CONCENTRATED FORCE IN THREE-DIMENSIONAL WEDGE

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ABSTRACT

This thesis solves the problem of a concentrated force acting at an interior point of a three-dimensional wedge with an angle $\alpha = \frac{180}{n}$, n = 2, 3, ..., and mixed boundary conditions of zero shearing stresses and zero normal displacements on both sides of the wedge.

The solution of the problem is based on the principle of superposition and the previously solved problem of the half-space with the same boundary conditions. A half-space with two symmetrically placed forces will have zero shearing stresses and zero normal displacements on the boundary, if the forces are opposite in direction when perpendicular to the boundary, or if the forces have the same direction when they are parallel to the boundary.

Tables for displacements and stresses for a wedge of 60° are given.

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NOTATION

(I) , (J)	The half-planes determined by the sides of the wedge.
α	The angle of the wedge.
X,Y,Z and X',Y',Z'	Two coordinate systems such that (I) lies in the plane Z = 0, (J) lies in the plane Z' = 0 and they are related as follows: $x' = x \cos\alpha + z \sin\alpha$ $y' = y$ $z' = -x \sin\alpha + z \cos\alpha$
i, j, k	Unit vectors in the direction of X,Y,Z respectively.
i', j', k'	Unit vector in the direction of X',Y',Z' respectively.
G	Modules of rigidity.
ν	Poisson's ratio.
F	Galerkin vector.
F_t	Galerkin vectors t = 1,2, ...
F_x, F_y, F_z	Components of the Galerkin vector.
B_x, B_y, B_z,	Papkovitch functions.
u	Displacement vector.
u_x, u_y, u_z	Components of displacement vector referred to the X,Y,Z-coordinate system.
$u_{x'}$, $u_{y'}$, $u_{z'}$	Components of displacements vector referred to the X',Y',Z'-coordinate system.
σ_{xx}, σ_{yy}, σ_{zz}	Normal components of stress referred to X,Y,Z-coordinate system.
$\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{z'z'}$	Normal components of stress referred to X',Y',Z'-coordinate system.

$\sigma_{xy}, \sigma_{xz}, \sigma_{yz}$	Shearing components of stress referred to X,Y,Z-coordinate system.
$\sigma'_{xy}, \sigma'_{xz}, \sigma'_{yz}$	Shearing components of stress referred to X',Y',Z'-coordinate system.
Δ	Laplace's operator.
$\delta_x, \delta_{xx}, \delta_{xy}$, etc.	Partial derivatives with respect to subscript variables.
A_t	The position of $\underline{F}_t$, $t = 1,2,\ldots$
R	Distance from the origin to (x,y,z).
R_t	Distance from A_t to (x,y,z) referred to X,Y,Z-coordinate system.
R'_t	The same distance R_t referred to X',Y', Z'-coordinate system.
r_1	Distance from $(0,0,c)$ to (x,y,z).
r_2	Distance from $(0,0,-c)$ to (x,y,z).

CHAPTER I

INTRODUCTION

1. The Problem of The Thesis and The Method of Solution.

This thesis aims at solving the problem of a concentrated force acting at an interior point of a homogeneous isotropic wedge with an angle α and with mixed boundary conditions of zero normal displacements and zero shearing stresses on both sides of the wedge.

Solving the problem consists in finding the position, magnitude and direction of forces which should be placed outside and around the wedge, such that when superposed with the given concentrated force acting at an interior point of the wedge, they will produce the desired boundary conditions on both sides of the wedge.

2. Historical Background.

For a homogeneous isotropic solid of indefinite extent, the fundamental solution in the linear theory of elasticity for concentrated force problems was given by Lord Kelvin in 1848. His solution gives the stresses and the displacements at any point in a homogeneous isotropic solid; caused by a concentrated force acting at an interior point of the solid (1) [*]. By differentiation, integration and superposition an infinite class of solutions known as nuclei of stain, can be derived from Kelvin's solution (5).

In 1935, R. D. Mindlin used the Galerkin vector representation of the nuclei to solve the problem of a concentrated force in the interior of a semi-infinite elastic solid (2), (3). In 1953 he showed that the solutions he had found priviously could be derived directly by combining the Papkovitch functions and Green's analysis (7). Since then this new approach, which proved to be very effective, has been used for all kinds of concentrated force problems. In 1955, L. Rogved (8) solved the problem of a concentrated force, acting in the interior of a semi-infinite solid with a fixed plane boundary, using the Papkovitch functions and in 1956 W. Hijab (9) solved the mixed boundary value problem of the half space.

3. Galerkin Vector.

The basic equation of the linear theory of elasticity is the equation of equilibrium, which, in terms of Galerkin vector notaion, can be written as

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[*] Numbers inside parenthesis refer to References at the end of the thesis.

$$G(\Delta + \frac{1}{1 - 2\nu} \text{ grad div})\underline{u} = 0 \qquad [1]^{*}$$

where G is the modules of rigidity, ν is Poisson's ratio, Δ is Laplace's operator and

$\underline{u} = \underline{i}u_x + \underline{j}u_y + \underline{k}u_z$ is the displacement vector.

The body forces have been neglected in the derivation of equation [1] and we assume zero body forces throughout this thesis.

Equation [1] is obtained by the substitution of Hooke's Laws, which state the relationship between the stresses and the displacements in an ideally elastic isotropic solid, into the equilibrium equations expressing the condition of zero resultant force on any element of the elastic solid. Hooke's Law is given by the following equations (6):

$$u_x = \frac{1}{2G(1 + \nu)}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})],$$

$$u_y = \frac{1}{2G(1 + \nu)}[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})], \qquad [2]$$

$$u_z = \frac{1}{2G(1 + \nu)}[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})],$$

where σ_{xx}, σ_{yy} σ_{zz} are the components of the normal stress.

Solving a problem in elasticity consists in finding a vector function which will produce the desired conditions on the boundary and satisfy the equilibrium equation [1].

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* Numbers inside the square brackets refer to the equations.

The Galerkin vector $\underline{F}$ with scalar components and the Galerkin functions are of great help in solving equation [1]. The displacement vector $\underline{u}$ can be derived from the Galerkin vector by:

$$2G\underline{u} = [2(1 - \nu)\Delta - \text{grad div}]\ \underline{F}, \qquad [3]$$

where $\underline{F} = \underline{i}F_x + \underline{j}F_y + \underline{k}F_z$, F_x, F_y, F_z are the Galerkin functions. For $\underline{u}$ to satisfy the equilibrium equation [1] the Galerkin vector should satisfy the biharmonic equation $\Delta\Delta\underline{F} = 0$.

4. Stresses and Displacements.

The Galerkin vectors for a single force placed at the origin are:

$$\underline{F} = \underline{i}F_x = \underline{i}\ R \qquad \text{(single force in the x-direction),}$$

$$\underline{F} = \underline{j}F_y = \underline{j}\ R \qquad \text{(single force in the y-direction),} \qquad [4]$$

$$\underline{F} = \underline{k}F_z = \underline{k}\ R \qquad \text{(single force in the z-direction),}$$

where $R = \sqrt{x^2 + y^2 + z^2}$ is the distance from the origin to an arbitrary point in space (6).

The components u_x, u_y, u_z of the displacement vector $\underline{u}$ are computed from equation [3]. The components of the normal stresses σ_{xx}, σ_{yy}, σ_{zz} and of the shearing stress σ_{xy}, σ_{xz}, σ_{yz} can be obtained from the displacements and equation [2].

a) <u>For a single force in the x-direction</u>

$\underline{F} = \underline{i}F_x = \underline{i}\ R$:

$$2Gu_x = 2(1-\nu)\Delta F_x \quad - \delta_{xx}F_x = \frac{3-4\nu}{R} + \frac{x^2}{R^3}$$

$$2Gu_y = \quad - \delta_{yx}F_x = \frac{xy}{R^3}$$

$$2Gu_z = \quad - \delta_{zx}F_x = \frac{xz}{R^3}$$

$$\sigma_{xx} = (2-\nu)\delta_x\Delta F_x \quad - \delta_{xxx}F_x = -\frac{(1-2\nu)x}{R^3} - \frac{3x^3}{R^5}$$

$$\sigma_{yy} = \nu\delta_x\Delta F_x \quad - \delta_{yyx}F_x = \frac{(1-2\nu)x}{R^3} - \frac{3xy^2}{R^5} \qquad [5]$$

$$\sigma_{zz} = \nu\delta_x\Delta F_x \quad - \delta_{zzx}F_x = \frac{(1-2\nu)x}{R^3} - \frac{3xz^2}{R^5}$$

$$\sigma_{xy} = (1-\nu)\delta_y\Delta F_x \quad - \delta_{xyx}F_x = -\frac{(1-2\nu)y}{R^3} - \frac{3x^2y}{R^5}$$

$$\sigma_{yz} = \quad - \delta_{yzx}F_x = -\frac{3xyz}{R^5}$$

$$\sigma_{xz} = (1-\nu)\delta_2\Delta F_x \quad - \delta_{xzx}F_x = -\frac{(1-2\nu)z}{R^3} - \frac{3x^2z}{R^5}$$

b) <u>For a single force in the y-direction</u>

$\underline{F} = \underline{j}F_y = \underline{j}\ R$:

$$2Gu_x = \quad - \delta_{xy}F_y = \frac{yx}{R^3}$$

$$2Gu_y = 2(1-\nu)\Delta F_y \quad - \delta_{yy}F_y = \frac{3-4\nu}{R} + \frac{y^2}{R^3}$$

$$2Gu_z = \quad - \delta_{zy}F_y = \frac{yz}{R^3}$$

$$\sigma_{xx} = \nu\delta_y\Delta F_y - \delta_{xxy}F_y = \frac{(1-2\nu)y}{R^3} - \frac{3yx^2}{R^5} \qquad [6]$$

$$\sigma_{yy} = (2-\nu)\delta_y\Delta F_y - \delta_{yyy}F_y = -\frac{(1-2\nu)y}{R^3} - \frac{3y^3}{R^5}$$

$$\sigma_{zz} = \nu\delta_y\Delta F_y - \delta_{zzy}F_y = \frac{(1-2\nu)y}{R^3} - \frac{3yz^2}{R^5}$$

$$\sigma_{xy} = (1-\nu)\delta_x\Delta F_y - \delta_{xyy}F_y = -\frac{(1-2\nu)x}{R^3} - \frac{3y^2x}{R^5}$$

$$\sigma_{yz} = (1-\nu)\delta_z\Delta F_y - \delta_{zyy}F_y = -\frac{(1-2\nu)z}{R^3} - \frac{3y^2z}{R^5}$$

$$\sigma_{xz} = -\delta_{xyz}F_y = -\frac{3xyz}{R^5}$$

c) <u>For a single force in the z-direction $\mathbf{F}=\underline{k}\,F_z=\underline{k}\,R$:</u>

$$2Gu_x = -\delta_{xz}F_z = \frac{xz}{R^3}$$

$$2Gu_y = -\delta_{yz}F_z = \frac{yz}{R^3}$$

$$2Gu_z = 2(1-\nu)\Delta F_z - \delta_{zz}F_z = \frac{3-4\nu}{R} + \frac{z^2}{R^3}$$

$$\sigma_{xx} = \nu\delta_z\Delta F_z - \delta_{xxz}F_z = \frac{(1-2\nu)z}{R^3} - \frac{3zx^2}{R^5}$$

$$\sigma_{yy} = \nu\delta_z\Delta F_z - \delta_{yyz}F_z = \frac{(1-2\nu)z}{R^3} - \frac{3zy^2}{R^5} \qquad [7]$$

$$\sigma_{zz} = (2-\nu)\delta_z\Delta F_z - \delta_{zzz}F_z = -\frac{(1-2\nu)z}{R^3} - \frac{3z^3}{R^5}$$

$$\sigma_{xy} = -\delta_{xyz}F_z = -\frac{3xyz}{R^5}$$

$$\sigma_{yz} = (1-\nu)\delta_y\Delta F_z - \delta_{yzz}F_z = -\frac{(1-2\nu)y}{R^3} - \frac{3yz^2}{R^5}$$

$$\sigma_{xz} = (1-\nu)\delta_x\Delta F_z - \delta_{xzz}F_z = -\frac{(1-2\nu)x}{R^3} - \frac{3xz^2}{R^5}$$

The above forces have magnitude $8\pi(1 - \nu)$ in the positive direction. To obtain the solution for a force of magnitude P we multiply each equation by the "force adjustment" factor $\frac{P}{8\pi(1 - \nu)}$ [11].

