions of this research are presented in chapter 7.

CHAPTER 2

LITERATURE REVIEW

A. Introduction

Most geotechnical analyses adopt a deterministic approach in analyzing the stability of soil slopes. However, this area of geotechnical engineering presents various types and levels of uncertainties. Many researchers invested in this area in order to realistically account for the uncertainties that affect the risk of failure of slopes. One of the main sources of uncertainty is the spatial variability of soil properties even within homogeneous layers as a result of depositional and post depositional processes (Lacasse and Nadim, 1996). Accordingly, numerous studies have been undertaken in recent years to develop a probabilistic slope stability analysis that deals with the uncertainties of soil properties in a systematic manner (Alonso, 1976; Vanmarcke, 1977b; Li and Lumb, 1987; Christian et al., 1994; Griffiths and Fenton, 2004).

In what follows is a literature review section that covers some of the work done on spatial variability in slope stability analyses that are based on traditional limit equilibrium methods (LEM) and finite element methods (FEM) which are used either in their simplistic or more advanced forms (Random Finite Element Method, RFEM).

B. Studies involving LEM with random field theory

Most traditional limit equilibrium methods (LEM) do not consider the effect of spatial variability; however, some researchers investigated the impact of spatial variability of soil properties on the slope analysis by combining the LEM with the random field theory. The theory of random fields (Varmarcke 1977a, 1977b, 1983) is a common approach for modeling the spatial variability of soil properties.

1. Li and Lumb (1987)

Li and Lumb (1987) conducted one of the earliest probabilistic slope stability analyses combining LEM with the random field theory. The Morgenstern and Price method (1965) was adopted in the slope stability analysis. Li and Lumb implemented the First Order Second Moment Method (FOSM) for computing and analyzing the reliability of slopes. The authors considered spatial variability in their study by defining an isotropic correlation structure. There results indicate that the probability of failure of the slope is sensitive to the scale of fluctuation. As result, they recommended that the reliability analyst must exert additional efforts in estimating the scale of fluctuation. Assuming perfect correlation in soil properties leads to an overestimation of the probabilities of failure. Nevertheless, Li and Lumb showed that the deterministic critical slip surface and the surface associated with the minimum reliability index are located closely to each other. Hence, the recommendations for the analysis were to start the analysis with an initial guess based on the deterministic critical slip surface. Then a deeper search should be conducted ending up with the critical slip surface having minimum reliability index.

2. Christian et al. (1994)

Christian et al. (1994) adopted the First Order Second Moment Method (FOSM) to evaluate the reliability index of slopes and they chose a well-known case history which is James Bay Embankments to perform the analysis on. Spatial variability and local averaging were both considered in the probabilistic analysis. Bishop's method of slices and the Morgenstern and Price method were used to evaluate the factor of safety of the dyke. The authors report difficulties in identifying the autocorrelation distance and bias error of the models used to predict the factor of safety. They state that even though bias constitutes a significant contributor to the overall uncertainty, it is often ignored from the analysis. The authors recommended that the engineers should carefully rely on their judgment when it comes to the bias contribution estimation.

3. Malkawi et al. (2000)

Malkawi et al. (2000) investigated the effect of the method used to perform the LEM analysis on the reliability of homogeneous and layered slopes. The methods used were Bishop, Ordinary Method of Slices (OMS), Janbu, and Spencer's methods. The reliability was assessed using the First Order Second Moment method (FOSM) and Monte Carlo Simulation method (MCSM). The authors accounted for spatial variability in their analysis and studied the effect of uncertainty of each soil property on the calculated factor of safety by varying the coefficient of variation of these properties. The major findings of the analysis of homogenous slopes were that both OMS and Bishop lead to the same reliability index regardless of the reliability method used. On the other hand, Janbu and Spencer's model exhibit some differences in the resulting reliability index. When Janbu's models are combined with the FOSM, a slight overestimation of the reliability index is witnessed whereas the MCSM is the method that overestimates the reliability index in the case of Spencer's model.

The authors also concluded that the reliability index is sensitive to the sample size of soil properties and that more than 700 samples should be used to perform the needed analysis. Consequently, they found that FOSM requires fewer calculations and computing time compared to MCSM. However, with the help of computers in data handling and speed, MCSM proved to be a powerful and an effective method for the probabilistic reliability analysis of slope analyses.

4. **E*l-Ramly et al. (2002)***

El- Ramly et al. (2002) conducted a practical probabilistic slope stability analysis based on Monte Carlo Simulation by developing a simple spreadsheet using the well-known software Microsoft Excel 97 and @Risk. The analysis is illustrated by analyzing the dykes of the James Bay hydroelectric project. The authors modeled the geometry, soil properties, stratigraphy, and slip surface in an excel spreadsheet. The Bishop method is used for the determination of the deterministic factor of safety. The uncertainties in input parameters are modeled statistically by representative probability distributions. The variances of the soil parameters are evaluated using judgment. The spatial variability of soil parameters was characterized by an isotropic autocorrelation distance assuming exponential autocovarience functions. Finally, the authors investigated the efficiency of the analysis by comparing the results by those obtained using First Order Second Moment Method (FOSM). The authors concluded that ignoring spatial variability of soil properties and assuming perfect correlation can significantly overestimate the probability of failure of slopes.

5. *Low (2003)*

Low (2003) implemented Spencer's method both deterministically and probabilistically in a spreadsheet platform. The author accomplished the search of the noncircular failure surface by using Spencer's method involving spatially correlated normal and lognormal variates. Then, he extended the use of the deterministic approach to the probabilistic analysis (FOSM) in order to calculate the reliability index without involving complex concepts. Both the probabilities of failures and probability density functions obtained showed a good agreement with those obtained by the Monte Carlo Simulation method.

6. *Babu and Mukesh (2004)*

Babu and Mukesh (2004) investigated the effect of spatial variation of soil strength on slope reliability for a simple cohesive soil slope. They defined the geometry, stratigraphy, and soil parameters of the slope. Moreover, they assumed an isotropic correlation structure. The authors calculated the factor of safety using Bishop's method. Next, they calculated the probability of failure of the slope by using the First Order

Second Moment Method. After that, they repeated the same procedure stated above but by assuming an anisotropic correlation structure by defining both vertical and horizontal correlation distances.

The authors concluded that not only the coefficient of variation of soil parameters and the correlation distance can affect the probability of failure of the slope, but also the mean factor of safety can affect the probability of failure. Additionally, the authors found that there is a significant need to include an anisotropic correlation structure in the probabilistic slope stability analysis. Performing reliability analysis by assuming that the correlation distance is the same in both horizontal and vertical directions leads to an overestimation of probability of failure of slopes.

7. Cho (2007)

Cho (2007) conducted a probabilistic slope stability analysis through a numerical procedure based on Monte Carlo Simulation (MCS) that considers the spatial variability of the soil properties based on local averaging. Hassan and Wolff (1999) concluded that the deterministic critical failure surface is not necessarily the failure surface with the highest probability of failure. Based on this finding, Cho (2007) adopted the First Order Reliability Method (FORM) to determine the critical probabilistic failure surface. Moreover, the author used FORM to identify the input parameters that have the greatest impact on the failure probability and Spencer's method to calculate the reliability index. The author concluded that the location of the critical probabilistic surface was somewhat different than the location of the critical deterministic surface. Furthermore, the probability of failure decreases with a decrease in the scale of fluctuation and vice versa. In addition to that, he deduced that the assumption of the isotropic field is conservative and the sensitivity of the unit weight is relatively small compared to those of cohesion and the angle of the internal friction. Finally, Cho (2007) found that in the case of small scale of fluctuation, a low probability of failure is obtained. Hence, more realizations are needed to conduct MCSM.

8. Cho (2010)

All the above case studies used the traditional LEM combined with the random field theory to calculate the probability of failure by taking spatial variability into consideration. This traditional analysis considers the influence of the random field along the predetermined critical surface. Cho (2010) proposed his method by using the Karhunen – Loeve Expansion Method that is independent of the division of slices in the sliding mass in order to be able to calculate the shear strength at any location along the trial slip surface. The author considered the strength reduction method in the calculation of Bishop's factor of safety. Conversely, he based his probabilistic analysis on a search algorithm that can find the surface with the minimum reliability index. The author illustrated his approach by analyzing a one layered with ø = 0.

Cho (2010) deduced that, the critical failure surface identified by the search algorithm always gives smaller factors of safety compared to that obtained from fixed critical surface. In contrast, the probability of failure that comprises all potential failure surfaces is greater than that obtained from the fixed critical surface and the relative difference between the two probabilities decreases when the autocorrelation distance increases.

9. Wang et al. (2011)

Wang et al. (2011) conducted a probabilistic slope stability analysis based on MCS by using Simulation Subset in order to improve the efficiency and resolution of the MCS. The analysis was implemented using a spreadsheet package that was used to explore the effect of spatial variability on the probability of failure of slopes. The methodology is illustrated through applying it to a cohesive slope and the deterministic factor of safety was calculated using the Ordinary Method of Slices. The results were validated by comparing the results with those obtained from other reliability methods. Wang et al. (2011) modeled the undrained shear strength by a lognormal random field and by an isotropic correlation structure using an exponential auto covariance function.

The authors found that if the spatial variability is ignored, the probability of failure is significantly overestimated particularly when the effective correlation length is smaller than the slope height. Moreover, they concluded that the variance of the factor of safety is overestimated when the spatial variability is ignored. This variance overestimation results in over conservative designs for cases where FS is taken to be greater than 1.0. Further, they deduced that when the spatial variability is considered, the critical slip surface varies spatially. Thus, the critical probabilistic surface should be investigated by conducting a search algorithm method to get the surface with the minimum reliability.

C. Studies Involving Random Finite Elements Method (RFEM)

All of the above studies have combined the limit equilibrium method (LEM) with the random field theory. However, the influence of the random field is only taken into account along the failure line and is therefore one-dimensional. Thus, to overcome this limitation, some investigators pursued a more rigorous method of probabilistic geotechnical analysis in which nonlinear finite- element methods are combined with random field generation techniques. This method is called Random Finite Element Method (RFEM). It captures the effect of soil spatial variability well where it fully accounts for spatial correlation and averaging. It is also a powerful slope stability analysis tool that does not require priori assumptions related to the shape or location of the failure mechanism. The following studies presented the probabilistic slope stability analysis based on Random Finite Element Method.

1. Griffiths and Lane (1999)

In this paper, the authors studied several examples of slope stability problems using the finite elements method to be compared against other solution methods. The paper included illustrations of the failure surface which indicate that the failure surface occurred naturally within the soil zones where the shear strength mobilized didn't sufficiently sustain the shear stresses applied. This aspect of the problem was investigated through the analysis of an undrained clay slope with a thin weak layer. The ratio of the layers' undrained shear strengths was varied. The FE analysis encountered a discontinuity in the failure surface whereby both a circular and non-circular mechanisms were captured. Analyzing the same problem using LEM wouldn't have allowed such a response (free occurrence of the failure surface) to be modeled realistically. The FS in the LEM methods was always found to be greater or equal to the FS resulting from FE but never less. This illustrates the FE advantages over LEM.

Finally, the authors stated that FE in conjunction with an elastic-perfectly plastic (Moh-Coulomb) stress-strain method has been shown to be a reliable and robust method for assessing the FS.

2. Griffiths and Fenton (2000)

Griffiths and Fenton (2000) conducted a Random Finite Element probabilistic analysis for studying the effect of the spatial correlation length on the probability of failure of the slope. This paper also involved a parametric study to highlight the influence of the scale of fluctuation and coefficient of variation of the shear strength on the stability of the slope. Griffiths and Fenton analyzed an undrained clay slope.

The results showed that the probability of failure increases as the coefficient of variation of the undrained shear strength increases. For coefficients of variation in the range of 0< COV< 0.5, an increase in the probability of failure resulted as the ratio of the correlation length to the slope height increased. However, at COV values that were higher than 0.5, the probability of failure decreased with the increase in the ratio of the correlation length to the slope height. The assumption of perfect correlation was shown to overestimates the probability of failure for low values of the coefficient of variation and for slopes with high factor of safety (FS > 1.4); however, it underestimates the probability of failure for high values of the coefficient of variation and for slopes with low factors of safety (FS < 1.40).

3. Griffiths and Fenton (2004)

In this paper, Griffiths and Fenton investigated the probability of failure of a cohesive slope using both the simple and more advanced probabilistic approaches under undrained conditions.

In the simple probabilistic approaches, the authors treated the undrained shear strength of the cohesive slope as a simple random variable and both spatial correlation and local averaging are ignored. The probability of failure in this simple methodology was estimated as the probability that the shear strength would fall below a critical value (associated with FS=1) based on a log-normal probability distribution. As expected, relatively larger probabilities of failure (p_f) were recorded for cases involving lower factors of safety. The FS considered was based on the value that would have been obtained if the same slope was analyzed deterministically with shear strength equal to the mean value considered in the probabilistic approach.

The results of the simple approach led to high probabilities of failure with a mean factor of safety = 1.47 which practically contradicted the common knowledge about slopes with FS=1.47 which rarely fail. To overcome this problem, the authors proposed two factorization methods that were used to reduce the mean value of the undrained shear strength. Hence, an increase in the strength reduction factor reduces the probability of failure to an acceptable value.

In the single random variable approach whereby simple probabilistic methods are applied, only homogeneous slopes are considered. That is, the correlation length assigned to the undrained shear strength is infinite. However, a more realistic model was analyzed in this paper in which the soil strength varies spatially within the slope in a correlated manner. A logical correlation length model was adopted which is the Markovian, an exponentially decaying correlation function:

$$\rho = e^{-(2\tau/\theta_{\ln C})} \qquad \text{(Eq. 2.1)}$$

where ρ=correlation coefficient

τ = absolute distance between two points in a random field

$\theta_{\ln C}$ = spatial correlation length

The correlation model assumes that soil samples that are close are more likely to be correlated than distant samples. Since field observations show that soil samples are correlated in the horizontal direction for much longer distances than that in the vertical direction, modeling the anisotropy in RFEM is important. The anisotropy in the soil properties is generally a result of the geological depositional environment that characterizes the structure and composition of the soil.

Griffiths and Fenton analyzed the same slope using the Random Finite-Element Method (RFEM). They combined elastoplastic finite-element analysis with random field theory using the local average subdivision method (Fenton and Vanmarcke 1990). The findings of the paper highlight the role of spatial averaging in enhancement the reliability of the slope. Spatial averaging was shown to reduce the mean and the variance of a log-normal point distribution, while preserving the median of the point distribution. Also, they concluded that the variance tends to zero and the mean tends to the median if significant levels of local averaging are applied.

Griffiths and Fenton begin and end their paper with the idea of RFEM overcoming LEM by the free occurrence of the failure surface by shape and location. Besides, the failure mechanism passes inevitably through elements of different strength values. Moreover, the finite element method (FE) seeks the weakest path along with the strength averaging stresses on the natural occurrence of the failure surface. If local averaging is included in the traditional methods, it has to be computed over a predetermined failure mechanism characteristic of the analysis method. The RFEM results show that smaller probabilities of failure were recorded when the spatial correlation concept alongside local averaging was adopted.

4. Chok et al. (2007)

In this study, the authors adopted the RFEM to conduct a parametric study on the reliability of a cohesive slope. The effect of spatial variability was modeled using the coefficient of variability (COV) and scale of fluctuation (ϴ) was the focus of this study. Various slope angles and depth factors were considered in the analysis which was conducted using Monte Carlo simulations allowing the probabilities of failure to be estimated for each case. The deterministic FS was also estimated based on the mean values of the undrained shear strength. The results indicated that the deterministic Factor of Safety (FS) is a poor indicator of the stability of a slope since high P_f could be recorded for slopes with high FS and certain COV and ϴ/H values. In addition, the more correlated the field, the higher p_f will be recorded which was explained by the fact that a more correlated field coupled with relatively high COV, lead to lower strength values which govern the reliability.

Moreover, the results show that the influence of ϴ/H on P_f is more significant for slopes with marginally stable slopes (FS closer to 1). Also, the probabilities of failure can either increase or decrease by approximately 50% as ϴ/H increases from 0.1 to 10. Whenever a perfectly correlated field is assumed i.e. spatial correlation is ignored, a slope stability analysis could overestimate or underestimate the probability of failure. Hence, any stability analysis shall consider spatial variability in the analysis.

5. Griffiths et. al (2009a)

In this study, the authors investigated the advantage of Random Finite Element Method (RFEM) over the traditional probabilistic method (FORM or MCS). The core of the paper aimed at investigating the influence of the spatial correlation length, local averaging and coefficient of variation of the strength parameters on the probability of failure of the slope. The results showed that simplified probabilistic analyses in which spatial variability of soil properties is not properly accounted for, can lead to unconservative estimates of the probability of failure if the coefficient of variation of the shear strength parameters exceeds a critical value (υ_{crit}). The authors also found that this value is influenced by slope inclination, mean factor of safety, and correlation between strength parameters. υ_{crit} was found to be lower for steeper slopes associated with low factors of safety than less steep slopes associated with higher factors of safety.

6. Griffiths et.al (2009b)

Griffiths et al. (2009) investigated the failure probability of 2-D and 3-D slopes using RFEM. Although it was commonly known that 2-D analyses constitute a conservative estimate of the factor of safety compared to 3-D slope analyses, this study revealed the contrary. In the deterministic analysis, factors of safety computed from the 2-D analyses lead to unconservative values and this was linked to the implicit assumption made in 2-D models about the third direction whereby infinite correlation is assumed to persist in this out-of plane direction. However, additional support is actually provided in 3-D analyses by the boundaries in the out of plane direction.

In the probabilistic analyses, results indicate that the 2-D analysis also ceased to be conservative when the out-of-plane direction was longer. The threshold length to slope height ratio $(L/H)_{3>2}$ above which 2-D would be unconservative was investigated together with the effect of the boundary condition in the out-of-plane direction (3-D analysis). Results indicate that rough boundaries resulted in higher $(L/H)_{3>2}$ ratios compared to smooth boundaries. The lowest value for this ratio was observed to be about 3. This value may be used as a conservative upper limit for the real cross-over length above which 2-D results would stop being conservative and eventually reliable.

It was also found that in slopes that are associated with higher length to height ratios, a maximum value of the probability of failure was observed when a worst case correlation length is assumed in the analysis. Conclusively, in the absence of good quality site-specific data, worst case spatial correlation length should be assumed to avoid unconservative analyses.

7. Griffiths et al. (2010)

The authors investigate the use of the Point Estimate Method (PEM), First Order Second Moment Method (FOSM), First Order Reliability Method (FORM), and Monte Carlo Simulations in slope problems involving spatial variability. These methods are used in combination with Limit Equilibrium Method (LEM) for studying the reliability of slopes. The authors performed the study by means of analyzing a hypothetical slope that has already been analyzed by other authors. The authors showed that LEM combined with 1D random field can give lower probabilities of failure than the RFEM and this is due to the fact that RFEM doesn’t require a priori assumptions for the shape or location of the failure mechanism. Moreover, the failure mechanism can freely occur through the weakest path mobilized in the random soil which overcomes the LEM analysis, whereby the location of the failure surface is fixed before the performance of the random field analysis.

