



AMERICAN UNIVERSITY OF BEIRUT

A QUEUEING MODEL OF TRAFFIC  
TO BALANCE COSTS AND EMISSIONS

by  
JOHNNY ELIE GEMAYEL

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submitted in partial fulfillment of the requirements  
for the degree of Master of Engineering Management  
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at the American University of Beirut

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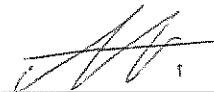
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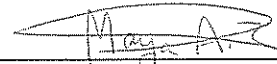
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# AN ABSTRACT OF THE THESIS OF

Johnny Elie Gemayel for Master of Engineering Management  
Major: Financial and Industrial Engineering

Title: A Queueing Model of Traffic to Balance Costs and Emissions

In this thesis, a highway system is analyzed based on established queueing theory models of traffic systems. Various model parameters are estimated from studies that utilize real data. Key Performance Indicators (KPIs), such as the throughput, the mean number of vehicles and average waiting time are calculated. We solve for the optimal number of lanes which minimizes the joint costs of building new roads and the costs of delays due to traffic. The model also incorporates the costs resulting from the environmental impact, in terms of carbon emissions, of road construction and traffic congestion. A sensitivity analysis is also presented, illustrating the most influential environmental parameters.

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# CHAPTER I

## INTRODUCTION

Queues occur ubiquitously, from transportation to communication and computer networks, to manufacturing, logistical and service systems. Queueing theory targets the design and control of queueing systems, for example, by determining the right level of resources (servers) and the right service discipline in a way that meets service requirements or minimizing cost (typically composed of the costs of waiting and the cost of providing service; e.g. Cooper, 1981, and Gross et al., 2008).

To the best of our knowledge, the literature on queueing systems seems to have ignored the effect of congestion on emissions, despite the natural link between these two factors. In a highway system for example, the emissions increase as cars spend more time in a traffic jam. Other transportation systems such as airplanes experience taxi delays on a runway, and ships experience queues in maritime passages (such as the Panama or Suez canals). Emissions have also been linked to congestion in computer networks. The Guardian (2008) reports that Google emits 1.5 million tons of carbon annually, which is slightly higher than the country of Laos. Much of the emissions are linked to the high utilization and congestion on Google's search engines.

This thesis develops queueing models that account for the important dependency between congestion and emissions. What is interesting about queueing systems is the strong sensitivity of their performance to operational adjustments. For instance, adding one more resource to a single server system could reduce delay by ten folds, in contrast to the two folds "linear" intuition (e.g. Gross et al., 2008). This is promising as simple yet highly inexpensive operational changes could reduce emissions significantly.

The focus of this thesis is solely on queueing in transportation networks, where various road costs are realistically approximated and a cost model is devised. It is to be noted that there are two kinds of traffic that are most commonly studied using queueing systems: Interrupted flows (where traffic is interrupted by signs, intersections, traffic signals, etc.) and uninterrupted flows (such as freeways and highways) (e.g. Elefteriadou, 2014). This thesis utilizes an uninterrupted flow approach which is commonly used to model highway road segments. Highway segments are usually accessed by several populated residential areas which results in an arrival process generated from the superposition of several arrival processes. In such a case, the Poisson process provides accurate approximations to the arrival process. (Whitt, 1982)

Previous work on modelling traffic in uninterrupted flows include Heidemann (1996) and Vandaele et al. (2000) who use basic queueing models to model traffic flows. These models were validated in Van Woensel and Vandaele (2006) with actual traffic data. Jain and Smith (1997) introduced state dependent queueing models to analyze traffic. These models were previously used to analyze pedestrian movements within critical facilities such as schools and hospitals in Yuhaski and Smith (1989), Cheah (1990), Smith (1991) and Cheah and Smith (1994).

None of the queueing traffic models in the literature account for emissions. This thesis presents what seems to be the first analytical queueing model that accounts for both congestion and emissions.

Recent operations management work on reducing emissions mainly focuses on single and multi-product inventory management. In single-product inventory management, Chen et al. (2013) use the Economic Order Quantity (EOQ) model to show how operational policies reduce emissions with minimal cost increase. Hua et al.

(2011) and Toptal et al. (2014) extend the EOQ model to account for carbon emissions policies. Other useful extensions were obtained in Benjaafar et al. (2013) and Song and Leng (2012). Recent multi-product inventory management accounting for emissions includes Zhang and Xu (2013) and Schaefer and Konur (2014). Our work in this thesis is similar to this stream of research in that we also seek to reduce emissions by operational adjustments.

The remainder of this thesis is organized as follows. In Chapter II, our cost model is presented, along with the theories and assumptions behind it. In Chapter III, the results of our cost model are analyzed and a sensitivity analysis is performed. Finally, we conclude in Chapter IV.

## CHAPTER II

### MODEL AND ASSUMPTIONS

This chapter presents the cost model and the main theoretical and empirical building blocks behind it. Section II.A presents a mathematical model showing the relationship between vehicle speed and carbon emissions. Section II.B reviews the queueing traffic model that we adopted from Jain and Smith (1997). Section II.C presents our base cost model, without accounting for carbon emissions. Finally in Section II.D, the cost model is extended to include costs due to carbon emissions.