5. Papkovitch Functions:

The following equations are given by R.D. Mindlin (4); they state the relationship between the Galerkin vector and the Papkovitch functions:

$$\underline{i}\, B_x + \underline{j}\, B_y + \underline{k}\, B_z = \frac{1 - \nu}{G} \Delta \underline{F}$$

$$\beta = \frac{1 - \nu}{G} (2 \operatorname{div} \underline{F} - \underline{r} \cdot \Delta \underline{F}) \qquad [8]$$

where B_x, B_y, B_z and β are the Papkovitch functions and $\underline{r} = \underline{i}x + \underline{j}y + \underline{k}z$ is the radius vector.

6. Half Space with Zero Normal Displacements and Zero Shearing Stresses at the Boundary.

The problem of a half space $z \geq 0$ and a force acting at (0,0,c), an interior point of the half space (Fig. 1), with mixed boundary conditions of zero normal displacements and zero shearing stresses on the plane z = 0, is solved by W. Hijab (9) in terms of Papkovitch function:

a) <u>For a force in the z-direction and of magnitude P</u>:

$$B_x = 0$$

$$B_y = 0$$

$$B_z = \frac{p}{4\pi G} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\beta = - \frac{c\,P}{4\,\pi\,G} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

where r_1 and r_2 denote respectively the distance from (0, 0, c) and (0, 0, -c) to an arbitrary point (x,y,z) in space

b) <u>For a force in the x-direction and of magnitude P</u>:

$$B_x = \frac{P}{4\,\pi\,G} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$B_y = 0$$

$$B_z = 0$$

$$\beta = 0$$

Using equations [8] we find the corresponding Galerkin vectors. For a force in the z-direction:

$$\underline{F} = \underline{k} \frac{P}{8\pi(1 - \nu)} (r_1 - r_2)$$

and for a force in the x-direction:

$$\underline{F} = \underline{i} \frac{P}{8\pi(1 - \nu)} (r_1 + r_2).$$

Apart from the force adjustment, we observe that the solution of the problem is obtained by superposing the nuclei in the z-direction and in the x-direction as follows:

$$\underline{k}\, r_1 - \underline{k}\, r_2$$

and

$$\underline{i}\, r_1 + \underline{i}\, r_2.$$

These results lead to the following principle:

> To have zero normal displacements and zero shearing stresses on a plane acted on by two single forces placed symmetrically with respect to the plane, forces perpendicular to the plane should be in opposite directions and forces parallel to the plane in the same direction (11).

