A SIMPLIFIED RATIONAL DESIGN METHOD
FOR
TORSION IN PRESTRESSED CONCRETE MEMBERS.

MUBADDA TEWFIC SUWAYDAN, B.C.E.

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Approved:

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Sami Abbud Klink
Adviser

American University of Beirut

Beirut , Lebanon

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ABSTRACT

This paper presents a rational study of prestressed concrete sections in bending, torsion and shear, based on the Maximum Stress Theory. It concludes with a suggested procedure for analysing such sections.

A general look at the problem of torsion in building construction, together with a review of important investigations and existing code requirements are given in Chapter I. This is followed in Chapter II by Theoretical Considerations based on the Maximum Stress Theory which are developed into two nondimensional relationships for prestressed concrete sections in bending, torsion and shear.

To check the validity of these relationships, a series of experiments was devised whereby prestressed test beams were loaded to failure with bending and torsion. These experiments are described in Chapter III and analysed in Chapter IV. The derived relationships were found to be valid.

Chapter V presents a suggested procedure for the analysis of problems of this kind and illustrates this procedure with a solved example. Chapter VI deals with suggested topics for further research in this field.

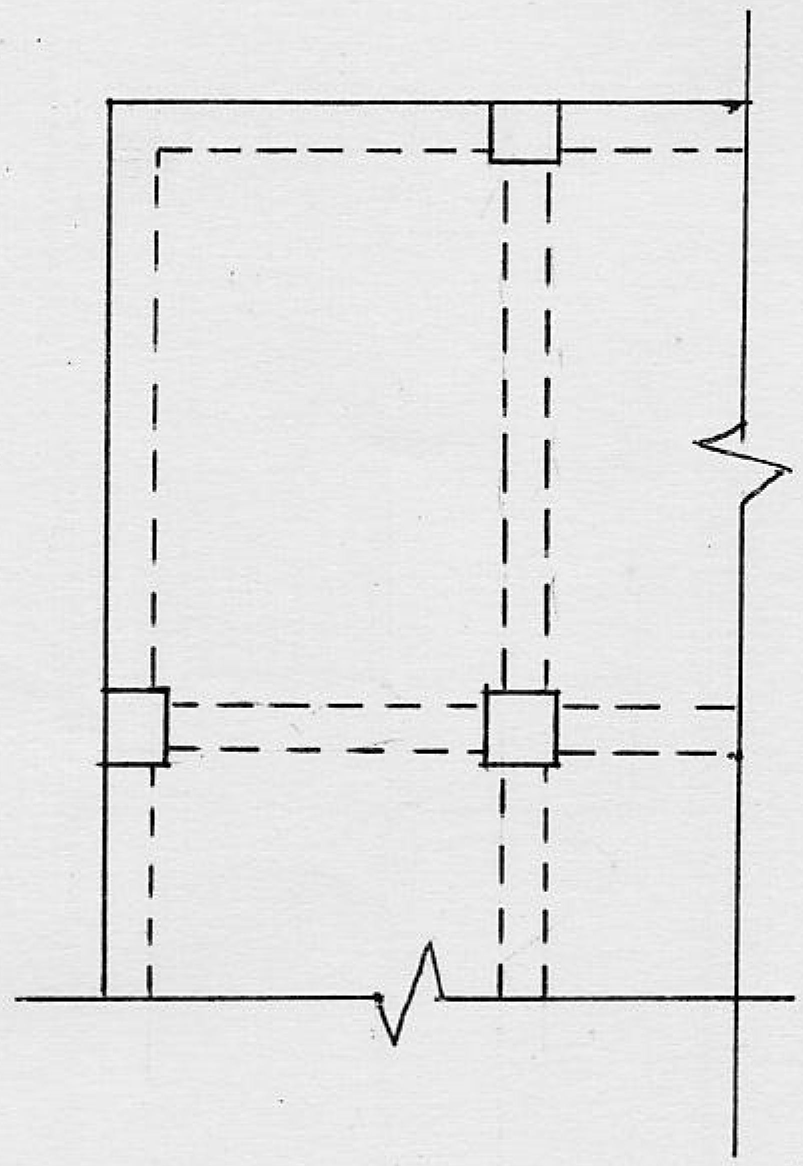
CHAPTER I

INTRODUCTION

Torsion is generally of secondary importance in prestressed or reinforced concrete structural members of buildings. It is therefore seldom mentioned in textbooks. When referred to, it is usually treated by means of approximate relationships. Few building codes make any reference to it. When confronted with a structure entailing torsional design, designers sometimes try to modify the basic layout of a structure to eliminate torsion. When it is not possible to eliminate torsion, approximate solutions are adopted leading to very large factors of safety.

The monolithic character of in situ concrete structure frequently results in torsion. Some such cases resulting in torsion are : floor beams joining onto a main beam at a point other than a column, asymmetry in the loading of a member. Torsion is inevitable in space frames. It also occurs when eccentric horizontal loading is applied to a structure. Figure 2 illustrates some of the above examples.

Sometimes twisting moments exercise a controlling influence over the design of structural members, i.e. edge beams, balcony girders, staircases without intermediate supports and curved and spiral staircases. The approximate design for stairs without intermediate supports generally results in heavy sections that defeat the architectural purpose of lightness. In screwed piles, torsion is predominant and would require quite heavy sections in plain reinforced concrete.


(a)

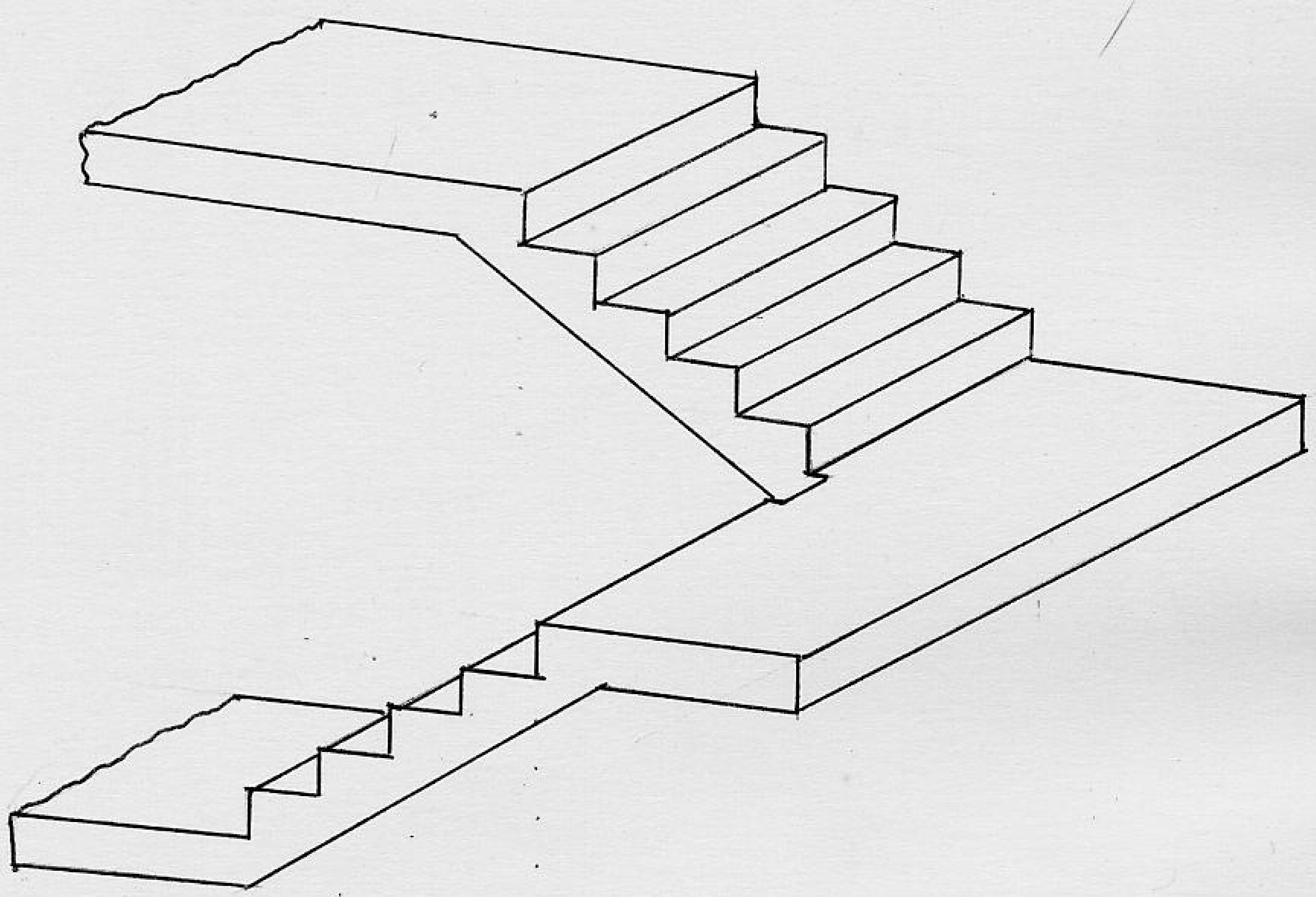

(b)

Fig. 1. Diagrams Indicating Two Cases of Structural Elements Where Torsion Has Controlling Influence on Design.

Torsion acts on a member by inducing shear stresses which result in diagonal tension. As concrete is relatively weak in tension, concrete members do not have any torsional rigidity or strength worth speaking of. Reinforcing steel in the form of transverse and longitudinal bars has to be introduced to take the tensile stresses induced in the member by the applied torque.

The situation is different in prestressed concrete as prestressed concrete has the inherent advantage of having an initial compressive stress that has to be overcome before the

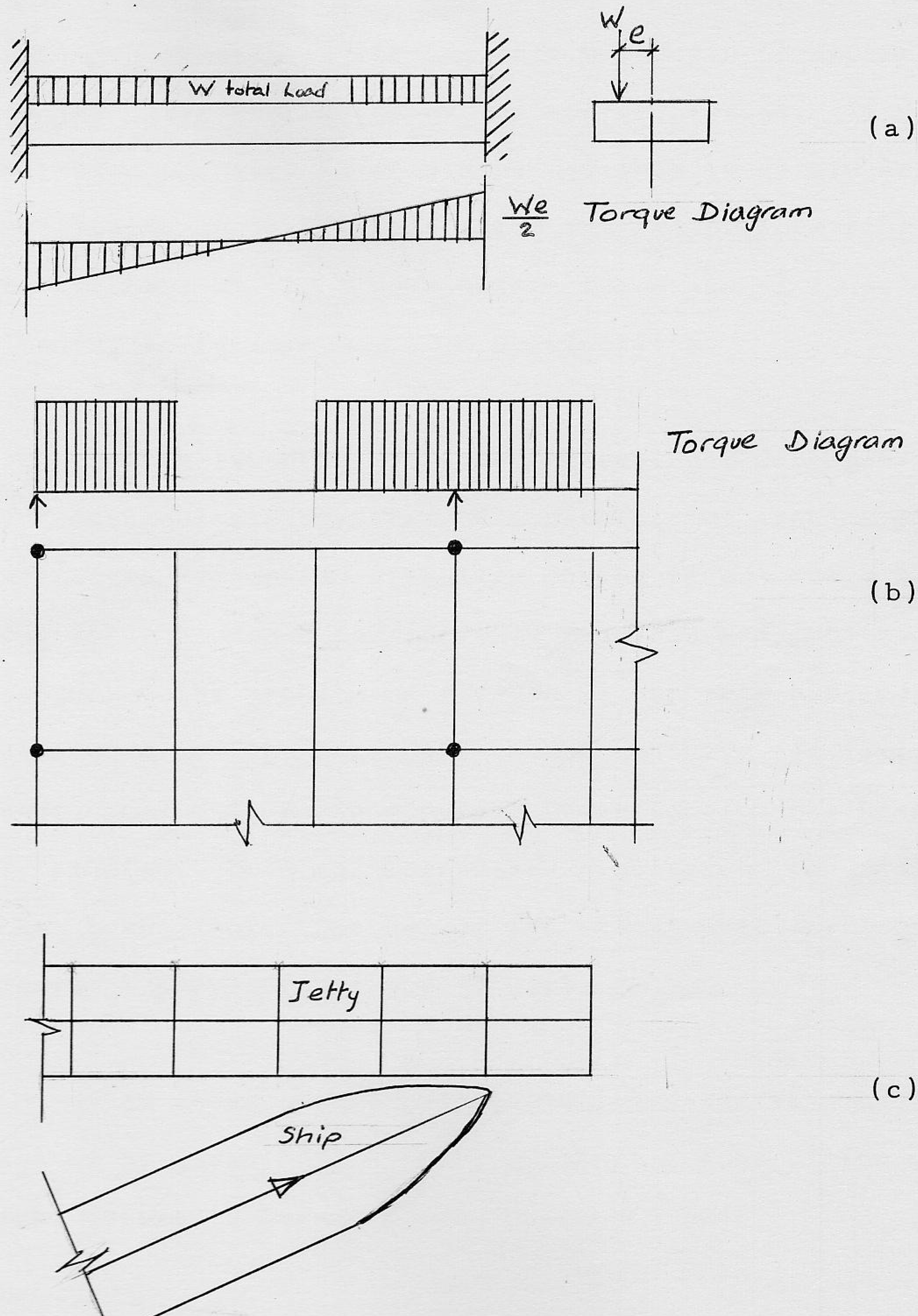


Fig. 2. Twisting Moments Applied to Structures by (a) Eccentric Loading on a Member, (b) Secondary Beams Joining Primary Beams at a Point Other Than a Column, (c) Eccentric Loading on a Structure.

diagonal tension in the concrete may begin to appear as a result of the shear introduced by the applied torque. If double prestressing is used it should be possible to attain an even higher resistance to torsion, as there are two prestressing forces to be overcome by the torque before the diagonal tensile stresses leading to failure begin to be effective.

Largely because of the increasingly daring use of concrete, and particularly prestressed concrete, the problem of torsion has attracted widespread attention during recent years which has consequently produced considerable research and publication on the subject. The problem of torsion in concrete has, however, been studied since the beginning of the century. It is felt[6], however, that only plain concrete under torsion has been exhaustively studied. Both reinforced and prestressed concrete under torsion have been studied but not to the extent enabling one to claim a thorough knowledge of the subject.

An index of all papers published on the subject is presented by Cowan[6]. A brief outline of some of the more important papers is herein quoted and described:

C. Bach and O. Graf: "Versuche über die Widerstandsfahigkeit von Beton und Eisenbeton gegen Verdrehung (Tests on the Resistance of Plain and Reinforced Concrete to Torsion)", Deutsher Ausschuss für Eisenbeton, Heft 16, (1912).

"Carried out at the laboratories of Stuttgart Institute of Technology, this was the first thorough investigation, and it has been the basis of much of the subsequent theoretical work. Specimens of square and rectangular cross-section were tested, reinforcement consisting of longitudinal bars, inclined bars and of longitudinal bars and spiral reinforcement. Longitudinal bars increased twisting moment by 9 per cent, spirals by a further 125 per cent. All photographs of spirally reinforced specimens show a continuous helical fracture, at right angles to the spiral reinforcement, and a very large increase in the angle of twist at failure."

T. Miyamato: "Torsional Strength of Reinforced Concrete", Concrete and Constructional Engineering, 22, (1927)

"86 circular specimens, with ten different combinations of reinforcement, were tested at the structural engineering laboratory of the Japanese Home Department. All specimens cracked at 45^{o} to the axis. The reinforcing bars were not fully stressed on failure, and they did not alter the angle of twist. Increases in ultimate strength of up to 118 per cent over plain concrete were obtained with 2 per cent of reinforcement".

E. Rausch: Berechnung des Eisenbetons gegen Verdrehung und Abscheren (Design of Reinforced Concrete for Torsion and Shear). Dissertation presented to the Technische Hochschule, Berlin, (1929).

"Derives an equation for the design of spiral reinforcement in circular and non-circular sections subjected to torsion and compares it with the experimental data of Bach Graf, and Mörsch."

P. Andersen: "Experiments with Concrete in Torsion". Proc. American Society of Civil Engineers, 60 (1934).

"Tests on 6 plain concrete circular specimens, and on 42 square specimens, reinforced with various combinations of longitudinal bars, circular hoops and circular spirals. Andersen appears to have been the first investigator to make measurement of concrete strains. On the basis of the tests he derives a formula which is similar to Rausch's, but better related to stress-distribution for rectangular specimens."

P. Andersen: "Rectangular Concrete Sections under Torsion". Journal of the American Concrete Institute, 9, (1937).

"Tests on a further 24 specimens with variation of concrete strength. It is shown that raising of compressive strength does not produce a proportional increase in torsional strength".

P. Andersen: "Design of Reinforced Concrete in Torsion." Proc. American Society of Civil Engineers, 63 (1937).

"Contains a further discussion of the Andersen formula showing that the strength of the reinforced concrete section, contrary to Rausch's theory, is approximately the strength of the plain concrete plus the additional strength of the reinforcement. This paper also contains a theoretical discussion of the various steps of design, including the application of the moment distribution method to space frames."

A. Ruchadze: "Torsion and Deformation by Shear Forces of an Elastic Beam, Consisting of Two Different Materials with Epitrochoidal Boundaries." Proceedings of the Mathematical Institute of Tbilissi, 1 (1937).
"An extension of St. Venant's torsion theory. It is claimed in the Russian summary that the method can be applied to reinforced concrete.

W.T. Marshall and N.R. Tembe: "Experiments on Plain and Reinforced Concrete in Torsion." Structural Engineer, 19 (1941).
"26 plain concrete specimens of circular, rectangular, T- and L-section and 24 rectangular and T-beams with longitudinal and stirrup reinforcement. Observes a plastic redistribution of stress in noncircular sections. A comparison is made with Rausch's formula, but the steel stress at failure was nowhere above the yield point, and failure appeared to be caused by a breakdown in bond."

W.T. Marshall: "The Torsional Resistance of Plastic Materials with Special Reference to Concrete." Concrete and Constructional Engineer, 39 (1944).
"Previous experimental work is analysed to show that consistent values for the ultimate torsional shear stress are obtained if concrete is assumed to be an ideal plastic material."

H. Nylander: "Vridnig ock Vridnigsinspänning vid Betongkonstruktioner (Torsion and Torsional Restraint of Concrete Structures). Statens Kommitte för Byggnadsforskning, Meddelanden Nr. 3 Stockholm 1945.

"Tests on 60 rectangular and T-shaped specimens, some reinforced, subjected to pure torsion, combined bending and torsion, combined shear and torsion and combined compression and torsion. In addition, two rigid frames consisting of three parallel beams connected by two cross girders at the ends were tested by applying two concentrated loads to the central beam. Nylander concludes that the stress distribution at failure is such that the concrete may be treated as a plastic material for the purpose of assessing ultimate torsional strength; this is particularly evident on T-sections. Transverse shear forces decrease torsional strength."

H.J. Cowan: "Elastic Theory for Torsional Strength of Rectangular Reinforced Concrete Beams." Magazine of Concrete Research, 2 (1950).

"The theories of Rausch and Andersen are examined, and a more accurate formula based on strain-energy considerations is derived."

H.J. Cowan: "The Strength of Plain, Reinforced and Prestressed Concrete under the Action of Combined Stresses, with Particular Reference to the Combined Bending and Torsion of Rectangular Sections." Magazine of Concrete Research, 5 (1952).

"A theory for the strength of concrete and reinforced concrete under combined stresses is advanced, which combines Rankine's maximum principle stress criterion and Coulomb's

internal friction criterion. Equations for the strength of plain, reinforced and prestressed concrete in combined bending and torsion are derived. The distinct difference between primary bending and primary torsion fracture is explained by the dual character of the criterion of failure. It also accounts for the increase in torsional strength resulting from the addition of bending."

K. Schaden: "Die Riss-und Bruchlast des auf reine Verdrehung beanspruchten Stahl-und Spannbetons (Cracking and failing loads of reinforced and prestressed concrete)." Osterreichische Bauzeitung, 8, (1953).
"A theoretical treatise based on principal stresses."

H.J. Cowan and S. Armstrong: "Experiments on the Strength of Reinforced and Prestressed Concrete Beams and of Concrete-encased Steel Joists in Combined Bending and Torsion," Magazine of Concrete Research, 7 (1955).
"32 rectangular beams were tested in combined bending and torsion, with various ratios ranging from pure torsion to pure bending. Deflections and strains in both the steel and the concrete were measured. Agreement between theory and experiment was generally good."

H.J. Cowan and S. Armstrong: "The Torsional Strength of Prestressed Concrete." Proc. World Conference on Prestressed Concrete, San Francisco, (1957).
"Experiments on 9 rectangular beams subjected to combined bending and torsion, both uniform and eccentric prestressing being used. Prestressing greatly increases the

torsional strength of concrete, but the failure is sudden and destructive."

R. Humphreys: "Torsional Properties of Prestressed Concrete," The Structural Engineer, 35 (1957).

"Reports tests on 94 uniformly prestressed rectangular concrete beams. Ultimate strength depended on the principal tensile stress and was not preceded by any appreciable cracking.

Akademiia Stroitel'stva i Arkhitektury SSSR. Institut Betona i Zhelezobetona (Russian Academy of Building and Architecture. Institute for Concrete and Reinforced Concrete). Trudy,5 (1959).

(i) N.N. Lessig: "Determination of the Load-bearing Capacity of Reinforced Concrete Elements with Rectangular Cross-section subjects to Flexure and Torsion."

(ii) Yu.V. Chinenkov: "Study of the Behaviour of Reinforced Concrete Elements in Combined Flexure and Torsion."

(iii) I.M. Lyalin: "Experimental Studies of the Behaviour of Reinforced Concrete Beams with Rectangular Cross-section subjected to the Combined Action of Transverse Shear, Flexural and Torsional Moment."

"A new theory based on ultimate strength considerations, which considers two cases: (a) the beam is over-reinforced in both the longitudinal and the transverse directions, the yield stress is not reached in any of the steel bars, and the beam fails through failure of the concrete in compression; and (b) the beam has an excess of either longitudinal or transverse reinforcement, and failure is initiated by yielding of the tension steel in one of the two directions. 72 beams were tested in combined bending and torsion, and the deflection and strain measurements are recorded."

R.P.M. Gardner: "The Behaviour of Prestressed Concrete I-beams under Combined Bending and Torsion." Cement and Concrete Association Tech. Rep. TRA/329 (1960).
"Reports on the second stage of the Cement and Concrete Association investigation: tests on 16 prestressed concrete I-beams, prestressed eccentrically, subjected to various ratios of combined bending and torsion, varying from T/M= 0 to T/M = 1. This is the first investigation on nonrectangular prestressed beams."

P. Zia: "Research in Torsion of Prestressed Members." Jnl. Prestressed Concrete Institute, 5 (1960).
"Tests on 68 prestressed specimens of rectangular, T-and I-shape, some with non-prestressed hoop reinforcement, the only investigation on record which examined the effect of web reinforcement in prestressed concrete. Zia proposes a Modified Cowan criterion, joining the intersection of the Coulomb limiting lines with the shear axis and the maximum tensile stress intercept with straight lines. The ultimate strength of a prestressed member with web steel is equal to the sum of the cracking moments of the member and the moment resisted by the web reinforcement."

N. Swamy: "The Behaviour and Ultimate Strength of Prestressed Con-

crete Hollow Beams under Combined Bending and Torsion." Magazine of Concrete Research, 14 (1962).

J.S. Reeves: "Prestressed Concrete Tee-Beams under Combined Bending and Torsion." Cement and Concrete Association Tech. Rep. TRA/364, (1962).

"44 beams were tested. Three criteria were examined: Octahedral stress criterion, maximum stress criterion with elastic stress distribution, and maximum stress criterion with plastic stress distribution. The last gave the best agreement. All beams were prestressed eccentrically, and a small amount of bending increased the torsional strength."

H. Gesund and L. Boston: "Ultimate Strength in Combined Bending and Torsion of Concrete Beams Containing only Longitudinal Reinforcement." Journal of the American Concrete Institute, 61, (1964).

"The authors tested a number of beams having varied reinforcement and concrete strength and arrived at a model of the failure mechanism whereby the longitudinal reinforcement acts about the hinge on the compression face of the beam to resist torsion. Dowel action by the longitudinal reinforcement is credited for an observed increase of strength of the beam against torsion."

H. Gesund, F. Schuette, C. Buchanan, and G. Gray: "Ultimate Strength in Combined Bending and Torsion of Concrete Beams Containing both Longitudinal and Transvere Reinforcement", Journal of the American Concrete Institute , 61, (1964).

The main conclusions of this investigation are that transverse reinforcement transforms torque on a reinforced concrete beam into additional bending moment and that the dowel action of the longitudinal reinforcement frequently provides greater torsional resistance than the transverse reinforcement.

R. Evans and S. Sarkar: "A Method of Ultimate Strength Design of Reinforced Concrete Beams in Combined Bending and Torsion." The Structural Engineer, 43, (1965).

The authors carried out a detailed study on hollow reinforced concrete beams under combined moments and torques. A simplified theoretical approach is presented for predicting the strength of reinforced concrete beams under combined loading.

The author has been unable to find published reference to any specification or code documents pertaining to the design of prestressed concrete under torsion or combined loadings. One paper[9] by G. Fisher and P. Zia presents a general review of twenty-two codes as regards torsion in plain and reinforced concrete. Of the twenty two codes reviewed, only eight were found to have reasonably satisfactory coverage on the subject. Those were the French (1960), Egyptian (1960), German (1959), Australian (1958), G.S.A. (1956), Polish (1956), Russian (1955) and Hungarian (1953) codes.

All the above mentioned codes with the exception of the Polish code calculate torsional shear stresses by formulae based on the elastic theory of St. Venant. They use formulae of the general form of

$$U = \frac{T}{bd^2} K \qquad \text{where } T \text{ is the torque}$$

with varying, or fixed, values of K.

The Polish code, however, uses a low value of K based on unquoted research. This is indicative of a plastic redistribution of shear stresses over the whole section.

Most of the codes in question generally specify the permissible torsional shear stresses to be the same as those for flexural shear. The German and Greek codes, however, specify lower permissible stresses. Further, with the exception of the Polish and Austrian codes, the others call for factors of safety of about 3 for low strength concrete to about 4.5 for high strength concrete. The Polish code, however, has factors of reserve strength which compensate for the low factor safety specified, (a little over 2).

As regards the design for torsional reinforcement, it is based in all codes on the elastic theory and is quite similar. The Australian code has been influenced by Cowan whereas Rausch is the influence for the German, Egyptian, Hungarian, Polish and Russian codes. Both Cowan's and Rausch's analyses may be represented [3] in the form of

$$T = \frac{\lambda\sqrt{2}\, A \cdot A_t \cdot t}{s}$$

where A = Area contained by the spiral reinforcement.

A_t = Cross sectional area of one spiral wire.

t = Maximum permissible stress in the spiral reinforcement.

s = Pitch of the spiral reinforcement parallel to the axis of the beam.

Rausch uses a value of $\lambda = 2$ whereas Cowan uses a corresponding value of $\lambda = 1.6$. Rausch further introduces a concept that the stress at a point is proportional to the distance of the point from the centroid of the section. Cowan's theory is based on total energy considerations and designs based on this theory tend to require more reinforcement than those based on Rausch's theory.

While it may be possible to suggest code specifications for prestressed concrete sections based on the existing reinforced concrete specifications, these will not be quite pertinent due to the inherent nature of prestressed concrete as regards homogenuity and strength.

Investigators have generally proposed solutions for the problem of torsion in reinforced concrete members based on one or combinations of more than one of the following failure theories: the maximum stress theory, the maximum strain theory, and the maximum shear theory. The author intends to develop a simple rational design approach to the problem of torsion in prestressed concrete members.based only on the maximum stress theory and to check its validity by experiment.

CHAPTER II

THEORETICAL CONSIDERATIONS

A material subject to combined stresses may be analyzed by choosing three mutually perpendicular planes at any point that are subject to normal stress only. These planes are traditionally named principal planes and the normal stresses to them, their principal stresses. Stresses acting on a point in a body may be reduced and combined to produce these principal stresses acting on their respective principal planes. The principal stresses thus obtained are the maximum and minimum stresses acting at that point in the body.

The traditional approach is to take an infinitesimal cube at that point such that the sides are parrallel to the coordinate system used and then to reduce the stresses acting on those sides to normal and tangential stresses. Calculation of the maximum and minimum normal and tangential stresses acting on the pertinent planes may then be carried out. A graphical approach to the problem of calculating the stresses required on any plane from those stresses given is presented in text books as the Mohr Circle. The above described maximum stress theory is used in this paper to calculate stresses producing a state of failure in prestressed concrete beams subject tp flexure, shear and torsion.

To be consistent with theory, an assumption is made to the effect that the concrete under study behaves as a homogeneous material. This is not rigorously true of prestressed concrete but the author believes it to be a justifiable assumption due to the prestress force introduced to the concrete by a relatively small percentage of high tensile strength steel.

Resistance to torsion by plain concrete sections is generally low. Torque applied to such sections induces principal shear and tensile stresses of equal value. The introduction of a compressive force increases the value of the principal shear that would induce the same tensile stress as before. The same section is thus capable of resisting a larger torque. An illustration of the above is made in Figure 3 for two dimensional stress.

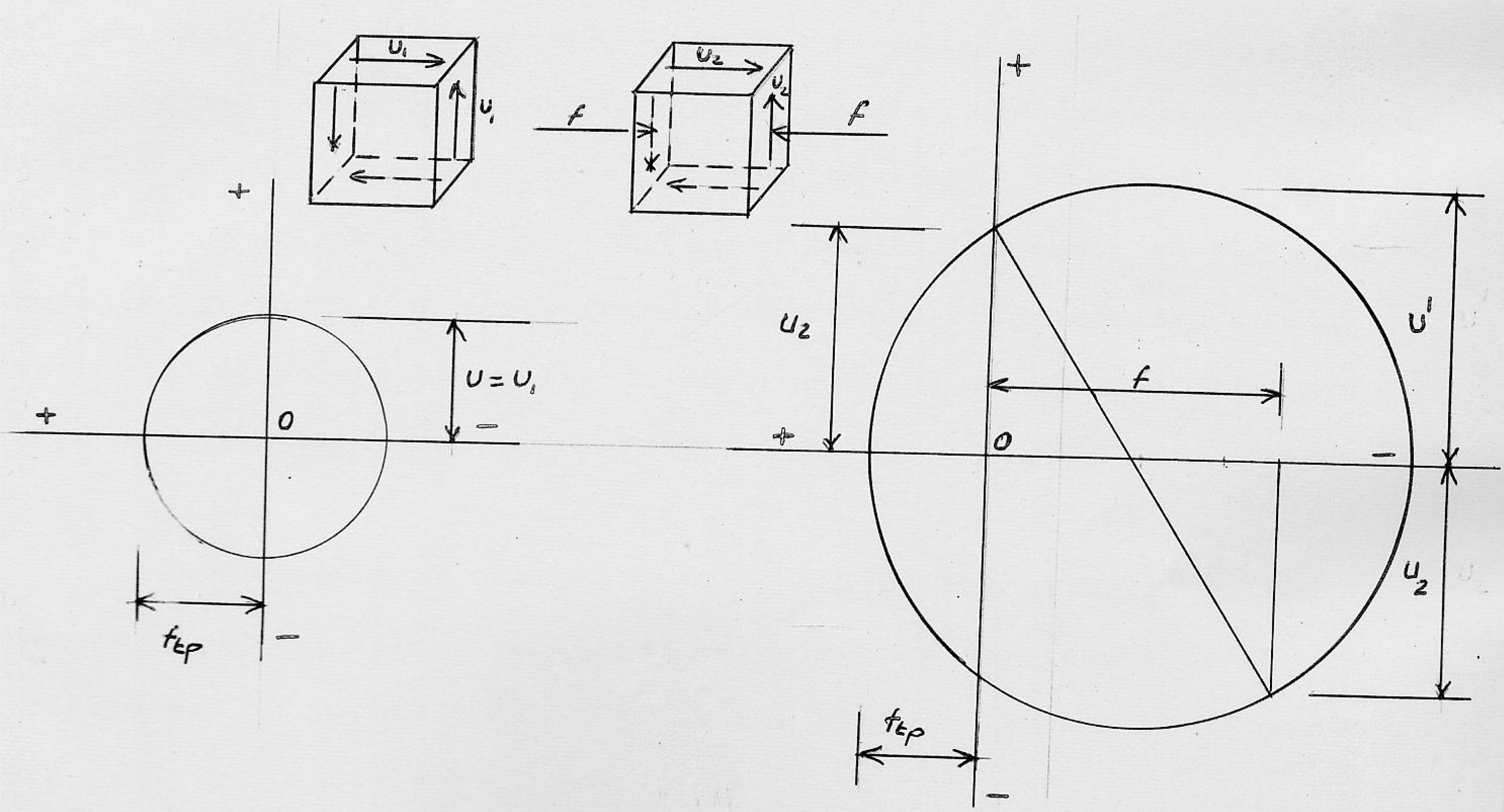


Fig. 3 Mohr Circle Illustration of Increase in Shear Resistance at a Point Due to the Introduction of Normal Stresses.