#### **A. Carbon Emissions and Vehicle Speed**

Literature studies show a parabolic relationship between the car speed and the carbon monoxide emissions (e.g. Sbayti et al., 2002; André and Rapone, 2009), where the emissions per kilometer tend to be large at low speeds, and then decrease at around 50 mph (80km/h) and after that the emissions increase again, as shown in Figure 1. The results of Figure 1 were obtained using a software simulator (Mobile5b) in Sbayti et al. (2002) for Beirut, Lebanon. The wide range of factors that affect car emissions as well as the different types of cars prevented previous literatures from obtaining a unified mathematical equation that links emissions to car speed.

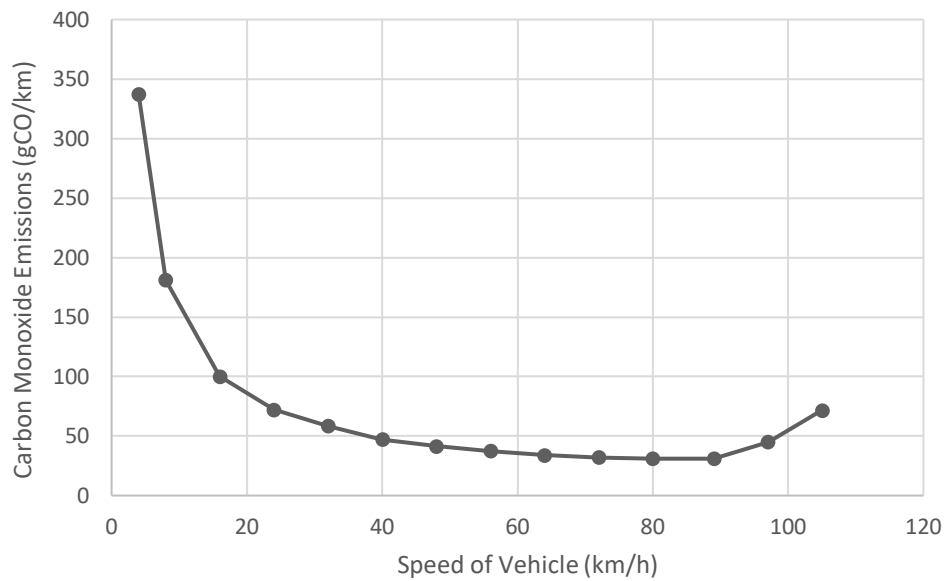


Figure 1: CO Emissions for a car on the road as a function of speed (Sbayti et. al., 2002)

Other than Carbon Monoxide (CO) emissions shown in Figure 1, cars also emit Hydrocarbons and Nitrogen Oxides, which have a similar emissions curve to the CO in Figure 1 (see Sbayti et al., 2002). However, we shall be focusing solely on CO emissions in this thesis, although our model can be extended to include other pollutants as well. In comparison to Carbon Dioxide gas (CO<sub>2</sub>), CO gas has lower greenhouse gas effects. However, its reactions that produce Methane and Ozone gases increase its greenhouse gas effect. The Global Warming Potential (GWP) of CO could reach up to 3 in the long-run, i.e. 1 kilogram of CO emitted is equivalent to emitting 3 kilograms of CO<sub>2</sub> (Fuglestvedt et al., 1996). This correlation between CO and CO<sub>2</sub> will be beneficial later on, where we will be calculating the carbon taxes due to car emissions, since carbon taxes are only applied to CO<sub>2</sub> emissions.

## B. *M/G/c/c* State-Dependent Queueing System (Jain and Smith, 1997)

For modelling and analyzing traffic flows, a state-dependent *M/G/c/c* queueing model is used in Jain and Smith (1997). The model assumes a Markovian (exponential) arrival rate. This assumption is utilized in many previous literatures for modelling traffic flows (e.g. Evans et al., 1964, Yeo and Weesakul, 1964, Heidemann, 1991, 1994, 1996, Vandaele et al., 2000 and Qin and Smith, 2001). The model also assumes a general and state-dependent service time distribution, with  $c$  servers and a limited capacity equal to the number of servers. Notice that the definition of the term capacity differs from the terminology used in transportation engineering, which typically refers to the expected departure rate (veh/h, e.g. Jain and Smith, 1997).

The term “state-dependent” accounts for the deterioration in the service rate of the road as the number of cars in the system increases. Furthermore, the model assumes that all the vehicle types are identical to the passenger car. We refer to the Highway Capacity Manual, (2000) for conversion factors of other vehicle types (Trucks, Buses, etc.).

The limiting probabilities of the *M/G/c/c* state-dependent models are given by:

$$P_j = \left[ \frac{[\lambda l / v_1]^j}{\prod_{m=1}^j m[(c+1-m)/c]} \right] P_0 \quad (1)$$

The probability of having an empty system,  $P_0$ , is given by:

$$P_0 = \left[ 1 + \sum_{i=1}^c \left[ \frac{[\lambda l / v_1]^i}{\prod_{m=1}^i m[(c+1-m)/c]} \right] \right]^{-1} \quad (2)$$

The reader is referred to Appendix 1 for a calculation algorithm of (1) and (2).

