DESIGN OF A REINFORCED CONCRETE ARCH BRIDGE ON THE BEIRUT RIVER

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1949

DESIGN OF A REINFORCED CONCRETE ARCH BRIDGE ON THE BEIRUT RIVER

by

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"This Thesis submitted to the Civil Engineering Faculty in Partial Fulfillment of the requirements for the Degree of Bachelor of Science in Civil Engineering"

American University of Beirut.

May 1949

ACKNOWLEDGMENT

The author gratefully acknowledges his indebtedness to Professor J. R. Osborn, Chairman of the Civil Engineering Department of the American University of Beirut, for assisting in several investigations, reading the text and making many useful suggestions.

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INTRODUCTION

Type of Bridge:- The bridge under study is of the open spandril type which is to replace an existing one, "Gisser-El-Basha", located on the outskirts of Beirut across the river.

The actual bridge is of the filled spandril type, built approximately 60 years ago during the Ottoman occupation.

Reasons for Selection:- The open spandril type was selected for two reasons: first; from rough estimates for the given span, it was found to be the most economical; and secondly, from the point of aesthetics it suits the particular conditions existing at the site, which will be discussed under the title Aesthetics.

Live Load:- This bridge will be designed for H 15 loading. Whenever the loaded length exceeds the 60 feet, the equivalent loading will be used which provides a uniform load of 480 pounds per linear foot of lane and a concentrated load, whose value shall be 13,500 pounds in computing bending moment stresses and 19,500 pounds in computing shearing stresses. The concentrated load shall be considered as uniformly distributed across the lane on a line normal to the center line of the lane.

Impact:- The live load will be increased by 30 per cent to cover impact stresses.

Wind Pressure:- Due to stability, low height, wide roadway and small area exposed to wind action, it is deemed unnecessary to consider any wind effect on the structure.

Dimensions:- Clear span 146 feet, Clear rise = 30.25 feet width of road way 20' - 00"; 2 footways, the width of each 05' - 00".

Application of General Assumptions:- The general assumptions and methods employed for reinforced concrete design are to be used in the design of this particular bridge and are as follows:-

1. Elastic modulus of concrete constant within limits of calculated stresses.
2. Steel reinforcement takes all tension.
3. Plane sections remain plane after bending.
4. Perfect adhesion between steel and concrete.

It has been proved that reinforced concrete structures of all classes erected to these accepted assumptions behave in a satisfactory way and give test results which agree with those calculated.

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Data and Specifications:- The columns supporting the deck are spaced 10'-00" center to center in one direction and 23'-00" center to center in the other direction. The transverse beams are placed one at each column and one at each mid-span of the supporting girders, thus making the distance center to center of beams 05'-00".

The general arrangement of beams is shown in Drawing No 1. A wearing surface of 30 pounds per square foot is to be included in the dead load on the slab.
The allowable unit stress of concrete is to be taken as 600 p.s.i. and 16,000 p.s.i. for steel. Considering the ration of modulus of elasticity of steel to that of concrete as 15, the ultimate compressive strength of concrete at age of 28 days will be 2000 p.s.i., K = 108, k = 0.379, j = 0.874, p = 0.0077.

The span length of the slab, the beams, and the girders will be taken as the distance center to center of supports. The resulting design is conservative, since good practice ordinarily permits the use of clear spans in case of continuous constructions.

Design of Deck Slab:- The slab is to be designed according to the following data:-

Span c. to c. of the supporting beams: 05'-00"

Wearing surface: 30 pounds per square foot.

Live load: H 15 truck loading.

Assuming an overall thickness of 7.50" and considering a 12" strip at right angles to the supporting beams.

The total dead load per square foot is 7.50/12 x 150 + 30 = 125 lbs.

the dead load moment $= \dfrac{wl^2}{10}$

$= \dfrac{125 \times 5 \times 5}{10} \times 12$

$=$ 3,750 in. lbs.

Live load Moment:-

The load on each rear wheel is 12,000 lbs; since the main reinforcement, in our case, will be parallel to the direction of the traffic the effective width will be given by the following formula: $E = 0.7l + T$

Where "E" is the effective width in feet.

Where "l" is the span of the slab in feet.

Where "T" is the width of the wheel in feet = 15/12 ft.

Thus $E = 0.7 \times 5.00 + 15/12 = 4.75$ ft.

According to the specifications of the American Association of State Highway Officials, the effective width shall have a maximum value of 7.00 feet. Since, however, this bridge carries two traffic lanes, the effective width cannot be greater than the sum of one-half the distance between wheels on one axle and one-half the distance between the adjacent wheels of two adjacent trucks. A rather improbable loading but required by the above specification.

therefore: $E = \frac{1}{2} (6 + 3) = 4.50$ feet.

The load on a unit width of slab is therefore 12000/4.50 equal to 2,670 lbs. In determining the maximum bending moment due to this concentrated load (2,670 lbs) placed at mid span, we shall take a condition midway between a fixed and free condition, that is, $\frac{3}{4}$ of free; therefore M = 3/16 Pl

Thus M = 3 x 2,670 x 5 / 16 x 12 = 30,000 in. lbs.

Summation of moments:-

Moment due to dead load	=	3,750 in lbs.
Moment due to live load	=	30,000 in lbs.
Impact Moment = 30,000 x 30%	=	9,000 in lbs.
Total	=	42,750 in lbs.

$$d = (M / Kb)^{\frac{1}{2}}$$

$$= (42,750 / 108 \times 12)^{\frac{1}{2}}$$

d = 5.75 in. say 6.00 in.

overall thickness = 6.00 in + 1.50 in. = 7.50"

$$As = M / fsjd$$

$$= 42,750 / 16,000 \times 0.874 \times 6$$

As = 0.51 sq. in.

An area of 0.52 sq. in. is furnished by $\frac{1}{2}$ in. round bars 4.50 in. center to center; these bars will be placed at the bottom throughout all the span. $\frac{1}{2}$ in. round bars at 9.00 in. center to center will be placed at the top throughout all the span; moreover at the supporting beams, at the top of the slab, $\frac{1}{2}$ in. round bars at 9.00 in. center to center will be provided 2.50 feet long.

All this top reinforcement will take care of the negative moment at the supports. In order to insure proper distribution of the concentrated loads and to provide for shrinkage, transverse $\frac{1}{2}$ in. round bars at 12 in. center to center will be placed directly on top of the longitudinal reinforcement.

Investigation for Web Reinforcement:-

$$v = V / \tfrac{7}{8} b' d$$

$$V = \tfrac{1}{2} (125 \times 5 + 1.30 \times 2670)$$
$$= 2,043 \text{ lbs.}$$

therefore, $v = 2,043 / \frac{7}{8} \times 12 \times 6 = 32.50$ p.s.i.

allowable unit shearing stress without any web reinforcement 40 p.s.i. , clearly, no web reinforcement is necessary.

Investigation for Bond Stress:-

$$u = V / \Sigma_o \times \tfrac{7}{8} \times d$$

$$\Sigma_o = 12 / 4.50 \times 1.571 = 4.20 \text{ in.}$$

therefore, $u = 2,043 / 4.20 \times \frac{7}{8} \times 6 = 90$ p.s.i.

allowable unit bond stress 90 p.s.i. clearly the adopted depth and reinforcement are satisfactory.

The arrangement of slab reinforcement is shown in Drawing No. 5.

Design of Transverse Beams: Since the slab and beams are to be poured at the same time and thoroughly bonded together, the latter may be designed as T-beams.

The transverse beams, of 32' span, with overhanging ends will be examined. These beams compared with the other transverse beams which are supported by the longitudinal ones receive

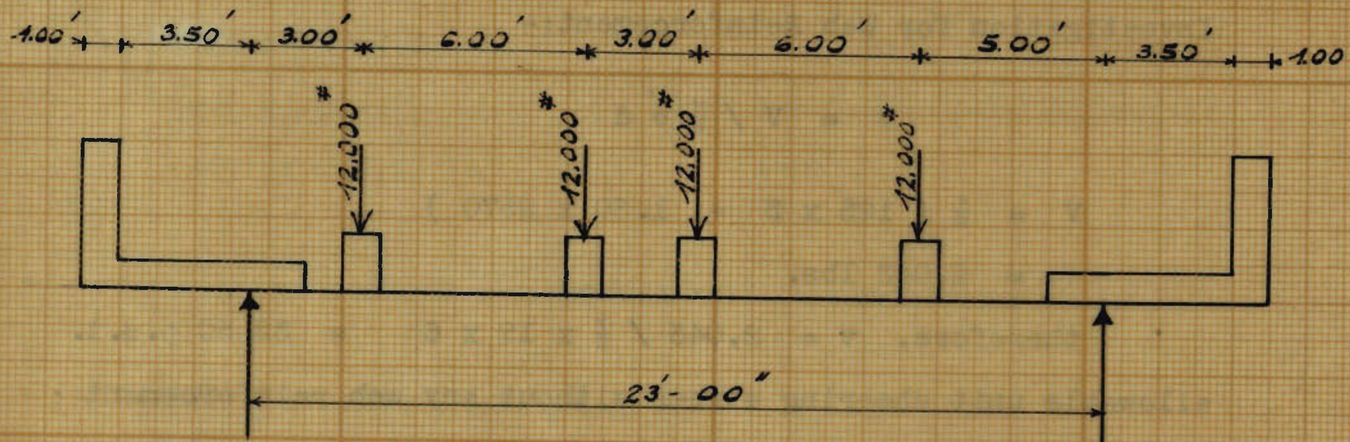

Fig. 1 - "Truck loading producing maximum shear"

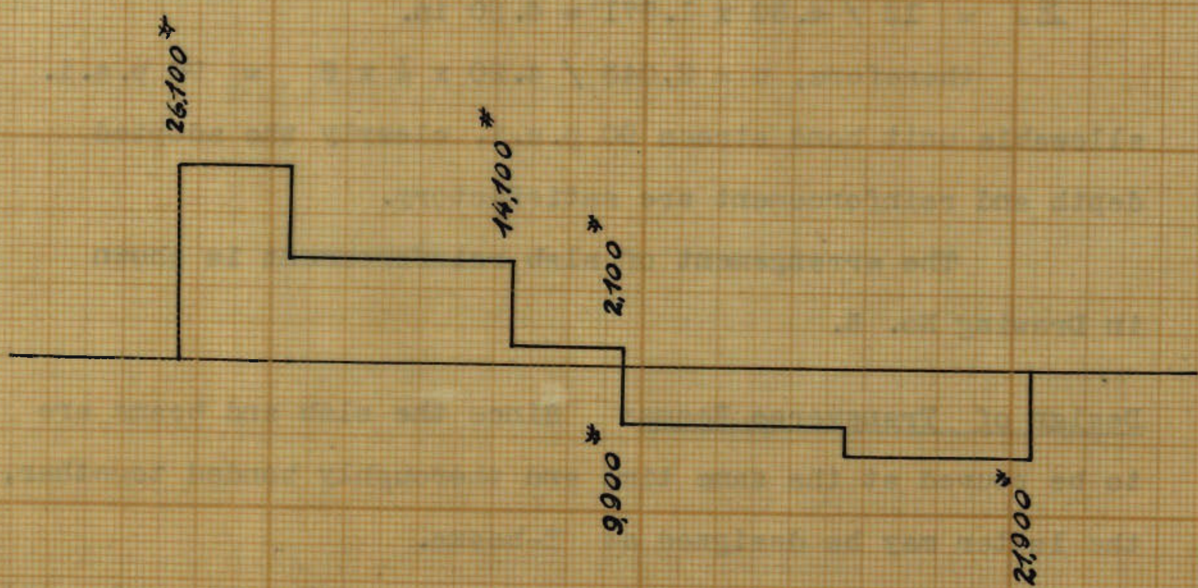

Fig. 2 - "Shear Diagram"

somehow larger shearing forces at the supports, whereas the maximum positive bending moment is larger only by 11 per cent. Assuming the stem depth of the beam to be 30 in. and 12 in. wide, the weight of the stem will be

30 x 12 x 150 / 144 = 370 lbs per linear foot.

Fig. 1. represents the truck loading when the transverse beam carries the rear wheels of two adjacent trucks. The wheels are placed in a such a way so as to produce the maximum possible shear at the left support.

$$R_L = 12{,}000\ (5 + 11 + 14 + 20)\ 1/23 = 26{,}100 \text{ lbs.}$$

$$R_R = 12{,}000\ (3 + 9 + 12 + 18)\ 1/23 = 21{,}900 \text{ lbs.}$$

In fig. 3 each of the concentrated loads due to the parapet is equal to 5.00' x 4.00' x 1.00' x 150 = 3,000 lbs. The uniform load for the pathway consists of the live load which is 112 lbs per square foot and its own weight.

Live load: 112 x 5	=	560
Weight of pathway : 16.50 / 12 x 150	=	206
Weight of the stem of the beam	=	370
Total		1136 lbs

The uniform load for the roadway consists of the slab weight and the stem of the beam.

From slab; 5x7.50x150 / 12	=	470
Weight of the stem of the beam	=	370
Total	=	840 lbs per linear foot.

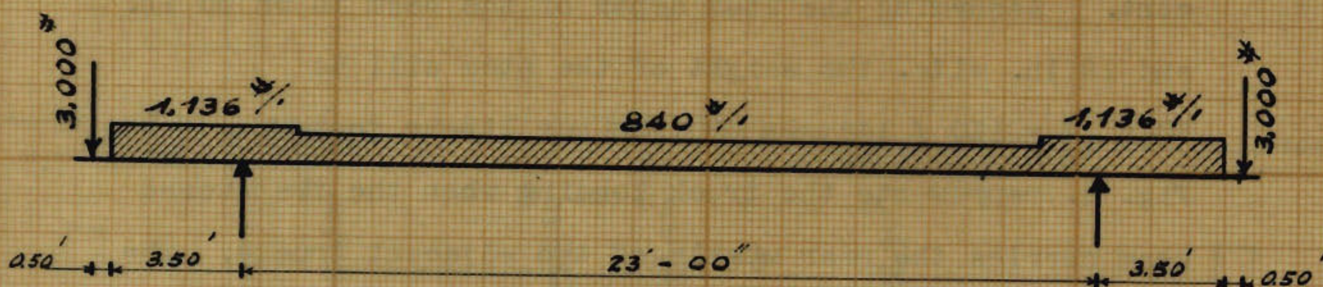

Fig. 3 - "Uniform load applied on the transverse beams"

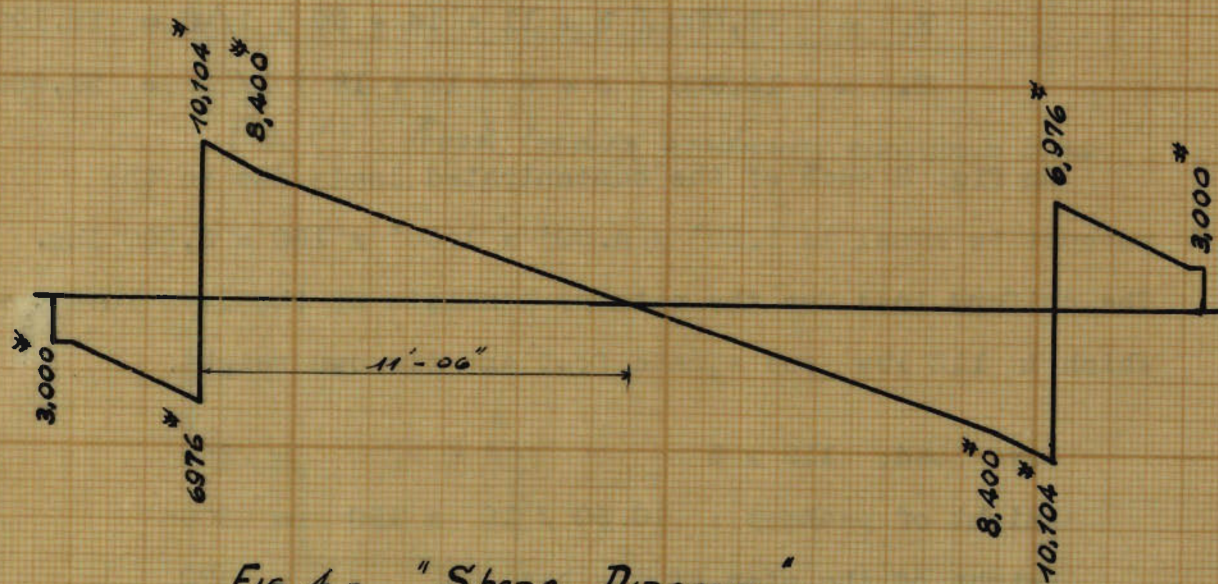

Fig. 4 - "Shear Diagram"

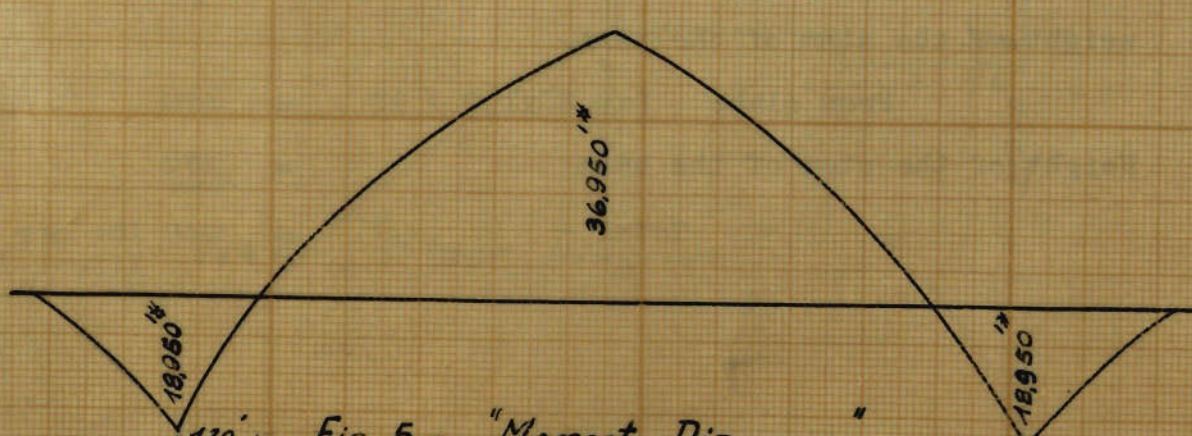

Fig 5 - "Moment Diagram".

$R_L = R_R = (3,000 + 1,136 \times 5 + 840 \times 10) = 17,080$ lbs.

Summation of Shearing Forces: -

Maximum Shear due to truck loading "Fig 2"	= 26,100
Shear due to impact = 26,100 x 30	= 7,900
Shear due to Dead Load "Fig 4"	= 10,104
Total Shear	44,104

$$d = 44104 / \tfrac{7}{8} \times 120 \times 12 = 36.50 \text{ in.}$$

Assuming two rows of steel bars;

Overall depth = 36.50 + 3.50 = 40 in.

The depth of the stem is 40 in. - 7.50 in = 32.50 in. instead of 30 in as it was assumed.

Let us examine the maximum bending moment produced in case that the beam receives the rear wheels of a single truck. For this purpose the wheels will be placed in a such a position so that the center of the span falls midway between the resultant of the truck loads and the load under which the moment is required, that is, wheel (a) in Fig 6.

Clearly, maximum bending moment $= \dfrac{10 \times 24,000 \times 10}{23} = 104,500$ ft. lbs.

When the beam receives the rear wheels of two trucks, the masimum bending moment will occur under wheel (b) "Fig 8". The wheels will be placed in a such a way so that the center of the span will be equidistant from the resultant of the truck loads and the wheel under which the bending moment is required see "Fig 8" .

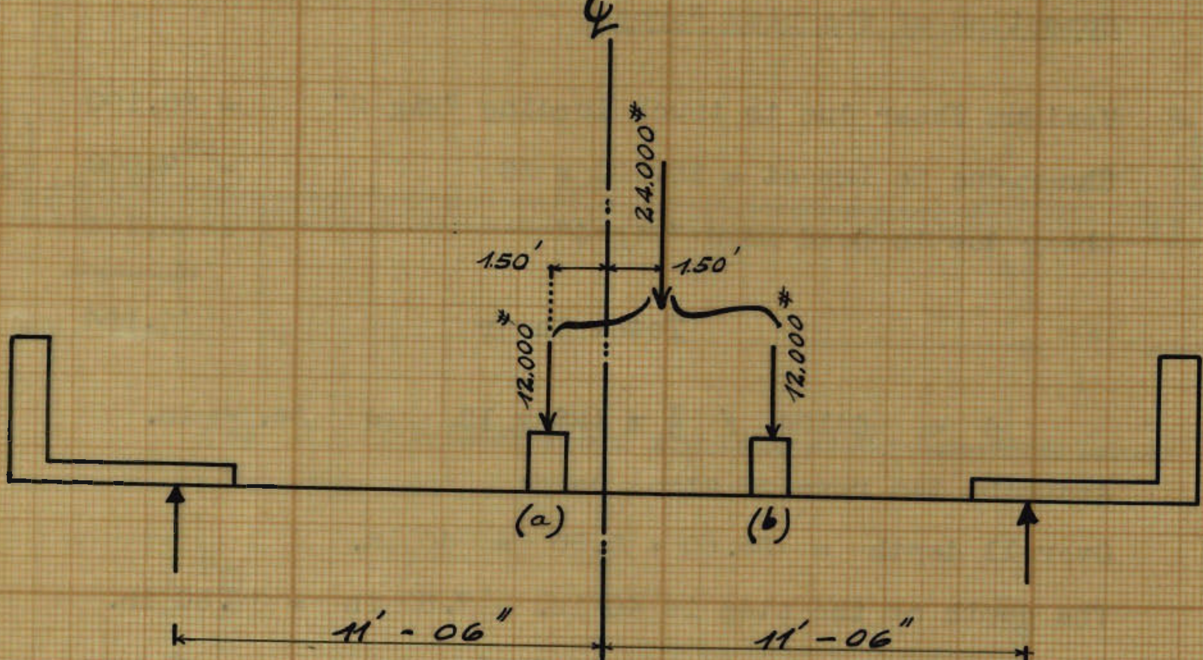

Fig. 6 - Single truck loading producing maximum moment

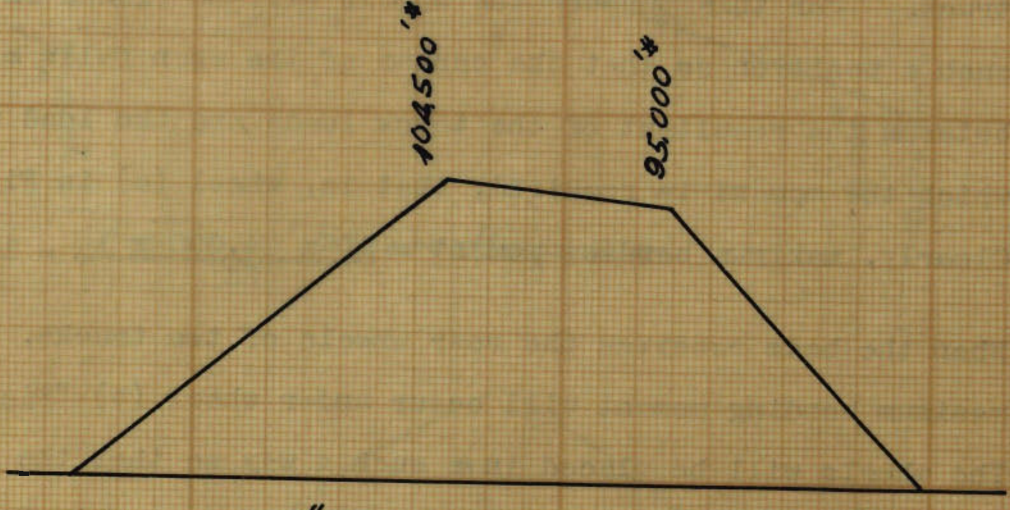

Fig. 7 - "Bending moment Diagram"

the maximum bending moment = $\frac{48,000 \times 10.75 \times 10.75}{28}$ - 12,006 x 6

= 170,000 ft. lbs.

Summation of Bending Moments:

Bending Moment due to dead load "Fig 5"	= 36,950
Bending Moment due to live load "Fig 9"	= 170,000
Bending Moment due to impact = 170,000 x 30%	= 51,000
Total maximum (+) bending moment	= 257,950 ft lbs.

$$A_s = M / f_s \left(d - \frac{t}{2} \right)$$

$$= 257,950 \times 12 / 16,000 \left(36.50 - \frac{7.50}{2} \right)$$

$$= 6.00 \text{ sq. in.}$$

8 round bars of 1 in. diameter will furnish an area of 6.28 sq. in; they will be placed in two rows 2 in. center to center. The lower row will be placed 2.50 in. from the bottom. 2 bars from the upper row will be bent at a distance of 3.00 feet from the support.

