



AMERICAN UNIVERSITY OF BEIRUT

A RELIABILITY-BASED DECISION FRAMEWORK FOR  
DESIGNING PILE-LOAD TEST PROGRAMS

by  
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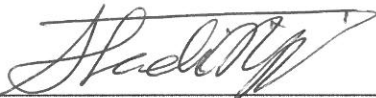
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# AN ABSTRACT OF THE THESIS OF

Youmna Hussein Abdallah for Master of Engineering  
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Title: A Reliability-Based Decision Framework for Designing Pile-Load test Programs

There is currently an inconsistency in the recommendations that are available in pile-design codes and practices regarding the required number of proof-load tests and the level of the proof loads for piles. This inconsistency has led to the implementation of unnecessarily costly pile load test programs in some cases and to insufficient or deficient load test programs in others. In both cases, the depletion of resources is the major outcome of the lack of rational methodologies for designing pile test programs.

In this thesis, first, we study the effect of choosing different proof-load test programs on the reliability of piles. This is achieved by utilizing a Bayesian approach to update the capacity distributions of piles at a site given the results of the proof-load test program. The results of the updating process constitute necessary input to a proposed rational decision framework; it is reliability-based, pre-posterior decision-making framework to allow for selecting the optimal pile-load test program that would result in the maximum expected benefit to a project, while maintaining a target level of reliability in the pile design at the site.

This proposed methodology is original, practical, and is based on site-specific information that is unique to any given project. In the final part of the thesis, the efficiency of the proposed decision framework is demonstrated by applying it on a practical design example.

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# CHAPTER 1

## INTRODUCTION

### **1.1 Background**

Pile load tests have proven to act as an efficient mean in reducing the uncertainties associated with pile capacity prediction. Traditionally, proof-load tests have been utilized to validate design methods and construction procedures in foundation engineering. In the allowable-stress design approach, the foundation is sized based on an empirical design method using a reduced factor of safety (typically 2.0) provided that it passes a proof-load test up to twice the design load (ASTM D1153 1994). Recently, and in the framework of reliability-based design, researches have shown that information from pile load tests may have a considerable effect on reducing the probability of failure, thus allowing for the use of lower factors of safety for the piles in a site. In many international design codes and practices that allow for the use of reduced factors of safety of different magnitudes, the proposed factors of safety are dependent on the number and type of pile load tests that are conducted in a given site. However, there is currently an inconsistency in the recommendations that are available in pile design codes and practices regarding the required number of proof-load tests and the level of the proof loads.

For example, some common recommendations from international pile design codes are summarized in Table 1.1. An investigation of the recommendations from different codes indicates a large variability in the correlation between the type and number of the specified pile load tests and the recommended reduced design factor of

safety. In addition to the variability between the recommendations of different design codes, a major drawback of any recommendation is that the designer does not have any indication of the inherent reliability/safety that is associated with the resulting design. This is due to the fact that the recommendations summarized in Table 1.1 are generally based on experience and are not associated with any robust reliability or risk analysis that supports their use and sheds light on the reliability of the resulting pile design.

In the last decade, some research efforts have targeted analyzing the impact of proof-load tests on the design of foundations in the framework of a reliability analysis. Examples include the work of Zhang and Tang (2002), Zhang (2004), Su (2006), Najjar and Gilbert (2009), and Park et al. (2011). Except for the study by Najjar and Gilbert (2009), current reliability analyses focus on utilizing results from proof-load tests to update the mean or median of the capacity distribution. Results from these reliability analyses indicate that the magnitude of the proof load has to be higher than the predicted mean capacity so that the updating process will have a significant effect on the reliability. As an example, Zhang (2004) recommends conducting 1 to 3 tests using proof loads that are larger than 1.5 times the predicted pile capacity (larger than 3 times the design load) so that the value of the proof-load test can be maximized.

Proof-load tests that are conducted up to 3 times the design load can be quite expensive and time consuming relative to the time scale of a given project. In addition, the likelihood of failing the pile during the test increases significantly as the proof-load level increases. For geotechnical engineering applications, the left-hand tail of the capacity distribution governs the probability of failure since the uncertainty in the capacity is generally larger than the uncertainty in the load. As a result, the reliability of a foundation is expected to be strongly affected by the presence of a lower-bound

capacity (Najjar and Gilbert 2009). This is clearly shown in Figure 1.1 which illustrates the effect of a lower-bound capacity on the probability of failure for a typical foundation (Najjar and Gilbert 2009).

Table 1.1 - Worldwide recommended safety factors for static-dynamic pile load tests programs

Country	S. F. w/o LT	S. F. with SLT	S. F. with DLT	Number of load tests required in a site and notes
China	1.65	1.60	1.60	pre-case pile (including steel pile)
	1.65	1.60	1.60	large-diameter cast-in-place pile
	1.67	1.62	1.62	wet work driven cast-in-place pile
	1.70	1.65	1.65	dry work driven cast-in-place pile
			SLTs on 1% of constructed piles (3 SLTs at least in a site if total quantities greater than 50 piles, 2 SLTs if total quantities is within 50 piles). DLTs on 5% of constructed piles (5 DLTs at least in a site).	
Europe		2.29		if one SLT is performed
EC7 2001		1.64		if SLTs greater than 5 are performed
			2.23	if two DLTs are performed
			1.95	if DLTs greater than 20 are performed
Europe		2.18		if number of tests is equal or less than 2.
EC7 2003		1.91		if number of tests is equal or greater than 20.
German		1.93		if 2 tests are performed
DIN 1054-2003		1.67		if tests greater than 4 are performed
India	2.5	2.5	2.5	1 to 2 % of total piles Minimum 2 piles for highway bridges
Japan	3.0	2.7	2.7	not specified, number load tests is not taken into account.
Kazakhstan	1.5	1.2	-----	SLTs on 1% of constructed piles (2 SLTs at least in a site)
Mexico	3.0	2.0	2.0 to 2.5	Between 1% to 5% depending the project. Minimum 3 piles. The specifications varie in each project- A global specification does not exist.
Netherlands	2.05		-----	pile design based on CPT
	1.78			pile design based on CPT
		1.71		
Norway	1.6	1.4	1.4	not specified
Singapore	3.0	2.0		when there is less certainty of the value of the ultimate capacity. where the ultimate has been determined by a number of loading tests or where they may be justified by local experience.
Sweden 1980	3.0	?		
			2.5	if 5% of piles are tested.
			2.0	if 25% of piles are tested.
Sweden 2000			2.0	if 25% of piles are tested.
			1.6	if 100% of piles are tested.
USA AASHTO 1992		2.0	2.25	
		1.9		if SLT and DLT are performed.
USA ASCE 1996	3.0			theoretical or empirical prediction
		1.6 to 1.9	1.7 to 2.0	in case of design capacity is 0.4 to 1.0 MN.
		1.8 to 2.2	2.0 to 2.4	in case of design capacity is over 1.0 MN.
USA IBC 2000		2.0	2.0	for design load greater than 0.4 MN.



The curves on Fig. 1.1 represent the case where the uncertainty in the capacity is relatively large compared to the uncertainty in the load. This example is representative of many geotechnical designs where the capacity is more uncertain than the load (McVay 2000; Kulhawy and Phoon 2002; Phoon et al. 2003; AASHTO 2004). The primary conclusion from Fig. 1.1 is that a lower-bound capacity can have a significant effect on the calculated reliability. For example, consider a typical case where the factor of safety is 3.0. If the lower-bound capacity is anything greater than 0.6, the probability of failure is reduced by more than an order of magnitude compared to the case where there is no lower bound.

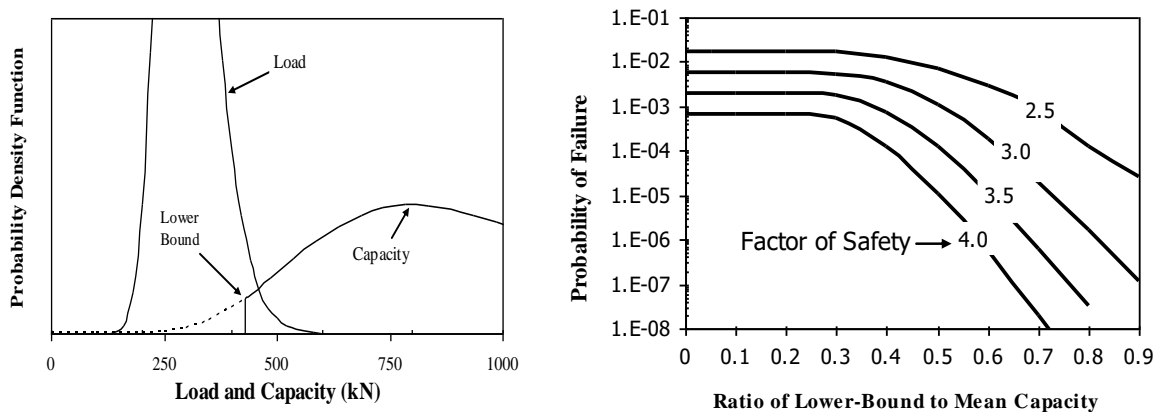


Figure 1. 1 - Effect of lower-bound capacity on the reliability (Najjar and Gilbert 2009)

When a limited number of proof-load tests are conducted on a small percentage of foundations at a site, Bayesian techniques can be used to update the probability distribution of the foundation capacity at the site. In the updating process, the results of proof-load tests are typically used to update the middle of the capacity distribution (mean or median). However, Bayesian techniques have been also utilized to update the lower-bound capacity (rather than the mean capacity) at the tail of truncated capacity distributions. Najjar and Gilbert (2009a) proved through an illustrative

example that running successful proof-load tests of relatively small magnitude (0.6 of the predicted capacity) on 3% of the piles at a site with 1000 piles resulted in a 30% reduction in the required median factor of safety while still maintaining the same level of reliability. The analysis assumes that all the piles survive the proof load tests and that the results of the load test program are used to update the lower-bound pile capacity. Using the updated lower-bound distribution, the median factor of safety required to achieve the desired reliability index of 3.0 was reduced from 3.2 to 2.5.

Results from previous studies show that different combinations of reduced factor of safety, proof load level, and number of positive proof load tests could be selected to achieve the desired level of reliability. For example, designers have the option of choosing test programs that are based on a few number of load tests that are conducted to a relatively high proof load level, or load tests that include larger number of proof tests that are conducted to a relatively smaller proof load level. There is a need for systematic and rational approaches that would allow for choosing the number of proof-load tests and the magnitude of the proof load that would maximize the value of any pile load test program.

## **1.2 Objectives**

The current research study aims at:

- (1) Formulating a robust mathematical code that is based on Bayesian techniques for updating the pile capacity distribution and the associated reliability given results from pile load tests.
- (2) Incorporating the lower-bound capacity in the probabilistic model and the updating process according to two different approaches: In the first approach, the lower- bound

capacity is defined as a lower-bound for the actual distribution of the pile capacity while in second approach the lower-bound capacity is defined a lower-bound for the distribution of the mean of the pile capacity.

(3) Investigating according to the two different probabilistic model approaches the effect of choosing different proof-load test programs on the reliability of pile design. In the analysis, the parameters that will be changed are the level of the proof load (relative to the design load), the number of proof-load tests, and the possible results of the proof-load tests.

(4) Studying the effect of the lower-bound capacity on the updating process for both approaches. Thus, the updating process for a given proposed load test program will first be conducted by updating the median capacity only, then, the analysis will be repeated for the case where the lower-bound capacity is updated only. Finally, the updating will be done for the two parameters simultaneously.

(5) Constructing a simple, logical, and practical decision-making framework for choosing the number of proof-load tests and the magnitude of the proof load that would maximize the value of information of a test program.

### **1.3 Thesis Organization**

The thesis will be comprised of seven chapters encompassing all the aspects of the study. A brief summary of the contents of each chapter is presented below:

(1) Chapter II: The formulation of the reliability problem according to two different approaches is described. This includes the characteristics of the design parameters involved in the probabilistic modeling of the load and the resistance as well as the details of the MATLAB code used to perform the Bayesian updating process.

- (3) Chapter III: Results of the Bayesian updating based on the first approach are presented and analyzed.
- (4) Chapter IV: Results of the Bayesian updating based on the second approach are presented and analyzed.
- (5) Chapter V: Based on the conclusions of previous chapters, a reliability-based decision tool is recommended for establishing a rational and practical decision-making framework for choosing the optimum testing program that will maximize the value of information at a given site.
- (6) Chapter VI: Conclusions and Future Work.

## CHAPTER 2

### FORMULATION OF THE PILE RELIABILITY PROBLEM

#### **2.1 General Probabilistic Form**

The main objective of the proposed study centers around updating the capacity distribution of piles at a site given results from a pile load testing program. The three parameters that will be assumed to define the pile capacity distribution are the mean capacity, the lower-bound capacity and the coefficient of variation. The incorporation of a lower-bound capacity in the probabilistic model of pile capacity distinguishes the work presented in this thesis from other studies in the literature. Two different approaches for incorporating the lower-bound capacity in the problem will be tested since there are currently two common schools for modeling the total uncertainty in pile capacity based on databases of pile load tests. The performance of the two approaches will be compared in Chapters 3 and 4 and a recommendation regarding the effectiveness of the two approaches will be presented.

#### **2.2 Approach 1 - Prior Statistics of the Pile Capacity**

In approach 1, the uncertainty in the pile capacity will be assumed to be modeled by a truncated lognormal distribution (Najjar 2005) as shown in Figure 2.1. The use of the lognormal distribution (rather than any other distribution) as a basis for the uncertainty in the capacity stems for the common use of the lognormal distribution in the published literature regarding the reliability-based design of piles coupled with the added advantage provided by the lognormal distribution in relation to it being

confined to positive numbers. The use of a “truncated” distribution allows for incorporating the lower-bound capacity, which has been shown by Gilbert et al. (2005) to provide a realistic representation of the left-hand tail of the capacity distribution for driven piles in sands and clays. The lower-bound capacity is a physical quantity that is predicted using models that take into consideration the pile dimensions and the properties of the soil (Gilbert et al. 2005). It is not a simple statistical parameter that is enforced on the capacity distribution.

In approach 1, both the mean capacity ( $r_{\text{mean}}$ ) and the lower-bound capacity ( $r_{\text{LB}}$ ) are assumed to be random variables (model parameters) following a lognormal distribution since both  $r_{\text{mean}}$  and  $r_{\text{LB}}$  cannot physically assume negative values. The prior statistics and probability distributions of the two parameters were determined based on several realistic assumptions and existing empirical models.

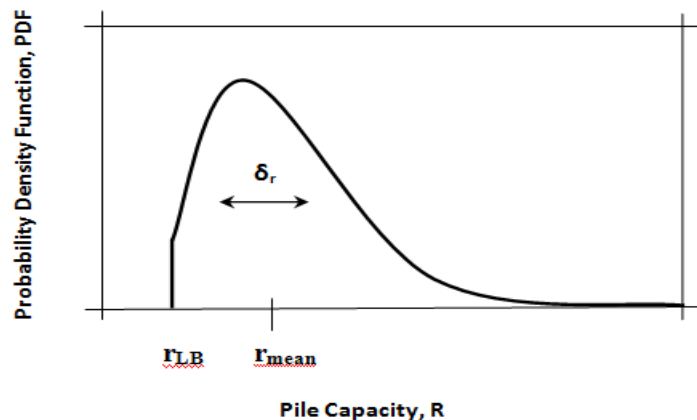


Figure 2. 1 - Parameters of Truncated Lognormal Pile Capacity Distribution

The Bayesian updating tool which will be discussed in the next section will allow for updating either or both of these two parameters given the results of pile load tests. More information about the prior statistics for the load and for each of the three statistical parameters describing the prior capacity distribution is provided below.

***a. The load (s): mean and coefficient of variation***

The load was assumed to follow a lognormal distribution with a coefficient of variation of 0.15 as is the convention. For comparison, the coefficients of variation specified by AASHTO (2004) to represent the uncertainty in bridge loads are 0.13 and 0.18 for the dead and live load respectively. For illustration and computational purposes, the mean load was assumed to take a value of 200 tons. In fact, a numerical estimate of the mean load is needed to illustrate the methodology presented in this paper for updating the pile capacity distribution using proof load tests. The results and conclusions will however be general and independent of the actual value of the mean load.

***b. The mean of the pile capacity ( $r_{mean}$ )***

It was assumed that the mean of  $r_{mean}$  could be estimated from databases of pile load tests as is conventionally done in evaluating the bias of pile capacity prediction models. The coefficient of variation of  $r_{mean}$  was assumed to be equal to 0.1 to account for systematic and random uncertainties in the determination of the soil properties at each test site in the database, uncertainties due to pile testing procedures and instrumentation, and uncertainties due to the interpretation of the pile capacity from the load-settlement curves of the pile tests in the database.

***c. The Lower-bound of the pile capacity ( $R_{LB}$ )***

With regards to the prior statistics of  $r_{LB}$ , it was assumed that the mean of  $r_{LB}$  is equal to about 0.5 of the mean of  $r_{mean}$ . This value is supported by the results presented in Gilbert et al. (2005) who show based on analyses of databases for driven piles in clays and sands that the ratio of the lower-bound capacity to the mean capacity for driven piles

could range from 0.4 to 0.9, with an average of about 0.55 to 0.60. The lower-bound capacities are computed using physical models (ex. Najjar 2005 and Gilbert et al. 2005) and are not based on statistical minimum values of pile capacity. The prior coefficient of variation in  $r_{LB}$  was assumed to be equal to 0.2 (Najjar and Gilbert 2009b) to account for (1) uncertainty due to spatial variability in the soil properties needed in the estimation of the lower-bound capacity and (2) uncertainty in the models available for predicting the lower-bound capacity.

***d. The coefficient of variation of the pile capacity ( $\delta_r$ )***

For simplicity, the coefficient of variation  $\delta_r$  will be assumed to be a deterministic parameter that is generally evaluated for different pile capacity prediction models using databases of pile load tests (ex. Barker et al. 1991, Withiam et al. 1997, Goble 1999, Liang and Nawari 2000, McVay et al. 2000, 2002 and 2003, Zhanget al. 2001, Kuo et al. 2002, Kulhawy and Phoon 2002, Phoon et al. 2003a and 2003b, Honjo et al. 2003, Paikowsky 2003, Withiam 2003 and Gilbert et al. 2005). As an example, Gilbert et al. (2005) report  $\delta_r$  values of 0.25 and 0.55 for the API (1993) method for driven steel pipe piles in clays and sands, respectively. Along the same lines, Zhang (2004) reports  $\delta_r$  values ranging from 0.21 to 0.57 for about 14 methods of pile capacity prediction. Table 2.1 summarizes the statistical parameters used in the reliability assessments conducted in this thesis with regards to approach 1.



Table 2. 1 - Statistics of Model Parameters - Approach 1

Design Parameter	Mean, $\mu$	Coefficient of Variation, $\delta$
Load, S	$\mu_S = 200$ tons	0.15
Mean of Pile Capacity, $r_{\text{mean}}$	FS. $\mu_S$	0.1
Lower-Bound of Pile Capacity, $r_{\text{LB}}$	x.FS. $\mu_S$	0.2
Coefficient of Variation of Pile Capacity, $\delta_r$	0.40	-

Note: FS is the mean factor of safety (ratio of mean capacity to mean load) and x is the ratio of the mean lower-bound capacity to the mean of the mean pile capacity.

### 2.3 Approach 1 - Probability Models

The model parameters to be updated based on proof-load test results are the mean and the lower-bound of the pile capacity at a given site. Given the mathematical complexities that are expected to exist in updating the probability density functions (PDFs) of the lower-bound and the mean of the pile capacity, a decision was made to model the two variables as discrete random variables rather than continuous variables. As a result, the lognormal distributions that model the uncertainties in  $r_{\text{LB}}$  and  $r_{\text{mean}}$  were replaced with probability mass functions (PMFs) that provided a simplified but accurate representation of the variation of the lognormal distribution. This representation is translated into a MATLAB code, and then, three MATLAB files were generated in order to update (1) prior PMFs of the mean of the pile capacity (2) prior PMFs the lower-bound capacity and (3) prior joint PMFs based on proof-load test scenarios. As shown in Table 2.1, the mean value of the pile capacity and the mean lower-bound capacity are both dependent on the mean factor of safety (FS). In addition, trial and error runs have shown that the mean design factor of safety will have a significant effect on determining the updated probability distributions. Thus, for the random variable to be updated, the range of values to be represented by the PMF modeling the random variable is selected based on the input value of FS so that to ensure a mathematically

adequate coverage of the corresponding probability density functions before and after the updating.

For almost all cases, the minimum value in the PMF range is determined as the mean value of the modeled random variable minus 4 standard deviations. Then, when the mean pile capacity is the parameter to be updated (first MATLAB file), the maximum value in the PMF range was chosen to be equal to the minimum value plus about 15 to 24 standard deviations, depending on the used FS. When the lower-bound capacity is the parameter to be updated (second MATLAB file), the maximum value in the PMF range will be equal to the minimum value plus 12 to 20 standard deviations. When the random variable is not the one to be updated, run trials showed that adding 8 standard deviations to the minimum value of the PMF ensures an adequate coverage of the corresponding probability density function.

Once the minimum and maximum values that define the range of the PMF were chosen, the range was divided into 45 equal intervals, resulting in a total of 45 values of  $r_{LB}$  or  $r_{mean}$  in the PMF. This number was chosen using trial and error (1) to ensure that the simplification that is brought by replacing the PDF with a PMF does not compromise the accuracy in modeling the uncertainty in  $r_{LB}$  and  $r_{mean}$  for both the prior and the updated distributions and (2) to minimize the computational effort as much as possible. It should be noted that runs based on a number of divisions that is greater than 45 did not show noticeable changes in the prior and updated reliability. Thus, for the specific mean load considered in this paper (200 tons), a fixed interval width of approximately 20 tons will be used in modeling the PMF for the mean capacity and about 15 tons for the lower-bound capacity.

As an example, required PMF range and resulting interval width for two typical factors of safety (2 and 3) are given in Table 2.2 where:

- $z$  stands for the number of standard deviations to be added to the minimum value in the selected range,
- $\sigma$  stands for the standard deviation,  $\sigma = \delta \cdot \mu$ , and
- the interval width  $w$  is calculated as  $w = z \cdot \sigma / 45$ .

Table 2. 2 - Examples of input data for generating PMF using MATLAB when the parameter to update is  
a) the mean of the pile capacity and (b) the lower-bound of the pile capacity

FS	$\sigma$ of $r_{\text{mean}}$ (tons)	$z$	$w$ (tons)
2	40	22	19.6
3	60	15	20

FS	$\sigma$ of $r_{\text{mean}}$ (tons)	$z$	$w$ (tons)
2	40	18	16
3	60	12	16

The values of the PMF range will be those corresponding to the centers of the corresponding  $w$  intervals, and the associated probabilities (probability mass densities, PMD) are calculated as the cumulative probabilities between the boundaries of the corresponding intervals. For example, for the mean pile capacity parameter:

- $\text{mean}_i = ((i-1)w + (i)w) / 2$
- $\text{PMD}(\text{mean}_i) = \Phi((i)w) - \Phi((i-1)w)$ ,  $\Phi(\cdot)$  referring here to the lognormal cumulative distribution function.

## 2.4 Approach 1 - Updating the Parameters of the Capacity Distribution

### *a. General concept of the updating*

In order to investigate the effect of choosing alternative proof-load test programs on the reliability of the pile, a MATLAB code was developed to return the updated PMFs and the probability of failure (or reliability index) based on input proof-load test programs and their results. In the analysis, the proof-load test program parameters that will be changed are:

1. The level of the proof load relative to the design load which is assumed as the mean load in this paper, and defined in the MATLAB code as  $proof = a \cdot DL$  where  $a$  will be taking values of 1, 1.5, 1.75, 2, 2.5, and 3 and  $DL$  is the design load.
2. The number of proof-load tests,  $n$  which refers to the total number of proof-load tests including the successful and failed tests. The term “successful” indicates that the pile passes the test without failure while the test is considered “failed” when pile failure occurs at the proof load level. In the code, the number of successful tests is referred to as  $k$  and the number of failed tests as  $(n - k)$ .
3. The magnitude of the design factor of safety, FS. In our analysis, we will be concerned with the following magnitudes: 1.75, 2, 2.25, 2.5, 2.75, and 3.

In order to isolate the effect of the lower-bound capacity from the mean capacity and to highlight the importance of the lower-bound capacity on the design of the piles at the site, the updating process for a given proposed load test program will be conducted according to three ways of analysis:

1. The updating process will first be conducted by updating the mean capacity only.
2. The analysis is repeated for the case where the lower-bound capacity is updated only.

3. Finally, the updating will be done for the two parameters simultaneously. In this case, the combined effect of the mean and the lower-bound is studied according to a joint probability model. The prior and updated joint PMFs of the mean and the lower-bound capacities in this case are used to calculate prior and updated marginal PMFs of the two parameters.

