OPTIMIZATION OF WATER ALLOCATION BY DYNAMIC PROGRAMMING

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OPTIMUM WATER ALLOCATION

JATALA

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AN ABSTRACT OF THE THESIS OF

Abdul Hamid Jatala for Master of Science in Agriculture
Major : Irrigation

Title: Optimization of water allocation by dynamic programming.

A methodology was prepared for developing benefit and cost functions and using these functions in dynamic programming analysis in order to determine water allocation based on a policy of optimum net benefits from geographic districts irrigated with deficient water supply.

Equations were developed to establish relationship between benefits and water supply. The main physical variables determining benefits and water requirements of a district were considered as soils, climate, crops, yields, and irrigated area.

Equations were described also for establishing a relationship between the cost of water conveyance and the conveyance capacity of the aqueduct. The main physical variables determining the costs were considered to be the volume of excavation, surface area for lining, land expropriation, the roads and control structures along the aqueduct.

These benefits and costs were expressed on an annual basis for comparison purposes.

Dynamic programming was then used as a procedure for determining proper allocations.

The use of the methodology was exemplified by taking three districts along the "Canal 900" in the Southern Beqa'a area of the Litani River Project of Lebanon.

It was concluded that the methodology is applicable to project areas variable in extent and physical properties.

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I. INTRODUCTION

Irrigation in many countries is an art as old as civilization, but for the modern world as a whole it is a science.

Irrigation enterprises in old times consisted of small areas of arable land along stream banks, where sufficient water was available during the growing season. Water was diverted by temporary dams and brought to the field through small ditches excavated by hand. The needs for additional food supplies and efforts for survival, as the population increased, necessitated an ever increasing expansion in irrigation. As the irrigable strips along the streams were fully utilized, conveyance of water to more distant areas became desirable, requiring more permanent, complicated, and expensive irrigation works. In order to finance, construct, and operate such large systems, it became necessary to adopt more comprehensive methods of project formulation, evaluation, and justification, in addition to the sophisticated construction and administration methods.

Among the questions involved in irrigation project formulation in the arid and semi-arid areas characterized by chronically deficient water supplies is the problem of water allocation to the potential users to be served by the distribution system. Decisions regarding water allocation may rest with agencies ranging from the private firm type to the national public authorities. However, insofar as the decision makers' interest lies in maximizing the net benefits of irrigation water as well as other resources, economics provides guidance and criteria for evaluating various aspects of a development project.

Unfortunately, economic analysis by direct comparisons of various water allocation policies may require a tremendous amount of computational work, because of the very large number of alternatives that must be considered.

In recent times, a dynamic programming method has been used to determine aqueduct capacity under a maximum or optimum net benefit policy. This method is believed to require fewer computations. However, there is still a lack of mathematical models which might be used to establish some of the benefit and cost functions necessary for the above mentioned method.

+The aim of this study is to formulate a general mathematical model for benefit and cost functions and for dynamic programming analysis that may be used for optimization of water allocation in irrigation projects. The expression, optimization of water allocation, is used to mean the process of apportioning irrigation water among various users to obtain the maximum net benefits from irrigation.

To exemplify the use of the model it is applied to three irrigation districts served by the "Canal 900" in the Southern Beqa'a area, which is a part of the overall Litani River development scheme in Lebanon.

II. REVIEW OF LITERATURE

Dynamic Programming

Dynamic programming was defined by Bellman (1957, pp vi-vii) as a mathematical theory designed to study the formulation, analysis, and computational treatment of multi-stage decision processes. Multi-stage decision processes are those in which a sequence of decisions must be made each of which affects the state of the underlying physical system and the choice of the subsequent decisions (Newman, 1963, pp 341).

Closely associated with dynamic programming is the principle of optimality which, according to Bellman (1957, pp 82), states that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision; Optimal policy being a sequence of admissible decisions that maximizes the utility, or value of any particular set of decisions.

Dynamic programming together with the principle of optimality have been applied to many areas of economics, industry, engineering, psychology, and mathematical physics (Newman, 1963, pp 341).

Optimum Benefit Analysis

Hall (1961, pp 1-12) used dynamic programming as the optimizing procedure for optimum benefit analysis for aqueduct capacity determination. The mathematical model used for this purpose was

$$V = \max \sum_{i=1}^{n} \left[vi\ (q_i) - C(q_1, q_2, q_3 \ldots\ldots q_n) \right]$$

subject to: $\sum_{i=1}^{n} q_i \leq Q$

where $v_i(q_i)$: The net beneficial return to the ith geographic district as a result of making available to that district a quantity of water q_i, exclusive of aqueduct costs incurred to bring water to the district.

Q : The maximum quantity of water that might be made available to n districts.

C : Total cost of the main aqueduct serving n districts.

V : The sum of the returns less total aqueduct costs.

The problem consisted in selecting the set of values q_i, $i = 1, 2, 3 \ldots\ldots n$, so as to maximize V which would determine not only the aqueduct capacity but also the appropriate water allocations to the particular districts.

The solution of the above equation, obtained by the dynamic programming method, is presented in the chapter on Methodology in this manuscript.

Benefit and Cost Concepts

The Subcommittee on Evaluation Standards (Anonymous, 1958, pp 5-33) of the U.S. Inter-Agency Committee on Water Resources developed a systematic framework for the economic analysis of water resources projects with the objective of establishing the fundamental economic principles and standards that would yield comparable estimates of benefits and costs, and would provide a proper basis for project formulation and selection. Some of the proposed concepts, relevant to this study, are presented below.

Terminology

Project: The term "project" was defined as any separable integral physical unit or several component and closely related units or features or system of measures, undertaken or to be undertaken within a specific area for the control and development of water and related land resources, which can be established and utilized independently or as an addition to an existing project, and can be considered as a separate entity for purposes of evaluation as recommended by the Subcommittee on Evaluation Standards (Anonymous, 1958, pp 7).

Primary Project Benefits: The value of products and services directly resulting from a project, net of all associated costs incurred in their realization, were treated as primary benefits attributable to the project (Anonymous, 1958, pp 8).

Attributable Secondary Benefits: The secondary benefits were considered from a national public point of view and were defined as the values added over and above the value of promary benefits (Anonymous, 1958, pp 8).

For purposes of economic evaluation from a national point of view, the project benefits would be the sum of the primary project benefits and the attributable secondary benefits.

Project Costs: The value of the goods and services used for the establishment, maintenance, and operation of the project together with the value of any induced adverse effects were termed as the project costs (Anonymous, 1958, pp 8).

Associated Costs: The value of the goods and services needed, over and above those included in the project costs, to make the immediate products or services of the project available for use or sale was denoted as the associated cost (Anonymous, 1958, pp 8).

General Measurement Standards

The Subcommittee on Evaluation Standards (Anonymous, 1958, pp 17) recommended that, by applying measurement principles and standards for economic analysis, such as those for prices, interest, period of analysis, and other factors, the benefits and costs of a project should be evaluated in monetary terms and reduced to a common time basis for comparison. It was further recommended that the benefits and costs be expressed in terms of average annual value over the selected period of analysis.

Price Levels: Prices are used to express the magnitude of benefits and costs in common terms. The Subcommittee on Evaluation Standards (Anonymous, 1958, pp 19-22) proposed that the prices expected to prevail at the time benefits are realized or costs incurred should be used for comparison purposes.

Ghahraman (1958, pp 148-150) reported three other methods of handling price levels; 1) a single future price level based on the prices of a particular year with general conditions similar to the expected period of the economic life of the project, 2) the average of prices over a period of years with the assumption that the previous prices considered would be a reasonable guide to prices over the life of the project, 3) prices current at the time of investigation, allowing for expected changes in prices of specified goods and services.

Regan and Timmons (1954, pp 1-15) observed that after using a projected prices base for several years, analysis of major flood control structures, navigation, and other related purposes was changed back to a current-price base. The justification offered was the need for greater emphasis on real or purchasing power values. Current prices were claimed to be more satisfactory for reflecting such relationships than price projections based on a substantially lower general price level than that currently prevailing.

Interest Rate: It was recommended by the Subcommittee on Evaluation Standards (Anonymous, 1958, pp 22-24) that estimates of benefits and costs accruing at various times should be made comparable by adjustment to a uniform time basis through the use of projected long-term interest rates. In the absence of such rates, the use of average rate of return; i.e., yield, on long-term government bonds was suggested.

Period of Analysis: The Subcommittee (Anonymous, 1958, pp 25-26) proposed that the maximum period of analysis should be taken as the expected economic life of the project or 100 years, whichever is shorter. All investment costs, less the expected salvage value at the end of the period of analysis, must be amortized within this period.

Measurement of Irrigation Benefits

The Subcommittee on Evaluation Standards (Anonymous, 1958, pp 36-37) stated that agricultural benefits frim irrigation development include reductions in production costs and increases in the value of agricultural production after allowance for associated costs. A similar view was expressed by Jones and Miller (1962, pp 65-70) who stated in more specific terms that the returns attributable to irrigation result from increased yields or increased quality of product.

A practical method for measuring irrigation benefits, called the crop-budget approach, was described by Stewart (1964, pp 107-126). According to this approach, a working number of representative farm situations should be selected for pertinent surveys, the findings of which would be added together by weighting, to estimate project benefits. Some economic study of the project area should be made to establish basis for projections related to the local economy. For analytical purposes, all farms may be treated as cash-crop farms although cropping patterns and forage prices would be estimated on the basis of livestock projected for the area and of off-farm markets for crops not fed to livestock.

Measurement of Irrigation Costs

Ghahraman (1958, pp 151-155) grouped irrigation costs into two classes on the basis of the time of occurrence; 1) investment costs defined as the costs which occur at the outset of the project and are necessary for the establishment of the project, and 2) operation, maintenance, and replacement costs which occur throughout the life of the project.

The Subcommittee on Evaluation Standards (Anonymous, 1958, pp 36-37) recommended the procedure for measuring costs by comparing anticipated conditions with and without the project in order to indicate the increased investments required for land preparation, water distribution structures, livestock, buildings, machinery, and local government services. The associated costs were suggested to be measured in terms of increased operating costs for production, interest on investment, maintenance, depreciation of equipment, property taxes, and family expenses.

Regan and Timmons (1954, pp 10) specified the associated costs as the cost of land leveling and other "on-site" development costs.

As noted previously, irrigation benefits, before allowing for associated costs, are directly measured on annual basis. For purposes of comparison, the costs should also be converted to annual basis. Ghahraman (1958, pp 152-155) listed the components of annual costs as annual interest on initial investment, annual amortization allowance for initial investment, annual operation and maintenance costs, and taxes and insurance that may affect the net benefit.

Linsley and Franzini (1964, pp 360-374), giving an example on cost calculations, combined the annual interest and amortization allowance into capital recovery factor.

Estimation of Water Delivery Requirements

Consumptive Use

Consumptive use, or evapo-transpiration was described by Israelsen and Hansen (1962, pp 231-265) as the sum of two terms; 1) transpiration, which is water entering plant roots and used to build plant tissue or being passed through leaves of the plant into the atmosphere, 2) evaporation, which is water evaporating from adjacent soil, water surfaces, or from the surfaces of leaves of the plant.