A sign convention for the stresses is adopted as follows: A stress acting normal to a plane and towards it is denoted as a negative stress while that acting normal to a plane and away from it is denoted as a positive stress. Stresses acting along two parallel planes such that they tend to produce a clockwise couple are denoted as positive shearing stresses while those tending to produce a counter clockwise couple are denoted as negative shearing stresses.

In prestressed concrete members elastic failure is usually signified by the cracking of a section, i.e. by having the principal tensile stress approach that of the ultimate tensile strength of the concrete in question. The principal attention is directed towards tensile stresses as the compressive stresses do not approach those of failure before the tensile stresses achieve that state. It is also assumed that, following general code requirements and directions for elastic design of prestressed concrete members, the compressive stress is still on the reasonably straight section of the stress-strain curve for concrete in compression. The exception to the above is the case where the prestressing force induces extremely high stresses on the section to cause a compressive failure upon the addition of flexure stresses. This, however, is rarely the case.

Any section of a structural member having external forces inducing flexural, shear and torsional stresses may generally be analyzed at the points where the various stresses interplay to produce principal tensile stresses.

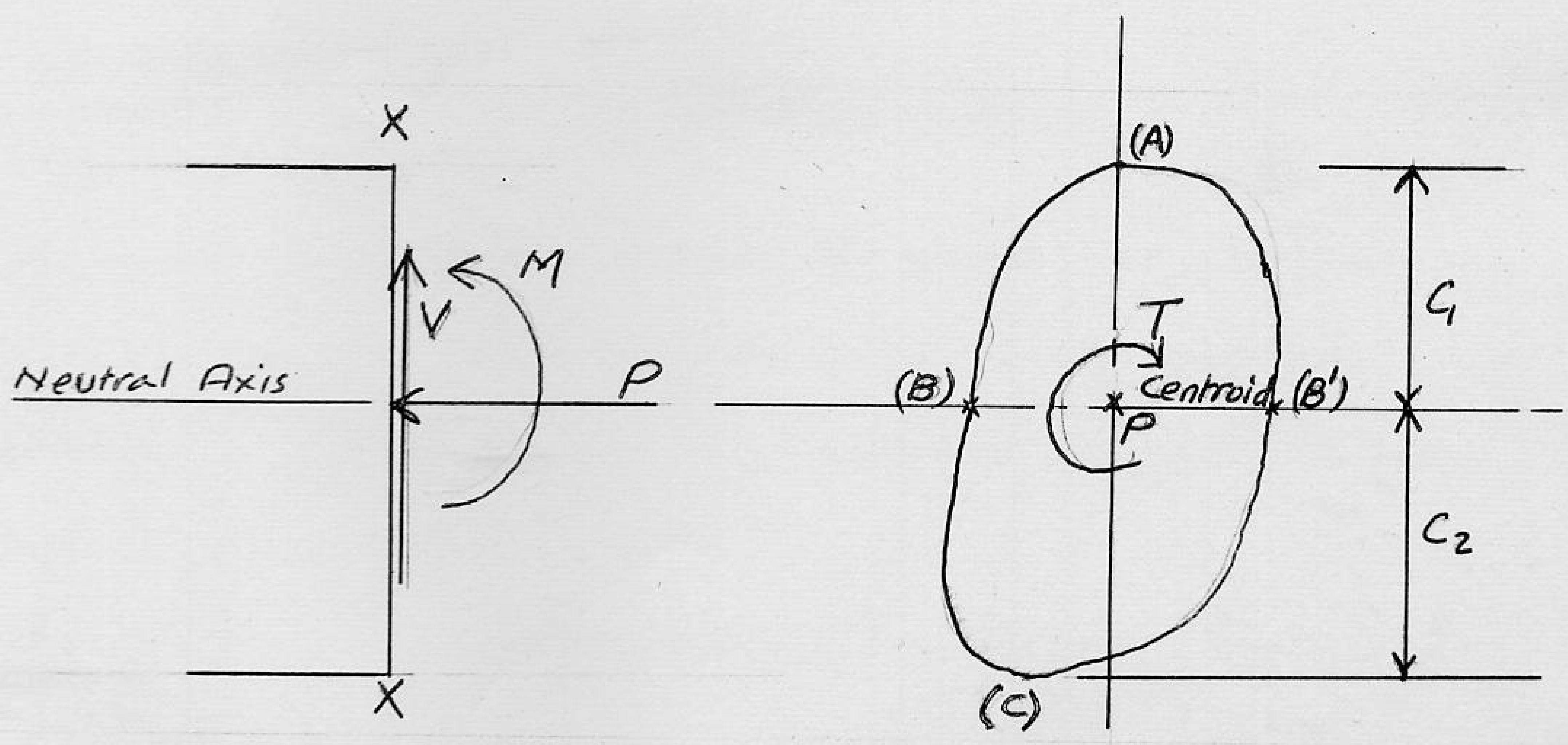


Fig. 4- An Uncracked Section of a Prestressed Member

Consider the uncracked section shown in Fig. 4.-

Let P denote the prestressing force acting on the section. It is shown to be acting at the neutral axis as any eccentricity effect could be assimilated into the moment M acting on the section. V is the vertical shear acting on the section and T is the torque induced on the section by the external forces acting on the member. The points considered are: (A) & (C) at the extremities of the section in a normal direction to the neutral axis where flexural, torsional shear and prestress stresses act with no horizontal shear stresses induced by V; and (B) and (B')on the neutral axis where flexural stresses are non existent but where those of horizontal shear are at a maximum. At (B) the torsional and shear stresses are additive while at (B') they are subtractive. The prestress force acting at the centroid of the section induces equal stresses at all points on the section. Note that the maximum v occurs at B and B' only for ordinary structural shapes.

The stresses acting at point A are:

f_c :compressive stresses due to the moment M, normal to the section and parallel to the longitudinal axis of the member.

f_p :compressive stress due to the prestress force P acting normally to section XX.

u :shearing stress, due to the torque T, acting tangentially along section XX.

At point C, the stresses acting are:

f_t :tensile stress due to the moment M, normal to the section and parallel to the longitudinal axis of the member.

f_p : compressive stress due to the prestress force P, acting normally on section XX.

u :shearing stress, due to the torque T, acting tangentially along section XX.

At point B the stresses acting are:

f_p: compressive stress due to the prestress force P, acting normally on section XX.

u : shearing stress, due to the torque T, acting tangentially along section XX.

v : shearing stress, due to the shear V, acting vertically along the section XX.

The stresses at B' are lower than those at B.

f_p : is the simplest to compute. It is $\frac{P}{A}$ where A is the area of the section XX.

f_c : is computed by $\frac{M \cdot c_1}{I}$ where c_1 is the distance between point (A) and the neutral axis and I is the moment of inertia of the section XX about its neutral axis.

f_t : is computed by $\frac{M \cdot c_2}{I}$ where c_2 is the distance between point (C) and the neutral axis.

v : is the stress equal to $\frac{VQ}{Ib}$ where Q is the statical moment of the area above (or below) the neutral axis, and b is the width of the section at the neutral axis.

u : is computed for any section of homogeneous material as $\frac{Tr}{J}$ where J is the polar moment of inertia for the section and r is the distance of the point in question from the centroid of the section.

For rectangular sections, symmetrical about the neutral axis and having an axis of symmetry perpendicular to the neutral axis. The above values are simple to obtain.

If b is the width of the section and d its depth and $d > b$, then

$$\left.\begin{aligned} f_p &= \frac{P}{bd} = \frac{Pd^2}{12I} & v &= \frac{VQ}{Ib} = \frac{Vd^2}{8I} \\ f_c &= f_t = \frac{Md}{2I} & u &= \frac{TK}{bd^2} = \frac{TKd}{12I} \\ K &= 3 + 1.8\frac{d}{b} \end{aligned}\right\} \quad \text{--(1)}$$

The value of K thus computed[8] is within 4% of the theoretically accurate one.

Computations for f_{tp}, the principal tensile stress, at the various points follow from the maximum stress theory:

At Point (A): $$f_{tp} = \sqrt{\left(\frac{f_p+f_c}{2}\right)^2 + (\gamma U)^2} + \frac{f_p+f_c}{2} \qquad (2)$$

At Point (B): $$f_{tp} = \sqrt{\left(\frac{f_p}{2}\right)^2 + (U+V)^2} + \frac{f_p}{2} \qquad (3)$$

At Point (C): $$f_{tp} = \sqrt{\left(\frac{f_p+f_t}{2}\right)^2 + (\gamma U)^2} + \frac{f_p+f_t}{2} \qquad (4)$$

Where γ is the ratio of U at the middle of the short side to U at the middle of the long side of the section[6].

Observation of the values obtained for f_{tp} at points (A) and (C) indicates that, of the two, $f_{tp}\big|_C$ tends to approach f_{tu}, the cracking or ultimate strength of the concrete in tension, at an earlier stage than $f_{tp}\big|_A$. Hence only the points (B) and (C) will be studied hereafter.

The maximum value of f_{tp} at points (B) and (C) will therefore be:

$$f_{tp}\Big|_C = \sqrt{\left(\frac{\frac{Pd^2}{12I}+\frac{Md}{2I}}{2}\right)^2 + \left(\frac{\gamma Tkd}{12I}\right)^2} + \frac{\frac{Pd^2}{12I}+\frac{Md}{2I}}{2} \qquad (5)$$

$$f_{tp}\Big|_C = \frac{1}{4I}\left\{\sqrt{\left(Md+\frac{Pd^2}{6}\right)^2 + \left(\frac{\gamma Tkd}{3}\right)^2} + \left(Md+\frac{Pd^2}{6}\right)\right\} \qquad (6)$$

$$f_{tp}\Big|_B = \sqrt{\left(\frac{Pd^2}{24I}\right)^2 + \left(\frac{Vd^2}{8I}+\frac{Tkd}{12I}\right)^2} + \frac{Pd^2}{24I} \qquad (7)$$

$$f_{tp}\Big|_B = \frac{1}{4I}\left\{\sqrt{\left(\frac{Pd^2}{6}\right)^2 + \left(\frac{Vd^2}{2}+\frac{Tkd}{3}\right)^2} + \frac{Pd^2}{6}\right\} \qquad (8)$$

at certain values and combinations of M, T and V, the values of $f_{tp}\big|_B$ and $f_{tp}\big|_C$ may be made to approach that of f_{tu}.

To obtain a nondimensional form of relationship the results shown above in equations (6) and (8) have to be related to a certain torque and/or moment. The particular torque and moments chosen are those pertaining to a section just before cracking. Thus, for a section having a prestressing force acting at the centroid of the section, M_c will denote the moment under which this section will crack if no torque acts on the section and T_c will denote the torque under which a section will crack if no moment acts on the section. In other words M_c is that moment corresponding to a $\frac{T}{M}$ ratio of zero and T_c is that torque corresponding to a $\frac{T}{M}$ ratio of infinity at cracking.

In this analysis the behaviour of concrete in tension is considered to be similar to that of concrete in compression as regards its stress-strain relationship.[7]

The stress-strain relationship is considered for concrete in tension to be parabolic[4,7,14], as is shown in Fig. 5, with the tensile strength attaining a maximum value of f_{tu}. If (a) in a tensile stress-strain diagram is equal to (a) in compressive stress-strain diagram then its value changes only with the strength of concrete, becoming smaller with an increase in concrete strength.[9]

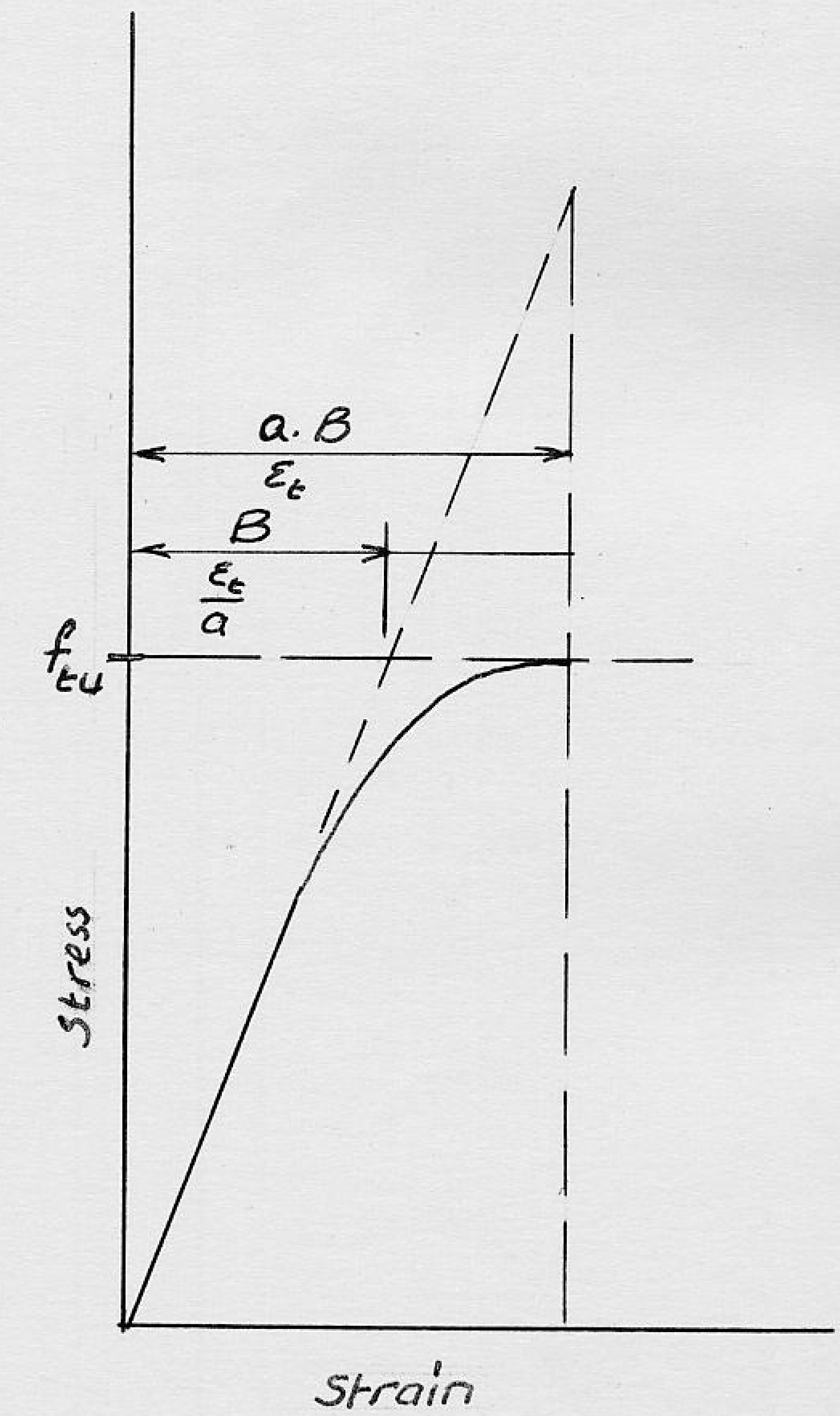


Fig. 5 - Stress Strain Diagram for Concrete in Tension.

Consider the stress diagram shown for an uncracked section in Fig. 6. This is due to the prestressing force and the loading of a structural member. The section remains plane under loading[15] and therefore the strain line is straight while that of the stress is parabolic in the tensile region and straight in the compressive region. This is consistent with the consideration that the stress-strain diagram of concrete in tension is parabolic and the f_c value is low in the uncracked section when compared with f'_c, the ultimate strength of concrete in compression.

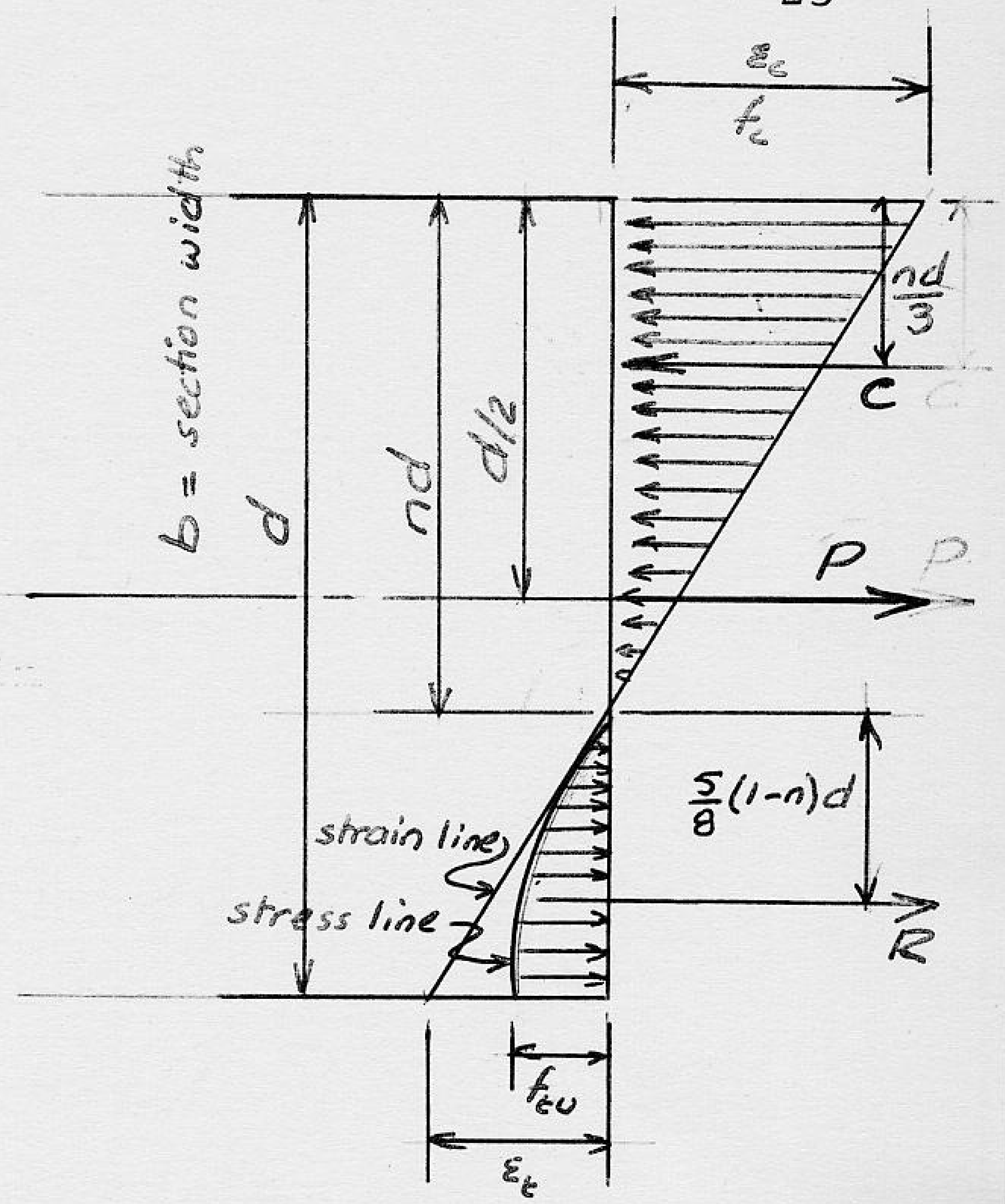


Fig. 6- Stress and Strain Diagram for an Uncracked Section Under Prestress.

$$\frac{\varepsilon_c}{\varepsilon_t} = \frac{n}{n-1} \qquad (9)$$

$$f_c = E_c\,\varepsilon_c \qquad (10)$$

$$f_{tu} = \frac{E_t\,\varepsilon_t}{a} \qquad (11)$$

Equation (11) is easily proved by considering similar triangles in Fig.5. E_c and E_t are the modulii of elasticity of concrete in compression and tension respectively. $E_t = \frac{f_{tu}}{B}$ which, by the above consideration of similarity between the stress-strain relationship of concrete in compression and tension, is equal to E_c.

Therefore

$$\left.\begin{aligned} f_c &= E_c\,\varepsilon_c \\ f_{tu} &= \frac{E_c\,\varepsilon_t}{a} \end{aligned}\right\} \qquad (12)$$

$$\frac{f_c}{f_{tu}} = \frac{E_c\,\varepsilon_c}{\frac{E_c\,\varepsilon_t}{a}} = a\,\frac{\varepsilon_c}{\varepsilon_t} = a\,\frac{n}{n-1} \qquad (13)$$

and

$$n = \frac{f_c}{f_c + a f_{tu}} \qquad (14)$$

Taking $\sum F = 0$ on the section of Fig. 6:

$$P + R = C$$

$$P + \frac{2}{3} b(1-n) d f_{tu} = \frac{1}{2} f_c n b d$$

$$P = \frac{1}{2} f_c n b d - \frac{2}{3} b d f_{tu} + \frac{2}{3} n b d f_{tu}$$

$$P = \frac{1}{2} A f_{tu} \left[\frac{a n^2}{1-n} + \frac{4}{3}(n-1) \right] = \frac{1}{2} A f_{tu} \alpha \qquad (15)$$

where $\alpha = \frac{a n^2}{1-n} - \frac{4}{3}(1-n)$

$$f_{tu} = 2P / \alpha A \qquad (16)$$

Taking $\sum M \Big|_{centroid} = 0$ just before cracking:

$$M_c = C\left(\frac{d}{2} - \frac{nd}{3}\right) + R\left[nd + \frac{5d}{8}(1-n) - \frac{d}{2}\right]$$

$$M_c = \frac{Cd}{6}(3-2n) + \frac{Rd}{8}(3n+1)$$

$$M_c = \frac{d}{6}(3-2n)\left(\frac{1}{2} f_c n b d\right) + \frac{d}{8}(3n+1)\left[\frac{2}{3} b d f_{tu}(1-n)\right]$$

$$M_c = f_c \frac{I}{d}(3n - 2n^2) + f_{tu} \frac{I}{d}(1 + 2n - 3n^2)$$

$$M_c = f_{tu} \frac{I}{d} \left[\frac{a n^2}{1-n}(3-2n) + (1 + 2n - 3n^2) \right] = f_{tu} \frac{I}{d} \beta \qquad (17)$$

where $\beta = \frac{a n^2}{1-n}(3-2n) + (1 + 2n - 3n^2)$

$$f_{tu} = \frac{M_c d}{I \beta} \qquad (18)$$

In deriving T_c for a rectangular section under the action of a centroidal prestressing force P, a Prandtl roof is considered, i.e. redistribution of the tensile stresses over the whole section with $f_t = f_{tu}$.[7]

A short review of the Membrane Analogy as originally developed by L. Prandtl will be given here.[15] Prandtl's analogy establishes certain relations between the deflected surface of a uniformly loaded membrane and the distribution of stresses in a bar under torque. The differential equation of the deflected surface has the same form as the equation which determines the stress distribution over the cross section of a twisted bar. If (S) is the tensile force per unit length of the boundary line of the membrane, (p) the lateral pressure per unit area, (G) the modulus of elasticity in shear and (θ) the angle of twist per unit length of the bar, then the two differential equations are identical if

$$\frac{p}{S} = 2G\theta$$

If this condition is fulfilled then:

1) the tangent to a contour line at any point of the deflected membrane gives the direction of the shearing stress at the corresponding point in the cross section of the twisted bar.

2) The maximum slope of the membrane at any point is equal to the magnitude of the shearing stress at that point.

3) Twice the volume included between the surface of the deflected membrane and the plane of its outline is equal to the torque acting on the twisted bar.

In deformation beyond the elastic limit of a material one has a roof of a constant maximum slope as shown in Fig. 7.

In a section ($b \times d$) having a prestressing force of P and torque T_c acting on it:

$$f_{tu} = \sqrt{\left(\frac{f_p}{2}\right)^2 + u^2} + \frac{f_p}{2} \quad --- (19)$$

where U is the shear stress due to T_c.

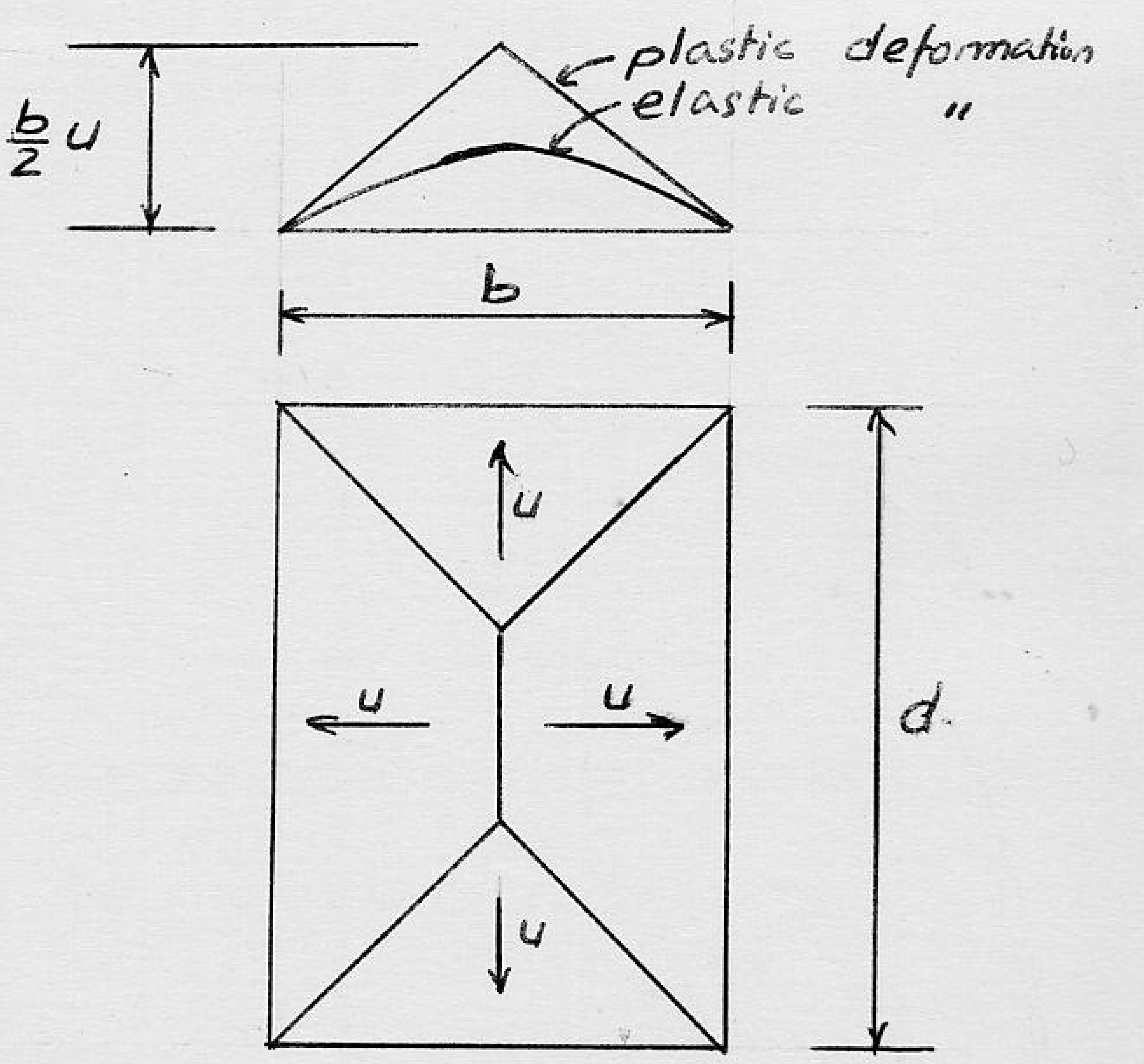


Fig. 7- Elastic and Plastic Prandtl Roofs for a Section

The volume of the space between the plastic deformed roof and the plane of its boundary is:

$$\text{Volume} = \frac{b^2}{3}\cdot\frac{b}{2}u + \frac{b^2}{4}(d-b)u = \frac{1}{2}u\left(\frac{bd^2}{2} - \frac{b^3}{6}\right)$$

$$= \frac{1}{2}uN \quad \text{where } N = \frac{bd^2}{2} - \frac{b^3}{6}$$

The torque, T_c, is equal to twice the volume

$$\text{i.e.} \quad T_c = uN \quad \text{------------------ (20)}$$

Substituting in (19):

$$f_{tu} = \sqrt{\left(\frac{f_P}{2}\right)^2 + \left(\frac{T_c}{N}\right)^2} + \frac{f_P}{2} \quad \text{---------- (21)}$$

One may now develop non-dimensional equations in terms of either of M_c or T_c,

Equating the formulae (6) and (18) for point C:

$$\frac{1}{4I}\left\{\sqrt{\left(Md + \frac{Pd^2}{6}\right)^2 + \left(\frac{\gamma Tkd}{3}\right)^2} + \left(Md + \frac{Pd^2}{6}\right)\right\} = \frac{M_c d}{I\beta}$$

$$\left(Md + \frac{Pd^2}{6}\right)^2 + \left(\frac{\gamma Tkd}{3}\right)^2 = \left[\frac{4M_c d}{\beta} - \left(Md + \frac{Pd^2}{6}\right)\right]^2$$

$$\left(\frac{\gamma Tkd}{3}\right)^2 = \frac{16 d^2 M_c^2}{\beta^2} - \frac{8d}{\beta}\left(Md + \frac{Pd^2}{6}\right)M_c$$

dividing throughout by $\dfrac{16 d^2 M_c^2}{\beta^2}$

$$\left(\frac{\gamma k\beta}{12}\right)^2\left(\frac{T}{M_c}\right)^2 + \frac{\beta}{2}\left(\frac{M + Pd/6}{M_c}\right) = 1 \quad \text{-------- (22)}$$

Equating the formulae (8) and (18) for point B:

$$\frac{1}{4I}\left\{\sqrt{\left(\frac{Pd^2}{6}\right)^2 + \left(\frac{Vd^2}{2} + \frac{Tkd}{3}\right)^2} + \frac{Pd^2}{6}\right\} = \frac{M_c d}{I\beta}$$

$$\left(\frac{Pd^2}{6}\right)^2 + \left(\frac{Vd^2}{2} + \frac{Tkd}{3}\right)^2 = \left[\frac{4dM_c}{\beta} - \frac{Pd^2}{6}\right]^2$$

$$\left[\frac{Vd^2}{2} + \frac{Tkd}{3}\right]^2 = \left(\frac{4d}{\beta}\right)^2 M_c^2 - \frac{4}{3}\frac{d^3}{\beta}PM_c$$

dividing throughout by $\left(\dfrac{4d}{\beta}M_c\right)^2$

$$\left(\frac{\beta}{4}\right)^2\left[\frac{Vd}{2M_c} + \frac{TK}{3M_c}\right]^2 + \frac{\beta}{12}\frac{Pd}{M_c} = 1 \quad \text{---------- (23)}$$

The form of equations (22) and (23) may be changed by introducing T_c for M_c. T_c and M_c may be related by combining equations (18) and (21) thus :

$$T_c = \sqrt{\frac{M_c P d^3 N^2}{12 I^2 \beta} + \frac{M_c^2 d^2 N^2}{I^2 \beta^2}}$$

The relationships shown in (22) and (23) are for cracking conditions. A section, however, is safe if the equality is not attained, i.e.:

$$\left(\frac{\gamma K \beta}{12}\right)^2 \left(\frac{T}{M_c}\right)^2 + \frac{\beta}{2}\left(\frac{M + Pd/6}{M_c}\right) \leq 1 \qquad (24)$$

$$\left(\frac{\beta}{4}\right)^2 \left[\frac{Vd}{2M_c} + \frac{TK}{3M_c}\right]^2 + \frac{\beta}{12}\frac{Pd}{M_c} \leq 1 \qquad (25)$$

It must be kept in mind that equation (24) refers to point (C) and that equation (25) refers to point (B) in Fig. 4. This entails that V and T have the same sign, that M is positive and that P is negative to conform with the sign convention adopted.

