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RELIABILITY ANALYSIS OF REINFORCED CONCRETE BRIDGES

by

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AN ABSTRACT OF THE THESIS

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Empirical expressions for estimating the wheel load distribution and live-load bending moment are typically specified in highway bridge codes such as the AASHTO procedures. The objective of this study is to assess the reliability levels that are inherent in concrete slab bridges that are designed based on the simplified empirical live load equations in the AASHTO LRFD procedures. To achieve this objective, typical one and multi lane straight bridges with different span lengths were modeled using finite-element analysis (FEA) subjected to HS20 truck loading, tandem loading, and standard lane loading per AASHTO LRFD procedures. The FEA results were compared with the AASHTO LRFD moments in order to quantify the biases that might result from the simplifying assumptions adopted in AASHTO. A reliability analysis was conducted to quantify the reliability index for bridges designed using AASHTO procedures. To reach a consistent level of safety for one lane and multi lane bridges, the live load factor in the design equation proposed by AASHTO LRFD needs to be revised by increasing the live load factor depending on the number of lanes. The results will provide structural engineers with more consistent provisions to design concrete slab bridges or evaluate the load-carrying capacity of existing bridges.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The design of highway bridges in the United States conforms to the American Association of State Highway and Transportation Officials AASHTO standard specifications for highway bridges Specs or AASHTO load and resistance factor design LRFD bridge design specifications. The analysis and design of any highway bridge must consider live loads such as HS20 (truck or lane) or HL93 (combination of truck or tandem with lane loading). To analyze and design reinforced concrete slab bridges, AASHTO specifies a distribution width for live loading that simplifies the two-way bending problem into a beam or one-way bending problem.

Empirical expressions for estimating the wheel load distribution and live load bending moment are typically specified in highway bridge codes such as the AASHTO LRFD. These equations do not take into account the many factors that govern the actual live load such as the transverse position of a truck or tandem on a specific lane, leading to either over-estimation or under-estimation of the bending moment obtained using finite element analysis. In addition, results from finite element analyses show that by alternating the position of the truck loads transversely, the resulting bending moments tend to increase as the applied live loads come closer to the transverse edge of a bridge (Mabsout et al., 1997; Mabsout et al., 2004).

Reliability analysis has been proven to be an effective tool for developing and assessing new and existing design codes. AASHTO LRFD code was calibrated to create new load and resistance factors to reach a pre-selected safety target based on a reliability analysis using the basic design equation (Nowak, 1999):

$$\Sigma \gamma_i X_i < \phi R_n \qquad (1)$$

Where γ_i represents a set of load factors that are greater than one and that are applied to the different load effects X_i, while ϕ represents a resistance factor that is generally less than one and that is multiplied by the nominal resistance R_n.

1.2 Research Objectives/ Aims

The aim of this research is to evaluate the current method used by AASHTO LRFD to determine the effects of live loads. The following steps show the methods used in this analysis to achieve the aforementioned objective.

First, a finite element analysis is used to evaluate numerically the maximum bending moments of single span, one or multilane bridges, with different span lengths and various slab thicknesses subjected to AASHTO LRFD live loads. Next, the bending moments are calculated using the simplified AASHTO LRFD provisions. The ratio of the FEA moments to the LRFD moments (α_{LL}) is then quantified for the different bridge cases analyzed.

The second step is to define the statistical characteristics of the different load effects and resistance as per Nowak (1995). This is followed by a reliability analysis that is aimed at quantifying the reliability levels that are inherent in the traditional LRFD design methodology as per the load and resistance factors that are recommended by AASHTO LRFD. The quantification of the reliability level is accomplished using Monte Carlo simulations whereby the reliability index of the bridge design is evaluated for the different bridges analyzed. The reliability analysis is then repeated while correcting the nominal LRFD live load moments to account for the more representative moments that were obtained from the finite element analysis.

The final step involves proposing modifications to the live load factors of the AASHTO LRFD equation to achieve a target reliability index of 3.5 for all the concrete slab bridges analyzed in this study.

1.3 Literature Review

1.3.1 AASHTO Live Load Model

The design of concrete slab bridges is influenced by the direction of traffic, with the longitudinal direction of traffic usually considered for designing the main reinforcement. The procedure for the design of concrete slab bridges presented by AASHTO, which was developed in the 1940's, was based on the studies of Westergaard and Jensen.

AASHTO standard specifications (2002) provides an empirical method for determining the bending moment and shear due to live load. Section 3.24.3.2 in the AASHTO standard specifications provides empirical equations for determining the bending moment and shear. Equations 2a through 2d show the empirical equations provided by AASHTO standard specifications.

$$M = 900S \; for \; S < 50 \; ft \qquad (2a)$$

$$M = 1000(1.3S - 20) \;\; for \; S > 50 \; ft \qquad (2b)$$

In SI units, the equations above are equivalent to:

$$M = 13{,}500S \; for \; S < 15 \; m \qquad (2c)$$

$$M = 1000 \; (19.5S - 90) \;\; for \; S > 15 \; m \qquad (2d)$$

Where M is the bending moment per unit width in lb – ft/ft or N-m/m; S is the span length in feet or meters.

AASHTO standards also specify a design live load model, HS20, for the purpose of calculating the effects of live load. The model that produces the maximum effect will be considered as the governing model. Figures 1.1, 1.2, and 1.3 show the HS20 live load models.

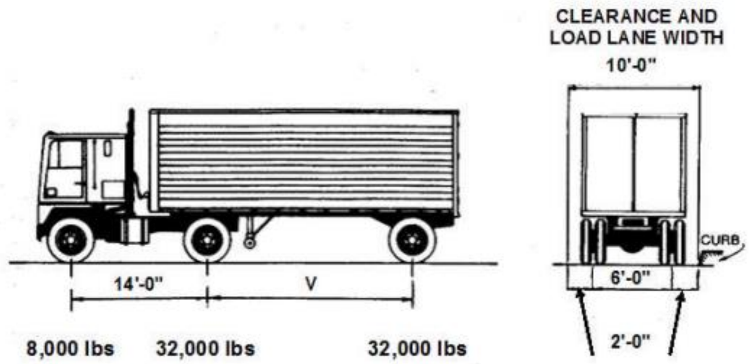


Figure 1.1 HS20-44 Design Truck Model (source: AASHTO Standard Specifications for Highway Bridges, 2002)

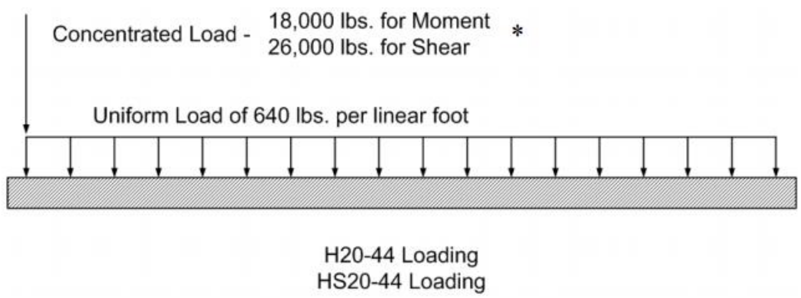


Figure 1.2 HS20 Lane Load (Source: AASHTO Standard Specifications for Highway Bridges, 2002)

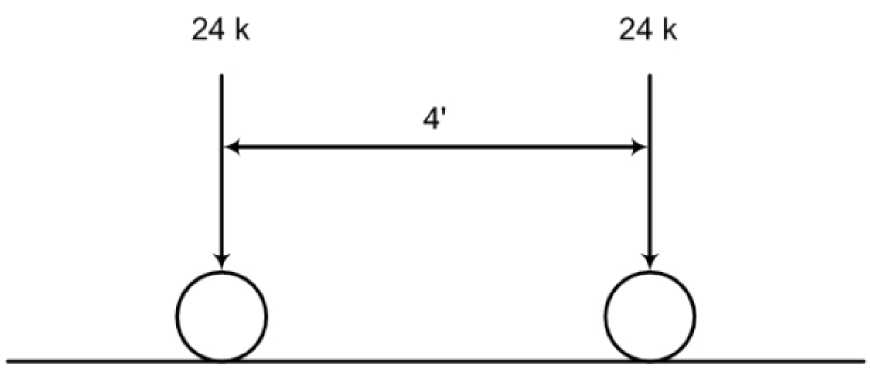


Figure 1.3 HS20 Military load (Source: AASHTO Standard Specifications for Highway Bridges, 2002)

Based on the Ontario Highway Bridge Design Code (1979), Kulicki et al. (1993) conducted a probabilistic study to determine a suitable model for the live load. Five live load cases were selected for the determination of the live load model. The resulting moments produced by each case were compared with the moment produced by the exclusion truck. The moments produced by a tandem plus a uniform load or an HS20 design truck plus a uniform load gave good ratios when the moment produced by the exclusion truck is divided by the resulting moment of the aforementioned case. This loading case was then adopted by the AASHTO LRFD code. Figures 1.4, 1.5, and 1.6 show the different live load cases selected for this study while Figure 1.7 shows the ratio of the moment caused by the exclusion truck divided by the moment produced by each case for a simply supported center line.