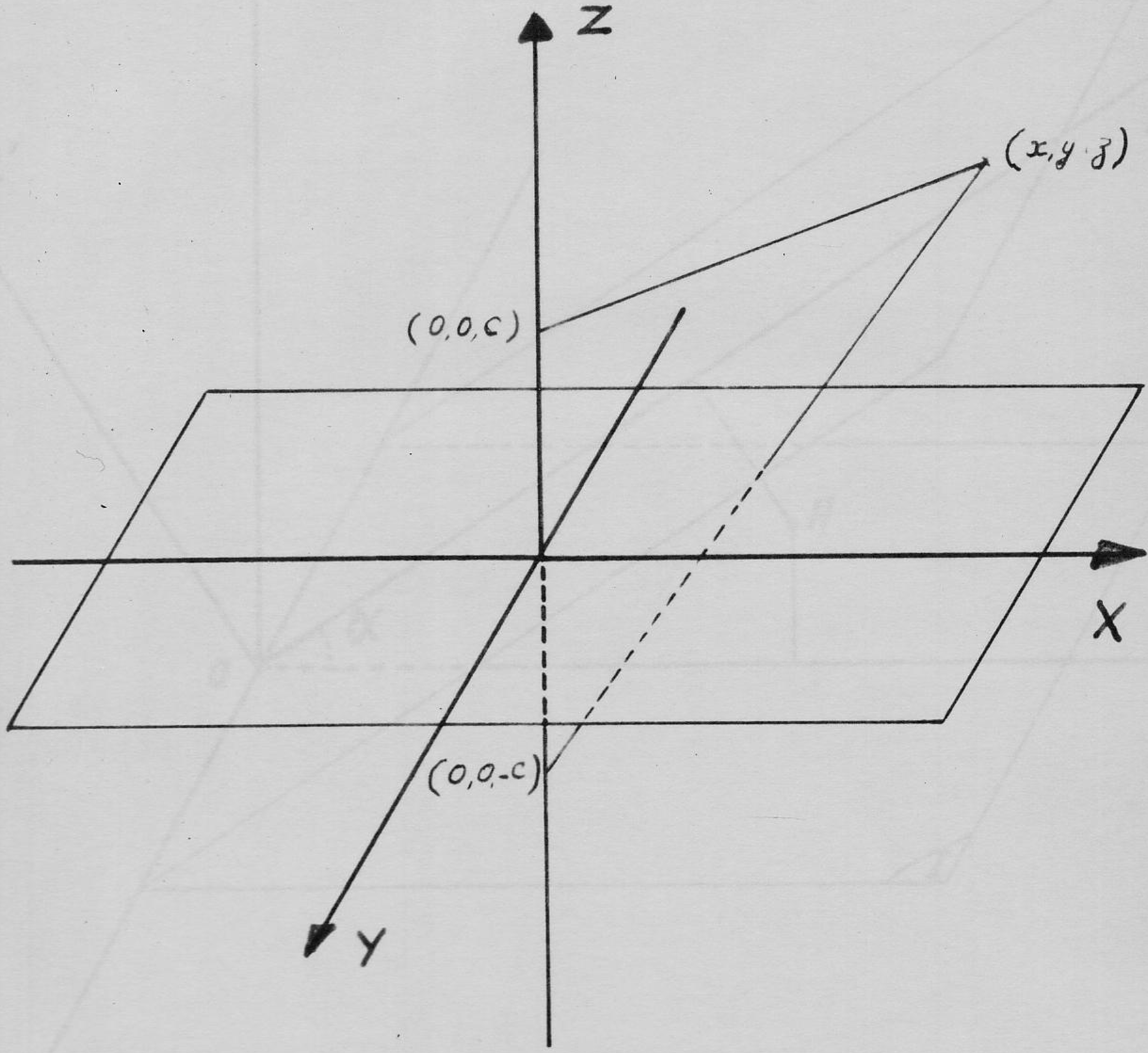

Fig 1: The Half-Space

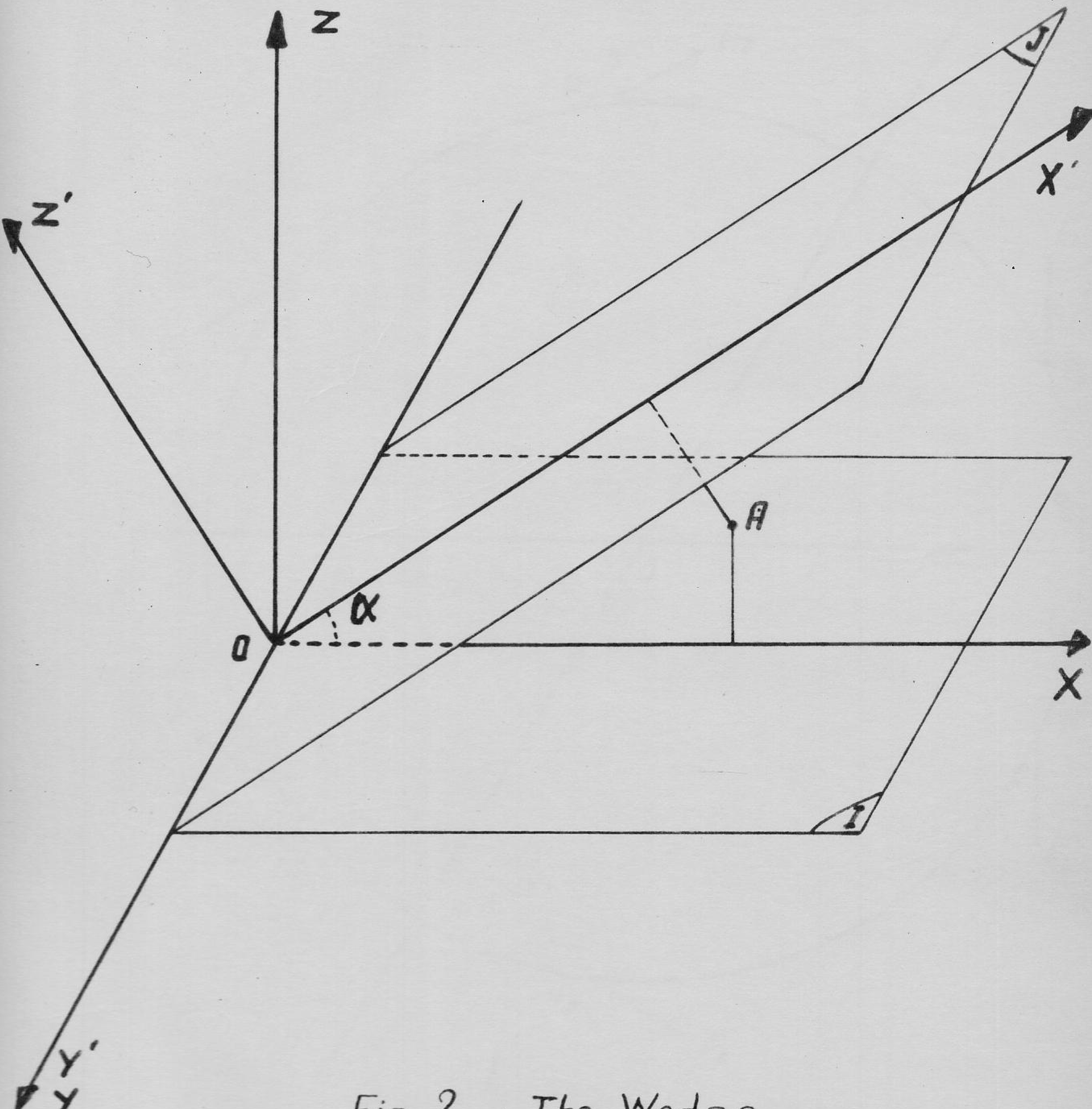

Fig. 2 : The Wedge

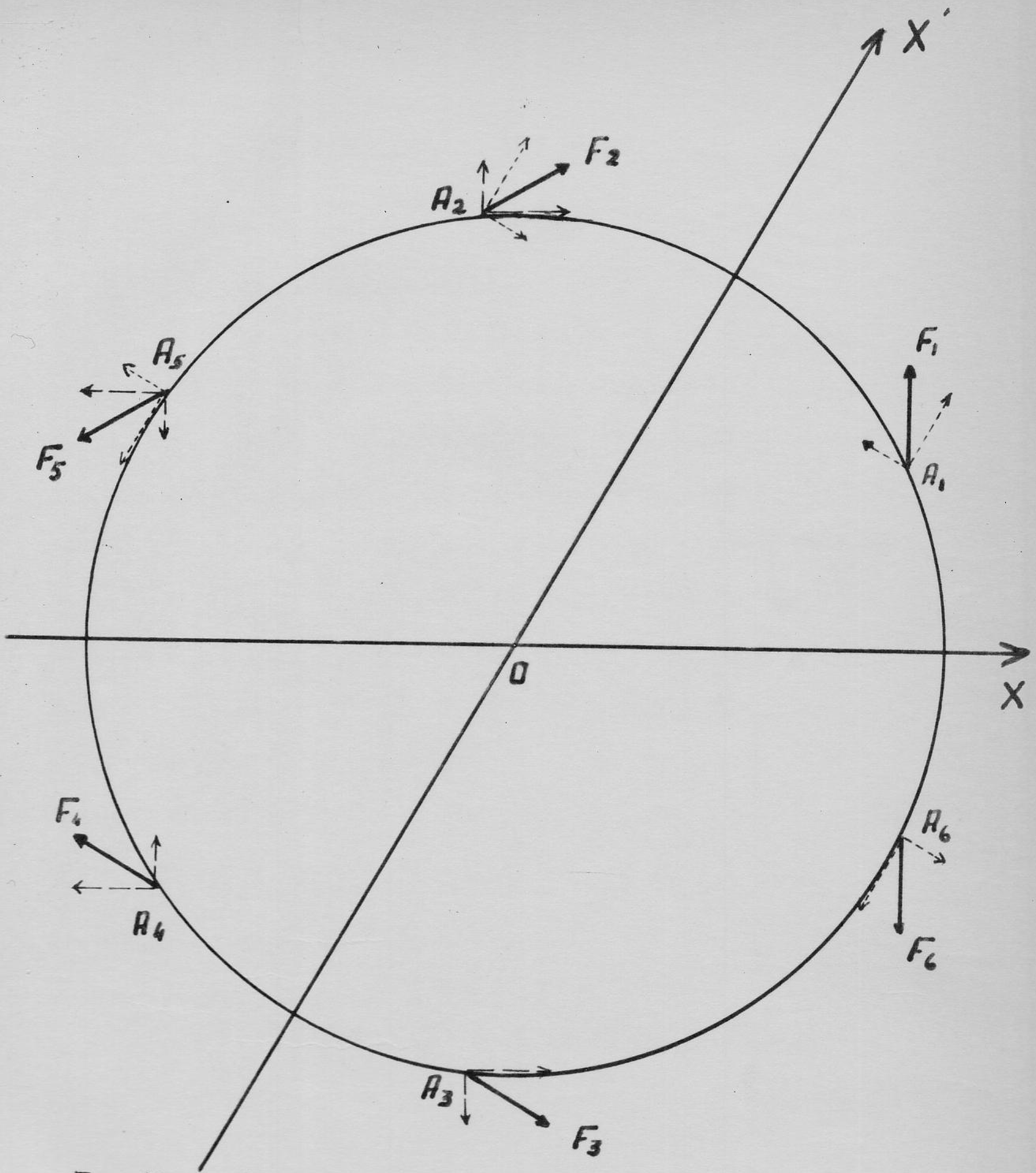

Fig. 3 : Cross section of a Wedge of 60°
Normal to its edge

⟶ Force F; ——→ : Components of F ∥ and ⊥ to X
- - - - → : " " " " " " " X'

CHAPTER II

FORCE PARALLEL TO THE EDGE OF THE WEDGE

1. Preliminary Considerations.

We have a wedge with an angle of α degrees, $0 < \alpha < 180°$, and a concentrated force acting at a point inside the wedge. We define the positive direction of the angle α to be the counter clockwise direction. We denote by (I) the half-plane determined by the side of the wedge where α is equal to zero, that is the initial side, and by (J) the half-plane determined by the other side of the wedge, that is the terminal side.

We choose two coordinate systems, X, Y, Z and X', Y', Z' as follows (Fig. 2):

a) X, Y, Z-coordinate System. We let the Y-axis to coincide with the line determined by the edge of the wedge and point in the direction of advance of a right-hand screw when its head is rotated in the positive direction of α. We denote by Y = 0 the plane containing the point of application of the concentrated force and orthogonal to the Y-axis. We let the X-axis to coincide with the line of intersection of (I) and the plane Y = 0, and point towards the opening of the wedge. We let the Z-axis to be the normal to (I) at the point O, where the X-axis and the Y-axis intersect, and point in the direction of a right-hand screw when its head is rotated from the positive Y-axis to the positive X-axis. The axes X, Y, and Z thus obtained are orthogonal, hence they determine a cart esian coordinate system with origin at O.

b) X', Y', Z'-Coordinate System. We let Y'-axis to coincide with the Y-axis. We let the X'-axis coincide with the line of intersection of (J) and the plane Y = 0 as defined above, and point towards the opening of the wedge . We note that (J) and the plane Y = 0 are orthogonal since the Y-axis belongs to (J) and is normal to the plane Y = 0. We let the Z'-axis to be the normal to (J) at the point O' where the X'-axis and the Y'-axis intersect, and point in the direction of a right-hand screw when its head is rotated from the positive Y'-axis to the positive X'-axis. We note that O' is the same point O, since O and O' both belong to the plane Y = 0 and to the Y-axis. The axes X', Y' and Z' thus obtained are orthogonal, hence they determine a cart esian coordinate system with O as the origin.

Given any point A in space we associate with A two sets of coordinates (x,y,z) and (x', y', z'), where the first set refers to X, Y, Z-coordinate system and the second to the X', Y', Z'-coordinate system as defined above. To differentiate between the two coordinate sets, we add a prime " ' " sign at the end of the parenthesis of the coordinate set which refers to X', Y', Z'-coordinate system.

The following equations give the relation between (x,y,z) and (x',y',z')'.

$$x' = x\cos\alpha + z\sin\alpha$$

$$y' = y$$

$$z' = -x\sin\alpha + z\cos\alpha\ , \qquad [9]$$

or

$$x = x'\cos\alpha - z'\sin\alpha$$

$$y = y'$$

$$z = x'\sin\alpha + z'\cos\alpha\ .$$

The purpose behind the use of two coordinate systems is to simplify the proofs involved and the results stated in this thesis.

2. <u>Decomposition of the Concentrated Force</u>.

Any vector in space is completely defined if we know the point at which it is applied and its three components in three linearly independent directions. The unit vectors $\underline{k}$, $\underline{k'}$, $\underline{j}$ parallel to Z, Z' and Y-axis respectively form a linearly independent set for $0 < \alpha < 180°$. The Z', X' and X-axis lie in the plane Y = 0, and Z' form an angle of $90 + \alpha$ degrees with the positive X-axis. The component of the unit vector <u>k</u>' along the X-axis is $k' \sin \alpha$. For $0 < \alpha < 180°$, $k' \sin \alpha \neq 0$, which implies that <u>k</u>' is independent of <u>k</u> and <u>j</u> for $0 < \alpha < 180°$. Hence <u>k</u>', <u>k</u>, <u>j</u> form a base for the three dimensional space.

We decompose the concentrated force into three components, one parallel to the Z-axis, one parallel to the Z'-axis and one parallel to the Y-axis.

This choice of the decomposition of the concentrated force into three components will help us to consider only two special cases while solving the problem:

(i) the special case when the concentrated force is parallel to the edge of the wedge;

(ii) the special case when it is perpendicular to one of the sides of the wedge, for example perpendicular to (I). The case when the force is perpendicular to (J) is similar to the second case and can be discussed in a similar way.

3. <u>The Points of Application of the Superposed Forces</u>.

In solving the problem we will be concerned with the points of application of the forces to be used. These points should be symmetrically situated with respect to both half-planes (I) and (J) of the wedge. By this we mean that if we consider any one of the two half-planes of the wedge, we will have an equal number of points on both sides of the half-plane considered, such that they will be symmetrically situated in pairs with respect to the half-plane considered.

We denote by A_1 the position of the concentrated force inside the wedge. We denote by A_2 the point symmetrical to A_1 with respect to (J). We can also say that A_2 is the image of A_1 in (J), (J) being considered as a mirror. (The reader can refer to Fig. 3 for the discussion here, although Fig. 3 illustrates the special case of a wedge with an angle of 60°). We denote by A_3 the point symmetrical to A_2 with respect to (I), that is A_3 is the image of A_2 in (I), where (I) is being considered as a mirror. We denote by A_4 the image of A_3 in (J) and by A_5 the image of A_4 in (I). We continue this process of taking images of image points alternatively once in (J) and once in (I) and we denote by

A_{2t} the image of A_{2t-1} in (J)

and by [10]

A_{2t+1} the image of A_{2t} in (I)

where $t = 1,2, \ldots$.

As the product of two reflections in two intersecting mirrors is a rotation through twice the angle between them and in the direction of the angle measured from the first mirror that is the mirror in which the first reflection takes place, to the second mirror, that is the mirror in which the second reflection takes place, we can give the points A_t's a new meaning.

The point A_3 is the product of the two reflections of the point A_1, first in (J) then in (I). Hence A_3 can be obtained by rotating A_1 through twice the angle between (J) and (I) in the clockwise direction since by our definition of the half-planes (I) and (J) in section 1 of this chapter, a rotation from (J) to (I) is in the clockwise direction. The angle A_1OA_3, then, measures 2α in the clockwise direction.

