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Undergraduate Thesis for the degree of

B.Sc. in Civil Engineering

DESIGN OF TWO REINFORCED CONCRETE

HIGHWAY BRIDGES.

Contents: 1. - T - BEAM BRIDGE - span 40'.

2. - OPEN SPANDREL ARCH BRIDGE - span 120'.

by

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PREFACE

This thesis is an attempt to present in a complete form and in a comprehensive sequence of steps the design of two reinforced concrete bridges:- a T-Beam girder bridge and an open-spandrel arch bridge. The reader will discover that a particular location was not in the author's mind, and consequently no survey was made of a proposed bridge site. The author's main purpose was to design an economical superstructure for a given span and a given rise. In view of the fact that no wide rivers or valleys exist in Lebanon that require very long-spanned bridges, two spans were chosen with dimensions that are frequently encountered in bridge design in this country.

The bridges are of reinforced concrete because this type is now generally used for highways. Concrete may be used in locations where it would not be possible to build steel structures or ordinary masonry arches. Furthermore, with the present prices of materials in the market, a reinforced concrete structure would be more economical in the ordinary case. This requires great care in construction and is of a more permanent character with very little maintenance work.

The author has not made his main aim to arrive at satisfactory and working results, although this has been attained. It has been his main concern to analyze and design the structures according to standard methods and with purposeful steps based on scientifically established and experimentally tested theory.

A summary of the theory and methods of analysis is given before each design. For more detailed information the books mentioned in the Bibliography should be consulted.

The author wishes to express his thanks and appreciation to Prof. J.R. Osborn, Chairman of the Civil Engineering Department, under whose supervision and guidance he was able to carry this work; he acknowledges his indebtedness to Prof. Osborn for his helpful suggestions. The author also thanks the Civil Engineering Department where he found encouragement and all the necessary conveniences to complete this thesis.

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INTRODUCTION

Reinforced concrete highway bridges may be classed in several distinct groups: (1) slab bridges; (2) T-Beam girder bridges; (3) Through girder bridges; (4) Arch bridges; (5) Other small structures such as culverts, trestles, etc.

In Lebanon and Syria it has been found, from the standpoint of economy, that for spans of 25 ft. to 60 ft., where headroom is not limited, the T-Beam bridge is the best; and for spans greater than 60 ft., where solid foundations are available, the openings are high and the structures carry very heavy traffic, the re - inforced concrete arch bridge is best adapted.

I therefore propose to design two bridges: a T-Beam bridge with a span of 40 ft., and an open spandrel arch bridge with a span of 120 ft.

ANALYSIS OF THE FORMULAS USED IN THE DESIGN

The stresses in reinforced concrete structures are to be calculated upon the basis of the following assumptions:

1.- Calculations are to be made with reference to working stresses and safe loads.

2.- A plane section before bending remains plane after bending.

3.- The modulus of elasticity of concrete in compression is constant and the distribution of stresses in beams is rectilinear.

4.- In calculating the moment of resistance in beams the tensile stresses in the concrete are neglected.

5.- Adhesion between concrete and reinforcing steel is assumed as perfect, and concrete and steel are therefore stressed in proportion to their moduli of elasticity.

6.- The ratio of the modulus of elasticity of steel to the modulus of elasticity of concrete is taken $n = 15$.

7.- Initial stresses in reinforcement due to the contraction of the concrete is neglected in the design of the T-beam bridge but special consideration is given to rib shortening when the arch bridge is designed.

Standard Notation.

1.- Rectangular beams:

f_s = tensile unit stress in steel

f_c = compressive unit stress in concrete

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

$N = E_s/E_c$

M_s = Moment of resistance relative to steel

M_c = Moment of resistance relative to concrete

M = Moment of resistance or bending moment in general.

A = steel area

b = breadth

d = depth

k = ratio of depth of neutral axis to depth d

j = ratio of lever arm to resisting couple to d

jd = arm of resisting couple

$p = A/bd$ = steel ratio (not in percentage)

R = coefficient of strength

2.- T-beams:

b = width of flange

b' = width of stem or web

t = thickness of flange

s = t/d = thickness of slab divided by depth of beam.

3.- Beams reinforced for compression:

M_1 = Moment which concrete can carry

M_2 = extra moment to be carried by steel

A_{s1} = area of tensile steel required to develop moment M_1

A_{s2} = area of additional tensile steel necessary to develop moment M_2

A'_s = total area of compressive steel

4.- Shear and bond:

V = total shear

v = maximum shearing unit stress

u = bond stress per unit area of bar

o = sum of perimeters of bars.

s' = spacing of bars

5.- Columns:

P = the strength of reinforced concrete column for stress f_c

P' = Reduced strength of column.

6.- Arch rib: See page

Formulae used :

The complete derivation of these formulae are found in any concrete text book. I shall only give the results of those derivations which I shall need in my design.

1.- Rectangular beams:

$$M_s = Tjd = f_sAjd = f_spjbd^2 = R_sbd^2 \qquad (1)$$

$$M_c = Cjd = \tfrac{1}{2}f_cbkd.jd = \tfrac{1}{2}f_ckjbd^2 = R_cbd^2 \qquad (2)$$

$$d = \sqrt{M/Rb} \qquad (3)$$

The value of R for the given f_s and f_c is found from Table XXII in Masonry Structures by Spalding.

$$f_s = \frac{M}{Ajd} \; ; \qquad f_c = \frac{2 f_s p}{k} \qquad (4)$$

$$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} \qquad (5)$$

If a value of "p" less than that given by (5) is used the steel determines the strength of the beam, while if "p" is greater, then the concrete will determine the strength of the beam.

2.- T-Beams:

There are two cases: (1) when the neutral axis is in the flange; (2) when the neutral axis is in the web. Only Case II will be needed in my design.

Case II:

$$b'd = \frac{V}{jv} = \text{section determined according to shear.} \qquad (6)$$

$$A_s = \frac{M}{f_s jd} = \text{area of steel required} \qquad (7)$$

$$k = \frac{pn + \frac{1}{2}(t/d)^2}{pn + t/d} \qquad (8)$$

$$f_c = \frac{f_s k}{n(1 - k)} \qquad (9)$$

3.- Beams reinforced for compression:

$$M_2 = M + M_1 \qquad (10)$$

$$A_{s1} = \frac{M_1}{f_s jd} \qquad (11)$$

A_{s2}

$$A_{s2} = \frac{M_s}{f_s jd} \quad (12)$$

$$A'_s = A_{s2} \times \frac{1 - k}{k - (d'/d)} \quad (13)$$

4.- Shear and bond:

$$V' = V - V_c \quad (14)$$

$$v = \frac{V'}{bjd}$$ unit shear to be carried by stirrups. (15)

$$s' = \frac{P}{v'b}$$ = spacing of stirrups (16)

$P = A_v \times f_s$ = capacity of one striupp to carry shear. (17)

$$\sum o = \frac{V}{ujd}$$ = required perimeter of bars (18)

5.- Columns:

$$f_c = 300 + (0.10 + 4p)f'_c$$ where f'_c = 2000 psi. (19)

$$P = f_c((A + (n - 1)A'_s)) \quad (20)$$

$$P'/P = 1.33 - \frac{h}{120R}$$ (reduction formula) (21)

6.- Arch rib:

The formulae used for the design of the arch rib are given on a separate sheet on page

R E I N F O R C E D   C O N C R E T E

T- B E A M   B R I D G E

The bridge will consist of a reinforced concrete slab supported on several reinforced concrete beams running between abutments.

Clear span = 40 ft.

Roadway = 25 ft.

Sidewalks = 2 x 5' = 10 ft.

Total width = 35 ft.

The sidewalks will be cast as slabs projecting outside like balconies.

<u>Outline of design</u>: In all problems of design in engineering the general method is to start from the top of the structure and proceed until the bottom is reached. The following will be the steps of procedure in this design:-

1.- Design of slab.

2.- Design of Intermediate beams.

3.- Design of sidewalks.

4.- Design of outside rectangular beams.

5.- Design of abutments.

To the design will follow drawings and estimates of quantities with their approximate prices.

DESIGN OF SLAB

The slab will be reinforced on top and bottom and will be assumed as continuous. Its span will be taken 5 ft. from beam to beam. The rear wheels of the auto truck will determine the section.

The effective width of slab for moment is:

$$e = 2/3(l+c) = 2/3(5 + 1) = 4 \text{ ft.}$$

where l = span in feet = 5 ft.;

c = width of tyre = 0.3 m. = 1 ft.

The coefficient of impact is given by:

$$I = 1 + \frac{0.4}{1 + 0.2\,l} + \frac{0.6}{1 + P/S}$$

where P/S is the ratio of dead load of bridge to maximum L.L. it can carry.

Assuming $P/S = \frac{160}{56}$; and knowing $L = \frac{40}{3.28} = 12.2$ m.

we find I = 1.17 say 1.20 or 20% impact.

The weight acting at the center is $\frac{6 \times 2240}{4.00} = 3360$ lbs.

Assume d = 6" + (1 1/2" covering)

wt. of concrete = $\frac{12 \times 7.5}{144} \times 150 = 93.5$ lbs./ft. say 95 lbs./ft.

wt. of earth = $\frac{8 \times 120}{12} = 80$ lbs./ft.

Total w = 80 + 95 = 175 lbs./ft.

For uniform load $M = 1/10\, wl^2$

For concentrated load take a condition midway between a fixed and a free condition. Take 3/4 of free condition; hence M = 3/16 PL.

$$M \text{ (total)} = \frac{wl^2}{10} + \frac{3\,PL}{16} + \frac{0.20 \times 3\,PL}{16}$$

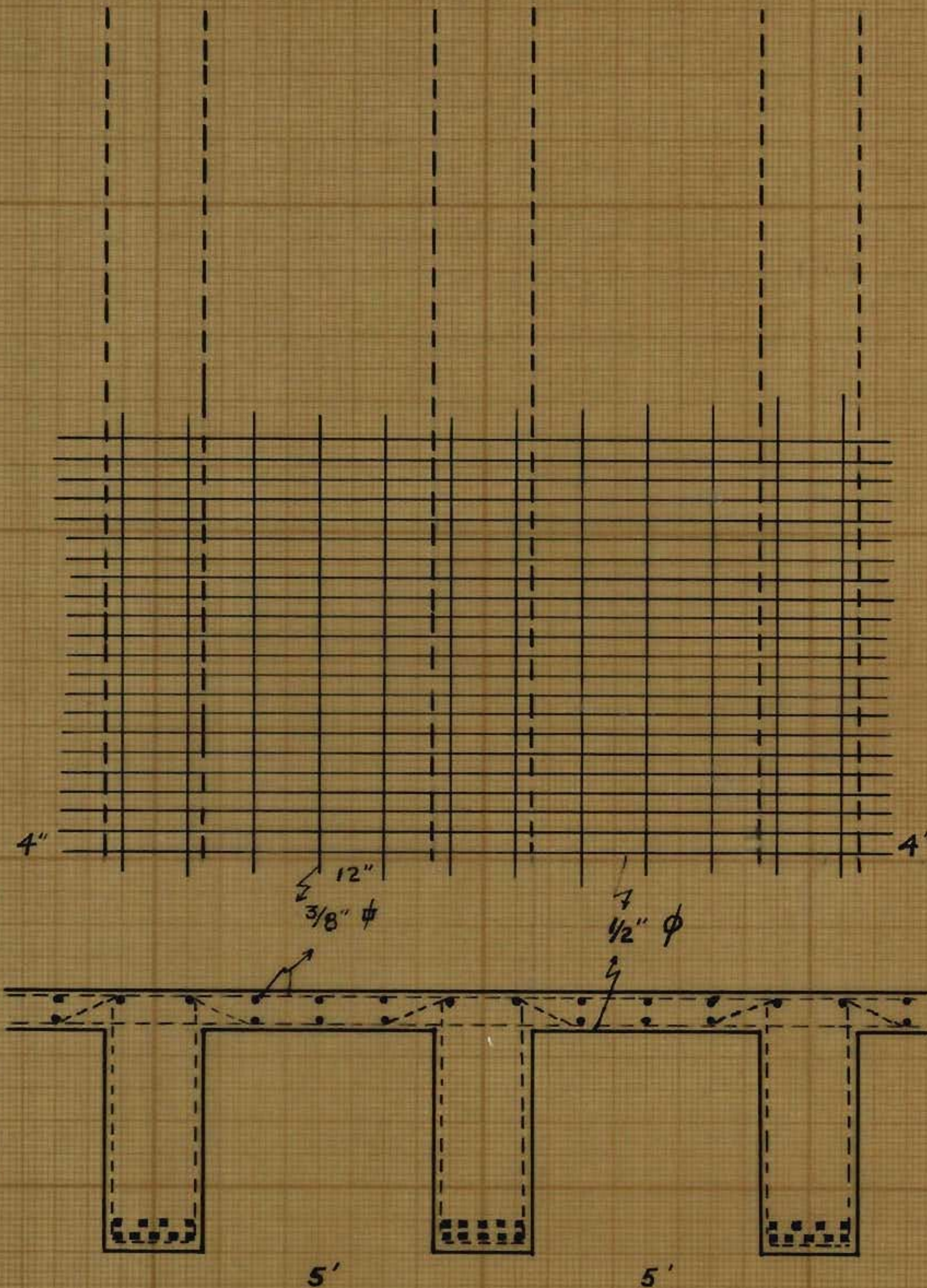


Fig. 1

$$M\ (total) = \frac{175 \times 5^2 \times 12}{10} + \frac{3 \times 3360 \times 5 \times I2}{16} + \frac{.20\ (L.L)}{1}$$

$$= 5250 + 37800 + 7750 += 50800 \text{ in-lbs.}$$

$M = Rbd^2$; f_s = I8000 p.s.i. ; f_c = 700 p.s.i. ; n = 15 ; R = II2

$$d^2 = \frac{M}{R\ b} = \frac{50800}{II2 \times I2} = 37.8$$

$$d = 6.I5" + (I.35" \text{ cov.}) = 7.5" \text{ total}$$

from tables p = 0.0072

$$A_s = pbd = 0.0072 \times I2 \times 6.I5 = 0.532 \text{ sq. in.}$$

THis is the area of steel per foot width required to develop this slab. Use 3 - 1/2 " round bars (= 0.59 sq. in.) 4" c. to c. Bars 3/8 " sq. spaced about I2" c. to C. will be placed near the tpp and bottom of the slab and running parallel to the beams. This reinforcement is to assist on distributing the loads and to provide for temperature changes.

The effective width for shear is taken equal to that for moment with a minimum of 3 ft. and a maximum of 6 ft. Taking this into consideration it is found that the slab thickness with its reinforcement is more than enough to take care of the shear.

DESIGN OF INTERMEDIATE BEAMS

The bending moment in one beam due to live load of:

$$p = (820 - 4 \times L) = (820 - 4 \times 128) = 769 \text{ Kg./m}^2$$

PLus 20% impact is:

$$M_L = \frac{wl^2}{8} = \frac{769 \times .0929 \times 2.2 \times 1.2 \times 5 \times \overline{42}^2}{8} = 208000 \text{ ft.lb}$$

The end shear due to the same uniform live load plus 30% imp.

is:
$$V_L = \frac{wl}{2} = \frac{769 \times 0.0929 \times 2.2 \times 5 \times 1.2 \times 42}{2} = 16300 \text{ lbs.}$$

The proportion of the front and rear wheels of the auto truck carried by one joist is: $\frac{(5)}{(6)} = 0.83$. To have maximum moment for concentrated live load we have the arrangement shown in the figure.

The auto truck gives larger values for both shear and moment than those given by the uniform load; therefore these values will be used.

The dead load shear and moment cannot be determined untill the weight of the beam is known. The stem of the beams will be assumed to be 18" wide and 40" deep and will weigh:

$$\frac{18 \times 40 \times 150}{144} = 750 \text{ lbs./ ft.}$$

Thus the dead load will consist of:

wearing surface = 5 x 80 = 400 lbs./ ft.

slab = 5 x 95 = 485

beam = 750

total 1625 lbs./ ft.

The dead load bending moment is:

$$M_D = \frac{wl^2}{8} = \frac{1625 \times \overline{42}^2}{8} = 358000 \text{ ft. lbs.}$$

The dead load shear is:

$$V_D = \frac{wl}{2} = \frac{1625 \times 42}{} = 34100 \text{ lbs.}$$

The total bending moment is:

$$M = 212000 + 358000 = 570000 \text{ ft. lbs.}$$

The total shear is:

$$V = 21700 + 34100 = 55800 \text{ lbs.}$$

The slab acts as the flange of a T-beam. The width on each side which may be considered as effective is: 4 x 7.5 = 30, making a total of 2 x 28 ± 18 = 74" assuming the stem to be 18" wide. This value is greater than the distance center to center of the beams; so the distance center to center of beams (= 60") will be used, provided it is not greater than 1/4 of the span or 40/4 = 10 ft. = 120".

The minimum section if fully reinforced for shear is:

$$b'd = \frac{V}{j.v} = \frac{55800}{.90 \times 120} = 518 \text{ sq. in.}$$

assuming j = .90; using b' = 18", d would equal 28.8". The depth should not be much less than 1/12 of the span = 40". Therefore take d = 38".

$$s = t/d = \frac{7}{38} = .184$$

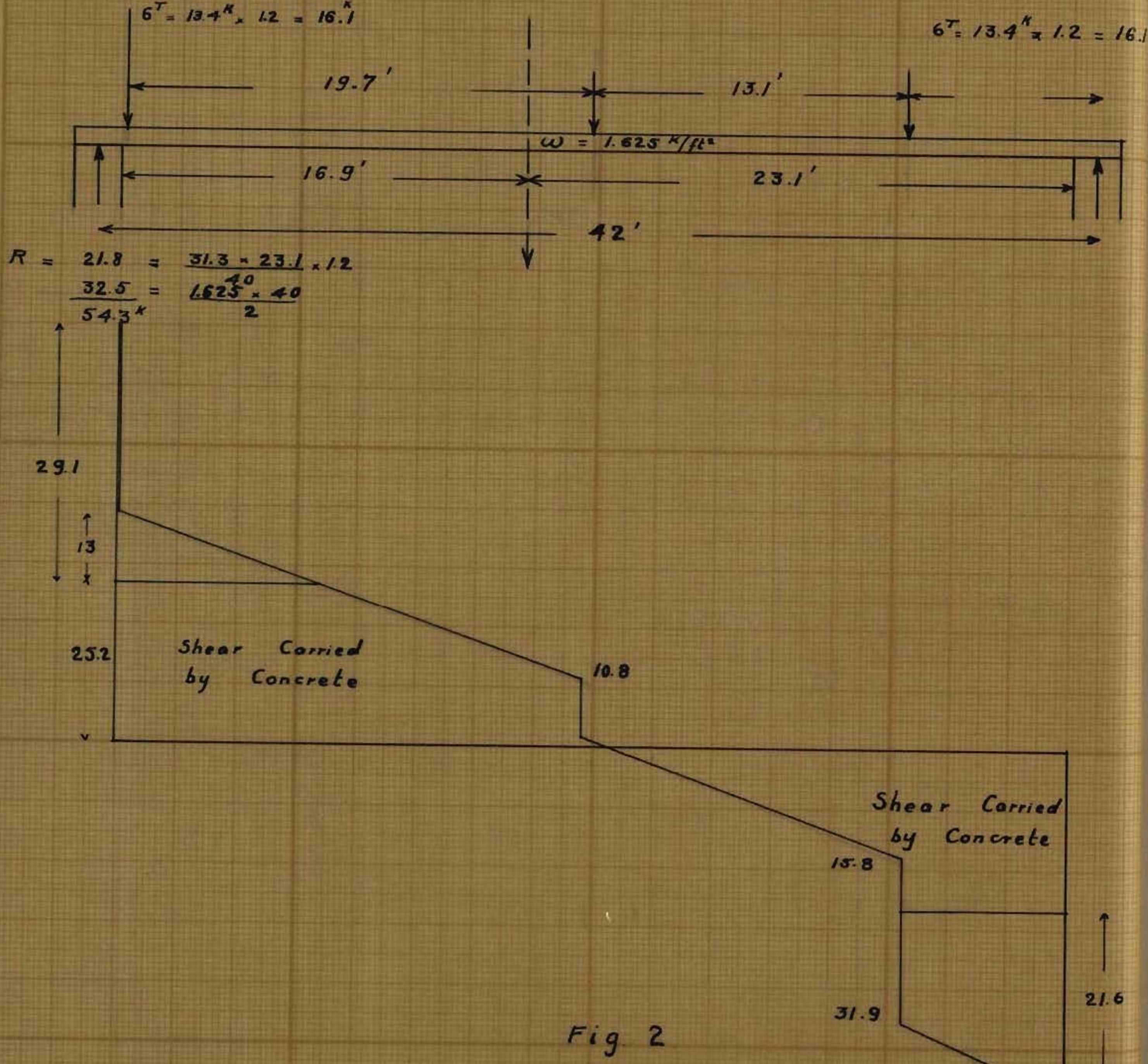
$$pn = .0072 \times 15 = .108$$

from the table : j = .90 (we assumed j = .90)

The area of steel required is:

$$A = \frac{M}{f_s.j.d} = \frac{570000 \times 12}{18000 \times .92 \times 38} = 10.9 \text{ sq. in.}$$

Use 9 - 1 1/8 " sq. bars = 11.39 sq. in.



6T = 13.4K x 1.2 = 16.1K
6T = 13.4K x 1.2 = 16.1
19.7'
13.1'
ω = 1.625 K/ft²
16.9'
23.1'
42'
R = 21.8 = 31.3 x 23.1 / 40 x 1.2
32.5 = 1.625 x 40 / 2
54.3K
29.1
13
25.2
Shear Carried by Concrete
10.8
Shear Carried by Concrete
15.8
31.9
21.6
46.8

Fig. 2

Now let us revise and see if f_c does not exceed 700 p.s.i. by finding p and k and substituting in

$$f_c = \frac{f_s k}{n(1 - k)}$$

$$p = \frac{A}{bd} = \frac{11.39}{60 \times 38} = 0.005$$

$$k = \frac{pn + \frac{1}{2}(t/d)^2}{pn + t/d} = \frac{0.005 \times 15 + 0.0168}{0.005 \times 15 + 0.184} = 0.355$$

$$f_c = \frac{18000 \times 0.355}{15(1 - 0.355)} = 662 \text{ p.s.i.}$$

The assumed section will be adopted. No revision in weight is necessary.

<u>Stirrups:</u> The shear diagram is as shown in figure 2.

The shear carried by the concrete alone is:

$$V_c = vbjd = 40 \times 18 \times 0.92 \times 38 = 25.2$$

This leaves at the left support a shear of 29.1 kips to be carried by the stirrups. A double prong 3/8" round stirrup can take:

$$P = 2 \times 0.11 \times 16000 = 3520 \text{ lbs.}$$

Unit shear:

$$v' = \frac{V}{bjd} = \frac{29100}{18 \times 0.92 \times 38} = 46.2 \text{ p.s.i.}$$

Change of shear per inch run at the extreme left:

$$bv' = 46.2 \times 18 = 830$$

spacing:

$$\frac{P}{v'b} = \frac{3520}{830} = 4.24 \text{ "}.$$

Maximum spacing = 0.45 x 38 = 17"

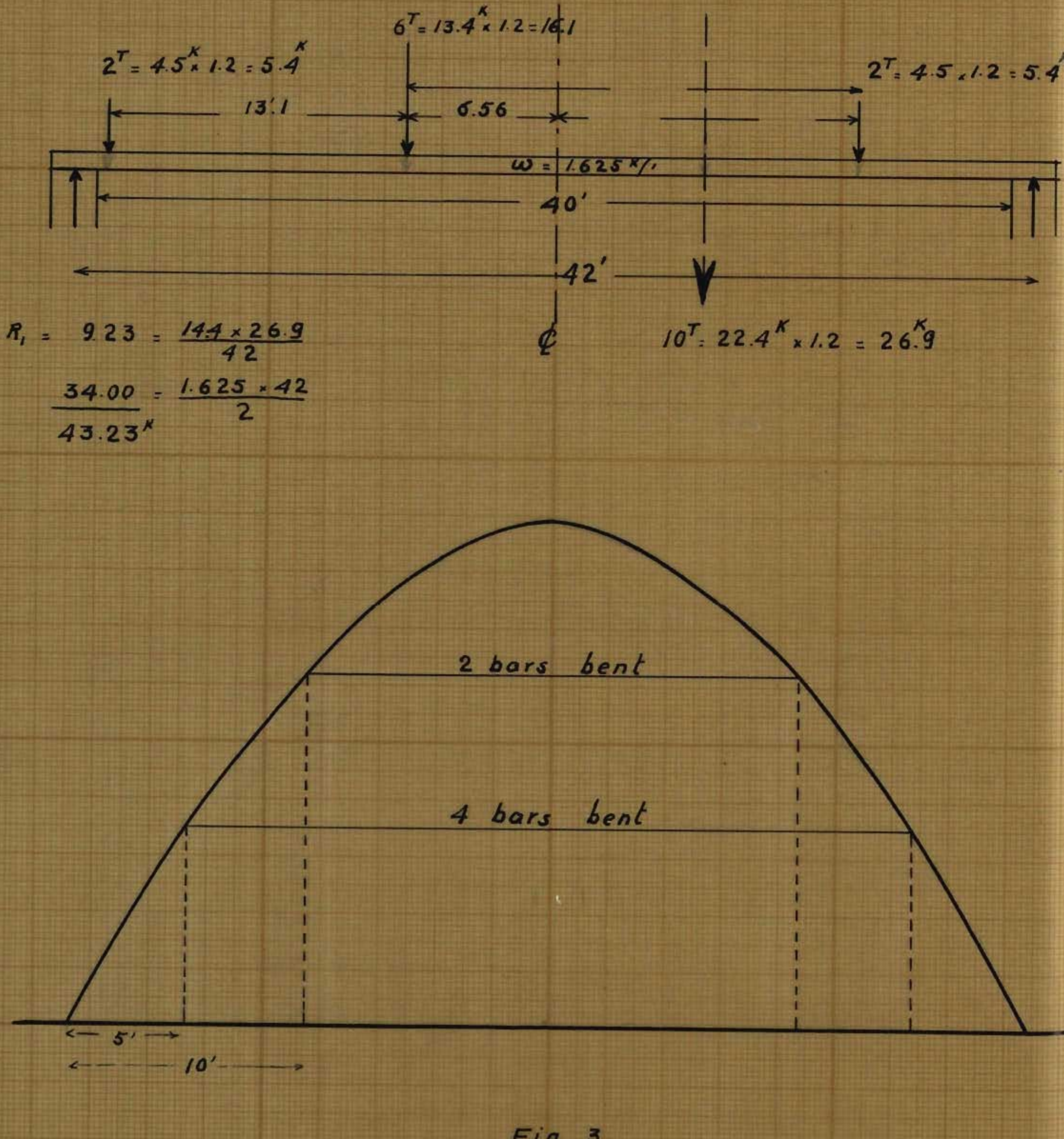

2T = 4.5K × 1.2 = 5.4K
6T = 13.4K × 1.2 = 16.1
2T = 4.5 × 1.2 = 5.4K
13.'1
6.56
ω = 1.625 K/'
40'
42'
℄
10T = 22.4K × 1.2 = 26.9K
R1 = 9.23 = 14.4 × 26.9 / 42
34.00 = 1.625 × 42 / 2
43.23K
2 bars bent
4 bars bent
5'
10'

Fig. 3

At the two extreme ends use a spacing of 4 ". Increase this gradually to 15 ".

