A REINFORCED CONCRETE ARCH BRIDGE

WITH A PAPER

ON A NEW METHOD OF SOLUTION OF INDETERMINATE PROBLEMS

BY

NAZIH J. TALEB

1950

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INTRODUCTION

The design of an economical fixed arch bridge by the usual method is a long and troublesome job. The method used in this thesis is a good and short practical one that utilizes some tables that are nothing but the evaluation of the usual summations, but in order not to loose the practice in designing arches by the exact summations, I have used the tables for the preliminary calculations then checked the final design by the summation method.

N. J. Taleb.

Beirut, June, 1950.

NOTATION

M = The bending moment at any point on the arch span due to any form of loading.

M_c = The bending moment at the crown due to any form of loading.

M_{cT} = The bending moment at the crown due to temperature change.

M_{cs} = The bending moment at the crown due to "arch shortening."

M_s = The bending moment at the springing due to any form of loading.

M_{sT} = The bending moment at the springing due to temperature change

M_{ss} = The bending moment at the springing due to "arch shortening".

H = The horizontal thrust due to any form of loading.

H_T = The horizontal thrust due to temperature change.

H_s = The horizontal thrust due to "arch shortening".

R_c = The vertical shear at the crown.

R_s = The vertical reaction at the left hand support

R_B = The vertical reaction at the right hand support.

N = The normal thrust at any point on the arch

x, y = The horizontal and vertical coordinates at the arch centre line, with centre of coordinates at the crown.

E = Young's modulus of elasticity for concrete.

I = The moment of inertia of the arch rib at any point.

I_c = The moment of inertia of the arch rib at the crown.

I_s = The moment of inertia of the arch rib at the springing.

x = The coefficient of expansion of concrete.

ϕ = The angle between the tangent to the arch centerline and the horizontal.

NOTATION (Cont.)

Δy = the vertical movement of the arch at the crown.

Δx = The horizontal movement of the arch at the crown.

θ = The angular movement of the arch at the crown.

d_s = The length of an infinitesimal portion of the arch curve.

A = The cross - sectional area of the arch rib at any point.

b = The width of the arch rib.

l = The length of the arch span measured from the centre line at the springing.

r = The rise of the arch measured from the centre line at the springing to the centre line at the crown.

W_c = The total dead load plus half the distributed live load at the crown

W_s = The total dead load plus half the distributed live load at the springing.

W_l = The uniformly distributed live load.

W = The total dead load plus half the distributed live load at any point.

CHAPTER I

CONCRETE AND REINFORCED CONCRETE ARCH BRIDGES

The development of concrete and reinforced concrete increased considerably by the use of arches for bridges. An arch bridge is subject mostly to comp. stresses for which concrete is particularly adapted. With steel reinforcement to take care of any possible tension a concrete arch bridge becomes superior to stone or brick arch where tension must be avoided. Arch bridges may have as long a span as 558 ft. or more.

Advantages of Arch Construction.-

1. Permanency. The arch gains in strength with time.
2. Aesthetic appearance.
3. Small cost of upkeep.
4. Less vibration and noise for arches have large masses.
5. Clearance for navigation.

Comparative Costs.- The cost of a bridge depends upon several factors and local conditions such as the ground. When hard foundation is not far from the surface of the ground, it is evident that an arch bridge is the most economical. But, in bad ground, the advantages of arch bridges are reduced due to the extra cost of foundation that should be designed to provide for horizontal thrust besides the vertical pressure.

Classification of Arch Bridges.-

According to their method of design the arch bridges may be divided into three-hinged, two-hinged, one-hinged, and fixed or hingeless arches.

According to their method of construction they may be divided into filled spandrel and open spandrel arches.

Filled Spandrel Arches.- In this type of construction, the space between the extrados and the roadway is filled with earth that is properly rolled and tamped in order to support the roadway. Spandrel walls are used on both sides of the roadway.

Open Spandrel Arches.- In this type of arches the fill above the arch ribs is omitted and the construction consists of(a) arch ribs (b) a system of vertical supports above the arch ribs, or (c) a horizontal floor construction carrying the roadway and supported by the vertical supports.

The economical advantages of this type over the Filled Spandrel Type are:

1. The dead load is reduced by omiting the fill so that the arches and the foundation may be made lighter.

2. The arches do not need to be made the full width of the bridge. Two or more independent narrow ribs may be used.

3. The independent ribs may be made deep enough to reduce the tensile stresses quite a lot.

4. The ribs may be made of rich concrete properly reinforced with consequent reduction in cost. Apart from economical advantages the open spandrel arch bridge has a nicer look than the filled type.

Arch Ribs.- The arch rib may be a barrel rib extending the full width of the bridge same as in filled spandrel arches. (see fig.1)

A more economical construction is fulfilled by using two seperate ribs whose combined width is smaller than that that would have been required for full barrel arches (fig. 2)

In very wide bridges three separate ribs may be used, a wide one in the center and two narrow ones at the sides. In case (as in the above) the width of the rib is large as compared to its depth, no lateral bracing is required.

Vertical Supports.- The load from the floor is transmitted to the arch ribs by means of vertisal supports that should be so designed as to transmit properly and uniformly the load to the arch ribs. The type of such supports depends on the type of arch used. For barrel or wide ribbed types, cross walls are being used (fig.1) while columns are used in the ribbed type of arches and are being erected at the center of the rib.

@" To distibute the load over the arch rib, reinforcement should be used on the top and bottom so as to take any tensible stress in case of cross bending of the ribs. Also stifenning cross ribs are often used over the rib between the columns. As the columns and walls are poured seperately from the arch, dowels should be provided in the arches of same number and size as used for column or wall reinforcement. A proper seat also should be provided in the arch rib with a horizontal bed to receive the column or wall."

Floor Construction.- The floor may rest on vertical supports all over the rib (fig.2) or the roadway in the central part of the span may rest directly on the rib (fig. 1)

In case cross walls are used as the vertical supports, the floor consists of:

1. Arches spanning between cross walls.

2. Slabs spanning between cross walls.

@" When type two is used, the spacing of the walls should be made small enough to permit the use of a slab thickness not larger than 8 inches. The main

@" Concrete-Plain and Reinforced" by Taylor, Thompson Smulski.

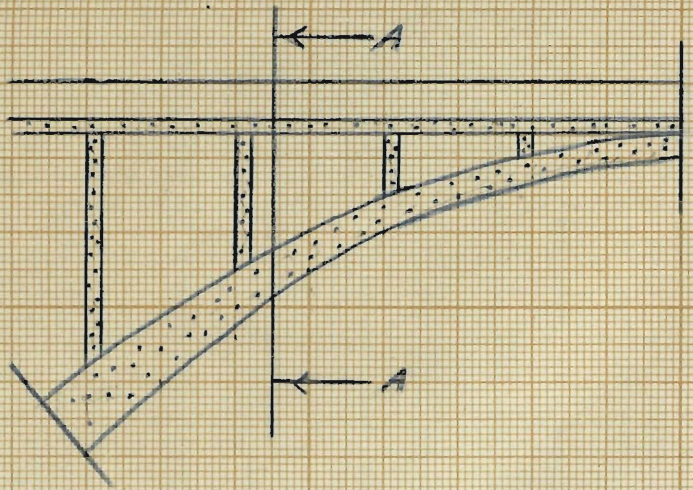


Fig. 1

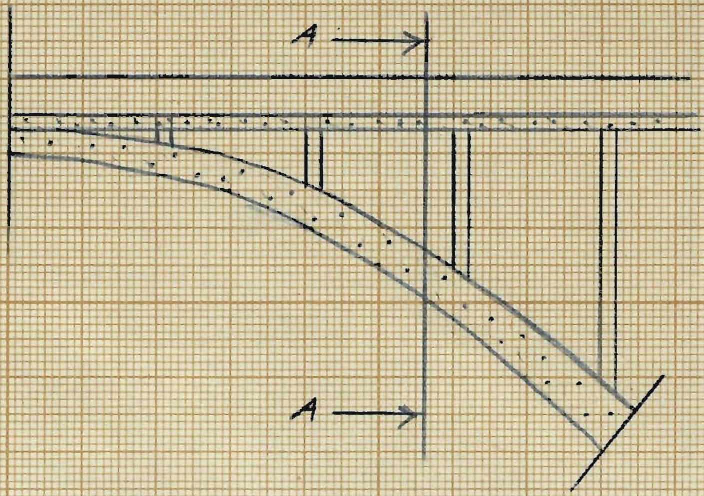
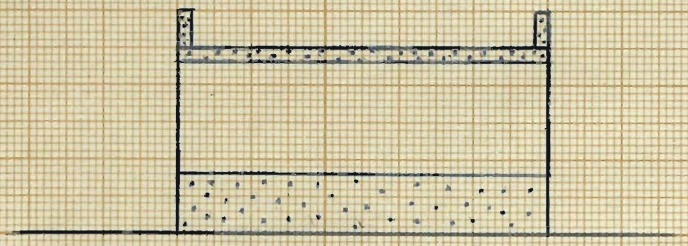
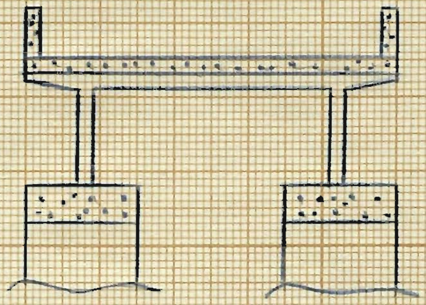


Fig. 2



SECTION A-A



SECTION A-A

The main reinforcement consists of bars running longitudinally with the bridge, when the vertical support consists of independent columns, the floor construction may consist of (1) cross beams running between columns and slab spanned between them, (2) stringers running longitudinally, supported by cross beams, and a slab running between stringers, the cross beams often being cantilevered out to support the sidewalk, and(3) flat slab construction where beams and girders are omitted and, instead a massive slab is supported on columns with enlarged heads."

USE OF REINFORCEMENT IN CONCRETE ARCHES.

Concrete, plain or reinforcement may be used in arches. Plain concrete should be used only where the resultant stresses are compressive. This happens in long massive arches. Yet some do not recommend the use of plainconcrete because in most cases reinforced concrete arches may be built cheaper and are more able to resist unexpected stresses due to any disarrangement of foundation or any tensile stresses due to any cause. Another advantage of reinforced arches is that the allowable unit compressive stresses in reinforced concrete is larger than for plain concrete. When narrow ribs are used, they should befully reinforced in the same manner as recommended for columns.

Reinforcement consists of usual bars placed symmetrically at the intrados and extrados, with tie bars between. Sometimes it is used near the lower face at the crown and then bent up near the top face at the springing. But this is not recommended by some authors. The amount of the longitudinal reinforcement usually ranges from $\frac{1}{4}$ - 1% of the cross section of the arch with some extra steel at points of maximum moments.

Spiral Reinforcement of Arches.- Sometimes it is desirable in arches consisting of seperate ribs to increase the compression strength to reduce the rib section. This is fulfilled by the use of spiral reinforcement which increases considerably the allowable unit compressive stress in concrete. To get the largest benefit from spiral reinforcement it is used in the highly compressed sections and the cross section of the rib is made I shaped in some places to increase the moment of inertia of the section.

ALLOWABLE UNIT STRESSES IN AN ARCH@

Allowable Compression Stresses.- The allowable unit stresses in an arch should not exceed the values given in the table below.

Describtion	Concrete or Nearly Concentric Load	Thrust and Bending Moment	
		$e < 1/6$ h.	$e > 1/6$ h.
Plain Concrete	$0.18f'_c$ *	$0.21f'_c$	
Reinforced concrete, min. p= 0.01	$0.225f'_c$	$0.265f'_c$	$0.315f'_c$

As far as compressive stresses are concerned, the stress conditions are the same as in columns.

* f'_c = Ultimate comp. strength of concrete at 28 days tested in cylinders.

@ "Concrete- Plain and Reinforced" by Taylor, Thompson, and Smulski.

CHAPTER II

THEORY OF ARCHES-DERIVATION OF THE FORMULAE FOR FIXED ARCHES.

Arch Action And Advantages.- Arch action is governed by the same rules of mechanics as ordinary beam action. An arch is nothing but a curved beam. A beam is subjected at all sections to shear and bending moment.

A hinged arch is a curved beam hinged at its two ends. When loaded vertically, shear and bending moment exist as well as horizontal thrusts that compress the arch inward thus decreasing the bending moment. This reduction in bending moment is one of the two advantages of a hinged arch. The second is that the whole section is subjected to compression stresses due to the thrust which reduces the tensile stresses produced in the section by the bending moment. A curved beam built into solid supports that prevent the ends from spreading or rotating is a fixed arch. The benefits derived from this are the same as those of the hinged arch. In addition, the resulting bending moments are smaller.

*Deflection of Arch under Different types of Loading.- There is a considerable difference between the manner of deflection of an arch under vertical loading and that of a beam.

A beam subjected to vertical loading always deflects downward. After deflection, all points on the axis of the beam(except the supports) are below the original position of the axis of the unloaded beam.

An arch subjected to vertical loading deflects downward throughout only for loads extending over the whole span. For partial loading part of the axis deflects downward and the balance deflects upwards. Fig. 3 shows in exaggerated form the shape assumed by the arch for different types of partial loading.

*" Concrete- Plain and Reinforced " by Taylor, Thompson and Smulski.

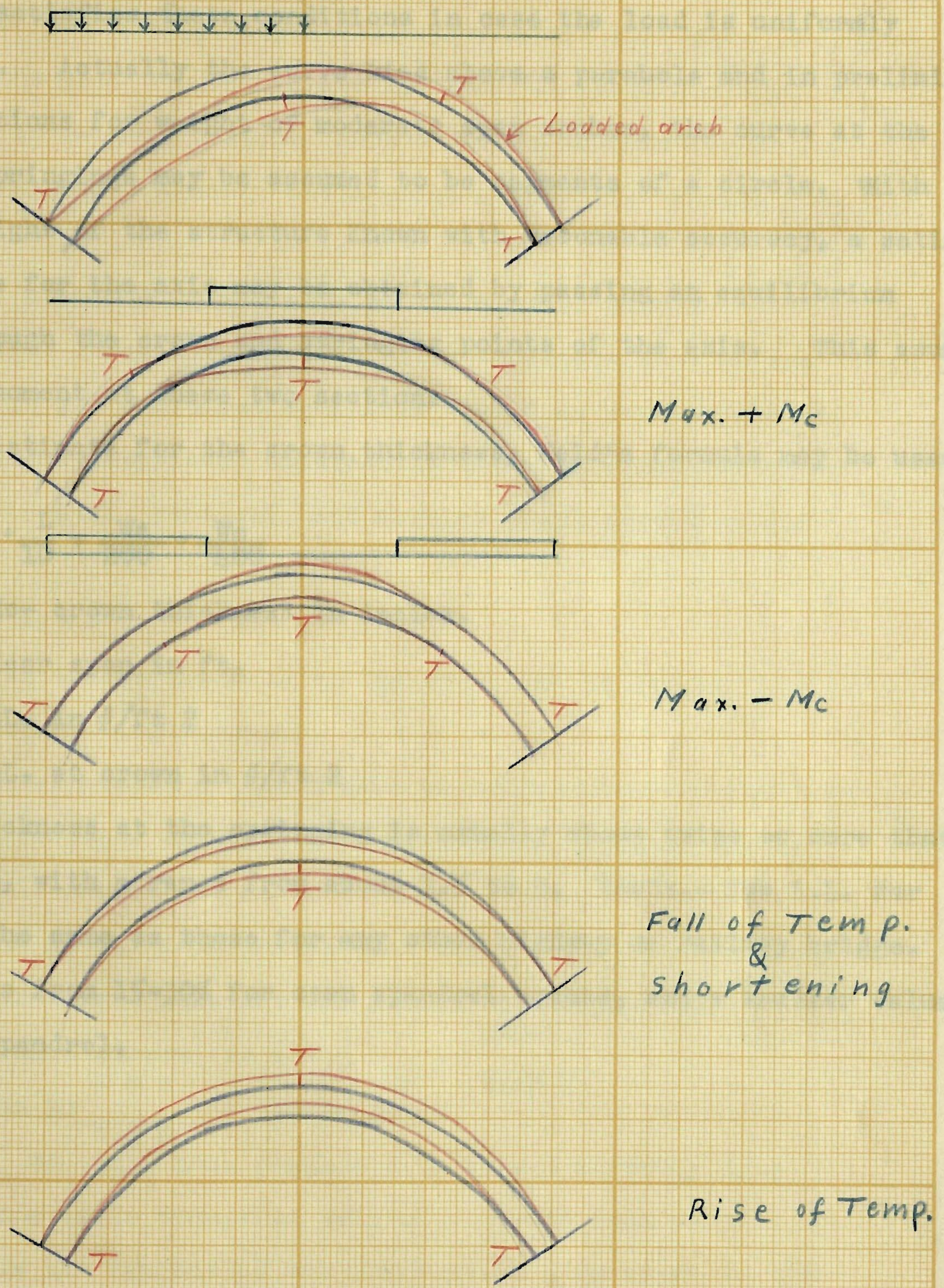
The most unfavorable loadings for an arch are-sided loadings shown in fig.3. The loaded part of the arch moves downward while the unloaded part moves up. The points of maximum tension, i.e., where cracks may be expected, are marked by T.

The partial loading shown in Fig. 3, producing maximum tension at the crown pushes the crown downward and the haunches outward. The exaggerated deflection is shown in the figure.

The loading producing maximum negative bending moment at crown shown in Fig. 3 has the opposite effect to that in the previous case. The arch is pushed downward at the haunches and forced up at the crown.

Arch Analysis.- A hingeless arch is indeterminate to the third degree In order to avalyze the stresses, the relation between the elastic deformations of the arch and the internal and external stresses should be considered. All different methods used are really different ways of arranging the same fundamental equations. These equations may be derived either by the method of least work or by the equations expressing the deflections of a curved beam. The labor of an exact analyses of the dimensions is so great that it is desirable to have available some simple method of arriving at a trial section that will require little if any change upon closer study. Mr. V.S. Cochrane has developed a simple and speedy method of applying the elastic theory to symmetrical hingeless arches which is somewhat approximate but sufficiently accurate for the final design of structures of moderate span where great refinement is not attempted. Mr. Whitney has prepared a similar adaptation of the elastic theory.

Proportions.- The axis (center line of the ring) should conform very closely to the dead load line of resistance, thus eliminating as far as possible bending under the permanent and major part of the load and reducing stresses to the least possible.



Deflection of Arch under Different Types of Loading.

FIG. 3

A parabola satisfies these conditions in case the load is uniformly distributed. Actually the curve lies above a parabola and in preliminary computations for weight of moderate span arches, the curve at the crown and springings may be assumed to be segments of a circle. With the dead weight of the structure Known with resonable accuracy, a satisfactory curve for the axis may be obtained by passing an equilibrium polygon through the crown and springing points of the axis. This asumes no bending moment at these two sections.

As an estimate for the crown thickness, Weld's formula may be used.

$$d_c = \sqrt{L} + \frac{L}{10} + \frac{W_L}{200} + \frac{W_c}{400}$$

where dc- crown thickness in incles.

L = clear span in ft.

WL = L.L. in #/ft 2

Wc = D.L. at crown in #/ft 2

The thickness at the springing is usually about twice or more that at the crown, with a range from about 1.5 to 3. loads.- The L.L. for arches are the same as those for any other highway or railway brigdge. Impact varies from 15-30% for open spandrel arches, and a smaller value for filled spandrel.

*

[or more that at the crown, with a range from about I.5 to 3.

Loads.- The L.L. for arches are the same as those for any other highway or railway bridge. Impact varies from I5-30 per cent for open spandred arches, and a smaller value for filled spandred.]

ANALYSIS OF THE ARCH BY THE ELASTIC THEORY

An arch with fixed ends is statically in determinate to the third degree. To get the three equations needed we have to resent to the following assumptions, considering the arch as cut at the crown, the horizontal, vertical, and angular displacement should be zero i.e.

$\Delta x = 0 \qquad \Delta y = 0 \qquad \Delta\phi = 0$

Our plan will be to cut the arch at the crown and determine the above mentioned displacements.