The arrival rate of cars to the road segment in study is  $\lambda$ . Let  $l$  denote the length of the

road segment in study and let  $v_l$  be the speed of a car travelling alone on the highway, which is assumed to be the speed limit of the road. Let  $P_j$  be the probability of having  $j$  cars in the system, where  $j=1, \dots, c$  and  $c$  is the capacity of the road (and the number of servers and capacity in the  $M/G/c/c$  model), which is given by:

$$c = k \times N \times l \quad (3)$$

In equation (3),  $k$  is the jam density per lane, or the maximum number of cars that a road lane can accommodate. Let  $N$  be the number of lanes of the road segment in study, which will be the main decision variable in our cost model.

The road segment under study along with the parameters explained earlier are shown in Figure 3, where  $\mu(j)$ , as defined in Jain and Smith (1997), is the state-dependent service rate of the road segment, which deteriorates as the number of cars  $j$  in the system increases. The state-dependent service rate is given by:

$$\mu(j) = \frac{v_j}{v_1} = \frac{(v_1/c)(c + 1 - j)}{v_1} = \frac{c + 1 - j}{c} \quad (4)$$

Here,  $v_j$  is the average speed when  $j$  cars are present in the system, which is assumed to decrease linearly as the number of cars in system  $j$  increases, as shown in Figure 2. In this work, we implemented a linearly decreasing model for  $v_j$ .



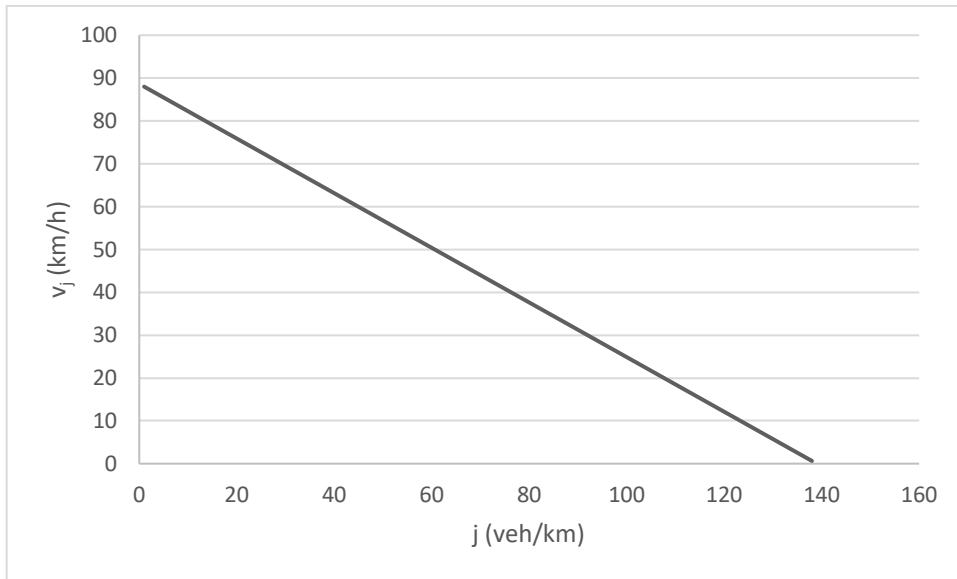


Figure 2. Speed vs. Jam Density (Jain and Smith, 1997)

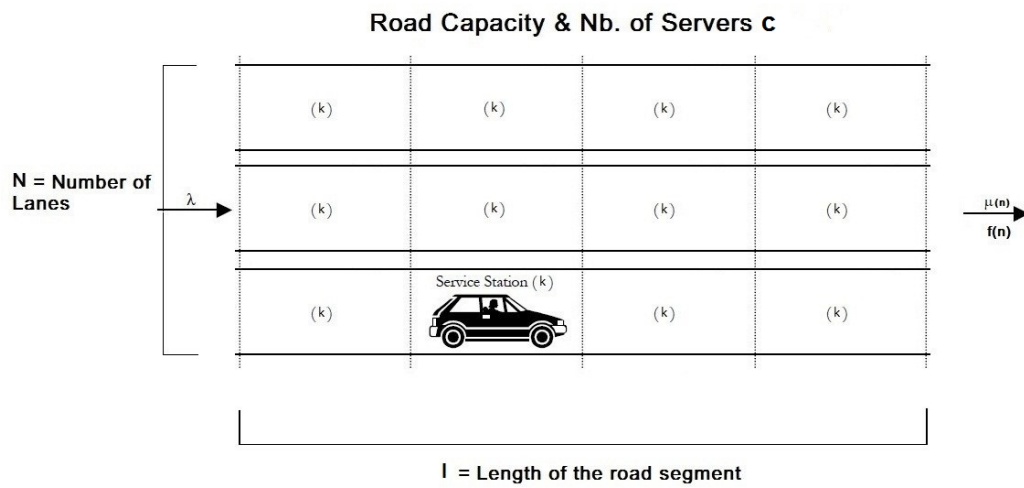


Figure 3: Road segment used for traffic analysis using the  $M/G/c/c$  model

### C. Our Cost Model without Carbon Emissions

We adopt the following model for the expected cost (\$/km.h):

$$EC(N) = C_S \cdot N + C_W \cdot L \quad (5)$$

This expected cost is composed of two main components,

1. The cost of providing service,  $C_S \cdot N$ , is the cost of building new lanes to accommodate for more vehicles per hour, where  $C_S$  is the hourly cost of providing service and  $N$  is the number of lanes.
2. The cost of waiting,  $C_W \cdot L$ , is the cost of delays on the road due to traffic congestion, where  $C_W$  is the cost of one car to wait one hour on the highway, and  $L$  is the expected number of vehicles on the highway.