From "Fig. 5" maximum negative moment = 18,950 ft lbs. This moment occurs at the support and the beam at this place is considered as an inverted rectangular beam.

$$A_s = M / f_s j d$$

$$= 18,950 \times 12 / 16,000 \times 0.874 \times 36.50$$

$$= 0.45 \text{ sq. in.}$$

The two round bars of 1 in. diam. plus 1 round bar of 1 in. diam. which will be placed at the top extending throughout all the span for holding in place the stirrups will supply

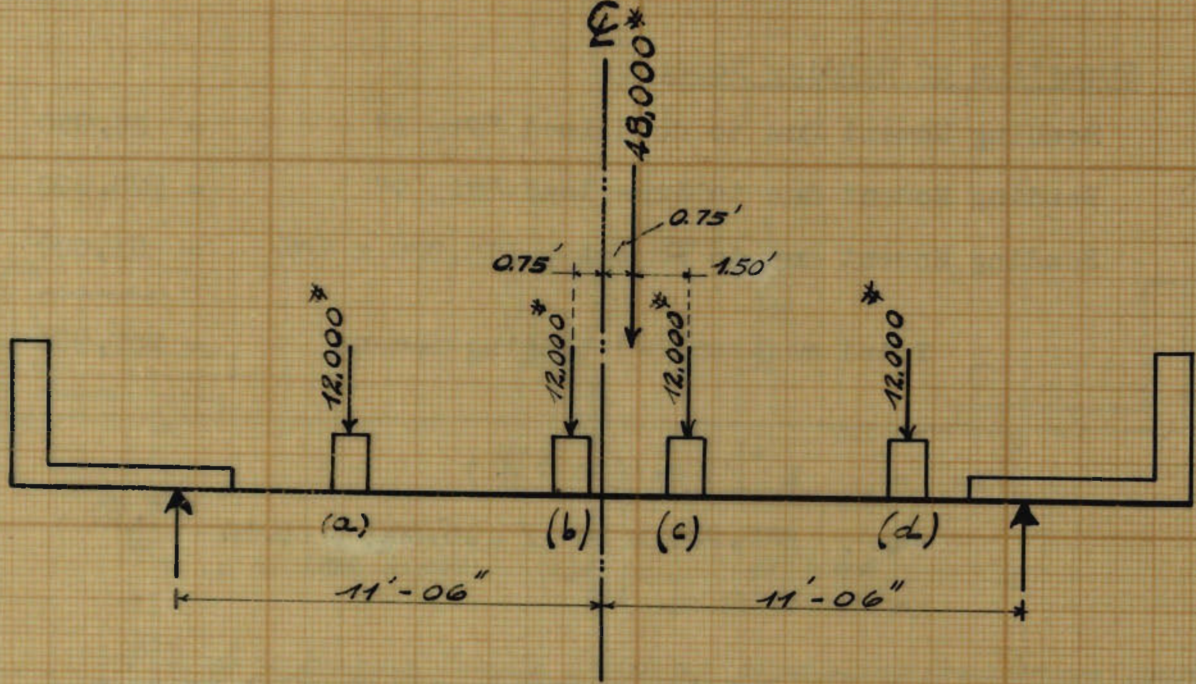

Fig. 8 - "Double truck loading producing maximum bending moment"

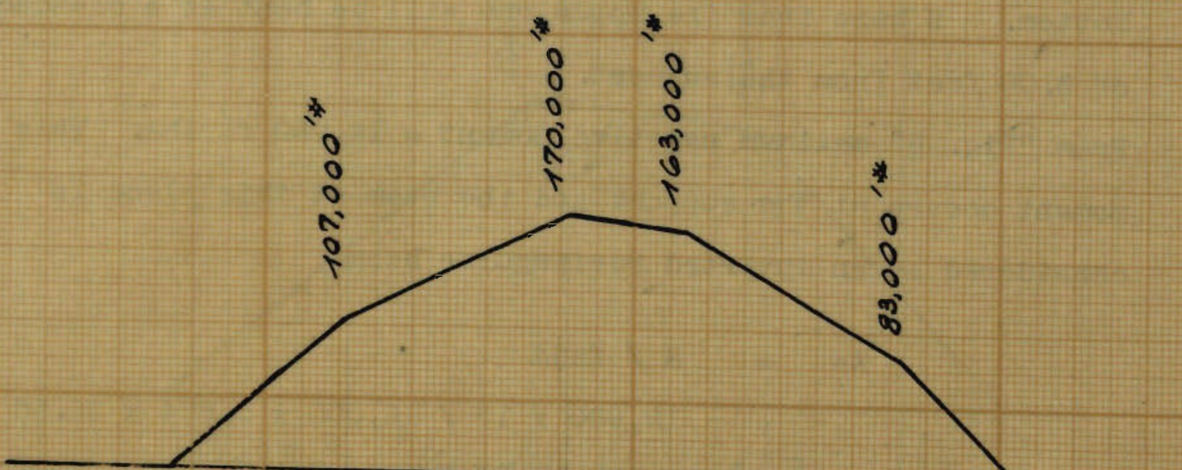

Fig. 9 - "Bending moment Diagram"

TABULATED FORM N° 1

Sections	D.L. Shear	L.L. Shear	Impact	Total Shear "V"	Unit Shear Stress $v = \frac{V}{7/8\ b'd}$	Unit Shear Stress taken by concrete V_c	Unit Shear Stress taken by stirrups $V-V_c$	Spacing of 1/2" stirrups $S = \frac{A_v f_v}{(V-V_c)b'}$	Maximum Allowable spacing
End of span	3,000 lbs "Fig.4"	-	-	3,000 lbs	8 p.s.i.	40 p.s.i.	-	-	$\frac{36.50}{2}$ = 18 in
Left Support	10,104 lbs "Fig.4"	26,100 lbs "Fig.2"	7,900 lbs	44,104 lbs	120 p.s.i.	40 p.s.i.	80 p.s.i.	12 in.	18 in.
5 ft. to the right of left Support	6,000 lbs "Fig.4"	14,100 lbs "Fig.2"	4,250 lbs	24,350 lbs	65 p.s.i.	40 p.s.i;	25 p.s.i.	40 in.	18 in.

A_v = 4 x 0.196 = 0.784 sq. in.

f_v = 16,000 p.s.i.

b' = 12 in.

d = 36,50 in.

The first stirrup will be placed at the end, the spacing thereafter till the support will be 18 in. The spacing from the support till 5 ft. towards the center will be 12 in. and thereafter till the center will be 17.30 in.

an area of 2.35 sq. in. which is much larger than the requiredone.

Investigation for Bond Stress:-

$u = V / \Sigma_o \frac{7}{8} d$

The maximum shear which occurs at the support is 44,104 lbs
At the support we shall be having 6 round bars of 1 in. diameter providing a perimeter of 15.70 in.

therefore $u = 44,104 / 15.70 \times \frac{7}{8} \times 36.50 = 85$ p.s.i.

the allowable bond stress is 90 p.s.i. therefore the section is satisfactory.

Design of Longitudinal Beams:- The span of each beam is 10'-00" center to center of columns. The concentrated load applied to mid-span is equal to the reaction of the transverse beam which is caused by the truck loading and the dead load consisting of the slab weight and the weight of the transverse beam.

Reaction due to truck loading "Fig 2"	= 26,100
Reaction due to dead load"Fig. 4"(6,976+10,104)	= 17,080
Reaction due to impact = 26,100 x 30%	= 7,440
Concentrated load	= 50,620 lbs.

The stem of the beam for archintectural purposes at the supports, that is, the columns, will have a depth of 50.50 in and at the center 32.50 in.

average depth = 32.50 + 50.50 / 2 = 41.50 in.

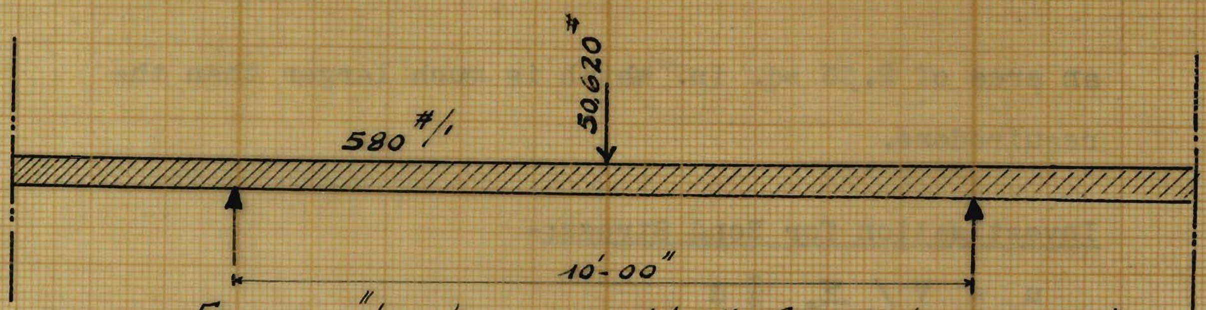

Fig. 10 - "Loading received by the longitudinal beams."

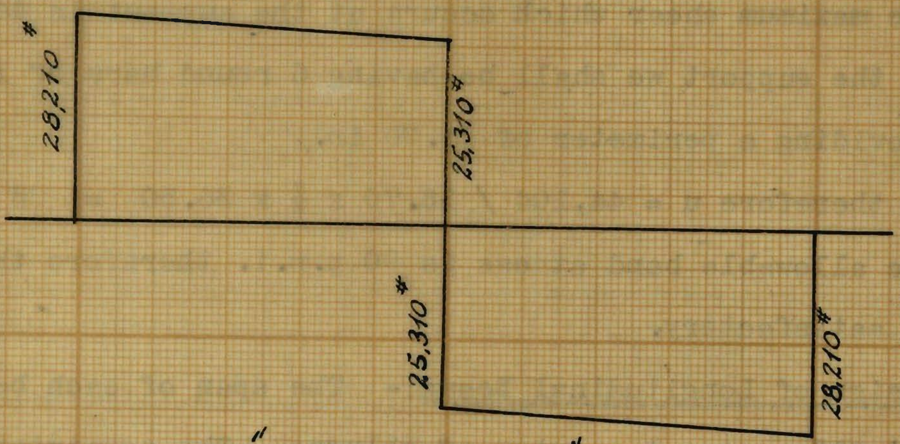

Fig 11 - "Shear Diagram."

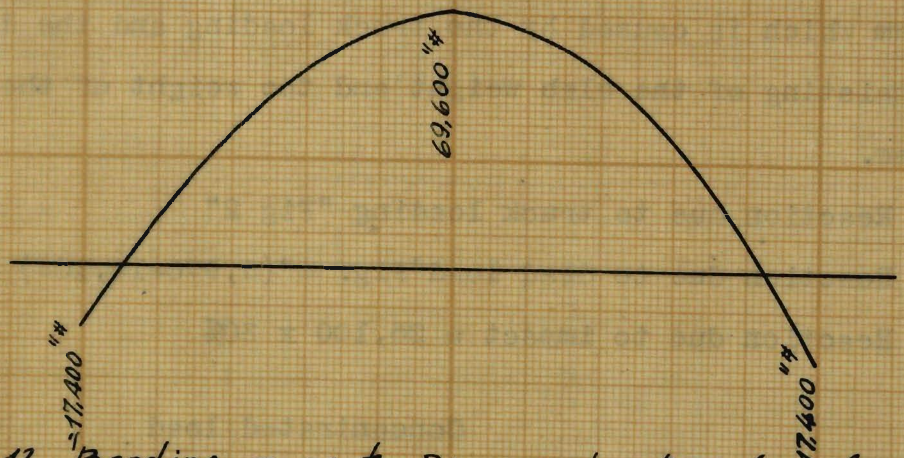

Fig 12. "Bending moment Diagram due to uniform load"

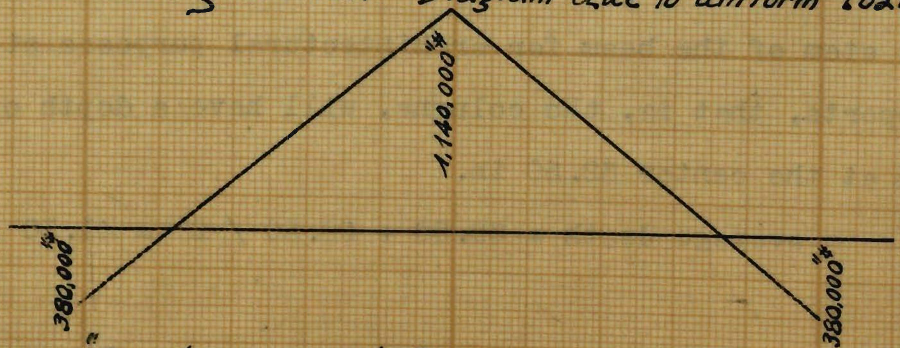

Fig 13 - "Bending moment Diagram due to concentrated load" if the width of the beam is 12 in.

Total weight of the stem of the beam =

41.50/12 x 1.00' x 10.00' x 150 = 5,800 lbs

Maximum Shear = Reation = $\frac{1}{2}$(50,620 + 5,800) = 28,210 lbs.

see Fig 11, for the shear diagram

The beam however can resist a shear of v x $\frac{7}{8}$ x b' x d, that is 120 x $\frac{7}{8}$ x 12 x 36.50 = 46,000lbs. Clearly the section of the beam is safe.

For this beam a condition midway between a fixed and free condition will be taken.

Maximum Positive Bending Moment:-

Moment due to uniform dead load = $wl^2/10 = \frac{580 \times 10 \times 10 \times 12}{10}$

= 69,600 in lbs. "Fig 12"

Moment due to the concentrated load = 3/16 Pl

$= \frac{3 \times 50,620 \times 10 \times 12}{16}$

= 1,140,000 in lbs "Fig 13"

Moment due to impact = 30%(3x26,100x10x12 / 16)

= 174,000 in lbs.

69,600 in lbs "Fig 12"
1,140,000 in lbs "Fig 13"
174,000 in lbs

1,383,600 in.lbs.

Total positive bending moment = 1,383,600 in lbs.

N.B. 26,100 lbs is a part of the concentrated load due to the truck loading.

$$As = M / fs\ (d-\frac{t}{2})$$

$$As = 1,383,600 / 16,000(36.50-\frac{750}{2})$$

$$= 2.65 \text{ sq. in.}$$

6 round bars of $\frac{3}{4}$ in. diameter will supply an area of 2.65 sq. in. They will be placed in two rows at 2 in. center to center. 2 of them will be bent at a distance of 3.00 ft. from the support. These two bars will not be extended beyond the supports, but they will be introduced inside the columns for about 2.50 feet so as to eliminate the effect of a loaded beam on an unloaded one. 1 round bar of 1 in. diam. will be placed at the top throughout the span for holding the stirrups properly.

Maximum Negative Bending Moment:-

Moment due to uniform dead load = $1/40\ wl^2$

$$= \frac{1580x10x10x12}{40} = 17,400 \text{ in lbs}$$

Moment due to concentrated load = 1/16 Pl

$$= \frac{50,620x10x12}{16} = 380,000 \text{ in lbs}$$

Moment due to impact = 30% 1/16 Pl

$$= (\frac{26,100x10x12}{16})30\% = 58,000 \text{ in lbs}$$

Total negative bending moment = 455,400 in lbs.

This negative bending moment occurs at the support, at that place the beam is considered as an inverted rectangular beam.

$$As = M / fs\ j\ d$$

$$As = \frac{455400}{16,000x0.874x54.50} = 0.60 \text{ sq. in.}$$

The supplied area is equal to 1.665 sq. in .

<u>Investigation for Web Reinforcement</u>:-

$$v = V/\tfrac{7}{8}b'd$$

At the center V = 25,310 lbs.

Therefore $v = \frac{25,310}{\frac{7}{8}x12x36.50} = 66$ p.s.i.

Clearly, the shear to be absorbed by the stirrups

= 66-40 = 26 p.s.i.

$$\text{spacing} = \frac{av\ fv}{(v-v_c)b'}$$

Using round bars of ½ in. diameter as stirrups with four legs;

$$s = \frac{4x0.196x16,000}{26x12} = 40 \text{ in.}$$

According to the joint Code specifications maximum allowable spacing = d/2, that is, the spacing should not exceed the 18 in.

The first stirrup will be placed at the support and the spacing thereafter will be 15 in.

Investigation for Bond Stress:-

$$u = \frac{V}{\Sigma_o \frac{7}{8} d}$$

$$u = \frac{28,210}{(4x2.356)\frac{7}{8}x54.50} = 63 \text{ p.s.i.}$$

The allowable bond stress is 90 p.s.i.

Reinforcement details concerning the transverse beams and longitudinal ones are shown in Drawing No. 5.

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Data and Specifications:- The columns supporting the deck are spaced 10'-00" center to center in one direction and 23'-00" center to center in the ohter.

Though the columns among them differ in slenderness, for architectural purposes the same cross-section, (o1'-06")x(o1'-06"), will be adopted for all of them.

The columns will be supplied with longitudinal reifnorcement and lateral ties; the adopted column cross-section will be verified using the Joint Code "1940" Specifications; ultimate strength of concrete (=f'c) = 2,000 p.s.i.; allowable unit tensile stress in longitudinal reinforcement 16,000 p.s.i.

Verification:- The column rearest to the approach of the bridge will be considered; its unsupported length 23'-00" is the longest one.

Actual Load acting at the bottom of the Column:-

Dead Load from deck	=	50,800 lbs.
Live Load "From Fig. 2"	=	26,100 lbs.
Impact: = 26,100 x 30%	=	7,830 lbs.
Column Weight = 1.50x1.50x23x150	=	7,750 lbs.
Total	=	92,480 lbs.

Carrying Capacity of Column:-

For short column, the Joint Code (1940) specifies for the safe axial load.

$$P = 0.18\, f'c A_g + 0.8\, A_s f_s$$

providing 8 round bars of $\frac{3}{4}$ in diameter as longitudinal reinforcement; the supplied area (=As) is 3.53 sq. in. which is a little bit larger than 1 per cent of the gross area of of concrete section.

Thus, P = o.18x2,000x1.50x1.50x144+0.80x3.53x16,000

= 116,500+45,500

P = 162,000 lbs.

The joint Code limits a short column to one whose unsupported length is not greater than 10 times the least lateral dimension.

Clearly, the column under consideration is a long one, since the ratio of its unsupported length to its lateral dimension is larger than 10, that is, 23.00/1.50 = 15.35.

For long columns, the Joint Code gives the working load as $P\ (1.30 - 0.03\ \frac{h}{d})$.

in which P is the total safe load on a column of the same section where the h/d ratio is less than 10, where d is the least dimension of the column and h the unsupported length of the column.

$$\text{therefore, } P' = P(1.30 - 0.03\frac{h}{d})$$

$$= 162{,}000\ (1.30 - 0.03 \times 15.35)$$

$$P' = 136{,}000 \text{ lbs.}$$

Evidently the adopted cross section is satisfactory.

There is no rational method of determining the size of steel used for a lateral tie. A safe rule to follow is to use round steel bar of such a diameter that the area of its section is not less than 2 per cent of the section of longitudinal reinforcement held in place by the tie. Round bars of $\frac{3}{8}$ in. will be used as lateral ties with a spacing of 10 in. This spacing does not exceed 16 bar diameters or 48 tie diameters or the column dimension.

The reinforcement arrangement of columns is shown in Drawing No. 5.

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STANDARD NOTATION

The principal symbols used in the previous discussions concerning the Deck and Column Design are collected here, for the convenience of reference.

a	=	Side of column parallel to principal beam.
Ag	=	Overall or gross area of concrete section.
As	=	Effective cross-sectional area of metal reinforcement in tension., in beams, and of longitudinal bars in columns.
Av	=	Total area of web reinforcement in tension.
b	=	Width of a rectangular beam or width of flange of T- beam.
b'	=	Width of stem of T-beam.
d	=	Depth from compression surface of beam or slab to center of longitudinal tension reinforcement.
Ec	=	Modulus of elasticity of concrete in compression = 2,000,000 p.s.i.
Es	=	Modulus of elasticity of steel in tension = 30,000,000 p.s.i.
f_s	=	tensile unit working stress in longitudinal reinforcement in beams and compressive unit working stress in longitudinal bars in columns.
f_v	=	Tensile unit stress in web reinforcement.
h	=	unsupported length of column.
j	=	Ration of lever arm of resisting couple to depth d.
jd	=	d-z = arm of resisting couple.

k = ratio of depth of neutral axis to depth d.

K = $\frac{1}{2}f_c kj$ or $pf_s j$ in rectangular beams.

l = span lenth of beam or slab.

M = Bending moment or moment of resistance.

n = Es/Ec = ration of modulus of elasticity of steel to that of concrete.

Σ_o = Sum of perimeters of bars in one set.

p = Ratio of effective area of tension reinforcement to effective of concrete in beams = As/bd

p_g = Ratio of total effective reinforcement in member subject to compression to gross concrete section.

P' = Total safe axial load on long column.

P = Total axial load on column.

s = Spacing of web members, measured at the plane of the lower reinforcement and in the direction of the longitudinal axis of the beam.

t = thickness of flange of T- beam.

u = bond stress per unit of area of surface of bar = 90 p.s.i.

v = Shearing unit stress.

V = Total shear.

v_c = shearing unit stress absorbed by concrete = 0.02f'c = 40 p.s.i.

w = Uniformly distributed load per unit lf length of beam or slab.

z = Depth from compression surface of beam or slab to resultant of compressive stresses.

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A R C H R I B S

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NOTATION:-

I_c = moment of inertia of section at the crown;

I_x = moment of inertia of section at an intermediate point x;

I_s = moment of inertia of section at the springing;

ϕ_s = angle between the section and the vertical at the springing.

ϕ = angle of inclination of intermediate section;

n = $\frac{I_c}{I_c \cos\phi_s}$

r = rise of arch;

l = span of arch;

d_c = depth of section at crown;

d_x = depth of section at intermediate point;

g_c = unit dead load at crown, lbs per lin. ft. of arch;

g_s = unit dead load at springing, lbs per lin. ft. of arch;

m = $\frac{g_s}{g_c}$

X and Y = cordinates referred to left support

X_s and Y_s = location of elastic center with reference to left support;

d_s = length of a division of the arch;

A_x = area of average section in each division of the arch;

x, y = coordinates of the center of the division of the arch referred to axes through elastic center;

M_s = static bending moment of loads considering arch as cantilevered at right support.

H = horizontal thrust at both supports for vertical loads;

The sign of horizontal forces acting inwards, such as horizontal thrust due to downward loads; is accepted as minus. The sign of horizontal forces acting outward, such as thrust due to rib shortening and fall of temperature is taken as plus.

M_A = bending moment at left support;

V_A = vertical reaction at left support;

M = auxiliary bending moment;

M_x = bending moment at any section of the arch.

Characteristics of Arch Action:- The main characteristic of arches is the presence of a horizontal thrust at the support, because the supports prevent the straightening of the arch under the action of vertical loads. This horizontal thrust produces compressive stresses throughout all the arch sections.

The second characteristic of the arch action is that the horizontal thrust produces negative bending moments which counteract the positive ones caused by the loads. Thus the static bending moment due to the load is considerably reduced by the bending moment due to the horizontal thrust. These characteristics are common to fixed and hinged arches. The difference between hinged and fixed arches is that in hinged arches at the hinges there is no bending moment and the thrust consequently is applied at the center of the hinge. In fixed arches all sections are subjected to bending moment whereas the point of application of the thrust at the springing is different for different positions of loading.

Linear Variations of a Fixed Arch:- The following effects, producing deformation in all arches having less than three hinges, are important and require to be given consideration in design.

(1) Effect of Temperature Changes:- When an arch is subjected to a change in temperature it undergoes an alteration in length. The abutments being immovable, the span between them remains unchange. It therefore results that the arch exerts either a thrust or a pull upon the abutments accordingly as the length of the arch is increased or decreased. Actually, of course, the arch will never exert a pull on the abutments, since the thrust due to the vertical loading will always be the greater force, causing an opposite thrust to be exerted upon the arch by the abutments. The pull will reduce the thrust and the resultant moment induced in the arch needs to be added algebraically to those produced by the supeimposed loading.

(2) Effect of Shrinkage:- Contraction of the arch results due to its shrinkage during setting and hardening, it is necessary to include this arch shortening in the calculations for the arch. This may be done by adding this latter shortening to that produced by the maximum fall in atmospheric temperature.

(3) Effect produced by the Settlement of Abutment when the Horizontal Thrust from the Arch is brought upon them:- This settlement, although not causing the arch to shorten, produces moments and stresses in it of the same sense. However, there is not sufficient reliable data for computing the induced stresses and moments.

(4) Effect of Rib Shortening:- The thrust acting on the arch compresses the arch ring. If free to move, the compressed arch would assume the shape of an arch with shorter span. Since the arch is not free to move, the span of the compressed arch remains the same as before compression and the shortened arch rib must adapt itself to the larger span by spreading. The crown is lowered and the arch bends. The maximum negative bending moment acts at the springing, where it produces tension at the top and maximum positive bending moment is at the crown where it produces tension at the bottom.

Clearly, the effect of rib shortening is similar to that of temperature fall.

For live load the effect of rib shortening is small and it can be neglected without any appreciable error. For dead load the effect of rib shortening is appreciable and it must be computed.

Though the arch axis will be made to coincide with the line of pressure for the dead load so that there will be no bending moment due to thrust, since the thrust will be acting centrally, however, this does not take into account the effect of the rib shortening due to the thrust which needs to be computed separately.

Data and Specifications for the ribs of the present Bridges:-

The arch will consist of two separate parallel fixed ribs.

theoretical span, l = 150'-00"

theoretical rise, r = 31'-09"

Uniform Live Load including impact = 70 lbs. per square foot.

Concentrated Live Load for moment = 13,500 lbs. per linear foor of lane

Changes of Temperature, = $\pm 25^{\circ}F$.

Shrinkage equivalent to a fall of temperature of $15^{\circ}F$.

Reinforcement: 1 percent of concrete area

Stresses in concrete:-

Direct stress and bending, small eccentricity,

f_c = 530 p.s.i.

" " " large eccentricity,

f_c = 630 p.s.i.

Stresses in ~~concrete~~ steel:- 16,000 p.s.i.

E_c = 2,000,000 p.s.i. a = 0.0000055 hence Ea = 11

Methods of Designing Fixed Arch Bridges:-

In designing arch bridges the following problems must be solved:

First, the curvature of the arch axis must be determined.

Second, preliminary dimensions, at crown and springing must be determined and the arch rib will be designed by the approximate method.

Third, complete analysis of the arch rib should be made on the basis of the Elastic Theory.

Fourth, after the bending moments and thrusts are determined maximum stresses at various sections should be computed.

Critical Cross-sections:- The depths of the cross-sections are measured upon lines drawn at right angles to the tangent of the arch axis. The dimensions of the cross-sections are not constant throughout the length of the arch but are smallest at the crown and increase gradually toward the springing.

A depth of 03'-00" is adapted at the crown. For average conditions n should be 0.30

$$\text{that is, } \frac{I_c}{I_s \cos\phi_s} = \frac{d_c^3}{d_s^3 \cos\phi_s} = 0.30$$

$d_c = 3^3 = 27$

$\cos\phi_s = 0.74$ (From Drawing No.2)

therefore, $d_s = \left(\frac{27}{0.74 \times 0.30}\right)^{1/3} = 5.00$ feet.

The depths of the intermediate cross-sections being computed from formula $\frac{(1+\tan\phi_x)^{1/6}}{\left[1-(1-n)4\frac{x^2}{l^2}\right]^{1/3}} d_c$ are tabulated in the

Tabulated Form No.2 The values of $\tan\phi_x$ are determined from the Arch layout, Drawing No.2 .

When the variation of the magniture of the cross-sections at the intermediate points is made according to the above formula, only three critical sections need to be examined. If the stresses at these are satisfactory, the other intermediate sections are also safe. The critical cross-sections are at: (1) The Springing: (2) The Quarter Point and (3) The Crown.

<u>Critical Positions of Live Load</u>:- The maximum bending moments at the critical cross-sections are produced not by a live load extending over the whole length of the arch span but by different positions of partial loadings.

TABULATED FORM N° 2

Ordinate Letter	A,A'	B,B'	C,C'	D,D'	E,E'	F,F'	G,G'	H,H'	I,I'	S,S'
Hgt of Arch ord=K r	31.42'	30.54'	29.08'	26.95'	24.30'	29.92'	16.84'	11.98'	6.43'	00.00
$\tan\phi$	0.078	0.156	0.236	0.319	0.405	0.495	0.590	0.690	0.796	0.913
$\tan^2\phi$	0.0061	0.0243	0.0557	0.102	0.164	0.245	0.348	0.476	0.634	0.835
$(1+\tan^2\phi)^{1/6}$	1.001	1.004	1.009	1.016	1.025	1.037	1.051	1.067	1.085	1.106
$\left[1-(1-n)4\frac{x^2}{l^2}\right]^{1/3}$	0.9976	0.9906	0.9785	0.9612	0.9379	0.9077	0.8693	0.8203	0.7577	0.6694
$d=\frac{(1+\tan^2\phi)^{1/6}}{\left[1-(1-n)4\frac{x^2}{l^2}\right]^{1/3}}d_c$	3.02'	3.03'	3.09'	3.17'	3.28'	3.43'	3.63'	3.90'	4.31'	5.00'

r (= rise) = 31.75 feet; $n = \frac{I_c}{I_s \cos\phi_s} = 0.30$

The positions of the live load producing maximum bending moments at the critical sections are given on Fig. 14.

Dead Load supported by the Arch Rib:- The dead load consists of the weight of paving, the weight of floor construction supporting the roadway, the weight of the vertical supporting members, that is, the columns and the weight of the arch ribs. The weight of the roadway and floor construction is constant throughout the whole length of the arch, and the variable items are the weights of the vertical supports (columns) which increase towards the springing because of the increasing height and the weight of the arch rib. This dead load is concentrated at the points of application of the columns. These concentrated loads will be used when determining the line of pressure. One-half of the dead load will come upon each rib.

Determination of Arch Axis:- The arch axis if properly designed, is a continuous consistent curve, the raduis of curvature being maximum at crown and diminishing towards the springings.

The most economical shape of the arch axis is that which coincides with the line of pressure for dead load.

A very near approximation to this ideal curve is obtained by adapting the curve whose equation is:-

$$Y = \frac{r}{m-1} \left(\cosh \frac{2pX}{l} - 1\right)$$

$$p = \log_e \left(m - (m^2 - 1)^{1/2}\right)$$

$$m = \frac{q_s}{q_c}$$

This equation is derived on the assumption that the variation in loading between crown and springing is proportional to Y. From Fairhurst Table No.1 is reproduced, this table gives the ordinate values for setting out the arch curve.