Let fig. 4 represent a curved beam whose curvature is small in propostion to its depth so that the length of all fibers may be considered equal. Assuming that ab is fixed and that dc rotates through an angle ϕ. The deformation that takes place at a distance e from the neutral axis equals $e \cdot \Delta\phi$. Unit deformation is $\dfrac{e\Delta\phi}{\Delta s} = \delta$ calling f, the unit stress, we get $E = \dfrac{f}{\delta} = \dfrac{f.\Delta s}{e.\Delta\phi}$

But $f = \dfrac{Me}{I}$. So $E = \dfrac{M\Delta s}{I\Delta\phi}$ Which gives

$\Delta\phi = \dfrac{M\Delta s}{EI}$. We could have reached this result directly from the fact that the angle change in axis of a loaded beam at any point P from the tangent at any other point A is equal to the area of the moment diagram between P and A divided by EI.

so,

* not considered

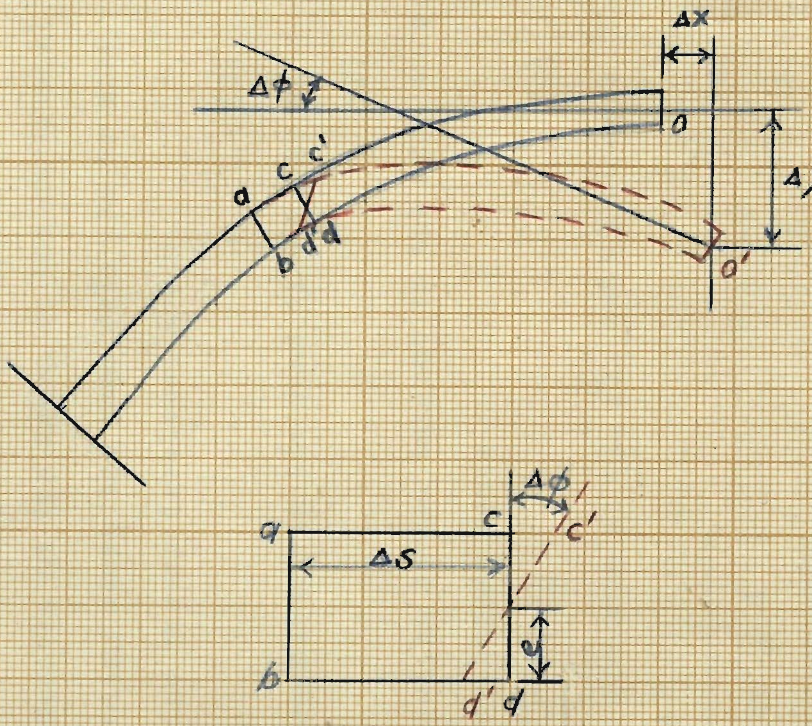


Fig. 4

$$\Delta\phi = \frac{\Delta Am}{EI} = \frac{M \cdot \Delta S}{EI}$$

again $\Delta Y = x \cdot \Delta\phi = \dfrac{-x M \Delta S}{EI}$

since ΔY is negative.

Similarly $\Delta x = \dfrac{YM\Delta S}{EI}$

With the origin of coordinates at the crown c, the horizontal movement of C due to bending bears the same relation to each cantilever. Then from the theory developed above,

$$\sum_c^A \Delta X = -\sum_c^B \Delta X \qquad (1)$$

The changes in ΔY are equal and in the same direction, so

$$\sum_c^A \Delta Y = \sum_c^B \Delta Y \qquad (2)$$

also the changes in direction of the tangent to the axis at c are equal but opposite in direction, hence

$$\sum_c^A \Delta\phi = -\sum_c^B \Delta\phi \qquad (3)$$

Substituting the values for each of ΔX, ΔY, and $\Delta\phi$ we get:

$$\sum_c^A \frac{MY\Delta S}{EI} = -\sum_c^B \frac{My\Delta S}{EI} \qquad (4)$$

$$\sum_c^A \frac{Mx\Delta S}{EI} = \sum_c^B \frac{Mx\cdot\Delta S}{EI} \qquad (5)$$

$$\sum_c^A \frac{M\Delta S}{EI} = -\sum_c^B \frac{M\Delta S}{EI} \qquad (6)$$

Denoting $\sum_c^A M$ as $\sum M_L$ and $\sum_c^B M$ as $\sum M_R$, deviding the arch ring into divisions such that ΔS is a constant, and eliminaing the constant E, we get :

$$\sum M_L \frac{Y}{I} = -\sum M_R \frac{Y}{I} \qquad (7)$$

$$\sum M_L \frac{x}{I} = \sum M_R \frac{x}{I} \qquad (8)$$

$$\sum \frac{M_L}{I} = -\sum \frac{M_R}{I} \qquad (9)$$

Considering the left half of the arch as a free body, Mc is +ve (It produces tension in the bottom fiber). Similarly for Vc, Hc produces compression.

Hence for any section

$$M_L = M_c + H_c \cdot Y + V_c \cdot X - m_L$$

Where m is the bending moment at the section due to the external load.

Similarly for the right half

$$M_R = M_c + H_c y - V_c x - m_R.$$

Substituting in equations (7), (8), & (9), and combining the terms:

$$2H_c \sum \frac{Y^2}{I} + 2M_c \sum \frac{Y}{I} - \sum m_L \frac{Y}{I} - \sum m_R \frac{Y}{I} = 0 \qquad (10)$$

$$2V_c \sum \frac{x^2}{I} - \sum m_L \frac{x}{I} + \sum m_R \frac{x}{I} = 0 \qquad (11)$$

$$2H_c \sum \frac{Y}{I} + 2M_c \sum \frac{1}{I} - \sum \frac{m_L}{I} - \sum \frac{m_R}{I} = 0 \qquad (12)$$

considering the application of load on the left half of the arch only, the terms containing mR disappear. Combining equations (10) & (12) ,

$$H_c = \frac{\sum \frac{my}{I} \sum \frac{1}{I} - \sum \frac{m}{I} \sum \frac{y}{I}}{2\left[\sum \frac{Y^2}{I} \sum \frac{1}{I} - \left(\sum \frac{Y}{I} \right)^2\right]} \qquad (13)$$

$$M_c = \frac{\sum \frac{m}{I} - 2H_c \sum \frac{y}{I}}{2 \sum \frac{1}{I}} \qquad (14)$$

equation (11) gives

$$V_c = \frac{\sum \frac{mx}{I}}{2 \sum \frac{x^2}{I}} \qquad (15)$$

We may shift our coordinate axis to the Elastic center so that

$$\sum \frac{Y}{I} = 0 .$$

This simplifies the above expressions but more work has to be done in computing the coordinates of this Elastic center.

Comment on the above equations.- The above equations have been derived on the basis that ΔS is constant. Some prefer to divide the arch asis in such a way that $\frac{\Delta S}{I}$ is constant. This is done by plotting I against ΔS then constructing isosceles similar triangles by making their sides respectively parallel as shown in Fig.

We can still simplify our fundamental equations by dividing the arch axis in such a way that $\Delta S . \frac{Y}{I}$ is constant. This is done thus:

$$\frac{\Delta SY}{I} = \frac{\Delta S}{\frac{I}{Y}}$$

This is done by plotting $\frac{I}{Y}$ against ΔS Then constructing similar isosceles triangles by making their sides respectively parallel as shown. in fig. 5

If $\frac{\Delta S}{I}$ is constant,

$$Hc = \frac{n \sum my - \sum m \sum y}{2\left[(\sum Y)^2 - n \sum Y^2 \right]}$$

$$Vc = \frac{\sum mx}{2 \sum x^2}$$

$$Mc = \frac{\sum m + 2\ Hc \sum Y}{2\ n}$$

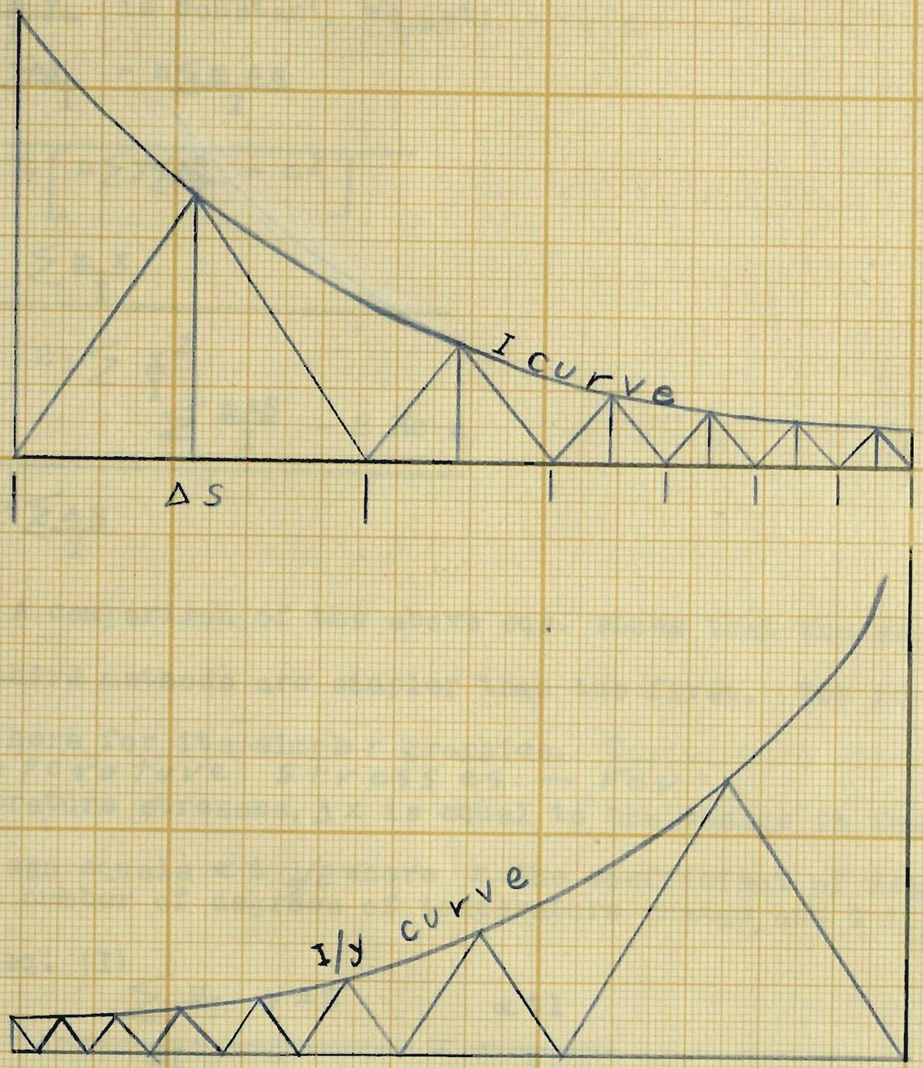


Fig. 5

In this case all y s are measured downward from the axis through the crown and are considered as - ve, n equals the number of divisions in one half of the arch.

If $\frac{\Delta S\, y}{I}$ is constant, we get:

$$Hc = \frac{\sum m \sum \frac{\Delta S}{I} - n \sum \frac{m\,\Delta S}{I}}{2\left[n \sum y \sum \frac{\Delta S}{I} - n^2 \right]}$$

$$Vc = \frac{\sum \frac{m\,x}{I}}{2 \sum \frac{x^2}{I}}$$

$$Mc = \frac{\sum \frac{m \Delta S}{I} - 2\,Hc\,n}{2 \sum \frac{\Delta S}{I}}$$

A comparison of the above eqs. shows that the second , and the third methods are simpler than the first. Yet it will be used here for its simpler graphics. Temperature Stresses.- For
Temperature Stresses.— For
temperature stresses, Δx is equal to the change in length of the half span equals $\alpha t \; l/2$ where α=the coefficient of linear expansion, t the number of degrees of temperature change, and l the span. then from eq. (I)

$$\sum_c^A \Delta x = \frac{\sum M_L \; y \Delta S}{EI} = \frac{\alpha t l}{2} \qquad (16)$$

also, $\therefore \Delta \phi = 0$

$$\sum \frac{M_L}{I} = 0 \qquad (17)$$

There being no external loads, $m = 0$, and from symmetry $Vc = 0$, hence, $M = Mc + Hcy$.

Substituting the value of M in the above eqs.,

$$M_e\sum\frac{Y}{I} + H_e\sum\frac{Y^2}{I} = \frac{\alpha t l}{2}\cdot\frac{E}{\Delta S} \tag{18}$$

$$M_c\sum\frac{1}{I} + H_c\sum\frac{Y}{I} = 0 \tag{19}$$

These give #

$$H_{cT} = \frac{\mp\alpha t\ l\ E\sum\frac{1}{I}}{2\Delta S\left[\sum\frac{Y^2}{I}\sum\frac{1}{I} - \left(\sum\frac{Y}{I}\right)^2\right]} \tag{20}$$

$$M_{cT} = \frac{-\alpha t\ l\ E\sum\frac{Y}{I}}{\text{same denom. as (20)}} \tag{21}$$

$$M = M_c + H_c Y$$

Shrinkage.- It hasthe same effect upon the arch as the fall of temp. It is often taken care of by adding 15° F to the assumed fall of temp. It should be noticed that when the arches are built in transverse strips, the arch is not closed until most of the strips have set and thereby undergone the largest part of the shrinkage. The arch as a whole is then effected only by the additional shrinkage of the cured concrete.

Rib shortening .- A thrust throughout the arch producing an average stress on the concrete equal to C_a P.s i . would shorten the arch span an amount equal to $\frac{C_a}{E}\,l$ ie. δl if the arch and the abutments were not fixed. Since they are fixed, and the arch cannot shorten, there is a tensi~~b~~le stress developed.

\# +for fall in temp.

\# -for rise in temp.

action is similar to that of fall in temp. The resulting Hc may be found by substituting $\frac{Ca\ell}{E}$ for $\alpha t \ell$ of eqs. (20), & (21).

$$Hcs = \frac{+Ca\ell \sum \frac{I}{I}}{\text{Denom. of } (20)} \tag{22}$$

$$Mcs = \frac{-Ca\ell \sum \frac{Y}{I}}{\text{Denom. of } (20)} \tag{23}$$

$M = Mcs + Hcs \cdot Y$

$Ca = \frac{N}{A}$

$N = H \cos. \phi + R \sin. \phi$

In case the line of pressure for D. L. + $\frac{I}{2}$ L.L.

coincides with the center line of the arch,

$N = H \sec. \phi$ or $R \operatorname{cosec} \phi$

So, $Ca = \frac{H \sec. \phi}{A}$

ARCH CURVE*

The most economical shape of arch is that whose center line coincides with the line of pressure for D.L. + $\frac{I}{2}$ Dist. L.L. An approximation to this ideal curve is derived below. See fig (6)

The normal thrust is the resultant of the shear and the horizontal thrust, and for the pressure curve to follow the center line of the arch,

$$\frac{R}{H} = \tan. \phi = \frac{dy}{dx} \tag{24}$$

Differentiating $\frac{R}{H}$ with respect to x (H constant) we obtain

$$\frac{d^2 Y}{dx^2} = \frac{I}{H} \cdot \frac{dR}{dx}$$

$\frac{dR}{dx}$ = rate of change of shear = W

* "Arch Design Simplified" By Fairhurst

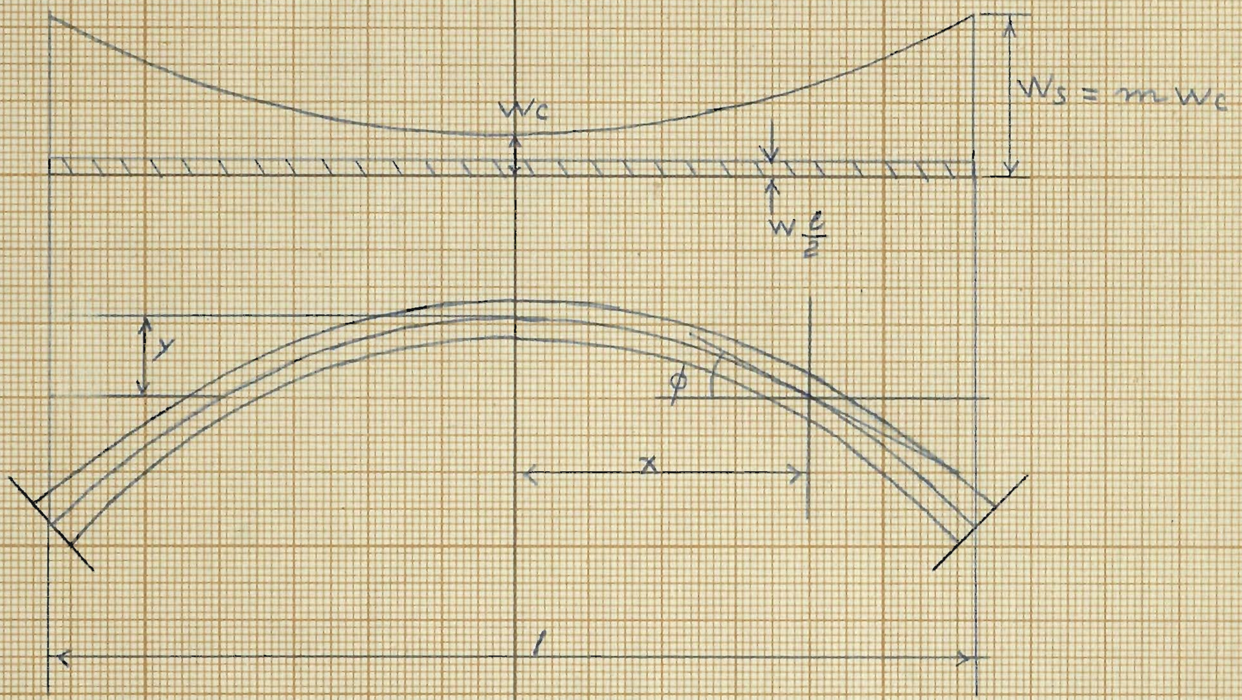


Fig 6

$$\therefore \frac{d^2 Y}{dx^2} - \frac{W}{H} = 0 \qquad (25)$$

Assuming that the load varies from the crown to the springing in proportion to the length of the ordinate of the arch center line,

$$W = Wc + \frac{Y}{r} (Ws - Wc)$$

Let $Ws = mWc$, then $W = Wc (I + \frac{m-I}{r} Y)$ (26)

Substituting the value of W in (25) we obtain

$$\frac{d^2 Y}{dx^2} - \frac{Wc}{H} \frac{m-I}{r} Y = \frac{Wc}{H} \qquad (27)$$

Solving this diff. eq. we find

$$y = C_I \cdot e^{ux} + C_2 e^{-ux} - \frac{r}{m-I} \qquad (28)$$

Where $u = \left(\frac{Wc}{H} \frac{m-I}{r} \right)^{\frac{I}{2}}$

$Y = 0$ When $x = 0$, and $\frac{dy}{dx} = 0$ when $x = 0$

$$C_I + C_2 = \frac{r}{m-I} \quad \& \quad C_I - C_2 = 0$$

$$\therefore C_I = C_2 = \frac{r}{2(m-I)}$$

$$\therefore Y = \frac{r}{m-I} \left[\frac{I}{2} (e^{ux} + e^{-ux}) - I \right]$$

$$\therefore Y = \frac{r}{m-I} \left[\cosh. ux - I \right] \qquad (28)\ a$$

When $x = \frac{l}{2}$, $Y = r$, and solving for H, we obtain

$$H = \frac{m-I}{4 P^2} \times \frac{Wc\ l^2}{r} \qquad (29)$$

Where $P = \log_e (m - \sqrt{m^2 - I})$

and $u = \frac{2P}{l}$

Substituting the value of u in (29) we obtain

$$Y = \frac{r}{m-I} \times \left(\cosh 2P \frac{x}{l} - I \right) \qquad (30)$$

(II)

Putting the value of Y in (26) we have

$$W = W_c \cosh 2P \frac{x}{l} \qquad (31)$$

Integrating W between X = 0 and x we find the shear R at X,

$$R = \frac{W_c l}{2P} \cdot \sinh. \ 2P \frac{x}{l} \qquad (32)$$

Shear at springing $R_s = \frac{W_c l}{2P} \sqrt{m^2 - 1}$ (32a)

$$\tan. \phi = \frac{R}{H} = \frac{2r}{l} \cdot \frac{P}{m-1} \cdot \sinh. \ 2P \frac{x}{l} \qquad (33)$$

$$\tan. \phi_S = 2 \frac{r}{l} \ \frac{P\sqrt{m^2-1}}{m-1} \qquad (33a)$$

$$\cos. \phi_S = \frac{1}{(1 + \tan^2 \phi_s)^{1/2}} \qquad (33b)$$

Calculations based upon the arch curve derived from formula (30) are found to possess a high degree of accuracy and very many examples have been worked out to test its practical use. When the arch design has been completed, it is desirable in the case of large spans that a final arch center line be obtained from the results to insure that the line of normal thrust for dead load plus half live load will coincide with the arch curve, and will produce no moments apart from those due to arch shortening. It will rarely be necessary to recalculate the arch stresses using the final arch shape as a basis, as the calculations derived from the tables using the appropriate m value will have been based on a shape of arch sufficiently near to the final shape to obviate so much additional labor.

Fairhurst Transformation of Arch Formulae.—

In the previous article we expressed y in terms of x thus rendering possible the integration of the summations of eqs. (I3) (I4). & (I5).