Accordingly, three separate MATLAB files will be created; the corresponding algorithms will be based on the Bayesian updating mathematical tool as described in the next section.

### ***b. Bayesian Updating***

When a limited number of proof-load tests are conducted on a small percentage of foundations at a site, Bayes' Theorem (Eq. 2.1) could be used to update the probability distribution of the model parameters for a given set of data such that:

$$f_{\vec{\Phi}|\vec{\varepsilon}}(\vec{\Phi}|\vec{\varepsilon}) = \frac{L(\vec{\varepsilon}|\vec{\Phi})f_{\vec{\Phi}}(\vec{\Phi})}{\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} L(\vec{\varepsilon}|\vec{\Phi})f_{\vec{\Phi}}(\vec{\Phi})d\Phi_1 \dots d\Phi_n} \quad (2.1)$$

Where  $f_{\vec{\Phi}|\vec{\varepsilon}}(\vec{\Phi}|\vec{\varepsilon})$  and  $f_{\vec{\Phi}}(\vec{\Phi})$  are the updated (given the new data  $\vec{\varepsilon}$ ) and prior joint distributions of the model parameters,  $\vec{\Phi}$ ,  $L(\vec{\varepsilon}|\vec{\Phi})$  is the likelihood function, and  $\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} L(\vec{\varepsilon}|\vec{\Phi})f_{\vec{\Phi}}(\vec{\Phi})d\Phi_1 \dots d\Phi_n$  is a normalizing constant. The assumption that the prior distributions of the mean and the lower-bound  $r_{LB}$  are modeled using probability mass functions instead of probability density functions facilitates the solution of Equation 2.1.

For illustration purposes, consider the case where the lower-bound is to be updated, if “n” proof-load tests are conducted using a proof-load level  $r_{proof}$ , and if all

the piles are able to withstand the proof load (n in this context refers to the number of positive tests), the prior probability distribution of the lower-bound capacity can be updated such that:

$$P''_{r_{LB}}(r_{LB}) = \frac{\left( \frac{\Phi\left(\frac{\ln r_{proof} - \lambda_R}{\zeta_R}\right) - \Phi\left(\frac{\ln r_{LB} - \lambda_R}{\zeta_R}\right)}{1 - \Phi\left(\frac{\ln r_{LB} - \lambda_R}{\zeta_R}\right)} \right)^n P'_{r_{LB}}(r_{LB})}{\sum_{i=1}^m \left( \frac{\Phi\left(\frac{\ln r_{proof} - \lambda_R}{\zeta_R}\right) - \Phi\left(\frac{\ln r_{LB,i} - \lambda_R}{\zeta_R}\right)}{1 - \Phi\left(\frac{\ln r_{LB,i} - \lambda_R}{\zeta_R}\right)} \right)^n P'_{r_{LB}}(r_{LB,i})} \quad (2.2)$$

where  $P''_{r_{LB}}(r_{LB})$  and  $P'_{r_{LB}}(r_{LB})$  are the prior and updated lower-bound probability mass functions respectively and  $\lambda_R$  and  $\zeta_R$  are the parameters of the lognormal distribution which are calculated as a function of the mean and coefficient of variation of the resistance. The updated distribution of the lower-bound capacity is then used to calculate an updated estimate of the reliability of the foundations at the site. It should be noted that Equation (2.2) is only illustrative since it assumes that  $r_{mean}$  is deterministic and  $r_{LB}$  is the random parameter that is being updated. In reality,  $r_{mean}$  in this thesis is assumed to be also a random parameter that follows a given PMF. As a result, Equation 2.2 needs to be amended to take that into consideration by adding the contribution of all possible values of  $r_{LB}$  (in the likelihood function and in the normalizing constant) and weighing them by their respective probabilities (evaluated from the prior PMF of  $r_{mean}$ ). The same principal is used to update  $r_{mean}$  instead of  $r_{LB}$  and in updating  $r_{mean}$  and  $r_{LB}$  together.

In MATLAB, a double loop operation system was built in order to account for the contribution of all the possible values of both the mean and the lower-bound capacity to the likelihood and the normalizing constant. For the case of updating the lower-bound, the likelihood, defined as the probability of getting a certain proof-load test result  $\varepsilon$  given a certain lower-bound value  $j$  will be calculated in the inner loop as:

$$\bullet \quad p(\varepsilon/LB_j) = \sum_{i=1}^{i=45} p(\varepsilon / LB_j / mean_i) \times p(mean_i) \quad (2.3)$$

Note that, for each modeled random parameter, the prior and updated PMFs will be associated with 45 specific values of the parameter. The normalizing constant will be then calculated in the outer loop as:

$$\bullet \quad k = \sum_{j=1}^{j=45} p(\varepsilon / LB_j) \times p(LB_j) \quad (2.4)$$

- For the joint probability problem, the prior joint probabilities or PMDs are defined as  $p(mean_i, LB_j) = p(mean_i) \times p(LB_j)$  then we will be updating them using Bayes' theorem by weighing with probabilities of all possible values of mean and lower-bound.

### ***c. Formulation of the Reliability Problem***

For the case where a truncated lognormal distribution is used to model the capacity,  $r$ , and a conventional lognormal capacity is used to model the load,  $s$ , the probability of failure  $p_f$  could be calculated as:

$$p_f = \int_0^{\infty} \left( \frac{\Phi\left(\frac{\ln s - \lambda_R}{\zeta_R}\right) - \Phi\left(\frac{\ln r_{LB} - \lambda_R}{\zeta_R}\right)}{1 - \Phi\left(\frac{\ln r_{LB} - \lambda_R}{\zeta_R}\right)} \right) \left( \phi\left(\frac{\ln s - \lambda_S}{\zeta_S}\right) \frac{1}{s} \right) ds = \Phi(-\beta) \quad (2.5)$$

where  $\Phi()$  is the standard normal cumulative distribution function,  $\phi()$  is the standard normal probability density function, and  $\beta$  is the reliability index. The probability of failure in Equation (2.3) is for one combination of  $r_{LB}$  and  $r_{mean}$  and is calculated using numerical integration. For the case where  $r_{LB}$  and  $r_{mean}$  are random model parameters, the total probability of failure will be obtained using the theorem of total probability by incorporating all the probability of failure for all combinations of  $r_{LB}$  and  $r_{mean}$ .

## **2.5 Approach 2 - Prior Statistics of the Pile Capacity**

In approach2, the uncertainty in the pile capacity is modeled by a conventional lognormal distribution (not a truncated distribution) with (1) a deterministic coefficient of variation that represents the uncertainty due to spatial variability in pile capacity in a given site and (2) an uncertain mean capacity that incorporates the model uncertainty of the pile capacity prediction method. This model for the uncertainty of pile capacity was adopted by Zhang (2004) and is based on the principal of isolating the model uncertainty (reflected in the mean resistance) from the uncertainty in the pile capacity due to spatial variability in a given site (reflected in the coefficient of variation of the pile capacity at the site). The model adopted by Zhang (2004) does not include the lower-bound capacity in its formulation.

Since the concept of the lower-bound capacity as presented in Gilbert et al. (2005) and Najjar and Gilbert (2009) is targeted primarily at reducing the uncertainty in the pile capacity predictions of available models, the lower-bound capacity in approach 2 in this thesis will be incorporated in the distribution of the mean capacity ( $r_{mean}$ ) by truncating the left-hand tail of the distribution of the mean at the assumed lower-bound capacity. Similar to approach 1, the lower-bound capacity ( $r_{LB}$ ) in approach 2 will be



assumed to be a random variable to account for uncertainties in the models used to predict the lower bound and to account for the effect of spatial variability on the predicted lower-bound capacity.

As a result, both the mean capacity ( $r_{\text{mean}}$ ) and the lower-bound capacity ( $r_{\text{LB}}$ ) are assumed to be random variables (model parameters) following a lognormal distribution. The coefficient of variation of the mean of the pile capacity  $r_{\text{mean}}$  will be assumed to be equal to 0.4 while the coefficient of variation of the lower-bound capacity ( $r_{\text{LB}}$ ) is assumed to be equal to 0.2. What remains is the coefficient of variation of the distribution of the pile capacity ( $\text{cov}_r$ ), which reflects the uncertainty due to spatial variability of the pile capacity in a given site. Based on data presented in Zhang and Tang (2002) from pile load tests that were conducted in the same site (done for several sites), it could be shown that the coefficient of variation of pile capacities in a given site is expected to be between 0.1 and 0.2, with the upper bound being a more realistic and conservative estimate of the uncertainty. In this thesis, the base case that will be investigated will involve a ( $\text{cov}_r$ ) of 0.2. The sensitivity of the results to the assumed value of ( $\text{cov}_r$ ) will be also be investigated. Table 2.3 summarizes the prior statistics corresponding to approach 2.

Table 2. 3 - Statistics of Model Parameters - Approach 2

<b>Design Parameter</b>	<b>Mean, <math>\mu</math></b>	<b>Coefficient of Variation, <math>\delta</math></b>
Load, S	$\mu_S = 200$ tons	0.15
Mean of Pile Capacity, $r_{\text{mean}}$	FS. $\mu_S$	0.4
Lower-Bound of Pile Capacity, $r_{\text{LB}}$	x.FS. $\mu_S$	0.2
Coefficient of Variation of Pile Capacity, $\delta_r$	0.2	-

*Note: FS is the mean factor of safety (ratio of mean capacity to mean load) and x is the ratio of the mean lower-bound capacity to the mean of the mean pile capacity.*

A comparison between the probabilistic models adopted in approach 1 and approach 2 for modeling the uncertainty in the pile capacity is presented in Fig. 2.2. The main differences between the two approaches are the following:

1. In approach 1, the model uncertainty in pile capacity prediction models is considered the basis for the uncertainty in the *pile capacity*, whereas in approach 2, the model uncertainty is assumed to be representative of the uncertainty in the *mean pile capacity*.
2. In approach 1, the uncertainty in the pile capacity resulting from spatial variability of soil properties with a given site is assumed to be implicit in the coefficient of variation of the pile capacity, whereas in approach 2, spatial variability is explicitly accounted for by assuming that the pile capacity distribution in a given site is modeled by a coefficient of variation that models the uncertainty due to spatial variability.
3. In approach 1, the lower-bound capacity is used to truncate the *pile capacity* distribution while in approach 2, the lower-bound capacity is used to truncate the tail of the distribution of the *mean pile capacity*. The other sources of uncertainties (those related to the determination of the soil properties, to pile testing procedures and instrumentation, and to the interpretation of the pile capacity from the load-settlement curves) are accounted for through the coefficient of variation of the pile capacity ( $\delta_r = 0.2$ ). Prior statics data of the lower-bound are the same as in the first approach.

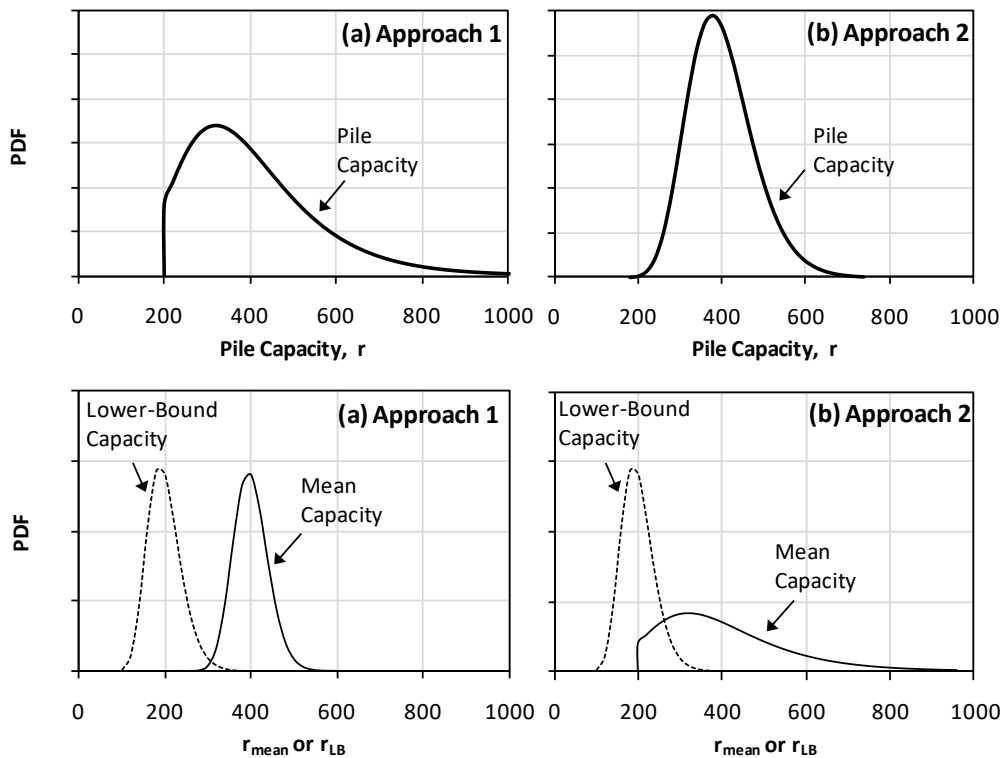


Figure 2. 2 - Probabilistic models of pile capacity for (a) Approach 1, and (b) Approach 2

It should be noted that the probabilistic model for pile capacity as assumed in approach 2 is relatively similar to the model adopted by Zhang et al. (2004), where the mean of the capacity distribution is updated based on proof-load tests results. In one of the example, Zhang et al. (2004) assume that the *mean pile capacity* for the case where the pile capacity is predicted using SPT-based methods (Meyerhof 1976) is modeled by a coefficient of variation of 0.5 while the within-site variability of the *pile capacity* was assumed to be 0.2. As mentioned previously, previous studies (including the work done by Zhang and his colleagues) do not incorporate the lower-bound capacity into the pile capacity mode. The incorporation of the lower-bound capacity in this study represents an essential improvement in the model relative to available models. The current work will assess the effectiveness of each of the two approaches with regards to the results of

the updating exercise to decide on the superiority of one method with regards to the other. The importance of incorporating a lower-bound in such type of reliability problems for both approaches will be emphasized.

## **2.6 Approach 2 - Probability Models**

As in the first approach, the model parameters to be updated based on proof-load test results are the mean and the lower-bound of the pile capacity at a given site using probability mass functions (PMFs). The same code will be used for establishing the new MATLAB algorithms for approach 2. Three new MATLAB files were established to conduct the reliability calculations according to the new approach for the purpose of updating (1) the prior PMFs of the mean of the pile capacity (2) the prior PMFs of the lower-bound capacity and (3) prior joint PMFs based on proof-load test scenarios. Several runs were done to determine the number of standard deviations to be added to the first value in the range of the values of the random parameter to be updated. As stated before, these numbers will be dependent on the used factor of safety. Table 2.4 shows the number of standard deviations to be added to the first value in the PMF range when updating the mean, then the lower-bound for factors of safety of 2 and 3. In terms of the number of divisions in the range of the parameter to be updated, 90 divisions ( $n_1=90$ ) will be assigned to the distribution of the mean capacity and 45 divisions ( $n_2=45$ ) for the distribution of the lower-bound. Thus, for the specific mean load considered in this paper (200 tons), a fixed interval width of approximately 20 tons will be adopted for the mean capacity and about 10-15 tons for the lower-bound capacity, given that these numbers satisfy both the accuracy in the prior and updated reliabilities and the adequacy in modeling uncertainties in the lognormal distributions.

Table 2. 4 - Examples of input data for generating PMFs using MATLAB when the parameter to be updated is (a) the mean of the pile capacity and (b) the lower-bound of the pile capacity

FS	$\sigma$ of $r_{\text{mean}}$ (tons)	z	w (tons)
2	160	12	21
3	240	8	21

FS	$\sigma$ of $r_{\text{mean}}$ (tons)	z	w (tons)
2	40	10	9
3	60	10	13

## 2.7 Approach 2 - The Updating Process

As previously stated, the updating of the pile capacity distribution will be using Bayesian technique, where the formulation of the updating exercise is based on Equation 2.1. The main difference between approaches 1 and 2 is the fact that the lower-bound in approach 1 is used to truncate the pile capacity distribution, whereas in approach 2, the lower bound is used to truncate the distribution of the mean of the pile capacity. This difference affects the formulation and solution of the Bayesian updating process, as will be reported in the following sections. In approach 1, the likelihood of observing a set of test results is calculated using the distribution of pile capacity which is truncated at the lower-bound capacity, whereas in approach 2, the likelihood is calculated from the non-truncated capacity distribution which does not explicitly include the effect of the lower-bound capacity. The likelihood is indirectly affected by the presence of the lower-bound capacity which only affects the distribution of the mean pile capacity.

For the case of updating the *mean* of the pile capacity, prior and updated PMFs will be associated with 90 specific values of the model parameter. Since the lower-bound capacity is used to truncate the distribution of the mean pile capacity in approach 2, the two parameters are considered to be statistically correlated and not independent.

For each possible value of the lower-bound capacity in the PMF of the lower bound, the probability mass function of the mean pile capacity will be conditional on the lower bound. Thus, in order to get the prior probability of each mean capacity value ( $i$ ), we have to account for all the possible values of the lower-bound according to:

- $$\text{Prior probability } (mean_i) = \sum_{j=1}^{j=45} p(mean_i/LB_j) \times p(LB_j) \quad (2.6)$$

Where

- $$p(mean_i/LB_j) = \frac{\phi((i)w) - \phi((i-1)w)}{1 - \phi(LB_j)}$$

Then, in order to get the updated PMDs of the mean, the likelihood and the normalizing constant are calculated so that all the mean and lower-bound values are taken into consideration through a double loop operation system.

For the case of updating the lower-bound, the effect of all the values of the mean on a certain lower-bound value will be taken into account in the likelihood function. In fact, probabilities of failure and success of a pile load test scenario depend on the statistical characteristics of the pile capacity distribution, and thus on the value of the mean. Accordingly, the likelihood expression for updating a determinate value of the lower-bound will take into consideration all the possible values of the mean by weighing with all their corresponding probabilities.

## CHAPTER 3

### RESULTS AND ANALYSIS – APPROACH 1

#### 3.1 Introduction

Based on the mathematical formulation devised in the previous chapter, approach one for modeling the pile capacity distribution will be utilized in this chapter, together with a Bayesian updating scheme, to update the prior pile capacity distribution based on results from proof load testing programs. The target is to investigate the effect of choosing alternative proof-load test programs on the reliability of the pile design and the required factors of safety. The parameters that will be changed in the analysis are:

- (1) the level of the proof load,  $r_{\text{proof}}$  (relative to the design load which is assumed as the mean load in this thesis,  $\mu_S = 200$  tons),
- (2) the number of proof-load tests, and
- (3) the magnitude of the design factor of safety.

To isolate the effect of the lower-bound capacity from the mean capacity, the updating process for a given proposed load test program will first be conducted by updating the mean capacity only. The analysis is repeated for the case where the lower-bound capacity is updated only. Finally, the updating will be done for the two parameters simultaneously.

Given the properties of the statistical parameters shown in Table 2.1 in the previous chapter, it could be shown that the required mean factor of safety would have to be around 3.0 to achieve atypical target reliability index of 3.0 for the piles at the site.

If proof load tests are to be conducted on a limited number of piles at the site, the required mean factor of safety could be reduced provided that the majority of the tests are successful.

### **3.2 Illustration of the Updating Process**

To illustrate the updating process utilized in approach 1, it is assumed that 15 statistically independent proof-load tests of up to 2 times the design load are conducted on 15 piles that are designed and constructed at a reduced mean factor of safety of 2. If the tests were successful, the results of the load test program could be used to update the capacity distribution of the piles at the site. This is illustrated in Figs. 3.1a, b, and c where the results of the testing program are used to update the probability mass functions of the mean capacity alone, the lower-bound capacity alone, and the joint PMF of the mean and the lower-bound capacity, respectively.

Results in Fig. 3.1 indicate that the impact of the successful proof load tests is to shift the distributions of both the mean capacity and the lower-bound capacity to the right. In other words, the probabilities of relatively low values of the mean and lower-bound capacities decrease, while the probabilities of the higher values increase as a result of the updating process. The shifting of the mean and the lower-bound capacity to the right is expected to be translated into improvements in the reliability index and reductions in the probabilities of failure of the piles at the site, thus allowing for the utilization of lower factors of safety for a given level of reliability.

Further analysis of the data on Fig. 3.1 indicates that when the updating process is conducted on the joint PMF of  $r_{\text{mean}}$  and  $r_{\text{LB}}$ , the major thrust of the updating process is on updating the lower-bound capacity rather than the mean. This observation could be explained by two facts. First, the uncertainty in the prior distribution of  $r_{\text{LB}}$



( $\delta_{LB}=0.2$ ) is larger than the uncertainty in the prior distribution of  $r_{mean}$  ( $\delta_{mean}=0.1$ ). This makes the lower-bound capacity a more favorable parameter for updating. Second, the likelihood function is expected to be more sensitive to changes in the lower-bound capacity (clearly illustrated in Fig. 2.1) than the mean capacity, particularly for values of  $r_{LB}$  that exceed 0.4 to 0.5 of the mean capacity, as is the case in this problem.

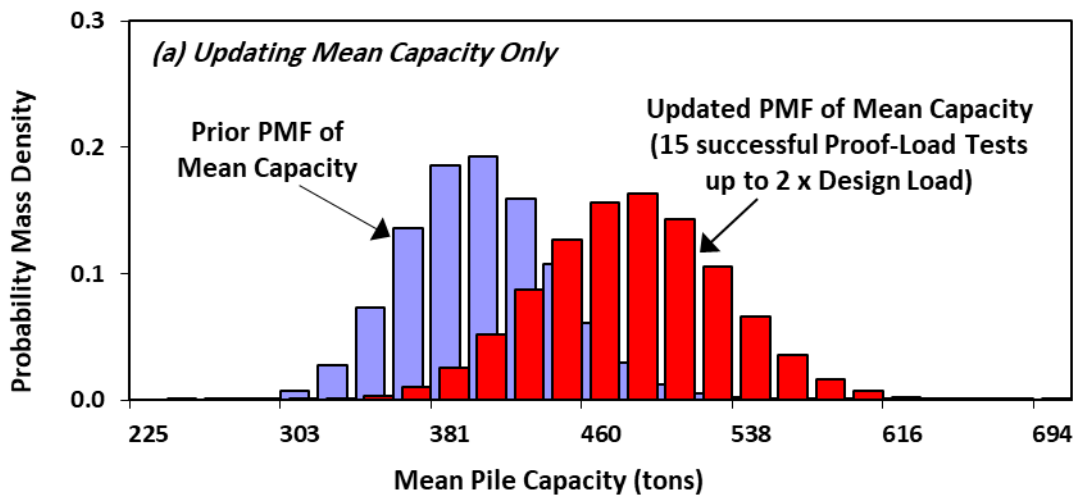


Figure 3.1. a - Updating the Probability Mass Functions of  $r_{mean}$  and  $r_{LB}$  (15 proof load tests,  $r_{proof} = 2 \times \text{Design Load}$ ,  $FS_{mean} = 2.0$ ) – Updating Mean Capacity only

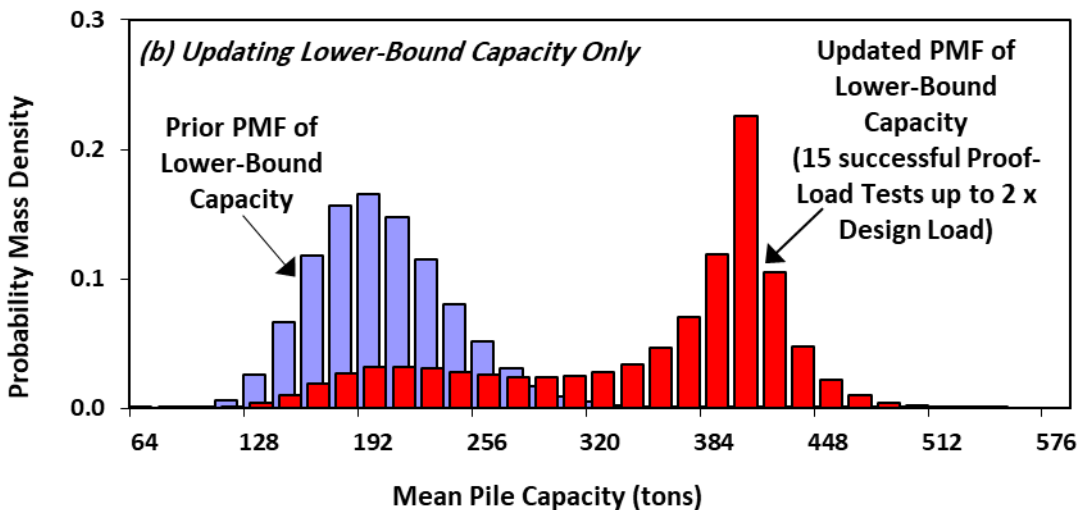


Figure 3.1. b - Updating the Probability Mass Functions of  $r_{mean}$  and  $r_{LB}$  (15 proof load tests,  $r_{proof} = 2 \times \text{Design Load}$ ,  $FS_{mean} = 2.0$ ) – Updating Lower-Bound Capacity only