Similarly the point A_5 is the product of the two reflections of the point A_3, first in (J) then in (I). Hence A_5 can be obtained by rotating A_3 through twice the angle between (J) and (I) in the clockwise direction and the angle A_3OA_5 measures 2α in the clockwise direction.

For any t, by [10], A_{2t+1} is the product of the two reflections of A_{2t-1} first in (J) then in (I). Hence the angle $A_{2t-1}OA_{2t+1}$ measures 2α in the clockwise direction. By induction we get

$$\text{angle } A_1OA_{2t+1} = 2t\alpha \qquad [11]$$

in the clockwise direction.

We are interested in having an even number of these points A_t's, A_1 inclusive, so that the condition of symmetry with respect to (I) and (J), discussed at the beginning of this section, will be satisfied.

For some even number 2n, the process described above will cease producing distinct points A_t's, when the point A_{2n+1} coincides with the point A_1, that is when the angle A_1OA_{2n+1} is equal to 360° in the clockwise direction. By [11],

$$\text{angle } A_1OA_{2n+1} = 2n\alpha$$

in the clockwise direction. Hence

$$2n\ \alpha = 360$$

and

$$\alpha = \frac{180}{n} \ . \qquad [12]$$

We note that equation [12] imposes a condition on the angle α of the wedge, that α should be a divisor of 180. In this thesis we will only consider the angles α such that $\frac{180}{\alpha}$ is an integer.

The set of all points A_t (t = 1,2,... 2n) thus obtained form the symmetry group of the regular

2n - gon, $\{2n\}$, called the dihedral group D_{2n} generated by the two reflections, first in (J) and then in (I). All of these points lie on the circle with center at the origin and passing through A_1.

Hence given a wedge with an angle α and a point A_1 inside the wedge we will have 2n $(n = \frac{180}{\alpha})$ distinct images of A_1, A_1 inclusive, in both mirrors (I) and (J). These points will be such that if we take into consideration any one of the two half-planes (I) and (J) of the wedge, we will have an equal number of them placed on both sides of the half-plane such that they will be symmetrical in pairs with respect to the half-plane considered.

4. Single Force Parallel to the Edge of the Wedge.

The Galerkin vector for the single force parallel to the edge of the wedge, or in the Y direction, placed at an interior point A_1 is given by equations [4]

$$\underline{F}_1 = \underline{j}\, F_y = \underline{j}\, R_1$$

where A_1 has coordinates (a,0,b) referred to the X, Y, Z-coordinate system, and R_1 is the distance from A_1 to any point (x,y,z) in space.

Here after we will represent each point and each vector in space in terms of two coordinate systems to simplify the proofs. By the help of equations [9] we can find the coordinates of A_1 referred to the X', Y', Z'-coordinate system.

A_1: (a,0,b) referred to the X, Y, Z-coordinate system.

: $(a \cos\alpha + b \sin\alpha, 0, -a \sin\alpha + b \cos\alpha)'$ referred to the X', Y', Z'-coordinate system.

The prime " ' " sign will help us to differentiate between the two sets of coordinates and we won't need to mention each time to which coordinate system they refer.

The single force will also have two representations

$\underline{F}_1 = \underline{j}\, F_y = \underline{j}\, R_1$ referred to the X, Y, Z-coordinate system.

$\underline{F}_1 = \underline{j}'\, F_y = \underline{j}' R_1'$ referred to the X', Y', Z'-coordinate system,

where R_1' is the distance from A_1 to any joint $(x',y',z')'$ in space. We note that R_1 and R_1' actually are two representations of the same distance, the first referred to the X, Y, Z-coordinate system and the second with a prime " ' " sign, referred to the X', Y', Z'-coordinate system.

We are interested in having zero shearing stresses and zero normal displacement on the half-planes (I) and (J) of the wedge simultaneously.

To have the desired boundary conditions on the half-plane (J) we use the principle given in chapter I section 6, and superpose on $\underline{F}_1$ an $\underline{F}_2$ placed at A_2, the point symmetrical to A_1 with respect to (J) such that $\underline{F}_2$ is parallel to $\underline{F}_1$ and is in the same direction:

$$\underline{F}_2 = \underline{i}\, R_2 = \underline{i}'\, R_2',$$

where R_2 and R_2' are two representations of the same distance from A_2 to any point in space referred to X, Y, Z and X', Y', Z'-coordinate systems respectively.

Given an arbitrary point A with coordinates (x,y,z) and (x',y',z')', we replace Z by -Z in the coordinate set (x,y,z) to get the coordinates of the point symmetrical to A with respect to (I); and we replace Z' by -Z' in the coordinate set (x',y',z')' to get the coordinates of the point symmetrical to A with respect to (J). Hence A_2, the point symmetrical to

$$A_1:\ (a,0,b)$$

or

$$(a \cos\alpha + b \sin\alpha,\ 0,\ -a \sin\alpha + b \cos\alpha)'$$

with respect to (J) will have coordinates

$$A_2:\ (a \cos\alpha + b \sin\alpha,\ 0,\ a \sin\alpha - b \cos\alpha)'.$$

Using the transformation equations [9],

$$A_2: (a \cos 2\alpha + b \sin 2\alpha, 0, a \sin 2\alpha - b \cos 2\alpha).$$

We place at A_3, the point symmetrical to A_2 with respect to (I) a force

$$\underline{F}_3 = \underline{i} R_3 = \underline{i}' R_3',$$

where R_3 and R_3' represent the same distance from A_3 to any point in space.

A_3 being the point symmetrical to A_2 with respect to (I) will have coordinates

$$A_3: (a \cos 2\alpha + b \sin 2\alpha, 0, -a \sin 2\alpha + b \cos 2\alpha).$$

Using the transormation equations [9],

$$A_3: (a \cos 3\alpha + b \sin 3\alpha, 0, -a \sin 3\alpha + b \cos 3\alpha)'.$$

We proceed in this manner and place at each point A_t a force

$$\underline{F}_t = \underline{i} R_t = \underline{i}' R'_t,$$

where R_t and R_t' represent the same distance from A_t to any point in space referred to X,Y,Z and X',Y',Z'-coordinate systems respectively.

Thus, if t is even we place at

$$A_t: (a \cos t\alpha + b \sin t\alpha, 0, a \sin t\alpha - b \cos t\alpha)$$

$$: (a \cos(t-1)\alpha + b \sin(t-1)\alpha, 0, a \sin(t-1)\alpha - b \cos(t-1)\alpha)' \qquad [13]$$

a force

$$\underline{F}_{2t-1} = \underline{i}R_{2t-1} = \underline{i}'R'_{2t-1}$$

with

$$\underline{F}_{2t} = \underline{i}R_{2t} = \underline{i}'\ R'_{2t} \quad .$$

By equations [13], $\underline{F}_{2t-1}$ is placed at

$$A_{2t-1}:(a \cos(2t-1)\alpha + b \sin(2t-1)\alpha\ ,0,-a \sin(2t-1)\alpha + b \cos(2t-1)\alpha)'$$

and $\underline{F}_{2t}$ is placed at

$$A_{2t}:(a \cos(2t-1)\alpha + b \sin(2t-1)\alpha\ ,0,\ a \sin(2t-1)\alpha - b \cos(2t-1)\alpha\ )'.$$

Hence $\underline{F}_{2t-1}$ and $\underline{F}_{2t}$ are symmetrically situated with respect to (J). Also they are both parallel to the Y-axis, hence parallel to (J) and to each other, and have the same direction.

Next let us consider the half-plane (I) of the wedge. We group the forces $\underline{F}_t$, t = 1, 2, ... 2n in pairs as follows:

$$\underline{F}_1 = \underline{i}\ R_1 = \underline{i}'\ R'_1$$

with

$$\underline{F}_{2n} = \underline{i}\ R_{2n} = j'\ R'_{2n}\ ,$$

and

$$\underline{F}_{2t} = \underline{i}\ R_{2t} = \underline{i}'\ R'_{2t}$$

with

$$\underline{F}_{2t+1} = \underline{i}\ R_{2t+1} = \underline{i}'\ R'_{2t+1}$$

$$\underline{F}_t = \underline{i}\, R_t = \underline{i}'\, R'_t \;;$$

and if t is odd, we place at

A_t: $(a \cos(t-1)\alpha + b \sin(t-1)\alpha\ ,0,-a \sin(t-1)\alpha + b \cos(t-1)\alpha)$

$:(a \cos t\alpha + b \sin t\alpha\ ,0,-a \sin t\alpha + b \cos t\alpha\ )'$

a force

$$\underline{F}_t = \underline{i}\, R_t = \underline{i}'\, R'_t \;,$$

where

$$t = 1,\ 2,\ \ldots\ 2n \qquad (n = \frac{180}{\alpha}).$$

To prove that the resulting pattern of forces yields the desired boundary conditions of zero shearing stresses and zero normal displacements on both half-planes (I) and (J), we show that if we consider any one of the two half-planes of the wedge, we will have an equal number of these forces on both sides of the half-plane considered, such that they will be symmetrically situated in pairs with respect to the half-plane, their components parallel to it are in the same direction and their components perpendicular to it are in the opposite directions, and hence when superposed they will produce the desired boundary conditions on (I) and on (J).

Let us consider the half-plane (J) of the wedge. We group the forces $\underline{F}_t$, t = 1,2, ... 2n in pairs as follows:

where

$$t = 1, 2, \ldots (n-1).$$

The force $\underline{F}_1$ is placed at

$$A_1: (a, 0, b)$$

and $\underline{F}_{2n}$ is placed at, by equations [13],

$$A_{2n}: (a \cos 2n\alpha + b \sin 2n\alpha, 0, a \sin 2n\alpha - b \cos 2n\alpha);$$

by equation [12], $n = \frac{180}{\alpha}$ or $2n\alpha = 360$, hence

$$A_{2n}: (a, 0, -b) .$$

This implies that $\underline{F}_1$ and $\underline{F}_{2n}$ are symmetrically situated with respect to (I), they also have the same direction and are both parallel to (I).

The forces $\underline{F}_{2t}$ and $\underline{F}_{2t+1}$ are placed, by equations [13], respectively at

$$A_{2t}: (a \cos 2t\alpha + b \sin 2t\alpha, 0, a \sin 2t\alpha - b \cos 2t\alpha)$$

and

$$A_{2t+1}: (a \cos 2t\alpha + b \sin 2t\alpha, 0, -a \sin 2t\alpha + b \cos 2t\alpha).$$

Hence $\underline{F}_{2t}$ and $\underline{F}_{2t+1}$ are symmetrically situated with respect to (I), they also have the same direction and are both parallel to (I).

We have thus proved that the pattern of forces given by equations [13] will produce the desired boundary conditions of zero shearing stresses and zero normal displacements on both half-planes (I) and (J) of the wedge.

The displacements and the stresses produced by these forces can be calculated by the help of equations [6].

