DIAGNOSIS OF ERRORS IN FUNDAMENTAL OPERATIONS IN ARITHMETIC

(Based on Research in UNRWA/UNESCO Elementary Schools in Lebanon)

BY

FAWZI AHMED JAMAL

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BY
FAWZI AHMED JAMMAL

Approved:

Robert Young
Dr. Robert Young, Advisor

F R Kerr
Dr. Frederick Korr, Member of Committee

Yakub Namek
Dr. Yakub Namek, Member of Committee

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ABSTRACT

This study was made with the aim of helping the teacher of arithmetic locate the specific weaknesses of his pupils in the basic facts and in the four operations, through systematic and scientific means. To achieve this purpose, the writer constructed eleven tests covering all aspects of the subject under study. Tests 1-4 deal with the basic facts, test 5 consists of mixed sums and tests 6-11 deal with the four operations, using whole numbers.

The study was conducted on a random sample of pupils from six UNRWA/UNESCO elementary schools in Lebanon, chosen at random from about fifty such schools in the country. The tests, given in Arabic, were distributed as follows:

Tests 1-5 were given to grade 3.

Tests 5-9 were given to grade 4.

Tests 8A-11 were given to grades 5 and 6.

In grade 3 the pupils' knowledge of the basic number combinations was tested. In grade 4, the main purpose was testing the pupils' knowledge of the skills involved in the four operations (only easy steps in division are considered). In grades 5 and 6, the pupils' knowledge of the skills involved in multiplication and division were tested. The purpose of giving one or more of the tests to two or more different classes is to find a basis for comparison of the different classes. Aside from revealing information concerning the skills being tested, tests 6-11 also provide some, though very little, information concerning the pupils' knowledge of the basic number combinations.

The test results were tabulated in the most convenient form possible and the conclusions presented after each test. It is important to point out that more emphasis was placed on a qualitative rather than a quantitative analysis of the results. The reason for this was that the aim was to analyze and not to evaluate the pupils' knowledge of the skills involved in the tests used in this research. We were more concerned with the sums a pupil has wrong than with those he has right, no matter how rare the mistakes were. Due attention should be paid to the specific weaknesses of the individual pupil or group of pupils. Such individual difficulties are not easy to locate without the use of carefully constructed diagnostic tests encompassing every aspect of the subject.

As for the findings of this research, the writer thinks that tables 2-21 and the interpretation accompanying each table are worthy of study. Although space does not permit stating all the findings in a single paragraph, one may point out to the main types of errors revealed.

In tests 1-4, two main weaknesses were uncovered:

i) Difficulties in the zero-facts. The percentages of errors were 8.0%, 22.2%, 28.5%, and 29.3% in the zero-facts in tests 1, 2, 3, and 4 respectively.

ii) Difficulties due to lack of understanding of the relationships among the different number combinations in the respective tests.

As for the graded tests in the four operations, the common types of errors were*:

A. COMMON ERRORS IN ADDITION

1. Errors in addition combinations
2. Errors in bridging
3. Omitting one or more digits
4. Disregarding column position
5. Adding same digit in two columns.

B. COMMON ERRORS IN SUBTRACTION

1. Errors in subtraction combinations
2. Errors in bridging
3. Disregarding column position.

C. COMMON ERRORS IN MULTIPLICATION

1. Errors in multiplication combinations
2. Omitting digit in multiplier and/or multiplicand
3. Errors due to zero in multiplicand
4. Errors due to zero in multiplier
5. Errors in position of partial products
6. Errors in addition.

D. COMMON ERRORS IN DIVISION

1. Errors in the technique of division
2. Errors in division and/or multiplication combinations
3. Errors due to zero in the quotient

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*The types of errors are arranged in the descending order of importance.

4. Errors due to zero in the dividend
5. Using a remainder equal to or larger than divisor
6. Omitting final remainder
7. Errors in subtraction combinations.

It remains to be mentioned that tables 2-21 reveal, in a comprehensive form, that there is only a limited number of common types of errors which are responsible for the pupils' computational inaccuracies.

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In every classroom the arithmetic teacher finds pupils who at some stage of their work encounter difficulty and fail to make satisfactory progress. Some of them do not seem to be able to compute with speed and accuracy - some are slow but accurate, some are slow and inaccurate, and some are fast but inaccurate. These pupils are generally labelled 'stupid', 'lazy' or 'careless' by their teachers. This imprecise labeling is due to the fact that the schools or the teachers made little, if any, effort to go deep below the surfaces and search for the causes of difficulty. Besides, the pupils, regardless of their needs and individual differences in mental powers, learning rates, background, .. etc. are given the same kind of treatment. In so doing, such schools and teachers are strongly violating the findings of educational research which emphasize the nature of individual differences in their various aspects.

This failure to diagnose the difficulties of the pupils in arithmetic allows errors to accumulate and accompany the pupil to the higher grades. This failure is a consequence of the teachers' failure to diagnose the difficulties of their pupils and, on the basis of the results obtained, to do the necessary remedial work. Thus the modern teacher of arithmetic is in need of scientific tools and devices for making a diagnosis of ailments in the subject and for setting down the remedial work needed.

The main purpose of this study is to help teachers of arithmetic in the prevention, diagnosis and remedy of pupils' difficulties in the subject. To succeed in the prevention of difficulties in arithmetic, the teacher should be aware of the

various factors which enter into arithmetic ability. Thus in Chapter II we will survey and discuss the main factors which compose arithmetic ability in the light of the findings of educational research. However, in practice the arithmetic curriculum and methods of teaching in so many schools do not take these findings into due consideration. In other words our curriculum and the methods of teaching are generally lagging behind research findings. Thus prevention of errors up to a satisfactory degree is not so easy.

Therefore, there is urgent need on the part of the teacher of arithmetic to make use of diagnostic and remedial techniques in order to discover and remedy the difficulties of his pupils. For it should be remembered that if the pupil's difficulties are left to accumulate, the outcomes might be quite serious. The learning of arithmetic is similar to the climbing of a ladder and if one does not succeed in climbing the first steps then it is not possible for him to reach higher steps. If the child, therefore, experiences repeated failure in the subject during the first stages in learning the subject, he will probably develop negative attitudes towards the subject or even towards his teacher and the school as a whole.

The teacher can realize, therefore, the significance of diagnosing the difficulties of his pupils before it is too late or at least difficult to offer them adequate help. For this purpose the writer has constructed systematic diagnostic tests covering all the basic number combinations in the four processes and all the important steps in each process after the pattern

used by Schonell.[1] Instead of being satisfied by the old hit-or-miss procedure for spotting pupils' difficulties, the teacher has now at hand a scientific and systematic tool to locate any deficiency in the pupils' knowledge in every specific aspect of the process under consideration. Having located the pupils' difficulties, the teacher may now plan for the necessary remedial work for the various kinds of difficulties thus revealed. In so doing, the teacher of arithmetic is undertaking measures which lead to the prevention or at least to the minimizing of backwardness among his pupils.

This study was conducted in six UNRWA/UNESCO schools in Lebanon chosen at random from about fifty such schools in the country.

Several officials contributed in making this study a success. Especially, I wish to thank the officials of the UNRWA/UNESCO Institute of Education in Beirut, the Education Section at the Lebanon Office, the officers of the respective areas, and the headmasters and teachers of the schools I have visited, without whose great help and cooperation this study would not have been possible.

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[1] Schonell, Fred J., A) Diagnosis and Remedial Teaching in Arithmetic: C. Oliver and Boyd, Edinburgh: Tweeddale Court, 1957.

Schonell, Fred J., B) Diagnostic and Attainment Testing, Oliver and Boyd, Edinburgh: Tweeddale Court, 1963, 4th edition.

CHAPTER I

INTRODUCTORY CHAPTER

A. BACKGROUND STATEMENT

One of the main subjects in the elementary school is arithmetic. This subject is so important since it enters into all aspects of our lives and it forms the basis of our later studies in mathematics and science. No wonder, then, that schools ought to take great care in developing the methods of teaching arithmetic and revising the content of the arithmetic curriculum now and then. Research has contributed a lot concerning the improvement of both method of teaching and content of arithmetic curriculum. However, despite this fact, arithmetic still forms a great obstacle for the pupils of the elementary school. This remark applies to the elementary schools of the Arab countries as well as to foreign countries.

We can realize that the percentage of failure in the subject is fairly high relative to that in other subjects of the elementary school. The pupils' failure in arithmetic may lead to hatred of the subject, the teacher and the school as a whole. Not only this, but failure in arithmetic may lead to loss of confidence and to failure in other subjects and finally to dropping out of the school.

As a result of his contacts with UNRWA/UNESCO schools in Lebanon, the researcher noticed a few things which might throw some light on the problem of failure in

arithmetic in these schools. It should be pointed out that it is not the writer's aim to examine the significance of these practices and their effect on the quality of instruction provided for the children. These observations were, however, the key points or hints which stimulated this study the purpose of which has already been elaborated. The following points summarize the observations under discussion:

1. The number of pupils per class is very large - sometimes reaching up to sixty pupils per class- especially in the lower classes (first, second, and third).

2. Very little use is made of concrete materials and objects in the teaching of arithmetic despite the fact that the UNRWA/UNESCO Institute of Education (inservice teacher training centre) emphasizes the use of concrete materials for teaching arithmetic. In most cases the children are given the tables to memorize instead of being given the opportunity to build these tables by themselves by using concrete objects.

3. No grading is done within classes. Consequently, pupils of mixed abilities exist together in the same class, a fact which has many implications for the kind of instruction provided and its effectiveness.

4. Currently there exists no way of evaluating the fundamental weaknesses of pupils in arithmetic

except through the old hit-or-miss procedure. In other words, a systematic and scientific device for locating the pupils' weaknesses is lacking. It is not unfamiliar to notice that correction of excercises is done by the whole class by copying correct solutions from the blackboard.

5. No remedial work is planned at present since the teachers do not know the particular weaknesses of their pupils. The teachers find themselves in need of assistance in diagnosing pupils' weaknesses and planning remedial work. Consequently, negative attitudes towards the subject develop amongst the duller pupils, in particular, who are now unable to compete with the good pupils. Not only this, but also in the absence of planned remedial work, weaknesses are carried through the subsequent grades.

6. The pupils are automatically promoted in the elementary stage[x]. This made many pupils indifferent, as some teachers have reported, for they are promoted anyhow. This means that difficulties, instead of being remedied, accumulated from the first grade up to the sixth grade.

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[x] N.B. This was the practice at UNRWA/UNESCO schools in Lebanon when (in May 1965) the study was conducted there. The writer discovered later that this policy had been partially abandoned and it is now applied only in the lower three grades (grades 1-3).

B. STATEMENT OF THE PROBLEM

Thus, the large enrollment per class, neglect of concrete materials, the absence of grading within classes, the absence of a scientific and systematic procedure of diagnosing pupils' weaknesses, the lack of planned, or even any sort of, remedial work, and the policy of automatic promotion, all suggest that:

1. There is a need for diagnostic tests in arithmetic from grade 1 to 6.
2. There is a need to plan suitable remedial work for the different grades.

C. PURPOSE OF THE STUDY

The purpose of this study is three-fold:

1. Preparation of suitable diagnostic tests at different levels-grade 1 to 6.
2. Evaluation and interpretation of the various types of pupils' errors as revealed by the tests.
3. Suggestions, on the basis of test results, for remedial work.

D. METHODOLOGY OF THE STUDY

Both empirical field research and a review of previous research are used simultaneously. The diagnostic tests used are constructed by the investigator after the pattern developed by Schonell.[1]

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[1] Op. Cit., p.3, 1A.

A sample of six schools was chosen at random from about 50 UNRWA/UNESCO schools in the country. The class lists of these schools were then consulted at the UNESCO Headquarters in Beirut and the pupils were chosen at random. The following table shows the number of pupils chosen per class for each school.

TABLE 1

DISTRIBUTION OF PUPILS

NAME OF SCHOOL	GRADE 3		GRADE 4		GRADE 5		GRADE 6		TOTAL
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	
Jiddin Elementary School for Boys Rashidieh/Tyre	20	-	14	-	12	-	12	-	58
Malkieh Elementary and Preparatory School-Burg Hammoud/Beirut	9	9	6	6	-	11	-	12	53
Lajjoun Elementary School - Hadath	14	6	7	9	2	8	-	12	58
Askalan Elementary School for Girls Ein-El-Hilwa, Sidon	-	19	-	15	-	12	-	11	57
Tabarieh Elementary School - Wevil-Baalbeck	10	10	8	7	6	5	6	6	58
Jordan River Elementary and Preparatory School for Boys - Baddawi/Tripoli	18	-	15	-	12	-	11	-	56
TOTAL	71	44	50	37	32	36	29	41	
	115		87		68		70		340

These officials said that they are in need of this research which they hope would serve their in-service

teacher-training programs. The researcher excluded the government and private schools in Lebanon for two main reasons:-

a) UNRWA/UNESCO schools in Lebanon follow the same curriculum as Lebanese government schools. If government and private schools were included that would make the planned research very extensive and (may be) repetitious.

b) The researcher had been provided with all the facilities to carry out his research at UNRWA/UNESCO elementary schools in Lebanon.

CHAPTER II

RELATED RESEARCH FINDINGS

As has already been pointed out in Chapter I, both emperical field research and a review of educational research are used in this study. In this chapter, the investigator will survey as briefly as possible the educational research in the area of mathematical achievement. In other words we will see what answers research offers for such questions as:

1. What factors influence pupils' mathematical achievement?
2. Is there a need to group pupils according to their mathematical abilities?
3. What is the influence of understanding on mathematical achievement?
4. What are the most suitable methods of teaching Arithmetic?
5. What factors lie behind the pupils' backwardness in Arithmetic?

This chapter, therefore, gives the reader a quick glance at the related findings of educational research and at the same time forms the background for the whole study.

A. FACTORS IN ARITHMETICAL ABILITY

There are many factors which affect mathematical attainment. Educational research emphasizes the great role that intelligence, experience, level of maturation, computational accuracy, verbal ability, visual imagery, spatial ability, temperament and emotional attitudes play in this respect. Let

us now see what research says about the influence of these and other factors on mathematical attainment.

1. Intelligence

Barakat found that innate intelligence contributes most to mathematical attainment, and that:

> There is ample evidence for a specialized factor in mathematical ability, its components being mathematical reasoning and a symbolic factor allied to mechanical and rote memory.[1]

Kemp,[2] and Rose[3] emphasize that a pupil's success at any arithmetic topic is to a large extent determined by the pupil's I.Q.

Depending on several investigations, Schonell[4] claims that the average correlation coefficient between arithmetic - both mechanical and problem arithmetic and intelligence is 0.7 (for different age groups). This coefficient

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[1]Barakat, M.K., "Factors Underlying the Mathematical Abilities of Grammar School Pupils," Rev. Educ. Res. Vol. 27, 1957, p. 331.

[2]Kemp, Leslie C.D., "Environmental and Other Characteristics Determining Attainment in Primary Schools," Br. J. Educ. Psych., Vol. 24-25, 1955, pp.67-77.

[3]Rose, Alvin W., "Intelligence, Sibling Position, and Socio-Cultural Background as Factors in Arithmetic Performance," Arithmetic Teacher, Vol. 8, 1961, pp.50-56.

[4]Schonell, Fred J., Diagnosis and Remedial Teaching in Arithmetic, p. 6.

varies between:

0.7 and 0.85 for problem arithmetic

0.55 " 0.72 " mechanical "

0.65 " 0.78 " mental "

This implies that in the preparation of arithmetic syllabuses and in the application of teaching methods we need to take into full consideration the fact that general intelligence is a crucial factor in determining success in all forms of arithmetic-problem solving, mechanical and mental. Not only this, but also these figures mean that general intelligence contributes more towards ability in problem arithmetic than in mechanical arithmetic. In other words it is more difficult to produce improvement in problem arithmetic than in mechanical arithmetic. For "in solving an arithmetical problem, it is sorting out the data, making inferences, and selecting the appropriate method that require intelligence."[5]

2. Experience

The formation of number concepts, number vocabulary and spatial knowledge depends not only on intelligence but also on the nature and extent of the experience that the pupil has with a variety of concrete materials in all sorts of situations. This experience with

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[5] Ibid. p. 8

numbers bears heavily on the child's later success in arithmetic. Pupils should be given the opportunity to form meaningful number concepts for themselves through informal experiences with concrete objects.[6] It is very significant for the teacher to bear in mind that a child who can count up to 10 or even 20, may not be able to recognize 5 objects. This means that the meaning of numbers develops slowly, and that we should not go beyond the child's readiness for number work. Varied concrete number situations are, therefore, indispensable in the early stages of arithmetic teaching.

The pupils' concept of numbers may be consolidated by utilizing the children's immediate surroundings like their activities, the articles and objects they use, etc... Through such experience, children come to realize that 5 means a set of 5 things of any kind. They learn the meaning of "larger" and "smaller"; they learn that when a number of objects are added to a set, a larger set is formed and that when several objects are taken away, a smaller set is formed.

This informal work with numbers in varied concrete situations forms the basis of later formal

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[6]See: Marks. Purdey. Kinney, Teaching Arithmetic for Understanding, McGraw-Hill Book Company, Inc., New York, 1958, Chapters 5-7 and pp. 411-418.

work in addition and subtraction, and should be dwelt upon before stepping into the names of symbols, addition and subtraction signs and working with combinations. For the teacher has always to keep in mind that most of the difficulty in arithmetic is the result of partly understood concepts.

Activity work and projects give the pupil the background of understanding so valuable for success in arithmetic. Activity in measuring, weighing, shopping, is as important as the working of sums. All oral and problem arithmetic should be framed in terms of the experiences of the pupils, for this gives the pupil the chance to understand what the sums mean and make inferences from them in terms of familiar concepts. Besides, setting the problem in a real life situation which the child faces would help the child utilize his intelligence and feel the functional value of arithmetic.

Every step in mechanical arithmetic should be followed by a corresponding step in problem form and the numbers used in the problems should be small in order to avoid cumbersome computations.

3. Maturation

Increased maturity improves the chances of success in arithmetic, for maturation means an increase in the children's mental ages and experiences.

There is enough evidence to support the conclusion that mature children can assimilate more from any teaching than less mature ones, i.e. that the importance of intelligence and increases in intellectual maturation outweigh the importance of special experience in promoting the children's understanding of arithmetical concepts and processes. Washburne and his "Committee of Seven" found that:

> There is a stage in a child's mental growth, before which it is ineffective, if not futile, to teach the topic, but after which most children can learn it with reasonableness.[7]

4. Computational Accuracy

Besides intelligence and experience, a fair speed in the basic number combinations is important for arithmetic attainment. It is usual to find that some intelligent and experienced children fail to get the right answer to a certain sum or problem. This inaccuracy in calculation is not only due to emotional conditions or temperamental traits of the pupil, but is also, and mainly so, due to insufficient, properly planned practice with the four processes. This implies that, besides other factors, the amount of scientifically planned

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[7] Washburne, Carleton. (Chairman). "Mental Age and the Arithmetic Curriculum. A Summary of the Committee of Seven Grade Placement Investigations to Date," Rev. Educ. Res., Vol. 5, 1935. AMS. Reprint Company, New York 3, N.Y. (Reprint 1963), p. 14.

practice provided in the classroom is a very important factor in securing computational accuracy. It is essential to realize that understanding followed by practice pays more dividends than practice without understanding or understanding without a proper amount of practice. It is through repeated success with numbers that the child masters fundamental material and gains self-confidence.

Meddleton found that the use of scientifically compiled sheets of number combinations gives results in arithmetic attainment superior to the ordinary "laissez faire" procedures of presentation and practice. He also found that through systematically compiled material, most gain was made in division, and that "an appreciable amount of computational accuracy could be transferred to problem arithmetic."[8] Besides, this investigation showed that teachers of the control group tended to give more practice in addition and multiplication, and tended to neglect subtraction and division. Systematically prepared practice overcomes this tendency and provides adequate practice in the four processes.

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[8]Meddleton, Ivor G., "An Experimental Investigation into the Systematic Teaching of Number Combinations in Arithmetic," Rev. Educ. Res., Vol. 27, 1957, p. 332

There is enough evidence in the educational literature which emphasizes the value of combining experience and practice. For this reason, children in the lower elementary stage should be guided to build up, by themselves, the multiplication and division tables by using counters or other concrete objects such as beans, sticks, marbles, ... etc. Research shows that children who have had such number experiences are in general quicker and more accurate in the basic facts than those children who learn the tables blindly as presented by their teacher.[9]

5. Verbal Ability, Visual Imagery and Spatial Ability

Research studies have demonstrated the importance of a verbal factor in arithmetic, especially in problem arithmetic and mental arithmetic.[10] For this reason improvement in reading comprehension usually brings some improvement in problem solving. It should be remembered, however, that vocabulary grows only through usage. The child should be provided with chances to use arithmetic vocabulary in real situations; this helps him understand,

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[9]McConnell, T.R., "Discovery versus Authoritative Identification in the Learning of Children," University of Iowa Studies in Education, 9:11-62 No. 5 (Univ. of Iowa, 1954).

[10]Tracy, J.P., "The Relationship of Reading Skills to the Ability to Solve Problems," J. Educ. Res., Vol. 38, pp. 86-96, October, 1944.

assimilate and use that vocabulary. In this respect Schonell states that:

> Training in reading problems has been found to reduce premature generalization, to decrease errors due to incorrect transcription and to prevent loss of confidence when dealing with rather wordy problem.[11]

This is also emphasized in the findings of Balow.[12] (Verbal ability may be more important in our case since spoken Arabic is colloquial and written Arabic is classical).

Additionally, it has been found, that imagery plays some part in arithmetical ability, and that spatial ability, which is connected with visual imagery, plays some part in success in arithmetic specially mensuration and problem-solving.[13] Familiarity and experience with shapes, plane figures, angles, lines, ... etc. provide the basis for success in simple mensuration.

However, there is no doubt that the development of spatial ability in pupils is largely connected with the nature and extent of the training the

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[11] Schonell, Fred J., Diagnosis and Remedial Teaching in Arithmetic, p. 26.

[12] Balow, I. H., "Reading and Computation Ability as Determinants of Problem Solving," Arithmetic Teacher, Vol. 11, January, 1964, pp. 18-22.

[13] Morton, Dan M., "Number Forms and Arithmetical Ability in Children," Br. J. Educ. Psych., Vol. 6, Part I, pp.58-73.

pupil has had with shapes, size and position and units of weight and measures. This implies that, in order to develop spatial ability in our pupils, our arithmetic syllabus should provide them with rich experiences in practical work of all kinds - paper folding, models-making, work with tiles, blocks, ... etc.

6. Temperament and Emotional Attitudes

Research has shown that the popularity of arithmetic differs from age group to age group, from boys to girls, and from normal to dull pupils. However, the success which pupils experience in arithmetic largely determines their attitudes towards the subject.[14] Bright pupils like arithmetic whereas dull pupils dislike it. This is so because the first group experienced success in the subject whereas the second group did not - method and curriculum failed to provide success experiences with numbers for the dull pupils. Undoubtedly the attitudes of pupils towards arithmetic would improve with improvement in arithmetic textbooks and methods of teaching.

Pupils who have experienced failure or censure in number combinations in the initial stages of arithmetic instruction lose confidence and are

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[14]Lerch, Harold H., "Arithmetic Instruction Changes Pupils' Attitudes Toward Arithmetic," Arithmetic Teacher, Vol. 8, 1961, pp. 117-119.

more emotionally disturbed during arithmetic lessons and more apprehensive of their abilities than those who experienced success.

Cleveland[15] noted that there is some relationship between arithmetic achievement in fundamentals and personality adjustment; and this relationship is more when it comes to achievement in concepts and problem-solving. He also noted that the higher socio-economic environment of the school community has a positive relationship to achievement in the three aspects of arithmetic and at all I.Q. levels studied.

Barakat found that:

> "Character qualities correlate with mathematical attainments to the extent of 0.30"; and that "emotional instability seems to correlate most highly with inaccuracy of computation and lack of industry with inefficiency in mathematical reasoning."[16]

Thus, the emotional attitudes of the pupil towards the subject and towards the teacher and towards his own success or failure in the subject together with the pupil's temperament all bear a very important influence on arithmetical attainment.

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[15] Cleveland, G.A., "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement," Dissertation Abstracts, Vol. 22, January-March, 1962, pp. 2681-2682.

[16] Barakat, M.K., "Factors Underlying the Mathematical Abilities of Grammar School Pupils," (Ph. D. Thesis, Univ. of London, 1950) Summary in Br. J. Educ. Psych., Nov. 1951 p. 239.

If the pupil is upset for one reason or another, he can't concentrate on the sums he is working with and consequently he will most probably fail to get these sums right. But what are those factors which might influence the pupil's ability to apply himself effectively during his arithmetic lesson? Schonell sums up these factors as:

> Lack of interest, continuous failure, the effects of censure, lack of any sense of achievement, disturbing thoughts or emotional conditions due to anxiety about himself or family matters, worry about work missed during absence, confusion arising from insecure knowledge (sometimes due to bad teaching) an impulsive manner - these and other emotional states all exert a more powerful influence on attainments in arithmetic than on those in any other school subject.[17]

But how can we help the pupil develop favorable attitudes towards arithmetic or get rid of negative attitudes already formed? The key to the answer of this question is the teacher himself - his attitudes towards the subject and towards his pupils and the methods of teaching he uses. The arithmetic teacher should be aware of the attitudes of his pupils towards the subject and should strive to use teaching methods that help in developing favorable attitudes towards the subject. The pupils should be assured of a certain measure of success each at his own level of ability and understanding.

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[17] Schonell, Fred J., Diagnosis and Remedial Teaching in Arithmetic, p. 30.

The teacher should provide children with situations in which they learn

> to count, to use addition, subtraction, multiplication, and division facts; to use ordinal numbers; recognize groups; judge space; measure; weigh; recognize coins; etc...[18]

The 'number chart' helps the first graders see that numbers have meaning just as words do; it helps them develop the true meaning of number; it helps them capitalize on their ability to recognize groups of 5, and to realize that they can close their eyes and still "see" a mental picture of any number of 10 or less. The number chart helps children develop numerous essential learnings. Also the flannel board and the pocket-board are two of the best ways of helping children learn to recognize numbers and learn the language of mathematics.

The findings of Lyda emphasize that:

1. When meaningful methods of teaching arithmetic are used, changes in attitudes toward arithmetic take place. Negative attitudes become positive and the intensity of positive attitudes become enhanced.

2. Associated with meaningful methods of teaching arithmetic and changes in attitude are significant gains in arithmetic achievement, that is, in

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[18] Cooke, Ralph J., "Helping Children Build a Positive Attitude Toward Arithmetic Through its Mathematical Concepts," School Science and Mathematics, Vol. 54, 1954, p. 197.

arithmetical computation and reasoning.[19]

B. INTRA-CLASS GROUPING FOR ARITHMETIC INSTRUCTION

1. Purpose

There is a lot of debate among arithmetic teachers about the value of intra-class grouping for arithmetic instruction. Let us see, in brief, what research says about this problem.

What is the purpose of subgrouping? Lerch asserts that the purpose of subgrouping is:

> to provide for individual differences by bringing pupils of like arithmetic backgrounds, skills, understandings, and concepts together into small groups so that they can study and work effectively at rates and levels that are closer to their own abilities and characteristics.[20]

2. Advantages and Disadvantages

Subgrouping would make it easier for pupils of similar needs and abilities to communicate, about their problems, with each other and with their teachers. In the case of reluctant pupils, this practice will be more conducive to active participation. All procedures and processes can be more clearly communicated to the pupils; and

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[19]Lyda, Wesley J., and Morse, Evelyn Chyton, "Attitudes, Teaching Methods, and Arithmetic Achievement," Arithmetic Teacher, Vol. 10, 1963, p. 138.