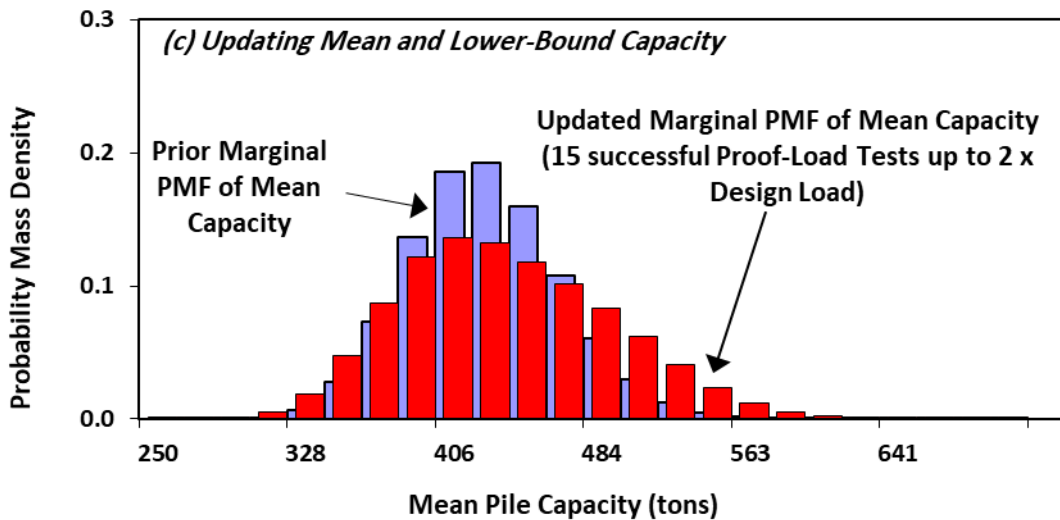
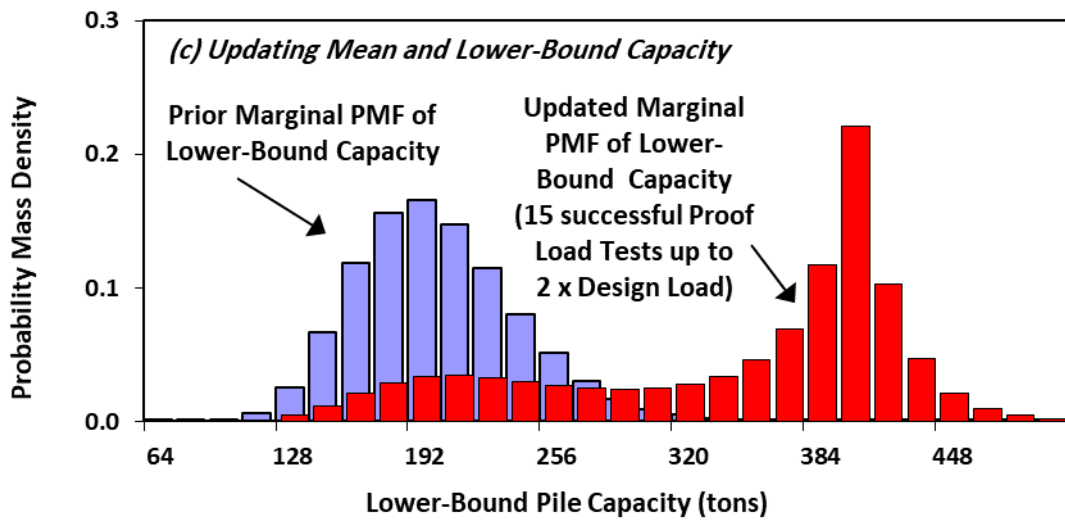


Figure 3.1. c - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $FS_{\text{mean}} = 2.0$ ) – Updating Mean and Lower-Bound Capacity

For the case considered in Fig. 3.1, the mean design factor of safety was assumed to be equal to 2.0. For the prior scenario (assuming no load tests are conducted), this relatively low factor of safety results in a relatively small and virtually unacceptable reliability index that is slightly less than 1.9. When 15 successful proof load tests with  $r_{\text{proof}}$  equal to twice the design load are conducted, the distribution of pile capacity at the site is updated through the PMFs of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  as indicated in Fig. 3.1. The positive

effect of the updating process is reflected in improved values of the reliability index as indicated in Fig. 3.2.

### **3.3 Updated Pile Reliability for Different Proof-Load Test Scenarios for $FS_{\text{mean}} = 2.0$**

For the specific case of the 15 proof load tests that are conducted to twice the design load and assuming a factor of safety of 2.0, results on Fig. 3.2 indicate that the reliability index increases from its prior value of 1.9 to values of about 2.2, 2.56, and 2.86 for cases where the mean capacity is updated alone, the lower-bound capacity is updated alone, and both the mean and the lower-bound capacity are updated together, respectively.

Results on Fig. 3.2 indicate that the effect of almost all the proof-load test programs is to increase the reliability compared to the case where no proof-load tests are conducted. As expected, the reliability index generally increases as the number of proof-load tests increases and as the proof-load level increases. Results on Fig. 3.2a indicate that utilizing the results of the proof-load tests to update  $r_{\text{mean}}$ , results in relatively small increases in the reliability index. For example, the reliability index increases from around 1.9 (for the case where no proof tests are conducted) to a maximum of about 3.0 (2.95) for the case where 30 tests are conducted to up to 3 times the design load. On the other hand, results on Fig. 3.2b indicate that updating the lower-bound capacity results in significant increases in the reliability index, with maximum values exceeding 6 for the largest number of tests and the highest proof-load levels.

These results are significant because they indicate that for the probabilistic model of the pile capacity that was adopted in approach 1, the results of a proof-load testing program could be more efficient at updating the lower-bound capacity than the

mean capacity. The results on Fig. 3.2c where the proof-load tests were used to update the joint PMF of the mean and lower-bound capacity confirm this observation since the updated marginal PMFs indicate that the lower-bound capacity governed the reliability index since it was the most affected by the updating process compared to the mean capacity.

A general comparison between the results on Figs. 3.2b and 3.2c indicates that updating  $r_{\text{mean}}$  and  $r_{\text{LB}}$  together (Fig. 3.2c) generally results in slightly higher values of the reliability index compared to the case where only  $r_{\text{LB}}$  is updated. However, this observation is reversed for the few cases where the calculated reliability index was very large (generally greater than 4.0), where higher reliability indices were calculated for the case where only  $r_{\text{LB}}$  was updated. From a physical standpoint, this observation might not be logical and is expected to be attributed to inaccuracies in the numerical computations and assumptions which could only be evident at such small values of the probability of failure and which are not expected to be relevant at typical target risk levels for foundation design (target reliability indices  $\sim 3.0$ ).

### **3.4 Factor of Safety vs Reliability for Different Test Programs**

Since the main objective of this thesis is to study the effect of choosing different proof-load test programs on the required factor of safety for piles, the target factor of safety needed to achieve target reliability indices of 2.5, 3.0, and 3.5 for the different proof-load testing programs considered in this study was calculated and plotted in Figs. 3.3a, 3.3b, and 3.3c, respectively. The results in Fig. 3.3 show that different combinations of factor of safety, proof-load level, and number of proof-load tests could be selected to achieve the desired level of reliability.

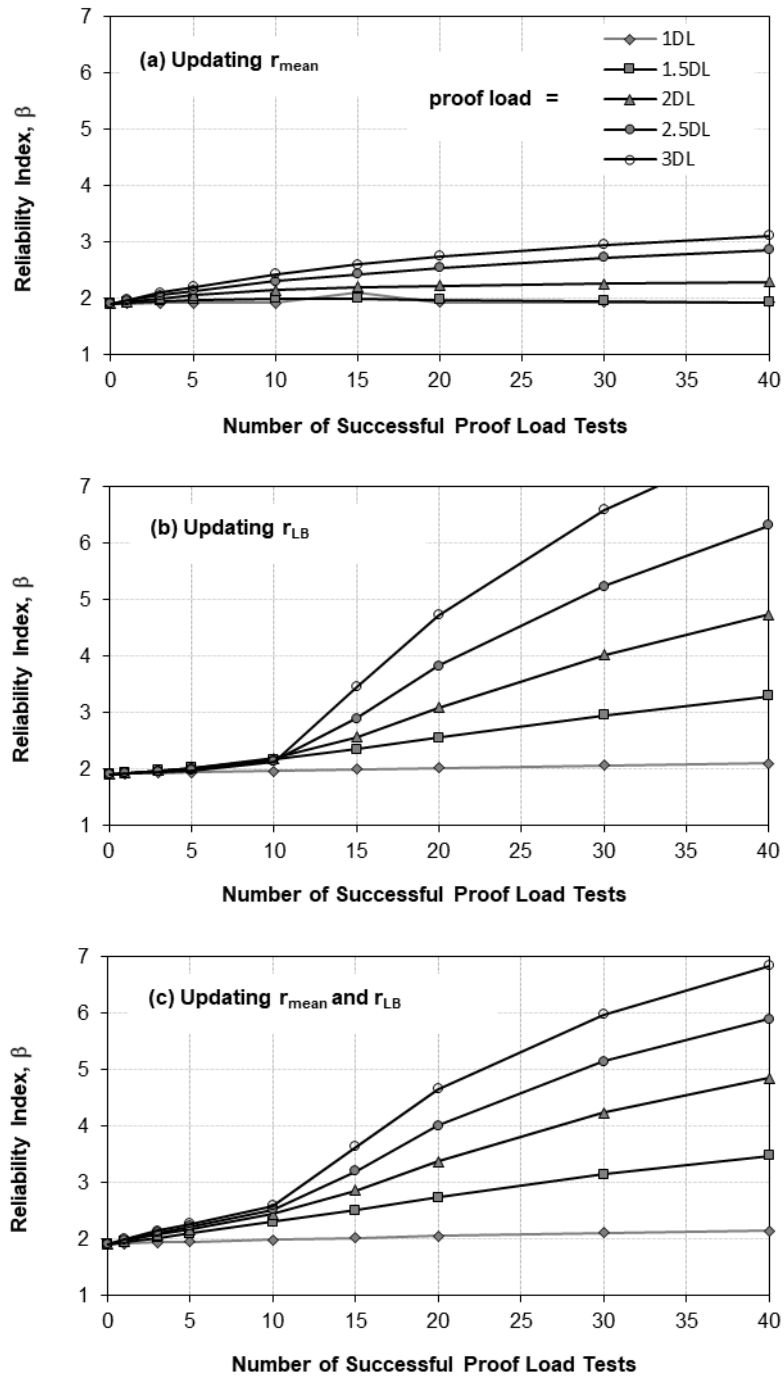


Figure 3. 2 - Effect of Load Test Programs on the Reliability of Pile Design ( $FS_{mean} = 2$ )

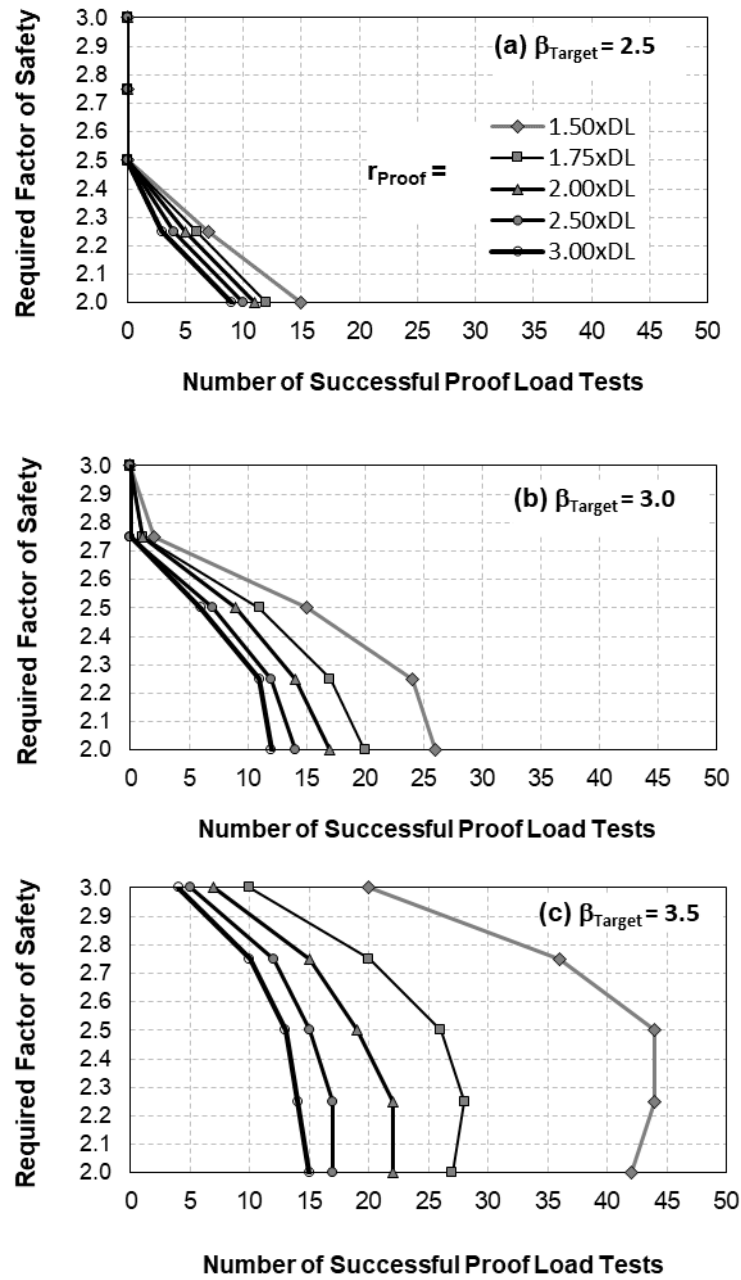


Figure 3.3 - Required Factor of Safety to Achieve a Target Reliability Level of  $\beta = 2.0, 3.0,$  and  $3.5$  for Different Load Testing Programs

For cases involving foundation systems that are redundant (example, large pile groups), it has been shown that the added redundancy allows for reducing the target reliability index of the individual foundation without compromising the reliability of the

foundation system. For a reduced reliability index of about 2.5, results on Fig. 3.3a indicate that no load tests are required to achieve the target reliability index if mean factors of safety that are greater than 2.5 are adopted. However, further reduction in the required factor of safety could be achieved with proof-load testing. For example, the factor of safety could be reduced to 2.0 by running 9 tests up to 3 times the design load, or 15 tests up to 1.5 times the design load.

For cases where the desired level of reliability is required to be higher than the typical acceptable reliability levels (example, sensitive structures, heavily loaded foundations with no redundancy, etc.), reliability indices that are in excess of 3.0 may be desired. Results on Fig. 3.3c indicate that if a target reliability level of 3.5 is desired, the required number of proof-load tests and the level of the proof loads will need to be higher compared to the previous cases where the reliability index was lower. As an example, one possible design scenario could involve the use of a factor of safety of 3.0. To achieve the desired reliability level with this design scenario, the designer has the option of using a test program consisting of 4 load tests up to 3 times the design load or 20 load tests up to 1.5 times the design load. Another design scenario could consist of using a reduced factor of safety of 2.0. In this scenario, the designer could choose a program consisting of 15 tests conducted up to 3 times the design load or 43 tests conducted up to 1.5 times the design load. Other combinations of design scenarios and load testing programs could also be selected to achieve the same reliability level.

### **3.5 Effect of Failures on the Updated Reliability**

In all the results and observations presented in the previous sections of this chapter, it was assumed that all the tested piles survived the proof-load tests. In reality,

a proof load testing program could witness a number of foundation failures during its implementation. The impact of these failures could be incorporated in the updating methodology presented in this thesis by modifying the likelihood function to reflect both survivals and failures. When a number of piles fail during a proof-load test program, the updated probability of failure is expected to increase compared to the case where all the piles survive the proof tests. With a large percentage of failed piles, the updated distributions of the mean pile capacity and lower-bound capacity could shift to the left, resulting in updated probabilities of failure that are even greater than the prior probability of failure (Zhang, 2004).

To study the impact of pile failures on the updated capacity distribution and resulting reliability index, the case is considered where 10 pile load tests are conducted resulting in either 0, 3, or 8 failures respectively. In this illustrative analysis, the proof load level is taken as 2 times the design load and the design factor of safety is considered to be 2.0. Results of the updating process indicate that the updated reliability index increases from 1.91 to 2.44 when all the tests are successful. When 3 out of 10 piles fail during the tests, the reliability index still increases compared to the prior value but only slightly, with a computed  $\beta$  of 2.11. When 8 tests are assumed to fail, results of the updating process indicate that  $\beta$  decreases to a low value of 1.74 (smaller than the prior  $\beta$ ) due to the effect of the pile failures.

To shed light on the mechanism behind the impact of failed tests on the reliability, the prior and updated marginal PMFs for the mean and the lower-bound capacity for the cases of 0, 3, and 8 failures are plotted on Figs. 3.4a, b, and c. Results on Fig. 3.4 indicate that the updated distributions for both the mean and the lower bound capacity shift to the left as the number of failures increase. For the case with 8 failures,



the updated PMFs of the mean and the lower bound shift to the left excessively, making the updated distribution fall to the left of the prior distribution. This explains the drop in the reliability index to values that are smaller than the prior in this particular case.

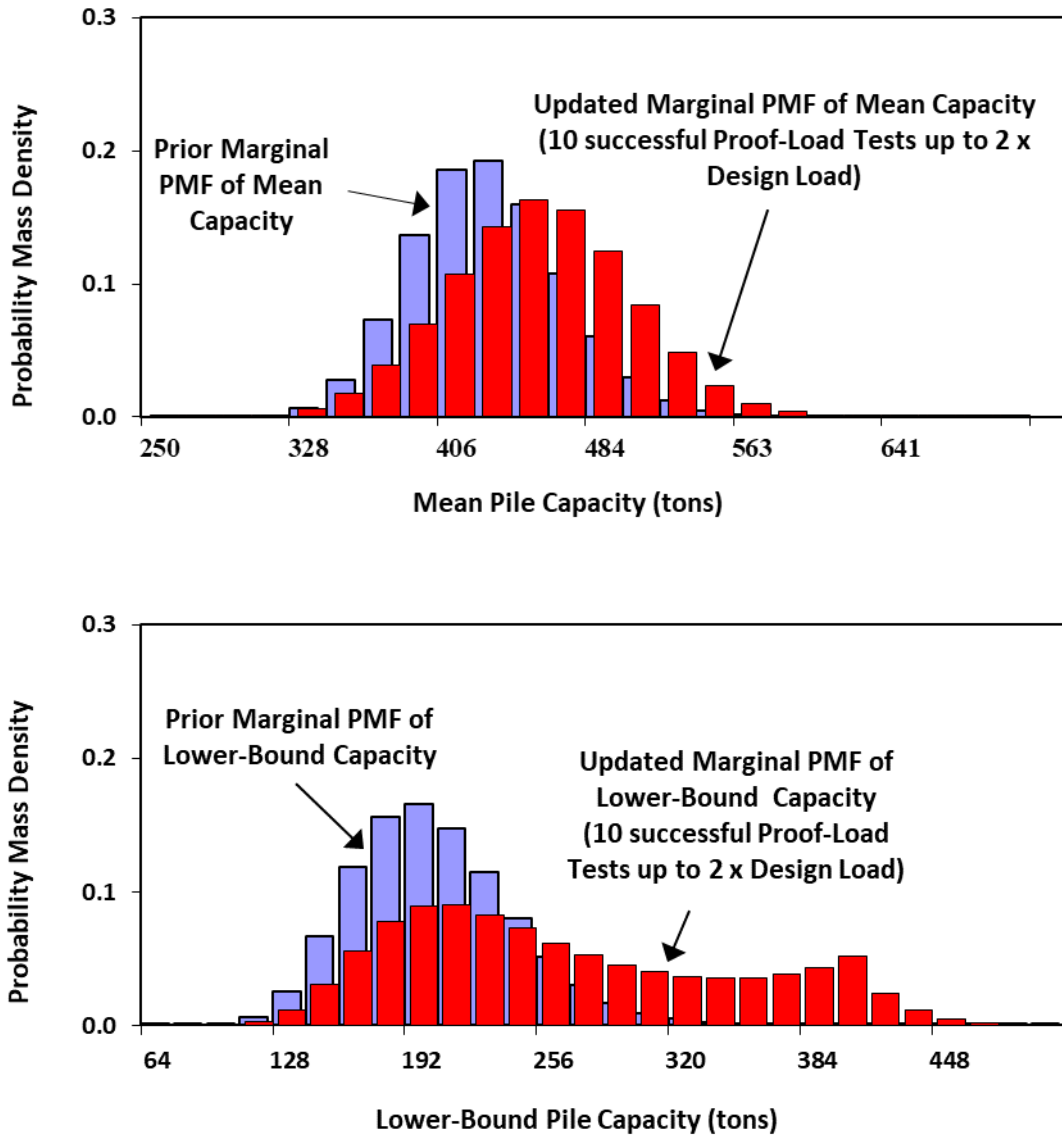


Figure 3.4. a - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  for 10 positive tests ( $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ )

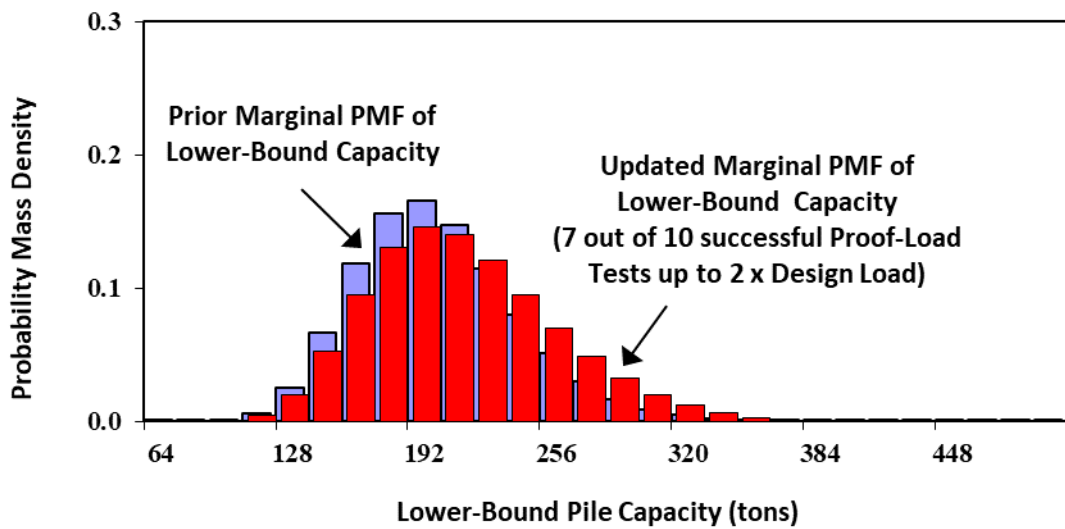
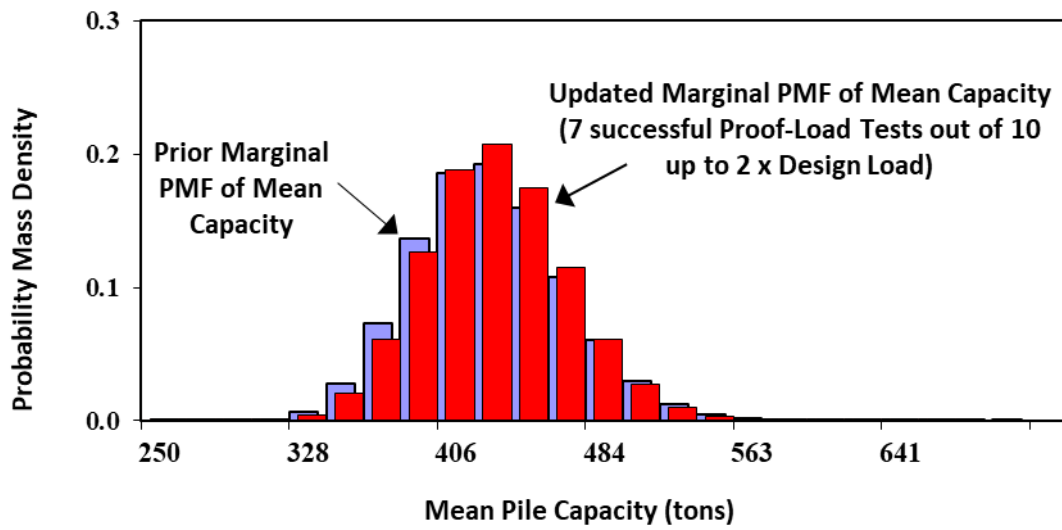