8. Jha and Ching (2013)

Jha and Ching (2013) used RFEM to analyze the effect of slope geometry, mean and coefficient of variation of the soil parameters, and the scale of fluctuation on the probability of failure. The authors statistically characterized the undrained shear strength of the soil layers by a random variable with a mean, coefficient of variation and lognormal distribution. The authors included spatial variability in their analysis in both in the vertical and horizontal directions. The vertical scale of fluctuation was back-calculated from a database of slope cases using the approach presented in Phoon and

Kulhawy (1999). As for the horizontal scale of fluctuation, it was assumed because it can't be calculated and it is usually greater than the vertical one.

The authors suggested equations that allow the designer to compute the statistical parameters of the factor of safety based on the shear strength and slope characteristics. They concluded that the ratio of the horizontal to the vertical scales of fluctuation has a slight effect on the mean and the coefficient of variation of the factor of safety. The authors' analysis showed that the mean factor of safety is always less than the deterministic one. It turned out that the difference between the mean and design FS is independent of the vertical and the horizontal scales of fluctuation but dependent on the coefficient of variation of the undrained shear strength. Whenever this COV is large, the reduction in the mean is more pronounced. On the other hand, the COV of the factor of safety was found to be always less than the coefficient of variation of the random field. Whenever the COV is large, the reductions in both the mean and the COV were more pronounced. Also, the reduction in the COV was more explicit when the ratio of the vertical scale of fluctuation to the length of the failure surface was small. The study ended up by a simplified equation to calculate the probability of failure for the undrained engineered slopes considering spatially variable shear strengths.

9. Huang et al. (2013)

Huang et al. (2013) performed three different types of analyses in their paper. First, they conducted a deterministic analysis in order to investigate the failure zones. Second, they conducted probabilistic analyses using Monte Carlo Simulation to investigate the probability density function of the factor of safety. Finally, they used the random finite element method to investigate the influence of spatial variability on the probability of failure of the slope. The authors analyzed a hypothetical slope analyzed by other investigators (Ching et al. 2009 and Low et al. 2011). They dealt with the reliability aspect of the problem as a system in which all potential slip surfaces are considered. The results showed that the probability of failure obtained by FEM is higher than that obtained by LEM and the probability of failure decreases with increasing spatial correlation length.

CHAPTER 3

RFEM SOFTWARE

A. Background

The random finite element method (RFEM) combines random field theory with finite element modeling in slope stability analysis and design. Since its inception by Fenton and Griffiths in 1992, the RFEM has been used by many researchers as a basic approach to incorporate spatial variability of soil properties in different geotechnical engineering applications (slope stability, foundations design, seepage through soils, etc.). Several software packages have incorporated the RFEM. Examples include “mrbear2d” that covers stochastic bearing capacity analysis for shallow foundation, “mrpill2d” that describes stochastic pillar analysis and “mrslope2d” that covers stochastic slope stability analysis.

Rslope2d is the software used in this study to assess the probability of failure of undrained slopes. The Software was developed by Griffiths and Fenton in 2011 and was subsequently updated to the latest version of 1.1.2 in 2012. The software is based on Monte Carlo simulations and takes into account different soil properties as input parameters. In each simulation, the mesh elements are assigned a property from the probability distribution that is selected at the input phase. After running the needed number of Monte Carlo simulations, the software yields the corresponding mean of the probability of failure and its standard deviation.

B. Input Parameters

The main input parameters required by the Rslope2d software are described below according to the definitions set by Griffiths and Fenton in their software guide. The corresponding assumptions made about the soil parameters and the soil geometry are stated and explained as follows.

1. Embankment Gradient

The software works on dividing the soil slope into finite elements where each finite element mesh is associated with a slope of gradient = run/rise (horizontal run / vertical fall). In this paper, two slope cases are considered: one with an angle of 26.6^o (run/rise=2:1), and the other with an angle of 45^o (run/rise=1:1).

2. Number of Finite Elements

The total number of elements in the x-directions is nx1+nx2+ny1*ngrad (ngrad being the horizontal projection of the slope) and the total number of elements in the y-direction is ny1+ny2 as it is shown in Fig. 3.1. The right and left edges are free to move vertically but constrained against horizontal movement, while the bottom is restrained in both directions. The maximum number of elements is 3620.

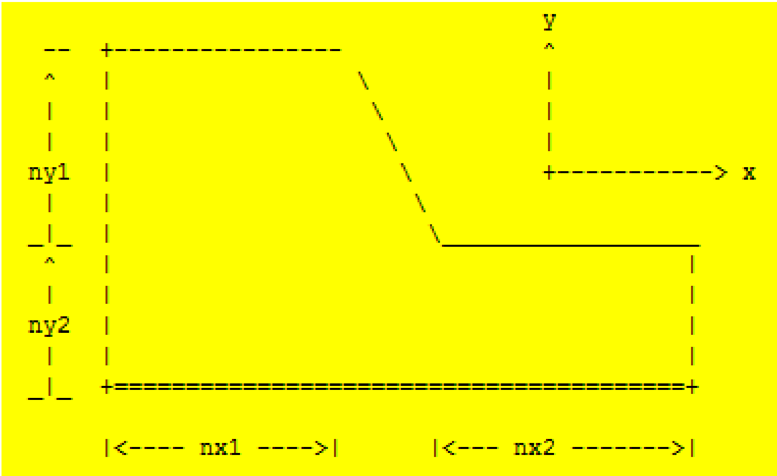

Figure 3.1: Total Number of Elements of the Slope – Taken from RSlope2d Help Manual.

3. Finite Element Size

A mesh is generated for the slope using 8-noded quadrilateral elements of equal size. The majority of the elements are square shaped and those adjacent to the slope are generated into triangles.

4. Number of Realizations

Rslope2d analyzes slope stability problems several times using Monte Carlo simulations. Each realization in the Monte Carlo process involves the same characteristics of the soil properties; however the spatial distribution varies from one realization to another. A sufficient number of realizations are required for the results to be accurate. The probability of failure is taken to be the number of failures/ total number of realizations assigned. The number of realizations *N* that are required for a target failure probability of 1% with a given degree of confidence is calculated using slope2d as described in Eq. (3.1):

$$N = \frac{z^2}{E\hat{}2} * \left(\frac{1-p}{p}\right) \qquad \text{(Eq. 3.1)}$$

Where z=1.96 for 95% confidence, E is the desired precision in the estimate of p which is the target probability, and 1-p is the target reliability. For example, the number of simulations required to estimate a 1% probability of failure with a 10% desired precision is:

$$N = \frac{1.96^2}{(10\%)\hat{}2} * \left(\frac{1-0.01}{0.01}\right) = 38032 \text{ simulations}$$

Since the reliability analyses that were conducted in this study involved different magnitudes of the probability of failure, and since the probability of failure was not known in advance, a preliminary analysis was conducted in Rslope2d using 1000 simulations to estimate the probability of failure. This preliminary estimate of the probability of failure was used as input to Equation 3.1 to estimate a more accurate number of simulations. Whenever the probability of failure is very low, even a first guess of 10000 simulations might result in a zero probability of failure. At this stage, 200000 simulations are assumed to guarantee a reasonable result.

5. Correlation Length

In random fields, the correlation length generally resembles the distance between two points over which the soil property could be assumed to be correlated. In the geotechnical literature, the lognormal distribution is typically adopted to characterize the uncertainty in soil properties (ex. undrained strength S_u). Since the logarithm of the lognormal distribution yields a normal (Gaussian) distribution, it is commonly assumed that the spatial correlation length is defined by the correlation distance for $\ln S_u$ and not for S_u. The spatial correlation length ($\theta_{\ln Su}$) represents the distance where the spatially random values of Su will tend to be significantly correlated in the normal field. In other words, a large value of $\theta_{\ln Su}$ implies smooth variation across the field, while a small value shows a ragged field of minimum correlations. For example, if the correlation length is 10 m, any two points within a distance of less than 10 m will have correlated soil properties, while any two points distant by more than 10 m would have statistically independent soil properties. Hence, neighboring elements are correlated to each other with varying degrees according to the spatial correlation lengths (Chok et al., 2007).

According to Griffiths and Fenton (2004), the magnitude of the spatial correlation length in the Gaussian field doesn’t differ much from that in the real space of Su. As a result, $\theta_{\ln Su}$ and θ_{Su} are interchangeable given their inherited uncertainty.

The random variables that constitute the finite element mesh in the RFEM software are produced using local averaging subdivision and are correlated with each other according to the Markovian correlation function which is exponentially decaying (Griffiths and Fenton 2004) as indicated in Eq. (3.2):

$$\rho = e^{-2\tau/\theta_{\ln Su}} \qquad \text{(Eq. 3.2)}$$

Where ρ is the correlation coefficient of the properties assigned between two points and τ is the absolute distance between two points in a random field. As mentioned by Griffiths and Fenton (2004), Eq. (3.2) would yield a value of 0.135 if computed between two points that are distant by θ_{lnSu}.

Fields with large correlation lengths are considered smooth, unlike fields with small correlation lengths which are considered to be erratic and unpredictable. The correlation length may differ in the two orthogonal directions (horizontal and vertical), but the correlation length in the horizontal direction is usually larger since soil is layered horizontally. This means that the soil properties will be smoother and more slowly varying horizontally.

For slope stability analyses, random finite element analyses indicate that the probability of failure of an undrained slope is generally governed by the normalized vertical correlation distance Θ_y, which is defined as the vertical correlation distance (δ_y) divided by the height of slope (H):

$$\Theta_y = \frac{\delta_y}{\mathrm{H}}. \qquad \text{(Eq. 3.3)}$$

It is worth noting that in all of the analyses conducted in this thesis using the RFEM, a slope height (H) of 20 m was adopted.

Fig. 4.2 shows two examples of isotropic random fields of S_u that are generated by RSlope2d for non-failing simulations of the Monte Carlo process for (a) Θ_{Su} = 0.25 (δ_y = 5m), and (b) Θ_{Su}=2 (δ_y = 40 m). Light and dark regions depict strong and weak soil properties respectively. The case analyzed corresponds to a 2:1 slope with an FS=1.5 (mean of S_u=103.5kPa and COV=0.5). Fig. 3.2(a) shows rapidly varying light and dark regions which reflect the small correlation length assigned to the slope. The higher the correlation length, the smoother becomes the change in the strength assigned for the elements (Fig. 3.2b).

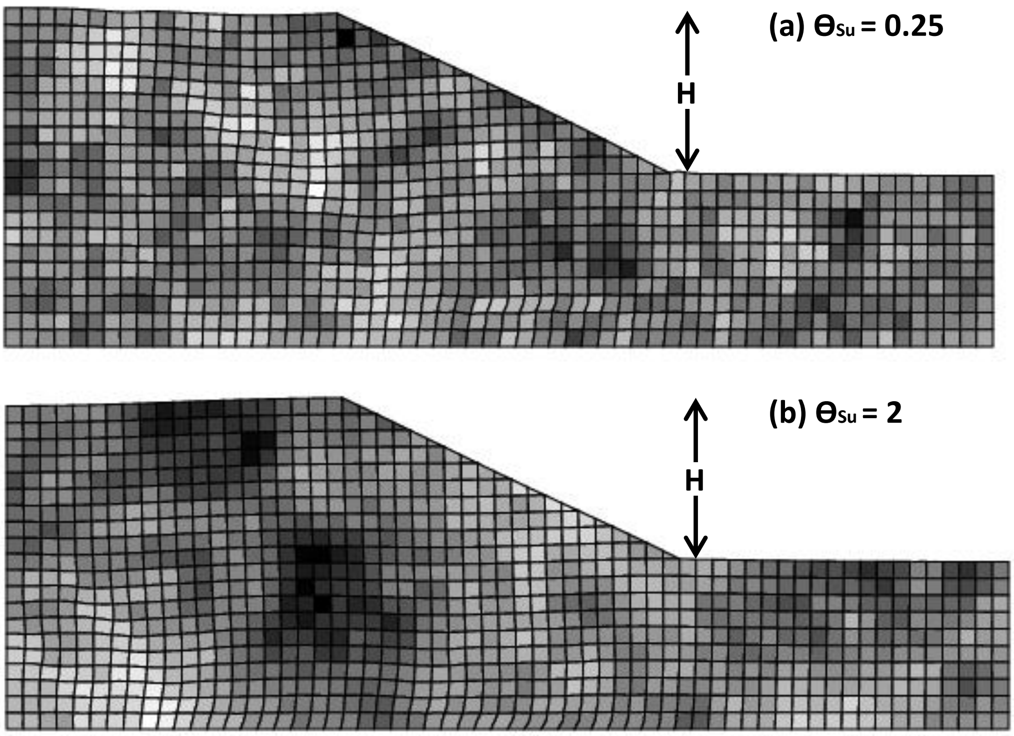

Figure 3.2: Random Fields Generated for FS=1.5, COV=0.5 and (a) Θ_{Su} = 0.2, and (b) Θ_{Su} = 2

Since soils tend to exhibit anisotropy, the spatial correlation lengths for Su will vary with direction, whereby longer correlation lengths are generally exhibited in the lateral/horizontal direction compared to the vertical direction. Anisotropy in soil properties is generally attributed to the depositional process and post depositional histories that characterize the formation of soils. To account for anisotropy in the RFEM, the following anisotropic autocorrelation model is generally adopted (Jha and Ching 2013):

$$\rho(\Delta x, \Delta z) = \exp\left(-\frac{2|\Delta x|}{\delta_x} - \frac{2|\Delta y|}{\delta_y}\right) \qquad \text{(Eq. 3.4)}$$

Where δ_x and δ_y are the horizontal and vertical correlation lengths respectively, and Δx and Δy are the horizontal and vertical distances between two points in space. Fig. 3.3 shows the same cases analyzed in Fig. 3.2 for (a) $\Theta_{Su} = 0.25$ (isotropic case) and for (b) $\Theta_y = 0.25$ and $\delta_x = 40$ m (anisotropic case). Clear differences are evident in the two random fields as a result of anisotropy. For the isotopic structure with a normalized correlation distance of $\Theta_y = 0.25$, the weak and strong zones vary rapidly along the vertical and horizontal directions (Fig. 3.3 (a)). For the anisotropic case with a horizontal correlation distance of 40m, the random field varies smoothly in the horizontal direction due to the higher correlation length adopted.

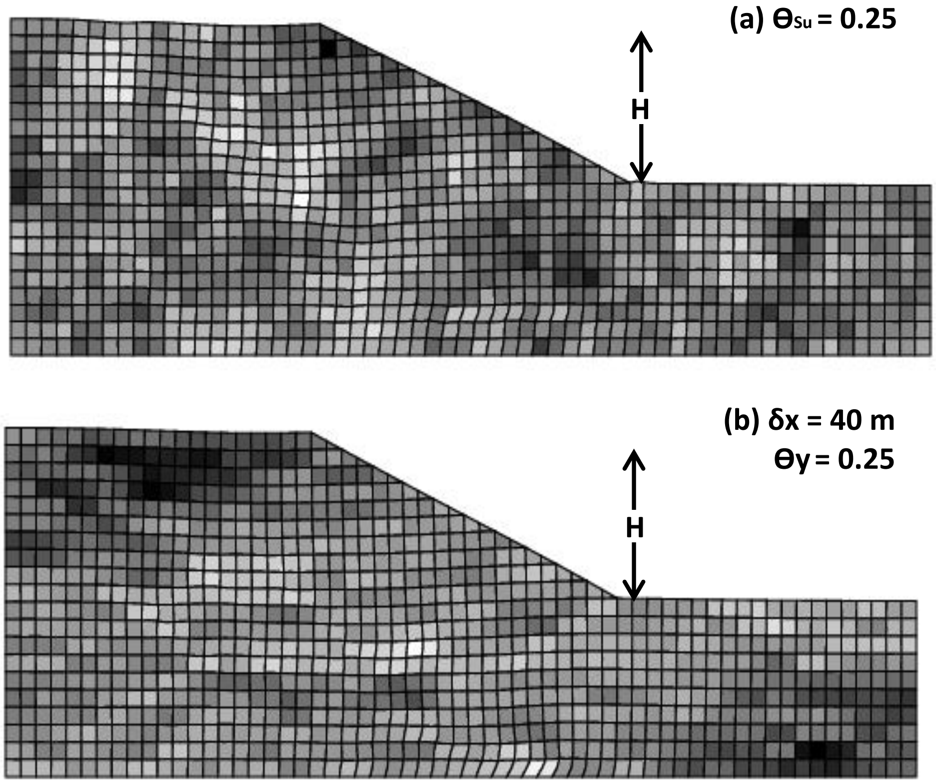

Figure 3.3: Random Fields Generated for FS=1.5, COV=0.5 and (a) Θ_{Su} = 0.25, (b) Θ_y = 2 and δ_x=40 m

6. Deterministic Strength Reduction Factors

Slope2d allows for determining the traditional factor of safety of the clayey slope and the probability of failure of the slope. The deterministic factor of safety (FS) is determined by the strength reduction method (Griffiths and Lane 1999) where the shear strength of all the elements is divided by a single strength reduction factor. Thus FS corresponds to the strength reduction factor causing the failure to occur. For the calculation of the probability of failure, Monte Carlo analyses are performed by simulating the soil properties from their associated random fields in the finite element mesh. Slope2d records all realizations where slope failures occur. The probability of failure is then estimated through the ratio of the number of failed simulations to the total number of simulations.

The slope stability analyses use an elastic-perfectly plastic stress-strain theory while adopting the Tresca failure criterion which is appropriate for undrained clayey slopes according to Griffiths and Fenton (2004). Whenever the Tresca failure criterion is violated, the attempt is to redistribute excess stress to neighboring elements that still have some reserve of strength. This is an iterative process that is repeated continuously until the Tresca criterion and global equilibrium are satisfied at all points within the mesh (Griffiths and Fenton, 2008). Griffiths and Fenton (2004) showed that 500 iterations are sufficient for the failure to occur for their analyzed slope. The same slope is adopted for the part of the analyses conducted in this study. For these cases, the same assumptions could be assumed to remain valid.

To illustrate the different failure modes that could exist in the RFEM, three realizations of failed slopes are presented in Fig. 3.4 assuming a FS=1.3 and COVSu=0.5. The failure cases correspond to different assumptions of the spatial correlation structure including (a) $\Theta_{Su} = 5$ (δ = 100 m), (b) $\Theta_y = 2$ and δ_x=100 m and (c) $\Theta_{Su} = 0.1$ (δ = 2m). The failure mechanism for relatively small correlation lengths (Fig. 3.4 (c)) lead to a preferred failure mechanism that tends to be global which means that it occupies wide zones of the soil field. As the spatial correlation length is increased in Fig. 3.4 (a), the preferred mechanism is attracted to “local pockets” of the weak soil and as a result occurs on a reduced width. As for Fig. 3.4 (b) that combines the horizontal and vertical correlation lengths from Fig.3.4 (a) and (c) respectively, it shows a complex failure mechanism that does not have a common shape; however, it covers a wide area while still having local failure mechanisms.