Evans (1962, pp 2-10) reviewed various methods of estimating consumptive use of water by crops, and classified them into five groups; 1) use of lysimeters, 2) correlation with simple climatic data, e.g. formulas developed by Thornwaite, Lowry and Johnson, and Blaney and Criddle, 3) energy budget equations, e.g. Penman equation, 4) mass transfer equations, e.g. Suerdrup, and Thornwaite and Holzman equations, and 5) correlation with evaporation from open pans, e.g. Taylor, Mech, and Cummings formulas.

The Blaney-Criddle formula has been widely used in Afghanistan, Colombia, Egypt, Greece, Jordan, Japan, Pakistan, Puerto Rico, Turkey, and U.S.A. Cavazza (1968, pp 14-26) mentioned Turc's formula which has found application in Northern Mediterranean countries. This formula also was developed by correlation of consumptive use with simple climatic factors; temperature and solar radiation.

Irrigation Requirements

The term irrigation requirements was defined by Blaney (1955, pp 341-345) as the amount of water, exclusive of precipitation, that is needed for the production of crops. It includes plant transpiration, evaporation, deep percolation and other economically unavoidable losses.

Linsley and Franzini (1964, pp 378-390) suggested that only the effective precipitation should be taken into consideration for estimations of irrigation requirements. It was proposed that acceptable estimates of effective precipitation may be made by assuming a linear variation from 100% effectiveness for the first 25 mm of rain in a month to zero effectiveness for all rain over 150 mm in a month.

Losses by deep percolation and surface run-off are incorporated in the term "water application efficiency", which was defined by Israelsen and Hansen (1962, pp 289) as the ratio of the quantity of water stored in the root zone to the quantity applied during an irrigation. Linsley and Franzini (1964, pp 384) reported a variation of 40 to 60% in water application efficiency. Matarrese (1968, pp 33-48), reviewing the results of various studies made on the subject, concluded that the said efficiency varied with the method of irrigation, type of soil, frequency of irrigation, and the volume of water applied. In general, the maximum value for sprinkler irrigation was found to be 84%, while the figures for surface irrigation methods varied from 34 to 60%.

Water Delivery Requirements

Water delivery requirements of a district consist of irrigation requirements and losses in the distribution system from the point of diversion to the individual farm outlets. According to Linsley and

Franzini (1964, pp 385), the losses in open channel distribution system would vary between 25 to 40% of the quantity of water delivered. These losses may be virtually eliminated by using a pipe system.

Conveyance Capacities

The design capacity of main canals (not to be confused with the optimum aqueduct capacity mentioned earlier) is influenced by the peak rate of water demand and the conveyance losses in addition to the extent of area served at the same time.

Cavazza (1968, pp 27-32) observed that the peak rate of water use occur at different times for different crops. However, on project areas under diversified crops, maximum demand for water occurs about the middle, or a little after the middle of the irrigation season. Houk (1962, pp 78) suggested that peak rate of water demand may be estimated by adding 10 to 15% to the average rate during the month of maximum use. It was also observed that the maximum monthly use forms 20 to 30% of the total seasonal delivery during a growing season of six to seven months.

On the subject of conveyance losses, Matarrese (1968, pp 36) reported Houk's findings that the major part of the conveyance losses consists of seepage losses which vary between 15 to 45% of the original quantity. Furthermore, these losses could be reduced to five percent if the canals were lined, and nearly eliminated if pipes were used for conveyance of water.

The commonly used formulas for calculating dimensions of conveyance channels are those of Bazin, Kutter, and Manning (King, 1954, pp 7/1-7/36).

Irrigation on Demand

In many countries particularly where sprinkler irrigation has become common, a new method of water distribution, called "distribution on demand", is replacing the rotational distribution in collective irrigation projects.

Matarrese (1963, pp 26-38) studied the rotational distribution of irrigation water in the Puglia region of Italy and found that 34.3% of the annual farm deliveries were not used by the farmers with a consequent loss of water. The renunciation of the use of water varied indirectly with daily evapotranspiration rates. Such losses could be eliminated by "distribution on demand" whereby the time of irrigation as well as the interval of irrigation is at the user's choice.

Bonnal (1963, pp 16-18) reported that annual water use on recent projects in France with distribution on demand, decreased to 30% of the water used in traditional systems of distribution by rotation. Besides, there was a considerable reduction in management, supervision, and maintenance requirements.

The backbone of the "distribution on demand" system is Clement's formula (Bonnal, 1963, pp 48-57 and Matarrese, 1966, pp 8-13) which is used to calculate the maximum number of farm outlets that would be opened simultaneously in a distribution network. Consequently, the design capacity of the network is increased as compared to the system of distribution by rotation.

III. METHODOLOGY

This chapter deals with the procedure for developing benefit and cost functions and using these functions in dynamic programming analysis in order to optimize water allocation to various districts fed by an aqueduct. First, certain assumptions are made regarding benefits and costs. Then, the equations for benefits and cost functions are described, followed by the dynamic programming procedure. Finally, an example is considered to show the application of the equations.

PART A. GENERAL FORMULATION

Assumptions

Project

Project was assumed to include water allocation to a number of irrigation districts from a limited water supply.

Returns

Returns were assumed to consist of the value of annual crop produce ready to be sold at the farm gate, net of production costs, excluding the cost of irrigation facilities and equipment. Furthermore, it was assumed that there is a direct relationship between returns per unit area of irrigated land and the level of irrigation until the irrigation requirements of a crop have been fully satisfied, so that the maximum returns per unit area occur at the full irrigation requirement level.

Benefits

Benefits were considered as the returns from irrigated land produce, less the returns from alternative dry land produce and the associated costs of irrigation.

Associated Costs of Irrigation

The following items were included in associated costs:

1. Annual allowance for investment, interest, and maintenance of irrigation equipment and facilities on the farm.
2. Annual allowance for investment, interest, maintenance, management, and operation of the distribution system within an irrigation district.
3. Annual allowance for costs attributable to the irrigation district for the provision of irrigation water at the head end of the main canal or aqueduct.

Project Costs

Project cost or simply "cost" was considered as the annual allowance for investment in main aqueduct, control structures and roads along the aqueduct, and land expropriated, as well as annual interest, maintenance, and management allowance for such conveyance.

Benefit Function

Estimation of Benefits

The procedure for estimating benefits was established by the following steps:

1. Estimating irrigable area of every major soil type in every district.
2. Projecting cropping patterns for every major soil type.
3. Estimating percent area under every crop on every major soil in an average year.
4. Estimating expected yields for every crop.
5. Estimating returns for every crop.
6. Calculating returns per typical hectare of every major soil type (on the basis of steps 3 and 5), both for irrigation and dry land conditions.
7. Estimating associated costs per hectare.
8. Calculating benefits per typical hectare of every major soil type.

This procedure contains an implicit assumption that the main factors affecting benefits of a district are soil type, climate, crops grown, and the area irrigated. The influence of all other factors would be lumped in the yields and prices selected.

Benefit Equation

On the basis of the procedure outlined above, a model for benefits per typical hectare of a soil type was developed as described below.

Let a = percent area assigned to a crop on a soil type in a district,

c_a = associated costs per hectare (which may or may not be related to soil type),

c_p = production costs per hectare of a crop (excluding associated costs and aqueduct costs),

P = price per unit of produce,

r = returns per hectare of a crop,

R_d = returns per typical hectare of a soil type under dry land conditions,

R_w = returns per typical hectare of a soil under irrigated conditions,

v_h = benefits per typical hectare of a soil type,

Y = yield of a crop

Y_d = dry land yield of a crop,

Y_w = irrigated yield of a crop,

then, in general the returns per hectare of a crop under any conditions would be

$$r = (Y.P - c_p).$$

The returns per typical hectare of a soil type under dry land conditions would be:

$$R_d = \frac{a_1 \, (Y_{d_1}.p_1 - C_{pd_1}) + a_2 \, (Y_{d_2}.p_2 - c_{pd_2}) + \ldots + a_n \, (Y_{dn}.p_n - C_{pdn})}{100}$$

$$= \frac{\sum a_i \, (Y_{di}.p_i - C_{pdi})}{100} \qquad (1)$$

where $i = 1, 2, 3, \ldots\ldots n$ dry land crops.

Similarly, returns per typical hectare of a soil type under irrigated conditions could be determined by:

$$R_W = \frac{\sum a_i \, (Y_{wi}.p_i - C_{pwi})}{100} \tag{2}$$

where i = 1, 2, 3, ------n irrigated crops,

Then, the benefit per typical hectare of a soil type would be

$$v_h = R_W - (R_d + C_a) \tag{3}$$

Water Delivery Requirement Equation

In order to calculate the water delivery requirement per typical hectare of a soil type, the following equation was developed.

Let a = percent area assigned to a crop on a soil type under consideration,

L_c = conveyance losses, expressed as a fraction,

L_f = farm losses, expressed as a fraction,

P_e = effective precipitation,

q_d = water delivery requirement per typical hectare of a soil type,

U = consumptive use of a crop per hectare of a soil type

then,

$$q_d = \frac{\dfrac{(a_1.U_1 + a_2.U_2 + a_3.U_3 + \ldots\ldots a_n U_n)}{100} - P_e}{(1-L_c)\,(1-L_f)}$$

$$= \frac{\dfrac{\sum (a_i \, . \, U_i)}{100} - P_e}{(1-L_c)\,(1-L_f)} \tag{4}$$

Benefit Function, v(q)

The benefit (v_h) and water delivery requirement (q_d) per typical hectare as well as the surface area of every major soil in every district can be determined by the above methods. The values of v_h and q_d are expected to be different for different soils. In addition, the extent of surface area of various soils will be different in different districts. Hence, the total benefits of one district are expected to be different from those of another district from similar water allocations.

The next step is to establish a relationship between the total benefits of a district and the quantity of water supplied to that district.

Let A_i = variable irrigated area of soil "i" in a district,

v = total benefits of the district,

v_{hi} = v_h for soil i,

q = quantity of water supplied to the district,

and q_{di} = q_d for soil i,

then

$$v = \sum (A_i \cdot v_{hi}) \tag{5a}$$

and

$$q = \sum (A_i \cdot q_{di}) \tag{5b}$$

where i = 1, 2, 3, ... n soils.

The desired benefit function, viz. v as a function of q, may be obtained graphically by plotting values of v and q for various values of A_i.

First, the soil with the highest benefit per typical hectare in the district may be considered as soil i = 1. Increasing values of A_1 are used to obtain corresponding values of v and q, which are then plotted. The soil with the next to the highest benefit per typical hectare, i.e., soil i = 2, is then considered. Values of v and q obtained at this stage for different A_2 values are added to the last values obtained for A_1 and then plotted as cumulative values of v and q.

The process is continued in this manner until all the irrigable area in the district is covered.

A smooth curve drawn through the plotted points would represent the benefit function, v(q) for the district under consideration.

Benefit functions for the other districts are obtained by using the same procedure.

Cost Function

An equation for project costs, as defined previously in this chapter, was developed on the assumption that the aqueduct is an open channel. Manning's equation was used to obtain hydraulic characteristics of the canal which in turn provide parameters for determining costs.

Hydraulic Equations

It was found desirable to formulate a "unit section" of a canal with unit bottom width and to use this as a proportionate section for calculating the volume of excavation and the area of canal lining.