<u>Bending of bars:</u> Maximum spacing for bending of bars at 45 degrees is:

$$s = \frac{45}{10 + \theta} d = \frac{45}{55} \times 38 = 31''$$

The following are a few points plotted on the Moment diagram:-

x	M
1.34	56.40 'k
2.00	79.65
4.00	145.52
6.00	204.88
8.00	257.84
10.00	304.40
12.00	347.50
14.44	384.00

Each bar carries a moment of:

$$M_s = A_s f_s jd = 1.27 \times 18 \times 0.92 \times 38$$

$$= \frac{800 \text{ "k}}{12} = 66.5 \text{ 'k}$$

2 bars carry 2 x 66.5 = 133.0 'k

4 bars carry 4 x 66.5 = 266.0 'k

<u>Bond:</u>

$$\Sigma_o = \frac{V}{ujd}$$

V at the end = 54300 lbs.

$$= \frac{54300}{80 \times .92 \times 38} = 19.5''$$

5 bars have a perimeter of 22.50. Bend 4 bars.

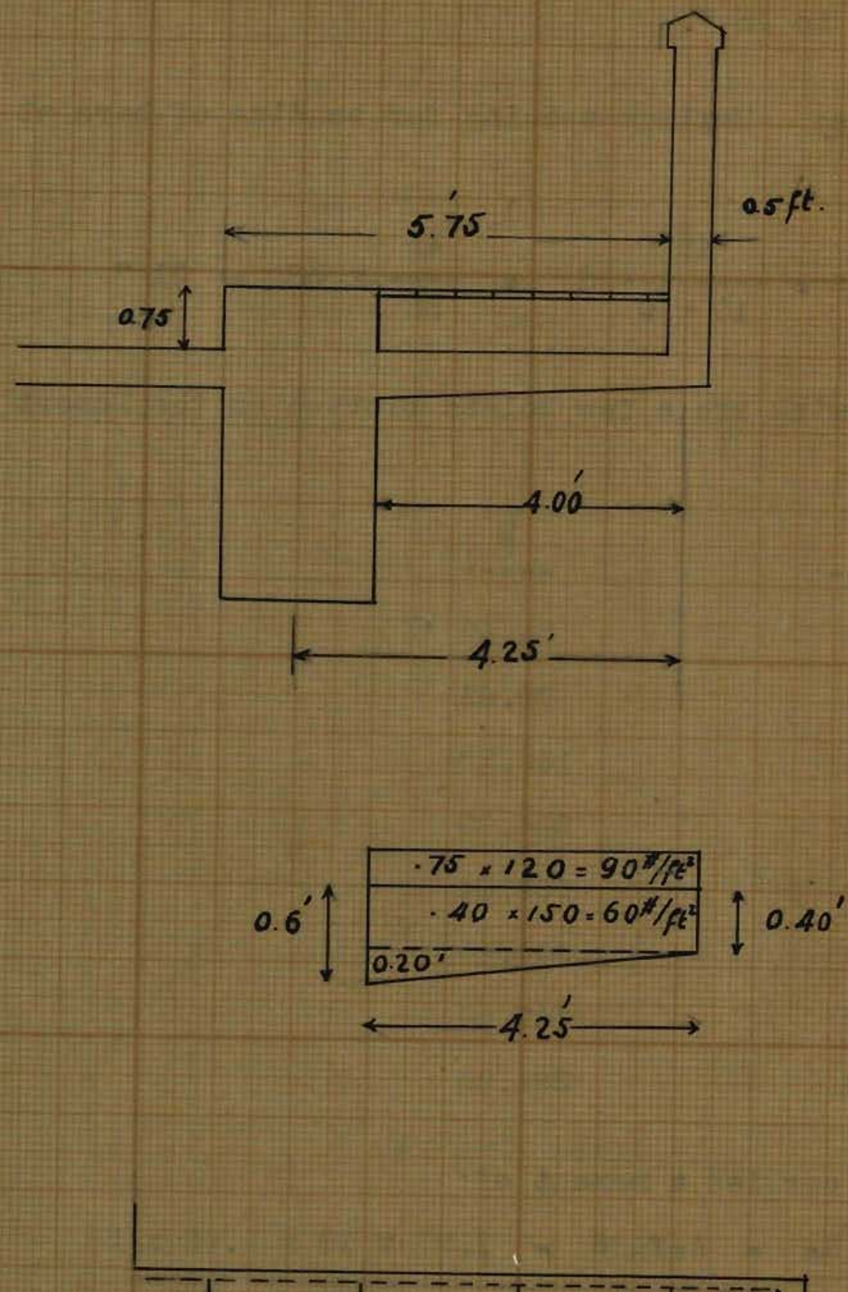

5.75'
0.5 ft.
0.75
4.00'
4.25'
·75 × 120 = 90#/ft²
·40 × 150 = 60#/ft²
0.6'
0.40'
0.20'
4.25'
1'
3/8 φ
1/2" φ

Fig. 4

DESIGN OF SIDEWALKS

The French system of loading specifies 400 kg/m^2 as the live load coming on sidewalks.

w_L = 400 kg/m^2 = 400 x 2.24 x 0.929 = 83 lbs/ft^2.

M_L = ½ x 83 x $(3.75)^2$ = 584

railing = 0.5 x 3.3 x 150 = say 300 lbs./ft.

M_{rail} = 300 x 4.00 = 1200

$$M_D = \frac{150 \times \overline{4.25}^2}{2} + \frac{0.20 \times 4.25}{2} \times 150 \times \frac{4.25}{3}$$

= 1350 + 90

= 1440 ft.lbs.

Total Moment = 584 1200 1440 = 3224 ft.lbs.

$$d = \sqrt{\frac{M}{R\ b}} = \frac{3224 \times 12}{112 \times 12} = 5.45\ "$$

5.45 + 1.75 (covering) =) 7.2 "

Assumed section was 0.6' x 12 = 7.2 " (No revision)

Reinforcement:

$$A_s = \frac{M}{f_s\ j\ d} = \frac{3224 \times 12}{18000 \times .88 \times 5.5} = 0.445 \text{ in.}^2$$

Use 2 - ½" square bars = 0.50 $in.^2$

Stirrups:

V_L = ½wl = ½ x 83 x 4.25 = 176

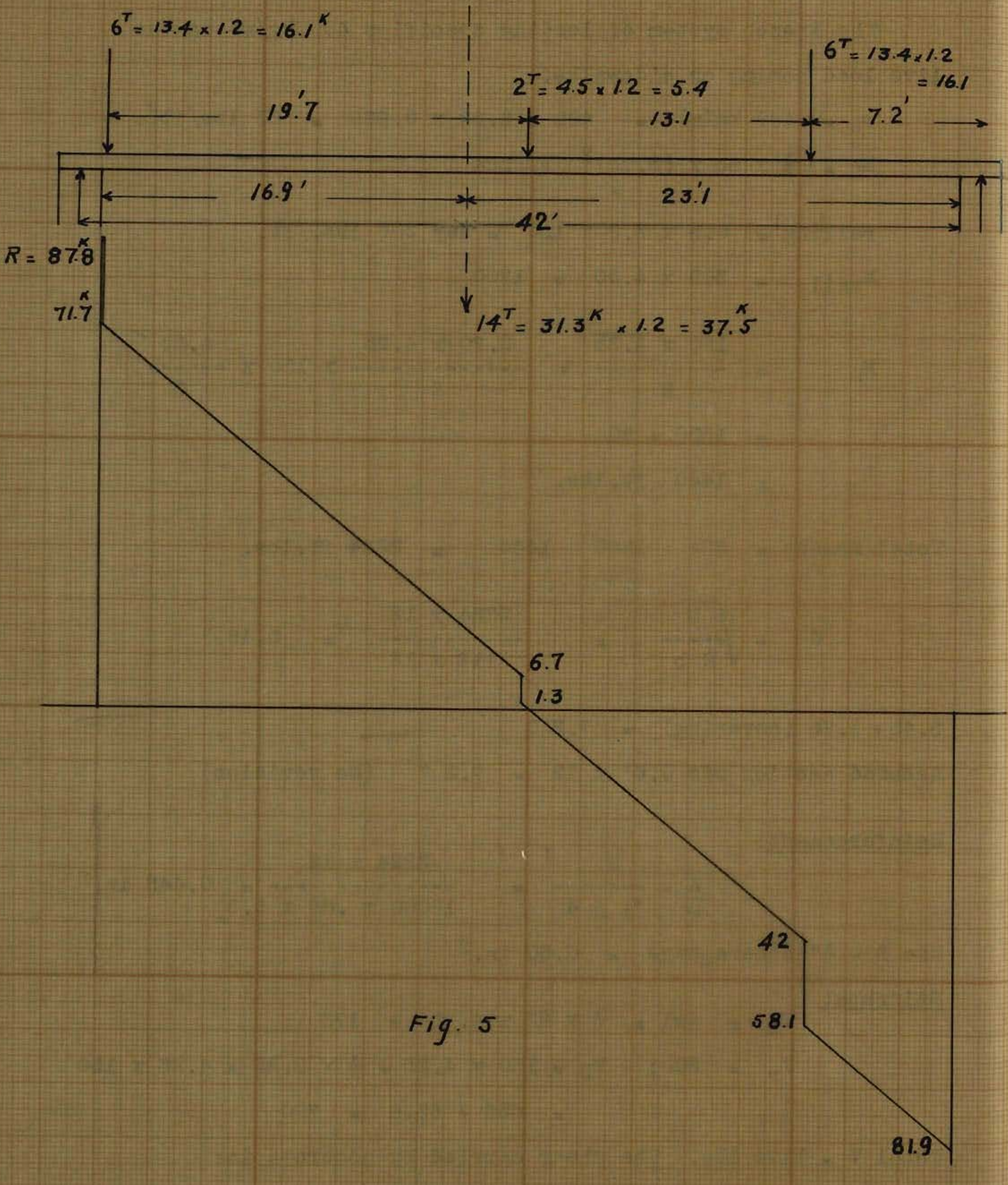
V_r = 300; V_D = 150 x 4.25 + ½ x 0.20 x 4.25 x 150

= 636 + 63.8 = 700

Total V = 1176 lbs. The shear carried by concrete is:

V_c = $f_v$bjd = 40 x 12 x 0.88 x 5.5 = 2320 lbs.

Theoretically no stirrups are necessary; however use stirrups with 1 ft. spacing (See diagram)



$6^T = 13.4 \times 1.2 = 16.1^K$
$6^T = 13.4 \times 1.2 = 16.1$
$2^T = 4.5 \times 1.2 = 5.4$
19.7'
13.1
7.2'
16.9'
23.1'
42'
$R = 87.8^K$
71.7^K
$14^T = 31.3^K \times 1.2 = 37.5^K$
6.7
1.3
42
58.1
81.9

Fig. 5

DESIGN OF OUTSIDE BEAMS

Specifications require that the live load used on intermediate beams be also used on outside beams.

Therefore M_L = 212000 ft.lbs.

V_L = 21700 lbs.

The dead load carried by the outside beam is:

wearing surface and filling =	90 x 4.25	= 382
slab = $\frac{1}{2}$(0.40 + 0.60) x 4.25 x 150		= 318
Beam (assumed)	60/12 x 150	= 1500
railing		= 300
		2500
half of roadway slab =	$\frac{1}{2}$ x 1625	= 812
		3312

Maximum D.L. bending moment:

$$M_D = \frac{1}{8} \times 3312 \times \overline{42}^2 = 735000 \text{ ft.lbs.}$$

Maximum D.L. shear:

$$V_D = \frac{1}{2} \times 3312 \times \overline{42}^2 = 69500 \text{ lbs.}$$

Total bending Moment:

$$M = M_L + M_D = 212000 + 734000 = 947000 \text{ ft.lbs.}$$

Total shear:

$$V = V_L + V_D = 21700 + 69500 = 91200 \text{ lbs.}$$

The width of the beam will be taken as 18 ", the same as the width of stem of the intermediate beams.

The minimum depth as determined by the bending moment is:

$$d = \sqrt{\frac{M}{R\ b}} \qquad \frac{947000 \times 12}{112 \times 18} = 66.5 \text{ "}$$

We want to limit the depth to 50 ". Therefore we have to reinforce for compression.

The moment which the concrete can carry is:

$$M_1 = 113 \times 24 \times \overline{47}^2 = 500{,}000 \text{ ft.lbs.}$$

$$M_2 = 947000 - 500000 = 447{,}000 \text{ ft. lbs.}$$

$$A_{s1} = \frac{500000 \times 12}{18000 \times 0.88 \times 47} = 8.1 \ \overline{\text{in}}^2$$

$$A_{s2} = \frac{447000 \times 12}{18000\ (47 - 2.5)} = 6.7 \ \overline{\text{in}}^2$$

$$\overline{14.8 \ \overline{\text{in}}^2}$$

A_s = 14.8 sq. in. Use 15 - 1" sq.bars (= 15 sq.in.)

$$A'_s = 6.7 \times \frac{1 - 0.369}{0.369 - \frac{2.5}{47}} = 6.7 \times \frac{0.631}{0.316} = 13.4 \ \overline{\text{in}}^2$$

(See reinforcement for compression)

in cross-section of T-Beam bridge)

Stirrups:

The shear diagram is shown in figure

Shear carried by concrete = V_c = vbjd = 40 x 24 x 0.88 x 60

= 50.7 lbs.

This leaves at the left support a shear of 71.7 - 50.7 = 21.0^k which is carried by stirrups.

A six-prong stirrup of 3/8 " dia. can take:

P = 6 x 0.11 x 16000 = 10560 lbs.

$$\text{Unit shear} = v' = \frac{V}{b\ j\ d} = \frac{50700}{24 \times 0.88 \times 60} = 40 \text{ p.s.i.}$$

Change of shear per inch run at the extreme left:

bv' = 40 x 24 = 960

Spacing = p/v'b = 10560/960 = 11 ".

Maximum allowable spacing = 0.45 x 60 = 27.0 "

(See diagram for spacings).

Bending of bars:

Maximum spacing for bending of bars at 45 degrees is

$$s = \frac{45}{10 + 9}\, d = \frac{45}{55}\, d = \frac{45}{55} \times 60 = 49 \text{ ".}$$

The following are a few points plotted on the Moment diagram.

x	1.34	4.00	6.00	8.00	10.00
M	106	277.2	372.4	494	573

(See diagram for bending of bars)

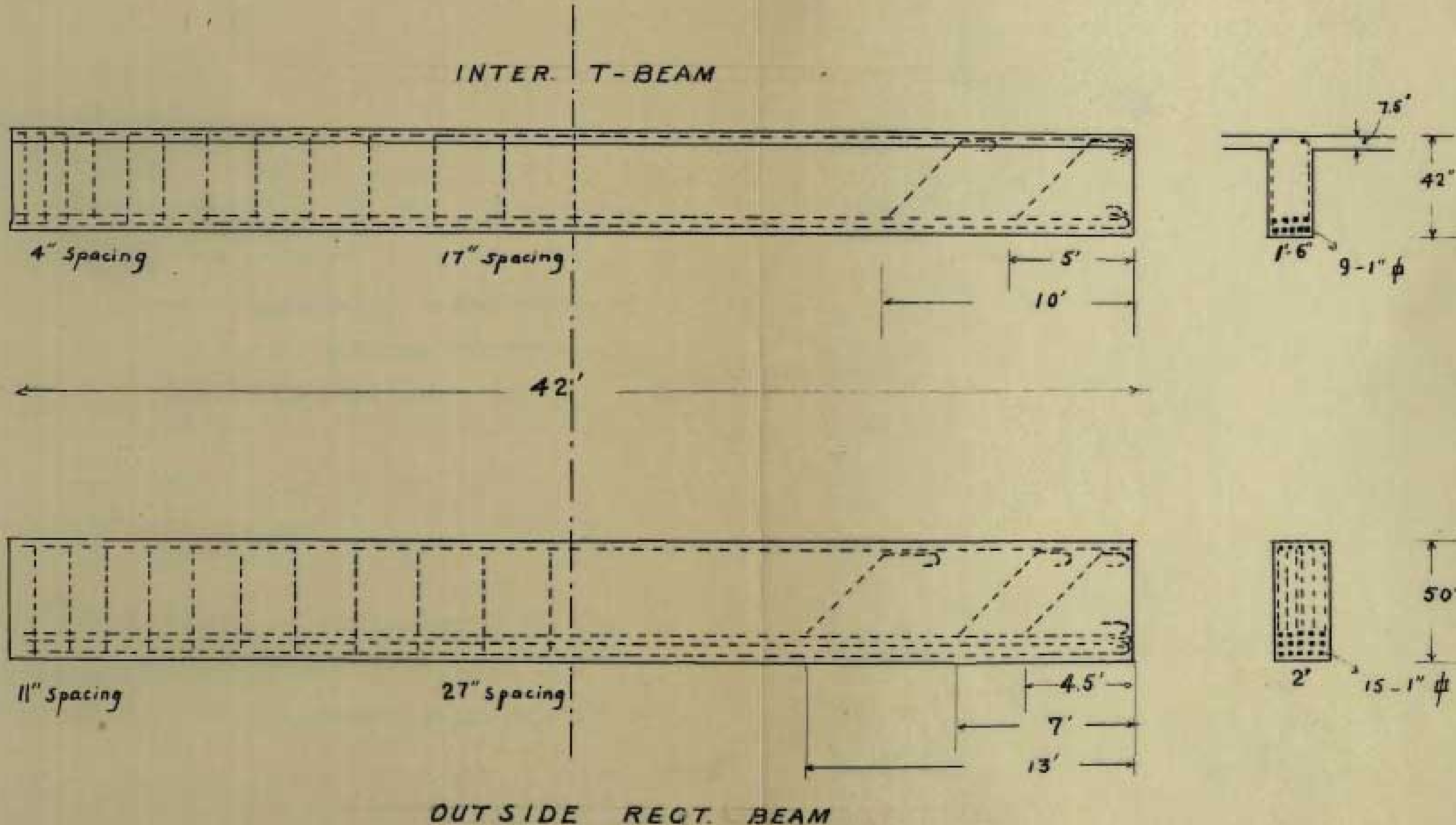
Bond:

$$\Sigma_o = \frac{V}{ujd} \qquad V = 87800 \text{ lbs.}$$

$$= 87800 \div (80 \times 0.88 \times 60) = 20.8 \text{ "}$$

5 bars give a perimeter of 22.50. Bend half the steel.

Fig. 7


INTER. T-BEAM
4" Spacing
17" Spacing
5'
10'
7.5"
42"
1'-6"
9-1" ϕ
42'
11" Spacing
27" Spacing
4.5'
7'
13'
50"
2'
15-1" ϕ
OUTSIDE REGT. BEAM

DESIGN OF ABUTMENTS

Reaction from interior beams = 3 x 55800 = 167400 lbs.

Reaction from outside beams = 2 x 91200 = 182400

Total reaction on abutment = 349800 lbs.

width of abutment is 30 ft.

$$\text{weight per foot} = \frac{349800}{30} = 11650 \text{ lbs.}$$

Traction:

20/100 x L.L. reaction.

L.L. reaction = 3 x 21700 = 65100

2 x 21700 = 43400

108500 ÷ 30' = 3617 lbs/ft.

Traction = 0.2 x 3617 = 783.4 lbs.

Since these reactions are concentrated, larger values of loading will be taken for the design, and more reinforcement will be placed under the beams so that the weight may be distributed evenly.

Take reaction per foot = 11650 #

traction per foot = 900 "

In calculating the surcharge we shall consider the L.L. which can come on the earth fill behind the abutment to a distance equal to the height of the abutment which we shall take as 21 ft.

In 21 ft. we can have 2 trucks side by side = 2 x 16^t

= 32^t

$$P = \frac{32 \times 2240}{21 \times 40} = 85.5 \text{ or } 0.85 \text{ ft. say 1 ft.}$$

of earth.

Pressure on stem = $w/2\,(h_2^2 - h_1^2)$ = $100/2\,(20^2 - 1^2) \times 0.22$

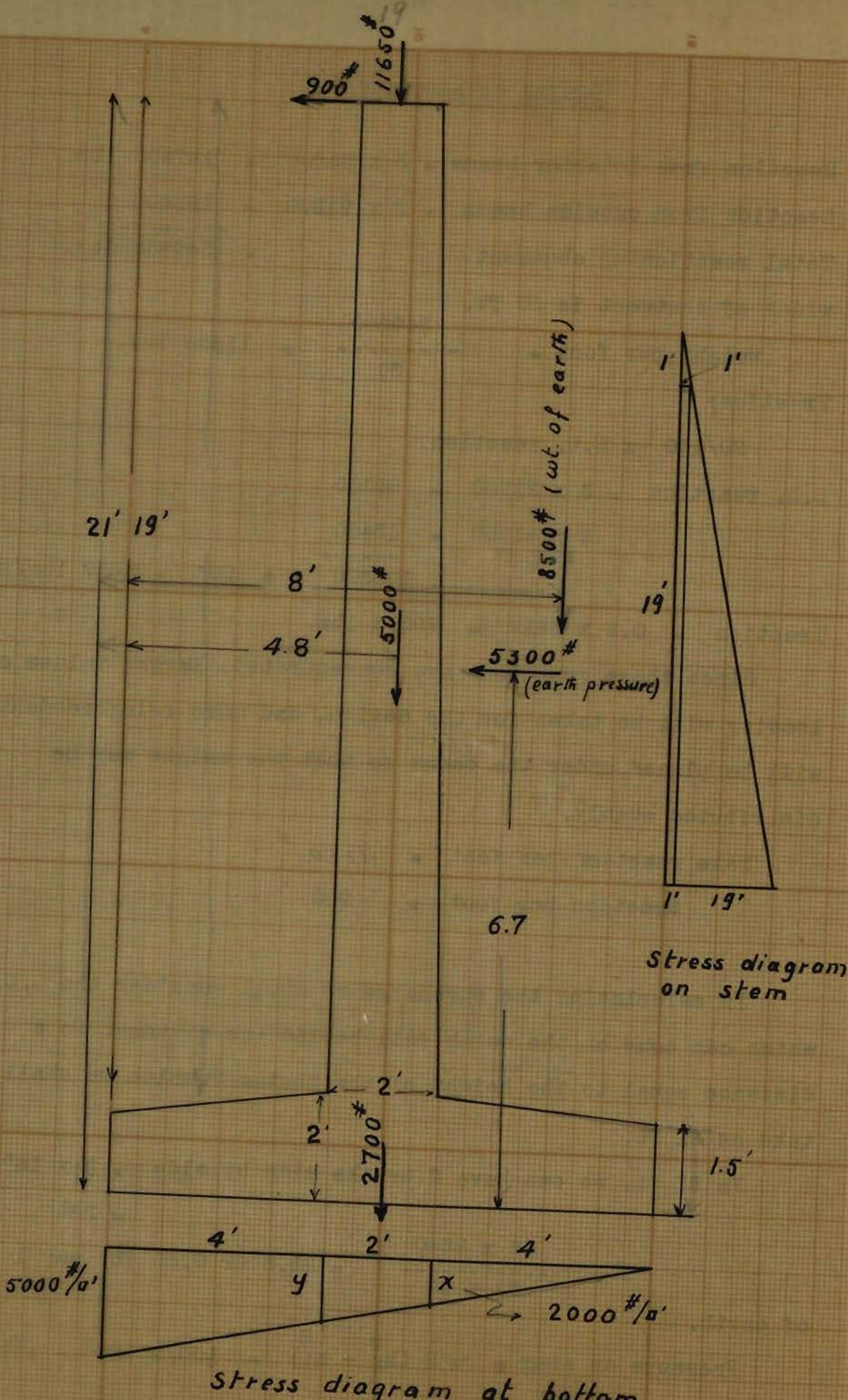

900#
11650#
21'
19'
8'
8500# (wt. of earth)
5000#
4.8'
5300#
(earth pressure)
6.7
2'
2'
2700#
1.5'
4'
2'
4'
5000#/□'
y
x
2000#/□'
Stress diagram at bottom
1'
1'
19'
1'
19'
Stress diagram on stem

Fig. 8

$$\text{Pressure on stem} = \frac{100}{2} \times 0.22\ (20^2 - 1^2)$$

$$= \frac{100}{2} \times 0.22 \times 399 = 4400 \text{ lbs.}$$

$$\bar{y} = \frac{1 \times 19 \times 9.5 + \frac{1}{2} \times \overline{19}^2 \times 19/3}{1 \times 19 + \frac{19 \times 19}{2}} = \frac{9.5 + 60.2}{1 + 9.5}$$

$$= \frac{69.7}{10.5} = 6.6'$$

$$M_1 = 900 \times 19 + 4400 \times 6.6 = 17100 + 29000 = 46100 \text{ ft.lbs.}$$

$$d = \sqrt{\frac{M}{R\,b}} = \sqrt{\frac{46100}{113 \times 12}} = \sqrt{410} = 20.4'' \text{ say } 2' \text{ with covering.}$$

When the traction is th the right M_2 equals 29000 - 17100 = 11900 ft.lbs.

$$A_{s1} = \frac{M_1}{f_s jd} = \frac{46100 \times 12}{18000 \times 0.88 \times 21} = 1.67\ \overline{\text{in}}^2 \text{ per foot.}$$

Use 2 - 1 " sq.bars per ft. ($= 2.00\ \overline{\text{in}}^2$)

$$A_{s2} = M_2 / f_s jd = \frac{11900 \times 12}{18000 \times 0.88 \times 21} = 0.428\ \overline{\text{in}}^2 \text{ per ft.}$$

Use 2 - ½" sq. bars per ft. = 0.5 sq. in.