CHAPTER III

FORCE PERPENDICULAR TO A SIDE OF THE WEDGE

1. Force Perpendicular to (I).

The Galerkin vector for a single force perpendicular to the half-plane (I) of the wedge placed at A_1 is given by the equations [4]:

$$\underline{F}_1 = \underline{k}\, F_z = \underline{k}\, R_1 \ .$$

Using the transformation equations [9] we get,

$$F_1 = \underline{i}'\, F_z \sin\alpha + \underline{k}'\, F_z \cos\alpha$$

$$= \underline{i}'\, R_1' \sin\alpha + \underline{k}'\, R_1' \cos\alpha \ .$$

To have zero shearing stresses and zero normal displacement on the half-plane (J), we superpose a force $\underline{F}_2$ placed at A_2 such that the component of $\underline{F}_2$ parallel to (J) is in the same direction as that of $\underline{F}_1$ and the component perpendicular to (J) is in the opposite direction to that of $\underline{F}_1$:

$$\underline{F}_2 = \underline{i}'\, R_2' \sin\alpha - \underline{k}'\, R_2' \cos\alpha \ .$$

Using the transformation equations[9],

$$\underline{F}_2 = \underline{i}\, R_2 \sin 2\alpha \; - \underline{k}\, R_2 \cos 2\alpha \; .$$

At A_3, the point symmetrical to A_2 with respect to (I), we place a force $\underline{F}_3$,

$$\underline{F}_3 = \underline{i}\, R_3 \sin 2\alpha \; + \underline{k}\, R_3 \cos 2\alpha$$

$$= \underline{i}'\, R_3' \sin 3\alpha \; + \underline{k}'\, R_3' \cos 3\alpha,$$

which has its component parallel to (I) in the same direction as that of $\underline{F}_2$ and its component perpendicular to (I) in the opposite direction to that of $\underline{F}_2$.

The presence of $\underline{F}_3$ will alter the boundary conditions on the half-plane (J), hence we superpose $\underline{F}_3$ by $\underline{F}_4$ placed at A_4,

$$\underline{F}_4 = \underline{i}'\, R_4' \sin 3\alpha - \underline{k}'\, R_4' \cos 3\alpha$$

$$= i\, R_4 \sin 4\alpha - \underline{k}\, R_4 \cos 4\alpha \; ,$$

which has its component parallel to (J) in the same direction as that of $\underline{F}_3$ and its component perpendicular to (J) in the opposite direction to that of $\underline{F}_3$. We proceed in this manner and if t is even, we place at

$$A_t : (a \cos t\alpha + b \sin t\alpha, 0, a \sin t\alpha - b \cos t\alpha)$$

$$: (a \cos(t-1)\alpha + b \sin(t-1)\alpha, 0, a \sin(t-1)\alpha$$

$$- b \cos(t-1)\alpha)'$$

a force

$$\underline{F}_t = \underline{i}\, R_t \sin t\alpha - \underline{k}\, R_t \cos t\alpha$$

$$= \underline{i}'\, R'_t \sin(t-1)\alpha - \underline{k}'\, R'_t \cos(t-1)\alpha \; ; \qquad [14]$$

if t is odd, we place at

$$A_t : (a \cos(t-1)\alpha + b \sin(t-1)\alpha \, , 0, -a \sin(t-1)\alpha + b \cos(t-1)\alpha)$$

$$:(a \cos t\alpha + b \sin t\alpha \, , 0, -a \sin t\alpha + b \cos t\alpha)'$$

a force

$$\underline{F}_t = \underline{i}\, R_t \sin(t-1)\alpha + \underline{k}\, R_t \cos(t-1)\alpha$$

$$= \underline{i}'\, R'_t \sin t\alpha + \underline{k}'\, R'_t \cos t\alpha$$

where

$$t = 1, 2, \ldots 2n \quad (n = \frac{180}{\alpha}).$$

We now prove that the resulting pattern of forces yields the desired boundary conditions on both half-planes (I) and (J) of the wedge.

Let us consider the half-plane (J) of the wedge. Using equations [14], we group the forces $\underline{F}_t$, t = 1,2, ... 2n in pairs as follows:

$$\underline{F}_{2t-1} = \underline{i}\, R_{2t-1} \sin(2t-2)\alpha + \underline{k}\, R_{2t-1} \cos(2t-2)\alpha$$

$$= \underline{i}'\, R'_{2t-1} \sin(2t-1)\alpha + \underline{k}'\, R'_{2t-1} \cos(2t-1)\alpha$$

placed at

$$A_{2t-1} : (a \cos(2t-2)\alpha + b \sin(2t-2)\alpha, 0, -a \sin(2t-2)\alpha + b \cos(2t-2)\alpha),$$

$$: (a \cos(2t-1)\alpha + b \sin(2t-1)\alpha, 0, -a \sin(2t-1)\alpha + b \cos(2t-1)\alpha)'$$

with

$$\underline{F}_{2t} = \underline{i}\, R_{2t} \sin 2t\alpha - \underline{k}\, R_{2t} \cos 2t\alpha$$

$$= \underline{i}'\, R'_{2t} \sin(2t-1)\alpha - \underline{k}'\, R'_{2t} \cos(2t-1)\alpha$$

placed at

$$A_{2t} : (a \cos 2t\alpha + b \sin 2t\alpha, 0, a \sin 2t\alpha - b \cos 2t\alpha)$$

$$: (a \cos(2t-1)\alpha + b \sin(2t-1)\alpha, 0, a \sin(2t-1)\alpha - b \cos(2t-1)\alpha)'.$$

The forces $\underline{F}_{2t-1}$ and $\underline{F}_{2t}$ are symetrically situated with respect to (J). Also they have their components parallel to (J) in the same direction, and their components perpendicular to (J) in opposite directions, hence when superposed they will produce zero shearing stresses and zero normal displacements on the half-plane (J).

Next let us consider the half-plane (I) of the wedge. Using equations [14], we group the forces $\underline{F}_t$, t = 1,2,... 2n in pairs as follows:

$$\underline{F}_1 = \underline{k}\, R_1$$

$$= \underline{i}'\, R_1' \sin\alpha + \underline{k}'\, R_1' \cos\alpha$$

placed at

$$A_1 : (a,\ 0,\ b)$$

$$: (a \cos\alpha + b \sin\alpha, 0, -a \sin\alpha + b \cos\alpha)'$$

with

$$\underline{F}_{2n} = \underline{i}\, R_{2n} \sin 2n\alpha - \underline{k}\, R_{2n} \cos 2n\alpha$$

$$= \underline{i}'\, R_{2n}' \sin(2n-1)\alpha - \underline{k}'\, R_{2n}' \cos(2n-1)\alpha$$

placed at

$$A_{2n} : (a \cos 2n\alpha + b \sin 2n\alpha, 0, a \sin 2n\alpha - b \cos 2n\alpha)$$

$$: (a \cos(2n-1)\alpha + b \sin(2n-1)\alpha, 0, a \sin(2n-1)\alpha$$

$$- b \cos(2n-1)\alpha)';$$

and

$$\underline{F}_{2t} = \underline{i}\, R_{2t} \sin 2t\alpha - \underline{k}\, R_{2t} \cos 2t\alpha$$

$$= \underline{i}'\, R_{2t}' \sin(2t-1)\alpha - \underline{k}'\, R_{2t}' \cos(2t-1)\alpha$$

placed at

$$A_{2t} : (a \cos 2t\alpha + b \sin 2t\alpha, 0, a \sin 2t\alpha - b \cos 2t\alpha)$$

$$: (a \cos(2t-1)\alpha + b \sin(2t-1)\alpha, 0, a \sin(2t-1)\alpha$$

$$- b \cos(2t-1)\alpha)'$$

with

$$\underline{F}_{2t+1} = \underline{i}\, R_t \sin 2t\alpha + \underline{k}\, R_t \cos 2t\alpha$$

$$= \underline{i}'\, R'_t \sin(2t+1)\alpha + \underline{k}'\, R'_t \cos(2t+1)\alpha$$

placed at

$$A_{2t+1}: (a \cos 2t\alpha + b \sin 2t\alpha\, , 0, -a \sin 2t\alpha + b \cos 2t\alpha)$$

$$: (a \cos(2t+1)\alpha + b \sin(2t+1)\alpha\, , 0, - a \sin(2t+1)\alpha + b \cos(2t + 1)\alpha)'$$

where

$$t = 1, 2, \ldots, (n - 1).$$

We first consider the pair $\underline{F}_1$ and $\underline{F}_{2n}$. By equation [12], $n = \frac{180}{\alpha}$ or $2n\alpha = 360$ hence

$$\underline{F}_{2n} = \underline{i}\, R_{2n} \sin 360 - \underline{k}\, R_{2n} \cos 360 = -\underline{k}\, R_{2n}.$$

and the point A_{2n} will have coordinates

$$A_{2n}: (a, 0, -b).$$

The forces $\underline{F}_1$ and $\underline{F}_{2n}$ are symmetrically situated also they are both perpendicular to (I) and are in opposite directions, hence when superposed they will produce the desired boundary conditions on (I).

We next consider the pairs $\underline{F}_{2t}$ and $\underline{F}_{2t+1}$ where $t = 1, 2, \ldots (n-1)$. It is clear from above that they are symmetrically situated with respect to (I), their components parallel to (I) are in the same direction and their components perpendicular to (I) are in opposite directions. Hence when superposed they will produce zero shearing stresses and zero normal displacements on (I).

Thus under the action of the totality of the forces given by equation [14] we will have the desired boundary conditions on (I) and on (J).

The displacements and stresses produced by these forces can by calculated by the help of equations [5] and [7].

2. Force Perpendicular to (J).

The case of a single force perpendicular to the half-plane (J) can be discussed in the same way as that of a single force perpendicular to (I).

We get the following result:

if t is even, we place at A_t a force

$$\underline{F}_t = \underline{i}\, R_t \sin(t-1)\alpha - \underline{k}\, R_r \cos(t-1)\alpha$$

$$= \underline{i}'\, R'_t \sin(t-2)\alpha - \underline{k}'\, R'_t \cos(t-2)\alpha \; ; \qquad [15]$$

if t is odd, we place at A_t a force

$$\underline{F}_t = \underline{i}\, R_t \sin(t-2)\alpha + \underline{k}\, R_t \cos(t-2)\alpha$$

$$= \underline{i}'\, R'_t \sin(t-1)\alpha + \underline{k}'\, R'_t \cos(t-1)\alpha \, .$$

CHAPTER IV

THE WEDGE WITH ANGLE 60°

1. Tables of the Forces and Their Application Points.

Given a wedge with an angle of 60° and a concentrated force $\underline{F}_1$ acting at an interior point $A_1(a, 0, b)$, we proceed in the manner described in Chapters II and III by placing forces, with appropriate directions, at different points A_t's, to have on both sides of the wedge zero shearing stresses and zero normal displacements.

For an angle of 60°, referring to Chapter II section 3, we use

$$2\left(\frac{180°}{60°}\right) = 6$$

forces, $\underline{F}_1$ inclusive, to obtain the desired boundary conditions on both sides of the wedge. The forces to be used and the coordinates of their points of application (Fig. 3) can be obtained by the help of equations [13], [14] and [15].