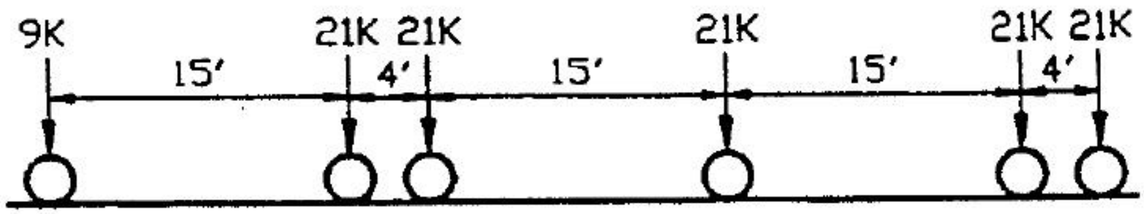


Figure 1.4 HTL-57 Loading Model (Source: Kulicki et al., 1993)

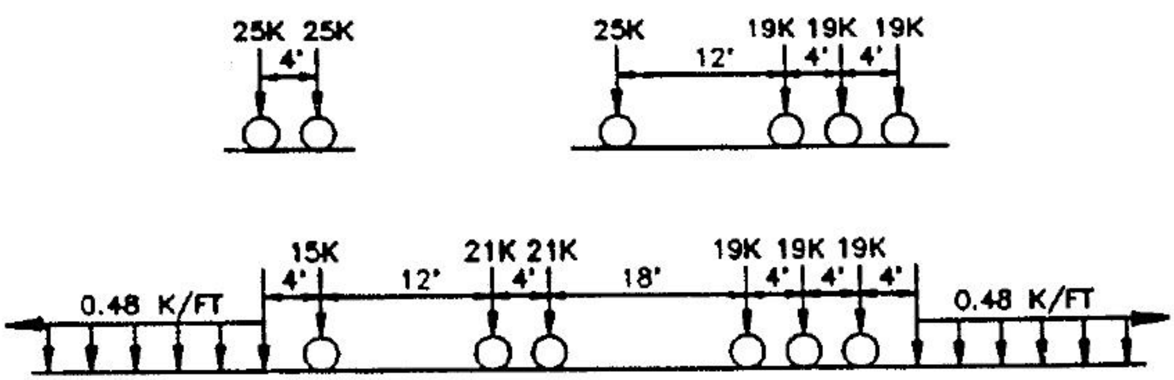


Figure 1.5 Family of Three Loads (Source: Kulicki et al., 1993)

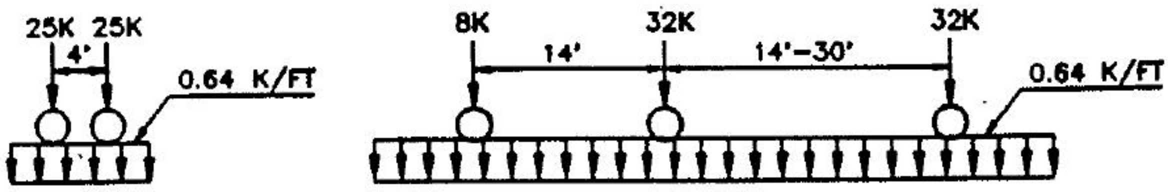


Figure 1.6 HL93 Loads (Source: Kulicki et al., 1993)

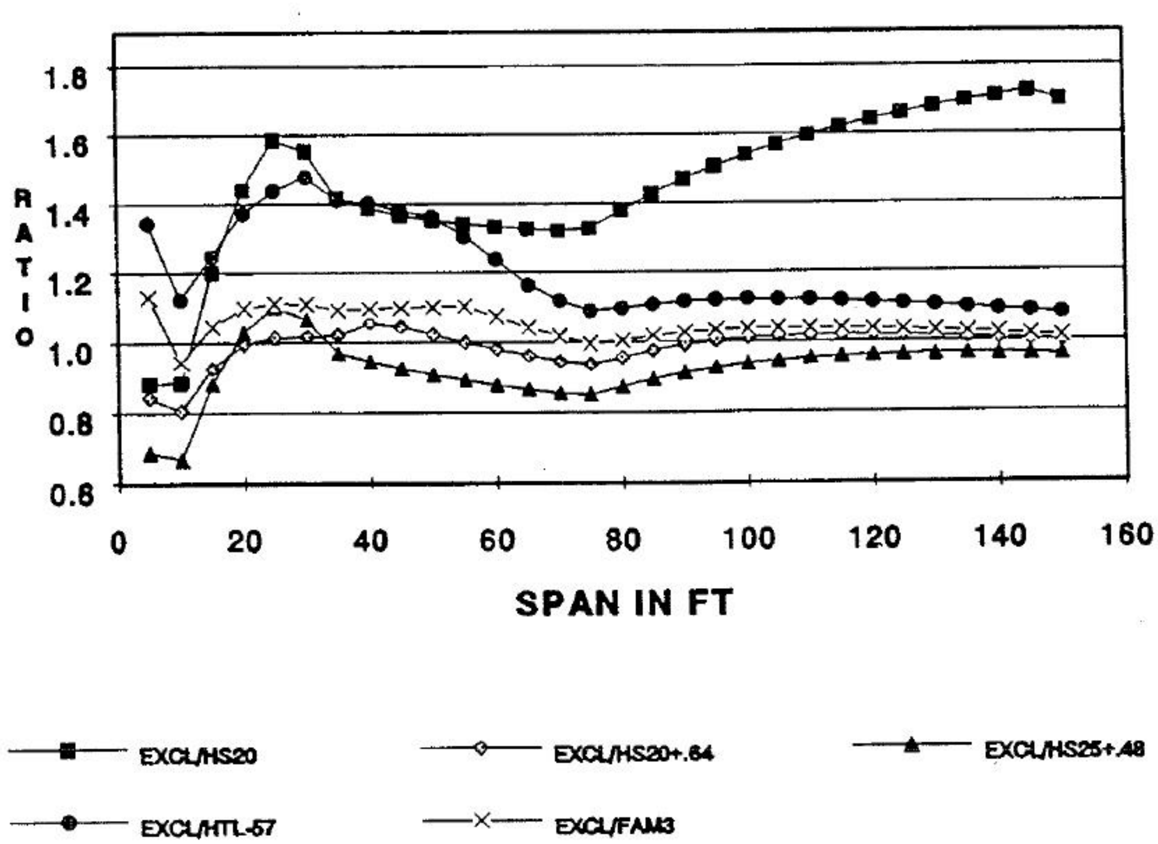


Figure 1.7 Exclusion Moment Divided by the Moment from each case (Source: Kulicki et al., 1993)

AASHTO LRFD (2014) adopted the live load model HL-93 proposed by Kulicki et al. (1993) for the purpose of calculating the effects of live loads. The AASHTO LRFD provision 4.6.2.3 provides a method for calculating the bending moment and shear due to live load. To determine the bending moment due to live load from this method, first the bending moment for a centerline of simply supported beam is found by applying the HL-93 load cases on the bridge span and determining which

HL-93 case is the most critical. Next the bending moment found in the previous step is divided by an equivalent width (E) of longitudinal strips per lane. The equivalent width equation used for determining the bending moment can also be used for determining the shear. The equations used to calculate the equivalent width for concrete slab bridges per AASHTO LRFD provisions is shown below.

$E = 10 + 5\sqrt{(L1xW1)}$ Equivalent width for one lane bridges (3a)

$E = 84 + 1.44\sqrt{L1xW1} \leq W/N_L$ Equivalent width for multilane bridges (3b)

Where E is the equivalent width for concrete slab bridges; L1 is the span length in feet taken to be the lesser of the actual span length or 60 ft; W1 is the modified edge to edge width of bridge taken to be equal to the lesser of the actual span width or 60 ft for multilane bridges or 30 ft for a single lane; W is the physical edge to edge width of the bridge; NL is the number of design lanes.

1.3.2 Calibrating AASHTO LRFD Using Reliability Analysis Method

Nowak et al. (1999) conducted a reliability analysis based on the specifications of the AASHTO standard and AASHTO LRFD. 200 representative bridges with different spans, lanes, and materials were selected for this study. The types of bridges considered in this study were reinforced concrete T-beam bridges, pre-stressed girder bridges, and steel girder bridges. The lengths of these bridges varied from 12 m to 60 m (40 ft to 197 ft) for pre-stressed girders and steel girders and from 12 m to 37 m (40 ft to 121 ft) for concrete T-beam bridges. The live load model considered in this study was the HS20 model and the newly proposed HL93 model. The effect of the live load models was considered per girder and thus multiplied by the girder distribution factor (GDF), a 0.5 reduction to take into account a single line of wheel load, and a multilane factor.