<u>Check for shear</u>:

$$V = 900 + 4400 = 5300 \text{ lbs.}$$

$$v = \frac{V}{bjd} = \frac{5300}{12 \times 0.88 \times 21} = 24 \text{ p.s.i. (safe)}$$

$$o = \frac{V}{ujd} = \frac{5300}{0.88x80x21} = 3.6"$$ (8.0" are provided)

__wt. of earth:__

w. = 4.25 x 20 x 100 = 8500 lbs.

stem = (1.5 x 19 + ½ x ½ x 19)150 = 5000 lbs.

arm = 4.8'

base = $(1.5 \times 10 + \frac{10+2}{2} \times 0.5) \times 150$ = 2700 lbs.

arm = 5.0'

P = $100/2 \times 0.22(\overline{22}^2 - 1^2)$ = 5300 lbs.

arm = 6.67'

M = 8500 x 8 = 68000
5000 x 4.8= 24000
2700 x 5 = 13500
11650 x4.75= 55400

160900 ft.lbs.

M = 900 x 21 + 5300 x 6.7 = 54500 ft.lbs.

F.S. for overturning = $\frac{160900}{54500}$ = 2.95

F.S. for sliding = $\frac{27850 \times 0.6}{6200}$ = 2.7

Max. Pressure:

$$a = \frac{160900 - 54500}{27850} = \frac{106400}{27850} = 3.8$$ (O.K.)

$$P = 2/3 \times R_V/a = \frac{2 \times 27850}{3 \times 3.8} = 4900 \text{ lbs.}$$

__Inner cantilever:__ Forces acting on cantilever are:

1) reaction of earth

2) weight of earth

3) weight of concrete.

$$M = 8500 \times 2 + 1.5 \times 4 \times 150 \times 2 + \tfrac{1}{2} \times \tfrac{1}{2} \times 4 \times 150 \times \frac{4}{3}$$

$$- \frac{2000 \times 4}{2} \times \frac{4}{3} = 13700 \text{ ft.lbs.}$$

$$d = \sqrt{\frac{13700 \times 12}{113 \times 12}} = \sqrt{121} = 11'' \text{ required. supplied } 21'' \text{ eff.}$$

$$A_s = \frac{13700 \times 12}{18000 \times 0.88 \times 21} = 0.495$$

Use 2 - ½ " sq. bars per ft. (= 0.50)

$$V = 8500 + \frac{1.5 \quad 2}{2} \times 150 \times 4 - \frac{2000 \times 4}{2} = 3000 \text{ lbs.}$$

$$v = \frac{3000}{12 \times 0.88 \times 21} = 13.5 \text{ p.s.i. (safe)}$$

Outer cantilever:

$$y = \frac{5000 \times 6}{10} = 3000 \text{ lbs/sq.ft.}$$

$$V = \frac{5000 \quad 3000}{2} \times 4 = 16000$$

$$v = \frac{16000}{12 \times 0.88 \times 21} = 72 \text{ p.s.i. (safe)}$$

Use stirrups of ⅜ " dia. with 1 ft. spacing.

$$M = 3000 \times 4 \times 2 + \frac{2000 \times 4}{2} \times 2 = 24000 + 8000 = 32000 \text{ ft.lbs.}$$

$$d = \sqrt{\frac{32000}{113}} = 16.8'' \text{ required. Supplied } 21''$$

$$A_s = \frac{32000 \times 12}{18000 \times 0.88 \times 21} = 1.15 \ \overline{\text{in}}^2$$

Use 3 - ¾" dia. round bars. (= 1.33 $\overline{\text{in}}^2$).

REINFORCED CONCRETE

ARCH BRIDGE

The bridge will consist of a floor system supported on columns resting on two arch ribs. There will be two approach spans on both sides of the amain span.

Dimensions:

main span = 120 '

approach spans = 30 ' each

Distance between columns in main span = 10 '

" " " " approach spans = 15 '.

roadway c. to c. of beams = 24 '.

sidewalks = 5 '

Total width of road = 34 '.

Outline of design:

The floor system will consist of a slab resting on two longitudinal outside beams that rest on columns, and a middle T-beam resting on Transverse beams that run between columns.

The following are the main steps in the design of the arch:

1) Design of main span:

a) Design of slab.

b) " " middle T-beam.

c) " " sidewalks.

d) " " outside longitudinal beams.

e) " " transverse beams.

f) " " columns.

2) Design of approach spans:

A a) Design of slab.

b) " " middle T-beam.

c) " " outside rectangular beams.

d) " " transverse beams.

e) " " columns

f) " " footings.

3) Design of parabolic arch:

a) Design of arch rib.

b) " " " abutments.

DESIGN OF SLAB

The ratio of the two sides is: $\frac{l_1}{l_2} = \frac{10}{12} = .835$

Therefore $\beta_1 = .500$ and $\beta_2 = .200$

Assume thickness of slab = 5.5" + 1.5" (cov.) = 7".

For uniform load the weight per foot run will consist of the weight of the concrete slab plus the weight of the road material which we shall consider 4" thick.

$$\text{wt. of conc.} = \frac{12 \times 7 \times 150}{114} = 90 \text{ lbs./ ft.}$$

$$\text{wt. of earth} = \frac{4'' \times 120}{12} = 40 \text{ lbs.: ft.}$$

total 130 lbs./ ft.

$$M_1 = \frac{130 \times \overline{10}^2}{10} = 130 \text{ ft. lbs.}$$

For concentrated load the effective width for the transverse direction is given by the equation:

$$e = 2/3\ l + c = \frac{20}{3} + 10 = 6.67$$

but the maximum is 6 ft.; therefore the load on a width of one foot will be:

$$P = \frac{6 \times 2240}{6} = 2240 \text{ lbs./ ft.}$$

For concentrated load take a condition midway between a fixed and free condition; take 3/4 of the free condition.

$$M_1 = \frac{3}{4} \times \frac{PL}{16} = \frac{3PL}{16}$$

$$M_1 = \frac{3 \times 2240 \times 10}{16} = 4200 \text{ ft. lbs.}$$

$$\text{Imp.} = .20 \times 4200 = 840 \text{ ft. lbs.}$$

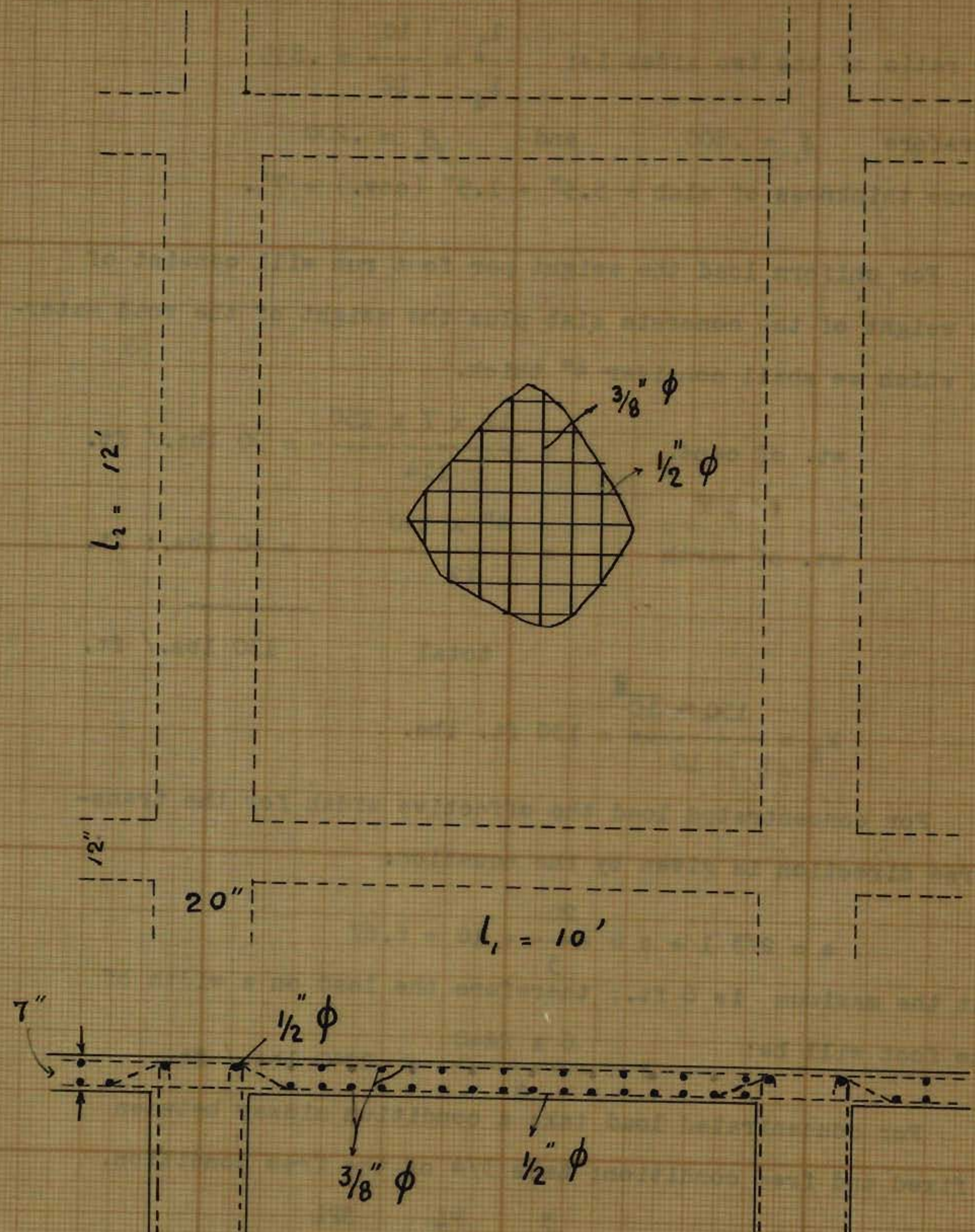


Fig. 9

$$\text{Total } M_I * = 1300 + 4200 + 840 = 6340 \text{ ft. lbs.}$$

$$\mu_1 = .500 \times 6340 = 3170 \text{ ft. lbs.}$$

$$d = \sqrt{\frac{3170 \times 12}{113 \times 12}} = \sqrt{28.1} = 5.3"$$

$$A_s = \frac{M}{f_s . j . d} = \frac{3170 \times 12}{18000 \times .88 \times 5.5} = .436$$

Use 2 - 1/2 " round bars per foot = .5 sq. in.

In sens l_2 :

$$M_2 = \frac{130 \times \overline{12}^2}{10} = 1870 \text{ ft. lbs. (unif.)}$$

Eff. width = e = 2/3 (L + c) = 2/3 (10 + 1)= 7.3

but the maximum value of e = 6 ft.

$$\text{therefore} \quad P = \frac{6 \times 2240}{6} = 2240 \text{ lbs.}$$

$$M_2 = \frac{3PL}{16} = \frac{3PL}{16} = \frac{3 \times 2240 \times 12}{16} = 5030 \text{ ff.lbs.}$$

$$\text{Imp.} = .20 \times 5030 = 1006 \text{ ft. lbs.}$$

$$\text{Total } M_2 = 1870 + 5030 + 1006 = 7906 \text{ ft. lbs.}$$

$$\mu_2 = M_2 \; .200 \times 7906 = 1581 \text{ ft.lbs.}$$

$$A_s = \frac{M_2}{f_s . j . d} = \frac{1580 \times 12}{18000 \times .88 \times 5.5} = .217 \text{ sq. in.}$$

Use 2 - 3/8 " round bars = .22 sq. in.

Total M_I * = 1300 + 4200 + 840 = 6340 ft. lbs.

μ_1 = .500 x 6340 = 3170 ft. lbs.

$$d = \sqrt{\frac{3170 \times 12}{113 \times 12}} = \sqrt{28.1} = 5.3"$$

$$A_s = \frac{M}{f_s \cdot j \cdot d} = \frac{3170 \times 12}{18000 \times .88 \times 5.5} = .436$$

Use 2 - 1/2 " round bars per foot = .5 sq. in.

In sens l_2 : $M_2 = \frac{130 \times \overline{12}^2}{10}$ = 1870 ft. lbs. (unif.)

Eff. width = e = 2/3 (L + c) = 2/3 (10 + 1)= 7.3

but the maximum value of e = 6 ft.

therefore $P = \frac{6 \times 2240}{6}$ = 2240 lbs.

$$M_2 = \frac{3PL}{16} = \frac{3PL}{16} = \frac{3 \times 2240 \times 12}{16} = 5030 \text{ ff.lbs.}$$

Imp. = .20 x 5030 = 1006 ft. lbs.

Total M_2 = 1870 + 5030 + 1006 = 7906 ft. lbs.

μ_2 = M_2 .200 x 7906 = 1581 ft.lbs.

$$A_s = \frac{M_2}{f_s \cdot j \cdot d} = \frac{1580 \times 12}{18000 \times .88 \times 5.5} = .217 \text{ sq. in.}$$

Use 2 - 3/8 " round bars = .22 sq. in.

MIDDLE T-BEAM

The uniform load coming from the slabs on the beam is:

$$w = p\frac{l_1 + l_2}{3} = 130 \times \frac{12 + 12}{3} = 1040 \text{ lbs./ft.}$$

But usually this formula gives safe values. Using the method of trpezoids and triangles and assuming in our case the 1/4 of each slab is carried by this beam, (which is on the safe side) we have:

$$w = \frac{10 \times 24}{4} \times 130 = 780 \text{ lbs./ft.}$$

assuming a beam of 25 x 12 = $\frac{25 \times 12}{144} \times 150 = 310$ lbs./ ft.

$$\text{Total } w = 780 + 310 = 1090 \text{ ft. lbs.}$$

$$M_w = \frac{wl^2}{10} = \frac{1090 \times \overline{10}^2}{10} = 10900 \text{ ft.lbs.}$$

For the concentrated load we shall assume that one wheel is at the middle third of the beam, and two wheels, one belonging to the same car and the other to the second one at 6 ft. from the beam. We shall assume that half of these wheel loads come on the beam distributed over a length of 6' from , since e = 6 ft.

$$w = \frac{26.8}{6} = 4.48 \text{ k/ft.}$$

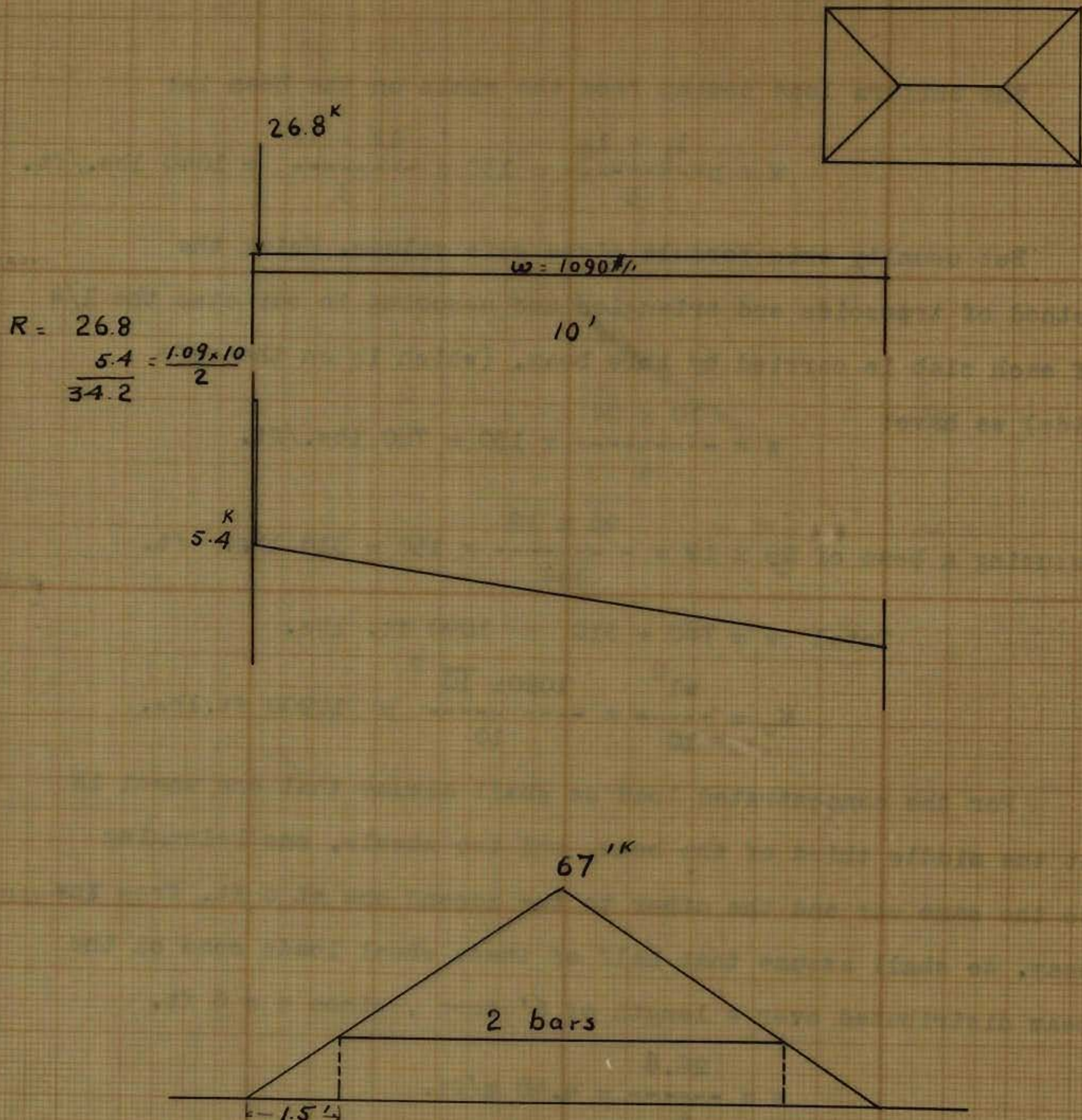
$$\text{Max. L.L Mom.} = 3/4\left(13.4 \times 5 - \frac{4.48 \times 3}{2}\right) = 43'^{k}$$

$$\text{Imp.} = .2 \times 43000 = 8600 \text{ ft. lbs.}$$

$$\text{Total } M = 10900 + 43000 + 8600 = 62500 \text{ ft. lbs.}$$

$$d = \sqrt{\frac{62500 \times 12}{113 \times 12}} = \sqrt{555} = 23.6''$$

$$A_s = \text{---------}$$



Middle T-Beam

Fig. 10

$$A_s = \frac{M}{f_s.j.d} = \frac{62500 \times 12}{18000(23.6 - 3)} = 2.02 \; \overline{in}^2$$

Use 4 - $\frac{7}{8}$" dia. bars ($= 2.4 \; \overline{in}^2$)

Stirrups: The shear diagram is as shown in figure 10

The shear carried by the concrete along is:

$$V_c = vbjd = 40 \times 12 \times 0.90 \times 25 = 10.8^k$$

This leaves at the left support a shear of 23.4^k to be carried by the stirrups. A double prong $\frac{3}{8}$" dia. stirrup can take:

$$P = 2 \times 0.11 \times 16000 = 3520 \text{ lbs.}$$

Unit shear:

$$v' = \frac{V}{b\;j\;d} = \frac{23400}{12 \times 0.90 \times 25} = 82 \text{ p.s.i.}$$

Change of shear per inch run at the extreme left:

$$bv' = 82 \times 12 = 985 \text{ lbs.}$$

Spacing:

$$\frac{P}{v'b} = \frac{3520}{985} = 3.58"$$

Maximum spacing = S = 0.45 x 25 = 11.2"

Use a spacing of 3.5" at the two extreme ends; then increase it gradually to 11" (see diagram)

Bending of bars: The moment diagram is shown in figure 10

One bar can take a moment of:

$$M_s = A_s f_s jd = 0.6 \times 18 \times 0.90 \times 25 = 24.3 \;'^k$$

Two bars can take 48.6 'k. Bend two bars at the end.

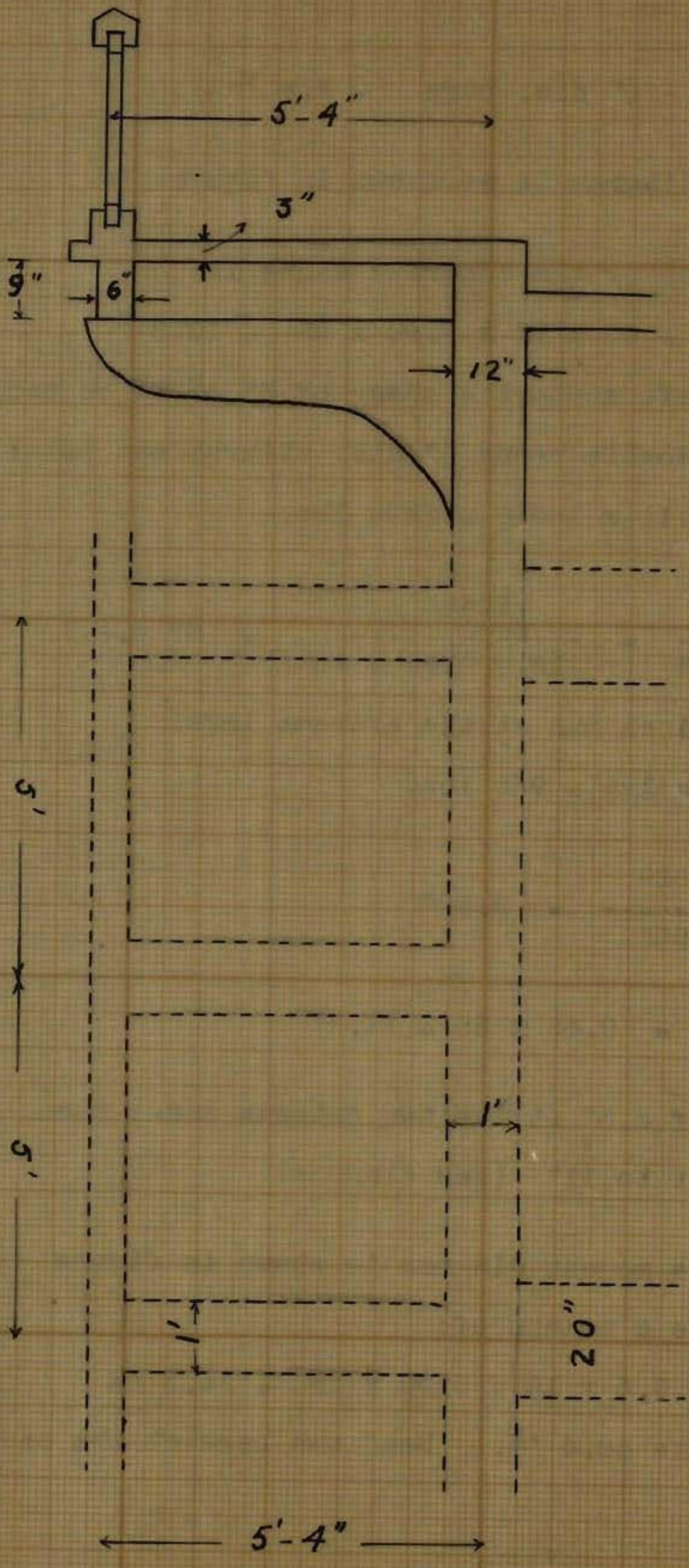


Fig. 11

DESIGN OF SIDEWALKS

Span 5 ft. c. to c. of beams.