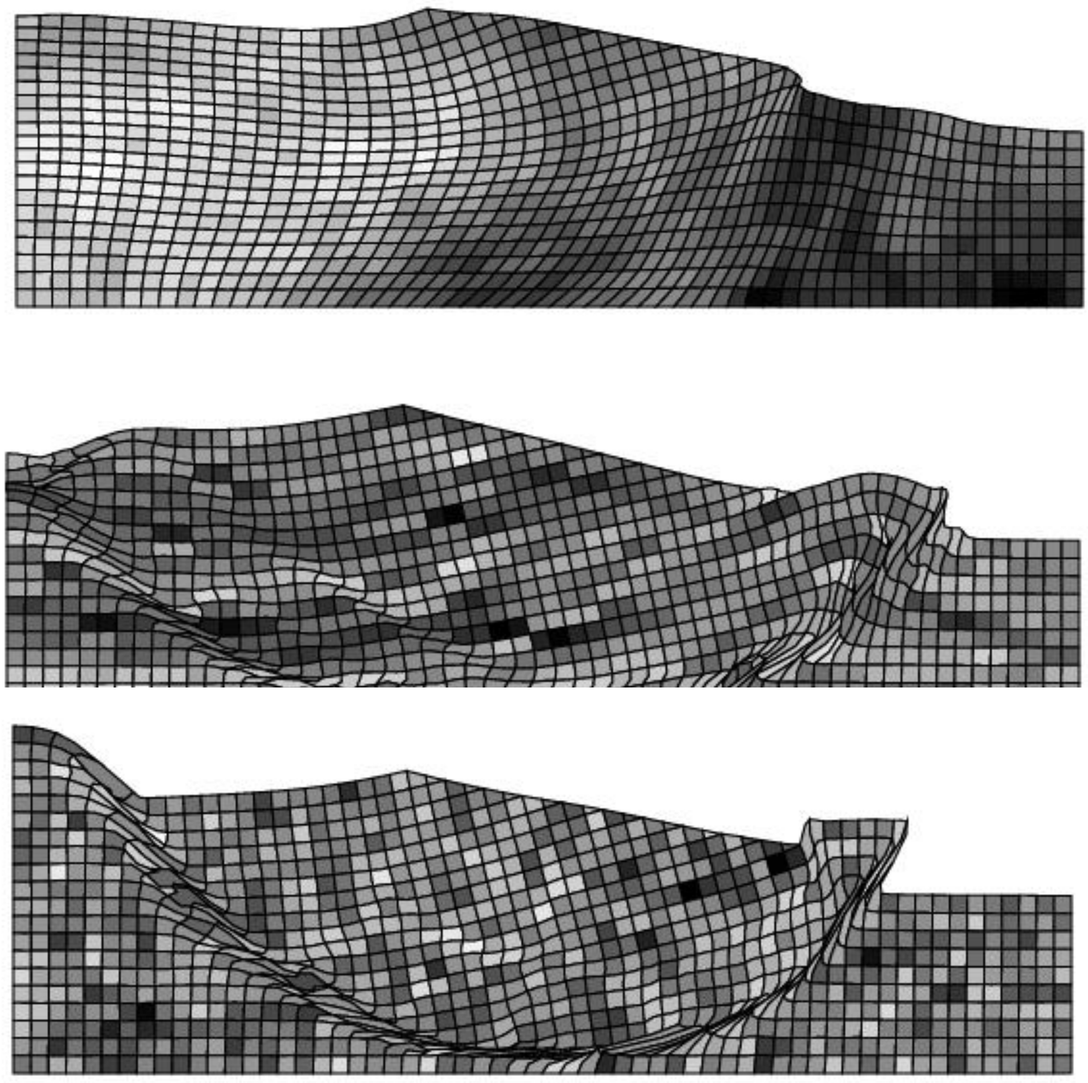

Figure 3.4: Failure Mechanisms Resulting from Rslope2d for FS=1.3, COV=0.5 for (a) $\Theta_{Su} = 5$, (b) $\Theta_y = 2$ and δ_x=100 m and (c) $\Theta_{Su} = 0.1$.

The failure mode associated with the RFEM realization for $\Theta_y = 2$ and δ_x=100 m is presented in Fig. 3.5 with a depiction of strain concentrations across two major planes. The same slope was analyzed in a Limit Equilibrium slope stability software (Talren) for the deterministic case. The resulting deterministic failure surface is presented in Fig. 3.6 and is shown to be close to one of the failure surfaces that were identified in the failure realization shown in Fig. 3.5.

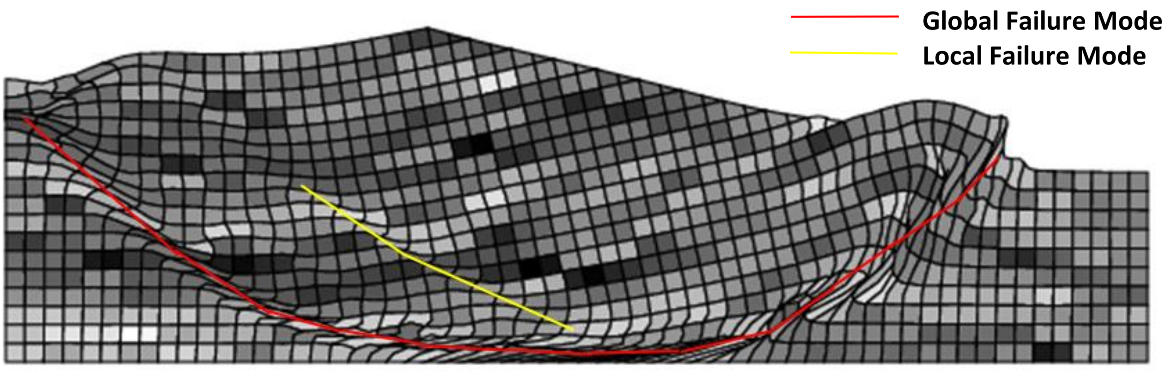

Figure 3.5: Failure Mechanisms Resulting from Rslope2d for FS=1.3, COV=0.5, δ_x=100 m and Θ_y = 2.

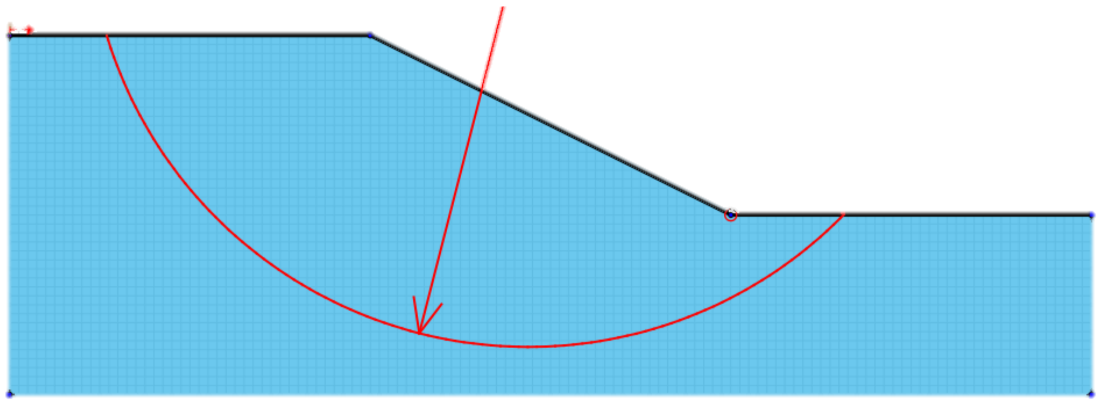

Figure 3.6: Failure Mechanisms Resulting from Rslope2d for FS=1.3 using LEM in Talren Software.

7. Stochastic Models for Su

RSlope2D incorporate four probability distributions that could be used to model the uncertainty in the undrained shear strength S_u. These include (a) the deterministic case where one parameter (basically the mean of Su) is required as input, (b) the normal distribution which requires two parameters (the mean and the standard deviation of Su) as input, (c) the lognormal distribution which also requires two parameters to be defined and (d) the 4-pramater bounded distribution which is defined by the lower bound a, the upper bound b, the location m, and the scale parameters. The distributions will be further described in section C.

8. Friction Angle

Since our study analyses undrained clayey slopes, a zero friction angle was assigned for the analysis conducted in this thesis.

9. Elastic Modulus and Poisson's Ratio

In this study, the elastic modulus which is the ratio of stress over strain is chosen to be deterministic with a value of 40,000 kPa. The Poisson ratio is taken as deterministic with a magnitude of 0.45.

C. Probabilistic Distribution of Shear Strength

1. Lognormal Distribution

In this study, all soil parameters are taken to be deterministic, except for the undrained shear strength which is assumed as the main source of uncertainty.

For the conventional case where the lower-bound shear strength is assumed to be zero, the undrained shear strength is defined as following a lognormal distribution with a mean and a standard deviation in accordance with the model described by Griffiths and Fenton (2004), and it is utilized in the RFEM to describe the random field. The probability density function (PDF) of the lognormal distribution is described by equation 3.5:

$$f(S_u) = \frac{1}{S_u \sigma_{\ln Su} \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln Su - \mu_{\ln Su}}{\sigma_{\ln Su}}\right)^2} \qquad \text{(Eq. 3.5)}$$

In equations 3.6 and 3.7, the standard deviation and mean of the equivalent normal distribution are defined as follows:

$$\sigma_{\ln S_u} = \sqrt{\ln\left\{1 + \left(\frac{\sigma_{Su}}{\mu_{Su}}\right)^2\right\}} \qquad \text{(Eq. 3.6)}$$

$$\mu_{\ln S_u} = \ln \mu_{Su} - \frac{\left(\sigma_{\ln S_u}\right)^2}{2} \qquad \text{(Eq. 3.7)}$$

The mean and the standard deviation are related by the dimensionless coefficient of variation COV, as represented by equation (Eq. 3.8):

$$\mathrm{COV} = \frac{\sigma_{S_u}}{\mu \mathrm{Su}} \qquad \text{(Eq. 3.8)}$$

The median of a log–normal distribution is given by Eq. 3.9 as follows:

$$median = \exp\left(\mu_{\ln S_u}\right) \qquad \text{(Eq. 3.9)}$$

2. Bounded Distribution

For cases where the undrained shear strength was assumed to be bounded at by a lower-bound value, a truncated lognormal distribution is generally adopted to incorporate the lower-bound strength. For that purpose, the left-hand tail of the undrained shear strength probability distribution is truncated at the value of the lower bound. This concept aims at reducing the likelihood of obtaining unrealistically small undrained shear strength values in the random finite element analysis. The lower bound undrained shear strength was represented by the remolded undrained strength and assumed to be equal to the mean of the undrained shear strength (undisturbed clay) divided by the sensitivity of the clay, taken in this study to be mainly between 1.5 and 4.0 (typical for natural clays). Conventional and truncated lognormal distributions for the case with a mean S_u of 100 kPa, a COV of 0.5, and a sensitivity of 2.0 (remolded S_u = 50 kPa) is presented in Fig. 3.7.

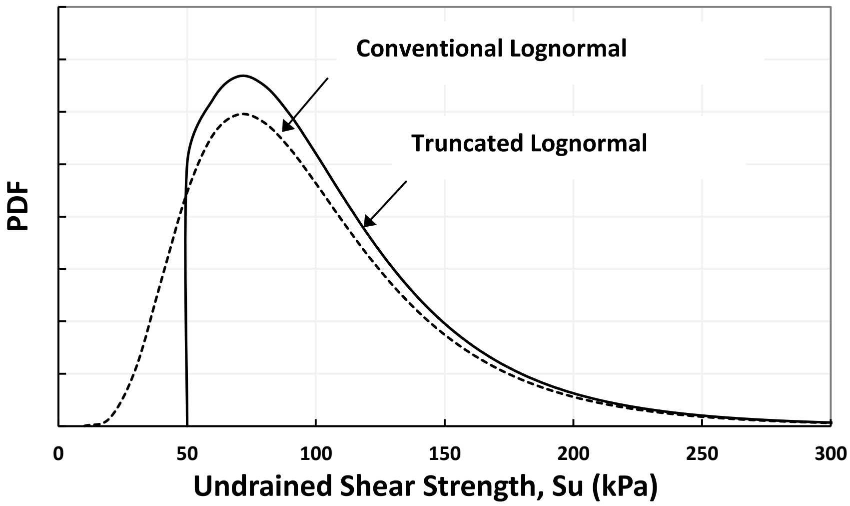

Figure 3.7: Conventional and Truncated Lognormal Probability Distributions for Su.

Given that a truncated lognormal distribution is not presented as an option for modeling the uncertainty in Rslope2D, the 4-parameter bounded distribution was used to model the truncated lognormal distribution. The bounded distribution can be established as shown in Equation 3.9 through a transformation between a standard normal variate G having a mean of 0 and a standard deviation of 1.0 to any random variable X that is bounded by [a b] interval:

$$X = a + \frac{1}{2}(b-a)\left[1 + \tanh\left(\frac{m + sG}{2\pi}\right)\right] \qquad \text{(Eq. 3.9)}$$

Where m, s, a and b are the location, scale, lower bound and upper bound parameters, respectively. The probability density function of the bounded distribution of X is:

$$f_X(x) = \frac{\sqrt{\pi}(b-a)}{\sqrt{2}s(x-a)(b-x)} e^{-\frac{1}{2s^2}\left[\pi \ln\left(\frac{x-a}{b-x}-m\right)^2\right]} \qquad \text{(Eq. 3.10)}$$

To match the PDF of the truncated lognormal distribution for any given case to that of an equivalent bounded distribution, the 4 parameters that describe the equivalent bounded distribution were determined through the Excel Solver. The solver seeks the parameters that would minimize the root-mean-square error between the cumulative distribution function (CDF) of the truncated lognormal distribution and the CDF of the equivalent bounded distribution.

As an example, Fig. 3.8 shows a comparison between the truncated lognormal CDF and the CDF of the equivalent 4-parameter bounded distribution. The example pertains to a case where the mean S_u is 90 kN/m^2, the COV of S_u is 0.3, and the clay has a sensitivity of 3 (lower bound is equal to 30 kPa). The comparison indicates that accurate and realistic representations of the truncated lognormal distribution could be obtained using the 4-parameter bounded distribution in Rslope2d.

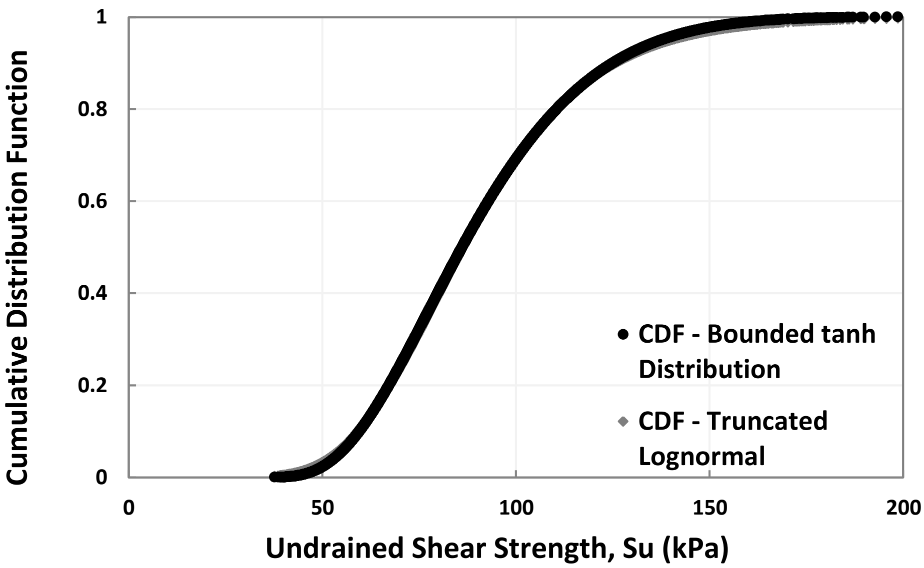

Figure 3.8: Truncated Lognormal and Bounded Tanh Cumulative Probability Distributions for Su.

3. Local Averaging

In the RFEM, all input parameters are defined at the point level. Although it is nearly impossible to quantify these statistics practically, they represent a fundamental baseline of the inherent soil variability. According to Griffiths and Fenton, this can be corrected by the use of local averaging and consideration of the sample size (Griffiths and Fenton, 2004). In each Monte Carlo simulation, each element possesses a property that remains constant during the single simulation based on the RFEM approach. The size of each finite element represents the “sample” which is used to discretize or transform continuous distributions into discrete parameters.

In the case where the point distribution is a normal distribution, local averaging could be shown to reduce the variance of the property but doesn’t affect its mean. For lognormal cases, local averaging reduces both the standard deviation and the mean. Griffiths and Fenton find this to be logically explained by the dependence of the mean of the lognormal distribution on the mean and variance of its normal distribution. The effect of local averaging (reduced mean and standard deviation) is more pronounced with larger elements and with simpler discretization of the slope problem.

The variance reduction factor due to local averaging γ, is defined as (Griffiths and Fenton, 2004):

$$\gamma = (\sigma_{lnSuA}/\sigma_{lnSu})^2 \qquad \text{(Eq. 3.11)}$$

Where σ_{lnSuA} is the standard deviation of the average soil property. For a square finite element of side k=α*θ_{lnSu}, it could be shown from (Vanmarcke, 1984) that the variance reduction factor for an anisotropic field is given by:

$$\gamma = \frac{4}{k^4}\int_0^k \int_0^k \exp\left(-\frac{2|\Delta x|}{\delta_x} - \frac{2|\Delta y|}{\delta_y}\right) * (k-x) * (k-y) dxdy \qquad \text{(Eq. 3.12)}$$

Numerical integrations of Eq. 3.12 assuming square finite elements of side=2 m and for the various correlation lengths adopted throughout this study, would lead to the variance values shown in Table 4.1. The results show that as the correlation length approaches the element size, the variance reduction becomes more significant.

The statistics of the underlying log field including local averaging are given by:

$$\sigma_{lnSuA} = \sigma_{lnSu\sqrt{\gamma}} \qquad \text{(Eq. 3.13)}$$

$$\mu_{lnSuA} = \mu_{lnSu} \qquad \text{(Eq. 3.14)}$$

Hence, the statistics of the lognormal field, averaged at the level of the element and mapped to the generated field are:

$$\mu_{SuA} = \exp(\mu_{lnSuA} + \frac{1}{2}\sigma_{lnSuA}{}^2) \qquad \text{(Eq. 3.15)}$$

$$\sigma_{lnSuA} = \mu_{SuA}\sqrt{\exp(\sigma_{SuA}{}^2) - 1} \qquad \text{(Eq. 3.16)}$$

Table 4.1: Variance Reduction Factor Computed for Square Finite Element of Side = 2m for the Various Correlation Lengths Considered throughout this study.

Horizontal Correlation Length δ_x (m)	Vertical Correlation Length δ_y (m)	Variance Υ
40	1	0.365
40	2	0.549
40	3	0.650
40	5	0.754
40	10	0.850
40	20	0.906
10	2	0.499
20	2	0.532
30	2	0.543
40	2	0.549
60	2	0.555
100	2	0.560

CHAPTER 4

RELIABILITY OF HOMOGENEOUS AND SPATIALLY VARIABLE CLAYEY SLOPES

A. Introduction

In a pioneering research study, Griffiths and Fenton (2004) incorporated the random finite element generation techniques along with local averaging theories to investigate the impact of spatial variability of clay properties on the reliability of spatially random slopes. The slope geometry that was adopted in their study is shown in Fig. 4.1 and entails a slope with a 2:1 slope angle and a 20-m slope height (H). The slope was assumed to be underlain by a 20-m thick clay layer with the same properties as the slope (unit weight of 20 kN/m^3). The clay layer is assumed to be underlain by a hard layer of infinite shear strength and stiffness. The slope investigated by Griffiths and Fenton (2004) was adopted in several other studies and was considered as the base case for the analyses conducted in the current research study.