[20]Lerch, Harold H., "Intra-Class Grouping for Arithmetic Instruction: Critique and Criteria," Arithmetic Teacher, Vol. 8, December, 1961, p. 404.

the pupils' skills and understanding may be more closely checked. It is now easier for each pupil to work at his rate and have successful arithmetical experiences at his own level of ability, and consequently this may result in favorable attitudes toward arithmetic. Pupils then may be shifted from one group to another according to their own arithmetical needs and abilities.

Research findings reject the criticism that subgrouping is analogous to a "caste system" or that it leads to the "disruption" of the unity of the class. Smith[21] found that intra-class ability grouping is a procedure which might be used to improve achievement in grades 2 through 5 of the elementary school.

Dewar[22] found that intra-class grouping is of benefit to the high and low groups but does not produce significantly better results for the middle pupils than the traditional method of whole class instruction. Besides, both pupils and teachers indicated that this was a valuable and worth-while means of conducting classes in arithmetic to provide

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[21]Smith, W.M., "The Effect of Intra-Class Ability Grouping on Arithmetic Achievement in Grades Two Through Five," Dissertation Abstracts, Vol. 21, July-September, 1960 p. 563.

[22]Dewar, J.A., "An Experiment in Intra-Class Grouping for Arithmetic Instruction in the Sixth Grade," Dissertation Abstracts, Vol. 22, January-March, 1962, p. 2247.

for individual differences in pupils' ability.

In a study conducted in Homewood Public School (Chicago) among 4th, 5th and 6th grades, Provus found that:

a) Children grouped as to ability were familiar with more arithmetic concepts than children not so grouped.

b) In no case were the children who were subject to grouping less proficient than the children who were not grouped.

c) A comparison of the bright, slow and average children shows that the more competent pupils profited most from ability grouping. The average children may have profited slightly, and the slow learners may have profited no more from ability grouping than they would have from a heterogeneous class. Furthermore, the bright and the average children who were in heterogeneous classes attained no greater knowledge of arithmetic concepts than the slow learners attained in these same classes. This condition prevailed when the intelligence quotient was held constant between groups.[23]

In a comparative study of pupil achievement in arithmetic in graded and non-graded primary schools, Hart found that the achievement of pupils in a non-graded school is significantly higher (0.02 level) than that of pupils in a graded system. He also noted that:

> A graded system tends to foster unrealistic standards and to be inflexible in meeting the needs of all pupils. A non-graded

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[23]Provus, Malcolm M., "Ability Grouping in Arithmetic," Elem. Sch. Journal, Vol. 60, April, 1960, p. 397.

> system can be flexible, fostering standards that challenge and interest the able learner without frustrating the slow learner. The teachers in a non-graded school find that they can be more effective and comprehensive in their teaching. Children appear to be happier and more secure without fear of retention and with competition controlled so that all pupils have a reasonable opportunity to succeed.[24]

Thus sectioning according to arithmetic ability is advisable. However, this grouping should be mobile, allowing pupils to move up or down according to the progress they show.[25]

C. UNDERSTANDING AND MATHEMATICAL ACHIEVEMENT

The new trend of teaching is to devote enough time to enable pupils to learn more effectively before engaging in practice in the particular topic. For premature drill makes little or no contribution to growth in quantitative thinking of the pupil.[26]

Instead of presenting the children with, say, the multiplication tables to be learnt, as the old method used to do, the new method allows every child to build his own multiplication tables from concrete material. This

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[24]Hart, Richard H., "The Non-Graded Primary School and Arithmetic," Arithmetic Teacher, Vol. 9, March 1962, p.133.

[25]Lerch, Harold H., "Intra-Class Grouping for Arithmetic Instruction: Critique and Criteria," Arithmetic Teacher, Vol. 8, December 1961, pp. 404-407.

[26]Brownell, W.A., and Chazel, C.B., "The Effects of Premature Drill on Third Grade Arithmetic," J. Educ. Res., 29 September, 1935, p. 26.

approach introduces meanings into numbers and consequently results in greater transfer of training.

There is conclusive evidence in the education research which proves that children who had the opportunity to discover the number relationships or methods and rules involved are superior in arithmetic achievement to children who were shown the relationship or told the method. This is so because the first group of children see sense in what they are doing, whereas the second group follows the rules blindly.

Harding and Bryant found that:

> Direct, first hand experiences with projects and enterprises in which children have a personal interest, proved more effective than vicarious experiences and drill procedures in developing the ability to solve problems as this ability was measured by the 'Arithmetic Reasoning' section of the achievement tests.[27]

This is also confirmed by the findings of Pace.[28]

This implies that teachers should first guide their children to acquire meanings in their study of arithmetic and then follow this by drill to increase, fixate, and maintain the efficiency and meanings acquired. However, teachers should bear in mind that too much practice may

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[27]Harding, L.S., and Bryant, I.P., "An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade," J. Educ. Res. 37 (January 1944) p. 335.

[28]Pace, Angelia, "Understanding and the Ability to Solve Problems," Arithmetic Teacher, Vol. 8, May 1961, p. 232.

lower achievement scores while too little practice may make reteaching necessary.[29]

Besides, practice would be most fruitful if every pupil is allowed to work at his own rate.[30]

D. METHOD OF TEACHING

Research tried to find answers to such questions as:

1) What is the best approach to teach the basic facts in arithmetic?

2) Are there basic facts which are inherently difficult to learn?

3) Which is better to add upwards or to add downwards?

4) What is the best method of subtraction- the method of decomposition or that of equal addition?

5) Does the "crutch" aid or hinder learning?

6) Does premature drill help?

Thiele noted that the generalization method (which stresses the importance of relationships among combinations) of teaching the 100 basic addition facts is superior to the drill method (which teaches each combination directly, separately and prevents discovery of relationships). He also found that children taught by the first method excelled those taught by the second method:

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[29]Shipp, Donald E., "An Experimental Study of Achievement in Arithmetic and the Time Alloted to Development of Meanings and Individual Pupil Practice," Dissertation Abstracts, Vol. 19, September 1958, p. 492.

[30]Moench, Laurel, "Individualized Practice in Arithmetic-A Pilot Study," Arithmetic Teacher, Vol. 9, October 1962, p. 328.

> in number of combinations known, regardless of intelligence level, and ability to extend their number knowledge to the addition of one-and two-place numbers as well as the addition of two place numbers.[31]

Thus there is conclusive evidence of the superiority of understanding over drill in teaching the basic addition facts. This would most probably hold true in the case of the basic facts in subtraction, multiplication and division, too.

Swenson[32] found that among the 100 basic addition facts, there are no facts which are "intrinsically" or "inherently" difficult. He noted that the difficulty ratings of addition combinations vary with learning method. On the other hand,

> educational research has shown that multiplication and division combinations differ in order of difficulty and that there is general agreement that facts such as 3 x 7 and 7 x 3 are not identical operations for a child. He experiences greater difficulty when the second term is larger than the first; and the larger the multiplier, and multiplicand, the harder the number fact becomes.[33]

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[31] Thiele, C. Louis, "Contribution of Generalization to the Learning of the Addition Facts," Rev. Educ. Res., Vol. 12, 1942, p. 394.

[32] Swenson, Esther J., "Difficulty Ratings of Addition Facts as Related to Learning Method," J. Educ. Res., Vol. 38, October 1944, pp. 31-85.

[33] Shaw, H., "The Development of the Child's Concept of Number," Educ. Rev., Vol. 18, 1960, p. 195.

Hightower[34] found no significant difference between pupils who were taught to add upwards and those taught to add downwards.

Schonell[35] advises teachers to teach upward addition first and later encourage pupils to use downward addition as a check.

There have been several studies about the methods of subtraction. The aim is to determine which is superior, the method of equal addition or the method of decomposition.

Rheins and Rheins[36] did not find a conclusive proof about the superiority of one method or the other.

Brownell and Moser conducted a study on 1400 third-grade pupils to determine the relative effect of a meaningful versus a mechanical approach when the decomposition method is taught and when the equal addition method is taught. The data provided by tests, interviews, and teachers' diaries revealed that:

a) decomposition taught meaningfully was the most successful method.

b) the equal addition method was difficult to rationalize.

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[34]Hightower, Howard W., "Effect of Instructional Procedures on Achievement in Fundamental Operations in Arithmetic," Educational Administration and Supervision, Vol. 40, October 1954, p. 336.

[35]Schonell, Fred J., Diagnosis and Remedial Teaching in Arithmetic, p. 47.

[36]Rheins, Gladys B., and Rheins, Joel J., "A Comparison of Two Methods of Compound Subtraction," Arithmetic Teacher, 2:63-69, October 1955.

c) when the teaching was mechanical, the equal addition method had some advantage.

d) the crutch was helpful when teaching the decomposition method and was discarded when teachers guided pupils away from it later.

e) pupils who had been taught meaningfully retained the learning longer and were better able to transfer this learning and

f) the most economical route to speedy and accurate computation was thru the use of rational procedures. This study emphasized that when methods are being compared, the extent of rationalization may well determine which method has the advantage.[37]

Thus there is a lot of controversy over this issue, and the true value of this method or that may probably be due to superior instruction and motivation rather than due to the method itself.[38]

In his investigation on the use of the "crutch", Brownell found that:

> There is "no relationship between the retention of the use of the "crutch" and chronological age, mental age, intelligence quotient, or separate measures of arithmetic ability."[39]

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[37] Brownell, William A., and Moser, Harold E., "Meaningful versus Mechanical Learning," Rev. Educ. Res., Vol. 21, 1951; p. 291-92.

[38] Hightower, Howard W., "Effect of Instructional Procedures on Achievement in Fundamental Operations in Arithmetic," Educational Administration and Supervision, Vol. 40, 1954, p. 347.

[39] Brownell, William A., "An Evaluation of an Arithmetic 'Crutch'," Rev. Educ. Res., Vol. 5, 1935, AMS Reprint Company, New York, 3, N.Y. (Reprinted 1963) p. 15.

And in a later study by Brownell, when the method of subtraction by decomposition alone was used, the "crutch" was found:

> to increase ease and efficiency in learning not only for the dullest children and those least capable in arithmetic but also for the brightest and the most capable as well. Most children tended to abandon the device when they no longer needed it, but the crutch was found useful later on in certain situations, when it was revived - situations in which the child was mastering a new complex task involving subtraction or regaining subtraction skill after a period of disuse.[40]

This finding was later confirmed by the results of a study conducted by Brownell and Moser.[41]

Thus, there is empirical evidence which proves that the "crutch" is a valuable aid in teaching subtraction, without any fear of making the pupils slaves to the "crutch" later on. Koenker goes even further when he defines the "crutch" as any meaningful aid which helps the child find the correct answer in arithmetical computations. He advises arithmetic teachers to use and encourage their pupils to use the "crutches" thus defined, because:

1) They make arithmetic more meaningful.
2) The immature learner usually cannot succeed without using them.
3) They make learning easier and more permanent.

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[40] Brownell, William A., "Borrowing in Subtraction," Rev. Educ. Res., Vol. 12, 1942. p. 394.

[41] Brownell, William A., and Moser, Harold E., "Meaningful versus Mechanical Learning," Rev. Educ. Res., Vol. 21, 1951. p. 291.

4) The child using the aid understands what he is doing.

5) And a child who successfully uses such meaningful aids becomes an independent worker who is able to discover the answer for himself.[42]

E. BACKWARDNESS IN ARITHMETIC

The experienced teacher realizes that arithmetic is one of the most difficult (if not the most difficult) subject in the elementary school. Inspite of the vast improvement in curriculum and methods that took place in the last few decades, arithmetic still forms an obstacle for many pupils. Indeed

> before failure of pupils was ruled out by progressive education, arithmetic caused more failure than any other school subject in the elementary curriculum. Today a high percentage of pupils show no enthusiasm for arithmetic.[43]

Research attributes this failure to a variety of reasons which may be grouped under three main headings:

1) Environmental causes - within the home and the school.

2) Intellectual causes.

3) Emotional causes.

1. Environmental Causes

Since arithmetic is a particulary difficult and abstract subject, the child needs a lot of

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[42]Koenker, Robert H., "The 'Crutch' in Arithmetic," Elem. Sch. J., Vol. 58, January 1958, p. 233.

[43]Wilson, G. M., "Why Do Pupils Avoid Mathematics in High School?," Arithmetic Teacher, Vol. 8, 1961, p. 168.

concrete number experiences before he can understand what they mean. And since counting is the basis of early number work, the child should have the chance for much counting using different kinds of material. This activity should continue for a sufficient period of time to enable pupils to have a real understanding of numbers before any attempt is made with formal work. Thus, deficiency in number experiences and early commencement of formal work with abstract numbers lead to backwardness in arithmetic.

Some children come to school with limited number experiences. In this case it is essential that the school provides compensation for the limitation in pre-school experiences. Not only this, but also there are slow or even dull pupils in every class. Such pupils need a prolonged activity work, with concrete objects before formal work is started. This practice leads to less failure and confusion and minimizes the emotional barriers created in the early stages of arithmetic teaching.

Home influences are important, too. The child who gets insufficient food of a nourishing kind has little mental energy and cannot concentrate during the arithmetic lesson. For such a child to provide him with more hours of sleep and nourishing food will serve him much better than individual

attention in class.

Absence from school is one of the most important causes of backwardness in arithmetic. This is so because every step in arithmetic is based on a previous step. Absence from school results in loss of instruction and a loss of sufficient practice in some operations and some essential steps. Not only this but also consequent failure and loss of confidence accentuate the effects of absence. Such pupils need individual help to convince them that they can easily catch up and overcome their apparent handicap.

Shifting from school to school has its effects, too, especially if different methods of teaching similar things are used, e.g. subtraction by equal addition and by decomposition. For this would confuse the pupil and make him feel at a loss. Not only this, but also sometimes there is lack of correspondence in syllabi from school to school. When this is the case, then the child who moves from one school to another may miss vital units of work. Hence, to avoid this cause of backwardness, more uniformity of procedure in sequence of arithmetic processes is needed.

Bad methods and attitudes of teaching contribute to backwardness in arithmetic. Over-explanation and

over-emphasis of mechanical work lead to negative results especially with dull pupils.

Syllabuses that are not 'life-centered' are dull and boring to the child. Much of the backwardness in arithmetic would be eliminated if our arithmetic syllabuses were limited to real-life problems.

Arithmetic teachers should be careful not to commence a new step before the previous one is mastered. It is unsound to introduce the pupil to the difficulties of 'bridging' in addition and subtraction before making sure that the pupil has mastered the 100 basic addition and the 100 basic subtraction facts as well as understood the meaning of place-value.

Careful grading of examples is essential to progress in arithmetic. Whenever a new step has been learnt, sufficient graded examples should be given. At first, computational difficulties should be minimized in order to allow the pupils to devote their attention to the method involved.

2. Intellectual Causes

Deficiency in general intelligence is the most obvious cause of backwardness in arithmetic especially when it comes to problem-solving.

Some pupils have weak memories for numbers, mainly as a result of lack of interest in arithmetic

and lack of confidence. Such pupils fail to recall a sequence of numbers correctly and consequently they are weak in oral or mental arithmetic.

Some pupils have a short span of concentration. This may be due to early failure in the subject or to lack of interest in the material or the reason may be physical or psychological or a combination of all of these. If the child's mind is wandering, the child may miss several figures in a column or add figures twice or carry when there is no carrying, ... etc. Such children should be provided with activities which interest them and be assured of a certain measure of success. This would help increase the intensity of attention during arithmetic lessons.

3. Emotional Causes

Emotional health is as important for success in arithmetic as is intellectual ability - if not more. Confusion, loss of self-esteem and self-confidence inhibit normal intellectual expression. Early failure produces confusion and later difficulties; continued failure breeds the feeling of complete inadequacy in the whole subject. Emotionally upset pupils need individual attention with quick success in arithmetic in order to regain self-confidence.

Some children are impulsive. They pay little attention to detail. They have the intellectual power to do the work, but they often do it in a hurry with the result of several inaccuracies. To minimize the errors of such pupils, they should be encouraged to report to their teachers after each sum has been completed, and they should build up a habit of checking their own work.

Lastly, we should not forget the positive role that a good class atmosphere plays in the learning process. A sympathetic, encouraging and stimulating atmosphere during the arithmetic lesson is conducive to greater progress in arithmetic than an atmosphere of censure, harshness and coercion.

We see, therefore, that the findings of educational research emphasize that intelligence is the main factor in mathematical ability, and that experience, with concrete materials, is essential for the child in order to develop meanings of numbers, mathematical concepts and to discover number relationships.

Besides, it was found that it is useless to try to teach the child something for which he is not ready yet. Mathematical achievement is not only dependent on the child's I.Q., maturation level or experience, but also on the child's computational

accuracy. Systematic practice, using well defined and systematically planned practice sheets, in the four processes, is essential for the child if a satisfactory degree of accuracy and speed of computation is to be achieved.

The child's verbal ability, visual imagery, spatial ability, temperament and emotional attitudes are very important especially for problem arithmetic which requires more than a mere knowledge of the four processes.

Research also indicates ample evidence in favor of intra-class grouping. There is also ample evidence in favor of understanding versus rote learning. Understanding followed by the proper amount of practice is found much more fruitful than practice without understanding or understanding without sufficient practice.

As for methods of teaching, research provides evidence that a meaningful approach of teaching arithmetic is indispensable for building up mathematical understandings and concepts. There is no difference between adding upwards or adding downwards. However, there is a lot of controversy over the issue of methods of subtraction. Some studies claim that the method of subtraction by equal additions is superior to that of subtraction by decomposition. Some studies claim the opposite,

and still other studies conclude that the true value of this method or that may probably be due to superior instruction and motivation rather than due to the method itself. However, the arithmetic teacher is advised to stick to one method only for fear of confusing the pupils if both methods are used simultaneously.

Investigations on the use of the "crutch" conclude that the "crutch" is very useful and that there is no fear of making the pupils slaves to the "crutch" later on.

Lastly we can see that there is a multiplicity of factors (intellectual, environmental and emotional) which interact and affect the pupil's arithmetic achievement positively or otherwise.

CHAPTER III

THE ARITHMETIC CURRICULUM AND PRACTICES OF TEACHING AT UNRWA/UNESCO ELEMENTARY SCHOOLS

A. CURRICULUM

As we have pointed out already, UNRWA/UNESCO schools in Lebanon follow the official Lebanese program. The following is a translation of the official Lebanese mathematics (arithmetic) curriculum in the elementary stage*:

First Year

- Study of numbers from 1 to 20, both reading and writing, and then numbers 20 to 100.
- Notion of addition and subtraction.
- Dividing a group of objects into two subgroups, then adding up the two subgroups together. The same process is repeated with groups of 11 and 12 things. From this should be concluded the sum of numbers below five, and later the sum of numbers less than 10.
- Addition without bridging. Addition tables (100 basic addition facts).
- Addition with bridging.
- Simple subtraction.
- Subtraction of numbers less than 100.
- Notion of multiplication and division.
- Counting groups of objects by twos; conclusion of the multiples of the first fifty numbers and taking an idea about even numbers. Later, reverse operations are done i.e. finding halves of even numbers less than 100.
- Counting objects by threes, fours, fives, and to conclude the way of multiplication by three, four and five.

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*Translated by the investigator from the Lebanese Official Curriculum.

- Then reverse operations are given to help children form an idea about division of numbers less than 50 by 3, 4, and 5.

Second Year

- During the first month, the requirements of the first year are reviewed.
- Reading and writing numbers from 1 to 1000.
- Addition of two numbers of three digits each.
- Subtraction of two numbers of three digits each.
- Multiplication of a number of three digits by 2, 3, 4, and 5.
- Reading and writing numbers up to 10.000.
- Excercises on the addition and subtraction of these numbers; addition and subtraction of decimal numbers made up of four digits.
- Multiplication by 6, 7, 8, and 9.
- Division of integeral and decimal numbers by 2, 3, and 4.
- Multiplication using a multiplier of two digits.
- Division of numbers by 5, 6, 7, 8, and 9.
- Simple excercises and problems on addition, subtraction, multiplication and division.

Third Year

- During the first month, the requirements of the second year are reviewed.
- Addition and subtraction of integral and decimal numbers of five or more digits.
- Multiplication using a multiplier of 3, 4, and 5 digits - multiplication balance.
- Division by a number of two digits; division by decimal numbers.

- Simple problems on the four operations.

- Notion of the metric system through practical and empirical procedures.

- Use of a ruler of two decimeters, use of the meter, decameter, hectometer, kilometer, gram, kilogram.

- The gram, kilogram.

- The liter, decaliter, hectoliter.

- The piaster and the pieces of money of 5, 10, 25, 50 and 100 piasters.

- Simple problems on the above information and on purchase price, selling price and gain.

- Simple, direct proportion.

Fourth Year.

- Reading and writing of numbers: decimal numbers, integral numbers.

- Magnifying or diminishing a number 10 times, 100 times.

- Addition: Notion - Addition of integral numbers, addition of decimal numbers. Arrangement of digits of numbers to be added - mental arithmetic.

- Subtraction: Notion - subtraction of integral and decimal numbers - arrangement of digits of numbers to be subtracted - exercises on speed of calculation.

- Multiplication: Notion - the terms used in multiplication, the multiplication tables - multiplication of integral numbers, multiplication balance - arrangement of partial products in long multiplication - mental arithmetic.

- Division: Notion - the terms used in division - division of integral and decimal numbers - division balance - presentation of the process (division) in writing - mental arithmetic.

- Divisibility by 2, 3, 5, and 9 and its application on the balances of multiplication and division.

- Units of time: the day - units larger than the day: the week, the month - the year. Units smaller than the day: the hour, the minute and the second. Some information about compound numbers.

- Simple problems involving compound numbers.

- Units of length: the meter - multiples and parts of the meter - measurements of length - features.

- Units of capacity: the liter - multiples and parts of the liter - units of capacity used for liquids and solids.

- Units of weight: the kilogram - its multiples - its parts - iron and brass units used - the box of weights - the balance - how to use the balance - the weight-bridge.

Introductory Information in Geometry

- The straight line - the broken line - the curved line - the vertical line - the horizontal line - the oblique line.

- The angle - the perpendicular lines.

- Usage of the ruler and the right angle.

- Parallel lines.

- The triangle and its perimeter.

- Quadrilaterals: the rectangle, square, rhombus, trapezium, parallelogram.

- Circumference of the circle; the circle, the radius, the diameter, the arc.

- How to draw a circle, the compasses, length of the circumference, the number π .

- Solids: the cube, the right prism, the parallelopiped.

- Fractions: Notion, representations, subtracting a fraction from a whole number, comparison of fractions with unity, simplification of fractions, reducing fractions to one common denominator, the four processes in fractions.

- The third simple rule, direct and inverse.

- Price expressed as a percentage.

Fifth Year

- Numeration - oral and writing.

- Roman numbers. Reading of decimal numbers.

- Addition: principles : properties of sums - addition of integral and decimal numbers - exercises on mental arithmetic.

- Subtraction: Principles: subtraction of integral and decimal numbers - problems involving addition and subtraction - exercises on mental arithmetic.

- Multiplication: principles: terms used in multiplication, multiplication of integers. Products of odd numbers, products of groups - multiplication of decimal numbers, exercises on mental arithmetic.

- Division: principles: terms used in division, division of integral and decimal numbers, properties of quotients, approximate quotients, practice on speed of calculation.

- Divisibility by 2, 3, 4, 5, and 9 - balance of multiplication by 9 - balance of division by 9 - indivisible numbers - finding the highest common factor and the lowest common multiple of two numbers.

- Fractions: principles: comparison of fractions, fractional numbers. Properties of fractions, simplification, reducing to a common denominator, the four operations in fractions. Decimal fractions. Squaring and extraction of square roots. Extracting the square root of an integral number, of a number near to 0.1, and of fractions.

The metric system: Theoretical and practical units - their multiples and parts. Units of length, area, volume and weight. Notion of density and specific gravity. Unit of work -unit of power.

- Compound numbers: Units of time. Subunits of time. Changing compound numbers into decimal numbers and vice versa. Notion of distance travelled and speed. Problems on the post office and distribution of messages.

- Proportionality of volumes: Ratio and proportion, the third rule, both simple and compound, rates, interest, discount, proportional division and mixtures and alloys.

Geometry:

Elementary notion of lines, straight lines, broken lines, curved lines, angles, perpendicular lines, parallel lines, the right-angled triangle, the circle and everything related to it, the protractor and the compasses.

- Measurement of arcs and angles, area of the circle and of the ring.
- Triangles: the base, heights and vertex, the right-angled triangle, the hypotenuse, area of the triangle.
- Quadrilaterals: the area of the rectangle, square, parallelogram, rhombus and trapezium.
- Regular and irregular polygons.
- Solids: the volume of the cube, parallelopiped, pyramid, cylinder, cone and sphere.

This, then, is the arithmetic curriculum at UNRWA/UNESCO schools in Lebanon. However, it remains to be noted that this curriculum is covered in 6 years and not in 5 years. For in UNRWA/UNESCO schools the elementary stage extends to 6 years instead of 5 as is the case in Lebanese Government schools.

B. METHODS OF TEACHING

In UNRWA/UNESCO schools in Lebanon, as in many other schools, educational practices lag behind the findings and recommendations of educational research. For one thing, the program itself is a traditional program, and the methods of teaching are traditional, too. The qualifications of the arithmetic teacher, the socio-economic conditions of the teacher, and the pupils, the policy of automatic promotion and the large enrollment per class, as well as other factors, all interact to produce the existing situation.

It is not uncommon to find that the rote method of learning is still the dominant one. The emphasis is still on rote learning and drill rather than on understanding. The basic facts in the four operations are still given to the child to commit to memory instead of providing these children with a variety of concrete objects that help them discover the number relationships and build up their own tables.

The great majority of teachers in UNRWA/UNESCO schools in Lebanon, had only a secondary education or even less when they were appointed. This implies that, educationally speaking, they were unqualified. Thus these teachers started teaching the way they were taught by their teachers so many years ago. To change this picture, there have been some attempts to raise the standard of the teachers through brief summer courses. However, this proved insufficient and improvement was intangible. But in the late fifties and

early sixties one could see the attempts of a few teachers to raise their qualifications by outside study. This movement is becoming stronger year after year. Hundreds of these teachers got their Lebanese, Syrian or Egyptian baccalaureat during their teaching experience. Not only this, but the opening of the Arab University in Beirut gave a greater opportunity for these teachers. Several dozen have already graduated from the Arab University and the Lebanese University with a "license" in humanities and several hundred more are on their way to be graduated. This does not solve the problem, for though the teacher has now a university degree, that degree in no way makes him a better teacher of arithmetic if it is a degree in social studies or in languages.