L.L. = 400 kgs/m^2 = 88 lbs/ft^2

Assumed D.L. = 2"+ 1" = 3" is the assumed depth

D.L. = 3/12 x 150 = 37.5 $lbs./ft^2$

Total w = 88 38 = 126 lbs/ft^2

$$M = 1/10\ wl^2 = 1/10 \times 126 \times \overline{5}^2 = 314 \text{ ft.lbs.}$$

$$d = \sqrt{\frac{M}{R\ \ b}} = \sqrt{\frac{314 \times 12}{113 \times 12}} = 1.68"$$

$$A_s = \frac{314 \times 12}{18000 \times 0.88 \times 2} = 0.12\ \overline{in}^2$$

Use $\frac{3}{8}$" dia. bars with a spacing of 7½" (= 0.176 $\overline{in}^2$/ft)

<u>Outside beam supporting the sidewalk slab</u>:

$$W_{(D.L. + L.L.)} = \tfrac{1}{2} \times 126 \times 5 + \frac{9}{12} \times \frac{6}{12} \times 150 = 370$$

370 + 300 (railing) = 670 lbs./ft

$$M = 1/10\ wl^2 = 1/10 \times 670 \times 5 \times 5 = 1600 \text{ ft.lbs.}$$

$$d = \frac{1680 \times 12}{113 \times 6} = 5.5"$$ (we have a total of 9") eff. - 7"

$$A_s = \frac{M}{f_s\ j\ d} = \frac{1680 \times 12}{18000 \times 0.88 \times 7} = 0.182\ \overline{in}^2$$

Use 2 - $\frac{3}{8}$" dia. bars (= 0.22 $\overline{in}^2$). Use also 2 - $\frac{3}{8}$" on top.

Stirrup spacing = 0.45 d = 0.45 x 7 = 3"

Design of Cantilevers:

M_c = Pl = 1670 x 5 = 8350 ft.lbs.

Assume d = 10 " and 3 ".at the end.

$$M = \frac{3 \times 5 \times 5}{12 \times 2} \times 150 \quad \frac{7}{12} \times 5 \times \frac{1}{2} \times 150 \times \frac{5}{3} = 834 \text{ ft.lbs.}$$

Total M = 8350 + 834 = 9184 ft.lbs.

$$d = \sqrt{\frac{9184 \times 12}{113 \times 12}} = 9" \quad \text{Total d will be 11".}$$

See diagram for stirrup spacing.

LONGITUDINAL OUTSIDE BEAMS

Uniform load from sidewalk = 314/2 = 157 lbs/ft say 160

$$\text{From slab} = \frac{1}{2} \times \frac{10 \times 12 \times 130}{10} = 390 \text{ lbs/ft.}$$

Total w = 160 + 390 + 340 = 890 lbs/ft

M_W = $1/10 \times 890 \times \overline{10}^2$ = 8900 ft.lbs.

L.L. moment is the same as that for the Intermediate beam.

M_{LL} = 43000 ft.lbs. ; Impact = 8600 ft.lbs.

Total = 43000 + 8600 + 8900 = 60500 ft.lbs.

$$d = \sqrt{\frac{60500 \times 12}{113 \times 12}} = 23.2" \text{ say } 25" \quad 2" = 27"$$

$$A_s = \frac{60500 \times 12}{18000 \times 0.88 \times 12} = 3.82" \text{ sq.in.}$$

Use 5 - 1" dia. bars (= 3.93)

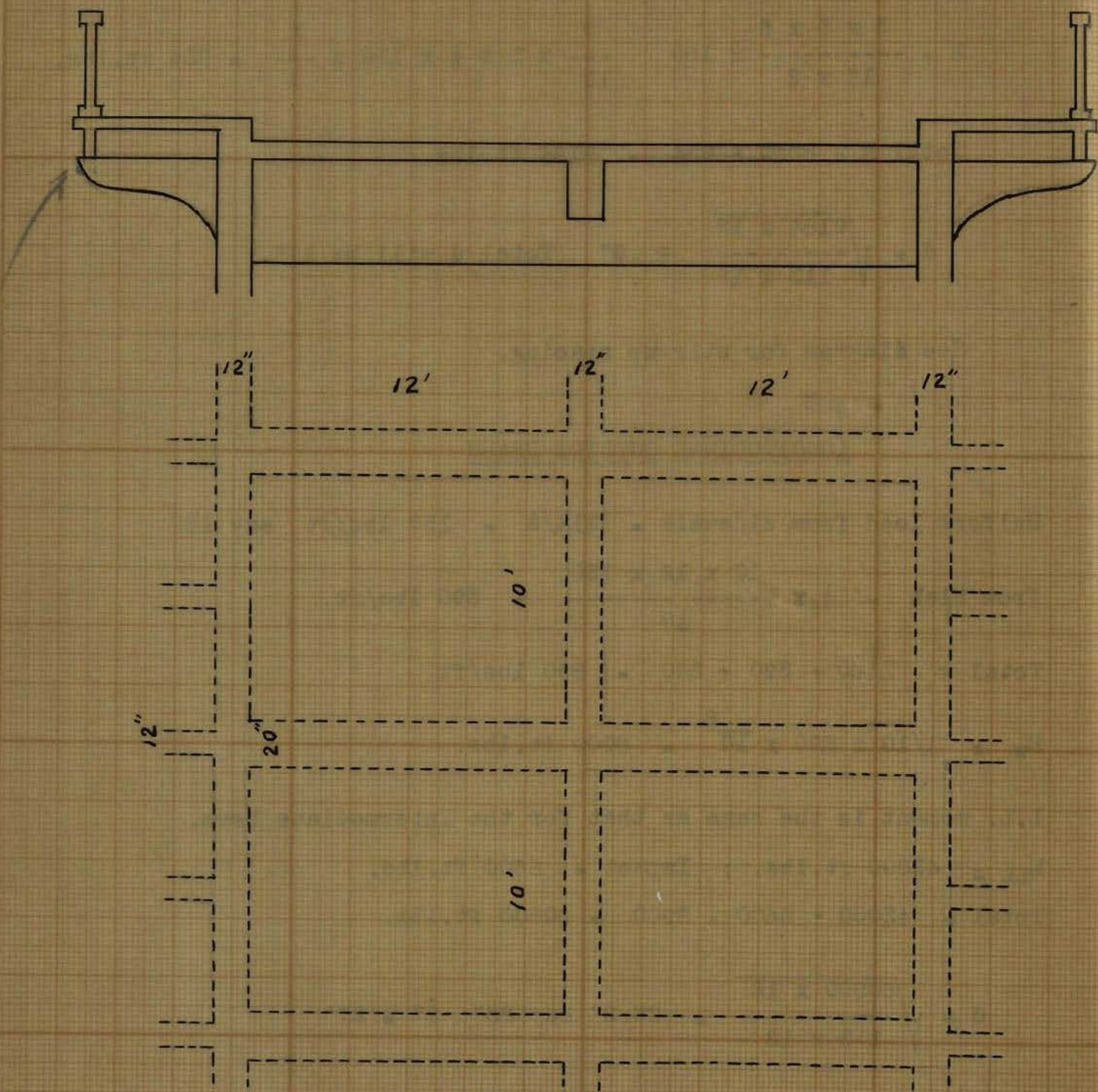


Fig. 12

Stirrups: Use same stirrups with same spacing as that used for the middle T-Beam.

Bending of bars: Bend only 2 bars out of 5 bars because we must not bend more than half the number of bars.

TRANSVERSE BEAMS

Uniform load from slab $= \frac{130\ (12 - 5) \times 5}{12} = 380$ lbs/ft.

D.L. of beam:

$$\frac{35 \times 20}{144} \times 150 = 730 \text{ lbs/ft.}$$

Total uniform load = 730 + 380 = 1110 lbs/ft.

Concentrated load from longitudinal interior beam :

$$\tfrac{1}{2} \times 890 \times 10 = 4550 \text{ lbs.}$$

D.L. moment $= 1/10\ wl^2 + 3/116\ Pl = 1/10 \times 1110 \times 24^2$
$+ 3/16 \times 4550 \times 24$

= 64000 + 20000 = 84000 ft.lbs.

Maximum L.L. moment occurs at the middle with the following symmetrical loading shown in the figure (assuming that the two trucks are almost touching each other).

L.L. Moment:
$26.8 \times 12 - 26.8 \times 4.28 = 207'^{k}$

Impact: $0.20 \times 207 = 41.4'^{k}$

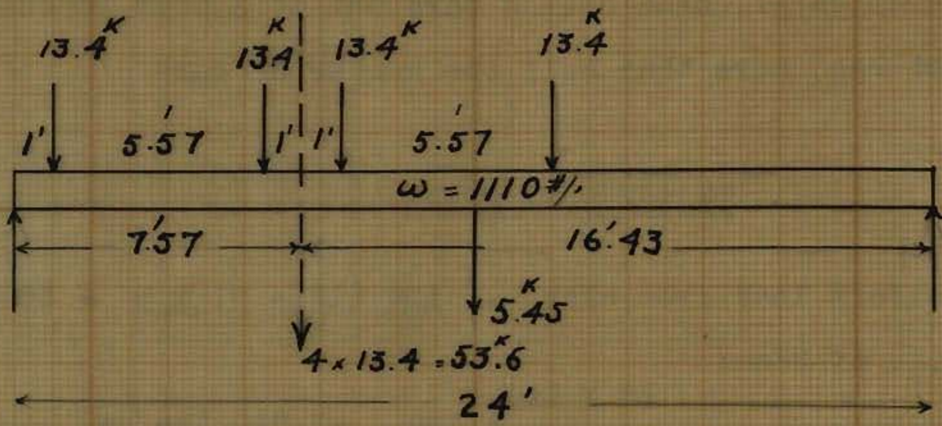
Total Moment = 84000 + 207000 + 41400 = 332400 ft.lbs.

$$d = \sqrt{\frac{332400 \times 12}{165 \times 20}} = 35'' \text{ say } 38''$$

$$A_s = \frac{M}{f_s(d - t/2)} = \frac{337000 \times 12}{18000(35 - 3)} = 7.2 \text{ sq. in.}$$

Use 8 - 1" sq. bars (= 8.00 sq. in.)

(For reinforcement see page , reinforcement for approaching spans)



Transverse beam.

Position of L.L. giving maximum reaction over the column.

Fig. 13

DESIGN OF COLUMNS

Weights coming on columns:

All the weights are transmitted to the columns through the longitudinal outside beams and the transverse beams.

D.L.

Longit. = wl = 890 x 10 = 8900 lbs.

Trans. = ½ x wl P/2 = ½x1110x24 + ½x5450

= 13300 + 2725 = 16025 "

Cantilever

$$5 \times \frac{10 \times 38}{2 \times 144} \times 150 \quad 1670 = 1800 \text{ "}$$

26725 lbs.

L.L.

When 4 rear wheels are on a transverse beam as near to one column as possible -

R_L = 4 x 13.4 x 16.43/24 = 36.6 ^{k}

Total R = 26725 + 36600 = 63325 lbs. say 63500 lbs.

Weight of spandrel walls = 5 x 12/12 x 10 x 150 = 7500 lbs.

Total weight on columns

63500 + 7500 = 71000 lbs.

Total D.L.

26725 + 7500 = 34225 "

Total L.L.

= 36600 "

Design of columns: W = 71000 lbs.

f_c = 300 (0.10 + 4 x 0.02) x 2000 = 660 p.s.i.

$$A = \frac{71000}{660(1 + 15 \times 0.02 - 0.02)} = 84.1 \text{ sq. in.}$$

P'/P = 1.33 - 12 x 12/(120 x 3.46) = 1.33 - 0.347

= 0.983

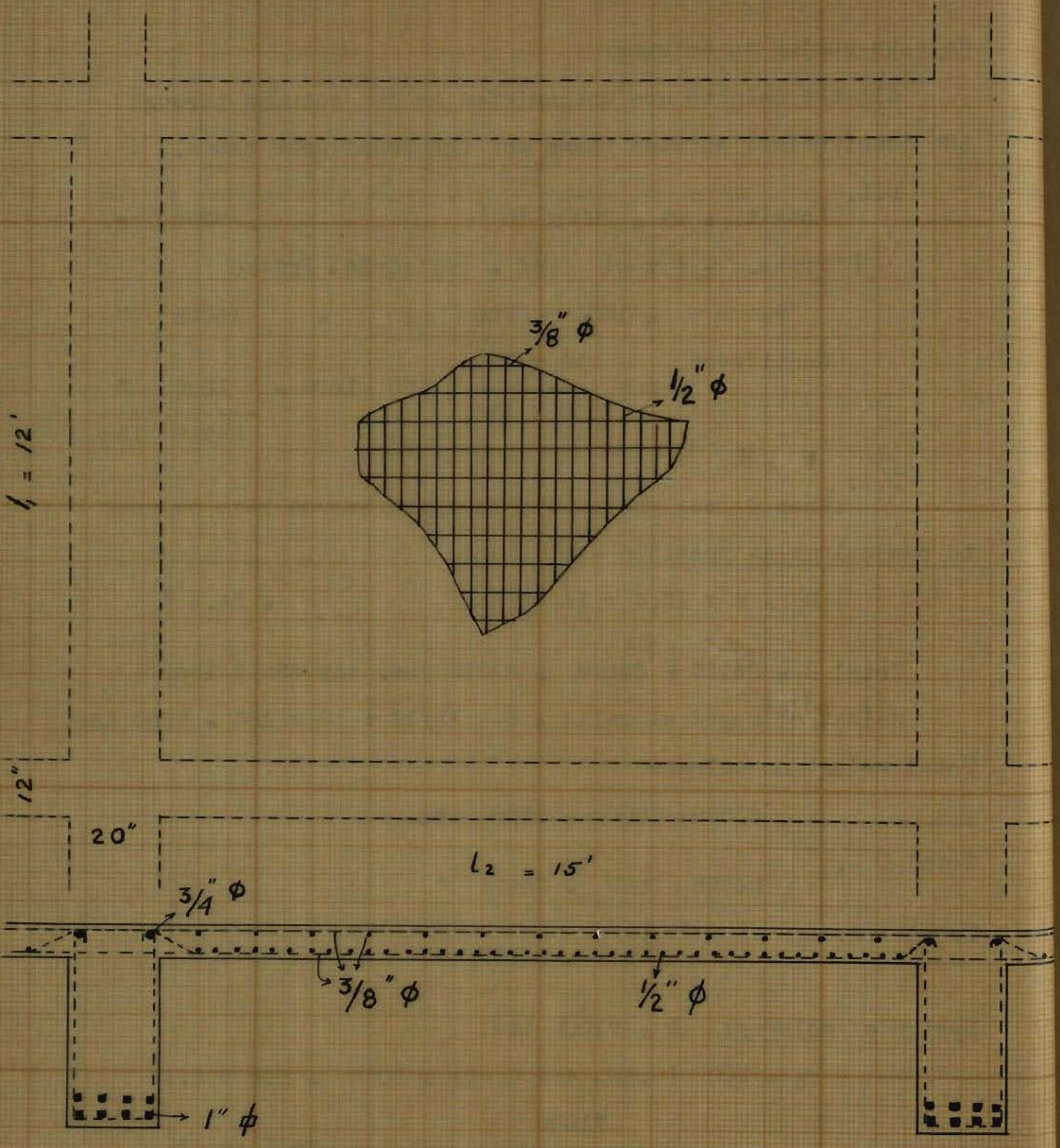


Fig 14

$$A = \frac{84.1}{0.983} = 86 \text{ sq. in.}$$

Columns with larger sections are used in order to be in good proportion with the rest of the structure.

A P P R O A C H   S P A N

DESIGN OF SSLAB

$$\frac{l_1}{l_2} = \frac{12}{15} = 0.8 \; ; \quad \beta_1 = 0.549 \text{ and } \beta_2 = 0.171$$

Assumed thickness = $8\frac{1}{2}$ " = $(7 + 1\frac{1}{2})$

$$\text{wt. of concrete} = \frac{12 \times 8.5}{144} \times 150 = 94 \text{ lbs/ft}^2$$

wt. of earth = 40 lbs/ft^2

Total weight = 134 lbs/ft^2

For uniform load

$$M_1 = 1/10\, wl_1^2 = 1/10 \times 134 \times \overline{12}^2 = 1920 \text{ ft.lbs.}$$

For concentrated load

$$P = \frac{6 \times 2240}{6} = 2240 \text{ lbs/ft.} \quad (e = 6')$$

$$M = 3/16 \times Pl = 3/16 \times 2240 \times 12 = 5040 \text{ ft.lbs.}$$

Impact = 0.20 x 5040 ft.lbs. = 1008 ft.lbs.

Total 7968 ft.lbs

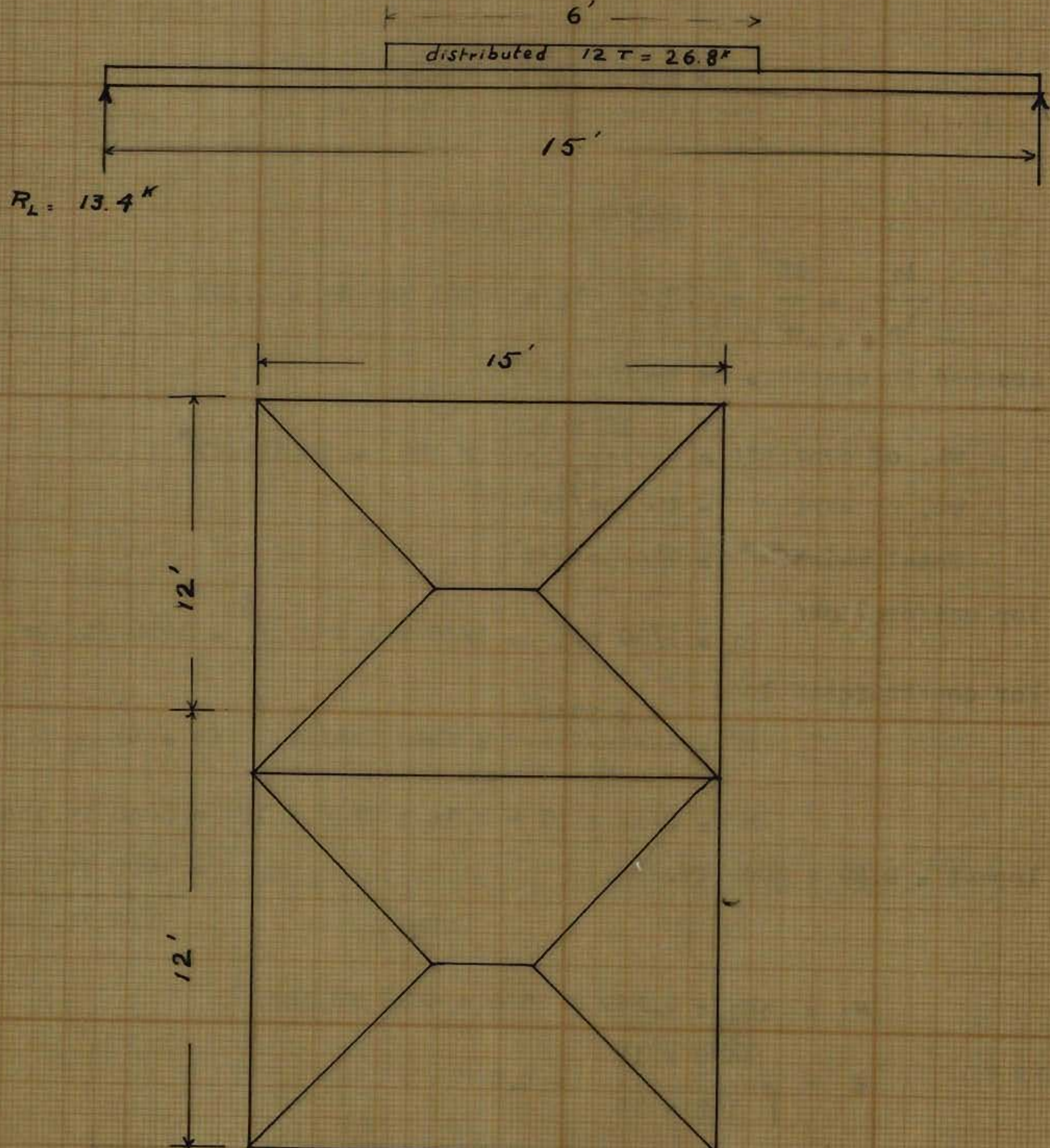
$$\mu_1 = {}_1M_1 = 0.549 \times 7968 = 4480 \text{ ft.lbs.}$$

$$d = \sqrt{\frac{4480 \times 12}{113 \times 12}} = 6.2 \text{ "}$$

A total thickness of $7\frac{1}{2}$ " will be satisfactory.

$$A_s = \frac{M}{f_s j d} = \frac{4480 \times 12}{18000 \times 0.88 \times 6.2} = 0.55 \text{ sq. in.}$$

Use 3 - $\frac{1}{2}$ " dia. bars per foot (= 0.59 sq. in.)



Middle T-Beam

Fig. 15

In direction l_2 :

$M_{2w} = 1/10 \times 134 \times \overline{15}^2 = 3020$ ft.lbs.

$e = 6$ ft.; $P = 2240$

$M_{2c} = 3/16 \times 2240 \times 15 = 6300$ ft.lbs.

Imp. $= 0.20 \times 6300 = 1260$ ft.lbs.

Total $= 10580$ ft.lbs.

$\mu_2 = \beta M_2 = 0.171 \times 10580 = 1810$ ft.lbs.

$$A_{s2} = \frac{M_2}{f_s\, j\, d} = \frac{1810 \times 12}{18000 \times 0.88 \times 6.2} = 0.221 \text{ sq.in.}$$

Use 2 - $\frac{3}{8}$" dia. bars per ft. (= 0.22 sq. in.)

MIDDLE T-BEAM

Area of trapezoid $= (l_2 - l_1/2)l_1/2 = (15 - 6)6 = 54 \text{ ft}^2.$

$$w = \frac{2 \times 54 \times 134}{15} = 965 \text{ lbs/ft} + 335 \text{ (assumed D.L. of beam)}$$

$= 1300$ lbs/ft.

$M_W = 1/10 \times 1300 \times \overline{15}^2 = 29200$ ft.lbs.

We have for the concentrated load:

$w = 26.8/6 = 4.48^k$

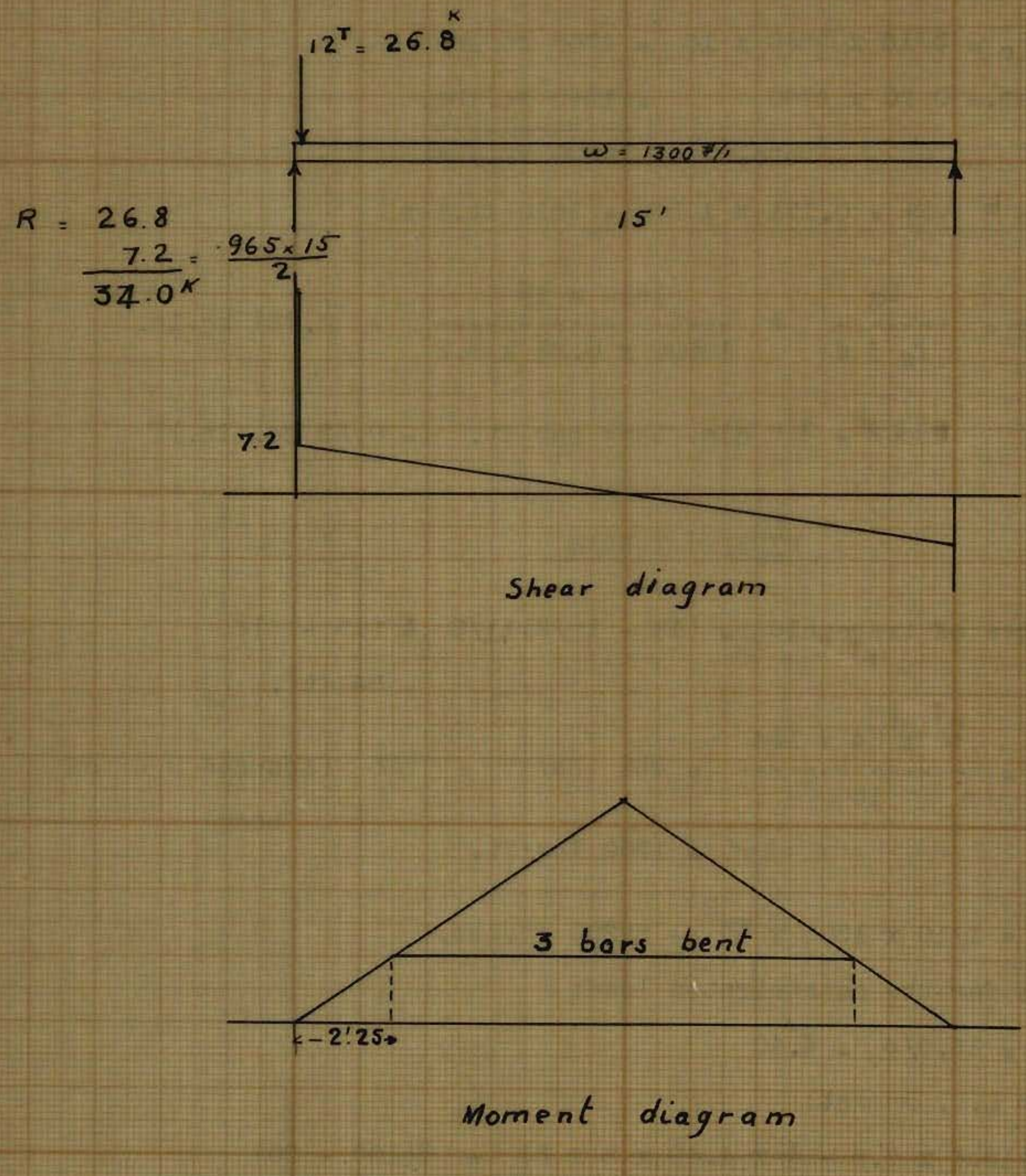
Maximum L.L. moment:

$\frac{8}{9}(13.4 \times 7.5 - \frac{1}{2} \times 4.48 \times 3 \times 3) = \frac{8}{9}(100 - 20)$

$= 60'^k = 60000$ ft.lbs.