Figure 3.4. b - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  for 10 tests with 3 failures ( $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ )

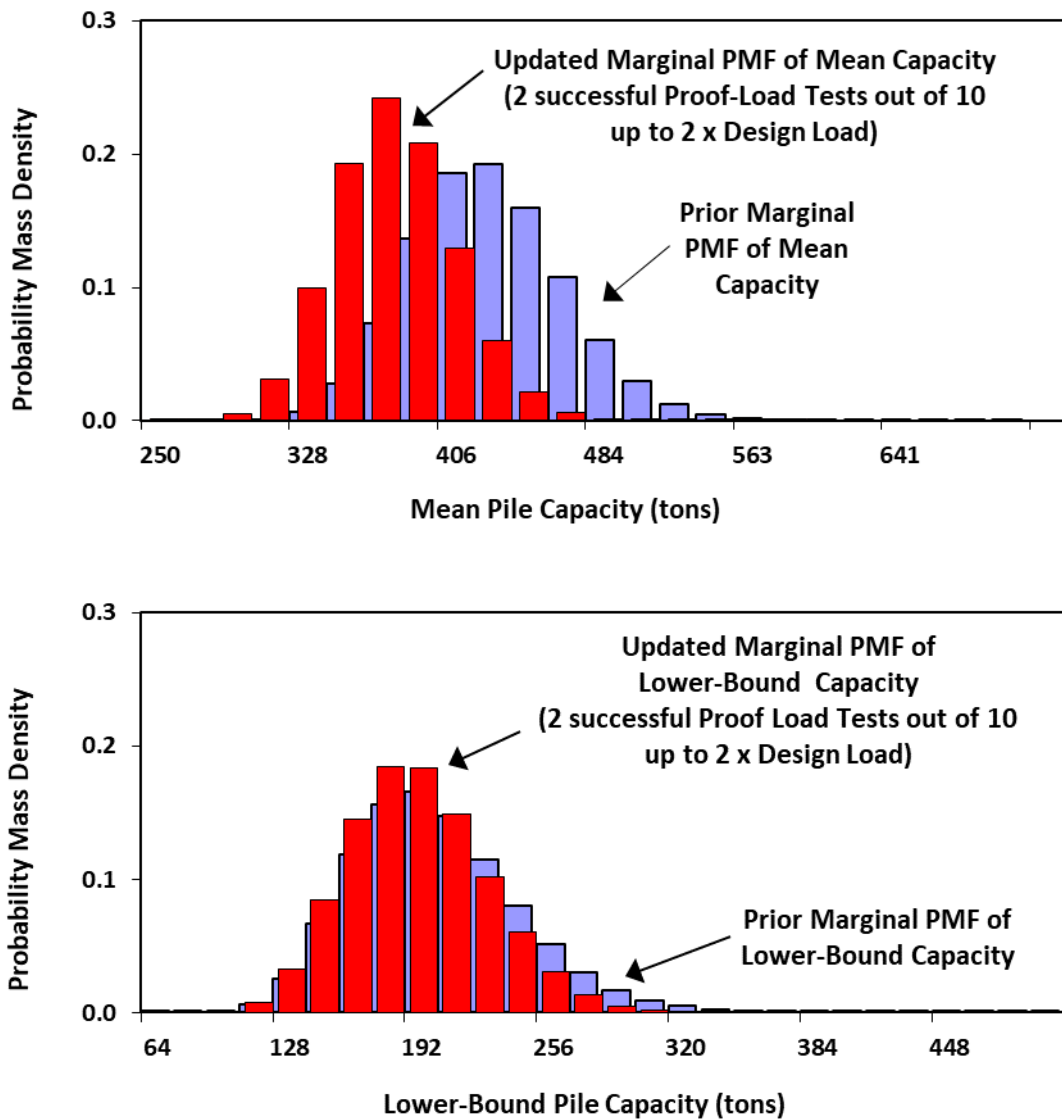


Figure 3.4. c - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  for 10 tests with 8 failures ( $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ )

The results on Fig. 3.4 illustrate the impact of failures on the updated capacity distributions for a particular case. To investigate the impact of different failure scenarios on the updated reliability of the pile design, several design scenarios (as reflected in the assumed mean factor of safety), several proof-load testing programs (as reflected in the number of proof-load tests), and several alternatives for the results of the proof-load tests (as reflected in the number of failed piles) were considered. The mean factor of

safety was varied from 2.0 to 3.0 and the updated reliability indices for test programs involving 5, 10, 20, and 30 proof-load tests that are conducted up to twice the design load was calculated. For each proof-load test program considered, the analysis was conducted for the cases where no failures occurred and for 5 other cases whereby a certain percentage of the test piles was assumed to have failed. The reliability indices that are associated with these cases are presented in Fig. 3.5 together with the reliability indices of the base case whereby no test program is implemented at the test site. A thorough analysis of the results on Fig. 3.5 leads to several interesting observations:

- as expected, for a given design scenario and a given proof-load test program, the updated reliability index was found to decrease as the percentage of failed piles increase,
- the magnitude of the relative decrease in the updated reliability index seems to decrease as the number of failed piles increases,
- the design scenarios that involve relatively large factors of safety generally suffer the most from the negative impact of the pile failures, and
- the percentage of failed piles that seem to result in updated reliability index that is almost equal to the prior reliability index (i.e., the proof-load test program becomes inefficient) seems to be in the range of 30 to 40% of the tested piles.

The above observations are significant in that they shed light on the impact of failures of proof-load tested piles on the updated reliability of the pile design. In the design phase of a project, and before the proof-load testing program is established, a designer has to consider all the possible scenarios that could occur with regards to the possible results of the proof load test program. The likelihood of occurrence of each possible test result could be evaluated using the prior distribution of the pile capacity at

the site. These likelihoods could be combined with the calculated updated reliability indices for the different test scenarios and utilized within a decision making framework at the design stage of the project to establish the load test program that would maximize the value of information of the test program.

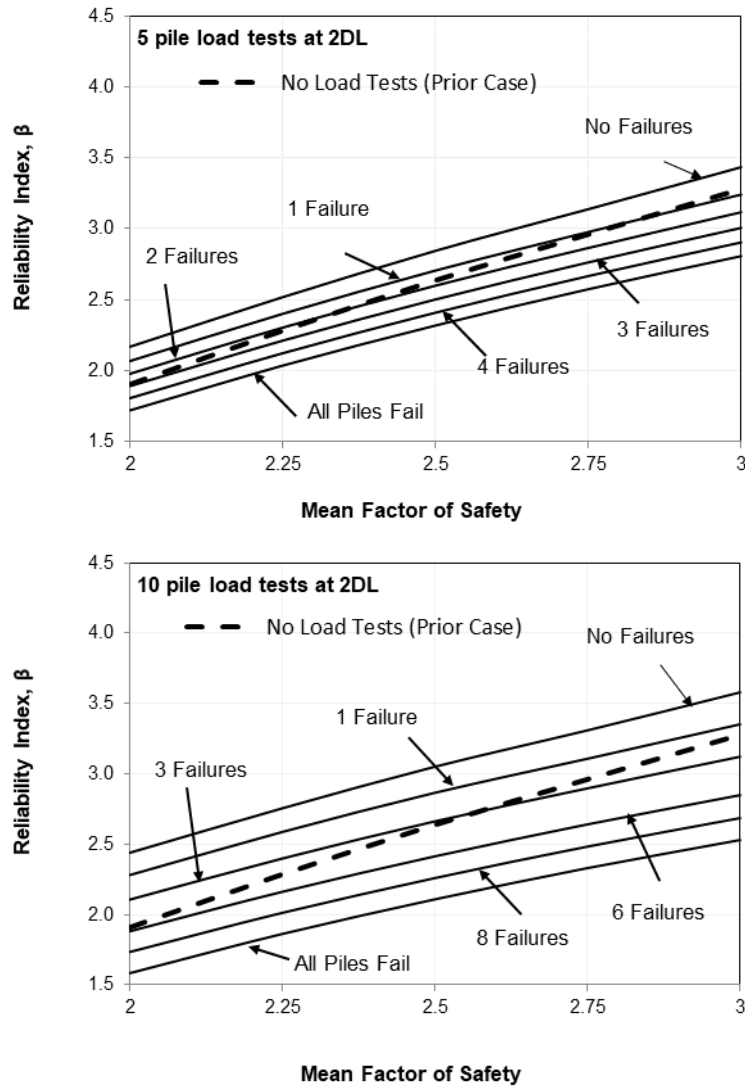


Figure 3. 5 - Effect of Pile Failures on the Updated Reliability Index

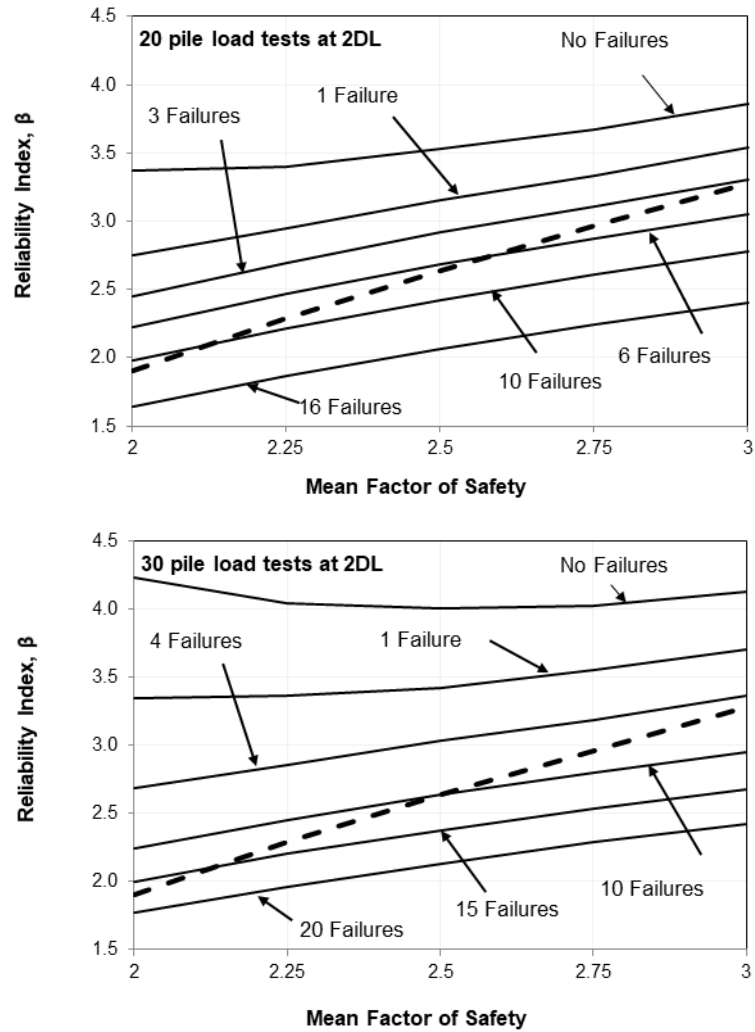


Figure 3.5 (continued) - Effect of Pile Failures on the Updated Reliability Index

### 3.6 Effect of the lower-bound to the mean capacity ratio

Since Approach 1 for updating the pile capacity distribution has been shown to affect the lower-bound capacity distribution significantly, it is necessary that the sensitivity of the results to the assumptions made in defining the prior lower-bound capacity distribution be studied. In particular, the sensitivity of the results to the assumed value of the ratio “x” of the mean of  $r_{LB}$  to the mean of  $r_{mean}$  is of importance. The results reported in the previous sections of this chapter are based on an “x” value of 0.5, which is a realistic estimate of the mean of the ratio of  $r_{LB}$  to  $r_{mean}$  as reported in the

published literature (ex. Gilbert et al. 2005 and Najjar and Gilbert 2009b). The effect of the ratio ( $x$ ) of the mean lower-bound to the mean pile capacity on the prior and updated reliability indices is illustrated on Fig. 3.6, where the ratio “ $x$ ” was taken as either 0.4, 0.5, or 0.6 for comparison. Results are shown for realistic mean factors of safety of 2.0 and 2.5 and for the common case where  $r_{proof}$  is taken as 2.0 times the design load. The curves on Fig. 3.6 show the variation of the reliability index with the number of pile load tests, which were assumed to be all successful in this sensitivity analysis.

Results of Fig.3.6 show that, prior to any updating, a higher value of the ratio “ $x$ ” results in a higher prior reliability index. This is expected since the probability of failure and the reliability index are governed by the left-hand tail of the capacity distribution (Najjar and Gilbert 2009), so a larger value for the lower-bound capacity will truncate the left hand tail of the capacity distribution closer to the mean, resulting in a lower probability of failure and thus a higher reliability index. Fig. 3.7, which is published in Najjar and Gilbert illustrates how the probability of failure of a pile is affected by the presence of a lower-bound capacity for different factors of safety. The prior reliability indices corresponding to the three values of “ $x$ ” are presented in Table 3.1. For  $FS_{mean} = 2$ , the prior  $b$  increases from 1.65 to 2.24 as “ $x$ ” is increased from 0.4 to 0.5. The equivalent increase in  $b$  for the case of  $FS_{mean} = 2.5$  is from 2.29 to 3.03.

Table 3. 1 - Prior Reliability Indices for Different Lower-Bound to Mean Capacity ratios

$x$	Prior $\beta$ for $FS_{mean} = 2$ (Fig. 3.6)	Prior $\beta$ for $FS_{mean} = 2.5$ (Fig. 3.6)
0.4	1.65	2.29
0.5	1.91	2.64
0.6	2.24	3.03

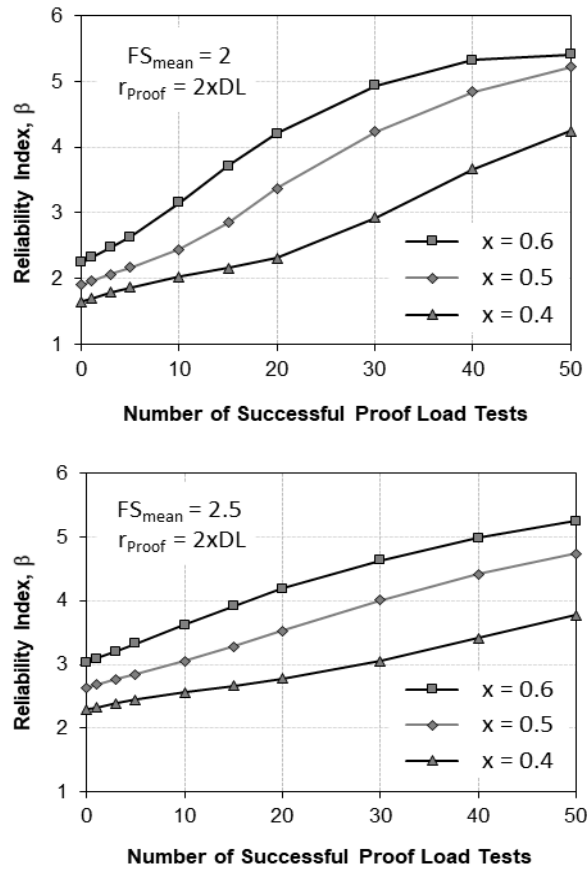


Figure 3. 6 - Effect of the Lower-bound to the Mean Capacity Ratio for  $FS = 2$  and  $FS = 2.5$

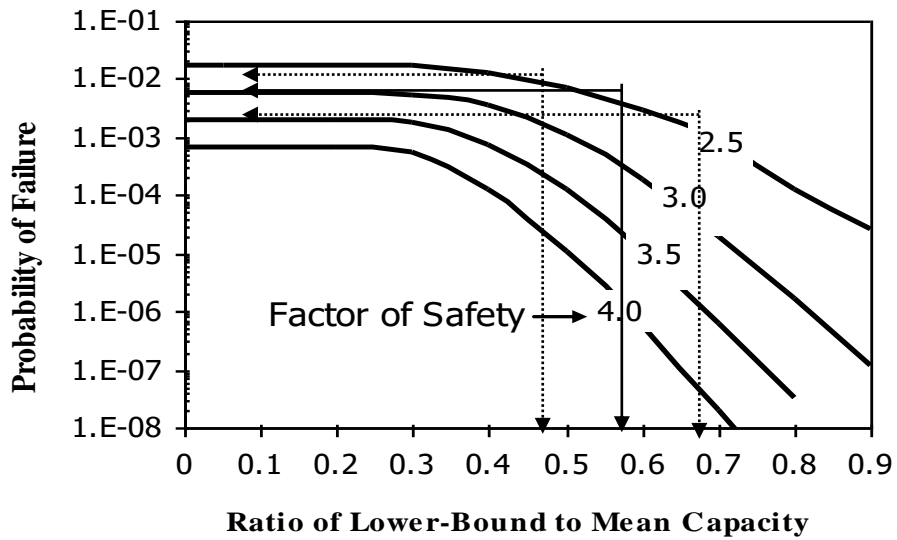


Figure 3. 7 - Effect of the Lower-bound to the Mean Capacity Ratio on the probability of failure (Najjar & Gilbert 2009)



The results on Fig. 3.6 indicate the ratio of the mean  $r_{LB}$  to the mean  $r_{mean}$  has a significant impact on the resulting updated reliability index for a given number of successful proof load tests. For a target reliability level, the required number of successful proof load tests is expected to differ significantly depending on the assumed value of “x”. This indicates that any effort that is aimed at recommending optimum load test programs for a given pile design scenario and soil conditions should take into consideration the ratio “x” as a major parameter that will affect the outcome of the decision making exercise.

### **3.7 Effect of uncertainty in the pile capacity distribution**

Another parameter that is also expected to play a role in the reliability-based updating using approach 1 is the coefficient of variation of the capacity distribution  $cov_R$ , which was taken as 0.4 in the previous sections of this chapter. The pile capacity is modelled using a truncated lognormal distribution with a coefficient of variation  $cov_R$ . Based on data collected from the literature (Zhang 2004), the  $cov_R$  is shown to generally range between 0.3 and 0.5 for different pile capacity prediction models and different soil types. In general, the smaller levels of uncertainty are for driven piles in clay while the highest levels of uncertainty are for driven piles in sands.

To investigate the sensitivity of the problem to the assumed  $cov_R$ , the variation of the reliability index with the number of pile load tests (assumed to be all successful) was plotted on Fig. 3.8 for  $cov_R$  of 0.3, 0.4, and 0.5. Results were produced for different proof load levels,  $r_{proof}$  starting from 1 times the design load to 3 times the design load. All results pertain to an assumed typical  $FS_{mean}$  of 2.0 (Fig. 3.8a) and  $FS_{mean}$  of 2.5 (Fig. 3.8b). Before discussing the results of the updating process, it is worth noting that for

the case of no load tests, an increase in the coefficient of variation of the pile capacity is expected to translate into an increase in the probability of failure and a decrease in the reliability index. This is clearly shown in Fig.3.8 for the cases where the number of tests was zero, where the reliability index was shown to decrease from 2.16 to 1.78 ( $FS_{mean}$  of 2.0) and from 2.94 to 2.47 ( $FS_{mean}$  of 2.5) as  $COV_R$  was increased from 0.3 to 0.5.

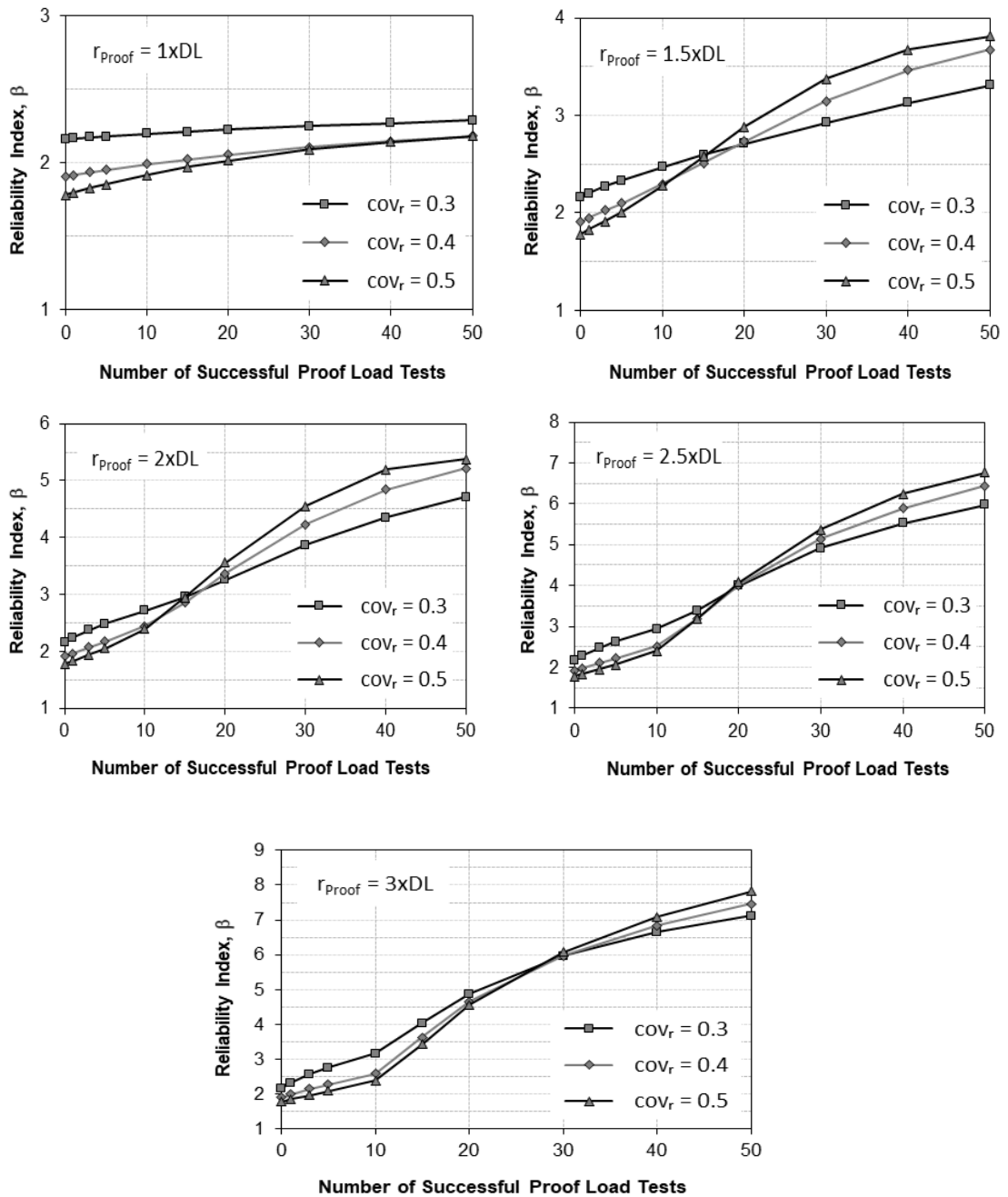


Figure 3.8. a - Effect of the uncertainty in the pile capacity for FS = 2

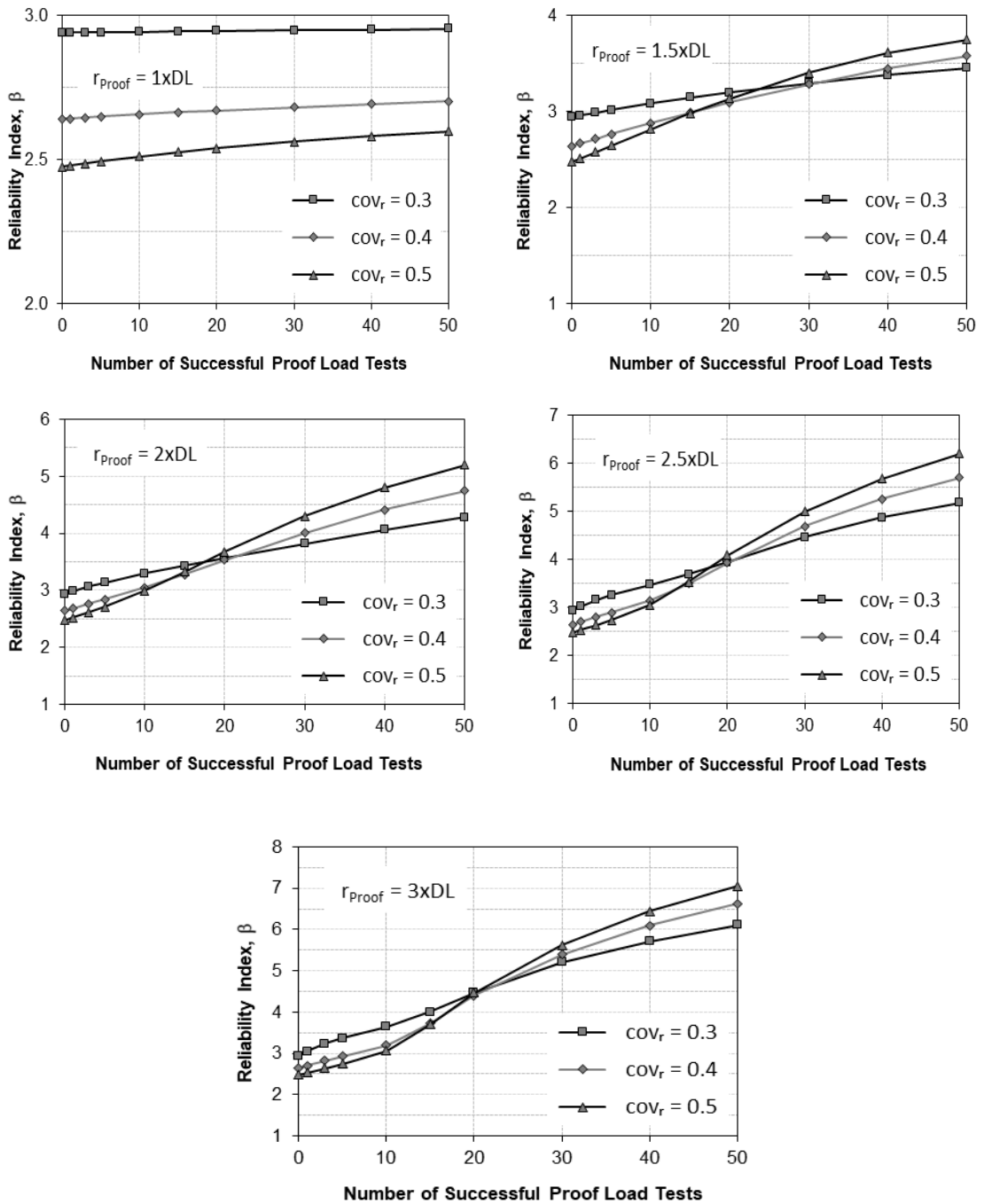


Figure 3.8. b - Effect of the uncertainty in the pile capacity for FS = 2.5

The results of the updating process on Figs. 3.8a and 3.8b lead to the following observation: when the number of successful tests is less than about 20, it seems that the

computed reliability index is the lowest for a  $\text{cov}_r = 0.5$  and the highest for a  $\text{cov}_r = 0.3$ , while the opposite is true when the number of successful proof load tests become larger than 20. In addition, the updating process seems to be more effective for the cases where the  $\text{cov}_r = 0.5$ , in the sense that the updated reliability index for these cases exhibited a relatively faster increase as the number of tests increased compared to the cases with the lower  $\text{cov}_r$  of 0.4 and 0.3. Despite this relatively faster increase, the fact that the curve corresponding to the  $\text{cov}_r = 0.5$  initiates from a lower prior reliability index compared to the lower  $\text{cov}_r$  dictates the observed behavior on Fig. 3.8, where the curve representing the variation of  $\beta$  with the number of tests for  $\text{cov}_r = 0.5$  was found to be initially lower than the other two curves, only to cross them as the number of tests increased.