The GDF equation for the AASHTO standard live load model was taken from the AASHTO code provisions. The GDF equations used to calculate the live load moment and shear found from the proposed HL93 live load model were taken from the work of Kulicki et. al (1993) knowing that these equations were not used in the AASHTO LRFD code. The multi-lane factors for each lane is determined from simulations. These simulations took into account the number of lanes and the traffic frequency. The resulting multilane factors are shown in table 1.1.

Table 1.1 Proposed multilane factors used for calibrating the LRFD code

ADTT	One lane	Two lanes	Three lanes	Four or more lanes
100	1.15	0.95	0.65	0.55
1000	1.20	1.00	0.85	0.60
5000	1.25	1.05	0.90	0.65

The impact (dynamic) load effect is considered as an equivalent static live load and is added to the static live load calculated from AASHTO standard specifications and AASHTO LRFD. The impact load factor is calculated from equations found in AASHTO standard code provisions and is proposed to be equal to 0.33 of the static live load (truck effect only) for the AASHTO LRFD code. For the nominal resistance, the equation used to calculate the nominal resistance relied mostly on the material strength and dimensions.

The statistical parameters used for calculating the dead load are as follows: the bias factor for factory made components is 1.03 and the COV is 0.08; the bias factor for cast-in-place components is 1.05 and the COV is 0.1. For the asphalt surface, the mean thickness is 75 mm (3 inches) and the COV is 0.25. The live load bias factors for shear and moment varied depending on the span length with a COV of 0.18 for the combined effect of static and dynamic load. Figures 1.8 and 1.9 show the bias factors for the live load effects with respect to the span length. The bias factors used for the resistance varied depending on the type of bridge from 1.05 to 1.165 with COV from 0.075 to 0.16. Table 1.2 shows the bias factors for the resistance.

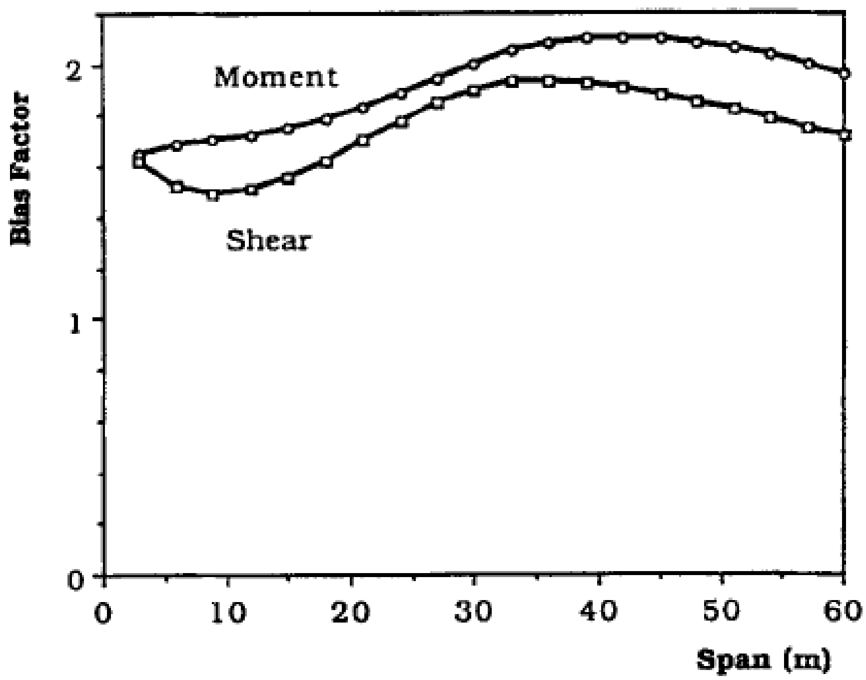


Figure1.8 bias factors for live load; AASHTO standard (Source: Nowak et al., 1999)

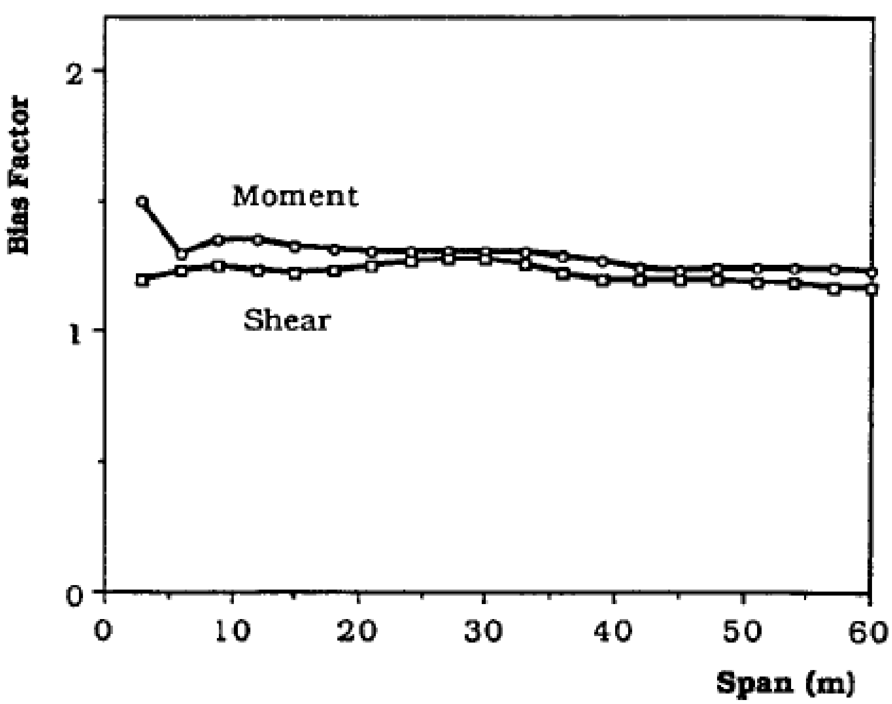


Figure 1.9 bias factors for live load; AASHTO LRFD (Source: Nowak et al., 1999)

Table 1.2 Resistance statistical parameters

Type of structure	Bias factor	Coefficient of variation
Non-composite steel girder		
Moment	1.11	0.115
Shear	1.14	0.12
Composite steel girders		
Moment	1.11	0.12
Shear	1.14	0.12
Reinforced-concrete T-beams		
Moment	1.14	0.13
Shear	1.165	0.16
Pre-stressed concrete girders		
Moment	1.05	0.075
Shear	1.165	0.16

The purpose of this study was to determine uniform safety levels for the load and resistance for the AASHTO LRFD code provisions. The safety levels of structures are measured by reliability indices (β) which are related to the probability of failure (p_f) through the inverse of the standard normal cumulative distribution function as: $\beta = -\Phi^{-1}(p_f)$. The reliability indices found in this study were used as a basis for selecting a target safety level. The target reliability index was taken as 3.5 which corresponds to a probability of failure of 2.3×10^{-4}.

Kulicki et al. (2007) implemented the AASHTO LRFD provisions in the original calibration of the AASHTO LRFD report conducted by Nowak et al. (1999) which was conducted prior to the publication of the AASHTO LRFD code. The provisions introduced to this study included changing the Annual Average Daily Truck Traffic (ADTT) from 1000 to 5000, the girder distribution factors, the calculations of the theoretical shear and bending moment, the dynamic load allowance, the multiple presence factors as well as many other factors and methods that were unavailable at the time of the calibration of the AASHTO LRFD. The results of this study are shown in Table 1.3 for moment and Table 1.4 for shear.

Table 1.3 Reliability indices corresponding to AASHTO LRFD, moment

Type of structure	Span range (ft.)	Spacing range(ft.)	Resistance factor	Reliability index range
Pre-stressed concrete	30-200	4-12	1.00	3.55-3.84
Reinforced concrete	30-120	4-12	0.90	3.54-3.97
Steel non-composite	30-200	4-12	1.00	3.46-3.71
Steel composite	30-200	4-12	1.00	3.58-3.80

Table 1.4 Reliability index corresponding to AASHTO LRFD, shear

Type of structure	Span range (ft.)	Spacing range(ft.)	Resistance factor	Reliability index range
Pre-stressed concrete	30-200	4-12	0.90	3.62-4.02
Reinforced concrete	30-120	4-12	0.90	3.53-3.95
Steel	30-200	4-12	1.00	3.70-4.03

1.3.3 Finite Element Analysis on Reinforced Concrete Bridges

Mabsout et al. (2004) conducted a study on concrete slab bridges using finite element analysis. The models used in this investigation were based on actual bridges located in different areas in the United States. These models simulated the different span lengths with various numbers of lanes and loading conditions. The loading conditions were an HS20 truck located at the center of one lane and an HS20 truck located at 1 ft. away from the edge of the lane. The resulting bending moments and edge beam moments were compared with the moments found from the AASHTO standard specifications and AASHTO LRFD specification. The results indicated that AASHTO codes tend to under-estimate or over-estimate the bending moment when compared with the bending moment found from the FEA models.

This study will concentrate on the results found by Nowak (1999), John M. Kulicki (2007), and Mabsout (2004) in order to find a consistent level of safety.