CHAPTER III

DESCRIPTION OF EXPERIMENTS

To check out the validity of equations (24) and (25) arrived at in Chapter II, it was decided to carry out some tests on prestressed concrete beams. These beams were to be tested to failure.

The discussion of Chapter II considers the concrete under prestress to be homogenous and does not consider the presence of shear reinforcement, be it in the form of stirrups or bent up bars. A further assumption is made this being that the area of prestressing wire is negligible in so far as its effect on the properties of the section are concerned. The test beams were designed with the above mentioned properties in mind, i.e. no shear or longitudinal reinforcement in the section under test. To determine the effect of the presence of the prestressing wires on the section two sets of beams were cast, one with the wires clustered at the centroid of the section and the other with the wires at the corners. Some researchers[10,11] have noticed that, for reinforced concrete test beams, the longitudinal reinforcing bars provide a kind of dowel action which tends to increase the torsional rigidity of the section. The beams having the prestress wires in the corners are intended to detect such dowel action, if any, produced by the prestressing wires.

High tensile strength 7mm prestressing wire was used for applying the prestressing force in the test beams. The wire was tested at the School of Engineering, A.U.B., Material Laboratories and was found to have a 0.2% strain yield strength of 200,000 psi and an ultimate strength of 228,000 psi. The stress-strain diagram obtained by the laboratory is reproduced in Appendix "A".

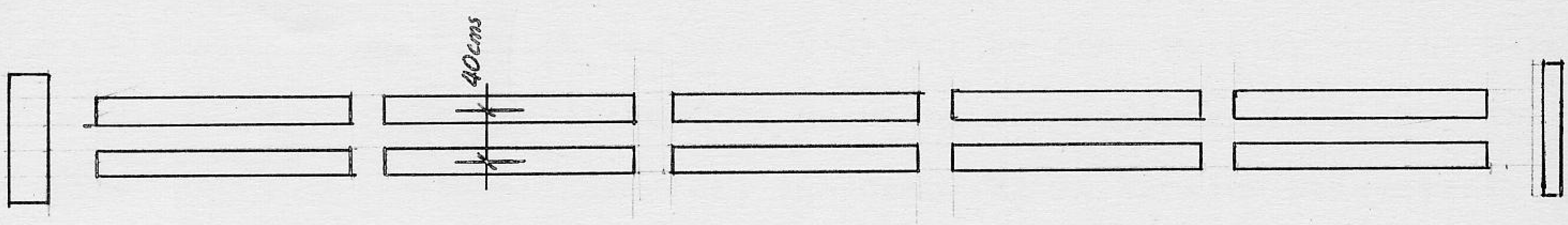
Chekka portland cement together with coarse and fine gravel from Nahr Ibrahim quarries and Khaldeh red sand were the ingredients used in the mix for the test beams, the torque arms and the casting bed bulkheads. The properties of the aggregate and the cement are tabulated for reference in Appendix "A". For the mix proportions an aggregate cement weight ratio of 5.3:1 was used, the aggregate proportions being: coarse gravel 43.5%, the fine gravel 23.5% and the sand 33.0% by weight. The water cement ratio was varied for the different pours.

The cement was mixed by a Richier Type 915 batching plant and a Type S840 E mixer and placed by a Pingon P.26 C-830 crane. It was vibrated in the forms by Vibroweken electrically motivated power vibrators.

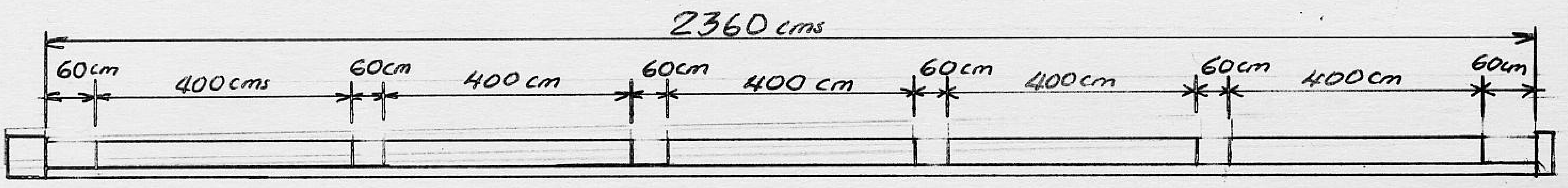
The stirrups and main reinforcement to the arms of the test beams, the ends of the test beams and the casting bed bulkheads were of commercially available mild and Tor reinforcing steel, having respectively yield strengths of 40,000 and 60,000 psi.

Test beams square in cross section were decided on as this is the section best suited of the rectangular sections to resist torques efficiently. A square section, 20 cm on a side, was thus chosen to facilitate handling. The length was chosen to be 4 meters for the same reason. A tentative concrete strength of 4000 psi at 28 days was used in the design calculations. The pertinent calculations are presented in Appendix "B".

The test beams were formed and cast in rows of five between two bulkheads 23.6 meters apart. The general agreement is shown in Fig.8.



(a)



(b)

Fig. 8 General Arrangement of Casting Beds and Bulkheads for the Prestressed Test Beams: (a) Plan, (b) Elevation.

Timber forms lined with 3mm plywood were used for the sides of the beams. The pouring of the concrete was arranged such that only one row of five beams was poured at one time. The forms were stripped from the first row and placed for the adjacent row. The prestressing wires were stressed on the morning of the pour. A Prescon prestressing jack was used for stressing the Ø 7mm wires.

Once the beams were removed from their beds they were spaced out on the ground next to the casting bed and the arms formed and cast about them.

Sampling of the concrete poured for testing purposes was by 6 inch x 12 inch cylinders and British Standards Tension Briquettes. Concrete was taken at the site of the beams or arms and poured into the cylinders. It was also placed in the briquettes as such with the exception that it had the large aggregates removed. Concrete in the cylinders was also vibrated with the same vibrator as that working on the beam's concrete. No curing for either the beams or the samples was made. The vibration of the cylinder samples, the sampling in tension briquettes, and the lack of curing are not standard testing practice but were effected to simulate actual beam conditions and strength.

The first set of five beams, T1, having their wires in the corners of the beam section, was cast on the 30th of April, 1966. Their wires were pretensioned to a load of 9000 lbs each, i.e. 150,000 psi. The water used for that mix was 55 liters of water for 100 kg of cement, i.e. a water cement ratio of 0.5, with a slump of $1\frac{1}{2}$ inches.

The second set of the beams, T2, having their wires clustered at the centroid of the section, was cast on the 1st May, 1966. Their wires were pretensioned to a load of 9,000 lbs, i.e. 150,000 psi. The water used, the slump, and the water cement ratio was the same as that of T1.

The wires for the T1 and T2 beams were released on the 5th of May, 1966, the beams of T1 and T2 having a strength of 2470 psi and 2370 psi respectively. Unfortunately the beams were largely firmly stuck to the casting bed as not enough form oil had been used. The result was a loss of four beams, two from each pour, due to cracking while being removed from the beds. When cutting the wires of the beams, the cutting was arranged such that the wires were cut in the sequence shown in Fig. 9. While releasing the load off the prestressing wires of T1 beams, it was noticed that the two top wires moved apart at the point of cutting about 5 cms more than the lower wires. No such occurence took place while releasing the load off the wires of T2 beams.

Beams T3, having the wires in the corners of the section, were cast on the 6th May, 1966, with a prestress-wire load of 10,000 lbs, i.e. 167,000 psi. The aggregate was wet as it had rained two days earlier and consequently it was difficult to estimate the water-cement ratio. 50 liters of water were, however, added to a mix having 100 kgs. of cement to obtain a slump of $2\frac{1}{2}$". The fourth and last set of beams, T4, having its wires clustered at the centroid of the section, was poured on the 7th May, 1966. Its wires were also stressed to 10,000 lbs, i.e. 167,000 psi, and 53 liters of water were added to a 100 kg of cement batch to produce a slump of $2\frac{1}{2}$".

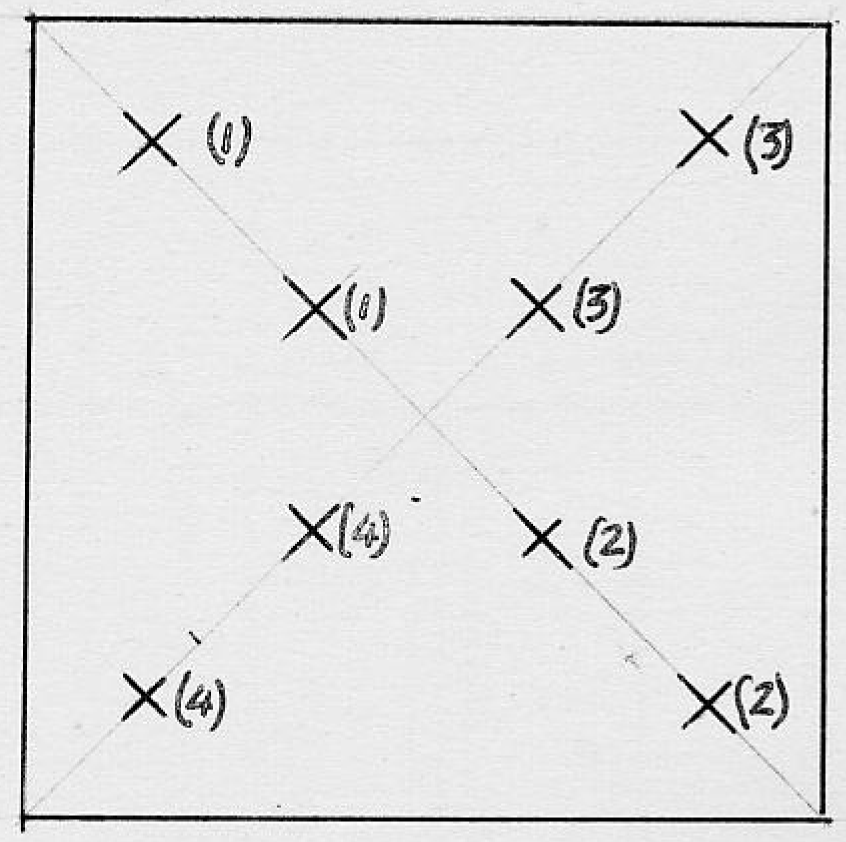
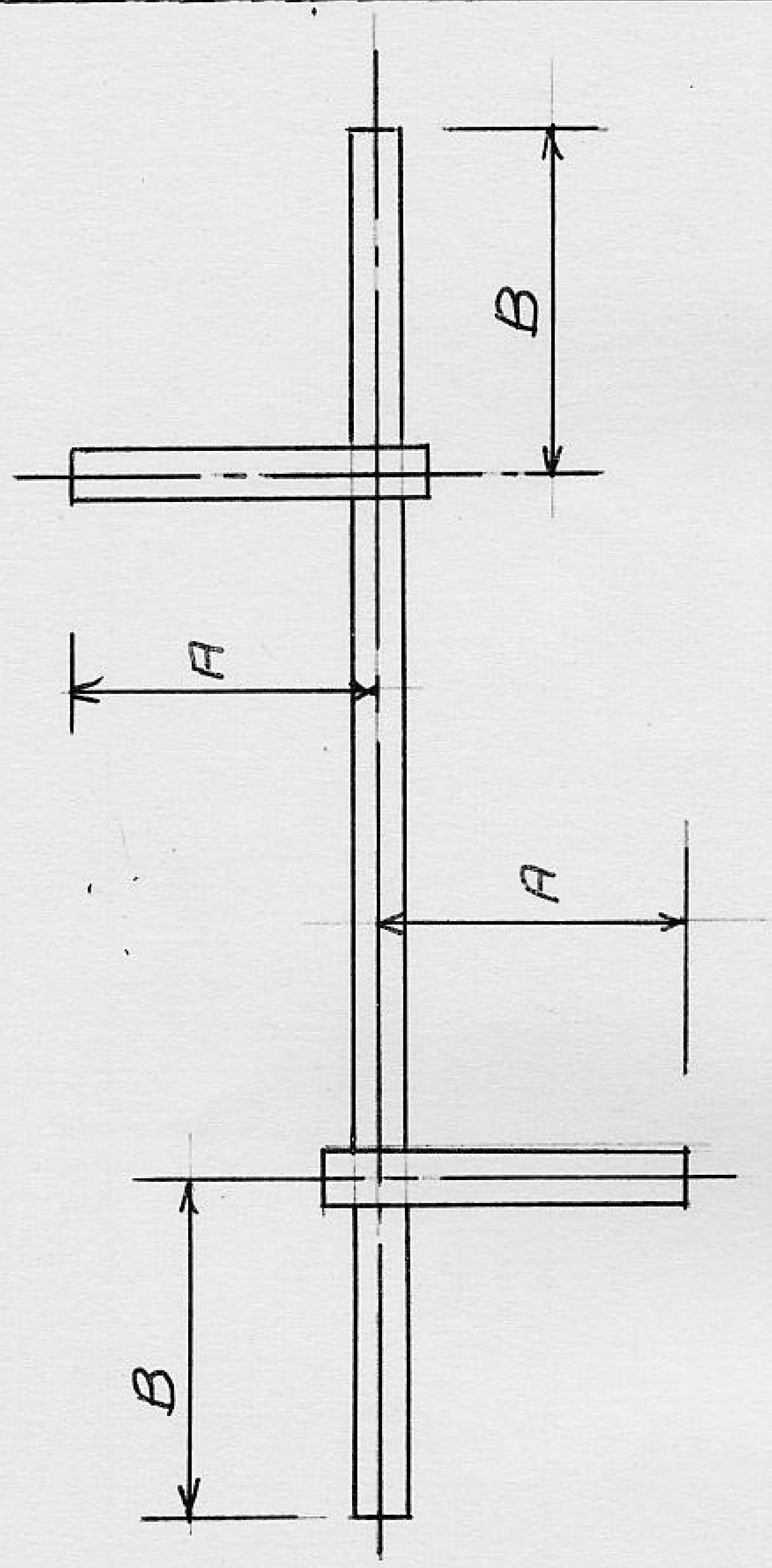


Fig. 9. Sequence of Cutting Wires to Transfer Prestress.

The wires for the T3 and T4 beams were released on the 13th May, 1966, with the concrete of the T3 beams having a strength of 2900 psi. No beams were lost this time while removing them from the bed. It was noticed, however, that the two top wires of the T3 beams moved apart about 5 cm more than the two bottom wires at the point of cutting. This did not happen to the beams of the T4 series.

The arms to the beams were cast in lots of six arms at a time. Concrete cylinders Ø 6 inches x 12 inches were taken as samples at the time of casting. Again these cylinders were not cured to simulate actual conditions of the arms. The arms were cast as follows:

DATA ON CASTING OF ARMS TO BEAMS			
BEAM	ARM POUR & DATE OF CASTING	A cms	B cms
T 1	A1 on 10/5-1966	100	90
T 1	A1 on 10/5-1966	100	90
T 1	A2 on 11/5-1966	60	90
T 2	A1 on 10/5-1966	100	90
T 2	A2 on 11/5-1966	100	90
T 2	A2 on 11/5-1966	60	90
T 2	A2 on 11/5-1966	60	90
T 3	A3 on 13/5-1966	100	30
T 3	A3 on 13/3-1966	100	25
T 3	A4 on 14/5-1966	60	90
T 4	A3 on 13/5-1966	100	30
T 4	A4 on 14/5-1966	100	25
T 4	A4 on 14/5-1966	60	90



The mix used for the arms cast onto the prestressed beams was the same as that of the beams themselves with a higher water cement ratio (0.55) and a higher slump (3inches).

Crushing of the cylinder samples and the tensile tests on the briquettes were carried out on an Avery 200 T crushing machine and an Avery Tension Machine at the School of Engineering Materials Laboratory.

The compression and tension strength results are tabulated in Appendix "A". While the compression test results seem to follow expected patterns, those of tension are haphazard. The author believes this to be due to the lack of curing which undoubtedly has produced shrinkage cracks resulting in a weaker tensile strength.

Testing of the beams was carried out as shown in the accompanying sketch, Fig. 10. The ends of the beams were held by $\frac{3}{4}$'' steel plate boxes having a 10cm long Ø 88mm shaft welded onto its outside face. SKF 6318 Ball Bearings were fitted onto this shaft and bore against a Ø 20mm bar loop fixed into the floor. The load onto the beam was applied vertically by two independently operated Origo Type 20-HL hydraulic jacks fitted with ENMF pressure gauges of 300 kg/cm^2 capacity graduated in 5kg/cm^2 units. Baty dial micrometer gauges, graduated in 0.001 inches, were used to read vertical and twist deflections of the beams under load. The jack pressure readings were taken in 5kg/cm^2 steps, the additional loads being applied at about three minute intervals. In testing all the beams except for Beams 6 and 7, both jacks were operated at the same time by two operators. Beams 6 and 7 were tested by operating one jack and taking the pressure gauge reading of the second

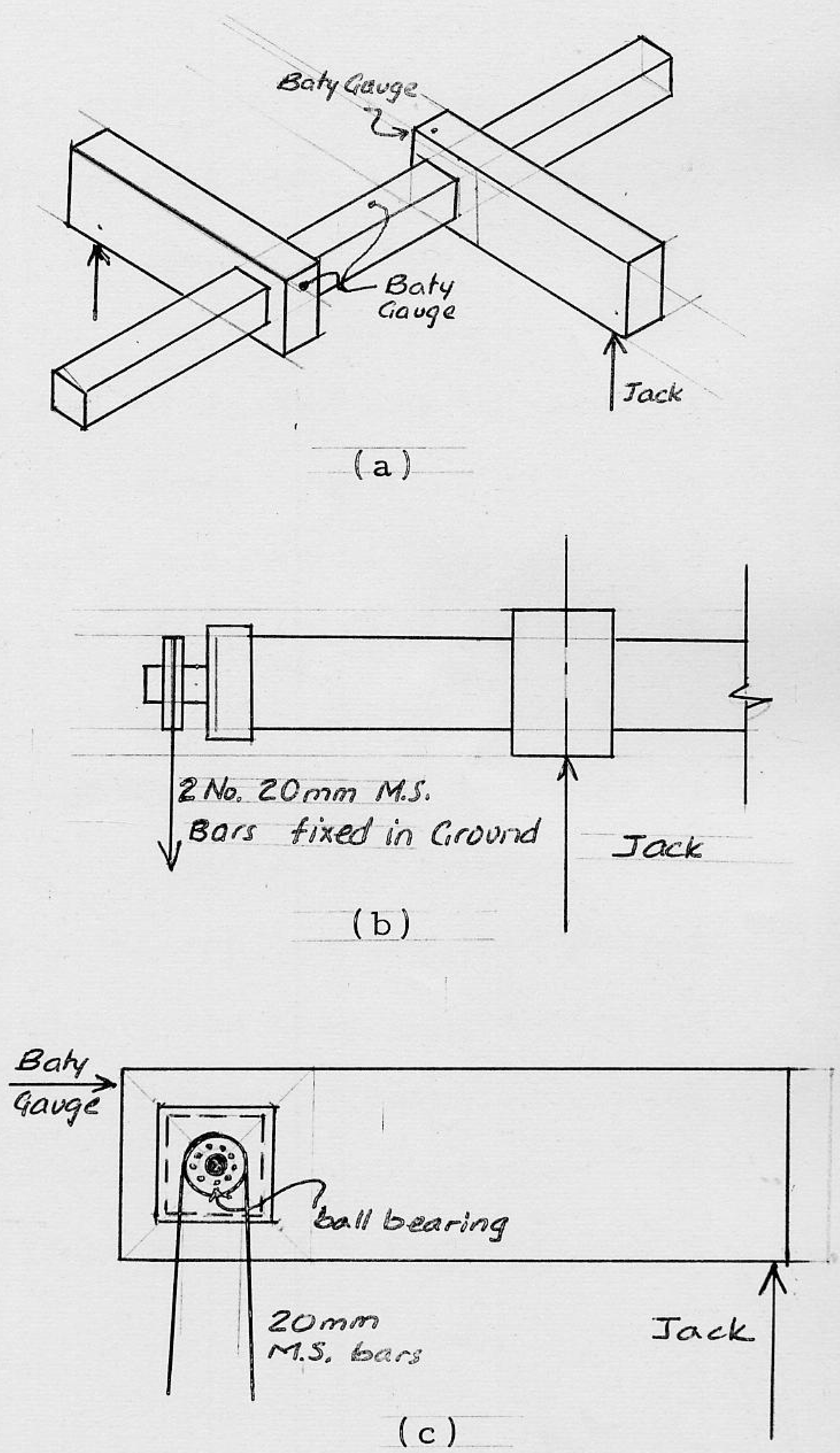
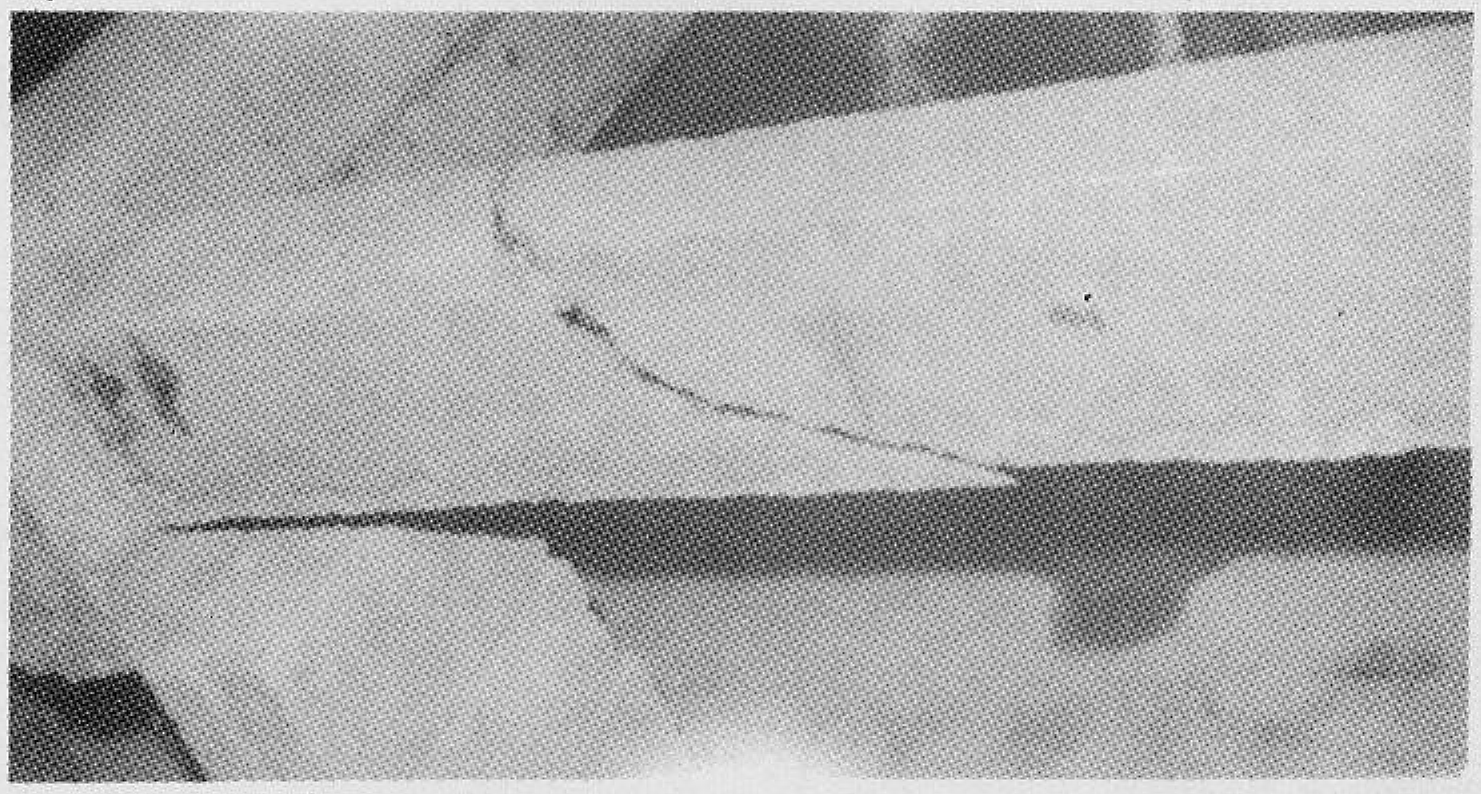


Fig. 10. Testing Set Up
(a) Perspective, (b) Side View of Part of Beam, (c) End View of Beam

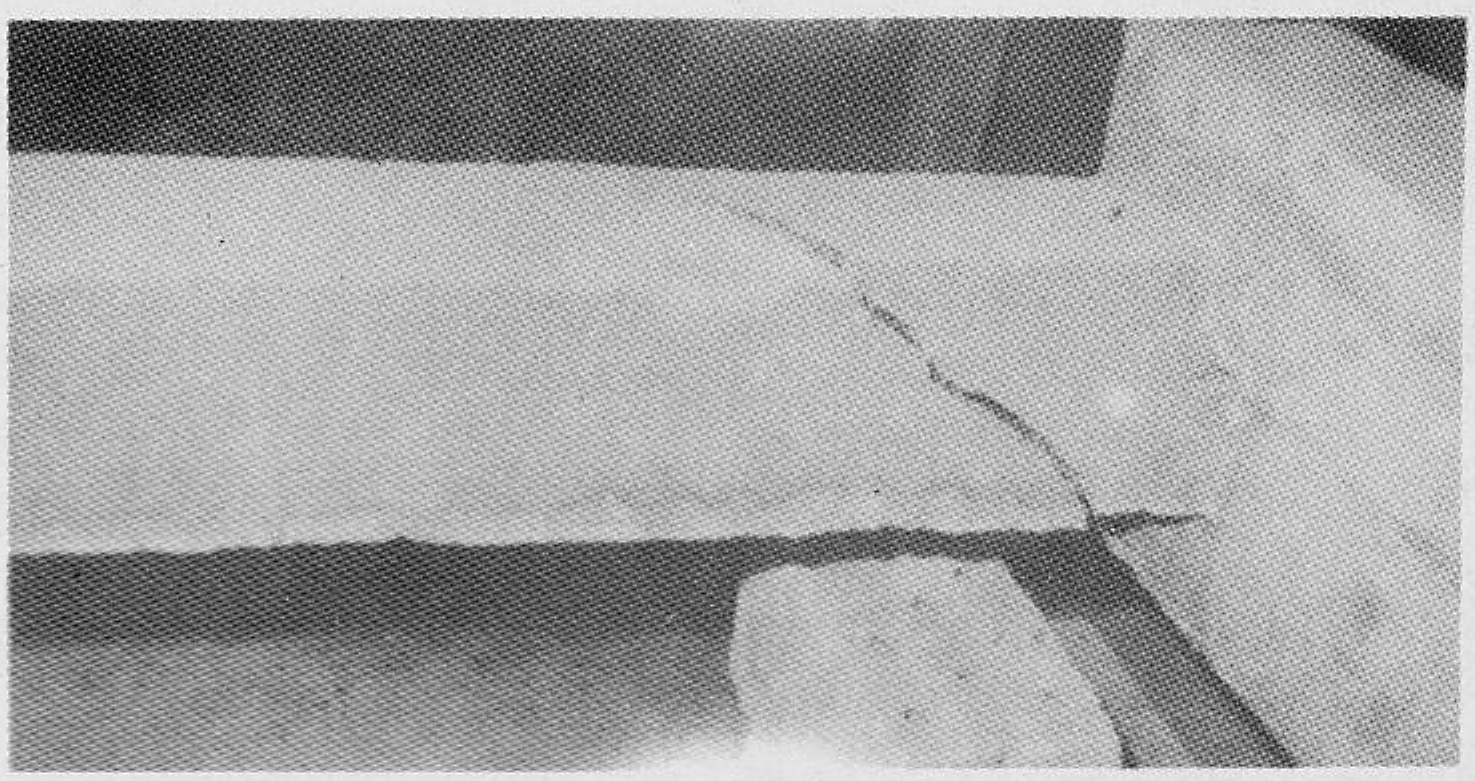
jack but not operating its pump.

Workers carried the beam to be tested to the testing bed and placed it on temporary supports. The steel brackets were then placed onto the beam ends and the ball bearings fitted on. The beam was then manually jockeyed to have the ball bearings fit under the bar straps fixed to the floor. The beam was lifted to bear against the straps and temporarily supported near the arms in that position. The two loading jacks were then placed in position and jacked up such that their readings were 10kg/cm2 (the first reading possible on the gauges). The Baty dial gauges were then fitted, one at the center of the beam to record vertical deflection, and one at the top of the back of either arm to record the horizontal movement of the top corner of the arm (see Fig. 11). The initial readings of these Baty gauges were then taken. Increments of 5kg/cm2 were then applied by the jacks and the corresponding Baty gauge readings recorded. This went on until cracks were observed on the beam In the cases where there was no torque applied,the gauge readings went up on further pumping and these were recorded with their respective Baty gauge values. In the cases where torque was applied the load dropped markedly as soon as cracks were observed and the Baty gauges were removed. Jacking went on, however, to produce more accentuated cracks. These cracks were recorded and, for the most part, photographed. They are recorded in Appendix "C". Failure generally occured just inside one of the two torque arms.

The two Origo jacks and their gauges were calibrated on an Olsen machine on the 21st and 27th May, 1966, i.e. just after the beginning and just before the end of this series of tests. The calibrations results are tabulated in Appendix "C".