However, serious attempts were then made to raise the standards of the teachers by acquainting them with educational ideas and practices. This is now being done through the UNRWA/UNESCO Institute of Education for inservice teacher-training. The Institute, which was founded in 1964, runs a course of two years for elementary school teachers with due emphasis on methods of teaching of elementary school subjects. A good team of experts writes the study assignments which are then circulated by the Institute through field representatives to the teachers joining the course. These representatives hold seminars with their groups of trainees every fortnight to discuss the assignments. The trainees then answer, at home, in writing the set of questions on

the concerned assignment and the answer papers are collected by the field representatives and sent to the INSTITUTE. Qualified correctors in the respective fields correct the answers and the INSTITUTE sends them back to the trainees through the field representatives. At the end of the first year the trainees attend a short summer course (2-3 weeks) where lecturers review with the trainees the assignments of the year in the respective subjects. Shortly after that general examinations are held. The same procedure is carried on throughout the second year. The trainees who pass the examinations are then certified.

Now to what extent does this program raise the standard of the teacher? More specifically, to what extent does this program improve the teaching methods of the teachers of arithmetic and raise their efficiency? Unfortunately no study has been made to compare the methods and efficiency of the teacher before joining the course with his methods and efficiency after graduation from the course. However, the general impression of the specialists at the INSTITUTE and the impression of the teachers themselves, is that the course was a real success. This course provided the trainees with opportunities which they never had before in terms of enlightenment of educational thought and practices. As a result of his close contact with the

Institute, the writer* received the same positive impression. Though the writer did not visit regular classes, he had the chance to evaluate the teachers' work through the demonstration lessons, some of them given under his guidance during the summer courses. The writer should point out that the majority of those lessons were model lessons in the full sense of the term.

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*N.B. Since October 1965, the writer has been a part-time corrector of answer papers in "Methods of Teaching Arithmetic" and lectured on the same subject in the summer courses of 1966, 1967, and 1968.

CHAPTER IV

DIAGNOSTIC TESTS

We have seen in chapter II that there is a multiplicity of factors influencing the pupil's attainment in arithmetic. Since a normal degree of general intelligence is essential for success in arithmetic, and because there is a wide range of innate intellectual power among pupils, we should always expect a wide range in arithmetic ability irrespective of other conditioning factors. Besides, emotional attitudes and tendencies are known to produce pronounced variations in the pupils' arithmetic attainment. In arithmetic, unlike reading or composition writing, any temporary lapse from stability and concentration on the part of the pupil will soon show in his work where final accuracy depends on accuracy in each step. We should not forget the negative effects of repeated absences, too. Thus, the interaction of innate and environmental factors produce children of varying abilities, and this presents for the teacher a great diversity of individual difficulties in arithmetic. These difficulties need to be detected by the teacher. Even if the teacher could know by himself the levels of his individual pupils, he still needs assistance in knowing exactly the nature, extent and causes of the pupils' errors in every particular aspect. When the pupils' specific weaknesses are spotted, the teacher can then reduce the extent of failure in his class by giving due consideration to each of these weaknesses. This can be satisfactorily done with the help of diagnostic tests in arithmetic.

A. AIMS OF DIAGNOSTIC TESTS

The specific purpose of a diagnostic test is to analyze the exact nature of the progress made by the pupil in each important aspect of the subject.

> The test takes into consideration all the vital skills involved in each important aspect and these are tested by a series of carefully graded examples to cover all important steps in the acquirement of the skill.[1]

Thus diagnostic tests aim to analyze and not to assess the ability of the pupils.

Although a teacher might be able to observe the accuracy, speed of work and method of work of his pupils and the extent of their arithmetical progress, he still needs a more analytic and systematic way to analyze their specific weaknesses. The carefully studied and constructed arithmetic diagnostic test would be the most effective tool to discover the specific weaknesses in arithmetic.

It should be clear that although achievement tests enable us to have a general estimate of the speed and accuracy of the pupil in a certain school subject, they can't be fully diagnostic. For example, a standardized achievement test in subtraction can give us a general estimate of the pupil's accuracy and speed, but it is not fully

> diagnostic in the sense that it presents all possible steps in subtraction arranged in order of difficulty, so that the exact level of the pupil's mastery of subtraction and the

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[1]Schonell, Fred J., Diagnosis and Remedial Teaching in Arithmetic, p. 76.

> exact location of his weakness can be discovered. It is just this objective that characterizes the diagnostic test; it aims at gathering information on all aspects of the subtraction process, irrespective of the speed factor, at finding out just what the pupil can do and the precise step at which his knowledge breaks down.[2]

B. CRITERIA OF GOOD DIAGNOSTIC TESTS

In order that diagnostic tests may be most effective, they should satisfy the following general criteria:-

1. There should be a careful analysis of the rules, principles, knowledge, or skills which the test is intended to measure.

2. A good diagnostic test is planned and constructed so that every rule or principle is adequately and equally tested by objective terms.

3. The test items are generally arranged in groups to facilitate the analysis and diagnosis.[3] And

4. The tests should suggest or provide remedial procedures for each error detected.

5. The tests should reveal the mental processes of the learner sufficiently to detect the points of error.[4]

C. THE TECHNIQUE OF DIAGNOSIS

The figure[5] below shows the five steps into which educational diagnosis falls. The first four levels, the

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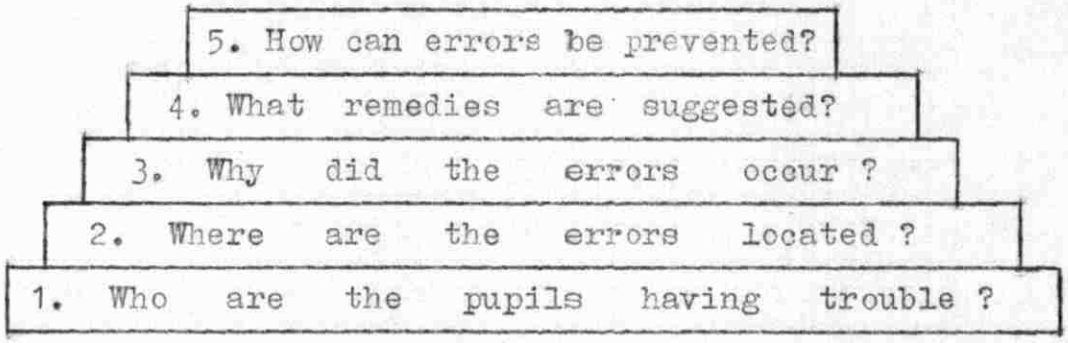
[2]Ibid., p. 77.

[3]Noll, Victor H., Introduction to Educational Measurement, Houghton Mifflin Company, Boston, 1957, p. 363.

[4]Lindquist, E. F. (Editor), Educational Measurement, American Council in Education, Washington, D.C., 1951, pp. 37-38.

[5]Ross, C. C., Stanley, J.C., Measurement in Today's Schools, 3rd ed; Englewood and Cliffs, N.Y., Prentice - Hall. INC., 1956, p. 332.

W's aim at corrective diagnosis, and the highest level aims at preventive diagnosis i.e. our immediate purpose is correction but our ultimate purpose is prevention.



General diagnosis, using achievement tests, is of very little help, for:

> It does not give the teacher any facts on the basis of which to determine the specific causes of superiority or inferiority, i.e., they do not supply the kind of information needed to make a detailed diagnosis of a particular pupil's deficiency; nor can the teacher determine on the basis of the test results the particular phases of the subject in which deficiencies exist.[6]

Analytical diagnosis on the other hand is the approach which enables us put our fingers on those pupils who need special teaching. For by using carefully constructed diagnostic tests which are based on a detailed analysis of the several skills which constitute the general ability under consideration we can locate the specific weaknesses of the pupils.

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[6]Brueckner, Leo J., and Melby, Ernesto, Diagnostic and Remedial Teaching, Houghton Mifflin Company, Boston, 1951, p. 124.

After selecting the pupils whom we think need remedial work, we locate their individual difficulties in each aspect of the activity measured. Those difficulties are thoroughly explored and put in table-form to make it easier for the teacher to study. However, the teacher must not be satisfied by obtaining the total score made by the pupils on a test. Rather, he must analyze the types of errors represented in every specific aspect of the test and he must examine the test papers to find out what types of errors are made by the pupil.

One of the very important advantages of such error analysis is that they reveal that a relatively few types of errors made over and over again are responsible for the performance of most pupils. This of course would direct teachers and pupils to focus their attention on a relatively few troublesome points and hence there will be better improvement.

Having found <u>where</u> the errors occured, we set ourselves to find out <u>why</u> they occured. The test-scores reveal to us the products of learning, but nothing about the learning process itself. The latter information has to be inferred from the measurement and other pertinent data such as the cummulative record of the pupil, home visits, interviewing the pupil, ... etc.

The best thing, however, is to ask the pupil to work aloud a number of significant examples from the diagnostic tests, including steps where he made errors, and then observe

the pupil's arithmetical methods very carefully. This will reveal the peculiar, round-about mental processes used by some pupils in working their sums.

When the cause or causes of difficulty have been found, an intelligent remedial program is planned either on a group basis or preferably on an individual basis.

However, we should not be satisfied with correcting errors. We should aim at preventing them from happening altogether. It is not unusual that analysis of the pupils' difficulties will suggest to us modification in school organization, curriculum, teaching aids and methods. This might be a great step towards preventing errors occuring under the existing program. A varied curriculum and differentiated methods of teaching to meet the varied needs of the pupils have proved to be a safe way out of many difficulties.

D. VALUES OF DIAGNOSTIC TESTS

Diagnostic tests serve so many purposes, chief of which are:

1. They reveal in a comprehensive way the exact level reached by the pupils and the precise nature of their difficulties.
2. They guide the teacher to the best way of overcoming the pupils' weaknesses.
3. They are more thoroughly analytical than most teachers are able to prepare.
4. They make the teachers aware of the important elements, necessary sequences, and difficulties of the process.

5. They save the teacher's time and energy in diagnosis and leave more for individual remedial work.

6. They help the pupil recognize his learning needs by systematically emphasizing his errors.

7. Remedial procedures are usually suggested or provided which save the teacher's time and also aid in systematizing the process.[7]

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[7]Lindquist, E.F. (Editor), Educational Measurement, p.37.

CHAPTER V

THE TESTS USED IN THIS RESEARCH

The primary purpose of these tests is to analyse the exact nature of the progress made by pupils in the fundamental processes in arithmetic. Once the specific weaknesses are located, the teachers may arrange for appropriate remedial work. Can the achievement tests, standardized or teacher-made, fulfill this purpose? No, for there are many differences between both kinds of tests which one may summarize as follows:

1. Diagnostic tests aim at discovering weaknesses in particular branches, especially fundamentals. Achievement tests aim at testing general ability over a wider field.

2. Diagnostic tests must be exhaustive on the section being tested. Achievement tests cannot be since they cover a wider range of operations.

3. Diagnostic tests should not be limited in time, but a note of the time taken by pupils to complete the test should be made. Achievement tests are usually set with a time limit.

4. Diagnostic tests should deal with particular sections of work e.g. division (short) with no bridging and no remainders; division (short) with remainders and no bridging, division (short) with bridging and no remainders; ... etc.

A. NATURE OF THESE TESTS

Test 1. Addition (100 basic number combinations)

Test 2. Subtraction (100 basic number combinations)

Test 3. Multiplication (100 basic number combinations)

Test 4. Division (90 basic number combinations)

Test 5. Miscellaneous (Difficult number combinations in the four processes together with the more difficult examples in multiplication and division by 10, 11, 12)

Test 6. Graded Addition.

Test 7. Graded Subtraction.

Test 8A & B Graded Multiplication.

Test 9. Graded Division (divisors from 2 to 12)

Test 10. Graded Long Division (easy steps)

Test 11. Graded Long Division (harder steps).

B. DETAILED DESCRIPTION OF TESTS 6 to 11[1]

Test 6. Graded Addition

In this test a complete inventory of all normal steps in ordinary addition has been provided. The steps have been graded in order of difficulty. Four examples are selected to test each step, and two of these units of four i.e. eight examples, appear in each of the seven lines of the test.

1st step[+]	Tens (under 20) in one line, units in the other; no carrying.
2nd step	Tens (under 20) in both lines; 0's introduced; no carrying.
3rd step	Tens (over 20) in both lines; no carrying.
4th step	Hundreds and tens in both lines; no carrying.
5th step	Units in one line, tens (under 20) in the other; carrying.

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[1]It should be stated again that these tests have been constructed by the investigator after the pattern used by Fred J. Schonell in his book: Diagnosis and Remedial Teaching in Arithmetic.

[+]Please refer to the test.

6th step Units in one line, tens (over 20) in the other; carrying.

7th step Tens in both lines, carrying in tens place.

8th step Tens in both lines, carrying in units place.

9th step Numbers over hundred in one or both lines, carrying in units, tens or hundreds place.

10th step Tens in both lines; carrying in both units and tens places.

11th step Column addition, 3 lines; numbers under 100; carrying.

12th step Hundreds, tens and units in both lines; carrying in two or three places.

13th step Column addition, 4 lines of 2 figures; 3 lines of 3 figures.

14th step Variations in column addition introducing difficult number combinations.

And at the end of the test there are two examples of addition in a horizontal setting.

Test 7. Graded Subtraction

This test aims at covering, by successive steps of increasing difficulty, all the major steps in the process of subtraction. Four examples are selected to test each step, and two of these units of four i.e. eight examples, appear in each of the seven lines of the test.

1st step[+] Tens and units in minuend; units in subtrahend; no borrowing.

[+]Please refer to test 7.

2nd step Tens and units in minuend and subtrahend; no borrowing.

3rd step Hundreds, tens and units in minuend and subtrahend, no borrowing.

4th step Numbers less than 20, unit digit in subtrahend less than unit digit in minuend; tens digit both unity.

5th step Tens in minuend; units in subtrahend; borrowing.

6th step Tens and units in minuend and subtrahend; borrowing in units.

7th step Hundreds, tens and units in minuend; borrowing in units.

8th step Borrowing in units and tens or borrowing in units and zero results in tens.

9th step Introduction of zero difficulty in units or tens.

10th step Examples of $\overset{0}{\underset{=}{1}}$ and $\overset{0}{\underset{=}{9}}$ difficulties

11th step Borrowing in tens place. Numbers over 100.

12th step Borrowing in hundreds, tens and units places.

13th step Advanced "0" difficulties and borrowing.

14th step More advanced "0" difficulties and borrowing.

Test 8A & B. Graded Multiplication

This test provides a gauge of achievement in multiplication, simple and compound. The test proceeds by steps of increasing difficulty, with four items of similar type to each step. The test has been split into two parts because it is too long for one paper to take. Six examples are alloted to each line on the first page of the test and three on the second page. Thus on the first page of the test a line contains four examples of one

step and two of the succeeding step, which is completed on the next line with two more examples.

1st step[+]	Simple multiplication, 2 figures in multiplicand; no carrying.
2nd step	Simple multiplication, 3 figures in multiplicand; no carrying.
3rd step	Simple multiplication, 2, 3, or 4 figures in multiplicand, "0" difficulties introduced; no carrying.
4th step	Simple multiplication, carrying into tens place.
5th step	Simple multiplication, multiplicand over 20; carrying into tens place.
6th step	Simple multiplication, numbers over 100. "0" difficulties; carrying.
7th step	Simple multiplication, 3 to 6 figures in multiplicand; carrying.
8th step	Compound multiplication, 2 figures in multiplicand and multiplier. Two examples; no carrying. Two examples; carrying.
9th step	Compound multiplication, 2 figures in multiplicand and multiplier. "0" difficulties, carrying.
10th step	Extension of "0" difficulties.
11th step	Compound multiplication, 3 figures in multiplicand; carrying.
12th step	Compound multiplication, 3 figures in multiplicand, tens figure "0"; carrying.
13th step	3 figures in multiplicand and multiplier.
14th step	4 figures in multiplicand, 3 figures in multiplier.

[+]Please refer to test 8A & B.

Test 9. Graded Division

This test consists of eleven steps of four examples each, i.e. forty-four simple examples in all, involving the use of divisors from 2 to 12. Adequate attention has been paid to the main zero-difficulties that occur in simple division. Examples are arranged four in a line.

1st step[+]	Divisor is contained a whole number of times in every figure of the dividend; no carrying; no remainders.
2nd step	Same as step 1, but with larger numbers; no carrying; no remainders.
3rd step	"0" in middle or at end of dividend; no carrying; no remainders.
4th step	Double zero at end of dividend; nought in quotient; no carrying, no remainders.
5th step	Divisor is not contained in 3rd figure of dividend (giving nought in quotient), but it is contained in last 2 figures of dividend; no carrying; no remainders.
6th step	Dividends under 100; remainders.
7th step	Similar to step 6.
8th step	3-figure dividends; no carrying; remainders.
9th step	3-figure dividends; larger divisors, carrying; remainders.
10th step	Larger dividends; carrying; remainders.
11th step	5-or 6-figure dividends; "0" difficulties; carrying; remainders.

[+]Please refer to test 9.

Test 10. Long Division

Tests 10 and 11 both deal with the most difficult process in elementary arithmetic, namely long division. Naturally the possible number of examples in relatively simple long division is exceedingly great, but an attempt has been made to compile two tests which will give a fairly comprehensive survey of the major long division difficulties. Four examples are alloted to each step and each step occupies a single line in the test.

1st step[+]	The simplest step in long division; the quotient which consists of one figure, is apparent at sight from the nature of the figures in divisor and dividend; no remainders.
2nd step	This step is only a very little different from step one; the quotient is apparent from the nature of the first figures of divisor and dividend, but there is a remainder.
3rd step	This step is a slight advance on step 2, as the quotient is not so apparent in two examples.
4th step	In this step the first figure of the divisor is not contained equally in the first figure of the dividend. There are single figure quotients and single figure remainders, with borrowing.
5th step	The first figure of the divisor in this step is not contained at all in the first figure of the dividend but in the first two figures; 3-figure dividend; single figure quotient; no remainders.
6th step	This is similar to step 5; but with remainders.
7th step	There are four figures in the dividend; 2 figures in the quotient; no remainders.

[+]Please refer to test 10.

8th step This is similar to step 7, but has single figure remainders.

9th step Quotients not always apparent. No remainders.

Test 11. Long Division

This test is a continuation of test 10 so that the steps follow on in approximate order of difficulty, from 9th step in test 10.

1st step[+] This step, while keeping two examples similar to those in step 8 of test 10, introduces a "0" into the quotient of the two remaining examples and has single figure remainders.

2nd step This step introduces trial divisors in all examples. There are single figure quotients and no remainders.

3rd step This is an extension of step 2 in so far as more trials are required to find the first quotient figure, and there are some remainders.

4th step In this step there are trial divisors, 2 figures in the quotient and no remainders in two sums.

5th step This step introduces 3-figure quotients, one with remainder and one with a "0" in the quotient.

6th step This step involves noughts in the quotient and some remainders.

[+]Please refer to test 11.

C. GENERAL INSTRUCTIONS FOR GIVING THE TESTS

It is advisable that these tests be given as early as possible in the school year in order to give the teacher a full and clear picture of where the weaknesses of his pupils lie. This helps him to dwell on these weak points before proceeding to a step for which his students are not ready or are ill-prepared.

Tests 1 thru 5 are given to grade 3. (age 8+)
Tests 5 thru 9 are given to grade 4. (age 9+)
Tests 8A & B thru 11 are given to grades 5 and 6*.
(age 10+, 11+)

The idea behind this overlap is to find a basis of comparison between every grade and the succeeding one- the next higher grade.

Although every group of these tests- (tests 1 thru 5), (tests 5 thru 9), (tests 8A & B thru 11)- can be given in one session of about 2½ hours, including intervals of rest, the teacher might find it more convenient to space these tests over several days or to give them separately whenever need calls for that. On the average, each test takes about 20 minutes.

D. MARKING AND SCORING THE TESTS

In marking each paper from tests 6-11, the teacher should put down the number of wrong sums in each half-row, on the opposite side of the row. This would enable him know where, in which step, the weaknesses of his pupils lie.

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*If the teacher wishes, he can give tests 1-9 to grade 4, and tests 1-11 to grades 5 and 6.

The scoring is one mark for each correct answer. The table below gives the maximum number of marks for each test. However, we should keep in mind that knowing the specific weaknesses of the pupil is much more important and revealing than the mere score of that pupil. In tests 6 thru 11, the teacher needs to examine each wrong sum to see the exact cause of the error, i.e. whether it is due to weakness in the basic facts or in bridging or in knowledge of place—value, ... etc. To complete the picture it is highly recommended that the teacher should ask those pupils who show marked weakness in a specific step to work a few sums aloud. This procedure would reflect to the teacher the mental process that produced the wrong answer. In Chapter VI we will see how to tabulate, analyse and interpret the results of these tests.

TEST	K I N D	MAX. SCORE
1	Addition	100
2	Subtraction	100
3	Multiplication	100
4	Division	90
5	Miscellaneous (1-4)	100
6	Graded Addition	58
7	Graded Subtraction	56
8A & B	Graded Multiplication	58
9	Graded Division	44
10	Long Division	36
11	Long Division	24
		761

E. THE USES OF THE RESULTS OF THESE TESTS

Although the primary aim of these tests is to help the arithmetic teacher diagnose the difficulties of his pupils through a scientific and systematic way, they can still be used for a variety of purposes. Schonell sums up these uses in the following eight points:

a) The tests are suitable for estimating accuracy in the fundamental combinations amongst pupils in infant classes.

b) The tests will provide useful information concerning all pupils in junior schools.

c) The tests are invaluable as a guide to both the attainments and the difficulties of dull pupils. They indicate starting points and curriculum objectives for the teacher.

d) With children backward in arithmetic, the tests isolate with certainty individual difficulties and reveal the lines along which remedial teaching should proceed.

e) Pupils coming to school from another area can be given the tests to ascertain where they shall be placed with regard to future arithmetic teaching.

f) The tests can be used as a basis for dividing pupils into arithmetic sets, where this method of organization in used.

g) Groups of pupils who are to be transferred to post-primary school can be tested prior to their final term in the junior school and given practice in the processes where weaknesses are still displayed.

h) Testing all pupils in a secondary modern school, but particularly the more backward ones, provides much useful information for teachers and prevents further work being attempted where the fundamentals are not still sufficiently well known.

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[2] Ibid., pp. 111 - 112.

F. ORAL INSTRUCTIONS

(To be read word by word)

"You have a piece of paper containing certain sums (addition, subtraction, multiplication, division or miscellaneous). You will find the first few sums very easy, but as you go down the paper the sums increase in difficulty. I want to know how many of these sums you can work correctly. I don't want you to hurry; take as much time as you need. You are free to omit any sum you cannot work, but try to work as many of the sums as you can."

"Start here and work across the paper"

(Teacher or proctor should show that on the paper)

"Show all your work on the paper"

"Do not look at the paper of your neighbor"

"When you have finished the whole paper, or those sums you could do, turn your paper face down and remain silent, till the whole group finishes."

"You are not allowed to ask any questions after you have started. If you have any questions ask them now."

(Pause. Teacher or proctor should answer the questions raised)

"Now you may start"

TEST 1* - ADDITION

These are all "add" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)
1 + 1 =	0 + 0 =	2 + 2 =	2 + 1 =	1 + 3 =
2 + 0 =	3 + 1 =	3 + 3 =	5 + 5 =	4 + 1 =
1 + 6 =	4 + 0 =	4 + 4 =	1 + 7 =	6 + 1 =
7 + 1 =	1 + 8 =	1 + 5 =	5 + 0 =	7 + 7 =
2 + 5 =	3 + 2 =	1 + 4 =	0 + 2 =	6 + 6 =
4 + 3 =	8 + 8 =	0 + 7 =	2 + 8 =	5 + 4 =
9 + 2 =	9 + 1 =	9 + 9 =	3 + 0 =	2 + 9 =
0 + 1 =	6 + 0 =	6 + 2 =	2 + 4 =	0 + 4 =
8 + 2 =	6 + 4 =	0 + 8 =	4 + 5 =	1 + 9 =
8 + 3 =	5 + 1 =	7 + 2 =	0 + 3 =	1 + 2 =
0 + 5 =	4 + 2 =	3 + 6 =	8 + 1 =	0 + 9 =
1 + 0 =	5 + 3 =	2 + 7 =	3 + 5 =	7 + 0 =
2 + 3 =	5 + 2 =	3 + 4 =	8 + 0 =	2 + 6 =
6 + 3 =	4 + 8 =	0 + 6 =	7 + 3 =	8 + 4 =
9 + 0 =	3 + 7 =	6 + 5 =	7 + 4 =	4 + 6 =
3 + 8 =	5 + 6 =	3 + 9 =	7 + 6 =	9 + 3 =
6 + 7 =	4 + 7 =	8 + 9 =	5 + 7 =	9 + 4 =
7 + 5 =	5 + 9 =	4 + 9 =	8 + 6 =	7 + 8 =
9 + 5 =	8 + 7 =	6 + 9 =	9 + 8 =	9 + 7 =
6 + 8 =	9 + 6 =	8 + 5 =	5 + 8 =	7 + 9 =

*The actual tests were given in Arabic.