To make possible the use of this table it is necessary to determine the value of m; thus the concentrated dead load at the crown and springing is replaced by a distributed load of an intensity equal to the concentrated loads divided by the spacing of concentrated loads.

The width of each rib is taken as 7.00 feet.

"q_c" Dead Load at crown per unit length of arch span:-

(1) parapet: 4.00 x 1.00 x 1.50 =	600 lbs.
(2) footway pavement: 6.00 x $\frac{16.50}{12}$ x 150 =	1,240 lbs.
(3) roadway slab: 10.00 x $\frac{7.50}{12}$ x 150 =	940 lbs.
(4) transverse beam: $\frac{1}{5.00}$(16.00x$\frac{32.50}{12}$x1.00x150) =	1,300 lbs.
(5) wearing surface : 16.00 x 30	480 lbs.
(6) lognitudinal beam: $\frac{1}{10.00}$(10.00x1.00x$\frac{41.50}{12}$x150) =	520 lbs.
(7) column weight: $\frac{1}{10.00}$(1.50x1.50x2.00x150) =	70 lbs.
(8) arch rib : $\frac{36.00}{12}$ x 7.00 x 150 =	3,150 lbs.
q_c =	8,320 lbs.

"q_s" Dead Load at springing per unit length of arch span:-

(1) parapet :	=	600 lbs.
(2) footway pavement :	=	1,240 lbs.
(3) roadway slab :	=	940 lbs.
(4) wearing surface:	=	480 lbs.
	Carried forward	3,260 lbs.

brought forward : 3,260 lbs.

(5) transverse beam : = 1,300 lbs.

(6) longitudinal beam : = 520 lbs.

(7) column weight: $\frac{1}{10.00}(27.00 \times 1.50 \times 1.50 \times 150) =$ 910 lbs.

(8) Arch rib : 5.00 x 7.00 x 150 = 5,250 lbs.

$g_s =$ 11,240 lbs

$$m = \frac{g_s}{g_c} = \frac{11,240}{8,300} = 1.50 \text{ appr.}$$

The ordinate values as tabulated in the Tabulated Form No.2 have been obtained by extrapolation from Table No.1

<u>Graphical Determination of Line of Pressure and Horizontal Thrust due to Dead Load</u>:- The points of application of load coincides with the location of columns. These concentrated loads will be computed first and then the line of pressure will be determined.

The weight due to the floor construction will be the same for all the columns:

<u>Weight due to floor construction</u>:-

(1) parapet: 4.00 x 1.00 x 10.00 x 150 = 6,000 lbs.

(2) footway pavement : $6.00 \times \frac{16.50}{12} \times 10.00 \times 150$ = 12,400 lbs.

(3) roadway slab : $10.00 \times \frac{7.50}{12} \times 10.00 \times 150$ = 9,400 lbs.

(4) wearing surface : 16.00 x 10.00 x 30 = 4,800 lbs.

(5) transverse beam : $2 \times 16.00 \times \frac{32.50}{12} \times 1.00 \times 150$ = 13,000 lbs.

(6) Longitudinal beam: $10.00 \times 1.00 \times \frac{41.50}{12} \times 150$ = 5,200 lbs.

Total = 50,800 lbs.

<u>Computation of Concentrated Loads</u>:-

Starting from the springing;

From floor construction:	=	50,800 lbs.
From arch rib : 12.50 x 7.00 x 4.25 x 150	=	56,000 lbs.
Column weight: 23.00 x 1.50 x 1.50 x 150	=	7,750 lbs.
First concentrated load	=	114,550 lbs.
From floor construction:	=	50,800 lbs.
From arch rib: 11.75 x 7.00 x 3.75 x 150	=	46,300 lbs.
Column weight: 17.00 x 1.50 x 1.50 x 150	=	5,750 lbs.
Second concentrated load	=	102,850 lbs.
From floor construction:	=	50,800 lbs.
From arch rib: 11.25 x 7.00 x 3.45 x 150	=	40,750 lbs.
Column weight: 11.50 x 1.50 x 1.50 x 150	=	3,900 lbs.
Third concentrated load	=	95,450 lbs.
From floor construction:	=	50,800 lbs.
From arch rib: 10.75 x 7.00 x 3.20 x 150	=	36,200 lbs.
Column weight: 7.50 x 1.50 x 1.50 x 150	=	2,530 lbs.
Fourth concentrated load	=	89,530 lbs.
From floor construction:	=	50,800 lbs.
From arch rib: 10.375 x 7.00 x 3.10 x 150	=	33,700 lbs.
Column weight: 4.00 x 1.50 x 1.50 x 150	=	1,350 lbs.
Fifth concentrated load	=	85,850 lbs.
From floor construction:	=	50,800 lbs.
From arch rib: 10.125 x 7.00 x 30.75 x 150	=	32,700 lbs.
Column weight: 2.00 x 1.50 x 1.50 x 150	=	675 lbs.
Sixth concentrated load	=	84,175 lbs.

From floor construction:	=	50,800 lbs.
From arch rib: 10.00 x 7.00 x 3.05 x 150	=	32,100 lbs.
Column weight: 1.00 x 1.50 x 1.50 x 150	=	337 lbs.
Seventh concentrated load	=	83,237 lbs.

All these concentrated loads are shown in Drawing No.2.

A force polygon for one-half of the arch is drawn.

A convenient pole distance will be selected and a funicular polygon will be drawn.

When the arch axis for the left side is drawn, the pole distance is placed to the right of the force polygon. The resulting funicular polygon is concave as shown in Drawing No.2. The end rays of the funicular polygon are extended till intersection. A vertical line is drawn through this point of intersection, this line indicates the position of the resultant of forces on the left half of the arch. A horizontal line is drawn through the crown. This line is the outside line of the line of pressure at the crown. This line is extended to intersection with the resultant. This new point of intersection is connected with the springing. The line thus obtained is the end line of the line of pressure at the springing.

Parallel lines to these two lines are drawn at the ends of the force polygon, the horizontal line at the bottom and the inclined at the top. The point of intersection of these two lines gives the pole for the line of pressure.

The horizontal thrust due to dead load is equal to the horizontal distance of the new pole from the force polygon, measured to the same scale as used in drawing the force polygon; its magnitude is 780,000 lbs.

With the new pole a new funicular polygon is drawn which is the desired line of pressure. The outside rays of this pass through the springing and crown, respectively.

As it is shown in Drawing No.2; the arch axis coincides with the line of pressure.

Design of Arch Rib by the Approximate Method:- The approximate method given below is based upon the same elastic theory as used in the exact method. This method has been evolved partly by Dr. Ing. Faber and partly by Strässner. Subsequently this method with modifications was embodied in a paper presented by Mr. Charles S. Whitney before the American Society of Civil Engineers.

Section at Springing,

f_c = 630 p.s.i.

cos = 0.740

Effect of Dead Load:- The arch axis is assumed to coincide with the line of pressure; therefore, the dead load does not produce any bending moment in the arch.

$$H_d = C_d \, g_c \, \frac{l^2}{r}$$

The constant C_d corresponding to m = 1.50 is found from diagram No.3, and it is 0.135

$$\text{therefore, } H_d = 0.135 \times 8320 \times \frac{150 \times 150}{31.75}$$

$$H_d = - \ 798{,}000 \text{ lbs.}$$

Uniform Live Load at Springing:- The uniform live load including impact is equal to 70 lbs. per square foot; therefore, per linear foot of arch rib live load plus impact is 70 x 16.00 = 1,120 lbs. The constants corresponding to m = 1.50 and u = 0.30, are found from Diagrams 8 and 9.

Maximum Positive Bending Moment:

$M_s = C_s wl^2$

$= 0.0235 \times 1{,}120 \times 150 \times 150$

$M_s = +\ 593{,}000$ ft.lbs.

Corresponding Thrust:

$H_s = C_{(hs)} wl \frac{l}{r}$

$= 0.0905 \times 1{,}120 \times \frac{150 \times 150}{31.75}$

$H_s = -\ 72{,}000$ lbs.

Maximum Negative Bending Moment:

$M_s = -\ C_{(-s)} wl^2$

$= -\ 0.0202 \times 1{,}120 \times 150 \times 150$

$M_s = -\ 510{,}000$ ft. lbs.

Corresponding Thrust:

$M_s = C_{(-hs)}\ wl\ \frac{l}{r}$

$= 0.0364 \times 1{,}120 \times \frac{150 \times 150}{31.75}$

$H_s = -\ 28{,}900$ lbs.

Knife Edge Load at Springing:- The Knife Edge Load for moment is 13,500 lbs per linear foot of lane. Since the lane has got a width is 9.00 feet, for every rib the knife edge load will be equivalent to $\frac{13{,}500 \times 10}{9} = 15{,}000$ lbs. Including the impact, the knife edge load becomes 15,000 x 1.30 = 19,500 lbs.

The constants corresponding to m = 1.50 and n = 0.30 are found from Tables 2, 3, 4 and 5 by extrapolation

Maximum Positive Bending Moment:

$M = \frac{Pl}{100} \times K$

$= \frac{19{,}500 \times 150 \times 6.95}{100}$

$M = +\ 204{,}000$ ft. lbs.

Corresponding Thrust:

$H = \frac{P}{10} \times \frac{l}{r} \times C$

$= \frac{19{,}500 \times 150 \times 2.30}{10 \times 31.75}$

$H = -\ 21{,}200$ lbs.

Maximum Negative Bending Moment: Corresponding Thrust

$M = \frac{Pl}{100} K$

$= \frac{19,500 \times 150 \times 8.18}{100}$

$M = -239,000$ ft. lbs.

$H = \frac{P}{10} \times \frac{l}{r} \times C$

$= \frac{19,500 \times 150 \times 0.5245}{10 \times 31.75}$

$H = -4,720$ lbs.

Location of Elastic Center:- Let Y_e be the vertical distance of elastic center from the crown:

$$Y_e = C_c r$$

The constant C_e corresponding to m = 1.50 and n = 0.30 is obtained from Diagram 1.

$$Y_e = 0.237 \times 31.75 = 7.525$$

Let X_s and Y_s be the location of elastic center with reference to the left support:

$$X_s = 75.00 \text{ ft. and } Y_s = 31.75 - 7.525 = 24.225 \text{ ft.}$$

Effect of Rib Shortening:- The thrust produced by the rib shortening acts through the horizontal axis passing through the elastic center and is equal to $\frac{I_c}{A_{av.} r^2} \times \frac{1}{C_h} \times H_d$

The constant C_h corresponding to m = 1.50 and n = 0.30 is obtained from Diagram 2.

Assuming the average depth of the arch rib to be 3.50 ft.

$$A_{av.} = 3.50 \times 7.00 = 24.50 \text{ sq. ft.}$$

This thrust produces a negative bending moment at the springing.

Negative Bending Moment: Corresponding Thrust

$M_s = 11,850 \times 24.225$

$M_s = -287,000$ ft. lbs.

$H = \frac{I_c \times H_d}{A_{av.} r^2 C_h}$

$= \frac{15.75 \times 798,000}{24.50 \times 31.75 \times 31.75 \times 0.043}$

$H = +11,850$ lbs.

Effect of Temperature Rise:- The thrust produced by the temperature rise acts through the horizontal axis passing through the elastic center. This resulting horizontal thrust will produce a positive bending moment at the springing.

The horizontal thrust is given by the following formula

$$H = \frac{aEI_c t^\circ}{r^2 C_h}$$

Positive Bending Moment:

$M_s = 14,440 \times 24.225$

$M_s = + 349,000$ ft. lbs.

Corresponding Thrust:

$$H = \frac{aEI_c t^\circ}{r^2 C_h}$$

$$= \frac{11 \times 144 \times 15.75 \times 25}{31.75 \times 31.75 \times 0,043}$$

$H = - 14,400$ lbs.

Effect of Temperature Fall and Shrinkage:- The thrust produced by the temperature fall and shrinkage acts through the horizontal axis passing through the elastic center. This resulting horizontal thrust will induce a negative bending moment at the springing. The horizontal thrust is given by the following formula.

$$H = \frac{aEI_c t^\circ}{r^2 C_h}$$

For fall of temperature plus shrinkage; $t^\circ = 40^\circ F$.

Negative Bending Moment:

$M_s = 23,000 \times 24,225$

$M_s = -560,000$ ft. lbs.

Corresponding Thrust:

$$H = \frac{aEI_c}{r^2 C_h}$$

$$= \frac{11 \times 144 \times 15.75 \times 40}{31.75 \times 31.75 \times 0.043}$$

$H = +23,000$ lbs.

TABULATED FORM N° 3

Summary of Bending Moments and Thrusts at the Springing

Type of Loading	Positive Bending Moments		Negative Bending Moments	
	Horizontal Thrusts	Bending Moments	Horizontal Thrusts	Bending Moments
Dead Load	-798,000 lbs	nill.	-798,000 lbs	nill.
Uniform Live Load	-72,000 lbs	593,000 ft.lbs	-28,900 lbs	510,000 ft.lbs
Knife Edge Load	-21,200 lbs	204,000 ft. lbs	-4,720 lbs	239,000 ft.lbs
Rib Shortening	-	-	+11,850 lbs	287,000 ft.lbs
Temperature Rise	-14,400 lbs	349,000 ft.lbs	-	-
Temperature Fall and Shrinkage	-	-	+23,000 lbs	560,000 ft.lbs
Total	-905,600 lbs	1,146,000 ft.lbs	-796,770 lbs	1,596,000 ft.lbs

Investigation for Stresses at Springing:- From the Tabulated Form No. 3, the maximum bending moment and the corresponding thrust will be selected.

maximum bending moment = 1,596,000 ft. lbs.

corresponding horizontal thrust = - 796,77o lbs.

$$e \text{ (= eccentricity)} = \frac{1,596,000}{796,776} = 2.00 \text{ feet.}$$

$$f_c = \frac{NK}{ba}$$

Assuming 1.50 in. net thickness of concrete for protection of reinforcement against rusting and fire and p = 1% :

$$\text{then } \frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{5.00 \times 12} = 0.04$$

$$\frac{e}{a} = \frac{2.00}{5.00} = 0.40$$

$$np = 15 \times 0.01 = 0.15$$

"From Design of Concrete Structures by L.C. Urquhart and C. E. O'rourke, Diagram 16 appendix D page 552, for the above data K = 2.90 and k = 0.63"

$$N \text{ (= normal thrust)} = \frac{\text{horizontal thrust}}{\cos\phi_s}$$

$$N = \frac{796,770}{0.746} = 1,0575,000 \text{ lbs.}$$

$$\text{therefore, } f_c = \frac{1,075,000 \times 2.90}{7.00 \times 5.00 \times 144} = 620 \text{ p.s.i.}$$

$$f_s = nf_c \left(\frac{d}{ka} - 1\right)$$

$$d = 5.00 - 1/12\ (1.50 + 3/8 + 0.50) = \mathbf{4.80} \text{ ft.}$$

$$\text{therefore } f_s = 15 \times 620 \left(\frac{4.80}{0.63 \times 5.00} - 1\right) = 500 \text{ p.s.i.}$$

Clearly, the unit stresses are within the limits

Section at Quarter Point

f_c = 530 p.s.i.

$\cos\phi_{1/4}$ = 0.926

Effect of Dead Load:- The horizontal thrust is constant throughout all the arch sections; it has been found to be - 798,000 lbs.

Uniform Live Load at Quarter Point:- The uniform live load including impact is 1,120 lbs. per linear foot of arch rib.

The constants corresponding to m = 1.50 and n = 0.30, are found from Diagrams 6 and 7.

Maximum Positive Bending Moment:

$M_{1/4} = C_{1/4}\, wl^2$

$= 0.0074 \times 1{,}120 \times 150 \times 150$

$M_{1/4}$ = 186,500 ft. lbs.

Corresponding Thrust:

$H_{1/4} = C_{(h\frac{1}{4})}\, wl\, \frac{l}{r}$

$= 0.0344 \times 1{,}120\, \frac{150 \times 150}{31.75}$

$H_{1/4}$ = - 27,350 lbs.

Maximum Negative Bending Moment:

$M_{1/4} = C_{(-1/4)}\, wl^2$

$= 0.0081 \times 1{,}120 \times 150 \times 150$

$M_{1/4}$ = - 205,000 ft. lbs.

Corresponding Thrust:

$H_{1/4} = C_{(-h\,1/4)}\, wl\, \frac{l}{r}$

$= 0.0925 \times 1{,}120 \times \frac{150 \times 150}{31.75}$

$H_{1/4}$ = - 73,500 lbs.

Knife Edge Load at Quarter Point: The knife edge load including impact becomes 19,500 lbs per arch rib.

The constants corresponding to m = 1.50 and n = 0.30 are obtained from Tables 2, 3, 6 and 7 by extrapolation

Maximum Positive Bending Moment:

$$M = \frac{Pl}{100} \times K$$

$$= \frac{19{,}500 \times 150 \times 5.223}{100}$$

$$M = + 153{,}000 \text{ ft. lbs.}$$

Corresponding Thrust:

$$H = \frac{P}{10} \times \frac{l}{r} \times C$$

$$= \frac{19{,}500 \times 150 \times 1.28}{10 \times 31.75}$$

$$H = - 11{,}820 \text{ lbs.}$$

Maximum Negative Bending Moment:

$$M = \frac{Pl}{100} \times K$$

$$= \frac{19{,}500 \times 150 \times 2.343}{100}$$

$$M = - 68{,}500 \text{ ft. lbs.}$$

Corresponding Thrust:

$$H = \frac{P}{10} \times \frac{l}{r} \times C$$

$$= \frac{19{,}500 \times 150 \times 2.48}{10 \times 31.75}$$

$$H = - 23{,}000 \text{ lbs.}$$

Effect of Rib Shortening: The resulting horizontal thrust from rib shortening is + 11,850 lbs. This thrust induces a positive bending moment at the quarter point.

The moment arm is 24.30 - 24.225 = 0.075 ft.

therefore; M = 11,850 x 0.075 = + 890 ft. lbs.

Effect of Temperature Rise:- The resulting horizontal thrust from temperature rise is - 14,400 lbs.

This thrust induces a negative bending moment at the quarter point. The moment arm = 0.075 ft. therefore, M = 14,400 x 0.075 = -1,080 ft. lbs.

Effect of Temperature Fall and Shrinkage:- The thrust produced by the temperature fall and shrinkage is + 23,000 lbs. This thrust induces a positive bending moment at the quarter point. The moment arm is 0.075 ft.

therefore, M = 23,000 x 0.075 = + 1,725 ft. lbs.

Investigation for Stresses at Quarter Point:- From the Tabulated Form No. 4, the maximum bending moment and the corresponding thrust will be selected.

TABULATED FORM Nº 4

Summary af Bending Moments and Thrusts at the Quarter Point

Type of Loading	Positive Bending Moments		Negative Bending Moments	
	Horizontal Thrusts	Bending Moments	Horizontal Thrusts	Bending Moments
Dead Load	-798,000 lbs	nill.	-798,000 lbs	nill.
Uniform Live Load	-27,350 lbs	186,000 ft.lbs	-73,500 lbs	205,000 ft.lbs
Knife Edge Load	-11,820 lbs	153,000 ft.lbs	-23,000 lbs	68,500 ft.lbs
Rib Shortening	+II,850 lbs	890 ft.lbs	-	-
Temperature Rise	-	-	-I4,4oo lbs	I,080 ft.lbs
Temperature Fall and Shrinkage	+23,000 lbs	I,725 ft.lbs	-	-
Total	-802,320 lbs	34I,6I5 ft.lbs	-908,900 lbs	274,580 ft.lbs

maximum bending moment = 341,615 ft. lbs.

corresponding thrust = - 802,320 lbs.

$$e \ (= \text{eccentricity}) = \frac{341,615}{802,320} = 0.425 \text{ ft.}$$

Assuming 1.50 in. net thickness of concrete for protection of reinforcement against rusting and fire and p = 1%:

$$\text{then, } \frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{3.28 \times 12} = 0.064$$

$$\frac{e}{a} = \frac{0.425}{3.28} = 0.13$$

$$np = 15 \times 0.01 = 0.15$$

"From Design of Concrete Structures by L.C. Urquhart and C.E. O'rourke, Diagrams 16 and 17 appendix D, for the above data K = 1.45. Throughout all the section there is only compression"

$$N \ (= \text{normal thrust}) = \frac{\text{horizontal thrust}}{\cos\phi_{1/4}}$$

$$N = \frac{802,320}{0.926} = 870,000 \text{ lbs.}$$

$$\text{maximum } f_c = \frac{870,000 \times 1.45}{3.28 \times 7.00 \times 144} = 380 \text{ p.s.i.}$$

Obviously, the maximum unit compressive stress is much less than the allowable one.

Section at Crown

$$f_c = 530 \text{ p.s.i.}$$

$$\cos\phi = 1.00$$

Effect of Dead Load:- The horizontal thrust due to dead load has been found to be - 798,000 lbs. This thrust does not produce any bending moment.

Uniform Live Load at Crown:- The uniform live load including impact is 1,120 lbs. per linear foot of arch rib. The constants corresponding to m = 1.50 and n = 0.30 : are obtained from Diagrams 4 and 5.

Maximum Positive Bending Moment :

$M_c = C_{(c)} wl^2$

$= 0.0048 \times 1,120 \times 150 \times 150$

$M_c = + 121,000$ ft. lbs.

Corresponding Thrust:

$H_c = C_{(hc)} wl \frac{l}{r}$

$= 0.0625 \times 1,120 \times \frac{150 \times 150}{31.75}$

$H_c = - 49,600$ lbs.

Maximum Negative Bending Moment

$M_c = C_{(-c)} wl^2$

$= 0.00415 \times 1,120 \times 150 \times 150$

$M_c = - 104,500$ ft. lbs.

Corresponding Thrust

$H_c = C_{(-hc)} wl \frac{l}{r}$

$= 0.0645 \times 1,120 \times \frac{150 \times 150}{31.75}$

$H_c = - 51,150$ lbs.

Knife Edge Load at Crown:- The knife edge load including impact is 19,500 lbs. per arch rib.

The constants corresponding to m = 1.50 and n = 0.30 are obtained from Tables 2, 3, 8 and 9 by extrapolation.

Maximum Positive Bending Moment:

$M = \frac{Pl}{100} \times K$

$= \frac{19,500 \times 150 \times 4.457}{100}$

$M = + 130,000$ ft. lbs.

Corresponding Thrust:

$H = \frac{P}{10} \times \frac{l}{r} \times C$

$H = \frac{19,500 \times 150 \times 2.54}{10 \times 31.75}$

$H = - 23,400$ lbs.

Maximum Negative Bending Moment:

$M = \frac{Pl}{100} \times K$

$= \frac{19,500 \times 150 \times 1.04}{100}$

$M = -30,400$ ft. lbs.

Corresponding Thrust:

$H = \frac{P}{10} \times \frac{l}{r} \times C$

$= \frac{19,500 \times 150 \times 1.28}{10 \times 31.75}$

$H = - 11,850$ lbs.

Effect of Rib Shortening:- The resulting horizontal thrust from rib shortening is + 11,850 lbs. This thrust induces a positive bending moment at the crown. The moment arm is 7,525 feet.

therefore, M = 11,850 x 7.525 = + 89,250 ft. lbs.

Effect of Temperature Rise:- The resulting horizontal thrust from temperature rise is - 14,400 lbs. This thrust induces a negative bending moment at the crown, The moment arm is 7,525 feet.

therefore, M = 14,400 x 7,525 = - 108,500 ft. lbs.

Effect of Temperature Fall and Shrinkage:- The thrust produced by the temperature fall and shrinkage is + 23,000 lbs. This thrust induces a positive bending moment at the crown. The moment arm is 7,525 feet.

therefore, M = 23,000 x 7.525 = + 173,200 ft. lbs.

Investigation for Stresses at Crown:- From the Tabulated Form No.5 the maximum bending moment and the corresponding thrust will be selected.

maximum bending moment = 513,450 ft. lbs.

corresponding thrust = - 836,150 lbs.

$$e\ (=\text{eccentricity}) = \frac{513,450}{836,150} = 0.615\ \text{ft.}$$

Assuming 1.50 in. net thickness of concrete for protection of reinforcement against rusting and fire and p = 1% :

$$\text{then, } \frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{3.00 \times 12} = 0.07$$

$$\frac{e}{a} = \frac{0.615}{3.00} = 0.205$$

$$np = 15 \times 0.01 = 0.15$$

TABULATED FORM No 5

Summary of Bending Moments and Thrusts at Crown

Type of Loading	Positive Bending Moments		Negative Bending Moments	
	Horizontal Thrusts	Bending Moments	Horizontal Thrusts	Bending Moments
Dead Load	-798,000 lbs	nill.	-798,000 lbs	nill.
Uniform Live Load	-49,600 lbs	I2I,000 ft.lbs	-5I,I50 lbs	I04,500 ft.lbs
Knife Edge Load	-23,400 lbs	I30,000 ft.lbs	-II,850 lbs	30,400 ft.lbs
Rib Shortening	+II,850 lbs	89,250 ft.lbs	-	-
Temperature Rise	-	-	14,400 lbs	I08,500 ft.lbs
Temperature Fall and Shrinkage	+23,000 lbs	I73,200 ft.lbs	-	-
Total	-836,I50 lbs	5I3,450 ft.lbs	-875,400 lbs	243,400 ft.lbs

"From Design of Concrete Structures by L.C. Urquhart and C. E. O'rourke, Diagrams 16 and 17 appendix D, for the above data K = 1.80"

N (= normal thrust) = horizontal thrust

$$\text{therefore, } f_c = \frac{836{,}150 \times 1.80}{3.00 \times 7.00 \times 144} = 500 \text{ p.s.i.}$$

From the Diagrams it will be noticed that the unit tensile stress absorbed by the steel is very small.

Clearly the maximum stresses are within the limits.

Design of Arch Rib by the Exact Method:-

A fixed arch is a statically indeterminate structure with three statically indeterminate values. These are:

1. Horizontal Thrust, H
2. Vertical Reaction, V_A
3. Auxiliary Bending Moment, M.

To compute the stresses in the arch rib it is necessary to determine these three statically indeterminate values. Knowing these three values the bending moment and shear at any section of the arch rib can be computed by statics.

For the derivation of the formulae for these three statically indeterminate values refer to Concrete Plain and Reinforced, Vol. 2 Chapter 8, by Taylor, Thomson and Smulski. The derivation of these formulae is based on the elastic theory.

After the arch axis is drawn and the dimensions are selected, the bending moments and thrusts are computed using the statically indeterminate values, finally the stresses are computed.

The final analysis should include:

Dead Load.

Live Load, consisting of uniform and a concentrated load.

Rib Shortening.

Effect of Changes of Temperature

Effect of Shrinkage.

Determination of Elastic Center of Arch:- After the shape of the arch is determined and the thickness of arch sections selected the Elastic center is found as follows:-

1. A system of coordinates with a center at the left support is adapted.
2. The arch rib is divided into a number of divisions. While the divisions may be of any length, the work is simplified by making the projection on the X - axis constant.

   The span is divided into 20 equal parts, that is, 10 divisions for each half of the arch rib.
3. The values of X and Y, relating to the left support, for the center of each division are found.
4. The thickness of arch rib at the center of each division is scaled The width is 7.00 feet.
5. The value of Y_s is found by tabulating values as shown in the Tabulated Form No.6.... In this Form it should be noted, that since the values of $\frac{I_c}{I_x \cos\phi_x}$ are already worked out the values of $\frac{I_c d_s}{I_x}$ are obtained by multiplying $\frac{I_c}{I_x \cos\phi_x}$ by the horizontal length of the division dx, because $\frac{dx}{\cos\phi_x} = ds$.