Table I gives the coordinates of the points of application of the forces to be used. Table II gives the forces to be used for $\underline{F}_1$ parallel to the edge of the wedge. Table III gives the forces to be used for $\underline{F}_1$ perpendicular to (I) and Table IV gives the forces to be used for $\underline{F}_1$ perpendicular to (J).

2. <u>Tables of The Displacements and Stresses</u>.

The displacements and the stresses produced by the forces $\underline{F}_1$, $\underline{F}_2$, $\underline{F}_3$, $\underline{F}_4$, $\underline{F}_5$ and $\underline{F}_6$ are calculated by the help of equations [5], [6] and [7] and the results are stated in tables I, II, III and IV.

Tables V and VI give the displacements and stresses for a force in Y-direction placed at the interior point A_1 of the wedge of 60° with zero normal displacements and zero shearing stresses on both sides of the wedge, in terms of X,Y,Z and X', Y', Z'-coordinates respectively. To check that on (I) we have the desired boundary conditions we use table V and substitute zero for Z. For (J) we use table VI and substitute zero for Z'. We will get as expected zero shearing stresses and zero normal displacements on (I) and on (J).

Tables VII and VIII give the displacements and stresses for a force in Z-direction placed at the interior point A_1 of the wedge of 60°, with zero normal displacements and zero shearing stresses on (I) and (J), in terms of X, Y, Z and X', Y', Z'-coordinates respectively.

Tables IX and X give the displacements and stresses for a force in Z'-direction placed at A_1 with zero normal displacements and zero shearing stresses on (I) and on (J) in terms of X, Y, Z and X', Y', Z'-coordinates.

Substituting zero for Z in Tables VII and IX we will get the desired boundary conditions on (I) for the respective forces. Substituting zero for Z' in the tables VIII and X we will get the desired boundary conditions on (J) for the respective forces.

A factor of $\frac{P}{8\pi(1-\alpha)}$ is omitted throughout the tables V, VI, VII, VIII, IX and X.

TABLE I

POINTS OF APPLICATION OF THE FORCES

	X,Y,Z-coordinates		X',Y',Z'-coordinates
A_1:	$(a, 0, b)$	:	$(\frac{a}{2} + \frac{b\sqrt{3}}{2}, 0, -\frac{a\sqrt{3}}{2} + \frac{b}{2})'$
A_2:	$(-\frac{a}{2} + \frac{b\sqrt{3}}{2}, 0, \frac{a\sqrt{3}}{2} + \frac{b}{2})$	:	$(\frac{a}{2} + \frac{b\sqrt{3}}{2}, 0, \frac{a\sqrt{3}}{2} - \frac{b}{2})'$
A_3:	$(-\frac{a}{2} + \frac{b\sqrt{3}}{2}, 0, -\frac{a\sqrt{3}}{2} - \frac{b}{2})$	:	$(-a, 0, -b)'$
A_4:	$(-\frac{a}{2} - \frac{b\sqrt{3}}{2}, 0, -\frac{a\sqrt{3}}{2} + \frac{b}{2})$	:	$(-a, 0, b)'$
A_5:	$(-\frac{a}{2} - \frac{b\sqrt{3}}{2}, 0, \frac{a\sqrt{3}}{2} - \frac{b}{2})$	:	$(\frac{a}{2} - \frac{b\sqrt{3}}{2}, 0, \frac{a\sqrt{3}}{2} + \frac{b}{2})'$
A_6:	$(a, 0, -b)$	:	$(\frac{a}{2} - \frac{b\sqrt{3}}{2}, 0, -\frac{a\sqrt{3}}{2} - \frac{b}{2})'$

TABLE II

FORCES TO BE USED FOR $\underline{F}_1$ PARALLEL TO THE EDGE OF THE WEDGE

In X, Y, Z-coordinates		In X', Y', Z'-coordinates
$\underline{F}_1 = \underline{i}\, R_1$	=	$\underline{i}'\, R'_1$
$\underline{F}_2 = \underline{i}\, R_2$	=	$\underline{i}'\, R'_2$
$\underline{F}_3 = \underline{i}\, R_3$	=	$\underline{i}'\, R'_3$
$\underline{F}_4 = \underline{i}\, R_4$	=	$\underline{i}'\, R'_4$
$\underline{F}_5 = \underline{i}\, R_5$	=	$\underline{i}'\, R'_5$
$\underline{F}_6 = \underline{i}\, R_6$	=	$\underline{i}'\, R'_6$

TABLE III

FORCES TO BE USED FOR $\underline{F}_1$

PERPENDICULAR TO (I)

	In X, Y, Z-coordinates	In X', Y', Z'-coordinates
$\underline{F}_1 =$	$\underline{k}\, R_1$	$= \underline{i}' \frac{\sqrt{3}}{2} R'_1 + \underline{k}' \frac{1}{2} R'_1$
$\underline{F}_2 =$	$\underline{i} \frac{\sqrt{3}}{2} R_2 + \underline{k} \frac{1}{2} R_2$	$= \underline{i}' \frac{\sqrt{3}}{2} R'_2 - \underline{k}' \frac{1}{2} R'_2$
$\underline{F}_3 =$	$\underline{i} \frac{\sqrt{3}}{2} R_3 - \underline{k} \frac{1}{2} R_3$	$= - \underline{k}' R'_3$
$\underline{F}_4 =$	$-\underline{i} \frac{\sqrt{3}}{2} R_4 + \underline{k} \frac{1}{2} R_4$	$= + \underline{k}' R'_4$
$\underline{F}_5 =$	$-\underline{i} \frac{\sqrt{3}}{2} R_5 - \underline{k} \frac{1}{2} R_5$	$= -\underline{i}' \frac{\sqrt{3}}{2} R'_5 + \underline{k}' \frac{1}{2} R'_5$
$\underline{F}_6 =$	$- \underline{k}\, R_6$	$= -\underline{i}' \frac{\sqrt{3}}{2} R'_6 - \underline{k}' \frac{1}{2} R'_6$

TABLE IV

FORCES TO BE USED FOR $\underline{F}_1$

PERPENDICULAR TO (J)

	In X, Y, Z -coordinates		In X', Y', Z'-coordinates
$\underline{F}_1 =$	$-\underline{i}\,\frac{\sqrt{3}}{2} R_1 + \underline{k}\,\frac{1}{2} R_1$	=	$\underline{k}'\, R'_1$
$\underline{F}_2 =$	$\underline{i}\,\frac{\sqrt{3}}{2} R_2 - \underline{k}\,\frac{1}{2} R_2$	=	$-\underline{k}'\, R'_2$
$\underline{F}_3 =$	$\underline{i}\,\frac{\sqrt{3}}{2} R_3 + \underline{k}\,\frac{1}{2} R_3$	=	$\underline{i}'\,\frac{\sqrt{3}}{2} R'_3 - \underline{k}'\,\frac{1}{2} R'_3$
$\underline{F}_4 =$	$+\underline{k}\, R_4$	=	$\underline{i}'\,\frac{\sqrt{3}}{2} R'_4 + \underline{k}'\,\frac{1}{2} R'_4$
$\underline{F}_5 =$	$-\underline{k}\, R_5$	=	$-\underline{i}'\,\frac{\sqrt{3}}{2} R'_5 - \underline{k}'\,\frac{1}{2} R'_5$
$\underline{F}_6 =$	$-\underline{i}\,\frac{\sqrt{3}}{2} R_6 - \underline{k}\,\frac{1}{2} R_6$	=	$-\underline{i}'\,\frac{\sqrt{3}}{2} R'_6 + \underline{k}'\,\frac{1}{2} R'_6$

TABLE V

Displacements and Stresses for a Force Parallel to the Edge of the Wedge Referred to X, Y, Z- Coordinate System

$$2Gu_x = y(x-a)\left[\frac{1}{R_1^3} + \frac{1}{R_6^3}\right] + y\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3} + \frac{1}{R_3^3}\right] + y\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3} + \frac{1}{R_5^3}\right].$$

$$2Gu_y = (3-4\nu)\left[\frac{1}{R_1}+\frac{1}{R_6}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right] + y^2\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}+\frac{1}{R_2^3}+\frac{1}{R_3^3}+\frac{1}{R_4^3}+\frac{1}{R_5^3}\right].$$

$$2Gu_z = y\left[\frac{z-b}{R_1^3} + \frac{z+b}{R_6^3}\right] + y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} + \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} + \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right].$$

$$\sigma_{xx} = (1-2\nu)y\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}+\frac{1}{R_2^3}+\frac{1}{R_3^3}+\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + 3y(x-a)^2\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}\right] - 3y\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right] - 3y\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_4^5}+\frac{1}{R_5^5}\right].$$

$$\sigma_{yy} = -(1-2\nu)y\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}+\frac{1}{R_2^3}+\frac{1}{R_3^3}+\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] - 3y^3\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}+\frac{1}{R_2^5}+\frac{1}{R_3^5}+\frac{1}{R_4^5}+\frac{1}{R_5^5}\right].$$

$$\sigma_{zz} = (1-2\nu)y\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}+\frac{1}{R_2^3}+\frac{1}{R_3^3}+\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] - 3y\left[\frac{(z-b)^2}{R_1^5}+\frac{(z+b)^2}{R_6^5}\right] - 3y\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5}+\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right] - 3y\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5}+\frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right].$$

$$\sigma_{xy} = -(1-2\nu)(x-a)\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] - 3y^2(x-a)\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}\right] - (1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - 3y^2\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right] - (1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] - 3y^2\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^5}+\frac{1}{R_5^5}\right].$$

$$\sigma_{yz} = -(1-2\nu)\left[\frac{z-b}{R_1^3}+\frac{z+b}{R_6^3}\right] - 3y^2\left[\frac{z-b}{R_1^5}+\frac{z+b}{R_6^5}\right] - (1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - 3y^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] - (1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}+\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right] - 3y^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5}+\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].$$

$$\sigma_{zx} = -3(x-a)y\left[\frac{z-b}{R_1^5}+\frac{z+b}{R_6^5}\right] - 3\left(x-\frac{b\sqrt{3}}{2}+\frac{a}{2}\right)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] - 3\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5}+\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].$$