TEST 2 - SUBTRACTION

These are all "subtract" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)
3 - 2 =	1 - 1 =	4 - 2 =	5 - 4 =	0 - 0 =
5 - 3 =	3 - 3 =	5 - 1 =	4 - 4 =	8 - 1 =
6 - 6 =	4 - 3 =	3 - 1 =	2 - 1 =	6 - 5 =
5 - 2 =	6 - 1 =	9 - 1 =	7 - 7 =	9 - 8 =
5 - 5 =	7 - 1 =	6 - 3 =	2 - 2 =	4 - 1 =
6 - 2 =	8 - 4 =	9 - 9 =	7 - 6 =	6 - 4 =
8 - 7 =	9 - 0 =	7 - 5 =	6 - 0 =	8 - 6 =
9 - 5 =	3 - 0 =	8 - 2 =	9 - 3 =	7 - 2 =
10 - 7 =	10 - 1 =	9 - 6 =	5 - 0 =	10 - 2 =
12 - 6 =	7 - 3 =	4 - 0 =	8 - 8 =	1 - 0 =
10 - 4 =	2 - 0 =	7 - 4 =	9 - 7 =	8 - 0 =
9 - 2 =	9 - 4 =	10 - 5 =	7 - 0 =	8 - 5 =
11 - 2 =	10 - 6 =	12 - 8 =	14 - 7 =	12 - 5 =
8 - 3 =	10 - 8 =	11 - 5 =	10 - 9 =	11 - 8 =
10 - 3 =	12 - 4 =	11 - 7 =	16 - 8 =	11 - 9 =
12 - 7 =	12 - 5 =	13 - 6 =	11 - 4 =	18 - 9 =
11 - 3 =	15 - 8 =	15 - 7 =	11 - 6 =	12 - 3 =
14 - 8 =	15 - 9 =	13 - 8 =	15 - 6 =	13 - 5 =
14 - 6 =	13 - 4 =	17 - 8 =	16 - 9 =	13 - 7 =
17 - 9 =	14 - 5 =	14 - 9 =	13 - 9 =	16 - 7 =

TEST 3 - MULTIPLICATION

These are all "multiply" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)
1 x 3 =	2 x 2 =	1 x 7 =	2 x 1 =	1 x 4 =
5 x 1 =	2 x 5 =	1 x 6 =	2 x 8 =	1 x 5 =
4 x 1 =	2 x 3 =	1 x 8 =	3 x 2 =	2 x 9 =
5 x 4 =	2 x 7 =	4 x 4 =	6 x 1 =	5 x 2 =
3 x 1 =	1 x 9 =	3 x 3 =	3 x 5 =	2 x 6 =
1 x 2 =	4 x 3 =	6 x 2 =	5 x 5 =	6 x 4 =
3 x 6 =	6 x 6 =	2 x 4 =	4 x 5 =	6 x 3 =
6 x 5 =	4 x 2 =	3 x 4 =	5 x 3 =	3 x 9 =
4 x 8 =	5 x 6 =	5 x 8 =	5 x 7 =	3 x 8 =
5 x 9 =	1 x 1 =	6 x 8 =	6 x 9 =	6 x 7 =
4 x 9 =	3 x 7 =	4 x 7 =	4 x 6 =	7 x 1 =
7 x 2 =	9 x 2 =	8 x 2 =	9 x 1 =	0 x 5 =
8 x 1 =	8 x 3 =	7 x 3 =	4 x 0 =	7 x 5 =
9 x 9 =	0 x 3 =	9 x 5 =	0 x 6 =	0 x 4 =
0 x 0 =	0 x 9 =	8 x 4 =	9 x 0 =	7 x 4 =
8 x 8 =	2 x 0 =	8 x 9 =	9 x 8 =	7 x 0 =
8 x 6 =	0 x 8 =	7 x 6 =	1 x 0 =	5 x 0 =
8 x 7 =	7 x 9 =	3 x 0 =	9 x 6 =	7 x 7 =
8 x 0 =	6 x 0 =	7 x 8 =	8 x 5 =	0 x 2 =
9 x 7 =	9 x 3 =	0 x 1 =	0 x 7 =	9 x 4 =

TEST 4 - DIVISION

These are all "divide" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)
6 ÷ 2 =	12 ÷ 2 =	9 ÷ 3 =	4 ÷ 2 =	15 ÷ 5 =
20 ÷ 5 =	25 ÷ 5 =	8 ÷ 2 =	15 ÷ 3 =	14 ÷ 2 =
10 ÷ 2 =	18 ÷ 2 =	12 ÷ 6 =	21 ÷ 3 =	10 ÷ 5 =
8 ÷ 4 =	36 ÷ 6 =	30 ÷ 5 =	20 ÷ 4 =	12 ÷ 4 =
40 ÷ 5 =	12 ÷ 3 =	6 ÷ 1 =	3 ÷ 3 =	8 ÷ 1 =
24 ÷ 6 =	9 ÷ 1 =	5 ÷ 1 =	36 ÷ 4 =	6 ÷ 3 =
18 ÷ 3 =	30 ÷ 6 =	16 ÷ 4 =	7 ÷ 1 =	16 ÷ 2 =
3 ÷ 1 =	24 ÷ 4 =	18 ÷ 6 =	28 ÷ 4 =	2 ÷ 1 =
42 ÷ 6 =	35 ÷ 5 =	24 ÷ 3 =	32 ÷ 4 =	5 ÷ 5 =
2 ÷ 2 =	27 ÷ 3 =	4 ÷ 4 =	45 ÷ 5 =	6 ÷ 6 =
48 ÷ 6 =	1 ÷ 1 =	54 ÷ 6 =	4 ÷ 1 =	0 ÷ 9 =
14 ÷ 7 =	16 ÷ 8 =	32 ÷ 8 =	18 ÷ 9 =	28 ÷ 7 =
9 ÷ 9 =	49 ÷ 7 =	72 ÷ 9 =	35 ÷ 7 =	54 ÷ 9 =
48 ÷ 8 =	0 ÷ 1 =	21 ÷ 7 =	0 ÷ 7 =	40 ÷ 8 =
24 ÷ 8 =	0 ÷ 5 =	63 ÷ 9 =	0 ÷ 6 =	64 ÷ 8 =
0 ÷ 8 =	56 ÷ 7 =	36 ÷ 9 =	8 ÷ 8 =	0 ÷ 3 =
45 ÷ 9 =	0 ÷ 4 =	56 ÷ 8 =	7 ÷ 7 =	0 ÷ 2 =
42 ÷ 7 =	27 ÷ 9 =	81 ÷ 9 =	72 ÷ 8 =	63 ÷ 7 =

TEST 5 - MISCELLANEOUS

There are FOUR kinds of sums here, "add", "subtract", "multiply", "divide". Work across the page.

(a)	(b)	(c)	(d)	(e)
3 + 8 =	7 x 6 =	42 ÷ 7 =	6 + 7 =	27 ÷ 3 =
12 - 7 =	5 x 0 =	7 x 9 =	9 + 5 =	15 - 7 =
18 - 9 =	13 - 6 =	1 x 0 =	63 ÷ 7 =	5 + 6 =
4 x 6 =	8 x 7 =	15 - 8 =	5 + 9 =	11 - 6 =
36 ÷ 9 =	64 ÷ 8 =	9 + 6 =	13 - 8 =	3 + 9 =
7 x 7 =	3 x 0 =	0 ÷ 5 =	14 - 6 =	15 - 9 =
8 + 5 =	4 + 9 =	9 x 6 =	48 ÷ 8 =	8 x 0 =
13 - 5 =	17 - 8 =	7 + 4 =	6 x 9 =	8 + 6 =
8 x 5 =	5 - 5 =	13 - 4 =	7 + 6 =	63 ÷ 9 =
0 ÷ 8 =	16 - 9 =	9 x 3 =	15 ÷ 5 =	0 x 2 =
16 - 7 =	11 - 7 =	6 x 0 =	7 x 8 =	5 + 8 =
6 x 7 =	0 x 1 =	4 ÷ 4 =	0 x 5 =	4 x 9 =
7 ÷ 7 =	17 - 9 =	9 + 4 =	54 ÷ 9 =	36 ÷ 4 =
13 - 9 =	9 + 7 =	0 ÷ 7 =	11 - 8 =	54 ÷ 6 =
84 ÷12 =	11 x 7 =	88 ÷ 11 =	121 ÷ 11 =	12 x 6 =
96 ÷12 =	60 ÷12 =	10 x 12 =	11 x11 =	12 x 4 =
108 ÷12 =	120 ÷12 =	11 x 12 =	132 ÷ 12 =	77 ÷ 11 =
12 x 7 =	72 ÷12 =	12 x 8 =	144 ÷12 =	12 x11 =
11 x 8 =	110 ÷11 =	11 x 9 =	132 ÷11 =	99 ÷11 =
11 x10 =	12 x12 =	48 ÷12 =	12 x 9 =	12 x 5 =

TEST 6 - GRADED ADDITION

These are all "add" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
13 + 5	15 + 4	6 + 11	4 + 12	10 + 13	12 + 17	15 + 12	16 + 10
45 + 23	36 + 53	24 + 71	37 + 40	234 +125	356 + 42	503 +264	362 + 417
7 + 16	15 + 6	9 + 19	17 + 9	37 + 7	48 + 5	6 + 39	8 + 58
57 + 61	86 + 53	94 + 74	70 + 81	34 + 17	29 + 58	24 + 59	37 + 48
301 +804	207 + 49	83 +492	725 +634	36 + 87	28 + 96	67 + 79	54 + 87
64 53 + 46	38 18 + 93	56 47 + 65	49 78 + 66	597 +687	963 +528	675 +867	925 + 469
65 77 42 + 58	78 53 69 + 85	377 164 +289	236 357 +779	37 206 374 + 9	405 903 67 +218	3 51 29 417 + 36	781 264 478 537 + 269

3 + 6 + 4 + 9 + 7 + 8 + 5 =

7 + 3 + 9 + 0 + 8 + 7 + 6 + 5 + 8 + 7 =

TEST 7 - GRADED SUBTRACTION

These are all "subtract" sums. Work across the page.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
58 − 3	39 − 7	64 − 2	76 − 6	77 − 45	59 − 38	98 − 10	87 − 23
269 − 135	789 − 537	598 − 362	958 − 436	16 − 14	18 − 13	19 − 10	17 − 15
61 − 3	83 − 4	56 − 7	74 − 8	54 − 29	42 − 15	88 − 39	36 − 27
351 − 24	463 − 37	296 − 59	587 − 68	415 − 26	364 − 57	78 − 69	357 − 289
860 − 820	254 − 54	70 − 43	240 − 226	180 − 71	230 − 29	350 − 21	780 − 779
436 − 285	369 − 174	586 − 391	257 − 164	348 − 279	741 − 357	7235 − 3576	8112 − 6798
600 − 394	507 − 298	800 − 393	706 − 199	891 − 405	706 − 308	305 − 109	7078 − 5980

TEST 8 A - GRADED MULTIPLICATION

These are all "multiply" sums. Work across the page and put your work on the page.

(a)	(b)	(c)	(d)	(e)	(f)
33 x 2	51 x 5	42 x 4	71 x 6	734 x 2	512 x 3
711 x 5	634 x 2	30 x 7	202 x 4	500 x 8	7010 x 6
16 x 7	18 x 9	17 x 8	19 x 5	79 x 6	87 x 8
93 x 7	68 x 4	108 x 6	107 x 9	3070 x 11	9006 x 8
273 x 12	6275 x 11	86394 x 12	638597 x 9	63 x 21	32 x 43
68 x 37	89 x 56	70 x 98	60 x 54	67 x 50	87 x 90
90 x 90	7860 x 400	90 x 100	1000 x 80	348 x 97	450 x 75

TEST 8 B - GRADED MULTIPLICATION

These are all "multiply" sums. Work across the page and put your work on the page.

(a)	(b)	(c)
705 x 39	870 x 56	308 x 46
609 x 87	508 x 40	807 x 70
326 x 471	407 x 609	657 x 380
6753 x 207	7060 x 805	

TEST 9 - GRADED DIVISION

These are all "divide" sums. If there is any remainder, write it down. Work across the page.

(a)	(b)	(c)	(d)
4) 88	3) 69	2) 24	5) 555
3) 639	2) 446	4) 484	2) 2824
3) 609	4) 408	2) 840	3) 960
2) 600	3) 900	9) 1800	8) 1600
6) 1212	7) 2163	9) 3654	8) 5624
5) 37	7) 64	9) 57	8) 71
6) 50	4) 93	7) 51	9) 86
4) 127	8) 169	7) 148	5) 156
8) 519	7) 614	11) 865	12) 753
3) 235	7) 654	5) 4827	6) 5179
5) 35256	8) 36821	7) 63030	9) 506764

TEST 10 - LONG DIVISION

These are all "divide" sums. Put your work on the page. Where there is a remainder, write it down. Work across the page.

(a)	(b)	(c)	(d)
20) 40	34) 68	31) 93	21) 84
42) 85	22) 69	36) 77	20) 88
27) 99	43) 89	30) 84	21) 76
33) 74	43) 93	23) 78	24) 57
21) 147	32) 128	53) 159	42) 168
67) 139	73) 155	93) 187	82) 248
36) 1224	53) 1219	47) 1974	94) 2914
61) 1412	97) 5438	76) 4793	84) 2614
57) 3420	64) 3200	48) 2400	81) 4050

TEST 11 - LONG DIVISION

These are all "divide" sums. Put your work on the page. Where there is a remainder, write it down. Work across the page.

(a)	(b)	(c)	(d)
27) 1628	67) 2149	53) 2497	86) 5169
18) 72	43) 172	39) 234	34) 102
29) 232	17) 113	39) 328	73) 441
35) 875	56) 2408	63) 1965	24) 553
25) 18275	45) 13545	38) 11379	76) 17556
19) 15333	83) 7060	31) 19538	67) 20708

CHAPTER VI

INTERPRETATION OF THE RESULTS

Before considering the interpretation of the results of these tests, it should be mentioned again that these tests are essentially for diagnostic purposes. The aim is not to know how well a pupil can do a test or how backward he is, but to know exactly the nature of his specific weaknesses in the subject and the reasons behind them. Thus it is the qualitative rather than the quantitative aspect of these tests that provides us with the required information.

However, although diagnostic tests emphasize qualitative examination, they also provide quantitative estimates. If the teacher wishes, he can note the time taken by each pupil to finish the test involved and note the number of examples worked correctly. This data could be used for purposes of comparing other pupils of similar ages.

To achieve the purpose of knowing the nature of errors made by the pupil and why they are made, it is essential to familiarize ourselves with the common types of errors. This means that there is a need to compile schedules of the common types of errors in the four processes as reflected by the diagnostic tests.

Concerning tests 1-5 there is only one type of error, namely lack of knowledge of the basic facts. The writer has tabulated the results of tests 1-5 in such a way that the reader can see at a glance the frequency as well as the percentage of errors in each fact. Similarly, in tests 6-11, the results have been tabulated

in such a way that the common types of errors in every step together with their frequencies and percentages are shown.

In the following schedules the reader finds the common types of errors that the tests revealed together with concrete examples of each type.

SCHEDULE A

Common Errors in Addition (Test 6)

1. Errors in Addition combinations e.g.

207 49 357	37 48 86	9 19 29	34 17 53	86 53 159	10 13 20	37 40 70

2. Errors in bridging[+] e.g.

38 18 93 139	56 47 65 158	49 78 66 183	377 164 289 1830	13 5 28	15 6 111	48 5 113	207 49 2416	37 7 114	8 58 146

234 125 449	503 264 857	725 634 1449	597 687 11184

3. Omitted one or more digits e.g.

503 264 763	12 17 17	28 18 93 146	675 867 942	405 903 67 218 293	356 42 98

[+]This includes errors such as:

a) Forgetting to add carried number.
b) Carrying wrong number.
c) Carrying where there was nothing to carry.
d) Adding carried number twice.

4. Disregarded column position e.g.

356	6	15	4	70	83	7	24	107	70
42	11	4	12	81	492	16	71	49	81
128	8	10	7	16	215	14	77	616	88

5. Added same digit in two columns e.g.

6	13	15	4
11	5	4	12
77	68	59	56

SCHEDULE B

Common Errors in Subtraction (Test 7)

1. Errors in subtraction combinations e.g.

36	83	741	54	296	19	98
27	4	357	29	59	10	10
5	78	364	23	238	00	80

2. Errors in bridging[+]

83	74	88	463	78	357	70	350	780
4	8	39	37	69	289	43	21	779
81	74	51	434	11	132	33	331	19

8112	600	891	7078	507	600	507	240
6798	394	405	5980	298	394	298	226
2686	300	494	2918	211	306	309	20

[+]This includes errors such as:

1. Not allowing for having borrowed.
2. Errors due to zero in minuend.
3. Subtracting minuend from subtrahend.
4. Failure to borrow, giving zero as an answer.

350	357	350	800	305	706	706	800
21	289	21	393	109	199	308	393
330	78	339	507	106	517	308	307

706	8112
199	6798
407	1414

3. Disregarded column position e.g.

58	39	64	76	77	39
3	7	2	6	45	7
25	02	42	10	3322	42

SCHEDULE C

Common Errors in Multiplication (Test 8A & B)

1. Errors in multiplication combinations e.g.

18	93	450	17	7010	202	711
9	7	75	8	6	4	5
153	741	2250	134	28060	848	2155
		4550				
		47750				

2. Errors due to zero in multiplier e.g.

90	1000	407	76	657	407
90	80	609	50	380	609
810	8000	36063	335	5256	3663
		24020		10710	24420
		2436263		24966	28083

3. Errors due to zero in multiplicand e.g.

108	9006	3070	450	202	7010	108
6	8	11	75	4	6	6
1008	73028	307	225	880	42600	1080
		37	315			
		677	3375			

9006	108	807	609	308
8	6	70	87	46
768	608	60090	4803	228
			5202	152
			56823	17480

4. Omitting digit in multiplier e.g.

32	63	705	326
43	21	39	471
126	123	6345	326
			2282
			23146

5. Omitting digit in multiplicand e.g.

348	407
97	609
2436	63
432	000
6756	2442
	244263

6. Errors in position of partial products e.g.

6275	63	657	407	6753
11	21	380	609	207
6275	63	0000	3663	47271
6275	126	52560	0000	00000
12550	189	19710	24420	135060
		72270	28085	182331

7. Errors in addition e.g.

86394	348
12	97
172788	2436
863940	3122
1016728	33656

SCHEDULE D

Common Errors in Division (Tests 9-11)

1. Errors in the technique of division e.g.

22	506	106	1006
5) 37	8) 519	4) 88	4) 484
		40	100
		48	384
		48	24
		00	

885	77	44	33
4) 93	21) 147	32) 128	53) 159
32			
61			
32			
29			
20			
9			

9	61	75	12
47) 1974	42) 85	3) 639	9) 86
188	24	21	9
009	61	52	16
		15	18
		43	2

2. Errors due to zero in the quotient e.g.

```
   22          39          46          73
6) 1212     7) 2163     9) 3654     8) 5624
   12          21          36          56
   0012        0063        0054        0024
    12          63          54          24
    00          00          00          00

   751         462         904         5637
5) 35256    8) 36821    7) 63030    9) 506764
   35          32          63          95
   0025        048         00030       56
     25          48            28        54
     006         0021          2         27
      5            16                     27
      1            5                      0064
                                            63
                                            1

    6           31          63          6
27) 1628    45) 13545   31) 19538   86) 5169
   162         135         186         516
   0008        00045        93         0009
                45          93
                00          008
```

3. Errors due to zero in dividend e.g.

```
   3000        2000        23          12
2) 600      9) 1800     3) 609      4) 408
   6           18          6           4
   0000        0000        009         0008
    000         00          9           8
                            00          00
```

```
   6                5
57)3420          64)3200
  342              32
  000              000
```

4. Errors in division and multiplication combinations e.g.

```
   81          800          11          06          3
8)169       9)1800       4)88        7)64       23)78
  16                                    63         66
   9                                     1         12
   8
   1

   4
82)248
  248
  000
```

5. Using a remainder equal to or larger than divisor e.g.

```
   8          89          22          210         71
7)64       7)654       3)69        3)900       6)50
  56         56          6           6          42
   8         94          09          3           8
             63          6           3           6
             31          3           00          2
                                     00
```

```
   6          813         32          254         6
9)86       7)654       36)1224     76)4793     21)147
  54         56          108         152        126
  32          9          144        3273         21
              7           72         380
             24           72        2893
             21
              3
```

```
    0411
47) 1974
    188
    0094
     47
     47
     47
     00
```

```
    13
27) 99
    27
    72
    71
    01
```

```
    71
29) 232
    203
    029
     29
     00
```

6. Omitting final remainder e.g.

```
    31
4) 127
```

```
    21
8) 169
```

```
    21
7) 148
```

```
    31
5) 156
```

7. Errors in subtraction combinations e.g.

```
    62
12) 753
    72
    33
    24
    11
```

```
    3
27) 99
    81
    19
```

```
    2
73) 155
   146
     8
```

```
    2
33) 74
    66
    12
```

```
    23
53) 1219
   106
   259
   159
   100
```

```
    516
97) 5438
    485
    158
     97
     618
     582
      36
```

TABLE 2

PUPILS' PERFORMANCE IN TEST 1 (BASIC ADDITION FACTS) Grade 3, 115 pupils.

C	N	%*	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
0+0	-	-	1+0	8	7.0	2+0	12	10.4	3+0	9	7.9	4+0	9	7.8	5+0	10	8.7	6+0	7	6.2	7+0	8	7.0	8+0	6	5.4	9+0	8	7.1
0+1	10	8.8	1+1	2	1.7	2+1	7	6.1	3+1	5	4.3	4+1	4	3.5	5+1	5	4.4	6+1	5	4.3	7+1	4	3.5	8+1	3	2.7	9+1	3	2.6
0+2	10	8.7	1+2	4	3.6	2+2	3	2.6	3+2	5	4.3	4+2	6	5.3	5+2	3	2.7	6+2	5	4.4	7+2	3	2.7	8+2	3	2.6	9+2	5	4.4
0+3	12	10.9	1+3	4	3.5	2+3	1	0.9	3+3	3	2.6	4+3	3	2.6	5+3	5	4.4	6+3	2	1.8	7+3	1	0.9	8+3	5	2.4	9+3	3	2.8
0+4	7	6.3	1+4	6	5.2	2+4	3	2.7	3+4	4	3.6	4+4	4	3.5	5+4	3	2.6	6+4	5	4.4	7+4	1	0.9	8+4	2	1.8	9+4	6	5.7
0+5	10	8.8	1+5	6	5.2	2+5	6	5.2	3+5	2	1.8	4+5	2	1.8	5+5	3	2.6	6+5	3	2.7	7+5	4	3.6	8+5	5	4.9	9+5	4	3.7
0+6	10	9.1	1+6	6	5.2	2+6	4	3.1	3+6	4	3.5	4+6	2	1.8	5+6	3	2.7	6+6	4	3.5	7+6	3	2.8	8+6	4	3.8	9+6	5	4.8
0+7	10	8.7	1+7	6	5.2	2+7	2	1.8	3+7	5	4.5	4+7	4	3.7	5+7	4	3.8	6+7	7	6.4	7+7	4	3.5	8+7	3	2.8	9+7	7	6.7
0+8	9	8.0	1+8	5	4.3	2+8	5	4.3	3+8	3	2.7	4+8	4	3.6	5+8	7	6.8	6+8	5	4.7	7+8	3	2.9	8+8	7	6.1	9+8	3	2.9
0+9	9	8.0	1+9	3	2.7	2+9	5	4.5	3+9	9	8.4	4+9	2	1.9	5+9	7	6.4	6+9	3	2.9	7+9	2	2.0	8+9	7	6.8	9+9	8	7.0

INTERPRETATION OF THE RESULTS

a) The most frequent errors were in the zero-facts (0+1, 0+2, 0+3, ..., 0+9) - about 8.6% of those who tried these combinations had them wrong.

b) Pupils made more errors in the combinations: 0+1, 0+2, 0+3, ..., 0+9, than in the combinations: 1+0, 2+0, 3+0, ..., 9+0. (8.6% versus 7.5%).

c) In those combinations involving the sum of a number and itself (i.e. 0+0, 1+1, 2+2, ... 8+8, 9+9), the most frequent errors were in the combinations 9+9 (7.0%) and 8+8 (6.1%).

d) Looking down the columns headed with the facts 1+0, 2+0, 3+0, ..., 9+0, it can be seen that, in general, more errors were made in the above-mentioned facts than in the subsequent facts in the respective columns. Similarly, looking along the rows with the entries 0+1, 0+2, ... 0+9, it can be seen that more errors were made in the above-mentioned facts than in the subsequent facts in the respective rows.

*In tests 1 thru 5, the percentage of errors was calculated by considering only the number of pupils who had the combination wrong out of those who tried that combination. For example, if 10 pupils omitted the combination 9+6, and 5 pupils had it wrong, then the percentage of error is $\frac{5}{(115-10)} \times 100 = \frac{5}{105} \times 100 = 4.8\%$

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

e) For the pupil, the combinations (a+b), (b+a) are two different ones. From the table, note the following pairs of combinations:

Combination	2+1, 1+2	3+2, 2+3	3+7, 7+3	3+9, 9+3	4+7, 7+4	9+4, 4+9	6+7, 7+6	9+7, 7+9	8+9, 9+8	0+3, 3+0
Percentage of Error	6.1, 3.6	4.3, 0.9	4.5, 0.9	8.4, 2.8	3.7, 0.9	5.7, 1.9	6.4, 2.8	6.7, 2.0	6.8, 2.8	10.9, 7.9

f) The fact that the percentage of errors in (a+b) combinations are not systematic with those in [(a+1)+b] combinations implies that the pupils were not adequately trained to see the relationships between numbers. From the table, note the following pairs of combinations:

Combination	1+1, 2+1	1+3, 2+3	6+4, 7+4	8+4, 9+4	2+5, 3+5	5+7, 6+7	8+7, 9+7	8+8, 9+8	4+8, 5+8	3+9, 4+9	4+9, 5+9	7+9, 8+9
Percentage of Error	1.7, 6.1	3.5, 0.9	4.4, 0.9	1.8, 5.7	5.2, 1.8	3.8, 6.4	2.8, 6.7	6.1, 2.9	3.6, 6.8	8.4, 1.9	1.9, 6.4	2.0, 6.8

g) These two schedules might strongly suggest that the pupils were not adequately trained to see the relationships between numbers.

TABLE 3

PUPILS' PERFORMANCE IN TEST 2. (BASIC SUBTRACTION FACTS) Grade 3, 115 Pupils.

C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
0-0	-	-	1-0	28	24.6	2-0	27	23.7	3-0	25	21.9	4-0	22	19.3	5-0	23	20.0	6-0	24	20.9	7-0	27	23.7	8-0	26	23.0	9-0	26	22.6
			1-1	5	4.3	2-1	5	4.3	3-1	8	7.0	4-1	8	7.0	5-1	7	6.1	6-1	10	8.9	7-1	10	8.7	8-1	8	7.0	9-1	10	8.7
						2-2	7	6.1	3-2	4	3.5	4-2	4	3.5	5-2	9	7.8	6-2	9	7.8	7-2	10	8.8	8-2	10	8.7	9-2	13	11.5
									3-3	7	6.1	4-3	9	7.8	5-3	4	3.5	6-3	12	10.4	7-3	8	7.0	8-3	10	8.8	9-3	9	7.8
												4-4	7	6.1	5-4	6	5.2	6-4	8	7.0	7-4	9	7.9	8-4	9	7.8	9-4	12	10.5
															5-5	6	5.2	6-5	5	4.3	7-5	2	1.7	8-5	11	9.7	9-5	8	7.0
																		6-6	7	6.1	7-6	7	6.1	8-6	11	9.7	9-6	12	10.4
																					7-7	11	9.6	8-7	8	7.0	9-7	10	8.8
																								8-8	10	8.8	9-8	6	5.2
																											9-9	9	7.8
10-1	7	6.1	11-2	9	7.9	12-3	17	16.0	13-4	12	11.0	14-5	14	13.7	15-6	18	16.8	16-7	17	17.0	17-8	19	17.8	18-9	15	13.5			
10-2	10	8.7	11-3	15	13.3	12-4	11	9.9	13-5	21	19.8	14-6	15	13.5	15-7	21	19.1	16-8	11	9.8	17-9	14	13.3						
10-3	9	8.0	11-4	13	11.6	12-5	14	12.5	13-6	14	12.5	14-7	11	9.7	15-8	18	16.2	16-9	20	19.0									
10-4	8	7.1	11-5	12	10.6	12-6	10	8.9	13-7	17	16.5	14-8	15	13.5	15-9	17	15.9												
10-5	4	3.5	11-6	15	13.8	12-7	15	13.3	13-8	16	15.0	14-9	12	11.9															
10-6	10	8.8	11-7	17	15.2	12-8	12	10.5	13-9	14	14.1																		
10-7	9	8.0	11-8	21	18.7	12-9	13	11.4																					
10-8	6	5.4	11-9	17	15.3																								
10-9	8	7.1																											

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

INTERPRETATION OF THE RESULTS

a) About 22.2% of the pupils made mistakes in the zero-facts i.e. 1-0, 2-0, 3-0,....,9-0. The highest percentage of errors was in the zero-facts.

b) The percentage of errors in subtracting a number from itself ranges from 4.3% (in the case of 1-1) to 9.6% (in the case of 7-7).

c) The percentage of errors in subtraction of numbers from 1 to 5 (average 5.6%) is lower than the percentage of errors in subtraction of numbers from 6 to 9 (average 8.0%) and this in turn is lower than the percentage of errors in subtraction of numbers from 10 to 18 (average 12.5%). However, subtracting a number from 10 seems to be relatively simpler than subtracting a number from 6, 7, 8 or 9.

d) Note the following pairs of combinations:

Combination	5-2, 5-3	6-1, 6-5	7-2, 7-5	9-1, 9-8	11-2, 11-9	11-3, 11-8	12-3, 12-9	13-5, 13-8	13-6, 13-7	17-8, 17-9
Percentage of Error	7.8, 3.5	8.9, 4.3	8.8, 1.7	8.7, 5.2	7.9, 15.3	13.3, 18.7	16.0, 11.4	19.8, 15.0	12.5, 16.5	17.8, 13.3

This implies that pupils were not adequately trained to see the relationships between numbers, i.e. to see that if a+b = c, then a = c-b, and b= c-a.

e) Note the following pairs of combinations:

Combination	10-9, 11-9	10-8, 11-8	15-8, 16-8	16-8, 17-8	10-7, 11-7	13-7, 14-7	14-7, 15-7	7-5, 8-5	10-5, 11-5	12-5, 13-5	13-5, 14-5
Percentage of Error	7.1, 15.3	5.4, 18.7	16.2, 9.8	9.8, 17.8	8.0, 15.2	16.5, 9.7	9.7, 19.1	1.7, 9.7	3.5, 10.6	12.5, 19.8	19.8, 13.7

This implies that pupils were not adequately trained to see the relationships between numbers, i.e. to see the relationship between (a-b) and [(a+1)-b] combinations.

f) Besides the zero-facts, the most difficult combinations seem to be those involving the subtraction of a number from 11, 12, ..., 18.

g) Besides the zero-facts, the most difficult fifteen combinations seem to be the following:

Combination	13-5	15-7	16-9	11-8	17-8	16-7	15-6	13-7	15-8	12-3	15-9	11-9	11-7	13-8	11-6
Percentage of Error	19.8	19.1	19.0	18.7	17.8	17.0	16.8	16.5	16.7	16.0	15.9	15.3	15.2	15.0	13.8

TABLE 4

PUPILS' PERFORMANCE IN TEST 3 (BASIC MULTIPLICATION FACTS) Grade 3, 115 Pupils.