The mean number of cars in the system is calculated as shown in (6), where  $c$  is the maximum number of cars in the system and  $P_j$  is the probability of having  $j$  cars in the system, which is given in (1).

$$L = \sum_{j=0}^c j \cdot P_j \quad (6)$$

In our  $M/G/c/c$  system,  $L$  can also be seen as the average number of busy servers.

### D. Our Cost Model with Carbon Emissions

In this section, we extend the model in Section II.C to account for carbon emissions, by adding two factors to the cost to the model of (5):

1. The emissions cost of providing service, which captures the emissions resulting from adding new lanes, and evaluates their cost based on the carbon tax, which

is the tax that some developed countries are currently imposing on activities that lead to carbon dioxide emissions.

2. The emissions cost of waiting, which evaluates the costs of the emissions due to traffic congestion.

Our updated cost model becomes:

$$ECT(N) = C_S \cdot N + C_W L + C_t N L_c + C_t I(v) L \quad (7)$$

In (7), the following parameters are introduced:

- (i)  $C_t$  is the carbon tax or cost of emitting 1 kg of CO<sub>2</sub>. This carbon tax varies for different countries
- (ii)  $L_c$  represents the carbon emissions due to the addition of one lane-km of road
- (iii)  $I(v)$  is the hourly rate of CO emissions per vehicle as a function of the vehicle's average speed  $v$ , and is obtained using (8), where  $H(v)$  represents the carbon monoxide emissions per kilometre divided by the time needed to traverse 1 km of road length ( $I/v$ ) to obtain the hourly rate of CO emissions, and multiplied by  $CO2_e$ , the Global Warming Potential (GWP) of CO, i.e. the kgCO<sub>2</sub> equivalency of emitting 1 kg of CO gases, which typically has a value of 3 as discussed in Section II.C (unit-less).

$$I(v) = H(v) \cdot v \cdot CO2_e \quad (8)$$

The function  $H(v)$  was modelled by fitting a curve to the data obtained from studies that analyse carbon emissions for different vehicle speeds (e.g. Sbayti et al., 2002, André and Rapone, 2009). Figure 4 shows the actual data obtained from Sbayti et al. (2002), and the fitted  $H(v)$ . In general, we observe that  $H(v)$  has the following form:

$$H(v) = \frac{A}{v} + B \times v^m, \quad (9)$$

where  $A$ ,  $B$  and  $m$  are positive constants.

The parameters of (9) were estimated based on a least-square fit to the data of Sbayti et al. (2002), where  $A$  is equal to 866.8,  $B$  is  $2.45 \times 10^{-5}$  and  $m$  is 3. The root mean square (RMS) error resulting from modelling  $H(v)$  against the actual function was 497, which is considered acceptable and thus the approximation is valid.

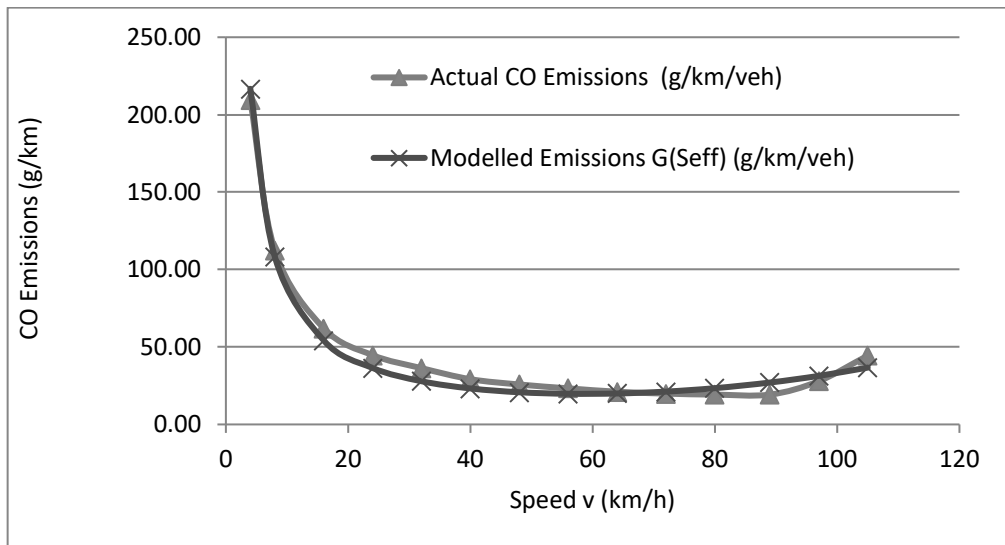


Figure 4. Actual vs Modelled Emissions Function

# CHAPTER III

## NUMERICAL RESULTS AND INSIGHTS

This section analyses the cost model developed in Chapter II and presents insights. Section III.A describes our approach for estimating the parameters realistically based on data from specialized studies in the literature. Section III.B evaluates the cost model without accounting for the costs from carbon emissions, which are later added to the model in Section III.C. Section III.D shows a sensitivity analysis on the various parameters of the cost model and presents insights.