TABLE VI

Displacements and Stresses for a Force Parallel to the Edge of the Wedge Referred to X', Y', Z'-Coordinate System

$$2Gu_{x'} = y'\left(x' - \frac{a}{2} - \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3}\right] + y'(x'+a)\left[\frac{1}{R_3'^3} + \frac{1}{R_4'^3}\right] + y'\left(x' - \frac{a}{2} + \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4'^3} + \frac{1}{R_5'^3}\right].$$

$$2Gu_{y'} = (3-4\nu)\left[\frac{1}{R_1'} + \frac{1}{R_2'} + \frac{1}{R_3'} + \frac{1}{R_4'} + \frac{1}{R_5'} + \frac{1}{R_6'}\right] + y'^2\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3} + \frac{1}{R_3'^3} + \frac{1}{R_4'^3} + \frac{1}{R_5'^3} + \frac{1}{R_6'^3}\right].$$

$$2Gu_{z'} = y'\left[\frac{z' + \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_1'^3} + \frac{z' - \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_2'^3}\right] + y'\left[\frac{z'+b}{R_3'^3} + \frac{z'-b}{R_4'^3}\right] + y'\left[\frac{z' - \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_5'^3} + \frac{z' + \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_6'^3}\right].$$

$$\sigma_{x'x'} = (1-2\nu)y'\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3} + \frac{1}{R_3'^3} + \frac{1}{R_4'^3} + \frac{1}{R_5'^3} + \frac{1}{R_6'^3}\right] - 3y'\left(x' - \frac{a}{2} - \frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_1'^5} + \frac{1}{R_2'^5}\right] - 3y'(x'+a)^2\left[\frac{1}{R_3'^5} + \frac{1}{R_4'^5}\right] - 3y'\left(x' - \frac{a}{2} + \frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_5'^5} + \frac{1}{R_6'^5}\right].$$

$$\sigma_{y'y'} = -(1-2\nu)y'\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3} + \frac{1}{R_3'^3} + \frac{1}{R_4'^3} + \frac{1}{R_5'^3} + \frac{1}{R_6'^3}\right] - 3y'^3\left[\frac{1}{R_1'^5} + \frac{1}{R_2'^5} + \frac{1}{R_3'^5} + \frac{1}{R_4'^5} + \frac{1}{R_5'^5} + \frac{1}{R_6'^5}\right].$$

$$\sigma_{z'z'} = (1-2\nu)y'\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3} + \frac{1}{R_3'^3} + \frac{1}{R_4'^3} + \frac{1}{R_5'^3} + \frac{1}{R_6'^3}\right] - 3y'\left[\frac{\left(z' + \frac{a\sqrt{3}}{2} - \frac{b}{2}\right)^2}{R_1^5} + \frac{\left(z' - \frac{a\sqrt{3}}{2} + \frac{b}{2}\right)^2}{R_2'^5}\right] - 3y'\left[\frac{(z'+b)^2}{R_3'^5} + \frac{(z'-b)^2}{R_4'^5}\right] - 3y'\left[\frac{\left(z' - \frac{a\sqrt{3}}{2} - \frac{b}{2}\right)^2}{R_5'^5} + \frac{\left(z' + \frac{a\sqrt{3}}{2} + \frac{b}{2}\right)^2}{R_6'^5}\right].$$

$$\sigma_{x'y'} = -(1-2\nu)\left(x' - \frac{a}{2} - \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_1'^3} + \frac{1}{R_2'^3}\right] - 3y'^2\left(x' - \frac{a}{2} - \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_1'^5} + \frac{1}{R_2'^5}\right] - (1-2\nu)(x'+a)\left[\frac{1}{R_3'^3} + \frac{1}{R_4'^3}\right] - 3y'^2(x'+a)\left[\frac{1}{R_3'^5} + \frac{1}{R_4'^5}\right] - (1-2\nu)\left(x' - \frac{a}{2} + \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_5'^3} + \frac{1}{R_6'^3}\right] - 3y'^2\left(x' - \frac{a}{2} + \frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_5'^5} + \frac{1}{R_6'^5}\right].$$

$$\sigma_{y'z'} = -(1-2\nu)\left[\frac{z' + \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_1'^3} + \frac{z' - \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_2'^3}\right] - 3y'^2\left[\frac{z' + \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_1'^5} + \frac{z' - \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_2'^5}\right] - (1-2\nu)\left[\frac{z'+b}{R_3'^3} + \frac{z'-b}{R_4'^3}\right] - 3y'^2\left[\frac{z'+b}{R_3'^5} + \frac{z'-b}{R_4'^5}\right] - (1-2\nu)\left[\frac{z' - \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_5'^3} + \frac{z' + \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_6'^3}\right] - 3y'^2\left[\frac{z' - \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_5'^5} + \frac{z' + \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_6'^5}\right].$$

$$\sigma_{x'z'} = 3\left(x' - \frac{a}{2} - \frac{b\sqrt{3}}{2}\right)y'\left[\frac{z' + \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_1'^5} + \frac{z' - \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_2'^5}\right] + 3(x'+a)y'\left[\frac{z'+b}{R_3'^5} + \frac{z'-b}{R_4'^5}\right] + 3\left(x' - \frac{a}{2} + \frac{b\sqrt{3}}{2}\right)y'\left[\frac{z' - \frac{a\sqrt{3}}{2} - \frac{b}{2}}{R_5'^5} + \frac{z' + \frac{a\sqrt{3}}{2} + \frac{b}{2}}{R_6'^5}\right].$$

TABLE VII

Displacements and Stresses for a Force in Z-Direction, that is Perpendicular to the Half-Plane (I), Referred to X, Y, Z-Coordinate System

$$2Gu_x = (x-a)\left[\frac{z-b}{R_1^3} - \frac{z+b}{R_6^3}\right] + \frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R_2}+\frac{1}{R_3}\right] + \frac{\sqrt{3}}{2}\left(x^2+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] + \frac{1}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R_4}+\frac{1}{R_5}\right] - \frac{\sqrt{3}}{2}\left(x+a+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + \frac{1}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right].$$

$$2Gu_y = y\left[\frac{z-b}{R_1^3} - \frac{z+b}{R_6^3}\right] + \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] + \frac{1}{2}y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + \frac{1}{2}y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right].$$

$$2Gu_z = (3-4\nu)\left[\frac{1}{R_1}-\frac{1}{R_6}\right] + \left[\frac{(z-b)^2}{R_1^3} - \frac{(z+b)^2}{R_6^3}\right] + \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} + \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + \frac{1}{2}(3-4\nu)\left[\frac{1}{R_2}-\frac{1}{R_3}\right] + \frac{1}{2}\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^3} - \frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^3}\right] - \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} + \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right]$$
$$+ \frac{1}{2}(3-4\nu)\left[\frac{1}{R_4}-\frac{1}{R_5}\right] + \frac{1}{2}\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^3} - \frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^3}\right].$$

$$\sigma_{xx} = (1-2\nu)\left[\frac{z-b}{R_1^3} - \frac{z+b}{R_6^3}\right] - 3(x-a)^2\left[\frac{z-b}{R_1^5} - \frac{z+b}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^3\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right]$$
$$- \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^3\left[\frac{1}{R_4^5}+\frac{1}{R_5^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right]$$
$$- \frac{3}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].$$

$$\sigma_{yy} = (1-2\nu)\left[\frac{z-b}{R_1^3} - \frac{z+b}{R_6^3}\right] - 3y^2\left[\frac{z-b}{R_1^5} - \frac{z+b}{R_6^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - 3\frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y^2\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{3}{2}y^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right]$$
$$- \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y^2\left[\frac{1}{R_4^5}+\frac{1}{R_5^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right] - \frac{3}{2}y^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].$$

$$\sigma_{zz} = -(1-2\nu)\left[\frac{z-b}{R_1^3} - \frac{z+b}{R_6^3}\right] - 3\left[\frac{(z-b)^3}{R_1^5} - \frac{(z+b)^3}{R_6^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5} + \frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right] - \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{3}{2}\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{R_2^5} - \frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{R_3^5}\right]$$
$$- \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right] + \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5} + \frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right] - \frac{1}{2}(1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right] - \frac{3}{2}\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{R_4^5} - \frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{R_5^5}\right].$$

$$\sigma_{xy} = -3(x-a)y\left[\frac{z-b}{R_1^5} - \frac{z+b}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)y\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2 y\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right] - \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5} - \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)y\left[\frac{1}{R_4^3}+\frac{1}{R_5^3}\right]$$
$$+ \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2 y\left[\frac{1}{R_4^5}+\frac{1}{R_5^5}\right] - \frac{3}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5} - \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].$$

$$\sigma_{yz} = -(1-2\nu)y\left[\frac{1}{R_1^3}-\frac{1}{R_6^3}\right] - 3y\left[\frac{(z-b)^2}{R_1^5} - \frac{(z+b)^2}{R_6^5}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5} + \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] - \frac{1}{2}(1-2\nu)y\left[\frac{1}{R_2^3}-\frac{1}{R_3^3}\right] - \frac{3}{2}y\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5} - \frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right]$$
$$+ \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5} + \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right] - \frac{1}{2}(1-2\nu)y\left[\frac{1}{R_4^3}-\frac{1}{R_5^3}\right] - \frac{3}{2}y\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5} - \frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right].$$

$$\sigma_{zy} = -(1-2\nu)(x-a)\left[\frac{1}{R_1^3}-\frac{1}{R_6^3}\right] - 3(x-a)\left[\frac{(z-b)^2}{R_1^5} - \frac{(z+b)^2}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3} + \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5} + \frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] - \frac{1}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}-\frac{1}{R_3^3}\right]$$
$$- \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5} - \frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3} + \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_4^5}\right] + \frac{3}{2}\sqrt{3}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5} + \frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right] - \frac{1}{2}(1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}-\frac{1}{R_5^3}\right]$$
$$- \frac{3}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5} - \frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right].$$

TABLE VIII

Displacements and Stresses for a Force in the Z-Direction, that is Perpendicular to the Half-Plane (I), Referred to X', Y', Z'-Coordinate System

$$2Gu_{x'} = \frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R'_1}+\frac{1}{R'_2}\right] + \frac{\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] + \frac{1}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] + (x'+a)\left[\frac{z'+b}{R'^3_3}+\frac{z'-b}{R'^3_4}\right] - \frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R'_5}+\frac{1}{R'_6}\right] - \frac{\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right] + \frac{1}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right].$$

$$2Gu_{y'} = \frac{\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y'\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] + \frac{1}{2}y'\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] + y'\left[\frac{z'+b}{R'^3_3}+\frac{z'-b}{R'^3_4}\right] - \frac{\sqrt{3}}{2}y'\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right] + \frac{1}{2}y'\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right].$$

$$2Gu_{z'} = \frac{\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}+\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] + \frac{1}{2}(3-4\nu)\left[\frac{1}{R'_1}-\frac{1}{R'_2}\right] + \frac{1}{2}\left[\frac{\left(z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^3_1}-\frac{\left(z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^3_2}\right] - (3-4\nu)\left[\frac{1}{R'_3}-\frac{1}{R'_4}\right] - \left[\frac{(z'+b)^2}{R'^3_3}-\frac{(z'-b)^2}{R'^3_4}\right] - \frac{\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}+\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right]$$
$$+(3-4\nu)\left[\frac{1}{R'_5}-\frac{1}{R'_6}\right] + \frac{1}{2}\left[\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^3_5}-\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^3_6}\right].$$