Manning's equation states that

$$q = \frac{1}{n} R^{2/3} A s^{1/2} \tag{6}$$

where q is the flow rate (discharge) in cubic meters per second,

n is the Manning's coefficient of roughness, R is the hydraulic radius in meters,

A is the cross-sectional area in square meters,

and s is the slope of the canal in meters per meter.

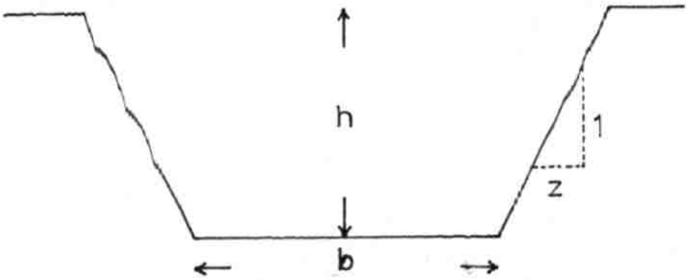

Figure 1. Cross-section of a canal.

The cross-section area (A) and the hydraulic radius (R) in equation (6) are calculated from canal geometry (Figure 1):

$$A = h\ (b + zh) \qquad (7)$$

where h is the depth of flow in meters,

b is the bottom width in meters,

and z is the side slope.

$$P = b + 2h\sqrt{1 + z^2} \qquad (8)$$

where P is the wetted perimeter.

Since $R = \frac{A}{P}$

from equations (7) and (8):

$$R = \frac{h\ (b + zh)}{b + 2h\ \sqrt{1 + z^2}} \qquad (9)$$

Let $$j = \frac{h}{b}$$

then, from equation (7),

$$\begin{aligned} A &= h\ (b + zh) \\ &= bj\ (b + zbj) \\ &= b^2.j\ (1 + zj) && (10a) \\ &= b^2.\ F_1\ (j,z) && (10b) \end{aligned}$$

where $F_1(j,z)$ is a function of j and z,

and, from equation (8),

$$\begin{aligned} P &= b + 2h\ \sqrt{1 + z^2} \\ &= b + 2bj\ \sqrt{1 + z^2} \\ &= b\ (1 + 2j\ \sqrt{1 + z^2}\ ) && (11) \end{aligned}$$

Now, equation (9) can be written as

$$\begin{aligned} R &= \frac{h\ (b + zh)}{b + 2h\sqrt{1+z^2}} \\ &= \frac{b^2\ j\ (1 + zj)}{b\ (1 + 2j\sqrt{1+z^2})} \\ &= b\ \frac{j\ (1 + zj)}{1 + 2j\ \sqrt{1+z^2}} \\ &= b.\ F_2\ (j\ .\ z) && (12) \end{aligned}$$

where $F_2\ (j.z)$ is another function of j and z.

Let $$K = \frac{1}{n}\ R^{2/3}\ A = \frac{q}{\sqrt{s}} \qquad (13)$$

then from equations (10b) and (12),

$$\begin{aligned} K &= \frac{1}{n}\left[b.F_2\ (j.z)\ \right]^{2/3} \quad \left[b^2.F_1\ (j,z)\right] \\ &= \frac{1}{n}\ .\ b^{8/3}\ .\ F_3\ (j,z) && (14) \end{aligned}$$

Where F_3 (j,z) is also a function of j and z.

The concept of the "unit section" as defined earlier can now be used advantageously.

For $b = 1$, the corresponding $K = K_1$ for the "unit section" is obtained from equation (14):

$$K_1 = \frac{1}{n} \cdot F_3\ (j,z)$$

From equations (14) and (15):

$$\frac{K}{K_1} = b^{8/3}$$

$$b = \left(\frac{K}{K_1}\right)^{3/8}$$

Since, from equation 13, $K = \frac{q}{\sqrt{s}}$

$$b = \left(\frac{q}{K_1\ \sqrt{s}}\right)^{3/8} \qquad (15)$$

Equation (15) may be used to determine the required channel bottom width for a given flow rate and canal slope. It also forms the basis for calculating volume of excavation shown in the following section.

The values of the ratio of depth of water flow to bottom width (j), side slope (z), coefficient of roughness (n), and the slope of the channel (s) are established by on-site technical and economic investigations, which are beyond the scope of this study. Assuming that these values have been established for a channel under consideration, the area of the "unit section", A_1, the wetted perimeter, P_1, the hydraulic radius, R_1, and the factor K_1 are then calculated as below:

From equation (10a)

$$A_1 = j\,(1 + zj) \tag{16}$$

From equation (11)

$$P_1 = 1 + 2j\,\sqrt{1 + z^2} \tag{17}$$

and, from equation (13)

$$K_1 = \frac{1}{n}\left(\frac{A_1}{P_1}\right)^{2/3} A_1$$

Values of K_1 and s are then substituted in equation (15) to obtain:

$$b = a\,.\,q^{3/8} \tag{18}$$

where $$a = \left(\frac{1}{K_1\,\sqrt{s}}\right)^{3/8}$$

For any desired discharge, q, the corresponding bottom width, b of the canal is found from equation (18).

Volume of Excavation

There are two elements of excavation per meter length of canal, A and A_0 (Figure 2).

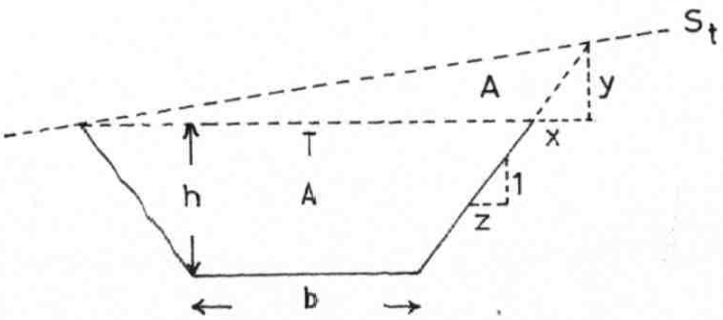

Figure 2. Excavation parameters of the cross-section of a canal.

The calculation of **A** has been described in equation (10a). The corresponding equation for A_0 is developed below.

Let T = top width of canal,

S_t = transversal slope of the natural terrain,

x = zy

then,

$$T = b + 2zh$$

$$= b\ (1 + 2zj)$$

and

$$\frac{Y}{T + x} = S_t$$

so that

$$Y = \frac{S_t . T}{1 - S_t z}$$

$$= \frac{S_t \ . \ b(1+2zj)}{1 - S_t z}$$

Therefore,

$$A_0 = \frac{T \ . \ Y}{2}$$

$$= \frac{b(1+2zj)}{2} \quad \frac{S_t \ . \ b(1+2zj)}{1 - S_t z}$$

$$= b^2 \ . \ \frac{S_t (1+2\ zj)^2}{2\ (1-S_t z)}$$

For a unit section, the corresponding element is A_{01} where

$$A_{01} = \frac{S_t\ (1+2zj)^2}{2\ (1-S_t z)}$$

The volume of excavation, E_1 per meter length of the unit section is

$$E_1 = A_1 + A_{01}$$

The volume of excavation, E in cubic meters per meter length of any section is

$$E = b^2 . E_1 \tag{19}$$

Where b is given by equation (18) for a desired value of discharge, q.

Area of Lining

The surface area for lining, P in square meters per meter length is

$$P = b . P_1 \tag{20}$$

where $P_1 = (1 + 2j\sqrt{1+z^2})$

Annual Costs of Aqueduct, c(q)

The annual total costs of conveyance are the annual allowance for the cost of excavation, lining, control structures, roads along the canal, and the land expropriated, plus the annual cost of management, maintenance, and operation of the system.

The cost of excavation, C_E for a canal reach L, is

$$C_E = E . c_e . L \tag{21}$$

Where E is the volume of excavation (equation 19),

c_e is the excavation cost per cubic meter volume per meter length of the reach,

and L is the length of the reach in meters.

The cost of lining, C_L for the reach is

$$C_L = P . t . c_1 . L \tag{22}$$

Where P is the perimeter from equation (20),

t is the thickness of lining in meters,

and c_l is the lining cost per meter thickness per meter length.

The investments in control structures, roads, and land expropriation, C_g, may be expressed as a percent of $(C_E + C_L)$.

Then, the total investment, C_T is:

$$C_T = C_E + C_L + C_g \tag{23}$$

The annual amortization allowance for the total investments including the interest is:

$$C_A = C_T \frac{i\ (1 + i)^n}{(1 + i)^n - 1} \tag{24}$$

Where i is the interest rate,

and n is the period of analysis in years.

Hence, the annual total costs as a function of the discharge delivered are

$$C\ (q) = C_A + C_M \tag{25}$$

Where C_M is the annual cost of management, maintenance, and operation of the aqueduct, usually expressed as a percent of C_T.

Unit of Discharge in Benefit and Cost Functions

It should be noted here that "q" in the benefit function, v(q), was a quantity of water annually delivered to a district, while in the cost function, C(q), it was considered as a flow rate, which is the discharge capacity of the reach L of the district concerned. This may not be a confusing point, however, because the discharge capacity is designed on the basis of the quantity of water to be delivered during the peak use part of the irrigation season.

Dynamic Programming Analysis

The equation developed by Hall (1961, pp 1-12) for optimal aqueduct capacity determination was used for optimal water allocation. This equation is written as

$$V = \max \sum_{i=1}^{n} \left[v_i(q_i) - C\,(q_1,\ q_2,\ q_3\ \ldots .q_n) \right]$$

$$\text{subject to}:\ \sum_{i=1}^{n} q_i \leqslant Q \qquad (26)$$

Solution of the above equation was obtained by the dynamic programming method as described below.

The districts fed by the aqueduct under consideration are numbered, beginning with the district at the end of the aqueduct, sequentially in order, back to the head end of the aqueduct. A reach of the aqueduct, designated as reach number one carries water from the head gate of district two to the head gate of district one.

Suppose that after district two has been allotted water, a variable quantity, $o \leq q \leq Q$, is available for transmission through reach one to district one. The problem is to determine the value of q_1 as a function of q, such that the net benefit is a maximum. If this maximum net benefit is designated as f_1, then;

$$f_1\ (q) = \max \left[v_1(q_1) - L_1\ C_1(q) \right] \qquad (27)$$

$$o \leqslant q_1 \leqslant q$$

$$o \leqslant q \leqslant Q$$

L_1 being the length of reach one and $c_1(q)$ is the average annual cost per meter length of the aqueduct for reach one as a function of the required capacity for delivering q_1.

The allocation to the first district is optimized as a function of the available supply after district two has been served. The next step is to optimize allocations for both districts one and two. In order to deliver water to districts one and two, it must be conveyed from the headgate of district three to the headgate of district two through reach number two. Suppose that a variable quantity of water q, $0 \leq q \leq Q$, is available after the needs of district three have been met under an optimum policy. The problem, then, is to determine the values of q_1 and q_2 as a function of q, such that the total net benefit, from both districts one and two is maximum.

At this point, the principle of optimality is introduced, which dictates that whatever the decision regarding the allocation q_2, the remaining quantity $(q-q_2)$ must be used in an optimum fashion to obtain optimum allocation.