A new term, n, is introduced and equals $\frac{I_c}{I_s \cdot \cos \phi_s}$ Fairhurst performed those intergrations and expressed everything as functions of m, n, & a, position of the loads. Then he prepared all the necessary tables for the design thus greatly simplifying the engineer's work.

The necessary portions of the mentioned tables are found on page ( ).

Point Load P at any Position.-

$$H = \frac{Pl}{r} \cdot f(m,n,a)$$

$$Mc = Pl \cdot F(m,n,a)$$

$$Rc = P \cdot \phi(n,a)$$

Temp. Effects.-

$$H_t = \alpha t E \left(\frac{m-I}{r}\right)^2 \quad Ic \; \frac{f_3(n)}{f_5(m,n)}$$

$$McT = -\alpha t E \left(\frac{m-I}{r}\right) I_c \cdot \frac{f_4(m,n)}{f_5(m,n)}$$

Functions f_3, f_4, f_5 are numbers only i.e. dimensionless.

Arch Shortening.-

$$Hs = -H \left(\frac{m-I}{r^2}\right)^2 \left(\frac{I_c^2}{I2\, b^2}\right)^{\frac{1}{3}} \beta \; \frac{f_3(n)}{f_5(m,n)}$$

M

$$Mcs = H \left(\frac{m-I}{r}\right) \left(\frac{I_c^2}{I2b^2}\right)^{\frac{1}{3}} \beta \; \frac{f_4(m,n)}{f_5(m,n)}$$

Where $\beta = \left[I + \frac{I6}{9}\left(\frac{r}{I}\right)^2 - (I-n)\left\{\frac{I}{9} + \frac{I6}{5}\left(\frac{r}{I}\right)^2\right\}\right]$

Uniform Dist. L.L. over whole Span.-

$$H_L = \frac{W_\ell \cdot l^2}{r} \cdot f_1(m,n)$$

Mcl($\frac{W}{2}$ produces no moment) $= W_\ell \cdot l^2 f_2(m,n)$

Uniform Dist L.L. over Portion of Span to Produce Max. Mom.-

$$Mcl\ (\text{max.}) = W_l \cdot l^2 \cdot f_8\ (m,n)$$

$$\text{Corresponding } H_l = W_l \cdot \frac{l^2}{r} \cdot f_9\ (m,n)$$

TABLE No. 1

Arch Functions

[" Height of arch ordinates and the angles between the tangent to the arch Center Line and the Horizontal."]

Ordinate number		1,1'	2,2'	3,3'	4,4'	5,5'	6,6'	7,7'	8,8'	9,9'	10,10'
$m=2$	K_1	0.9913	0.9651	0.9209	0.8580	0.7753	0.6712	0.5441	0.3917	0.2114	0.0000
	K_2	0.3479	0.7018	1,0679	1.4526	1.8625	2,3047	2.7870	3.3177	3.9059	4.5621
$m=3$	K_1	0.9922	0.9686	0.9284	0.8705	0.7929	0.6933	0.5686	0.4148	0.2272	0.0000
	K_2	0.3123	0.6344	0.9762	1.3485	1.7628	2.2319	2.7706	3.3956	4.1264	4.9858

Arch centre line laid out to coincide with line of pressure for D.L. plus $\frac{1}{2}$ L.L.. Apart from Arch shortening and temp. Effects, there are no moments acting with this load.

Ht of arch ordinate $= K_1 r \quad \tan \phi = k_2 \cdot \frac{r}{l}$

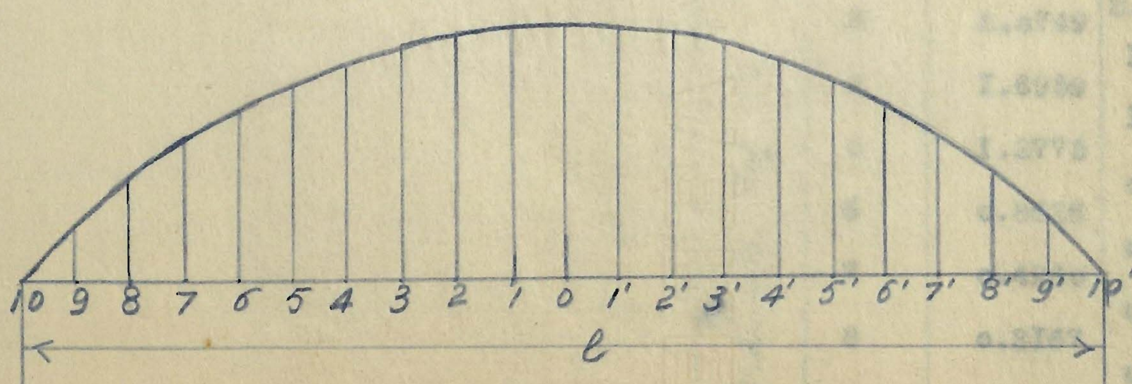


Table No. 2

ARCH FUNCTIONS

[Horizontal Thrust and Reactions for D.L plus $\frac{I}{2}$ L.L.]

$H = \frac{W_c l^2}{Ior} \cdot f_6 \quad R_A = R_B = W_c\, l.\, f_7$

m	2	3
f_6	I.44I4	I.609I
f_7	0.6576	0.8023

TABLE No. 3

m = 2

HORIZONTAL THRUST
for load P at any ord.

$H = \frac{Pl}{Ior} \times$ Table coeff.

n		. I8	. 20
Ordinate Ref. No.	0	2.635I	2. 6206
	I	2.5686	2. 5552
	2	2.3755	2. 3653
	3	2.0749	2. 0693
	4	I.6959	I. 6954
	5	I.2775	I. 28I6
	6	0.8626	0. 8696
	7	0.4960	0. 5035
	8	0.2I67	0. 2222
	9	0.0502	0. 0523

TABLE No. 4 m = 2

CROWN MOMENT
for load P at any ord.
$Mc = \frac{Pl}{Ioo} \times$ (table coefficient)

ord.	0	I	2	3	4	5	6	7	8	9
m=.18	4.434	2.25o	.68I	-.322	-.837	-.968	-.834	-.562	-.275	-.o69
m=.20	4.458	2.272	.698	-.3I3	-.836	-.975	-.845	-.576	-.284	-.o72

TABLE No. 5

SPRINGING MOMENT for load P at any ord.

$M_s = \frac{Pl}{Ioo} \times$ Table coeff.

n		m = 2		m = 3	
		.I8	.2o	.I8	.2o
Ord. Ref. No	9	-4.4oI	-4.374	-4.376	-4.348
	8	-7.336	-7.26o	-7.297	-7.I4o
	7	-8.667	-8.553	-8.44I	-8.327
	6	-8.496	-8.367	-8.4o9	-8.267
	5	-7.o83	-6.969	-6.6I3	-6.5o7
	4	-4.787	-4.7II	-4.238	-4.I72
	3	-I.99o	-I.966	-I.378	-I.367
	2	+ .92I	.888	I.59I	I.523
	I	3.6o5	3.52I	4.275	4.I74
	o	5.785	5.664	6.46o	6.322
	I'	7.267	7.I27	7.937	7.78o
	2'	7.952	7.8I5	8.6o2	8.45o
	3'	7.844	7.726	8.456	8.325
	4'	7.o3I	6.947	7.58o	7.486
	5'	5.697	5.65I	6.I67	6.II3
	6'	4.o8o	4.o69	4.I67	4.I69
	7'	2.463	2.473	2.689	2.699
	8'	I.I2o	I.I36	I.239	I.256
	9'	.267	.276	.292	.3o2

T A B L E No. 6

Quarter Point Moment for Load P at any ordinate

$M_{l/4} = \frac{Pl}{100} \times$ Table coefficient

	m=2		m=3	
n	.I8	.2o	.I8	.2o
9	o.I27	o.I33	o.I24	o.I3o
8	o.598	o.6I6	o.586	o.6o3
7	I.52I	I.55o	I.489	I.5I8
6	2,96I	3.000	2,9o7	2.946
5	4.958	5.000	4.874	4.92o
4	2.52o	2.56o	2.4I5	2.454
3	o.633	o.665	o.5o4	o.536
2	-.738	-o.7I8	-o.885	-o.865
I	-I.643	-I.637	-I.8o2	-I.796
o	-2.I44	-2.I53	-2.3o8	-2.3I6
I′	-2.3I2	-2.334	-2.47I	-2.493
2′	-2.223	-2.255	-2.37o	-2.4oI
3′	-I.95I	-I.99o	-2.o8o	-2.II9
4′	-I.57I	-I.6I2	-I.677	-I.7I7
5′	-I.I52	-I.I9o	-I.23I	- I.27o
6′	-o.752	-o.782	-o.8o6	- o.836
7′	-o.4I5	-o.437	-o.446	-o.469
8′	-o.I74	-o.I86	-o.I87	-o.I99
9′	-o.o39	-o.o42	-o.o42	-o.o45

Ord. Ref. No.

T A B L E No. 7

	n	.o.I8	0.2o
m = 2	f_8	.45o	.455
	f_9	-.o2I	-.o23
m = 3	f_8	.455	.458
	f_9	.o33	.o3o

MAX. Crown MOM. and corresponding H- thrusts for distributed L.L.

mcL(pos) $\frac{Wl.l^2}{Ioo}$. f_8 , corresp. HcL $\frac{Wl.l^2}{Ior}$ .f_9

max. (neg.) crown mom. for unif. dist. L.L. has the same numerical value as max. (pos.) mom. Corresponding H- Thrust has aso same numerical value for max. (neg.) and (pos.) moments, but with a difference in sign.

T A B L E No. 8

Max. Crown mom and Corresp. H- Thrusts for Dist. L.L.

Same as what was said for table no. 7 except that f_{10} & f_{11} were substituted for f_8 & f_9.

	n	0.I8	0.20
m = 2	f_{10}	.7II	.722
	f_{11}	-.3I3	-.3I2
m = 3	f_{10}	.73o	.74I
	f_{11}	-.3I9	-.3I7

Max. SPRINGING MOM. AND CORRESP. H-THRUSTS FOR DIST. L.L.

Same as table No. 7 with f_{12} & f_{13} substituted for f_8 & f_9

TABLE No. 9

n		.I8	.20
f_{12}	m = 2	2.42o	2.387
f_{13}		.3o2	0.3oo
f_{12}	m = 3	2.5o3	2.468
f_{13}		.3I9	.3I7

Table No. Io

H-THRUST AND MOMENT AND ARCH SHORTENING

	n	.I8	.2o
m = 2	f_3	.7267	.7333
	f_4	.I576	.I6I5
	f_5	.o294	.o3o7
m = 3	f_4	.2987	.3o6I
	f_5	.III8	.II66

$$H_1T = (t-t_0)\,\alpha\,E\,\frac{(m-I)^2}{r^2}\,Ic\,\frac{f_3}{f_5}$$

$$McT = -(t-t_0)\,\alpha\,E\,\frac{m-I}{r}\,Ic\,\frac{f_4}{f_5}$$

$$Hs = -H\frac{(m-I)^2}{r^2}\left(\frac{Ic^2}{I2b^2}\right)^{\frac{1}{3}}\beta\,\frac{f_3}{f_5}$$

$$Mcs = H\frac{(m-I)}{r}\left(\frac{Ic^2}{I2b^2}\right)^{\frac{1}{3}}\beta\,\frac{f_4}{f_5}$$

$$\beta = \left[I + \frac{I6}{9}\left(\frac{r}{l}\right)^2 - (I-n)\left\{\frac{I}{9} + \frac{I6}{I5}\left(\frac{r}{l}\right)^2\right\}\right]$$

USUAL PROCEDURE IN ARCH DESIGN

I. The proper type of construction must be selected .

2. The arc axis is laid out according to some equation such as

$$y = \frac{8\ r\ l}{6+5r}\ (\ 3c^2\ \ 10\ \ c^4 r\ )\quad \text{where } c = \frac{x}{l}$$

3. Preliminary dimensions for crown and springings are determined as was mentioned on page (   ).

4. The arch so determined is analyzed by the elastic theory for max. stresses in the steel and in the concrete. In most arches the max. stresses occur either at the crown or at the springinging although where the ratio of L.L. to D.L is large the max. stresses may be found in the haunch. For aesthetic reasons the arch ring must gradually increase in thickness from crown to springing. Such a ring has a thickness much greater than required over the greater part of the distance between crown and springing. For this reason an investigation of the crown and springing sections is usually sufficient.

When the stresses are either too small or too large, proper change in the dimensions of the arch section or the amount of rei-forcement should be made.

The design of symmetrical fixed arch using the elastic theory is very long. Fairhurst's evaluation of the àlastic theory integrals simplified the work considerably. Besides that he found a quick method for the selection of an economical arch curve and form.

I am going to design the arch first and compute the stresses a according to Fairhurst method. The design will then be checked by the Elastic Theory which means that the arch will be analyzed twice.

Analytic Method of Determining the Line of Pressure.- #
Knowing the magnitude and the positions of dead Loads at the various divisions, the line of pressure may be determined analytically by computing the static beding movements of the loads for each division point, considering the arch as a simply supported beam, and by dividing these beding moments byt he horizontal thrust. The result gives at the respective points the vertical distance of the line of pressure from a base line passing through the springings. The horiontal thrust is found by dividing the static bending moment in the center by the rise.

This method follows directly from the requirement that for fixed loads there should be no bending moment at any point in the arch. To make this possible the positive static bending moment due to the loads must be balanced by the negative " " " " " " horizontal thrust. The horizontal thrust applied at the springing and its bending moment equals Hy, that is the thrust times the vertical distance of the point at the arch axis from the springing. By equting the static bending to Hy the value of Y may be found.

The amount of work may be reduced by finding the increments of the static bending moments starting from the crown. These divided by the horizontal thrust give the depth of the line of pressure below the crown. The procedure is shown in the table on the following page. The loads and dimensions usedin the table are taken from fig.(7) the values a_1, a_2, a_3, a_4 are distances of the centers of gravity of loads in each division from their left end. ~~cent~~

CONCRETE PLAIN AND REINFORCED BY TAYLOR; THOMPSN, & SMULSKI

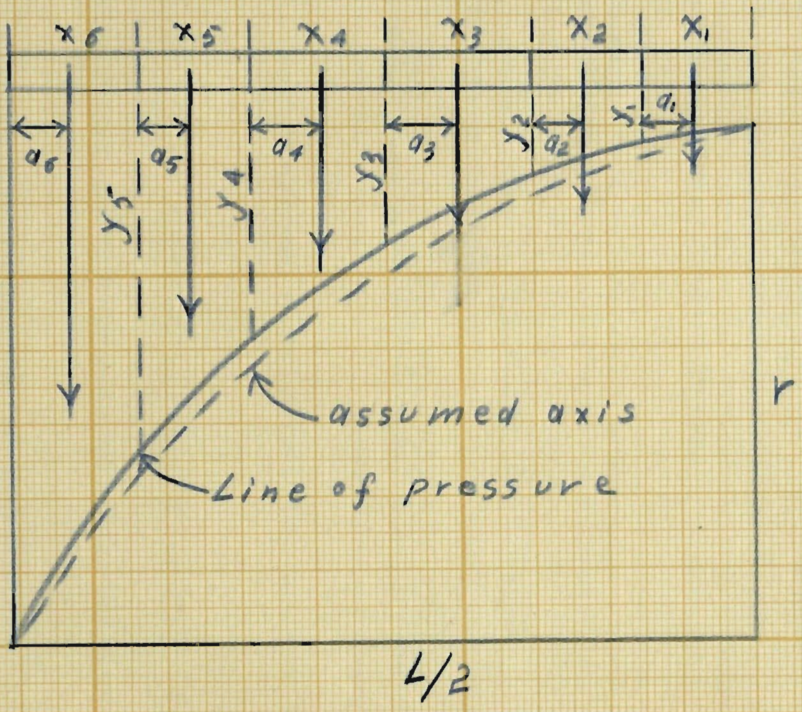

x6
x5
x4
x3
x2
x1
a6
a5
a4
a3
a2
a1
y5
y4
y3
y2
assumed axis
Line of pressure
r
L/2

Fig. 7

The loads P_1 to P_6 are dead loads. The arch is shown as divided into six divisions. Usually a larger number of divisions is advisable. the ordinates ofthe line of pressure referred to an axis passing through the crown are given in column (b) in the table.

LINE OF PRESSURE FOR SYMMETRICAL DEAD LOAD.

Points	Loads	Distance of load.	P_nA_m	Length of Div.	$P_1+P_2+\dots P_n)x_n$	M_{n-1}	Bending Moments M_m (4)+(6)+(7)	$Y'_n = \frac{M_m}{H}$ (8) / H.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	P_1	a_1	P_1a_1	x_1	0	0	$M_1=P_1a_1$	$Y'_1 = M_1/H$
2	P_2	a_2	P_2a_2	x_2	P_1x_1	M_1	$M_2=P_2a_2+P_1x_2+M_1$	$Y'_2 = M_2/H$
3	P_3	a_3	P_3a_3	x_3	$(P_1+P_2)x_3$	M_2	$M_3=P_3a_3+(P_1+P_2)x_3+M_2$	$Y'_3 = M_3/H$
4	P_4	a_4	P_4a_4	x_4	$(P_1+P_2+P_3)x_4$	M_3	M_4	$Y'_4 = M_4/H$
5	P_5	a_5	P_5a_5	x_5	$(P_1+P_2+P_3+P_4)x_5$	M_4	M_5	$Y'_5 = M_5/H$
A	P_6	a_6	P_6a_6	x_6	$(P_1+P_2+P_3+P_4+P_5)x_6$	M_5	M_A	$Y'_A = M_A/H$

A is the springing.

The value of H in the last column equals bending moment at springing divided by the rise or $H = \frac{M_A}{r}$.

Values in Col. (8) are obtained by adding items of Col. (4), (6), and (7).

Values in Col. (7) are equal to the values in Col. (8) for the previous point.

CHAPTER III

TYPICAL EXAMPLE

Design a bridge that is supposed to replace the Pasha Bridge on Beirut River.

According to the previous chapters, and to preliminary approximate computations of costs, and to some other considerations, I decided to design a reinforced concrete open-spandrel arch bridge.

Clear span = 121′ rise = 30′

L.L is H 20 Loading i.e. :

Dist. L.L. 70 lbs./□′

Concent. L.L. = 2000 lbs/ ft. of width.

Fs = 16000 f′c = 2000 fc = 650

Side walks = 5′ Railings = 1′

Total width = 2(10 + 5 + 1) = 32′

CHAP. III a. DESIGN OF DECK

Design of slab.-

Let columns be spaced at 6′ c to c .

$R = 1000 + \frac{6 \times 70}{2} = 1210$ lbs. See fig. (8)

L.L. Mom. $= 12\left(1210 \times 3 - \frac{70 \times 3^2}{2}\right) = 39780$ in - lbs

I mp. $\frac{50}{121 + 125} =$ 20.3 percent $= \frac{8,080}{47,860}$

Assume an 8″ slab (most economical)

$\left(\frac{8}{12} \times 1 \times 1\right) 150 = 100$ lbs / ft.

Wearing $\frac{30}{130}$

D.L. Mom. $= \frac{(130 \times 6^2)\, 12}{80} = 5,880$

L.L + I mp. Mom. $= \frac{47,860}{53,740}$

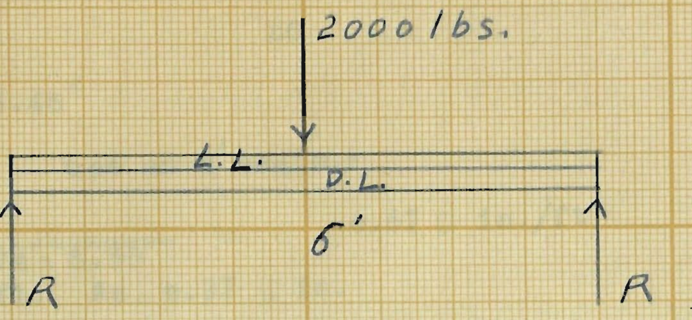


Fig. 8

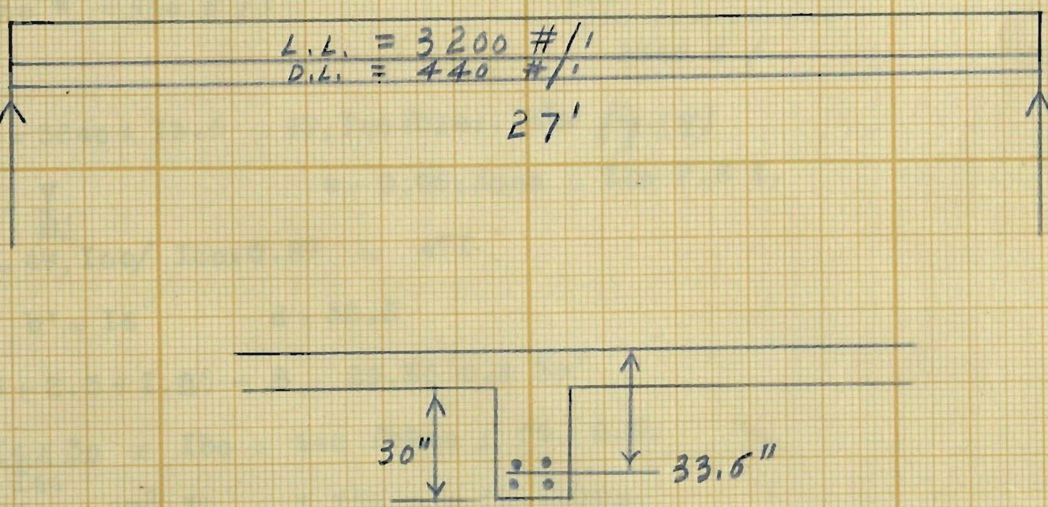


Fig. 9

$d = \sqrt{\dfrac{5374o}{I2 \times Io8}} = 6.43''$

use an 8'' scale.