1.4 Thesis Organization

This thesis is divided into 4 chapters. The first chapter contains the research aims and objective as well as the background studies. The second chapter is separated into two sections: the first section contains a method used by AASHTO LRFD to design a reinforced concrete bridge, while the second part contains the finite element procedure used in this study to determine the bending moment resulting from the live loads. The third chapter of this study looks into the reliability analysis performed using the available statistics and the results obtained from the previous chapter. The last chapter contains the results of this study as well as conclusions and discussions.

CHAPTER 2

STRUCTURAL ANALYSIS OF CONCRETE SLAB BRIDGES

2.1 Introduction

In this chapter, the live load models proposed in AASHTO LRFD and the AASHTO LRFD design procedure are investigated and compared with results from three dimensional analysis using the finite element analysis of several proposed bridge models.

2.2 Bridge Properties

The bridge models considered for this study have distinct geometry, number of lanes, thicknesses, and physical properties. These properties are discussed in this section.

First, span lengths and thickness for each slab were taken as follows (thickness were determined for each span in order to take the deflection into account):

- 24 feet of span length and 18 inches of slab thickness
- 36 feet of span length and 21 inches of slab thickness
- 46 feet of span length and 24 inches of slab thickness
- 54 feet of span length and 27 inches of slab thickness

For the number of lanes, the following were taken into consideration:

- 14 feet for one lane bridges (12 feet of lane and 1 feet width of slab on each side)
- 24 feet for two lane bridges (12 feet for each span)
- 36 feet for three lane bridges (12 feet for each span)
- 48 feet for four lane bridges (12 feet for each span)

The physical properties (for design purposes) were taken as: Compressive strength $f'_c = 4000$ psi; Modulus of elasticity E_c: $3.60 * 10^6$ psi; and Poisson's ratio ν: 0.2. The unit weight of concrete (reinforced) was taken as 0.145 kcf (AASHTO LRFD, Table 3.5.1-1). The unit weight of the wearing surface was taken as 0.14 kcf.

2.3 Design Using AASHTO LRFD

AASHTO LRFD provisions propose a design equation that represents a set of factored loads that will be taken into consideration and are not allowed to surpass the nominal resistance, as shown in equation (1) in the previous chapter.

The next section will discuss the applied live loads and dead loads considered for this study as well as the method used to calculate the nominal resistance based on equation (1).

2.3.1 Loading Models

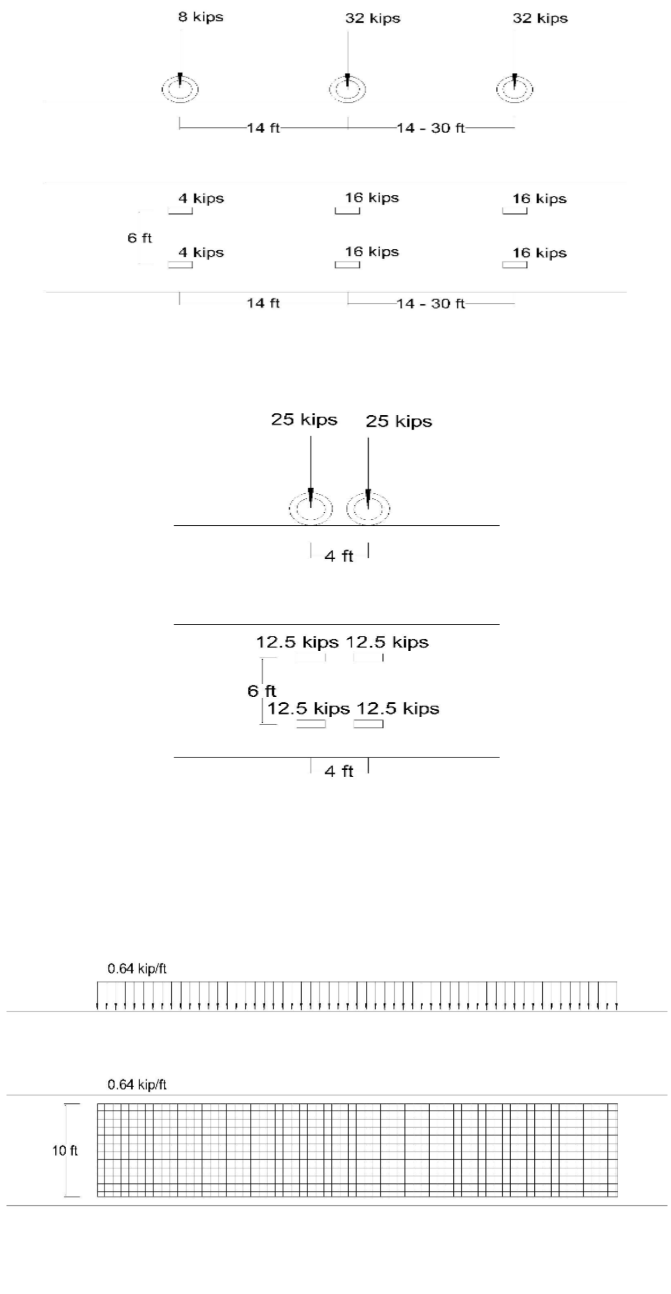
In this study, the analysis is limited to the bending moment due to the different types of loads that could affect the design of a reinforced concrete slab bridge. It is important to note that the main reinforcement direction in these type of bridges is the longitudinal direction. The bending moment due to dead loads includes the bending moment due to the slab's own weight DC and the bending moment due to the wearing surface weight DW. To determine the stress due to slab weight, the thickness of the slab was multiplied by the unit weight of the concrete (0.145 kcf). Similarly, the stress due to the wearing surface was calculated by multiplying the thickness of the wearing surface (0.25 ft) by the unit weight (0.14 kcf). The nominal bending moment due to the components of the dead load was then determined based on the simply supported equation. The resulting bending moment for the dead load components are shown in Table 2.1.

Table 2.1 Bending Moments Due to Dead Weight

Lanes	Span length (ft)	Slab thickness (ft)	Moment DC (kip-ft/ft)	Wearing surface thickness (ft)	Moment DW (kip-ft/ft)
one	24	1.5	15.66	0.25	2.52
	36	1.75	41.11	0.25	5.67
	46	2	76.71	0.25	9.2575
	54	2.25	118.92	0.25	12.7575
two	24	1.5	15.66	0.25	2.52
	36	1.75	41.11	0.25	5.67
	46	2	76.71	0.25	9.2575
	54	2.25	118.92	0.25	12.7575
three	24	1.5	15.66	0.25	2.52
	36	1.75	41.11	0.25	5.67
	46	2	76.71	0.25	9.2575
	54	2.25	118.92	0.25	12.7575
four	24	1.5	15.66	0.25	2.52
	36	1.75	41.11	0.25	5.67
	46	2	76.71	0.25	9.2575
	54	2.25	118.92	0.25	12.7575

In order to determine the bending moment due to live load, AASHTO LRFD proposes a live load model HL93 which consists of either an HS20 truck load combined with a lane load or a tandem load combined with a lane load. The HS20 truck load consists of 3 axle truck point loads, with the separation distance between the first two axles taken to be 14 ft while the separation distance for the last two axles taken to be between 14 to 28 ft depending on the span length and which axle distance give the most critical effect on the bridge. The separation distance between the tire loads was taken as 6 ft.

The first pair of axle point loads are taken to be 4 kip while the other two axel point loads were taken as 16 for each point load. For the tandem load, ASSHTO LRFD proposes a 2 axel tandem truck with a separation distance of 4 ft between axles and 6 ft transversely between tire loads. The axle loads for the tandem case were taken as 12.5 kip for each tire load. As for the lane load, it was taken as a distributed load along the span length with a 10 ft width and a load magnitude of 0.64 kip/ft. Figure 2.1 shows the HL93 loading setup.



Figures 2.1 HL93 Load Models (a) HS20 Truck Load Distribution (b) Tandem Load Distribution (c) Lane Load Distribution

The resulting bending moment calculated from the AASHTO LRFD provisions is then divided by the equivalent width E for each span. The equivalent width could be determined from equations (4a) and (4b) such that:

$E = 10 + 5\sqrt{L_1 W_1}/12$ For single-lane bridges (4a)

$E = 84 + 1.44\sqrt{L_1 W_1}/12$ For multiple-lane bridges (4b)

Where:

L_1 = span length in feet, the lesser of the actual span or 60 feet.

W_1 = edge-to-edge width of bridge in feet taken to be the lesser of the actual width or 60 feet for multi-lane loading, or 30 feet for single-lane loading.

For the impact (dynamic) load, AASHTO LRFD defines the ratio of the impact load allowance as 33% of the static moment of the truck or tandem components of the static live load M_{LL} only. Table 2.2 shows the resulting live load bending moments from the AASHTO LRFD provisions.