C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
0x0	1	0.9	1x0	26	26.6	2x0	30	29.8	3x0	25	26.4	4x0	33	31.8	5x0	28	28.6	6x0	24	26.4	7x0	31	30.7	8x0	23	24.2	9x0	30	29.4
0x1	22	23.6	1x1	11	10.1	2x1	11	9.6	3x1	9	7.9	4x1	6	5.2	5x1	8	7.0	6x1	11	9.6	7x1	8	7.8	8x1	6	5.7	9x1	11	10.5
0x2	24	25.3	1x2	8	7.0	2x2	4	3.5	3x2	11	9.6	4x2	12	10.4	5x2	10	8.8	6x2	8	7.0	7x2	13	12.4	8x2	14	13.3	9x2	14	13.3
0x3	34	33.0	1x3	4	3.5	2x3	12	10.4	3x3	12	10.4	4x3	16	14.2	5x3	14	12.5	6x3	18	16.1	7x3	18	17.5	8x3	14	13.3	9x3	20	21.8
0x4	33	32.0	1x4	6	5.2	2x4	12	10.4	3x4	14	12.5	4x4	16	13.9	5x4	17	14.9	6x4	15	13.5	7x4	16	16.3	8x4	20	20.0	9x4	19	20.2
0x5	28	26.7	1x5	7	6.1	2x5	8	7.0	3x5	14	12.3	4x5	16	14.0	5x5	10	8.7	6x5	17	15.7	7x5	19	18.6	8x5	21	22.8	9x5	20	19.8
0x6	33	32.0	1x6	6	5.2	2x6	10	8.8	3x6	24	21.4	4x6	17	16.5	5x6	13	11.7	6x6	19	16.7	7x6	26	27.4	8x6	30	31.2	9x6	26	26.6
0x7	24	25.8	1x7	5	4.3	2x7	7	6.2	3x7	12	11.3	4x7	19	18.3	5x7	19	17.6	6x7	26	25.2	7x7	23	24.8	8x7	28	29.8	9x7	33	25.9
0x8	29	30.5	1x8	6	5.2	2x8	16	13.9	3x8	15	14.3	4x8	18	16.4	5x8	20	18.3	6x8	25	24.0	7x8	28	29.8	8x8	37	39.8	9x8	33	35.2
0x9	30	29.4	1x9	5	4.3	2x9	10	8.8	3x9	22	20.4	4x9	19	19.1	5x9	14	13.1	6x9	29	28.2	7x9	32	34.0	8x9	28	30.2	9x9	23	22.8
Average Percentage of Errors:					5.7			8.7			13.3			14.2			12.5			17.3			21.0			22.9			21.8

INTERPRETATION OF THE RESULTS

a) The zero-facts are the most difficult facts. About 28.5% of the pupils had these facts wrong.

b) The combinations 0x1, 0x2, ..., 0x9 and the combinations 1x0, 2x0, ..., 9x0 are of the same difficulty. In the former the average percentage of errors is 28.7% and in the latter 28.2%.

c) The percentage of errors in multiplying a number by itself ranges from 3.5% (in the case of 2x2), to 39.8% (in the case of 8x8).

d) Note that, excluding the zero facts, the multiplication facts become more and more difficult (i) as the numbers involved become larger and larger, and (ii) as the second number becomes larger and larger.

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

e) Note the following pairs of combinations:

Combination	1x9, 9x1	2x9, 9x2	3x7, 7x3	6x8, 8x6	8x9, 9x8	2x7, 7x2	3x6, 6x3	5x9, 9x5	7x9, 9x7
Percentage of Error	4.3, 10.5	8.8, 13.3	11.3, 17.5	24.0, 31.2	30,2, 35.2	6.2, 12.4	21.4, 16.1	13.1, 19.8	34.0, 25.9

This implies that, for the pupil, the combinations (axb), and (bxa) are two different ones.

f) Note the following pairs of combinations:

Combination	2x2, 2x3	3x5, 3x6	3x8, 3x9	5x4, 5x5	6x2, 6x3	6x6, 6x7	7x5, 7x6	8x1, 8x2	8x3, 8x4	8x5, 8x6	8x7, 8x8
Percentage of Error	3.5, 10.4	12.3, 21.4	14.3, 20.4	14.9, 8.7	7.1, 16.1	16.7, 25.2	18.4, 27.4	5.7,13.3	13.3, 20.0	22.8, 31,2	29.8, 39,8

The fact that the percentages of errors in (axb) combinations are not systematic with those in a(b+1) combinations implies that the pupils were not adequately trained to see the relationships between numbers.

8x8, 8x9	9x2 9x3	9x5, 9x6	9x7, 9x8	9x8, 9x9
39.8, 30.2	13.2, 21.8	19.8, 26.6	25.9, 35,2	35.2, 22.8

g) Besides the zero-facts, the fifteen most difficult combinations seem to be the following :

Combination	8x8	9x8	7x9	8x6	8x9	7x8	8x7	6x9	7x6	9x6	9x7	6x7	7x7	6x8	9x9
Percentage of Error	39.8	35.2	34.0	31.2	30.2	29.8	29.8	28.2	27.4	26.6	25.9	25.2	24.8	24.0	22.8

TABLE 5

PUPILS' PERFORMANCE IN TEST 4. (BASIC DIVISION FACTS) Grade 3, 115 Pupils.

C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
0÷1	27	30.0	1÷1	17	17.4	2÷1	26	25.5	3÷1	22	21.2	4÷1	31	31.6	5÷1	23	21.2	6÷1	22	19.8	7÷1	22	21.0	8÷1	29	26.4	9÷1	23	21.3
0÷2	22	26.8	2÷2	27	27.0	4÷2	14	12.2	6÷2	19	16.5	8÷2	26	22.6	10÷2	19	16.7	12÷2	19	16.7	14÷2	24	21.0	16÷2	25	23.8	18÷2	36	31.8
0÷3	24	28.2	3÷3	37	35.6	6÷3	30	27.6	9÷3	19	16.7	12÷3	25	22.6	15÷3	26	23.2	18÷3	30	28.6	21÷3	33	29.4	24÷3	29	29.6	27÷3	31	32.4
0÷4	24	29.0	4÷4	24	24.8	8÷4	29	25.4	12÷4	25	22.3	16÷4	18	17.7	20÷4	27	24.3	24÷4	28	28.0	28÷4	24	24.5	32÷4	26	27.0	36÷4	33	31.4
0÷5	25	28.7	5÷5	28	27.8	10÷5	29	25.4	15÷5	28	24.6	20÷5	19	16.5	25÷5	15	13.2	30÷5	30	26.8	35÷5	23	23.0	40÷5	27	24.6	45÷5	29	30.2
0÷6	25	29.2	6÷6	26	26.6	12÷6	36	31.2	18÷6	29	30.6	24÷6	27	25.2	30÷6	21	20.0	36÷6	31	28.0	42÷6	32	32.3	48÷6	27	28.8	54÷6	27	29.0
0÷7	27	30.7	7÷7	24	29.6	14÷7	24	25.2	21÷7	26	30.0	28÷7	22	25.0	35÷7	22	25.6	42÷7	22	27.2	49÷7	21	24.4	56÷7	25	29.8	63÷7	25	30.8
0÷8	26	30.6	8÷8	24	29.0	16÷8	32	34.0	24÷8	25	29.8	32÷8	25	26.6	40÷8	24	28.6	48÷8	26	30.2	56÷8	24	30.4	64÷8	24	29.8	72÷8	24	29.6
0÷9	26	31.0	9÷9	25	27.8	18÷9	28	31.4	27÷9	24	29.3	36÷9	25	30.6	45÷9	22	27.2	54÷9	29	33.7	63÷9	27	33.4	72÷9	22	26.5	81÷9	21	26.2

INTERPRETATION OF THE RESULTS

a) 29.3% of the pupils made errors in the zero-combinations: 0÷1, 0÷2, ..., 0÷9.

b) 22.8% of the pupils made errors in the combinations: 1÷1, 2÷1, ..., 9÷1.

c) 22.8% of the pupils made errors in the combinations: 1÷1, 2÷2, 3-3, ..., 9÷9.

d) 20.9% of the pupils made a mistake in dividing a number* by 2

e) 28.3% " " " " " " " " " " " 3

f) 25.1% " " " " " " " " " " " 4

g) 23.6% " " " " " " " " " " " 5

h) 28.0% " " " " " " " " " " " 6

i) 29.7% " " " " " " " " " " " 7

j) 29.8% " " " " " " " " " " " 8

k) 29.6% " " " " " " " " " " " 9

*A number different from zero.

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

l) Remarks (d) to (k) suggest that, in general, the division facts become more and more difficult as the divisor becomes larger and larger.

m) On the other hand, division by 5 seems to be easier than division by 3. This might mean that pupils do more grouping by fives than by threes.

n) Note the following pairs of combinations:

Combination	6÷3, 6÷2	10÷5, 10÷2	12÷6, 12÷2	16÷8, 16÷2	20÷5, 20÷4	30÷6, 30÷5	42÷7, 42÷6
Percentage of Error	27.6, 16.5	25.4, 16.7	31.2, 16.7	34.0, 23.8	16.5, 24.3	20.0, 26.8	27.2, 32.3

This means that the pupils were not adequately trained to see that if $a = b.c$, then $b = \frac{a}{c}$ and $c = \frac{a}{b}$.

o) The fifteen most difficult combinations seem to be:

Combination	3÷3	16÷8	54÷9	63÷9	27÷3	42÷6	18÷2	36÷4	18÷9	12÷6	63÷7	18÷6	56÷8	48÷8	45÷5
Percentage of Error	35.6	34.0	33.7	33.4	32.4	32.3	31.8	31.4	31.4	31.2	30.8	30.6	30.4	30.2	30.2

p) Comparing the results of tests 1, 2, 3, 4, it is seen that the pupils' performance gets lower and lower in that same order. One of the reasons for this is that the basic facts are taught in the above order itself and that, as research indicates, teachers tend to provide more practice in addition and multiplication than in subtraction and division. This might also be due to an intrinsic difficulty in the respective tests.

TABLE 6

PUPILS' PERFORMANCE IN TEST 5 (MIXED SUMS) Grade 3, 115 Pupils.

C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
3+8	9	7.8	7x6	34	32.0	42÷7	34	31.5	6+7	15	15.0	27÷3	35	31.2
12-7	17	14.8	5x0	36	31.6	7x9	39	37.2	9+5	26	22.8	15-7	22	19.6
18-9	23	20.0	13-6	20	17.7	1x0	31	27.6	63÷7	27	26.4	5+6	15	13.4
4x6	21	19.3	8x7	30	27.8	15-8	21	18.7	5+9	15	13.3	11-6	19	17.0
36÷9	38	35.8	64÷8	41	40.1	9+6	14	13.1	13-8	20	18.5	3+9	11	10.1
7x7	31	29.5	3x0	34	31.4	0÷5	45	41.3	14-6	21	20.2	15-9	23	21.1
8+5	16	14.8	4+9	12	11.5	9x6	39	38.6	48÷8	30	31.0	8x0	34	32.4
13-5	21	19.6	17-8	24	24.2	4+7	12	12.4	6x9	35	37.6	8+6	19	20.0
8x5	23	23.4	5÷5	26	26.8	13-4	20	20.0	7+6	18	18.9	63÷9	30	33.0
0÷8	37	37.0	16-9	22	23.2	9x3	29	30.2	14-5	15	15.8	0x2	28	28.9
16-7	26	26.3	11-7	20	21.5	6x0	28	29.4	7x8	26	29.6	5+8	20	22.0
6x7	31	33.0	0x1	28	30.4	4÷4	25	27.5	0x5	26	28.6	4x9	27	30.4
7+7	23	25.6	17-9	18	20.2	9+4	15	16.7	54÷9	23	28.4	36÷4	24	29.3
13-9	20	22.0	9+7	16	18.8	0÷7	26	31.8	11-8	17	21.0	54÷6	21	26.2
84÷12	32	80.0	11x7	31	43.6	88÷11	29	67.5	121÷11	22	78.6	12x6	37	55.3
96÷12	28	82.4	60÷12	29	83.0	10x12	29	57.0	132÷12	22	96.0	12x4	26	42.6
108÷12	25	80.9	120÷12	25	86.2	11x12	32	78.0	11x11	38	86.4	77÷11	25	80.6
12x7	27	55.0	72÷12	25	100.0	12x8	25	57.0	144÷12	17	81.0	12x11	29	83.0
11x8	23	49.0	110÷11	17	81.0	11x9	19	41.4	132÷11	21	100.0	99÷11	24	92.5
11x10	25	61.0	12x12	26	70.2	48÷12	22	91.6	12x9	22	52.4	12x5	23	51.0

INTERPRETATION OF THE RESULTS

a) Consider the following combinations:

Combination	% of Error in Test 1	% of Error in test 5
3+8	2.7	7.8
8+5	4.9	14.8
4+9	1.2	11.5
9+7	6.7	18.8
9+6	4.8	13.1
4+7	3.8	12.4
9+4	5.7	16.7
6+7	6.4	15.0
9+5	3.7	22.8
5+9	6.4	13.3
7+6	2.8	18.9
5+6	2.7	13.4
8+6	3.8	20.0
5+8	6.8	22.0

Combination	% of Error in Test 2	% of Error in Test 5
18-9	13.5	20.0
16-7	17.0	26.3
13-9	14.1	22.0
13-6	12.5	17.7
17-8	17.8	24.2
16-9	19.0	23.2
11-7	15.2	21.5
17-9	13.3	20.2
13-4	11.0	20.0
13-8	15.0	18.5
14-6	13.5	20.2
15-9	15.9	21.1

Combination	% of Error in Test 3	% of Error in Test 5
4x6	16.5	19.3
7x7	24.8	29.5
6x7	25.2	33.0
7x6	27.4	32.0
5x0	28.6	31.6
3x0	26.4	31.4
0x1	23.6	30.4
7x9	34.0	37.2
9x6	26.6	38.6
9x3	21.8	30.2
6x0	26.4	29.4
6x9	28.2	37.6
8x0	24.2	32.4
0x2	25.3	28.9
4x9	19.1	30.4

Combination	% of Error in Test 4	% of Error in Test 5
36÷9	30.6	35.8
0÷8	30.6	37.0
7÷7	29.6	25.6
64÷8	29.8	40.1
42÷7	27.2	31.5
0÷5	28.7	41.3
63÷7	30.8	26.4
54÷9	33.7	28.4
54÷6	29.0	26.2

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

It is seen that the pupils responded differently when the same test contained mixed sums than when all the sums were of the same type. Thus pupils could not observe the bonds carefully. Probably the pupils did not have sufficient practice in working out a test or an exercise of mixed sums in order to get into the habit of carefully observing the bonds between numbers.

b) If we consider the last thirty combinations - in the bottom six rows - we notice that:

1. 86.8% of the pupils made mistakes in division by 12.
2. 60.2% " " " " " " multiplication by 12.
3. 83.4% " " " " " " division by 11.
4. 71.5% " " " " " " multiplication by 11.

The reason for this very high percentage of error is that the pupils did not deal with the tables of 11 and 12 - as reported by the pupils and the teachers as well.

c) From (b) above we could conclude that division by 11 and by 12 seem to be more difficult than multiplication by 11 and by 12; also division by 11 and by 12 seem to be equally difficult and multiplication by 11 and by 12 seem to be equally difficult.

TABLE 7

(MIXED SUMS)

PUPILS' PERFORMANCE IN TEST 5.

Grade 4, 87 Pupils.

C	N	%	C	N	%	C	N	%	C	N	%	C	N	%
3+8	8	9.2	7x6	18	21.4	42÷7	17	21.2	6+7	12	14.3	27÷3	26	32.2
12-7	12	13.8	5x0	21	24.2	7x9	25	30.1	9+5	15	17.9	15-7	9	10.8
18-9	16	18.4	13-6	10	11.9	1x0	16	18.8	63÷7	20	27.0	5+6	4	4.8
4x6	13	15.1	8x7	20	23.8	15-8	9	10.8	5+9	4	4.7	11-6	12	14.7
36÷9	29	34.5	64÷8	21	28.4	9+6	9	10.8	13-8	11	13.6	3+9	6	7.3
7x7	16	18.6	3x0	19	22.9	0÷5	17	21.8	14-6	13	16.1	15-9	13	16.0
8+5	7	8.1	4+9	9	10.7	9x6	18	23.1	48÷8	20	28.6	8x0	17	20.8
13-5	11	12.8	17-8	12	15.4	4+7	5	6.3	6x9	15	20.0	8+6	10	12.5
8x5	12	14.8	5÷5	21	27.3	13-4	9	11.7	7+6	10	12.8	63÷9	15	21.4
0÷8	26	32.5	16-9	12	15.6	9x3	15	19.7	14-5	12	15.4	0x2	15	18.5
16-7	15	18.1	11-7	9	11.8	6x0	14	18.4	7x8	16	20.5	5+8	7	8.8
6x7	18	22.2	0x1	9	11.7	4÷4	18	24.4	0x5	10	12.8	4x9	14	18.2
7÷7	20	25.6	17-9	17	22.7	9+4	5	8.4	54÷9	17	24.0	36÷4	14	19.2
13-9	11	14.3	9+7	9	12.0	0÷7	19	26.8	11-8	13	18.1	54÷6	8	11.7
84÷12	28	52.0	11x7	16	24.6	88÷11	25	42.4	121÷11	23	50.0	12x6	18	30.5
96÷12	23	50.0	60÷12	19	47.5	10x12	20	35.7	132÷12	17	44.7	12x4	19	34.0
108÷12	12	33.3	120÷12	13	38.3	11x12	25	41.6	11x11	27	55.0	77÷11	18	42.0
12x7	19	36.6	72÷12	11	40.8	12x8	23	50.0	144÷12	16	50.0	12x11	21	44.6
11x8	10	21.3	110÷11	16	48.5	11x9	9	20.4	132÷11	17	50.0	99÷11	15	41.6
11x10	22	46.8	12x12	27	61.4	48÷12	15	65.3	12x9	17	39.5	12x5	13	30.2

(Key: C = Combination; N = Number of Errors; % = Percentage of Error).

INTERPRETATION OF THE RESULTS

Combinations*	Percentage of Error Grade 3	Grade 4
Addition	15.4	9.9
Subtraction	20.1	14.8
Multiplication	29.4	19.9
Division	31.4	25.4
Division by 11	83.4	45.8
Multiplication by 11	71.5	36.3
Division by 12	86.8	46.9
Multiplication by 12	60.2	40.4
Average	49.8	30.0

This schedule shows that:

a) The frequency of errors made by Grade 4 on this test is remarkable. However, this might be due to carelessness in observing the bonds in every combination (especially in the first 14 rows) and not to a lack of mastery of these combinations, as has been noticed in tests 1-4 and test 5 for Grade 3.

b) Grade 4 is in a better position on this test than Grade 3. This difference is especially obvious in the combinations involving division and multiplication by 11 and by 12.

c) Multiplication by 11 and by 12 seem to be easier than division by 11 and by 12 for both Grade 3 and Grade 4.

d) Besides, multiplication by 11 and by 12 seem to be equally difficult; and division by 11 and by 12 seem to be equally difficult.

*The different combinations in the first 14 rows are considered first and then the last 6 rows (multiplication and division by 11 and by 12.

TABLE 8

COMPARISON OF PUPILS' PERFORMANCE IN TESTS 1-4

Grade 3; 115 Pupils.

Number of Combinations Worked	T E S T '1'				T E S T '2'				T E S T '3'				T E S T '4'			
	Number of Pupils whose % of Error is:				Number of Pupils whose % of Error is:				Number of Pupils whose % of Error is:				Number of Pupils whose % of Error is:			
	10% or Less	11%-20%	21%-30%	More than 30%	10% or Less	11%-20%	21% - 30%	More than 30%	10% or Less	11%-20%	21% - 30%	More than 30%	10% or Less	11%-20%	21% - 30%	More than 30%
80-100	98	8	-	2	84	16	2	11	60	15	5	20	48	9	1	25
60-79	2	1	1	2	-	-	1	-	3	1	-	-	1	6	-	2
40-59	-	-	-	-	-	-	-	1	3	-	2	3	4	3	1	3
39 or Less	-	-	-	1	-	-	-	-	1	1	1	-	2	5	3	2

INTERPRETATION OF THE RESULTS

1. Consider the first row:

   a) The majority of pupils worked between 80-100 combinations (80-90 in test 4). In terms of percentage, this corresponds to 94%, 98%, 87% and 72% of the pupils.

   b) Of those pupils who worked between 80-100 combinations, the majority had only 10 percent or less of the combinations wrong.

   c) However, it is also observed that as we move from test 1 to test 4, the working accuracy of the pupils decends in that same order. In other words, the percentage of pupils having 10% or less of the combinations wrong decends from 85% in test 1, to 73% in test 2, to 52% in test 3, to 42% in test 4.

   d) It is important to note that the percentage of pupils who had more than 30% of the combinations wrong increases as we move from test 1 to test 4. This percentage rises from 1.8% in test 1, to 9.6% in test 2, to 17.4% in test 3, to 21.8% in test 4.

2. If we consider the remaining three rows, it is seen that nearly all pupils worked all the combinations in test 1-3, whereas in test 4, 7% worked between 60-79 combinations, 9.6% worked between 40-59 combinations, and 10.4% worked 39 combinations or less.

3. Three groups of pupils can be distinguished from this table:

   Group 1. This group includes those pupils who make a large number of errors (more than 30%) in tests 1-4. Some of the pupils in this group are fast (top row), others are very slow (lower two rows). The dull, and the less competent pupils fall in this group.

   Group 2. This group includes those pupils whose accuracy is satisfactory, but who are slow in responding to the number combinations. (See the few pupils in the second row).

   Group 3. This group includes those pupils who have good working accuracy in tests 1-4, but who make certain special errors. (See top row, those pupils who made 10% or less of the combinations wrong.)

TABLE 9

INDIVIDUAL PUPIL'S PERFORMANCE IN TESTS 1-5

Grade 3, 115 Pupils.

P	Test 1		Test 2		Test 3		Test 4		Test 5	
	W	L	W	L	W	L	W	L	W	L
1	1	-	8	-	4	35	3	52	5	79
2	-	-	1	-	7	25	18	38	5	65
3	19	-	9	-	25	1	5	44	9	55
4	-	-	-	-	-	73	4	72	7	71
5	-	-	12	-	4	62	4	66	18	60
6	-	-	10	-	17	41	21	45	8	86
7	2	-	1	-	75	1	9	58	6	69
8	1	-	9	15	5	20	7	53	5	67
9	1	-	-	-	3	-	3	40	5	50
10	1	-	1	-	2	-	3	-	3	28
11	-	-	3	1	16	56	8	61	19	35
12	-	-	-	-	1	-	1	-	5	21
13	21	24	42	-	96	-	90	-	71	28
14	-	-	4	1	4	59	4	74	4	84
15	3	-	1	-	2	54	5	58	8	63
16	-	-	1	-	7	68	5	80	7	73
17	1	3	6	-	10	38	6	55	23	62
18	-	-	16	-	15	20	4	69	17	81
19	1	-	1	-	-	-	-	-	5	22
20	-	-	-	-	1	-	1	-	8	14
21	-	-	1	-	7	-	3	-	12	22
22	3	-	8	-	20	1	73	-	30	31
23	19	4	15	-	38	-	44	-	70	-
24	2	40	37	19	1	59	71	3	27	47
25	2	-	-	-	1	-	1	-	1	15
26	-	-	1	-	1	-	10	-	20	-
27	-	-	-	-	9	8	10	31	2	33
28	5	1	-	-	19	9	6	-	19	14
29	-	20	12	3	31	1	63	1	28	41
30	5	-	1	-	28	-	13	1	30	14
31	3	-	12	12	41	5	31	10	33	31
32	43	16	20	-	49	-	76	-	60	-
33	1	4	23	8	34	8	64	-	79	8
34	-	-	-	-	3	-	2	-	12	14
35	-	-	10	-	-	-	13	-	36	-
36	1	-	11	4	20	-	12	26	14	46
37	1	-	12	4	1	-	16	-	27	18
38	30	40	7	-	51	-	23	49	64	-
39	-	-	1	-	1	-	-	-	6	12
40	-	-	2	6	10	-	8	23	8	30
41	1	-	1	-	-	-	1	30	7	25
42	32	40	34	55	18	56	50	34	5	49
43	1	-	3	4	6	-	-	-	5	30
44	-	-	4	4	16	-	10	29	11	46
45	-	-	1	-	-	-	-	-	19	12
46	-	-	-	-	3	-	5	-	13	15
47	-	-	1	-	1	-	21	-	18	31
48	-	-	-	-	12	-	3	2	16	33
49	-	-	-	-	1	-	-	-	3	28
50	-	-	-	-	13	-	5	-	9	25
51	-	-	-	-	-	-	-	5	7	22
52	1	1	-	-	2	-	-	-	19	1
53	-	-	1	-	5	-	-	-	5	23
54	15	1	15	1	39	-	56	1	66	5
55	-	3	3	-	18	1	50	5	35	15
56	-	-	-	-	5	-	4	40	5	36
57	1	-	1	2	22	-	8	10	20	26
58	11	16	16	35	60	20	72	9	84	5
59	2	4	35	-	78	-	82	-	78	-
60	1	-	-	-	1	4	4	2	11	20
61	4	-	11	-	14	12	3	64	11	40
62	2	-	5	-	16	-	15	-	14	26
63	9	-	15	-	17	4	26	22	19	34
64	1	-	5	-	21	-	10	-	18	23
65	-	-	4	5	-	-	5	37	6	47
66	7	-	1	2	9	-	1	-	19	4
67	-	-	-	-	10	-	3	-	14	7
68	8	36	92	2	24	57	82	-	69	17
69	1	-	-	-	1	-	9	-	6	28
70	-	-	-	-	1	-	-	-	1	19
71	16	-	92	-	65	-	70	-	76	-
72	-	-	1	-	-	-	-	-	-	26
73	19	1	79	-	80	1	64	1	98	-
74	-	-	12	4	1	-	9	3	3	28
75	-	-	-	2	1	-	4	-	5	28
76	-	-	1	-	1	-	2	-	8	15
77	1	-	-	-	1	-	10	-	1	42
78	13	-	45	-	58	-	90	-	96	-
79	-	-	4	-	15	12	66	-	60	-
80	87	-	99	-	80	-	87	-	95	-
81	2	-	1	-	-	-	2	1	6	30
82	2	-	2	1	1	-	2	-	2	30
83	1	-	5	15	28	6	79	-	62	18
84	-	-	15	-	7	-	19	-	7	19
85	25	7	96	-	62	-	80	-	20	70
86	25	28	85	-	70	1	79	2	93	-
87	1	6	36	1	67	2	88	2	97	-
88	4	-	5	-	39	-	59	-	48	39
89	-	-	24	-	48	-	80	-	83	5
90	2	-	3	-	5	-	2	35	3	65
91	2	-	1	-	-	-	10	-	6	16
92	-	-	7	-	1	-	2	38	4	29
93	3	-	1	-	5	-	5	45	5	60
94	-	-	-	-	-	-	3	-	1	36
95	11	-	9	15	10	58	32	57	11	84
96	-	-	2	-	3	6	4	-	7	15
97	3	-	4	6	6	-	3	19	4	67
98	1	-	1	-	-	-	12	-	3	20
99	-	-	-	-	-	-	1	-	2	21
100	-	-	1	-	2	13	2	18	20	6
101	1	11	10	-	1	25	6	-	6	30
102	-	-	2	-	-	-	5	-	6	14
103	1	-	-	-	-	-	1	-	3	15
104	1	-	2	-	5	-	6	3	4	23
105	-	-	-	-	-	-	1	-	-	21
106	2	-	13	-	13	-	9	1	27	26
107	-	-	9	-	1	-	-	-	3	15
108	3	-	3	4	20	-	10	24	1	56
109	1	-	5	-	5	-	9	5	17	18
110	1	-	1	-	12	-	-	-	7	7
111	-	-	1	9	1	-	-	-	3	36
112	-	-	-	-	-	-	2	-	5	19
113	1	-	1	-	1	-	-	-	15	20
114	-	-	1	5	1	-	8	-	7	24
115	-	-	1	-	1	-	-	1	1	25

INTERPRETATION OF THE RESULTS

a) In general, the pupils' knowledge of the basic addition and basic subtraction facts is better than their knowledge of the basic multiplication and basic division facts.

b) All pupils showed more errors in Test 5 than in Tests 1-4.

c) Some pupils are extremely weak all through the Tests 1-5. Examples: pupils 13, 23, 32, 38, 42, 68, 71, 73, 78, 80, 85, 86.