$As = \dfrac{53,74o}{I6ooo \times o.874 \times 6.35} =$ o.6I □ in./ft

use 3- 5/8 ϕ / ft. As = o.9I □ in.

$\frac{I}{2}$ of steel is bent at I'-4'' from beams. $\frac{I}{2}$'' ϕ placed at 6'' transversally

on top of longitudinal bars.

Transverse T- Beams.-

D.L. from slab = I × 6 × .66 × I5o = 6oo

D.L. from wearing surface = I × 6 × 3o = I8o

L.L. (unif.) = I × 6 × ~~33o~~ 7o = 42o

L.L. (concent.) = 2ooo

assume W = 44o # /' 44o

3640 # /

R = V = 364o × I3.5 = 49,Ioo lbs. See fig.(9)

$b'd = \dfrac{V}{vj}$ v = 0.06 × 2ooo = I2o P.S i.

b'd = 49,Ioo/ I2o × 0.87 = 47I

Let b' = I4'' d = 33.6

33.6 + 2.5 + I.25 - 8 = 29.35 i.e 3o''

$W = \dfrac{I4 \times 3o}{I44} \times I5o = 44o$ lbs / ft. O.K.

$M = \dfrac{364o \times 27^2}{Io} \times I2 = 3,I8o,ooo$ in-lbs.

$As = \dfrac{3,I8o,ooo}{I6,ooo \times .87 \times 33.6} =$ 6.8 in^2.

use 6-I $\frac{I}{4}$'' ϕ As = 7.36 in.^{2}

use 3-$\frac{5}{8}$'' ϕ on top of beam.

Max. l of bars here = 6.5 m. = 2I'

$v = \dfrac{49,Ioo}{I4 \times 0.87 \times 33.6} =$ II9.6 p.s.i O.K

Let all the shear be taken by the stirrups.

A_v for $\frac{I}{2}''\phi$ stirrups with 6 branches = I.I8 in^2.

$$S = \frac{A_v f_v jd}{V} = \frac{I.I8 \times 20,000\ (33-\frac{8}{2})}{49,Ioo} = I3.9 \text{ in. say } I4 \text{ in.}$$

Put 4 stirrups @ I4 in. at supports and the rest @ 2 ft.

Design of columns.-

(Highest column) Load transmitted from projecting part:

Let t = 6″

$(2\times\frac{6}{I2} + 3.75\times I)\times I\times I5o = 7I5$ lbs/′I

7I5 × 6 = 429o lbs. D.L.

I × 6 × I3o = 78o lbs. L.L.

5o7o lbs.

49Ioo lbs.

54I7o lbs.

Try I4 × I8″ column.

P = 0.I8 f′c Ag + 0.8 fs As = 9I, 000 + 32,000 = I23, 000 lbs.

$$L/r = \frac{3o\times I2}{I4} = 25.7$$

P = I23,ooo(I.3-0.03 × 25.7) = 65,2oo lbs.

$$\text{D.L. of col.} = \frac{I2 \times I8}{I44} \times I5o \times 3o = 676o \text{ lbs.}$$

65200

54I7o

65200 > 6o,93o lbs.

As = I percent Ag = I2 × I8 × $\frac{I}{Ioo}$ = 2.I6 in^2

use 6-$\frac{3}{4}''\phi$ As = 2. 65 in^2

use 3/8″ ϕ ties @ 5″ spacing.

Max. S = I6 × 3/8 = 6"

CHAP. III(b)

CHART FOR ARCH DESIGN BY THE FIRST METHOD

Name	Location	Type	Span		Rise	
			Axis	Clear	Axis'	Clear
Open Spandrel ' Bridge '	Nahr Beirut	Open-Spandrel with 2 ribs 5.5 ft. wide at 27 ft apart	124,75 ft.	121 ft.	29.58 ft.	30' ft.

Thickness		Concrete		Properties		Dist. L.L.	Concent. L.L.
Crown	Springing	Mix	Lim.S	E	X		
2.5'	5'	$1:2\frac{1}{2}:4$	650p.s.i.	288×10^{6} lb/ft²	6×10^{-6} /F°	W=70 lbs/ft²	2000 lbs/ft. width

Dead Load plus half uniformly Dist L.L

(Moments due to arch shortening only)

(1) H- Thrust $(H_d+\frac{1}{2}L) = \frac{W_c\, l^2}{10\, r} f_6 = 419{,}000$ lbs.

(2) $R_A = R_B = W_c B \cdot L_7 = 5588 \times 124.75 \; 0.6431 = 449{,}000$ lbs.

$$\beta = \left[1+\frac{16}{9}\left(\frac{r}{l}\right)^2 - (1-n)\left\{\frac{1}{9}+\frac{16}{15}\left(\frac{r}{L}\right)^2\right\}\right] = 0.962$$

$$m = \frac{W_s^*}{W_c^*} = \frac{10450}{5588} = 1.9$$

$$I_c = \frac{5.5\times(2.5)^3}{12} = 7.16 \text{ ft}^4$$

$$I_s = \frac{5.5\times(5)^3}{12} = 57.4$$

$\tan \phi_s$ (table) $= 1.12$

$$\cos\phi_s = \frac{1}{(1+\tan^2\phi_s)^{\frac{1}{2}}} = 0.67$$

$$n = \frac{I_c}{I_s \cdot \cos\phi_s} = 0.19$$

*See next page.

C H A R T (continued)

Find Wc.-

D.L. of Arch	$2.5 \times 5.5 \times I \times I5o$	=	I875 lbs/I
D.L. of scale	$I6 \times 0.66 \times I \times I5o$	=	I6oo
D.L. of Wearing	$I6 \times I \times 3o$	=	48o
D.L. of beam	$\frac{I4 \times 3o}{I44} \times I5o \times \frac{I4}{I2}$	=	5I3
L.L. (Dist.)	$\frac{I}{2} \times I6 \times I \times 7o$	=	56o
Railings	$3.75 \times I \times I5o$	=	56o
	Wc	=	5588 lb/I

Find Ws.-

D.L. of scale		=	I6oo
Wearing		=	48o
Beams		=	5I3
L.L. (Dist.)	$2 \times 56o$	=	II2o
Railings		=	56o
D.L. of Arch	$\frac{5 \times 5.5 \times I \times I5o}{0.68}$	=	6I5o
			Io45o lb/I

Assume nom $\cos \phi_s = 0.68$

Tan. $\phi_s = 4.5I97 \times r/1 = I.I2$

cos. $\phi_s = 0.68$ as assumed.

CHART (Continued)

Ord. No.	I,I′	2,2′	3,3′	4,4′	5,5′	6,6′	7,7′	8,8′	9,9′	Io,Io′
Ht of arch ord. $K_1 r$	29.3,	28.55	27.23	25.35	22.88	19.81	16.04	11.52	6.22	0.000
tan $K_2 r/l$	.0835	.1695	.256	.348	.446	.551	.664	.788	.924	1.078
$\tan^2\phi$	.0069	.0287	.0651	.121	.199	.304	.441	.621	.851	1.16
$(I+\tan^2\phi)^{\frac{1}{6}} = A$	1.001	1.005	1.009	1.020	1.031	1.045	1.062	1.085	1.101	1.135
$I-(I-n)\,4x^2/l^2]^{\frac{1}{3}} = \beta$	.991	.986	.971	.951	.925	.890	.842	.782	.699	.575
$= \frac{A}{B} \times$ dc	2.52	2.54	2.59	2.68	2.78	2.93	3.14	3.46	3.93	4.95

C H A R T (Continued).

LOADS. ECT.	C R O W N 1	 2
E T C.	MAX. + ve MOM.	CORRESP) THRUST.
D. L. + ½ L.L.	N il	$H_D + {}^1/_2 L$ (as above) +419,000
DIST. L)L.	$+ M_{c1} = \frac{Wl^2}{100} \times f_8$ $= \frac{15 x 70 x 124.75^2 x .452}{100}$ $= +73800$	$H_{c1} = \frac{Wl.l^2}{lor} \times f_8 =$ $= \frac{15x70x124.75^2 x -.027}{10x\ 29.58.}$ $= -1488$
CONCENT. L)L.	$+ M_{cp} = \frac{P.l}{100} x F$ $= \frac{15x2000x124.75}{100} \times 4.426$ $= +166,000$	$H_{cp} = \frac{P.l.}{lor} x f$ $= \frac{15x2000x124.75x2.623}{10x29.58}$ $= +33180$
TEMP.	Fall of 30°F $M_{cT} = -(\Delta t \alpha E \frac{m-1}{r} I_c \times \frac{f_4}{f_5}$ $= \frac{40x6x288x.9}{29.58x.0216} x7.16$ $= +101,300$	Fall of 40° F. $H_T = -\Delta t \alpha E \frac{(m-1)^2}{r^2} I_c . \frac{f_3}{f_5}$ $= \frac{40x6x288x.9^2 x7.16x.730}{29.58^2 \times .0216}$ $= -15450.$
SHRINKAGE	Equivalent to fall of 20°F. $M_{csh} = +$ + 50650	Equivalent to fall of 20°F. $H_{sR} = H_T$ (above) $\times \frac{20}{40}$ $= -7725.$
ARCH SHORTENING)	$M_{cs} = H_D + {}^1/_2 L \frac{(m-1)}{(r)}$ $\left(\frac{I_c^2}{12b^2}\right)^{\frac{1}{3}} \beta . \frac{f_4}{f_5}$ $= \frac{419,000x.9}{29.58} \left(\frac{9.16^2}{12x5.5^2}\right)^{\frac{1}{3}}$ $x \frac{.962x.1452}{.0216}$ $= 42900$	$H_s = -H_D + \frac{1}{2}L \frac{(m-1)^2}{(r)^2} x \left(\frac{Ic^2}{12b^2}\right)^{\frac{1}{3}}$ $x \frac{f_3}{f_5}$ $= 419,000x.9^2 \left(\frac{7.16^2}{12x5.5^2}\right)^{\frac{1}{3}}$ $x \frac{.962x.730}{29.58^2 x.0216}$ $= -6540.$
S U M.	+ 434,650 lb✓.	+ 420,980 lbs.

Using 20- $1^1/_4$ ⌀ $N.nP_g = 0.19$

$e = 434,650/420,980 = 1.033$ ft. $e/a = 1.033/2.5 = 0.413$ Let $d' = 2''$

$d'/a = 0.07$ $K\surd = 2.77$ $f_c = 420,980x2.77/2.5x5.5x144 = 589$ P.S.I.

√ Tension over part of the section (for 1.033 > 2.5/6)

C H A R T (Continued)

LOADS ETC.	QUARTER POINT 3 MAX. + ve MOM.	4 CORRESP. THRUST	5 MAX. - ve MOM.	6 CORRESP. THRUST
D.L. + $^1/_2$ LL.	Nil	as Col. 2 +419,000 lbs.	Nil	as Col. 2 + 419,000 lbs.
DIST. L.L.	$M_{Ql} = \frac{Wl.l^2}{100} \times f_{10}$ (f_{10} = 0.974) = +189,700	$H_{Ql} = \frac{Wl.l^2}{10r} \times f_{11}$ (f_{11} = -0.313) = -17,600	-Col. 3 = -119,700	- Col. 4 + 17,600
CONCENT. L.L.	$M_{Qp} = \frac{Pl}{100} \times F_a$ (F_a = 4.987) = + 189,700	$H_{Qp} = \frac{Pl}{10r} \times f_Q$ (f_Q = 1.2755) = + 16,310	$- M_{Qp} = \frac{Pl}{100} \times F_Q$ (F_Q = 2.339) = - 88900	$H_{Qp} = \frac{Pl}{10r} \times f_Q$ (f_Q = 2.557) = +32,700
TEMP.	Fall of 40° F $M_{Qt} = M_{ct} + H_T \times Y$ (Y = 6.71) = -2500	Fall of 40° F As Col. 2 - 15450	- Col .3 + 2500	- Col. 4 + 15450
SHRINKAGE	Equivalent to fall of 20°F $M_{Qsh} = M_{Qt} \times \frac{20}{40}$ = - 1250	Col. 2 - 7725	Col. 3 - 1250	Col. 2 - 7725
ARCH SHRINKAGE	$M_{Qs} = M_{cs} + H_{s}.y$ = 1000	Col. 2 - 6540	Col. 3 - 1000	Col. 2 - 6540
S U M.	+304,650 lb.ft.	+ 387,995 lbs.	- 208,350 lb.ft.	+470,485 lbs.

Using 10-1 $^1/_4$" Ø P_g = 0.083 Let $d' = 2''$ $\frac{d'}{a} = \frac{2}{32}$ = 0.06

$e = \frac{304,650 \times 0.911}{387,995} = 0.716'$ e/a = 0.716/2.78 = 0.257 So $K^{\sqrt{}} = 2.35$.

$f_c = \frac{387,995 \times 2.35}{0.911 \times 2.78 \times 5.5 \times 144}$ = 455 P. S. I.

√. Tension over part of the section. for 0.716 > 2.78/6

CHART (Continued)

SPRINGING

LOADS ETC.	7 MAX + Ve MOM	8 CORRESP. THRUST	9 MAX - Ve MOM	10 CORRESP. THRUST
D. L. + ½ L. L.	NIL	as Col. 2 +419,000	NIL	Col. 2 + 419,000
DIST. LL. L. L.	$Msl = \frac{Wl.\ell^2}{100} \times f_{12}$ (f_{12} = 2.396) = +402,000	$Hsl = \frac{Wl.\ell^2}{10r} \times f_{13}$ (f_{13}=0.300) =+16960	- Col. 7 = = -402,000	- Col.8 =-16,960
CONCENT L. L.	$Msp = \frac{Pl}{100} \times Fs$ (Fs = 7.919) = +301,000	$Hsp = \frac{Pl}{10r} \times fs$ (fs= 2.3657) =+30,350	$-Msp = \frac{Pl}{100} \times Fs$ (Fs= 8.633) =-328,000	$Hsp = \frac{Pl}{10r} \times fs$ (fs = 0.4976) =+6375
TEMP.	Rise of 40°F $Ms_t = Mc_t + H_t \times K$ =+356, 500	- Col. 2 = + 15450	- Col. 7 = - 356,500	Col. 2 = -15450
SHRINKAGE	$Mssh = -Ms_t \times \frac{20}{40}$ =-178,250	Col. 2 = -7725	Col. 7 = -178,250	Col. 2 = -7725
Arch. SHORTENING.	Mss = Mcs + Hs r = -150,700	Col. 2 = -6540	Col. 7 = -150,700	Col. 2 = -6540
S U M	+730,550 Lb.	+ 467,495 Lbs.	-1,415,450Lb.Ft.	+378,700 Lbs.

Using 22 - 1¼" ∅ n.Pg=0.1o2 Let d' = 2" d'/a = 2/60 = 0.03.

$e = \frac{1,415,450 \times 0.67}{378,700} = 2.51'$ e/a=2.51/5 = 0.501 So K^* = 3.84

$f_c = \frac{378,700 \times 3.84}{0.67 \times 5 \times 5.5 \times 144} = 547$ P.S.i.

*. Tension over part of the section. for 2.51 > 5/6)

CHAP. III (C)

ANALYSIS OF THE DESIGNED ARCH BY THE ELASTIC THEORY (SECOND METHOD).

The dimensions of the Arch that was designed by Fairhurst method will be considered now as the preliminary dimensions for the second design then the streses will be calculated by the aid of the Elastic Theory and a Comparison made between th e two results.

I will first proceed to determine the arch axis analytically and see whether it agrees with that of the arch already designed or not.

Determination of the Line of Pressure Analytically

Pt. (1)	Loads* Pn. (2)	αn (3)	Pn αn (4)	Length of Division (5)	$(P_1+P_2+\ldots Pn)Xn$ (6)	Mn. (7)	Y (8)	Actually (9)
1	38,660 #√	3.92	151,700	6.24		151700	0.34	0.29
2	39,380	2.96	116,500	6.24	241,500	509700	1.16	1.03
3	39,690	3.08	122,200	6.24	487,500	1119400	2.56	2.35
4	40,480	3.12	126,200	6.24	734,000	1979600	4.54	4.23
5	41,420	3.	124,300	6.24	989,000	3092900	7.10	6.70
6	42,500	3.2	136,000	6.24	1246,000	4474900	10.25	9.77
7	44,800	3.1	139,000	6.24	1511,000	6124900	14.00	13.54
8	47,200	3.1	146,300	6.24	1791,000	8062200	18.40	18.10
9	51,260	3.2	164,000	6.24	2085000	10311200	23.60	23.36
10	58,500	3.16	185,000	6.24	2405,000	12901200	29.58	29.58

$$H = \frac{12,901,200}{29.58} = 436,000 \#$$

*. Those are the loads due to the dead load of the rib plus the Dist.L.L.

It is seen that the arch so gotten (analytically) is a little bit less than that we got by Fairhurst equation. It means that our negative moments are a little bit increased and so the unit stresses area little bit more than those that we get using Fairhurst method.