(a)



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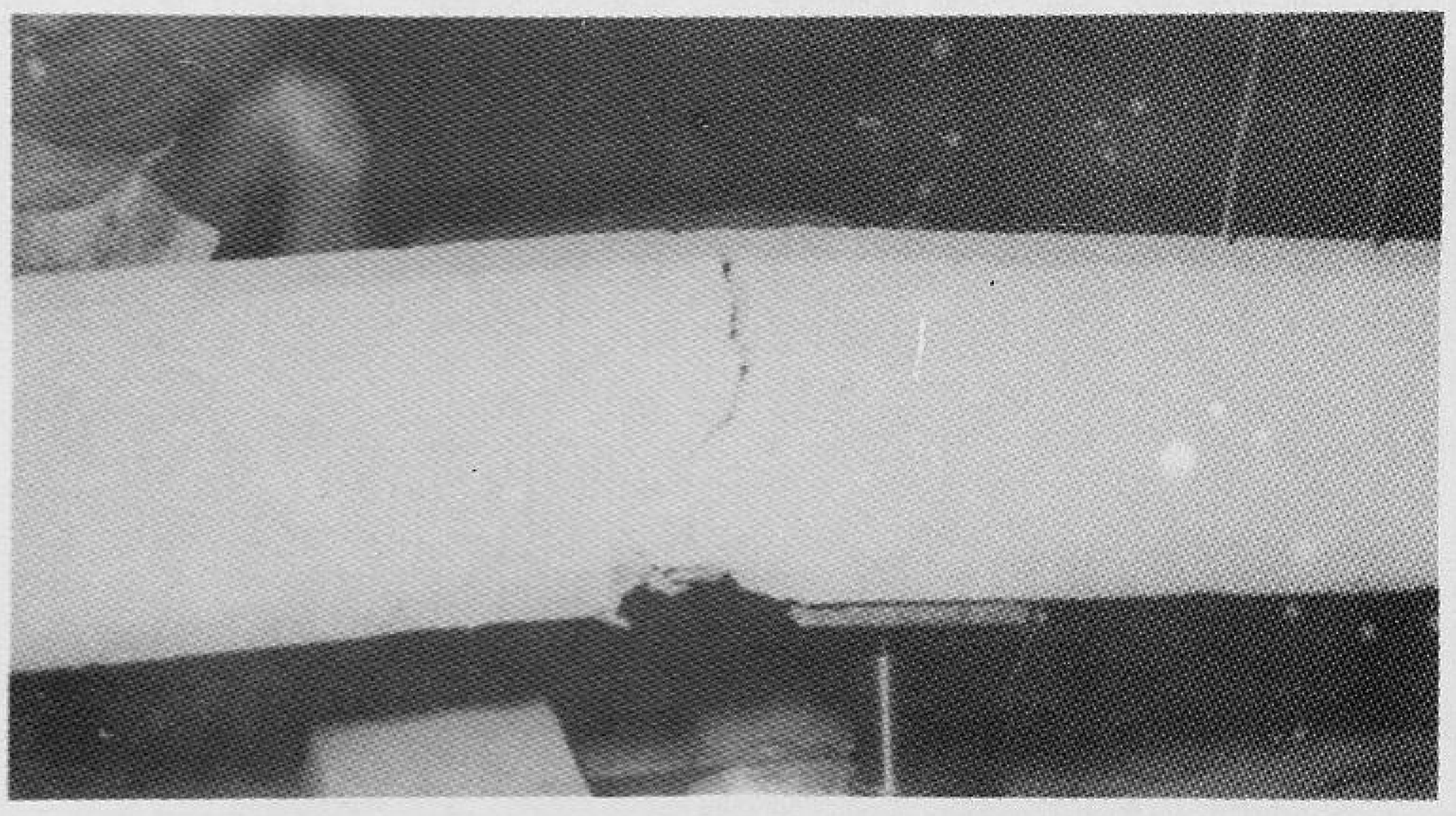
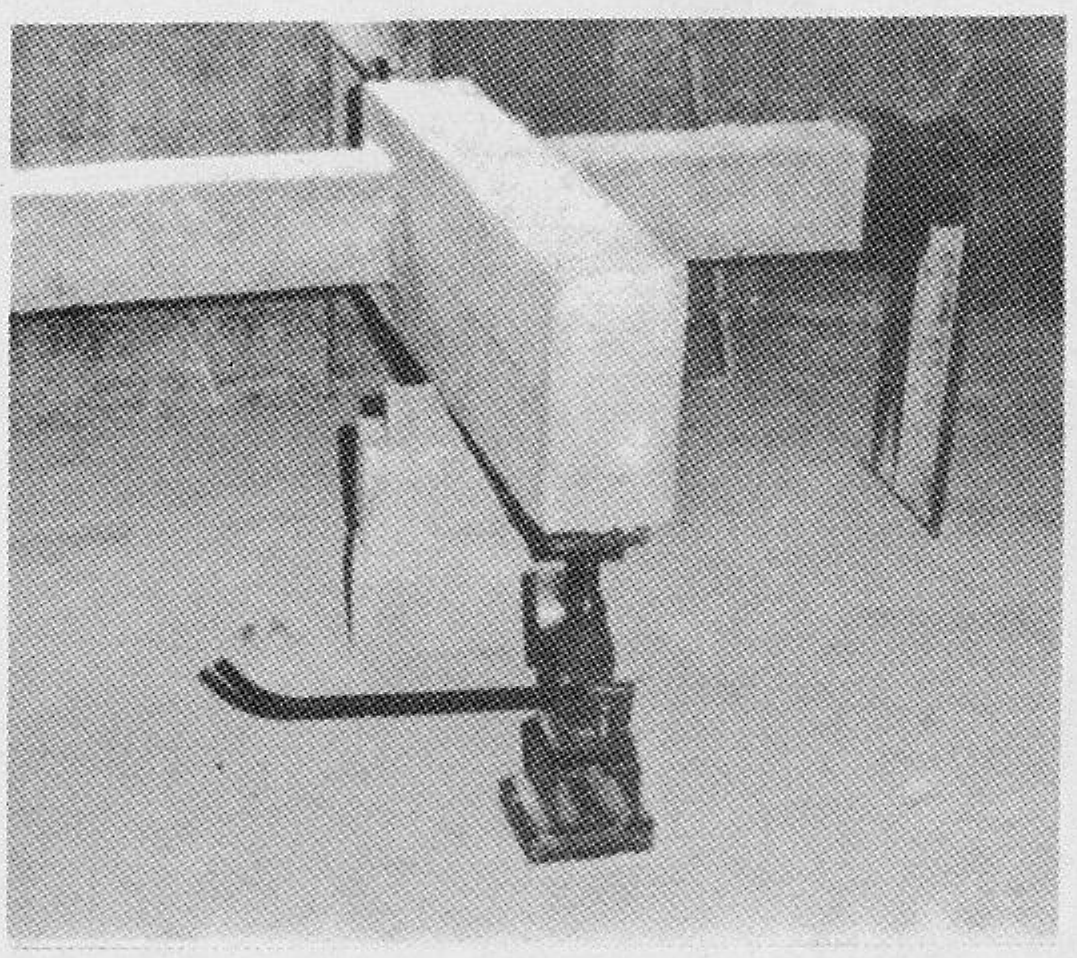


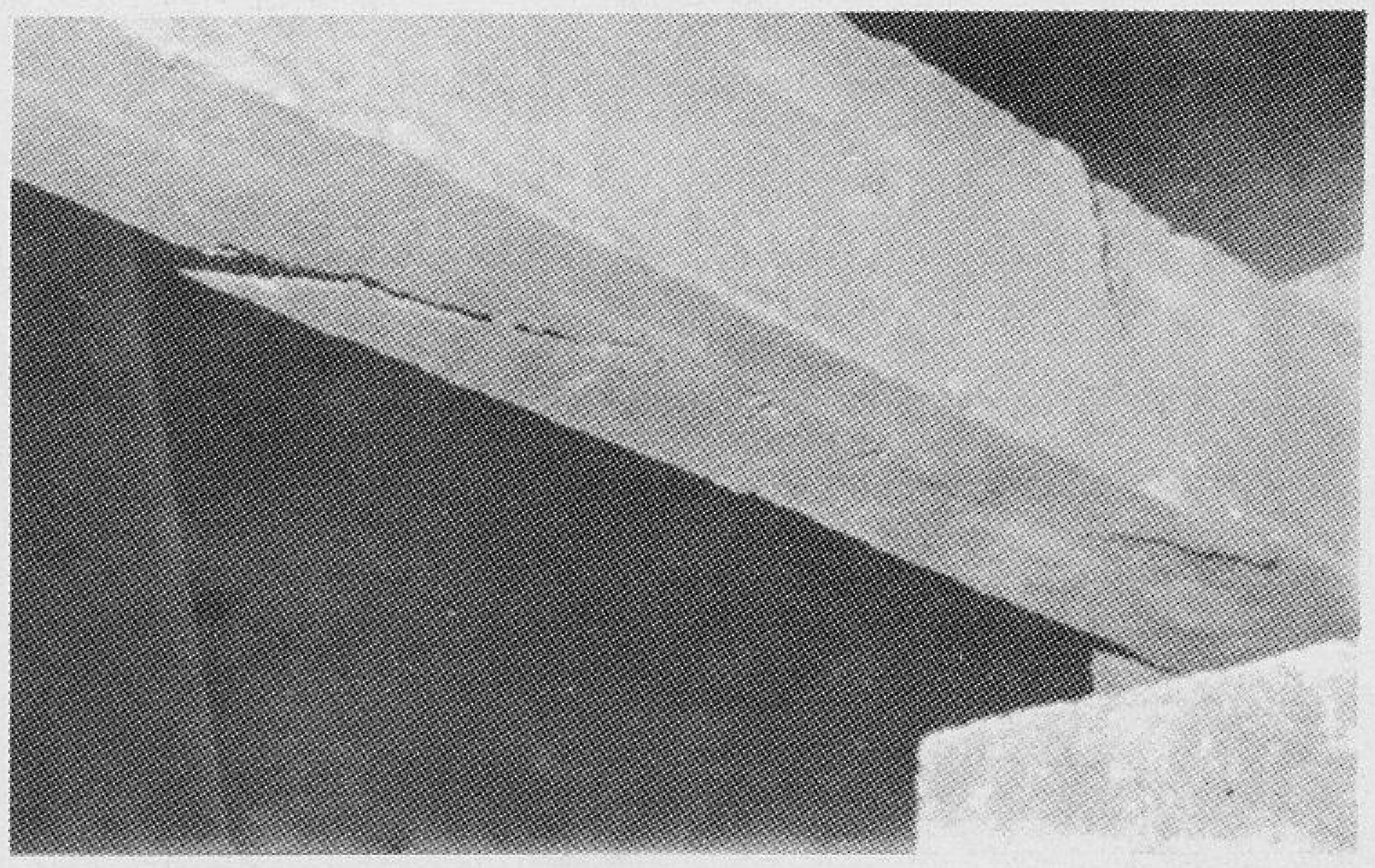
(d)

Plate I : (a), (b) and (c) Typical Top and Side Views of a Torsional Failure

(d) Pure Flexural Failure Showing a Secondary Compression Failure.



(a)



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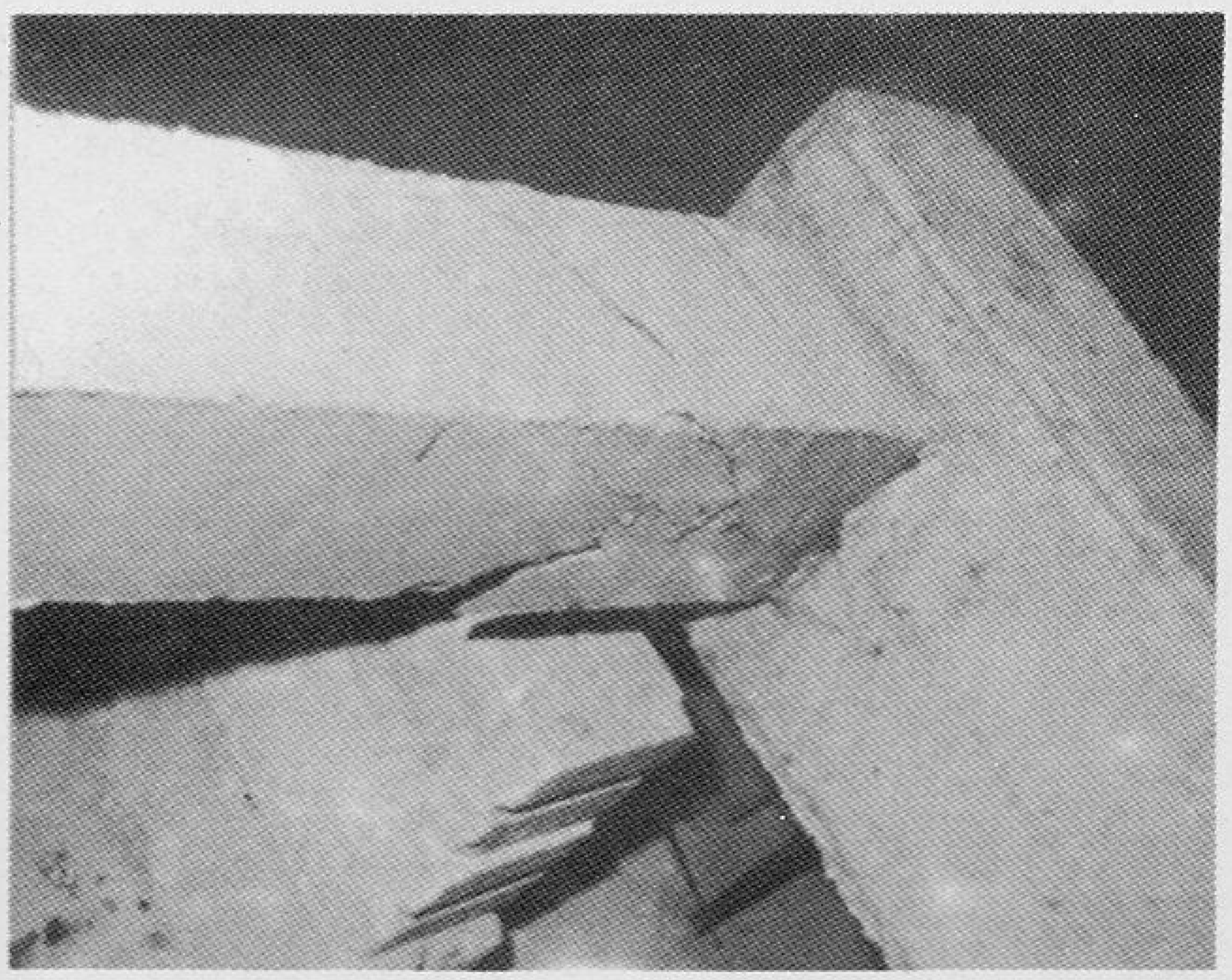
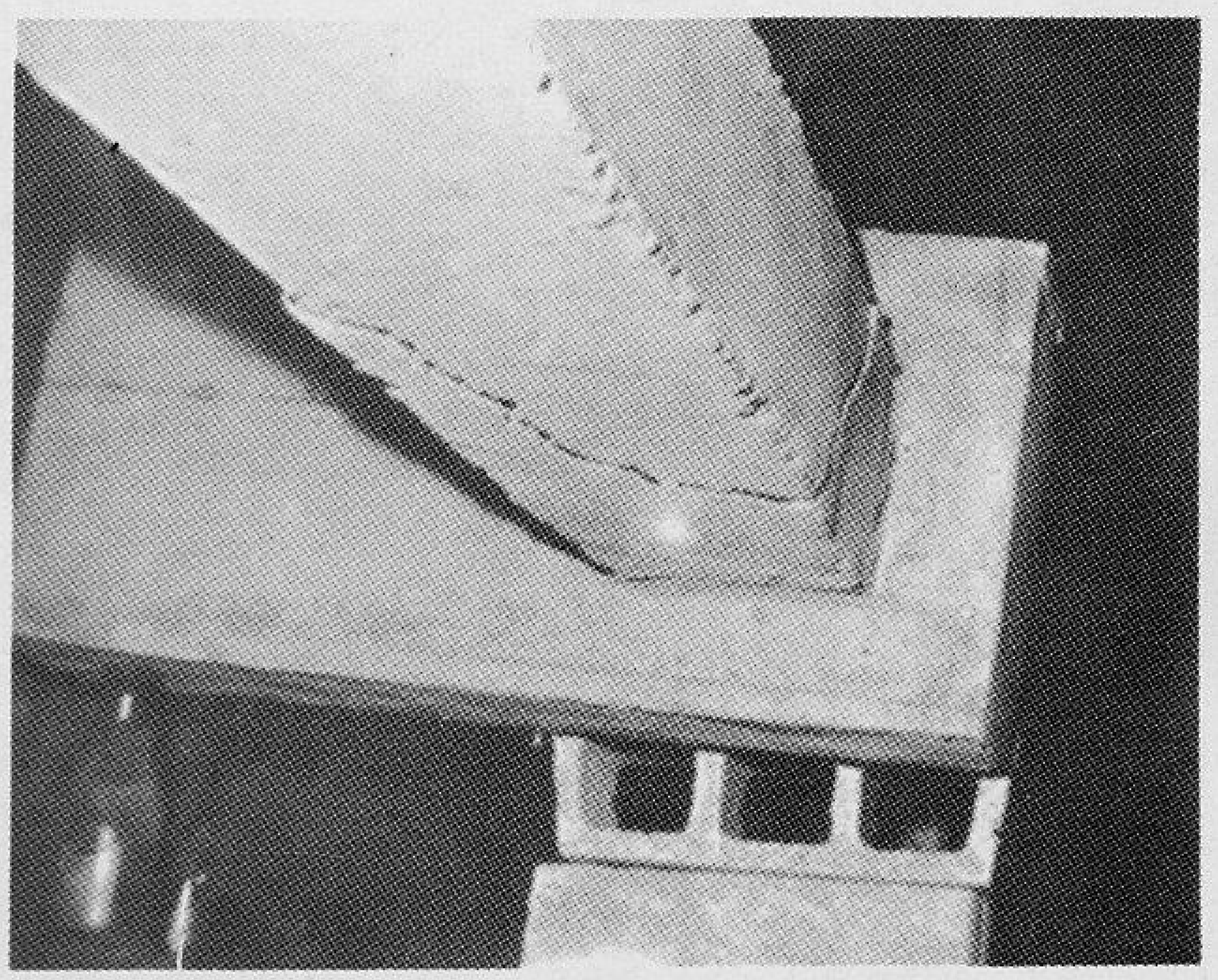


(c)

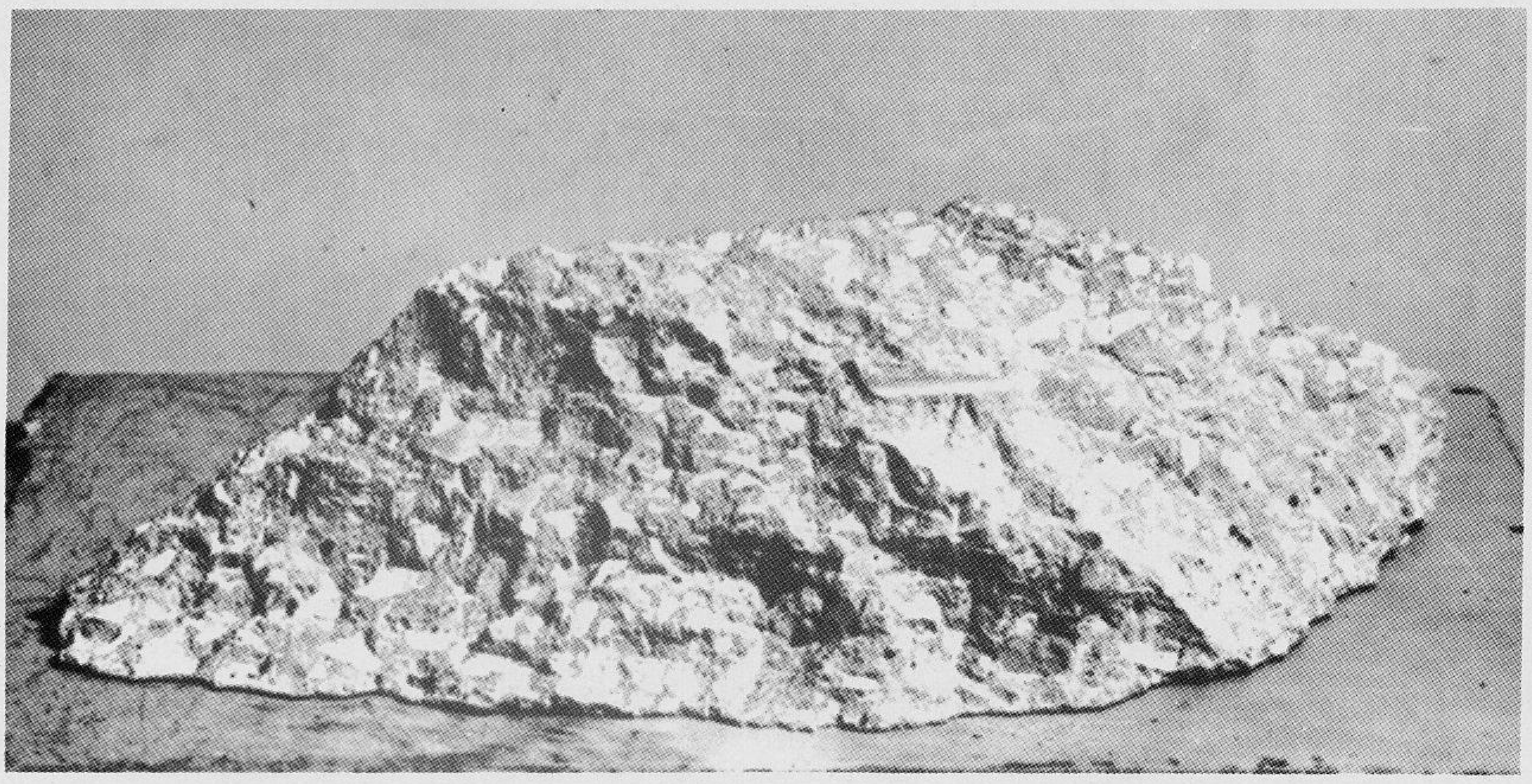


(d)

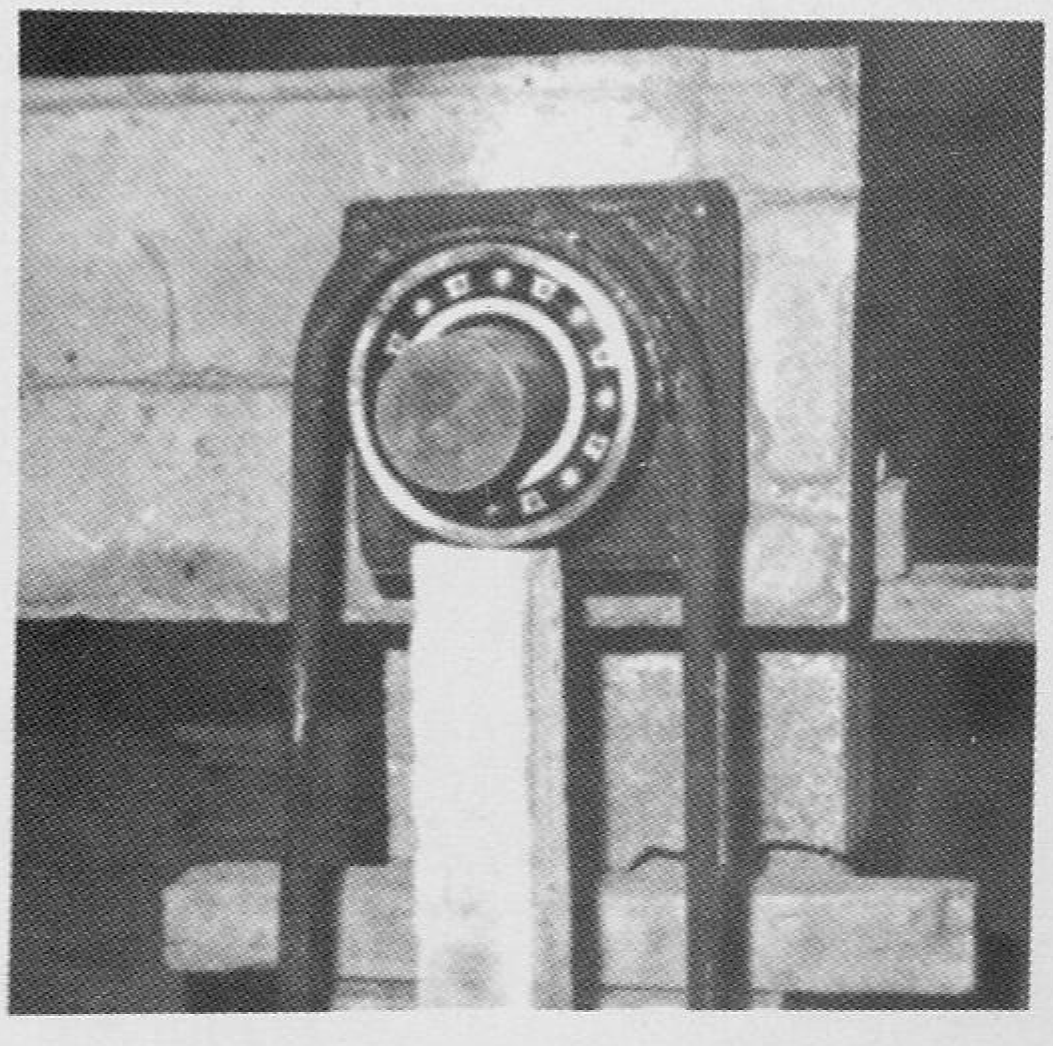
Plate II : (a) Testing Set up Showing a Jack Operating on a Beam.

(b), (d) Typical Hinge Formation in a Torsional Failure.

(c) "Garbled" Failure – Neither Flexural nor Torsional.



(a)



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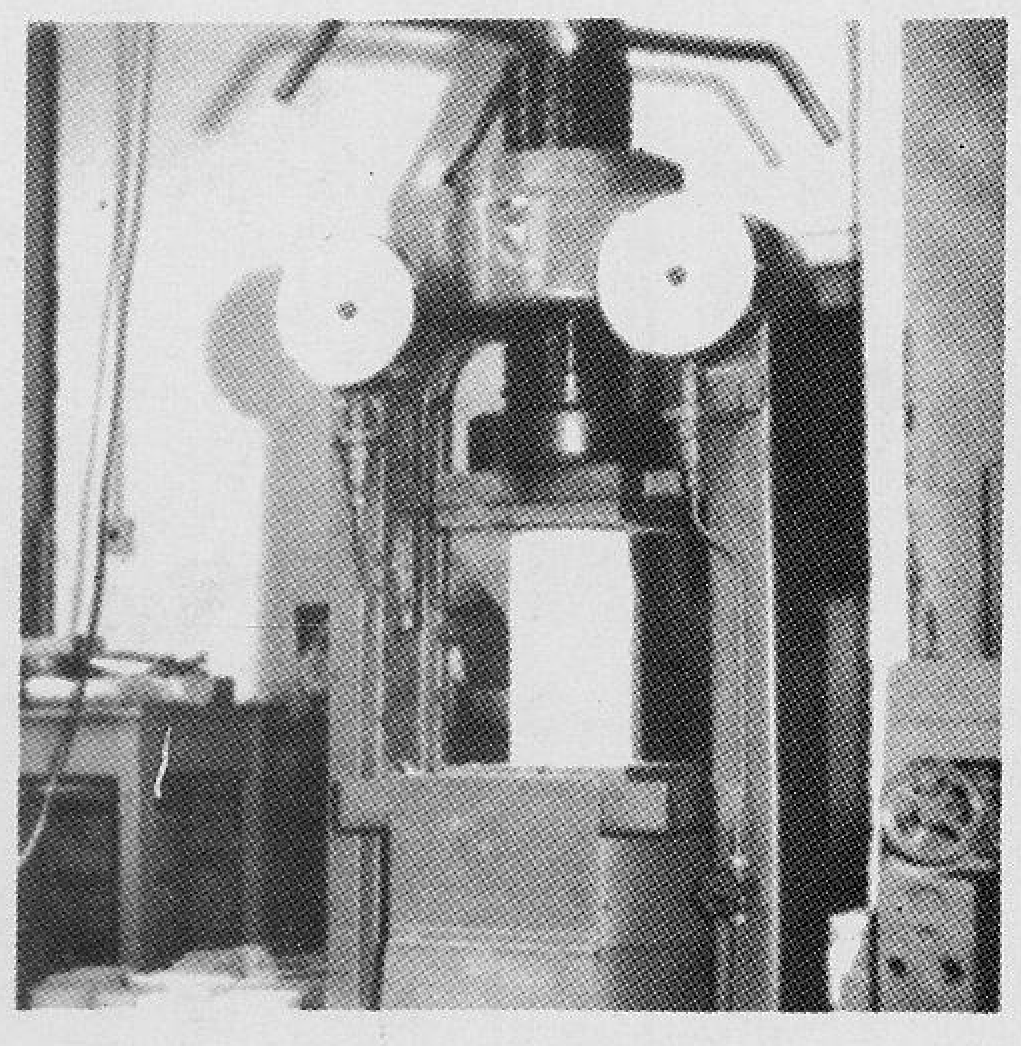


(c)

Plate III : (a) Corner Wedge Split Off from Beam after Torsional Failure.

(b) End of Beam Being Tested.

(c) Test Cylinder Being Crushed.

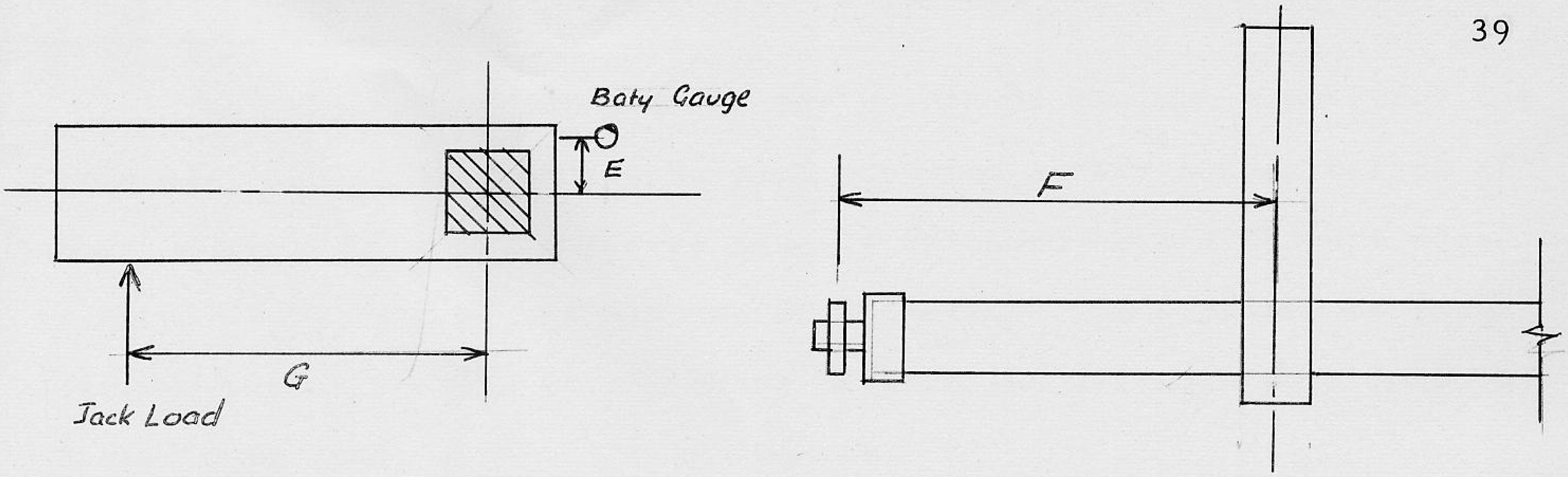


Fig. 11- Elevation of Arm ( ) and Plan of Arm and End of Beam ( ) Showing Dimensions Tabulated in Appendix "C" (2).