It could thus be concluded that for a certain factor of safety, the improvement in the updated pile reliability relative to the prior reliability is greater for greater degrees of uncertainty in the prior pile capacity distribution. An explanation to this observation from the Bayesian updating perspective could be that the high uncertainty associated with the prior pile capacity distribution will allow for the data to be more effective at updating the distribution.

### **3.8 Conclusions**

Results of the analysis conducted in this chapter prove that proof-load testing may be very efficient in improving pile reliability which indicates for engineers the importance of incorporating programs about pile-load testing programs in the prior stage of the design.

In general, the impact of conducting a number of successful proof-load tests is to shift the distributions of the mean capacity and lower-bound capacity to the right,

resulting in an improved reliability index and a reduced probability of failure. The impact of the proof-load tests increases as the number of proof-tested piles increases and as the level of the proof-load tests increases. In addition, the higher the required target level of reliability, the more the successful tests that are needed and the higher the associated safety factors.

The positive impact of proof-load test programs was found to decrease when the results indicated a number of failed piles. The percentage of failed piles that seem to result in an updated reliability index that is almost equal to the prior reliability index (i.e., the proof load test program becomes inefficient) seems to be in the range of 30 to 40% of the tested piles.

Note that the analysis developed in this chapter are based on the statistical model of approach 1 according to which the pile capacity distribution is assumed to have a coefficient of variation of 0.4 and a lower-bound capacity characterized by cov of 0.2 and a mean value of 0.5 times the mean pile capacity. Extra analyses were introduced at the end of the chapter to see how changes in some assumptions as the lower-bound to mean capacity ratio and the cov of the pile capacity may affect the prior and updated reliability. Corresponding results show that these two parameters have an effect on the prior reliability and on the value of information to add to the prior reliability from a certain proof-lad test program. Interest about these observations will be dependent on the kind of data available for a specific pile design project.

## CHAPTER 4

### RESULTS AND ANALYSIS – APPROACH 2

#### **4.1 Introduction**

Approach two will be utilized in this chapter to update the pile capacity distribution based on results from proof-load testing programs. In approach 2, the lower-bound capacity is used to truncate the left-hand tail of the distribution of the mean pile capacity as indicated in Chapter 2 of this thesis. As was the case for approach 1, the target of the analysis is to investigate the effect of choosing alternative proof-load test programs on the reliability of the pile design and the required factors of safety. The updating process for a given proposed load test program will first be conducted by updating the mean capacity only, then the lower-bound capacity then the two parameters simultaneously.

#### **4.2 Illustration of the Updating Process**

In the base case analysis, the coefficient of variation of the capacity distribution  $COV_R$ , which reflects within-site variability, will be assumed to be equal to 0.2 as recommended by Zhang et al. (2002), while the coefficient of variation of the mean capacity will be assumed to be equal to 0.4 to reflect model uncertainty in the pile prediction method. Assuming 15 statistically independent proof-load tests of up to 2 times the design load on 15 piles that are designed and constructed at a reduced mean factor of safety of 2, and assuming all the tests are successful, the prior distributions are

updated as shown in Figs 4.1a, b, and c. The results of the testing program are used to update the probability mass functions of the mean capacity alone, the lower-bound capacity alone, and the joint PMF of the mean and the lower-bound capacity, respectively.

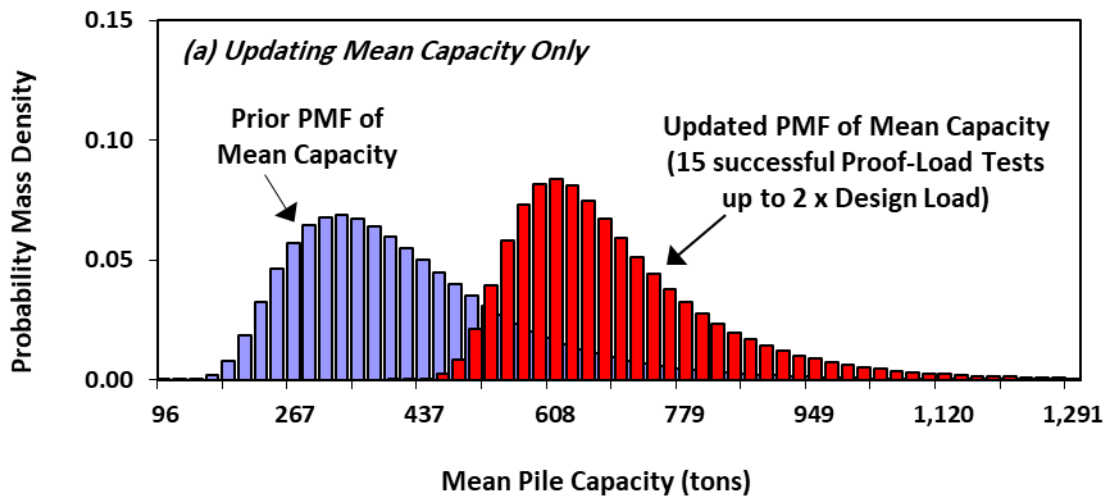


Figure 4.1. a - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ ) – Updating Mean Capacity only

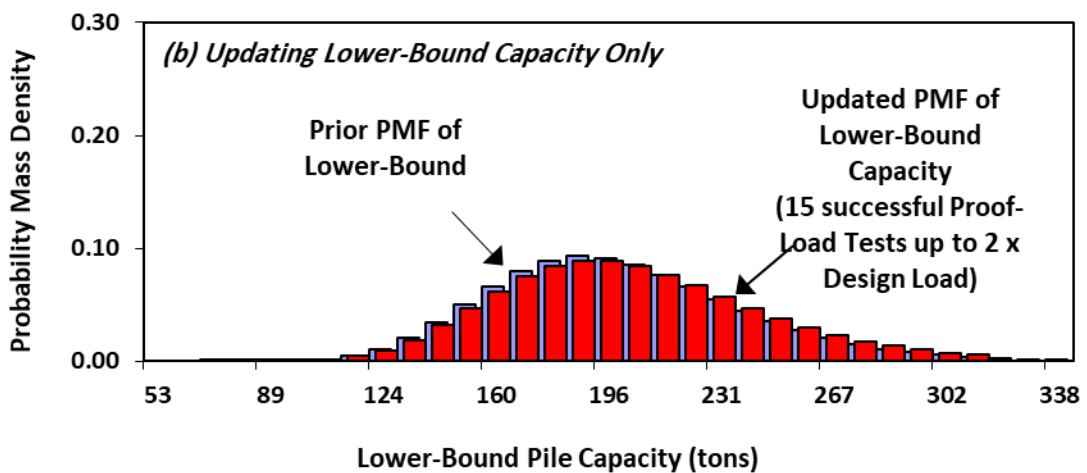


Figure 4.1. b - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ ) – Updating Lower-Bound Capacity only

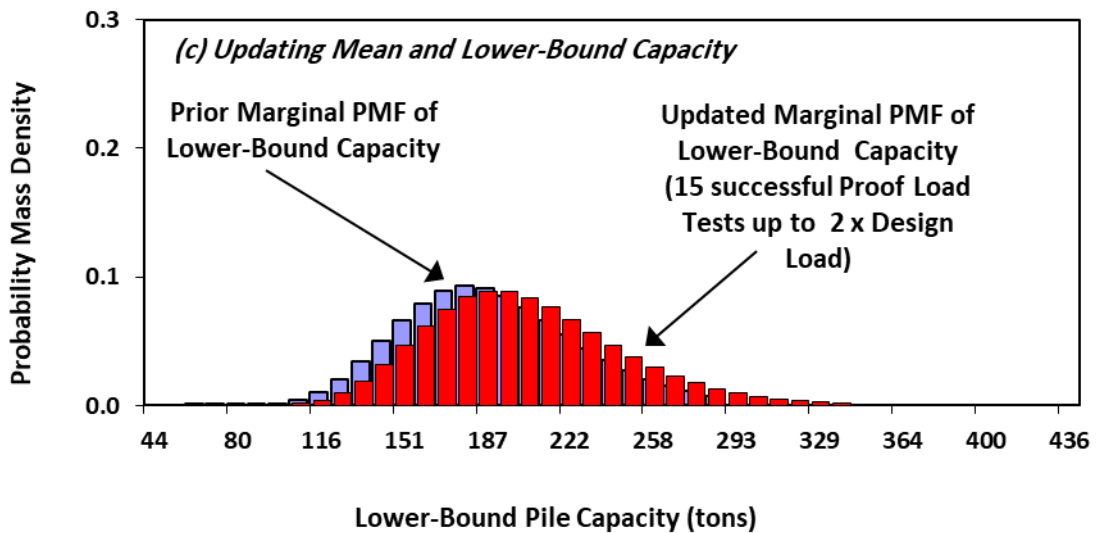
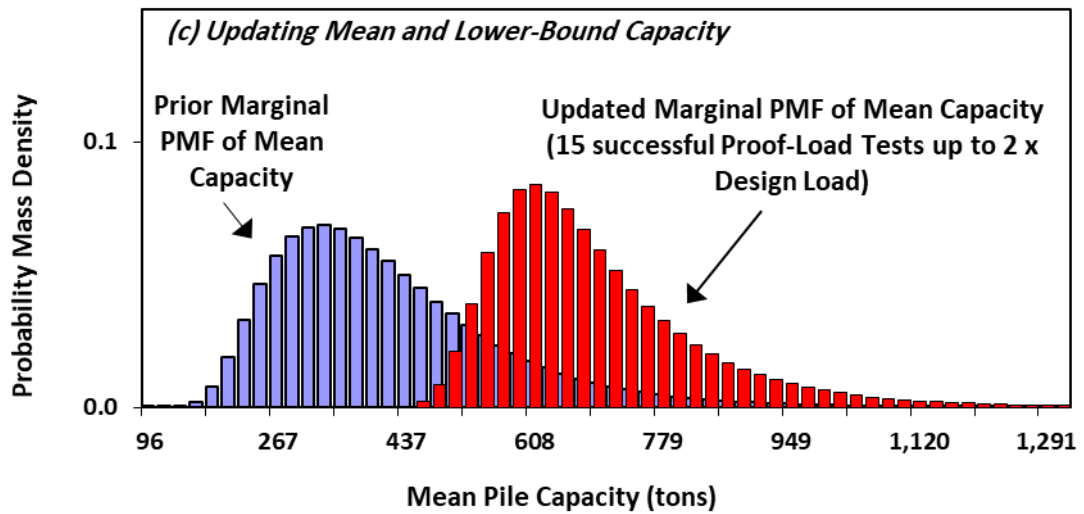


Figure 4.1. c - Updating the Probability Mass Functions of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $\text{FS}_{\text{mean}} = 2.0$ ) – Updating Mean and Lower-Bound Capacity

The results as presented in Fig. 4.1 are interesting since they indicate that the effect of conducting proof load tests in approach 2 is concentrated on the distribution of the mean pile capacity and negligible with regards to the distribution of the lower-bound capacity. For the case of 15 successful proof load tests of up to twice the design load, the updated PMF of the mean pile capacity shifted significantly to the right, indicating that the updated values of the mean pile capacity given the results of the assumed proof load test



program are much higher than the prior values. In fact, there is almost no overlap between the prior and updated PMFs of the mean pile capacity for the example under consideration. On the other hand, the lower-bound capacity distribution did not show any significant sensitivity to the results of the proof load testing program. This observation may be explained by the fact that the uncertainty in the prior distribution of  $r_{\text{mean}}$  ( $\delta_{\text{mean}} = 0.4$ ) is larger than the uncertainty in the prior distribution of  $r_{\text{LB}}$  ( $\delta_{\text{LB}} = 0.2$ ) which makes the mean the parameter with the higher potential to be updated. In addition, the fact that the lower-bound is truncating the distribution of the mean increases this potential by eliminating the low values of the mean in the distribution.

In order to get a more detailed representation of the combined effects of updating both the mean and the lower-bound, the prior and updated probabilities are plotted as joint probability mass functions as shown in Figure 4.2. The representation of the prior and updated distributions in three-dimensional PMFs is important for approach 2, since the two parameters of interest ( $r_{\text{LB}}$  and  $r_{\text{mean}}$ ) are correlated and not statistically independent. The correlation is related to the fact that the lower-bound capacity  $r_{\text{LB}}$  in approach 2 truncates the lower-tail of the mean capacity distribution  $r_{\text{mean}}$ . This renders the conditional probability distribution of  $r_{\text{mean}}$  with respect to  $r_{\text{LB}}$  sensitive to assumed values of  $r_{\text{LB}}$ . This is clearly illustrated in Figs. 4.3a and b, which show the prior and updated conditional PMF of  $r_{\text{mean}}$  for two specific values of the lower-bound capacity (mainly 204.4 and 337.8 tons). It is interesting to note that although the lower-bound capacity is clearly shown to truncate the distribution of the mean capacity in the prior distributions, the fact that the updating process shifted the distribution of the mean to the right and did not affect the distribution of the lower-bound renders the lower-bound capacity ineffective in the updated probability distributions (Fig. 4.3a). This observation

is important because it indicates that for approach 2, the impact of the lower-bound capacity on the reliability after updating is expected to be relatively weak, contrary to the results presented in Chapter IV for approach 1.

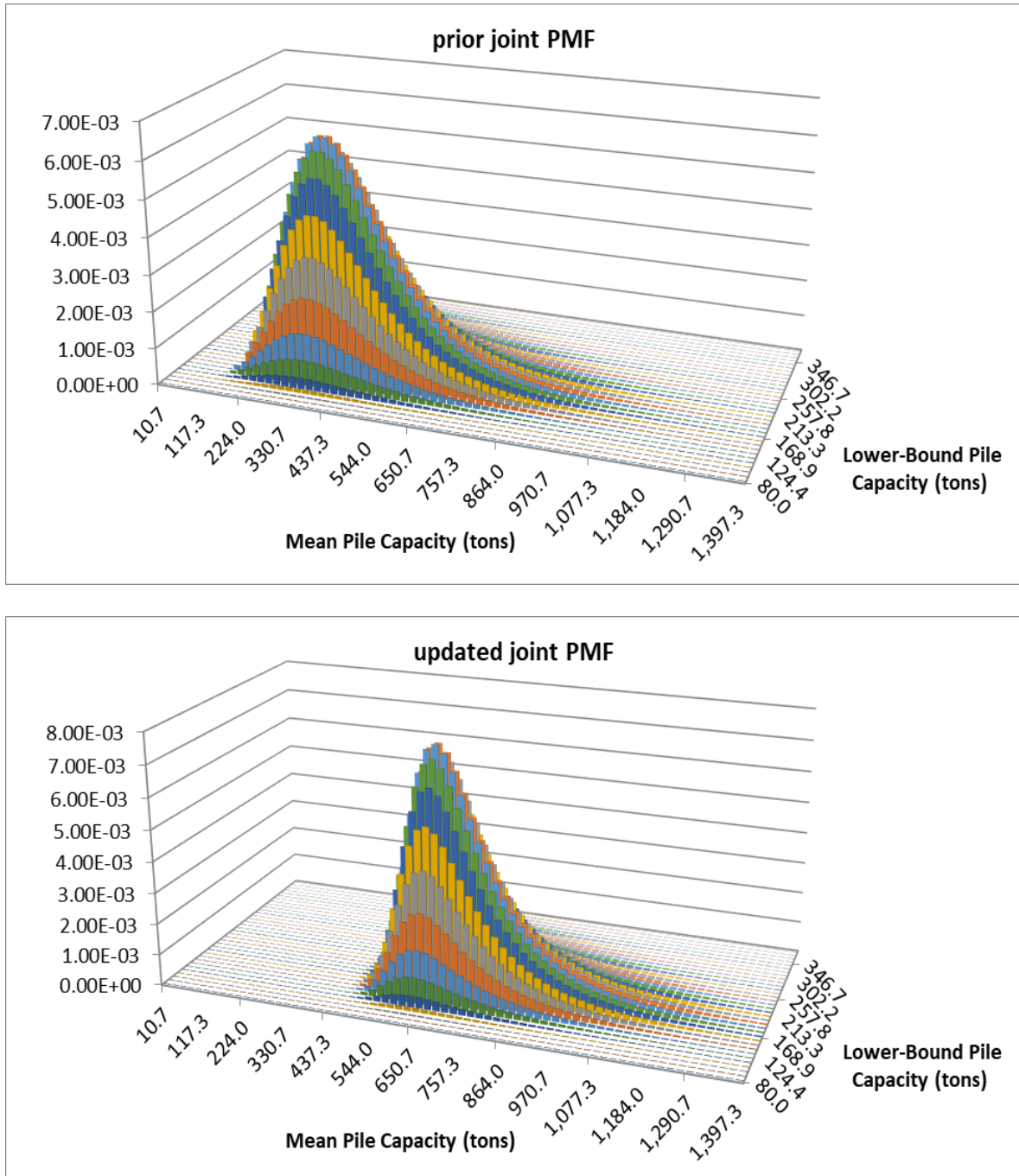


Figure 4. 2 - Updating the Joint Probability Mass Functions (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $FS_{\text{mean}} = 2.0$ )

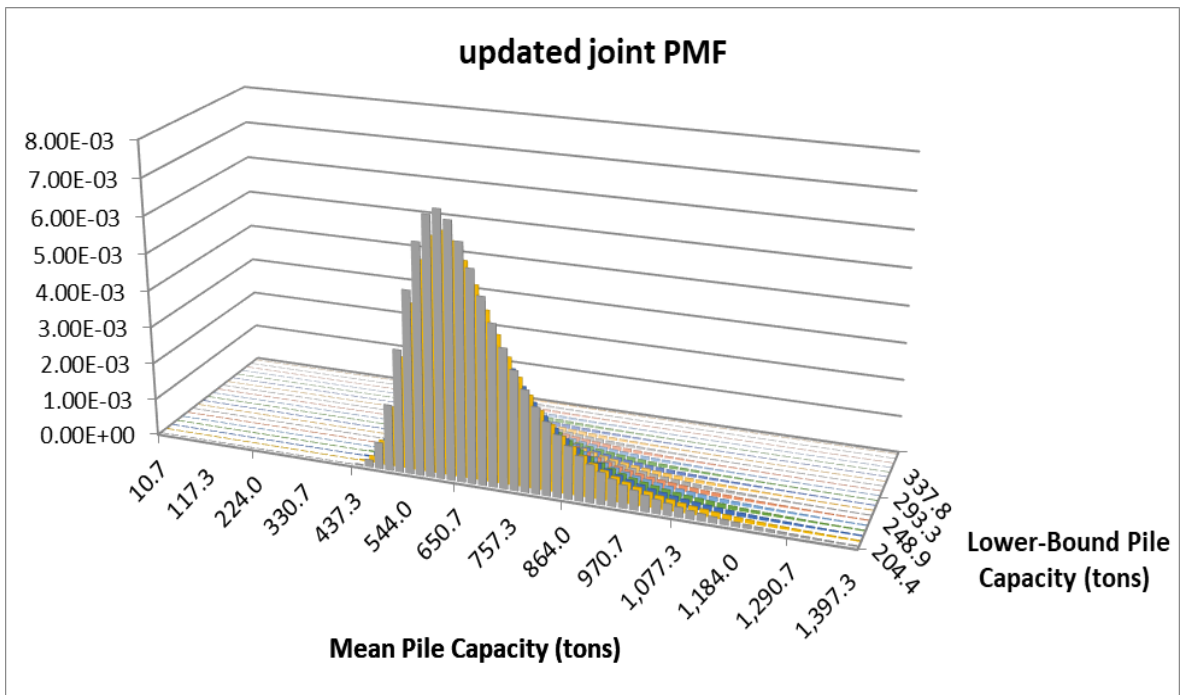
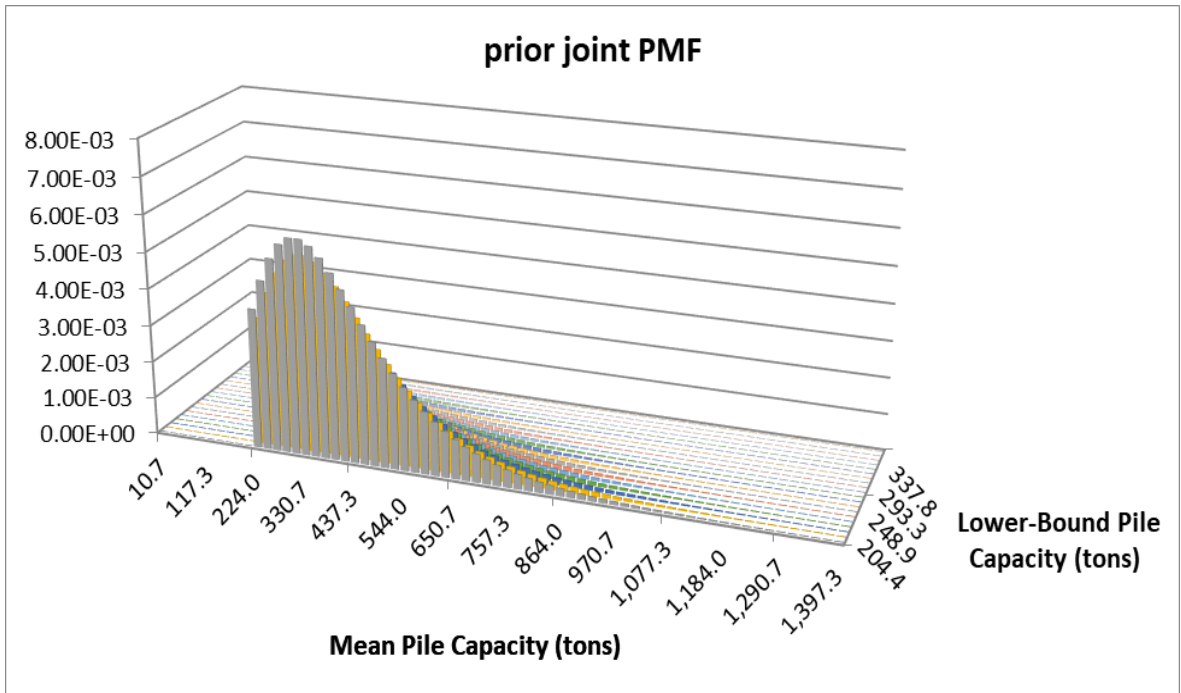