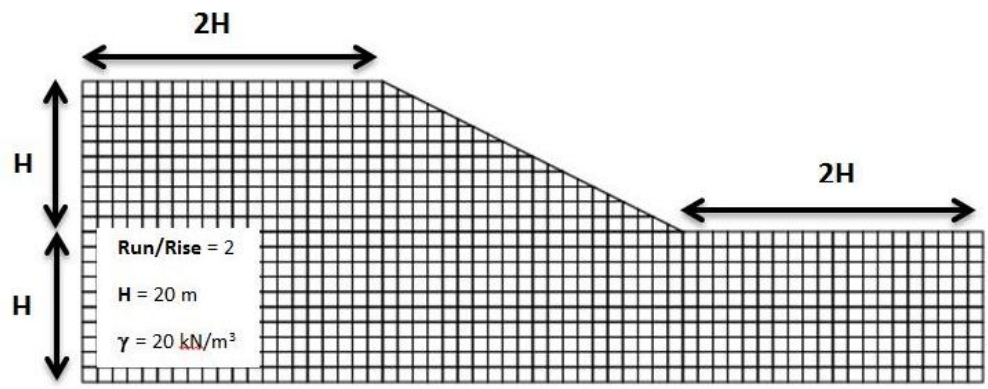

Figure 4.1: Geometry of the slope analyzed by Griffiths and Fenton (2004).

B. Reliability of Homogeneous Slope – Simplified Analytical Solution

To illustrate the impact of spatial variability on the reliability of the considered slope, Griffiths and Fenton (2004) analyzed the slope initially with the assumption that it is a homogeneous slope with uncertain undrained shear strength (Su). In other words, they assumed that Su in the different clay elements is constant in any given realization of Su. In this case, the probability of failure of the slope could be easily computed without the need for any sophisticated finite element analysis or random field theory. The probability of failure could be computed by determining the probability that the undrained shear strength (Su) will fall below a predetermined critical value of Su that would cause failure of the slope from a deterministic perspective. This critical value is the minimum value of Su that will result in a FS of 1, with the factor of safety being predicted using simplified analytical slope stability methods. Quantitatively, the probability of failure is equal to the area of the probability density function of the lognormal distribution characterizing $Su_{critical}$ and is given by:

$$p_f = p(Su < Su_{critical}) = \Phi\left\{\frac{\ln(Su_{critical}) - \mu_{lnSu}}{\sigma_{lnSu}}\right\} \qquad \text{(Eq. 4.1)}$$

Where Φ=cumulative standard normal distribution function.

For the case of a homogeneous slope, Fig. 4.2 shows the different probabilities of failure that were computed for a range of factors of safety and COVs considered in the analysis (Griffiths and Fenton, 2004). The reported factors of safety correspond to the factors of safety that would have been obtained in a deterministic analysis where the undrained shear strength is taken as the mean of Su. The undrained shear strength COV values that were adopted by Griffiths and Fenton (2004) ranged from 0 to 8. It should be noted that the realistic upper bound of COV for practical field applications involving the undrained strength of clays is around 0.5.

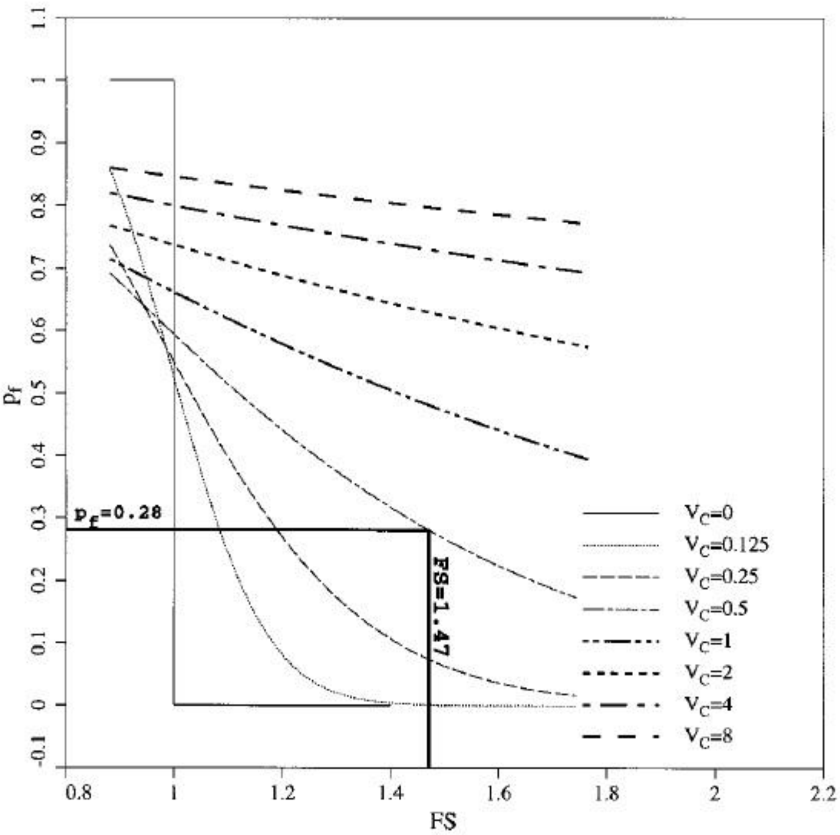

Figure 4.2: Griffiths and Fenton (2004) results – p_f versus FS for various COVs.

Results on Fig. 4.2 indicate that the probability of failure of the homogeneous clayey slope is highly dependent on the coefficient of variation of Su and on the assumed factor of safety. For factors of safety that are relatively small (FS ~ 1.25), the probability of failure ranges from 5% to 45% for a realistic range of COV_{Su} of 0.125 to 0.50. For a higher and more representative FS of 1.5, these probabilities of failure reduce to almost zero for the COV of 0.125 and 28% for the case with the higher COV of 0.5.

C. Reliability of Homogeneous Slopes – RSlope2d Solution

In an initial attempt to utilize the Rslope2D Software in this research study, the same slope that was analyzed by Griffiths and Fenton (2004) (Fig. 4.1) was adopted and an effort was made to compute the probability of failure of the slope assuming a homogeneous random field. In the analysis, Su was assumed to follow a conventional lognormal distribution with a mean value that would produce a target deterministic factor of safety and a COV (designated by V in Figs. 4.2 and 4.3) that varies between 0 and 0.5. The condition of a homogeneous random field of Su was enforced by adopting horizontal and vertical correlation distances that were very large (assumed to be 1000 m in this illustrative example). In each simulation level, the Su in any given element would randomly be selected from the lognormal distribution assuming a correlation of 1000m in the random field. The output of each particular case (a given FS and COV) is quantified by a probability of failure that was calculated by the software as the number of failed slopes divided by the total number of Monte Carlo simulations assigned at the input level. The calculated probabilities of failure are presented in Fig. 4.3. As expected the results were almost identical to the results presented in Fig. 4.2 for the homogeneous case analyzed by Griffiths and Fenton (2004) using the simplified analytical approach. These results indicate the validity of the random finite element model that was adopted in this study and provide confidence in applying this model for more realistic cases that involve random fields that do not necessarily have to be homogenous. Such analyses will be conducted in the following section.

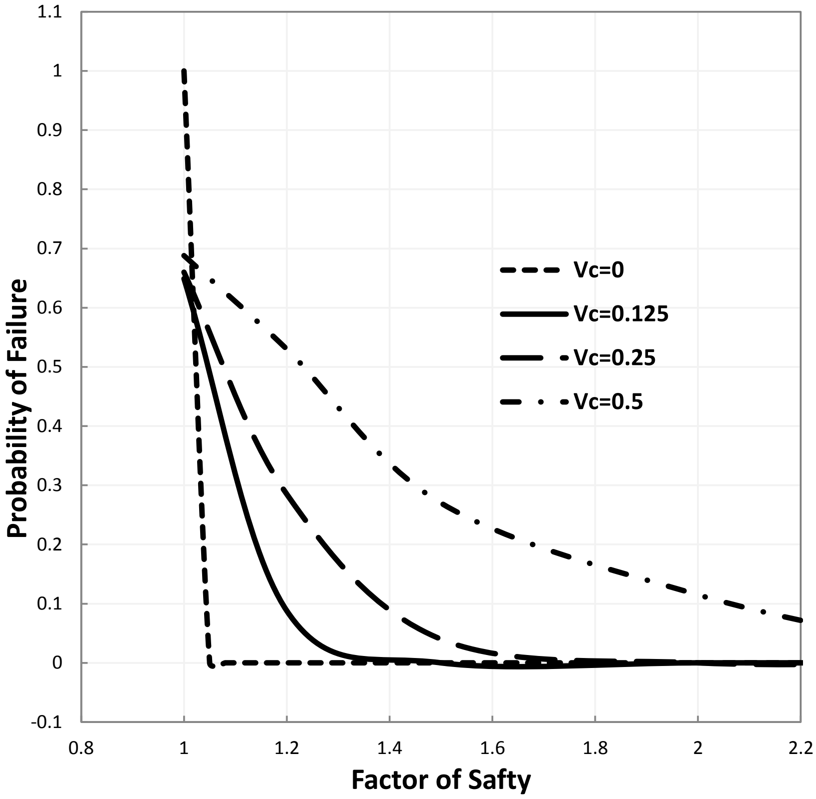

Figure 4.3: Replicated results – p_f versus FS resulting from RSlope2d.

D. Reliability of Homogeneous Slopes - Effect of Lower-Bound Shear Strength

In the previous section, the Su was characterized by a lognormal distribution having specific mean and COV. In this section, the impact of including a lower-bound estimate of the undrained shear strength within the formulation of the reliability problem will be investigated in the context of a homogeneous slope.

Gilbert et al. (2005) and Najjar and Gilbert (2009) adopted a lognormal distribution that is truncated at a lower-bound value to model the uncertainties in the factor of safety of a drained clayey slope and in the capacity of deep foundations in sands and clays, respectively. In these studies, a truncated lognormal distribution is convenient because the parameters describing the distribution are the same as those of the non-truncated distribution with the addition of one extra parameter, the lower-bound value. Chapter 3 explains the inability of the user to characterize Su by a truncated lognormal in RSlope2d. Also it explains how a bounded distribution could be matched to represent the uncertainty of a truncated lognormal distribution.

The major objective of this section is to investigate the effectiveness of introducing a lower bound in the probabilistic model describing the uncertainty in the undrained shear strength of clay in slope stability analyses. The lower bound is represented by the remolded undrained shear strength of the clay, which is a well-known property that could be determined using information about the sensitivity of the clay as follows:

$$LB = \frac{\mu_{Su}}{Clay\ Sensitivity} \qquad \text{(Eq. 4.2)}$$

Where, μ_{Su} represents the undisturbed undrained shear strength of the clay.

The random finite element method is utilized to quantify the probability of failure of homogeneous undrained slopes that are designed with different deterministic factors of safety with and without a lower-bound undrained shear strength. The RFEM is conducted for the case of a “homogeneous” slope (1000 m correlation length). For each case, the analysis is repeated for a coefficient of variation (COV) of 0.3 and 0.5 in the undrained shear strength and deterministic factors of safety ranging from 1.15 to 2.0. Clay sensitivity was considered between 1.5 and 3.5. Fig. 4.4 shows the results of this analysis

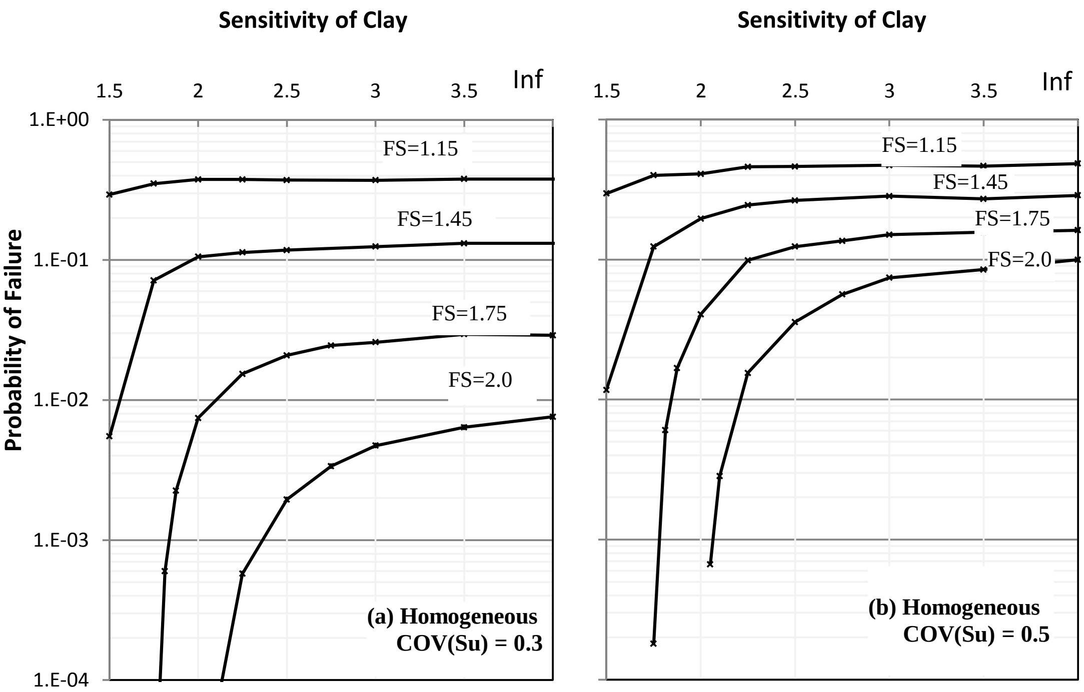

Figure 4.4: Probabilities of Failure Computed at Various Sensitivies for Different FS and COVs - Homogeneous Case

An analysis of the results on Fig. 4.4 indicates that (1) for the conventional case where no lower-bound shear strength is included in the analysis (shown as sensitivity = inf), as the factor of safety increases from 1.15 to 2.0, the probability of failure of the slope decreases (as expected) from 38.2% to 1.4% for COV = 0.3 and from 59.4% to 10% for COV = 0.5, (2) for the cases were lower-bound shear strengths that are a function of the sensitivity of the clay are incorporated in the analysis, the probability of failure decreases as the sensitivity of the soil decreases, and (3) the effect of incorporating the lower-bound Su on the probability of failure becomes more significant as the factor of safety increases and as the COV of Su increases.

A more detailed analysis of the reliability analysis conducted for the homogenous slopes shows that for the relatively smaller safety factors (less than 1.5), no effect of the lower-bound shear strength was evident except for cases with sensitivities that are less than 2.0. For the cases with higher factors of safety (greater than 1.5), the lower-bound shear strength started to have an impact on the probability of failure at sensitivities as high as 3.0. As an example, consider the case where the factor of safety is equal to 1.75 and the COV = 0.3. The curves on Fig. 4.4a indicate that the probability of failure decreases from 4.2% (no lower bound) to 2% for the case with a lower bound defined by a sensitivity of 2.5. The probability of failure decreases further to 0.74% for the case with a sensitivity of 2. Similar reductions are observed for the case of a COV of 0.5 whereby the probability of failure decreases from 17.4% for the case with no lower bound to 12.4% and 4.0% for cases with sensitivities of 2.5 and 2.0, respectively. These results indicate that the effect of the lower bound increases for higher COVs and for higher factors of safety.

E. Reliability of Spatially Random Slopes

The assumption that the spatial variability model of the undrained shear strength can be expressed by a homogeneous field is not realistic. Phoon (1995) report that the undrained shear strength of clays can be described by spatially random fields that are characterized by vertical and horizontal correlation distances that range between δ_y of 0.8m to 6.1m (mean ~ 2.5m) and δ_x of 20m to 70m (mean ~ 45m). As a result, the probability of failures that were calculated in Section 4.3 assuming a homogeneous isotropic field with very large correlation lengths may not be realistic. In this section, the reliability of the example clayey slope will be assessed using the RFEM assuming a realistic random field for Su.

1. Reliability of Spatially Random Slope – No Lower-Bound Shear Strength

To investigate the impact of a realistic random field of Su on the reliability, the same slope (Fig. 4.1) was analyzed in RSlope2D for the same conditions that were adopted in the homogeneous case except for assigning realistic spatial correlation lengths to the Su field. For this purpose, an anisotropic random field was adopted with a horizontal correlation length (δ_x) and a vertical correlation length (δ_y) that are equal to 40 m and 2 m, respectively. These values are in line with the average correlation lengths that were reported in Phoon (1995). The reliability calculations were made assuming different design factors of safety for Su COVs of 0.3 and 0.5. The resulting probabilities of failure are presented in Fig. 4.5 together with the results obtained for the homogeneous case that was assumed in Section 4.3.

Results of Fig 4.5 show that for both the homogeneous and spatially variable (SV) cases, the probability of failure decreases with the increase of FS which is a behavior that was already explained in section 4.1. However, results show a major difference in the magnitude of this decrease whereby the probability of failure in the case of spatially variable slopes decreases dramatically with small increases in the factor of safety. As an example, the probability of failure for a COV of 0.3 decreases from 15.9% to 0.2% as the factor of safety increases slightly from 1.15 to 1.30. A similar significant reduction in the probability of failure (from 70% to 3.7%) is witnessed for the case of a COV = 0.5 as the factor of safety increases from 1.15 to 1.45.

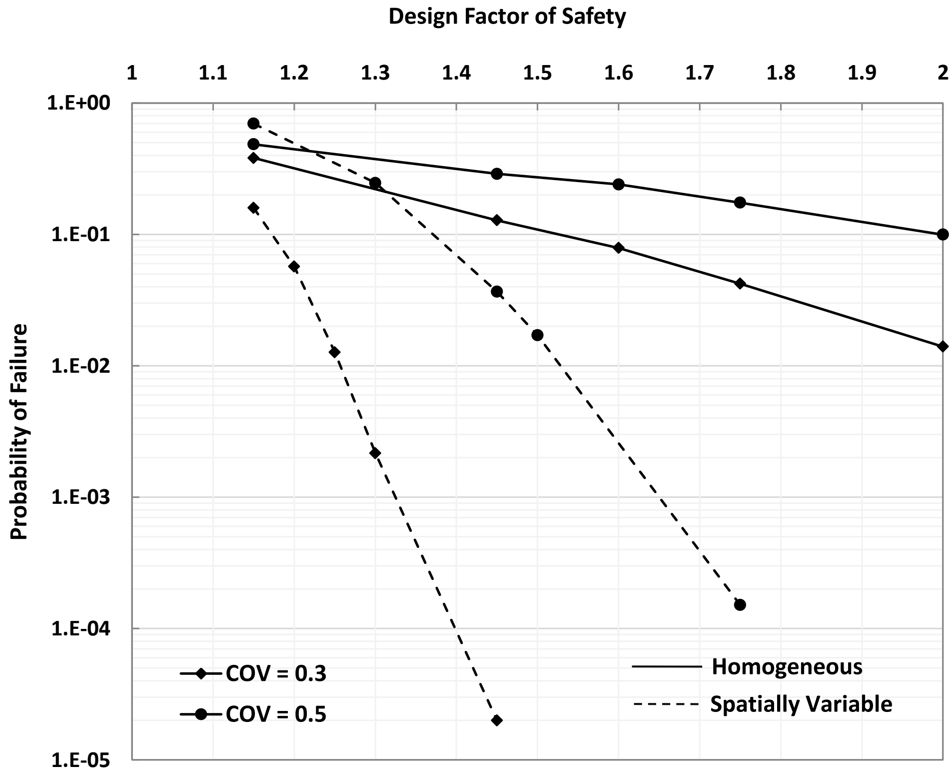

Figure 4.5: Probabilities of Failure Computed with No LB Su for Different FS and COVs - Homogeneous and Spatially Variable Cases.