### A. Parameters Estimation

In this section, the different parameters introduced in Chapter II will be realistically estimated. Starting with the parameters of equation (1), the arrival rate of cars to the road segment  $\lambda$  ranges from 1000 to 6000 vehicles per hour (veh/h) in Jain and Smith (1997), and is assumed to be 4000 veh/h as a base case in our cost model. The road segment length  $l$  is assumed to be 1 km for generality. The speed of a car travelling alone on the highway,  $v_l$ , practically ranges from 60 to 120 km/h, and is assumed to be 88 km/h as a base case following Jain and Smith (1997).

Regarding the jam density  $k$  in equation (3), its value ranges from 115 to 165 veh/lane-km according to Jain and Smith (1997). It is assumed that  $k$  equals 138 veh/lane-km as a base case.

As for the parameters of our cost model in equation (7), the hourly cost of providing service,  $C_s$  is calculated as the present cost of building one kilometre of a new

lane, divided by the expected lifetime of that lane (\$/lane.km.h). According to Litman (2009), expansion of urban highways in the US typically costs around 6 to 14 million US Dollars per lane-mile including land acquisition price. Also, the lifetime of the lane is around 20 years. Converting to metric units, the lane costs are \$3.75 to \$8.75 million US Dollars per lane-km. Assuming the costs lie in exactly the middle of the range specified (\$6.25 million) and accounting for the time-value of money over 20 years, with a 0.5% monthly-compounded interest rate, the total cost of \$6.25 million is equivalent to paying equal monthly instalments given by:

$$A = 6.25 \times \frac{1.005^{240} \times 0.005}{1.005^{240} - 1} = 44,780\$ \quad (10)$$

Dividing the above by 30 days and then 24 hours, we get an hourly cost of 62.19\$ per lane-km. The cost of waiting,  $C_w$ , is estimated using the national average hourly wage. According to Yaacoub and Badre (2011), the average monthly wage for Lebanese citizens in a study done in 2007 (excluding construction industry) was 690,000 Lebanese Pounds, or 457.71\$ as per the currency equivalence at the time of publication of this thesis. Assuming an average of 40 working hours per week, or around 160 working hours a month, the Lebanese get paid an average of 2.86\$ an hour. Such a wage however is too low in practice, thus we shall choose the cost to be 5\$/veh.h as a base case.

Regarding the carbon tax  $C_t$ , its average is 1.2 cents for each kg of CO<sub>2</sub> emitted (Dhar, 2011), which will be our base case value.

Finally, regarding the carbon emissions due to the addition of one lane-km of road  $L_C$ , and according to Park et al. (2003), it is approximated that the construction of a

4-lane, 1 km road would emit 2438.5 tons of CO<sub>2</sub> during its 20-year life span. This leads to an approximate hourly rate of 3.47 kgCO<sub>2</sub> per lane-km.

## B. Results of Cost Model without Carbon Emissions

Utilizing the base parameter values estimated in Section III.A, the limiting probabilities in (1) and (2) are obtained using MATLAB according to the algorithm in Appendix 1. The expected cost was calculated and plotted in Figure 5 against the number of lanes (varying from 1 to 10 lanes), which is considered a decision variable in this case.

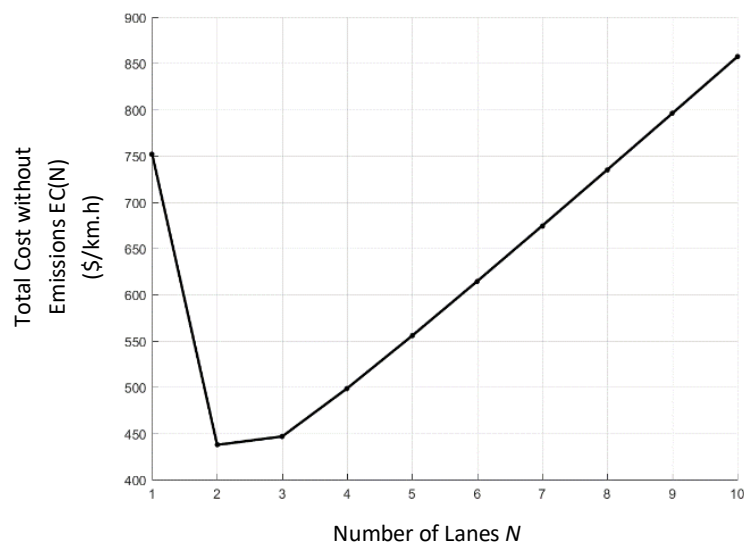


Figure 5. Total Cost vs. Number of Lanes without Emissions

The optimal number of lanes which minimizes the cost in Figure 5 is at  $N = 2$ , where the minimum cost is 437.58\$/km.h.