Figure 4.3. a - Prior and Updated Distributions of the Mean for a Lower-Bound Value of 204.4 tons (15 proof load tests,  $r_{proof} = 2 \times \text{Design Load}$ ,  $FS_{mean} = 2.0$ )

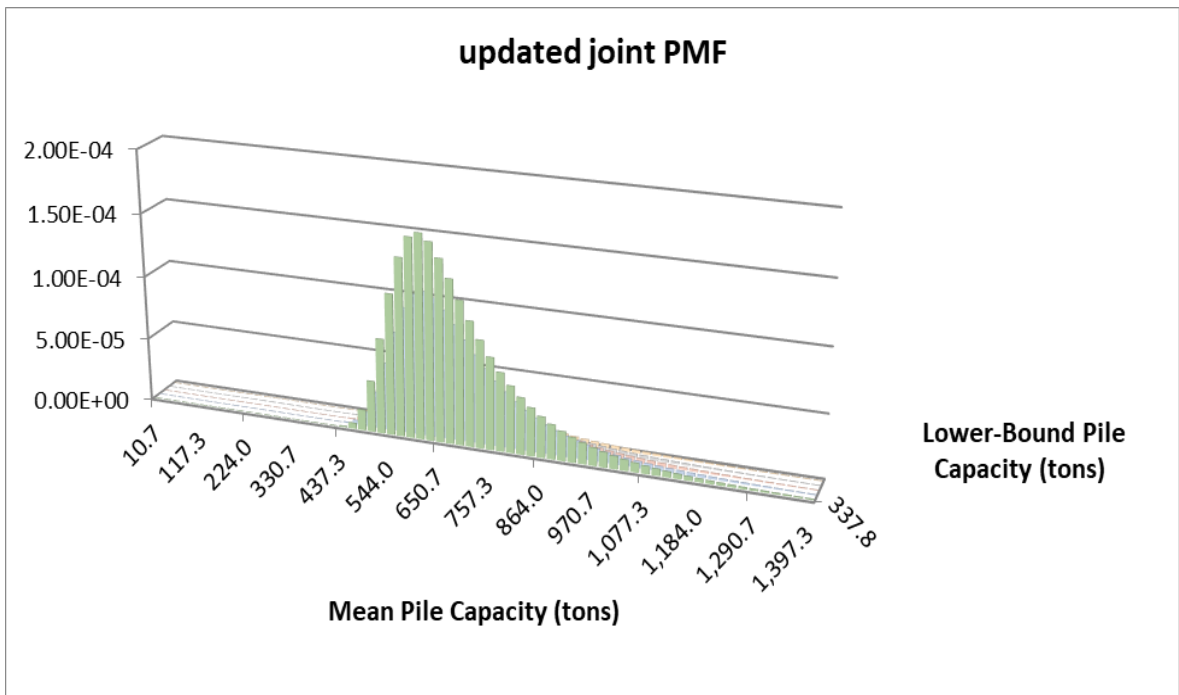
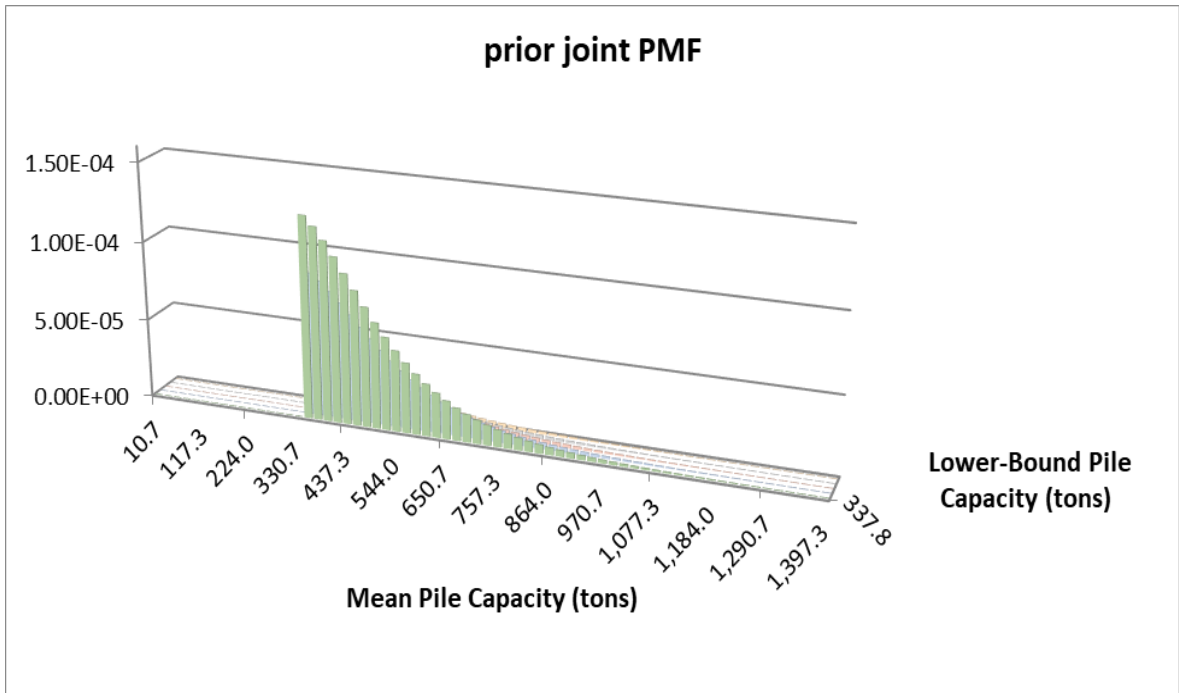


Figure 4.3. b - Prior and updated distributions of the mean for a lower-bound value of 337.8 tons (15 proof load tests,  $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $FS_{\text{mean}} = 2.0$ )

### 4.3 Updated Pile Reliability for Different Proof-Load Test Scenarios for $FS_{\text{mean}} = 2.0$

The effect of the updating process on the reliability index of piles tested at different proof load levels and for different assumed number of tested piles is investigated on Fig. 4.4 For the specific case of the 15 proof load tests that were conducted to twice the design load and assuming a factor of safety of 2.0 (corresponding to results on Figs.5.1 to 5.3), the reliability index increases from its prior value of 1.59 to 4.08 for the case where the mean capacity is updated alone and the case where both the mean and the lower-bound capacity are updated together. When updating the lower-bound alone, the reliability index remains almost constant. Combinations of high proof-load levels and high numbers of tests result in updated reliabilities as high as 5 and 5.5; however, the significant improvement in the reliability, especially for high proof-load level, is achieved at the first 3 to 5 positive tests.

The main observations that could be made based on the results presented in the previous paragraphs are summarized below:

- (1) the lower-bound capacity does not seem to play a role in improving the reliability index in approach 2.
- (2) The impact of the load test program is reflected almost completely in the updated distribution of the mean pile capacity.
- (3) The updated reliability index seems to be very sensitive to the initial 5 proof load tests. The sensitivity of the reliability index to the number of tests seems to decrease as the number of successful proof load tests is increased further. For example, for  $FS = 2$  and proof load levels of 2 times the design load, the reliability increases from 1.59 to 2.74 at the first positive test then to 3.4 at 3

positive tests. Then, the rate of improvement in reliability decreases. The reliability reaches 3.66 at 5 tests and 4.08 at 15 tests.

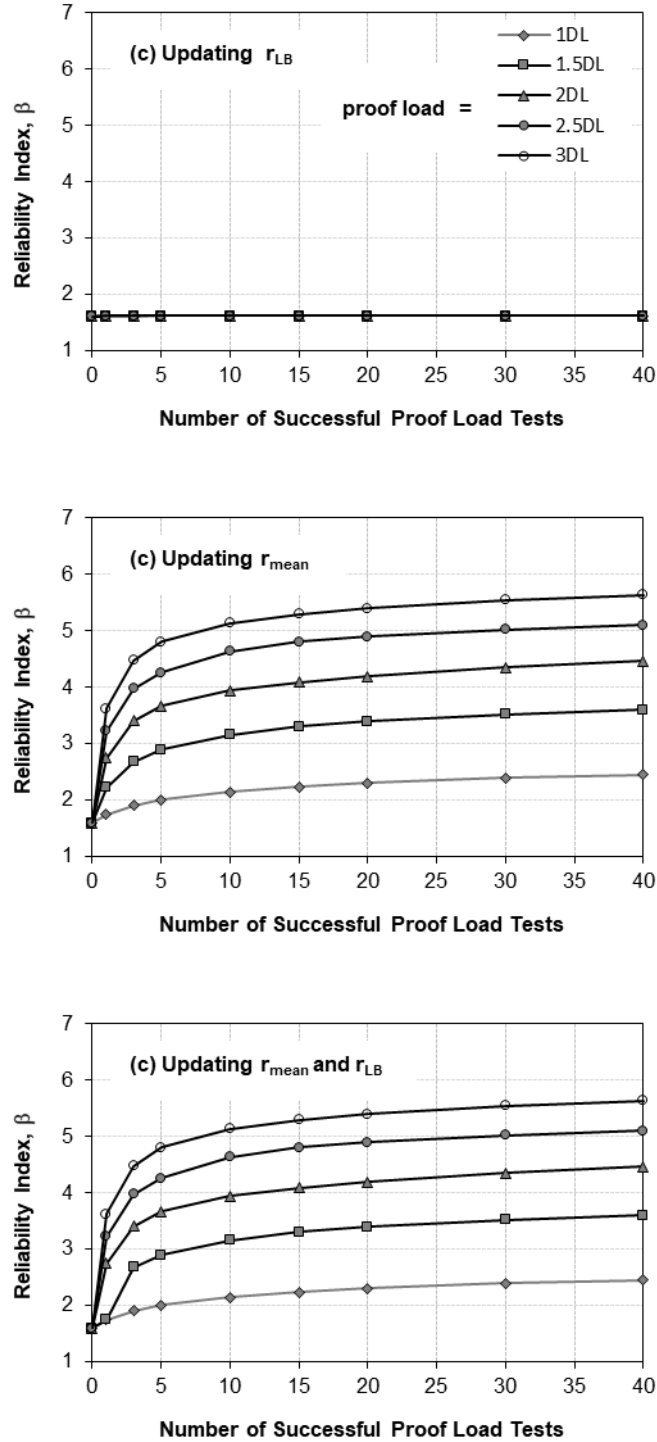


Figure 4. 4 - Effect of Load Test Programs on the Reliability of Pile Design ( $FS_{mean} = 2$ )

These results indicate that for the probabilistic model of the pile capacity that was adopted in this approach, the mean capacity governs the reliability index since it was the major parameter to be affected by the updating process whereas the effect of the lower-bound capacity on the updating seems to be negligible. This observation constitutes the major difference between approach 2 and approach 1, where the results of the probabilistic model of the first approach were affected significantly by the lower-bound capacity. Moreover, the results of the second approach as compared to those of the first approach indicate a high potential of saving in the number of positive tests required to achieve a typical target reliability index. This will be further examined in future sections of this chapter.

#### **4.4 Factor of Safety vs Reliability for Different Test Programs**

Figs. 4.5a, 4.5b, and 4.5c show plots for different combinations of design factor of safety, proof-load level, and number of proof-load tests required to achieve the desired level of reliability of 2.5, 3, and 3.5. As shown in the Fig. 4.5, no more than few positive tests are required to achieve these levels of reliability according to the second approach.

To get a target reliability index of 3.0 for relatively high proof-load levels (2.5 or 3 times the design load), no more than one positive proof-load test is required even for a low factor of safety as 2. On the other hand, when the proof-load level is 1.5 times the design load, 2 positive tests are required if we choose to use a factor of safety of 3, and 7 tests if we choose to minimize the factor of safety to 2. Comparing results of the two approaches show a large margin of minimizing the required number of tests in this approach relative to the first one. For example, according to the results of the first approach, in order to minimize the factor of safety to a value of 2 when the target

reliability is 3, one possible proof-load test program is 12 tests up to 3 times the design load, or even 27 tests up to 1.5 x design load.

For a relatively low target reliability index of 2.5 (generally for redundant pile foundation systems), results show that for any proof-load level greater than 1.5 times the design load, no more than one positive proof-load test is required. In addition, no tests are required when the factor of safety is 3. On the other hand, for a relatively higher target reliability index of 3.5, a reduced design factor of safety of 2 could be adopted with 28 successful low level proof-load tests of 1.5 times the design load. If the proof-load level is increased slightly to 1.75DL, the number of positive proof-load tests decreases to 9 if a reduced factor of safety of 2 is adopted. For the same case, the number of successful tests with proof load level of 1.75DL could be reduced further to 4 tests, if a factor of safety of 3 is adopted. The above observations are limited to cases where all the tests are positive. The effects of failures will be studied in the next section.

#### **4.5 Effect of Failures on the Updated Reliability**

The effect of pile failures on the updated marginal PMFs of  $r_{\text{mean}}$  and  $r_{\text{LB}}$  is studied in Figures 4.6a and 4.6b, respectively. For illustration purposes, results are shown for the typical case where a reduced factor of safety of 2 is adopted and a total of 10 tests that are conducted at a proof level of 2 times the design load are considered in the analysis. The prior and updated PMFs of the mean capacity for cases with no failures, 3 failures (out of 10) and 8 failures (out of 10) are presented in Fig. 4.6a., while the prior and updated PMFs of the lower-bound capacity are presented in Fig. 4.6b.



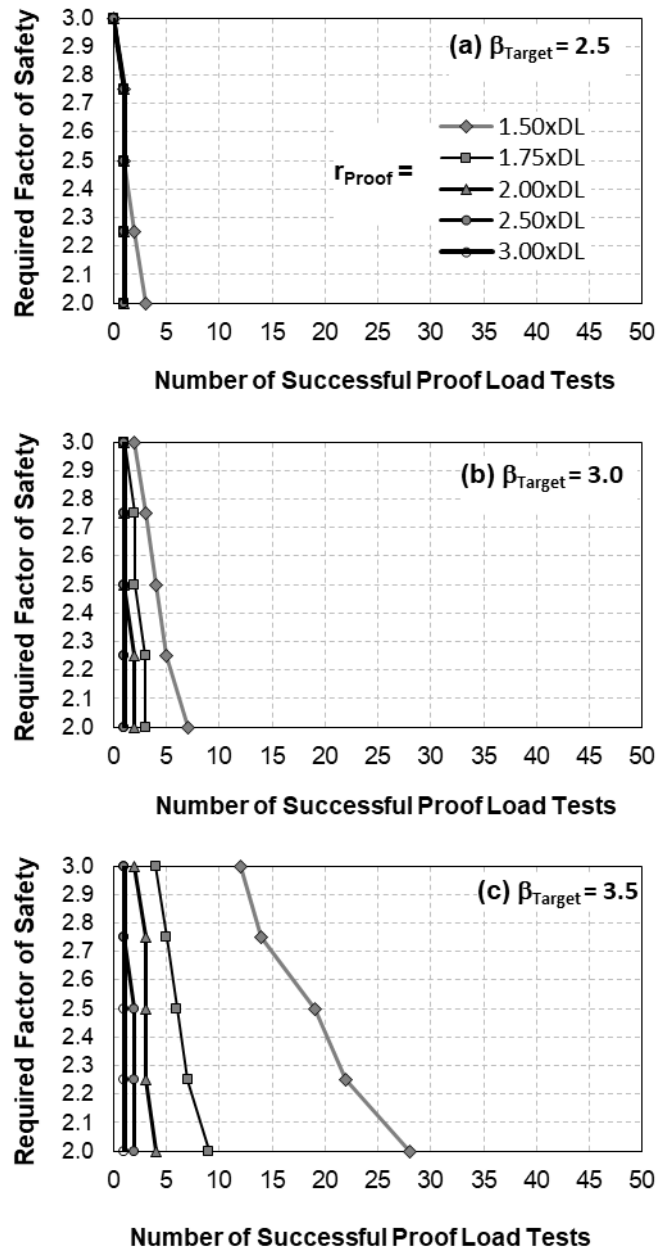


Figure 4. 5 - Required Factor of Safety to Achieve a Target Reliability Level of  $\beta = 2.5, 3.0,$  and  $3.5$  for Different Load Testing Programs

Results on Figs 4.6a (prior and updated mean capacity) lead to the following observations: (1) when all the tests are successful (no failures), the updated PMF of the mean capacity shifts to the right indicating higher reliability level in the updated scenario. (2) when 3 out of a total of 10 piles are assumed to fail, the updated PMF of

the mean capacity is still shifted to the right relative to the prior distribution, but to a much lesser degree in comparison to zero failures. More importantly, the uncertainty in the mean pile capacity seems to be reduced significantly compared to the prior distribution. (3) for the case with 8 pile failures (out of 10), the updated PMF of the mean capacity indicates that the position of the PMF is almost similar to the position of the prior, but the uncertainty in the updated mean capacity is reduced significantly compared to the prior. For the lower-bound capacity, results on Fig. 4.6b indicate that the lower-bound capacity is not very sensitive to the updating process and to the number of failed piles.

The impact of failures on the updated PMF of the mean capacity is translated into an impact on the updated reliability index as indicated in Fig. 4.7 which shows the variation of the updated reliability index with the number of failed piles and the assumed factor of safety for tests conducted at twice the design load. For illustration, the analysis is conducted for the cases of 5 tests, 10 tests, 20 tests, and 30 tests, respectively. Results on Fig. 4.7 indicate that failures reduce the updated reliability indices compared to the 100% successful case, with the reduction in the reliability index being more significant as the number of failed tests increase. It could also be noted that the effect of the design factor of safety on the updated reliability for an assumed number of failed piles seems to be relatively negligible.

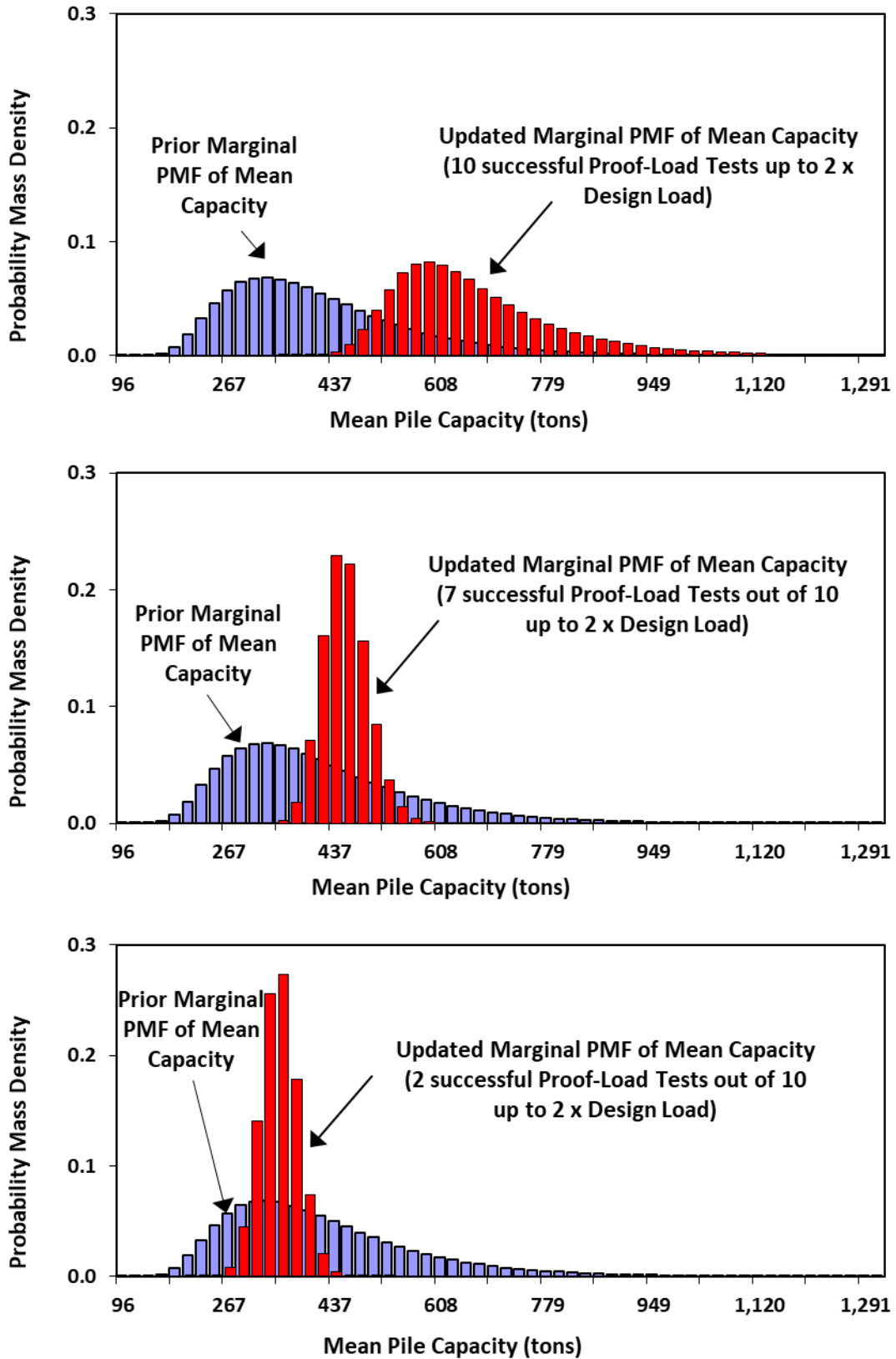


Figure 4.6. a - Effect of Failures on Marginal PMF of  $r_{\text{mean}}$  for 10 tests ( $r_{\text{proof}} = 2 \times \text{Design Load}$ ,  $FS_{\text{mean}} = 2.0$ )

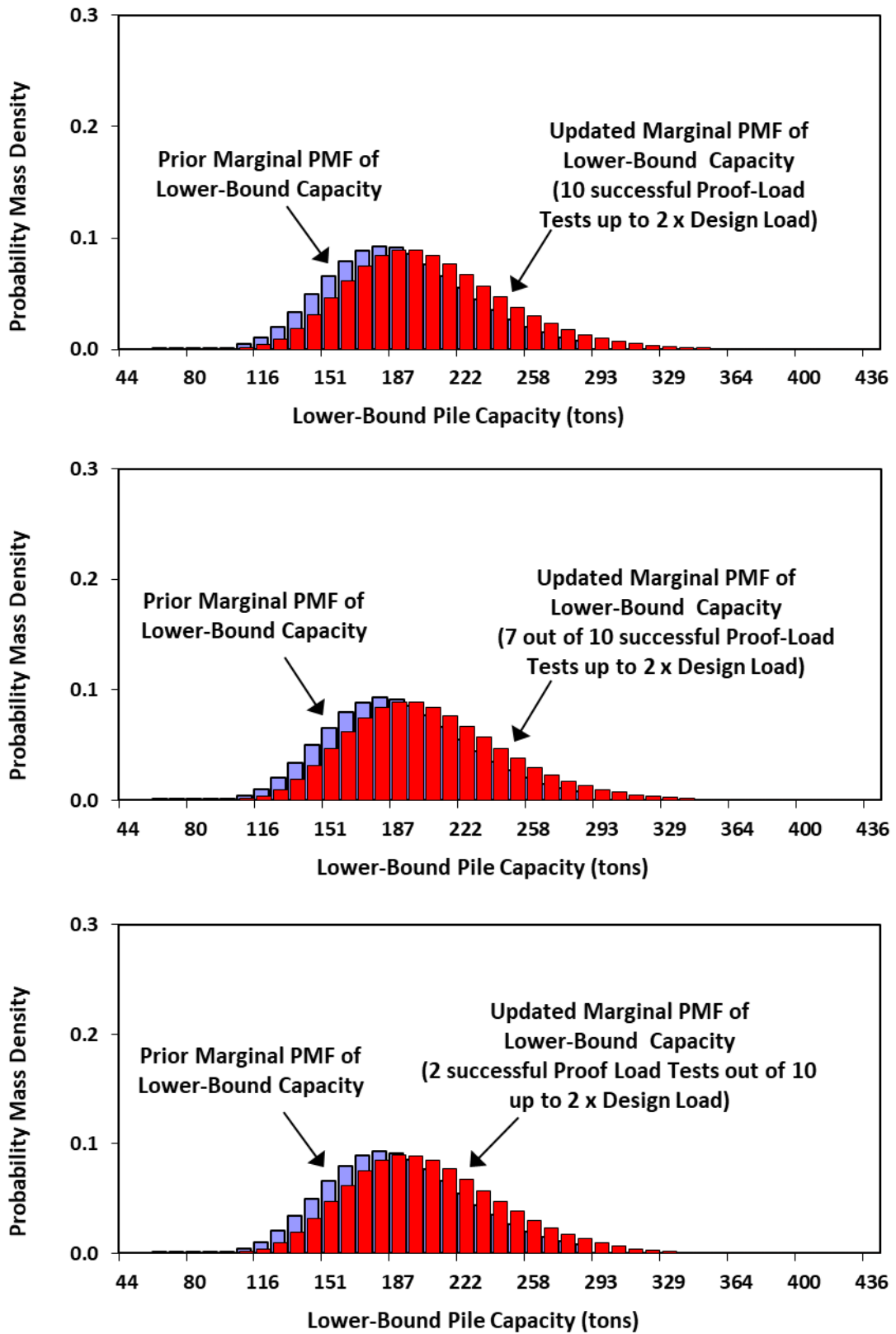


Figure 4.6. b - Effect of Failures on Marginal PMF of  $r_{LB}$  for 10 tests ( $r_{proof} = 2 \times \text{Design Load}$ ,  $FS_{mean} = 2.0$ )

A thorough investigation of the results presented in Fig. 4.7 indicates that for approach 2, the proof-load test program seems to become inefficient at a percentage of failed piles of about 60 to 80% of the tested piles. At this percentage of failed piles, the updated reliability index becomes almost equal to the prior reliability index. For comparison, the equivalent percentage of failed piles in approach 1 was 30-40% of the total number of tested piles. These results are interesting since they indicate that for approach 2, the pile load testing program will have a benefit even if 60 to 80% of the piles fail. This could be explained by studying the updated PMFs of the mean pile capacity in Fig. 4.6a. These PMFs indicate that even when the majority of piles fail, the left hand tail of the updated capacity distribution which governs the reliability does not extend to low values of capacity as is the case for the prior distribution. This could be further explained by noting that the level of the proof load chosen in this illustrative example is in the 400 tons (twice the design load). Even if some piles fail at this proof load, the Bayesian approach still shows improvements in the left hand tail of the distribution since from a statistical perspective, a failed proof load test indicates that the capacity of the pile is less than the proof load level (400 tons). This means that the mean pile capacity could have assumed values that are very close to 400 tons even if the piles failed during the test. These values that are close to the value of the proof load (although smaller than the proof load) could be considered to be an improvement in comparison to very low values that exist in the tail of the prior distribution and which lead to very low reliability indices in the prior distribution.