$$\sigma_{x'x'} = -\frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] - \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^3\left[\frac{1}{R'^5_1}+\frac{1}{R'^5_2}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] - \frac{3}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_2}\right] - (1-2\nu)\left[\frac{z'+b}{R'^3_3}-\frac{z'-b}{R'^3_4}\right] + 3(x'+a)^2\left[\frac{z'+b}{R'^5_3}-\frac{z'-b}{R'^5_4}\right]$$
$$+\frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right] + \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^3\left[\frac{1}{R'^5_5}+\frac{1}{R'^5_6}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right] - \frac{3}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_6}\right].$$

$$\sigma_{y'y'} = \frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] - \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y'^2\left[\frac{1}{R'^5_1}+\frac{1}{R'^5_2}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] - \frac{3}{2}y'^2\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_2}\right] - (1-2\nu)\left[\frac{z'+b}{R'^3_3}-\frac{z'-b}{R'^3_4}\right] + 3y'^2\left[\frac{z'+b}{R'^5_3}-\frac{z'-b}{R'^5_4}\right]$$
$$-\frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right] + \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y'^2\left[\frac{1}{R'^5_5}+\frac{1}{R'^5_6}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right] - \frac{3}{2}y'^2\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_6}\right]$$

$$\sigma_{z'z'} = \frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] - \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_1}+\frac{\left(z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_2}\right] - \frac{1}{2}(1-2\nu)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] - \frac{3}{2}\left[\frac{\left(z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{\quad}-\frac{\left(z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{\quad}\right]$$
$$+(1-2\nu)\left[\frac{z'+b}{R'^3_3}-\frac{z'-b}{R'^3_4}\right] + 3\left[\frac{(z'+b)^3}{R'^5_3}-\frac{(z'-b)^3}{R'^5_4}\right] - \frac{\sqrt{3}}{2}(1-2\nu)\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right] + \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_5}+\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_6}\right] - \frac{1}{2}(1-2\nu)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right]$$
$$-\frac{3}{2}\left[\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{R'^5_5}-\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{R'^5_6}\right].$$

$$\sigma_{x'y'} = -\frac{\sqrt{3}}{2}(1-2\nu)y'\left[\frac{1}{R'^3_1}+\frac{1}{R'^3_2}\right] - \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2 y'\left[\frac{1}{R'^5_1}+\frac{1}{R'^5_2}\right] - \frac{3}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y'\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_1}-\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_2}\right] + 3(x'+a)y'\left[\frac{z'+b}{R'^5_3}-\frac{z'-b}{R'^5_4}\right] + \frac{\sqrt{3}}{2}(1-2\nu)y'\left[\frac{1}{R'^3_5}+\frac{1}{R'^3_6}\right]$$
$$+\frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2 y'\left[\frac{1}{R'^5_5}+\frac{1}{R'^5_6}\right] - \frac{3}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y'\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_5}-\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_6}\right].$$

$$\sigma_{y'z'} = -\frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y'\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_1}+\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_2}\right] - \frac{1}{2}(1-2\nu)y'\left[\frac{1}{R'^3_1}-\frac{1}{R'^3_2}\right] - \frac{3}{2}y'\left[\frac{\left(z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_1}-\frac{\left(z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_2}\right] + (1-2\nu)y'\left[\frac{1}{R'^3_3}-\frac{1}{R'^3_4}\right] + 3y'\left[\frac{(z'+b)^2}{R'^5_3}-\frac{(z'-b)^2}{R'^5_4}\right]$$
$$+\frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y'\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_5}+\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_6}\right] - \frac{1}{2}(1-2\nu)y'\left[\frac{1}{R'^3_5}-\frac{1}{R'^3_6}\right] - \frac{3}{2}y'\left[\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_5}-\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_6}\right].$$

$$\sigma_{z'x'} = -\frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_1}+\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_2}\right] - \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z'+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_1}+\frac{z'-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_2}\right] - \frac{1}{2}(1-2\nu)\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_1}-\frac{1}{R'^3_2}\right] - \frac{3}{2}\left(x'-\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_1}-\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_2}\right]$$
$$+(1-2\nu)(x'+a)\left[\frac{1}{R'_3}-\frac{1}{R'_4}\right] + 3(x'+a)\left[\frac{(z'+b)^2}{R'^5_3}-\frac{(z'-b)^2}{R'^5_4}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^3_5}+\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^3_6}\right] + \frac{3\sqrt{3}}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R'^5_5}+\frac{z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R'^5_6}\right]$$
$$-\frac{1}{2}(1-2\nu)\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R'^3_5}-\frac{1}{R'^3_6}\right] - \frac{3}{2}\left(x'-\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z'-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R'^5_5}-\frac{\left(z'+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R'^5_6}\right].$$

TABLE IX

Displacements and Stresses for a Force in the z'-Direction, that is Perpendicular to the Half-Plane (I), Referred to X, Y, Z-Coordinate System

$$2Gu_x = -\frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R_1}+\frac{1}{R_6}\right] - \frac{\sqrt{3}}{2}(x-a)^2\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{1}{2}(x-a)\left[\frac{z-b}{R_1^3}-\frac{z+b}{R_6^3}\right] + \frac{\sqrt{3}}{2}(3-4\nu)\left[\frac{1}{R_2}+\frac{1}{R_3}\right] + \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{1}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + \left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right].$$

$$2Gu_y = -\frac{\sqrt{3}}{2}(x-a)y\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{1}{2}y\left[\frac{z-b}{R_1^3}-\frac{z+b}{R_6^3}\right] + \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{1}{2}y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right].$$

$$\begin{aligned}2Gu_z = &-\frac{\sqrt{3}}{2}(x-a)\left[\frac{z-b}{R_1^3}+\frac{z+b}{R_6^3}\right] + \frac{1}{2}(3-4\nu)\left[\frac{1}{R_1}-\frac{1}{R_6}\right] + \frac{1}{2}\left[\frac{(z-b)^2}{R_1^3}-\frac{(z+b)^2}{R_6^3}\right] + \frac{\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] - \frac{1}{2}(3-4\nu)\left[\frac{1}{R_2}-\frac{1}{R_3}\right] - \frac{1}{2}\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^3}-\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^3}\right]\\ &+(3-4\nu)\left[\frac{1}{R_4}-\frac{1}{R_5}\right] + \left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^3}-\frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^3}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{xx} = &\frac{\sqrt{3}}{2}(1-2\nu)(x-a)\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{3\sqrt{3}}{2}(x-a)^3\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z-b}{R_1^3}-\frac{z+b}{R_6^3}\right] - \frac{3}{2}(x-a)^2\left[\frac{z-b}{R_1^5}-\frac{z+b}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^3\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right]\\ &- \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + (1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right] - 3\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{yy} = &-\frac{\sqrt{3}}{2}(1-2\nu)(x-a)\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{3\sqrt{3}}{2}(x-a)y^2\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z-b}{R_1^3}-\frac{z+b}{R_6^3}\right] - \frac{3}{2}y^2\left[\frac{z-b}{R_1^5}-\frac{z+b}{R_6^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y^2\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right]\\ &- \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + \frac{3}{2}y^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + (1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right] - 3y^2\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{zz} = &-\frac{\sqrt{3}}{2}(1-2\nu)(x-a)\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{3\sqrt{3}}{2}(x-a)\left[\frac{(z-b)^2}{R_1^5}+\frac{(z+b)^2}{R_6^5}\right] - \frac{1}{2}(1-2\nu)\left[\frac{z-b}{R_1^3}-\frac{z+b}{R_6^3}\right] - \frac{3}{2}\left[\frac{(z-b)^3}{R_1^5}-\frac{(z+b)^3}{R_6^5}\right] + \frac{\sqrt{3}}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right]\\ &- \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5}+\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right] + \frac{1}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right] + \frac{3}{2}\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{R_2^5}-\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{R_3^5}\right] - (1-2\nu)\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^3}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^3}\right]\\ &- 3\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^3}{R_4^5}-\frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^3}{R_5^5}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{xy} = &\frac{\sqrt{3}}{2}(1-2\nu)y\left[\frac{1}{R_1^3}+\frac{1}{R_6^3}\right] + \frac{3\sqrt{3}}{2}(x-a)^2y\left[\frac{1}{R_1^5}+\frac{1}{R_6^5}\right] - \frac{3}{2}(x-a)y\left[\frac{z-b}{R_1^5}-\frac{z+b}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)y\left[\frac{1}{R_2^3}+\frac{1}{R_3^3}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{1}{R_2^5}+\frac{1}{R_3^5}\right]\\ &+ \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}-\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] - \frac{3}{2}\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)y\left[\frac{z+\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_4^5}-\frac{z-\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_5^5}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{yz} = &\frac{3\sqrt{3}}{2}(x-a)y\left[\frac{z-b}{R_1^5}+\frac{z+b}{R_6^5}\right] - \frac{1}{2}(1-2\nu)y\left[\frac{1}{R_1^3}-\frac{1}{R_6^3}\right] - \frac{3}{2}y\left[\frac{(z-b)^2}{R_1^5}-\frac{(z+b)^2}{R_6^5}\right] - \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)y\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + \frac{1}{2}(1-2\nu)y\left[\frac{1}{R_2^3}-\frac{1}{R_3^3}\right]\\ &+ \frac{3}{2}y\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5}-\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right] - (1-2\nu)y\left[\frac{1}{R_4^3}-\frac{1}{R_5^3}\right] - 3y\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5}-\frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right].\end{aligned}$$

$$\begin{aligned}\sigma_{zx} = &\frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z-b}{R_1^3}+\frac{z+b}{R_6^3}\right] + \frac{3\sqrt{3}}{2}(x-a)^2\left[\frac{z-b}{R_1^5}+\frac{z+b}{R_6^5}\right] - \frac{1}{2}(1-2\nu)(x-a)\left[\frac{1}{R_1^3}-\frac{1}{R_6^3}\right] - 3(x-a)\left[\frac{(z-b)^2}{R_1^5}-\frac{(z+b)^2}{R_6^5}\right] - \frac{\sqrt{3}}{2}(1-2\nu)\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^3}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^3}\right]\\ &- \frac{3\sqrt{3}}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)^2\left[\frac{z-\frac{a\sqrt{3}}{2}-\frac{b}{2}}{R_2^5}+\frac{z+\frac{a\sqrt{3}}{2}+\frac{b}{2}}{R_3^5}\right] + \frac{1}{2}(1-2\nu)\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_2^3}-\frac{1}{R_3^3}\right] + \frac{3}{2}\left(x+\frac{a}{2}-\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z-\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_2^5}-\frac{\left(z+\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_3^5}\right]\\ &- (1-2\nu)\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{1}{R_4^3}-\frac{1}{R_5^3}\right] - 3\left(x+\frac{a}{2}+\frac{b\sqrt{3}}{2}\right)\left[\frac{\left(z+\frac{a\sqrt{3}}{2}-\frac{b}{2}\right)^2}{R_4^5}-\frac{\left(z-\frac{a\sqrt{3}}{2}+\frac{b}{2}\right)^2}{R_5^5}\right].\end{aligned}$$

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