For any given factor of safety, a comparison between the probabilities of failure calculated for the random field and the homogeneous slope indicates that p_f of the spatially random field is always lower than the p_f of the homogeneous slope. The significant reductions in the probability of failure for the case of a spatially random field with relatively small correlation distances is attributed to variance reduction that occurs due to spatial averaging along the failure plane and which is not present in the case of a "homogenous" slope. This variance reduction reduces the uncertainty in the undrained shear strength resulting in significant reductions in the number of failure cases in the Monte Carlo simulations, especially for cases with larger FS. This indicates that the inclusion of spatial correlation and local averaging lead to smaller probabilities of failure. Thus, simplified probabilistic analyses that assume fully correlated random fields of Su might lead to conservative estimates of the probability of failure. The effect is more pronounced at lower factors of safety or at higher COVs.

From a reliability-based design perspective, the results on Figure 4.5 could be used to back-calculate the design factor of safety that is required to achieve a given target level of reliability for the slope design. For example, consider the case where a probability of failure of 5% is targeted. Based on the results of Fig. 4.5, the required factor of safety would have to be equal to 1.72 (COV = 0.3) and 2.4 (COV = 0.5) for the homogeneous slope assumption. For the more realistic anisotropic random field option, the required factors of safety are reduced to 1.2 and 1.42, respectively. These results are important since they point to the importance of modeling the random field of Su in the design of undrained clayey slopes.

The probabilities of failure that were calculated in this study for the anisotropic random field of Su with δ_x = 40 m and δ_y = 2 m were compared on Fig. 4.5 with published results from Griffiths and Fenton (2004) for isotropic random fields of various correlation lengths ($\delta_x = \delta_y$ = 10m, 20 m, and 80m). For all factors of safety considered, results indicate that the considered anisotropic random field yields probabilities of failure that are smaller than the isotropic fields. The smaller probabilities of failure for the anisotropic field are a result of the small vertical correlation length of 2.0m that was adopted in the anisotropic field. It is well known that the vertical correlation length plays a significant role in the process of variance reduction due to averaging. Since vertical correlation lengths are more likely to be between 1.0m and 6.0m, the probabilities of failure that are associated with the anisotropic random field adopted in this thesis are expected to be more realistic than the conservative probabilies of failure calculated in the isotopic fields analyzed by Griffiths and Fenton (2004).

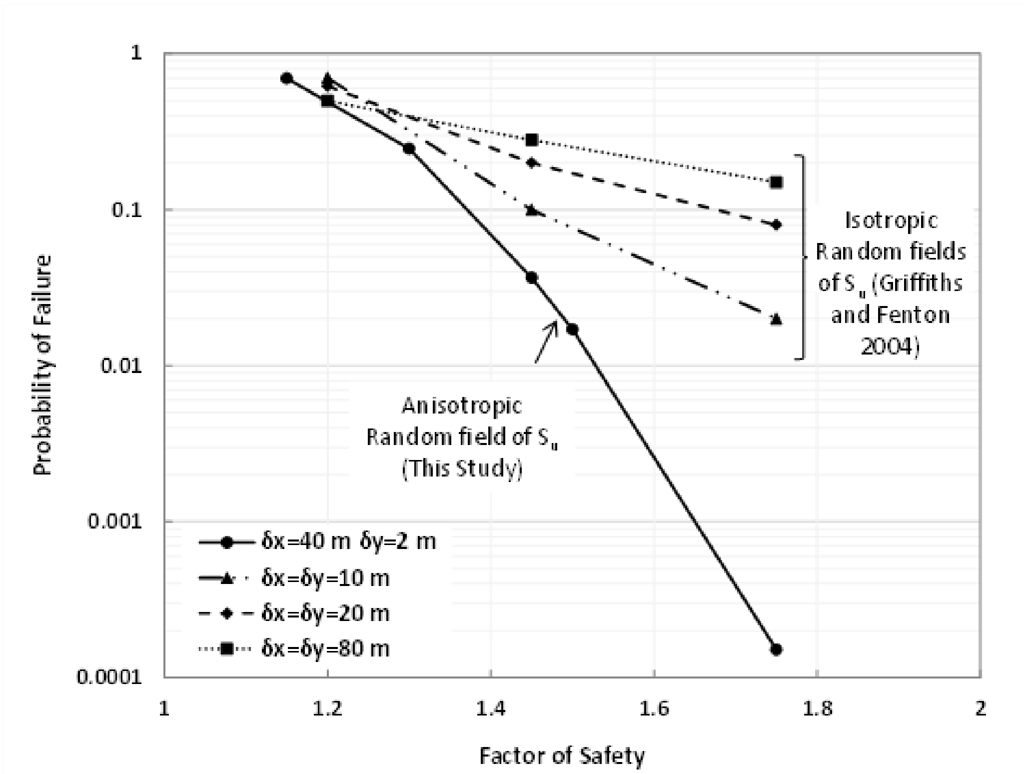

Figure 4.6: Probabilities of Failure Computed with No LB Su for Different FS and correlation length; COV = 0.5.

2. Reliability of Spatially Random Slope – With Lower-Bound Shear Strength

In this section, the effect of incorporating a lower-bound undrained shear strength in the stochastic model of the random field of Su is investigated. For this purpose, the anisotropic random field adopted in the previous section was used with the addition of a lower-bound shear strength that is dictated by sensitivities ranging from 1.5 to 4.0 as indicated in Fig. 4.7.

With the incorporation of a lower-bound shear strength in the spatially random field, results on Fig. 4.7 indicate that the probability of failure is highly sensitive to the presence of the lower bound. This is reflected in the sharp reductions that are observed in the probability of failure for all factors of safety even at high values of sensitivity. For illustration, consider the case with a factor of safety of 1.3. Results on Fig. 4.7a with a COV of 0.3 reflect a 4.5 fold reduction in the probability of failure for clay with a sensitivity of 2.25 and two orders of magnitude reduction in the probability of failure for the case with a lower-bound shear strength that is given by a sensitivity of 1.75. Similar observations are made in Fig 4.7b for a COV of 0.5 in the undrained shear strength whereby the probability of failure for the case with an FS of 1.3 exhibits a 5 fold reduction in the probability of failure (from 24.6% for the case with no lower bound to 4.6%) for clay with a sensitivity of 2.25, and more than two orders of magnitude reduction in the probability of failure (from 24.6% to 0.04%) for the case with a lower-bound shear strength that is given by a sensitivity of 1.75.

The results are important since they indicate that the incorporation of a physical lower-bound shear strength that is based on the remolded undrained shear strength of a clay could have a significant impact on the reliability of a slope, particularly for spatially variable slopes with realistic correlation lengths in the lateral and vertical directions (see Fig. 4.7).

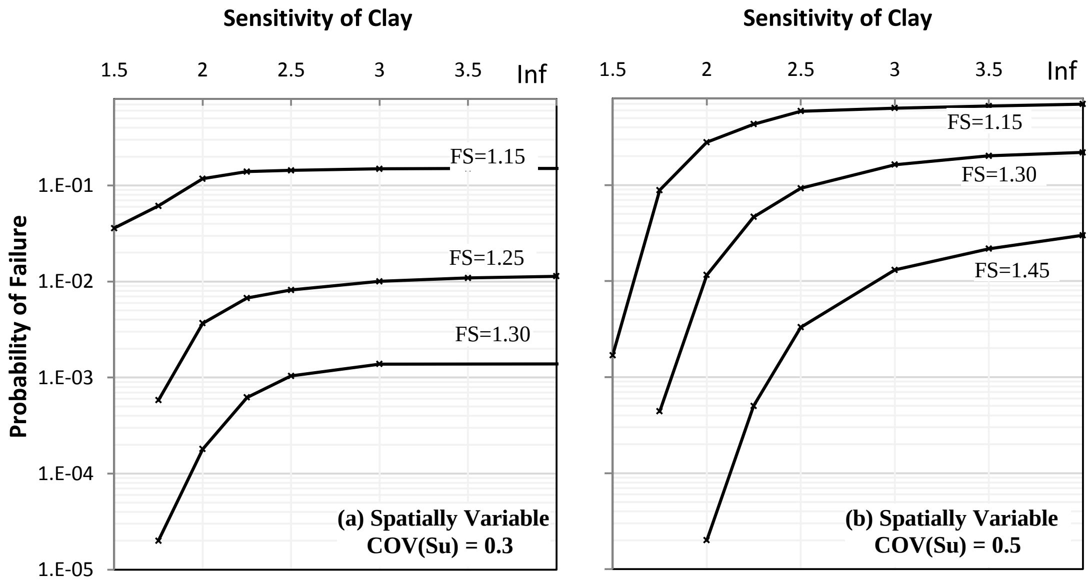

Figure 4.7: Probabilities of failure resulting at various sensitivies for different FS and COVs – Spatially Variable case

The truncation of the probability distribution of the undrained shear strength by the lower-bound strength reduces the uncertainty in the Monte Carlo simulations by eliminating unrealistically low shear strength values that are theoretically lower than the lower-bound shear strength. This reduces the number of cases that fail in the Monte Carlo simulations thus reducing the probability of failure of the slope. It could thus be concluded that incorporating a lower-bound shear strength in the reliability analysis of an undrained slope by truncating the distribution of S_u at the lower tail is analogous (in its effect) to reducing the total uncertainty in the undrained shear strength via spatial averaging.

CHAPTER 5

SENSITIVITY OF THE SLOPE RELIABILITY TO THE RANDOM FIELD OF THE UNDRAINED STRENGTH

A. Introduction

Results in Chapter 4 indicated that the correlation lengths that describe the random field of Su could have a significant impact on the reliability of clayey slopes. In this chapter, the sensitivity of the probability of failure to the vertical and horizontal correlation lengths will be investigated. In the sensitivity analysis, the normalized vertical correlation distance Θ_y was varied from 0.1 to 1.0, while the horizontal correlation distance δ_x was varied from 20m to 100m. These ranges are expected to be indicative of the variation of practical correlation distances for the undrained shear strength of clays in geotechnical practice (Phoon 1995, Phoon and Kulhawy 1999, and Jha and Ching 2013). It should be noted that for the example clay slope that was adopted in this study (slope height = 20m), the range of normalized vertical correlation length correspond to actual correlation distances of 2m to 20m.

Since the main objective of this research study is to investigate the impact of incorporating a lower bound shear strength on the reliability, a set of reliability analyses were conducted whereby lower bounds that are associated with different clay sensitivities were included as part of the random field. These analyses were aimed at investigating whether the impact of the lower bound on the reliability is sensitive to the assumed correlation lengths.

Finally, the analyses that are conducted in this chapter are repeated for the case of a slope with a 1:1 angle of inclination rather than a 2:1 angle. The objective of this set of analyses is to quantify any impact that the slope angle could have on the reliability of the slope for the same design factor of safety and random field properties. It is expected that the slope angle could have an impact on the reliability since it may affect the location of the potential failure surfaces which define the level of variance reduction exhibited in different random fields.

B. Sensitivity of the Reliability of 2:1 Clayey Slopes to the Random Field of S_u

As mentioned earlier in the previous section, various FS and COVs of Su were analyzed for different clay sensitivities. The first case investigated in this section represents that of a 2:1 run/rise slope or equivalently a 26.7^{o} slope angle.

1. Effect of the Correlation Length – Case of No Lower-Bound

The 2:1 slope presented in Fig. 4.1 is further analyzed in order to illustrate the sensitivity of the reliability of the slope to the spatial variability of S_u for the conventional case that does not include a lower-bound shear strength in the formulation of the random field of Su. In a first attempt to show this effect, the horizontal correlation length is fixed at δ_x = 40 m while the vertical correlation length is varied from 2 to 20

m. The sensitivity of the probability of failure to the assumed vertical correlation distance is presented in Fig. 5.1 for the case involving a COV Su 0f 0.5 and for different assumed factors of safety.

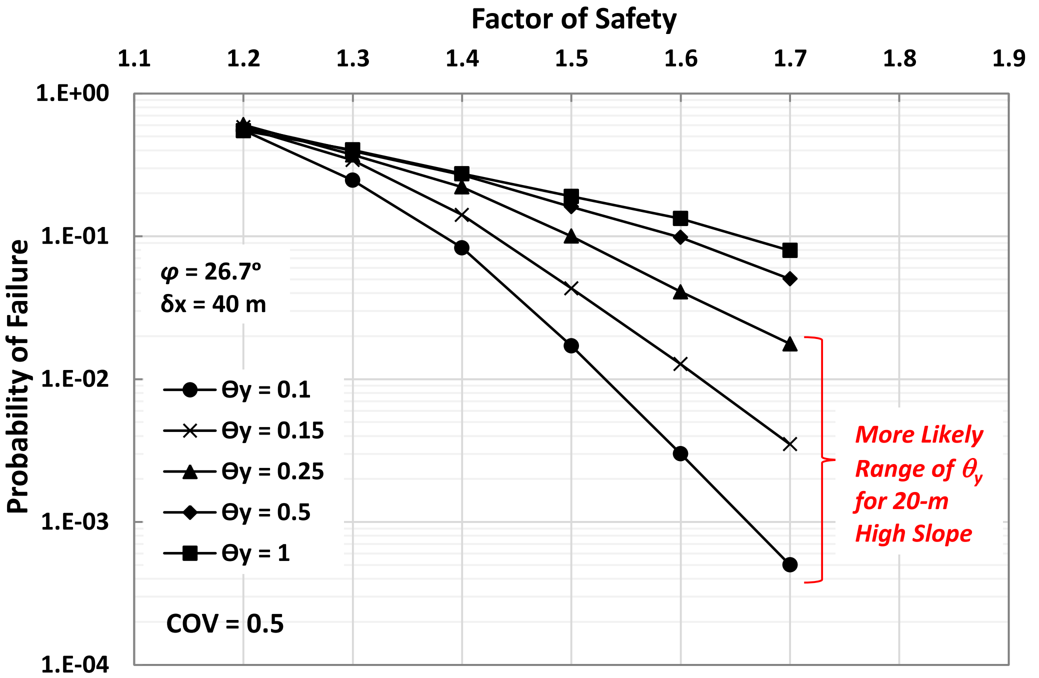

Fig 5.1: Probabilities of Failure versus Sensitivity of Clay for Different Vertical Correlation Length; FS and COV Held Constant

Results on Fig. 5.1 indicate that the assumed vertical correlation distance could have a significant impact on the probability of failure of the slope, particularly for larger factors of safety. For example, the probabilities of failure that is associated with a typical design factor of safety of 1.5 decreases from 20% for the case with a large Θ_y of 1.0 to a value as low as 2% for the case with a Θ_y of 0.1. This order of magnitude decrease in the probability of failure as Θ_y decreases from 1.0 to 0.1 is a direct result of variance reduction in the undrained shear strength due to averaging along the failure surfaces. It should be noted that for the particular case of the slope analyzed in this study (H = 20 m), the normalized vertical correlation distance Θ_y is more likely to be between 0.1 and 0.3 (marked in red in Fig. 5.1), since the likely range of correlation distances for S_u is between 2m and 6m (Phoon 1995).

2. Effect of the Correlation Length – with Lower-Bound Shear Strength

To investigate the impact of incorporating a lower-bound shear strength in the different S_u random fields, the probability of failure of the 2:1 slope was calculated for two design factors of safety (FS = 1.3 and FS = 1.5) assuming clay with sensitivities ranging from 1.5 to 4.0. A clay with a sensitivity as large as 4.0 is expected to have a relatively small remolded strength, yielding lower-bound values that are too small to have an impact on the reliability of the slope. On the other hand, the left hand tails of the probability distribution of Su for clays with sensitivities that are close to 1.5 will be significantly truncated due to the presence of the lower-bound undrained shear strength leading to increased reliability levels (smaller probabilities of failure) for the slopes under consideration.

The calculated probabilities of failure are presented in Fig. 5.2 as a function of the sensitivity of the clay. As expected, results indicate that the probability of failure of the slope decreases as the sensitivity of the clay decreases. Irrespective of the factor of safety, as the vertical correlation length is decreased, the effect of the lower bound seems to become more pronounced. Consider the example of FS =1.3. At a sensitivity equal to 3, the case of θ_y=1 (δ_y=20 m) resulted in a probability of failure of 34.8%

compared to 23.9% for the case of θ_y= 0.15 (δ_y=3 m). If the sensitivity is decreased further to a value of 1.75, p_f decrease slightly to a value of 7.4% at θ_y=1 compared to a very significant decrease to 0.19% at θ_y=0.15. Finally, a comparison between Figs. 5.2a (FS = 1.3) and 5.2b (FS = 1.5) indicates that the positive effect of the lower-bound shear strength on the reliability is exhibited at larger sensitivities for the case of FS = 1.5, irrespective of the correlation length adopted.

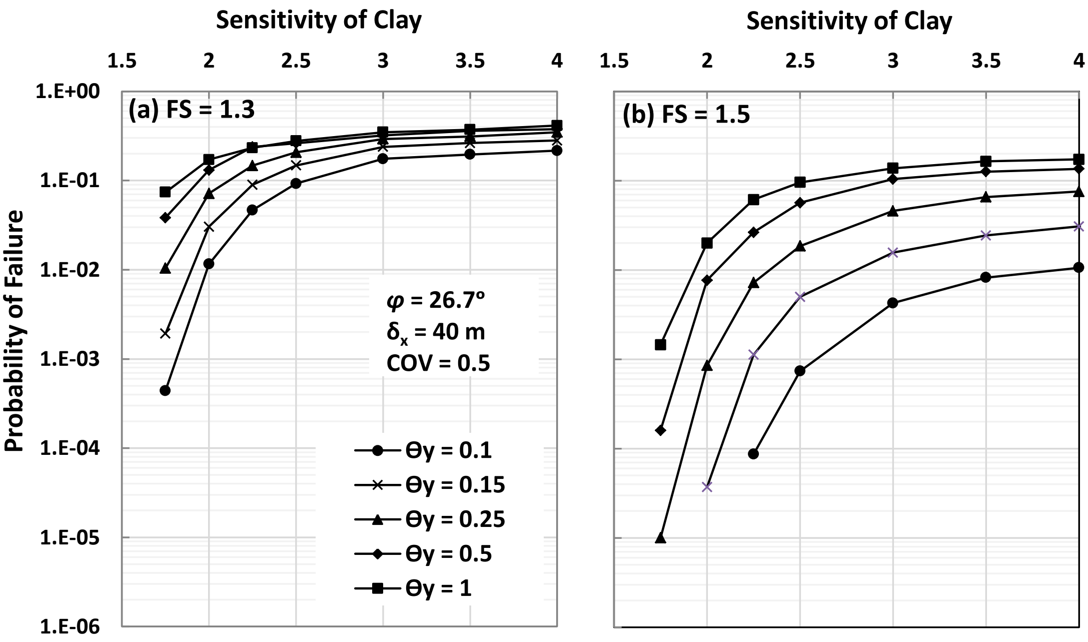

Fig 5.2: Probabilities of failure Versus Sensitivity of Clay for Different Vertical Correlation Lengths for (a) FS = 1.3 and (b) FS = 1.5.