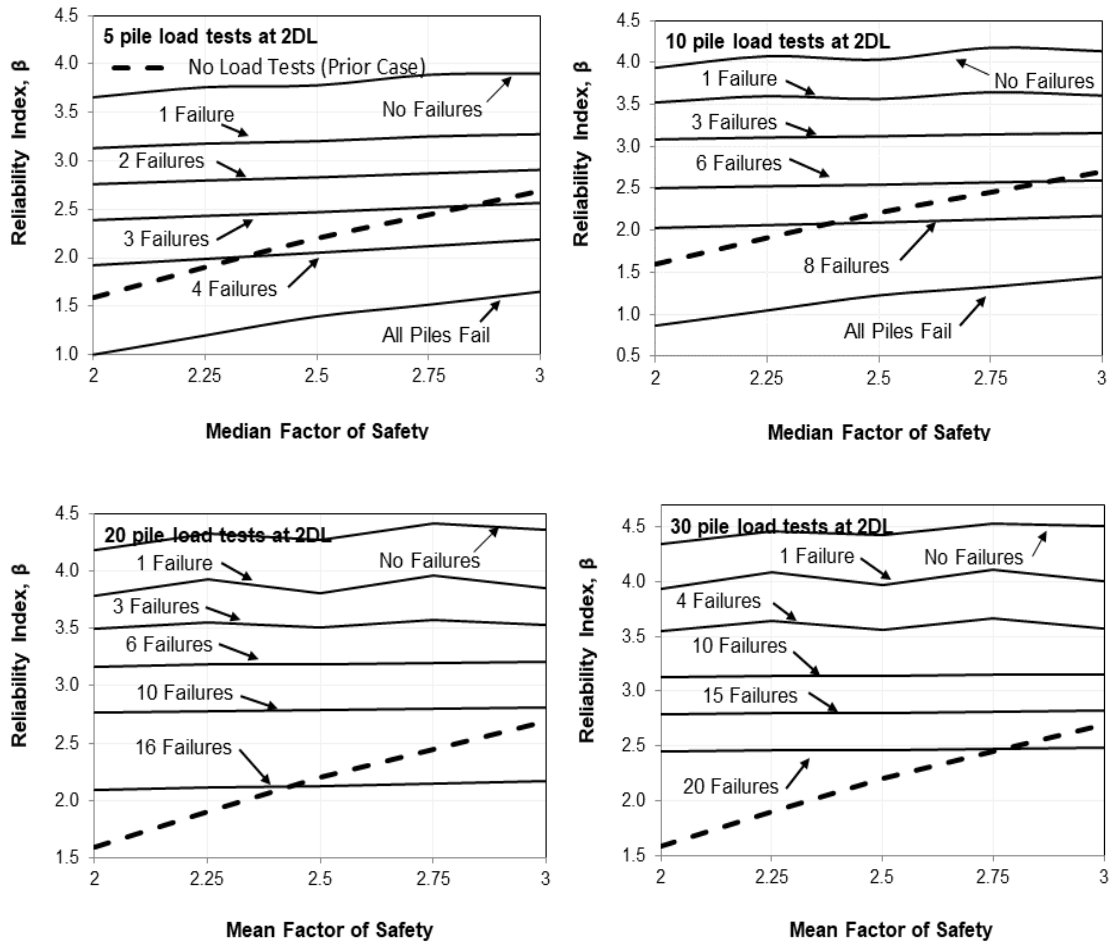


Figure 4.7 - Effect of Pile Failures on the Updated Reliability Index

#### 4.6 Effect of the lower-bound to mean capacity ratio

Results of approach 2 have shown that the effect of the lower-bound capacity on the updating process is negligible. These results were based on an assumed lower-bound to mean capacity ratio ( $x$ ) of 0.5. In order to illustrate the importance of the effect of this ratio on the results, the sensitivity of the results to the assumed value of  $x$  was studied.

The effect of the ratio ( $x$ ) on the prior and updated reliability indices is illustrated on Fig. 4.8, where the ratio “ $x$ ” was taken as 0.4, 0.5, or 0.6 for comparison. The curves on Fig. 4.8 show the variation of the reliability index with the number of

pile load tests, which were assumed to be all successful for a proof-load level of 2.0 times the design load.

Results show that the updated reliability index corresponding to a certain proof-load test scenario is independent of the ratio ( $x$ ). This ratio will only play a role into determining the prior reliability where the prior reliability indices corresponding to  $x = 0.4, 0.5,$  and  $0.6$  are respectively 1.43, 1.59, 1.78 for  $FS = 2$  and 1.98, 2.2, 2.44 for  $FS = 2.5$ .

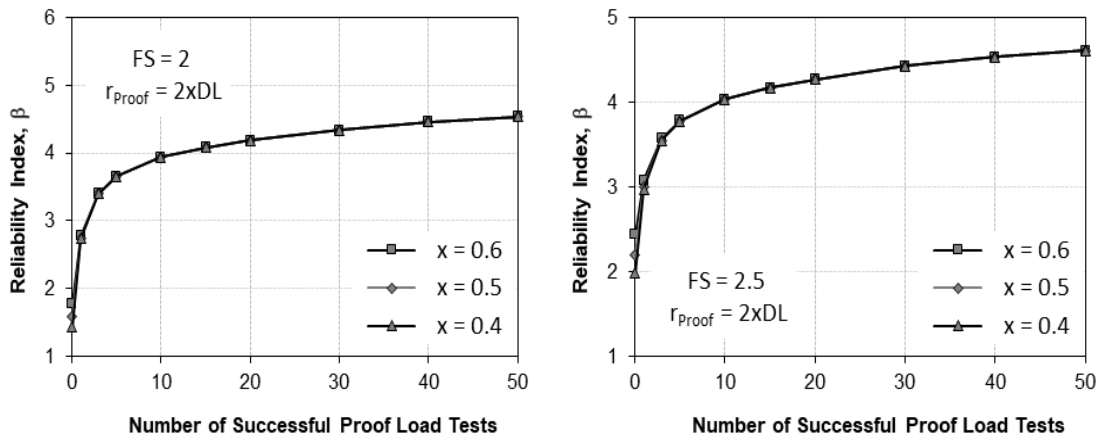


Figure 4. 8 - Effect of the Lower-bound to the Mean Capacity Ratio for  $FS = 2$  and  $FS = 2.5$

#### 4.7 Effect of uncertainty in the distribution of the mean of the pile capacity

Another parameter that is also expected to play a role in the reliability-based updating using approach 2 is the coefficient of variation of the mean pile capacity, which was taken as 0.4 in the previous sections of this chapter.

To investigate the sensitivity of the problem to the assumed  $COV_R$ , the variation of the reliability index with the number of pile load tests (assumed to be all successful) was plotted on Fig. 4.9 for  $COV_R$  of 0.3, 0.4, and 0.5. Results were produced for different proof load levels,  $r_{proof}$  starting from 1 times the design load to 3 times the design load.

All results pertain to an assumed typical  $FS_{\text{mean}}$  of 2.0 (Fig. 4.9a) and  $FS_{\text{mean}}$  of 2.5 (Fig. 4.9b). Before discussing the results of the updating process, it is worth noting that for the case of no load tests, an increase in the coefficient of variation of the pile capacity is expected to translate into an increase in the probability of failure and a decrease in the reliability index. This is clearly shown in Fig, 4.9 for the cases where the number of tests was zero, where the reliability index was shown to decrease from 1.64 to 0.97 ( $FS_{\text{mean}}$  of 2.0) and from 2.45 to 2.00 ( $FS_{\text{mean}}$  of 2.5 ) as  $COV_R$  was increased from 0.3 to 0.5.

The results of the updating process on Figs. 4.9a and 4.9b indicate that for a certain factor of safety, the improvement in the updated pile reliability relative to the prior reliability is insensitive to the uncertainty in the prior mean pile capacity distribution. Some sensitivity of the updated reliability index to the coefficient of variation of the mean pile capacity is observed at very small number of load tests due to the effect of the prior reliability index. Also, at a relatively high number of proof load tests, the updated reliability index for the large prior COV of 0.5 seems to be slightly higher than the smaller COVs. This is related to the Bayesian updating which affects parameters with higher prior uncertainty levels more than parameters with smaller uncertainty levels.



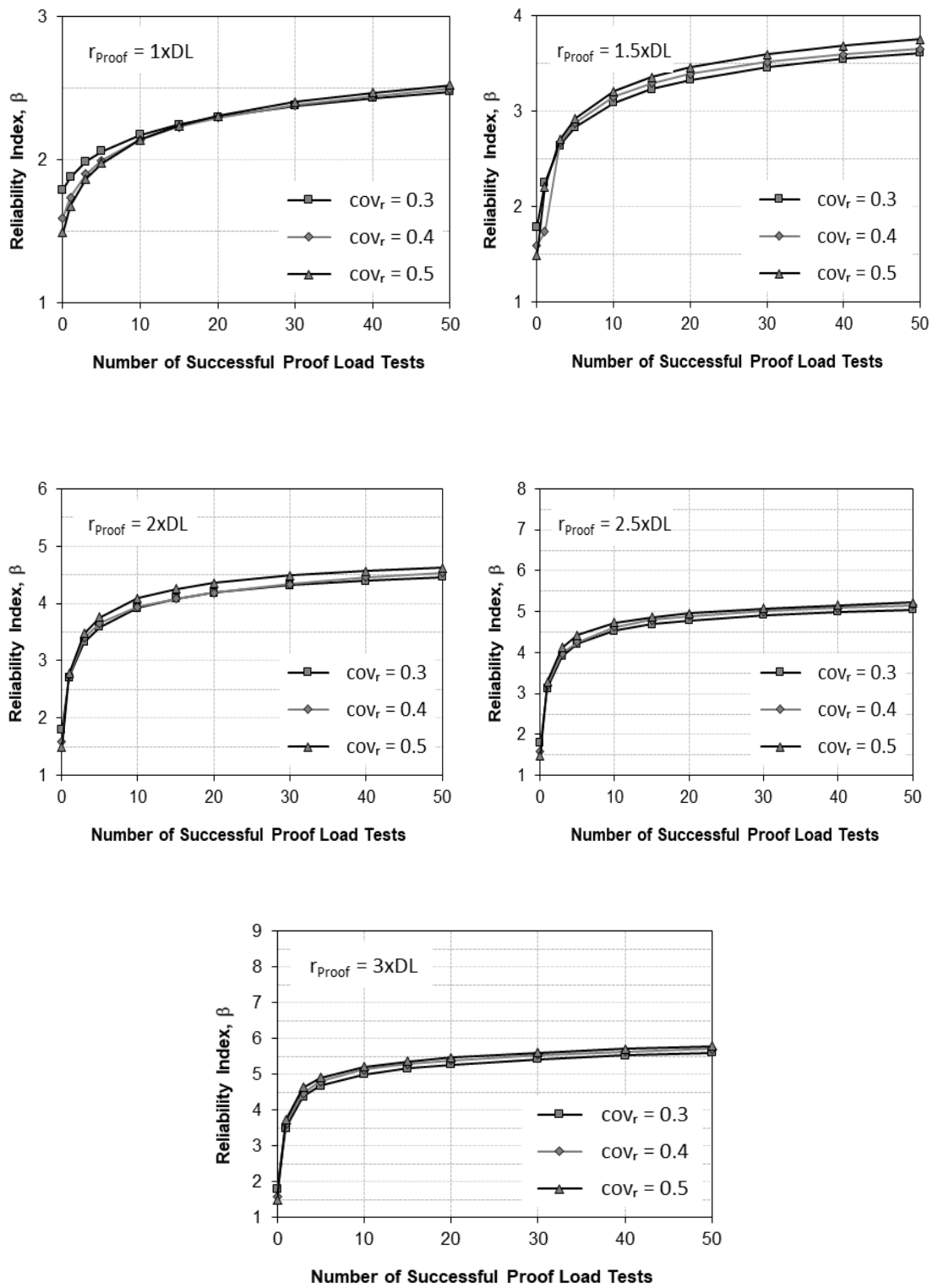


Figure 4.9. a - Effect of the Uncertainty in the Mean Pile Capacity for FS = 2

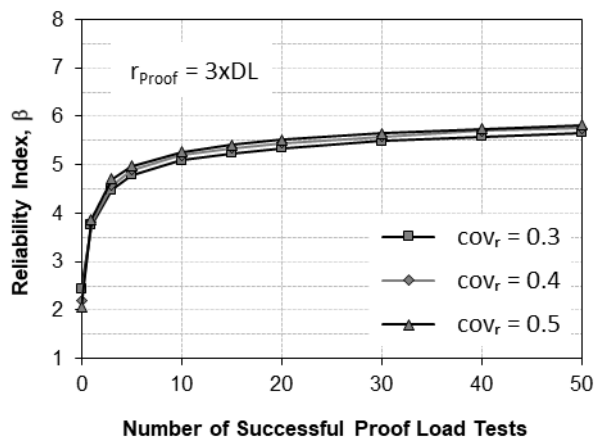
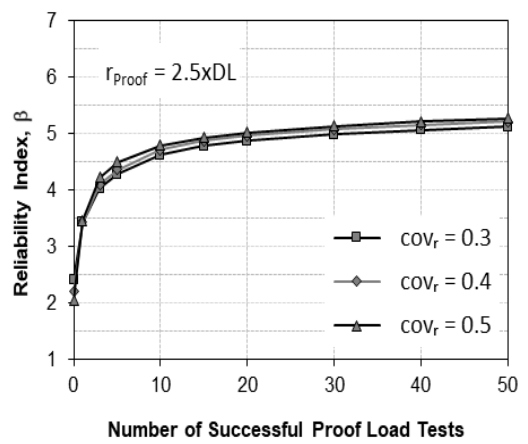
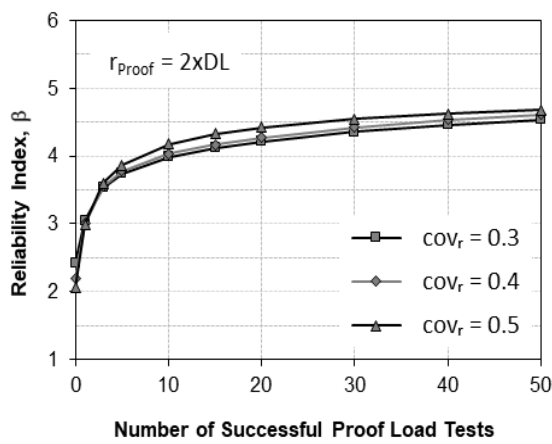
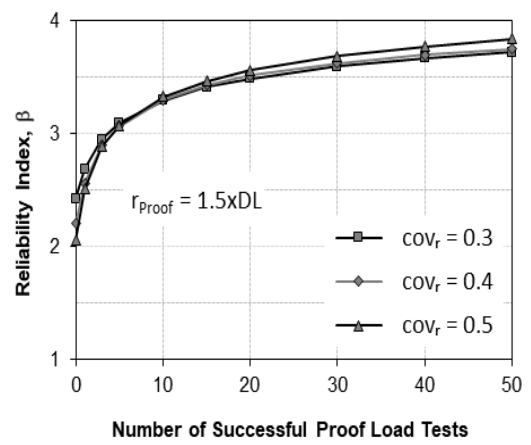
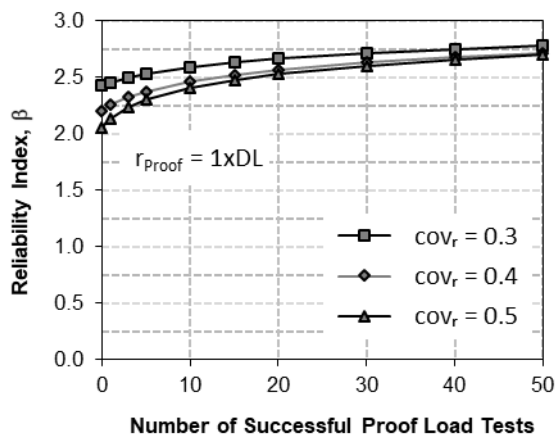


Figure 4.9. b - Effect of the Uncertainty in the Mean Pile Capacity for FS = 2.5

#### 4.8 Comparing Results for Approach 1 and Approach 2

Based on the results presented in Chapter 3 for approach 1 and this chapter for approach 2, it could be observed that although the two approaches are common approaches for modelling uncertainty in pile capacity and although the two approaches result in relatively similar reliability indices for the prior analysis, the two methods show very distinct trends in the updated distributions and the corresponding updated reliability indices.

Two illustrate the difference between the two methods, graphs showing the variation of the reliability index with the number of successful tests for the case of  $FS = 2$  and proof load levels of 1.5, 2, and 2.5 the design load are presented in Fig. 4.10 for approaches 1 and approach 2. Results on Fig. 4.10 indicate that the updating process for approach 2 (with  $cov_r = 0.2$  which is the case analysed in this chapter) is more effective than approach 1 with regards to the resulting reliability level. It is clear that in approach 2, significant improvements in the reliability index are obtained at a relatively small number of tests, with the rate of improvement decreasing dramatically as the number of tests is increased. On the other hand, in approach 1, the increase in the reliability index for a relatively small number of piles is very slow. Contrary to approach 2, the rate at which the reliability index increases with the number of tests increases as the number of tests increases. At a relatively large number of tests (approximately 30 to 40), the two curves converge indicating that the two methods become equally effective at updating the reliability. The number of tests at which the two curves converge seems to decrease at the level of the proof load increases (see Fig. 4.10).

This observed behaviour is mainly attributed to the fact that in approach 1, the main thrust of the updating process is the lower-bound capacity while in approach 2, the

main thrust of the updating is the mean of the capacity distribution. The slow rate of improvement in approach 1 is due to the fact that the lower-bound capacity does not affect the reliability except after a threshold value (see Chapter 3) after which the updated reliability index becomes very sensitive to the value of the lower-bound. For the relatively lower number of tests, although the lower-bound is updated, the values of the updated distribution do not cross the threshold value at which these values become effective. This causes a relatively slow rate of improvement in the reliability index as the number of tests increase. On the other hand, results from approach 2 indicate that the distribution of the mean capacity is very sensitive to the number of tests even when the number of tests is very small (1 to 3 tests). The major improvements are obtained at the lower range of the number of proof tests with improvement levelling out at higher number of tests.

A thorough investigation of the curves on Fig. 4.10 indicates that the starting or prior reliability indices for approaches 1 and 2 (with typical  $\text{cov}_r = 0.2$ ) are not exactly identical. To provide a one to one comparison between approaches 1 and 2, the case where a lower within-site variability that is reflected with a  $\text{cov}_r = 0.1$  is considered for approach 2 and the analysis repeated for all the cases analyzed on Fig. 4.10. The curves corresponding to this case were added to Fig. 4.10 for comparison with the base case in approach 2 where the  $\text{cov}_r = 0.2$ . An investigation of the curves on Fig. 4.10 leads to the following observations: (1) the prior reliability indices for approach 1 and approach 2 with a reduced  $\text{cov}_r = 0.1$  show a perfect match, (2) although the two approaches start from the same point, the results of the updating process indicated an immediate shift in behaviour for cases with proof load tests, with the shift being the most evident for very small number of tests (1 to 5 tests), and (3) a comparison between

the results of approach 2 for  $cov_r = 0.1$  and  $cov_r = 0.2$  indicate that the updated reliability index are sensitive to the assumed value of the within-site variability in pile capacity. As expected, the case with the smaller within site variability ( $cov_r = 0.1$ ) resulted in higher updated reliability indices compared to the more realistic base case with a  $cov_r$  of 0.2. Since published data show that the  $cov_r$  is most likely to be close to 0.2 than 0.1, the results of the base case of  $cov_r$  of 0.2 will be assumed to be more representative of realistic within site variability data.

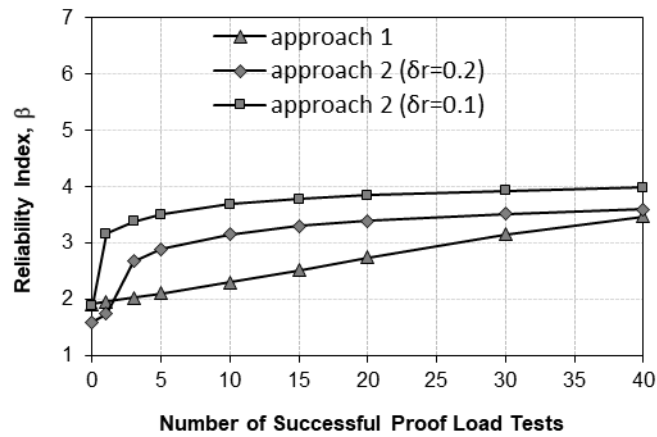


Figure 4.10. a - Comparing Updated Reliability for Different Approaches for FS = 2 and  $r_{proof} = 1.5 \times$  Design Load

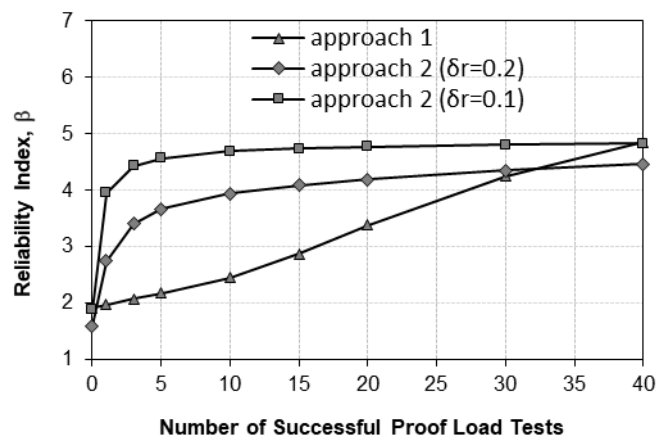


Figure 4.10. b - Comparing Updated Reliability for Different Approaches for FS = 2 and  $r_{proof} = 2 \times$  Design Load

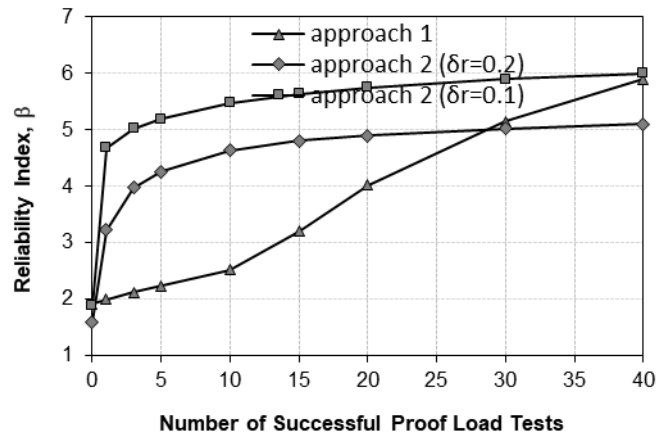


Figure 4.10. c - Comparing Updated Reliability for Different Approaches for FS = 2 and  $r_{proof} = 2.5 \times$  Design Load

#### 4.9 Conclusions

In this Chapter, the results pertaining to the case where the prior capacity distribution was modeled using approach 2 were presented. These results indicated that the distribution of the mean capacity is the most affected by the updating process and that significant improvements in the updated reliability index are obtained with a relatively small number of tests, with the improvements leveling out at higher number of tests. A comparison between the results from approach 1 and approach 2 indicate that for a given number of successful proof load tests, the updated reliability indices from approach 2 are higher than those obtained from approach 1, as long as the number of tests is less than 30 to 40 tests.