The test readings are also tabulated in Appendix "C". The actual loads are introduced instead of the pressure gauge readings. Also, rather than tabulate the horizontal deflections of the beam arms, the writer has tabulated the average of the two arm horizontal readings.

No claim is being made by the author to the effect that the tests carried out as described above were sufficiently accurate to be considered conclusive. For one thing, no strain gauges were introduced onto the prestressing wire or in the concrete to determine actual prestress and so only approximate values for the prestressing force are available. For another, in applying the loads the jacks were impossible to coordinate to the extent that there was certainty of the equal application of load. What was more important, however, was that in the attempt to get 5kg/cm2 increments on the gauges the jacks were frequently going above the desired reading temporarily by as much as 5kg/cm2. Further, cracks were noticed in general when the concrete beam gave with a bang and the gauges on the jacks lost considerable pressure.

The position of the arms at the points of maximum moment for the large part of the beam series perhaps produced stress concentration at those points. Further, stirrups usually extended beyond the arm edge by about 10 cm towards the midspan of the beam. The presence of these stirrups probably affected the crack pattern and strength when failure occured just next to the face of the arm.

It is also probable that the accuracy of the Baty gauges was not consistent with the mode of measurement, i.e. the distance of the gauge point from the center line of the beam was continually changing and the gauge point was moving against the concrete surface which was sure to have surface undulations which would effect the accuracy of the readings.

The above mentioned inaccuracies would result in errors of the order of:

1) Lack of strain gauges would probably entail an error in $P = \pm$ 5%. This, however, introduces errors of only $\pm$ 2-3% in the values observed for moments and torques as shown by calculation.
2) The irregular manual jacking resulting in higher temporary gauge readings would result in errors of up to +10% in the load on the beams. This introduces an error of up to +20% in the moment and up to + 10% in the torque.
3) Where secondary compressive failures were observed cracks may have occured earlier than actually detected. The error entailed, however, cannot be great, being possibly of the order of − 5% in the torque and − 10% inthe moment values.
4) The stress concentration at the arms is for the large part offset by the presence of stirrups.
5) The vertical movement of the beam and the irregularities of the

concrete surface are expected to have introduced an error into the angle of twist observation of up to ± 100% in the elastic range and up to ± 20% for the plastic range.

The tests, however, are believed to be indicative as regards the type of failure to be expected at different torque to moment ratios and as regards the value at cracking failure of the torques and moments.

CHAPTER IV

ANALYSIS OF TEST RESULTS

The test results of Chapter III were analysed to obtain the actual torques and moments at cracking. The torques were estimated in a straight forward manner to be the jack force multiplied by the distance between the jacks and the centerline of the beam less the torque due to the dead weight of the arm cast onto the beam. The maximum moment on the beam is that at the arm. Any point between the arms has a smaller moment due to the dead weight of the beam. In estimating this moment use must be made of the distance between the centerline of the arm and an end point of the beam. Because of the loose box connection at the end of the beam neither the ball bearing nor the end of the beam were considered to be the end point. The distance between the ball bearing and the end of the beam varied between 10 cm and 14 cms. The end point was considered to be the point 10cms inside the ball bearing center. This end point was used for the concentrated moments and the dead load of the beam moment. The latter assumed the dead load to extend fully between end points, the error thus introduced being very small. The following table contains the value of T and M at cracking for the various beams tested. A sample calcu-

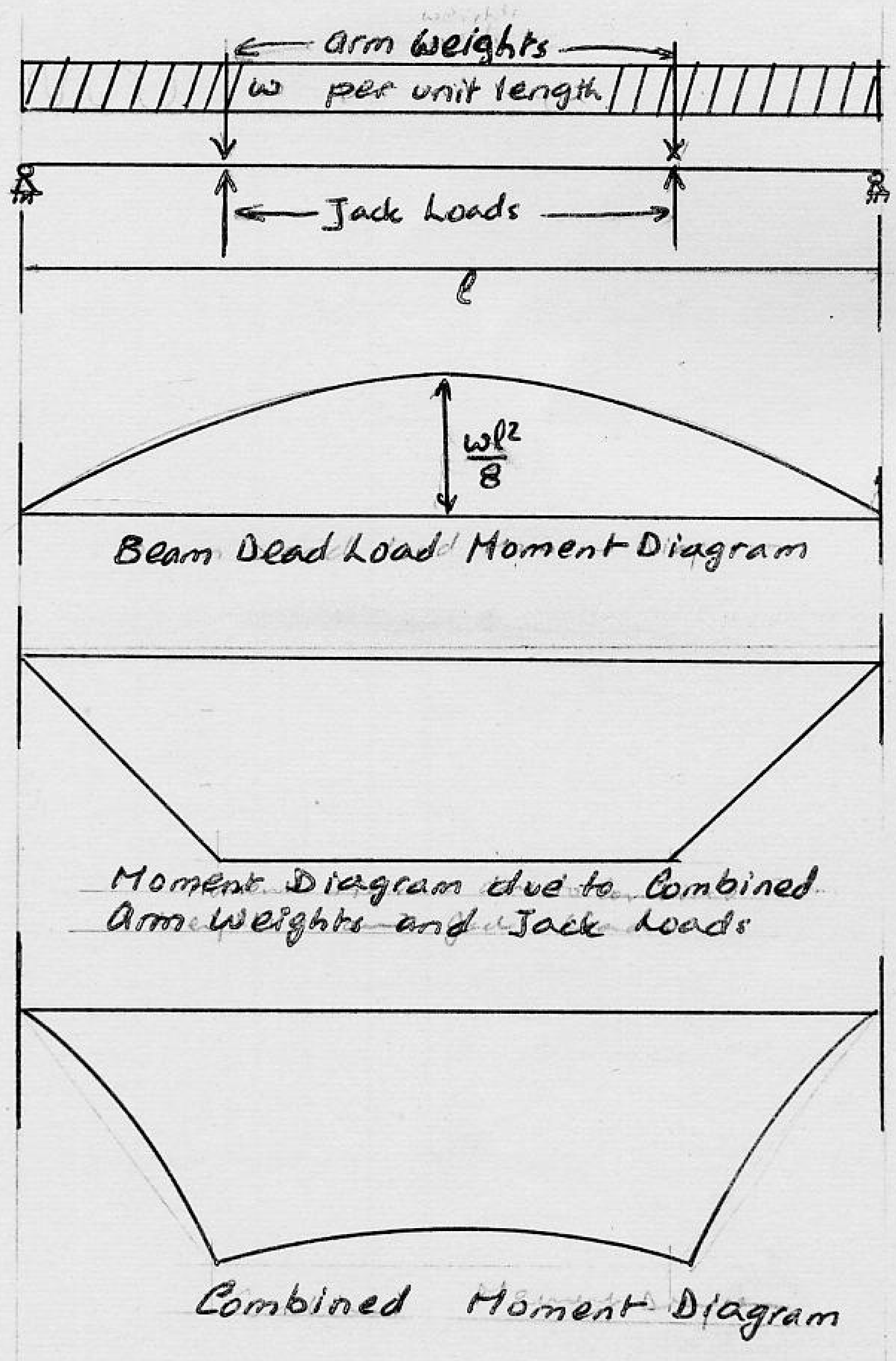


Fig. 14- Moment Diagrams for Test Beams.

lation is shown in Appendix "D".

BEAM NO.	T/M	MOMENT Kg.-m	TORQUE Kg.-m	BEAM NO.	T/M	MOMENT Kg - m	TORQUE Kg - m
3	0	850	-	12	0.4	532	213
5	2.1	227	481	13	5.5	142	782
6	1.9	290	547	14	5.0	173	866
7	0.8	587	483	15	3.7	137	509
8	0.8	605	483	16	3.4	185	620
9	1.2	444	529	Test Beams 1,2,4,11 were discarded as per Appendix "C".			
10	1.2	444	540				

Before it was possible to proceed any further the values of α and β as defined in equations (15) to (18) of Chapter II had to be obtained. The value of (a), however, had to be ascertained. Two values were tried and used in trial calculations. They were for a=1.3 and 2.0. The former value follows from the assumption made in Chapter II that the stress-strain relationship for concrete under tension is similar to that for concrete under compression. Fergusson[9] reproduces stress-strain curves for concrete under compression as found by Hognestad and reported in the ACl Jour-

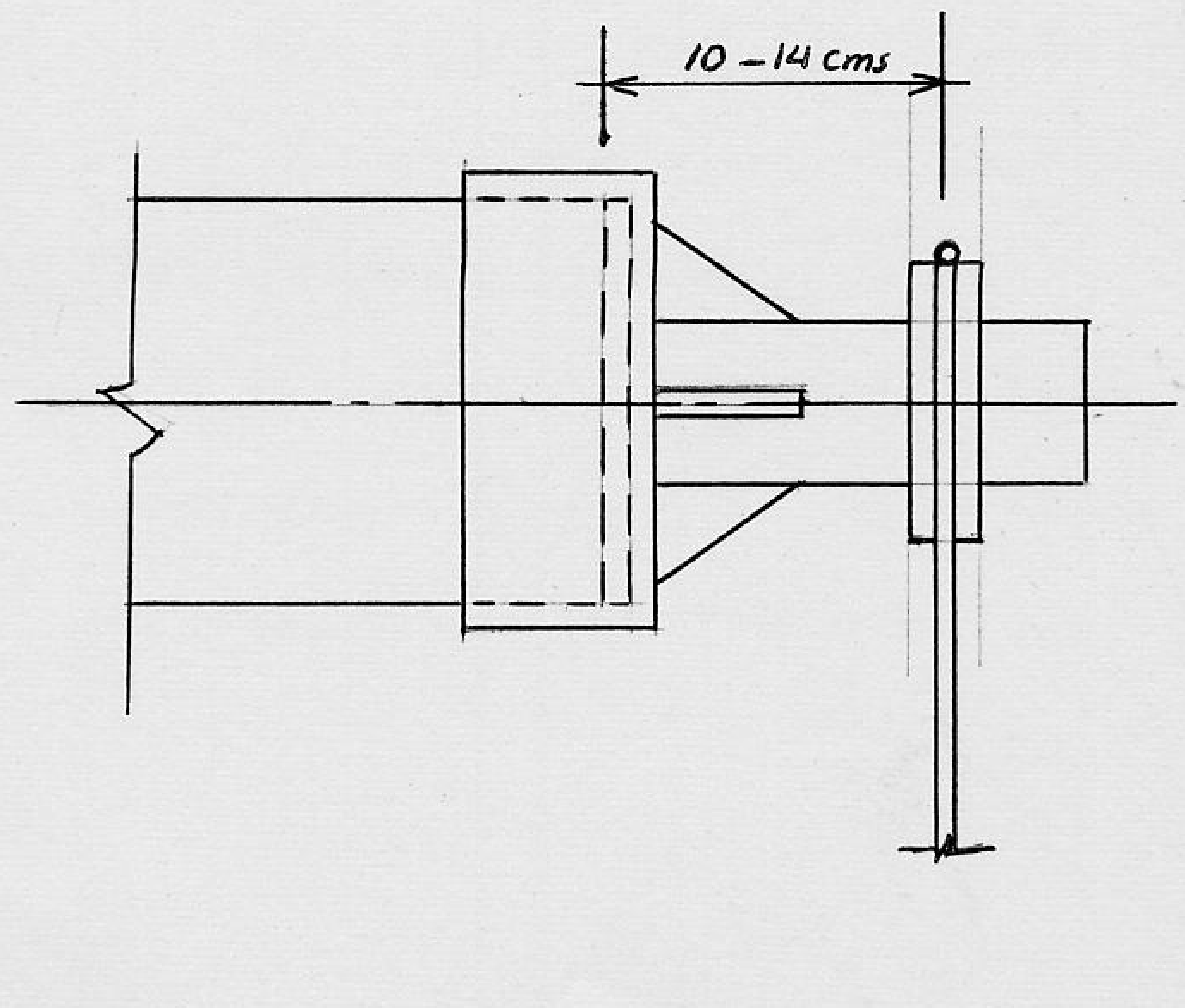


Fig.15- End Box Connection to Test Beams.

nal, 27, Dec. 1955. These show that (a) has a value of approximately 1.3 for concrete having an f'_c of 4000 psi and higher. The second value of (a) = 2.0, is that used by Evans[7] in the evaluation of his experiments on hollow reinforced concrete beams. The values of α and β are presented for both the above values of (a) for n =0.70 to 0.90 in Appendix "D".

The torsional factor for square beams K is 4.8 and is plugged in as such. In the absence of strain gauges attached to the prestressing strands a factor must be used to estimate the prestressing force after losses. This factor was chosen to be 0.85, representing a 15% loss of prestress. Next a factor was assumed for the ratio f_{tu}/f'_c thus enabling the evaluation of f_{tu}. With f_{tu} and P thus known α and β may be evaluated. Knowing β enables one to calculate M_c for the beam. Then, with all the beam constants known, the pertinent values of M and T are calculated by means of equations (24) and (25). A check was made to see the correspondence of the values thus calculated with those observed. Adjustments to the ratio of f_{tu}/f'_c were made accordingly and the calculations repeated until the observed values checked out reasonably well with those calculated. It was found that for (a) being 1.3 and 2.0 the values calculated for M and T were the closest to those observed at f_{tu} =0.1 f'_c. Those for a=1.3 corresponded more closely, however, with the observed values at a T/M ratio of 0, and were therefore chosen as the applicable values of (a) and f_{tu}/f'_c. The values of M and T for a=1.3 and f_{tu} = 0.1 f'_c are listed in the table below:

BEAM POUR	BEAM NO.	T/M	MOMENT Kg - m		TORQUE Kg - m		% ERROR
			OBSERVED	COMPUTED	OBSERVED	COMPUTED	
T2 & T4 Beams	3	0	850	820	-	-	+3.6
	5	2.1	227	262	481	550	-13.4
	7	0.8	587	493	483	395	+19.1
	10	1.2	444	410	540	492	+ 8.5
	12	0.4	532	680	213	272	-21.7
	14	5.0	173	121	866	605	+43.0
	16	3.4	185	170	620	578	+ 8.8
T1 & T3 Beams	6	1.9	290	280	547	532	+ 3.6
	8	0.8	605	532	483	427	+13.7
	9	1.2	444	403	529	485	+10.2
	13	5.5	142	118	782	650	+20.4
	15	3.7	137	172	509	636	-20.3

Values calculated for a=1.3, $f_{tu} = 0.1 f'_c$

A sample calculation for one beam together with a table showing the various canstants and values used for each beam are presented in Appendix "D".

The observed values of torques and moments, with the exception of those of Test Beam No.14, are all within ± 20% of the corresponding computed values. Considering the experimental error involved this percentage error is reasonable for the subject matter. The large error of Beam No.14 may be explained by the fact that the jack load on the beam at cracking was actually less than that recorded. The beam cracked upon reaching a jack load of 2300 lbs. The failure was so sudden and quick that there was no time to observe the deflection readings. If the next lower jack load observed (2100 lbs) is used, the following more manageable results are obtained.

	Observed	Computed	% Error
Moment (Kg - m)	146	116	26
Torque (Kg - m)	780	620	26

To represent the derived equations and the observed values graphically a beam of the same section having an initial prestress of 40,000 lbs and an f'_c at the testing date of 250 kg/cm2 was used for the same values of the T/M ratios observed. The values thus computed were adjusted by the percentage errors in the observed values and both were plotted in Fig. 16.

Using the algebraic percentages of error as tabulated above and calculating the algebraic average of errors for both the beams with the prestressing wires at the centroid and at the corners it is seen that for the former the algebraic percentage scatter is +7% while for the latter it is +5.5%.

One may conclude the following from a study of the test results as analysed above:

1. A study of Fig.16 and the observed crack patterns tabulated in Appendix "C" would lead to the following conclusions:
   For a T/M ratio of 0 to about 0.25 failure is flexural.
   For a T/M ratio of about 0.25 to 1.0 failure is garbled, beginning as flexural but, once cracks form, ending up as torsional.
   For a T/M ratio over 1.0 failure is torsional.

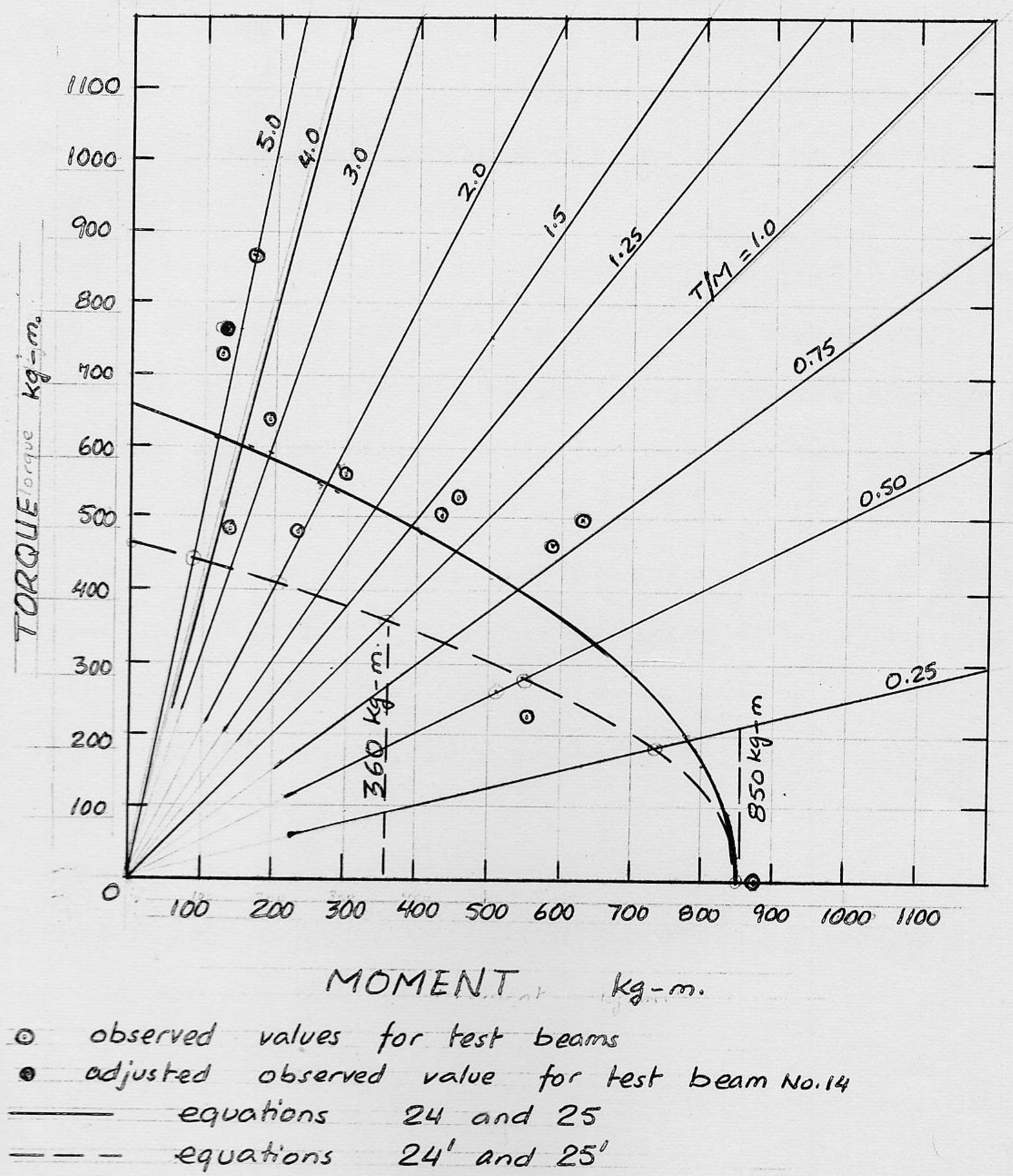


Fig. 16- Graphical Representation of Equations (24), (25), (24'), (25') as Applied to the Test Beams with the Observed Results of the Test Beams.

2. No dowel action due to the presence of the prestressing wires at the corners of the section was noted.
3. Equation (24) is satisfactorily borne out by the experiments.
4. Cleavage, (torsional) failures occured suddenly whereas flexural failures occured in two stages: an initial tensile failure followed by either a cleavage or a secondary compression failure.
5. For square and near square sections equation (24) applies to the exclusion of (25) while for rectangular sections equation (25) deserves checking and would produce the controlling torques and moments for deep rectangular sections.
6. For a T/M ratio of 0 to 0.1 the moment capacity of a section is not affected by the torque and the torque may be neglected.
7. For a T/M ratio of 3 to ∞ the moment acting on the section need not be considered and the design could be made for the torque and shear only.
8. The pertinent factor of safety for moments may be safely applied to the torque involved in a T/M ratio of up to 1.0. For higher T/M ratios it is suggested that the factor of safety for torsion be increased due to the sudden, and sometimes explosive nature, of a cleavage or torsional failure.
9. A prestressed concrete member without web reinforcement fails abruptly under torsion so that the elastic or cracking torque and the ultimate torque are one and the same.

CHAPTER V

A DESIGN PROCEDURE & ILLUSTRATIVE EXAMPLE

The author would like to present a design procedure for prestressed beams under loading conditions producing torques, moments and shears. This is based on equations (24) and (25) as borne out by the experiments described and analysed in Chapters III and IV. For easy reference equations (24) and (25) are now reproduced.

$$\left(\frac{\gamma K \beta}{12}\right)^2 \left(\frac{T}{M_c}\right)^2 + \frac{\beta}{2}\left(\frac{M + Pd/6}{M_c}\right) \leq 1 \qquad (24)$$

$$\left(\frac{\beta}{4}\right)^2 \left[\frac{Vd}{2M_c} + \frac{TK}{3M_c}\right]^2 + \frac{\beta}{12}\frac{Pd}{M_c} \leq 1 \qquad (25)$$

The assumption is made that the Torque, Moment, and Shear values (T,M,V) have already been modified by a suitable uniform factor of safety prior to their being plugged into the equations. The author believes that a further factor of safety should be introduced to the torque and shear values, however, Any increase of $\sqrt{2}$ is proposed. Equations (24) and (25) modified by this increase would be:

$$2\left(\frac{\gamma K \beta}{12}\right)^2 \left(\frac{T}{M_c}\right)^2 + \frac{\beta}{2}\left(\frac{M + Pd/6}{M_c}\right) \leq 1 \qquad (24')$$

$$2\left(\frac{\beta}{4}\right)^2 \left[\frac{Vd}{2M_c} + \frac{Tk}{3M_c}\right]^2 + \frac{\beta}{12}\frac{Pd}{M_c} \leq 1 \qquad (25')$$

For a given moment, torque and shear acting on a beam the designer will be able to evaluate T/M. An estimate of the ratio of M to that moment M_0 having a T/M ratio of zero is possible to make from Fig. 16. The section is designed for M_0 and checked by equations (24) and (25) or by equations (24') and (25') if the additional shear and torsion factor is adopted.

<u>Example (1)</u> Design a prestressed concrete beam for the following and check by equations (24') and (25').

T = 4000 kg-m
M = 4000 kg-m } These values include a uniform load factor.
V = 1000 kg

f'_c = 400 kg/cm^2 a=1.3

f_{tu} = 40 kg/cm^2 d=2b

$f_p \leq$ 100 kg/cm^2

<u>Solution:</u>

From Fig. 16 $\frac{M}{M_o} = 0.43$ for $\frac{T}{M} = 1$ $\left(\begin{matrix} M = 360 \text{ kg-m} \\ M_o = 850 \text{ kg-m}\end{matrix}\right)$

$$M_o = \frac{4000}{0.43} = 9300 \text{ kg-m.}$$

Try a section 25 cms by 50 cms with P= 100,000 kgs.

$$I = \frac{25 \times 50^3}{12} = 260{,}000 \text{ cm}^4 \qquad A = 25 \times 50 = 1250 \text{ cm}^2$$

$$\frac{P}{A} \pm \frac{Md}{2I} = -\frac{100{,}000}{1250} \pm \frac{9300 \times 100 \times 50}{2 \times 260{,}000} = -80 \pm 90 \text{ kg/cm}^2$$

$$f_c = -170 \text{ kg/cm}^2$$

$$f_t = +10 \text{ kg/cm}^2$$

The section can resist the moment of 9300 kg - m.

$$\frac{2P}{A} = 160 \text{ kg/cm}^2 \qquad \alpha = \frac{160}{40} = 4.00 \qquad \beta = 6.61$$

$$K = 3.00 + 1.80 \times \frac{50}{25} = 6.6 \qquad \gamma = 0.63 \qquad M_c = 13{,}750 \text{ kg-m.}$$

Substitute in equation (24'):

$$2\left(\frac{6.6 \times 6.61 \times 0.63}{12}\right)^2\left(\frac{4000}{13750}\right)^2 + \frac{6.61}{2}\left(\frac{4000 - 100{,}000 \times 0.50/6}{13750}\right)$$

$$= 0.89 - 1.04 = -0.15 < 1$$

Substitute in equation (25'):

$$2\left(\frac{6.61}{4}\right)^2\left[\frac{1000 \times 0.50}{2 \times 13750} + \frac{4000 \times 6.6}{3 \times 13750}\right]^2 - \frac{6.61}{12} \times \frac{100{,}000 \times 0.50}{13{,}750}$$

$$= 2.37 - 2.01 = 0.36 < 1$$

The section is oversafe.

Two alternatives are open for a final solution:

a) reduce the prestress.

b) reduce the section dimensions.

If (a) is to be applied:

assume P is reduced to 70,000 kgs.

$$\frac{2P}{A} = 112 \text{ kg/cm}^2 \qquad \alpha = \frac{112}{40} = 2.80 \qquad \beta = 5.41 \qquad M_c = 11,250 \text{ kg-m.}$$

Substitute in equation (24):

$$2\left(\frac{6.6 \times 5.41 \times 0.63 \times 4000}{12 \times 11250}\right)^2 + \frac{5.41}{2}\left(\frac{4000 - 70,000 \times 0.50/6}{11250}\right)$$

$$= 0.89 - 0.44 = 0.45 < 1$$

Substitute in equation (25):

$$2\left(\frac{5.41}{4}\right)^2\left[\frac{1000 \times 0.50}{2 \times 11250} + \frac{4000 \times 6.6}{11250 \times 3}\right]^2 - \frac{5.41}{12} \times \frac{70,000 \times 0.50}{11250}$$

$$= 2.35 - 1.40 = 0.95 < 1$$

A section of 25 cms x 50 cms with a prestressing force of 70,000 kgs meets the requirements.

One may reduce the section to a smaller size by the same procedure. The most economical of the two solutions will then be adopted.

CHAPTER VI

DISCUSSION AND SUGGESTED FURTHER RESEARCH IN THIS FIELD OF STUDY

By comparison with the vast amount of experimental evidence on the behaviour of concrete in bending, compression and transverse shear, the attention given to torsional strength is slight.

This investigation is based on assumptions that require verification. It is also not related to actual construction practice in some aspects.

A glaring assumption in need of verification is that the concrete stress-strain relationship in tension is parabolic. That concrete in tension is plastic[7] has been investigated and proven by torsion failure tests. A great amount of work, however, remains to detect the degree of plasticity and its variation with the strength of concrete. It is known that the higher the strength of concrete, the more brittle it is[9]. Once the particular stress-strain relationship of concrete in tension is known, equivalent geometric relationships could be established along the lines of the Whitney equations for concrete in ultimate compressive strength. Further, observe that, setting T=0 in equation (24) does not result in $M = M_c$ unless $\beta - \alpha = 2$. This indicates that the assumption that the stress-strain for concrete in tension is parabolic is an approximation at best as $\beta - \alpha \neq 2$ except for special cases of (n) and (a). This aspect of the problem could do with further study.

The compressive strength of concrete may readily be measured by means of cylinders or cubes. No such quick or reliable method is in use for the determination of the tensile strength of concrete. Professor I. Rubinsky[13] has proposed a method that is claimed to give a reliable assessment of the concrete tensile strength of a bar sample. The writer feels that the method proposed is not very handy and does not lend itself easily to construction practice.

Prestressed concrete members nearly always have shear reinforcement and/or longitudinal non-stressed reinforcing steel. These are bound to have an effect on the torsional strength of the members. There is, however, only one investigation[6] on record where use of torsional reinforcement was made in addition to prestressing. Investigations on reinforced concrete[10,11] indicate that longitudinal reinforcing bars induce a dowel action that strengthens a section against torsion while shear reinforcement reduces the strength of a member against flexure. Surely the topic is important enough to warrant many more investigations into the behaviour of prestressed concrete under such conditions.

Double prestressing should increase a section's resistance to torsion much more than single prestressing. The case of double prestressing is frequently applied in bridge decks where torsion is a factor to be considered in the design. Analysis of double prestressing could follow the same general discussion presented in Chapter II.

Finally, construction makes use of Tee, L, box and other non rectangular sectioned members that are prestressed. It is possible to apply the membrane analogy to extend the discussion of this investigation to that of such shapes[15] but such a method is rather tedious and is better suited to laboratory work. Bach[6] has presented a method for the evaluation of the torsional rigidity of such non rectangular sections. Inspite of the fact that non rectangular sections are very frequently made use of in design, the author has found record of only two investigations on the subject[6], one is by R.P.M. Gardner (1960) and the other by P.Zia (1960). Both are mentioned in Chapter I of this paper.

One can only conclude that there is a very wide scope for further study and investigation on the subject of torsion in prestressed concrete. The work already carried out is a good start but should not be considered as anything but a beginning.