The effect of the horizontal correlation length δ_x is investigated in Fig 5.3 for the 2:1 slope assuming a constant θ_y=0.1 while varying δ_x from 20m to 100m. Results are shown for the case of a COV Su of 0.5 and for factors of safety of 1.3 and 1.5. Results on Fig. 5.3a for FS = 1.3 indicate that the probability of failure is significantly affected by the presence of the lower-bound irrespective of the assumed horizontal correlation distance. In fact, results show that the relationship between the probability of failure and the sensitivity of the clay is not significantly affected by the adopted horizontal correlation length. For the case of FS = 1.5 (Fig. 5.3b), the relationship between p_f and clay sensitivity seems to be more affected by variations in δ_x, where higher probabilities of failure were observed for higher horizontal correlation distances. However, a comparison between Figs. 5.2 and 5.3 indicates with certainty that the vertical correlation length governs the probability of failure.

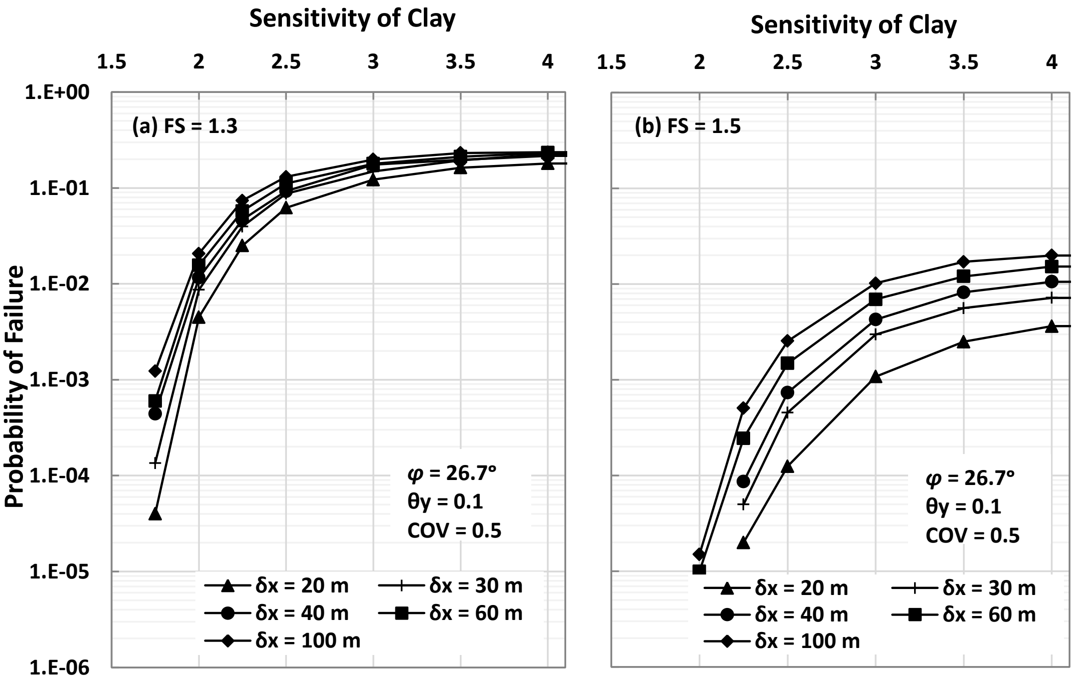

Fig 5.3: Probabilities of Failure versus Sensitivity of Clay for Different Vertical Correlation Lengths (a) FS =1.3 and (b) FS = 1.5

Griffiths et al. (2009) attribute the smaller probabilities of failure that were observed for cases with smaller correlation lengths to the fact that relatively smaller correlation lengths of S_u lead to weak and strong zones of soil that vary rapidly over short distances of the slope. In these cases, the preferred failure mechanism tends to be global and occupies wider zones of the soil field, leading to significant variance reduction along the failure zones due to averaging. On the other hand, as the correlation lengths increase, the failure mechanism is attracted to "local pockets" of weak soil and occurs on a reduced width. If the failure mechanism occurs locally, it has more opportunities to fail at different locations along the slope length which naturally leads to higher p_f. Figs. 5.4 and 5.5 show failure mechanisms associated with two cases of smaller (δ_x= 40 m, δ_y=2m) and larger (δ_x=100 m, δ_y=2m) correlation lengths of Su. The failure surface traverses a wider zone of the soil slope for the case of lower correlation distances (Fig 5.4) and a much shorter and localized failure surface for the case of high correlation lengths (Fig. 5.5).

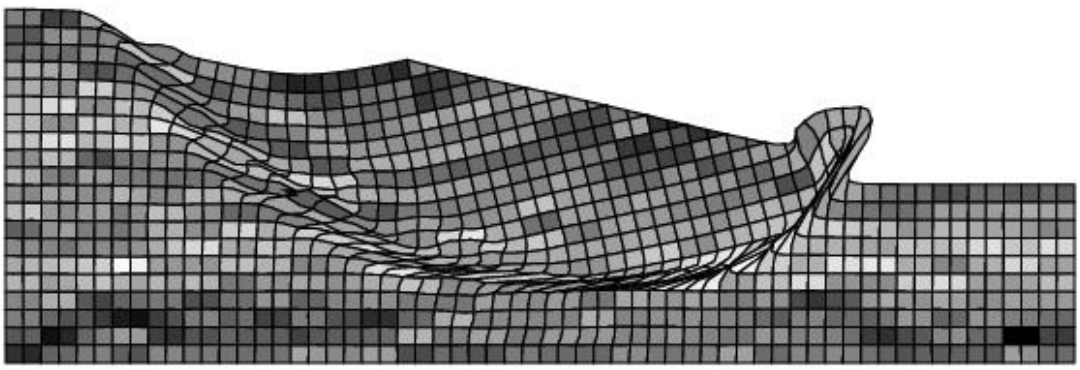

Fig 5.4: Failure Mechanism for the case of δ_x=40 m, δ_y=2m – FS=1.3 and COV=0.5

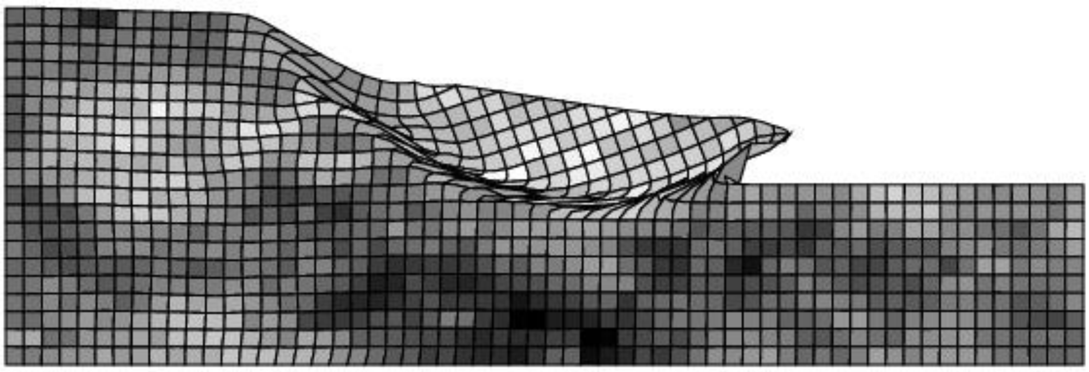

Fig. 5.5: Failure Mechanism for the case of δ_x=100 m, δ_y=2m – FS=1.3 and COV=0.5

3. Effect of the COV of S_u on the Reliability of the Slope

The effect of the COV of Su on the reliability of spatially random slopes is illustrated in Fig. 5.6 for cases involving a 2:1 slope with a representative horizontal correlation distance δ_x = 40 m and a variable normalized vertical correlation length θ_y that varies from 0.1 to 1.0. For the two cases where the COV was varied from 0.3 to 0.5, results on Fig. 5.6 indicate the probability of failure of the slope for the case where no lower bounds are included in the analysis is highly sensitive to the COV of Su, with significantly lower probabilities of failure exhibited in the case of COV = 0.3. The difference in the probability of failure for the two COV cases increases for cases with larger design factors of safety and smaller vertical correlation distances.

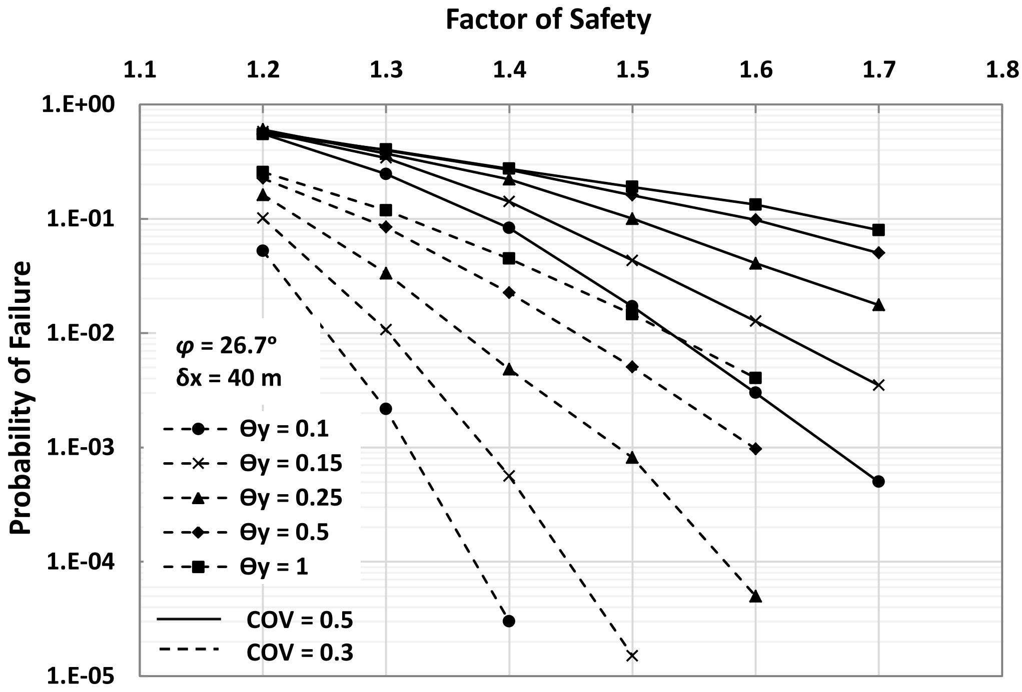

Fig 5.6 Probabilities of Failure versus Sensitivity of Clay for COV Su = 0.3 and 0.5.

When a lower-bound shear strength is included in the analysis, a comparison between the cases where the COV = 0.3 and 0.5 (Fig. 5.7) indicates that that for clays with high sensitivities (lower bound does not affect the reliability), the probabilities of failure for the cases involving a COV of 0.3 are relatively small compared to the case with a COV of 0.5. However, in the case where the COV of Su is taken as 0.5, the effect of the lower-bound shear strength is exhibited early on at higher values of clay sensitivity. More importantly, the lower bound in the case of a COV of 0.5 is more effective at reducing the probability of failure for any vertical correlation distance and any clay sensitivity. These results are important since they indicate that the incorporation of the lower-bound capacity in the reliability of spatially random slopes increases the reliability of the slope, particularly for cases studies that exhibit large variabilities in the undrained shear strength Su.

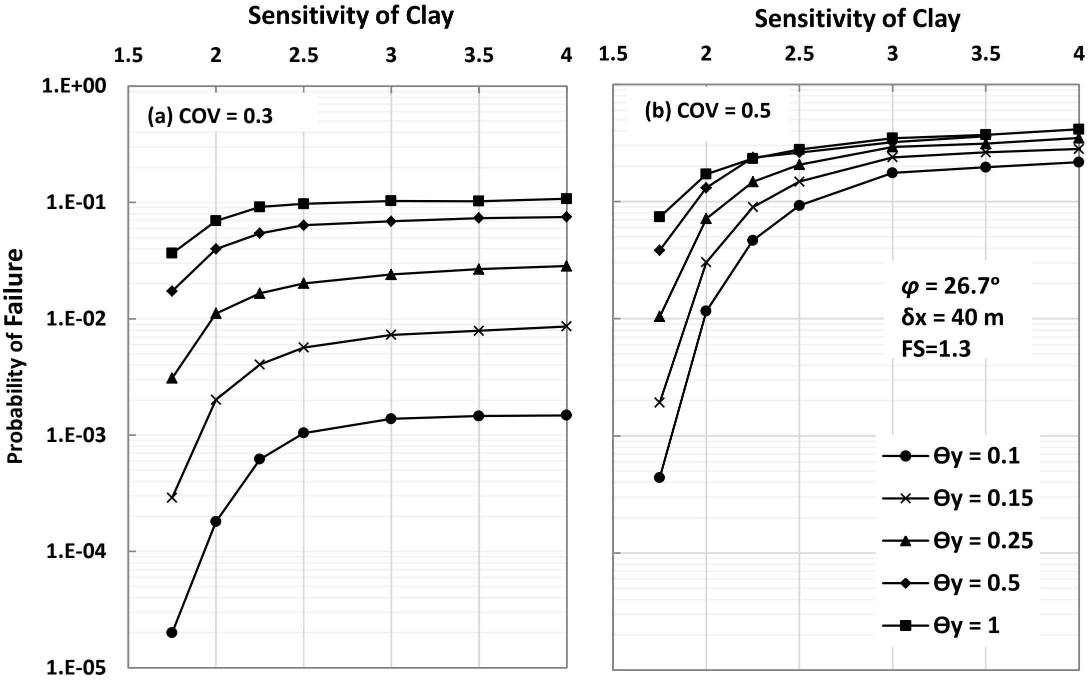

Fig 5.7 Effect of Lower-Bound Shear Strength on the Probability of Failure for (a) COV = 0.3 and (b) COV = 0.5.

C. Effect of the Inclination Angle of the Slope on the Reliability

In the previous analyses that were conducted in this study, a slope with an angle of inclination of 2:1 was used as a basis for studying the impact of the lower-bound shear strength and the correlation distances on the reliability of spatially variable undrained slopes. In all the analyses, the factor of safety was varied by varying the mean of the undrained shear strength of the slope while keeping the geometric characteristics of the slope fixed (H = 20m, slope angle is 2:1). From a deterministic design perspective, the factor of safety of an undrained slope is dependent on the undrained shear strength and unit weight of the clay, in addition to the angle and height of the slope. As a result, the same factor of safety could be obtained for two different slope angles by changing the value of the undrained shear strength, despite the fact that the two critical failure surfaces for the two slopes will be different.

From a reliability-based design perspective, it is hypothesized that the probability of failure of two slopes that are designed with different angles of inclination but have the same factor of safety could be different. The hypothesis is based on the fact that although the two slopes have similar factors of safety, they may have failure surfaces of different lengths and orientations. As a result, it is expected that if the two slopes are analyzed in the context of the RFEM, even of identical correlation distances are assigned to the two slopes, the resulting probability of failure could be different due to the different magnitudes of variance reduction along the different failure surfaces. As a result, the findings of the previous sections which were based on a slope angle of 2:1 might not be generalized to any other slope orientation even if the slopes are designed to the same factor of safety.

To test the above hypothesis, the reliability analyses that were conducted in the previous section were repeated for the case of a slope angle of 1:1 (i.e. 45 degrees instead of 26 degrees). Fig. 5.8 shows the geometry of the 45 degree slope. The height of the slope is maintained at 20 m and the unit weight is maintained at 20 kN/m^3.

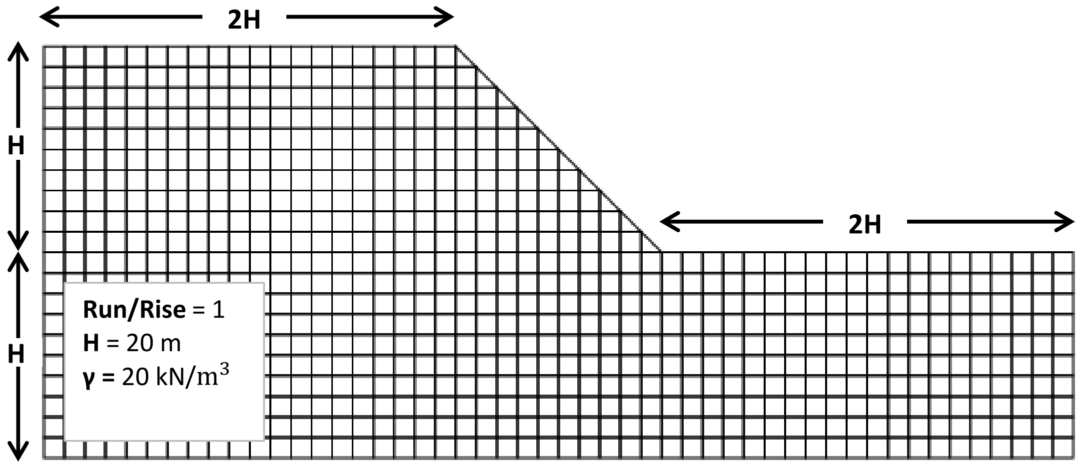

Figure 5.8: Geometry of the slope analyzed for a 1:1 Slope Angle.

The mean undrained shear strength was computed for every factor of safety considered in the first set of analyses. As mentioned above, the main objective behind conducting the same analyses on another slope angle was to investigate the likelihood of obtaining the same probabilities of failure when considering the same factors of safety.

1. Effect of the Slope Angle – Case of No Lower-Bound

The variations of the probability of failure of the slope with the design factor of safety is presented on Fig. 5.9 for the 1:1 slope and the 2:1 slope for comparison. Results are presented for the case of a COV of Su of 0.5, a typical horizontal correlation distance of 40m, and a variable normalized vertical correlation distance θ_y without incorporating the lower-bound shear strength in the analysis.

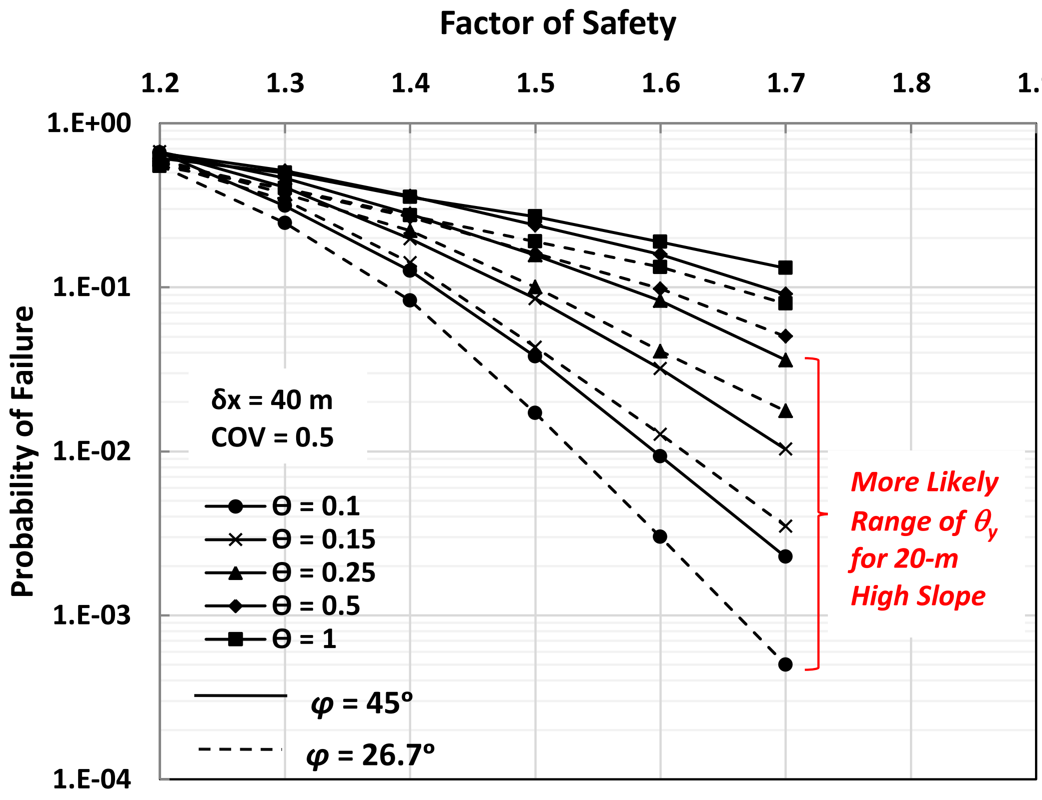

Figure 5.9: Probabilities of Failure versus FS of Clay for Different FS and Vertical Correlation Lengths for Slope Angles of 26.7° versus 45°.