If $f_2(q)$ is defined as the maximum net benefit from allocations to districts one and two out of the quantity available just downstream of the headgate of district three, then

$$f_2(q) = \max_{\substack{0 \leq q_2 \leq q \\ 0 \leq q \leq Q}} \left[v_2(q_2) - L_2\, C_2(q) + f_1(q-q_2) \right] \qquad (28)$$

The cost of delivering $(q-q_2)$ to district one has been accounted for in $f_1(q)$ which is now $f_1(q-q_2)$. Thus, only the additional cost $L_2\ C_2\ (q)$ is considered at the second stage of the analysis.

The same reasoning would be applied to district one, two, and three together when the allocation is to be made to the three districts out of a variable quantity available at the headgate of district four.

This analysis is carried out over all districts in sequential order so that

$$f_n\ (q) = \max \left[v_n\ (q_n) - \frac{1}{n}\ c_n(q) \pm f_{n-1}\ (q-q_n) \right] \qquad (29)$$

$$0 \leqslant q_n \leqslant q$$

$$0 \leqslant q \leqslant Q$$

Solution of equation 27 yielded the maximum net benefit $f_1(q)$ obtainable from district one as well as the optimum water allocation $q_1(q)$ as a function of the available water supply. Solution of equation 28 led to the maximum net benefit from district one and two together, and optimal allocation $q_2(q)$. The process is continued until $f_n(q) = V(q)$ is obtained from equation 29, giving water allocations for all districts under an optimum net benefit policy.

PART B - APPLICATION TO A PROJECT

In order to show the application of the procedure developed for determining benefit and cost functions and their subsequent use in dynamic programming analysis to obtain an optimal water allocation, an example was taken of three irrigation districts to be fed by the "Canal 900" in the Southern Beqa'a area, which is a part of the Litani River Project in Lebanon.

Description of the Study Area

The three districts selected for this study have already been delineated in the preliminary plan of the Litani River Project (Saliba, 1964, pp 187-189). For purposes of analysis in this study, the three districts are numbered sequentially in order starting with the last downstream district.

District One

This district covers the villages of Hosh-Harimi, Jazireh, Magdal Anjar (partly), Marj (partly), and Wakf. The total irrigable area of this district is 2,330 ha, out of which 330 ha are irrigated by local wells. The rest, 2,000 ha, will be irrigated by the "Canal 900" as envisaged in the preliminary plan mentioned earlier.

District Two

The second district consists of the villages of Ghazzeh, Khiara, Khiara Atika, and Sultan Yaqoub. The total irrigable area is 2,495 ha of which 1,800 ha will be irrigated by the "Canal 900".

District Three

The third district constitutes a major part of the village of Jib-Jannine. The "Canal 900" will irrigate 800 ha out of 1,120 ha of the total irrigable area.

Water Resources

A limited area in each district, as mentioned above, is already irrigated by local wells. The major source of irrigation water, however, is the Karaoun reservoir from which water will be pumped into the "Canal 900" (Zambarakji, 1967). The headgate for district three will be at 17.5 km from the head end of the canal, for district two at 25.5 km, and for district one at 31.5 km. At each headgate, there will be a pumping station for drawing water from the canal to feed the distribution system within the district.

Distribution System and Method of Irrigation

The distribution system will consist of underground asbestos cement pipes. The size of the distribution conduits is designed to meet the requirements of irrigation on demand, by the sprinkler method of irrigation (Saliba, 1964, pp 178-189).

According to the preliminary plan, there will be a turnout (hydrant) for every 2.5 ha and water will be delivered with a pressure of 2.5 atm. The total head to be made available at the pumping station of district one will be 35 m, while for districts two and three it will be 33 m. The time of irrigation is planned to be 18 hours per day.

Premises of the Problem

It was assumed in this study that there is a deficient water supply available for the three districts, not exceeding 20 million cubic meters actually distributable in a normal year. The problem was to apportion this quantity in such a manner as to obtain the maximum total net benefit from the three districts.

Secondly, the organization handling the irrigation project was considered as a semi-autonomous institution, so that the secondary benefits were not attributable to the project. Furthermore, irrigation structures and equipment would not be subject to any taxes or insurance. It was also assumed that funds will be made available by the state to finance the project.

Thirdly, it was assumed that all prices relative to this study are expected to change in equal proportions and in the same direction so that the use of a currently prevalent price level is advantageous. Benefits and costs were estimated beginning with their full occurrence.

Fourthly, all farms were treated as owner-operated.

Benefit Functions, $v_i(q_i)$

Irrigable Areas According to Soil Types

A soil survey was carried out in 1957 by the "Groupe Français du Litani" (G.F.L., 1957, pp 17-37 and 140). On the basis of the findings of the survey, this organization prepared a soil map, classified by the Litani River Authority (Office National du Litani) as map No. G.F.L. 7. In order to estimate the irrigable area of the soils of the three districts, the soil map G.F.L. 7 and a preliminary irrigation map, denoted by the Litani River Authority (L.R.A.) as IR-BS-306, were made available by the L.R.A. By superposing the two maps and by planimetry, the desired areas were estimated as shown in Table 3 in Appendix A.

Projected Cropping Patterns

The G.F.L. (1957, pp 44-56) had proposed crop rotations according to soil type, based on future demand and supply conditions of various farm products. Taking into account these projections, a projected annual cropping pattern was prepared for every soil found in the three districts as shown in Tables 4 and 5 in Appendix A.

Projected Returns of Various Crops

Various reports and publications were consulted on this subject, specially Anonymous (1963), Anonymous (1965, pp 1-16), Anonymous (1967, pp 1-37), Boyaji (1967), Sa'ab (1965, pp 1-74), Terzibachian (1966, pp 1-106), and Ward (1959, pp 425-441). On the basis of these reports, returns per hectare were calculated for the dry land as well as irrigated crops and are shown in Tables 6 and 7 in Appendix A.

Returns Per Typical Hectare (R_w & R_d)

The projected cropping patterns of Tables 4 and 5 and returns per hectare of Tables 6 and 7 were used to calculate returns per typical hectare of the soils as shown in Tables 8 and 9, all in Appendix A.

Water Delivery Requirements (q_d)

The G.F.L. (1957, pp 138-139) had calculated the consumptive use requirements according to soil type by using the Blaney-Criddle formula and weighted crop coefficients.

Applying a farm efficiency of 85% ($1 - L_f = 0.15$) and a conveyance efficiency of 95% ($1 - L_c = 0.05$), the consumptive use (U) figures were divided by 0.80 ($0.85 \times 0.95 = 0.80$) to obtain water delivery requirements (q_d) of various soils (Table 10 of Appendix A).

Associated Costs (C_a)

Associated costs were estimated as average per hectare and per cubic meter. Three components of the associated costs were considered; 1) costs at the source of water; 2) cost of the distribution network; and 3) cost of the mobile equipment. The data on investment costs of various structures and equipment were taken from the preliminary plan of the Litani River Project (Saliba, 1964, pp 185-199).

Cost at the Source of Water: The part of investments in the Karaoun dam and reservoir allocated to the annual supply of 58 million cubic meters of irrigation water for the whole area of Litani River Project is L.L. 20 million. Assuming the economic life of the installation as 50 years, rate of interest as 4½%, and the annual maintenance, management, and administration costs as three percent of the initial investment, the annual costs per cubic meter will be:

$$\frac{20 \times 10^6 \; (0.0506 + 0.03)}{58 \times 10^6} = 0.02779 \text{ L.L./m}^3$$

where 0.0506 is the capital recovery factor.

From the Karaoun reservoir, water is pumped into the "Canal 900". The investment in the pumping station is L.L. 4.4 millions. Assuming a life period of 25 years and the rest of the conditions the same as in the case of the dam, the annual cost of the pumping station per cubic meter of water:

$$\frac{4.4 \text{ X } 10^6 \ (0.0674 + 0.03)}{58 \text{ X } 10^6} = 0.00739 \text{ L.L.}/\text{m}^3$$

The annual cost of power consumption:

$$\frac{0.82 \text{ X } 10^6}{58 \text{ X } 10^6} = 0.01414 \text{ L.L.}/\text{m}^3$$

The annual total cost per cubic meter of water:

$$0.02779 + 0.00739 + 0.01414 = 0.04932 \text{ L.L.}/\text{m}^3$$

It was assumed that the conveyance loss from the source of water to the headgate of a district would be ten percent, so that one cubic meter actually distributed would cost ten percent more at the source of water. Hence, the annual cost at the source:

$$0.04932 + (1 + 0.10) = \underline{\underline{0.05425}} \text{ L.L.}/\text{m}^3$$

<u>Cost of the Distribution Network:</u> The distribution network will consist of the main pumping station at the headgate of district, a reservoir, underground asbestos cement pipes carrying water to individual plots and the related control structures.

A pressure reservoir catering to the annual needs of 2,050 ha costs L.L. 250 thousands. Assuming a life period of 50 years, rate of interest 4½%, and the other costs occurring annually as 2½% of the initial investment, the annual cost of the reservoir:

$$\frac{25 \text{ X } 10^4 \ (0.0506 + 0.025)}{2,050} = 9.22 \text{ L.L./ha}$$

The annual cost of the district pumping station, assuming a life period of 25 years, rate of interest 4½%, and the other costs occurring annually as three percent of the initial investment of L.L. 850 thousands for pumping 11.4 million cubic meters.

$$\frac{85 \text{ X } 10^4 \ (0.0674 + 0.03)}{11.4 \text{ X } 10^6} = 0.00727 \text{ L.L./m}^3$$

The annual power consumption, charging 0.055 L.L./Kwh

$$\frac{14 \text{ X } 1 \text{ X } 35}{3600} \text{ X } 0.055 = 0.00749 \text{ L.L./m}^3$$

Where 35 is the average lift in meters for one cubic meter per second discharge and 14 is a conversion factor (Saliba, 1964, pp 183).

The investment in the conduits and other structures is 1,080 L.L./ha on the average. Assuming a life period of 25 years, rate of interest 4½% and the costs occurring annually as two percent of the initial investment, the annual cost:

$$1,080 \ (0.0674 + 0.02) = 94.73 \text{ L.L./ha}$$

Thus, the annual cost of the distribution network

$$9.22 + 94.73 = \underline{\underline{103.95}} \text{ L.L./ha}$$

plus $0.00727 + 0.00749 = \underline{\underline{0.01476}} \text{ L.L./m}^3$

<u>Cost of the Mobile Equipment:</u> The investment in the mobile equipment, e.g. sprinklers, laterals, couplers is estimated to be 850 L.L./ha. Assigning an economic life of 15 years, rate of interest

eight percent, and the other costs occurring annually as four percent of the initial investment, the annual cost of the mobile equipment:

$$850\ (0.1168 + 0.04) = \underline{\underline{133.28}}\ \text{L.L./ha}$$

Thus, the annual associated costs, excluding labor has been already included in production costs:

$$103.95 + 133.28 = 237.23\ \text{L.L./ha} \ \ldots\ldots\ C_{a1}$$

$$\text{plus} \quad 0.05425 + 0.01476 = 0.06901\ \text{L.L.}/m^3 \ldots\ldots C_{a_2}$$

Benefit Functions, $v_i(q_i)$

Having calculated the returns per typical hectare of various soils under irrigated as well as under dry land conditions (Tables 8 and 9 in Appendix A) and the associated costs, benefits of three districts were determined for variable irrigated areas as shown in Tables 11, 12, and 13 in Appendix A.