Table 2.2 Static (M_{LL}) and Dynamic (M_{IL}) Nominal Live Load Moments

Lanes	Span length (ft)	Slab width (ft)	Governing M_{LL} source	M_{LL} (kip-ft/ft)	M_{IL} (kip-ft/ft)	$M_{(LL+IL)}$ (kip-ft/ft)
one	24	14	Tandem	34.95	9.84	44.79
	36	14	Tandem	49.44	13.13	62.57
	46	14	Truck	60.68	18.93	79.61
	54	14	Truck	69.84	18.77	88.61
two	24	24	Tandem	29.97	7.69	37.66
	36	24	Tandem	47.85	10.23	58.08
	46	24	Truck	63.19	14.71	77.90
	54	24	Truck	75.82	14.57	90.39
three	24	36	Tandem	28.13	7.92	36.05
	36	36	Tandem	44.49	12.71	57.20
	46	36	Truck	60.36	18.17	78.53
	54	36	Truck	77.1	18.77	95.87
four	24	48	Tandem	26.74	7.53	34.27
	36	48	Tandem	42	11.16	53.16
	46	48	Truck	59.77	18.00	77.77
	54	48	Truck	77.1	19.22	96.32

Given the nominal dead load and live load moments, the AASHTO LRFD design equation can be applied to calculate the nominal moment resistance R_n for each bridge such that:

$$Rn = \frac{1.25M_{DC}+1.5\ M_{DW}+1.75\ (M_{LL}+M_{IL})}{0.9} \quad (5)$$

2.4 Finite Element Analysis

2.4.1 Loading Models for FEA

The load model used in the FEA method is the HL93 loading model proposed by AASHTO LRFD. To simulate this load model, the live loads were assumed as either a combination of HS20 trucks and lane loads or tandem trucks with lane loads. The maximum moment developed by the HS20 truck or tandem loads are calculated based on several truck positions. Trucks were placed at different distances on the bridge to produce the maximum effect. The most critical effects were produced when the truck was centered on each lane or at one feet away from the edge of the bridge. The centered case is assumed when the truck is centered on a lane and has a separation distance of 6 feet between two side by side trucks in the multi-lane case.

In the edge case, the first truck is placed at one foot away from the edge while the next truck is placed at four feet away from the previous truck (this applies to multiple presence case in which multiple trucks are present side by side on a multi-lane bridge). Figures 2.2 and 2.3 show a typical setup for an HS20 truck or tandem truck for the centered and edged cases, respectively.

(a) Centered Truck Loading (b) Edge Truck Loading

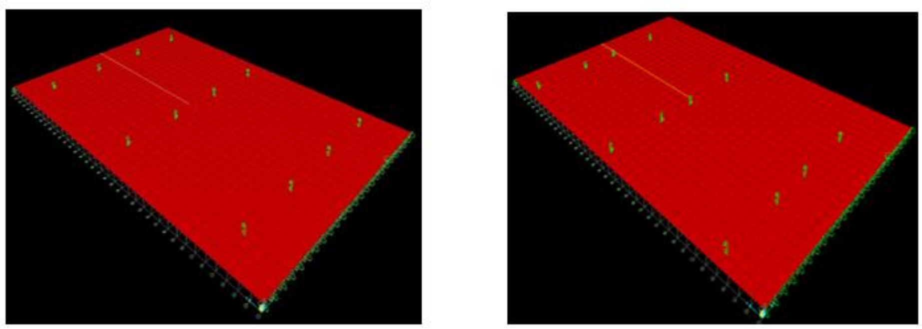


Figure 2.2 Typical Two Lane Concrete Slab Bridge with Tandem Loads

(b) Centered Tandem Loading (b) Edge Tandem Loading

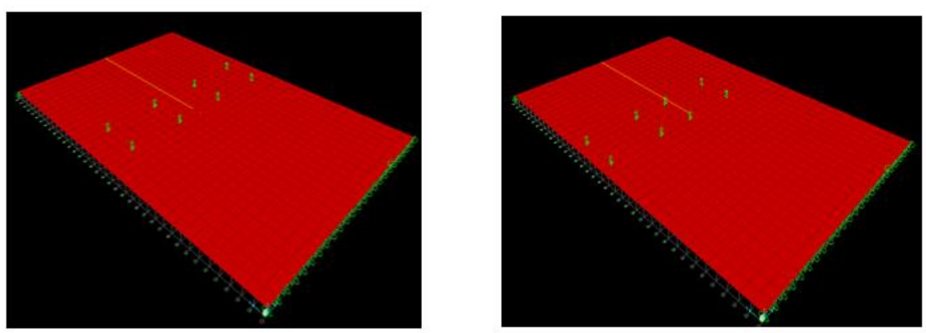


Figure 2.3 Typical Concrete Slab Bridges with HS20 Truck Loading

2.4.2 Finite Element Modeling and Results

The finite element method was used to investigate the effects of live loads on concrete slab bridges using the SAP2000 FEA computer software. The bridges are modeled as simply supported slabs divided into shell elements. The area of each shell element was taken as 1 square feet. The properties and dimensions proposed in section 2.2 were used to simulate the bridge models. Live load models from the previous section were applied on these bridges to determine the maximum effect. The resulting bending moments from the finite element analysis are presented in Table 2.3 for all of the bridge cases analyzed. These results indicate that the maximum moment based on a combination of tandem loads and lane loads governs in short spans (24 and 36 ft), while the maximum moment found from the combination of the HS20 truck loads and lane load governs in the longer spans. Results also show that the moments that were calculated for the edge loading case are generally larger than the moments calculated for a centered loading case. This applies to shorter and longer spans, respectively.

To allow for a one to one comparison between the FEA bending moments and the bending moments determined from the simplified AASHTO LRFD method, it was assumed that the largest value of the maximum longitudinal moment governs the span. The maximum bending moments that were calculated using the simplified AASHTO LRFD procedures are presented in Table 2.3. These results indicate that for a single lane, the moments calculated using the simplified LRFD procedure for shorter spans deviate from the FEA moments for both the centered and edge loading cases. For the case of one lane, the ratio between the FEA moments and the AASHTO moments (α_{LL}) ranges from 0.67 to 1.04. In the case of two lanes, the two moments tend to be closer to each other with α_{LL} ranging from 0.98 to 1.14. For the three lane bridges, the resulting α_{LL} ranges between 1.11 to 1.27 while for the four lane case it ranges from 1.18 to 1.22.

Table 2.3 FEA Maximum Longitudinal Moment FEA vs AASHTO LRFD

Lane	**Span length (ft)**	**FEA center Kip-ft/ft**	**FEA edge Kip-ft/ft**	**LRFD Kip-ft/ft**	α_{LL} **(FEA/LRFD)**
One	24	23.65	24.21	34.95	0.69
	36	38.45	39.1	49.44	0.79
	46	54.49	56.91	60.68	0.94
	54	69.35	72.91	69.84	1.04
Two	24	27.29	29.34	29.97	0.98
	36	44.72	47.07	47.85	0.98
	46	63.26	67.86	63.19	1.07
	54	80.63	86.52	75.82	1.14
Three	24	30.75	35.69	28.13	1.27
	36	44.71	49.18	44.49	1.11
	46	63.26	70.44	60.36	1.17
	54	85.81	90.05	77.1	1.17
Four	24	27.15	31.57	26.74	1.18
	36	44.92	50.76	42	1.21
	46	66.82	73	59.77	1.22
	54	85.78	92.56	77.1	1.20

CHAPTER 3

STRUCTURAL RELIABILITY ANALYSIS

3.1 Introduction

Modern structural design is based on structural reliability. Structural reliability can be defined as the probability that the structural capacity of a member or system will exceed the applied load effect. Structural reliability relies on the accurate estimation of uncertain variables which may include a structure's geometry, material properties, fabrication, and load models. In this study, structural reliability is measured by a reliability index which is a measurement of structural safety.

3.2 Performance Function and Probability of Failure

The performance function describes the boundary between acceptable and unacceptable performance of a structure. The performance function can be described as a function (g) in terms of the load and resistance as shown in equation (6):

$$g(R, Q) = R - Q \qquad (6)$$

Where R is the resistance and Q is the load.

When g(R, Q) is more than zero, this represents a safe behavior, while g(R,Q) < 0 represents an unsafe behavior. The probability of failure in this case can be described as the probability that the structural member's limit state (g) is less zero such that:

$$P_f = P(\, R - Q < 0 \,) = P(g < \, 0) \qquad (7)$$

Where P_f is the probability of failure and R and Q are random variables that are represented by continuous probability distributions that are described using cumulative distribution functions (CDF) $F_x(x)$. The first derivative of $F_x(x)$ is called the probability density function (PDF) that is represented by $f_x(x)$. Figure 3.1 shows the PDF of the continuous random variables R, Q, and g.