As expected, results on Fig. 5.9 confirm the hypothesis that the probability of failure of a spatially random clayey slope is affected by the angle of inclination of the slope even for the same factor of safety. The steeper slope angle resulted in higher probabilities of failure for the same factor of safety. For instance, at an FS=1.5 and θ_y=0.1, the 2:1 slope yields a failure probability 1.7 % while the 1:1 slope yields a slightly higher probability of failure of 4.3%. As the factor of safety increases, the difference in the probabilities recorded for the different slope angles increases. For the same example of θ_y=0.1, as FS increases to 1.7, the p_f values become 0.05 % and 0.227 % for the 26.7° and 45° slopes, respectively. It should be noted that as the vertical correlation distance increases, the calculated probabilities failure become less sensitive to the angle of the slope provided that the factor of safety is held constant.

The sensitivity of the probability of failure to the angle of inclination of the slope could be attributed to the averaging effect that occurs along the critical slip surface and the associated variance reduction that governs the probability of failure. Vanmarcke (1983) proposed a practical approximation for the variance reduction factor that accompanies the averaging process as follows:

$$\Gamma^2(\mathrm{L_a}) = \delta_\mathrm{v}/\mathrm{L_a} \qquad \text{for } L_a > \delta_v \qquad \text{(Eq.5.1)}$$

Where, δ_v is the vertical scale of fluctuation (estimated as the correlation distance) and $\mathrm{L_a}$ is the length of the averaging interval which designates the failure surface length in the case of a soil slope. Although this equation considers only variance reduction due to the vertical correlation component of the random field, it could explain the small increases that were observed in the probabilities of failure between the two slope angles. For the shallower 2:1 slope, the length of the failure surface for the deterministic case could be estimated to be around 122 m, while the length of the failure surface in the 1:1 slope was around 104 m. The 2:1 slope angle is hypothesized to have resulted in lower probabilities of failure due to the higher variance reduction due to averaging along its relatively longer failure surfaces.

2. *Effect of the Slope Angle – with a Lower-Bound Shear Strength*

The analysis conducted in Section 5.3.1 is repeated for the cases where the lower-bound capacity is included in the random field of S_u. The analysis is conducted for the case of the 1:1 slope angle and the resulting probabilities of failure assuming clay sensitivities ranging from 1.5 to 4 and design factors of safety of 1.3, 1.4 and 1.5 are presented with the results obtained from the shallower 2:1 slope in Fig. 5.10. The results on Fig. 5.10 pertain to the a COV=0.5 for Su and correlation lengths of (a) δ_x=40m and θ_y =0.1, (b) δ_x =40m and θ_y =0.15m, (c) δ_x =100m and θ_y = 0.1 and (d) δ_x =40 m and θ_y =1.0.

The results on Fig. 5.10 indicate that for all the correlation cases adopted, the 1:1 slope (45^o angle) show higher probabilities of failure compared to the 2:1 slope (26.7^o angle) for any fixed factor of safety. The difference in the probabilities of failure is found to be the highest for cases with the less correlated S_u structure (δ_x=40m and θ_y =0.1) and the largest design factor of safety of 1.5. On the other hand, the smallest differences are for highly correlated S_u structures (δ_x=40m and θ_y =1.0) and the lowest factors of safety. More importantly, the difference in the calculated probabilities of failure for the two slopes tends to become significant as the clay sensitivity decreases. For example, for the case of (δ_x=40m and θ_y =0.1) and the largest design factor of safety of 1.5, the probabilities of failure for the case of a sensitivity of 4.0 increases slightly from 1.0% to 2.5% when the slope angle is increased from 2:1 to 1:1. On the other hand, the probability of failure increases by almost an order of magnitude (0.01% to 0.1%) for the case of a clay sensitivity of 2.25. For the more correlated Su structures

(δ_x=40m and θ_y =1.0). The difference between the pf calculated in the 2:1 slope and 1:1 slope still increases with smaller sensitivities at a smaller scale.

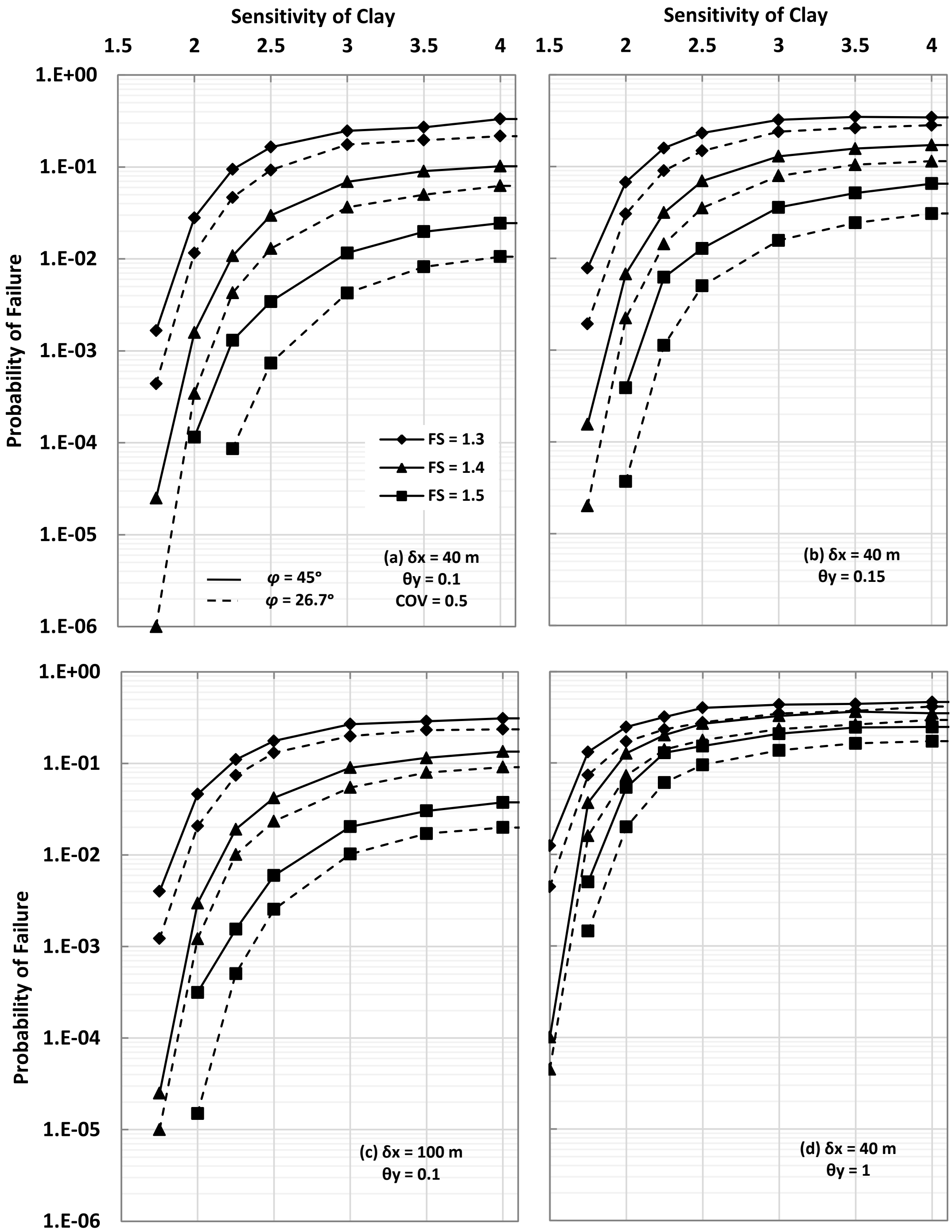

Figure 5.10: Probabilities of failure versus sensitivities for 1:1 and 2:1 slope angles and FS=1.3, 1.4 and 1.5 (a) δ_x=40m and δ_y=2m, (b) δ_x =40m and δ_y=3m, (c) δ_x =100m and δ_y= 2m and (d) δ_x =40 m and δ_y =20 m.

CHAPTER 6

DESIGN RECOMMENDATIONS

The results presented in Chapter 5 indicate that the probability of failure of an undrained clayey slope is expected to be affected by the design factor of safety, the properties of the random field of S_u (COV, θ_y, and δ_x), the sensitivity of the clay, and the geometry of the slope. It is thus anticipated that different factors of safety will be required to yield a target level of reliability in the design of the slope.

In this chapter, an attempt is made to recommend factors of safety that would result in target probabilities of failure for spatially random clayey slopes for the case where the COV of Su is equal to 0.5. Required factors of safety are recommended for a practical target probability of failure of 1%. The sensitivity of the resulting factors of safety to the assumed target probability of failure, vertical and horizontal correlation distances and inclination of the slope is investigated. The required factors of safety were calculated by utilizing the results of the reliability analyses conducted in Chapter 5 in addition to several additional analyses which were required to supplement the existing data to be able to produce the recommended design curves.

D. Required Factors of Safety for 2:1 Slope and Target p_f of 1%

The factors of safety that are required to achieve a target probability of failure of 1% for the 2:1 slope for cases where the random field of Su is defined by an average horizontal correlation distance of 40m were calculated and plotted in Fig. 6.1 for different normalized vertical correlation lengths. The results were plotted in the form of curves that relate the required factor of safety to the clay sensitivity for a fixed vertical correlation length. These curves allow the user to the select the design factor of safety needed to achieve slope designs with a probability of failure of 1% given information regarding the vertical correlation length and the sensitivity of the clay.

Results on Fig. 6.1 indicate that design factors of safety in the range of 1.5 (θ_y = 0.1) to 1.9 (θ_y = 0.5) are required to achieve the desired reliability level for cases involving clays of relatively high sensitivity, where the impact of the lower-bound shear strength on the reliability is expected to be minimal. More importantly, results indicate that the required factor of safety decreases as the sensitivity of the clay decreases. For example, for a typical normalized vertical correlation distance of θ_y = 0.1, the required factor of safety decreases from about 1.5 for the case where the sensitivity of the clay is high (~ 5.0) to a value of 1.3 for the case with a typical clay sensitivity of 2.0. For more correlated random fields of Su (example θ_y = 0.5), a more significant reduction in the required FS is observed with FS values decreasing from 1.9 to 1.5 as the clay sensitivity is decreased from around 5.0 to 2.0.

The results point to the importance of including a lower-bound shear strength that is based on the clay sensitivity in the slope stability design problem. Information about the sensitivity of a clay can be obtained using simple and inexpensive field or laboratory tests (ex. field vane, torvane, minvane, and triaxial tests) or even from correlations with other index properties (ex. the liquidity index). By including the remolded undrained shear strength of the clay as a lower-bound for the uncertainty in

Su, reduced factors of safety could be adopted in the design of a slope without sacrificing any additional risk. In other words, for a given slope design, slopes with steeper angle or larger heights could be adopted while maintaining the target reliability level for the slope design.

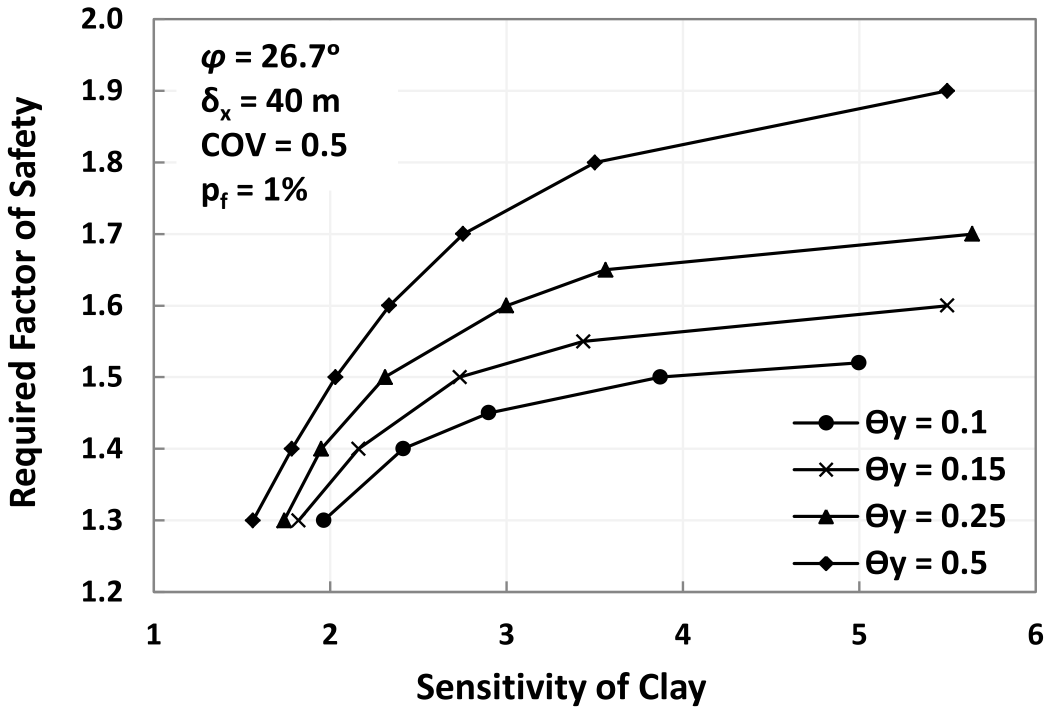

Figure 6.1: Required Factors of Safety versus Sensitivity for 2:1 Slope with δ_x = 40 m and COV=0.5 (design charts for target Probability of Failure of 1 %).

The analysis presented in Fig. 6.1 was repeated for the cases where the vertical correlation distance was fixed at θ_y = 0.1 while varying the horizontal correlation distance from □$_y$ = 30m to 100m. The resulting design factors of safety that would yield the target probability of failure of 1% are presented in Fig. 6.2.

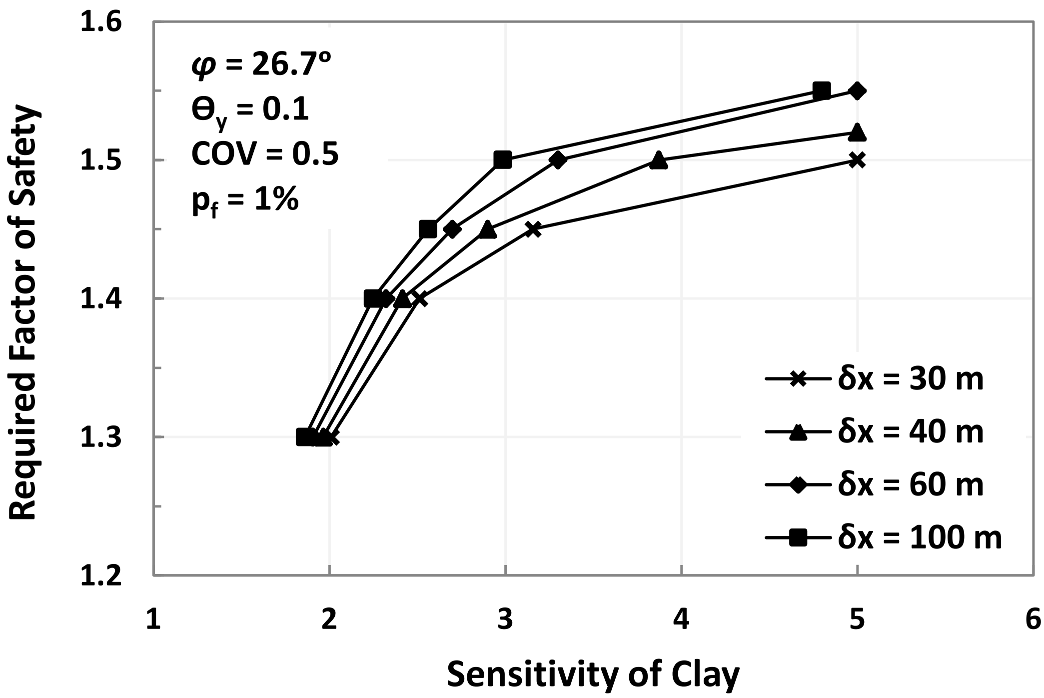

Figure 6.2: Required Factors of Safety versus sensitivity for 2:1 Slope with Θ_y = 0.1 and COV=0.5 (design charts for target Probability of Failure of 1 %).

Compared to the results of Fig. 6.1 where the vertical correlation length was varied, it could be observed that the required factor of safety on Fig. 6.1 is not sensitive to horizontal correlation distance as much as it was for the vertical correlation distance. For example, for the case of clays with relatively high sensitivities (minimal lower-bound effect), the required FS is shown to vary in the narrow range of 1.5 to 1.55 as the horizontal correlation distance is varied from 30m to 100m. This range of FS is much smaller than the range of 1.5 to 1.9 which was observed for the case where the vertical correlation distance was varied between 2m and 10m.

As was the case in Fig. 6.1, results in Fig. 6.2 also point to the important role that the lower-bound could play in decreasing the required factor of safety. In fact, it could be concluded from the data on Fig. 6.2 that factors of safety as small as 1.3 could be used to achieve the desired reliability level if the clay has a sensitivity in the order of 2.0. This observation is unaffected by the value of the horizontal correlation distance adopted in the design (see Fig. 6.2).

E. Effect of Target Reliability Level on the Required Factors of Safety

To illustrate the impact of the target reliability level on the required factors of safety, cases where a probability of failure of 5% is targeted were analyzed for the 2:1 slope for the realistic case of δ_y = 40m. The required factors of safety for cases involving normalized vertical correlation distances of 0.15 and 0.50 were determined and plotted on Fig. 6.3 together with associated results pertaining to the cases where the lower probability of failure of 1% was targeted.

As expected, results on Fig. 6.3 indicate that lower factors of safety are needed to ensure slope designs with probabilities of failure of 5%. For the case of clays with high sensitivity (minimal lower bound effect), the required factors of safety decrease from 1.9 to 1.7 (for θ_y = 0.15) and from 1.6 to 1.5 (for θ_y = 0.5) as the target probability of failure is increased from 1% to 5%. For cases with relatively lower clay sensitivities (ex. sensitivity = 2.0), the reduction in the magnitude of the required factor of safety for higher probabilities of failure (p_f = 5%) becomes less. It is worth noting that even for a relatively high target probability of failure of 5%, the required factor of safety is observed to be very sensitive to the assumed value of the clay sensitivity, with significant possible reductions in the required factor of safety due to the incorporation of a relatively high lower-bound shear strength (low sensitivity) in the reliability problem.