TABULATED FOR N° 6 - FINDING ELASTIC CENTER Y_s

Point	Y	X	$\frac{I_c}{I_x \cos\phi_x}$	$\frac{I_c ds}{I_x}$	$Y \frac{I_c ds}{I_x}$	y	$y \frac{I_c ds}{I_x}$	x
0	31.75	75.00	-	-	-	+7.75	-	00.00
1	31.65	71.25	0.963	7.225	222.95	+7.65	55.25	± 3.75
2	31.00	63.75	0.914	6.860	213.00	+7.00	48.00	±11.25
3	29.85	56.25	0.907	6.800	203.00	+5.85	39.70	±18.75
4	28.25	48.75	0.840	6.300	178.00	+4.25	26.80	±26.25
5	25.75	41.25	0.756	5.670	146.00	+1.75	9.90	±33.75
6	22.75	33.75	0.757	5.680	129.25	-1.25	-7.10	±41.25
7	18.90	26.25	0.745	5.590	105.80	-5.10	-28.50	±48.75
8	14.35	18.75	0.564	4.230	60.80	-9.65	-40.90	±56.25
9	9.25	11.25	0.436	3.270	30.25	-14.75	-48.25	±63.75
10	3.25	3.75	0.331	2.485	8.75	-20.75	-51.60	±71.25
S	00.000	00.00		54.110	1297.80	-24.00	+ 3.30	±75.00

$$Y_s = \frac{\sum_0^{l/2} Y \frac{I_c ds}{I_x}}{\sum_0^{l/2} \frac{I_c ds}{I_x}} = \frac{1297.80}{54.11} = 24.00 \text{ feet.}$$

The values of $\frac{1}{\cos\phi_x}$ are obtained from Drawing No. 2. The values of x and y in the Tabulated Form No. 6 refer to the system of coordinates with an origin at the elastic center.

Since the sum of $y \cdot \frac{I_c ds}{dx}$ is only + 3.30 instead of 0 the work is quite accurate.

Effect of Dead Load: As the arch axis coincides with the line of pressure for the dead load, the latter does not induce any bending moment. The horizontal thrust for dead load, from Drawing No. 2, is -780,000 lbs.

Formulae:- The formulae for the exact analysis of an arch are the following ones:-

Horizontal Thrust Due to Vertical Loads:-

$$H = \frac{\sum_{-l/2}^{l/2} M_s\, y\, \frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} y^2 \frac{I_c ds}{I_x} + \sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}}$$

This value is negative. $\sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}$ may be made $\frac{I_c l}{A_{av.}}$

Vertical Reaction Due to Vertical Loads:-

$$V_A = -\frac{\sum_{-l/2}^{l/2} M_s\, x\, \frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} x^2 \frac{I_c ds}{I_x}}$$

Since M_s is negative, V_A is positive.

Auxiliary Bending Moment:-

$$M = \frac{\sum_{-l/2}^{l/2} M_s \frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} \frac{I_c ds}{I_x}}$$

Since M_s is negative, M is positive.

Bending Moment at left Support:-

$$M_A = M - V_A \frac{l}{2} - H_A Y_s$$

Bending Moment at Any Section of the Arch with ordinates x and y:

$$M_x = M + V_A x + H_A y + M_s$$

In the above equations M_s is the static bending moment of the vertical loads at the various points, obtained for each point by multiplying the loads to the left of it by their distance from the point under consideration.

Determination of the denominators for H, V_A and M:-

From the given formulae it can be seen that the denominators for H, V_A and M do not depend upon the character of the loading and they are functions of the shape and dimensions of the arch rib. Hence, they will be determined first.

Denominator for H:-

$$\text{the denominator} = \sum_{-l/2}^{l/2} y^2 \frac{I_c ds}{I_x} + \frac{I_c l}{A_{av.}}$$

Average thickness, perpendicular to the arch axis is assumed to be 3.50 ft.

TABULATED FORM N° 7

Denominators for H, V_A and M.

Section	y	$\frac{I_c ds}{I_x}$	$y \frac{I_c ds}{I_x}$	$y^2 \frac{I_c ds}{I_x}$	$\bar{x}$	x^2	$x^2 \frac{I_c ds}{I_x}$
I	+7.65	7.225	55.25	422.25	±3.75	I4.00	I03.00
2	+7.00	6.860	48.00	336.00	±II.25	I26.50	869.00
3	+5.85	6.800	39.70	232.00	±I8.75	35I.50	2390.00
4	+4.25	6.300	26.80	II3.90	±26.25	689.00	4400.00
5	+I.75	5.670	9.90	I7.30	±33.75	II39.00	6450.00
6	-I.25	5.680	-7.I0	8.85	±4I.25	I700.00	9650.00
7	-5.I0	5.590	-28.50	I45.50	±48.75	2375.00	I3250.00
8	-9.65	4.230	-40.90	394.50	±56.25	3I65.00	I3400.00
9	-I4.75	3.270	-48.25	7II.50	±63.75	4065.00	I3300.00
I0	-20.75	2.485	-5I.60	I070.00	±7I.25	5070.00	I2600.00
		54.II0		345I.80			76,4I2.00

From the Tabulated Form No.7; $\sum_{0}^{l/2} y^2 \frac{I_c ds}{I_x}$ = 3451.80

therefore, $\sum_{-l/2}^{l/2} y^2 \frac{I_c ds}{I_x}$ = 2 x 3451.80 = 6903.60

$\frac{I_c l}{A_{av.}} = \frac{7.00 \times 3.00 \times 3.00 \times 3.00 \times 150.00}{12 \times 7.00 \times 3.50}$ = 96.50

Denominator = 7,000

Denominator for V_A :-

The denominator = $\sum_{-l/2}^{l/2} x^2 \frac{I_c ds}{I_x}$

From the Tabulated Form No.7 $\sum_{0}^{l/2} x^2 \frac{I_c ds}{I_x}$ = 76,412

therefore, $\sum_{-l/2}^{l/2} x^2 \frac{I_c ds}{I_x}$ = 2 x 76,412 = 152,824

Denominator for M :-

the denominator = $\sum_{-l/2}^{l/2} \frac{I_c ds}{I_x}$

From the Tabulated Form No.7; $\sum_{0}^{l/2} \frac{I_c ds}{I_x}$ = 54.11

therefore, $\sum_{-l/2}^{l/2} \frac{I_c ds}{I_x}$ = 54.11 x 2 = 108.22

Effect of Live Load:- As it has been already mentioned the live load consists of a uniform load 1,120 per linear foot of arch and of a concentrated one 19,500 lbs per arch rib. The above values include also the impact.

The denominators of the statically indeterminate values being constant for any character of loading they have been already computed. The numerators for the statically indeterminate values depend upon the position of the live load s on the arch.

As it will be explained later, it is sufficient to determine the numerators for the live load extending over the whole span, using Tabulated Form No. 8, and for the live load extending over 5/8 of the span length, using Tabulated Form No. 9. By proper combination of these values it is possible to get statically indeterminate values for the most unfavorable positions of the live load producing maximum stresses at the springing, quarter point and crown.

After the statically indeterminate values are determined, bending moments and thrusts are found at the critical sections.

Partial Loadings Producing Maximum Bending Moments:- The following loadings produce maximum stress at the critical three cross-sections of the arch ribs.

(see Fig. 14)

At Left Springing:- Maximum positive bending moment is produced by loading scheme 1. Maximum negative bending moment is produce by loading scheme 2.

At Left Quarter Point:- Maximum positive bending moment is produced by loading scheme 2. Maximum negative bending moment is produced by loading scheme 1.

At Crown:- Maximum positive bending moment is produced by loading scheme 3. Maximum negative bending moment is produced by loading scheme 4.

To find the statically indeterminate values, H, V_A and M, for each of the above conditions, it is sufficient to find the values of a load extending over the whole span of the arch and for a load extending over 5/8 of the span of the arch rib. The values for

the other ca

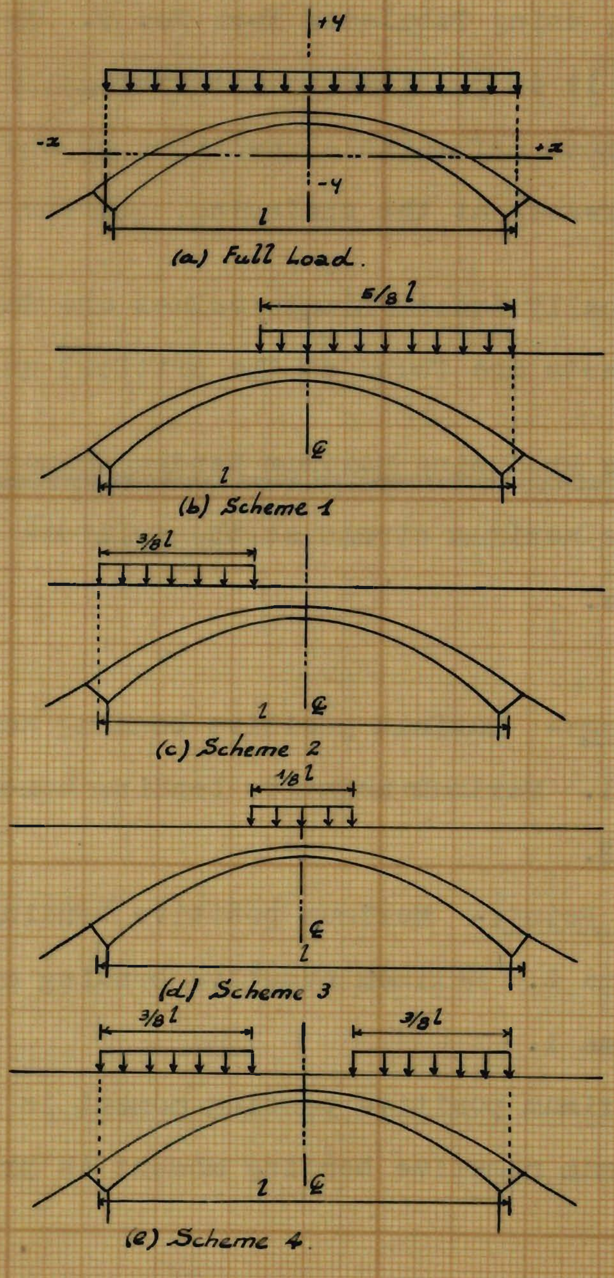

Fig. 14 - Position of Loading for maximum bending moments.

the other cases may then be found by combining the values for these two loading schemes in the way that is given below.

Notation:-

Let M = auxiliary moment fall load.

M_1 = auxiliary moment for left springing, loading scheme 1;

M_2 = auxiliary moment for left springing, loading scheme 2;

M_3 = auxiliary moment for left springing, loading scheme 3;

M_4 = auxiliary moment for left springing, loading scheme 4;

V_A = vertical reaction at left springing, full load;

V_{A1} = vertical reaction at left springing, loading scheme 1;

V_{A2} = vertical reaction at left springing, loading scheme 2;

V_{A3} = vertical reaction at left springing, loading scheme 3;

V_{A4} = vertical reaction at left springing, loading scheme 4;

H = horizontal thrust, full load

H_1 = horizontal thrust, loading scheme 1;

H_2 = horizontal thrust, loading scheme 2;

H_3 = horizontal thrust, loading scheme 3;

H_4 = horizontal thrust, loading scheme 4;

w = uniformly distributed load per unit of length = 1,120 lbs.

l = span of arch

Full Loading:- The statically indeterminate values for load extending over the whole span may be found as follows:-

TABULATED FORM No. 8

Numerator for H and M in case of Full Loading

Section	x	y	$x^2 \frac{I_c ds}{I_x}$	$x^2 y \frac{I_c ds}{I_x}$
1	3.75	7.65	103.00	788
2	11.25	7.00	869.00	6,080
3	18.75	5.85	2,390.00	13,900
4	26.25	4.25	4,400.00	18,700
5	33.75	1.75	6,450.00	11,250
6	41.25	-1.25	9,650.00	-12,050
7	47.75	-5.10	13,250.00	-67,550
8	56.25	-9.65	13,400.00	-129,200
9	63.75	-14.75	13,000.00	-196,250
10	71.25	-20.75	12,600.00	-260,175
			76,412.00	-614,507

The numerators are obtained from the Tabulated Form No. 8, whereas the denominators have been already computed.

Horizontal Thrust : "H" :-

numerator = - 614,507 w

denominator = 7,000

therefore $H = -\frac{614,507w}{7,000} = -87.75$ w.

Auxiliary Moment "M" :-

$$\text{numerator} = w\left[\sum_0^{l/2} x^2 \frac{I_c ds}{I_x} + \left(\frac{l}{r}\right)^2 \sum_0^{l/2} \frac{I_c ds}{I_x}\right]$$

denominator = 108.22

$$\text{therefore, } M = \frac{(76,412 + (\frac{150}{2})(\frac{150}{2}) \times 54.11)w}{108.22}$$

M = 3,520 w

Vertical Reaction "A_V" :-

$V_A = \frac{1}{2}wl = \frac{1}{2}w\,150 = 75$ w.

Loading Scheme 1:- The statically indeterminate values, H, V_{A1} and M_1 will be found as follows:-

The numerators will be obtained from the Tabulated Form No. 9.

Horizontal Thrust: "H_1" :-

numerator = - 453,070 w

denominator = 7,000

therefore, $H_1 = \frac{-453,070w}{7,000} = -64.80$ w

Vertical Reaction "V_{A1}" :-

numerator = 3,696,370 w

denominator = 152,824

therefore, $V_{A1} = \frac{3,696,370w}{152,824} = 24.20$ w

TABULATED FORM N°9

Numerator for H_1 ; M_1 and V_{A1} "Loading Scheme N°1"

Section	x	$\frac{1}{8}l+x$	$(\frac{1}{8}l+x)^2$	$\frac{7}{2}(\frac{1}{8}l+x)^2$	$\frac{I_c ds}{I_x}$	$\frac{I_c ds}{2I_x}(\frac{1}{8}l+x)^2$	$x\frac{I_c ds}{2I_x}(\frac{1}{8}l+x)^2$	y	$y\frac{I_c ds}{2I_x}(\frac{1}{8}l+x)^2$
2'	-II.25	7.50	56.25	28.I3	6.860	I93	-2I70	+7.00	I350
I'	- 3.75	I5.00	225.00	II2.50	7.225	8I0	-3030	+7.65	6200
I	+ 3.75	22.50	507.00	253.50	7.225	I833	6870	+7.65	I4030
2	+II.25	30.00	900.00	450.00	6.860	3085	34700	+7.00	2I600
3	+I8.75	37.50	I405.00	702.50	6.800	4775	89500	+5.85	27900
4	+26.25	45.00	2030.00	I0I5.00	6.300	6400	I68000	+4.25	27200
5	+33.75	52.50	2755.00	I377.50	5.670	7820	264000	+I.75	I3700
6	+4I.25	60.00	3600.00	I800.00	5.680	I0220	422500	-I.25	-I2800
7	+48.75	67.50	4550.00	2275.00	5.590	I2700	620000	-5.I0	-64750
8	+56.25	75.00	5625.00	28I2.50	4.230	II900	670000	-9.65	-II5000
9	+63.75	82.50	6800.00	3400.00	3.270	IIII5	7I0000	-I4.75	-I64000
I0	+7I.25	90.00	8I00.00	4050.00	2.485	I0050	7I6000	-20.75	-208500
						80,90I	3,696,370		-453,070

Auxiliary Moment "M_{A1}" :-

numerator = 80,901 w

denominator = 108.22

therefore $M_{A1} = \frac{80,901w}{108.22} = 747.50\ w$

Loading Scheme 2:- Knowing the indeterminate values for full load and for scheme 1, the indeterminate values for scheme 2 may be obtained by simple subtraction.

Horizontal Thrust "H_2" :-

$H_2 = H - H_1$

$= (87.75 - 64.80)w$

$H_2 = 22.95w$

Ausiliary Bending Moment "M_2"

$M_2 = M - M_1$

$= (3,520 - 747.50)w$

$M_2 = 2,772.50w$

Vertical Reaction "V_{A2}" :-

$V_{A2} = V_A - V_{A1}$

$= (75 - 24.20)w$

$V_{A2} = 50.80w$

Loading Scheme 3:- The statically indeterminate values for scheme 3, in which the loading extends on both sides of the crown for a distance equal $\frac{1}{8}l$, are obtained as follows.

Horizontal Thrust "H_3" :-

$H_3 = H_1 - H_2$

$= (64.80 - 22.95)w$

$H_3 = 41.85\ w$

Auxiliary Bending Moment "M_3" :-

$$M_3 = 2M, - M - \frac{15}{128} wl^2$$

$$= 2 \times 747.50w - 3,520w - \frac{15}{128} w\ 150 \times 150$$

$$M_3 = 605w.$$

Vertical Reaction V_{A3} :-

$$V_{A3} = \frac{1}{8} wl = \frac{1}{8} w\ 150 = 18.75\ w.$$

<u>Loading Scheme 4</u>:- The Loading in this scheme extends on each side of the arch from the springing for a distance equal to 3/8 l. The statically indeterminate values are obtained as follows:

Horizontal Thrust "H_4" :-

$$H_4 = H - H_3$$

$$= (87.75 - 41.85)w$$

$$H_4 = 45.90w$$

Auxiliary Bending Moment: "M_4"

$$M_4 = M - M_3$$

$$= (3520 - 605)w$$

$$M_4 = 2,915w$$

Vertical Reaction V_{A4} :-

$$V_{A4} = \frac{3}{8} w = \frac{3}{8} w\ 150 = 56.25w.$$

TABULATED FORM No. 10

Statically Indeterminate Values for All Schemes of Loading

Type of Loading	H in pounds	V_A in pounds	M in ft. lbs.
Full Load	87.75w = -98,300	75w = 84,000	3,520w = 3,940,000
Scheme 1	64.80w = -72,600	24.20w = 27,100	747,50w = 837,500
Scheme 2	22.95w = -25,700	50.80w = 57,000	2,772.50w=3,110,000
Scheme 3	41,85w = -47,000	18.75w = 21,000	605w = 677,500
Scheme 4	45.90w = -51,500	56,25w = 63,000	2,915w = 3,260,000

Bending Moments for the Uniform Live Load:- Using the values given in the Tabulated Form No. 10, the bending moments at the springing, quarter point and crown are found as follows:-

Full Loading:-

Bending Moment at Springing:

$$M_A = M - V_A \frac{l}{2} - H Y_s$$

$$= 3,940,000 - 84,000 \times \frac{150}{2} + 98,300 \times 24.00$$

$$M_A = -800 \text{ ft. lbs.}$$

Loading Scheme 1:-

Bending Moment at Springing.

$$M_A = M_1 - V_{A1} \frac{l}{2} - H_1 Y_s$$

$$= 837,500 - 27,100 \times \frac{150}{2} + 72,600 \times 24.00$$

$$M_A = + 547,400 \text{ ft. lbs.}$$

Bending Moment at Quarter Point.

The bending moment at any point is given by the following formula:

$$M_x = M_1 + V_{A1}x + H_1y + M_s$$

$$M_{1/4} = 837,500 + 27,100\ (-\frac{150}{4}) - 72,600 \times (24.30 - 24.00) + 0$$

$$M_{1/4} = -\ 200,530 \text{ ft. lbs.}$$

Loading Scheme 2:-

Bending Moment at Springing.

$$M_A = M_2 - V_{A2}\ \frac{l}{2} - H_2Y_s$$

$$= 3,110,000 - 57,000 \times \frac{150}{2} - 25,700 \times 24.00$$

$$M_A = -\ 547,000 \text{ ft. lbs.}$$

Bending Moment at Quarter Point.

The bending moment at any point is given by the following formula:

$$M_x = M_1 + V_{A1}x + H_1y + M_s$$

$$M_{1/4} = 3,110,000 + 57,000 \times (-\frac{150}{4}) - 25,700(24.30-24.00) - \frac{1}{2}1,120 \times \times (\frac{150 \times 150}{4 \times 4})$$

$$M_{1/4} = 178,800 \text{ ft. lbs.}$$

Loading Scheme 3:-

Bending Moment at Crown:

$$M_x = M_3 + V_{A3}x + H_3y + M_s$$

$$M_c = 677,500 - 0 - 47,000(31.75-24.00) - \frac{1}{2}x1,120x(\frac{150x150}{8})^2$$

$$M_c = +\ 116,250 \text{ ft. lbs.}$$

Loading Scheme 4:

Bending Moment at Crown

$$M_x = M_4 + V_{A4}x + H_4y + M_s$$

$$M_c\ 3,260,000 + 0 - 51,500(31.75-24.00) - \frac{3}{8} \times 150x1,120x\frac{5}{16}x150$$

$$M_c = -\ 94,130 \text{ ft. lbs.}$$

TABULATED FORM No. 11

Maximum Bending Moments due to Uniform Live Load

SPRINGING			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
547,400 ft.lbs	-72,600 lbs	547,000 ft.lbs	-25,700 lbs.
QUARTER POINT			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
178,800 ft.lbs	-25,700 lbs	200,530 ft.lbs	-72,600 lbs.
CROWN			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
116,250 ft.lbs	47,000 lbs	94,130 ft.lbs	-51,500 lbs.

Effect of Rib Shortening:- The horizontal Thrust due to Rib Shortening is found from the following formula:

$$H = \frac{\sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}}{\sum_{-l/2}^{l/2} y^2 \frac{I_c dx}{I_x} + \sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}} H_d$$

$\sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}$ may be replaced by $\frac{I_c l}{A_{av}}$.

numerator = 96.50 x 780,000

denominator = 7,000.00

therefore, $H = \frac{96.50 \times 780,000}{7,000}$

$H = + 10,750$ lbs.

Effect of Temperature Changes and Shrinkage:-

$$H = \frac{a E l I_c}{\sum_{-l/2}^{l/2} y \frac{I_c ds}{I_x} + \sum_{-l/2}^{l/2} \frac{I_c dx}{A_x}} \times (\pm t^\circ)$$

$$= \frac{11 \times 144 \times 150.00 \times 7.00 \times 3.00 \times 3.00 \times 3.00}{12 \times 7,000} \times (\pm t^\circ)$$

$H = - 535 \times (\pm t^\circ)$

Rise of temperature $t = + 25^\circ F$

therefore, corresponding H = - 535 x 25 = - 13,400 lbs.

Fall of temperature plus shrinkage = - 40°F therefore, Corresponding H = - 535 x (-40) = + 21,400 lbs

TABULATED FORM No. 12

Bending Moments and Corresponding Thrusts

due to Rib Shortening and Temperature Changes.

	H	Springing y = -24.00'	Quarter Pt. y = 0.30'	Crown. y = 7.75'
Rib Shortening	10,750 lbs	-258,500	+3,230	+83,500
Temp. Rise	-13,400 lbs	+322,000	-4,025	-104,000
Fall plus shrinkage	21,400 lbs	-514,000	+6,425	+166,000

Effect of Knife Edge Load:- The effect of Knife Edge Load will be studied by the use of influence lines. Influence lines for the three statically indeterminate values H, V_A and M are drawn from the Tabulated Forms 13, 14 and 15. In addition influence lines for bending moments at the critical cross-sections are drawn from the Tabulated Forms 16, 17 and 18.

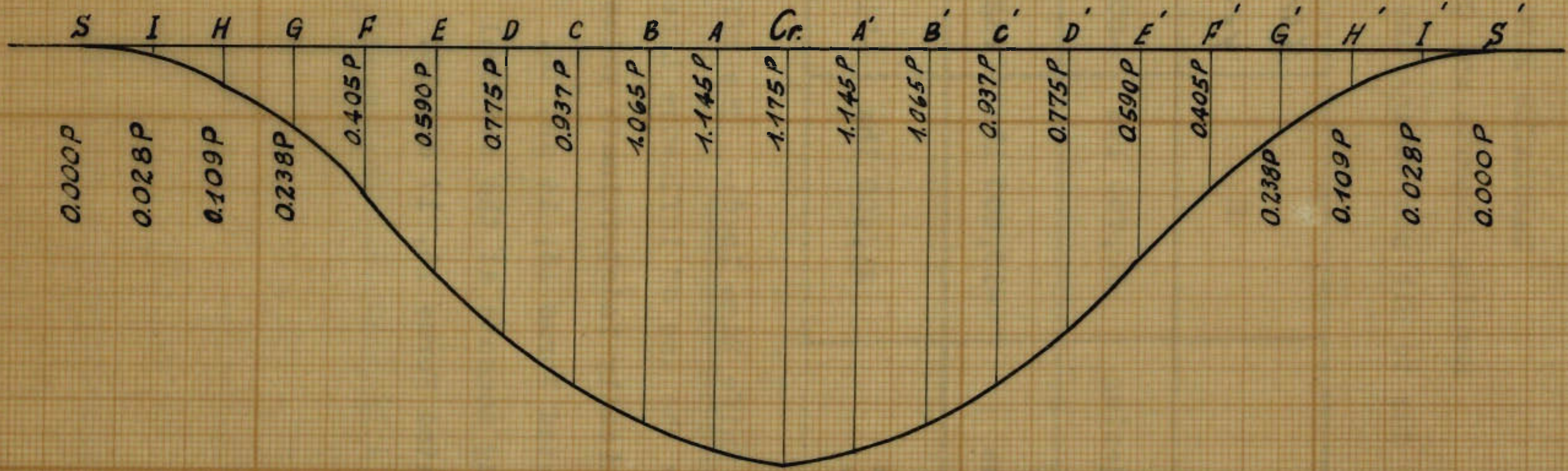

FIG. 15 - Influence Line for horizontal Thrust "H".