APPENDIX "A"

PROPERTIES OF MATERIAL USED IN TEST BEAMS

1. Properties of Ø 7mm high tensile wire:

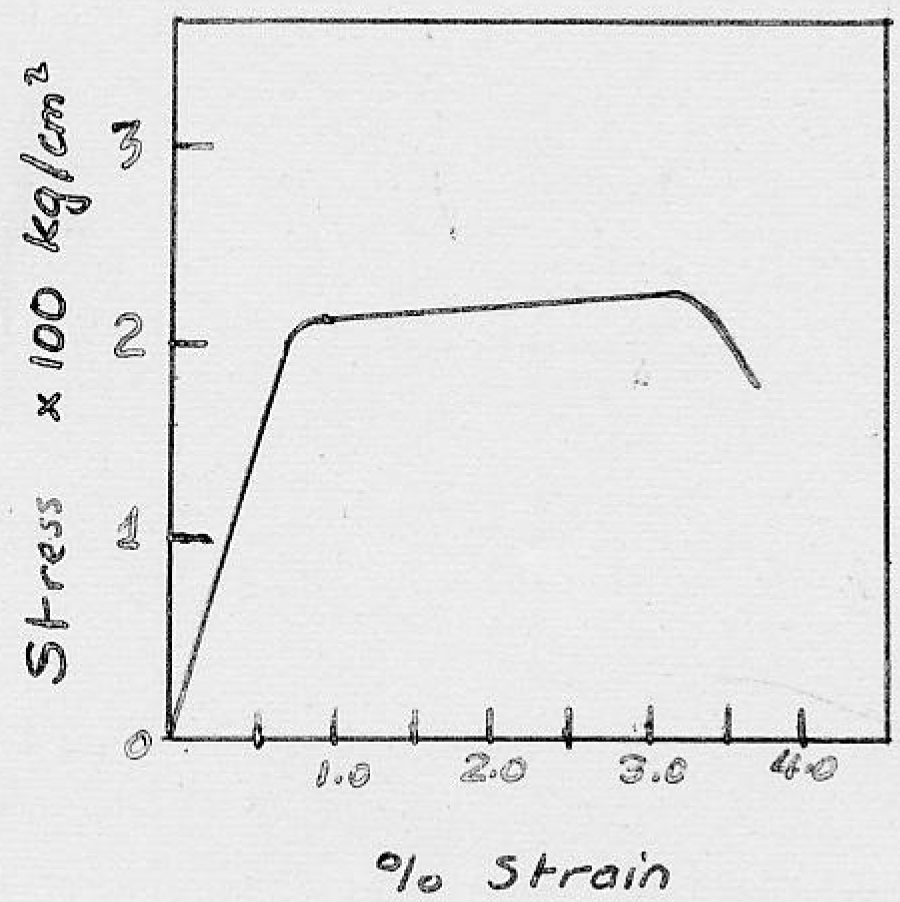
   Yield stress at 0.2% strain: 200,000 psi

   Ultimate stress: 228,000 psi

   Stress at rupture: 178,000 psi

   Elongation % on 61 cm. gauge length: 3.61

   Cold Bend: No crack appeared



2. Properties of Concrete Aggregate:

	Coarse Gravel	Fine Gravel	Sand
Finess Modulus	6.84	2.52	1.45
Absorption	1.00%	2.02%	0.57%
Specific Gravity	2.69	2.69	2.65
Unit Weight-Loose	87.9lb/cu.ft	85.7lb/cu.ft	95.6 lb/cu.ft
Rodded	104.7lb/cu. ft	100.6lb/cu.ft	105.2 lb/cu.ft
Caorimetric Test	-	-	500 ppm
Abrasion by L.A. Machine	24.26%	-	-
Gradation % Passing Sieve			
1½"	100	100	100
¾"	93.0	100	100
3/8"	22.7	99.3	100
4	0.29	46.2	100
8	0.06	2.31	99.9
16	0.05	0.12	99.5
30	0.04	-	98.4
50	0.03	-	56.1
100	0.02	-	1.03
200	0.01	-	0.23

3. Mix Gradations:

Seive No.	% Passing
$1\frac{1}{2}$"	100
3/4"	96
3/8"	66
4	44
8	34
16	33
30	33
50	19
100	0.4
200	0.1

The gradation curve is shown in Fig.13-

4. Concrete Strength in compression:

The graphs shown in Fig. 12 indicate the cylinder crushing strength for the various testing dates for the four series of test beams: The test figures are as follows:

(T1) cast on 30/4-1966	(T2) cast on 1/5-1966
2470 psi at 5 days 3400 psi) 3530 psi)at 24days 3880 psi) 3280 psi at 25 days 3610 psi) 4250 psi)at 26 days	2370 psi at 4 days 3460 psi) 4060 psi)at 22 days 4600 psi) 3610 psi) 4460 psi)at 25 days 3390 psi) 4170 psi) 4100 psi)at 26 days 3960 psi) 2470 psi)

* Poor Capping Failure - Disregarded

(T3) cast on 6/5-1966	(T4) cast on 7/5-1966
2900 psi at 7 days 3610 psi at 17days * 2970 psi) 3610 psi) at 19 days 3950 psi) 3850 psi) 3570 psi) at 21 days 3960 psi) 3070 psi) at 22 days	3250 psi at 16 days 3360 psi) * 2610 psi) at 17 days 3230 psi) 3680 psi at 20 days 3460 psi) 3180 psi) at 21 days

* Poor capping Failure-Disregarded

A1 on 10/5	A2 on 11/5
3390 psi) 3680 psi) at 14 days 4100 psi)	3250 psi) 3390 psi) at 12 days 3530 psi)

A3 on 13/5	A4 on 14/5
2830 psi) 2970 psi) at 14 days 2970 psi)	2470 psi) 2900 psi) at 10 days 3110 psi)

5. Concrete Strength in Tension:

(T1) cast on 30/4-66	(T2) cast on 1/5-66
120 psi	245 psi
240 psi at 24 days age	240 psi at 22 days age
210 psi	280 psi
200 psi	360 psi
220 psi at 25 days age	280 psi at 25 days age
220 psi	215 psi
170 psi	270 psi
160 psi at 26 days age	240 psi at 26 days age
160 psi	360 psi
150 psi	270 psi

(T3) cast on 6/5-66	(T4) cast on 7/5-66
125 psi	130 psi at 16 days age
170 psi at 17 days age	140 psi
150 psi	150 psi
130 psi	265 psi at 17 days age
145 psi at 19 days age	170 psi
155 psi	180 psi
140 psi at 21 days age	190 psi at 20 days age
125 psi	170 psi
120 psi at 22 days age	145 psi at 21 days age
100 psi	200 psi

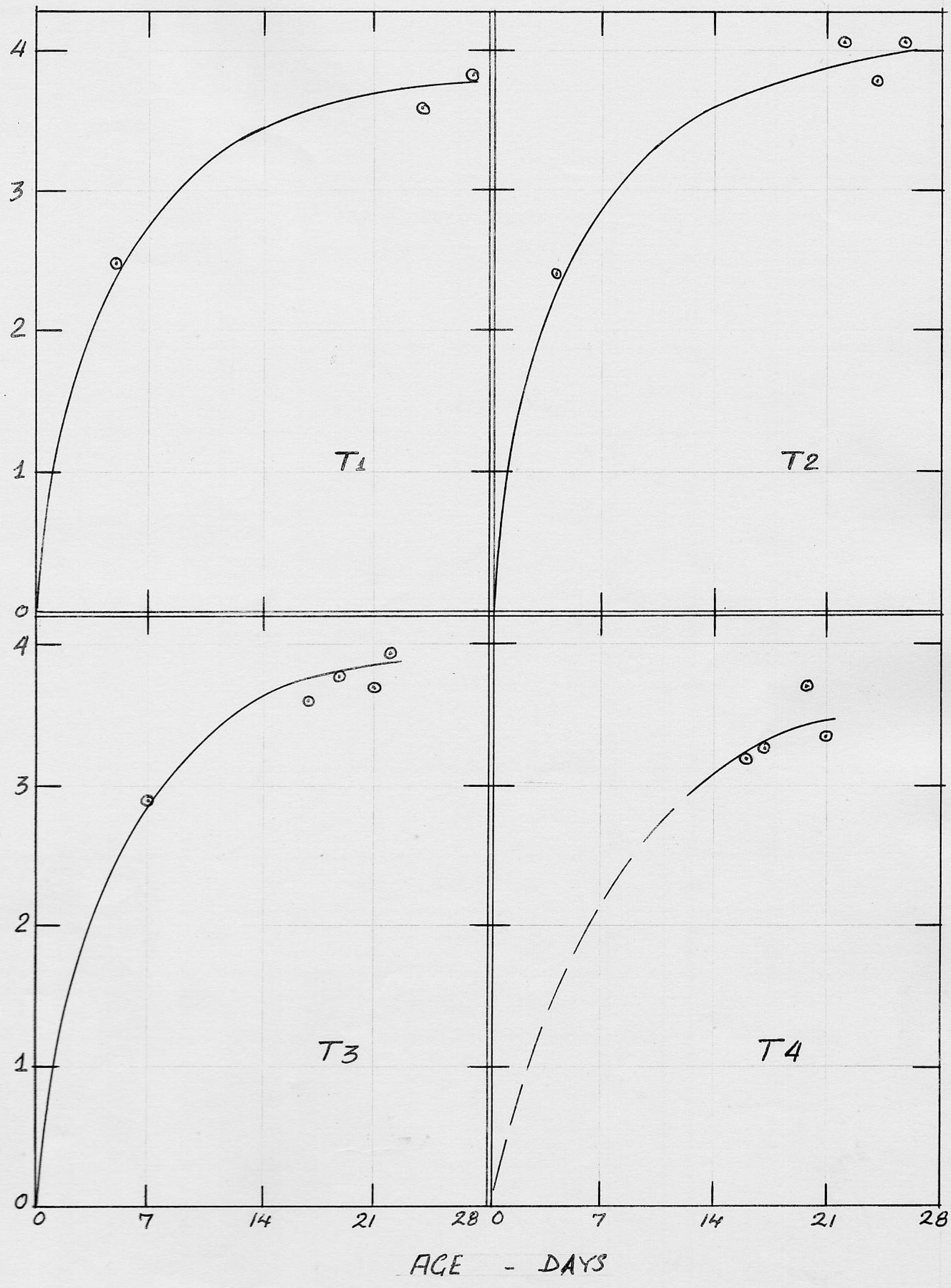


Fig. 12- Crushing Strength of Pours T1, T2, T3,and T4.

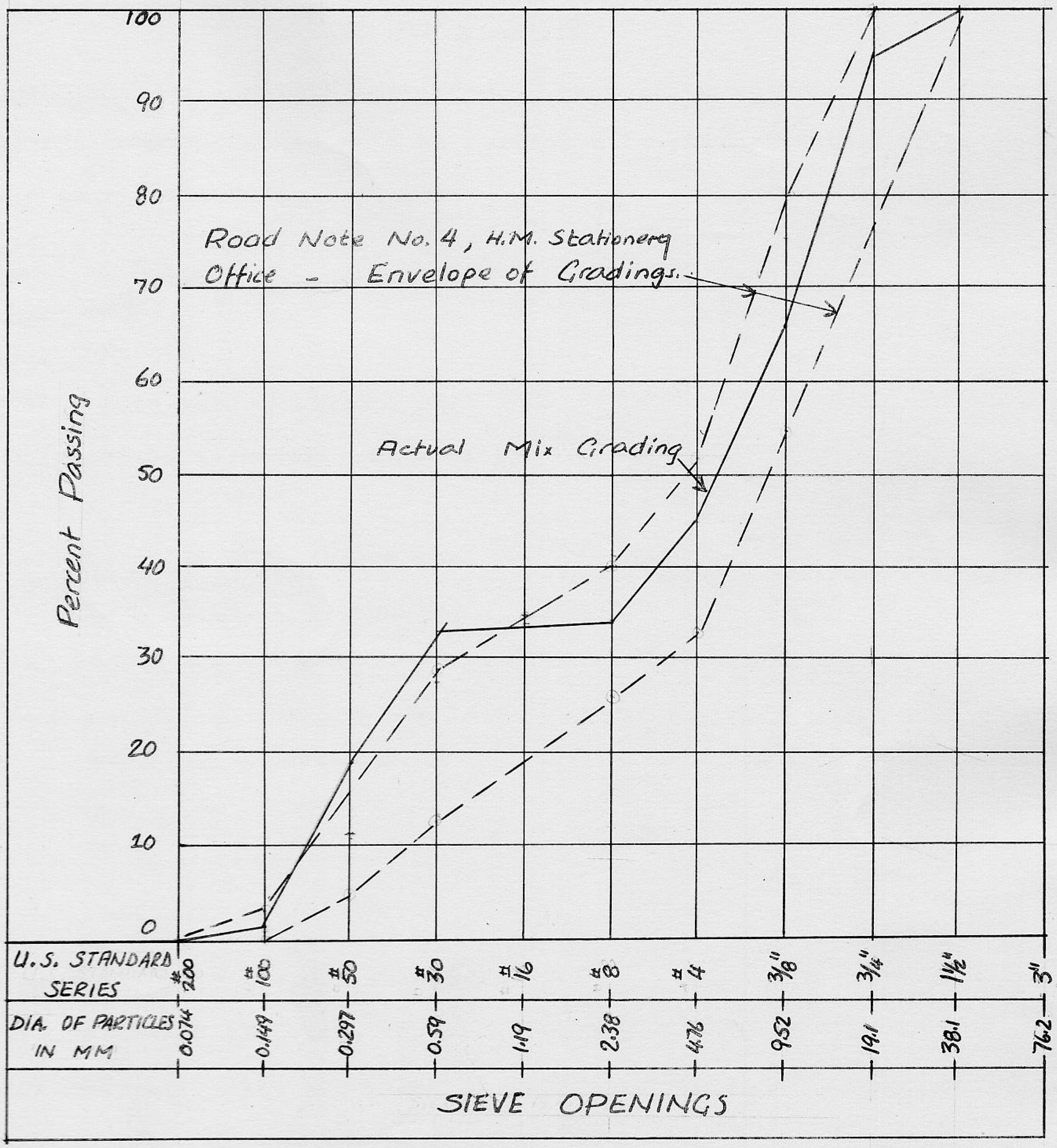


Fig. 13- Gradation of Concrete Mix Used for Test Beams and Torque Arms.

APPENDIX "B"

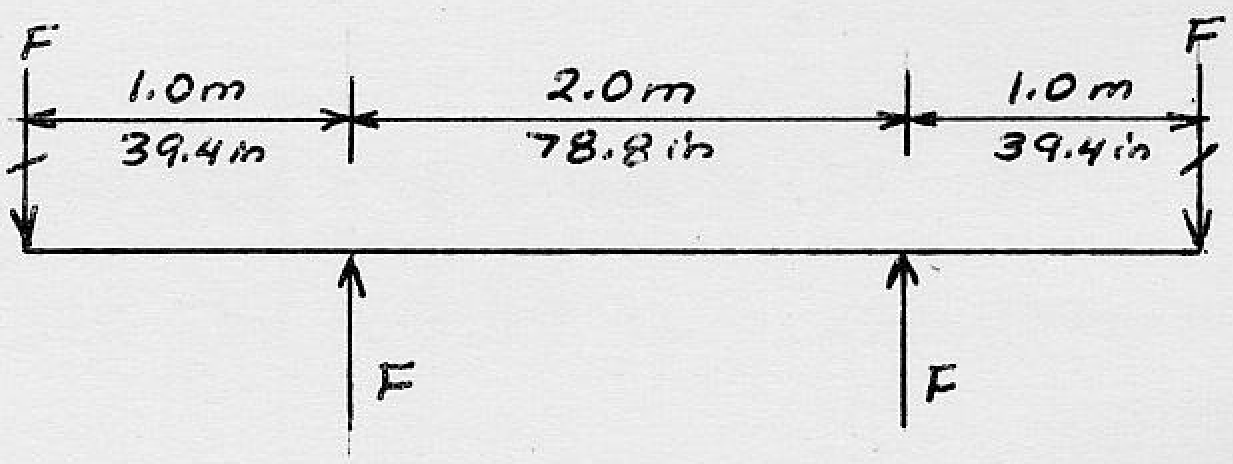
DESIGN OF TEST BEAMS

Approximate design of prestressed 20 cm x 20 cm beams:

ACI 1963 code complied with as regards allowable stresses.

b=20 cm = 7.9 inches

d=20 cm = 7.9 inches



f'_c = 4000 psi

f_{tu} = 0.1 f'_c

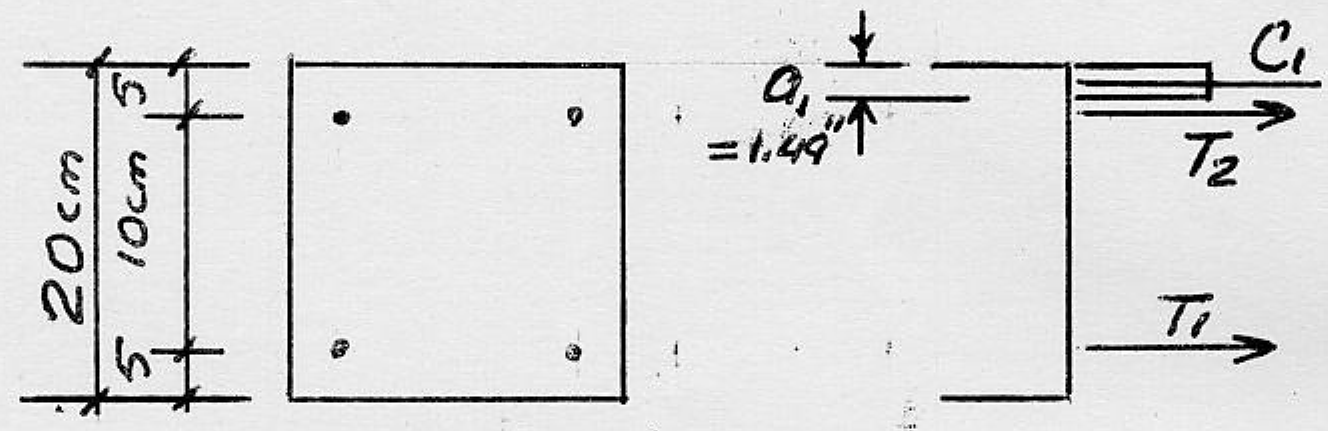
Initial prestress = 150,000 psi

4 Ø 7mm wires used for prestressing

$A_s = 4 \times 0.06\ in^2 = 0.24\ in^2$

$P_i = 0.24 \times 150{,}000 = 36{,}000$ lbs

$P = 0.85 \times 36{,}000 = 30{,}600$ lbs



CASE I

$$\frac{P}{A} = \frac{30{,}600}{7.9 \times 7.9} = 490\ psi$$

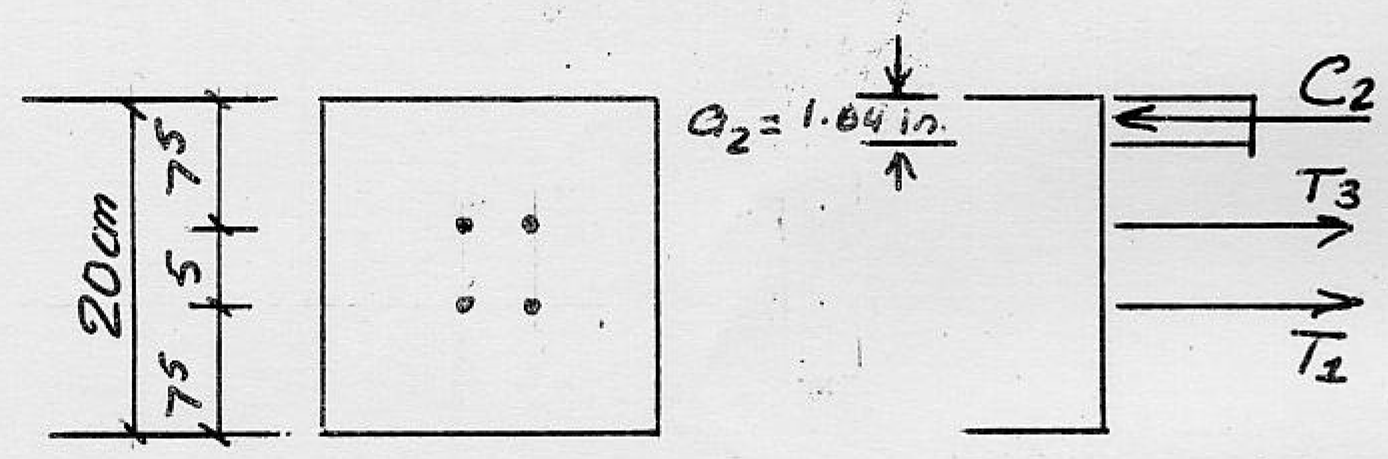
$$\frac{Md}{2I} - \frac{P}{A} = \frac{M \times 3.95 \times 12}{7.9^4} - 490 = 0.1 \times 4000 = 400 psi$$

$$M = \frac{890 \times 7.9^4}{3.95 \times 12} = 72600\ in\text{-}lbs.$$

$$f_c = 890 + 490 = 1380\ psi.$$

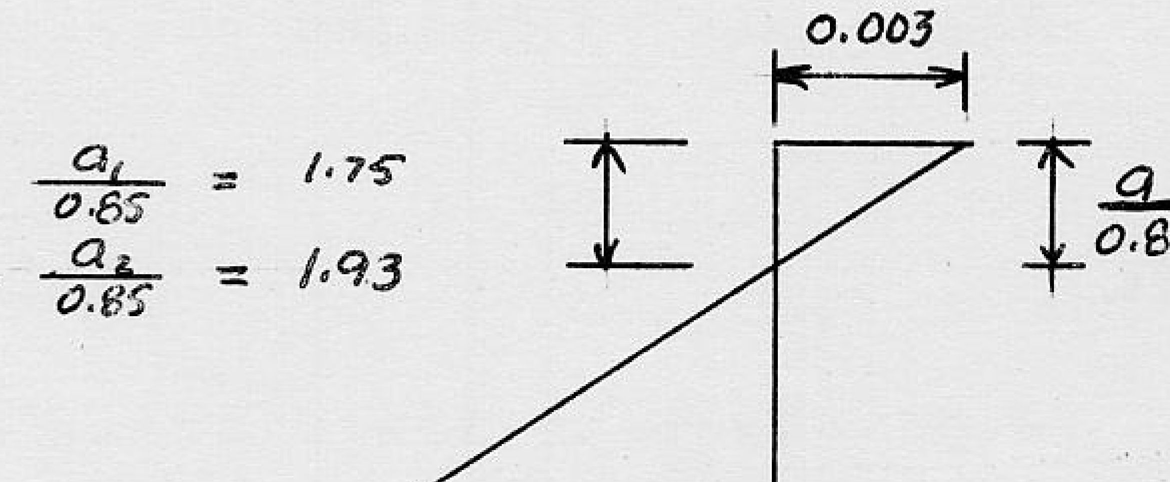
$$M = F_{lbs} \times 39.4\ in$$

$$F = \frac{72600}{39.4} = 1840\ lbs$$



CASE II

For the ultimate failure load two cases, (I) with the wires in the corner, and (II) with the wires in the center of the section are considered.



STRAIN LINE FOR CASES I & II

$T1 = 2 \times 0.06 \times 200{,}000 = 24{,}000$ lbs

$$T2 = 2 \times 0.06 \times 0.85 \times 150{,}000 + \frac{1.96-1.75}{1.75} \times 0.003 \times 29 \times 10^6 = 16{,}000 \text{ lbs}$$

$$T3 = 2 \times 0.06 \times 0.85 \times 15{,}000 + \frac{2.95-1.93}{1.93} \times 0.003 \times 29 \times 10^6 = 20{,}000 \text{ lbs}$$

C1 = 40,000 lbs

C2 = 44,000 lbs

$$a_1 = \frac{40,000}{7.9 \times 4000 \times 0.85} = 1.49 \text{ inches}$$

$$a_2 = \frac{44,000}{7.9 \times 4000 \times 0.85} = 1.64 \text{ inches}$$

$$M_u\Big|_{case\ I} = 24,000 \text{ lbs} \times 5.25 \text{ inches} + 16,000 \text{ lbs} \times 1.2 \text{ in.} = 145,000 \text{ in-lbs}$$

$$M_u\Big|_{case\ II} = 24,000 \text{ lbs} \times 4.11 \text{ inches} + 20,000 \text{ lbs} \times 2.13 \text{ in.} = 141,000 \text{ in-lbs}$$

$$F_{u\ case\ I} = \frac{145,000}{39.4} = 3700 \text{ lbs}$$

$$F_{u\ case\ II} = \frac{141,000}{39.4} = 3600 \text{ lbs}$$

$$v = \frac{3700}{7.9 \times 7.9} = 59 \text{ psi} < 125 \text{ psi allowable}$$

Ø 6mm stirrups @ 6 in centers will be placed in the last meter length from either end to counteract bursting tension and to stiffen that section of the beam.

A check on end anchorage length reveals the following:

ω_1, the allowable bond stress, = 0.1 f'_c = 400 psi

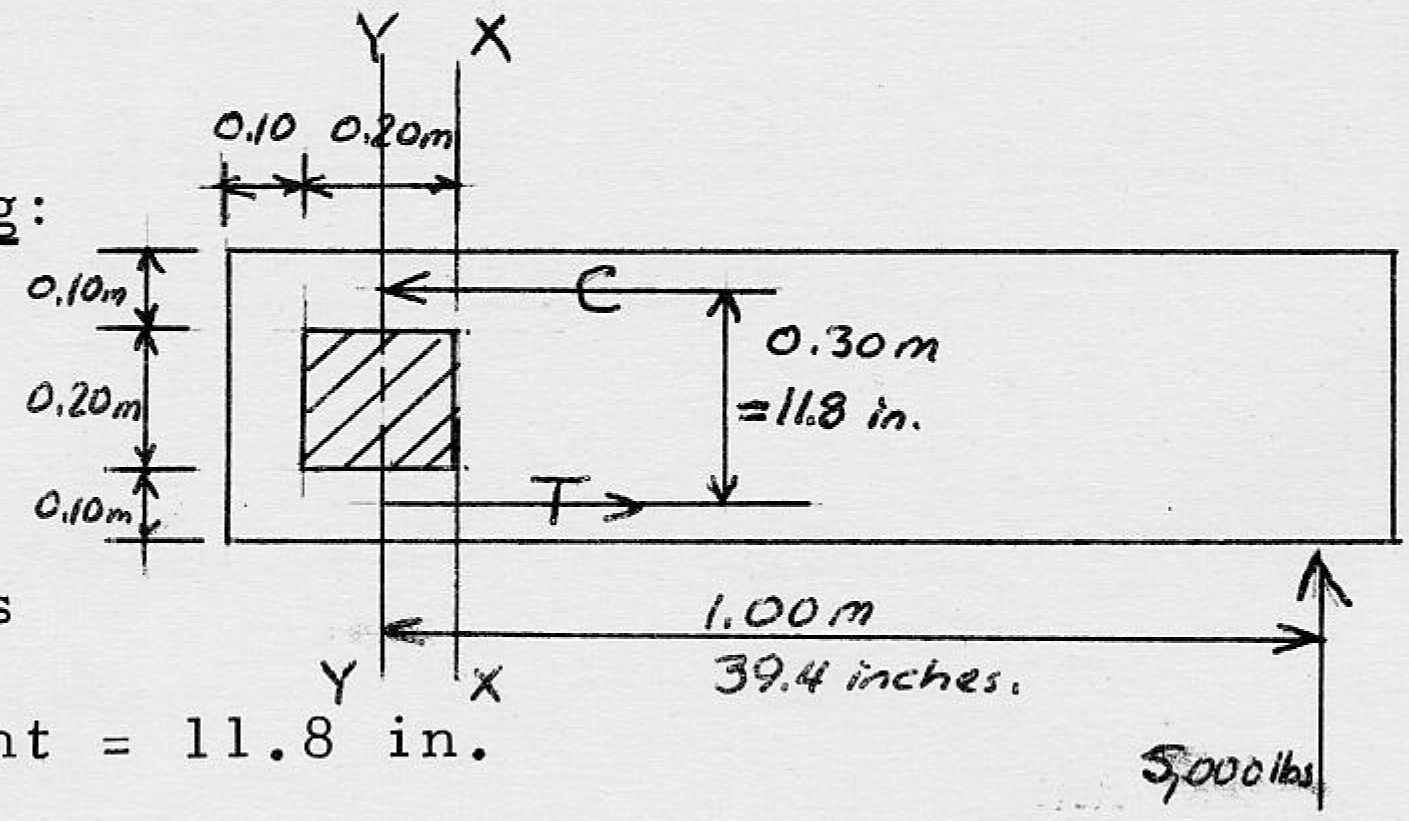
The bond length required to balance stresses caused by the cracking moment = X''

$$X'' = \frac{0.276 \times 150,000}{4 \times 400} = 25.9 \text{ inches}$$

The additional bond length required to balance stresses caused by the ultimate moment M_u = L'', the wires attaining yield stress only.

$$L'' = \frac{0.276\ (200-150)\ 1000}{4 \times 200} = 17.2 \text{ inches}$$

The total bond length required therefore to prevent bond failure prior to ultimate failure of the wire in tension = 43.1 inches. Only 40 inches of this is provided. As it is anticipated that the beam will yield in a secondary compression fashion and not in ultimate tensile failure of the prestressing wires, the bond length provided may be considered sufficient for the purpose.



Design of Torque Arm, 1 Meter Long:

f'_c = 4,000 psi

b = 7.9

M_{YY} = 5000 x 39.4 = 197,000 in-lbs

Arm for tensile steel reinforcement = 11.8 in.

$$T = C = \frac{197,000}{11.8} = 16,700 \text{ lbs}$$

$$A_s = \frac{16,700 \text{ lbs}}{25,000 \text{ psi}} = 0.67 \text{ in}^2$$

Section XX will be safer as M is less and the full section is working.

$$V = \frac{5000}{7.9 \times 13.5} = 47 \text{ psi} < 125 \text{ psi allowed}$$

Place nominal stirrups only.

APPENDIX "C"

RECORD OF TEST OBSERVATIONS

1. Calibration of the Gauges on the Origo Jacks by the Olsen Machine at the Materials Laboratory, School of Engineering, A.U.B.