The benefit figures were then plotted against water delivery requirements on semi-log paper to obtain the functions $v_i(q_i)$ for each district (Figure 3 and Table 1 in Chapter IV).

Cost Function, C(q)

The cost of construction of concrete-lined canals for variable discharge was taken from experience in Jordan[1]. The said cost was increased by 20% to cover the costs of land expropriation, roads and control structures along the canal. The period of analysis was assumed to be 50 years, rate of interest 4½%, the other costs occurring

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[1] Personal communication from Prof. Salim W. Macksoud.

annually as 2½% of the initial investment. The annual cost per meter length of canal for various discharge capacities was calculated as presented in Table 14 in Appendix A.

Annual cost values were plotted against discharge, q on semi-log paper to obtain the cost function, C(q) as shown in Figure 4 and Table 1 in Chapter IV.

In order to determine the discharge capacity of the canal required for delivering a certain quantity of water annually during 18 hours of irrigation per day, the peak monthly delivery requirements were taken as 15% of the annual delivery requirement and a conveyance loss of ten percent was added. For example, the discharge capacity required for delivering one million cubic meters annually (Table 1) is:

$$\frac{(1 \times 10^6)\ (0.15)}{(1-0.10)\ (30 \times 18 \times 3600)} = 0.11\ m^3/s$$

Dynamic Programming Analysis

Dynamic programming analysis was carried out in three stages as shown in Tables 15, 16, and 17 in Appendix A; stage one for district one alone, stage two for districts one and two together, and stage three for the three districts together. The final allocations giving maximum net benefits are summarized in Table 2 in Chapter IV.

IV. RESULTS AND DISCUSSION

This chapter deals with the results obtained by the application of the procedure, described in the previous chapter, for the systematic development of benefit and cost functions for the three districts under consideration and for the use of these functions in dynamic programming analysis for optimal water allocation.

For convenience of presentation, however, only the benefit and cost functions and the final allocations to the three districts are presented here with appropriate comments on the rest of the results which were considered as of transitory nature and are presented in Appendix A.

Estimated Annual Benefits

Cropping Patterns

The information available on the study area showed that there is hardly any tendency towards the adaptation of dry land crops according to soil type. It was realistic therefore, to project similar cropping patterns for all soils of the area (Table 4 in Appendix A).

Under irrigation, however, the difference in the suitability of various crops to certain soils is more prominent. The projections in Table 5 took account of this specialized use of soil.

Annual Returns

Annual returns shown in Tables 6 and 7 do not include land rent. As mentioned previously, the farms of the study area were considered as owner-operated so that land rent would not be paid by the owner.

Secondly, there was no major difference in the proximity of various land areas to large population centers so as to influence the land rent.

Thirdly, it was argued that the difference in land rent under dry land and irrigated conditions is due to the introduction of irrigation. The use of irrigation water enables production of high value crops and increase in yields over dry land conditions, thereby increasing the value of land. The cost of introducing irrigation water, however, is taken care of under other cost items.

On the other hand, family labor was counted as a cost in Tables 6 and 7 in order to expose the difference in labor requirements of dry land and irrigated crops. Otherwise, it would have been assumed that the family labor, previously underemployed in dry land farming, would be fully employed in irrigated farming, thus excluding most of the labor costs.

Comparison of the costs of dry land and irrigated wheat (Tables 6 and 7, respectively) shows a decrease of labor costs from L.L. 94 to L.L. 24 per hectare with the introduction of irrigation. This is expected due to the higher use of mechanization the cost of which appears under the equipment and services item.

Water Delivery Requirements

As seen from Table 10 in Appendix A, the peak water requirements occur in May. This is contrary to the general belief that the peak use of irrigation water takes place about the middle or just after the middle of the irrigation season, which, in the study area, extends from April to October. However, this phenomenon does not affect the estimation of benefit and cost functions, because the importance of peak water use lies in its magnitude rather than in its time of occurrence.

Benefit and Cost Functions

The total benefits of all the three districts were found to be increasing with an increase in the quantity of water delivered as shown by Figure 3. The rate of increase of benefits, however, was not regular. It tends to be decreasing. This is due to the fact that as the irrigated area of a district was increased, there was a change from the more productive soil (with higher value crops) to the less productive ones which caused a decrease in returns per typical hectare. Also, the associated costs per hectare were relatively constant. The net effect was a decrease in benefits per hectare, hence the decreasing rate of increase in total benefits.

If a detailed design of the distribution network for a variable irrigated area in a district were made, it would have shown a large variation in associated costs per hectare due to differences in location of different areas with respect to the head gate. In that case, the variation in the rate of change in total benefits with

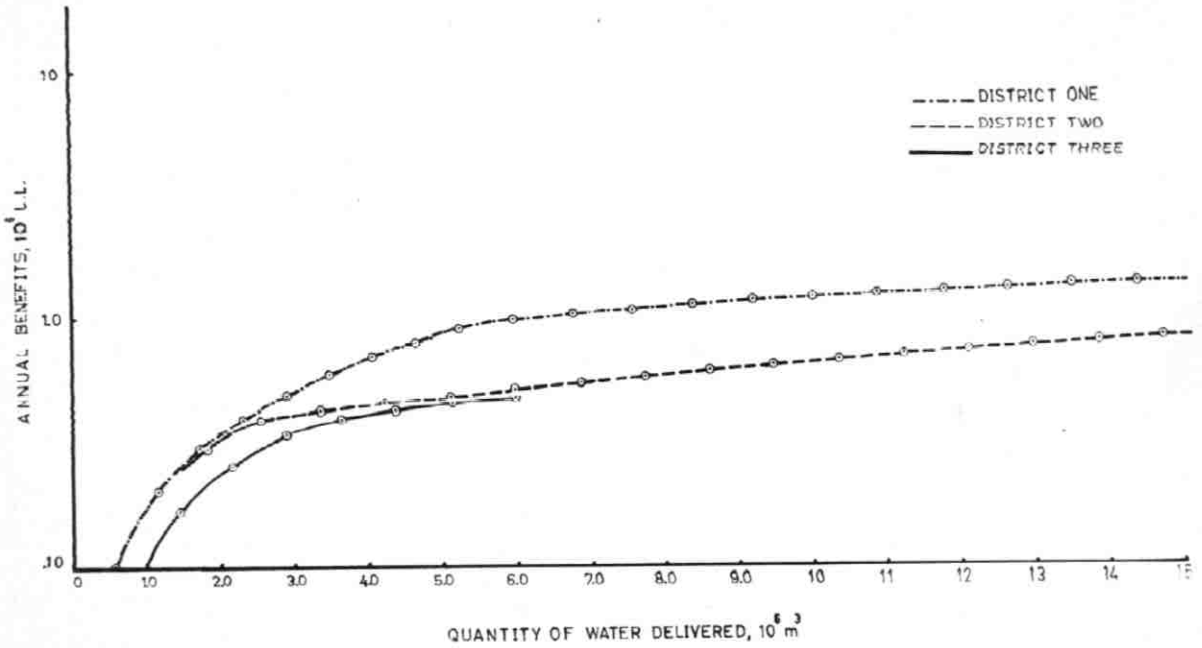

Figure 3. Benefit functions, $v_i(q_i)$ of district one, two, and three.

increasing quantity of water delivered would be more prominent. Unfortunately, the information required for such an analysis was not available. Therefore, the associated costs per hectare and per cubic meter of water were assumed as constant.

The values of the benefit functions, $v_i(q_i)$ of the three districts along with the increasing units of water delivered listed in Table 1, were read from Figure 3.

Figure 4 shows a more pronounced behavior of the cost function. As the discharge capacity required to deliver a variable quantity of water increases, the total cost of the aqueduct increases with a gradually decreasing rate of increase. The explanation is found in equation 18, which indicates that the increase in discharge (q) does not require an identical increase in canal dimensions (b). The latter increases with a power of 3/8 only, so that the relative cost of construction and lining decreases with increasing units of discharge capacity.

A variable proportion of the investments for land expropriation, roads, and control structures would further contribute to the variation in the annual costs of the aqueduct. For the sake of simplicity, however, these costs were taken as a constant proportion of the costs of construction and lining.

A detailed study of the terrain in which the aqueduct would be constructed might indicate some differences in construction costs of various reaches. In that case, each district would have its own cost function.

The values of the cost function, $C(q_i)$ in Table 1, were read

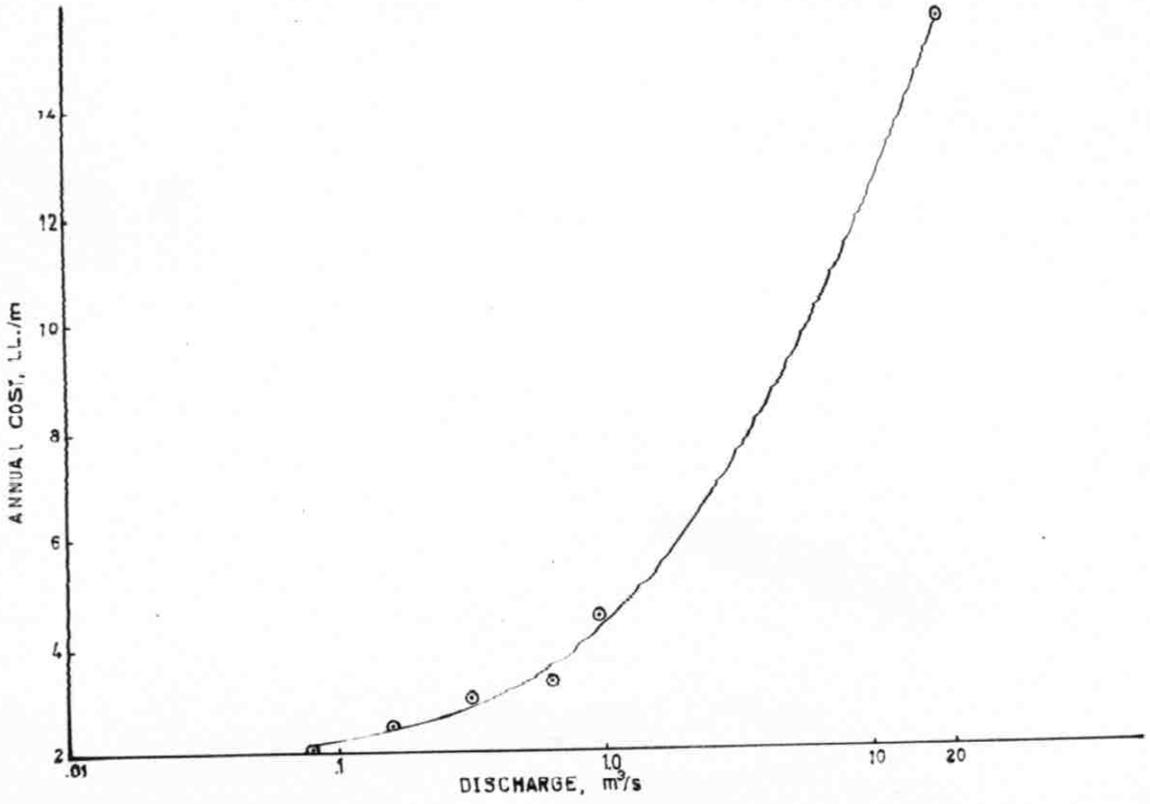

Figure 4. Cost function, C (q_i).