Point	d	$I_c = \frac{5.5d^3}{12}$ =.458d^3	$(\frac{d}{2} - d^1)^2$ (d^1=2 H¹)	$\frac{5.5d^3}{12}(\frac{d}{2} - d^1)^2$ .14Is	I=Ic+14Is	Y	$\frac{Y}{I}$	$\frac{1}{I}$	X
1	2.52¹	7.39	1.19	2.84	10.23	0.28	.027	.098	6.24
2	2.54	7.55	1.20	2.86	10.41	1 .03	.099	.096	12.48
3	2.59	8.00	1.26	1.50	9.50	2.35	.247	.106	18.72
4.	2.68	8.85	1.37	1.63	10.48	4.23	.403	.095	24.96
5	2.78	9.89	1.49	1.77	11.66	6.70	.575	.086	31.20
6	2.93	11.60	1.66	1.98	13.58	9.77	.72	.074	37.44
7	3.14	14.25	1.96	2.34	16.59	13.54	.80	.060	43.68
8.	3.46	19.05	2.44	6.40	25.45	18.06	.71	.039	49.92
9	3.93	27.80	3.20	8.38	36.18	23.36	.65	.028	56.16
0	4.95	55.60	5.39	14.10	69.70	29.58	.42	.014	62.37
							4.671	.696	

						Unit P at O					P at 2			
Pt.	X	Y	$\frac{1}{I}$	$\frac{y}{I}$	y^2/I	$\frac{x^2}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	m/I	$\frac{mx}{I}$	$\frac{my}{I}$
1	6.24	.28	.098	.028	0.00	3.81	6.24	0.61	3.81	0.17				
2	12.48	1.03	.098	.099	0.10	14.90	12.48	1.19	14.8	1.23				
3	18.72	2.35	.106	.247	0.58	37.30	18.72	1.98	37.10	4.63	6.24	.66	12.30	1.55
4	24.96	4.23	.095	.403	1.70	59.30	24.96	2.37	59.30	10.10	12.48	1.18	29.50	5.60
5	31.20	6.70	.086	.575	3.85	84.00	31.20	2.68	83.70	17.90	18.72	1.61	50.20	10.80
6	37.44	9.77	.074	.720	7.05	102.00	37.40	2.76	103.30	27.00	24.96	1.85	69.30	18.05
7	43.68	13.54	.060	.820	11.00	107.20	43.70	2.62	114.30	35.90	31.20	1.87	81.80	25.40
8	49.92	18.06	.039	.710	12.80	97.20	49.90	1.95	97.3	35.50	37.40	1.45	72.50	26.20
9	56.16	23.36	.028	.650	15.80	88.50	56.16	1.57	88.3	36.50	43.70	1.22	68.60	28.50
10	62.4	29.58	.014	.420	12.30	54.60	62.37	.87	54.3	26.20	49.90	0.70	43.70	20.70
			.696	4.670	64.68	648.81		18.40	656.21	195.13		10.54	427.90	136.20

According to formulas (13),(14) and (15) for Mc,Hc and Rc we get,

	P at O	P at 2
	Hc = 1.07	Hc = .989
	Mc = 6.02	Mc = .942
	Rc = .505	Rc = .33

	P at 4				P at 6				P at 8			
Pt.	$\frac{m}{I}$	m	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{Mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$
1												
2												
3												
4												
5	.537	6.24	16.70	3.59								
6	.925	12.48	34.60	9.00								
7	1.120	18.72	49.00	15.30	6.24	.375	16.40	5.12				
8	.975	24.96	48.70	17.70	12.48	.487	26.80	8.89				
9	.875	31.20	49.20	20.30	18.72	.524	29.50	12.20	6.24	.175	9.85	4.06
10	.524	37.40	32.70	15.70	24.96	.350	21.80	10.50	12.48	.175	11.00	5.26
	4.956		230.90	81.59		1.736	94.50	36.71		.350	20.85	9.32

P at 4: Hc = .726, Mc = -1.31, Rc = .18

P at 6: Hc = .376, Mc = -1.28, Rc = 0.073

P at 8: Hc = .105, Mc = -0.455, Rc = .016

	Pat 4				Pat 6				Part. 8			
Pt.	$\frac{m}{I}$	m	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{Mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$
1												
2												
3												
4												
5	.837	6.24	16.70	3.59								
6	.925	12.48	34.60	9.00								
7	1.120	18.72	49.00	15.30	6.24	.375	16.40	5.12				
8	.975	24.96	48.70	17.70	12.48	.487	26.80	8.89				
9	.875	31.20	49.20	20.30	18.72	.524	29.50	12.20	6.24	.175	9.85	4.06
10	.524	37.40	32.70	15.70	24.96	.350	21.80	10.50	12.48	.195	11.00	5.26
	4.956		230.90	81.59		1.736	94.50	36.71		.350	20.85	9.32

Hc = .726
Mc = -1.31
Rc = .18

Hc = .376
Mc = -1.28
Rc = 0.073

Hc = .105
Mc = -0.455
Rc = .016

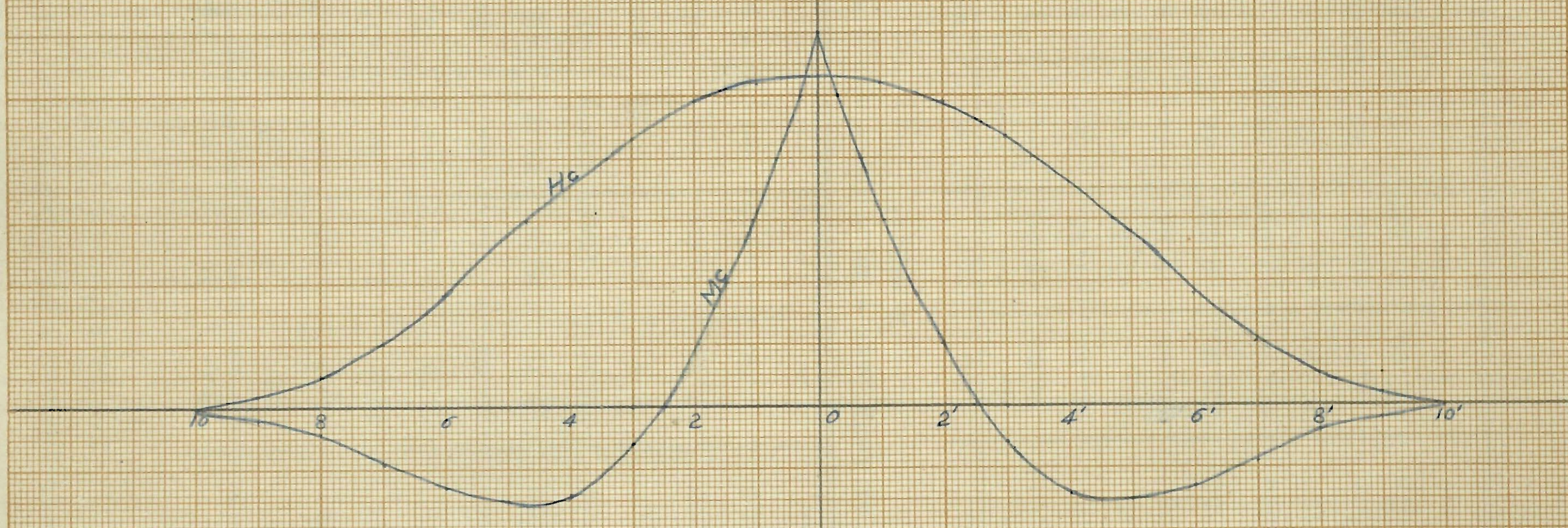
Resumé for Mc, Hc, Rc.

Unit Load at	Mc	Hc	Rc
10'	0.00	.000	.00
8'	-.455	-.105	-.016
6'	-1.28	.376	-.073
4'	-1.31	.726	-.18
2'	.942	.989	-.33
0'	6.02	1.07	.505
2	.942	.989	.33
4	-1.31	.726	.18
6	-1.28	.376	.073
8	-.455	.105	.016
T 10	.00	.00	.00

INFLUENCE LINES FOR Mc & Hc

Mc: 1 vert. □ = 1 #'

Hc: 1 Horiz. □ = 0.2 #



$+M_{1/4}$ for left portion = $Mc + Hcy + Vc \cdot X - m$

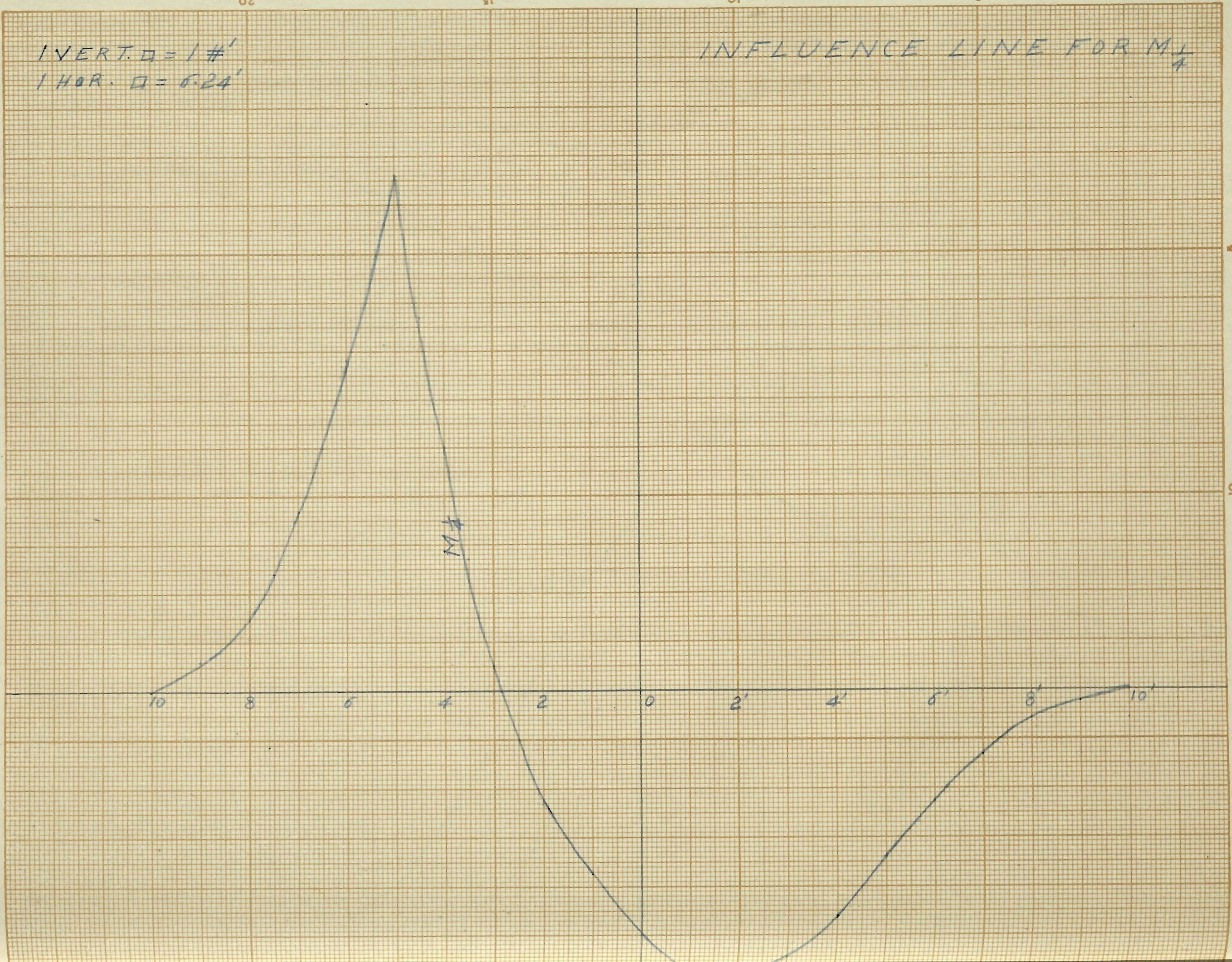
$y = 6.421'$ $X = 31.21'$

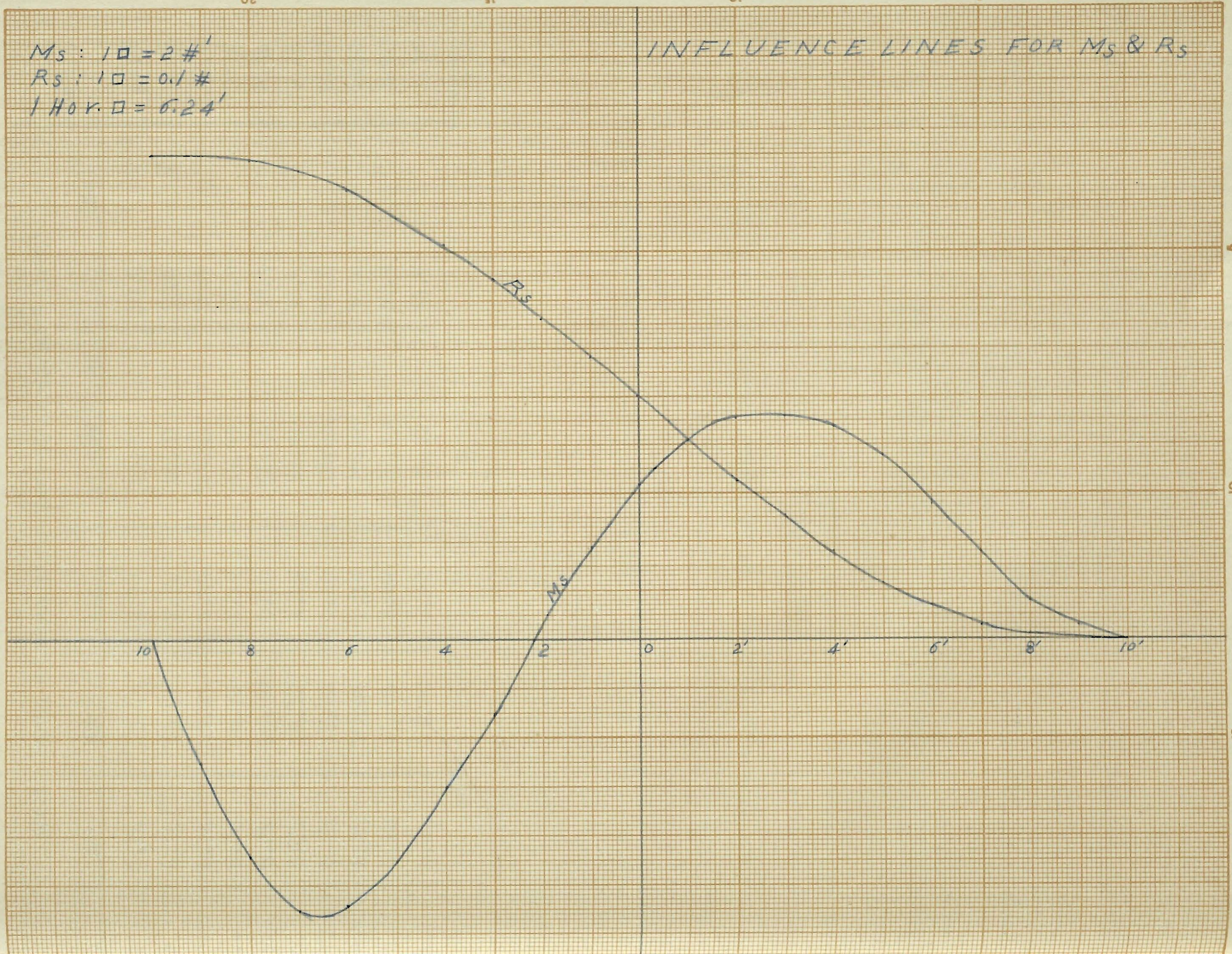
Unit Pat.	Hc	Hcy	Vc	Vcx	Mc	m	$M_{1/4}$
10'	ø	ø/	ø	ø	ø	ø	ø
8'	.105	.67	-.016	-.50	-.455	--	-.28
6'	.367	2.42	-.07	-2.28	-1.28	--	-1.14
4'	.726	4.66	-.18	-5.61	-1.31	--	-2.26
2'	.989	6.35	-.33	-10.30	.94	--	-3.01
0	1.07	6.87	.505	15.80	6.02	31.2	-2.51
2	.989	6.35	.330	10.30	.942	18.72	-1.13
4	.726	4.66	.18	5.61	-1.31	6.24	+2.72
6	.376	2.42	.073	2.28	-1.28	--	+3.42
8	.105	.675	.016	.50	- .455	--	+ .72
10	--	--	--	--	--	--	--

Ms of left part = Mc + Hcy + V_cx - m

Y =29.3' **X =62.4'**

Unit Pat.	Hc	Hcy	Vc	Vcx	Mc	m	Ms
10'	--	--	--	--	--	---	--
8'	.105	3.08	-.016	-1.00	-.455	--	1.62
6'	.376	11.10	-.07	-4.37	-1.28	--	5.45
4'	.726	21.30	-.18	-11.20	-1.31	--	8.79
2'	.989	29.00	-.33	-20.60	0.94	---	9.34
0	1.07	31.40	.505	31.50	6.02	62.4	6.52
2	.989	29.00	.330	20.60	.942	50.40	.54
4	.726	21.30	.18	11.20	-1. 31	37.5	-6.31
6	.376	11.10	.073	4.37	-1.28	25.0	-10.81
8	.105	3.08	.016	1.00	- .455	12.48	-8.86
10	---	---	---	---	---	---	---


INFLUENCE LINE FOR M 1/4
1 VERT. □ = 1 #'
1 HOR. □ = 6.24'
M 1/4
10
8
6
4
2
0
2'
4'
6'
8'
10'


INFLUENCE LINES FOR Ms & Rs
Ms : 1□ = 2#'
Rs : 1□ = 0.1 #
1 Hor. □ = 6.24'
Rs
Ms
10
8
6
4
2
0
2'
4'
6'
8'
10'

ΔX is lenth of each strip i.e. 1/10 (1/2) = 6.24'

Pt	ΔX	Δh	$14A_s$	$A = \Delta h + 14\,A_s$	$(\Delta x)^2$	$\frac{(\Delta x)^2}{A}$
1	6.24	13.85	2.38	16.23	38.9	2.39
2.	6.24	13.95	2.38	16.33	38.9	2.38
3.	6.24	14.25	1.19	15.44	38.9	2.51
4..	6.24	14.72	1.19	15.93	38.9	2.44
5.	6.24	15.30	1.19	16.49	38.9	2.36
6.	6.24	16.12	1.19	17.31	38.9	2.25
7.	6.24	17.26	1.19	18.45	38.9	2.10
8.	6.24	19.08	1.19	20.27	38.9	1.92
9.	6.24	21.61	2.62	24.23	38.9	1.60
10.	6.24	27.20	2.62	29.82	38.9	1.30
						21.25.

MAX. CROWN MOMENT

Mc due to D. L. + $\frac{1}{2}$ L.L. dist.-

Mom. due to $\frac{1}{2}$ L.L. = 70x15x0.25x6.24 = -1640 lb.'

Since net area of influence line = 0.25 □ s & 1 □ = 6.24 lb.'

Mom. due to D. L. of deck:-

A = 0.5 □ s 1 □ = 6.24

Mc due to D. L. of deck = 3153 x 0.5 x 6.24 = -9860

Mom due to D. L. of rib* -34060

Σ Mc = -45560 lb.'

Mc due to dist. L.L. .- From influence lines for Mc we get,

A = 13.5 □ s 1 □ = 6.24 lb.'

Mc = 70x15x13.5x6.24 = + 88,500 lb.'

Mc due to concent. L.L. .- Max. Ord. = 6 lb.'

Mc = 2000 x 15x6 = + 180,000 lb.'

- It is seen that max. +ve Mc exists when the arch is loaded over the middle quarter of the span i.e. on 1/8th the span on each side of the crown.

Temp. effect. - equation (20) gives Mct = +104,600 lb.'

Shrinkage effect .- it equals $\frac{1}{2}$ the temp. effect i.e. + 52,300 lb.'

Shortening effect - Ca = H sec. Ø/A

H.= 403,612 + 71,000# + 31,800# = 506,412 lbs.

$$Ca = \frac{506,412 \times 1.49}{5.5 \times 5} = 27,400 \text{ lb/ft}^2$$

$$Mcs = \frac{27,400 \times 124.8 \times 4.67}{6.24 \times 46.5} = 54,700 \text{ lb - ft.}$$ See eq. (23)

---

* See the table on the next page

#. See corresp. Thrust at crown.

Mc Due to D. L. of Rib.

Pt.	Ord.	W. Lbs.	Mom.
.05 l/2	+ 4.8	12,460	+ 59,900
.15	+ 2.0	13,180	+ 26,360
.25	0	13,490	----
.35	-1.08	14,280	- 15,820
.45	-1.5	15,220	- 22,830
.55	-1.4	16,300	- 22,810
.65	-1.06	18,600	- 19,550
.75	-.55	21,000	- 11,540
.85	-0.3	25,060	- 7,510
.95	-0.1	32,300	- 3,230

$\frac{1}{2} \sum$ Mc = - 17,030

$\sum$ Mc = - 34,060 lb.'

THRUST AT CROWN CORRESP. MAX. + VE MOM. c

Thrust due to D. L. + $\frac{1}{2}$ L.L. (Dist.) .- Find the parts of Ws & Wc due to D. L. only (without arch rib itself)

D. L. of slab = 1600** lb. / ft. of width.

D. L. of wearing 480

D. L. of beams 513

D. L. of railing 560

3153 lb./ ft. width.

From influence lines A = 64 □ s 1 □ = 1.25

Hc = 3153 x 54 x 1.25 = 213,000 lbs.

Hc due to $\frac{1}{2}$ L.L. = 35,500# lbs.

Hc due to rib D. L. = 155,112**

Hc " " D.L. + $\frac{1}{2}$ L.L. = 403,612 lbs.

Hc due to dist. L.L. .- A = 54 □ s 1 □ = 1.25

Hc. = 70 x 15 x 54 x 1.25 = 71,000 lbs.

Hc due to concent. L.L. .- Max. Ord. = 5.3 □ s 1 □ = 0.2

Hc. = 2000 x 15 x 5.3 x .2 = 31,800 lbs.

Temp. effect .- See eq. (20)

$$Hc_T = \frac{30 \times 6 \times 10^{-6} \times 288 \times 10^{6} \times 124.8 \times .696}{6.24(129.36 \times .696 - 43.7)} = 15,560 \text{ lbs.}$$

It is - ve for rise in temp.

**. See next page.

\# . It is half the 71,000 lbs. See Hc for dist. L.L.

D. L. of Rib And Hc. Due to it.

Pt.	av.d.	L	W. lbs.	Ord.	Mom.
.05 1/2	2.51	6	12460	1.04	13,000
.15	2.53	6.3	13180	1.02	13,400
.25	2.57	6.35	13490	.92	12,400
.35	2.64	6.55	14280	.80	11,400
.45	2.73	6.75	15220	.64	9,700
.55	2.86	6.9	16300	.44	7,150
.65	3.04	7.4	18600	.30	5,490
.75	3.30	7.7	21000	.14	2,870
.85	3.70	8.2	25060	.06	1,500
.95	4.44	8.8	32300	.02	646
			181,890		77,556
					2
					155,112

Shrinkage effect .- it is half that due to temp. i.e. = -7780 lbs.

Shortening effect .- See eq. (22)

$$Hcs = \frac{27,400 \times 124.8 \times .696}{6.24 \times 46.56} = 8220 \text{ lbs.}$$

RESUMÉ FOR CROWN.