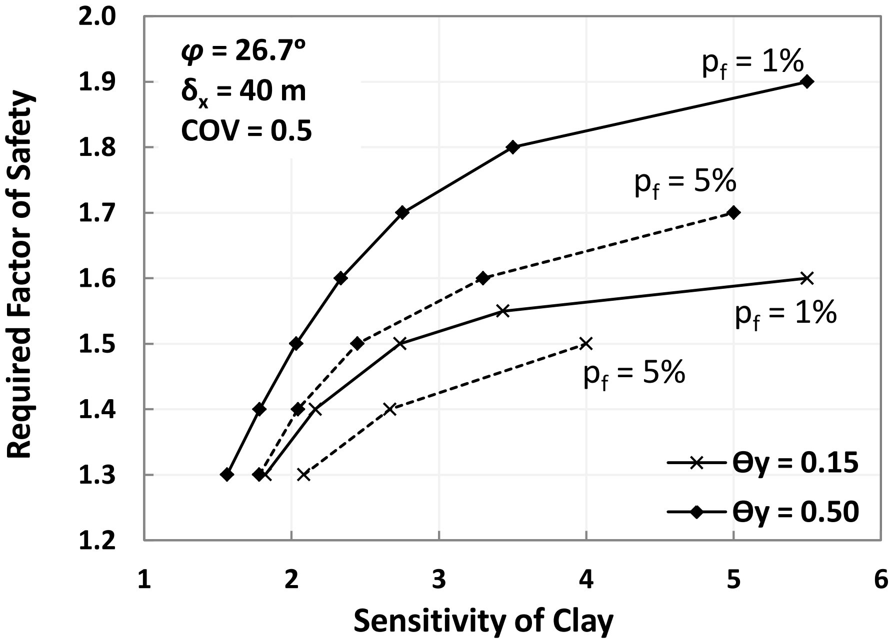

Figure 6.3: Required Factors of Safety versus sensitivity for 2:1 Slope with δ_x = 40m and COV=0.5 (design charts for target Probabilities of Failure of 1% and 5%).

F. Effect of Slope Angle on the Required Factors of Safety

Since the probability of failure of a slope was shown in Chapter 5 to be slightly sensitive to the adopted slope angle, required factors of safety were determined and compared for the 2:1 slope and the 1:1 slope in Figs. 6.4 and 6.5. In Fig. 6.4, required factors of safety are shown for a fixed δ_x of 40m, for the two cases with a vertical normalized correlation distance θ_y of 0.1 and 0.25. In Fig. 6.5, similar results are presented for the case with a fixed $\theta_y = 0.1$ and δ_x of 40m and 100m. All results are shown for the case where the COV of Su = 0.5.

Results on Figs. 6.4 and 6.5 indicate that the slope angle has an slight effect on the required factor of safety, particularly for cases involving clays of high sensitivity (greater than 3.0) where the required factor of safety is found to increase by an increment of 0.1 (1.5 to 1.6 for $\theta_y = 0.1$ and 1.7 to 1.8 for $\theta_y = 0.25$) for the steeper 1:1 slope. The differences in the required factors of safety become less as the clay sensitivity decreases to about 2 where the difference between the required factors of safety in the two slopes becomes small, particularly for the higher θ_y of 0.25. Similar results are observed in Fig. 6.5 where the horizontal correlation distance is increased from 40m to 100m, with relatively small differences being observed in the required factors of safety, particularly for high sensitivity levels.

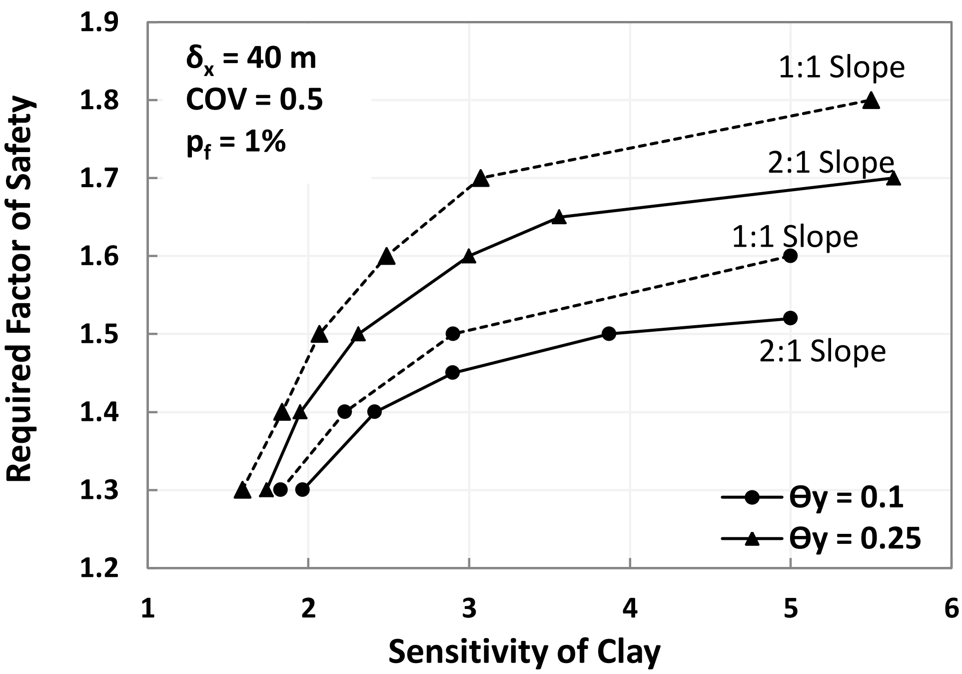

Figure 6.4: Required Factors of Safety versus sensitivity for 1:1 and 2:1 Slopes with δ_x = 40m and COV=0.5 (design charts for target Probabilities of Failure of 1%).

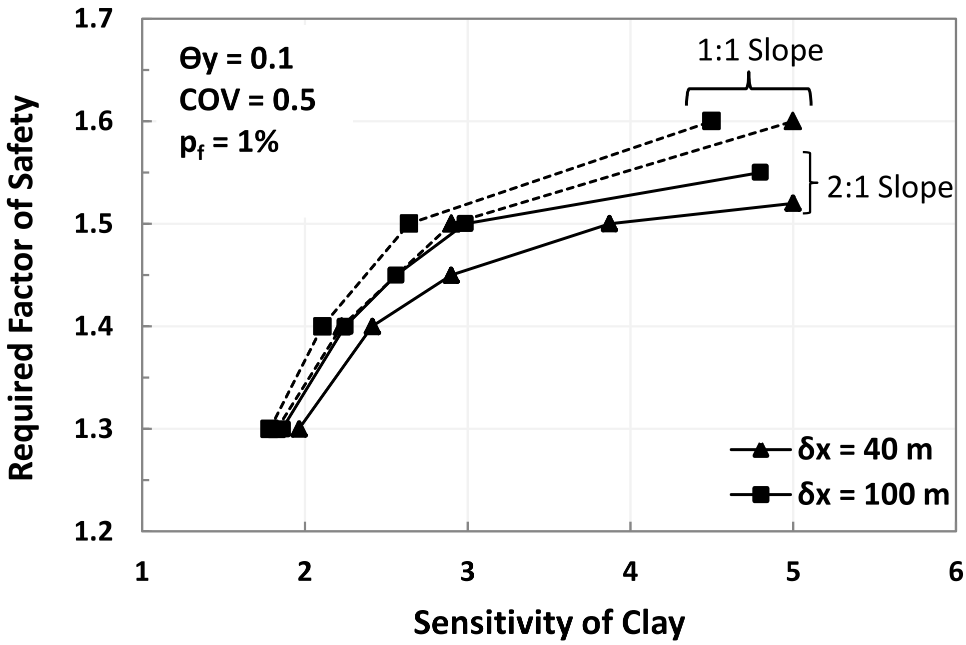

Figure 6.5: Required Factors of Safety versus sensitivity for 1:1 and 2:1 Slopes with $\theta_y = 0.1$ and COV=0.5 (design charts for target Probabilities of Failure of 1%).

CHAPTER 7

CONCLUSIONS

The RFEM was utilized in this study to investigate the effectiveness of incorporating a physical lower-bound shear strength in the probability model describing the uncertainty in the undrained shear strength of a clayey slope. The basis for the existence of the lower bound shear strength is the fact that the remolded undrained shear strength of a clay constitutes the minimum possible value of strength for that particular clay. The remolded strength is reflected in the sensitivity of the clay which generally varies from 1.5 to 4 in clays of low to average sensitivity.

In addition, RFEM was performed to investigate the sensitivity of the reliability of the slope to the random field characterizing the spatial variability of S_u. Typical ranges of the correlation length were considered while mapping anisotropic fields to the generated mesh. The anisotropy was represented by horizontal correlation lengths ranging from 20 to 100 m and vertical correlation lengths ranging from 2 to 20 m. Results of the reliability analysis in which the mean, COV and correlation lengths of the undrained shear strength in addition to the sensitivity of the clay were varied lead to the following major conclusions:

1. For the example undrained slope problem that was utilized in this paper, slopes that are spatially variable have a smaller probability of failure compared to slopes that are homogeneous, assuming a given design factor of

safety. As the correlation distances decrease, the probabilities of failure of the slope were also found to decrease. This result is expected given the spatial averaging and variance reduction that are expected to occur in the spatially variable slope.

2. The incorporation of a lower-bound shear strength in the reliability analysis could have a significant effect on the risk of failure of the slope. The effect is confined to reducing the probability of failure and is more significant in cases involving spatially random soils with relatively higher factors of safety and higher COVs of S_u. The reduction in the probability of failure increases as the sensitivity of the clay decreases and could reach orders of magnitude for sensitivities in the order of 1.5. This reduction in the probability of failure in the presence of a lower-bound shear strength is attributed to eliminating unrealistically low shear strength values that are theoretically lower than the lower-bound shear strength. This reduces the number of cases that fail in the Monte Carlo simulations thus reducing the probability of failure of the slope.
3. The effect of the lower-bound Su is more pronounced whenever spatial variability is increased and that is due to the rapidly changing weak and strong zones of soil over short distances of the slope field for which the preferred failure mechanism tends to be global (occupies wide zones of the soil field). As the correlation length is increased, the preferred mechanism is attracted to "local pockets" of the weak soil and as a result occurs on a reduced width. If the failure mechanism occurs locally, it has more opportunities to fail at different locations along the slope length direction which naturally leads to higher pf.

4. The geometry effect was analyzed by changing the slope angle. It could be said that the increase in the slope angle leads to slightly higher probabilities of failure and this can be attributed to the averaging effect that occurs along the critical slip surface. Smaller lengths of the failure surface were obtained for the steeper slope angle, leading to smaller variance reduction in the COV characterizing the average undrained shear strength.
5. Finally, it could be concluded based on the reliability analyses conducted in this paper that for a given target reliability level, the required factor of safety of a slope could be decreased as the lower-bound shear strength is increased in the reliability analysis and as the slope angle and correlation lengths are decreased.

It should be noted that in all the analyses conducted in this research study, the only source of uncertainty that was incorporated in the analysis was the uncertainty in the undrained shear strength s_u. In addition, no effort was made to incorporate the effect of the model uncertainty in the reliability analysis. The model uncertainty is always present in geotechnical engineering problems and is expected to add to the total uncertainty in the slope stability predictions. Combining model uncertainty and uncertainty due to spatial variability in soil properties is the subject of another research study that is currently being implemented at the American University of Beirut.

Appendix A – Results of Reliability Analysis for 1:1 Slope

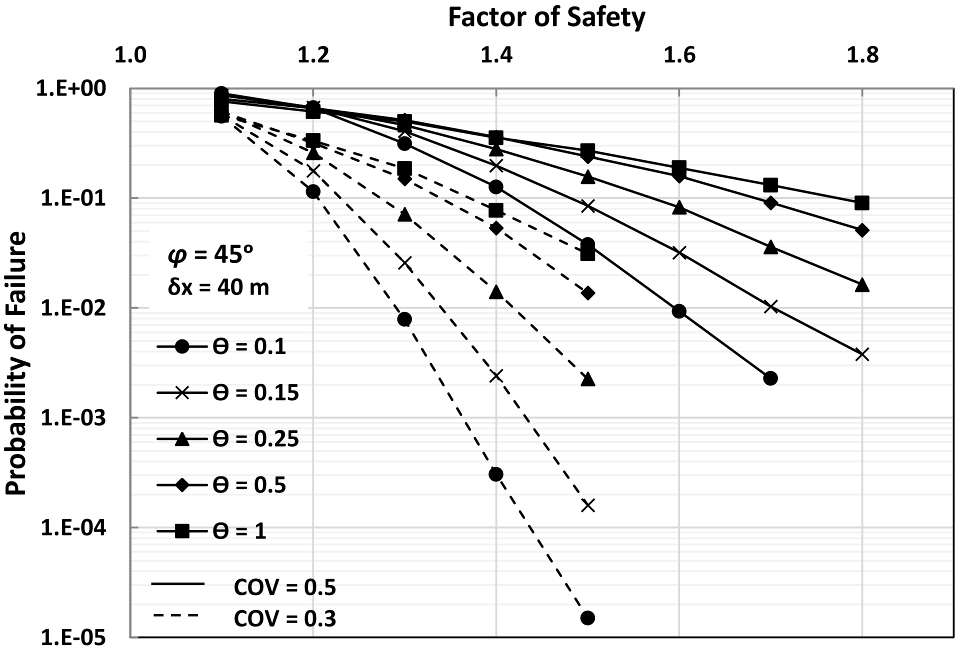

Fig A-1 Probabilities of Failure versus FS of Clay for Different Vertical Correlation Lengths; FS =1.3 and COV = 0.3 and 0.5.

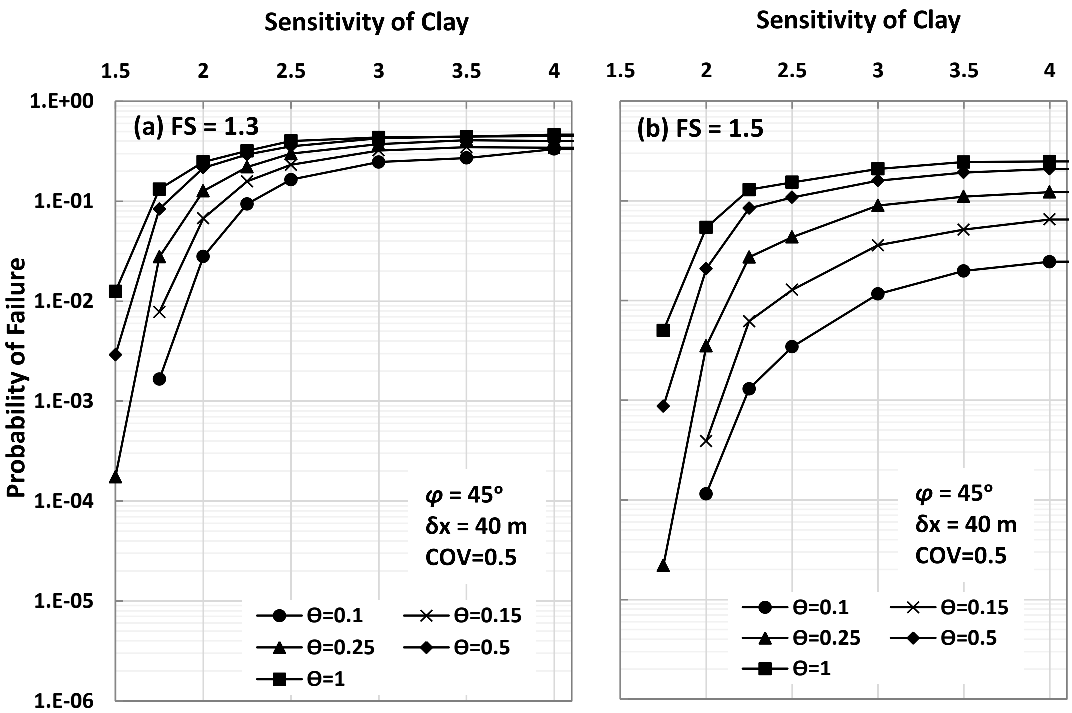

Figure A-2: Probabilities of Failure versus Sensitivity of Clay for COV=0.5, δ_x=40 m,

δ_y varied between 2 and 20m; FS = (a) 1.3 and (b) 1.5.

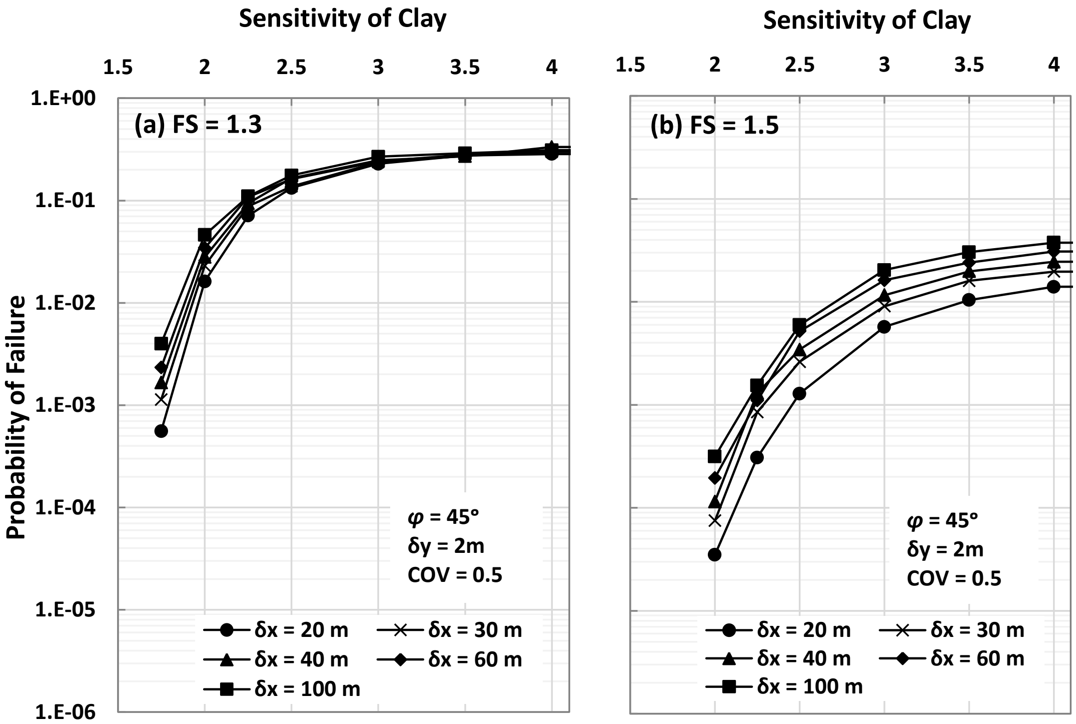

Figure A-3: Probabilities of Failure versus Sensitivity of Clay for COV=0.5, δ_y= 2 m,

δ_x varied between 20 and 100 m; FS = (a) 1.3 and (b) 1.5.

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