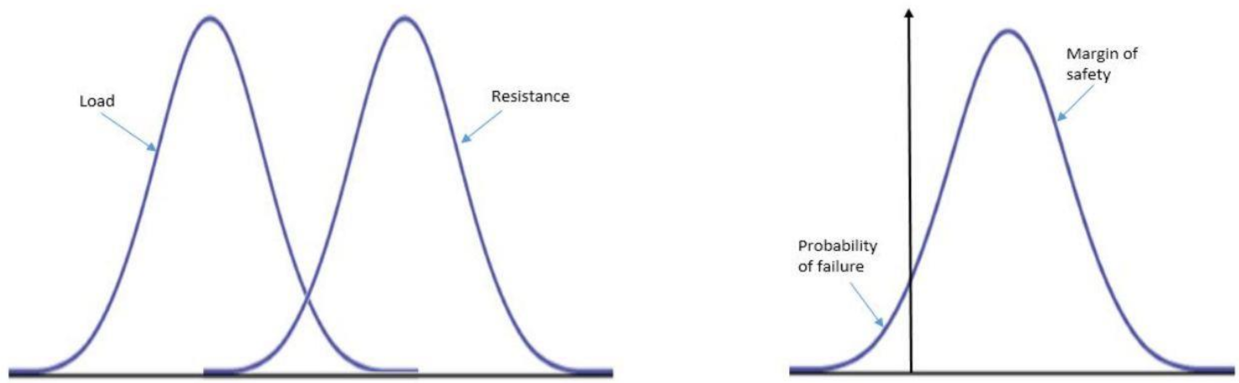


Figure 3.1 PDFs of load, resistance, limit state function

Because the probability of failure represents the probability that the resistance is less than the load, and since the load and resistance are continuous random variables, the probability of failure can be defined as all possible combinations of Q = q_i and R < q_i ; thus the probability of failure can be formulated as:

$$P_f = \sum P\,(Q = qi \cap R < qi) = \sum P\,(R < Q|\,Q = qi)\,P(Q = qi) \qquad (8)$$

And in integral form, the probability of failure can be defined as:

$$P_f = \int_{-\infty}^{\infty} F_R(x_i) f_Q(x_i) dx_i \qquad (9)$$

Where F_R is the CDF of the resistance and f_Q is the PDF of the load.

3.3 Reliability Index

The reliability index is a measure of structural safety. Assuming that the performance equation is normally distributed, the reliability index can be determined using the following equation:

$$\beta = -\Phi^{-1}\,(P_f) \qquad (10)$$

Where $-\Phi^{-1}$ is the inverse of the standard normal cumulative distribution function.

One method used to calculate the reliability index is the first order second moment method (Nowak and Collins, 2000). The first order refers to the first derivative in the Taylor series, the second moment refers to the second moment of the random variables. The first moment is the expected value and the second moment is the variance. The reliability index equation in this case is as follows:

$$\beta = \frac{\mu g}{\sigma g} = \frac{(g(\mu_{x1},\ldots,\mu_{xn}) + \sum_{i=1}^{n}(x_i - \mu_{xi})\frac{dg}{d_{xi}})}{\sqrt{\sum_{i=1}^{n}\sigma_{xi}{}^2 \, (\frac{dg}{d_{xi}})^2}} \tag{11}$$

$\frac{dg}{dxi}$ is evaluated at μ_{xi}, x_i are statistically independent variables.

When R and Q are normally distributed, the reliability index equation becomes:

$$\beta = \frac{\mu g - \mu Q}{\sqrt{\sigma R^2 + \sigma Q^2}} \tag{12}$$

Ali: This equation is also not correct. Please correct it. Also, you need to use subscripts correctly in the equations. It's not the "R" and "Q" that are squared!! It's the sigma. You need to use subscripts correctly in all the equations in the thesis.

And when R and Q are lognormally distributed the reliability index could be evaluated as follows:

$$\beta = \frac{\ln\left(\frac{\mu g}{\mu Q}\right)\sqrt{\frac{VQ^2+1}{VR^2+1}}}{\ln((VQ^2+1)(VR^2+1))} \tag{13}$$

Define V… and reformat the equation.

Another method that can be used to determine the reliability index when the solution is not in closed form is the Monte-Carlo simulation method. The principle behind the Monte-Carlo simulations is that it draws random samples from known populations and observes the behavior of a statistic in an artificial world which simulates the real world in all relevant respects. This artificial world consists of mathematical procedures for generating sets of numbers that resemble samples of data drawn from the true population. This artificial world is used to conduct multiple trials of the statistical procedure to investigate how that procedure behaves across samples (Christopher Z. Mooney, 1997).

The application of Monte-Carlo simulations in bridges can be summed up as (Kullicki et al. 2007):

1. Assume that the effect of the load is normally distributed.

2. Assume that the effect of the resistance is log-normally distributed
3. Generate random numbers based on the reported statistics for the load and resistance. The random numbers are reconstituted and chosen from reconstituted load and resistance distributions.
4. Next, the performance function is evaluated. When the value of the performance function is below zero, then this particular case fails.
5. The above step is iterated multiple times and the total number of failures are then counted and a failure rate is determined.
6. Using the failure rate calculated from the previous step, the reliability index is calculated using the inverse of the standard normal cumulative distribution of the failure rate.

3.4 STATISTICAL MODELS

The statistical parameters (bias λ and the coefficient of variation V) for the bending moments due to slab weight DC were adopted from (Nowak, 1995) as $\lambda_{DC} = 1.05$ and $V_{DC} = 0.1$, and bending moments due to wearing surface DW were taken as $\lambda_{DW} = 1$ and $V_{DW} = 0.25$. The bias factor is defined as the ratio of the mean of a given variable to the nominal value of that variable. As a result, the mean values for DC and DW for all bridges considered can be defined from λ_{DC} and λ_{DW} together with the nominal values shown in Table (2.1).

The bias factors λ_{LL} of the static live load moments (M_{LL}) were presented by Nowak (1995) and are dependent on the number of lanes and span lengths. The coefficient of variation of M_{LL} has been found to be constant with a value of 0.12 (Nowak ,1995). The mean and standard deviation of the total live load $M_{(LL+IL)}$ were determined from equations (14a) and (14b) by combining the statistics of the static live load and the impact load. In equation (14b), the values 0.12 and 0.8 represent the coefficients of variation of the static live load and dynamic load, respectively (kulicki et al., 2007). The resulting means and standard deviations of the total live load for the cases analyzed in this study are presented in Table 3.1 together with the corresponding estimates of the coefficient of variation of the $M_{(LL+IL)}$.

$$Mean\ M_{(LL+IL)} = \lambda_{LL} * M_{(LL+IL)nominal} \tag{14a}$$

$$\text{Standard Deviation of } \mathrm{M(LL + IL)} = \sqrt{\left(0.12\lambda_{LL}M_{LL}\right)^2 + \left(0.8M_{IL}\right)^2} \tag{14b}$$

The bias factor and the COV of the resistance (moment capacity) for reinforced concrete bridge slabs was taken as 1.14 and 0.13, respectively.

In this analysis, the moments due to slab weight, wearing surface, and total live load were assumed to be continuous normal distributions, while the moment capacity was assumed to be lognormally distributed (Kulicki et al., 2007).

Table 3.1 Live load statistical parameters

No. of lanes	Span (ft)	λ_{LL}	V_{LL}	Mean of $M_{(LL+IL)}$	Standard deviation $M_{(LL+IL)}$	$V_{(LL+IL)}$
One	24	1.38	0.12	61.80	9.77	0.158
	36	1.39	0.12	86.97	13.36	0.154
	46	1.37	0.12	109.01	18.14	0.166
	54	1.36	0.12	120.5	18.85	0.156
Two	24	1.16	0.12	43.68	7.43	0.170
	36	1.19	0.12	69.11	10.66	0.154
	46	1.19	0.12	92.70	14.83	0.160
	54	1.18	0.12	106.65	15.85	0.149
Three	24	1.16	0.12	41.81	7.45	0.178
	36	1.19	0.12	68.07	11.99	0.176
	46	1.19	0.12	93.46	16.90	0.181
	54	1.18	0.12	113.12	18.56	0.164
Four	24	1.16	0.12	39.75	7.08	0.178
	36	1.19	0.12	63.26	10.76	0.170
	46	1.19	0.12	92.54	16.74	0.181
	54	1.18	0.12	113.66	18.86	0.166

3.5 Reliability Analysis

The method used in this reliability analysis is the Monte-Carlo simulation method. The performance equation used for defining the failure limit is shown in equation (15):

$$g = R - DC + DW + (LL + IL) \quad (15)$$

Where R, DC, DW, and (LL+IL) were assumed to be random variables as discussed in the previous section. The probability of failure P_f was determined from the Monte-Carlo simulations by counting the realizations with ($g < 0$) and dividing them by the total number of simulations (1,000,000 simulations). The reliability index β, which is a measure of structural safety, was then calculated using equation (10).