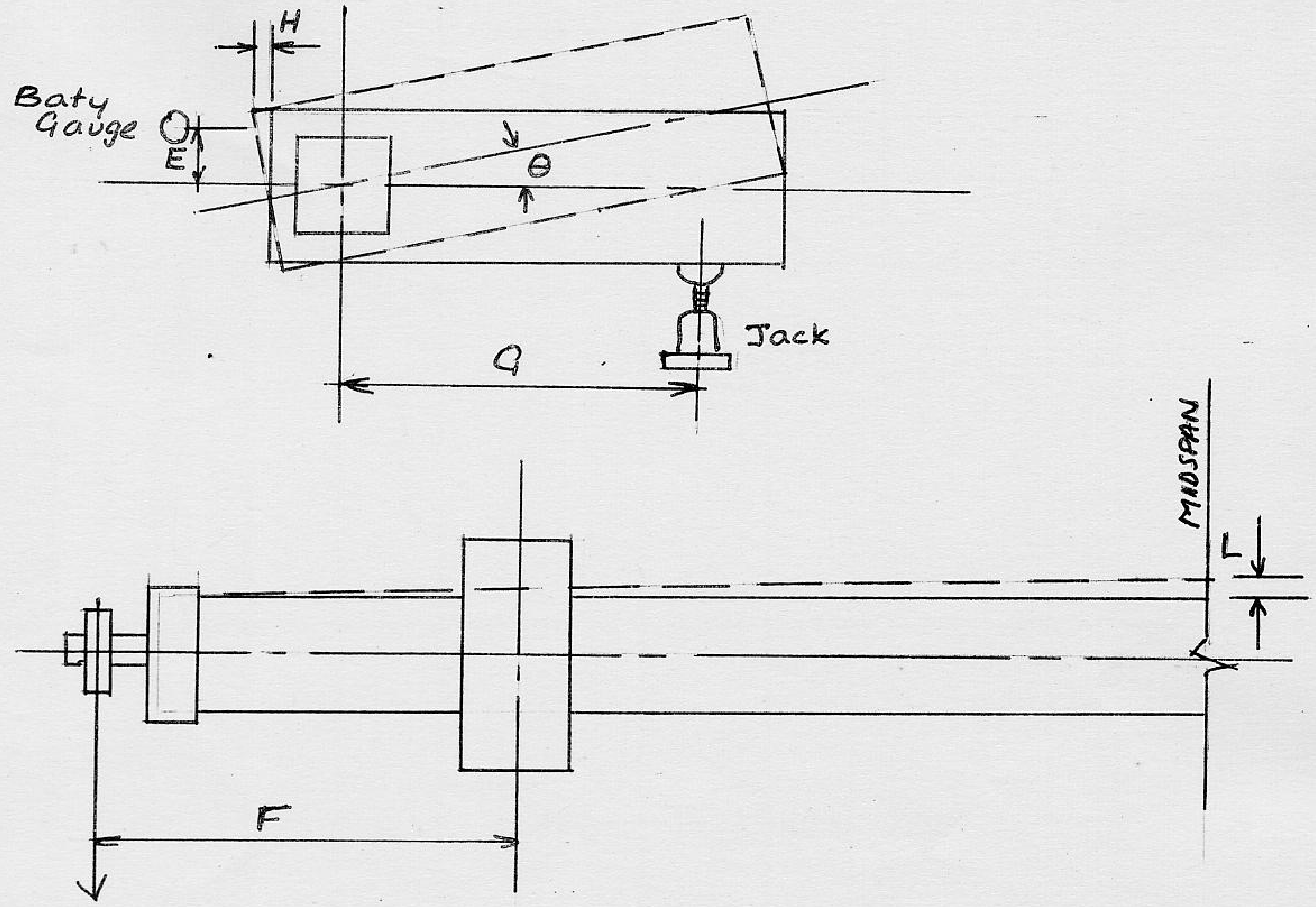
21st May, 1966

ENMF Gauge Reading kg/cm^2	Jack A lbs	Jack B lbs
10	170	170
20	500	530
30	900	960
40	1300	1350
50	1650	1700
60	2000	2020
70	2400	2440
80	2750	2760
90	3060	3080
100	3420	3530

27th May, 1966:

ENMF Gauge Reading	Jack A	Jack B
10	100	280
20	500	620
30	850	1050
40	1240	1420
50	1600	1750
60	1900	2100
70	2250	2500
80	2620	2800
90	2950	3150
100	3320	3510

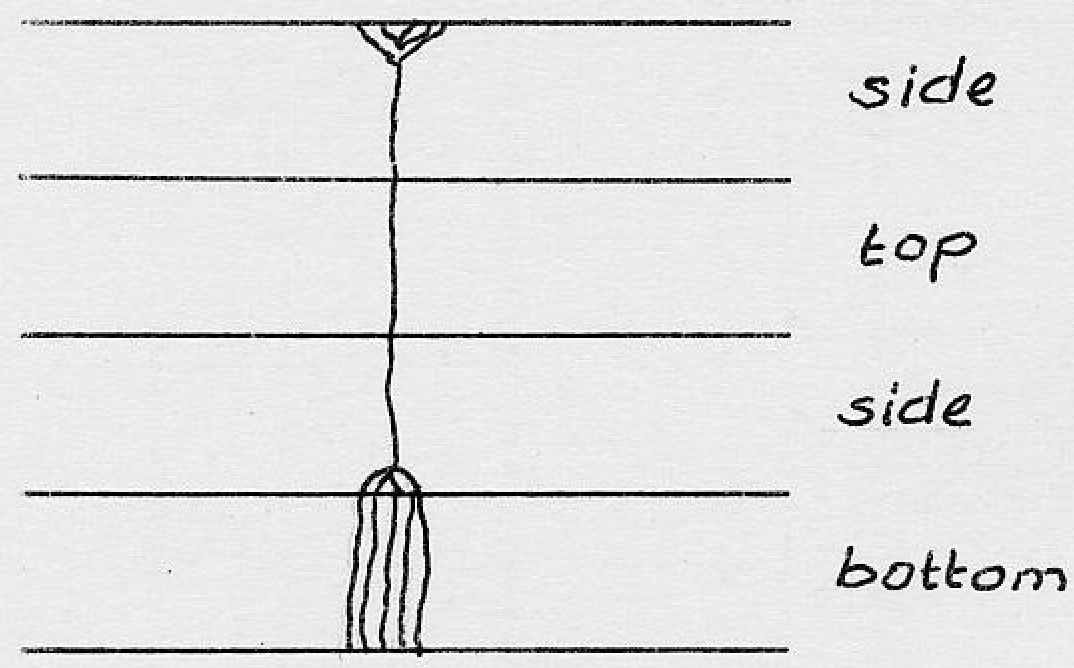
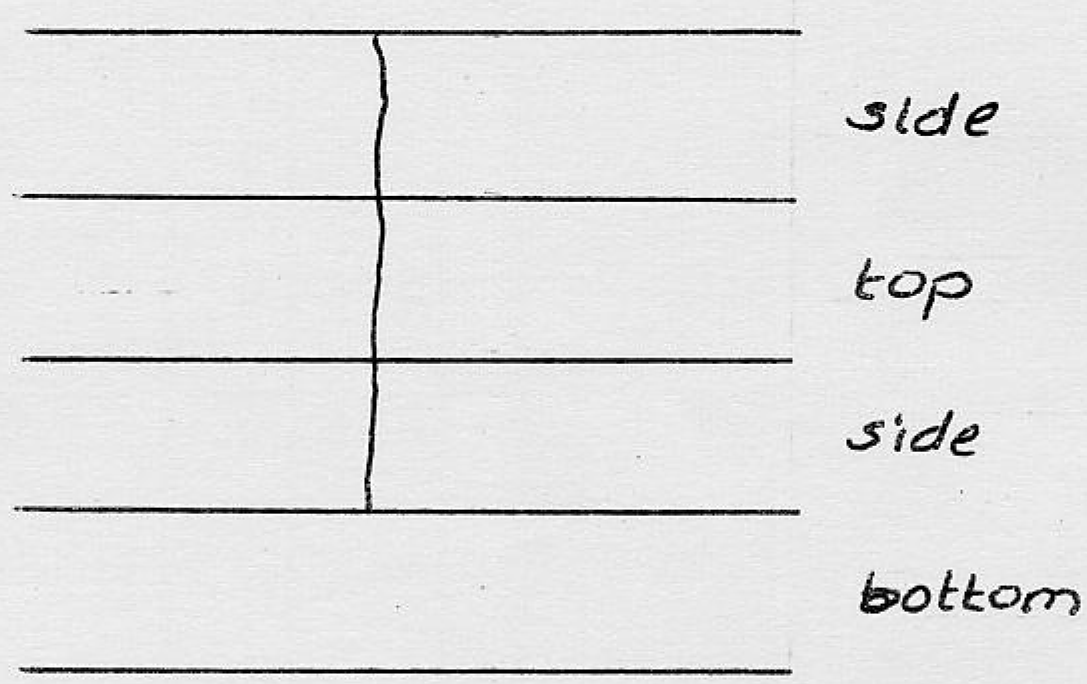
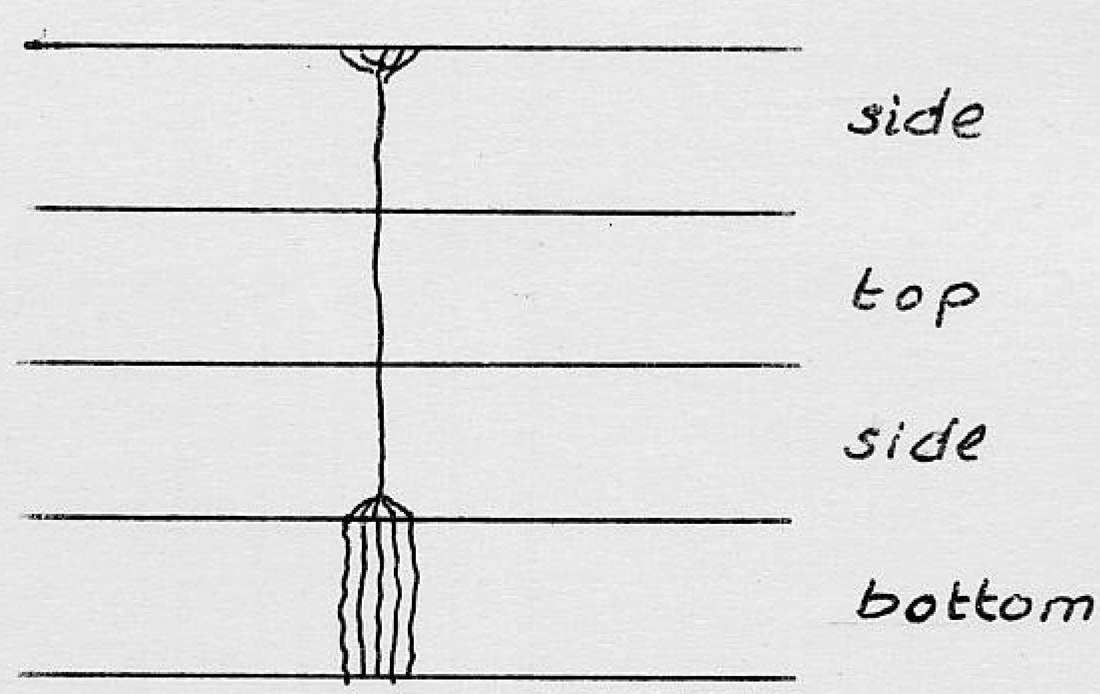
2- <u>Tabulation of Test Observations</u>



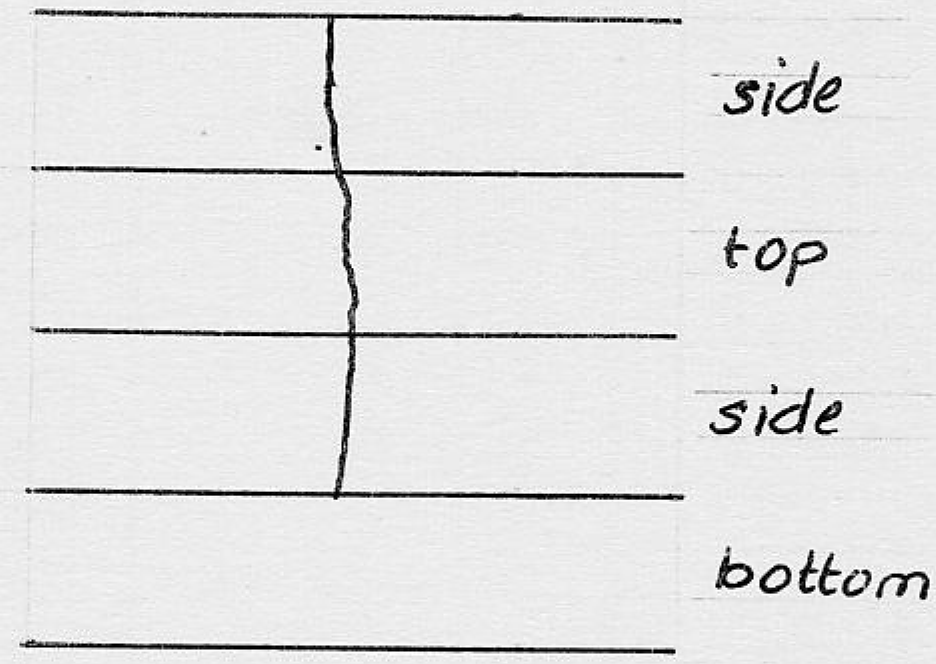
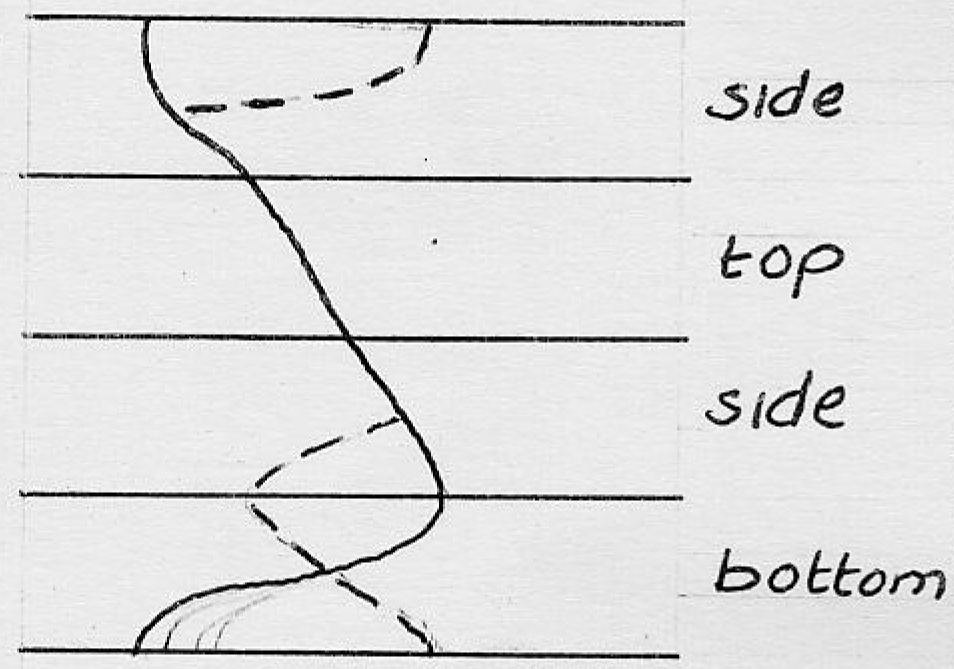
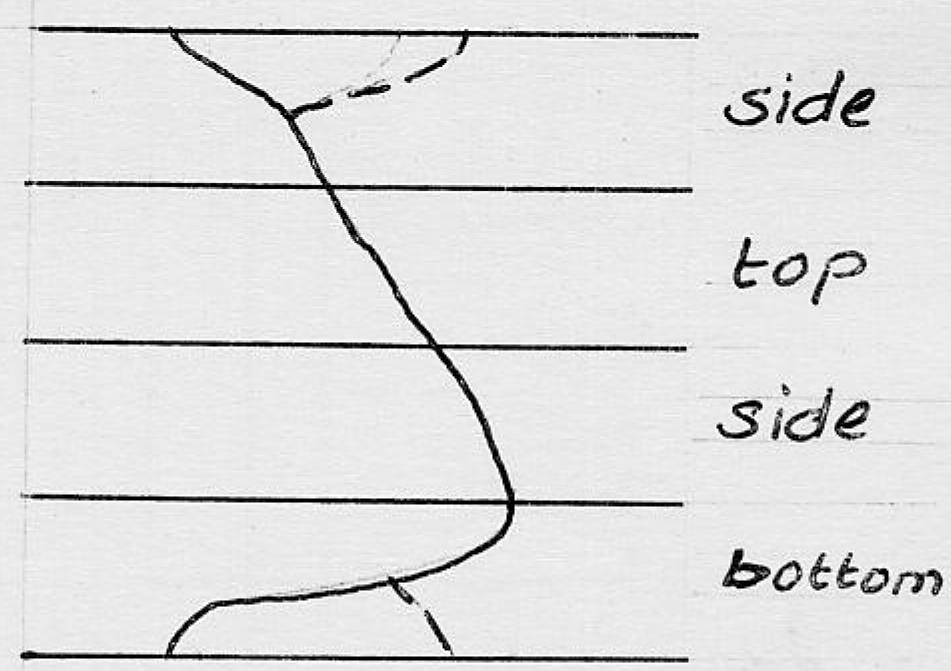
Test Beam Number	Marking of Test Beam Pour	Date of Test	F cms	G cms	Position of Temp. Support	$\frac{G}{F}$	E cms	Jack Load lbs	L in	H in	Ø rad. x 10^3	Comments	Developed Diagram of Cracks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

The tabulation of test observations does not fit into a single page width. It was therefore split into two parts, columns (1) through (12) in one part and columns (13) and (14) in the other.

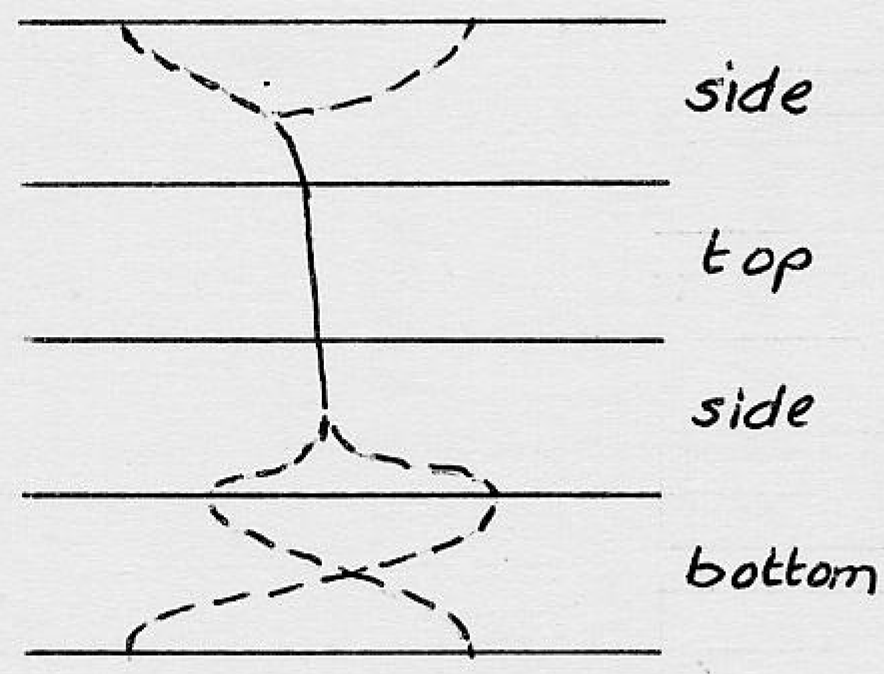
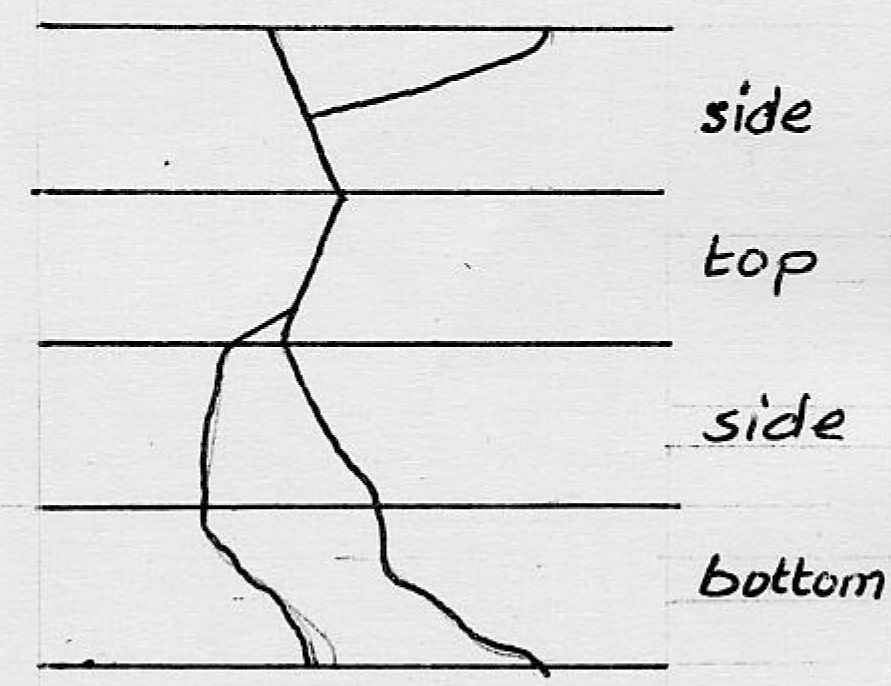
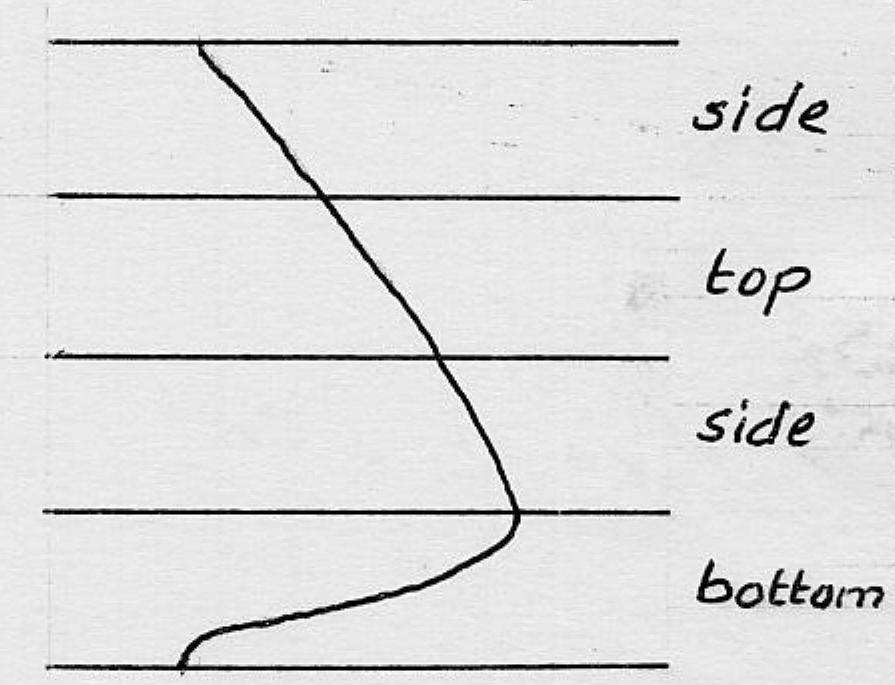
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	T4	20th May, 1966	100	0	-	0	--	c.1600	-	-	-
								c.3400			
								730	0.040	-	-
2	T3	21st May, 1966	100	0	-	0	-				
								1350	0.316	-	-
								830	0.172	-	-
								1160	0.206	-	-
								1350	0.248	-	-
								1630	0.284		
3	T4	21st May, 1966	110	0	-	0	-	1860	0.318	-	-
								2020	0.356	-	-
								2230	0.420	-	-
								2440	0.475	-	-
								2600	0.603	-	-
								2760	0.712	-	-

<table>
<tr><th>(1)</th><th>(13)</th><th>(14)</th></tr>
<tr><td>1</td><td>Initial tension cracks on top face at between two readings occured at jack positions. Further loading led to a secondary compression failure at the same point.</td><td>
</td></tr>
<tr><td>2</td><td>Initial tension cracks on top face at point of load application just above a jack load of 1250 lbs. Cracks opened up with further jacking but did not lead to a secondary compression failure.</td><td>
</td></tr>
<tr><td>3</td><td>Initial tension cracks on the upper face appeared at the point of the application of load at a load of 2230 lbs. Secondary compression failure of the lower face occured at 2760 lbs at the same point.</td><td>
</td></tr>
</table>

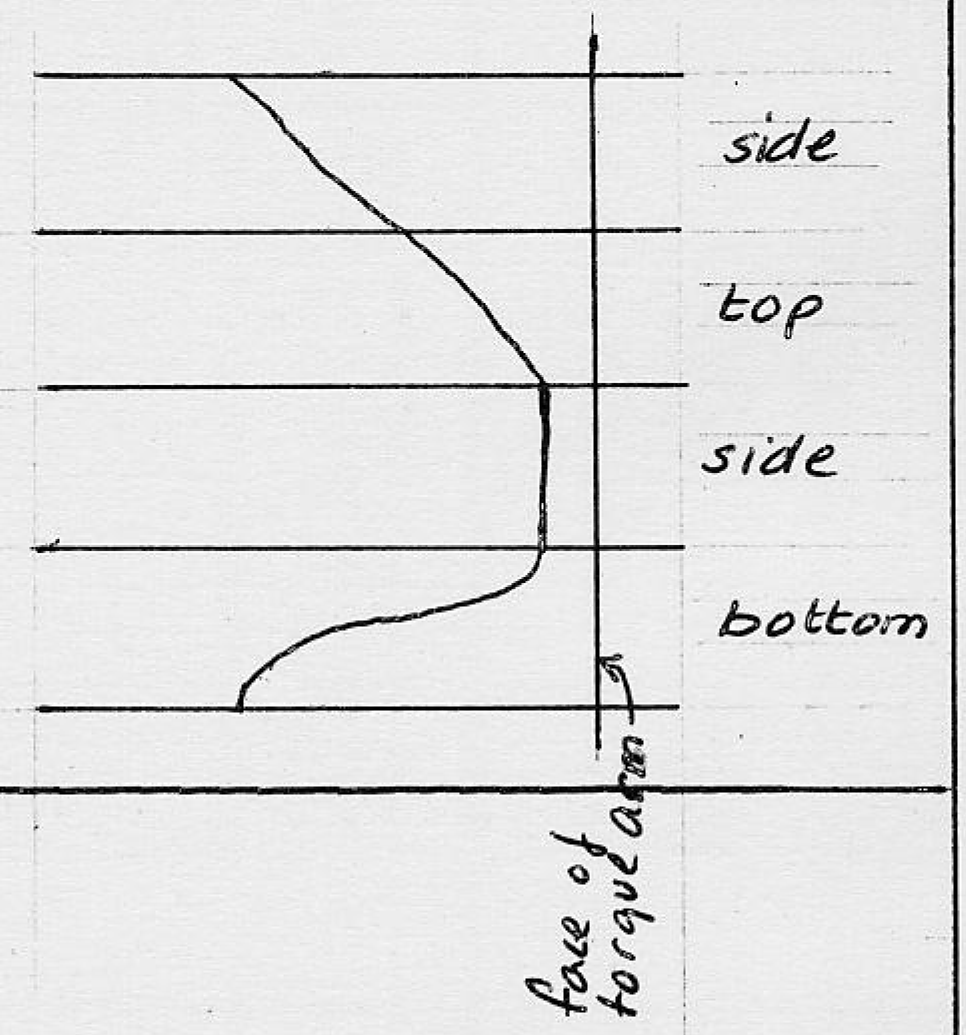
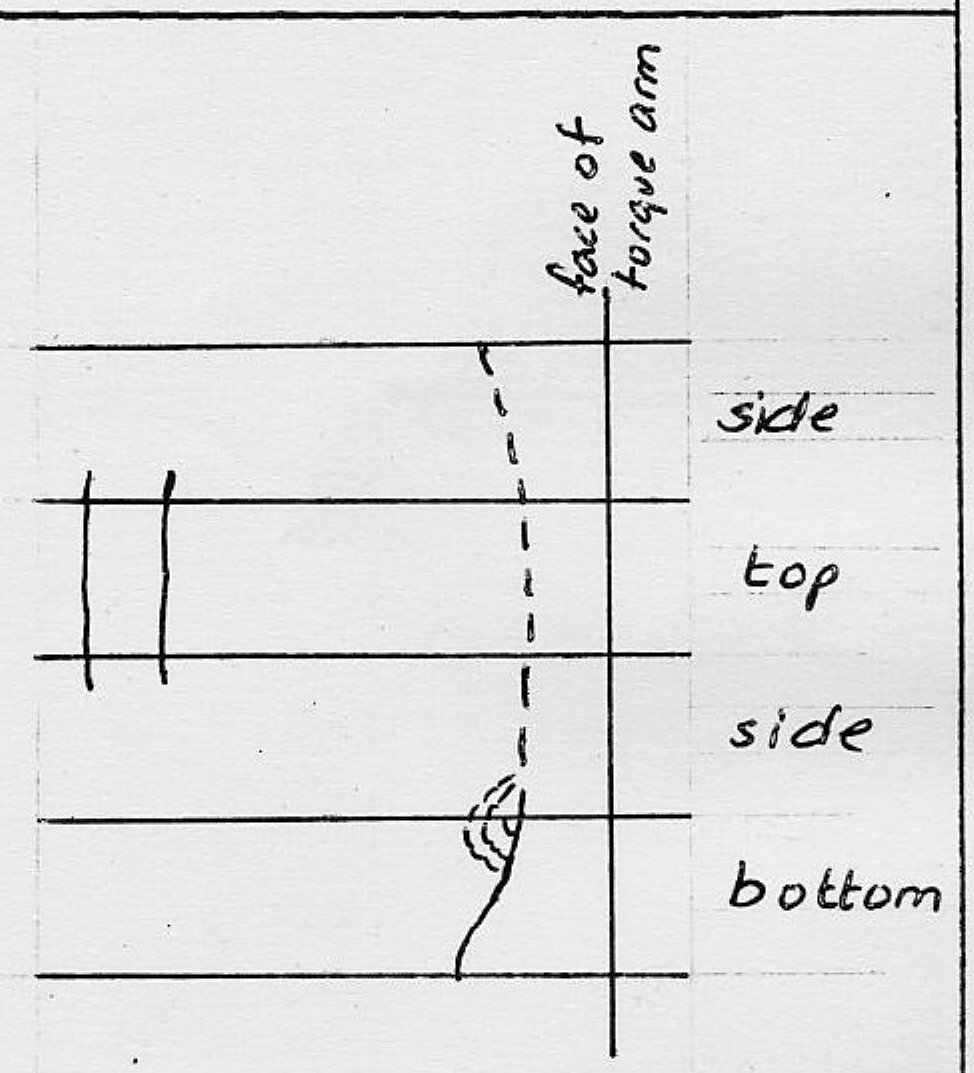
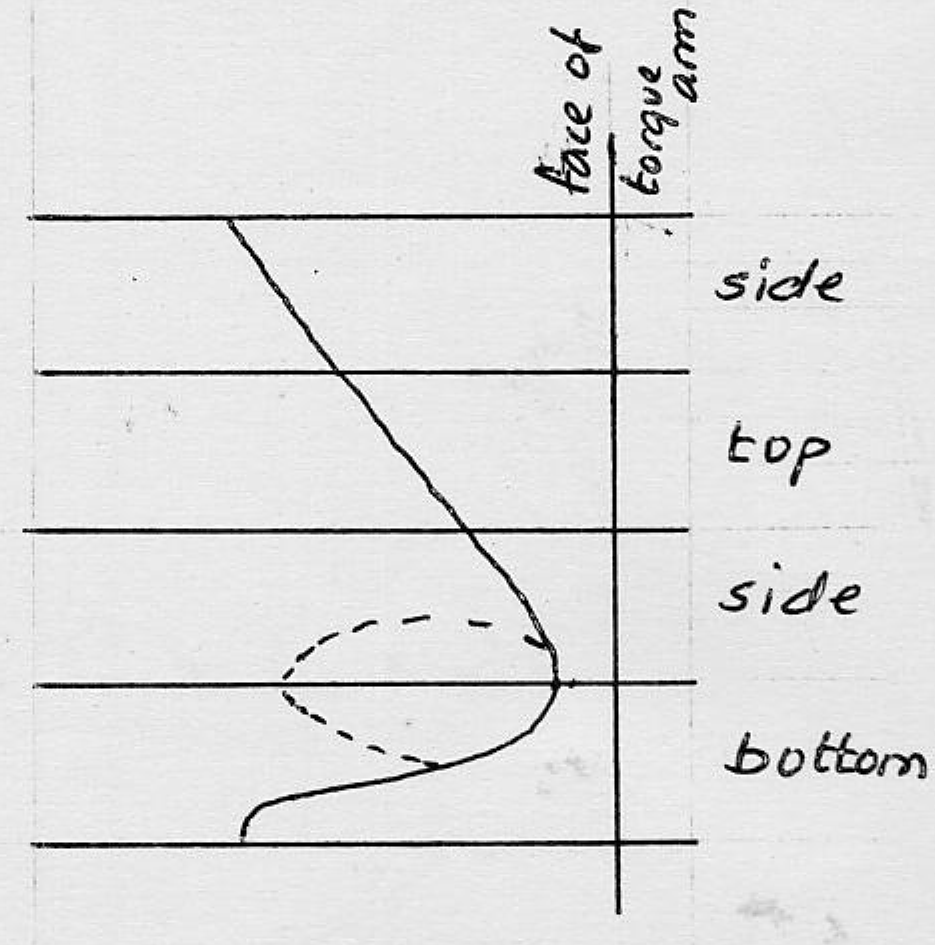
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	T3	23rd May, 1966	110	0	-	0	-	830	0.098	-	-
								1160	0.211	-	-
								1350	0.288	-	-
5	T2	23rd May, 1966	104	94	At arms at centerline	0.9	19	530	0.001	0.000	0.00
								740	0.005	0.002	0.10
								960	0.010	0.004	0.21
								1160	0.092	0.008	0.42
								1350	0.132	0.109	5.74
6	T1	24th May, 1966.	104	94	At arms at centerline	0.9	19	170	0.003	0.000	0.00
								340	0.008	0.003	0.16
								510	0.013	0.005	0.26
								720	0.022	0.010	0.53
								930	0.040	0.013	0.68
								1120	0.052	0.012	0.63
								1320	0.135	0.024	1.26
								1500	0.175	-	-

(1)	(13)	(14)
4	Initial tension cracks on top face of beam at the point of application of load of 1350 lbs. No load other than dead load recorded by jackgauges after cracking occured. Cracks opened up with further jacking but did not lead to a secondary compression failure on the lower beam face.	
5	Cracking occured on the top and sides at a load of 1350 lbs. just inside of one arm. Further jacking led to the development of a"hinge"on the bottom face and later to the splitting off of a wedge (A) of the beam corner as shown in the sketch. Dotted lines indicate secondary cracks. Torsional failure features.	
6	Cracking occured at 1500 lbs. jack load just inside arm of torque on the top and sides of the beam. Further jacking resulted in the formation of a"hinge"on the lower face which in turn led to the splitting of a corner wedge having a crown at the prestressing wire. Torsional failure.	