(Key: P = Pupils; W = Number of Combinations Wrong; L = Number of Combinations Left Out).

TABLE 10

PUPILS' PERFORMANCE IN TEST 6. (GRADED ADDITION) Grade 4; 87 Pupils.

STEP	N				%				C				%				B				%				B % C				%				A	%	O	%	D	%
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4						
I	11	3	2	3	12.6	3.5	2.3	3.5	6	1	-	-	6.9	1.2	-	-	-	1	-	-	-	1.2	-	-	-	-	-	-	-	-	-	-	4	4.6	3	3.5	1	1.2
II	7	1	1	4	8.0	1.2	1.2	4.6	8	-	-	1	9.2	-	-	1.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	1.2	-	-
III	4	1	1	3	4.6	1.2	1.2	3.5	2	1	-	-	2.3	1.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2.3	1	1.2
IV	13	6	2	1	15.0	6.9	2.3	1.2	11	5	-	-	12.6	5.7	-	-	-	-	1	-	-	-	1.2	-	-	-	-	-	-	-	-	-	-	2	2.3	1	1.2
V	9	4	4	1	10.3	4.6	4.6	1.2	10	3	-	-	11.5	3.5	-	-	4	1	-	-	4.6	1.2	-	-	2	-	-	-	2.3	-	-	-	-	-	-	-	1	1.2
VI	9	2	1	4	10.3	2.3	1.2	4.6	9	1	-	-	10.3	1.2	-	-	2	-	-	1	2.3	-	-	1.2	-	1	-	-	-	1.2	-	-	-	-	-	-	-	-
VII	10	1	2	4	11.5	1.2	2.3	4.6	9	-	1	-	10.3	-	1.2	-	-	-	-	1	-	-	-	1.2	-	-	-	-	-	-	-	-	-	-	-	-	2	2.3
VIII	9	3	-	6	10.3	3.5	-	6.9	8	2	-	-	9.2	2.3	-	-	-	1	1	2	-	1.2	1.2	2.3	1	-	-	-	1.2	-	-	-	-	-	-	-	-	-
IX	16	6	2	2	18.4	6.9	2.3	2.3	12	1	-	-	13.8	1.2	-	-	10	2	-	-	11.5	2.3	-	-	2	1	-	-	2.3	1.2	-	-	-	-	-	-	1	1.2
XX	14	3	1	5	16.1	3.5	1.2	5.8	12	3	-	-	13.8	3.5	-	-	5	-	-	1	5.7	-	-	1.2	1	-	1	-	1.2	-	1.2	-	-	-	-	-	-	-
XI	19	11	5	3	21.8	12.6	5.8	3.5	14	4	-	-	16.1	4.6	-	-	13	3	-	-	15.0	3.5	-	-	-	4	1	-	-	4.6	1.2	-	-	-	6	7.0	-	-
XII	18	3	3	3	20.7	3.5	3.5	3.5	14	2	1	-	16.1	2.3	1.2	-	12	2	-	-	13.8	2.3	-	-	-	-	-	-	-	-	-	-	-	-	1	1.2	-	-
XIII	20	14	8	6	23.0	16.1	9.2	6.9	18	4	1	-	20.3	4.6	1.2	-	21	5	1	-	24.3	5.8	1.2	-	3	1	1	2	3.5	1.2	1.2	2.3	-	-	4	4.6	-	-
XIV	26	17	8	4	30.0	19.5	9.2	4.6	21	5	1	-	24.2	5.8	1.2	-	20	3	-	-	23.0	3.5	-	-	10	1	-	1	11.5	1.2	-	1.2	-	-	13	15.0	-	-
A	12				17.9																																	
B	21				31.8																																	

INTERPRETATION OF THE RESULTS

a) The percentage of pupils who made only one mistake in steps I - XIV predominates.

b) The majority of errors in the respective steps is due to errors in the basic addition combinations.

c) Those who made only one mistake in the addition combinations are considerably greater than those who made 2, 3, or 4 mistakes. This might mean that those mistakes are due to carelessness, rather than due to insufficient knowledge of these facts.

d) The pupils know how to add using bridging fairly well.

e) Horizontal addition - operations A, and B - seems not so easy for the pupils. Probably practice in horizontal addition is insufficient.

*In tests 6-11, the percentage of error was calculated by considering the number of pupils who had the sum wrong out of the total number of pupils.

(Key: N = Number of Errors; C = Errors in Basic Addition Facts; B = Errors in Bridging; A = Adding same Digit in Two Columns; D = Disregarding Column Position; % = Percentage of Error).

TABLE 11

PUPILS' PERFORMANCE IN TEST 7. (GRADED SUBTRACTION) Grade 4; 87 Pupils.

STEP	N				%				C				%				B				%				B & C				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	8	1	2	9	9.2	1.2	2.3	10.3	7	1	1	6	8.0	1.2	1.2	6.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	8	4	-	6	9.2	4.6	-	6.9	7	4	-	5	8.0	4.6	-	5.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
III	14	2	1	6	16.1	2.3	1.2	6.9	13	2	1	5	15.0	2.3	1.2	5.8	1	-	-	-	1.2	-	-	-	-	-	-	-	-	-	-	-
IV	6	2	1	6	6.9	2.3	1.2	6.9	6	1	1	5	6.9	1.2	1.2	5.8	-	1	1	-	-	1.2	1.2	-	-	-	-	-	-	-	-	-
V	16	3	4	18	18.4	3.5	4.6	20.6	13	2	-	2	15.0	2.3	-	2.3	7	2	3	14	8.0	2.3	3.5	16.1	1	-	-	-	1.2	-	-	-
VI	16	5	2	16	18.4	5.8	2.3	18.4	15	3	-	-	17.2	3.5	-	-	7	3	2	12	8.0	3.5	2.3	13.8	-	1	1	-	-	1.2	1.2	-
VII	12	8	2	18	13.8	9.2	2.3	20.6	14	3	-	-	16.1	3.5	-	-	6	-	2	14	6.9	-	2.3	16.1	4	-	1	2	4.6	-	1.2	2.3
VIII	25	5	2	16	28.8	5.8	2.3	18.4	12	2	-	-	13.8	2.3	-	-	17	2	2	14	19.5	2.3	2.3	16.1	1	-	1	1	1.2	-	1.2	1.2
IX	7	13	3	5	8.0	15.0	3.5	5.8	6	2	1	-	6.9	2.3	1.2	-	5	15	1	2	5.8	17.2	1.2	2.3	-	-	-	-	-	-	-	-
X	7	2	3	15	8.0	2.3	3.5	21.8	2	-	-	-	2.3	-	-	-	7	1	3	17	8.0	1.2	3.5	19.5	-	-	-	1	-	-	-	1.2
XI	12	3	3	14	13.8	3.5	3.5	16.1	11	1	1	1	12.6	1.2	1.2	1.2	4	3	2	10	4.6	3.5	2.3	11.5	1	1	-	-	1.2	1.2	-	-
XII	15	13	8	10	17.2	15.0	9.2	11.5	10	5	1	-	11.5	5.8	1.2	-	14	6	5	8	16.1	6.9	5.8	9.2	3	1	-	-	3.5	1.2	-	-
XIII	11	5	11	16	12.6	6.9	12.6	18.4	4	3	-	-	4.6	3.5	-	-	9	7	10	13	10.3	8.0	11.5	15.0	1	1	-	-	1.2	1.2	-	-
XIV	19	4	6	11	21.8	4.6	6.9	12.6	7	1	-	-	8.0	1.2	-	-	11	4	7	9	12.6	4.6	8.0	10.3	3	-	-	-	3.5	-	-	-

INTERPRETATION OF THE RESULTS

a) The percentage of pupils who made four mistakes in every step is fairly high.

b) A conceivable number of errors in the respective steps is due to errors in the basic subtraction combinations.

c) In steps X-XIV, it is seen that the percentage of pupils who made 3 or 4 mistakes is high and that the majority of these mistakes is due to bridging. Thus knowledge of bridging, as a whole, is still insufficient as can be seen from the column headed "B".

(Key: N = Number of Errors; C = Errors in Basic Subtraction Facts; B = Errors in Bridging; % = Percentage of Error).

TABLE 12

PUPILS' PERFORMANCE IN TEST 8A & B. (GRADED MULTIPLICATION) Grade 4; 87 Pupils.

STEP	N				%				C				%				E.R				%				E.D				%				O.R &/O.D				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	8	1	1	12	9.2	1.2	1.2	13.8	7	1	1	–	8.0	1.2	1.2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	1.2	–	–	–
II	9	2	–	14	10.3	2.3	–	16.1	6	2	–	2	6.9	2.3	–	2.3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	3	–	–	1	3.5	–	–	1.2
III	15	1	2	19	17.2	1.2	2.3	21.8	14	–	3	6	16.1	–	3.5	6.9	–	–	–	–	–	–	–	–	1	1	–	–	1.2	1.2	–	–	–	–	–	–	–	–	–	–
IV	15	3	4	15	17.2	3.5	4.6	17.2	11	2	3	1	12.6	2.3	3.5	1.2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	1	1.2	–	–	1.2
V	13	5	7	16	15.0	5.8	8.0	18.4	15	4	6	3	17.2	4.6	6.9	3.5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
VI	14	7	15	20	16.1	8.0	17.2	23.0	9	8	3	2	10.3	9.2	3.5	2.3	–	1	–	–	–	1.2	–	–	9	1	7	2	10.3	1.2	8.0	2.3	8	–	–	–	9.2	–	–	–
VII	21	18	11	16	24.2	20.7	12.6	18.4	25	3	3	–	28.8	3.5	3.5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	11	1	2	4	12.6	1.2	2.3	4.6
VIII	20	9	3	22	23.0	10.3	3.5	25.3	11	3	1	–	12.6	3.5	1.2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	1	7	2.3	–	1.2	8.0
IX	15	5	4	21	17.2	5.8	4.6	24.2	11	4	1	–	12.6	4.6	1.2	–	–	1	–	–	–	1.2	–	–	1	1	–	–	1.2	1.2	–	–	1	–	–	7	1.2	–	–	8.0
X	14	13	1	22	16.1	15.0	1.2	25.3	11	–	–	2	12.6	–	–	2.3	10	8	1	1	11.5	9.2	1.2	1.2	6	6	–	–	6.9	6.9	–	–	1	–	–	3	1.2	–	–	3.5
XI	26	17	7	30	30.0	19.5	8.0	23.0	22	3	2	–	25.3	3.5	2.3	–	–	–	–	–	–	–	–	–	4	1	–	–	4.6	1.2	–	–	6	4	1	4	6.9	4.6	1.2	4.6
XII	16	15	5	34	18.4	17.2	5.8	39.2	13	6	1	8	15.0	6.9	1.2	9.2	1	1	–	–	1.2	1.2	–	–	7	1	1	3	8.0	1.2	1.2	3.5	2	1	–	6	2.3	1.2	–	6.9
XIII	24	14	36		27.6	16.1	41.5		15	1	2		17.2	1.2	2.3		4	1	–		4.6	1.2	–		2	1	–		2.3	1.2	–		15	4	8		17.2	4.6	9.2	
XIV	18	34			20.7	39.2			9	3			10.3	3.5			2	2			2.3	2.3			2	–			2.3	–			5	8			5.8	9.2		

A				%				P.P				%			
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	1	–	–	–	1.2	–	–	–
6	1	–	1	6.9	1.2	–	1.2	–	–	–	–	–	–	–	–
–	–	–	1	–	–	–	1.2	–	–	1	–	–	–	1.2	–
3	–	–	1	3.5	–	–	1.2	2	–	–	–	2.3	–	–	–
18	2	–	–	20.7	2.3	–	–	3	2	1	–	3.5	2.3	1.2	–
6	2	–	–	6.9	2.3	–	–	8	1	1	3	9.2	1.2	1.2	3.5
3	1	–	–	3.5	1.2	–	–	3	3	–	–	3.5	3.5	–	–
1	–	–	–	1.2	–	–	–	4	2	–	–	4.6	2.3	–	–
17	1	–	–	19.5	1.2	–	–	6	1	–	1	6.9	1.2	–	1.2
13	–	1	–	15.0	–	1.2	–	4	1	–	–	4.6	1.2	–	–
16	–	–		18.4	–	–		13	2	1		15.0	2.3	1.2	
5	1			5.8	1.2			3	2			3.5	2.3		

INTERPRETATION OF THE RESULTS

a) The majority of errors is in the basic multiplication facts; and

b) those who made only one mistake in the multiplication facts are considerably greater than those who made 2, 3 or 4 mistakes. This might be due to carelessness or due to insufficient knowledge of these facts.

c) Next in importance are errors due to omission of a digit from multiplier and/or multiplicand.

d) Note that some pupils, though not many, made mistakes in the position of partial products; and besides,

e) some made mistakes, though mainly one mistake, in adding partial products.

f) On the whole, it seems safe to state that the major difficulty of the pupils is lack of perfect knowledge of basic multiplication facts.

(Key: N = Number of Errors; C = Errors in Multiplication Facts; E.R= Errors due to Zero in Multiplier; E.D = Errors due to Zero in Multiplicand; O.R = Omitting Digit in Multiplier; O.D = Omitting Digit in Multiplicand; A = Errors in Addition; P.P = Errors in Position of Partial Products; % = Percentage of Error).

TABLE 13

PUPILS' PERFORMANCE IN TEST 8A & B. (GRADED MULTIPLICATION) Grade 5; 68 Pupils.

STEP	N				%				C				%				E.R				%				E.D				%				O.R & O.D				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	9	2	1	1	13.3	2.9	1.5	1.5	8	2	1	-	11.8	2.9	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	1	1	2	2	1.5	1.5	2.9	2.9	-	1	2	1	-	1.5	2.9	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
III	7	1	5	2	10.3	1.5	7.4	2.9	7	1	4	1	10.3	1.5	5.9	1.5	-	-	-	-	-	-	-	-	1	-	-	-	1.5	-	-	-	-	1	-	-	-	1.5	-	-
IV	12	4	-	3	17.7	5.9	-	4.4	13	3	-	2	19.1	4.4	-	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
V	20	11	4	3	29.4	16.2	5.9	4.4	18	10	3	2	26.5	14.7	4.4	2.9	-	-	-	-	-	-	-	-	1	-	-	-	1.5	-	-	-	-	-	-	-	-	-	-	-
VI	12	4	6	5	17.7	5.9	8.8	7.4	7	6	4	1	10.3	8.8	5.9	1.5	-	-	-	-	-	-	-	-	4	-	1	-	5.9	-	1.5	-	3	-	-	-	4.4	-	-	-
VII	29	12	5	2	42.6	17.7	7.4	2.9	33	2	1	-	48.5	2.9	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	-	5.9	-	-	-
VIII	12	10	3	2	17.7	14.7	4.4	2.9	13	4	1	-	19.1	5.9	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1.5	-	-	-
IX	16	6	3	3	23.6	8.8	4.4	4.4	16	3	2	-	23.6	4.4	2.9	-	1	1	-	-	1.5	1.5	-	-	2	-	-	-	2.9	-	-	-	1	-	-	-	1.5	-	-	-
X	17	7	1	4	25.0	10.3	1.5	5.9	16	-	2	-	23.6	-	2.9	-	10	1	-	-	14.7	1.5	-	-	2	1	-	-	2.9	1.5	-	-	2	-	-	-	2.9	-	-	-
XI	23	18	4	2	33.8	26.5	5.9	2.9	20	8	1	-	29.4	11.8	1.5	-	-	-	-	-	-	-	-	-	4	-	-	-	5.9	-	-	-	5	1	-	-	7.4	1.5	-	-
XII	14	12	6	9	20.6	17.7	8.8	13.3	16	7	5	2	23.6	10.3	7.4	2.9	1	1	-	-	1.5	1.5	-	-	4	-	-	1	5.9	-	-	1.5	-	2	-	-	-	2.9	-	-
XIII	16	16	13		23.6	23.6	19.1	-	13	3	4		19.1	4.4	5.9		2	-	-		2.9	-	-		3	-	-		4.4	-	-		7	2	-		10.3	2.9	-	
XIV	14	13			20.6	19.1			7	6			10.3	8.8			-	-			-	-			3	-			4.4	-			1	-			1.5	-		

A				%				P.P				%			
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	-	-	-	1.5	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	1.5	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	1.5	-	-	-	-	-	-	-	-	-	-	-
2	1	-	-	2.9	1.5	-	-	-	-	-	-	-	-	-	-
2	-	-	-	2.9	-	-	-	3	-	-	-	4.4	-	-	-
11	2	-	-	16.2	2.9	-	-	4	-	-	-	5.9	-	-	-
2	1	-	-	2.9	1.5	-	-	8	2	-	-	11.8	2.9	-	-
1	-	-	-	1.5	-	-	-	1	1	1	-	1.5	1.5	1.5	-
-	-	-	-	-	-	-	-	1	-	-	-	1.5	-	-	-
13	-	-	-	19.1	-	-	-	2	1	-	-	2.9	1.5	-	-
3	-	-	-	4.4	-	-	-	-	-	-	1	-	-	-	1.5
6	-	-		8.8	-	-		13	3	1		19.1	4.4	1.5	
1	-			1.5	-			7	1			10.3	1.5		

INTERPRETATION OF THE RESULTS

a) The majority of errors is in the basic multiplication facts, and

b) those who made only one mistake in the multiplication facts are considerably greater than those who made 2, 3 or 4 mistakes. This might be due to carelessness or due to insufficient knowledge of these facts.

c) In the remaining columns the percentage of those who made only one mistake in every step predominates. The conclusion is that knowledge of the technique of multiplication is satisfactory.

d) Comparing grades 4 and 5 it is seen that grade 5 made fewer mistakes in every respect than grade 4 - except that the percentage of those who made one mistake in the multiplication in grade 5 is relatively higher than their counterpart in grade 4.

(Key: N = Number of Errors; C = Errors in Basic Multiplication Facts; E.R = Errors due to Zero in Multiplier; E.D = Errors due to Zero in Multiplicand; O.R = Omitting Digit in Multiplier; O.D = Omitting Digit in Multiplicand; A = Errors in Addition; P.P = Errors in Position of Partial Products; % = Percentage of Error).

TABLE 14

PUPILS' PERFORMANCE IN TEST 8A & B. (GRADED MULTIPLICATION) Grade 6; 70 Pupils.

	N				%				C				%				E.R				%				E.D				%				O.R & O.D				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	3	-	-	-	4.3	-	-	-	3	-	-	-	4.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	1	1	-	-	1.4	1.4	-	-	1	1	-	-	1.4	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
III	5	1	-	1	7.1	1.4	-	1.4	6	-	-	1	8.6	-	-	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-	-
IV	8	4	1	-	11.4	5.7	1.4	-	7	3	1	-	10.0	4.3	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-	-
V	9	3	1	2	12.9	4.3	1.4	2.9	9	3	1	2	12.9	4.3	1.4	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
VI	15	5	3	3	21.4	7.1	4.3	4.3	14	2	4	1	20.0	2.9	5.7	1.4	-	-	-	-	-	-	-	-	2	-	1	-	2.9	-	1.4	-	4	-	-	-	5.7	-	-	-
VII	43	7	1	-	61.5	10.0	1.4	-	36	2	-	-	51.5	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	-	-	-	10.0	-	-	-
VIII	15	5	3	-	21.4	7.1	4.3	-	8	3	2	-	11.4	4.3	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	7.1	-	-	-
IX	11	4	1	1	15.7	5.7	1.4	1.4	12	2	-	1	17.2	2.9	-	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-
X	15	3	2	1	21.4	4.3	2.9	1.4	9	2	-	-	12.9	2.9	-	-	2	1	-	-	2.9	1.4	-	-	2	1	-	-	2.9	1.4	-	-	2	-	-	-	2.9	-	-	-
XI	19	10	3	-	27.2	14.3	4.3	-	12	9	-	-	17.2	12.9	-	-	-	-	-	-	-	-	-	-	2	-	-	-	2.9	-	-	-	1	-	-	-	1.4	-	-	-
XII	11	6	2	5	15.7	8.6	2.9	7.1	11	5	1	4	15.7	7.1	1.4	5.7	-	-	-	-	-	-	-	-	4	-	1	-	5.7	-	1.4	-	-	-	-	-	-	-	-	-
XIII	24	12	5		34.4	17.2	7.1		12	2	2		17.2	2.9	2.9		1	-	-		1.4	-	-		4	-	-		5.7	-	-		9	-	-		12.9	-	-	
XIV	20	5			28.6	7.1			8	3			11.4	4.3			-	-			-	-			4	-			5.7	-			2	-			2.9	-		

	A				%				P.P				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
III	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
IV	2	-	-	-	2.9	-	-	-	-	-	-	-	-	-	-	-
V	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
VI	-	-	-	-	-	-	-	-	3	-	-	-	4.3	-	-	-
VII	13	-	-	-	18.6	-	-	-	-	-	-	-	-	-	-	-
VIII	5	1	-	-	7.1	1.4	-	-	2	-	-	-	2.9	-	-	-
IX	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-
X	2	-	-	-	2.9	-	-	-	4	1	-	-	5.7	1.4	-	-
XI	9	2	-	-	12.9	2.9	-	-	-	-	-	-	-	-	-	-
XII	1	-	-	-	1.4	-	-	-	1	-	-	-	1.4	-	-	-
XIII	13	-	-		18.6	-	-		11	2	-		15.7	2.9	-	
XIV	3	-			4.3	-			7	-			10.0	-		

INTERPRETATION OF THE RESULTS

a) The majority of errors is in the basic multiplication combinations and

b) those who made only one mistake in the multiplication combinations are considerably greater than those who made 2, 3, or 4 mistakes. This might be due to carelessness or to insufficient knowledge of these facts.

c) In the remaining columns the percentage of those who made only one mistake in every step predominates. The conclusion is that knowledge of the technique of multiplication is satisfactory.

d) Comparing the results of grades 4, 5 and 6 we see that grade 6 made the least errors in every respect.

(Key: N = Number of Errors; C = Errors in Basic Multiplication Facts; E.R = Errors due to Zero in Multiplier; E.D = Errors due to Zero in Multiplicand; O.R = Omitting Digit in Multiplier; O.D = Omitting Digit in Multiplicand; A = Errors in Addition; P.P = Errors in Position of Partial products; % = Percentage of Error).

TABLE 15

PUPILS' PERFORMANCE IN TEST 9. (GRADED DIVISION) Grade 4; 87 Pupils.

STEP	N				%				T				%				Q				%				Z				%				M &/D				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	5	7	3	22	5.8	8.0	3.5	25.3	2	2	2	20	2.3	2.3	2.3	23.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	6	-	-	4.6	6.9	-	-
II	10	7	2	23	11.5	8.0	2.3	26.6	-	1	2	21	-	1.2	2.3	24.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10	6	-	-	11.5	6.9	-	-
III	9	4	5	25	10.3	4.6	5.8	28.8	2	-	-	22	2.3	-	-	25.3	-	-	-	-	-	-	-	-	1	4	2	1	1.2	4.6	2.3	1.2	8	2	-	1	9.2	2.3	-	1.2
IV	12	15	6	24	13.8	17.3	6.9	27.6	2	1	2	21	2.3	1.2	2.3	24.2	-	-	-	-	-	-	-	-	4	1	2	-	4.6	1.2	2.3	-	12	11	3	3	13.8	12.6	3.5	-
V	7	5	10	52	8.0	5.8	11.5	59.8	4	-	5	23	4.6	-	5.8	26.6	4	3	7	24	4.6	3.5	8.0	27.6	1	-	-	-	1.2	-	-	-	4	3	-	-	4.6	3.5	-	-
VI	19	4	7	31	21.8	4.6	8.0	35.6	2	1	2	23	2.3	1.2	2.3	26.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	1	2	-	9.2	1.2	2.3	-
VII	15	7	6	32	17.3	8.0	6.9	36.8	4	4	2	27	4.6	4.6	2.3	31.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	1	-	-	10.3	1.2	-	-
VIII	5	3	7	28	5.8	3.5	8.0	32.3	4	-	3	24	4.6	-	3.5	27.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	2	-	-	4.6	2.3	-	-
IX	11	6	10	22	12.6	6.9	11.5	25.3	8	3	7	20	9.2	3.5	8.0	23.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	1	-	-	6.9	1.2	-	-
X	10	7	3	25	11.5	8.0	3.5	28.8	5	2	3	21	5.8	2.3	3.5	24.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	2	1	-	6.9	2.3	1.2	-
XI	5	3	5	28	5.8	3.5	5.8	32.3	1	1	1	21	1.2	1.2	1.2	24.2	2	-	4	6	2.3	-	4.6	6.9	-	-	-	-	-	-	-	-	3	1	1	-	3.5	1.2	1.2	-

	U				%				O				%				S				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
III	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
IV	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
V	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
VI	10	-	-	-	11.5	-	-	-	4	1	5	6	4.6	1.2	5.8	-	1	-	-	-	1.2	-	-	-
VII	8	4	1	-	9.2	4.6	1.2	-	-	1	4	-	-	1.2	4.6	-	4	-	1	-	4.6	-	1.2	-
VIII	-	-	-	-	-	-	-	-	3	1	1	3	3.5	1.2	1.2	-	2	1	1	-	2.3	1.2	1.2	-
IX	1	4	1	-	1.2	4.6	1.2	-	1	-	-	-	1.2	-	-	-	-	2	-	-	-	2.3	-	-
X	8	1	-	-	9.2	1.2	-	-	-	-	-	-	-	-	-	-	3	-	-	-	3.5	-	-	-
XI	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1.2	-	-	-

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 3, or 4 mistakes outweighs by far the percentage of those who made 1 or 2 mistakes in every step.

b) The main source of errors is lack of knowledge of the technique of division. The percentage of those who made 3 or 4 mistakes in the technique outweighs by far the percentage of those who made only 1 or 2 mistakes. It can be safely said that around 25% of the 4th graders do not master the division skills involved in Test 9.

c) Next in importance are errors due to insufficient knowledge of the basic division and basic multiplication combinations. However, it should be noted that those who made only one mistake here predominate. This implies that the mistake might be due to carelessness or due to lapses of thought.