TABULATED FORM No 13

ORDINATES OF INFLUENCE LINE FOR "H"

$$H = \frac{\sum_{x_1}^{l/2} (x-x_1) y \frac{I_c ds}{I_x}}{\sum_{-\frac{l}{2}}^{l/2} y^2 \frac{I_c ds}{I_x} + \frac{I_c l}{A_{av.}}}$$

Section	$\frac{I_c ds}{I_x}$	y	$y\frac{I_c ds}{I_x}$	x	Load at End of 1st division $x_1 = 67.50$		Load at End of 2nd division $x_1 = 60.00$		Load at End of 3rd division $x_1 = 52.50$		Load at End of 4th division $x_1 = 45.00$		Load at End of 5th division $x_1 = 37.50$		Load at End of 6th division $x_1 = 30.00$		Load at End of 7th division $x_1 = 22.50$		Load at End of 8th division $x_1 = 15.00$		Load at End of 9th division $x_1 = 7.50$		Load at End of 10th division $x_1 = 0.00$	
					$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)y\frac{I_c ds}{I_x}$
1'	7.225	7.65	55.25	3.75																			3.75	207
2'	6.860	7.00	48.00	11.25																	3.75	180	11.25	540
3'	6.800	5.85	39.70	18.75															3.75	149	11.25	447	18.75	745
4'	6.300	4.25	26.80	26.25													3.75	100	11.25	302	18.75	503	26.25	705
5'	5.670	1.75	9.90	33.75											3.75	37	11.25	112	18.75	186	26.25	260	33.75	334
6'	5.680	-1.25	-7.10	41.25									3.75	-27	11.25	-80	18.75	-133	26.25	-186	33.75	-240	41.25	-293
7'	5.590	-5.10	-28.50	48.75							3.75	-107	11.25	-321	18.75	-535	26.25	-750	33.75	-961	41.25	-1,175	48.75	-1,390
8'	4.230	-9.65	-40.90	56.25					3.75	-153.50	11.25	-460	18.75	-767	26.25	-1,075	33.75	-1,380	41.25	-1,690	48.75	-1,990	56.25	-2,300
9'	3.270	-14.75	-48.25	63.75			3.75	-181.50	11.25	-542.50	18.75	-905	26.25	-1,265	33.75	-1,630	41.25	-1,990	48.75	-2,350	56.25	-2,720	63.75	-3,080
10'	2.485	-20.75	-51.60	71.25	3.75	-193.50	11.25	-580.00	18.75	-967.50	26.25	-1355	33.75	-1,740	41.25	-2.130	48.75	-2,510	56.25	-2,900	63.75	-3.290	71.25	-3,680
$\sum_{x_1}^{l/2} (x-x_1) y \frac{I_c ds}{I_x}$						-193.50		-761.50		-1,663.50		-2827		-4,120		-5,413		-6,551		-7,450		-8,025		-8,212
$\frac{\sum_{x_1}^{l/2}(x-x_1)y\frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} y^2 \frac{I_c ds}{I_x} + \frac{I_c l}{A_{av.}}} = \frac{\sum_{x_1}^{l/2}(x-x_1)y\frac{I_c ds}{I_x}}{7,000}$						-0.0277		-0.109		-0.238		-0.405		-0.590		-0.775		-0.937		-1.065		-1.145		-1.175

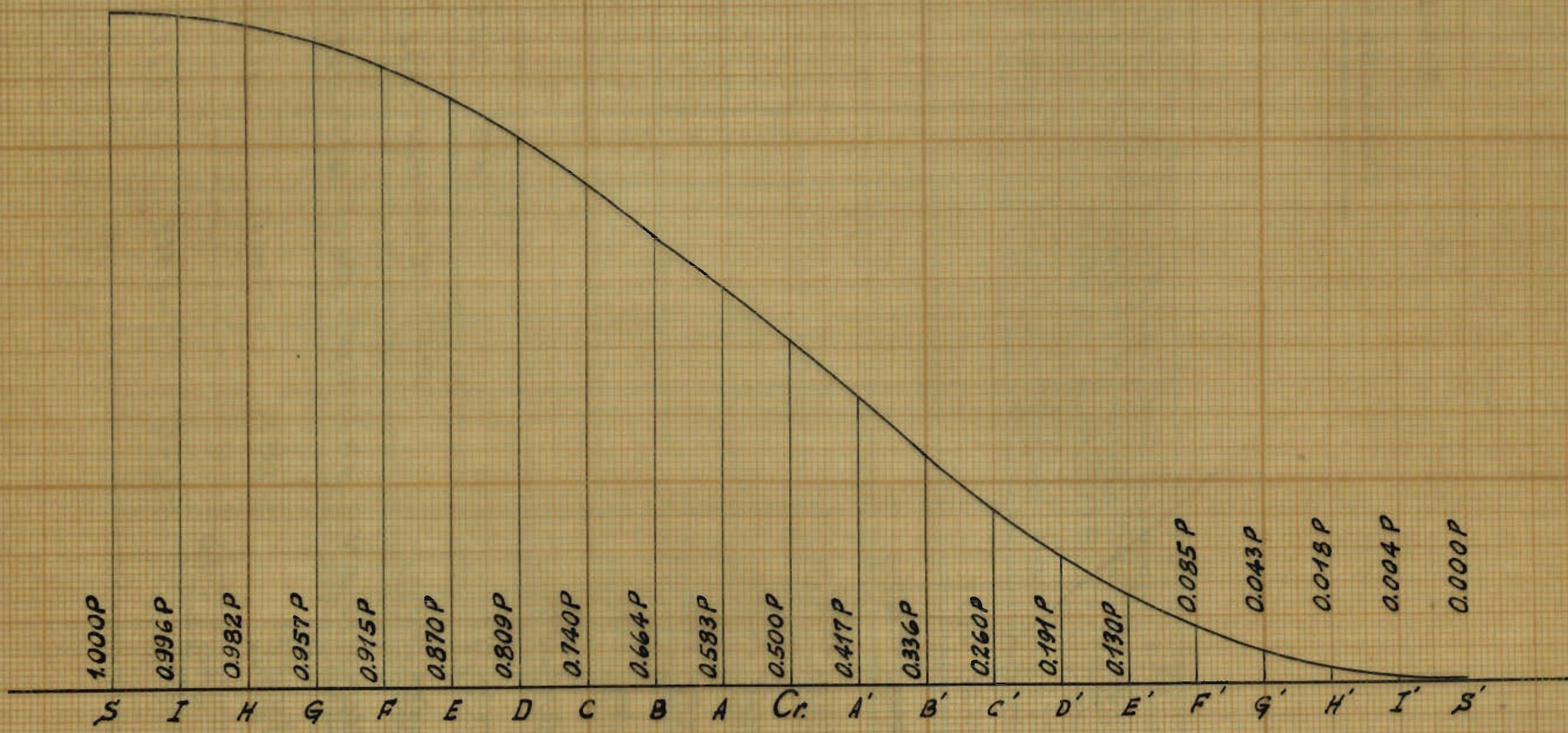

Fig. 16.- Influence Line for Vertical Reaction "V_A"

TABULATED FORM N° I4

ORDINATES OF INFLUENCE LINE FOR "V_A"

$$V_A = \frac{\sum_{x_1}^{l/2} (x-x_1)x \frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} x^2 \frac{I_c ds}{I_x}}$$

Section	x	$x\frac{I_c ds}{I_x}$	Load at End of 1st division $x_1 = 67.50$		Load at End of 2nd division $x_1 = 60.00$		Load at End of 3rd division $x_1 = 52.50$		Load at End of 4th division $x_1 = 45.00$		Load at End of 5th division $x_1 = 37.50$		Load at End of 6th division $x_1 = 30.00$		Load at End of 7th division $x_1 = 22.50$		Load at End of 8th division $x_1 = 15.00$		Load at End of 9th division $x_1 = 7.50$		Load at Center $x_1 = 0.00$	
			$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)x\frac{I_c ds}{I_x}$
1'	3.75	27.10																			3.75	102
2'	11.25	77.25																	3.75	289	11.25	870
3'	18.75	127.50															3.75	480	11.25	1,435	18.75	2.390
4'	26.25	165.50													3.75	620	11.25	1,865	18.75	3,100	26.25	4,350
5'	33.75	197.00											3.75	715	11.25	2,150	18.75	3.580	26.25	5,015	33.75	6,450
6'	41.25	235.00									3.75	880	11.25	2,640	18.75	4,410	26.25	6,165	33.75	7,920	41.25	9,700
7'	48.75	272.00							3.75	1,020	11.25	3,060	18.75	5,100	26.25	7,130	33.75	9.175	41.25	11,230	48.75	13,250
8'	56.25	239.00					3.75	895	11.25	2.690	18.75	4.480	26.25	6,270	33.75	8.080	41.25	9,850	48.75	11,650	56.25	13,450
9'	63.75	209.00			3.75	783	11.25	2.360	18.75	3,920	26.25	5.500	33.75	7,070	41.25	8,650	48.75	10,200	56.25	11,750	63.75	13.350
10'	71.25	177.00	3.75	663	11.25	1,999	18.75	3,320	26.25	4,650	33.75	5,970	41.25	7,300	48.75	8,630	56.25	9,950	63.75	11,300	71.25	12,600
$\sum_{x_1}^{l/2}(x-x_1)x\frac{I_c ds}{I_x}$				663		2,782		6,575		12,280		19,890		29,095		39,670		51,265		63,689		76,512
$\frac{\sum_{x_1}^{l/2}(x-x_1)x\frac{I_c ds}{I_x}}{152,824}$				0.00435 0.99565		0.0182 0.9818		0.043 0.957		0.085 0.915		0.130 0.870		0.191 0.809		0.260 0.740		0.336 0.664		0.417 0.583		0.500

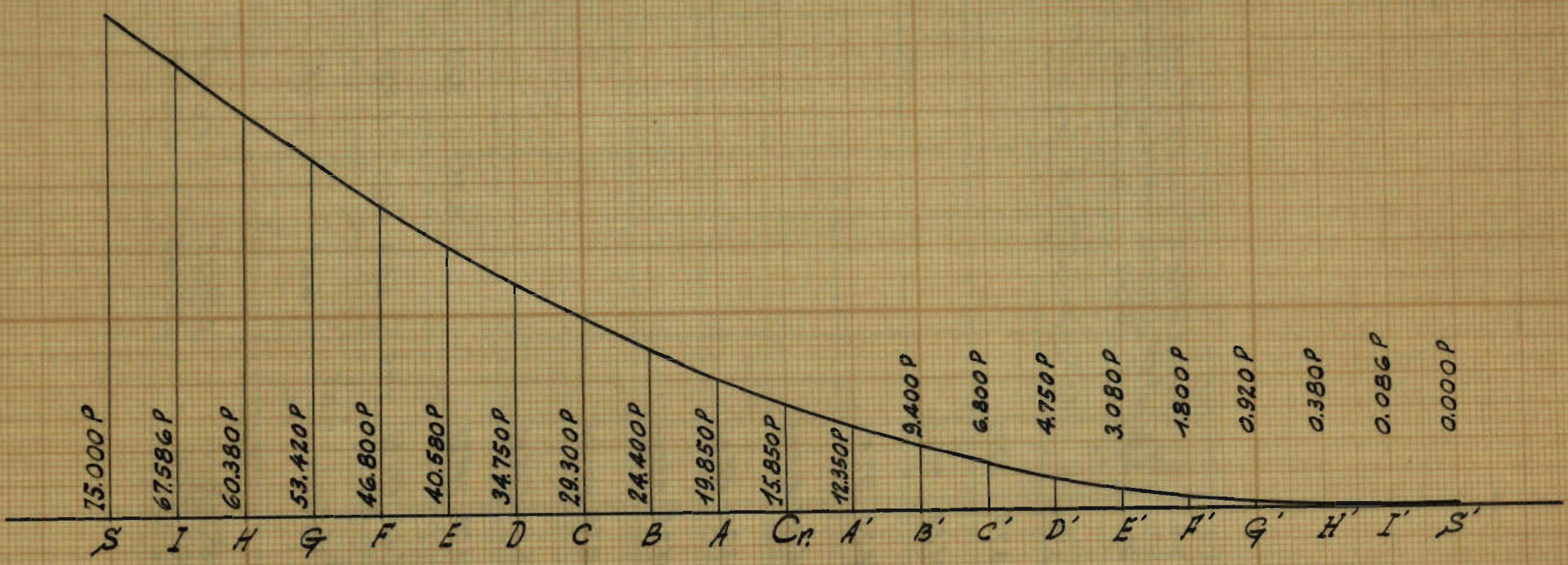

Fig. 17.- Influence Line for Auxiliary Bending Moment. "M"

TABULATED FORM N° I5

ORDINATES OF INFLUENCE FOR AUXILIARY MOMENT "M"

$$M = \frac{\sum_{x_1}^{l/2} (x - x_1) \frac{I_c ds}{I_x}}{\sum_{-l/2}^{l/2} \frac{I_c ds}{I_x}}$$

Section	x	$\frac{I_c ds}{I_x}$	Load at End of 1st division $x_1 = 67.50$		Load at End of 2nd division $x_1 = 60.00$		Load at End of 3rd division $x_1 = 52.50$		Load at End of 4th division $x_1 = 45.00$		Load at End of 5th division $x_1 = 37.50$		Load at End of 6th division $x_1 = 30.00$		Load at End of 7th division $x_1 = 22.50$		Load at End of 8th division $x_1 = 15.00$		Load at End of 9th division $x_1 = 7.50$		Load at Center $x = 0.00$	
			$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$	$x-x_1$	$(x-x_1)\frac{I_c ds}{I_x}$
1'	3.75	7.225																			3.75	27.10
2'	11.25	6.860																	3.75	25.70	11.25	77.25
3'	18.75	6.800															3.75	25.50	11.25	76.60	18.75	127.50
4'	26.25	6.300													3.75	23.70	11.25	71.00	18.75	118.00	26.25	165.00
5'	33.75	5.670											3.75	21.20	11.25	63.75	18.75	106.50	26.25	148.50	33.75	191.50
6'	41.25	5.680									3.75	21.30	11.25	64.00	18.75	106.50	26.25	149.00	33.75	192.00	41.25	235.00
7'	48.75	5.590							3.75	21.00	11.25	63.00	18.75	105.00	26.25	146.50	33.75	189.00	41.25	231.00	48.75	273.00
8'	56.25	4.230					3.75	15.85	11.25	47.65	18.75	79.50	26.25	111.00	33.75	143.00	41.25	175.00	48.75	206.00	56.25	238.00
9'	63.75	3.270			3.75	12.25	11.25	36.80	18.75	61.25	26.25	86.00	33.75	110.50	41.25	135.00	48.75	159.50	56.25	184.00	63.75	208.00
10'	71.25	2.485	3.75	9.32	11.25	28.00	18.75	46.50	26.25	65.15	33.75	83.80	41.25	102.50	48.75	121.00	56.25	139.50	63.75	158.50	71.25	177.00
$\sum_{x_1}^{l/2} (x-x_1) \frac{I_c ds}{I_x}$				9.32		40.25		99.15		195.15		333.60		514.20		739.45		1015.50		1,340.30		1,719.35
$\frac{\sum_{x_1}^{l/2} (x-x_1) \frac{I_c ds}{I_x}}{108.22}$				0.086 67.586		0.38 60.38		0.92 53.42		1.80 46.80		3.08 40.58		4.75 34.75		6.80 29.30		9.40 24.40		12.35 19.85		15.85

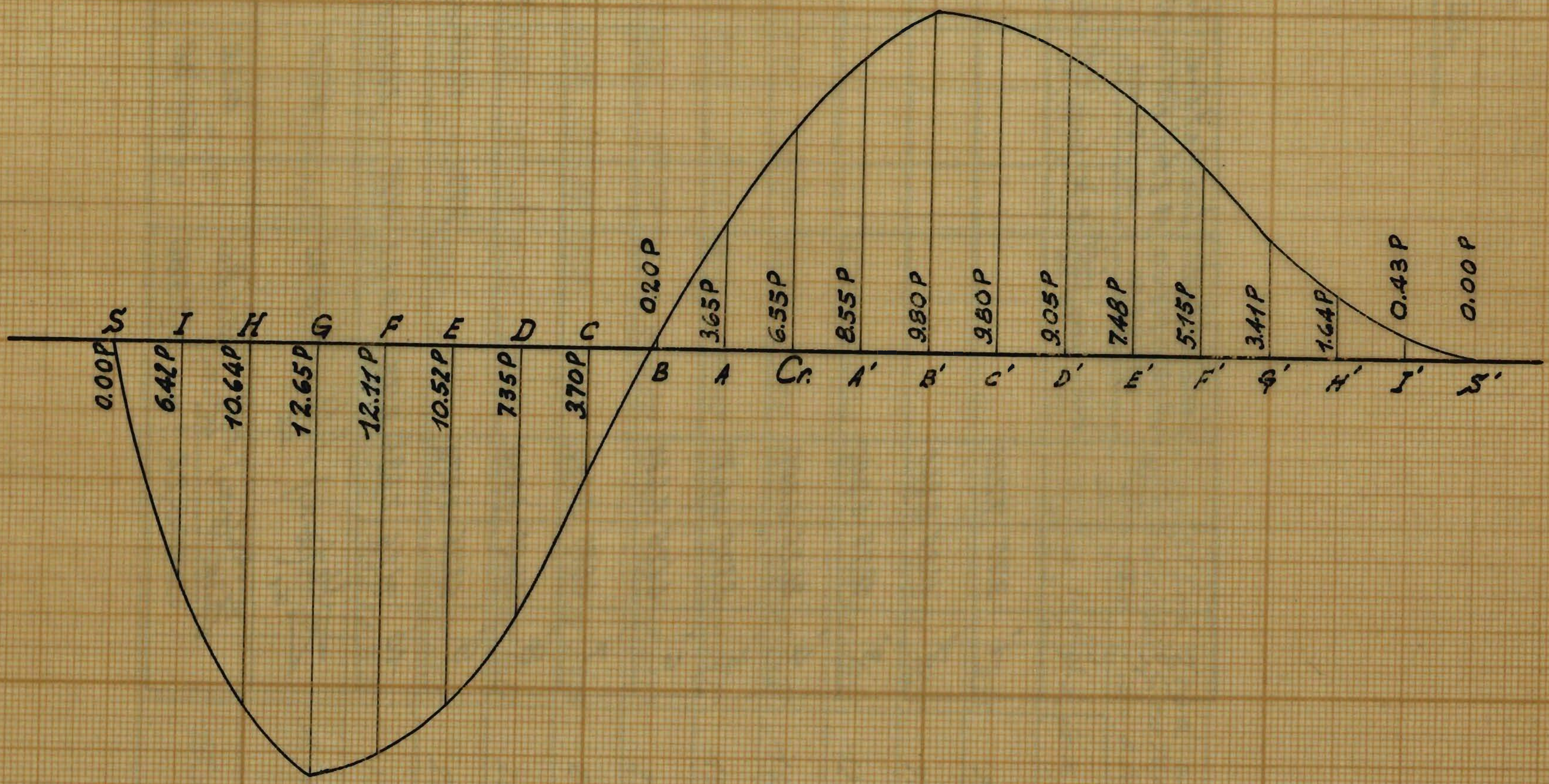

Fig. 18 - Influence Line for Bending Moment at Springing.

TABULATED FORM N° I6

INFLUENCE LINE FOR BENDING MOMENT AT LEFT SUPPORT "M_A"

Section	$-H$ "Tab. Form. 13"	$-H\,Y_s$	V_A "Tab. Form N°14"	$\frac{1}{2}V_A$	M "Tab. Form 15"	$M - V_A\frac{l}{2} - HY_s$	x_1	$\frac{1}{2}-x_1$	$1\,V_A$	Section	$M_A(x_1)+lV_A(x_1) - (\frac{l}{2}-x_1)$
Cr	1.175	28.20	0.500	37.50	15.85	+6.55	0	75	75.00	Cr.	+6.55
A'	I.I45	27.50	0.4I7	31.30	I2.35	+8.55	7.50	67.50	62.60	A	+3.65
B'	I.065	25.60	0.336	25.20	9.40	+9.80	I5.00	60.00	50.40	B	+0.20
C'	0.937	22.50	0.260	I9.50	6.80	+9.80	22.50	52.50	39.00	C	-3.70
D'	0.775	I8.60	0.I9I	I4.30	4.75	+9.05	30.00	45.00	28.60	D	-7.35
E'	0.590	14.15	0.I30	9.75	3.08	+7.48	37.50	37.50	I9.50	E	-I9.52
F'	0.405	9.72	0.085	6.37	I.80	+5.I5	45.00	30.00	I2.74	F	-I2.II
G'	0.238	5.7I	0.043	3.22	0.92	+3.4I	52.50	22.50	6.44	G	-I2.65
H'	0.I09	2.62	0.0I82	I.36	0.38	+I.64	60.00	I5.00	2.72	H	-I0.64
I'	0.0277	0.667	0.00,435	0.326	0.086	+0.427	67.50	7.50	0.652	I	-6.42
S'	0	0	0	0	0	0	75.00	0	0	S	0

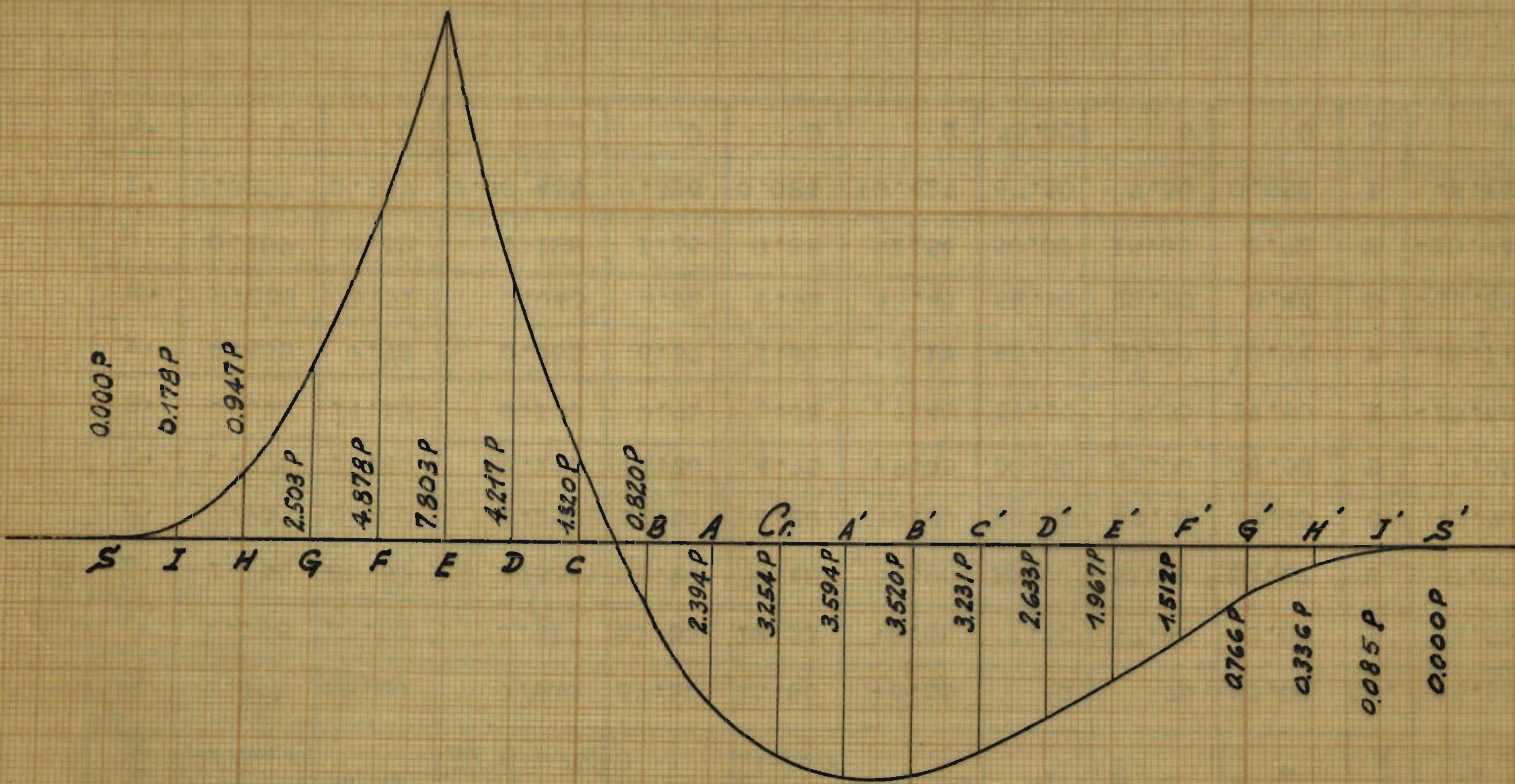

FIG. 19 - Influence Line for Bending Moment at Quarter Point.

TABULATED FORM № 17

INFLUENCE LINE FOR BENDING MOMENT AT QUARTER POINT :-

At Quarter point x_n = 37.50 y_n = 0.30

Section	M	V_A	$V_A x_n$	H	Hy_n	$M_{x_n} = M + V_A x_n + Hy_n$	Section	M	V_A	$V_A x_n$	H	$H y_n$	x	$M_s = x_n - x$	$M_{x_n} = M + V_A x_n + Hy_n + M_s$
Cr.	15.85	0.500	-18.75	1.175	-0.354	-3.254	Cr.	15.85	0.500	-18.75	1.175	-0.354	0.000	0.000	-3.254
A'	12.35	0.417	-15.60	1.145	-0.344	-3.594	A	19.85	0.583	-21.90	1.145	-0.344	-7.50	0.000	-2.394
B'	9.40	0.336	-12.60	1.065	-0.320	-3.520	B	24.40	0.664	-24.90	1.065	-0.320	-15.00	0.000	-0.820
C'	6.80	0.260	-9.75	0.937	-0.281	-3.231	C	29.30	0.740	-27.70	0.937	-0.281	-22.50	0.000	+ 1.320
D'	4.75	0.191	-7.15	0.775	-233	-2.633	D	34.75	0.809	-30.30	0.775	-0.233	-30.00	0.000	+4.217
E'	3.08	0.130	-4.87	0.590	-0.177	-1.967	E	40.58	0.870	-32.60	0.590	-0.177	-37.50	0.000	+7.803
F'	1.80	0.085	-3.19	0.405	-0.122	-1.512	F	46.80	0.915	-34.30	0.405	-0.122	-45.00	-7.50	+4.878
G'	0.92	0.043	-1.615	0.238	-0.0712	-0.766	G	53.42	0.957	-35.90	0.238	-0.0712	-52.50	-15.00	+2.503
H'	0.38	0.0182	-0.683	0.109	-0.0327	-0.336	H	60.38	0.982	-36.90	0.109	-0.0327	-60.00	-22.50	+0.947
I'	0.086	0.00435	-0.163	0.0277	-0.0083	-0.085	I	67.586	0.996	-37.40	0.0277	-0.0083	-67.50	630.00	+0.178
S'	0.000	0.000	0.000	0.000	0.000	0.000	S	75.00	1.000	-37.50	0.000	0.000	-75.00	-37.50	0.000

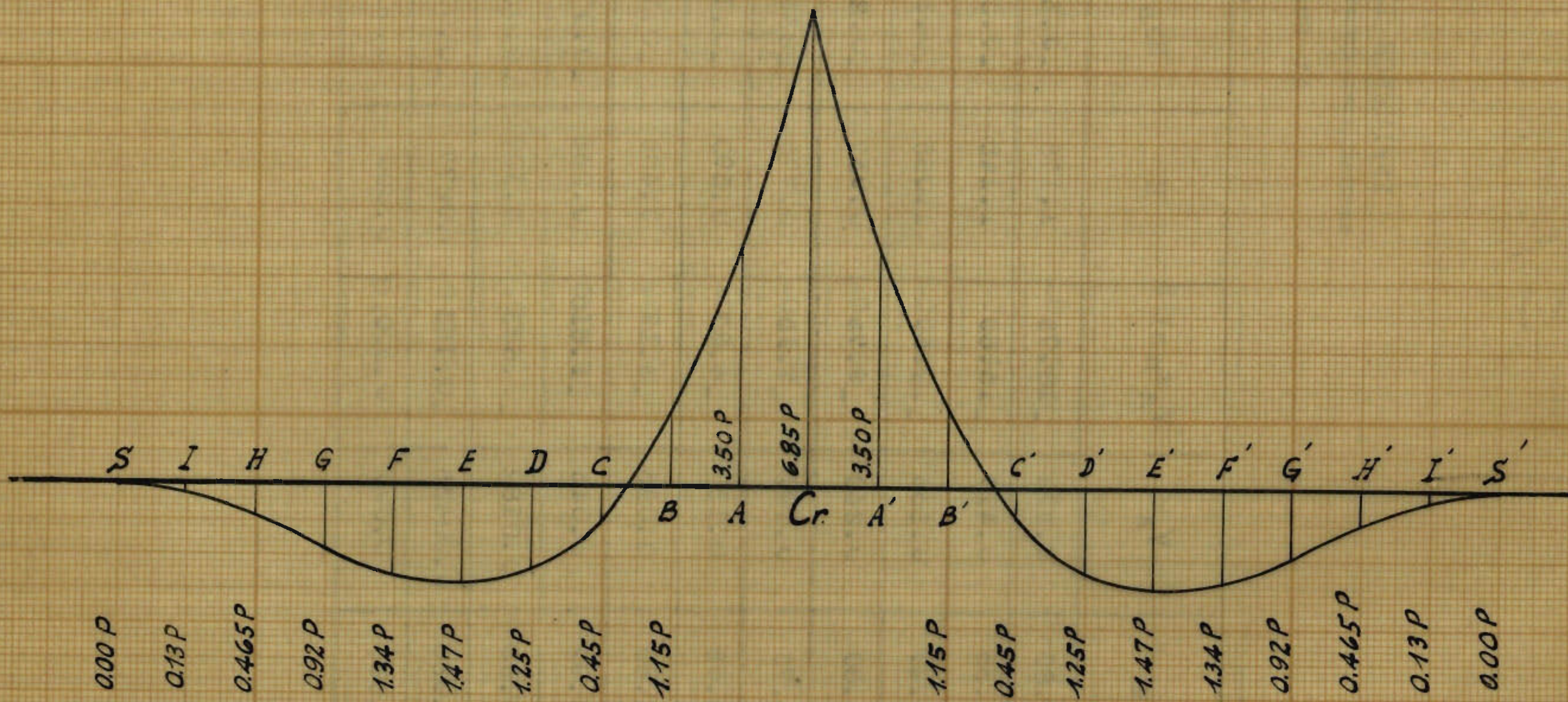

Fig. 20 - Influence Line for Bending Moment at Crown.