	Max. +ve Mom. (1)	Corresp. Thrust. (2)
D. L. + $\frac{1}{2}$ L.L.	-45,560	+403,612
Dist. L.L.	+88,500	+71 ,000
Cocent. L.L.	+180,000	+31, 800
Temp.	+1o4,600	-15, 560
Shrinkage	+52 ,300	- 7, 780
Shortening	+54, 700	- 8, 220
Σ	434,540	474, 852

STRESSES IN CONCRETE AND STEEL AT THE CROWN

Using 20 - 1 $\frac{1}{4}''$ Ø $\quad nP_g = 0.19$

$e = 434{,}540/474{,}852 = 0.916 > 2.5/6$ i.e. Tension occurs on part of the section.

$e/a = 0916/2.5 = 0.366 \quad$ Let $d' = 2 \quad d'/a = 0.07 \;\therefore K=2.50$

$f_c = \frac{NK}{ba}$ where $K = \frac{2k}{k^2+2nP_g k - nP_g}$

$f_c = \frac{474{,}852 \times 2.5}{2.5 \times 5.5 \times 144} = 600$ P.S.i. which is 1.8% greater than that found by the first method on page ( ).

From the above value of K if we substitute for it 2.5 we get:

$k = 0.696$

$f_s = nf_c\left(\frac{d}{ka} - 1\right) = 15 \times 600\left(\frac{2.34}{2.5 \times .696} - 1\right) = 3160$ P.S.I.

MOM. $\frac{1}{4}$ DUE TO D.L. OF RIB.

Ft.	Ord.@	W lbs.	Mom.
.05	- 2.2	12460	- 27430
.15	- 1.5	13180	- 19780
.25	- .5	13490	- 6765
.35	+ 1.4	14280	+ 20000
.45	+ 3.8	15220	+ 57900
.55	+ 4.1	16300	+ 67000
.65	+ 2.6	18600	+ 48400
.75	+ 1.2	21000	+ 25200
.85	+ .5	25060	+ 12530
.95	+ 1.5	32300	+ 4850
.05'	-2.2	12460	- 27400
.15'	- 3.05	13180	- 40200
.25'	- 2.6	13490	- 35100
.35'	- 2.5	14280	- 35700
.45'	- 2.0	15220	- 30440
.55'	-1.45	16300	- 23620
.65'	- .9	18600	- 16720
.75'	- .45	21000	- 9470
.85'	- .20	25060	- 5012
.95'	- .05	32300	- 1615
			- 43400 lb. ft. = ΣM

@ It refers to the ordinates of the influence lines for $M\frac{1}{4}$

MAX. QUARTER POINT + VE MOM.

$M_{1/4}$ due to D.L. + 1/2 L.L. (Dist.) .-

Mom. due to 1/2 L.L. = 70 x 15 x 19 x 3.12 = +62,400 lb.ft.

Since the net area of the influence lines is 19 squares.

Where one square = 3.12 lb. ft.

Mom. due to D.L. of deck. = 3153 x 14.5 x 3.12 -142,500

Since A = 14.5 □ s.

Mom. due to D.L. of rib√ = -43,400

Total M 1/4 = -123,500 lb.ft.

M 1/4 should have been zero. We shall consider it with the - ve moments for safety.

M 1/4 due to dist. L.L. .- From influence lines for M1/4 we get:

A = 27.5 □ s. 1 □ =3.12 lb.ft.

M 1/4 = 70 x 15 x 27.5 x 3.12 = √ +90,300 lb.ft.

M 1/4 due to concent. L.L. .-

Max. Ord. = 10.6 □ s. 1 □ = 1/2 lb.ft.

$M_{1/4}$ = 2000x15x10.6 x 0.5 = +159,000 lb.ft.

It is seen that to get max. - ve mom. on the left 1/4 POINT, the arch should be loaded on its right 0.65 of span i.e. at its 5/8 th. of span. To get max. + ve mom., the left 3/8 th. of span should be loaded.

+ ve mom. due to temp. effect.- $M_{aT} = M_{cT} + H_{cT} \times Y$

= 104,600 - 15,560 x 6.71 = - 200 lb.ft.

+ve Mom. due to shrinkage effect.—

$M_{sh} = M_{aT}/2$ = -200/2 = -100,

Shortening effect.- $M_{as} = M_{cs} + H_s y$ = 54,700 - 8,220 x 6.71 = - 450.

√ See next page.

MAX. QUARTER POINT - ve MOM.

$M_{1/4}$ due to dist. L.L. .- From influence lines for $M_{1/4}$ we get;

A = 42 □ s   1 □ =3.12

$M_{1/4}$ = 70x15x42x3.12 = - 137,500 lb. ft.

$M_{1/4}$ dut to concent. L.L. .-

Max. Ord. = 6.1 □ s   1 □ =0.5

$M_{1/4}$ = 2000x15x6.1x0.5 = -91,500 lb. ft.

Temp. effect .- Same as in the previous article with the sign changed i.e. +200

Shrinkage effect. - Same as in the previous article i.e. -100

Shortening effect. - Same as in the previous article i.e. -450

Temp. effect .- $H_{1/4}$ =- Hc i.e. + 15,560 lbs.

Shrinkage effect .- Same as that of crown i.e. .- 7780 lbs.

Shortening effect.- Same as that of crown i.e. .- 8220 lbs.

THRUST AT $\frac{1}{4}$th. POINT CORRESP. MAX. + ve $M_{1/4}$

Thrust due to D.L. + $\frac{1}{2}$ L.L. (Dist.) .- same as that of the crown i.e. + 403,612 lbs.

Thrusts dueto dist. L.L. .- The influence lines for $M_{\frac{1}{4}}$ and H_c show that the dist. L.L. should be just on the left $\frac{1}{3}$rd. of the arch.

A = 12 □ s 1 □ = 1.25

$H_{\frac{1}{4}}$ = 70x15x12x1.25 = + 15,750 lbs.

Thrust due to concent. L.L. .- The concent L.L. should be put on the left $\frac{1}{4}$th. point of the arch.

Ord. = 2.6 □ s. 1 □ =0.2

$H_{\frac{1}{4}}$ =2000 x15 x 2.6 x 0.2 = + 15600 lbs.

Temp. effect. - Same as that at crown i.e. - 15,560 lbs.

Shrinkage effect.- Same as that at crown i.e. 7,780 lbs.

Shortening effect.-Same as that at crown i.e. 8,220 lbs.

THRUST AT $\frac{1}{4}$th. POINT CORRESP. MAX. - ve $M_{\frac{1}{4}}$

Thrust due to D.L. + $\frac{1}{2}$ L.L. (Dist.) .- Same as that of the crown i.e. 403,612 lbs.

Thrusts due to dist. L.L. - The right $\frac{5}{8}$th. of span should be loaded (See influence lines for $M_{\frac{1}{4}}$ & Hc)

A = 40 □ s 1 □ = 1.25

$H_{\frac{1}{4}}$ = 70x15x40x1.25 = 52,500 lbs.

Thrust due to concent. L.L. .- The concent. L.L. should be placed at a point $\frac{1}{10}$ th. of the span to the right of the crown.

Corresp. Ord. = 5.1 □ s 1 □ = 0.2

$H_{\frac{1}{4}}$ = 2000x15x5.1x0.2 = 30,600 lbs.

R E S U M E'

FOR

QUARTER POINT

	Max. +ve Mom. 3	Corresp. Thrust. 4	Max. - ve Mom. 5	Corresp. Thrust. 6
D. L. ½ L.L.	0	as col. 2 403,612	-123,500	Col. 2 403,612
Dist. L.L.	90,500	15 ,750	-137,500	52 ,500
Concent. L.L.	159,000	15,600	- 91,500	30,600
Temp.	- 200*	Col. 2 -15,560*	-Col. 3 200	-Col. 4 15,560
Shrinkage	- 100	-7780	-100	- 7,780
Shortening	- 450	Col. 2 -8,220	Col. 3 -450	Col. 2 - 8,220
Σ	248,750	403, 402	352,850	486,272

CONCRETE AND STEEL STRESSES AT THE ¼ POINT

Using 10- $1\frac{1}{4}$" ϕ $\quad n.P_g = 0.083 \quad$ Let $d' = 2"$, $\frac{d'}{a} = 2/32 = 0.06$.

$e = \frac{352{,}850 \times 0.911}{486{,}272} = 0.661' > 2.78/6$ i.e. tension occus on part of the section. $e/a = 0.661/2.78 = 0.238$

$K. = 2.20 \qquad f_c = \frac{486{,}272 \times 2.2}{0.911 \times 2.78 \times 5.5 \times 144} = 530$, P.S. I. which is 14% greater than that found by the first method on page (   )

Since $K = 2.20 \qquad k = 0.84$

So $f_s = 15 \times 530 \left(\frac{2.62}{0.84 \times 2.78} - 1 \right) = 980$ P.S.I.

MAX. SPRINGING POINT +VE MOM.

M_s due to D.L. + $\frac{1}{2}$ L.L. (Dist.).-

Mom. due to $\frac{1}{2}$ L.L., A = -22.3 □ s   1 □ = 12.48

M_s = 70 x 15 x 22.3 x 12.48 =-   - 292,009 lbs.

Mom. due to D.L. of deck: A = 6.7 □ s   1□ = 12.48

M_s = 3153 x 6.7 x 12.48 =   + 263,000 lb'

Mom. due to D.L. of rib *   - 96,000 lb'

$\sum M_s$ =   - 125,000 lb'

This will be considered with the negative M_s

Ms due to dist. L.L. .- A = 32 □ s.   1 □ = 12.48 lb'.

M_s = 70x15 (32x12.48) = 419,000 lbs. & occurs when loaded over right 5/8th of span.

M_s due to concent L.L. .- Max. Ord. = 9.10 lb'.

M_s = 2000x15x9.10 = 273,000 lb'. & occurs when P is at 1/10th. of span. to the right of the crown.

Temp. effect .- $M_s = M_c + H_c \times r$

= - 104,600 +15,560 x29.58 = 356,400 lbs'.

Shrinkage effect . - It is half that of temp. effect with the sign changed i.e. - 178,200

Shortening effect .- M_s sh.= M_{cs}+H_{cs}.r = 54,700 - 8220x29.58 = -188,600lb'.

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* See table on next page.

M_s D U E To D.L. of RIB

Pt. :	Ord* :	W.lbs. :	Mom. :
.05 ½	5.2	12,460	64,400
.15	2.2	13,180	29,000
.25	:- 1.2	13,490	-16,200
.35	-4.8	14,280	-68,700
.45	- 7.8	15,220	-118,600
.55	-10.0	16,300	-163,000
.65	-11.	18,600	-206,300
.75	-10.2	21,000	-212,100
.85	- 7.4	25,060	-185,500
.95	- 3.0	32,300	- 96,900
.05	7.4	12,460	+92,400
.15	8.8	13,180	116,200
.25	9.7	13,490	131,200
.35	9.2	14,280	131,200
.45	8.2	15,220	124,900
.55	6.8	16,300	110,900
.65	4.4	18,600	81,800
.75	2.6	21,000	54,700
.85	1.1	25,060	27,600
.95	0.2	32,300	6,460
		Σ Ms =	-96, 000 lb'.

* It refers to the Ordinate of the influence lines for M_s

MAX. SPRINGING POINT - VE MOM.

M_s due to dist. L.L. .- A = 27.5 □ s   1 □ = 12.48

M_s = 70x15x27.5x12.48 = -360,000 lb.' & occurs when load is on left 3/8th. of span.

M_s due to concent. L.L. .- Max Ord. = 5.55 □ s   1 □ = 2.

M_s = 2000x15x5.55x2 = -333,000 lb.' & occurs when P is at 1/3 rd. of span to the left of the crown.

Temp. effect .- Same as in the previous article with the sign changed i.e. - 356,400 lb.'

Shrinkage effect .- Same as in the previous article i.e. - 178,200 lb.'

Shortening effect .- Same as in the previous article i.e. - 188,600 lb.'

THRUSTS AT SPRINGING CORRESP. MAX. + VE MOM.

H_s due to D.L. + $\frac{1}{2}$ L.L. .- same as that of the crown i.e. 403,612 lbs.

H_s due to dist. L.L. .- From the influence lines for M_s & H_c we get:

A = 40 □ s   1 □ = 1.25.

H_c = H_s = 40x1.25x70x15 = 52,600 lbs.

H_s due to concent L.L. .- A = 4.8 □ s   1 □ = 0.2

H_c = H_s = 2000 x 15 x 4.8 x 0.2 = 28,800 lbs.

Temp. effect .- same as that at crown with the sign changed i.e. 15,560 lb

Shrinkage effect . - Same as that at crown i. e. - 7780

Shortening effect .- Same as that at crown i. e. -8220

THRUSTS AT SPRINGING CORRESP. MAX . - VE MOM.

H_s due to D.L. + $\frac{1}{2}$ L.L. - Same as that at crown i.e. 403,612 lbs.

H_s due to dist. L.L. - From the influence lines for M_s & H_c we get:

A = 14 □ s 1, □ = 1.25

$H_c = H_s = 70 \times 15 \times 14 \times 1.25 = 18{,}400$ lbs.

H_s due to concent L.L. .- A = 1.4 □ s 1 □ = 0.2

$H_c = H_s = 2000 \times 15 \times 1.4 \times 0.2 = 84{,}00$ lbs.

Temp. effect - Same as that at crown i.e. - 15,560

Shrinkage effect .- Same as that at crown i.e. - 7780 lbs.

Shortening effect . - Same as that at crown i.e. -8, 220 lbs.

RESUMÉ
for
SPRINGINT POINT

	Max. +ve Mom. 7	Corresp. Thrust. 8	Max.-ve Mom. 9	Corresp. Thrust. 10
D.L. ½ L.L.	0	Col. 2 403,216	- 125,000	Col. 2 403,612
Dist.L.L.	419,000	52,600	- 360,000	18,400
Concent. L.L.	273,000	28,800	- 333,000	8,400
Temp.	356,400	15,560	- 356,400	-15,560
Shrinkage	-178,200	- 7,780	- 178,200	- 7,780
Shortening	-188,600	- 8,220	- 188,600	- 8,220
Σ	681,600 #'	484,572 #	1,541,200	398,852 #

CONCRETE AND STEEL STRESSES AT SPRINGING POINT

Using 22 - $1\frac{1}{4}$" ϕ $n p_g = 0.102$ Let $d' = 2"$; $\frac{d'}{a} = 0.03$.

$e = \frac{1,541,200 \times 0.67}{398,852} = 2.596'$ > than 5/6 i.e. tension occurs on part of the section.

$e/a = 2.596/5 = 0.52$ So $K = 3.99$

$f_c = \frac{398,852 \times 3.99}{0.67 \times 5.5 \times 144 \times 5} = 600$ P.S.I. which is 8.8% > than that found by the first method on page ( )

CHAP III (d)

DESIGN OF ABUTMENTS.

The abutments at each end of the bridge will be designed for both ribs together i.e. one single abutment is used for both ribs' ends. They will be designed to carry the fill and the forces transmitted to them from the deck slab and arch rib. The horizontal and vertical springing thrusts are those that correspond to the max.-ve springing moments.

The weight of the abutment itself as well as that of the fill is of prime importance as will be noticed from the design.

From a study of the locality of the bridge and from a geologic map, the abutments were seen to rest on a layer of sand and gravel which can carry a pressure of 4.5 - 5 tons/ft^2 approximately. It was also decided to have the abutments made of plain concrete with a stone finish. The stone face is of the projecting type.

A retaining wall of the cantilever type is built monolithically with the abutments to hold the back-fill of earth.

SHEAR AT SPRINGING

Unit P. at	R_c	R_s
10'	0.000	0.000
8'	-0.016	0.016
6'	-0.073	0.073
4'	-0.180	0.180
2'	-0.330	0.330
0	0.505	0.500
2	0.330	0.660
4	0.180	0.820
6	0.073	0.927
8	0.016	0.984
10	0.000	1.000

This table allows us to plot the influence lines far the shear at the springint.

R due to D.L. - = 363,780 Lbs

D. L. of rib = 395,000

758,780 lbs.

@ 2 = 379,390 lbs.

Also, from influence lines:

A = 100 □ s 1 □ = 0.624

R_s due to D.L. of Deck. = 3153 x 100 x 0.624 = 395,000/2

R_s due to L.L. .- R_s that corresponds max. - ve

M_s occurs when A = 67 □ s 1 □ = 0.624

R = 70x15x67x0.624 = 44,100 lbs.

This is the case for dist. L.L.. When the L.L. is concent., max. Ord. = 0.95 □ s

R_s = 2000x15x0.95 = 28,500 lbs.

Temp. effect .- R_s = H tan. Ø tan. Ø = 1.078

R_s = ± 15,560 x 1.078 = ±16,800 lbs.

Shrinkage effect = ½ x ± 16,800 = ±8400 lbs.

Shortening effect - - 8220 x 1.078 = -8850. lbs.

M_s due to D.L. of Dekc * = 263,000 Corresp. H_s = 213,000

M_s due to D.L. of rib = - 96 ,000 " H = 155,200

167,000 lbs' 368,112 lbs.

	M_s	Corresp. H_s	Corresp. R_s
D.L.	167,000	368,112	379,390
Dist. L.L.	-360,000	18,400	44,100
Cocent.L.L.	-333,000	08,400	28,500
Temp.	-356,400	15,500	16,800
Shrinkage	-178,200	-7,780	-8,400
Shortening	-188,600	-8,220	-8,850
	-1,249,200	394,472	451,540

*. See Max. Springint paint + Ve M_om.

DESIGN OF RETAINING WALL.

$$P = \tfrac{1}{2}\, wh\,(h + 2h') \times \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$Y = \frac{h^2 + 3h\,h'}{3(h + 2h')}$$

The fill is gravel and sand $\phi = 40^\circ$ $w = 110$ lbs./ ft^3

$h' = 70/110 = 0.64'$

From the above we get:- See fig. (10)

$$P = \tfrac{1}{2} \times 110 \times 32\,(32 + 2 \times .64) \times \frac{1 - .64}{1 + .64} = 12{,}830 \text{ lbs.}$$

$$Y = \frac{32^2 + 3 \times 32 \times .64}{3(32 + 1.28)} = 10.9'$$

Max. Mom. = 12,830 x 10.9 = 140,000 lb'

$$d = \sqrt{\frac{140{,}000 \times 12}{108 \times 12}} = 35.9''$$

use 38, thickness

$$As = \frac{140{,}000 \times 12}{16{,}000 \times 0.87 \times 35.9} = 3.36 \text{ in. square /ft.}$$

1 @ 1 1/8" ϕ = 0.994 in. sq.

3.36 / 0.994 = 3.38 bars / ft.