Impact: $0.20 \times 60000 = 12000$ ft.lbs.

Total Moment: $29200 + 60000 + 12000 = 101200$ ft.lbs.



Middle T-Beam.

Fig. 16

$$d = \sqrt{\frac{101200 \times 12}{113 \times 12}} = \sqrt{895} = 30''$$

$$A_s = \frac{M}{f_s(d - t/2)} = \frac{101200 \times 12}{18000 \times (30 - 3.5)} = 2.54 \ \overline{in}^2$$

Use 6 - ¾ " dia. bars (= 2.65)

Take total depth of beam = 33 ".

Stirrups: The shear diagram is shown in figure 16

The concrete can carry a shear of:

$$V_c = vbjd = 40 \times 12 \times 0.90 \times 30 = 13^k$$

The shear to be carried by the stirrups is 21^k

Use same spacing as that for the middle T-Beam in main span.

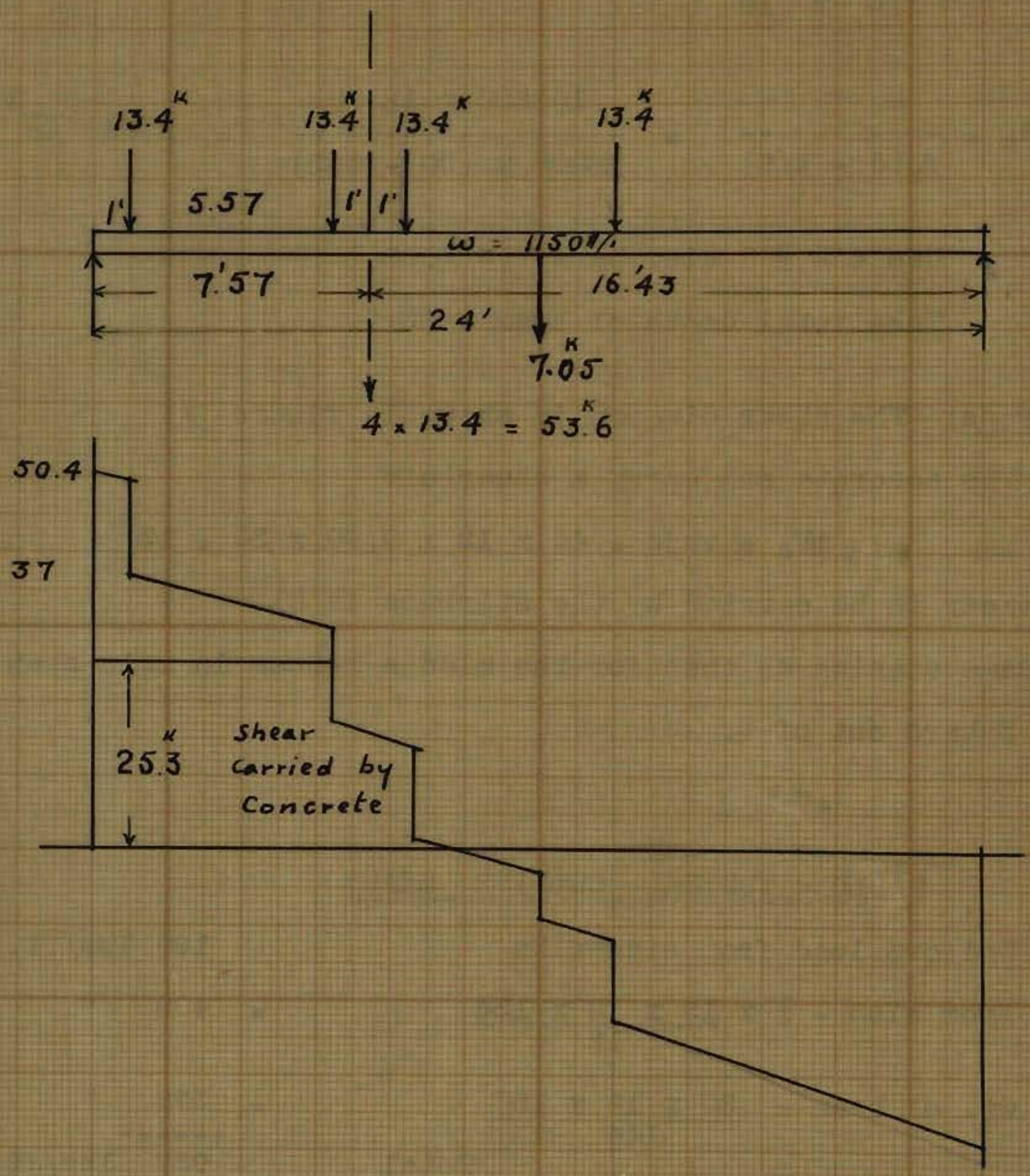
Bend 3 bars out of 6.

LONGITUDINAL OUTSIDE BEAMS

Uniform load from sidewalks	=	160 lbs/ft.
From slab = $\frac{1}{4} \times \frac{15 \times 12 \times 134}{15}$	=	402 "
wt. of beam = $\frac{30 \times 12 \times 150}{144}$	=	375 "
Total		937 lbs/ft.

$M_w = 1/10 \times 937 \times \overline{15}^2$	= 21100 ft.lbs.
L.L. moment same as for int. beam	= 60000 "
Impact	= 12000 "
	93100 ft.lbs.

$$d = \sqrt{\frac{93100 \times 12}{113 \times 12}} = 28.7'' \text{ say } 29''$$



Shear diagram for Transverse beams.

Fig. 17

$$A_s = \frac{93100 \times 12}{18000 \times 0.88 \times 15} = 4.71 \text{ in}^2$$

Use 6 - 1 " dia. bars. (= 4.71 in^2)

Stirrups:

The stirrups and the bending of bars will be exactly like those for the middle T-beam.

TRANSVERSE BEAMS

Uniform load from slab :

$$134 \times \frac{12 \times 6}{12 \times 2} = 402 \text{ lbs/ft.}$$

Load of beam = 750 "

Total uniform load 1150 lbs/ft.

Concentrated load from long. inter. beam:

$\frac{1}{2} \times 937 \times 15$ = 7050 lbs.

D.L. Moment: $1/10 \text{ wl}^2$ 3/16 Pl =

1/10 x 1150 x 24x24 3/16 x 7050 x 24

= 66000 + 31700 = 97700 ft.lbs.

L.L. Moment =207000 "

Impact = 41400 "

Total = 346100 ft.lbs.

Using concrete at 900 p.s.i. -

$$d = \sqrt{\frac{346100 \times 12}{165 \times 20}} = 35.3 \text{ " say } 36 \text{ " with covering} = 39 \text{ "}$$

$$A_s = \frac{M}{f_s(d - t/2)} = \frac{346100 \times 12}{18000(36 - 3\frac{3}{4})} = 7.15 \text{ sq.in.}$$

Use 8 - 1 " sq. bars (= 8.00 sq. in.)

Stirrups: The shear diagram is as shown in figure 17

Concrete can carry $V_c = v\ b\ j\ d = 40 \times 20 \times 0.88 \times 36 = 25.3^k$

The shear to be carried by stirrups is $50.4 - 25.3 = 25.1^k$

Using a double prong $\frac{3}{8}$" dia. stirrup, P = 3520 lbs.

Unit shear:

$$v' = \frac{V}{b\ j\ d} = \frac{25.1}{20 \times 0.88 \times 36} = 38.6 \text{ p.s.i.}$$

Change of shear per inch run at extreme left:

$$bv' = 20 \times 38.6 = 780 \text{ lbs.}$$

Spacing:

$$\frac{P}{v'\ b} = \frac{3520}{780} = 4.6\ "$$

Maximum allowable spacing = 0.45 x 36 = 16 "

Use a spacing of 4.5 " at the ends and increase it to 16 " at the middle.

Bending of bars:

Bend four bars as shown in the diagram. (Drawing of Cross section)

COLUMNS

Weight coming on columns:

D.L.

Longit. = w l = 937 x 15	= 14100	lbs.
Transverse = wl/2 P/2 = ½x24x1150 + ½x7050	= 17300	"
Cantilever	= 1800	"
Spandrel walls = $\frac{7 \times 12}{12}$ x 15 x 150	= 15700	"
Total =	48900	lbs
L.L. =	36600	"
Assumed wt. of column = 2 x 2 x 20 x 150 =	12000	"
Grand total =	97000	lbs
	say 100000	lbs

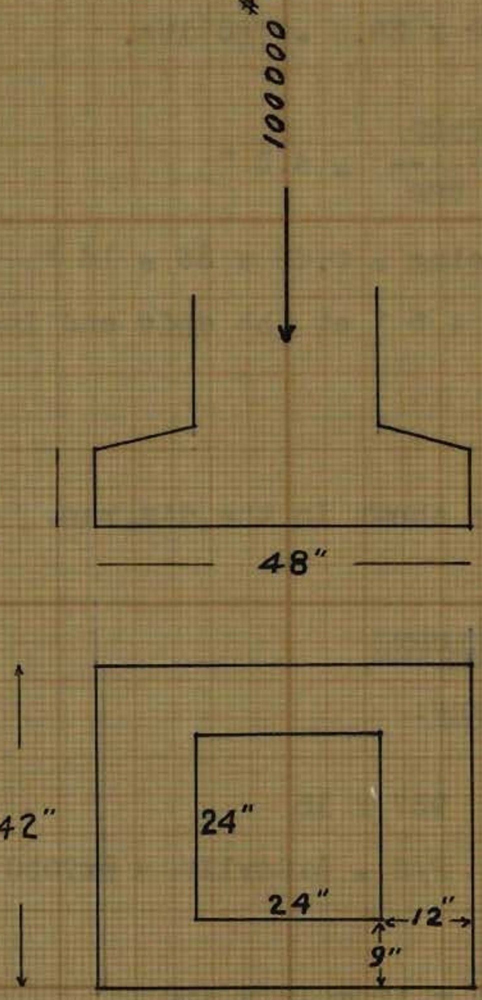


Fig. 18

Design of columns:

$$f_c = 660 \text{ p.s.i.}$$

$$A = \frac{100000}{660(1 + 15 \times 0.02 - 0.02)} = \frac{100000}{660 \times 1.28}$$

$$= 118 \text{ sq. in. say } 12" \times 12"$$

$$P'/P = 1.33 - \frac{12 \times 12}{120 \times 3.46} = 0.983$$

$$A = \frac{118}{0.983} = 120 \text{ sq. in.}$$

A larger section is used so that the columns may be in good proportion with the rest of the structure.

Design of footings: Load = 100000 lbs.

Assume soil to carry 4^t per sq. ft. = 8000 lbs/ft^2

$$\text{Area of footing} = \frac{100000}{8000} = 12.5 \text{ sq.ftn.}$$

Use an area of 3.5' x 4' = 14 sq. ft.

$$M_{xx} = \frac{8000}{2}(24 + 1.2 \times 0.75)\,\overline{0.75}^2 = 56000 \text{ ft.lbs.}$$

$$d = \frac{(A - a^2)w}{4as_s} = \frac{(2020 - 570)\,8000}{96 \times 80 \times 144} = 10.3" \text{ say } 11"$$

$$A_s = \frac{M}{f_s\, j\, d} = \frac{56000 \times 12}{} = 3.85 \text{ sq. in.}$$

Use 9 - 3/4" round bars, 4" c. to c. = 3.98□"

ANALYSIS OF THE SYMMETRICAL ARCH BY THE ELASTIC THEORY

General Discussion.

The fixed end reinforced concrete arch is a statically indeterminate structure. There are three unknowns at each support and therefore we need six equations. Statics gives us three equations and the remaining three are taken from the following conditions:

The change in span of the arch $= \Delta x = 0$

The vertical displacement at one end relative to the other $= \Delta y = 0$

The angle between the tangents to the arch axis at the two ends of the arch remains unchanged, or $\angle K = 0$

These three conditions must be true since the arch is fixed at the abutments.

Instead of actually finding the components of the reactions and the moments at the supports, it is simpler to take the origin at the crown and find the thrust, shear and moment at that point.

The analysis of an arch consists in finding the thrust, shear and bending moment at the crown and at intermediate sections in the arch rib, and then finding the resulting stresses.

However to find the stresses we should first assume the dimensions of the arch in order to be able to analyze it. So that our assumption may not be far from an economical and a safe design, and also, to save us the trouble of repeating several times the exact analysis of the arch by the long and tedious elastic theory, we first make a tentative design by using Cochraues formulas given on page(*) The common method of procedure is to select an arch ring whose dimensions are obtained by comparison with existing arches of similar span and loading conditions. This arch is then analyzed, and if the stresses are found to be within the allowable limits the design is considered satisfactory. If the stresses are too high or too low, changes are made until finally a design is obtained in which the stresses are satisfactory.

Notation.

s = length of a division of the arch ring measured along the arch axis.

n_h = number of divisions in one-half the arch.

l = span of arch axis.

C_a = average unit compression in concrete of arch ring due to thrust.

t_c = coefficient of linear temperature expansion.

t_D = number of degrees rise or fall in temperature.

E_c = modulus of elasticity of concrete.

At the crown, let

H_c = horizontal thrust.

V_c = vertical shear.

R_c = resultant of H_c and V_c

M_c = bending moment.

At any point on the arch axis, with coordinates x and y referred to the crown as origin, let

N = thrust (normal) or radial section.

S = shear or radial section.

R = resultant force on radial section, resultant of N and S.

x_o = eccentricity of thrust on section, or distance of N from the arch axis.

t = depth of section.

I = moment of inertia of section including steel

= $I_c + nI_s$

A = area of section including steel = $a_c + na_s$

P_o = steel ratio for total steel at section.

d' = embedment of steel from either upper or lower surface.

M = moment = Nx_o

m_L = moment at any point on right half of arch axis of all external loads between the point and the crown.

m_R = moment at any point on right half of arch axis of all external loads between the point and the crown.

m = moment at any point on either half of arch axis of all external loads (P_1 , P_2 , etc.) between the point and the crown.

<u>Formulas.</u>

We know from mechanics that for a curved beam shown in the fig. the following relations hold true.

$$\Delta_\phi = \int_A^B \left[\frac{Mds}{EI} + \frac{fds}{E\rho} \right]$$

$$\Delta_y = \int_A^B \left[-\frac{Mxds}{EI} + \frac{fdy}{E} - \frac{fxds}{E\rho} - \omega t dy \right]$$

$$\Delta_x = \int_A^B \left[\frac{Myds}{EI} + \frac{fdx}{E} + \frac{fyds}{E\rho} - \omega t dx \right]$$

Now, writing the three conditions: (at the crown)

$$\Delta y_L = \Delta y_R$$

$$\Delta x_L = -\Delta x_R$$

$$\Delta \phi_L = -\Delta \phi_R$$

with the three equations of statics, we can find H_c , V_c and M_c . To find the forces acting at any section we can write:

For the left side:

$$M_x = M_c + H_c y + V_c x + m_L$$

For the right side:

$$M_x = M_c + H_c y - V_c x + m_R$$

For the vertical loads we have also,

For the left side:

$$N_x = H_c \cos\alpha + (\textstyle\sum_c^D P - V_c) \sin$$

$$V_x = H_c \sin\alpha - (\textstyle\sum_c^D P - V_c) \cos$$

For the right side:

$$N_x = H_c \cos\alpha + (\textstyle\sum_c^D P - V_c) \sin$$

$$V_x = -H_c \sin\alpha + (\textstyle\sum_c^D P - V_c) \cos$$

Now when these equations are solved it will be seen that we shall have very general expressions for the thrusts, shears and moments at the crown and other sections. However, if our arch axis is devided in such a way that s/I is a constant throughout the whole length of the arch axis, the formulas become much simpler. Hool gives us a table of such simple formulas which we shall use in our design.

Temperature.

Considerable uncertainty exists regarding the exact nature of temperature changes in an arch and their relation to external temperature conditions. However, experiments have been carried out under the direction of the Engineering Experiment station at Ames, Iowa and in the bulletin which was published, the following conclusion was put:

"To render an arch structurally safe, provision should be made for stresses induced by a temperature variation of at least 40% F each way from an assumed temperature of no stress. Particular circumstances may demand a greater variation to prevent cracks but this remains largely a matter of judgment with the designing engineer." We shall therefore, consider a change of 40° F each way.

Stresses Due to Rib Shortening or Shrinkage Due to Setting.

When the concrete, in arches, shrinks due to setting, the arch ring is a little bit deformed in a manner similar to that resulting from a fall of temperature. As the concrete contracts, the steel is compressed until an equilibrium is established between the compression in the steel and the tension in concrete. This deformation induces bending stresses, tension on one side and compression on the other. These stresses should be combined with those due to loading, in a way to give the maximum stress which could ever occur both in the concrete and in the steel.

Reliability of the Elastic Theory.

In general, theory and experiment have agreed within practical limits. If any error occurs it is due chiefly to the inaccurate assumption that the entire section of the arch ring is effective and that the location of the neutral axis, due to bending only, does not change throughout the arch. This, however, is not correct. But to locate the curve of the neutral axis would be very difficult, so that the assumption is made in arch analysis that the arch axis passes through the neutral axis of each section.

FORMULAS USED IN THE DESIGN OF THE ARCH.

Loading:

$$H_c = \frac{n_h \Sigma (m_L + m_R) y - \Sigma (m_L + m_R) \Sigma y}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \qquad (1)$$

$$V_c = \frac{\Sigma (m_L - m_R) x}{2 \Sigma x^2} \qquad (2)$$

$$M_c = \frac{\Sigma (m_L + m_R) - 2 H_c \Sigma y}{2 n_h} \qquad (3)$$

$$M = M_c \quad H_c Y \quad V_c x \; - \; m_L \qquad (4)$$

$$M = M_c \quad H_c y - V_c x \; - \; m_R \qquad (5)$$

All values of m_L, m_R, x and y should be substituted as positive. All summations refer to one half of the arch axis. Positive value of V_c indicates that the line of pressure slopes upwa upward towards thelleft; a negative value, downwards towards the left. Positive value of M_c indicates that the thrust H_c acts above the arch axis. Signs preceding terms M_c and $V_c x$ in the above formulas depend upon the results of (2) and (3).

Temperature:

$$H_c = \frac{I}{s} \cdot \frac{t_c \, t_D \, l \, n_h \, E_c}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \qquad (6)$$

t_D should be inserted as for a ris; - for a drop.

$$M_c = - \frac{H_c \Sigma y}{n_h} \qquad (7)$$

$$M = M_c \quad H_c y \qquad (8)$$

The value of t_D should be inserted as plus for a rise of temperature; minus for a drop. Signs preceding H_c in formulas (7) and (8) depend upon the result of formula (6). Sign preceding M_c in formula (8) depends upon the result of formula (7). Thus for fall of temperature, thrust and moment are of opposite sign from those for a rise. 1 = span of arch axis.

Rib shortening:

$$H_c = -\frac{I}{s} \cdot \frac{c_a \, l \, n_h}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \qquad (9)$$

$$M_c = -\frac{H_c \Sigma y}{n_h} \qquad (10)$$

$$M = M_c \quad H_c y \qquad (11)$$

Values of moments and thrusts for rib shortening are of same sign as for temperature fall. l = span of arch axis.[1]

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(1) The formulas were taken from Hool, Reinforced Concrete Construction - Bridges and Culverts. (see Bibliography)

METHODS OF PROCEDURE IN ARCH-RING DESIGN

Steps.

The main stept to be followed in the design of our arch may be enumerated as follows:

Given span and rise:

1. Assume a thickness for the arch ring at the crown and at the springing, using empirical formulas, if desired, as an aid to the judgment.

$$t = \sqrt{1} - \frac{1}{10} - \frac{w}{100} - \frac{w'}{200}$$ (Weld's formula - See Spalding p.)

2. Draw your arch axis by using Cochraues formula for open spandrel arches:

$$y = z^2 h \quad \frac{1 - 1/6\ (g-1)\ z^2}{1 - 1/6\ (g-1)}$$

$$g = \frac{w_s}{w_c} = \frac{\text{wt. per foot at springing}}{\text{wt. per foot at crown}}$$ is very approximately equal to unity. This simplifies our equation to a parabola:

$$y = z^2\ h$$

where h = rise of arch

z = proportionate distance from crown to ordinate y in terms of the half span l_1.

This axis follows more or less the dead load line of thrust.

3. From Cochraues table (Table 24, p. 390 in Turneauve) find the thickness of the arch rib at different sections. Draw intrados and extrados. Find the equivalent I of various sections and draw a curve showing variation of I along the arch axis. This curve

will allow us to devide the arch axis into segments of length s, so that $\frac{s}{I}$ = constant for all.

4. Calculate H_c , V_c and M_c for unit load placed successively at the different load points (points where the columns rest on the arch rib.)

5. Draw influence lines for NK and NK' in the formulas:

$$f_c = \frac{NK}{bt} \qquad \text{and } f'_c = \frac{NK'}{bt}$$

where

$$\begin{matrix}(K)\\(K')\end{matrix} = \frac{1}{1 + np_o} \begin{matrix}(+)\\(-)\end{matrix} \frac{6x_o t}{t^2 + 12np_o r^2}$$

or in our case where n = 15

$$\begin{matrix}(K)\\(K')\end{matrix} = \frac{1}{1 + 15 \times p_o} \begin{matrix}(+)\\(-)\end{matrix} \frac{6}{1 + 28.8\ p_o} \times \frac{x_o}{t}$$

The ordinates of these influence lines will be directly proportional to the stresses as it can be seen. Therefore, we can see what loading will give maximum stress in a section, by merely studying the influence line.

6. Consider 2 kinds of loadings:

Loading No. 1 = The loading which gives maximum compression in the upper fibers of a section.

Loading No 2 = The loading which gives maximum compression in the lower fibers.

and stresses in upper and lower fibers under both loadings.

7. Find H_c and M_c for temperature and rib shortening, and combine the stresses which they cause in the four possible ways given below:

(a) No. 1 loading, fall of temperature and rib shortening.

(b) No. 1 " rise " " " " "

(c) No. 2 loading, fall of temperature and rib shortening.

(d) No. 2 " rise " " " " "

8. If for any section there is considerable tension for a given loading, draw influence lines for moment and thrust for that section. Find the L.L. which will give maximum moment and find the stress by the formula

$$f_c = \frac{M}{Lbt^2}$$

$$f_s = nf_c\left(\frac{d}{Kt} - 1\right)$$

This will give the time stress in that section.

9. If the stresses are within the allowable limits, the design is satisfactory. If not new dimensions must be assumed and all the problem repeated until a good design is obtained.

These stept will be clearer when we take the designing sheets separately.

Advantages of the method used.

There is another method called the direct load method, where the whole span, or half of it is loaded to give maximum stress. This might be right in particular cases, or it might be approximate. However, in the influence line method we are choosing the position of the live load which will give maximum stress. A quotation from Hool: " In fact, in the case of large and important structures, the only satisfactory way to analyze for maximum stress is by what might be called a unit load or influence line method."

Now another point is this: in my design I have found the moments and thrusts graphically, whereas Turneaure finds them by computation. (His method is also by influence lines.) However, in his computations any small mistake in one number will be multiplied many times in the procedure and we have less chances of having the correct answer. The graphical method which is as accurate as the computation method, (when drawn to a large scale) has the advantage of being less liable to mistakes, it is simpler and more understandable.