Table 1. Benefit and cost functions of districts one, two, and three.

q 10^6 m^3	q m^3/s	$C(q)^x$ 10^3 L.L/km	$v_1(q)$ 10^3L.L.	$v_2(q)$ 10^3L.L.	$v_3(q)$ 10^3L.L.
1	0.11	2.20	173	173	105
2	0.22	2.50	345	322	233
3	0.33	2.80	515	410	344
4	0.44	3.10	685	445	410
5	0.55	3.40	855	480	450
6	0.66	3.70	995	515	485
7	0.77	3.90	1050	550	
8	0.88	4.10	1105	585	
9	0.99	4.30	1160	620	
10	1.10	4.50	1210	655	
11	1.21	4.70	1250	690	
12	1.32	4.90	1290	725	
13	1.43	5.10	1325	760	
14	1.54	5.30	1360	790	
15	1.65	5.45	1370	810	
16	1.76	5.60			
17	1.87	5.75			
18	1.98	5.90			
19	2.09	6.05			
20	2.20	6.20			

x Applicable to all parts of the aqueduct.

from the curve in Figure 4 common to the three districts.

Final Allocations

Table 2 shows the final allocations to the three districts with maximum net benefits arrived at through dynamic programming analysis. It is seen from the table that, in order to obtain maximum net benefits, the first unit of water (one million cubic meters) should be allocated to district two; the next five units to district one; the seventh unit to district two; eighth to district one; the following three units to district three; the twelfth unit to district two; and so forth until all the twenty units are allocated as twelve units to district one, three to district two, and five to district three obtaining maximum net benefits of L.L. 1.969 millions per year.

Table 2. Final allocations of water to districts one, two, and three with optimum net benefit.

q 10^6 m^3	q_1 10^6 m^3	q_2 10^6 m^3	q_3 10^6 m^3	f_3 (q) 10^3 L.L.
1	0	1	0	117
2	1	1	0	269
3	2	1	0	432
4	3	1	0	592
5	4	1	0	752
6	5	1	0	914
7	5	2	0	1058
8	6	2	0	1190
9	6	2	1	1292
10	6	2	2	1416
11	6	2	3	1524
12	6	3	3	1607
13	6	3	4	1670
14	7	3	4	1718
15	8	3	4	1767
16	9	3	4	1817
17	10	3	4	1861
18	10	3	5	1899
19	11	3	5	1934
20	12	3	5	1969

V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

A methodology was worked out for developing benefit and cost functions and using these functions in dynamic programming analysis in order to determine water allocations based on a policy of optimum net benefit from geographic districts irrigated with a deficient water supply.

The main physical variables determining benefits from a geographic district were taken as the soil, climate, crop, yield, area irrigated, and the quantity of water supplied to the district.

It was assumed that yields are maximum when the level of irrigation meets the full water requirements of the irrigated crops. In the methodology, therefore, irrigated yields were taken at the full level of irrigation.

Crops were assumed to be a function of the soil because of the differences in adaptability of crops to different soils so that the type of soil would be a determining factor for the crops to be grown advantageously.

Considering only one level of irrigation, the extent of the area irrigated would be determined by the quantity of water supply and water requirements of crops determined by climatic factors.

Thus, the independent variables that would determine the benefits from a district were reduced to soil type and the quantity of water supplied.

Benefits were defined on an annual basis as the increased value of crop produce due to irrigation, less the associated costs of irrigation equipment and structures.

Equations were developed to determine the relationship between benefits and water delivery requirements of various soils taking into account the assumptions described above. Assuming a variation found in the type of soils of a district, a function may be prepared indicating the relationship between benefits and water supply for the whole district.

The project costs were defined as the annual costs of the aqueduct serving the districts. Equations were described to calculate the annual cost of an aqueduct as a function of discharge capacity.

Dynamic programming was used as a procedure to determine the optimal water allocation to various districts with the benefit and cost functions already prepared.

The application of the methodology was exemplified by taking the case of three geographic districts along the "Canal 900" in the Southern Beqa'a area of the Litani River Project.

Conclusions

The methods presented in this study for determining proper water allocation to various geographic districts along an aqueduct are a rational approach to rentable irrigation project planning. The procedure affords a way of maximizing net benefits from an irrigation water development project.

From the results and discussion it is concluded that the procedure is applicable to project areas variable in extent and physical properties.

The methods used in this study, however, are applicable to the planning of new development projects. They are not suited for determining allocations of water from existing aqueducts since the latter type of project may have originally been designed on other bases and as such may be tied up by previously existing technical, operational and legal matters.

Recommendations

It is recommended that a study should be carried out to apply the methods presented here on a different but larger area than that considered in this study. Furthermore, it would be interesting to consider the associated costs as a variable function of the irrigated area and the distance from the headgate of the district concerned.

In extensive projects a large amount of the computational work may be reduced by the use of electronic computers if available.

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APPENDICES

Appendix A

The following pages contain the tables showing results of various steps described in the chapter on Methodology in order to estimate annual benefits of the three irrigation districts with corresponding quantities of water delivery requirements. Tables containing dynamic programming analysis are also presented in this section.

Table 3. Irrigable area of districts one, two, and three according to soil type.

Soil		District 1	District 2	District 3
No.	Type	ha	ha	ha
I	Hydromorphous	910	250	-
II	Alluvial	110	120	350
III	Tirs[x]	500	-	-
IV	Tirsifié[x]	480	1360	-
V	Red soil	-	70	450
Total		2000	1800	800

x. English equivalent not available.

Table 4. Percentage (a_i) of typical hectare projected for dry land crops in the study area[x].

Crop	Barley	Fallow	Chick peas, lentils, and water melons	Wheat	Total
%	5	30	30	35	100

x. Applicable to all soils.

Table 5. Percentage (a_i) of typical hectare projected for irrigated crops according to soil types of the study area.

Crop	Soil I	Soil II	Soil III	Soil IV	Soil V
	%	%	%	%	%
Winter wheat	40	33	66	20	20
Spring wheat	-	-	-	20	-
Onions	-	-	33	-	-
Potatoes	40	33	33	-	-
Sugarbeet	40	33	-	-	-
Flax	-	-	-	20	20
Alfalfa	-	33	-	60	60
Short season forages	-	-	33	-	-
Fruit trees	20	-	-	-	-

Table 6. Annual returns per hectare of dry land crops of the study area.

Item	Barley	Chick-peas	Lentils	Water-melons	Wheat
Receipts					
Yield, kg/ha	1500	710	800	6000	1300
Price, L.L./kg	0.17	0.45	0.70	0.10	0.27
Receipts, L.L.	255	319	560	600	351
Straw, kg/ha	700	-	800	-	2050
Price, L.L./kg	0.035	-	0.04	-	0.035
Receipts, L.L.	24	20	32	-	72
Total receipts	279	339	592	600	423
Production costs					
Seeds, L.L.	20	67	126	16	44
Fertilizers, L.L.	-	-	-	-	24
Equipment and services, L.L.	24	15	17	52	23
Miscellaneous, L.L.	13	10	10	9	14
Labor, L.L.	91	90	191	141	94
Total costs	148	182	344	218	199
Returns, L.L./ha	131	157	248	382	224

Table 7. Annual returns per hectare of irrigated crops of the study area.

Item	Alfalfa	Onions	Potatoes	Sugar-beet	Wheat
Receipts					
Yield, kg/ha	80000	23000	16500	40000	2000
Price, L.L./kg	0.03	0.10	0.20	0.06	0.27
Receipts, L.L.	2400	2300	3300	2400	540
Straw, kg/ha	-	-	-	-	2000
Price, L.L./kg	-	-	-	-	0.037
Receipts, L.L.	-	-	-	-	74
Total receipts, L.L.	2400	2300	3300	2400	614
Production costs					
Seeds, L.L.	50	350	600	59	54
Fertilizers, L.L.	200	195	280	289	60
Pesticides, L.L.	-	-	50	24	-
Equipment and services, L.L.	150	47	79	131	60
Miscellaneous, L.L.	50	43	52	41	16
Labor, L.L.	200	635	259	702	24
Total costs, L.L.	650	1270	1320	1246	214
Returns, L.L./ha	1750	1030	1980	1154	400

Estimated returns of fruit trees = 1950 L.L./ha
flax = 500 L.L./ha
and short season forages = 500 L.L./ha

Table 8. Annual returns per typical hectare of dry land farming in the study area[x].

Crop	Returns L.L./ha	Area/typ. ha %	L.L./typ.ha Rd. L.L.
Barley	131	5	6.55
Chick peas	157	10	15.70
Lentils	248	10	24.80
Watermelons	382	10	38.20
Wheat	224	35	78.40
Fallow	-	30	-
Total	-	100	163.65

x. Applicable to all soils.

Table 9. Annual returns per typical hectare of irrigated soils of the study area.

Soil	Crop	Returns L.L./ha	Area/typ.ha %	L.L./typ.ha Rw.
I	Winter wheat	400	40	160
	Potatoes	1980	40	792
	Sugarbeets	1154	40	460
	Fruit trees	1950	20	390
		Returns per typical hectare = 1802		
II	Winter wheat	400	33	132
	Potatoes	1980	33	653
	Sugarbeets	1150	33	380
	Forages	1750	33	578
		Returns per typical hectare = 1743		
III	Winter wheat	400	66	264
	Onions	1030	33	340
	Potatoes	1980	33	653
	Short season forages	500	33	165
		Returns per typical hectare = 1422		
IV	Winter wheat	400	20	80
	Spring wheat	400	20	80
	Flax	500	20	100
	Forages	1750	60	1050
		Returns per typical hectare = 1310		
V	Winter wheat	400	20	80
	Flax	500	20	100
	Forages	1750	60	1050
		Returns per typical hectare = 1230		

Table 10. Monthly water delivery requirements (q_d) according to soil types of the study area.

Soil	Water measure	April	May	June	July	Aug.	Sept.	Oct.	Total
I	U, cm	5.8	8.9	5.8	5.5	6.4	7.5	6.6	46.5
	q_d, cm	7.2	11.1	7.2	6.9	8.0	9.4	8.2	58.0
	q_d, m^3/ha	720	1110	720	690	800	940	820	5800
II	U, cm	6.0	10.3	8.5	8.3	9.1	9.9	7.9	60.0
	q_d, cm	7.5	12.6	10.6	10.4	11.4	12.4	9.9	74.8
	q_d, m^3/ha	750	1260	1060	1040	1140	1240	990	7480
III	U, cm	6.1	11.5	9.6	9.5	10.2	10.1	8.0	65.0
	q_d, cm	7.6	14.4	12.0	11.9	12.8	12.6	10.0	81.3
	q_d, m^3/ha	760	1440	1200	1190	1280	1260	1000	8130
IV	U, cm	7.3	11.7	11.0	11.9	10.5	9.6	7.9	69.9
	q_d, cm	9.1	14.6	13.8	14.9	13.1	12.0	9.9	87.4
	q_d, m^3/ha	910	1460	1380	1490	1310	1200	990	8740
V	U, cm	7.5	10.1	8.6	9.5	9.2	7.5	5.7	58.1
	q_d, cm	9.4	12.4	10.8	11.9	11.5	9.4	7.1	72.5
	q_d, m^3/ha	940	1240	1080	1190	1150	940	710	7250

Table 11. Annual benefits of district one from variable irrigated area.