TABULATED FORM N° 18

INFLUENCE LINE FOR BENDING MOMENT AT CROWN

AT CROWN x_n = 0.00 : y = 7.75

Load at Section	M	V_A	$V_A x_n$	H	$H y_n$	$M_{x_n} = M + V_A x_n + H y_n$	Load at Section	M_{x_n}
Cr	15.85	0.500	0.000	1.175	-9.00	+6.85	Cr	+6.85
A'	12.35	0.417	0.000	1.145	-8.85	+3.50	A	+3.50
B'	9.40	0.336	0.000	1.065	-8.25	+1.15	B	+1.15
C'	6.80	0.260	0.000	0.937	-7.25	-0.45	C	-0.45
D'	4.75	0.191	0.000	0.775	-6.00	-1.25	D	-1.25
E'	3.08	0.130	0.000	0.590	-4.55	-1.47	E	-1.47
F'	1.80	0.085	0.000	0.405	-3.14	-1.34	F	-1.34
G'	0.92	0.043	0.000	0.238	-1.84	-0.920	G	-0.920
H'	0.38	0.0182	0.000	0.109	-0.845	-0.465	H	-0.465
I'	0.086	0.004,35	0.000	0.0277	-0.215	-0.129	I	-0.129
S'	0.000	0.000	0.000	0.000	0.000	-0.000	S	0.000

TABULATED FORM No. 19

Maximum Bending Moments and Corresponding

Thrusts due to Knife Edge Load

SPRINGING			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
M = 9.80 P =186,500 ft.lbs	H =-1.065 =20,300 lbs.	M =12.65 P =241,000 ft.lbs	H=-0.238 P =-4,600 lbs
QUARTER POINT			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
M = 7.803 P =148,500 ft.lbs	H=-0.59 P =-11,200 lbs	M = 3.594 P =68,250 ft.lbs	H =-1.145 P =-21,800 lbs
CROWN			
Maximum (+) Moment	Corresponding Thrust	Maximum (-) Moment	Corresponding Thrust
M = 6.85 P =131,000 ft.lbs	H=-1.175 P =-22,400 lbs	M = 1.47 P =28,000 ft.lbs	H= 0.59 P =-11,200 lbs

The maximum bending moments negative and positive at the springing, quarter point and crown and their corresponding thrusts are obtained from the tabulated forms 11, 12 and 19 and tabulated in the form N° 20 in which the total maximum bending moments positive and negative and their corresponding thrusts are computed.

TABULATED FORM N° 20

RESULTANT MAXIMUM BENDING MOMENTS AND CORRESPONDING THRUSTS.

LOADS	SPRINGING				QUARTER POINT				CROWN			
	Maximum (+) moment.	Corresponding Thrust.	Maximum (-) Moment	Corresponding Thrust.	Maximum (+) Moment	Corresponding Thust	Maximum (-) Moment	Corresponding Thrust.	Maximum (+) Moment	Corresponding Thrust.	Maximum (-) Moment	Corresponding Thrust.
DEAD LOAD	-	-780,000	-	-780,000	-	-780,000	-	-780,000	-	-780,000	-	-780,000
UNIF. LIVE LOAD	547,400	-72,600	547,000	-25,700	178,800	-25,700	200,530	-72,600	116,250	-47,000	94,130	-51,500
RIB SHORTENING	-	-	258,500	+10,750	3,230	+10,750	-	-	83,500	+10,750	-	-
TEMP. RISE	322,000	-13,400	-	-	-	-	4 052	-13,400	-	-	104,000	-13,400
TEMP. FALL + SHRINKAGE	-	-	514,000	+21,400	6,425	+21,400	-	-	166,000	+21,400	-	-
KNIFE EDGE LOAD	186,500	-20,300	241,000	-4,600	148,500	-11,200	68,250	-21,800	131,000	-22,400	28,000	-11,200
SUMMATIONS	1,055,900	-886,300	1,560,500	-778,150	336,955	-784,750	272,805	-887,800	496,750	-877,250	226,130	-856,100

Investigation for Stresses at Springing:- From the Tabulated From No. 20, the maximum bending moment and the corresponding thrust will be selected.

maximum bending moment = 1,560,500 ft. lbs.

corresponding horizontal thrust = 778,150 lbs.

$$e(= \text{eccentricity}) = \frac{1,560,500}{778,150} = 2.00 \text{ feet}$$

$$f_c = \frac{NK}{ba}$$

Assuming 1.50 net thickness of concrete for protection of reinforcement against rusting and fire and $p = 1\%$

$$\text{the } \frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{5.00 \times 12} = 0.05$$

$$\frac{e}{a} = \frac{2.00}{5.00} = 0.40$$

$$np = 15 \times 0.01 = 0.15$$

"From Design of Concrete Structures by L.C. Urquhart and C. E. O'rourke, Diagram 16 appendix D page 552, for the above data $K = 2.90$ and $k = 0.63$"

$$N\ (=\text{normal thrust}) = \frac{\text{horizontal thrust}}{\cos\phi}$$

$$\text{therefore, } N = \frac{778,150}{0.740} = 1,055,000 \text{ lbs}$$

$$fc = \frac{1,055,00 \times 2.90}{7.00 \times 5.00 \times 144} = 605 \text{ p.s.i.}$$

$$f_s = nf_c\left(\frac{d}{ka} - 1\right)$$

$$d = 5.00 - \frac{1}{2}(1.50 + 3/8 + 0.50) = 4.80 \text{ ft.}$$

$$\text{therefore } f_s = 15 \times 605\left(\frac{4.80}{0.63 \times 5.00} - 1\right) = 490 \text{ p.s.i.}$$

Clearly, the unit stresses are within the limits.

Investigation for Stresses at Quarter Point:-

From the Tabulated Form No. 20,

maximum bending moment = 336,955 ft. lbs.

corresponding horizontal thrust = 784,750 lbs.

$$e = \frac{336,955}{784,750} = 0.43 \text{ ft.}$$

$$\frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{3.28 \times 12} = 0.064$$

$$\frac{e}{a} = \frac{0.430}{3.28} = 0.131$$

$$np = 15 \times 0.01 = 0.15$$

For the above data K = 1.45; throughout all the section there is only compression

$$N \text{ (= normal thrust)} = \frac{\text{horizontal thrust}}{\cos\phi}$$

$$\text{therefore } N = \frac{784,750}{0.926} = 850,000 \text{ lbs.}$$

$$\text{maximum } f_c = \frac{850,000 \times 1.45}{3.28 \times 7.00 \times 144} = 375 \text{ p.s.i.}$$

Investigation for Stresses at Crown:-

From the Tabulated Form No. 20;

maximum bending moment = 496,750 ft. lbs.

corresponding horizontal thrust = 817,250 lbs.

$$e = \frac{496,750}{817,250} = 0.61 \text{ ft.}$$

$$\frac{d'}{a} = \frac{1.50 + 3/8 + 0.50}{3.28 \times 12} = 0.07$$

$$\frac{e}{a} = \frac{0.61}{3.00} = 0.205$$

$$np = 15 \times 0.01 = 0.15$$

For the above data $K = 1.80$

normal thrust = horizontal thrust

therefore, $f_c = \frac{817,250 \times 1.80}{3.00 \times 7.00 \times 144} = 490$ p.s.i.

From the Diagrams of Design of Concrete Structures by L.C. Urquhart and C.E. O'rourke, appendix D, it will be noticed that the unit tensile stress absorbed by the steel is very small:

Clearly, the maximum stresses are within the limits.

Longitudinal Reinforcement:- The longitudinal reinforcement will consists of 19 bars 1 in. diameter at the top and bottom, at the crown, increasing to 32 top and bottom bars, 1 in. diameter at the springing. "See Drawing No. 5 for reinforcement details.

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D E S I G N
O F
A B U T M E N T

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Data and Specifications:- The abutment will serve as a support for the bridge and as a retaining wall; it will be constructed of cyclopean concrete (specific weight, 140 pounds per cubinc foot). Total height 49'-04"; the abutment is to sustain a bank of earth with a horizontal surcharge of 70 pounds per square foot which is equivalet to 0.70 ft of filling above the top of the wall. The safe bearing pressure on the foundation bed, which consists of coarse sand and gravel, is 4.50-5.00 Tons per square foot.

The weight of the retained fill is 100 pounds per cubic foot; the angle of repose is 33 40'

Application of Fundumental Principles:- The procedure in the design is to select a tentative section. In the present case the tentative dimensions are shown in Drawing No. 4. During the procedure, an investigation first should be made with regard to the stability of the abutment and secondly it is desirable to arrange the width of base so that the normal resultant force from all the external permanent and non-permanent loadings passes approximately through the center of gravity of the base. This ensures an evenly distributed pressure upon the ground under the condition of loading causing settlement.

The effect of the live load, so far as foundation pressures are concerned, is of secondary importance, since the assumed maximum superimposed loading rarely, if ever, comes upon the structure.

In any case, the whole of the superimposed loading is much smaller from the dead weight of the structure, which is permanently present. However it is customary to include the effect of the maximum superimposed loading and to ensure that the maximum pressure under this condition does not exceed the safe bearing capacity of the ground.

Design of the Abutment:- Referring to Fig. 21 it will be seen that the forces coming upon the abutment from the arch consist of the horizontal thrust, vertical reaction applied at the point around which moments are taken so as to make the design safer and bending moment at springing, if any.
On the abutment itself the principal load is the weight of earth above it and its own weight. The earth filling behind the abutment also acts upon it in the opposite direction to that of the horizontal arch thrust.
The magnitude and position of its resultant may be found by Rankines Formula.

The above forces will now be calculated and moments taken about point T (Fig. 21) of the abutment in order to find the position of the upward resultant of the pressure from the ground under the abutment.

(a) Foundation Pressure under Dead Load only.

Vertical Reaction from Arch Ribs:- Lower part of ribs: 2(6.50x5x7x150)	= 68,200 lbs.
From the exterior half-panel of deck slab = 620 x 32	= 19,800 lbs.
From Drawing No. 2 2 x 114,550	= 229,100 lbs.
Carried forward	317,100 lbs.

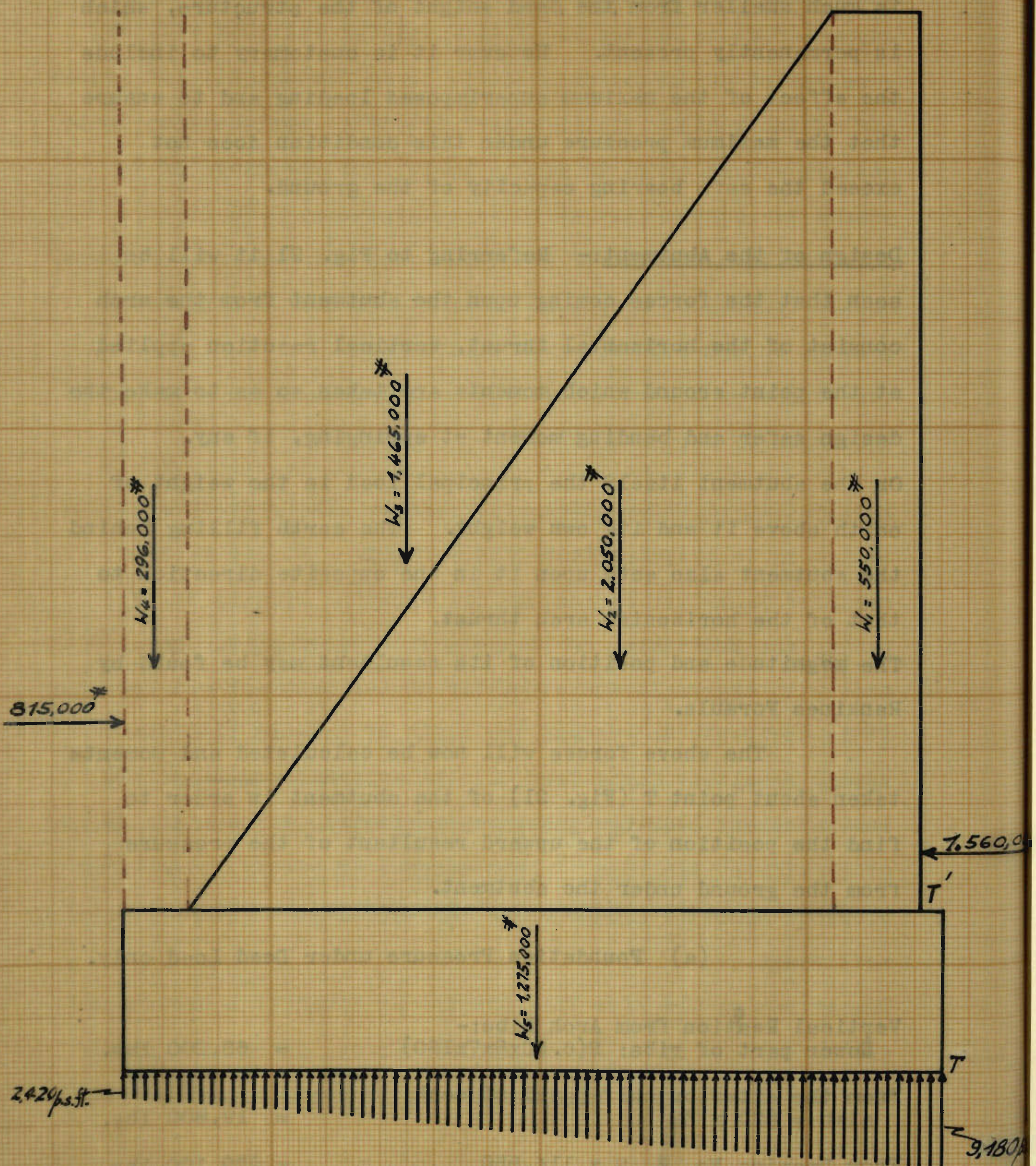
W4 = 296,000#
W3 = 1,465,000#
W2 = 2,050,000#
W1 = 550,000#
815,000#
7,560,0
T'
W5 = 1,275,000#
T
2,420 p.s.ft.
9,180

FIG. 21.

Brought forward.			317,100 lbs.
From Drawing No 2	2 x 102,850	=	205,700 lbs.
= = =	2 x 95,450	=	190,900 lbs.
= = =	2 x 89,530	=	179,060 lbs.
= = =	2 x 85,850	=	171,700 lbs.
= = =	2 x 84,175	=	168,350 lbs.
= = =	2 x 83,237	=	166,474 lbs.
			1,399,284 lbs.

Summation of Vertical Forces:-

From arch ribs	=	1,399,284
$W_1 = 4.00 \times 41.83 \times 23.50 \times 140$	=	550,000
$W_2 = (30.00 \times 41.83 \times 23.50 \times 140)\frac{1}{2}$	=	2,050,000
$W_3 = (30.00 \times 41.83 \times 23.50 \times 100)\frac{1}{2}$	=	1,465,000
$W_4 = (3.00 \times 41.83 \times 23.50 \times 100)$	=	296,000
$W_5 = 38.00 \times 32.00 \times 7.50 \times 140$	=	1,275,000
TOTAL	=	7,035,284 lbs.

Summation of Stabilizing Moments (Moments around Point T):-

(W_1) $550,000 \times 3.00$	=	1,650,000
(W_2) $2,050,000 \times (5.00 + \frac{30.00}{3})$	=	31,000,000
(W_3) $1,465,000 \times (5.00 + \frac{2}{3}\,30.00)$	=	36,700,000
(W_4) $296,000 \times 36.50$	=	10,800,000
(W_5) $1,275,000 \times 19.00$	=	24,300,000
Arch Thrusts : $2 \times 780,000 \times 10.25$	=	16,000,000
TOTAL		120,450,000 ft lbs.

Overturning Moment:-

The total active pressure of the earth is calculated from Rankine's formula ($H = \frac{wxhxhxb}{2} K$)

w = weight of earth per cubic foot = 100 lbs.

h = maximum height of earth = 49'-04"

l = length of the abutment = 23'-06"

K = constant depending upon the angle of repose, in the present case is 0.286

Total Earth Pressure = $\frac{100x49.33x49.33x0.286}{2}$ = 815,000 lbs.

This horizontal pressure acts at 49.33/3 = 16.40 feet above the bottom of the abutment.

Overturning moment = 815000 x 16.40 = 13,350,000 ft lbs

stabilizing moment = 120,450,000 ft lbs.

overturning moment = 13,350,000 ft lbs.

resultant moment = 107,100,000 ft lbs.

To find the position of the resultant upward component from point T of the abutment, the resultant moment will be divided by the sum of the vertical forces.

that is, $\frac{107,100,000 \text{ ft. lbs.}}{7,035,000 \text{ lbs.}}$ = 15.30 ft.

Clearly, the resultant upward force falls within the middle third since 15.30 feet is greater than 38/3 ft.

The eccentricity e of the resultant from the center line of the base is thus (38/2 - 15.30) = 3.70 ft.

The maximum pressure at the outside edge of theabutment is thus W/A + Wec/I

that is, maximum pressure = $\frac{7,035,284}{38x32} + \frac{7,035,284x3.70x19x12}{32x38x38x38}$

= 5,800 + 3,380

= 9,180 lbs. per square foot.

Evidently the maximum possible pressure is within the limits.

Factor of safety against overturning:-

Stabilizing moment = 120,450,000 ft. lbs.

Overturning moment = 13,350,000 ft lbs.

factor of safety = 120,450,000 / 13,350,000 = 9

Factor of safety against sliding:-

The coefficient of friction between the abutment and the foundation bed is presumed to be 0.60;

Resultant Vertical Force = 7.035,284 lbs.

Resultant Horizontal force = 815,000 - 780,000 x 2

= 745,000 lbs.

Factor of safety = 7,035,284 lbs x 0.6 / 745,000 lbs

= 5.65

Now a sectionX-X, at 07'-06" above the abutment base, will be passed and the part of the abutment above this section will be considered as a free body; see "Fig 21". The main purpose is to investigate wether the vertical component of the resultant pressure falls within the middle third.

Summation of Vertical Forces:-

From arch ribs		1,399,284 lbs
W_1	(Fig 21)	550,000 lbs.
W_2	(Fig 21)	2,050,000 lbs.
W_3	(Fig 21)	1,465,000 lbs.
	Total	5,464,284 lbs.

Summation of Stabilizing moments (Moments around point T'):-

(W_1)	$= 550{,}000 \times 2.00$	=	1,100,000
(W_2)	$= 2{,}050{,}000 \times (4 + \frac{30}{3})$	=	28,800,000
(W_3)	$= 1{,}465{,}000 \times (4 + \frac{2}{3}30)$	=	35,200,000
From arch thrusts :	$2 \times 780{,}000 \times 2.75$	=	4,300,000
			69,400,000 ft.lbs.

Overturning Moment :-

Total active pressure of earth = $\frac{w \times h \times h \times b}{2} K$

the height being 41' - 10"

Total Earth Pressure = $\frac{100 \times 41.83 \times 41.83 \times 23.50 \times 0.286}{2}$ = 590,000 lbs.

This horizontal pressure acts at $\frac{41.83}{3}$ = 13.94 feet above the point T'

Overturning Moment = 590,000 x 13.94 = 8,250,000 ft. lbs.

Stabilizing Moment =	69,400,000 ft. lbs.
Overturning Moment =	8,250,000 ft. lbs.
Resultant Moment =	61,150,000 ft. lbs

To find the position of the resultant upward component from point T' of the abutment, the resultant moment will be divided by the sum of the acting vertical forces.

that is, $\frac{61{,}150{,}000 \text{ ft. lbs.}}{5{,}464{,}284 \text{ lbs.}}$ = 11.30 feet.

Clearly, the resultant upward force falls at the edge of the middle third since 11.30 feet = $\frac{34.00 \text{ feet}}{3}$

The maximum pressure at point T' is $2 \frac{W}{A}$

That is, maximum pressure = $\frac{2 \times 5,464,284}{34.00 \times 23.50}$

= 13,700 lbs. per square foot

The allowable bearing pressure of cyclopean concrete is 36000 pounds per square foot, therefore the maximum possible stress is within the limits.

The unit shear of cyclopean concrete being 40 p.s.i., the allowable shearing force = 144 x 3400x23.50x40 = 4,600,000 lbs. the actual shearing force = 780,000x2-590,000 = 970,000 lbs. Evidently, the adopted section is safe against shearing stress.

If any, other section is passed through any position and the abutment is studied as a free body above this particular section, the resultant force will always fall within the middle third. Whereas the compressive and shearing stresses on the concrete mass will always be much less than the allowable one.

(b) Foundation Pressures, taking into account Suprimposed Loading:-

The arch horizontal thrust is maximum when the bridge is loaded throught all its span with the uniform live load and the knife edge load is at midspan.

Vertical Reaction from Arch Ribs:-

From Dead Load	=	1,399,284
From Uniform Live Load : 2 x 1120 x 75	=	168,000
From the Knife Edge Load : $\frac{2 \times 19000}{2}$	=	19,000
		1,586,284 lbs.

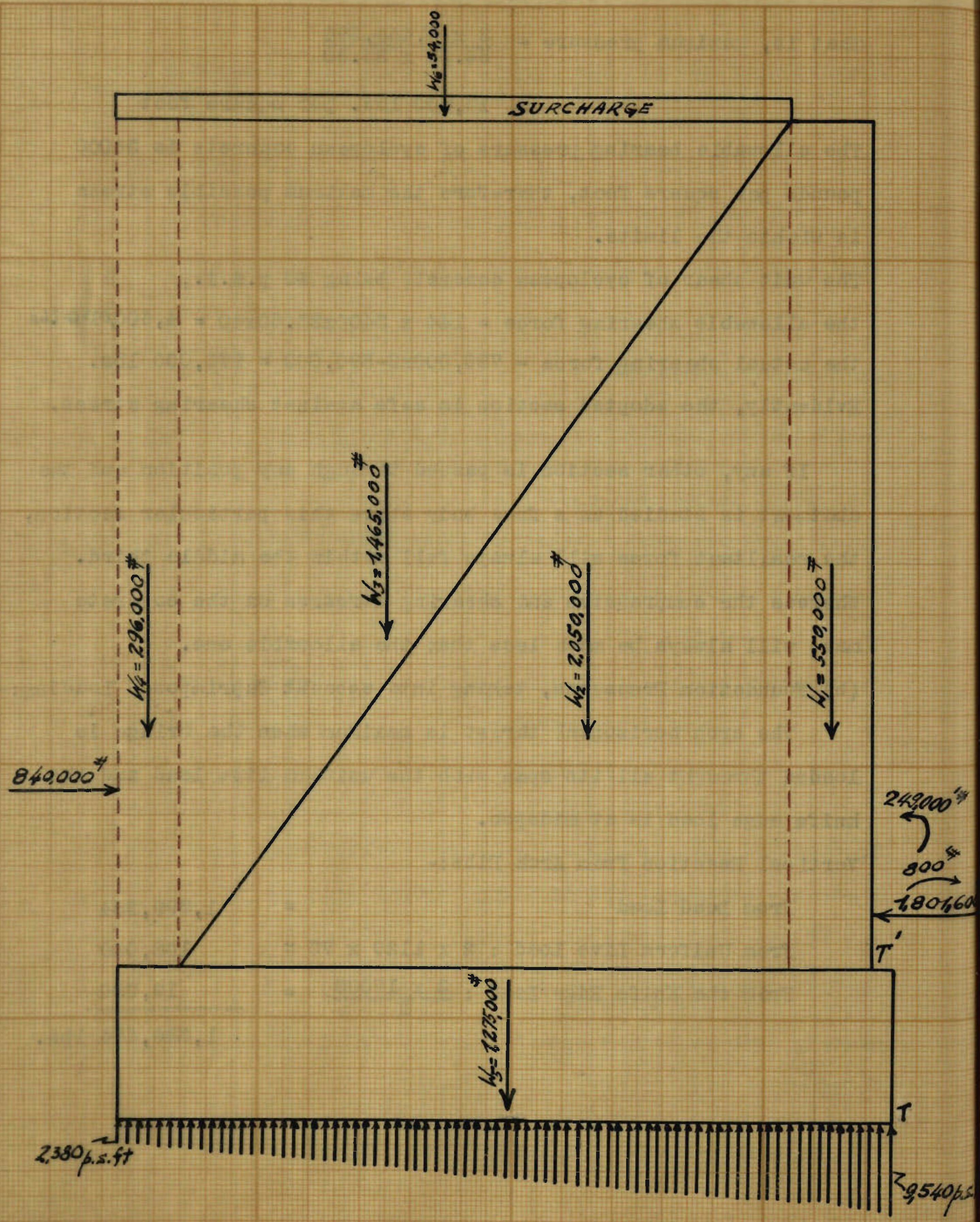
W6=54,000
SURCHARGE
W3=1,465,000#
W4=296,000#
W2=2,050,000#
W1=550,000#
840,000#
242,000'#
800#
1,801,60
T'
W5=1,275,000#
T
2,380 p.s.ft
9,540 p.s.

FIG. 22

Arch Horizontal Thrusts:-

From Dead Load = 2 x 780,000	=	1,560,000
From Uniform Live Load (see Tabulated Form No.10)		
= 2 x 98,300	=	196,600
From Knife Edge Load (see Fig.No.15)		
= 2 x 1.175 x19,000=		45,000
Total	=	1,801,600 lbs

Summation of Vertical Forces acting at the Foundation Bed:-

From arch ribs (Fig.22)	=	1,586,284
W_1 (Fig.22)	=	550,000
W_2 (Fig.22)	=	2,050,000
W_3 (Fig.22)	=	1,465,000
W_4 (Fig.22)	=	296,000
W_5 (Fig.22)	=	1,275,000
W_6 (Fig.22)=		54,000
Total		7,276,284 lbs.

Summation of Stabulizing Moments (Moments around point T):- The Knife Edge Load placed at mid-span to produce the maximum horizontal thrust, produces a moment at the springing which has got a stabilizing effect.

Moment due to Knife Edge Load: 2x19,000x6.55 (see Fig.18) = 249,000

(W_1)	550,000 x 3.00	=	1,650,000
(W_2)	2,050,000 x $(5.00 + \frac{30.00}{3})$	=	31,000,000
(W_3)	1,465,000 x $(5.00 + \frac{2}{3}\,30.00)$	=	36,700,000
(W_4)	296,000 x 36.50	=	10,800,000
(W_5)	1,275,000 x 19.00	=	24,300,000
(W_6)	54,000 x 21.50	=	1,160,000
Arch Thrust :	1,801,600 x 10.25	=	18,500,000
	Total	=	124,359,000

Overturning Moment :-

Total active Pressure of Earth = $\frac{w \times h \times h \times b \times K}{2}$

the height being (49'-04") + 0.70 ft. = 50.03 feet.

Total Earth pressure = $\frac{100 \times 50.03 \times 50.03 \times 23.50 \times 0.286}{2}$ = 840,000 lbs.

This horizontal pressure acts at $\frac{50.03}{3}$ = 16.68 feet above the bottom of the abutment.

The bridge when entirely loaded with a uniform live load, a moment at springing is produced which has got an overturning effect.