DESIGN

D.P. No. 1 Gives the dead load line of thrust. The dead load includes all the weight coming on the rib through the columns, plus the weight of the arch rib. As we should expect from Cochraues formula the dead load line of thrust follows nearly the arch axis.

D.P. No. 2 This sheet gives a table of the values of I at different points on the axis ($I = I_c + I_s$) and shows the graphical way of deviding the arch axis into segments so that $\frac{s}{I}$ is constant. First the arch axis is developped into a straight line. Then, the I curve is drawn. Now draw a trial line oa and complete the isoceles triangle oab. Then, draw parallels to oa and ab forming successive isoceles triangles. If the end of the last triangle coincides with the end of the arch axis, the divisions are all right. If not, another trial is necessary. Usually three or four trials are necessary.

D.P. No. 3 Gives the computations needed ind etermining the moments and thrusts at the crown for a load of unity placed successively at the load points L_1 , L_2 , L_3 .... L_6

x and y refer to the centers of the sections determined in D.P.Z. The quantity m (cautelever moment) at a given section for a load of unity at a given load point is equal to the value of x for the section in q uestion minus the value of x for the given load point. The values of the moments and thrusts were determined by the formula given on p.

D.P. No.4 Gives computations and graphical work required in determining the moments and thrusts at the various sections due to the above loadings. For example, consider a unit load at L_4. The values of H_c, V_c and x_o (at the crown) for this loading are known from D.P. No. 3. These val ues determine the magnitude, direction and point of application of the reaction at the crown. Now the force polygone AOC is constructed by putting AB = V_c ; BO = H_c and AC = 1. Then a line parallel to AO is drawn through the point of application at the crown until it intersects the line of action of L_1. Through this point of intersection another line is drawn parallel to BO and the equilibrium polygon for unit load at L_4 is complete.

Now the bending moment due to unit load at L_4 at any section can be calculated by scaling the vertical ordinate from the arch axis to this equilibrium polygon, and multiplying it by H_c.

D.P.No.5 Influence lines for moment and thrust could be drawn now. But usually maximum stress does not occur for maximum moment or maximum thrust. The true maximum occurs for loadings that make the algebraic sum of the stresses due to moment and thrust a maximum. Therefore, it is better to draw directly influence lines for outer fiber stress.

However, before the influence lines are drawn we don't know whether we should use the formulas of case I (namely $f_c = \frac{NK}{bt}$ and $f'_c = \frac{NK'}{bt}$ for compression over the entire cross section) or case II ($f_c = \frac{M}{Lbt^2}$ tension in part of the section). We shall assume first that case I applies for all sections. But if a great tension is obtained for a given loading, we shall draw influence lines separately for moment and thrust and find stresses as shown in D.P. No. 10.

The following formulas refer to case I; for n = 15

$$\begin{pmatrix} K \\ K' \end{pmatrix} = \frac{1}{1 + 15p_o} \begin{pmatrix} + \\ - \end{pmatrix} \frac{6}{1 + 28.8p_o}$$

D.P.No.6 Shows the influence lines for NK and NK' since these quantities are proportional to f_c and f'_c .

D.P.No.7 Computations giving maximum stresses assuming case I to apply at all sections. We consider the live loads which make the algebraic sum of the products of NK ordinates with the loading, a maximum. At the crown values are doubled because only half of the arch has been considered.

D.P.No.8 Gives the total maximum stresses assuming case I to apply, and considering the effect of temperature and rib shortening. We have considered four conditions of loading as indicated. The last lolumn gives the sections and loadings that need to be considered for a case II distribution of stress.

D.P.No.9 Takes up the cases where case II should be applied and finds the stresses by first finding total values of moment and thrust and then using the formula:

$$f_c = \frac{M}{Lbt^2}$$

where L is found from diagrams 3 to 6 in Hool vol. III.

D.P.No.10 Gives the influence lines for moment and thrust at the different sections.

D.P.No.11 Gives the design of the abutment. The two most extreme conditions are those loadings which cause maximum compression in the upper and lower fibers of the arch at the springing section. The moments and thrusts for this section are given on page . For every case the eccentricity e is found

$$e = \frac{M}{N}$$

From the center of the springing section e is laid off upward or downward (as the case may be) and the resultant of N and V obtained. This resultant is combined with the weight of the abutment (whose section is assumed) and the weight of the superstructure coming through the end column on the abutment. The resultant force should fall in the middle third of the base of the abutment.

Then EF is drawn perpendioulat to this pressure and the maximum pressure is given by:

$$\text{Max. pressure} = \frac{4EF - 6EG}{EF^2} \times R =$$

DEAD LOAD.

Weight of arch rib.

L_0	=	7 x 4.20 x 3	11800 Lbs.	8850
L_1	=	13 x 3.5 x 4 x 150 =	27300	20400
L_2	=	12 x 3.3 x 4 x 150 =	23800	17800
L_3	=	11.4x 3.2 x 4 x 150 =	21800	16300
L_4	=	10.6x 3.1 x 4 x 150 =	19700	14750
L_5	=	9.2x 3 x 4 x 150 =	16500	12400
L_6	=	5 x 3 x 4 x 150 =	9000 x 2	6750 x 2

Weight of columns.

L_0	=		4000 Lbs.
L_1	=	21 x 1.5 x 1.5 x 150 =	7100
L_2	=	13.6x 1.2 x 1.2 x 150 =	2940
L_3	=	8 x 1 x 1 x 150 =	1200
L_4	=	4 x .8 x .8 x 150 =	384
L_5	=	3 x 5 x 1 x 150 =	2250
L_6	=	3 x 5 x 1 x 150 =	1500 x 2

Combined wright of column and rib.

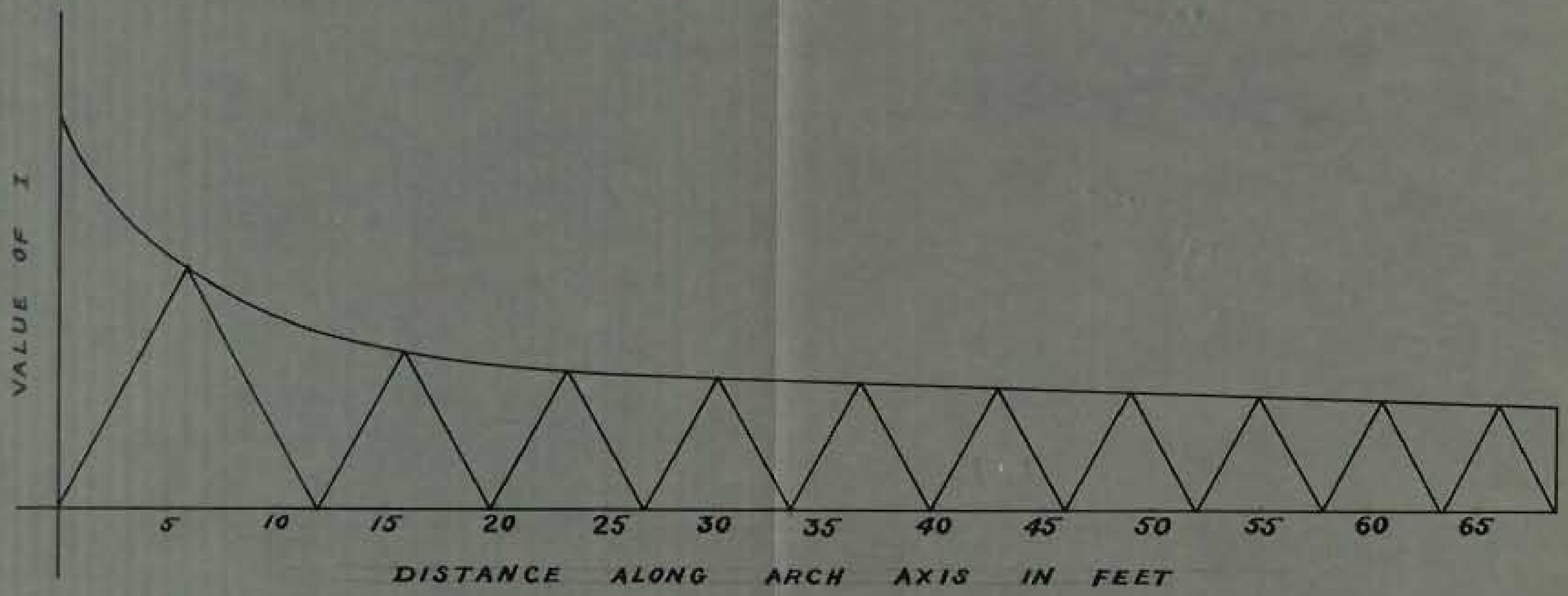
Supposed as acting under the column.

L_0	=	8850 + 4000	=	12850 Lbs.
L_1	=	20400 + 7100	=	27500
L_2	=	17800 + 2940	=	20740
L_3	=	16300 + 1200	=	17500
L_4	=	14750 + 384	=	15150
L_5	=	12400 + 2250	=	14650
L_6	=	13500 + 3000	=	16500

Total dead loads at the load points.

L_0	=	12850 + 17100	=	29950 Lbs.
L_1	=	27500 + 34200	=	61700
L_2	=	20740 + 34200	=	54950
L_3	=	17500 + 34200	=	51700
L_4	=	15150 + 34200	=	49350
L_5	=	14650 + 34200	=	48850
L_6	=	16500 + 34280	=	50700

Connected with Designing Plate I
(see drawings)



DIVISION OF ARCH RING FOR CONSTANT S/I

Measured length of half arch axis = 68.8 ft.

Rods - 1" square - 6 rods per foot width.

steel area = 3 x 6 = 18 sq. in.

$$P_o = \frac{18}{3 \times 3x144} = 0.0139 \text{ (at crown)}$$

$$A_s = \frac{18}{144} = 0.125 \text{ sq. ft.}$$

$$14a_s = 14 \times 0.125 = 1.75$$

$$I_s = \frac{(n-1)\ a_s \times d_1^2}{144} = \frac{14 \times 18 \times d_1^2}{144} = 1.75\ d_1^2$$

$$I_c = \frac{bd^3}{12} = \frac{3}{12} \times d^3 = \frac{d^3}{4}$$

Moment of Inertia Curve.

	d_s/d_c	d	d_1	d_1^2	I_s	I_c	I
0.00	1.000	3.00	1.37	1.88	3.28	6.75	10.03
0.05	1.007	3.02	1.38	1.90	3.32	6.90	10.22
0.15	1.021	3.062	1.406	1.97	3.44	7.17	10.61
0.25	1.035	3.11	1.43	2.05	3.58	7.52	11.10
0.35	1.049	3.142	1.441	2.07	3.62	7.76	11.38
0.45	1.063	3.19	1.47	2.16	3.78	8.10	11.88
0.55	1.077	3.22	1.48	2.22	3.88	8.35	12.23
0.65	1.095	3.287	1.518	2.31	4.03	8.87	12.90
0.75	1.145	3.432	1.591	2.54	4.44	10.05	14.49
0.85	1.245	3.735	1.742	3.04	5.32	13.00	18.32
0.95	1.406	4.22	1.980	3.92	6.85	18.70	25.55
1.00	1.500	4.5	2.12	4.50	7.85	22.70	30.55

Designing Plate II

DETERMINATION AT MOMENTS AND THRUSTS AT CROWN
(Unit Load at Definite Points.)

Pt	x	y	x	y	unit load at L_1 m	mx	my	unit load at L_2 m	mx	my	unit load at L_3 m	mx	my
L_1	50												
L_2	40												
L_3	30												
L_4	20												
L_5	10												
L_6	0												
1	55.3	25.4	3060	646	5.3	293	141	15.3	846	389	25.3	1400	643
2	46.3	19.4	2340	376				8.3	401	161	18.3	885	354
3	42.4	15.0	1800	225				2.4	102	36	12.4	526	185.6
4	36.6	11.1	1340	123							6.6	242	732.2
5	30.8	8.0	950	64							0.8	24.6	6.4
6	25.1	5.3	630	28.1									
7	19.4	3.1	376	19.6									
8	13.7	1.6	188	2.6									
9	8.1	0.6	65.5	0.4									
10	2.7	0.2	7.6	0.0									
		89.7	10757.1	1484.7	5.3	293	141	26.0	1349	586	63.4	3077.6	1262.

Pt	unit load at L_4 m	mx	my	unit load at L_5 m	mx	my	unit load at L_6 m	mx	my
1	35.3	1950	896	45.3	2501	1151	55.3	3060	1408
2	28.3	1365	549	38.3	1850	742	46.3	2340	935
3	22.4	950	335.5	32.4	1372	485	42.4	1800	635
4	16.5	607	184	26.6	972	295	36.6	1340	805
5	10.8	336	86.4	20.8	640	166.4	30.8	950	245.4
6	5.1	128	27	15.1	379	80.0	25.1	630	133
7				9.4	182	29.1	19.4	376	60
8				3.7	71.6	5.9	13.7	188	21.9
9							8.1	65.5	4.9
10							2.7	7.6	0.5
	118.5	5336	2077.9	191.6	7967.8	2954.4	282.4	10757.1	3849.7

Load at L_1.

$$H_c = \frac{n\ my - m\ y}{2\ n\ y - (\ y)^2} = \frac{10 \times 141 - 5.3 \times 89.7}{2 \times\ 10 \times 1484.7 - (89.7)^2} = 0.07$$

$$V_c = \frac{mx}{2\ x^2} = \frac{293}{2 \times 10757.1} = \frac{293}{21514} = .0136$$

$$M_c = \frac{m - 2H_c\ y}{2n} = \frac{5.3 - 2 \times 0.07 \times 89.7}{20} = -.365$$

$$x_o = \frac{M_c}{H_c} = \frac{-.365}{0.07} = -5.22$$

Unit load at L_2.

$$H_c = \frac{10 \times 586 - 26 \times 89.7}{13394} = \frac{3530}{13394} = .264$$

$$V_c = \frac{1349}{21514} = 0.0616$$

$$M_c = \frac{26 - 2 \times .264 \times 89.7}{20} = \frac{-21.4}{20} = 1.07$$

$$x_o = \frac{-1.07}{264} = -4.05$$

Unit load at L_3.

$$H_c = \frac{10 \times 1262.2 - 63.4 \times 89.7}{13394} = \frac{6932}{13394} = 0.51$$

$$V_c = \frac{3077.6}{21514} = .143$$

$$M_c = \frac{63.4 - 2 \times 0.51 \times 89.7}{20} = \frac{-28.2}{20} = -1.41$$

$$x_o = \frac{-1.41}{0.51} = -2.76$$

Unit load at L_4.

$$H_c = \frac{10 \times 2077.9 - 1185 \times 89.7}{13394} = \frac{10159}{13394} = 0.76$$

$$V_c = \frac{5336}{21514} = .248$$

$$M_c = \frac{118.5 - 2 \times 0.76 \times 89.7}{20} = \frac{-17.5}{20} = -0.86$$

$$x_o = \frac{-0.86}{0.76} = -1.13$$

Unit load at L_5.

$$H_c = \frac{10 \times 2954.4 - 191.6 \times 89.7}{13394} = \frac{12394}{13394} = 0.925$$

$$V_c = \frac{7967.6}{21514} = .37$$

$$M_c = \frac{191.6 - 2 \times 0.925 \times 89.7}{20} = \frac{25.6}{20} = 1.28$$

$$x_o = \frac{1.28}{0.925} = 1.38$$

Unit load at L_6.

$$H_c = \frac{10 \times 3849.7 - 2824 \times 89.7}{13394} = \frac{13197}{13394} = 0.985$$

$$V_c = \frac{10757.1}{21514} = 0.5$$

$$M_c = \frac{282.4 - 2 \times 0.985 \times 89.7}{20} = \frac{105.4}{20} = 5.27$$

$$x_o = \frac{5.27}{0.985} = 5.35$$

Designing Plate III

MOMENTS AND THRUSTS IN

CROWN AND SPRINGING

(unit loads at definite points.)

Load point.	Crown			Springing.		
Left half	V_d	$M = H_c V_d$	N	V_d	$M = H_c V_d$	N
L_0						0.728
L_1		- 0.365	0.07	- 110.3	- 7.73	0.765
L_2		- 1.07	0.264	- 36.7	- 9.66	0.86
L_3		- 1.41	0.51	- 14.6	- 7.45	0.972
L_4		- 0.86	0.76	- 4.1	- 3.12	1.07
L_5		- 1.28	0.925	- 1.2	- 1.11	1.095
L_6		- 5.27	0.985	- 4.8	- 4.73	1.04
Right half.						
L'_5				- 7.3	- 6.75	0.9
L'_4				- 9.2	- 7.00	0.7
L'_3				- 10.4	- 5.30	0.454
L'_2				- 11.5	- 3.03	0.223
L'_1				- 13.6	- 0.95	0.058

Unit Load at L_1

Pt.	t	P.	Left Half $\frac{x_0}{t} = \frac{MN}{Nt}$	K	NK	K'	NK'	Right Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	3.00	0.014	-1.74	8.126	-0.568	6.474	+0.453					
Spring	4.5	0.0104	-2.2	10.995	-8.4	9.255	+7.09	-3.66	17.765	+1.03	16.035	-0.93
1/4	3.2	0.013	+3.06	14.136	+ .82	12.464	- .722	-1.03	5.316	- .372	3.644	+ .255

Unit Load at L_2

Pt.	Left Half $\frac{x_0}{t}$	K	NK	K'	NK'	Right Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	-1.35	6.486	-1.71	4.834	+1.275					
Spring	-2.49	12.365	-10.625	10.635	+9.15	+2.98	14.665	+3.28	12.935	-2.89
1/4	-4.22	19.236	+ 3.94	17.564	-3.59	-1.19	6.036	-1.57	4.364	+1.14

Unit Load at L_3

Pt.	t	P.	Left Half $\frac{x_0}{t}$	K	NK	K'	NK'	Right Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	3.00	0.014	-0.92	4.686	-2.38	3.034	+1.545					
Spring	4.5	0.0104	-1.71	8.765	-8.51	7.035	+6.83	-2.88	12.766	+5.79	11.035	-5.01
1/4	3.2	0.013	+2.53	11.836	+7.28	10.164	-6.25	-1.13	5.756	-3.0	4.084	+2.13

Unit Load at L_4

Pt.	Left Half $\frac{x_0}{t}$	K	NK	K'	NK'	Right Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	-0.376	2.406	-1.83	0.754	+0.573					
Spring	-.645	3.845	-4.11	2.115	+2.26	+2.22	13.365	+9.35	11.635	-8.15
1/4	+0.72	3.925	+3.98	2.04	-2.07	-1.06	5.456	-4.32	3.784	+3.0

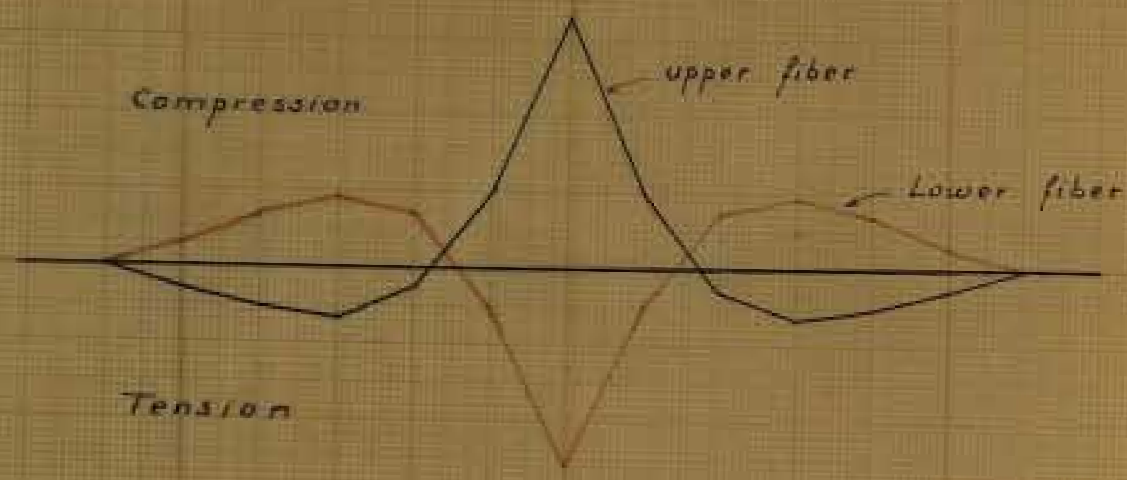
Unit Load at L_5

Pt.	t	P.	Left Half $\frac{x_0}{t}$	K	NK	K'	NK'	Right Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	3.00	0.014	+0.46	2.756	+2.55	1.104	-1.02					
Spring	4.5	0.0104	+0.4	8.715	+3.97	0.985	-1.08	+1.7	8.715	+7.85	6.985	-6.28
1/4	3.2	0.013	-.218	1.786	-1.98	.114	+.128	- .905	4.77	-4.74	3.104	+3.08

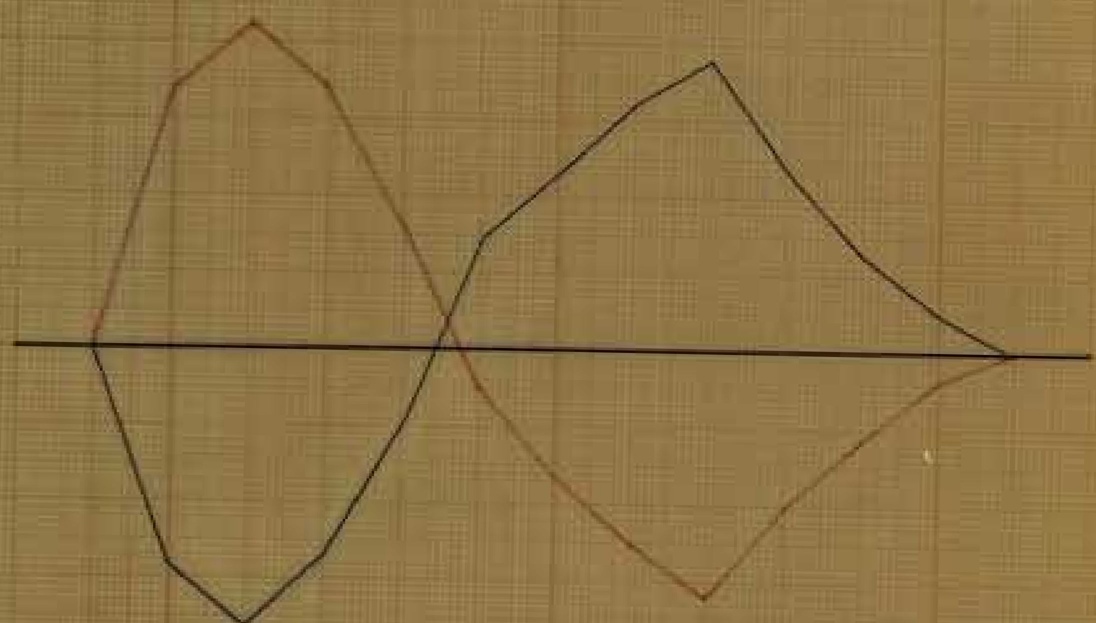
Unit Load at L_6

Pt.	Left Half $\frac{x_0}{t}$	K	NK	K'	NK'
Crown	+1.78	8.296	+8.15	6.644	-6.55
Spring	+1.02	5.585	+5.582	3.855	-4.02
1/4	- .655	3.698	-4.08	2.024	+2.24

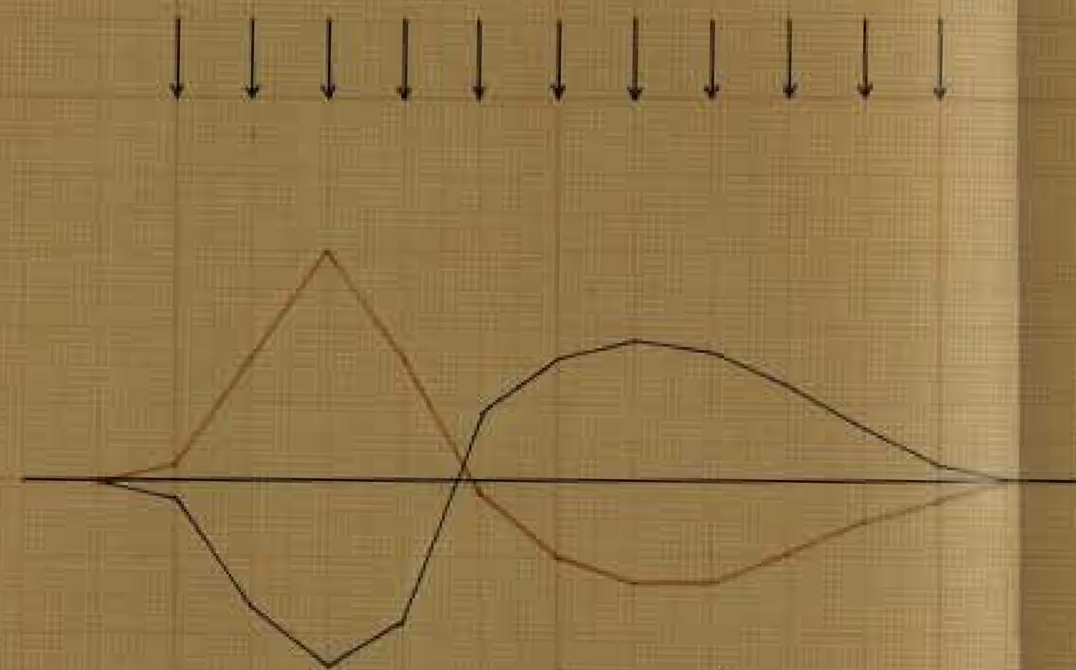
Designing Plate V



Crown



Springing



Quarter point

Designing Plate VI

MORE ACCURATE CALCULATION OF STRESSES

Crown. Loading No. 1. (maximum comp. in upper fiber.)