12/ 3.38 = 3.55''

use 1 1/8'' ϕ @ $3\tfrac{1}{2}$'' C to c.

$$v = \frac{12{,}830}{12 \times 0.87 \times 35.9} = 34.3 \text{ P. S. I.} \quad \text{O.K.}$$

Consider a section at 22' from the top:

$$P = \tfrac{1}{2} \times 110 \times 22 \times 23.28 \times 0.22 = 6200 \text{ lbs.}$$

$$Y = \frac{22^2 + 3 \times 22 \times 0.64}{3(23.28)} = 7.5'$$

M = 6200x 7.5 x 12 = 558,000 lb''

$$d = \sqrt{\frac{558{,}000}{12 \times 108}} = 20.7''$$ use 23'' actual d is 29.7'' - 2 = 27.7''

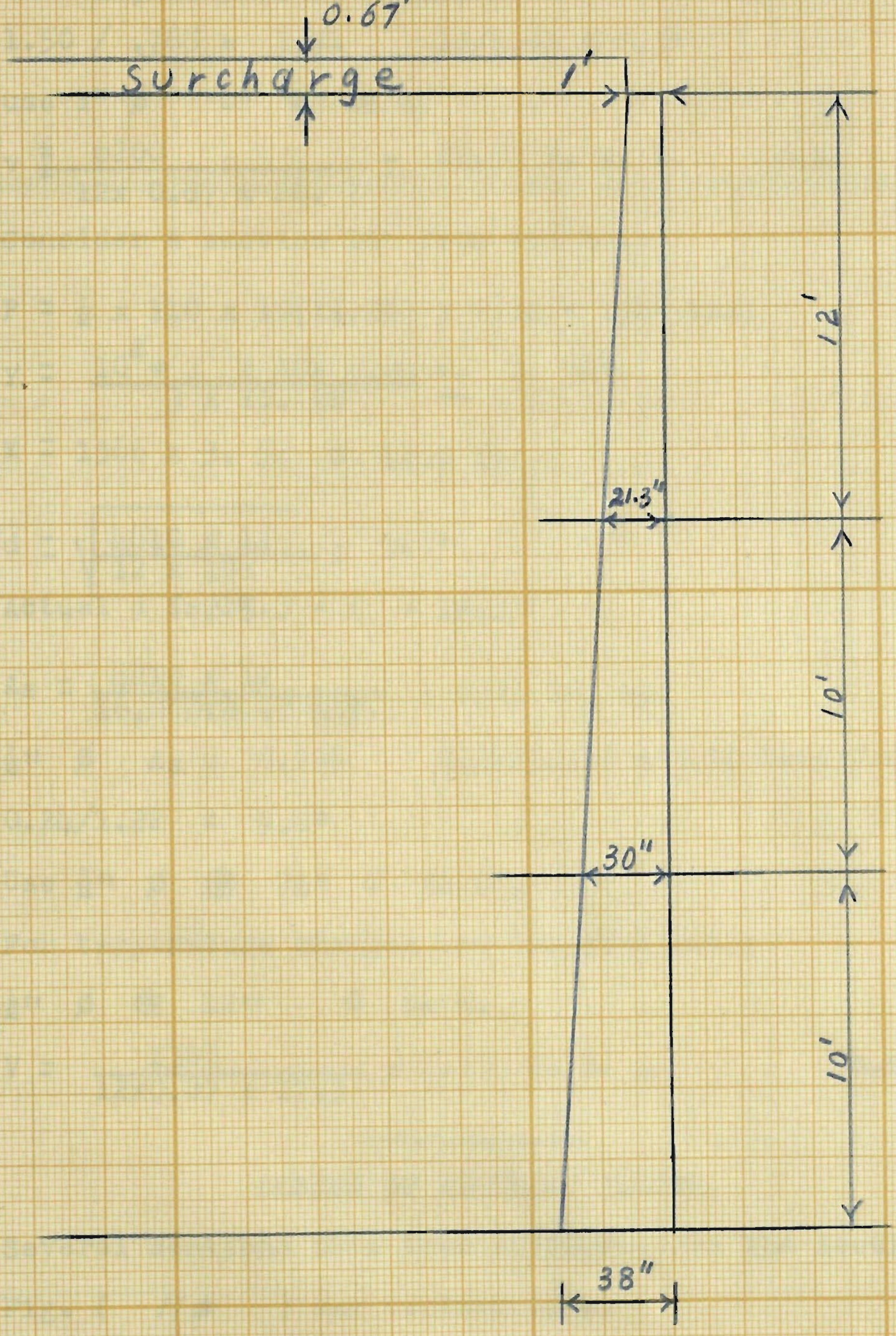


Fig. 10

$$A = \frac{558,000}{16,000 \times 0.87 \times 27.7} = 1.50 \text{ in. sq., } 1''\text{Ø} \qquad A_s = 0.785 \text{ in.sq.}$$

$1.50 / .785 = 1.91 \qquad 12/1.91 = 6.3''$

use 1" Ø 6" c to c.

$$v = \frac{6200}{12 \times 0.87 \times 26.7} = 22.3 \text{ P. S. I.} \qquad \text{O.K.}$$

Consider a section 10' from the top:

$P = \frac{1}{2} \times 110 \times 10(11.28) \times 0.22 = 1366$ lbs.

$$Y = \frac{10^2 + 3 \times 10 \times 0.64}{3 \times 11.28} = 3.52'.$$

$M = 1366 \times 3.52 = 4810$ lb'

$$d = \sqrt{\frac{4810 \times 12}{12 \times 108}} = 6.7''$$

Actual d is $21.3 - 2 = 19.3''$

$$A_s = \frac{4810 \times 12}{16,000 \times 0.87 \times 19.3} = 0.24 \text{ in. sq.}$$

$\frac{1}{2}''$ Ø $A_s = 0.196 \qquad 0.24/0.196 = 1.22$ bars / ft.

$0.24/1.22 = 9.8''$

Use $\frac{1}{2}''$ Ø @ $9\frac{1}{2}''$ C to C .

For temperature changes use longitudinally

$\frac{1}{2}''$ Ø @ 12" C to C.

$$V = \frac{1366}{12 \times 0.87 \times 17.3} = 7.59 \text{ P. S. I.} \qquad \text{O.K.}$$

DESIGN OF ABUTMENT PROPER.

Several sections have been assumed before the section shown in Fig. (11) was decided upon.

$M_s = 2 \times 1,249,200^* = 2,498,400$ lb'

$H_s = 2 \times 394,472^* = 788,944$ lbs.

$R_s = 2 \times 451,540^* = 903,080$ lbs.

$P = \frac{1}{2} \times 110 \times 46.6 \times 47.9 \times 0.22 = 26,900$ lb/1

$\times 36 = 970,000$ lbs.

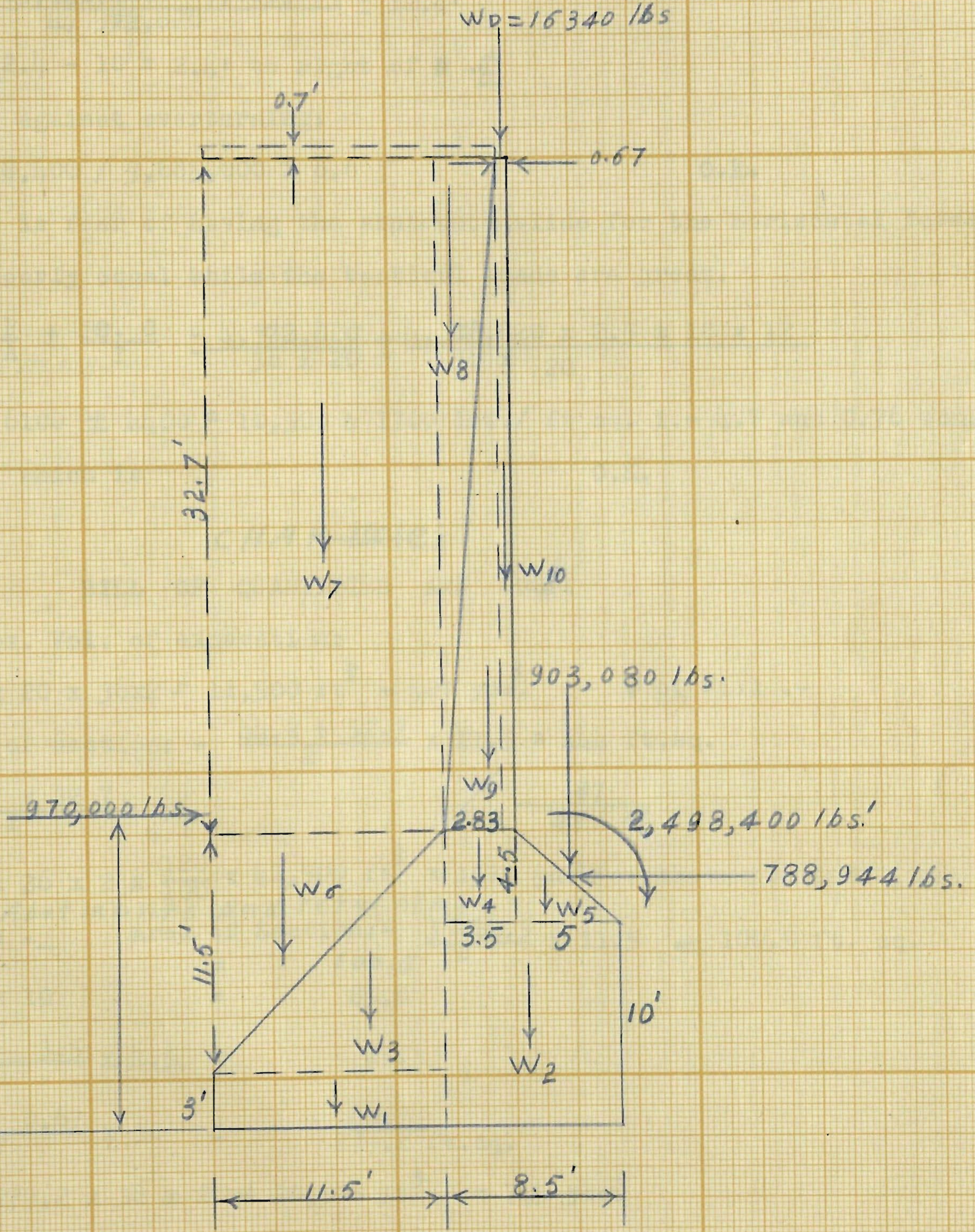
$$Y = \frac{46.5^2 + 3 \times 46.5 \times 0.7}{3 \times 47.9} = 15.7'$$

* See the table found on Page (64)

STABILYZING LOADS AND MOMENTS.

	Loads in bls.		Lever arm.	Mom. Lb'
W^1	3 x 11.5 x 36 x 150 *	= 186,00 0	05.75	1,065,000
W^2	8.5 x 10 x 5400	= 459,000	15.75	7,210,000
W^3	11.5 x 11.5 x.5 x 5400	=3,580,000	07.70	2,760,000
W^4	3.5 x 4.5 x 5400	=0,085,000	13.30	1,130,000
W^5	4.5 x 2.5 x 5400	=0,060,800	16.67	1,015,000
W^6	11.5 x 11.5 .5 x 3960#	=0,262,000	03.85	1,010,000
W^7	32.7 x 11. 5 x 3960	=1,495,000	05.75	8,580,000
W^8	2.83 x 32.7 x .5 x3960	=0,184,000	12.40	2,280,000
W^9	2.83 x 16 x 5400	=0,244,000	13.30	3,250,000
W^{10}	0.67.x32 x 5400	=0,115,600	14.67	1,700,000
W_0		=0,016,340	14.67	00,240,000
R_s		=0,903,080	17.50	15,800,000
	Σ W =	4,372,820		
P		=0, 976,000	15.70	15,200,000
				02,498,400
M_s				63,522,400 lb'.

*. 36 x 150 = 5400
#. 36 x 110 = 3960



Wp = 16340 lbs
0.7'
0.67
W8
32.7'
W7
W10
903,080 lbs.
W9
970,000 lbs
2.83
2,498,400 lbs.'
4.5
788,944 lbs.
W6
W4
W5
3.5
5
11.5'
10'
W3
W2
3'
W1
11.5'
8.5'

Fig. 11

Overturning Moments:-

$788,944 \times 12.25 = 9,685,000$ lb'

$$\bar{X} = \frac{63,522,400 - 9,685,000}{4,372,820} = 12.4'$$

$C = 12.4 - 10 = 2.4'$ to right of ℄

F. S. against overturning:

$63,738,400 / 9,685,000 = 6.6$ O.K.

There is fear of having the abutments slide for the horizontal forces are nearly equal while the vertical loads are great.

$$S = \frac{P}{A} \pm \frac{Pe.\ C}{I} = \frac{4,372,820}{36 \times 20} \pm \frac{4,372,820 \times 2.5 \times 10 \times 12}{36 \times 20}$$

$= 6100 \pm 4400 = 10,500$ & 1700 lbs./ ft sq. i.e 4.7 and 0.76 tons/ft^2

which is O.K.

CHAP ~~III~~ (e)

BILL OF QUANTITIES AND COST.

Approx. Vol. of excavation:

$= 8 \times 20 \times 36 \times 2 = 11,500$ ft^3 $= 426$ yds^3. V of Fill .-

Area of section: - $\frac{44.2 + 32.7}{2} \times 11.5 = 444$ ft.sq.

$\frac{2.83 \times 32.7}{2} = 47$ 47

491 ft.sq.   491 □'

$491 \times 36 \times 2 \times \frac{120}{100} = 42,500$ ft^3 $= 1575$ yds^3. V of Abutments and Retaining

V of Abutments and retaining walls .- Area of Section:- $\frac{3 + 14.5}{2} \times 11.5 = 100.5$ ft. sq.

	=	100.5
8.5×10	=	85.0
$\frac{8.5 + 3.5}{2} \times 4.5$	=	27.
$\frac{3.5 + 0.67}{2} \times 32$	=	66.7
		279.2 ft.sq.

$V = 279.2 \times 36 \times 2 = 20,100$ ft^3 $= 745$ yds^3.

The area of the stones that cover the abutments is 540 yds.sq.

Deck - Amount of Concrete needed.

Area of Section (excluding columns, ribs, and transverse beans.):

Slale $= 30 \times \frac{8}{12} = 20$ ft. sq. 20

Sidewalks $= 12 \times \frac{1}{2} =$ 06

Railings $= 3.75 \times 1 \times 2 =$ 07.7

Longit. beans $= 1.7 \times \frac{18}{12} \times 2 =$ 05.1

38.6

$38.6 \times 133' = 5120$ ft.^{3}

Transverse Beans = $\left(\frac{30 \times 14}{144} \times 25\right) 23 = 1680$ ft. sq.

columns.-

A = $18 \times 14/144 = 1.75$ ft. sq.

h'	V ft^3
30	52.5
28	49.0
22.6	39.5
18	31.5
14	24.5
1o.8	18.9
7.6	13.3
5.4	9.5
4.0	7.0
2.6	4.6
2.6	4.6
1.0	1.8

$256.7 \times 2 = 513.4$ ft.^{3}

Volume of Arch Ribs.-

From page (51), W of ribs = 181,890 lbs. i.e. V = 1210 ft.^{3}

1210 x 2 = 2420 ft.^{3}

Ribs = 2420 ft.^{3}

Slab = 5120 "

Beams = 1680 "

Colums = 0513 "

9733 " = 326 yds.^{3}

The arch ribs, columns, and railings have all to be plastered which means an area of 1028 yds. sq.

The sidewalks have to be paved with concrete tiles. Area = 147 yds.^{2}
Kerbs are 264 yds long. They have to be of good precast concrete.
The Road way has to be asphalted with Idealit No. 5

Y

SCHEDULE OF REINFORCEMENT.

Member	No. of	Bar	No. in each.	Total No.	Diam.	l'	Total L.	Weights.
Arch Rib	2	a	44	88	$1\frac{1}{4}$"	21.5	1892'	
		b	32	64	$1\frac{1}{4}$	22.5	1440	
		c	20	40	$1\frac{1}{4}$	20.5	820	
		d	32	64	$1\frac{1}{4}$	17	1085	
		e	40	80	$1\frac{1}{4}$	21.5	1720 6957	29,000 lbs.
Slab	1							
Longitudinal Bars				1980	5/8	6	11880	12,400
Transversal Bars.				528	$\frac{1}{2}$	16.4	8450	5,660
Transversal Beams.	23		6	138	$1\frac{1}{4}$	21'	2900	12,050
	23		3	69	5/8	21	1450	1,510
Longitundinal Beans	44		4	176	5/8	8	1408	1,460
	44		2	88	$\frac{1}{2}$	6	528	353
Columns	46				3/4		480	730
Railing					$\frac{1}{2}$		1600	1,070
T I E S								
Ribs					5/8		11,600	11,640
Columns					3/8		7,670	2,920
Transversal Beans.					$\frac{1}{2}$		7,350	49, 30
								83, 713
Add 3% arbitrarily								2, 500
								86. 213 Kips.

BILL OF QUANTITIES AND COSTS.

	: Amount	: Cost/Unit	: Total Cost.
Excavation	426 yds^3	1.90	810
Fill	1575 yds^3	1.90	3, 000
Abutments.	745 yds^3	30.00	22, 350
Stones	540 yds^2	15.00	08, 100
Deck & Ribs	326 yds^3	60.00	19, 560
Steel	86.2 Kips	110 .00	09, 500
Plastering	1028 $yds.^2$	2. 0	02, 056
Tiles	147 yds^2	4. 0	00, 588
Kerb	264 yds	1. 0	00, 264
Asphalt	293 $yds.^2$	2. 0	00, 586
			66, 814 lbs.
Contractor's Profit: 15 %			10, 000
			76, 814 lbs. Lebanese

Horizontal area of the bridge = 32 x 132 ft. sq.

76, 814 / 32 x 132 = 18.2 lb / ft. sq. = 196 lbs. / m^2

which means twice as much as 1 m^2 of one good story building costs.

CHAP. IV

A NEW METHOD OF SOLUTION OF INDETERMINATE PROBLEMS.

HISTORY OF THE METHOD.- In the summer of 1948 after completing my Junior Year in Engineerings , I have found a new method of solution for indeterminate problems in Mechanics of Materials. In the scholastic year 1948-1949 a copy of the solution was presented to Prof. J. R. Osborn, Head of the engineering Department and another to Prof. R.W. Sloane, Head of the Physics Dep. They both have studied the method carefully then gave their final answers saying that the method was correct.

Prof. Sloan then sent a copy to the Institute of Civil Engineers of London. They answered him saying that Presentation of the method was recorded and that a copy of it was put in the Library of the Institute.

I have presented them a copy to Dr. Raif Abu -Lameh, Minister of Education in Lebanon and another to President Penrose who sent me a kind letter expressing his feelings towards the Deed of that Method.

A Civil Engineer by the name of Shukri Ghibreel published the news in the Lebanese News Papers.

This was reported to the Outlook review that published it also. Prof. Sloan is about to publish it for me in some American and English Scientific Magazines.

A METHOD OF SOLUTION OF CONTINUOUS BEAM PROBLEMS

NAZIH TALEB

(Communicated by Robert W. Sloane, Ph.D., A.M.I.E.E.)

Summary of Procedure

All moments and reactions at the supports are found in terms of the slopes at the end of a span, and the values as found from the spans on either side of a support are equated. ~~The equated.~~ The equations are solved for the slopes. The values thus found are substitued in the expressions for the reactions and moments.

Notation:

M = End moment.

R = Reaction. At a support between two spans, each R refers to the reaction due to one span.

W = Uniform load per unist of length.

P. = Concentrated load.

L = Length of span.

M = Slope of the elastic curve at the end of a span.

E = Modulus of elasticity of the material of which the span is composed

X = Moment of inertia of the cross-section of the span with respect to the neutral axis.

Formulae for the special cases shown in Figures 1 - 4.

I. Fig.12

$$M = \frac{wL^2}{8} - \frac{3\,mEI}{L}$$

$$R_1 = \frac{3}{8}wL + \frac{3\,mEI}{L^2}$$

$$R_2 = wL - R_1$$

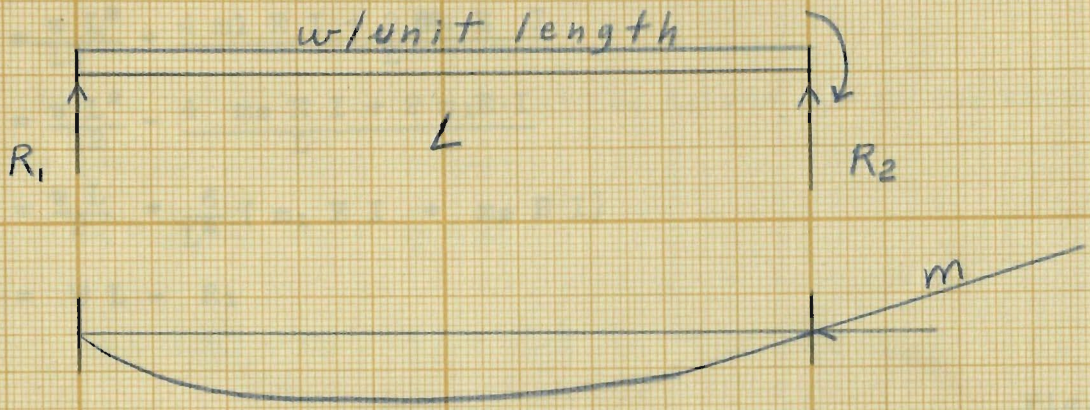


Fig. 12

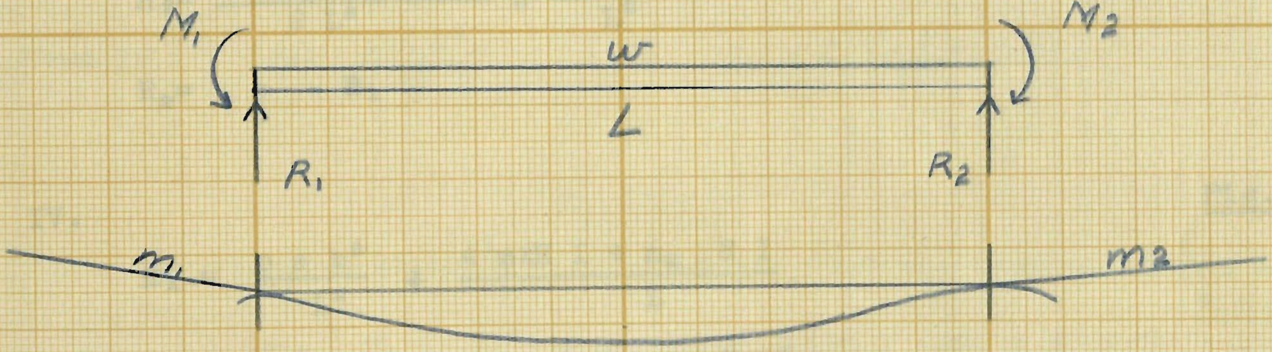


Fig. 13

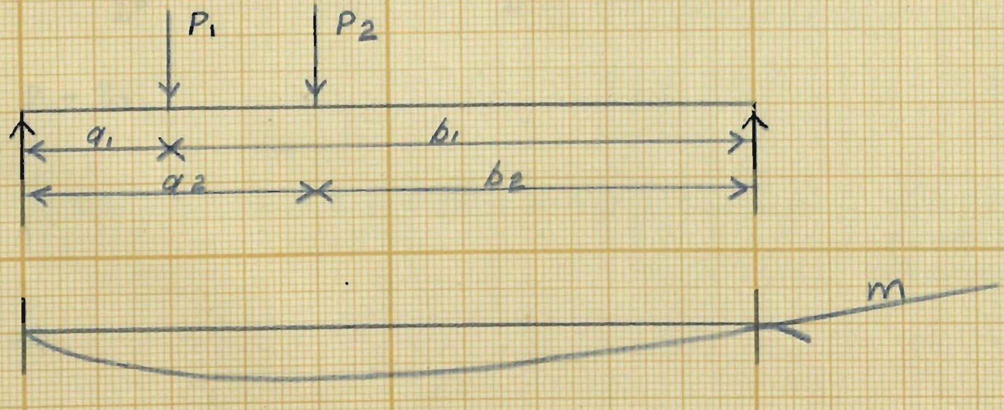


Fig. 14

Fig.13

II.