(Key: N = Number of Errors; T = Errors in the Technique of Division; Q = Errors due to Zero in Quotient; Z = Errors due to Zero in Divisor; M &/D = Errors in Basic Multiplication and Basic Division Facts; U = Errors due to Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder; S = Errors in Basic Subtraction Facts; % = Percentage of Error).

TABLE 16

PUPILS' PERFORMANCE IN TEST 9. (GRADED DIVISION) Grade 5; 68 Pupils.

STEP	N				%				T				%				Q				%				Z				%				%				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	7	1	2	3	10.3	1.5	2.9	4.4	-	-	1	3	-	-	1.5	4.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	1	-	-	8.8	1.5	-	-
II	8	2	1	5	11.8	2.9	1.5	7.4	-	-	-	5	-	-	-	7.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	2	-	-	11.8	2.9	-	-
III	9	4	2	6	13.3	5.9	2.9	8.8	-	-	-	5	-	-	-	7.4	-	-	-	-	-	-	-	-	3	3	-	1	4.4	4.4	-	1.5	7	2	1	-	10.3	2.9	1.5	-
IV	7	6	2	6	10.3	8.8	2.9	8.8	-	2	-	5	-	2.9	-	7.4	-	-	-	-	-	-	-	-	-	1	-	1	-	1.5	-	1.5	8	4	-	-	11.8	5.9	-	-
V	3	4	4	44	4.4	5.9	5.9	64.7	1	-	2	11	1.5	-	2.9	16.2	4	3	2	31	5.9	4.4	2.9	45.6	-	-	-	-	-	-	-	-	1	-	-	-	1.5	-	-	-
VI	11	11	4	12	16.2	16.2	5.9	17.7	2	1	-	8	2.9	1.5	-	11.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	4	2	-	11.8	5.9	2.9	-
VII	15	6	6	12	22.0	8.8	8.8	17.7	1	-	2	9	1.5	-	2.9	13.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11	1	-	-	16.2	1.5	-	-
VIII	9	2	2	13	13.3	2.9	2.9	19.1	-	-	-	10	-	-	-	14.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	1	-	-	10.3	1.5	-	-
IX	8	14	8	17	11.8	20.6	11.8	25.0	3	2	4	12	4.4	2.9	5.9	17.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13	4	1	-	19.1	5.9	1.5	-
X	14	10	4	12	20.6	14.7	5.9	17.7	4	2	-	11	5.9	2.9	-	16.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	4	-	-	13.3	5.9	-	-
XI	11	3	5	25	16.2	4.4	7.4	26.8	6	2	1	8	8.8	2.9	1.5	11.8	6	2	7	10	8.8	2.9	10.3	14.7	-	-	-	-	-	-	-	-	5	2	-	-	7.4	2.9	-	-

	U				%				O				%				S				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	-	-	10.3	1.5	-	-	4	1	2	1	5.9	1.5	2.9	1.5	5	1	-	-	7.4	1.5	-	-
10	2	1	-	14.7	2.9	1.5	-	2	1	-	2	2.9	1.5	-	2.9	4	-	-	-	5.9	-	-	-
1	1	-	-	1.5	1.5	-	-	2	-	2	2	2.9	-	2.9	2.9	-	-	-	-	-	-	-	-
4	2	2	-	5.9	2.9	2.9	-	-	-	-	-	-	-	-	-	9	4	-	-	13.3	5.9	-	-
10	4	-	-	14.7	5.9	-	-	-	-	-	-	-	-	-	-	2	1	-	-	2.9	1.5	-	-
2	1	-	-	2.9	1.5	-	-	1	-	-	-	1.5	-	-	-	1	-	-	-	1.5	-	-	-

(Key: N = Number of Errors; T = Errors in the Technique of Division; Q = Errors due to Zero in Quotient; Z = Errors due to Zero in Dividend; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder; S = Errors in Subtraction Combinations; % = Percentage of Error).

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 3 or 4 mistakes is less than the percentage of those who made 1 or 2 mistakes except in steps V, VIII, IX, XI.

b) The main source of errors is lack of knowledge of the technique of division. The percentage of those who made 3 or 4 mistakes in the technique outweighs by far the percentage of those who made only 1 or 2 mistakes. It can be safely said that around 10% of the 5th graders do not master the division skills involved in Test 9.

c) Next in importance are errors due to inadequate knowledge of the basic division and basic multiplication combinations. However, it should be noted that those who made only one mistake here predominate. This implies that the mistake might be due to carelessness or due to lapses of thought.

d) Comparing grades 4 and 5 it can be said that the 5th graders are better off in the skills involved in Test 9 - notice the difference in the percentage of errors in the technique of division in both cases.

TABLE 17

PUPILS' PERFORMANCE IN TEST 9. (GRADED DIVISION) Grade 6; 70 Pupils.

STEP	N				%				T				%				Q				%				Z				%				D &/M				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	4	1	-	2	5.7	1.4	-	2.9	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	1	-	-	5.7	1.4	-	-
II	9	-	-	2	12.9	-	-	2.9	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	-	-	-	12.9	-	-	-
III	9	1	-	2	12.9	1.4	-	2.9	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-	9	-	-	-	12.9	-	-	-
IV	7	1	-	2	10.0	1.4	-	2.9	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-	-	6	1	-	-	8.6	1.4	-	-
V	5	2	6	33	7.1	2.9	8.6	47.2	-	-	-	3	-	-	-	4.3	5	1	5	32	7.1	1.4	7.1	45.8	-	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-
VI	16	6	1	3	22.8	8.6	1.4	4.3	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	3	1	1	12.9	4.3	1.4	1.4
VII	12	3	-	2	17.2	4.3	-	2.9	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	1	-	-	7.1	1.4	-	-
VIII	11	-	-	2	15.7	-	-	2.9	1	-	-	2	1.4	-	-	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	7.1	-	-	-
IX	19	8	8	2	27.2	11.4	11.4	2.9	6	1	1	2	8.6	1.4	1.4	2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16	-	-	-	22.8	-	-	-
X	15	10	3	5	21.4	14.3	4.3	7.1	4	-	-	3	5.7	-	-	4.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	12	3	-	-	17.2	4.3	-	-
XI	16	10	11	16	22.8	14.3	15.7	22.8	2	1	-	3	2.9	1.4	-	4.3	7	4	9	7	10.0	5.7	12.9	10.0	-	-	-	-	-	-	-	-	16	3	-	-	22.8	4.3	-	-

U				%				O				%								%			
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	4.3	-	-	-	4	-	-	-	5.7	-	-	-	6	-	-	-	8.6	-	-	-
5	-	-	-	7.1	-	-	-	-	-	-	-	-	-	-	-	6	-	-	-	8.6	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	7.1	-	-	-
7	2	-	-	10.0	2.9	-	-	-	-	-	-	-	-	-	-	13	2	1	-	18.6	2.9	1.4	-
11	5	1	-	15.7	7.1	1.4	-	-	-	-	-	-	-	-	-	8	1	-	-	11.4	1.4	-	-
8	-	-	-	11.4	-	-	-	-	-	-	-	-	-	-	-	3	-	-	-	4.3	-	-	-

(Key: N = Number of Errors; T = Errors in the Technique of Division; Q = Errors due to Zero in Quotient; Z = Errors due to Zero in Dividend; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder; S = Errors in Subtraction Combinations; % = Percentage of Error).

INTERPRETATION OF THE RESULTS

a) Knowledge of the technique of division and of the basic multiplication combinations seem to be the two main sources of errors.

b) It can be safely said that the 6th graders master the division skills involved in Test 9 satisfactorily

c) Comparing grades 4, 5 and 6, it is seen that

   1. Grade 6 made the least number of errors as a whole and Grade 4 made the greatest number of errors.

   2. Grade 6 made the least number of errors in the technique of division (around 3%) whereas Grade 4 made the greatest number of errors (around 25%).

d) The percentages of errors in the basic division and basic multiplication facts are nearly the same in the three grades. This implies that carelessness or more lapses of thought are probably the main causes of errors in this respect.

TABLE 18

PUPILS' PERFORMANCE IN TEST 10. (GRADED DIVISION) Grade 5; 68 Pupils.

STEP	N				%				T				%				D &/M				%				U				%				O				%				S				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	1	2	4	11	1.5	2.9	5.9	16.2	-	1	5	12	-	1.5	7.4	17.7	1	1	-	-	1.5	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	10	6	1	15	14.7	8.8	1.5	2.2	4	-	1	13	5.9	-	1.5	19.1	6	1	-	-	8.8	1.5	-	-	5	1	-	-	7.4	1.5	-	-	-	-	-	1	-	-	-	1.5	3	-	-	-	4.4	-	-	-
III	14	3	5	16	20.6	4.4	7.4	23.6	1	-	1	14	1.5	-	1.5	20.6	12	2	-	-	17.7	2.9	-	-	3	2	-	-	4.4	2.9	-	-	-	-	1	-	-	-	1.5	-	7	1	-	-	10.3	1.5	-	-
IV	11	4	2	19	16.2	5.9	2.9	28.0	-	-	1	14	-	-	1.5	20.6	7	1	-	-	10.3	1.5	-	-	5	1	-	1	7.4	1.5	-	1.5	1	1	1	1	1.5	1.5	1.5	1.5	8	-	-	-	11.8	-	-	-
V	8	8	1	14	11.8	11.8	1.5	20.6	-	-	1	13	-	-	1.5	19.1	8	-	-	-	11.8	-	-	-	7	2	-	-	10.3	2.9	-	-	-	-	-	-	-	-	-	-	4	-	-	-	5.9	-	-	-
VI	10	6	-	17	14.7	8.8	-	25.0	-	1	-	13	-	1.5	-	19.1	4	4	-	1	5.9	5.9	-	1.5	2	1	-	-	2.9	1.5	-	-	-	-	-	2	-	-	-	2.9	4	-	-	-	5.9	-	-	-
VII	14	9	7	21	20.6	13.3	10.3	30.8	8	2	4	18	11.8	2.9	5.9	26.4	9	4	-	-	13.3	5.9	-	-	6	1	-	-	8.8	1.5	-	-	-	-	-	-	-	-	-	-	4	4	-	-	5.9	5.9	-	-
VIII	10	10	5	19	14.7	14.7	7.4	28.0	6	2	-	18	8.8	2.9	-	26.4	5	6	-	-	7.4	8.8	-	-	4	2	-	-	5.9	2.9	-	-	-	-	-	-	-	-	-	-	8	1	-	-	11.8	1.5	-	-
IX	6	1	1	16	8.8	1.5	1.5	23.6	1	-	-	15	1.5	-	-	22.0	4	-	1	-	5.9	-	1.5	-	1	1	-	-	1.5	1.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 3 or 4 mistakes outweighs the percentage of those who made 1 or 2 mistakes.

b) The main source of difficulty is lack of knowledge of the technique of division. More than 20% of the pupils made 4 mistakes in the technique of division.

c) Errors due to multiplication and/or division facts, to using a remainder equal to or larger than divisor, and errors in subtraction facts do not seem to be serious since those who made only one mistake in every step predominate.

(Key: N = Number of Errors; T = Errors in the Technique of Division; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder; S = Errors in Subtraction Combinations; % = Percentage of Error).

TABLE 19

PUPILS' PERFORMACE IN TEST 10. (GRADED DIVISION) Grade 6; 70 Pupils.

STEP	N				%				T				%				D &/M				%				U				%				O				%				S				%			
	1	2	3		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	4	1	-	4	5.7	1.4	-	5.7	-	-	-	4	-	-	-	5.7	2	-	-	-	2.9	-	-	-	-	1	-	-	-	1.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
II	7	-	-	5	10.0	-	-	7.1	-	-	-	4	-	-	-	5.7	3	-	-	-	4.3	-	-	-	2	-	-	-	2.9	-	-	-	-	-	-	1	-	-	-	1.4	2	-	-	-	2.9	-	-	-
III	23	5	-	4	32.8	7.1	-	5.7	-	-	-	4	-	-	-	5.7	12	3	-	-	17.2	4.3	-	-	5	-	-	-	7.1	-	-	-	-	-	-	-	-	-	-	-	6	1	-	-	8.6	1.4	-	-
IV	16	4	2	6	22.8	5.7	2.9	8.6	-	-	-	4	-	-	-	5.7	6	1	-	-	8.6	1.4	-	-	4	-	-	2	5.7	-	-	2.9	1	-	-	-	1.4	-	-	-	10	2	1	-	14.3	2.9	1.4	-
V	7	-	-	5	10.0	-	-	7.1	-	-	-	4	-	-	-	5.7	4	-	-	-	5.7	-	-	-	2	-	-	1	2.9	-	-	1.4	-	-	-	-	-	-	-	-	1	-	-	-	1.4	-	-	-
VI	16	-	1	4	22.8	-	1.4	5.7	-	-	-	4	-	-	-	5.7	8	-	-	-	11.4	-	-	-	1	-	-	-	1.4	-	-	-	-	-	1	-	-	-	1.4	-	7	-	-	-	10.0	-	-	-
VII	16	7	9	7	22.8	10.0	12.9	10.0	7	3	1	7	10.0	4.3	1.4	10.0	15	4	-	-	21.4	5.7	-	-	10	1	-	-	14.3	1.4	-	-	-	-	-	-	-	-	-	-	4	1	-	-	5.7	1.4	-	-
VIII	19	6	12	10	27.2	8.6	17.2	14.3	7	3	3	6	10.0	4.3	4.3	8.6	15	5	1	-	21.4	7.1	1.4	-	11	2	-	-	15.7	2.9	-	-	-	-	-	-	-	-	-	-	11	2	1	-	15.7	2.9	1.4	-
IX	4	3	1	3	5.7	4.3	1.4	4.3	-	-	-	3	-	-	-	4.3	-	2	-	-	-	2.9	-	-	2	2	-	-	2.9	2.9	-	-	-	-	-	-	-	-	-	-	3	-	-	-	4.3	-	-	-

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 3 or 4 mistakes is lower than the percentage of those who made 1 or 2 mistakes.

b) The main source of difficulty is lack of knowledge of the technique of division. About 6% made 3 or 4 mistakes in the technique of division.

c) Errors due to multiplication and/or division facts, to using a remainder equal to or larger than divisor, and errors in subtraction facts, do not seem to be serious since those who made only one mistake in every step predominate.

d) Comparing grades 5 and 6, it is seen that grade 6 are better off in their knowledge of the division skills involved in Test 10.

e) Concerning the remaining types of errors, grades 5 and 6 are nearly in the same position. May be in both cases the error is due to carelessness or to lapses of thought.

(Key: N = Number of Errors; T = Errors in the Technique of Division; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder ; S = Errors in Subtraction Combinations; % = Percentage of Error).

TABLE 20

(GRADED DIVISION)

PUPILS' PERFORMANCE IN TEST 11.

Grade 5; 68 Pupils.

STEP	N				%				T				%				D &/M				%				U				%				Q				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	10	2	16	24	14.7	2.9	22.8	35.3	5	4	4	23	7.4	5.9	5.9	33.8	7	2	–	–	10.3	2.9	–	–	9	1	–	–	13.3	1.5	–	–	12	10	–	–	17.7	14.7	–	–
II	14	6	4	18	20.6	8.8	5.9	26.4	3	4	2	18	4.4	5.9	2.9	26.4	5	3	–	–	7.4	4.4	–	–	6	1	–	–	8.8	1.5	–	–	–	–	–	–	–	–	–	–
III	10	7	7	25	14.7	10.3	10.3	36.8	–	3	5	21	–	4.4	7.4	30.8	7	3	1	–	10.3	4.4	1.5	–	4	3	1	–	5.9	4.4	1.5	–	–	–	–	–	–	–	–	–
IV	14	6	6	23	20.6	8.8	8.8	33.8	6	4	3	22	8.8	5.9	4.4	32.4	12	1	–	–	17.7	1.5	–	–	4	1	–	–	5.9	1.5	–	–	–	–	–	–	–	–	–	–
V	9	13	10	22	13.3	19.1	14.7	32.4	13	2	3	21	19.1	2.9	4.4	30.8	9	2	–	–	13.3	2.9	–	–	7	–	–	–	10.3	–	–	–	12	–	–	–	17.7	–	–	–
VI	12	6	6	26	17.7	8.8	8.8	38.2	4	1	–	19	5.9	1.5	–	28.0	10	2	–	–	14.7	2.9	–	–	7	2	2	1	10.3	2.9	2.9	1.5	10	–	3	–	14.7	–	4.4	–

O				%				S				%			
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
–	–	–	–	–	–	–	–	7	–	–	–	10.3	–	–	–
–	–	–	–	–	–	–	–	2	–	–	–	2.9	–	–	–
–	–	1	–	–	–	1.5	–	4	2	–	–	5.9	2.9	–	–
–	–	–	–	–	–	–	–	5	–	–	–	7.4	–	–	–
–	–	–	–	–	–	–	–	9	–	–	–	13.3	–	–	–
1	–	–	–	1.5	–	–	–	10	–	–	–	14.7	–	–	–

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 3 or 4 mistakes outweighs the percentage of those who made 1 or 2 mistakes.

b) The main source of difficulty is lack of knowledge of the technique of division. About 34% made 3 or 4 mistakes in the technique of division. In other words, about 34% of the 5th graders still do not master the division skills involved in Test 11.

c) Errors due to multiplication and/or division facts, to using a remainder equal to or larger than divisor, and errors in subtraction facts do not seem to be serious since those who made only one mistake in every step predominate.

d) Comparing the results of grade 5 in Tests 9, 10 and 11, it is seen that these tests rank in difficulty in that ascending order.

(Key: N = Number of Errors; T = Errors in the Technique of Division; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; O = Omitting Final Remainder; S = Errors in Subtraction Combinations; % = Percentage of Error).

TABLE 21

PUPILS' PERFORMANCE IN TEST 11. (GRADED DIVISION) Grade 6; 70 Pupils.

STEP	N				%				T				%				D &/M				%				U				%				Q				%			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
I	7	22	21	12	10.0	31.4	30.0	17.2	10	3	4	4	14.3	4.3	5.7	5.7	11	2	-	-	15.7	2.9	-	-	9	3	-	-	12.9	4.3	-	-	11	28	-	-	15.7	40.0	-	-
II	14	2	-	5	2.0	2.9	-	7.1	-	-	-	4	-	-	-	5.7	9	-	-	-	12.9	-	-	-	10	1	-	-	14.3	1.4	-	-	-	-	-	-	-	-	-	-
III	18	9	2	8	25.8	12.9	2.9	11.4	2	-	1	3	2.9	-	1.4	4.3	11	6	1	-	15.7	8.6	1.4	-	5	2	1	-	7.1	2.9	1.4	-	-	-	-	-	-	-	-	-
IV	13	10	5	8	18.6	14.3	7.1	11.4	8	1	1	6	11.4	1.4	1.4	8.6	5	2	-	-	7.1	2.9	-	-	3	5	-	-	4.3	7.1	-	-	-	-	-	-	-	-	-	-
V	17	10	8	16	24.2	14.3	11.4	22.8	15	14	1	5	21.4	20.0	1.4	7.1	11	1	1	-	15.7	1.4	1.4	-	10	-	-	1	14.3	-	-	1.4	15	-	-	-	21.4	-	-	-
VI	17	9	12	10	24.2	12.9	17.2	14.3	7	4	1	2	10.0	5.7	1.4	2.9	12	3	-	-	17.2	4.3	-	-	9	4	1	-	12.9	5.7	1.4	-	18	5	1	-	25.8	7.1	1.4	-

O				%				S				%			
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
-	-	-	-	-	-	-	-	11	2	-	-	15.7	2.9	-	-
-	-	-	-	-	-	-	-	2	-	-	-	2.9	-	-	-
-	-	-	-	-	-	-	-	14	-	-	-	20.0	-	-	-
-	-	-	-	-	-	-	-	15	1	-	-	21.4	1.4	-	-
-	-	-	-	-	-	-	-	14	2	-	-	20.0	2.9	-	-
-	-	-	-	-	-	-	-	10	1	-	-	14.3	1.4	-	-

INTERPRETATION OF THE RESULTS

a) The percentage of those who made 1 or 2 mistakes outweighs the percentage of those who made 3 or 4 mistakes.

b) The main source of difficulty is lack of knowledge of the technique of division. About 4% made 3 or 4 mistakes in the technique of division.

c) Errors due to multiplication and/or division facts, to using a remainder equal to or larger than divisor, and errors in subtraction facts do not seem to be serious since those who made only one mistake in every step predominate.

d) Comparing the results of grades 5 and 6 in Test 11, it is seen that Grade 6 is much better off in their knowledge of the division skills involved in Test 11.

e) Concerning the remaining types of errors, grades 5 and 6 are nearly in the same position. May be in both cases the error is due to carelessness or to lapses of thought.

f) Comparing the results of Grade 6 in Tests 9, 10, and 11 it is seen that in Test 9 about 3% made 3 or 4 mistakes in the technique of division, in Test 10 about 6% and in Test 11 about 4%.

(Key: N = Number of Errors; T = Errors in the Technique of Division; D &/M = Errors in Division and Multiplication Combinations; U = Using a Remainder Equal to or Larger than Divisor; Q = Errors due to Zero in Quotient; O = Omitting Final Remainder; S = Errors in Subtraction Combinations; % = Percentage of Error).

CHAPTER VII

CONCLUSIONS AND SUGGESTIONS

In diagnosing the difficulties of the pupils in the fundamental number combinations and in the four operations, the investigator used suitable carefully constructed diagnostic tests in order to locate the specific weaknesses in all aspects of the subject. The test data have then been tabulated in the most convenient form, and these data have been carefully studied. This called not only for analysis of the test scores, but also for close scruting of the test papers themselves. In terms of figures, this meant examining the following total number of sums:

Test	Number of Sums
1 - 4	44,850
5	20,200
6	5,046
7	4,872
8A & B	11,925
9	9,900
10	4,968
11	3,312
Total	= 105,073 sums

Tests 6-11 involved a special difficulty of sifting the different types of errors into well-defined categories in order to get a clear picture of the pupils' common types of errors, and the position of these types relative to each other. We should always keep in mind that detailed diagnosis is the most important initial step towards preparation for corrective and remedial teaching. Once we have put our fingers on the specific weaknesses, their remedy becomes relatively easy.

Close study of tables 2-21 will reveal a wide range of arithmetical difficulties which can be classified into four groups. Three of these groups can be distinguished from observing the results of tests 1-4*.

Group 1.

This group includes those pupils who made a large number of errors in tests 1-4. Some of the pupils in this group are fast, others are very slow. Dull and less competent pupils fall into this group too.

Group 2.

This group includes those pupils whose accuracy is satisfactory, but who are slow in responding to the number combinations in tests 1-4.

Group 3.

This group includes those pupils who have good working accuracy in tests 1-4, but who make certain special individual errors.

Group 4.

This group includes those pupils whose test results show, not weakness in the basic facts, but insufficient knowledge of the skills involved in the various steps in the four processes i.e. in tests 6-11.

Now that the specific weaknesses of the pupils are known, remedial work should be planned to fit the needs of these afore-

*See Table 6 page 102.

mentioned groups. It is not the aim of this study, however, to do this extensive work*. While this study makes few remarks about the needs of the above-mentioned groups, suffice it to state that all of these groups need:

> the same kind of sympathetic, encouraging approach to produce early success and to dissipate the effects of past failure. For most of them this success will come from the systematic and frequent use in the early stages of scientifically compiled and carefully graded remedial material. This approach, using scientifically prepared material, and supplemented by play-way methods, will in almost all instances, bring marked improvement.[1]

Let us now turn our attention to the needs of the individual groups:

Group 1. Large Number of Errors in Basic Facts

The pupils in this group have not formed adequate number concepts. They have not formed meanings for the numbers with which they are working. They do not know, for example, that 7 means seven single units, that it also means 6+1, 5+2, 4+3, 3+4, 2+5, 1+6, 1+2+4, and so on. These pupils need an extensive period of experience with concrete material of varied kinds to form the needed number meanings and concepts.

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*This task will be left to those responsible for planning the necessary remedial work.

[1]Schonell, F.J., Diagnosis and Remedial Teaching. p. 133.

With these children, we should first dwell on the basic addition and subtraction combinations until satisfactory success is made.

Numbers below 20 should be dealt with first. Not only this, but also we should lead the child step by step up to the level of abstraction. Concrete materials like counters, blocks, beads, beans, ... etc. should be used first. Later, pictorial representations of concrete objects could be used followed by distributed written practice with the basic number facts and their higher decade extensions in order to extend and consolidate their previous understandings. Practice in the form of individual or group games using number combinations cards, multiplication and division cards, jig-saw cards and number charts, ... will re-kindle the pupils' interests and rebuild their persistance. Finally, a culminating activity for this group such as making a model post office, bazaar, ... etc. is most useful to show the pupils the everyday applications of the basic number facts.

Group 2. Accuracy but Low Speed in Basic Number Facts

The fact that the pupils in this group give correct responses indicates that the pupils have some understanding of number concepts. On the other hand, the fact that they are slow in responding to the number combinations suggests that they are still in need of both additional teaching, using concrete materials, and additional practice in the basic number facts and their extensions. Through real experience, these children will master the basic number combinations and understand the relationships underlying multiplication and division processes.

Group 3. Good Working Accuracy Accompanied by Individual Errors

The pupils in this group show particular special weaknesses. Examples:

a) Errors in the zero combinations such as 2+0, 5-0, 6x0, 0+7.

b) Errors in subtraction of like figures such as 3-3, 5-5, 9-9.

c) Errors in division of like figures such as 3÷3, 7÷7, 9÷9.

d) Errors in difficult combinations such as 7+9, 8x7, 9+8, 72÷8.

Such individual errors may be eliminated by practice both in the basic number combinations and in the applied form.

Group 4. Insufficient Understanding of the Four Processes

These are the pupils who know the basic combinations and their extensions fairly well, but who make some mistakes in the processes of addition, subtraction, multiplication and division because they have not yet mastered the idea of bridging in addition and subtraction, the idea of compound multiplication, or that of trial divisors in division. The deficiency might be due to several factors such as frequent absence from school, ineffective teaching, or hurrying the pupils along too quickly before they could really grasp the necessary understandings. This group is the easiest to deal with because they already have mastered the basic number facts. These pupils need remedial work with sets of examples at each level of difficulty in the process. As has already been pointed out, the difficulties of this group are best revealed through tests 6-11.