Upper fiber.

D.L.

			Comp.	tens.
L_1	=	.453 x 61700		- 27900
L_2	=	1.275 x 54940		- 70000
L_3	=	1.545 x 51700		- 80000
L_4	=	0.573 x 49350		- 28200
L_5	=	2.55 x 48850	+ 124500 x 2	
L_6	=	8.16 x 50700	+ 415000	
			+ 664000	- 206100 x 2 = 251800

L.L.

			Comp.	tens.
L_6	=	8.16 x 36600	+ 298000	
L_5	=	2.55 x 8410	+ 21400	
L_4	=	0.573x 3780		- 2160
			+ 319400	- 2160 = 317240

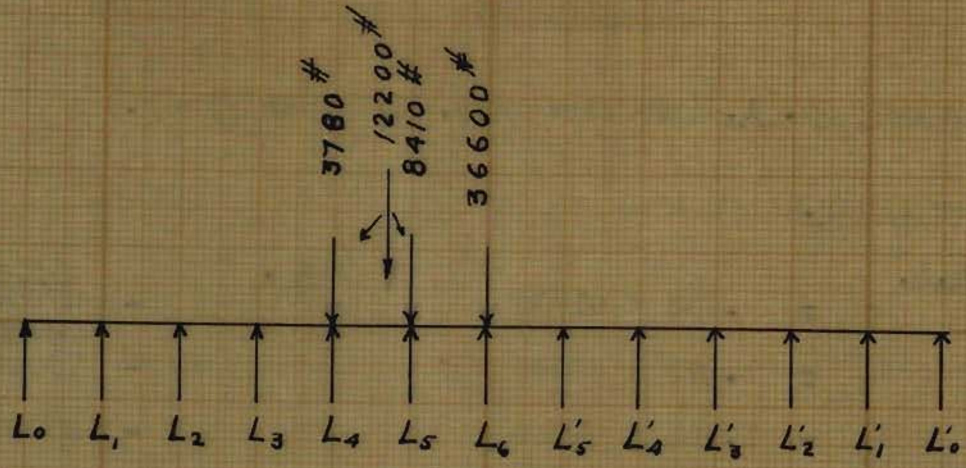
Stresses.

$$\text{D.L.} = \frac{+\ 251800}{3 \times 3 \times 144} = 195 \text{ Lbs./in}^2$$

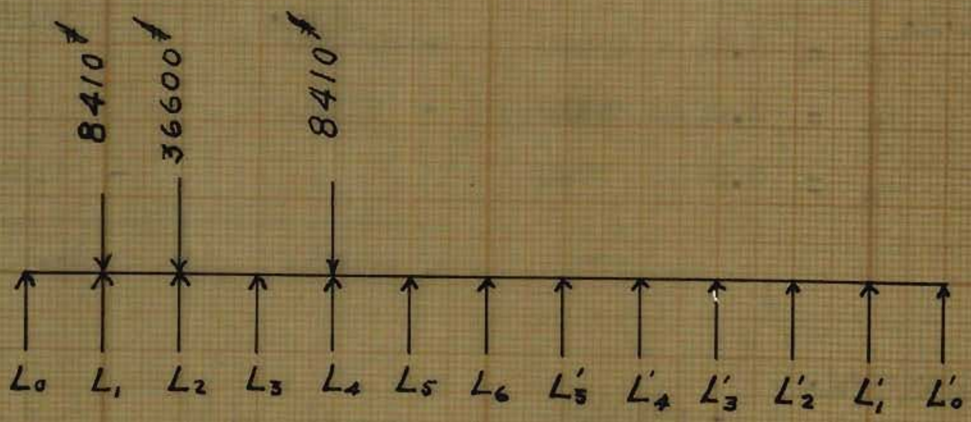
$$\text{L.L.} = \frac{317240}{3 \times 3 \times 144} = 244 \text{ Lbs./in}^2$$

$$\text{Total} = 440 \text{ Lbs./in}^2$$

Designing Plate VII (p. 59 - 67)



Crown - Loading No. 1



Crown - Loading No. 2

Fig. 19

Lower fiber.

L.L.			Comp.	tens.
L_6	=	6.55 x 36600		- 240000
L_5	=	1.02 x 8410		- 8600
L_4	=	1.83 x 3780	+ 6910	
			+ 6910	- 248600 = - 241700

Stress. = $\frac{241700}{3 \times 3 \times 144}$ = - 186 Lbs./in^2 tension

Net stress = 195 - 186 = + 9 Lbs./in^2 safe

Crown. Loading No. 2. (maximum comp. in lower fiber.)

Lower fiber.

D.L.			Comp.	Tens.
L_1	=	0.568 x 61700 x 2	+ 70500	
L_2	=	1.71 x 54940 x 2	+ 187500	
L_3	=	2.38 x 51700 x 2	+ 247000	
L_4	=	1.83 x 49350 x 2	+ 180000	
L_5	=	1.02 x 48850 x 2		- 99500
L_6	=	6.55 x 50700		-332000
			+ 685000	- 431500 = 253500

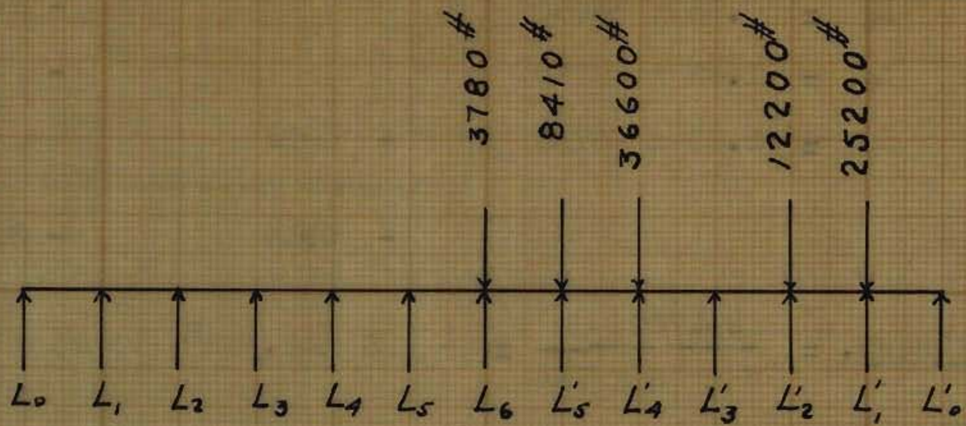
L.L.			Comp.
L	=	1.71 x 36000	+ 62500
L_4	=	1.83 x 8410	+ 15400
L	-	0.568 x 8410	+ 4780
			82680

Stresses.

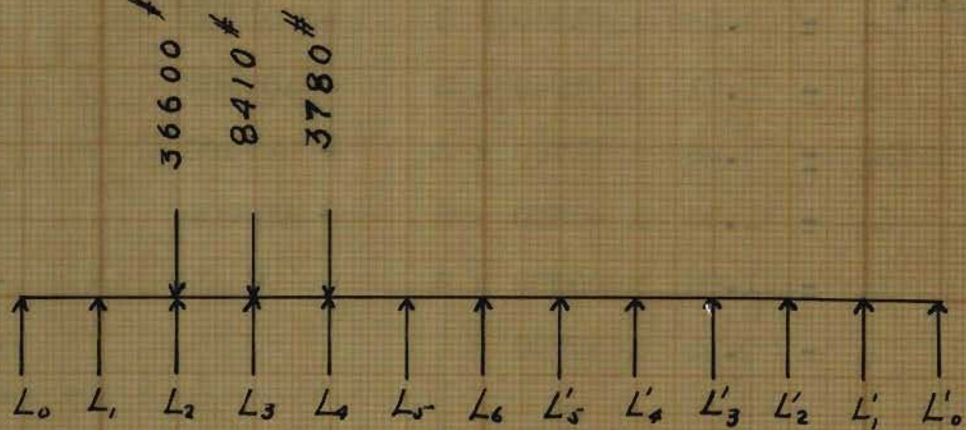
D.L. = $\frac{253500}{3 \times 3 \times 144}$ = 195 Lbs./in^2

L.L. = $\frac{82680}{3 \times 3 \times 144}$ = 64 Lbs./in^2

259 Lbs./in^2



Springing - Loading No 1



Springing - Loading No 2

Upper Fiber.

L.L.			tens.
L_1	=	0.453 x 8410	3810
L_2	=	1.275 x 36600	46600
L_4	=	0.573 x 36600	20900
			71310

$$\text{Stress} = \frac{71310}{3 \times 3 \times 14\ 4} = 55 \text{ Lbs./in}^2 < 195 \quad \text{very safe}$$

Net stress = 140 Lbs./in^2

Springing. Loading No. 1. (max. comp. in upper fibers.)

Upper fiber.

D.L.			Comp.	tens.
L_1	=	7.09 x 61700		- 437000
L_2	=	9.15 x 54940		- 502000
L_3	=	6.83 x 51700		- 353000
L_4	=	2.26 x 49350		- 111000
L_5	=	3.97 x 48850	+ 194000	
L_6	=	5.582 x 50700	+ 283000	
L_5	=	7.85 x 48850	+ 383000	
L_4	=	9.35 x 49350	+ 461000	
L_3	=	5.79 x 51700	+ 299000	
L_2	=	3.26 x 54940	+ 178000	
L_1	=	1.03 x 61700	+ 63500	
			+ 1861500	- 1403000 = 4585 00

L.L.

$L_6 = 5.582 \times 3780 = 21100$

$L_5 = 7.85 \times 8410 = 66000$

$L_4 = 9.35 \times 36600 = 342000$

$L_2 = 3.26 \times 12200 = 39700$

$L_1 = 1.03 \times 25200 = 25900$

494700

Stresses.

$$D.L. = \frac{458500}{3 \times 4.5 \times 144} = 236 \text{ Lbs./in}^2$$

$$L.L. = \frac{494700}{3 \times 4.5 \times 144} = 254 \text{ Lbs./in}^2$$

490 Lbs./in^2

Lower fiber.

D.L.

$L_6 = 4.02 \times 3780 = 15200$

$L_5 = 6.28 \times 8410 = 52800$

$L_4 = 8.15 \times 36600 = 298000$

$L_2 = 2.89 \times 12200 = 35200$

$L_1 = 0.93 \times 25200 = 23400$

424600

$$L.L. = \frac{424600}{3 \times 4.5 \times 144} = -218 \text{ tension}$$

net $= -218 + 166 = -52$ Lbs./in^2

Springing. Loading No. 2. (max. comp. in lower fibers.)

Lower fiber.

D.L.

					Comp.	tens.
L_1	=	8.4	x	61700	+ 518000	-
L_2	=	10.62	x	54940	+ 583000	
L_3	=	8.51	x	51700	+ 451000	
L_4	=	4.11	x	49350	+ 202000	
L_5	=	1.08	x	48850		- 52600
L_6	=	4.02	x	50700		- 204000
L_5	=	6.28	x	48850		- 307000
L_4	=	8.15	x	49350		- 402000
L_3	=	5.01	x	51700		- 259000
L_2	=	2.89	x	54840		- 159000
L	-	0.93	x	61700		- 57400

1754000 - 1441000 = 313000

L.L.

L_2 = 10.62 x 36600 = 390000

L_3 = 8.51 x 8410 = 71500

L_4 = 4.11 x 3780 = 15500

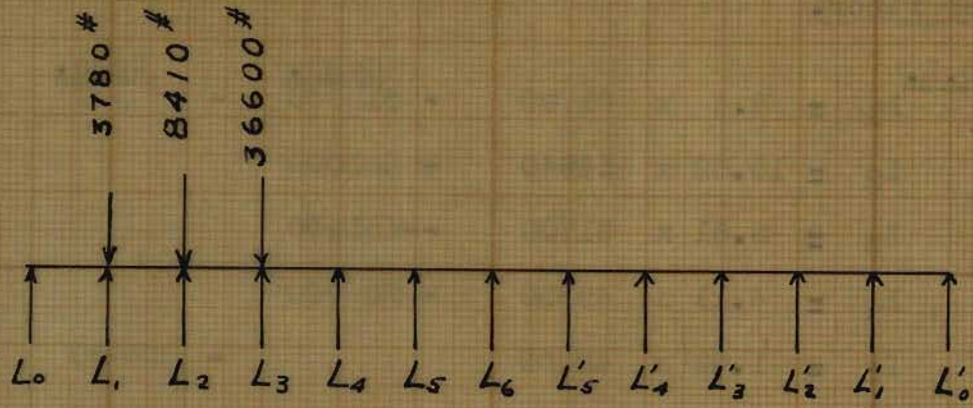
477000

Stresses.

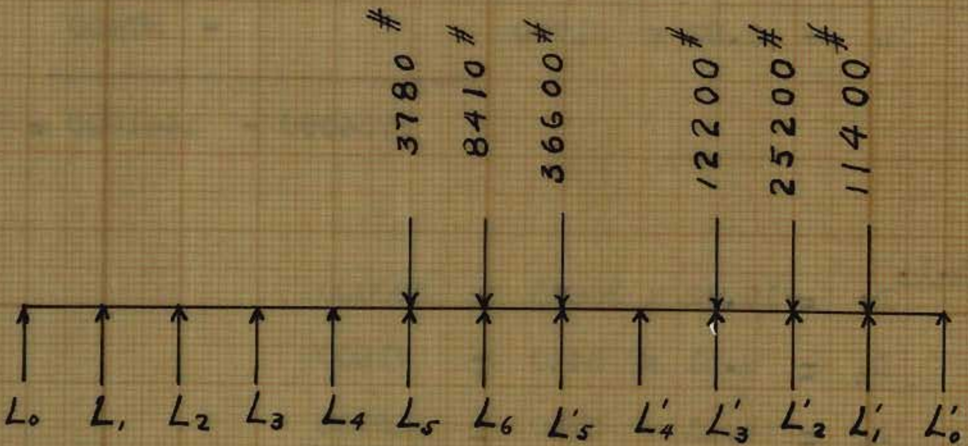
$$D.L. = \frac{313000}{3 \times 4.5 \times 144} = 166 \text{ Lbs./in}^2$$

$$L.L. = \frac{477000}{3 \times 4.5 \times 144} = 246 \text{ Lbs./in}^2$$

412 Lbs./in^2



Quarter point - Loading - N°1



Quarter point - Loading N°2

Upper fiber.

L.L.

L_2 = 9.15 x 36600 = 335000

L_3 = 6.83 x 8410 = 57400

L_4 = 2.26 x 3780 = 8550

400950

$$\frac{400950}{3.x\ 4.5 \times 144} = 206 \text{ Lbs./in}^2 \text{ tension}$$

net = - 206 - 236 = - 30 Lbs./in^2 O.K.

Quarter point. Stresses. Loading No. 1. (max. comp. in upper fibers.)

Upper fibers.

D.L.

	Comp.	tens.
L_1 = .82 x 61700 =	+ 50600	
L_2 = 3.94 x 54940 =	+216000	
L_3 = 7.28 x 51700 =	+376000	
L_5 =-1.26 x 48850 =	6150	
L_4 = 3.98 x 49350 =	+196000	
L_6 = 2.24 x 50700 =		- 113000
L_5 = 3.08 x 48850 =		- 150000
L_4 = 3.0 x 49350 =		- 148000
L_3 = 2.13 x 51700 =		- 110000
L_2 = 1.14 x 54940 =		- 63500
L_1 = .255 x 61700 =		- 15700
	+ 844750	- 599200 = 245550

L.L.

L_1 = .82 x 3780 = 3100

L_2 = 3.94 x 8410 = 33100

L_3 = 7.28 x 36600 = 266000

302200

Stresses.

$$D.L. = \frac{245550}{3.2 \times 3 \times 144} = 177 \text{ Lbs./in}^2$$

$$L.L. - \frac{302200}{3.2 \times 3 \times 144} = 218 \text{ Lbs./in}^2$$

395 Lbs./in^2

Lower fiber.

L_1 = .722 x 3780 = 2730

L_2 = 3.59 x 8410 = 30200

L_3 = 6.25 x 36600 = 228000

260930

$$\text{Stress} = \frac{260930}{3.2 \times 3 \times 144} = 189 \text{ tension}$$

Net = 250 - 189 = 61 Lbs./in^2 O.K.

Loading No. 2.(max. comp. in lower fiber.)

Lower fiber.

D.L.

			Comp.	tens.
L_1	=	.722 x 61700		- 44500
L_2	=	3.59 x 54940		- 197000
L_3	=	6.25 x 51700		- 323000
L_4	=	2.07 x 49350		- 102000
L_5	=	1.98 x 48850	+ 96600	
L_6	=	4.08 x 50700	+ 207000	
L_5	=	4.74 x 48850	+ 231000	
L_4	=	4.32 x 49350	+ 213000	
L_3	=	3.0 x 51700	+ 155000	
L_2	=	1.57 x 54940	+ 86100	
L_1	=	.372 x 61700	+ 23000	
			+ 1011700	- 666500 = 345200

L.L.

L_5	=	1.98 x 3780	=	7500
L_6	=	4.08 x 8410	=	34400
L_5	=	4.74 x 36600	=	173000
L_3	=	3.0 x 12200	=	36600
L_2	=	1.57 x 25200	=	39500
L_1	=	.372 x 11400	=	4240
				294240

<u>Stresses.</u>

$$D.L. = \frac{345300}{3.2 \times 3 \times 144} = 250 \text{ Lbs./in}^2$$

$$L.L. = \frac{294240}{3.2 \times 3 \times 144} = 213 \text{ Lbs./in}^2$$

Total 463 Lbs./in^2

<u>Upper fiber.</u>

L_5 = .126 x 3780 = 476

L_6 = 2.24 x 8410 = 18800

L_5 = 3.08 x 36600 = 112000

L_3 = 2.13 x 12200 = 26000

L_2 = 1.14 x 25200 = 28700

L_1 = .255 x 11400 = 2810

188786

$$\text{Stress} = \frac{188786}{3.2 \times 3 \times 144} = 136 \text{ Lbs./in}^2 \text{ tension}$$

Net = 177 - 136 = 41 Lbs./in^2 comp. O.K.

Designing Plate VIII

Pt	y	t	P_o	Temperature fall of 40° F, Moments and Thrusts for rise of 40° of opposite sign. H_c - 11400, M_c -102000: $H_c y$	M	N Scaled	Rib Shortening: M	N (-6630)	Fall of temperature and rib shortening $f_c = \frac{NK}{bt}$ $f'_c = \frac{NK'}{bt}$: M	N	$\frac{x_o}{t}$	K	K'	Stresses upper fiber	Stresses lower fiber	Rise of temp. and rib shortening. $\frac{-1 + .58}{+1 + .58} = -0.26$ Stresses upper fiber	Stresses lower fiber	Total maximum stresses Case I (a) Loading upper	(a) Loading lower	(b) Loading upper	(b) Loading lower	(c) Loading upper	(c) Loading lower	(d) Loading upper	(d) Loading lower	Section and loadings for which case I does not apply
crown	0	3.00	0.014		102000	- 11400	+ 59300	-6630	+161300	-18030	- 2.99	12.9	11.27	+158	-180	-40.2	-47.9	598	- 171	400	57	417	-40	219	188	a , c
spring	30	4.50	0.0104	- 34200	240000	- 7800	- 139000	-4530	-379000	-12330	+ .684	4.015	2.285	-254	+144	+67.5	-38.3	236	92	558	-90	-224	556	98	379	b , d
1/4	7.6	3.2	0.013	- 86500	15500	- 10300	+ 8990	-5980	+24490	-16280	- .47	2.886	1.214	+ 14.3	- 33.9	- 3.8	+ 9.0	409	27	391	70	55	429	37	472	

Loading.

a = No. 1 loading, fall of temp. and rib shortening.

b = " " " rise of temp. and rib shortening.

c = No. 2 loading, fall of temp. and rib shortening.

d. = " " " rise of temp. and rib shortening.

Temperature fall.

$$\frac{I}{S} = \frac{22}{11.9} = 1.85$$

$$H_c = \frac{-1.85 \times .000006 \times 40 \times 120 \times 10 \times 2000000 \times 144}{2 \quad 10 \times 1484.7 - (89.7)^2} = 11400\#$$

$$M_c = -\frac{H_c}{n} = -\frac{11400 \times 89.7}{10} = 102000 \text{ ft. lbs.}$$

Rib shortening.

Ca at crown. (assume uniform weight of 80 ft² lbs.)

assume L.L. on ½ of span.

on each column: L.L. = 10 x 12 x 80 = 9600 lbs

say 10000 lbs.

	D.L. =	D.L. + L.L.	
L_1	= 61700	61700 + 10000	= 71700
L_2	= 54940	54940 + 10000	= 64940
L_3	= 51750		= 61700
L_4	= 49350		= 59350
L_5	= 48850		= 58850
L_6	= 50700		= 60700

Ca at crown.

L_1 = 0.07 x 133400 = 93500

L_2 = 0.264 x 119880 = 21600

L_3 = 0.51 x 113400 = 57900

L_4 = 0.76 x 108700 = 82500

L_5 = 0.925 x 107700 = 99500

L_6 = 0.985 x 111400 = 109900

464900

$$Ca = \frac{464900}{93 \times 3 - 15 \times \frac{18}{144}} = \frac{464900}{10.88} = 42700 \text{ ft}^2\text{ lbs}$$

Ca at spring.

L_1 = ½ (0.765 + 0.058) = 0.411 x 133400 = 44900

L_2 = ½ (0.86 + 0.223) = 0.541 x 119880 = 65000

L_3 = ½ (0.972 + 0.454) = 0.713 x 113400 = 81000

L_4 = ½ (1.07 + 0.7) = 0.88 x 108700 = 95500

L_5 = ½ (1.095 + 0.9) = 0.888 x 107700 = 95700

L_6 = ½ (1.04 + 1.04) = 1.04 x 111400 = 116000

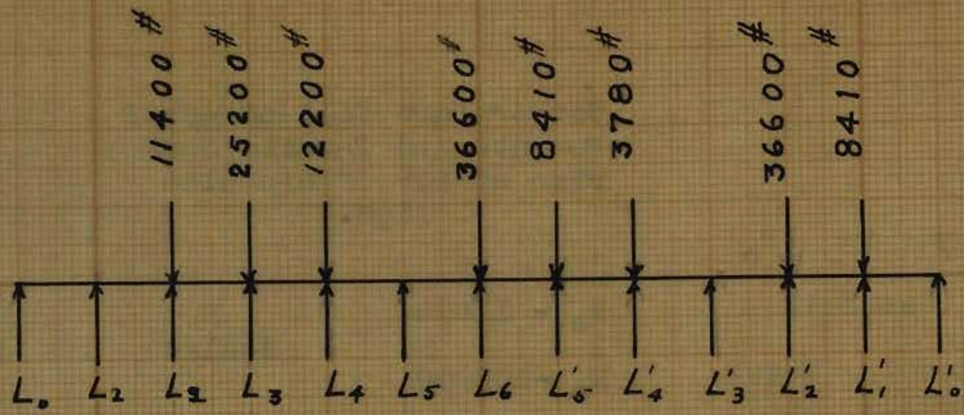
498100

$$Ca - \frac{498100}{3 \times 4.5 + 15 \times \frac{18}{144}} \quad \frac{498100}{15.38} - 32400 \text{ ft}^2 \text{ lbs.}$$

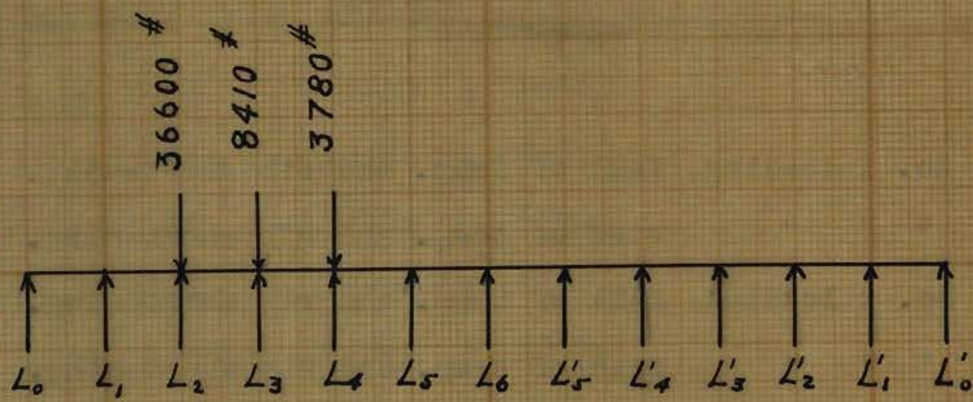
Average Ca = 42700 - 32400 = 75100 : 2 = 40000 ft² lbs

$$H = -\frac{1.85 \times 40000 \times 120 \times 10}{13394} = -6630 \text{ Lbs.}$$

$$\text{ratio } \frac{H_c \text{ rib}}{H \text{ temp.}} = \frac{6630}{11400} = 0.58$$



Crown (case a)



Springing (case c)

CALCULATION OF M AND N TO FIND

TOTAL MAXIMUM STRESSES FOR CASE II CONDITIONS.