Moment due to uniform Live Load	=	800
Moment due to Earth Pressure = 840,000 x 16.68	=	14,100,000
Total	=	14,100,800 ft.lbs.

Stabilizing Moment	=	124,359,000
Overturning Moment	=	14,100,800
Resultant	=	110,259,800 ft. lbs.

To find the position of the resultant upward component from T of the abutment, the resultant moment will be divided by the sum of the vertical forces.

that is, $\frac{110,259,800}{7,276,284}$ = 15.20 feet.

Clearly, the resultant upward force falls within the middle third since 15.20 feet is greater than $\frac{30 \text{ feet}}{3}$; The eccentricity e is thus $(\frac{38}{2} - 15.20) = 3.80$ feet. The maximum pressure at the outside edge of the abutment is $\frac{W}{A} - \frac{Wec}{I}$

that is, maximum pressure = $\frac{7,276,284}{38 \times 32} + \frac{7,276,284 \times 3.80 \times 19 \times 12}{32 \times 38 \times 38 \times 38}$

= 5,960 + 3,580

= 9,540 lbs per square foot.

Evidently, the maximum possible pressure is within the limits.

Again the part of the abutment above the section X - X will be considered as a free body "see Fig.22"

Summation of Vertical Forces:-

From arch ribs. (Fig.22)	1,586,284
W_1 (Fig.22)	550,000
W_2 (Fig.22)	2,050,000
W_3 (Fig.22)	1,465,000
W_6 (Fig.22) 23.50 x 0.70 x 30.00 x 100	49,000
Total =	5,700,284 lbs.

Summation of Stabilizing Moments (Moments around point T'):-

From arch thrusts: 1,801,600 x 2.75	= 5,000,000
(W_1) 550,000 x 2.00	= 1,100,000
(W_2) 2,050,000 x $(4.00 - \frac{1}{3} 30.00)$	=28,800,000
(W_3) 1,465,000 x $(4.00 - \frac{2}{3} 30.00)$	=35,200,000
(W_6) 49,000 x 19.00	= 930,000
Moment due to Knife Edge Load: 2 x 19,000 x 6.55	= 249,000
Total =	71,279,000 ft.lbs.

Overturning Moment:-

Total active pressure of earth = $\frac{w \times h \times h \times b \times K}{2}$

the height being 41'-10"

Total Earth Pressure = $\frac{100 \times 41.83 \times 41.83 \times 23.50 \times 0.286}{2}$ = 590,000 lbs.

This horizontal pressure acts at $\frac{41.83}{3}$ = 13.94 feet

above the point T'

Moment due to Earth Pressure : 599,000 x 13.94 = 8,250,000

Moment due to Uniform Live Load = 800

8,250,800 ft.lbs

Stabilizing Moment = 71,279,000

Overturning Moment = 8,250,000

63,029,000 ft. lbs.

To find the position of the resultant upward component from point T' of the abutment, the resultant moment will be divided by the sum of the acting vertical forces.

that is, $\frac{63,029,000}{5,700,284}$ = 11.10 feet.

Evidently, the resultant upward force falls a little bit beyond the middle third edge since 11.10 feet is less than $\frac{34.00 \text{ feet}}{3}$

The eccentricity e is thus $(\frac{34.00}{2} - 11.10)$ = 5.90 feet

The maximum pressures at the outside edges of the abutment are

$$\frac{W}{A} \pm \frac{Wec}{I}$$

that is, $\frac{5,700,284}{34 \times 23.50} \pm \frac{5,700,284 \times 5.90 \times 17 \times 12}{23.50 \times 34 \times 34 \times 34}$

7,140 $\pm$ 7,420

maximum compressive unit stress = 14,560 lbs per square foot.

maximum tensile unit stress = 280 lbs per square foot

This minute tensile stress can be resisted by the concrete mass without resulting fissures.

Concluding we may say that the adopted section for the present abutment is satisfactory.

AESTHETICS

For a bridge to satisfy the aesthetic taste, an attempt should be made to fulfill the points discussed briefly below:-

1) <u>The design of the bridge must suit the particular conditions existing at the site</u>. The fulfillment of these conditions is in the main an engineering one and is satisfied by most modern bridges.
For the given specifications, such as, span, width, traffic loading etc. the design adopted fulfills the requirements of the site from the point of view of cheap construction and availability of material and labor from the neighborhood.

2) <u>Suitability of the materials to the design adopted:</u>
The open spandril type, which is basically a slender structure with graceful lines, cannot be realized fully except by using reinforced concrete which is a smooth surfaced, plastic medium.

The arch rib due to its flatness, should not be faced with stone since this will lead to a false impression, i.e. a stone arch can never be as flat as the one designed. This smooth continuity of the arch provides a clean strong look which cannot be equalled by any other material.

3) The appearance of the bridge should fulfill its purpose and form of construction: The purely functional shape of an open spandril arch bridge, every member of which satisfies a structural purpose, is a logical justification of its function. The elimination of a wide field for decorative purposes emphasizes the mathematical lines and geometrical shapes used in the structure.

4) Proportions, masses and lines: The bridge is by nature beautiful and no attempt should be made to distort the lines and proportions in the general appearance. The lines of the columns are straight with no projecting elements or architectural stylistic features to disturb the equilibrium of two straight lines flowing into a semi-circular form.

The overchanging portions of the transverse beams have been moulded in a way as to cast a soft shadow on the surface below it.

The arch rib has also been recessed so as to provide a surface for shadow effect as well as to accomodate drainage.

The parapet has neither paneling nor openings so as to disturb the continuity, thus emphasizing the horizontal lines of the bridge.

5) Texture and Color of the material: Buildings in the neighborhood have plastered whitewashed walls and are not highly decorated. Consequently, the bridge surface will be

plastered all over except for certain areas discussed below so as to conform with the neighboring structures.

The exterior and interior plinth of the parapet will be bushhammered so as to provide a surface which is slightly darker from the untreated one and which also provides protection against weathering.

The part of the retaining wall which is exposed will be dressed with rock face stone which offers a rich contrast to the plain surface of the concrete.

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E S T I M A T E   O F   C O S T

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M A T E R I A L	QUANTITY	UNIT COST	ESTIM.COST	REMARKS
Excavation........	980 cu.yds.	L.L. 5	L.L. 2,940	All nature of soils..........
Cyclopean Concrete including all materials and labor for forms and pouring concrete.........	2,000 cu.yds.	L.L. 31	L.L. 62,000	50% hard and clean rubble stones. 3 bags of cement per cubic yard.....
Concrete for Deck Slab, Transverse Beams, Longitudinal Beams, Columns, Arch Ribs and Parapets including all materials and labor for forms and pouring concrete......	650 cu.yds.	L.L. 54	L.L. 35,100	
Reinforcement including price of steel and labor...	124 Kips	L.L. 275	L.L. 34,000	
			L.L. 134,040	

MATERIAL	QUANTITY	UNIT COST	ESTI.COST	REMARKS
			L.L. I34,040	
Plastering.........	2,760 sq.yds.	L.L. I.25	L.L. 3,500	6 bags of cement per cu. yard of sand...
Kerb Stone..........	II0 yards	L.L. I.40	L.L. L55	White dressed stone from the region....
Sand for footway and roadway...........	4I.50 cu.yds.	L.L. 4.60	L.L. I95	Fine grains ....
Asphalt "Idealite" 2 in. thickness	550 sq.yds.	L.L. I.70	L.L. 940	
			L.L. I38,830	
Contractor's Profit I5%...			L.L. 2I,000	
			L.L. I59,830	

BIBLIOGRAPHY

The following works of reference have been consulted:-

Arch Design by W. A. Fairhurst.

Concrete Plain and Reinforced Vol. II by Taylor, Thomson and Smulski.

Design of Concrete Structures by Urquhart and O'rourke.

Reinforced Concrete Bridge Design by Chettoe and Adams.

The Tables and Diagrams, at the end, have been reproduced from "Arch Design by W.A.FAIRHURST" and "Concrete Plain and Reinforced Vol. II by TAYLOR, THOMSON, and SMULSKI" respectively.

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T A B L E S

A N D

D I A G R A M S

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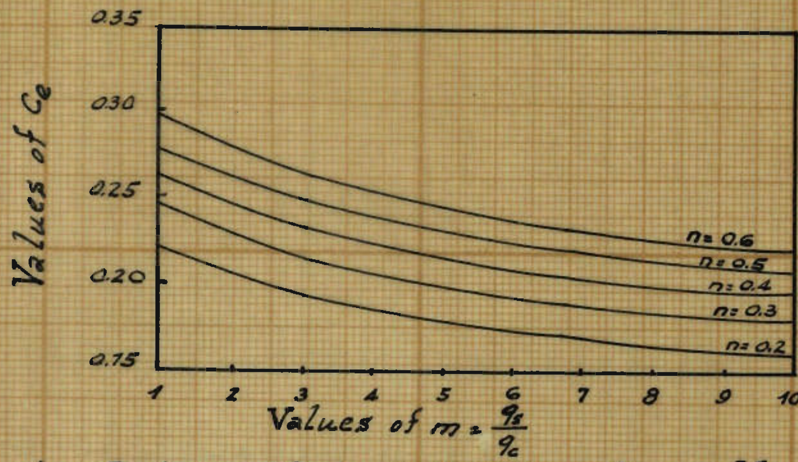
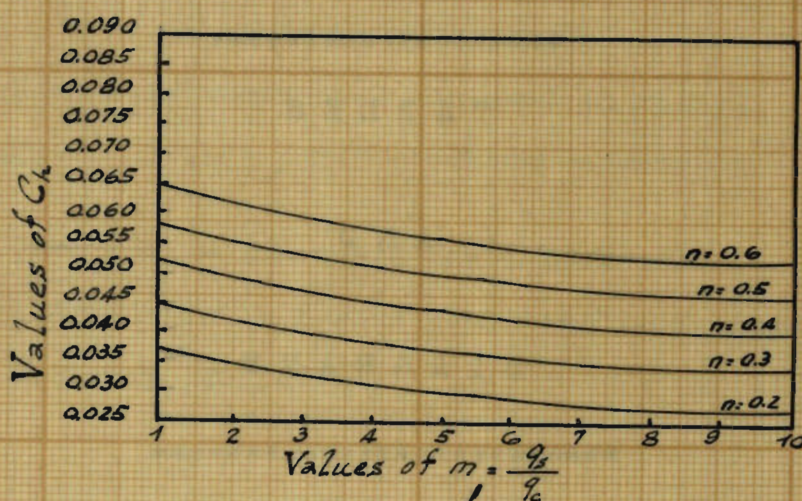
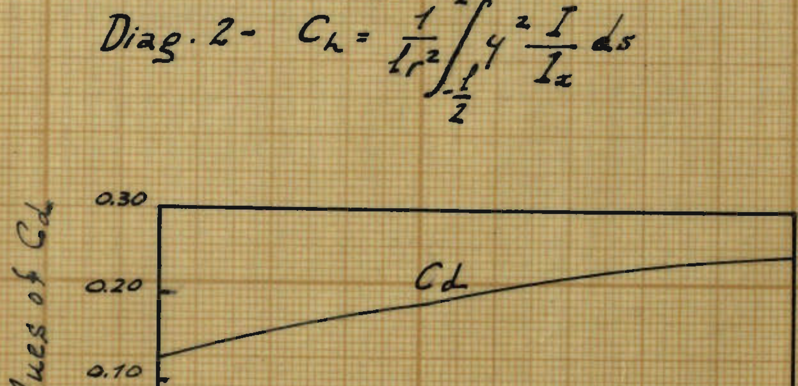


Diag. 1 - Distance Elastic Center from Crown $Y_e = C_e r$



Diag. 2 - $C_h = \frac{1}{l r^2}\int_{-\frac{l}{2}}^{\frac{l}{2}} y^2 \frac{I}{I_x} ds$



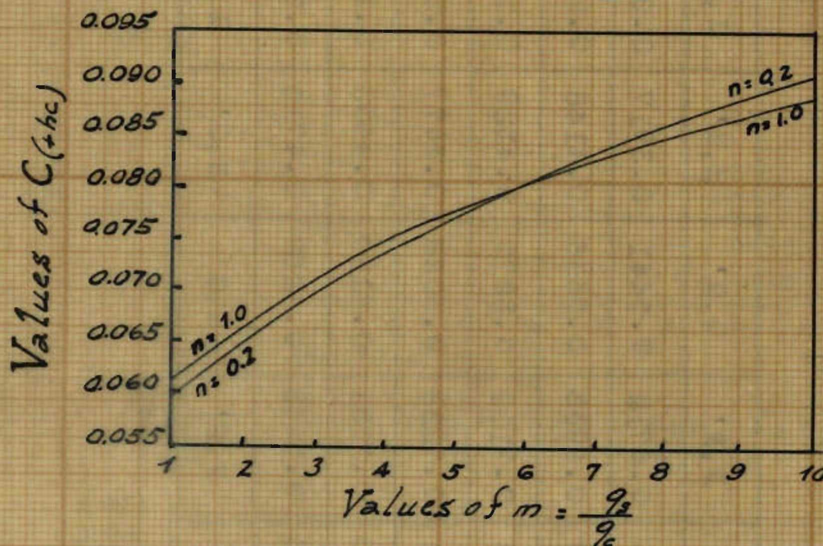
Diag. 3. - Constant C_d for Horizontal Thrust for Dead Load.

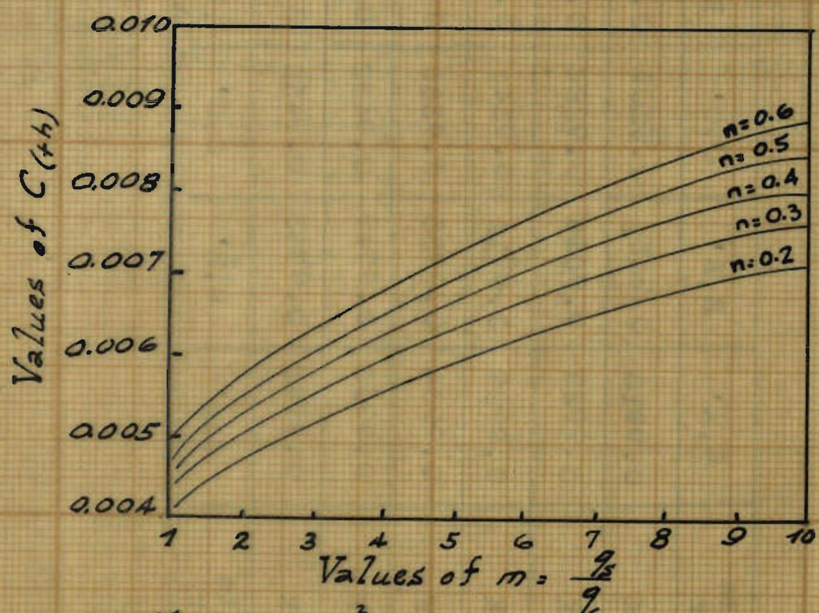
TABLE N° 1

HEIGHT OF ARCH ORDINATES

Ht. of Arch Ordinate = Kr

Ordinate Letter		A,A'	B,B'	C,C'	D,D'	E,E'	F,F'	G,G'	H,H'	I,I'	S,S'
m=2	K	0.9913	0.9651	0.9209	0.8580	0.7753	0.6712	0.5441	0.3917	0.2114	0.000
m=3	K	0.9922	0.9686	0.9284	0.8705	0.7929	0.6933	0.5686	0.4148	0.2272	0.000
m=4	K	0.9929	0.9712	0.9341	0.8799	0.8063	0.7102	0.5875	0.4329	0.2398	0.000
m=5	K	0.9934	0.9733	0.9385	0.8873	0.8170	0.7238	0.6028	0.4477	0.2503	0.000
m=6	K	0.9938	0.9749	0.9422	0.8935	0.8258	0.7351	0.6157	0.4603	0.2592	0.000
m=7	K	0.9942	0.9763	0.9452	0.8986	0.8333	0.7448	0.6268	0.4712	0.2670	0.000
Ordinate Letter		A,A'	B,B'	C,C'	D,D'	E,E'	F,F'	G,G'	H,G'	I,I'	S,S'





$M_c = C_{(+c)} w l^2 \quad H_c = C_{(+hc)} w l \frac{l}{r}$

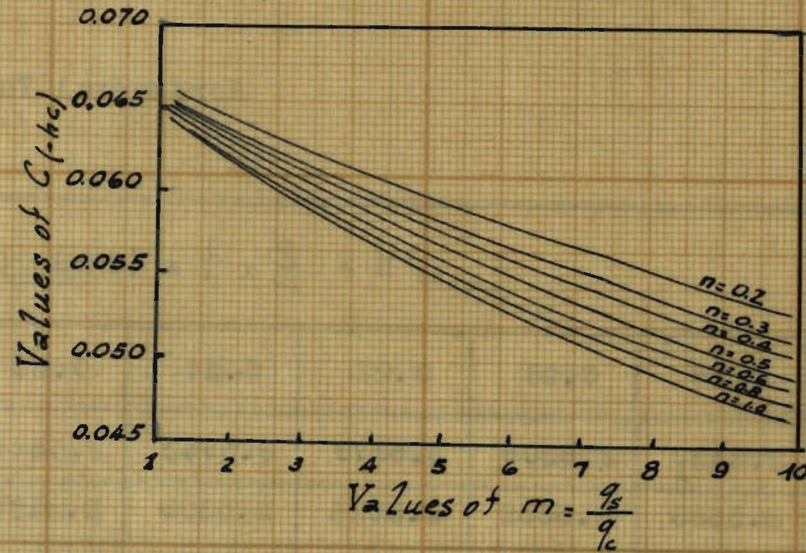
Diag. 4 - Coefficients for Maximum Positive Bending Moment and Corresponding Horizontal Thrust at Crown.

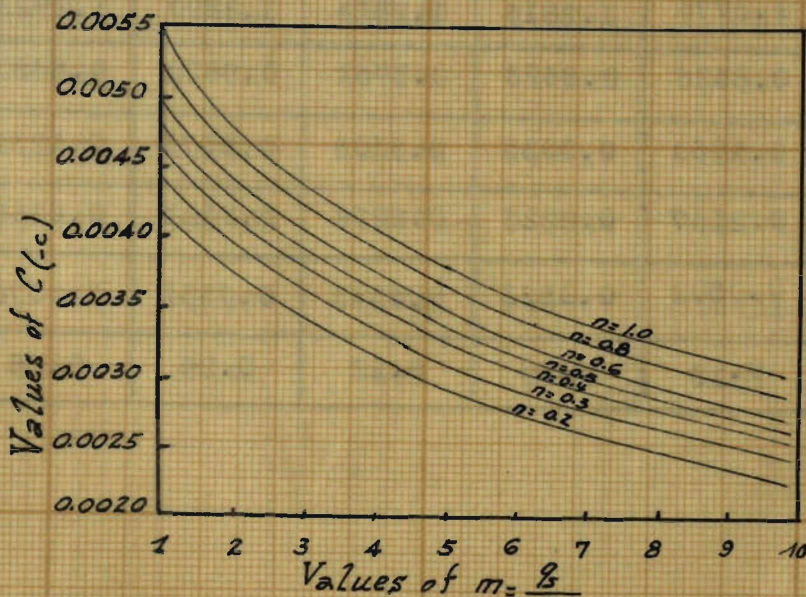
TABLE No 2

Horizontal Thrust for Load "P" at any ordinate .

$H = \frac{P}{10} \times \frac{1}{r} \times$ (Table Coefficient) $n = \frac{I_c}{I_s \cos \phi_s}$ m = 2

n		0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
Ordinate Ref. Letter	Cr.	2.6351	2.6206	2.6070	2.5943	2.5824	2.5711	2.5605	2.5458	2.5321	2.5194	2.5076	2.4966	2.4862	2.4765	2.4674
	A	2.5686	2.5552	2.5626	2.5309	2.5199	2.5096	2.4998	2.4862	2.4736	2.4619	2.4510	2.4408	2.4313	2.4224	2.4139
	B	2.3755	2.3653	2.3558	2.3468	2.3384	2.3305	2.3231	2.3126	2.3303	2.2941	2.2858	2.2780	2.2708	2.2639	2.2575
	C	2.0749	2.0693	2.0641	2.0592	2.0546	2.0502	2.0461	2.0404	2.0351	2.0302	2.0257	2.0214	2.0174	2.0137	2.0102
	D	1.6959	1.6954	1.6949	1.6945	1.6941	1.6937	1.6933	I.6928	I.6924	I.69I9	I.69I5	I.69II	I.6908	I.6905	I.6902
	E	I.2775	I.28I6	I.2854	I.2890	I.2923	I.2955	I.2985	I.3026	I. 3065	I.3I0I	I.3I34	I.3I65	I.3I94	I.322I	I.3247
	F	0.8626	0.8696	0.876I	0.8822	0.8880	0.8934	0.8985	0.9056	0.9I22	0.9I83	0.9239	0.9292	0.9342	0.9389	0.9433
	G	0.4960	0.5035	0.5I05	0.5I7I	0.5232	0.5290	0.5345	0.5422	0.5492	0.5558	0.56I9	0.5676	0.5729	0.5780	0.5827
	H	0.2I67	0.2222	0.2274	0.2322	0.2368	0.24II	0.245I	0.2508	0.2560	0.2608	0.2653	0.2695	0.2734	0.277I	0.2806
	I	0.0502	0.0523	0.0543	0.056I	0.0578	0.0595	0.06I0	0.063I	0.065I	0.0670	0.0687	0.0702	0.07I7	0.073I	0.0745
n		0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.5I	0.54





$M_c = -C_{(-c)} w l^2$, $H_c = -C_{(hc)} w l \frac{l}{r}$

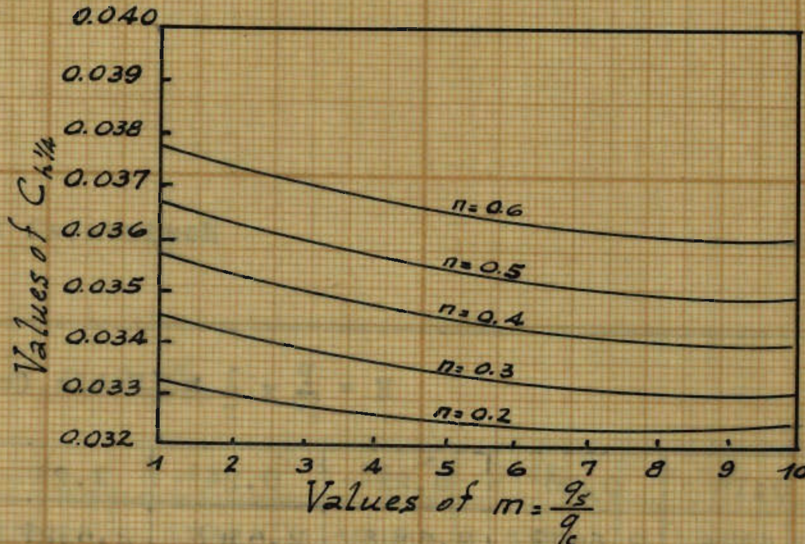
Diag. 5.- Coefficients for Maximum Negative Bending Moment and Corresponding Horizontal Thrust at Crown.

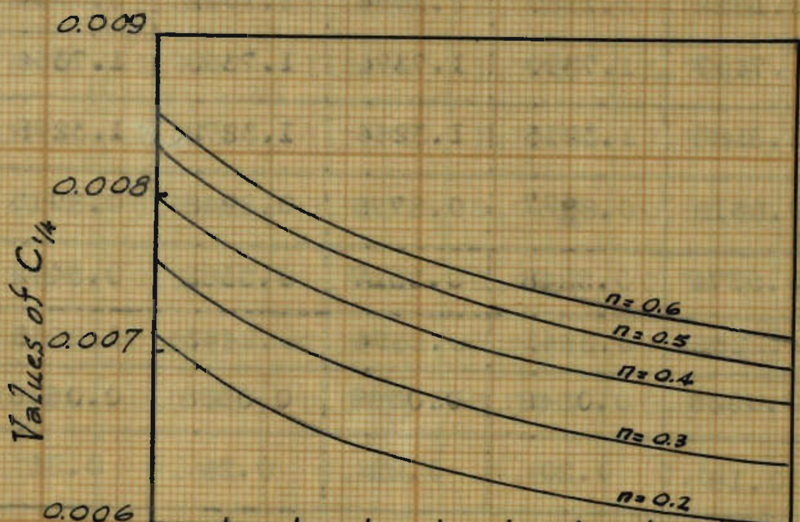
T A B L E N° 3

Horizontal Thrust for Load "P" at any ordinate.

$H = \frac{P}{10} \times \frac{l}{r} \times$ (Table Coefficient) $\quad n = \frac{I_c}{I_s \cos \phi_s}$ $\quad$ **m = 3**

Ordinate Ref. Letter	n	0;18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
	Cr.	2.6823	2.6659	2.6506	2.6363	2.6229	2.6103	2.5984	2.5819	2.5665	2.5525	2.5394	2.5271	2.5156	2.5049	2.4947
	A	2.8161	2.6008	2.5866	2.5733	2.5609	2.5492	2.5382	2.5228	2.5086	2.4955	2.4833	2.4719	2.4612	2.4512	2.4418
	B	2.4232	2.4113	2.4001	2.3897	2.3799	2.3708	2.3621	2.3501	2.3390	2.3287	2.3191	2.3102	2.3018	2.2940	2.2868
	C	2.1222	2.1151	2.1084	2.1022	2.0964	2.0909	2.8867	2.0786	2.0719	2.0658	2.0601	2.0548	2.0498	2.0451	2.0407
	D	1.7407	1.7390	1.7374	1.7360	1.7364	1.7333	1.7320	1.7303	1.7288	1.7273	1.7259	1.7248	1.7235	1.7224	1.7213
	E	1.3183	1.3215	1.3244	1.3272	1.3298	1.3322	1.3345	1.3377	1.3407	1.3434	1.3460	1.3484	1.3506	1.3527	1.3547
	F	0.8612	0.8697	0.8775	0.8848	0.8917	0.8981	0.9042	0.9127	0.9205	0.9277	0.9344	0.9407	0.9465	0.9520	0.9572
	G	0.5174	0.5248	0.5317	0.5382	0.5442	0.5499	0.5552	0.5627	0.5695	0.5759	0.5818	0.5873	0.5925	0.5973	0.6019
	H	0.2285	0.2341	0.2394	0.2443	0.2488	0.2531	0.2572	0.2628	0.2680	0.2729	0.2773	0.2815	0.2854	0.2891	0.2926
	I	0.0526	0.0548	0.0569	0.0588	0.0606	0.0623	0.0639	0.0661	0.0681	0.0700	0.0718	0.0734	0.0749	0.0764	0.0777
	n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54





$$M_{1/4} = C_{1/4}\, w l^2 \;,\quad H_{1/4} = C_{h1/4}\, w l \frac{l}{r}.$$

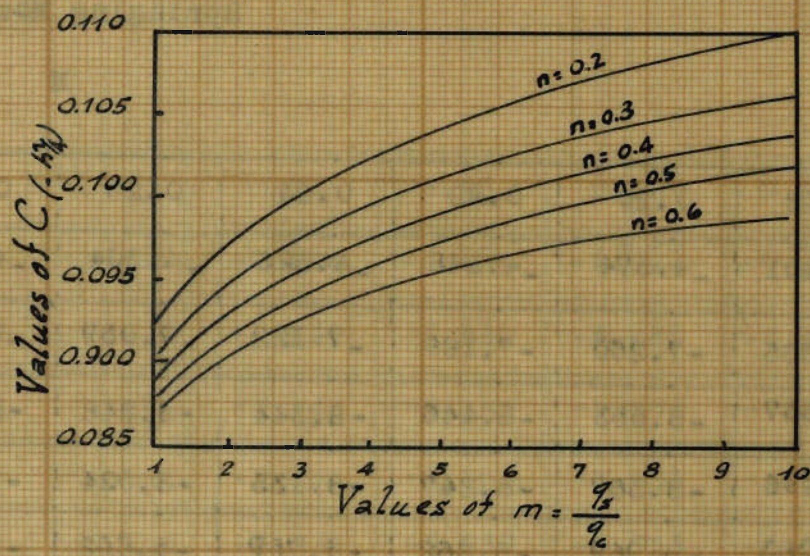
Diagr. 6. _ Coefficients for Maximum Positive Bending Moment and Corresponding Horizontal Thrust at Quarter Point.