The importance of diagnostic tests in arithmetic cannot be overemphasized. These tests reveal to the teacher of arithmetic the specific weaknesses of his pupils and enable him take adequate measures before commencing a new assignment. This means saving a

lot of time and effort and focussing attention on the specific difficulties of the pupils.

In this study, the investigator tried to give the reader an idea about related research findings before commencing the real part of the work. Due attention was given to the aims, methods and advantages of educational diagnosis. This was followed by a detailed description of the tests used in this research and instructions concerning their usage.

Special attention should be given to the results of this study. As has already been mentioned, the investigator put more emphasis on qualitative rather than quantitative analysis of the test data. This method of analysis enabled the researcher to spot the special weaknesses of the pupils in the various aspects of the subject and make adequate conclusions. Unfortunately, space does not permit restating the results of each test in this paragraph; and the writer is cautious not to give a general statement for that might be truly misleading. Therefore, it is highly recommended that the reader consult tables 2-21 and the interpretation accompanying each table. This would enable the reader to have a detailed picture of the pupils' performance in each test, and, may be, make extra conclusions.

APPENDIX A

ANSWERS TO THE TESTS USED IN THIS RESEARCH

Answers to Test 1

a	b	c	d	e
2	0	4	3	4
2	4	6	10	5
7	4	8	8	7
8	9	6	5	14
7	5	5	2	12
7	16	7	10	9
11	10	18	3	11
1	6	8	6	4
10	10	8	9	10
11	6	9	3	3
5	6	9	9	9
1	8	9	8	7
5	7	7	8	8
9	12	6	10	12
9	10	11	11	10
11	11	12	13	12
13	11	17	12	13
12	14	13	14	15
14	15	15	17	16
14	15	13	13	16

Answers to Test 2

a	b	c	d	e
1	0	2	1	0
2	0	4	0	7
0	1	2	1	1
3	5	8	0	1
0	6	3	0	3
4	4	0	1	2
1	9	2	6	2
4	3	6	6	5
3	9	3	5	8
6	4	4	0	1
6	2	3	2	8
7	5	5	7	3
9	4	4	7	3
5	2	6	1	3
7	8	4	8	2
5	7	7	7	9
8	7	8	5	9
6	6	5	9	8
8	9	9	7	6
9	9	5	4	9

Answers to Test 3

a	b	c	d	e
3	4	7	2	4
5	10	6	16	5
4	6	8	6	18
20	14	16	6	10
3	9	9	15	12
2	12	12	25	24
18	36	8	20	18
30	8	12	15	27
32	30	40	35	24
45	1	48	54	42
36	21	28	24	7
14	18	16	9	0
8	24	21	0	35
81	0	45	0	0
0	0	32	0	28
64	0	72	72	0
48	0	42	0	0
56	63	0	54	49
0	0	56	40	0
63	27	0	0	36

Answers to Test 4

a	b	c	d	e
3	6	3	2	3
4	5	4	5	7
5	9	2	7	2
2	6	6	5	3
8	4	6	1	8
4	9	5	9	2
6	5	4	7	8
3	6	3	7	2
7	7	8	8	1
1	9	1	9	1
8	1	9	4	0
2	2	4	2	4
1	7	8	5	6
6	0	3	0	5
3	0	7	0	8
0	8	4	1	0
5	0	7	1	0
6	3	9	9	9

Answers to Test 5

a	b	c	d	e
11	42	6	13	9
5	0	63	14	8
9	7	0	9	11
24	56	7	14	5
4	8	15	5	12
49	0	0	8	6
13	13	54	6	0
8	9	11	54	14
40	1	9	13	7
0	7	27	9	0
9	4	0	56	13
42	0	1	0	36
1	8	13	6	9
4	16	0	3	9
7	77	8	11	72
8	5	120	11	48
9	10	132	121	7
84	6	96	12	132
88	10	99	12	9
110	144	4	108	60

Answers to Test 6

a	b	c	d	e	f	g	h
18	19	17	16	23	29	27	26
68	89	95	77	359	398	767	779
23	21	28	26	44	53	45	66
118	139	168	151	51	87	83	85
1105	256	575	1359	123	124	146	141
163	149	168	193	1284	1491	1542	1394
242	285	830	1372	626	1593	536	2329
42	-	-	-	-	-	-	-
60	-	-	-	-	-	-	-

Answers to Test 7

a	b	c	d	e	f	g	h
55	32	62	70	32	21	88	64
134	252	236	522	2	5	9	2
48	79	49	66	25	27	49	9
327	426	237	519	389	307	9	68
40	200	27	14	109	201	329	1
151	195	195	93	69	384	3659	1314
206	209	407	507	486	398	196	1098

Answers to Test 8A

a	b	c	d	e	f
66	255	168	426	1468	1536
3555	1268	210	808	4000	42060
112	162	136	95	474	696
651	272	648	963	33770	72048
3276	69025	1036728	5747373	1323	1376
2516	4984	6860	3240	3350	7830
8100	3144000	9000	80000	33756	33750

Answers to Test 8B

a	b	c
27495	48720	14168
52983	20320	56490
153546	247863	249660
1397871	5683300	-

Answers to Test 9

a	b	c	d
22	23	12	111
213	223	121	1412
203	102	420	320
300	300	200	200
202	309	406	703
7 r = 2	9 r = 1	6 r = 3	8 r = 7
8 r = 2	23 r = 1	7 r = 2	9 r = 5
31 r = 3	21 r = 1	21 r = 1	31 r = 1
64 r = 7	87 r = 5	78 r = 7	62 r = 9
78 r = 1	93 r = 3	965 r = 2	863 r = 1
7051 r = 1	4602 r = 5	9004 r = 2	56307 r = 1

Answers to Test 10

a	b	c	d
2	2	3	4
2 r = 1	3 r = 3	2 r = 5	4 r = 8
13 r =18	2 r = 3	2 r =24	3 r =13
2 r = 8	2 r = 7	3 r = 9	2 r = 9
7	4	3	4
2 r = 5	2 r = 9	2 r = 1	3 r = 2
34	23	42	31
23 r = 9	56 r = 6	63 r = 5	31 r = 7
60	50	50	50

Answers to Test 11

a	b	c	d
60 r = 8	32 r = 5	47 r = 6	60 r = 9
4	4	6	3
8	6 r = 11	8 r = 16	6 r = 3
25	43	31 r = 12	23 r = 1
731	301	352 r = 3	231
807	85 r = 5	630 r = 8	309 r = 5

APPENDIX B

SOME NOTES ON THE USE OF THE TESTS

1. Who Should Take the Tests and When

The teacher should keep in mind the nature and purpose of each of these tests in order to make adequate use of them. Thus, if the teacher's aim is to find out the weaknesses of his pupils' in the basic number combinations, then tests 1-4 should be used. If on the other hand, the aim is to find out the pupils' weaknesses in one or more of the four operations, then the test(s) concerned with that operation should be given.

It might be preferable to give tests 1-4 to the 4th, 5th and 6th grades before giving them tests 6-11, in order to ascertain the pupils' knowledge of the basic facts. Instead of giving only tests 8A-11 to grades 5 and 6, the teacher might give tests 6 and 7 to these grades, too. However, this has not been done in the present study, for the study would then be too extensive.

The teacher is urged to give these tests as early as possible in the school year in order to discover the difficulties of his pupils and remedy them before starting a new assignment. Also these tests may be used during the year after completing a certain unit of work to ascertain the pupils' knowledge before carrying on further work. Although the tests might all be given in one session of about 2½ hours, it is preferable to give them at intervals in order not to overload the pupils and the teacher.

2. Marking and Scoring the Tests

In tests 1-5, there is only one type of error, namely, a lack of knowledge of the basic facts. In marking these test papers therefore, the arithmetic teacher is advised to put a small cross to the right of the wrong answer, and nothing opposite the right answer. This makes it easier for the teacher to see at a glance where the weaknesses of each pupil appear. To make the correction of papers easy, the teacher is advised to write the correct answers for every column on a strip of paper which is as long as the column and lay the strip over the corresponding column in such a way that the pupil's answers and the corresponding answers on the strip are opposite each other.

Tests 6-11 are graded tests. Each of these tests involves a certain number of well-defined steps, and four examples are used to test each of these steps. Thus, tests 6 and 7 are made up of 14 steps each. The examples on these steps are arranged in seven rows, each row containing eight operations. In other words, there are two steps in every row. In marking the tests, a cross is put to the right of each wrong answer, (nothing opposite a right answer), and then the number of wrong sums in every step is written in the margin adjacent to that step. For example, if the pupil has two sums wrong in the first step, then '2' is written opposite that step in the left margin. If the pupil has three

sums wrong in the second step then '3' is written opposite that step in the right margin, and so on. This would help the teacher see easily the number of errors made in every step by each pupil. To make the correction easier, one may use a sheet of cardboard that fits the test paper and have the cardboard perforated in such a way that the answers to the operations appear through the holes. If the correct answers are then written below the perforations, then the teacher may, at a glance, check the pupil's answers by fitting the sheet of cardboard over the test paper and comparing both answers.

In test 8A, every row contains six operations. This means that every row contains one step and a half. Test 8B is a continuation of test 8A, and there are three sums in every row. The correcting of test 8A & B and the recording of the number of wrong sums in every step is done as previously explained.

Tests 9-11 are concerned with division. Each row contains four operations, i.e. each row contains a single step. The correction of the test papers is done as explained above, and the number of wrong sums in every step is written in the left margin adjacent to that step.

3. Interpretation of the Results

After correcting the test papers, the teacher should not be satisfied by knowing the pupil's score. On the contrary, the teacher should be more concerned with the number of wrong operations, no matter how few these are, than with the number of

right operations. This implies that the next step, after correcting the papers, is to examine closely every wrong sum in every test paper in order to see where the pupil's weaknesses appeared i.e. in what basic facts (in the case of tests 1-5), or in what steps (in the case of tests 6-11) does the pupil encounter difficulty. If the teacher is working with individual pupils, then this analysis is a simple matter. If the teacher is instructing the whole class, then he needs to tabulate the results in a way similar to that which the researcher has done in tables 2-21. This would help the teacher see, in a comprehensive way, how many of his pupils share a common difficulty and what the common types of errors are in his class. Having thus grouped the pupils according to their difficulties, the teacher should start planning for remedial work according to the needs of each individual group*. It should be mentioned that in tests 6-11, those pupils who make only one mistake in every step (one operation wrong out of four) are not in need of help. Those who make two mistakes in every step are in a doubtful position and, therefore, they should be given a parallel test shortly after taking the first test to ascertain their actual difficulties. Those who make three mistakes in every step are definitely in need of help in one way or the other.

After subjecting the pupils to remedial work for a certain period of time, whether on an individual basis or a group basis,

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*A few suggestions have been made in Chapter VII concerning the remedial work needed in the various cases of difficulty.

the pupils are given the same tests (or preferably parallel tests in the case of tests 6-11) to see what weaknesses they still show. One last remark should be made, namely, that it is always advisable that the teacher ask those pupils having difficulties in a certain sum or step to work aloud one or two such sums in order to get a better understanding of the pupil's mental processes which resulted in the wrong answers.

It is hoped that these suggestions will complement the tests themselves by facilitating test scoring and interpretation. Subsequently, the teacher should then find himself in an advantageous position in which to procede with remedial work.

BIBLIOGRAPHY

A. BOOKS

1. Blair, GlennMyers. Diagnostic and Remedial Teaching. The Macmillan Company, New York, 1956.

2. Brueckner, Leo J., and Bond, Juy L. The Diagnosis and Treatment of Learning Difficulties. Appleton-Century-Crofts, Inc., New York, 1955.

3. Brueckner, Leo J., and Melby, Ernesto. Diagnostic and Remedial Teaching. Houghton Mifflin Company, Boston, 1951.

4. Fernald, Grace M. Remedial Techniques in Basic School Subjects. McGraw-Hill Book Company, Inc., New York, 1943.

5. Hickerson, J. Allen. Guiding Children's Arithmetic Experiences. Prentice-Hall, Inc., New York, 1953.

6. Hollister, G.E., and Gunderson, A.G. Teaching Arithmetic in Grades I and II. D.C. Health and Company, Boston, 1954.

7. Hunnicutt, C.W., and Iverson, W.J. Research in the Three R's. Harper and Brothers, Publishers, New York, 1958.

8. Law, Margaret. Number Building. Thomas Nelson and Sons, Copewood and Davis St., Camden 3, N. J., 1964.

9. Lindquist, E.F. (editor). Educational Measurement. American Council in Education, Washington, D.C., 1951.

10. Lyman, Howard B. Test Scores and What They Mean. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

11. Marks. Purdy. Kenny. Teaching Arithmetic for Understanding. McGraw-Hill Book Company, Inc., New York, 1958.

12. Noll, Victor H. Introduction to Educational Measurement. Houghton Mifflin Company, Boston, 1957.

13. Polkinghorne, M.I.R., and K.R. Fundamental Number Teaching. George G. Harrap and Co., Ltd., London, 1955.

14. Ross, C.C., and Stanley, J.C. Measurement in Today's Schools. 3rd ed., Englewood and Cliffs, Prentice-Hall, Inc., N. Y., 1956.

15. Schonell, Fred J. Diagnostic and Attainment Testing. 4th. ed. Oliver and Boyd, Edinburgh, Tweeddale Court, 1963.

16. _____, Diagnosis and Remedial Teaching in Arithmetic. Oliver and Boyd, Edinburgh, Tweeddale Court, 1957.

17. Spitzer, Herbart F. The Teaching of Arithmetic. 3rd. ed. Houghton Mifflin Company, Boston, 1961.

18. Traxler, Arthur E. The Use of Test Results in Diagnosis and Instruction in the Tool Subjects. Education Records Bureau 437 West 59th st., New York 19, N.Y. 1949.

B. ARTICLES

1. Alfred, Brother U. "Importance of Elementary Operations," Mathematics Teacher, 56:614-19, December, 1963.

2. Anderson, R.C. "The Case for the Non-Graded Homogeneous Grouping," Elementary School Journal, 62:193-98, January, 1962.

3. Ausubel, D.P. "Some Psychological and Educational Limitations of Learning by Discovery," Arithmetic Teacher, 11:290-302, May, 1964.

4. Balow, I.H. "The Effects of 'Homogeneous' Grouping in Seventh-Grade Arithmetic," Arithmetic Teacher, 11:186-91, March, 1964.

5. _____. "Reading and Computation Ability as Determinants of Problem Solving," Arithmetic Teacher, 11:18-22, January, 1964.

6. Barakat, M.K. "Factors Underlying the Mathematical Abilities of Grammar School Pupils," (PH.D. Thesis, University of London, 1950). Summary in British Journal of Educational Psychology, 51:239, November, 1951.

7. Bassham, A., Murphy, M., and Murphy. K. "Attitude and Achievement in Arithmetic," Arithmetic Teacher, 11:66-72, February, 1964.

8. Bernstein, A. "Library Research - A Study in Remedial Arithmetic," School Science and Mathematics, 59:185-95, March, 1959.

9. Brewer, E. "A Survey of Arithmetic Intra-Class Grouping Practices," Arithmetic Teacher, 13:310-14, April, 1966.

10. Brownell, W.A. "An Evaluation of an Arithmetic 'Crutch'," Review of Educational Research, 5:15, 1935 (Reprint 1963).

11. Brownell, W.A.,and Chazel, C.B. "The Effects of Premature Drill on Third Grade Arithmetic," Journal of Educational Research, 29:17-28, September, 1935.

12. Brownell, W.A., and Moser, H.E. "Meaningful versus Mechanical Learning," Review of Educational Research, 21:291-92, 1951.

13. Burns, P.C. "Intensive Review as a Procedure in Teaching Arithmetic," Elementary School Journal, 60:205-11, January, 1960.

14. _____. "Arithmetic Research that has Made a Difference," Elementary School Journal, 65:386-92, April, 1965.

15. Burns, P.C., and Dessart, D.J. "A Summary of Investigations Relating to Mathematics in Elementary Education-1964," School Science and Mathematics, 65:779-90, December, 1965.

16. Caldwell, E. "Group Diagnosis and Standardized Achievement Tests," Arithmetic Teacher, 12:123-25, February, 1965.

17. Careccio, J. "Helping Children Learn Multiplication Facts," Arithmetic Teacher, 9:149-51, March, 1962.

18. Chase, C.I. "Formal Analysis as a Diagnostic Technique in Arithmetic," Elementary School Journal, 61:282-86, February, 1961.

19. Cleveland, G.A. "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement," Dissertation Abstracts, 22:2681-82, January-March, 1962.

20. Cooke, R.J. "Helping Children Build a Positive Attitude Toward Arithmetic Through its Mathematical Concepts," School Science and Mathematics, 54:197-211, March, 1954.

21. Davis, O.L., Jr., and Tracy, N.H. "Arithmetic Achievement and Instructional Grouping," Arithmetic Teacher, 10:12-17, January, 1963.

22. Dawson, D.T., and Ruddell, A.K. "The Case for the Meaning Theory in Arithmetic," Elementary School Journal, 55:393-99, March, 1955.

23. Deans, E. "Grouping - an Aid in Learning Multiplication and Division Facts," Arithmetic Teacher, 8:27-31, January, 1961.

24. Dewar, J.A. "An Experiment in Intra-Class Grouping for Arithmetic Instruction in the Sixth Grade," Dissertation Abstracts, 22:2247, January-March, 1962.

25. _____. "Grouping for Arithmetic Instruction in Sixth Grade," Elementary School Journal, 63:266-69, February, 1963.

26. Durr, W.K. "The Use of Arithmetic Workbooks in Relation to Mental Abilities and Selected Achievement Levels," Journal of Educational Research, 49:561-71, April, 1958.

27. Dutton, W.H. "Measuring Attitudes Toward Arithmetic," Elementary School Journal, 55:24-31, September, 1954.

28. Faust, C.E. "A Study of the Relationship Between Attitude and Achievement in Selected Elementary School Subjects," Dissertation Abstracts, 23:2752-53, February, 1963.

29. Feldhausen, J., Check, J., and Klausmeier, H.J. "Achievement in Subtraction," Elementary School Journal. 61:322-27, March, 1961.

30. Feldhausen, J., and Klausmeier, H.J. "Achievement in Counting and Addition," Elementary School Journal, 59:388-93, April, 1959.

31. Flagg, E.B. "Developing Confident, Self-Reliant Learners in Arithmetic," School Science and Mathematics, 55:381-88, May, 1955.

32. Fuleher, H.L. "Improving Arithmetic Skills by Testing and Reteaching," Elementary School Journal, 53:402-09, March, 1953.

33. Gane, T.J. "Research should Guide Us," Arithmetic Teacher, 9:441-45, December, 1962.

34. Goldstein, A. "Does Homework Help? A Review of Research," Elementary School Journal, 60:212-25, January, 1960.

35. Guggenheim, F. "Classroom Climate and the Learning of Mathematics," Arithmetic Teacher, 8:363-67, December, 1961.

36. Gunderson, A.B. "Thought Patterns of Young Children in Learning Multiplication and Division," Elementary School Journal, 55:453-61, April, 1955.

37. Haggerty, J.B. "Projects Make Mathematics more Meaningful," Arithmetic Teacher, 8:172-75, 1961.

38. Hall, W.F., and Demarest, D. "Effect on Achievement Scores of a Change in Promotion Policy," Elementary School Journal, 58:204-08, January, 1958.

39. Hamilton, J.F. "Remedial Arithmetic in the Regular Classroom," School Science and Mathematics, 56:197-209, March, 1956.

40. Hammond, R.L. "Ability with the Mathematical Principles Governing the Operations of Addition, Multiplication, Subtraction and Division," Dissertation Abstracts, 23:2372-73, January, 1963.

41. Harding, L.W., and Bryant, I.P. "An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade," Journal of Educational Research, 37:321-37, January, 1944.

42. Hart, R.H. "The Non-Graded Primary School and Arithmetic," Arithmetic Teacher, 9:130-33, March, 1962.

43. Harvey, L.F. "Improving Arithmetic Skills by Testing and Reteaching," Elementary School Journal, 53:402-09, March, 1953.

44. Harvey, L.F., and Kyte, G.C. "Zero Difficulties in Multiplication," Arithmetic Teacher, 12:45-50, January, 1965.

45. Higgins, C., and Rush, R.R. "Remedial Teaching of Multiplication and Division: Programmed Textbook versus Workbook - A Pilot Study," Arithmetic Teacher, 12:32-38, January, 1965.

46. Hightower, H.O. "Effect of Instructional Procedures on Achievement in Fundamental Operations in Arithmetic," Educational Administration and Supervision, 40:336-48, October, 1954.

47. Ivey, J.O. "Computation Skills: Results of Acceleration," Arithmetic Teacher, 12:39-42, 1965.

48. Jackson, H.C. "Techniques for Drill in Arithmetic Fundamentals," Mathematics Teacher, 49:47-48, January, 1956.

49. Jarvis, O.T. "Time Allotment Relationships to Pupil Achievement in Arithmetic," Arithmetic Teacher, 10:248-50, May, 1963.

50. _____. "Boy-Girl Ability Differences in Elementary School Arithmetic," School Science and Mathematics. 64:657-59, November, 1964.

51. _____. "An Analysis of Individual Differences in Arithmetic," Arithmetic Teacher, 11:471-73, November, 1964.

52. Kemp, L.D. "Environmental and Other Characteristics Determining Attainment in Primary Schools," British Journal of Educational Psychology, 25:67-77, February, 1955.

53. Kersh, B.T. "Learning by Discovery: What is Learned?," Arithmetic Teacher, 11:226-32, April, 1964.

54. _____. "Learning by Discovery: Instructional Strategies," Arithmetic Teacher, 12:414-17, October, 1965.

55. Klausmeier, H.J., and Check, J. "Retention and Transfer in Children of Low, Average, and High Intelligence," Journal of Educational Research, 55:319-22, April, 1962.

56. Koch, E.A., Jr. "Homework in Arithmetic," Arithmetic Teacher, 12:9-13, January, 1965.

57. Koenker, R.H. "The 'Crutch' in Arithmetic," Elementary School Journal, 58:232-33, January, 1958.

58. Kowitz, G.T., and Armstrong, C.M. "The Effect of Promotion Policy on Academic Achievement," Elementary School Journal, 61:435-43, March, 1961.

59. Lerch, H.H. "Arithmetic Instruction Changes Pupils' Attitudes Toward Arithmetic," Arithmetic Teacher, 8:117-19, March, 1961.

60. _____. "Intra-Class Grouping for Arithmetic Instruction: Critique and Criteria," Arithmetic Teacher, 8:404-407, December, 1961.

61. Lyda, W.J., and Morse, E.C. "Attitudes, Teaching Methods, and Arithmetic Achievement," Arithmetic Teacher, 10:136-38, March, 1963.

62. Mazzei, R. "A Technique for the Prevention of Errors in Arithmetic," School Science and Mathematics, 59:493-97, June, 1959.

63. Marks, R.R. "The Testing of a Diagnostic and Remedial Teaching Technique in the Mastery of Certain Addition Facts," Dissertation Abstracts, 26:159, July, 1965.

64. Mc Connell, T.R. "Discovery versus Authoritative Identification in the Learning of Children," University of Iowa Studies in Education, 9:11-62, (University of Iowa, 1954)

65. Meddleton, I.G. "An Experimental Investigation Into the Systematic Teaching of Number Combinations in Arithmetic," British Journal of Educational Psychology, 26:117-27, February, 1956.

66. Moench, L. "Individualized Practice in Arithmetic - A Pilot Study," Arithmetic Teacher, 9:321-29, October, 1962.

67. Morton, D.M. "Number Forms and Arithmetic Ability in Children," British Journal of Educational Psychology, 6:58-73.

68. Pace, A. "Understanding and the Ability to Solve Problems," Arithmetic Teacher, 8:226-33, May, 1961.

69. Passy, R.A. "The Effect of Cuisenaire Materials on Reasoning and Computation," Arithmetic Teacher, 10:439-40, November, 1963.

70. _____. "Socio-Economic Status and Mathematics Achievement," Arithmetic Teacher, 11:469-70, November, 1964.

71. Pinney, G.C. "Grouping by Arithmetic Ability - An Experiment in the Teaching of Arithmetic," Arithmetic Teacher, 8:120-23, March, 1961.

72. Potmakin, C.C. "An Experiment in Individualized Arithmetic," Elementary School Journal, 64:155-63, December, 1964.

73. Provus, M.M. "Ability Grouping in Arithmetic," Elementary School Journal, 60:391-98, April, 1960.

74. Purdy, C.R., and Kenny, L.B.""Directing Learning in Arithmetic," Elementary School Journal, 54:285-90, January, 1954.

75. Rheins, G.B., and Rheins, J.J. "A Comparison of Two Methods of Compound Subtraction," Arithmetic Teacher, 2:63-89, October, 1955.

76. Rose, A.W. "Intelligence, Sibling Position, and Socio-Cultural Background as Factors in Arithmetic Performance," Arithmetic Teacher, 8:50-56, February, 1961.

77. Schell, L.M., and Burns, P.C. "Pupil Performance with Three Types of Subtraction Situations," School Science and Mathematics, 62:208-14, March, 1962.

78. Shaw, H., "The Development of the Child's Concept of Number," Educational Review, 18:195, 1960.

79. Shipp, D.E. "An Experimental Study of Achievement in Arithmetic and the Time Alloted to Development of Meanings and Individual Pupil Practice," Dissertation Abstracts, 19:492, September, 1958.

80. Shuster, A.H., and Pigge, F.L. "Retention Efficiency of Meaningful Teaching," Arithmetic Teacher, 12:24-31, January, 1965.

81. Smith, W.M. "The Effect of Intra-Class Ability Grouping on Arithmetic Achievement in Grades Two Through Five," Dissertation Abstracts, 21:563-64, July-September, 1960.

82. Steelman, M. "Improving Understanding of Number Concepts in Third Grade," Arithmetic Teacher, 9:37-40, January, 1962.

83. Swenson, E.J. "Difficulty Ratings of Addition Facts as Related to Learning Method," Journal of Educational Research, 38:81-85, October, 1944.

84. Thiele, C.L. "Contribution of Generalization to the Learning of the Addition Facts," Review of Educational Research, 12:394, 1942.

85. Treacy, J.P. "The Relationship of Reading Skills to the Ability to solve Arithmetic Problems," Journal of Educational Research, 38:86-96, October, 1944.

86. Van Engen, H. "The Child's Introduction to Arithmetic Reasoning," School Science and Mathematics, 55:358-63, 1955.

87. Washburne, C., (chairman). "Mental Age and the Arithmetic Curriculum: A Summary of the Committee of Seven Grade Placement Investigations to Date," Journal of Educational Research, 28:210-31, March, 1931.

88. Weaver, J.F. "Differentiated Instruction and School-Class Organization for Mathematical Learning within the Elementary Grades," Arithmetic Teacher, 13:495-506, October, 1966.

89. Wilson, G.M. "Why Do Pupils Avoid Mathematics in High School?," Arithmetic Teacher, 8:168, 1961.

90. Wozencraft, M. "Are Boys Better than Girls in Arithmetic?," Arithmetic Teacher, 10:486-90, December, 1963.