CHAPTER 4

ANALYSIS RESULTS AND DISCUSSION

4.1 Results of the Reliability Analysis

The first set of reliability analyses were conducted to assess the reliability levels that are inherent in concrete slab bridges that are designed in accordance with the current LRFD design equation which is based on a live load factor of 1.75. The results of this set of analyses are presented in Figure 4-1a and indicate that the reliability index β ranges from 2.6 to 3.0 for the cases involving single lane bridges and is slightly below 3.5 for the cases involving two-lane slab bridges. The results for the three lane bridges show that the reliability index is between 3.31 and 3.42 for short spans and between 3.34 and 3.37 for longer spans. For the four lane case, the results show that the reliability index is between 3.36 and 3.42. Results of the single lane concrete bridges reflect reliability levels that fall short of the target reliability index of 3.5 that was set by AASHTO LRFD (Nowak, 1995). On the other hand, the results of the two-lane, three-lane, and four lane bridges are closer to the target reliability level. Table 4.1 shows the reliability indices found for the different cases using the AASHTO LRFD live load factor =1.75 as well as the reliability indices for the revised AASHTO LRFD live load factor.

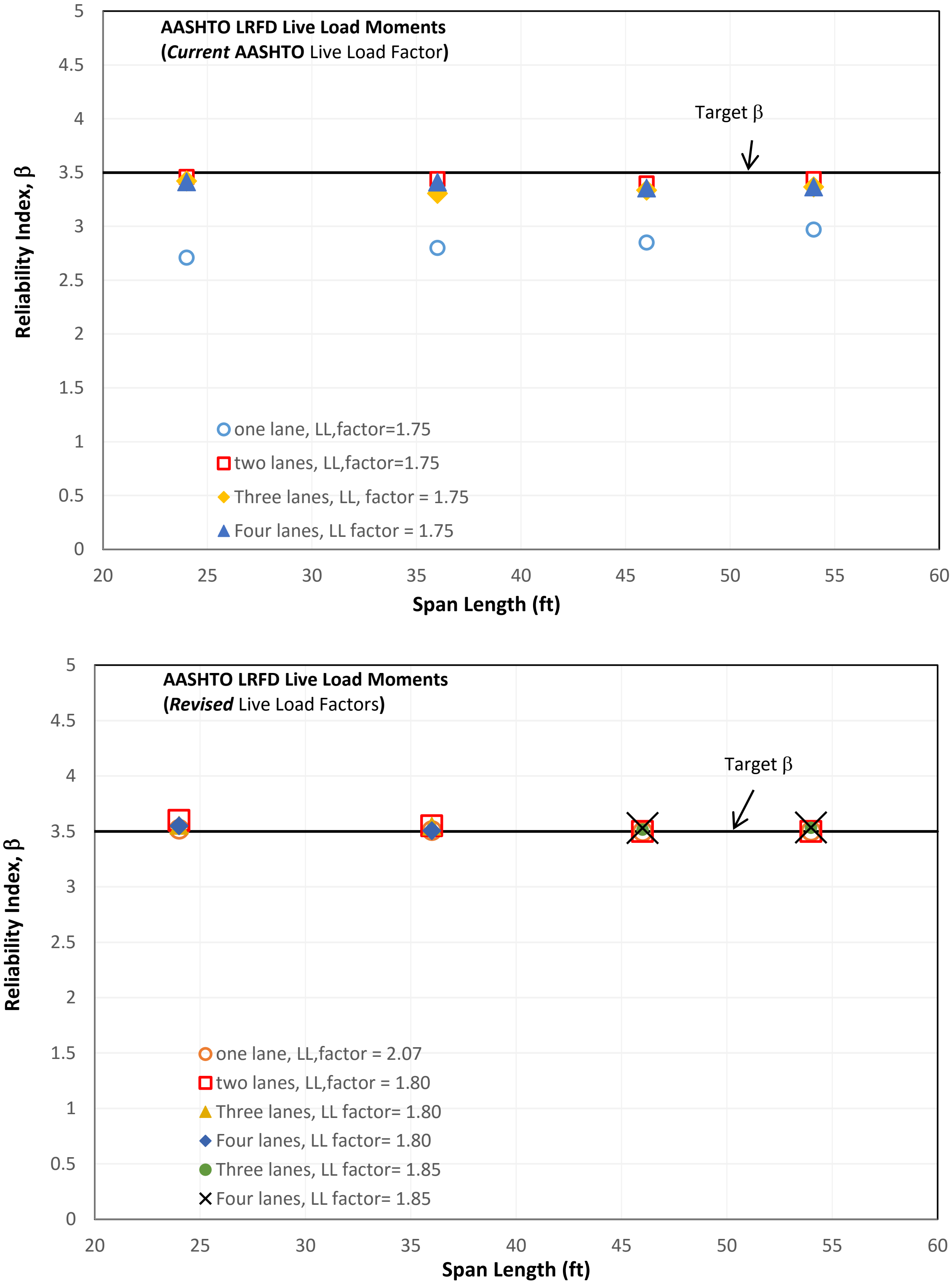


Figure 4.1 Reliability Indices for Concrete Slab Bridges per Simplified AASHTO LRFD Moments for (a) AASHTO Live Load Factors and (b) Revised Live Load Factors.

Results on Figure 4.1b point to the need for revising the AASHTO LRFD live load factors if a reliability index as high as 3.5 is to be targeted. This is particularly important for the case involving single lane bridges. As a result, the reliability analysis was repeated assuming different live load factors in an attempt to identify the factors that would ensure the desired level of reliability in the design. Results indicated that for single lane bridges, a live load factor that is as high as 2.07 is required to ensure that bridges with all span lengths would achieve a target reliability index of 3.5. For the two lane loading case, the LRFD load factor needs to be increased slightly from 1.75 to 1.8 to achieve the target reliability level. For the three lane and four lane bridges, the reliability indices found when the live load factor was increased to 1.80 showed that for the short spans, the reliability indices reached the target index while for the longer spans the live load factor had to be increased to 1.85 to reach the target index. The revised LRFD design equations for the single and double lane scenarios are presented in Equations (16), (17) and (18), respectively. The resulting reliability levels for the revised cases are presented in Figure 4.1b.

$\phi \mathrm{Rn} = (1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 2.07\ (\mathrm{MLL} + \mathrm{MIL}))$ For single lane bridges (16)

$\phi \mathrm{Rn} = (1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 1.8\ (\mathrm{MLL} + \mathrm{MIL}))$ For two lane, three lane and, four lane (short spans) bridges (17)

$\phi \mathrm{Rn} = (1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 1.85\ (\mathrm{MLL} + \mathrm{MIL}))$ For three lane and, four lane (long spans) bridges (18)

Table 4.1 Reliability Indices for Concrete Slab Bridges per Simplified AASHTO LRFD Moments for AASHTO Live Load Factors and Revised Live Load Factors.

Lane	Span length (ft)	Beta (LL factor= 1.75)	Beta (LL factor= 1.80)	Beta (LL factor= 1.85)	Beta (LL factor= 2.07)
One	24	2.71	-	-	3.52
	36	2.8	-	-	3.51
	46	2.85	-	-	3.50
	54	2.97	-	-	3.51
Two	24	3.46	3.6	-	-
	36	3.44	3.55	-	-
	46	3.4	3.50	-	-
	54	3.44	3.50	-	-
Three	24	3.42	3.56	-	-
	36	3.31	3.55	-	-
	46	3.34	-	3.52	-
	54	3.37	-	3.53	-
Four	24	3.42	3.55	-	-
	36	3.41	3.51	-	-
	46	3.36	-	3.53	-
	54	3.37	-	3.54	-

The results presented in Figure 4-1 pertain to bridge designs that are based on the live load moments M_{LL} that are calculated using the simplified procedure recommended by AASHTO LRFD. Results presented in Table 2.3 indicate that these M_{LL} values could deviate from the more representative finite element values, particularly for cases involving single lane bridges with relatively shorter spans (24ft and 36ft). To account for this discrepancy in the value of M_{LL}, the reliability analyses were repeated such that the simplified AASHTO live load moments were corrected by multiplying these moments by the ratio α_{LL} (see Table 2.3). For the cases involving single lane bridges with shorter spans (24, 36 ft) where the tandem/lane combination governed, an α_{LL} value of 0.74 (average of the two α_{LL} values for the two span lengths) was adopted. For a single lane with longer spans (46, 54 ft) where the HS-20 truck/lane combination governed, an α_{LL} value of 0.99 was adopted.

For the two lane bridge cases, the α_{LL} ratios of 0.98 and 1.11 were adopted for the shorter and longer spans, respectively. For the three lane bridge cases, the ratios were taken as 1.19 for short spans and 1.17 for long spans. As for the four lane bridge cases, this ratio was taken as 1.18 for short spans, and 1.21 for long spans.

To incorporate the ratio α_{LL} in the reliability analysis, the static live load moment that is based on the simplified AASHTO LRFD procedure was multiplied by α_{LL} as reflected in the modified performance function shown in Equation (19):

$$g = R - (DC + DW + \alpha_{LL} (LL) + IL) \quad (19)$$

The results of the reliability analysis that was conducted using the revised performance function that is presented in Equation (19) are shown in Figure 4-2. Results pertain to the conventional live load factor of 1.75 that is recommended by AASHTO LRFD. As expected, the calculated reliability indices for the single lane bridges with the shorter span lengths of 24 and 36 ft increased significantly compared with the earlier results (Figure 4.1). This increase in the reliability index (up to values of 3.8) is directly correlated to the smaller α_{LL} ratio (average of 0.74) which indicates that the simplified AASHTO LRFD procedure overestimated the maximum live load moments on the bridge. For the single lane bridges with the longer spans, the reliability indices were found to be still less than the target reliability index since the α_{LL} ratio for these cases was close to 1.0.