(All values in thousands)

Area ha	q m^3	(R_w-R_d) L.L.	Associated costs, Ca Ca_1 L.L.	 Ca_2 L.L.	 Ca L.L.	Benefits, V, L.L.
0.100	580	164	23.7	40.0	64	100
0.200	1160	327	47.4	80.1	128	199
0.300	1740	491	71.2	120.2	191	300
0.400	2320	655	94.9	160.3	255	400
0.500	2900	819	118.6	200.3	319	500
0.600	3480	982	142.3	240.4	383	599
0.700	4060	1146	166.1	280.5	447	699
0.800	4640	1310	189.8	320.5	510	800
0.900	5220	1474	213.5	360.6	574	900
1.000	5950	1632	237.2	411.0	648	984
1.100	6760	1764	261.0	467.0	728	1036
1.200	7573	1894	284.7	523.1	808	1086
1.300	8386	2024	308.4	579.3	888	1136
1.400	9199	2148	332.1	635.5	968	1180
1.500	10012	2268	355.8	691.6	1047	1221
1.600	10874	2385	379.6	751.2	1131	1254
1.700	11784	2500	403.3	812.0	1215	1285
1.800	12622	2615	427.0	872.0	1299	1316
1.900	13469	2730	450.7	930.4	1381	1349
2.000	14370	2845	474.5	992.7	1467	1378

Table 12. Annual benefits of district two from variable irrigated area.

(All values in thousands)

Area,	q	(R_w-R_d)	Ca			V
ha	m^3	L.L.	Ca_1 L.L.	Ca_2 L.L.	Ca L.L.	L.L.
0.100	580	164	23.7	40.1	64	100
0.200	1160	327	47.4	80.1	128	199
0.300	1804	488	71.2	124.6	196	292
0.400	2545	632	94.9	175.8	271	361
0.500	3359	775	118.6	232.0	351	424
0.600	4233	890	142.3	292.4	435	455
0.700	5107	1005	166.1	352.8	519	486
0.800	5981	1120	189.8	413.2	603	517
0.900	6855	1235	213.5	473.5	687	548
1.000	7729	1350	237.2	533.9	771	579
1.100	8603	1465	261.0	594.3	855	610
1.200	9477	1580	284.7	654.7	939	641
1.300	10351	1695	308.4	715.0	1023	672
1.400	11225	1810	332.1	775.4	1108	702
1.500	12099	1925	355.8	835.8	1192	733
1.600	12973	2040	379.6	896.2	1276	764
1.700	13847	2155	403.3	956.6	1360	795
1.800	14721	2265	427.0	1017.0	1444	821

Table 13. Annual benefits of district three from variable irrigated area.

(All values in thousands)

Area	q	$(R_w - R_d)$	Ca			V
ha	m^3	L.L.	Ca_1 L.L.	Ca_2 L.L.	Ca L.L.	L.L.
0.100	725	158	23.7	50.1	74	84
0.200	1450	315	47.4	100.2	148	167
0.300	2175	473	71.2	150.2	221	252
0.400	2900	631	94.9	200.3	295	336
0.500	3637	764	118.6	251.2	370	394
0.600	4385	871	142.3	302.9	445	426
0.700	5133	978	166.1	354.6	521	457
0.800	5881	1085	189.8	406.3	596	489

Table 14. Annual cost of aqueduct for various discharge capacities.

Discharge, m^3/s :	0.080	0.160	0.320	0.640	0.960	20.000
Construction costs, L.L./m :	23.22	27.52	32.68	36.98	49.88	172.00
Annual costs, L.L./m:	2.05	2.45	2.95	3.30	4.50	15.60

Table 15. Dynamic programming analysis, stage one.

$$f_1\ (q) = \max \left[v_1(q_1) - L_1\ C(q) \right]$$

$$0 \leqslant q_1 \leqslant q \leqslant Q$$

$$L_1 = 6.0 \text{ km}$$

q 10^6 m^3	$v_1(q_1)$ 10^3 L.L.	L_1 $C(q)$ 10^3 L.L.	f_1 $(q_1 q_1)$ 10^3 L.L.	$f_1(q)$ 10^3 L.L.	Allocation 10^6 m^3
1	173	13	160	160	$q_1 = 1$
2	345	15	330	330	$q_1 = 2$
3	515	17	498	498	$q_1 = 3$
4	685	19	666	666	$q_1 = 4$
5	855	20	835	835	$q_1 = 5$
6	995	22	973	973	$q_1 = 6$
7	1050	23	1027	1027	$q_1 = 7$
8	1105	25	1080	1080	$q_1 = 8$
9	1160	26	1134	1134	$q_1 = 9$
10	1210	27	1183	1193	$q_1 = 10$
11	1250	28	1222	1222	$q_1 = 11$
12	1290	29	1261	1261	$q_1 = 12$
13	1325	31	1294	1294	$q_1 = 13$
14	1360	32	1328	1328	$q_1 = 14$
15	1370	33	1337	1337	$q_1 = 15$

Table 16. Dynamic programming analysis, stage two.

$$f_2 (q) = \max \left[v_2(q_2) + f_1 (q-q_2) - L_2 C(q) \right]$$

$$0 \leqslant q_2 \leqslant q \leqslant Q$$

$$L_2 = 8,9 \text{ km}$$

q 10^6 m^3	q_2 10^6 m^3	$q-q_2$ 10^6 m^3	f_1 $(q-q_2)$ 10^3 L.L.	v_2 (q_2) 10^3 L.L.	L_2 $C(q)$ 10^3 L.L.	f_2 $(q_1$ $q_2)$ 10^3 L.L.	$f_2(q)$ 10^3 L.L.	Allocation 10^6 m^3
1	0	1	160	0	18	142		$q_1 = 0$
	1	0	0	173	18	155	155	$q_2 = 1$
2	0	2	330	0	20	310		
	1	1	160	173	20	313	313	$q_1 = 1$
	2	0	0	322	20	302		$q_2 = 1$
3	0	3	498	0	22	476		
	1	2	330	173	22	481	481	$q_1 = 2$
	2	1	160	322	22	460		$q_1 = 1$

Table 16 (Continued).

	3	0	0	410	22	388	
4	0	4	666	0	25	641	
	1	3	498	173	25	646	646	$q_1 = 3$
	2	2	330	322	25	627		$q_2 = 1$
	3	1	160	410	25	545		
	4	0	0	445	25	420		
5	0	5	835	0	27	805		
	1	4	666	173	27	812	812	$q_1 = 4$
	2	3	498	322	27	793		$q_2 = 1$
	3	2	330	410	27	713		
	4	1	160	445	27	578		
	5	0	0	480	27	453		
6	0	6	973	0	30	943		
	1	5	835	173	30	978	978	$q_1 = 5$
	2	4	666	322	30	956		$q_2 = 1$

Table 16 (Continued).

	3	3	498	410	30	878		
	4	2	330	445	30	745		
	5	1	160	480	30	610		
	6	0	0	515	30	485		
7	0	7	1027	0	31	996		
	1	6	973	173	31	1115		
	2	5	835	322	31	1126	1126	$q_1 = 5$
	3	4	666	410	31	1045		$q_2 = 2$
	4	3	498	445	31	912		
	5	2	330	480	31	779		
	6	1	160	515	31	644		
	7	0	0	550	31	519		
8	0	8	1080	0	33	1047		
	1	7	1027	173	33	1167		
	2	6	973	322	33	1262	1262	$q_1 = 6$

Table 16 (Continued).

	3	5	835	410	33	1212		$q_2 = 2$
	4	4	666	445	33	1078		
	5	3	498	480	33	945		
	6	2	330	515	33	812		
	7	1	160	550	33	677		
	8	0	0	585	33	552		
9	0	9	1134	0	34	1100		
	1	8	1080	173	34	1219		
	2	7	1027	322	34	1315	1349	$q_1 = 6$
	3	6	973	410	34	1349		$q_2 = 3$
	4	5	835	445	34	1240		
	5	4	666	480	34	1102		
	6	3	498	515	34	979		
	7	2	330	550	34	846		
	8	1	160	585	34	711		
	9	0	0	620	34	586		

Table 16 (Continued).

10	0	10	1183	0	36	1147		
	1	9	1134	173	36	1271		
	2	8	1080	322	36	1366		
	3	7	1027	410	36	1401	1401	$q_1 - 7$
	4	6	973	445	36	1382		$q_2 = 3$
	5	5	835	480	36	1279		
	6	4	666	515	36	1145		
	7	3	498	550	36	1012		
	8	2	330	585	36	879		
	9	1	160	620	36	744		
	10	0	0	655	36	619		
11	0	11	1222	0	38	1184		
	1	10	1183	173	38	1318		
	2	9	1134	322	38	1418		
	3	8	1080	410	38	1452	1452	$q_1 = 8$
	4	7	1027	445	38	1434		$q_2 = 3$

Table 16 (Continued).

	5	6	973	480	38	1415		
	6	5	835	515	38	1312		
	7	4	666	550	38	1178		
	8	3	498	585	38	1045		
	9	2	330	620	38	912		
	10	1	160	655	38	777		
	11	0	0	690	38	652		
12	0	12	1261	0	39	1232		
	1	11	1222	173	39	1356		
	2	10	1183	322	39	1466		
	3	9	1134	410	39	1505	1505	$q_1 = 9$
	4	8	1080	445	39	1486		$q_2 = 3$
	5	7	1027	480	39	1468		
	6	6	973	515	39	1449		
	7	5	835	550	39	1346		
	8	4	666	585	39	1212		

Table 16 (Continued).

	9	3	498	620	39	1079		
	10	2	330	655	39	946		
	11	1	160	690	39	811		
	12	0	0	725	39	686		
13	0	13	1294	0	41	1253		
	1	12	1261	173	41	1393		
	2	11	1222	322	41	1503		
	3	10	1183	410	41	1552	1552	$q_1 = 10$
	4	9	1134	445	41	1538		$q_2 = 3$
	5	8	1080	480	41	1519		
	6	7	1027	515	41	1501		
	7	6	973	550	41	1482		
	8	5	835	585	41	1379		
	9	4	666	620	41	1245		
	10	3	498	655	41	1112		
	11	2	330	690	41	979		

Table 16 (Continued).