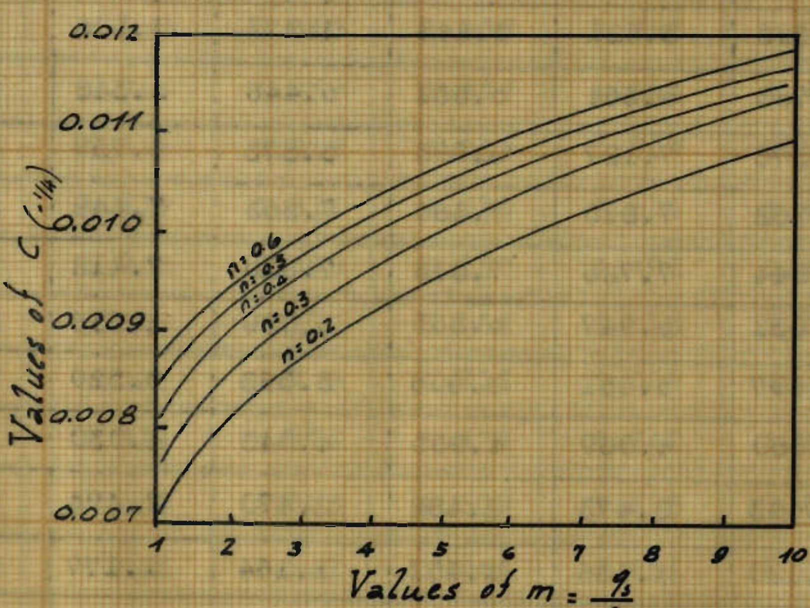
TABLE Nº 4

Springing Moment for Load "P" at any ordinate : m = 2

$$M = \frac{Pl}{100} \times \text{Table Coefficient.}$$

Ordinate Ref. Letter / n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
I	-4.40I	-4.374	-4.348	-4.326	-4.304	-4.282	-4.262	-4.234	-4.208	-4.I84	-4.I6I	-4.I42	-4.I22	-4.I03	-4.085
H	-7.336	-7.260	-7.I89	-7.I22	-7.059	-6.999	-6.942	-6.863	-6.789	-6.72I	-6.657	-6.597	-6.542	-6.489	-6.439
G	-8.667	-8.553	-8.446	-8.344	-8.248	-8.I57	-8.07I	-7.950	-7.838	-7.733	-7.635	-7.543	-7.458	-7.376	-7.300
F	-8.496	-8.367	-8.247	-8.I33	-8.024	-7.92I	-7.824	-7.687	-7.559	-7.439	-7.328	-7.223	-7.I24	-7.03I	-6.942
E	-7.083	-6.969	-6.862	-6.759	-6.662	-6.569	-6.48I	-6.356	-6.239	-6.I29	-6.027	-5.930	-5.839	-5.753	-5.67I
D	-4.787	-4.7II	-4.639	-4.569	-4.502	-4.438	-4.377	-4.290	-4.200	-4.I30	-4.056	-3.986	-3.9I9	-3.857	-3.797
C	-1.990	-1.966	-1.942	-1.918	-1.894	-I.872	-1.848	-1.8I4	-1.78I	-I.748	-I.7I6	-I.685	-I.657	-I.627	-I.600
B	0.92I	0.888	0.859	0.832	0.808	0.788	0.769	0.744	0.723	0.705	0.689	0.676	0.665	0.655	0.646
A	0.605	3.52I	3.443	3.37I	3.306	3.245	3.187	3.I09	3.037	2.972	2.913	2.857	2.807	2.762	2.7I8
Cr	5.785	5.664	5.55I	5.446	5.348	5.255	5.169	5.050	4.940	4.838	4.745	4.659	4.577	4.502	4.433
A'	7.276	7.127	6.996	6.873	6.759	6.652	6.549	6.408	6.276	6.I54	6.042	5.935	5.838	5.747	5.660
B'	7.952	7.8I5	7.686	7.563	7.448	7.340	7.237	7.093	6.956	6.835	6.7I9	6.6II	6.5I0	6.4I4	6.324
C'	7.844	7.726	7.616	7.510	7.4I2	7.3I6	7.226	7.I00	6.98I	6.870	6.766	6.669	6.577	6.49I	6.4I0
D'	7.03I	6.947	6.865	6.789	6.7I6	6.646	6.579	6.482	6.393	6.308	6.228	6.I92	6.08I	6.0I3	5.949
E'	5.697	5.65I	5.606	5.563	5.520	5.48I	5.44I	5.382	5.329	5.275	5.225	5.I78	5.I3I	5.087	5.045
F'	4.080	4.069	4.055	4.043	4.030	4.0I5	4.002	3.979	3.957	3.935	3.9I2	3.89I	3.870	3.849	3.830
G'	2.463	2.473	2.482	2.490	2.494	2.499	2.503	2.506	2.506	2.507	2.505	2.503	2.500	2.498	2.494
H'	1.I20	1.I36	1.I5I	1.I64	1.I77	1.I87	1.I96	I.209	I.2I9	I.227	I.235	I.24I	I.246	I.25I	I.255
I'	0.267	0.276	0.284	0.290	0.296	0.302	0.308	0.3I4	0.320	0.326	0.33I	0.334	0.338	0.34I	0.345
n	0.I8	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.5I	0.54





$$M_{1/4} = -C_{(-1/4)}\, w l^2 \;,\quad H_{1/4} = -C_{(h1/4)}\, w l \frac{l}{r}$$

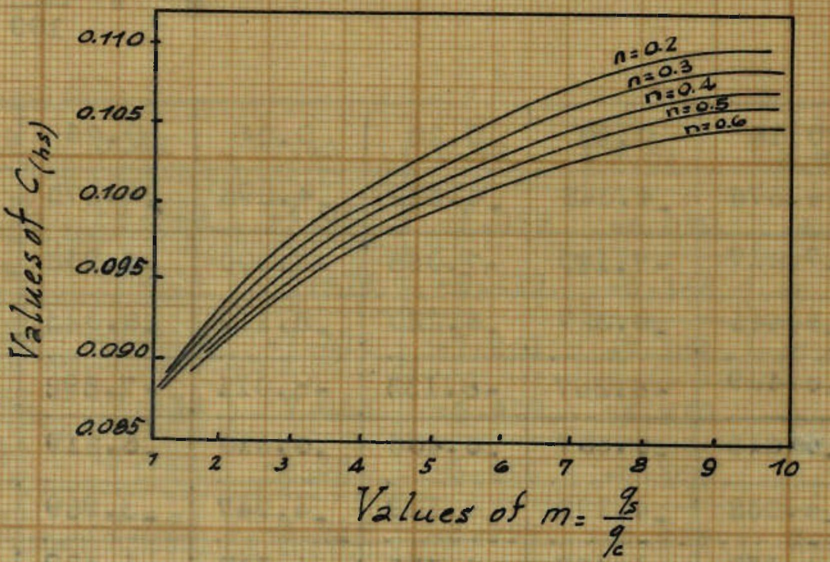
Diag. 7. – Coefficients for Maximum Negative Bending Moment and Corresponding Horizontal Thrust at Quarter Point.

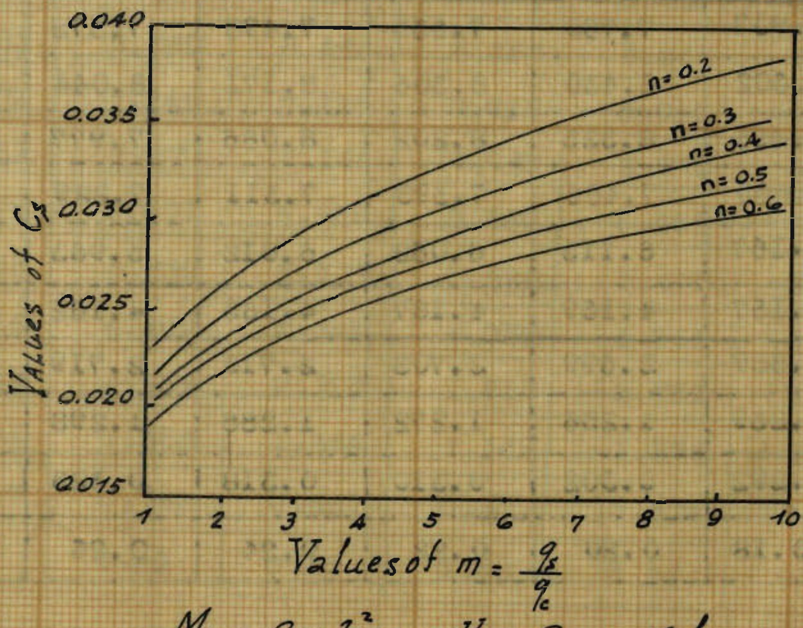
TABLE No 5

Springing Moment for Load "P" at any ordinate m = 3

$$M = \frac{Pl}{100} \times \text{Table Coefficient}$$

Ordinate Ref: Letter \ n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
I	-4.376	-4.348	-4.322	-4.298	-4.275	-4.253	-4.233	-4.204	-4.178	-4.153	-4.130	-4.109	-4.089	-4.070	-4.053
H	-7.217	-7.140	-7.068	-7.000	-6.938	-6.877	-6.820	-6.741	-6.667	-6.597	-6.534	-6.474	-6.419	-6.366	-6.316
G	-8.441	-8.327	-8.220	-8.119	-8.023	-7.933	-7.848	-7.728	-7.617	-7.514	-7.416	-7.326	-7.241	-7.162	-7.086
F	-8.409	-8.267	-8.135	-8.011	-7.892	-7.781	-7.675	-7.526	-7.387	-7.258	-7.138	-7.025	-6.919	-6.819	-6.725
E	-6.613	-6.507	-6.406	-6.310	-6.219	-6.133	-6.050	-5.932	-5.823	-5.719	-5.622	-5.531	-5.445	-5.364	-5.286
D	-4.238	-4.172	-4.109	-4.047	-3.989	-3.933	-3.879	-3.801	-3.727	-3.659	-3.593	-3.529	-3.470	-3.413	-3.360
C	-1.378	-1.367	-1.356	-1.342	-1.329	-1.316	-1.302	-1.279	-1.258	-1.235	-1.212	-1.190	-1.169	-1.148	-1.128
B	1.571	1.523	1.479	1.440	1.404	1.373	1.343	1.306	1.272	1.242	1.215	1.193	1.172	1.155	1.137
A	4.275	4.174	4.082	3.996	3.919	3.845	3.777	3.683	3.598	3.521	3.451	3.386	3.326	3.271	3.219
Cr	6.460	6.322	6.194	6.075	5.964	5.860	5.762	5.628	5.503	5.390	5.286	5.186	5.098	5.015	4.937
A'	7.937	7.780	7.635	7.498	7.372	7.252	7.139	6.982	6.837	6.703	6.580	6.464	6.357	6.256	6.161
B'	8.602	8.430	8.306	8.171	8.044	7.926	7.811	7.656	7.508	7.372	7.245	7.128	7.017	6.914	6.815
C'	8.456	8.325	8.202	8.086	7.977	7.872	7.772	7.635	7.504	7.383	7.270	7.164	7.065	6.970	6.882
D'	7.580	7.486	7.395	7.311	7.229	7.151	7.077	6.971	6.873	6.779	6.691	6.609	6.530	6.457	6.386
E'	6.167	6.113	6.062	6.012	5.963	5.917	5.872	5.806	5.745	5.685	5.630	5.577	5.525	5.476	5.430
F'	4.167	4.169	4.167	4.165	4.162	4.155	4.151	4.140	4.129	4.116	4.102	4.089	4.075	4.061	4.047
G'	2.689	2.699	2.708	2.715	2.719	2.723	2.726	2.728	2.727	2.726	2.724	2.720	2.717	2.712	2.708
H'	1.239	1.256	1.272	1.286	1.298	1.309	1.318	1.331	1.341	1.351	1.358	1.364	1.369	1.374	1.378
I'	0.292	0.302	0.310	0.318	0.325	0.331	0.337	0.344	0.350	0.357	0.362	0.367	0.371	0.374	0.377
n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54





$M_s = C_s w l^2, \quad H_s = C_{(hs)} w l \frac{l}{r}.$

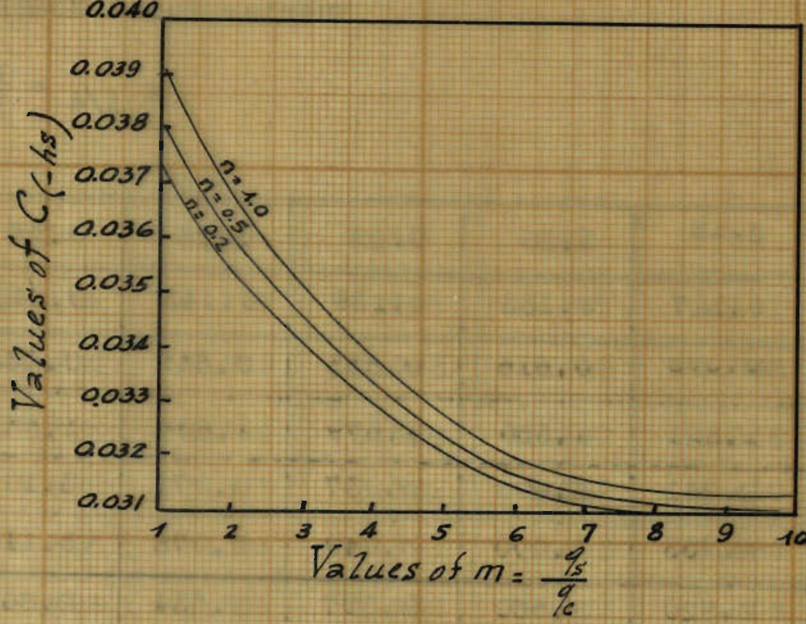
Diagr. 8.– Coefficients for Maximum Positive Bending Moment and Corresponding Thrust at Springing.

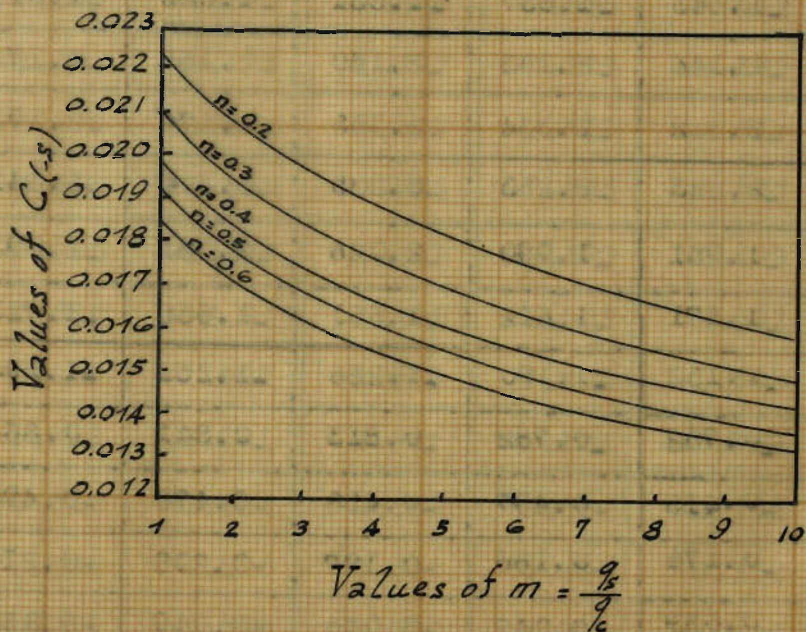
TABLE N° 6

Quarter Point Moment for Load "P" at any ordinate m = 2

$$M_{1/4} = \frac{Pl}{100} \times \text{Table Coefficient}$$

Ordinate Ref. Letter \ n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
I	0.127	0.133	0.139	0.143	0.148	0.153	0.158	0.164	0.169	0.174	0.179	0.183	0.187	0.191	0.195
H	0.598	0.616	0.633	0.649	0.664	0.678	0.692	0.711	0.728	0.744	0.759	0.773	0.785	0.798	0.809
G	1.521	1.550	1.579	1.606	1.631	1.656	1.679	1.711	1.740	1.768	1.794	1.818	1.840	1.862	1.881
F	2.961	3.000	3.037	3.072	3.105	3.137	3.167	3.209	3.248	3.286	3.319	3.352	3.382	3.410	3.438
E	4.958	5.000	5.040	5.078	5.115	5.150	5.183	5.230	5.275	5.315	5.354	5.391	5.425	5.458	5.488
D	2.520	2.560	2.597	2.634	2.669	2.703	2.735	2.779	2.823	2.863	2.902	2.938	2.973	3.005	3.036
C	0.633	0.665	0.696	0.725	0.754	0.781	0.808	0.846	0.882	0.917	0.950	0.982	1.012	1.041	1.069
B	-0.738	-0.718	-0.698	-0.680	-0.661	-0.641	-0.624	-0.598	-0.572	-0.548	-0.524	-0.501	-0.479	-0.457	-0.436
A	-1.643	-1.637	-1.631	-1.625	-1.617	-1.610	-1.602	-1.591	-1.581	-1.569	-1.557	-1.546	-1.535	-1.522	-1.511
Cr	-2.144	-2.153	-2.160	-2.167	-2.172	-2.178	-2.182	-2.187	-2.190	-2.194	-2.195	-2.197	-2.198	-2.197	-2.196
A'	-2.312	-2.334	-2.355	-2.374	-2.391	-2.406	-2.421	-2.442	-2.461	-2.478	-2.492	-2.507	-2.519	-2.530	-2.540
B'	-2.223	-2.255	-2.285	-2.314	-2.341	-2.365	-2.390	-2.424	-2.454	-2.483	-2.509	-2.534	-2.556	-2.577	-2.597
C'	-1.951	-1.990	-2.026	-2.061	-2.093	-2.126	-2.155	-2.197	-2.237	-2.273	-2.309	-2.341	-2.372	-2.400	-2.427
D'	-1.571	-1.612	-1.651	-1.688	-1.723	-1.756	-1.789	-1.835	-1.878	-1.918	-1.957	-1.993	-2.027	-2.061	-2.091
E'	-1.152	-1.190	-1.226	-1.261	-1.294	-1.325	-1.357	-1.401	-1.442	-1.483	-1.521	-1.556	-1.591	-1.623	-1.654
F'	-0.752	-0.782	-0.813	-0.841	-0.868	-0.895	-0.921	-0.959	-0.994	-1.029	-1.061	-1.092	-1.121	-1.150	-1.176
G'	-0.415	-0.437	-0.458	-0.478	-0.498	-0.516	-0.535	-0.562	-0.588	-0.612	-0.636	-0.659	-0.681	-0.702	-0.722
H'	-0.174	-0.186	-0.197	-0.208	-0.218	-0.229	-0.239	-0.254	-0.268	-0.282	-0.295	-0.308	-0.321	-0.332	-0.344
I'	-0.039	-0.042	-0.045	-0.049	-0.052	-0.055	-0.058	-0.062	-0.067	-0.071	-0.075	-0.080	-0.083	-0.087	-0.090
u	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54





$$M_s = -C_{(-s)} \omega l^2 \quad , \quad H_s = -C_{(-hs)} \omega l \frac{l}{r}$$

Diagr. 9. - Coefficients for Maximum Negative Bending Moment and Corresponding Thrust at Springing.

TABLE N° 7

Quarter Point Moment for Load "P" at any ordinate m = 3

$$M_{1/4} = \frac{Pl}{100} \times \text{Table Coefficient.}$$

Ordinate Ref. Letter	n = 0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54
I	0.124	0.130	0.135	0.140	0.145	0.149	0.153	0.159	0.164	0.170	0.174	0.178	0.182	0.185	0.188
H	0.586	0.603	0.619	0.635	0.649	0.663	0.675	0.694	0.710	0.726	0.740	0.754	0.765	0.777	0.788
G	1.489	1.518	1.546	1.572	1.598	1.621	1.644	1.675	1.704	1.730	1.756	1.779	1.800	1.821	1.840
F	2.907	2.946	2.983	3.018	3.052	3.082	3.112	3.154	3.194	3.230	3.263	3.295	3.325	3.353	3.379
E	4.879	4.920	4.960	4.997	5.033	5.067	5.100	5.146	5.189	5.230	5.269	5.305	5.339	5.371	5.402
D	2.415	2.454	2.491	2.528	2.562	2.595	2.627	2.673	2.716	2.755	2.794	2.831	2.865	2.898	2.929
C	0.504	0.536	0.566	0.597	0.626	0.653	0.679	0.719	0.755	0.790	0.824	0.856	0.887	0.916	0.944
B	-0.885	-0.865	-0.845	-0.826	-0.806	-0.787	-0.769	-0.741	-0.715	-0.690	-0.666	-0.641	-0.618	-0.595	-0.574
A	-1.802	-1.796	-1.789	-1.782	-1.773	-1.766	-1.758	-1.746	-1.733	-1.720	-1.707	-1.694	-1.681	-1.669	-1.656
Cr	-2.308	-2.316	-2.322	-2.328	-2.333	-2.337	-2.341	-2.344	-2.347	-2.349	-2.349	-2.348	-2.348	-2.346	-2.343
A'	-2.471	-2.493	-2.513	-2.531	-2.547	-2.563	-2.577	-2.596	-2.614	-2.629	-2.643	-2.655	-2.666	-2.676	-2.685
B'	-2.370	-2.401	-2.432	-2.460	-2.486	-2.511	-2.535	-2.567	-2.597	-2.625	-2.651	-2.674	-2.696	-2.715	-2.735
C'	-2.080	-2.119	-2.155	-2.189	-2.222	-2.254	-2.284	-2.325	-2.365	-2.401	-2.435	-2.467	-2.497	-2.525	-2.551
D'	-1.677	-1.717	-1.757	-1.793	-1.829	-1.864	-1.895	-1.941	-1.985	-2.026	-2.065	-2.101	-2.136	-2.168	-2.199
E'	-1.231	-1.270	-1.306	-1.342	-1.376	-1.409	-1.440	-1.485	-1.528	-1.568	-1.606	-1.642	-1.677	-1.710	-1.741
F'	-0.806	-0.836	-0.866	-0.895	-0.922	-0.950	-0.975	-1.014	-1.049	-1.084	-1.117	-1.149	-1.179	-1.208	-1.236
G'	-0.446	-0.469	-0.490	-0.511	-0.531	-0.551	-0.570	-0.598	-0.625	-0.651	-0.674	-0.698	-0.721	-0.743	-0.763
H'	-0.187	-0.199	-0.211	-0.223	-0.234	-0.244	-0.256	-0.271	-0.286	-0.300	-0.314	-0.328	-0.341	-0.353	-0.366
I'	-0.042	-0.045	-0.049	-0.053	-0.056	-0.059	-0.063	-0.067	-0.072	-0.076	-0.080	-0.084	-0.088	-0.093	-0.097
n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54

T A B L E N° 8

Crown Moment for Load "P" at any ordinate

$M_c = \frac{Pl}{100}$ x Table Coefficient : m = 2

Ordinate Ref. Letter															
n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.5I	0.54
Cr	4.434	4.458	4.48I	4.503	4.524	4.544	4.564	4.592	4.6I9	4.644	4.669	4.693	4.7I5	4.737	4.759
A	2.250	2.272	2.293	2.313	2.333	2.352	2.370	2.396	2.420	2.444	2.467	2.488	2.509	2.530	2.550
B	0.681	0.698	0.714	0.729	0.744	0.759	0.772	0.792	0.8II	0.829	0.846	0.863	0.879	0.895	0.9I0
C	-0.322	-0.3I3	-0.304	-0.296	-0.287	-0.280	-0.272	-0.26I	-0.25I	-0.24I	-0.232	-0.222	-0.2I4	-0.205	-0.I97
D	-0.837	-0.836	-0.836	-0.835	-0.834	-0.833	-0.832	-0.832	-0.83I	-0.830	-0.829	-0.828	-0.827	-0.827	-0.826
E	-0.968	-0.975	-0.982	-0.988	-0.994	-0.999	-I.005	-I.0I3	-I.020	-I.028	-I.035	-I.04I	-I.048	-I.054	-I.060
F	-0.834	-0.845	-0.857	-0.867	-0.877	-0.887	-0.896	-0.9I0	-0.923	-0.935	-0.947	-0.958	-0.969	-0.980	-0.989
G	-0.562	-0.576	-0.587	-0.598	-0.609	-0.6I9	-0.629	-0.644	-0.658	-0.67I	-0.684	-0.696	-0.708	-0.7I9	-0.730
H	-0.275	-0.284	-0.293	-0.30I	-0.309	-0.3I7	-0.324	-0.335	-0.345	-0.355	-0.364	-0.373	-0.382	-0.390	-0.398
I	-0.069	-0.072	-0.075	-0.079	-0.082	-0.085	-0.087	-0.091	-0.095	-0.099	-0.I02	-0.I06	-0.I09	-0.II2	-0.II5
n	0.I8	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.5I	0.54

TABLE N° 9

CROWN MOMENT FOR LOAD "P" AT ANY ORDINATE.

$M_c \frac{Pl}{100}$ x Table Coefficient **m = 3**

	n	0.I8	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.42	0.45	0.39	0.48	0.5I	0.54
Ordinate Ref. Letter	Cr	4.637	4.663	4.688	4.712	4.735	4.757	4.778	4.809	4.838	4.892	4.9I8	4.865	4.942	4.966	4.990
	A	2.445	2.469	2.492	2.5I4	2.536	2.556	2.576	2.604	2.63I	2.682	2.706	2.657	2.729	2.75I	2.772
	B	0.854	0.873	0.89I	0.908	0.925	0.94I	0.956	0.979	1.000	I.039	I.058	I.020	1.076	1.094	I.II0
	C	-0.I83	-0.I72	-0.I6I	-0.I50	-0.I40	-0.I3I	-0.I22	-0.I08	-0.096	-0. 072	-0.06I	-0.084	-0.050	-0.040	-0.030
	D	-0.736	-0.733	-0.73I	-0.728	-0.726	-0.724	-0.72I	-0.7I8	-0.7I5	-0.7I0	-0.707	-0.7I3	-0.705	-0.720	-0.700
	E	-0.906	-0.9I2	-0.9I6	-0.92I	-0.926	-0.930	-0.934	-0.940	-0.946	-0.956	-0.96I	-0.95I	-0. 966	-0.97I	-0.975
	F	-0.733	-0.746	-0.759	-0.77I	-0.782	-0.794	-0.804	-0.820	-0.834	-0.862	-0.875	-0.848	-0.887	-0.899	-0.9II
	G	-0.550	-0.562	-0.573	-0.584	-0.594	-0.604	-0.6I3	-0.627	-0.640	-0.664	-0.676	-0.653	-0. 687	-0.698	-0.708
	H	-0.274	-0.283	-0.292	-0.300	-0. 308	-0.3I5	-0. 323	-0.333	-0.343	-0. 36I	-0.370	-0.352	-0.379	-0.387	-0.395
	I	-0.068	-0.07I	-0.075	-0.078	-0.08I	-0.084.	-0.087	-0.09I	-0.095	-0.I02	-0.I05	-0.098	-0.I08	-0.II2	-0.II5
	n	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.42	0.45	0.39	0.48	0.5I	0.54