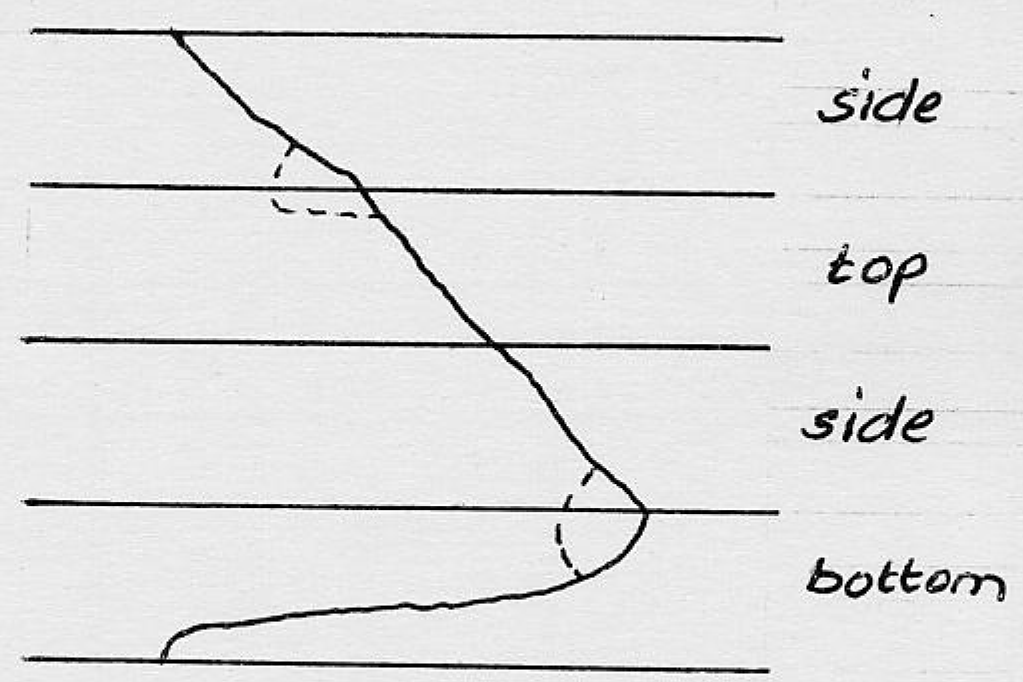
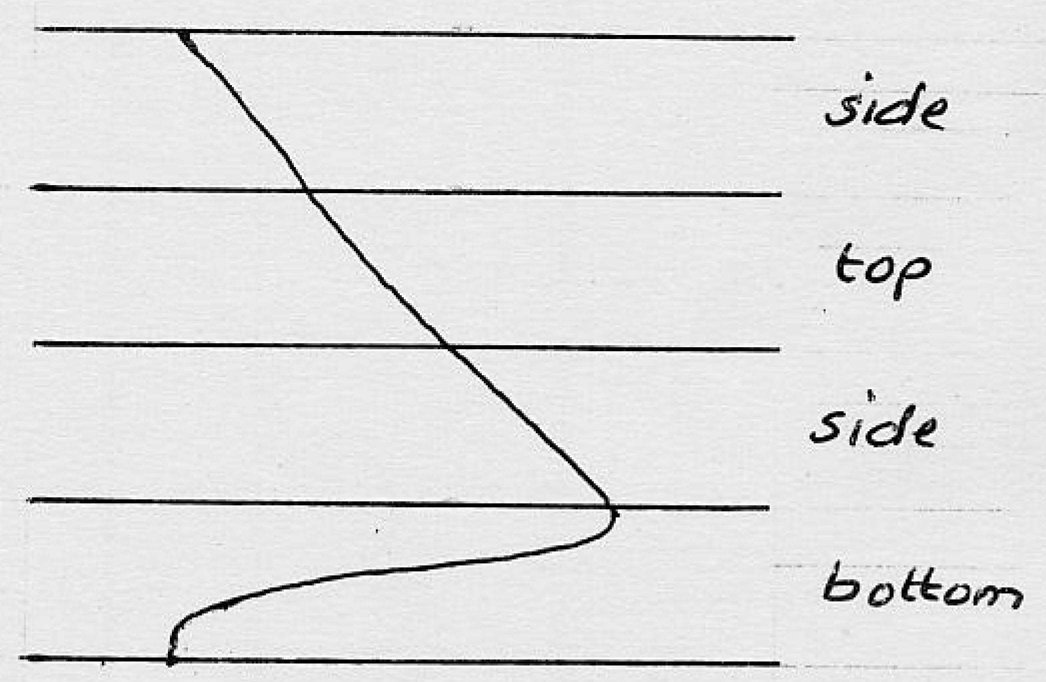
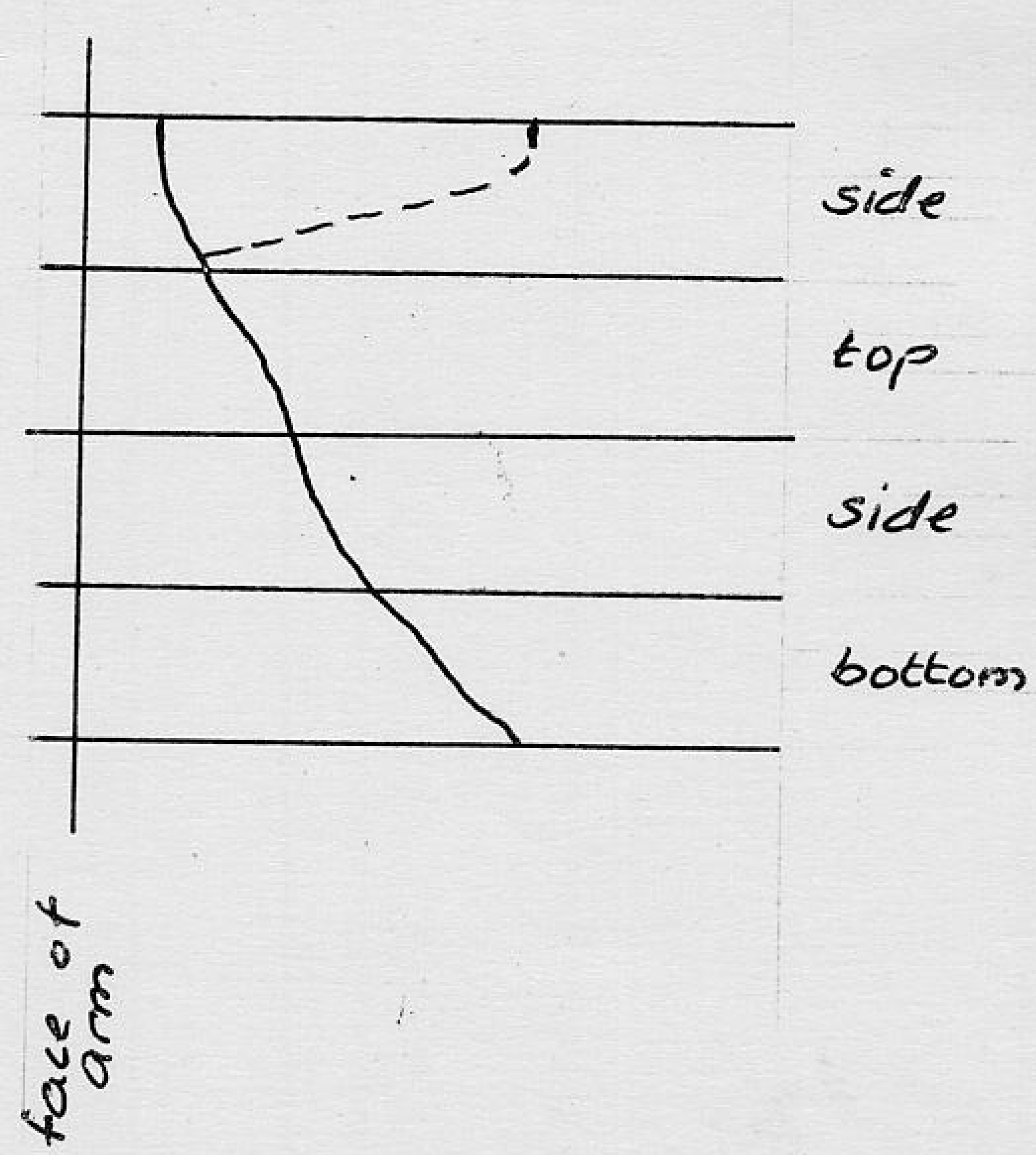
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
7	T4	24th May, 1966	104	56	On Beam at Outer Face of Arms	0.54	18	280	0.029	0.000	0.00
								350	0.030	0.001	0.06
								530	0.034	0.003	0.17
								750	0.044	0.007	0.39
								960	0.073	0.014	0.78
								1160	0.194	0.025	1.39
								1350	0.236	0.026	1.44
								1520	0.264	-	-
								1770	0.293	-	-
								1860	0.315	-	-
								2020	0.384	-	-
								1860	0.565	0.300	16.65
								1860	0.578	0.360	20.00
								1860	0.650	0.475	26.30
8	T3	25th May, 1966	104	55	On Beam at Outer Face of Arms	0.53	15	220	0.010	0.000	0.00
								400	0.010	0.001	0.07
								570	0.020	0.005	0.33
								790	0.028	0.007	0.47
								1000	0.086	0.016	1.07
								1190	0.147	0.017	1.14
								1470	0.190	0.016	1.07
								1630	0.218	0.020	1.33
								1800	0.243	0.023	1.54
								1890	0.278	0.025	1.67
								2060	0.304	0.030	2.00
								2060	0.462	0.172	11.50
9	T1	25th May, 1966	104	74	On Beam at Outer Face of Arms	0.71	16	220	0.037	0.000	0.00
								400	0.043	0.000	0.00
								580	0.053	0.004	0.25
								880	0.072	0.012	0.75
								1000	0.084	0.012	0.75
								1200	0.208	0.013	0.81
								1380	0.356	0.015	0.94
								1550	0.383	0.013	0.81
								1720	0.420	0.024	1.50
								1860	-	-	-

(11)	(13)	(14)
7	Initially a tension crack appeared on the top face perpendicular to the sides of the beam. This was at a loading of 2020 lbs. just inside the arm. Upon further jacking the load fell off to 1860 lbs. with the crack widening and then developing into the torsional failureform leading to corner wedges splitting off. Flexural failure leading to a torsional failure.	
8	Beam turned upside down to check if lower wires had slipped also. Results indicate non-slip of lower wires. A tension crack appeared on the top face close to the inside face of the arm and then developed down the sides in a diagonal fashion. Neither a full flexural nor a full torsional failure is indicated. Cracks began at a load of 2060lbs. per jack and were developed under this same force.	
9	Cracks began forming on the top face of the beam on the inside of the arm as the load reached 1860 lbs. per jack. No readings were taken prior to cracking at this load. Failure was of a torsional type with a"hinge"developing on the bottom face of the beam and corner wedges splitting off with further jacking.	

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
10	T2	26th May, 1966	102	74	At Arms at ℄	0.72	19	220	0.032	Defective Gauge	Defective Gauge
								400	0.041		
								580	0.051		
								790	0.068		
								1000	0.102		
								1190	0.269		
								1520	0.355		
								1720	0.380		
								1890	0.402		
11	T1	26th May, 1966						-	-	-	-
12	T2	26th May, 1966	103	27	At Arms at ℄	0.26	18	280	0.030	0.000	- 0
								450	0.036	- 0.002	- 0.10
								620	0.043	- 0.006	- 0.33
								840	0.066	- 0.010	- 0.58
								1050	0.118	- 0.008	- 0.50
								1240	0.177	- 0.007	- 0.39
								1420	0.205	- 0.001	- 0.05
								1580	0.228	0.002	+ 0.10
								1750	0.259	0.021	+ 1.16
								1920	0.277	0.037	+ 2.05
								2000	0.332	0.058	+ 3.23
								2100	0.451	-	-
13	T3	27th May, 1966	36	92	At Beam ℄ JUST INSIDE ARMS	2.55	18	280	0.084	0.000	0.00
								450	0.086	- 0.006	- 0.33
								620	0.090	- 0.012	- 0.67
								840	0.100	- 0.011	- 0.61
								1050	0.131	- 0.005	- 0.28
								1240	0.260	+ 0.057	+ 3.17
								1420	0.460	+ 0.079	+ 4.39
								1620	0.487	-	-
								1750	0.500	-	-
								2000	0.514	-	-
								2100	-	-	-
								2100	0.510	-	-

<table>
<tr><th>(1)</th><th>(13)</th><th>(14)</th></tr>
<tr><td>10</td><td>A torsional failure pattern began appearing on the top face of the beam at a load of 1890 lbs. Upon further jacking at a load of 1380lbs. deeper cracks developed at the edge of the arm on one vertical side with a "hinge" developing on the bottom face.</td><td></td></tr>
<tr><td>11</td><td>Beam cracked while being placed in position for testing. This was the beam at the end of which the wires of the T1 pour were cut and slipping on the top wires was observed.</td><td></td></tr>
<tr><td>12</td><td>Clear flexural cracks occured on the top face close to the midspan at a load of about 2100 lbs. These were followed by torsional cracks having angles greater than 45° with the sides upon further jacking and a force of 2100lbs. per jack. At the corner of the hinge on the bottom face a secondary compression zone appeared. The negative twist values are due to a Baty gauge not functioning properly.</td><td></td></tr>
<tr><td>13</td><td>A torsional failure pattern appeared at reaching a load of 2100 lbs. per jack. No deflection readings were made, however, at that load. It was possible with further pumping to maintain the load of 2100 lbs. per jack and to take deflection readings at the midspan. A corner wedge developed after further pumping and split off.</td><td></td></tr>
</table>

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
14	T4		37	92		2.49	18	280 450 620 840 1050 1240 1420 1680 1790 1920 2100 2300	0.053 0.063 0.069 0.075 0.090 0.113 0.220 0.248 0.255 0.273 0.290 -	0.000 0.006 0.012 0.016 0.030 0.040 0.052 0.061 0.062 0.072 0.080 -	0.00 0.33 0.67 0.89 1.66 2.22 2.89 3.39 3.44 4.00 4.45 -
15	T3		40	70		1.75	18	100 340 500 780 850 1040 1240 1420 1600 1750 1900	0.028 0.029 0.031 0.035 0.038 0.044 0.053 0.060 0.072 0.083 0.101	0.000 0.014 0.008 0.016 0.021 0.026 0.032 0.032 0.042 0.056 0.080	0.00 0.78 0.44 0.89 1.17 1.44 1.78 1.78 2.33 3.11 4.45
16	T4		40	70		1.75	18	100 300 500 780 850 1040 1240 1420 1660 1750 1900 2080 2250	0.046 0.049 0.056 0.064 0.075 0.084 0.097 0.107 0.118 0.136 0.151 0.158 -	0.000 0.004 0.008 0.016 0.016 0.022 0.026 0.032 0.035 0.041 0.055 0.061 -	0.00 0.22 0.44 0.89 0.89 1.22 1.44 1.78 1.94 2.28 3.05 3.39 -

(1)	(13)	(14)
14	A torsional failure pattern developed just inside one arm at a load of 2300 lbs. Cracking, however, was just after attaining this load per jack and no deflection readings were possible to take.	
15	A torsional failure pattern developed just inside one arm at a jack load of 1900 lbs. Just after deflection readings were taken the beam cracked as shown.	
16	A torsional failure pattern developed just inside the left arm at a jack load of 2250 lbs. No deflection readings were taken, however, before the cracks occured. The hinge showed up on one side. As the crack was close to the edge of the arm on the top face it was at a greater angle from the side than usual, probably due to the presence of stirrups in the beam at this point.	

APPENDIX "D"

CALCULATIONS FOR ANALYSIS OF TEST BEAMS

1- Evaluation of α and β :

$a = 1.3$			$a = 2.0$		
n	α	β	n	α	β
0.70	1.72	4.32	0.70	2.86	6.15
0.71	1.82	4.48	0.71	3.09	6.41
0.72	2.04	4.64	0.72	3.33	6.65
0.73	2.21	4.82	0.73	3.59	6.94
0.74	2.39	5.00	0.74	3.86	7.24
0.75	2.60	5.21	0.75	4.17	7.56
0.76	2.81	5.42	0.76	4.50	7.93
0.77	3.04	5.65	0.77	4.84	8.28
0.78	3.30	5.90	0.78	5.24	8.69
0.79	3.58	6.19	0.79	5.66	9.14
0.80	3.89	6.50	0.80	6.13	9.64
0.81	4.23	6.84	0.81	6.65	10.17
0.82	4.62	7.23	0.82	7.24	10.77
0,83	5.03	7.64	0.83	7.88	11.44
0.84	5.52	8.12	0.84	8.61	12.19
0.85	6.07	8.68	0.85	9.45	13.08
0.86	6.69	9.30	0.86	10.38	14.00
0.87	7.39	10.00	0.87	11.46	15.12
0.88	8.24	10.85	0.88	12.74	16,49
0.89	9.21	11.84	0.89	14.25	17.98
0.90	10.40	13.02	0.90	16.07	19.82

$$\alpha = \frac{an^2}{1-n} - \frac{4}{3}(1-n) \; , \quad \beta = \frac{an^2}{1-n}(3-2n) + (1+2n-3n^2)$$

2- Tabulation of Beam Factors: a=1.3

Beam	P kg	Pxd kg-m	2P/A kg/cm^2	f'_c kg/cm^2	f_{tu} kg/cm^2	α	β	M_c
3	15,500	3,100	77.5	225	22.5	3.45	6.08	910
5	13,900	2,780	69.5	274	27.4	2.54	5.15	940
6	13,900	2,780	69.5	260	26.0	2.67	5.28	915
7	15,500	3,100	77.5	232	23.2	3.34	5.94	925
8	15,500	3,100	77.5	260	26.0	2.98	5.59	970
9	13,900	2,780	69.5	267	26.7	2.60	5.21	925
10	13,900	2,780	69.5	277	27.7	2.51	5.12	945
12	13,900	2,780	69.5	277	27.7	2.51	5.12	945
13	15,500	3,100	77.5	267	26.7	2.90	5.51	980
14	15,500	3,100	77.5	239	23.9	3.24	5.87	935
15	15,500	3.100	77.5	274	27.4	2.83	5.44	995
16	15,500	3,100	77.5	239	23.9	3.24	5.84	930

3- Evaluation of γ :

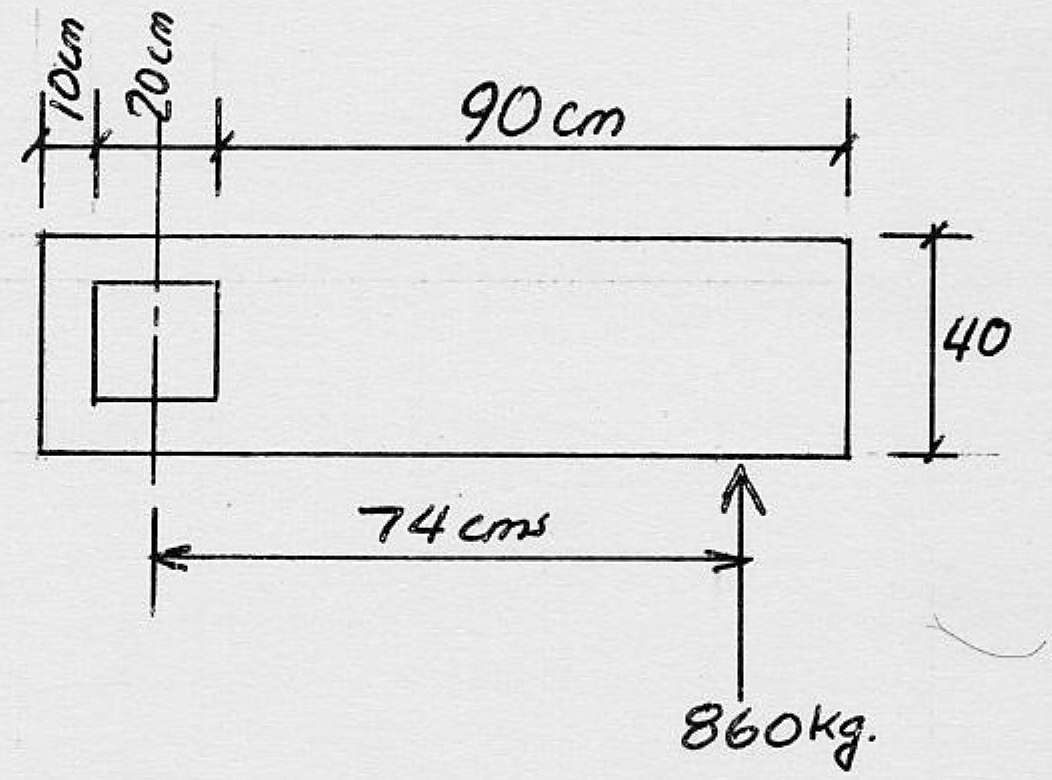
d/b	γ
1.0	1.00
1.2	0.90
1.4	0.81
1.6	0.74
1.8	0.68
2.0	0.63
2.5	0.52
3.0	0.45
5.0	0.27
10.0	0.14

4- Sample Calculation: Test Beam No. 10.

Test Data: as shown on the sketches below:

Cracking Load = 1890 lbs.= 860 kgs.

P = 13,900 kgs.



Dead Load of Prestressed Beam:

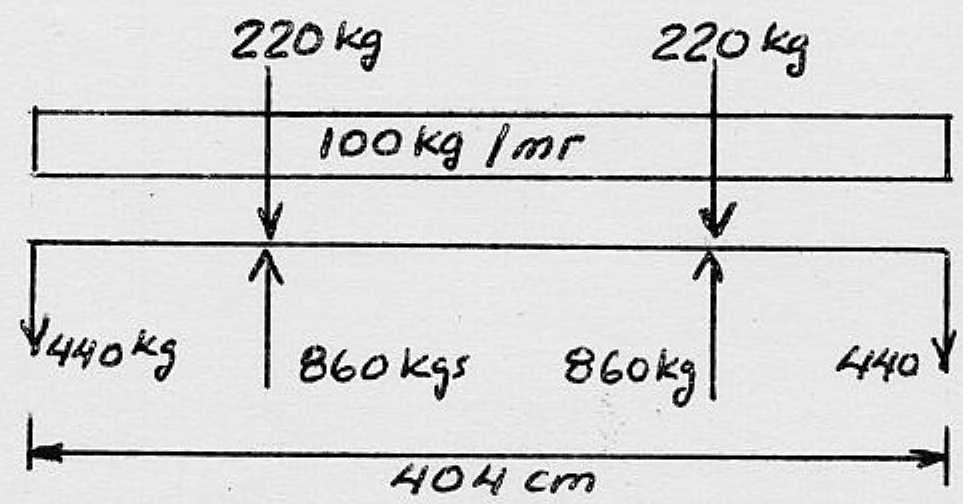
$0.2^2 \times 2500\ kg/m^3 = 100\ kg/mr.$

Dead Load of Arm:

(0.90x0.40x0.20+2x0.20x0.10x0.20

+0.10x0.40x0.20)2500=

180+20+20= 220 kgs.



LOAD DIAGRAM

Center of Gravity of Arm from

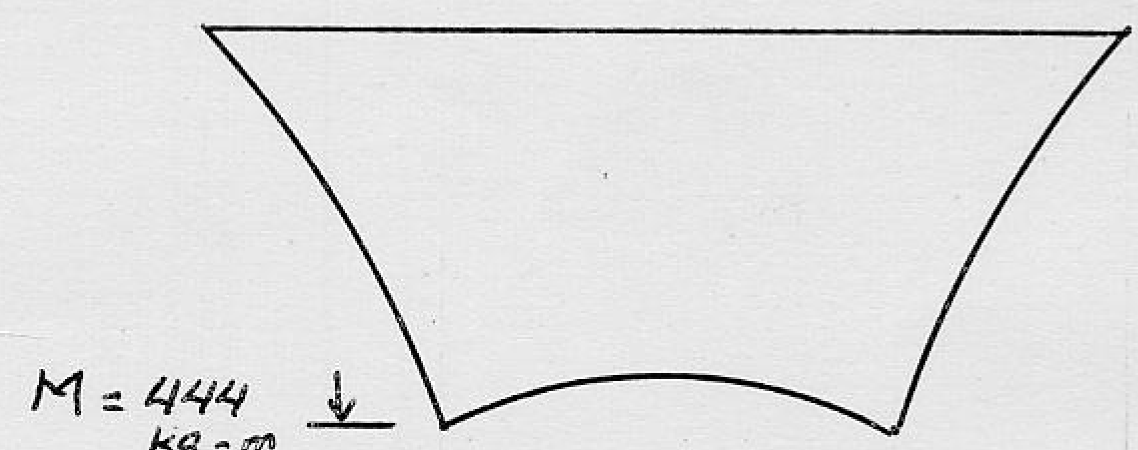
center line of Beam:

180x0.55= + 99

20x0 = 0

20x(-0.15)= - 3

220 kg 96 kg -m

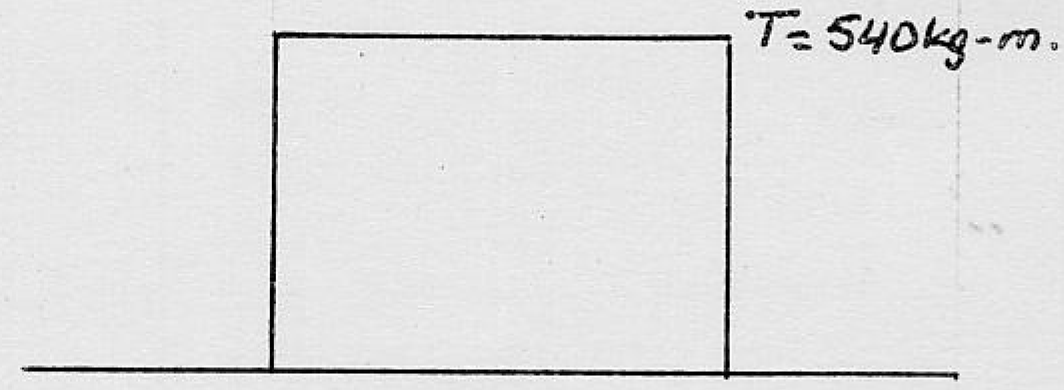


MOMENT DIAG.

$$\text{Center of Gravity} = \frac{96}{220} \times 100 = 43.6 \text{ cms.}$$

away from center line of beam.

Shear at Arm = 200 - 92 = 108 kgs.



TORQUE DIAG.

Moment at Arm:

440x0.92 + 100x0.92x0.46=444kg-m

Torque on Prestressed Beam:

- 96 kg-m+860x0.74= 540 kg-m

$T/M = \frac{540}{444} = 1.2$ $A = 20 \times 20 = 400\ cm^2$ $I = \frac{20 \times 20^3}{12} = 13,330\ cm^4$

$f_{tu} = 0.1 f'_c = 0.1 \times 277\ kg/cm^2 = 27.7\ kg/cm^2$

$\alpha = \frac{2P}{A} = 2.51$ $\beta = 5.12$

$M_c = \frac{I}{d}\beta = \frac{13,330x5.12}{20x100} x27.7 = 945$ kg -m.

$K = 3+1.8x1 = 4.8$

With the beam constants calculated above and substituting T=1.2 M, equations (24) and (25) reduce to the following:

$$\left(\frac{4.8x5.12}{12} x \frac{1.2M}{945}\right)^2 + \frac{5.12}{2}\left(\frac{M - 13,900x0.20/6}{945}\right) = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots a'$$

$$\text{and} \left(\frac{5.12}{4}\right)^2 \left[\frac{108x0.20}{2x945} + \frac{Tx4.8}{3x945}\right]^2 - \frac{5.12}{12} x \frac{13,900x0.20}{945} = 1 \ldots\ldots\ldots\ldots b'$$

Solving a' for M:

$(0.0026)^2 M^2 + 0.00271\ M - 2.255 = 0$

or $M^2 + 400\ M - 334000 = 0$

$M = -200 + \sqrt{374000} = -200 + 610 = 410$ kg-m

$T = 1.2x410 = 492$ kg-m

Solving b' for T:

$1.64\left[0.0114 + 0.001695\ T\right]^2 = 1.255 + 1 = 2.255$

$0.0114 + 0.001695\ T = \sqrt{\frac{2.255}{1.64}} = 1.173$

$T = \frac{1.162}{0.001695} = 686$ kg-m

$M = \frac{686}{1.2} = 571$ kg-m

Therefore the critical M and T are those calculate by equation (24), i.e. M = 410 kg - m

T = 492 kg - m

The percentage error in the experimental values is $+\frac{34x100}{410} = +8.5\%$

It should be noted that the shear in this case contributed very little towards the results and affected the result of equation (25) by only 1%.

APPENDIX "E"

NOMENCLATURE

A – Area of section or area inside helical reinforcement.

a – Factor indicating plasticity of concrete in tension.

$a_{1,2}$ – Depth of area of concrete section in compression as per ACI code.

B – Elastic strain of concrete in tensile failure.

b – Width of section.

C – Compression or compressive force.

d – Depth of a section.

E_c – Modulus of Elasticity of concrete in compression.

E_t – Modulus of Elasticity of concrete in tension.

F – Force.

f – Stress normal to a plane.

f_c – Compressive stress normal to a plane.

f'_c – Compressive stress in concrete at failure.

f_p – Compressive stress due to a prestressing force.

f_t – Tensile stress normal to a plane.

f_{tp} – Tensile stress on a principal plane.

f_{tu} – Tensile stress in concrete at failure.

G – Modulus of Elasticity in shear.

I – Moment of inertia of a section about the horizontal axis of symmetry.

J – Polar moment of inertia of a section about centroid.

K – Section factor used in evaluating torsional shear stresses.

M – Moment acting on a section.

M_c – Moment acting on a section at which cracks begin to appear.

M_u – Moment acting on a section at which section fails in flexure.

N – Factor in computing T_c

n – Fraction indicating ratio of depth in compression to total depth of section.

P — Prestressing force.

p — Lateral pressure per unit area.

Q — Statical moment of area.

R — Resultant of tensile normal stresses in a section.induced by a moment.

r — Distance between a point in a section and the centroid of the section.

S — Tensile force per unit length of the boundary line of the membrane.

s — Pitch of spiral reinforcement parallel to the axis of a beam.

T — Torque acting on a section.

T_c — Torque acting on a section at which cracks begin to appear.

u — Shearing stress due to torque.

V — Shearing force.

v — Shearing stress due to shearing force.

α — Non-dimensional factor as defined by equation (15).

β — Non-dimensional factor as defined by equation (17).

γ — Ratio of u at the middle of the short side to u at the middle of the long side of a section.

ε_c — Strain in concrete in compression.

ε_t — Strain in concrete in tension.

θ — Angle of twist per unit length of a bar.

λ — Factor used in computing the torque that may be resisted by a section having helical shear reinforcement.

APPENDIX "F"

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