### C. Results of Cost Model with Carbon Emissions

The cost including emissions was calculated and plotted in Figure 6 against the number of lanes. All the parameters were evaluated at their base values from Section III.A.

The optimal number of lanes which minimizes the cost in Figure 6 is at  $N = 2$ , where the minimum cost is 445.16\$/km.h, with an increase of 1.73% from the minimum cost excluding emissions evaluated earlier.

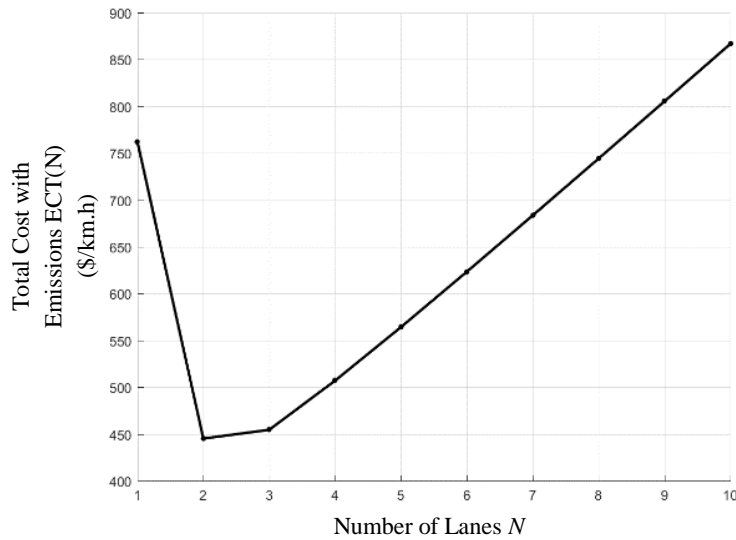


Figure 6. Total Cost vs. Number of Lanes with Emissions

### D. Sensitivity Analysis

The base case values estimated in Section III.A may vary in many circumstances, and their variation will cause a greater impact of emissions on the cost function, and even to the point of affecting the optimal solution.

Upon varying the parameters  $\lambda$ ,  $a$ ,  $C_s$ ,  $C_w$ ,  $C_t$  and  $L_c$ , the variable  $L_c$  had the least impact on the cost function, while variables  $\lambda$  and  $A$  affected the decision variable and forced increasing the number of lanes as the variables increase, and that is to maintain



an acceptable level of service on the road, which is indicated by the throughput of the system shown in (11) (Jain and Smith, 1997).  $P_{balk}$  is the probability of cars arriving at the road segment under study, but failing to enter the system since the capacity  $c$  was already reached. An acceptable level of service is assumed when  $\lambda_{eff}$  is at least 90% of  $\lambda$ . Although variables  $\lambda$  and  $a$  altered the optimal solution, this impact was obvious and the variables failed to produce any interesting results. The variables which yielded interesting results were  $C_s$ ,  $C_w$  and  $C_t$ , which happen to be costs that will vary according to countries, road types, obstacles, etc... Thus, varying these parameters around the base case is realistic, and yields practical results rather than results that may only occur in theory.

$$\Theta = \lambda_{eff} = \lambda(1 - P_{balk}) \quad (11)$$

Variation of the cost of providing service yielded interesting results, since changing  $C_s$  from 62.19 \$/lane.km.h to 52.5 \$/lane.km.h changes the decision variable  $N$  from 2 to 3 lanes as shown in Figure 7. The minimum cost corresponding to 3 lanes was 425.43\$/km.h.

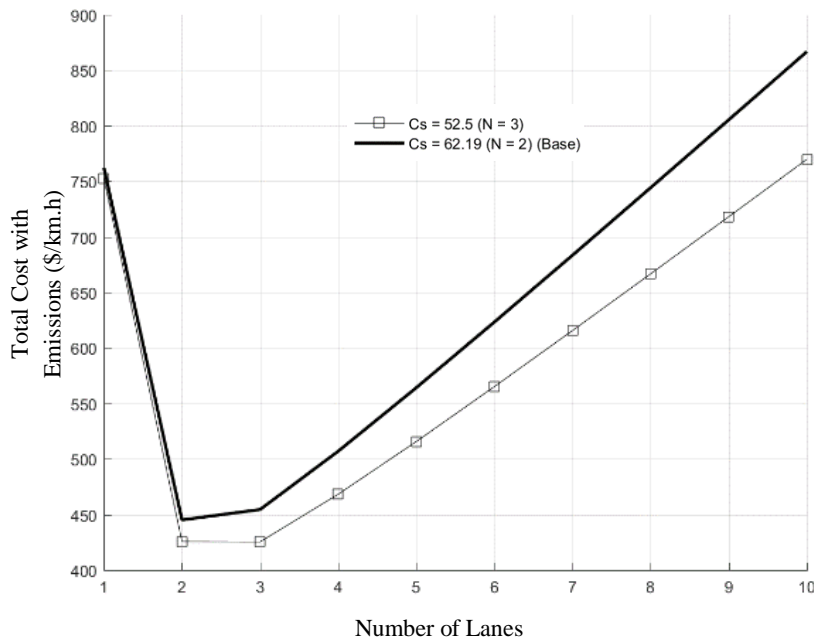


Figure 7: Changing  $C_s$  from 62.19 to 52.5 vs. Number of Lanes

Including emissions cost, the minimum cost occurs at 2 lanes instead of 3 lanes, and was 426.78\$/km.h. This shows how carbon emissions cost can change significant decision variables such as building a 2-lane road instead of a 3-lane road to reduce harmful emissions.