Crown - (case a)

M_c.

L_1 = - 0.365 x 61700 x 2 = - 45000

L_2 = - 1.07 x 54940 x 2 = -118000

L_3 = - 1.41 x 51700 x 2 = -146000

L_4 = - 0.86 x 45570 x 2 = -78400

L_5 = + 1.28 x 57260 x 2 = +146000

L_6 = + 5.27 x 87100 = +459000

-387400 + 605000 = 217600

Exact calculations for H_c.

L_1 = 0.07 x 131810 = 9220

L_2 = 0.264 x 157880 = 41700

L_3 = .51 x 128600 = 65500

L_4 = .76 x 114630 = 87000

L_5 = 0.925 x 106110 = 98900

L_6 = 0.985 x 87300 = 86000

367420

M_c = + 217600

H_c = + 367420

M_{tr} = + 161300

H_{tr} = - 18030

Designing Plate IX

(p. 69 - 72)

Totals.

$M = 217600 + 161300 = 378900$ ft. lbs.

$H = 367420 - 18030 = 349390$ lbs.

$$\frac{x_o}{t} = \frac{M}{Nt} \quad \frac{378900}{349390 \times 3} = 0.352$$

$P_o = 0.014$

$K = .72$

$L = .140$

$$f_c = \frac{M}{Lbt^2} = \frac{378900}{.140 \times 3 \times 3 \times 3 \times 144} = 695 \text{ Lbs./in}^2$$

say 700 Lbs./in² O.K.

$$f_s = nf_c \left(\frac{d}{Kt} - 1\right) = 15 \times 700 \quad \frac{2.875}{.72 \times 3} - 1$$

$15 \times 700 \ (1.28 - 1)$

$15 \times 700 \times .28 = 2830$ Lbs./in² O.K.

Springing. (case c)

Moments.

L_1	=	- 7.73	x	61700	=	- 477000	
L_2	=	- 9.66	x	91540	=	- 886000	
L_3	=	- 7.45	x	60110	=	- 448000	
L_4	=	- 3.12	x	53130	=	- 166000	
L_5	=	- 1.11	x	48850	=		+ 54200
L_6	=	+ 4.73	x	50700	=		+ 240000
L_5	=	+ 6.75	x	48850	=		+ 330000
L_4	=	+ 7.0	x	49350	=		+ 346000
L_3	=	+ 5.3	x	51700	=		+ 274000
L_2	=	+ 3.03	x	54940	=		+ 166000
L_1	=	+ 0.95	x	61700	=		+ 58600

$- 1977000 + 1418800 = -559000$

Thrust.

L_0	=	0.725	x	29950	=	21700
L_1	=	0.765	x	61700	=	47200
L_2	=	0.86	x	91540	=	78500
L_3	=	0.972	x	60110	=	58500
L_4	=	1.07	x	53130	=	57000
L_5	=	1.095	x	48850	=	53500
L_6	=	1.04	x	50700	=	52700
L_5	=	0.9	x	48850	=	43900
L_4	=	0.7	x	49350	=	34500
L_3	=	0.454	x	51700	=	23500
L_2	=	0.223	x	54940	=	12200
L_1	=	0.058	x	61700	=	3580
						487780

Shear.

L_1	=	.06	x	61700	=	3700	
L_2	=	.23	x	91540	=	21000	
L_3	=	.4	x	60110	=	24000	
L_4	=	.72	x	53130	=	38200	
L_5	=	.92	x	48850	=	45000	
L_6	=	1.05	x	50700	=	53200	
		.37	x	50700	=		+ 18800
L_5	=	.42	x	48850	=		+ 20600
L_4	=	.38	x	49350	=		+ 18700
L_3	=	.27	x	51700	=		+ 14000
L_2	=	.15	x	54940	=		+ 8200
L_1	=	.04	x	61700	=		+ 2400

- 185100 + 82700 = - 102400

M_s = - 559000

H_s = - 487780

M_{tr} = - 379000

H_{tr} = - 12330

<u>Totals.</u>

M_s = - 559000 - 379000 = - 938000

H_{tr} = 487780 - 12330 = 475450

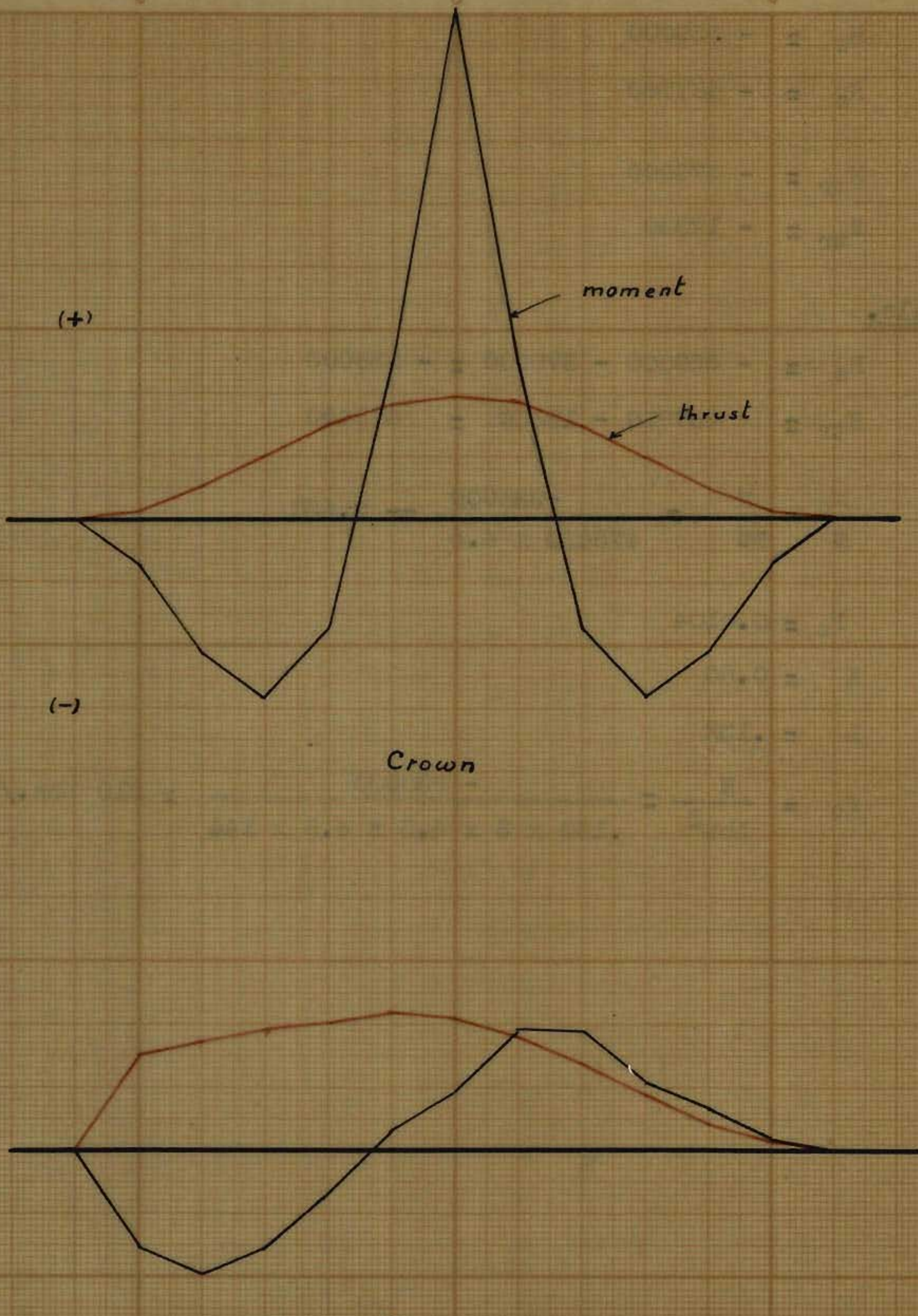
$$\frac{x_o}{t} = \frac{M}{Nt} = \frac{-938000}{475450 \times 4.5} = 0.438$$

P_o = 0.0104

K = 0.60

L = .132

$$f_c = \frac{M}{Lbt^2} = \frac{-938000}{.132 \times 3 \times 4.5 \times 4.5 \times 144} = 810 \text{ Lbs./in}^2$$



Designing Plate X

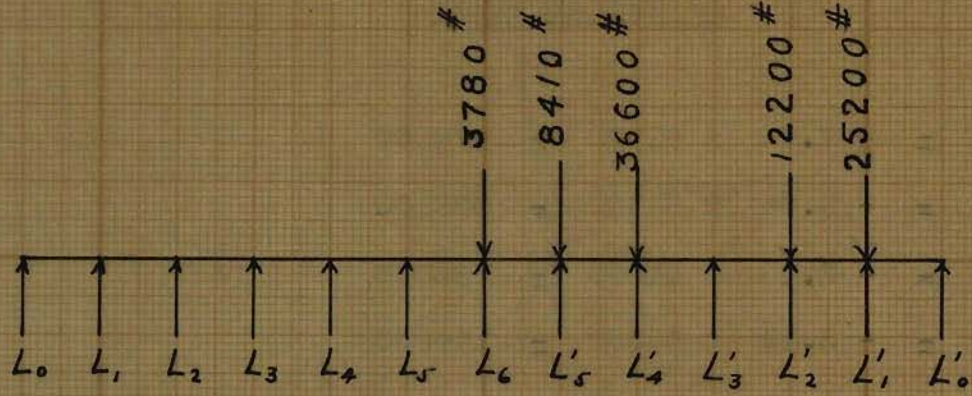
SPRINGING MOMENT, THRUST AND SHEAR
FOR MAXIMUM STRESS IN UPPER FIBER USED IN THE DESIGN OF THE ABUTMENT

L_1	=	− 7.73	x 61700	= − 478000		
L_2	=	− 9.66	x 54940	= − 530000		
L_3	=	− 7.45	x 51700	= − 386000		
L_4	=	− 3.12	x 49350	= − 154000		
L_5	=	+ 1.11	x 48850	=	+ 54300	
L_6	=	+ 4.73	x 54480	=	+ 257000	
L_5	=	+ 6.75	x 57260	=	+ 386000	
L_4	=	+ 7.0	x 85950	=	+ 601000	
L_3	=	+ 5.3	x 51700	=	+ 274000	
L_2	=	+ 3.03	x 67140	=	+ 203000	
L_1	=	+ 0.95	x 86900	=	+ 82500	
				− 1548000	+ 1857800	= 309800

For shear.

L_1 = .06 x 61700 =	3700		L_6 = .37 x 54480 =	20200	
L_2 = .23 x 54940 =	12600		L_5 = .42 x 57260 =	24000	
L_3 = .4 x 51700 =	20600		L_4 = .38 x 85950 =	32600	
L_4 = .72 x 49350 =	35500		L_3 = .27 x 51700 =	14000	
L_5 = .92 x 48850 =	45000		L_2 = .15 x 67140 =	10000	
L_6 = 1.05 x 54480 =	57100		L_1 = .04 x 86900 =	3500	
−	174500	+		104300	= 70200

In Connection with Designing Plate XI (p. 73 - 75)
(D.P. XI is in the drawings.)



Loading, giving maximum positive moment at springing.

(used in the design of the abutment)

<u>For thrust.</u>

$L_0 = 0.725 \times 29950 = 21700$

$L_1 = 0.765 \times 61700 = 47200$

$L_2 = 0.86 \times 54940 = 47300$

$L_3 = 0.972 \times 51700 = 50400$

$L_4 = 1.07 \times 49350 = 52600$

$L_5 = 1.095 \times 48850 = 53500$

$L_6 = 1.04 \times 54480 = 56600$

$L_5 = 0.09 \times 57260 = 51500$

$L_4 = 0.7 \times 85950 = 60000$

$L_3 = 0.454 \times 51700 = 23500$

$L_2 = 0.223 \times 67140 = 15000$

$L_1 = 0.058 \times 86900 = 4950$

484250

$$e = \frac{309800}{484250} = .64$$

DESIGN OF ABUTMENTS - MAIN ARCH

Weight coming over the abutment.

D.L.			
	From 15 ft. span =	48900 / 2 =	24450 lbs.
	From 10 ft. span =	34200 / 2 =	17100 "
	Columns - 2 x 2 30 x 150	=	18000 "
			59550 lbs.
L.L.			36600 "
			96150 lbs.
		say	100000 lbs.

(See design of the abutment by the graphical method).

ARTISTIC FEATURES IN THE DESIGN OF THE T-BEAM AND ARCH BRIDGES.

T-BEAM GIRDER BRIDGE: (See elevation)

The esthetic treatment of the T-Beam girder bridge is limited. It will have a pleasing appearance if it looks heavy and solid, and if the height of the bridge is not great in proportion to the span. My design is that of a simple bridge with girders that impress the observer with their solidity, strength and well-proportioned dimensions. No decorative features are included, because their absence rather than their presence contributes to the pleasing appearance of the bridge.

OPEN SPANDREL ARCH BRIIGE: (See elevation)

In the design of the open-spandrel arch bridge equal importance was given to economy and appearance. The structure as a whole gives the effect of solidity and usefulness. These two main characteristics have to remain the dominant features of the bridge, and therefore care was taken to avoid all exaggerated ornamentations that might obscure them.

The main consideration from the esthetic viewpoint should be the expressiveness and pleasing appearance of the concrete bridge. To obtain this result, a symmetrical arrangement was given to the structure as a whole, and a style was secured that would be in proper harmony with the material used.

Symmetry: The bridge is symmetrical about the center line.

The dominating span is the parabolic arch of 120 ft. length. There are two approach spans, one on each side, with two arch spans of 15 ft. span. Obviously, the mind of the observer should be impressed with the expressiveness of the main span rather than be diverted to the small arches on both sides of it. At the same time, the continuity of the structre should not be marred by a sharp demarcation. Having these two facts in mind, the rectangular columns on both sides of the main span were designed to emphasize the dominance of the middle span without causing a break in continuity, because the mind of the observer will still be impressed by the monolithic structure at which his eyes are gazing, since care has been taken to choose moderate dimensions for the rectangular columns. Moreover, the engaged columns will stress the unity that exists between the approach spans and the main span.

Style: The bridge should have a style that conforms with its surroundings. During the design of the bridge I had in mind the Lebanese environment with its mountainous sites. In such a rugged countryside a bridge should have a simple and bold outline so that the surrounding rocky cliffs may not dwarf it.

The arches and the columns convey this simplicity and boldness.

In addition to the harmony of the structure with its surroundings, there should also be harmony of form, dimensions and material.

a) Harmony of form: I have selected an open-spandrel arch instead of an earth-filled one, because the bridge is rather high and the span is not short; furthermore, it is desirable to reduce the weight of the structure.

The form of arch ring is a parabola, because it most closely follows the true pressure curve of equilibrium.

The approach spans consist of semi-circular arches, because it would be an awkward combination to have an arch with girder approaches.

The small arches in the main span that support the roadway harmonize with the other units in giving consistency to the structure. In the ordinary case it is better to have a fill b between the crown and the roadway, bat in this case, this distance is rather great. I have therefore used a three-centred arch and have thus avoided the fill that that would have conveyed the impression of heaviness. Furthermore, to avoid the use of a column resting exactly on the crown, the three-centred arch was resorted to.

b) Harmony of dimensions: The rise bears a reasonable r relation to the length of the span. The arch ring increases in thickness from crown to springing. The arches in the approach spans are of small dimensions so that their secondary importance in the structure may be apparent.

c) Harmony of material: One kind of material is used, namely concrete, because it is difficult to secure an appearance of balance when different materilas are used.

Unlike many bridges, no effort was made to have the bridge resemble a masonry construction. Concrete is plain, without joints and projections, and therefore the structure was also made plain.

Ornamentation:

a) general: The general appearance of the bridge is of utmost importance. The details are secondary, and even if all ornamentation is omitted, the structure will retain its beauty as long as the lines are graceful and pleasing.

It will be noticed that the only ornamentation I have ventured to use is the set of horizontal lines on the two rectangular columns and on the abutments. The purpose was simply to emphasize these parts for their functional role. Otherwise they would be dwarfed by the more harmonious arches and columns with which they are surrounded.

b) The railing: The railing is the part that is seen both from the street and from the outside. Some decoration that is carefully designed would here be permissible. Its function is protection of traffic and hence I have made it to appear strong enough for the purpose. Iron pipe railings would have appeared flimsy and would give the impression of cheapness. The railing is entirely made of concrete. It consists of a base, coping, and central portion or dado. The coping and the base are of the same width, whereas the dado is more slender, with vertical spacings in between. There are four posts that divide the railing into three panels above the main span and a panel over each approach span. The number of panels over the main span is uneven so that there will be a panel and not a post over the center of the span.

PRICES AND ESTIMATE OF COST

We shall consider local prices as they are at present (Spring 1946).

Excavation: 3 - 4 meters deep in earth mixed with rock costs 4.50 LLS per cubic meter.

Plain concrete:

Gravel	0.80 m^3 at 12 LLS/m^3	= 10.00 LLS
Sand	0.40 " " 6 "	= 3.00 "
Cement	7 bags at 5 LLS	= 35.00 "
Labor per cubic meter		= 40.00 "
		88.00 LLS/m^3

Reinforced concrete :

Iron	100 kgs/m^3 at 0.40 LLS/kg	= 40.00 "
		128.00 LLS/m^3

In the case of the arch rib add 10 LLS extra for labor making a total of say 140 LLS/m^3.

Roadway:

Blockage , gravel, idealit, rolling, watering, and levelling = 6 LLS/m^2

Engineering: 15% of cost.

<u>Estimate of quantities for T-Beam Bridge:</u>

EXcavation = 20 m^3

Reinforced concrete

Let us find the amount of concrete per foot run of the bridge

Slab of roadway 93.5 x 20 = 1870 lbs/ft. run

Sidewalk = 700 "

Reinforced concrete (cont'd)

Slab of roadway 93.5 x 25	= 2340 lbs/ft run
sidewalk	= 700 "
Int. T-Beams 4 x $\frac{18 \times 32}{144}$ x 150	= 2400 "
Outside Beams 2 x 2 x 68/12 x 150	= 3400 "
	8840 "

370000 lbs. = 42 x

Abutments:

stem = 5000

base = 2700

7700 x 35 = 270000 lbs.

Wings, same. = 270000 ""

Total = 910000 lbs x $\frac{1}{2.2 \times 1000 \times 2.5}$

= 165 cubic metres.

Roadway 25 x 40 = 1000 ft^2 = 90 m^2

Estimate of Cost:

Excavation = 20 at 4.5 ££S	= 90 ££S
Reinforced concrete = 165 at 130 ££S	= 21400 "
Roadway = 90 at 6 ££S	= 540 "
Total	22030 ££S
Engineering 15 % = 0.15 x 22030	= 3305
Grand Total	25335 ££S

Estimate of quantities of the arch bridge:

Excavation:

4 abutments of arch ribs	=	1770 cu. ft.
2 abutments of approach spans	=	1500 " "
4 footings of columns	=	56 " "
95 cu. m.	=	3326 " "

Plain concrete:

4 abutments of arch ribs = 700 x 4 = 2800 cu. ft.

or 79 cu. m.

Reinforced concrete:

½ cross-sectional area running throughout the bridge.-

railing - average - 1' x 3'	=	3 sq. ft.
beam under railing 0.75 x 0.5	=	0.38 "
slab of sidewalk 0.3 x 6	=	1.8 "
Outside beams and average of curtain walls 1 x 4	=	4.0 "
slab of roadway 8/12 x 12	=	8.0 "
Middle T-Beam 0.5 x 25/12	=	1.04 "
		18.22 " x 2 x 180
		= 6550 cu. ft.

Columns - L = 20 x 2 x 2	x 4	=	320 "
L_1 = 20 x 1½ x 1½	x 2	=	90 "
L_2 = 14 x 1½ x 1½	x 2	=	70 "
L_3 = 8 x 1 x 1	x 2	=	16 "
L_4 = 4 x 1 x 1	x 2	=	8 "
L_5 = 2x 2 x 2	x 2	=	16 "
Tie beams 1 x 0.5 x 24	x12	=	144 "
Footings 14 x 1	x 4	=	56 "
			7220 cu. ft.

Forward -	7270	cu. ft.
Abutments at approaching spans assumed as much as T-beam bridge abutments, 1540x2=3080	"	"
Transverse beams 20 x 38 x 1/144 x 24 x 14 = 1770	"	"
340 cu.m. =	12120	cu. ft.

Reinforced concrete arch rib:

3 x 4 x 136 x2 = 3260 cu.ft. = 100 cu.m.

Roadway:

180 x 24 = 4300 sq. ft. = 400 sq.m.

<u>Estimate of cost</u>:

Excavation 95 m^3 say 100 m^3 at 4.5 ££S	= 450	££S
Plain concrete 80 m^3 at 90 "	= 7200	"
Reinforced concrete 340 m^3 at 130 "	= 44000	"
Reinf. concrete arch rib 100 m^3 at 140 ££S	= 14000	"
Roadway 400 m^2 at 6 ££S	= 2400	"
Total =	68100	££S
Engineering 15 % = 0.15 x 68100	= 10200	"
Grand Total	= 78300	££S

CONCLUSION

The problem of most concern to many engineers as well as to Public Works and Construction officials is the construction, with the least amount of expense,of a bridge that will not fail. It must be admitted that this attitude is justified, since economy of material and expenditure are very desirable in all types of construction. However, a superstructure that merely stays on its abutments without falling does not fulfill all the essential requirements of a bridge.

The bridges designed in this thesis clearly indicate that architectural considerations have sometimes required larger sections for columns, and therefore economy of material was of secondary importance there. At other places, it was desired to impress the observer with the solidity of the structure, and hence heavier members were used.Nevertheless, the estimate of cost shows that the bridges may be erected with reasonable expenditure. The results obtained are even cheaper than the current cost of bridges in the country. This is due to the fact that the assumption in cubic meters of excavation was relatively small.

The author is of the opinion that he has arrived at working results that are acceptable from the standpoint of safety, economy and pleasing appearance.

SPECIFICATIONS

1.- Loading:

Two systems of loading have to be considered.

A) The roadway is designed to carry a uniform load of:

$$p = (820 - 4\ L)\ kgs/m^2$$ where L = span in meters, with a minimum of 500 kgs/m^2 for L larger than 80 m.

B) The roadway is designed to carry a system composed of two trucks, each having the following characteristics:

Total load ----------------	16 tons
Rear axle load ------------	12 tons
Front axle load -----------	4 tons
Total length --------------	10 meters
Total width ---------------	2.5 "
Distance c.L. to c.L. axles --	4.0 "
Distance c.L. to c.L. wheels -	1.7 "

We will assume travelling side by side, and in the same direction as many of these systems as the width of the roadway permits.

The coefficient of impact is given by:

$$I = 1 + \frac{0.4}{1 + 0.2\ L} \qquad \frac{0.6}{1 + 4P/S}$$

where L = span; P = total D.L.; S = total L.L.

The two systems of loading have to be considered, and whichever gives the biggest results shall govenr the design.

2.- Unit Stresses:

f_s = 18000 p.s.i.; f_c = 700 p.s.i.

n = 15 u = 80 p.s.i.

v = 40 p.s.i. without diagonal tension reinforcement

= 120 p.s.i. with diagonal tension reinforcement.

(The transverse beams and the arch rib in the arch design are to be cast in concrete at 900 p.s.i. in order not to have very heavy beams.)

3.- Dimensions:

Dimensions are given for each case of design . In addition:

Bituminous wearing surface will have a thickness of 2".

2" crown.

4.- Weight of materials:

Reinforced concrete - 2500 kgs/m^3 or 150 lbs/cu.ft.

Wearing surface - 120 lbs/cu.ft.

5.- Distribution of Concentrated Loads:

The distribution of concentrated loads on concrete slab shall be calculated as follows:

a) The distribution of concentrated wheel loads for bending moments in reinforced concrete slabs on longitudinal girders shall be calculated by the formula

$$e = 2(1 + c)/3$$

with a maximum limit of 6 ft. for "e", where e = effective width (distance that the load may be considered as uniformly distributed on a line down the middle of the slab parallel to the supports) See figure

b) The distribution of concentrated wheel loads for bending moments in reinforeed concrete slabs with transverse girders shall be calculated by the formula

$$e = 2l/3 + c \qquad (2)$$

with a maximum limit of 6 ft. for "e", where e = effective width, l = span, and c = width of tire of cheel as defined in para a). See figure

c) The effective width for shear in beams carrying concentrated loads shall be taken the same as for bending moment as calculated by formula (1) or formula (2), with a minimum effective width of 3 ft. and a maximum effective width of 6 ft.

6.- Temperature stresses:

Reinforced concrete arches, frames and other restrained structures shall be designed for a variation in temperature of ± 40 degrees F.

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