For the two lane bridges, results in Figure 4.2a indicate that the target reliability index was achieved for the shorter spans, but fell short of achieving a target reliability index of 3.5 for the longer spans for the case where the conventional AASHTO LRFD load factor of 1.75 was adopted.

The reliability indices for the three lane bridge cases fell below the target reliability index of 3.5 when the α_{LL} ratio was adopted. This is due to the fact that the moment found by the AASHTO LRFD is overestimating the moments for the three lane bridge cases. The same can be said about the four lane bridge cases in which the α_{LL} ratio was larger than 1.0. Table 4.2 shows the resulting reliability indices obtained when the α_{LL} ratio was applied.

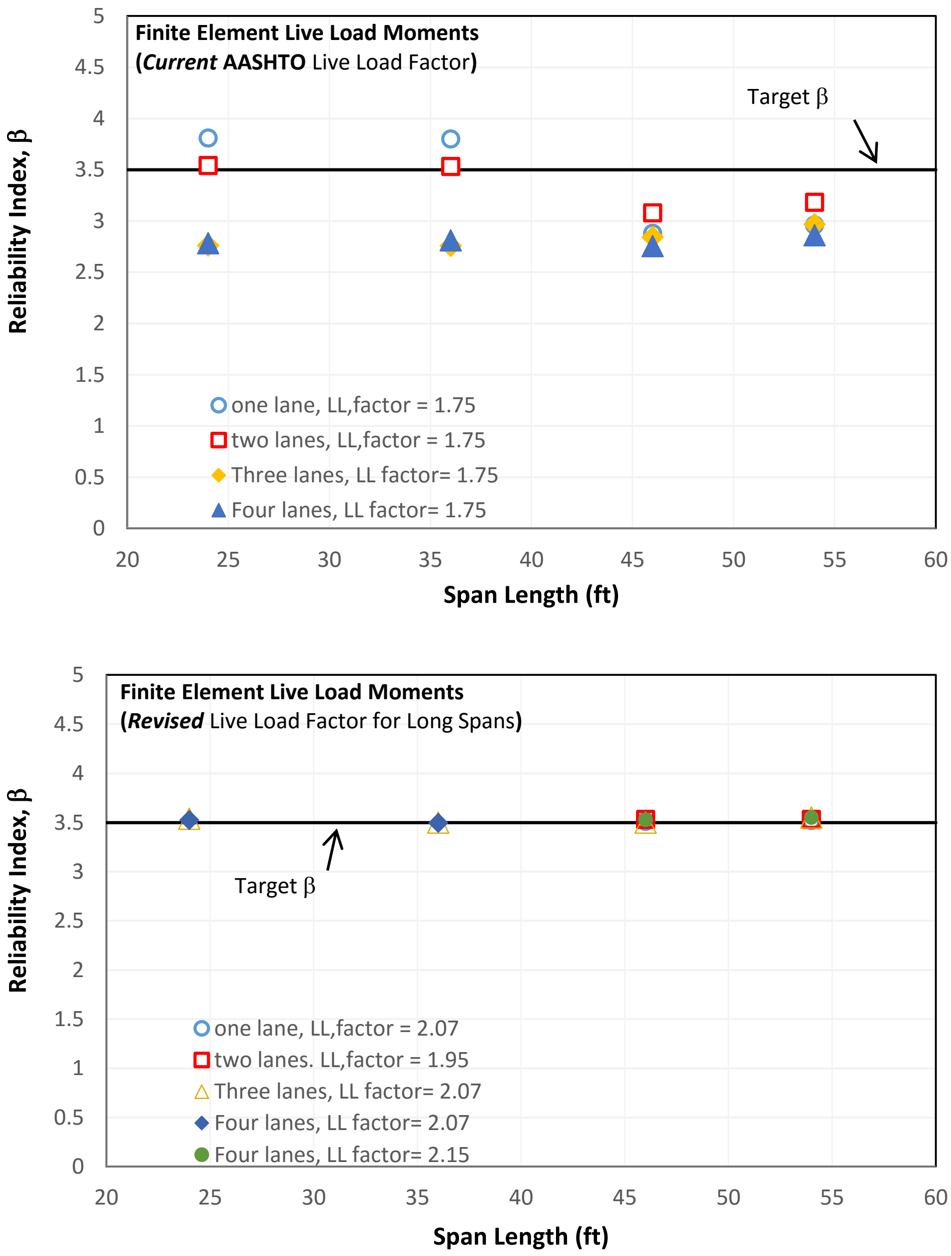


Figure 4.2 Reliability Indices for Concrete Slab Bridges where the AASHTO LRFD Live Load Moments are corrected using the FEA Results for (a) AASHTO Live Load Factors and (b) Revised Live Load Factors

Table 4.2 Reliability Indices for Concrete Slab Bridges where the AASHTO LRFD Live Load Moments are corrected using the FEA Results for AASHTO Live Load Factors and Revised Live Load Factors

Lane	Span length (ft)	Beta (LL factor= 1.75)	Beta (LL factor= 1.95)	Beta (LL factor= 2.07)	Beta (LL factor= 2.15)
One	24	3.81	-	-	-
	36	3.8	-	-	-
	46	2.88	-	3.51	-
	54	2.96	-	3.52	-
Two	24	3.54	-	-	-
	36	3.53	-	-	-
	46	3.08	3.53	-	-
	54	3.18	3.53	-	-
Three	24	2.76	-	3.54	-
	36	2.76	-	3.50	-
	46	2.84	-	3.50	-
	54	2.97	-	3.55	-
Four	24	2.78	-	3.53	-
	36	2.81	-	3.50	-
	46	2.75	-	-	3.53
	54	2.86	-	-	3.55

To ensure a target reliability index of 3.5 the LRFD live load factors need to be revised for the single lane, two lanes, three lanes, and four lanes bridge cases. Results from the reliability analysis indicated that for the one-lane case with longer spans, the live load factor has to be increased from 1.75 to 2.07, even if the live load moments are corrected based on the FEA results. As for the two lane bridge cases, the target reliability levels for the longer spans cases could be ensured with a revised live load factor of 1.95 as shown in Figure 4-2b. For the three lane bridge cases, the live load factor was increased to 2.07 to ensure that the indices are at the target level. For the four lane bridge cases, the reliability index for short spans reached the target level at LL factor of 2.07; while the longer spans required a 2.15 LL factor to ensure the target reliability index.

Thus, it is recommended that design Equations (20), (21), and (22) be used for the different lane cases.

$\phi \mathrm{Rn} = (1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 2.07\ (\mathrm{MLL} + \mathrm{MIL}))$ For single lane, three lanes, and short spanned four lanes bridge cases bridges (20)

$\phi \mathrm{Rn} = \left(1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 1.95\ (\mathrm{MLL} + \mathrm{MIL})\right)$ For two lane bridge cases (21)

$\phi \mathrm{Rn} = (1.25\ \mathrm{MDC} + 1.5\ \mathrm{MDW} + 2.07\ (\mathrm{MLL} + \mathrm{MIL}))$ For long spans four lane bridge cases (22)

4.2 Summary and Conclusions

The method used to calculate the bending moment in AASHTO LRFD tends to overestimate the live load moments for shorter spans in one and two lane bridges when compared to the moment obtained from the finite element analysis. For longer spans, the bending moment obtained from AASHTO LRFD provisions tends to slightly underestimate the moment when compared with the FEA moment for one and two lane reinforced concrete bridges. For three and four lane bridge cases, AASHTO LRFD tends to underestimate the bending moment when compared with the bending moments produced from the finite element models.

The reliability analysis performed in this study is used to check the level of safety for the reinforced concrete bridges that are designed with the AASHTO LRFD provisions. The results of the reliability analysis showed that the reliability index is slightly lower than the target reliability index for two lane, three lane, and four lane bridges. The reliability indices for one lane reinforced concrete bridges were considerably lower than the target reliability index.

To reach a consistent level of safety for one lane and multi-lane bridges, the live load factor in the design equation proposed by AASHTO LRFD needs to be revised by increasing the live load factor to 2.07 for one lane and 1.8 for two lanes, three lanes with short spans, and four lanes with short spans. While for three lanes and four lanes with longer spans the live load factor needs to be increased to 1.85 to ensure the target reliability index.

When the difference between the moments obtained from AASHTO LRFD and FEA is incorporated in the reliability analysis, the results showed acceptable target reliability levels for shorter span bridges and relatively inferior reliability indices for longer spans. To achieve the target reliability levels for these cases, the load factors in the AASHTO LRFD provisions needed to be increased to 2.07 for a single lane with longer spans, three lanes, and four lanes with short spans. For the case of two lanes with longer spans the live load factor must increase to 1.95, and for four lanes with longer spans it needs to be increased to 2.15.

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