Variation of  $C_w$  tends to significantly increase the cost function, especially when the number of lanes is less than 3. This is logical, since for a high waiting cost, each additional vehicle in the system will incur a high waiting cost which dominates the service cost (which is directly proportional to the number of lanes) at low number of lanes. At higher number of lanes however, the service cost dominates the waiting cost.

Also, increasing  $C_w$  from 5 to 6\$/veh.h changes the decision variable from the previously optimal value of 2 to 3 lanes. Similarly to the case with  $C_s$ , the decision variable depends on the emissions cost for  $C_w \approx 7$ \$/veh.h.

As for the carbon tax, all our previous analysis is based on a tax of 0.028 \$/kgCO<sub>2</sub> emitted. Since this cost is much lower than the service and waiting costs, it usually has a small impact on the cost function, excluding special cases similar to the ones stated earlier. However, this carbon tax is bound to increase in the future, as more efforts are put into decreasing emissions to mitigate global warming effects. This increase in carbon tax will produce more interesting results in the cost model developed, and will eventually be a main component to account for during road design.

Next, we perform a one-way sensitivity analysis using the spider plot and the tornado diagram to understand the effect of each variable on our cost model. The spider plot of our cost model is shown in Figure 8. Each variable shown in the plot is varied by -100% to +100% from its base value, and the percentage change in the optimum cost was plotted for each. The lines with the steepest slopes represent the most influential variables on the optimal cost. For the cost variables, the waiting cost  $C_w$  has the biggest influence on the optimal cost, followed by the service cost  $C_s$  and the carbon tax cost  $C_t$ . As for the non-cost variables,  $\lambda$ , which is a variable dependent on road utilization, is the most influential one, followed by  $a$  then  $k$ .

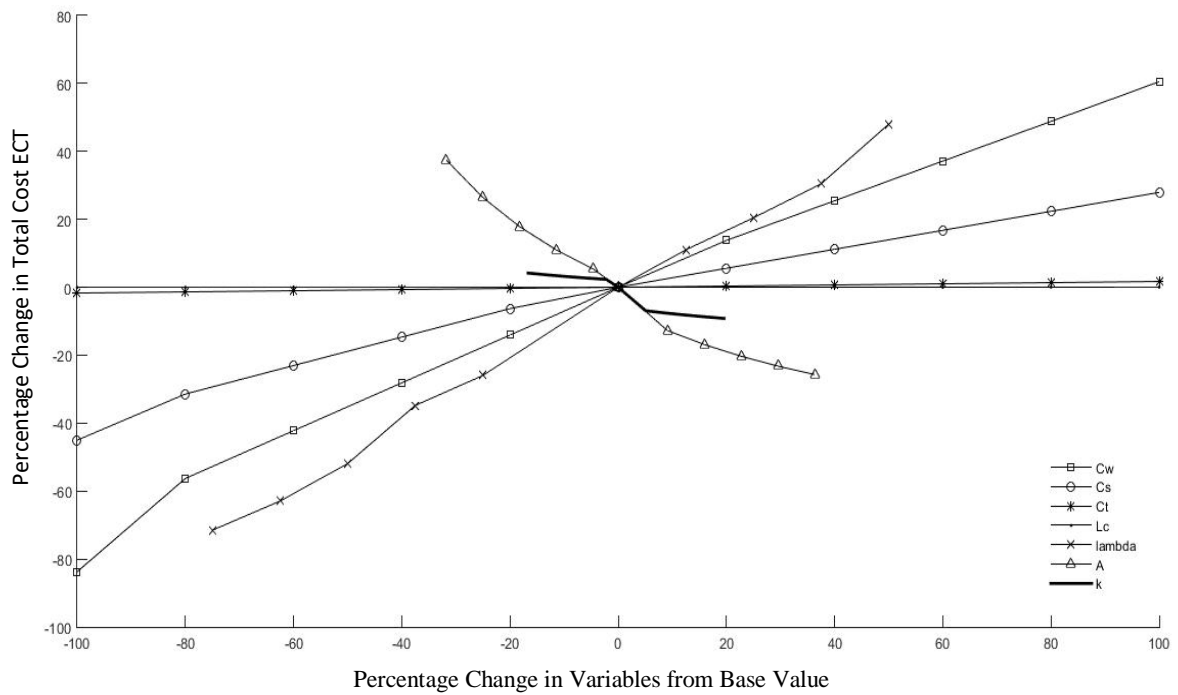


Figure 8: Spider Plot of our cost model

A tornado plot is also shown in Figure 9 in order to gauge the absolute effect of changing the model parameters. Each variable in our cost model was varied from its minimum value to its maximum. The ranges of the variables used for the tornado plot of Figure 9 are shown in Table 1 along with the impact on the optimal cost. The resulting impact on the optimal cost is then plotted, and the variables are sorted in decreasing order of influence on the cost model. As expected, the most influential variable was  $C_w$ , followed by  $\lambda$ ,  $C_s$ ,  $v$ ,  $k$  then  $C_t$ .