	12	1	160	725	41	844		
	13	0	0	760	41	719		
14	0	14	1328	0	42	1286		
	1	13	1294	173	42	1425		
	2	12	1261	322	42	1541		
	3	11	1222	410	42	1590	1590	$q_1 = 11$
	4	10	1183	445	42	1586		$q_2 = 3$
	5	9	1134	480	42	1572		
	6	8	1080	515	42	1553		
	7	7	1027	550	42	1535		
	8	6	973	585	42	1516		
	9	5	835	620	42	1413		
	10	4	666	655	42	1279		
	11	3	498	690	42	1146		
	12	2	330	725	42	1013		
	13	1	160	760	42	878		

Table 10 (Continued)

	14	0	0	790	42	742		
15	0	15	1337	0	44	1293		
	1	14	1328	173	44	1457		
	2	13	1294	322	44	1572		
	3	12	1261	410	44	1627	1627	$q_1 = 12$
	4	11	1222	445	44	1623		$q_2 = 3$
	5	10	1183	480	44	1619		
	6	9	1134	515	44	1605		
	7	8	1080	550	44	1586		
	8	7	1027	585	44	1568		
	9	6	973	620	44	1549		
	10	5	835	655	44	1446		
	11	4	666	690	44	1312		
	12	3	498	725	44	1179		
	13	2	330	760	44	1046		
	14	1	160	790	44	906		

Table 16 (Continued)

	15	0	0	810	44	766		
16	1	15	1337	173	45	1465		
	2	14	1328	322	45	1605		
	3	13	1294	410	45	1659		
	4	12	1261	445	45	1661	1661	$q_1 = 12$
	5	11	1222	480	45	1657		$q_2 = 4$
	6	10	1183	515	45	1653		
	7	9	1134	550	45	1639		
	8	8	1080	585	45	1620		
	9	7	1027	620	45	1602		
	10	6	973	655	45	1583		
	11	5	835	690	45	1480		
	12	4	666	725	45	1346		
	13	3	498	760	45	1213		
	14	2	330	790	45	1075		
	15	1	160	810	45	925		

Table 16 (Continued)

17	2	15	1337	322	46	1613		
	3	14	1326	410	46	1692		
	4	13	1294	445	46	1693		
	5	12	1261	480	46	1695	1695	$q_1 = 12$
	6	11	1222	515	46	1691		$q_2 = 5$
	7	10	1183	550	46	1680		
	8	9	1134	585	46	1673		
	9	8	1080	620	46	1654		
	10	7	1027	655	46	1636		
	11	6	973	690	46	1617		
	12	5	835	725	46	1514		
	13	4	666	760	46	1380		
	14	3	498	790	46	1242		
	15	2	330	810	46	1094		
18	3	15	1337	410	47	1700		
	4	14	1328	445	47	1726		

Table 16 (Continued)

	5	13	1294	480	47	1727		
	6	12	1261	515	47	1729	1729	$q_1 = 12$
	7	11	1222	550	47	1725		$q_2 = 6$
	8	10	1183	585	47	1721		
	9	9	1134	620	47	1707		
	10	8	1080	655	47	1688		
	11	7	1027	690	47	1670		
	12	6	973	725	47	1651		
	13	5	835	760	47	1548		
	14	4	666	790	47	1409		
	15	3	498	810	47	1261		
19	4	15	1337	445	48	1734		
	5	14	1328	480	48	1760		
	6	13	1294	515	48	1761		
	7	12	1261	550	48	1763	1763	$q_1 = 12$
	8	11	1222	585	48	1759		$q_2 = 7$

Table 16 (Continued)

	9	10	1183	620	48	1755		
	10	9	1134	655	48	1741		
	11	8	1080	690	48	1722		
	12	7	1027	725	48	1702		
	13	6	973	760	48	1685		
	14	5	835	790	48	1577		
	15	4	666	810	48	1428		
20	5	15	1337	480	50	1767		
	6	14	1328	515	50	1793		
	7	13	1294	550	50	1794		
	8	12	1261	585	50	1796	1796	$q_1 = 12$
	9	11	1222	620	50	1792		$q_2 = 8$
	10	10	1183	655	50	1788		
	11	9	1134	690	50	1772		
	12	8	1080	725	50	1755		
	13	7	1027	760	50	1737		
	14	6	973	790	50	1713		
	15	5	835	810	50	1595		

Table 17. Dynamic programming analysis, stage three.

$$f_3(q) = \max \left[v_3(q_3) + f_2(q-q_3) + L_3\, C(q) \right]$$

$$0 \leqslant q_3 \leqslant q \leqslant Q$$

$$L_3 = 17.5 \text{ km}$$

q $10^6\ m^3$	q_3 $10^6\ m^3$	$q-q_3$ $10^6\ m^3$	$f_2(q-q_3)$ 10^3 L.L.	$v_3(q_3)$ 10^3 L.L.	$L_3\, C(q)$ 10^3 L.L.	$f_3(q_1 q_2 q_3)$ 10^3 L.L.	$f_3(q)$ 10^3 L.L.	Allocation $10^6\ m^3$
1	0	1	155	0	38	117	117	$q_1 = 0$ $q_2 = 1$ $q_3 = 0$
	1	0	0	105	38	67		
2	0	2	313	0	44	269	269	$q_1 = 1$
	1	1	155	105	44	216		$q_2 = 1$
	2	0	0	233	44	189		$q_3 = 0$
3	0	3	481	0	49	432	432	$q_1 = 2$
	1	2	313	105	49	369		$q_2 = 1$
	2	1	155	233	49	339		$q_3 = 0$
	3	0	0	344	49	295		

Table 17 (Continued)

4	0	4	646	0	54	592	592	$q_1 = 3$
	1	3	481	105	54	532		$q_2 = 1$
	2	2	313	233	54	492		$q_3 = 0$
	3	1	155	344	54	445		
	4	0	0	410	54	356		
5	0	5	812	0	60	752	752	$q_1 = 4$
	1	4	646	105	60	691		$q_2 = 1$
	2	3	481	233	60	654		$q_3 = 0$
	3	2	313	344	60	597		
	4	1	155	410	60	505		
	5	0	0	450	60	390		
6	0	6	978	0	64	914	914	$q_1 = 5$
	1	5	812	105	64	853		$q_2 = 1$
	2	4	646	233	64	815		$q_3 = 0$
	3	3	481	344	64	761		

Table 17 (Continued)

	4	2	313	410	64	659		
	5	1	155	450	64	541		
	6	0	0	485	64	421		
7	0	7	1126	0	68	1058	1058	$q_1 = 5$
	1	6	978	105	68	1015		$q_2 = 2$
	2	5	812	233	68	977		$q_3 = 0$
	3	4	646	344	68	922		
	4	3	481	410	68	823		
	5	2	313	450	68	695		
	6	1	155	485	68	572		
8	0	8	1262	0	72	1190	1190	$q_1 = 6$
	1	7	1126	105	72	1159		$q_2 = 2$
	2	6	978	233	72	1139		$q_3 = 0$
	3	5	812	344	72	1084		
	4	4	646	410	72	984		

Table 17 (Continued)

	5	3	481	450	72	859		
	6	2	313	485	72	726		
9	0	9	1349	0	75	1274		
	1	8	1262	105	75	1292	1292	$q_1 = 6$
	2	7	1126	233	75	1284		$q_2 = 2$
	3	6	978	344	75	1247		$q_3 = 1$
	4	5	812	410	75	1147		
	5	4	646	450	75	1021		
	6	3	481	485	75	891		
10	0	10	1401	0	79	1322		
	1	9	1349	105	79	1375		
	2	8	1262	233	79	1416	1416	$q_1 = 6$
	3	7	1126	344	79	1391		$q_2 = 2$
	4	6	978	410	79	1309		$q_3 = 2$
	5	5	812	450	79	1183		

Table 17 (Continued)

	6	4	646	485	79	1052		
11	0	11	1452	0	82	1370		
	1	10	1401	105	82	1424		
	2	9	1349	233	82	1500		
	3	8	1262	344	82	1524	1524	$q_1 = 6$
	4	7	1126	410	82	1454		$q_2 = 2$
	5	6	978	450	82	1346		$q_3 = 3$
	6	5	812	485	82	1215		
12	0	12	1505	0	86	1419		
	1	11	1452	105	86	1471		
	2	10	1401	233	86	1548		
	3	9	1349	344	86	1607	1607	$q_1 = 6$
	4	8	1262	410	86	1586		$q_2 = 3$
	5	7	1126	450	86	1490		$q_3 = 3$
	6	6	978	485	86	1377		

Table 17 (Continued)

13	0	13	1552	0	89	1463		
	1	12	1505	105	89	1521		
	2	11	1452	233	89	1596		
	3	10	1401	344	89	1656	1670	$q_1 = 6$
	4	9	1349	410	89	1670		$q_2 = 3$
	5	8	1262	450	89	1623		$q_3 = 4$
	6	7	1126	485	89	1527		
14	0	14	1590	0	93	1497		
	1	13	1552	105	93	1564		
	2	12	1505	233	93	1645		
	3	11	1452	344	93	1703		
	4	10	1401	410	93	1718	1718	$q_1 = 7$
	5	9	1349	450	93	1706		$q_2 = 3$
	6	8	1262	485	93	1654		$q_3 = 4$
15	0	15	1627	0	95	1532		

Table 17 (Continued)

	1	14	1590	105	95	1600		
	2	13	1552	233	95	1690		
	3	12	1505	344	95	1754		
	4	11	1452	410	95	1767	1767	$q_1 = 8$
	5	10	1401	450	95	1759		$q_2 = 3$
	6	9	1349	485	95	1739		$q_3 = 4$
16	0	16	1661	0	98	1563		
	1	15	1627	105	98	1634		
	2	14	1590	233	98	1725		
	3	13	1552	344	98	1798		
	4	12	1505	410	98	1817	1817	$q_1 = 9$
	5	11	1452	450	98	1804		$q_2 = 3$
	6	10	1401	485	98	1788		$q_3 = 4$
17	0	17	1695	0	101	1594		
	1	16	1661	105	101	1665		

Table 17 (Continued)

	2	15	1627	233	101	1759		
	3	14	1590	344	101	1833		
	4	13	1552	410	101	1861	1861	$q_1 = 10$
	5	12	1505	450	101	1854		$q_2 = 3$
	6	11	1452	485	101	1836		$q_3 = 4$
18	0	18	1729	0	103	1626		
	1	17	1695	105	103	1697		
	2	16	1661	233	103	1791		
	3	15	1627	344	103	1868		
	4	14	1590	410	103	1897		$q_1 = 10$
	5	13	1552	450	103	1899	1899	$q_2 = 3$
	6	12	1505	485	103	1887		$q_3 = 5$
19	0	19	1763	0	106	1657		
	1	18	1729	105	106	1728		
	2	17	1695	233	106	1822		

Table 17 (Continued)

	3	16	1661	344	106	1899		
	4	15	1627	410	106	1931		$q_1 = 11$
	5	14	1590	450	106	1934	1934	$q_2 = 3$
	6	13	1552	485	106	1931		q_3 5
20	0	20	1796	0	108	1688		
	1	19	1763	105	108	1760		
	2	18	1729	233	108	1854		
	3	17	1695	344	108	1931		
	4	16	1661	410	108	1963		$q_1 = 12$
	5	15	1627	450	108	1969	1969	$q_2 = 3$
	6	14	1590	485	108	1967		$q_3 = 5$

Appendix B

Abbreviations

American Society of Agricultural Engineers	ASAE
American Society of Civil Engineers	ASCE
atmosphere (s)	atm.
centimeter	cm
cubic meter	m^3
cubic meter per second	m^3/s
dunum (= 1,000 square meters)	dun
Food and Agriculture Organization	FAO
for example	e.g.
Groupe Français du Litani	GFL
hectare (s)	ha
hour (s)	hr
journal	j.
kilometer (s)	km
kilowatt (s)	Kw
kilowatt hour (s)	Kwh
Lebanese Pound (s)	L.L.
meter (s)	m
millimeter (s)	mm
number	No.

Office National du Litani (Litani River Authority)	LRA
Organization for Economic Cooperation and Development	OECD
percent	%
that is	i.e.
United Nations Development Program	UNDP