$$M_1 = \frac{w L^2}{12} + \frac{4 m_1 E I + 2 m_2 E I}{L}$$

$$M_2 = \frac{w L^2}{12} - \frac{4 m_2 E I + 2 m_1 E I}{L}$$

$$R_1 = \frac{w L}{2} + \frac{6}{L^2}(m_1 E I + m_2 E I)$$

$$R_2 = W L - R_1$$

Fig.14

III.

$$M = \frac{P a b (2 a + b)}{2 L^2} - \frac{3 m E I}{L}$$

$$R_1 = \frac{P b^2 (3 a + 2 b)}{2 L^3} + \frac{3 m E I}{L^2}$$

$$R_2 = p - R_1$$

Fig.15

IV.

$$M_1 = \frac{p a b^2}{L^2} + \frac{4 m_1 E I + m_2 E I}{L}$$

$$M_2 = \frac{p a^2 b}{L^2} - \frac{4 m_2 E I + m_1 E I}{L}$$

$$R_1 = \frac{p b^2 (3a+b)}{L^3} + \frac{6 m_1 E I + 6 m_2 E I}{L^2}$$

$$R_2 = P - R_1$$

Use of Formulae

Example 1

If zero is substituted for the slopes m_1 and m_2 in the above formulae we get the standard forms:-

Form I $\quad M = \dfrac{w\ L^2}{8},\quad R_1 = \dfrac{3}{8}\, w\ L$

Form II $\quad M_1 = \dfrac{w\ L^2}{12} = M_2\ ,\quad R_1 = \dfrac{w\ L}{2}$

Form III $\quad M = \dfrac{p\ a\ b\ (\ 2a + b)}{2\ L^2},\qquad R_1 = \dfrac{P\ b^2\ (3a + 2b)}{2\ L^3}$

Form IV $\quad M_1 = \dfrac{p\ a\ b^2}{L^2},\qquad R_1 = \dfrac{p\ b^2\ (3a + 2b)}{L^3}$

Example 2

In the beam shown in figure, 5, to find the bending moment at B.

Fig. 16

$$M_{B_1} = \frac{80 \times 24^2}{8} - \frac{3\ m\ E\ I}{24} + \frac{500 \times 20 \times 4\ (2 \times 20 + 4)}{2 \times 24^2}$$

AND $\quad M_{B_2} = \dfrac{400 \times 4 \times 6(8 + 6)}{2 \times 10^2} + \dfrac{3\ m\ E\ I}{10}$ (by I and III)

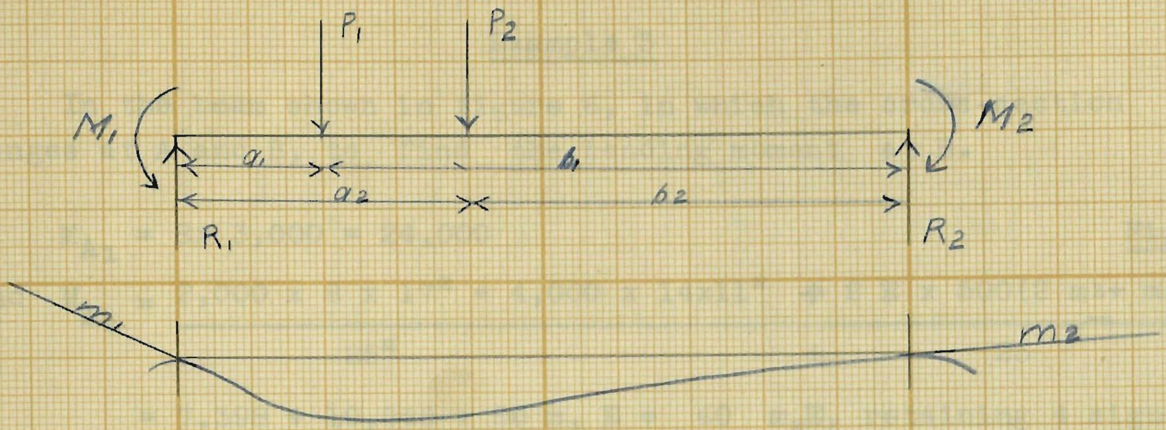
$M_{B_1} = M_{B_2}$

$\therefore$ $5,760 - 0.125\ m\ E\ I + 1528 = 672 + 0.3\ m\ E\ I$, retaining 4 significant figures.

$\therefore$ $0.425\ m\ E\ I = 6616$

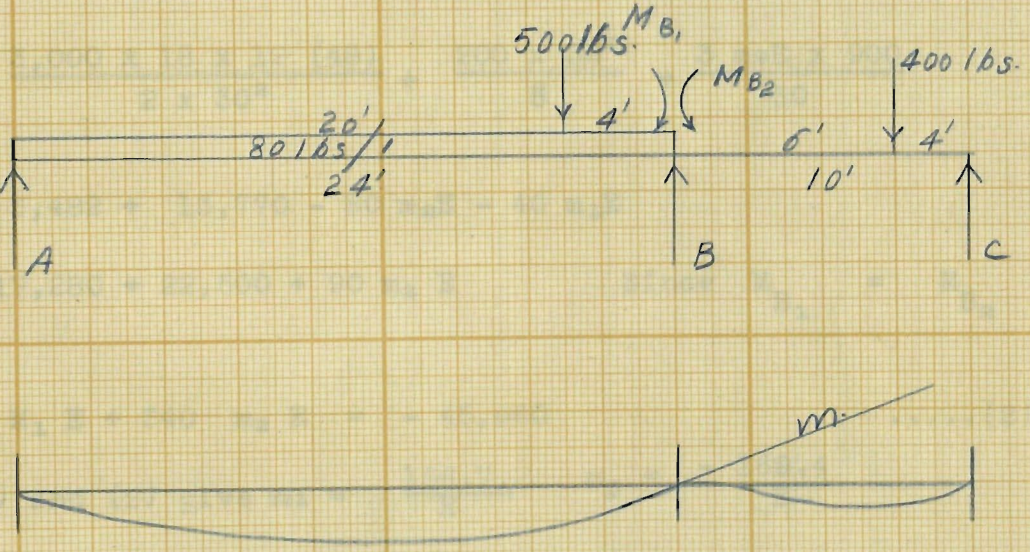
$\therefore$ $m = \dfrac{15,570}{E\ I}$

$\therefore$ $M_B = 672 + 0.3 \times 15,570 = 5,340$, correct to 3 significant figures.



P1
P2
M1
M2
a1
b1
a2
b2
R1
R2
m1
m2

Fig. 15



500 lbs.
MB1
MB2
400 lbs.
20'
4'
80 lbs/'
24'
6'
4'
10'
A
B
C
m

Fig. 16

Example 3

In the beam shown in figure 6, in which the cross-section changes at A and B, to find the bending moment at B.

$M_{A_1} = 8 \times 3{,}000 = 24{,}000$ Fig. 17

and $$M_{A_2} = \frac{2{,}000 \times 8 \times 17^2 + 4{,}000 \times 14 \times 11^2}{25^2} + \frac{2\,E \times 500(2\,m_1 + m_2)}{25}$$

$= 7{,}398 + 10{,}840 + 80\,m_1\,E + 40\,m_2 E$, retaining 4 significant figures.

$\therefore \quad 80\,m_1\,E + 40\,m_2\,E = 5{,}760$, since $M_{A_1} = M_{A_2}$ .....(1)

$$M_{B_1} = \frac{2{,}000 \times 64 \times 17 + 4{,}000 \times 14^2 \times 11}{25^2} - \frac{2\,E \times 500\,(2m_2 + m_1)}{25}$$

and $$M_{B_2} = \frac{3{,}000 \times 18 \times 12\,(48)}{2 \times 30^2} + \frac{200 \times 30^2}{8} + \frac{3\,m_2E \times 900}{30}$$

$\therefore \quad 3{,}482 + 13{,}770 - 80\,m_2E - 40\,m_1E$

$= 17{,}280 + 22{,}500 + 90\,m_2\,E$ Since $M_{B_1} = M_{B_2}$

$\therefore\ 80\,M_1\,E + 340\,m_2\,E = -45{,}060$ .....(2)

(1) and (2) give $m_1 = \dfrac{156.7}{E}$, $m_2 = -\dfrac{169.4}{E}$

$\therefore \quad M_B = M_{B_2} = 17{,}280 + 22{,}500 - 90 \times 169.4 = 24{,}500$ correct to 3 significant figures.

This method seems to the author to be shorter and more direct than the methods taught at present. He hopes it will be found to be of use.

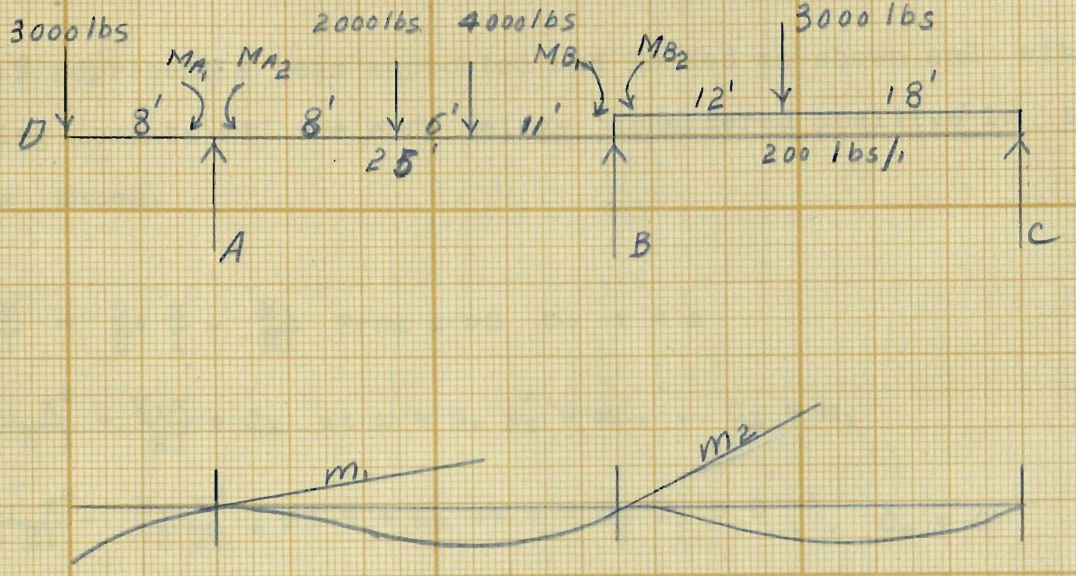

3000 lbs
2000 lbs
4000 lbs
3000 lbs
M_{A_1}
M_{A_2}
M_{B_1}
M_{B_2}
D
8'
8'
6'
11'
12'
18'
25'
200 lbs/'
A
B
C
m_1
m_2

Fig. 17

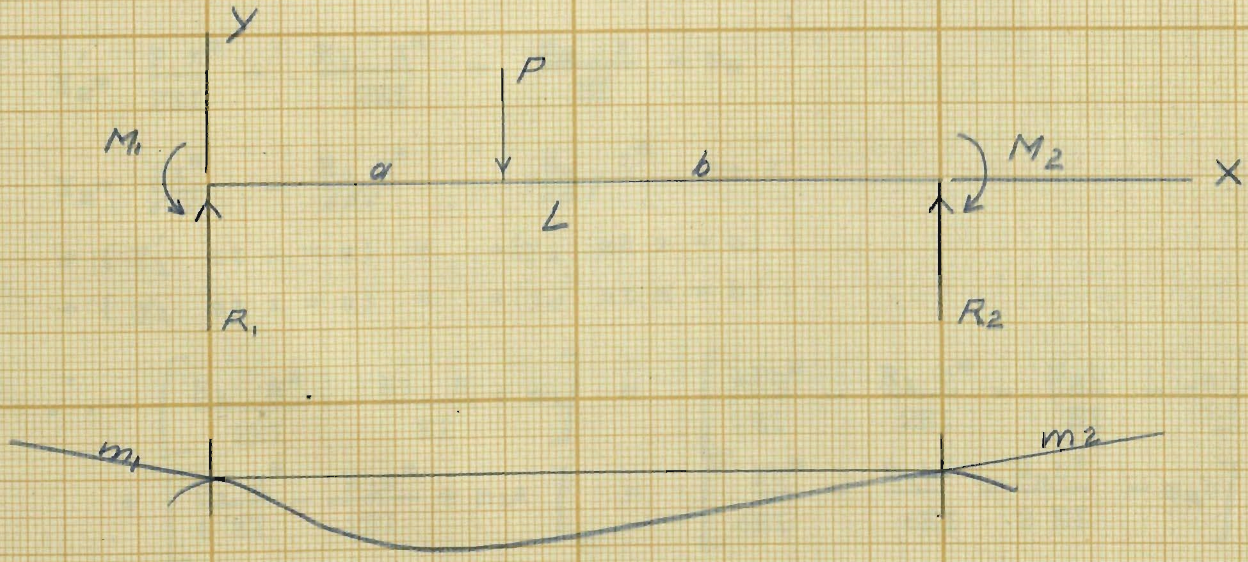

Y
P
M_1
a
b
M_2
X
L
R_1
R_2
m_1
m_2

Fig. 18

PROOF OF FORMULA IV

Let fig. 18 represent a beam supported and loaded as shown and with the bending moments at the ends beings considered positive in the directions shown.

$$Y_1'' = \frac{M}{EI} = \frac{R_1 x}{EI} - \frac{M_1}{EI} \quad \text{From } x = 0 \text{ to } x = a$$

$$Y_1' = \frac{R_1 x^2}{2EI} - \frac{M_1 x}{EI} + C_1 \quad \text{at } x = 0,\ y_1' = m_1 \therefore C_1 = m_1$$

$$Y_1 = \frac{R_1 x^3}{6EI} - \frac{M_1 x^2}{2EI} + m_1 x + C_2 \quad \text{at } x = 0,\ Y_1 = 0 \therefore C_2 = 0$$

Similarly if we take the right end as the origin, we get:

$$Y_2' = \frac{R_2 X^2}{2EI} - \frac{M_2 x}{EI} - m_2$$

$$Y_2 = \frac{R_2 x^3}{6EI} - \frac{M_2 x^2}{2EI} - m_2 x$$

Since $R_2 = P - R_1$, we get:

$$Y_2' = \frac{P x^2}{2EI} - \frac{R_1 x^2}{2EI} - \frac{M_2 x}{EI} - m_2$$

$$Y_2 = \frac{P x^3}{6EI} - \frac{R_1 x^3}{6EI} - \frac{M_2 x^2}{2EI} - m_2 x$$

$$+(Y_1' \text{ at } x = a) = -(y_2' \text{ at } x = b)$$

$$+(y_1 \text{ at } x = a) = +(y_2 \text{ at } x = b)$$

$$\therefore -\left[\frac{R_1 a^2}{2EI} - \frac{M_1 a}{EI} + m_1\right] = \left[\frac{Pb^2}{2EI} - \frac{R_1 b^2}{2EI} - \frac{M_2 b}{EI} - m^2\right] \quad (1)$$

$$+\left[\frac{R_1 a^3}{6EI} - \frac{M_1 a^2}{2EI} + m_1 a\right] = \left[\frac{Pb^3}{6EI} - \frac{R_1 b^3}{6EI} - \frac{M_2 b^2}{2EI} - m_2 b\right] \quad (2)$$

Substituting $M_2 = Pb + M_1 - R_1 l$ we get:

$$-\frac{R_1 a^2}{2EI} + \frac{M_1 a}{EI} - m_1 = \frac{Pb^2}{2EI} - \frac{R_1 b^2}{2EI} - M_2 - \frac{Pb^2}{EI} - \frac{M_1 b}{EI} + \frac{R_1 l b}{EI} \quad (1)$$

$$\frac{R_1 a^3}{6EI} - \frac{M_1 a^2}{2EI} + m_1 a = -m_2 B - \frac{Pb^3}{2EI} - \frac{M_1 b^2}{2EI} + \frac{R_1 l b^2}{2EI} + \frac{Pb^3}{6EI} - \frac{R_1 b^3}{6EI} \quad (2)$$

$$R_1 \left[\frac{b^2-a^2-2lb}{2\,EI}\right] = M_1 \left[\frac{-a-b}{EI}\right] + m_1 - m_2 - \frac{Pb^2}{2EI} \qquad (1)$$

$$R_1 = \frac{M_1 \left[\frac{-a-b}{EI}\right] + m_1 - m_2 - \frac{Pb^2}{2\,EI}}{\frac{I}{2EI}\left[b^2 - a^2 - 2ab - 2b^2\right] = -\frac{1}{2EI}(a+b)^2} \qquad (1)$$

$$R_1 \times \left[\frac{a^3}{6EI} - \frac{3lb^2}{6EI} + \frac{b^3}{6EI}\right] = M_1 \left[\frac{a^2-b^2}{2\,EI}\right] - m_2 b - m_1 a - \frac{Pb^3}{3EI} \qquad (2)$$

$$R_1 = \frac{M_1 \left[\frac{a^2-b^2}{2\,EI}\right] - \frac{Pb^3}{3EI} - m_2 b - m_1 a}{\frac{1}{6EI}\left[a^3 + b^3 - 3ab^2 - 3b^3\right] = \frac{I}{6EI}(a+b)^2\,(a-2b)} \qquad (2)$$

R_1 of (1) = R_1 of (2). Cross multiply we get:

$$M_1\,(2a^2 - 4b^2 - 2ab) + Pb^2\,(a-2b) - 2\,EI\,(am_1 + 2m_2 b - 2m_1 b - am_2)$$

$$= M_1\,(3a^2 - 3b^2) - 2\,Pb^3 + 6EI\,(-m_2 b - m_1 a)$$

$$M_1\,(a^2+2ab+b^2) = Pab^2 - 2Pb^3 - 2EIam_1 - 4EIm_2 b + 2EIam_2 + 4EIm_1 b + 2pb^3 + 6EIm_2 b + 6EIm_1 a$$

$$M_1 l^2 = P\,a\,b^2 + 4\,E\,I\,m_1 a + 4\,E\,I\,m_1 b + 2\,E\,I\,m_2 a + 2\,E\,I\,M_2 b$$
$$= P\,a\,b^2 + 4\,m_1 E\,I\,(a+b) + 2\,m_2 E\,I\,(a+b)$$

$$M_1 = \frac{Pab^2}{l^2} + \frac{4\,m_1 EI}{l} + \frac{2m_2 EI}{l} \qquad \text{Similarly} \quad M_2 = \frac{Pab^2}{l^2} - \frac{4m_1 EI}{l} - \frac{2m_2 EI}{l}$$

$$R_1 = \frac{Pb + M_1 - M_2}{l} \pm \frac{Pb^2(3a+b)}{l^3} \quad \frac{6m_1 EI + 6\,m_2 EI}{l^2}$$

The only expression that is a function of a and b is that which contains P.

This leads to the General Equation+

$$M_1 = \frac{\sum Pab^2}{l^2} + \frac{4m_1EI}{l} + \frac{2m_2EI}{l}$$

This corresponds to Formula IV. Formula II may be obtained from IV by intergration+

$M_1 = \frac{1}{l^2} \int_0^l w\,d\,x\,(l-x)^2 x$. Formula III may be obtained by setting one moment equal to zero, thereby obtaining an equation to eliminate one slope. Formula I may be obtained from III by integration. Other types of loading may be dealty with by integration by using formula IV.

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B I B L I O G R A P H Y

The books that were used as references are put here in order of the use derived from them:

1. " Arch Design Simplified " By Fairhurst.

2. " Concrete- Plain and Reinforced" By Taylor, Thompson, and Smulski

3. " Reinforced concrete Bridges" By Scott.

4. " Design of concrete Structures" By Urquhart and O'Rourke.