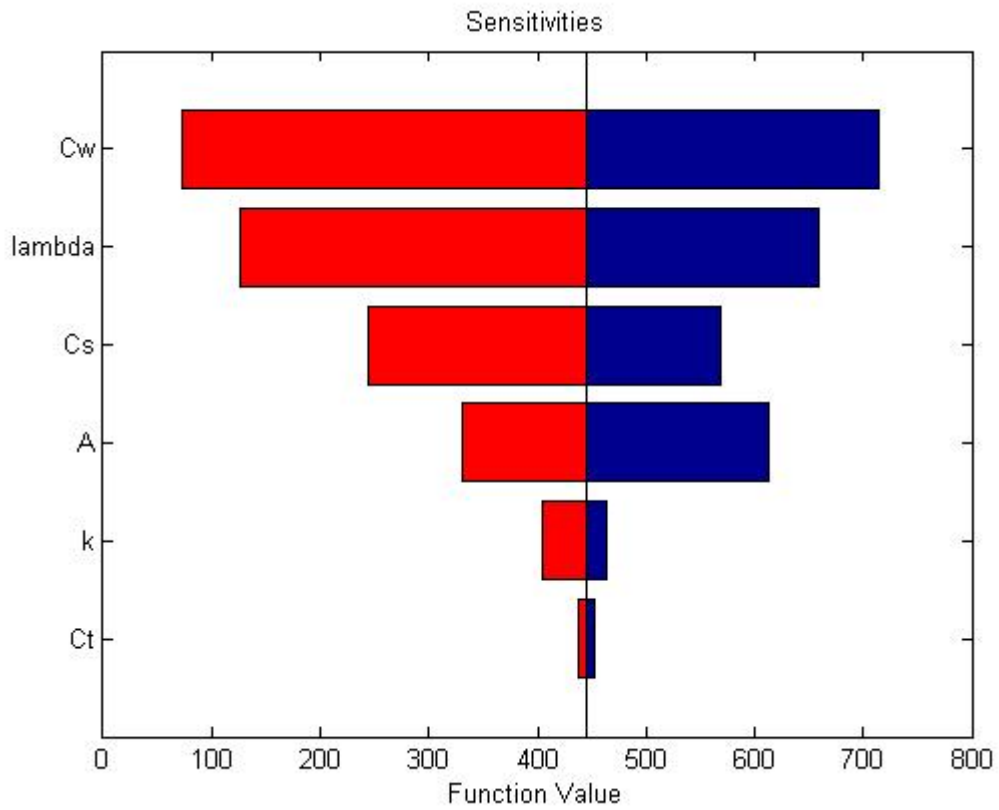


Figure 9: Tornado Plot for our Cost Model

Table 1: Ranges of Variables used for Tornado Plot

Variable	Base Value of Variable	Minimum Value of Variable	Maximum Value of Variable
$C_w$	5 \$/veh.h	0	10
$\lambda$	4000 veh/h	1000	6000
$C_s$	62.19 \$/lane.km	0	124.38
$v$	88 km/h	60	120
$k$	138 veh/lane.km	115	165
$C_t$	0.028 \$/kgCO <sub>2</sub>	0	0.056

## CHAPTER IV

### CONCLUSION

In this thesis, a road cost model with realistic parameters is developed based on well-established queueing models in traffic systems. The cost model mainly contributes to studies aiming to decrease greenhouse gases through improved design (e.g. by choosing the right number of lanes) and operational adjustments (e.g. setting the right speed limit). It was shown that the cost of emissions could be significant in some cases, and would affect road design and operational decisions. Our cost model is simple and easy to understand, with measurable inputs that can be estimated easily. This makes our model appealing for practical operations.

Future work could address developing similar queueing models that efficiently balance cost and emissions in other applications. Such models can be investigated in different contexts such as airport plane taxi traffic and port berthing ships traffic.

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## APPENDIX I

### MATLAB ALGORITHM FOR CALCULATION OF EQUATIONS (1) AND (2)

As a reminder, equation (1) and (2) state the following.

$$P_j = \left[ \frac{[\lambda l / v_1]^j}{\prod_{m=1}^j m[(c+1-m)/c]} \right] P_0$$
$$P_0^{-1} = 1 + \sum_{i=1}^c \left[ \frac{[\lambda l / v_1]^i}{\prod_{m=1}^i m[(c+1-m)/c]} \right]$$

To find  $P_j$  for all  $j$ , we have to find  $P_0$  first.  $P_0$  was found using an iterative algorithm in MATLAB as follows:

- Set  $b = \frac{\lambda l}{v_1}$
- Set  $P = 1$
- Set  $S = P$
- For  $j = 1$  to  $c$ 
  - Set  $P = P \times \frac{b}{j(c+1-j)}$
  - Set  $S = S + P$
- $P_0 = 1/S$