As a result, it is recommended that approach 2 be adopted in any decision making exercise that is aimed at providing a framework for selecting the number of proof load tests and the level of the proof load based on a reliability analysis. The results of this method are reliable and effective and are expected to lead to more economical proof load test programs for piles.

# CHAPTER 5

## DECISION MAKING

### 5.1 Introduction

In previous chapters, we have used results from different proof-load test scenarios to update the pile reliability according to two different approaches. Accordingly, we have been studying the effects of the number of the tested piles and the proof-load level on the updated reliability. In this chapter, we will be constructing, based on the second approach, a simple, logical, and practical decision-making framework for choosing the number of proof-load tests and the magnitude of the proof load that would maximize the value of information of a test program. Such analysis will study the possibility of using lower design factors of safety while incorporating pile-load test programs in the study and this will be based on the saving in total costs associated with each program.

### 5.2 Decision Making Framework

With the mathematical formulation devised in approach 2 for updating the probability of failure given results from proof load tests, a rational decision making framework could be envisaged to facilitate the choice of a load test program that has the maximum expected benefit to the project. The proposed framework is presented in this section and is supplemented with a practical example that illustrates its use.

In the context of a decision analysis, the main decision alternatives that are relevant to the problem at hand are (1) the proof load level  $r_{proof}$  and (2) the number of

proof load tests to be conducted,  $n$ . Since the decision regarding the optimal combination of proof load level and number of tests has to be made prior to conducting any real load testing in the site, the decision analysis will have to be based on a pre-posterior approach (Ang and Tang 1984) that incorporates the different possible outcomes of any given decision alternative. For example, consider one decision alternative (among several other alternatives) which entails conducting 3 proof load tests at 2 times the design load. The outcomes of such a test program could include four possible scenarios: (1) all the piles will fail, (2) all the piles will survive, (3) one pile fails and two survive, and (4) one pile survives and two fail. The likelihood of obtaining any of the different outcomes  $\theta$  can be determined using the prior knowledge about the pile capacity distribution at the site.

For each of the potential test outcomes that are associated with a given decision alternative, the updated reliability index could be evaluated. The resulting updated reliability index will depend on the outcome, with relatively high indices expected for cases involving positive tests and relatively low indices for cases involving failures. These reliability indices could be lower or higher than a target reliability index that is set for the piles in the project. In the decision making framework that is proposed in this paper, it will be assumed that the decision to be made following any potential test outcome should involve a pile design with a typical target reliability index of 3.0. As a result, outcomes where the updated reliability index is below 3.0 indicate that the allowable capacity per pile (design load per pile) will have to be reduced in light of the load test results. On the other hand, outcomes where the updated reliability index is above 3.0 allow for an increase in the allowable capacity (design load) per pile in comparison to the base case.



The allowable pile capacity (design load) could be calculated by utilizing the updated capacity distribution for that particular outcome and searching for the mean load (design load) that would ensure that the target reliability level is achieved. For example, for the test outcomes where the updating process resulted in reliability indices that are greater than 3.0, the mean of the load distribution could be increased systematically until the reliability index of 3.0 is achieved. The mean load that ensures a reliability index of 3.0 is then selected as the new design load for the piles at the site.

The consequences that are associated with each possible test outcome for any decision alternative could be quantified in light of the updated pile design load that ensures a  $\beta_{\text{target}}$  of 3.0. From a practical design standpoint, any increase or decrease in the allowable capacity per pile as a result of conducting the proof load tests can be translated to (1) reduction/increase in the total number of piles required to support the superstructure loads without changing the geometry of the piles, or (2) reduction/increase in the geometry of the piles (length and/or diameter) without changing the total number of piles required. In this paper, the consequences associated with the outcomes of any decision alternative are assumed to be reflected in the total number of required piles without resorting to any change in the pile length or diameter. Any attempt to revise the length or diameter of the pile design as a result of the updating process may be prohibitive since such revisions may require detailed knowledge regarding the contribution of skin friction and end bearing to the updated design load. In a pre-posterior decision analysis that is conducted in the design stage of the project, the potential outcomes from proof load tests do not include enough information to allow for revising the length or diameter of the piles with certainty.

Based on the above, the consequences of any potential test outcome will be reflected in the benefits/costs associated with reducing/increasing the required number of piles to support the superstructure load without changing the geometry of the piles under consideration. Inherent in this approach are the assumptions that the piles are perfectly plastic and are loaded equally in the vertical direction. Despite of these simplifying assumptions, this approach is considered practical and could be of value in making decisions at the design stage of the project and even at the construction stage by adopting a flexible construction sequence that will allow for adding or eliminating piles at the site depending on the results obtained from the proof load test program which could be implemented as the construction of the piles is underway.

To quantify the benefit associated with each possible test outcome in the decision analysis, one could calculate the required number of piles to support the superstructure load for each test outcome. For the case where the allowable capacity per pile increases due to successful test results, the required number of piles decreases and the opposite is true for outcomes in which the updated allowable capacity per pile decreases. The financial benefit is reflected in the cost savings associated with this reduction in the number of piles. On the other hand, there is a negative financial cost that is associated with the cost of conducting the load test program alternatives and the cost of replacing failed piles when relevant. The net benefit of any test outcome can be calculated by subtracting the benefits due to reducing (or cost due to increasing) the number of piles in the site from the costs associated with conducting the proof load tests including the cost of replacing failed piles. The net benefit corresponding to a potential outcome  $\theta_j$  of decision test alternative  $a_i$  could be expressed as  $B(a_i, \theta_j)$ .

Once the net benefit of all the test alternatives and their associated potential outcomes are calculated, the “expected” benefit  $E(B_{a_i})$  of each alternative load test program  $a_i$  can be calculated by multiplying the net benefit  $B(a_i, \theta_j)$  of each outcome by the likelihood of occurrence of that outcome  $P(\theta_j)$  and summing the contributions of all outcomes such that:

$$E(B_{a_i}) = \sum_{j=1}^n B(a_i, \theta_j) P(\theta_j) \quad (5.1)$$

The alternative pile testing program that has the highest expected benefit could then be selected as the test alternative that has the highest value.

### 5.3 Illustrative Decision Making Example

To illustrate the practicality and value of the proposed decision making framework, a practical design example that involves piles that are driven in a site consisting of medium dense sand is considered. The pile dimensions and soil profile utilized in the design example were adopted from Goble (1996). A similar design example was used by Najjar and Gilbert (2009b) to illustrate the importance of incorporating a lower-bound capacity in the design of driven piles. The pile design adopted in this paper consists of closed-ended steel pipe piles with an outside diameter of 355 mm and a length of 25 m. A simplified schematic of the soil profile at the site is shown in Fig. 5.1. The predicted nominal axial capacity of a single driven pile is 1.8 MN (as calculated by the API method-API 1993) and the lower-bound is calculated in Najjar and Gilbert (2009b) to be equal to 0.9 MN accounting to about 0.5 of the nominal axial capacity.

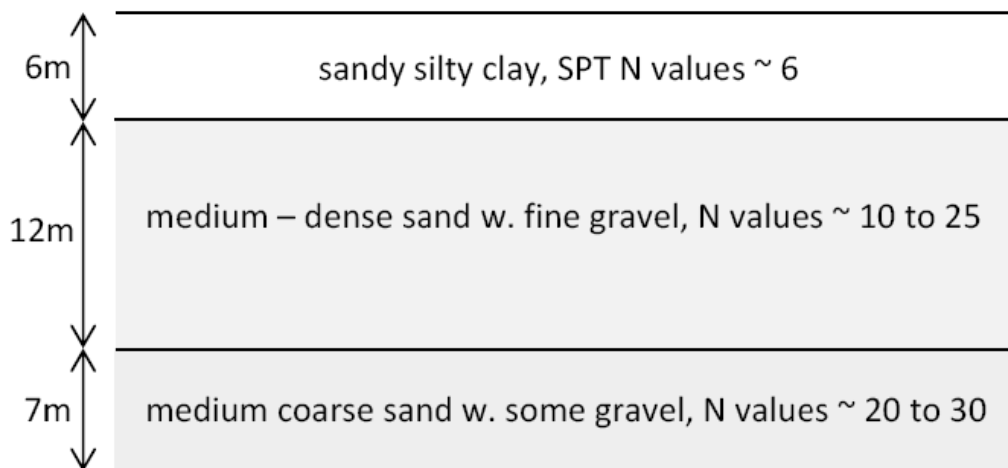


Figure 5. 1 - Illustrative soil profile for example case study (Goble 1996)

For the purpose of illustrating the decision making methodology, hypothetical cases that involve different superstructure loads will be adopted. For simplicity, it will be assumed that the superstructure load will be supported by a group of identical steel pipe piles that will share the superstructure load equally. In addition, it will be assumed that the piles are separated enough to eliminate any pile group effects that could affect the efficiency of the pile group.

Since the design example involves the case of driven steel pipe piles in sands, the coefficient of variation of  $r_{\text{mean}}$  was assumed to be equal to 0.5 (and not 0.4) as reported in Gilbert, Najjar, and Choi (2005). In addition, the predicted ultimate capacity of 1.8 MN was assumed to be equal to the mean of  $r_{\text{mean}}$  since the analysis of the database assembled in Gilbert, Najjar, and Choi (2005) for driven steel pipe piles in sandy soils indicate that the API (1993) method is relatively unbiased. Based on these assumptions, if no pile load tests are to be conducted, a reliability-based analysis indicates that a mean factor of safety in the order of 3.5 is required to achieve a target reliability index of 3.0 for the piles at the site. As a result, the allowable capacity

(design load) per pile could be estimated to be around 0.514 MN if no proof load tests are to be conducted at the site. If the total load that is applied by the superstructure  $S_{\text{super}}$  is given, the required number of piles could be calculated as:

$$N_{\text{required}} = S_{\text{super}} \text{ (MN)} / 0.514 \text{ (MN)} \quad (5.2)$$

It could be argued that the required number of piles  $N_{\text{required}}$  could be reduced assuming that a proof load pile testing program is implemented in the site. To limit the scope of the analysis, it will be assumed that the load testing program will be conducted on piles that have been designed with a reduced factor of safety of 2.0 as is the convention. The decision framework will be limited to determining the optimum proof load level and the optimum number of tests to be conducted. For simplicity, the number of pile load tests to be considered as decision alternatives is 1, 2, 3, 5, 7, and 10 proof tests to be conducted at load levels of 1.5, 2, and 2.5 times the design load or 0.75, 1.0, and 1.25 times the predicted capacity. These decision alternatives are presented in the context of a simplified decision tree in Fig. 5.2. Detailed calculations pertaining to the case of the test alternative that includes a proof load level of 2.0 times the design load are presented in Fig. 5.2. For each of the alternative load test programs, the likely outcomes and their associated likelihoods are presented.

For each of the test outcomes, the updated reliability indices were calculated and presented in Fig. 5.2. For the case of the 3 tests, the reliability indices range from a low value of 0.98 for the case with 3 pile failures to a high value of 3.5 for the case involving 3 pile successes. Following the results of the updating process, the updated allowable capacity per pile (design load per pile) is calculated for each outcome to ensure a  $\beta_{\text{target}}$  of 3.0 (see Fig. 5.2). As an example, the revised allowable capacities per pile for the different test outcomes for the case involving 3 proof load tests range from

0.47 MN to 1.03 MN for test outcomes with 3 failures and 3 successes, respectively. For comparison, the allowable pile capacity that results in a target reliability index of 3.0 for the case where no proof load tests are conducted is equal to 0.514 MN. Based on the updated allowable capacity (design load) per pile  $S_{design}$ , the required number of piles could be revised as:

$$N'_{required} = S_{super} \text{ (MN)} / S_{design} \text{ (MN)} \quad (5.3)$$

If the cost of conducting a proof load test ( $C_{r,proof}$ ) is assumed to be directly proportion to the magnitude of the proof-load level, and if the cost of replacing  $(n-k)$  failed piles is assumed to be simply equal to the actual cost of the failed piles ( $C_{pile}$ ), the net benefit of any test alternative and its associated outcomes could be calculated as:

$$B(a_i, \theta_j) = (N'_{required} - N_{required}) \cdot C_{Pile} - n \cdot C_{r,proof} - (n-k) \cdot C_{Pile} \quad (5.4)$$

Where  $C_{pile}$  is the cost of manufacturing and installing a closed-ended steel pipe pile with a diameter of 355 mm and a length of 25 m and  $C_{r,proof}$  is the cost of conducting a single proof load test.

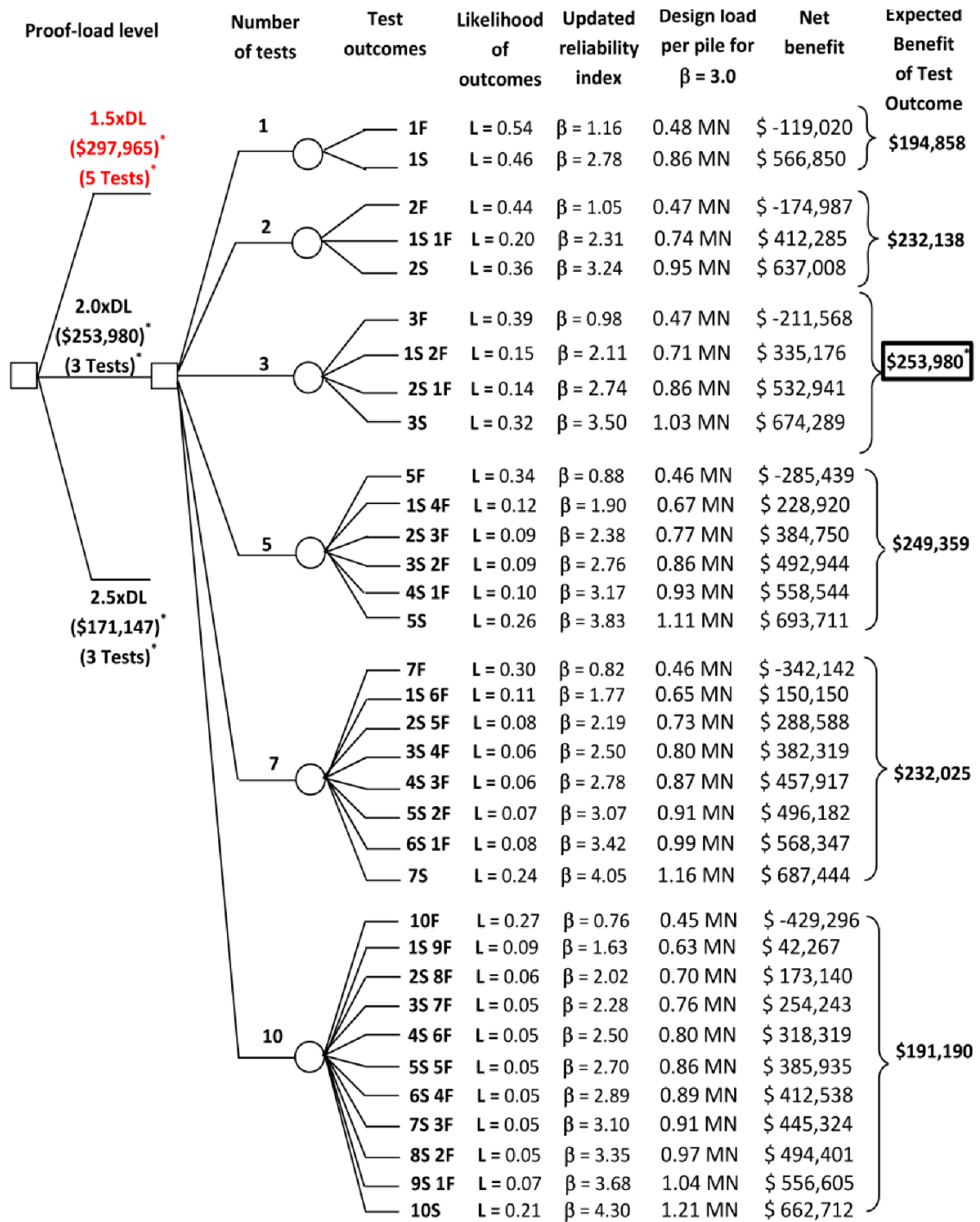


Figure 5. 2 - Proposed decision tree for choosing the best proof load test program

It should be noted that the choice of a  $\beta_{\text{target}}$  of 3.0 is in line with typical target reliability indices that are considered acceptable in the design of foundation systems. It could be argued that lower target reliability levels ( $\beta_{\text{target}}$  of 2.0 or 2.5) could be

considered acceptable for piles that carry the load as part of a system as is the case in the example presented in this paper (see Zhang 2004). The proposed decision making methodology allows for the selection of any target reliability index to be used as a basis for making the decision. It is expected that the choice of the target reliability index will have a significant impact on the optimal load test program.

For the example under consideration, it is assumed (as a base case analysis) that (1) the total superstructure load ( $s_{\text{super}}$ ) to be supported by the pile group is equal to 150 MN, (2) the cost of production and installation of a single closed ended steel pipe pile is \$ 5000, and (3) the cost of running a static proof load test on such piles is in the order of \$10 per kN of test load. Based on these realistic base case assumptions, the required number of piles  $N_{\text{required}}$  prior to conducting any proof load tests can be calculated to be around 292 piles and the net benefits of the different test outcomes for the different decision alternatives are presented in Fig. 5.2. The calculated values of the net benefit were used to calculate the “expected” net benefit of any decision alternative using Equation (5). The resulting values are presented in Fig. 5.2 for the case involving a proof load level of twice the design load. For this particular proof load level, the results on Fig. 8 indicate that the alternative pile testing program that has the highest expected benefit (\$ 253,980) is that corresponding to  $n = 3$ . When the same exercise was repeated for the other load test program alternatives that involve proof tests with smaller (1.5xDL) and larger (2.5xDL) proof load levels, the expected net benefits of the different decision alternatives changed and are presented in Fig. 5.3. The results indicate that the proof-load test program alternative that is based on conducting 5 proof load tests up to a proof load level of 1.5 times the design load yields the largest expected benefit (\$ 297,965) among all other test alternatives.



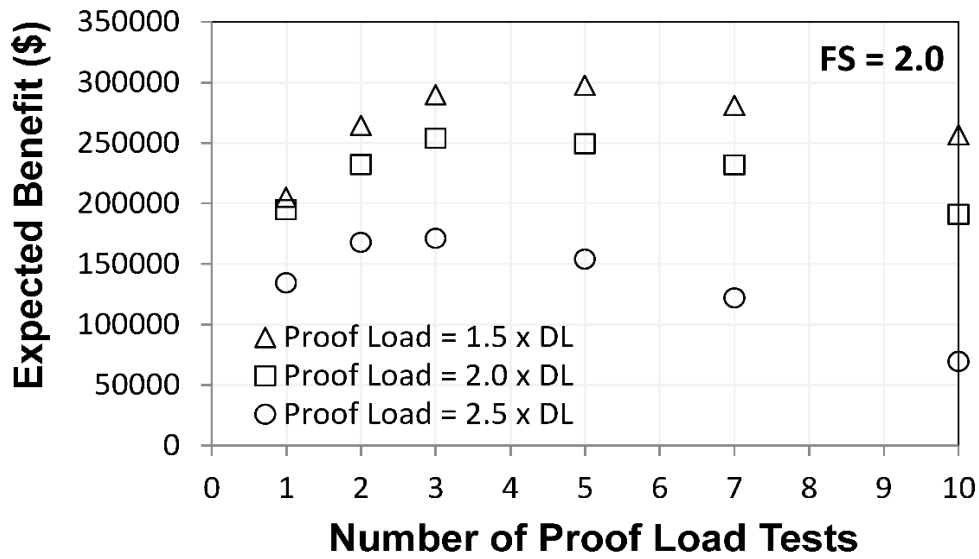


Figure 5.3 - Expected benefit of alternative proof load test programs

#### 5.4 Discussion

The illustrative example presented above indicates that the optimal proof load level for the case considered is only 1.5 times the design load. Given that a factor of safety of 2.0 was adopted in the decision making exercise, the optimal value of the proof load accounts to 0.75 times the mean pile capacity. This proof load value could be considered to be relatively smaller than that typically used in practical projects whereby a proof load level of 2 times the design load is usually adopted. The optimal proof load level is also considered to be much smaller than 3 times the design load (1.5 times the mean capacity) which has been shown by previous studies to have the highest impact in updating the reliability of piles at a site.

To investigate the reasons leading to the choice of the relatively smaller proof load level (0.75 times the mean capacity) as the optimal proof load, the following analysis was conducted and portrayed in Fig. 5.4. The analysis involves the two extreme proof load levels adopted in the case study ( $r_{proof} = 1.5DL$  and  $r_{proof} = 2.5DL$ ). For each

case, the factor of safety that is required to achieve a target reliability index of 3.0 given the results of the proof load test (after updating) was calculated for all the test alternatives considered (1, 2, 3, 5, and 7 tests) and for all possible test outcomes (including failures and successes). The resulting factors of safety (see Fig. 5.4), when compared to the factor of safety of 3.5 which is needed for the case where no tests are conducted, reflect the benefit of the different test alternatives and associated outcomes. Required factors of safety that are less than 3.5 indicate that the outcomes of the test alternative will have a beneficial effect on the design and vice versa.

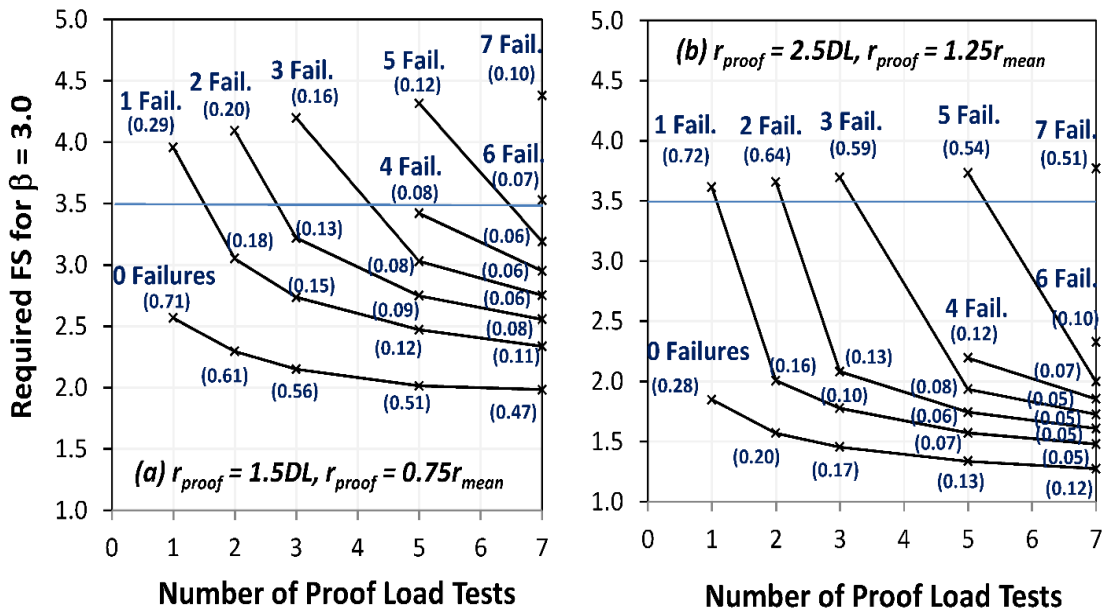


Figure 5. 4 - Comparison between results of cases with (a)  $r_{proof} = 1.5DL$  and (b)  $r_{proof} = 2.5DL$

A comparison between the required factors of safety for the two proof load test cases (Figs. 5.4a and 5.4b) indicates that for any given test alternative and test outcome, much smaller required factors of safety are needed for the case of the higher proof load in Fig. 5.4b. The relatively lower factors of safety reflect the added value of conducting proof load tests at a high level (2.5 times the design load) particularly for

cases involving positive tests. However, the factor that is not reflected in the required factor of safety and which will eventually render the smaller proof load level as the optimal proof load level is the likelihood associated with each test outcome. These likelihoods are shown on Fig. 5.4 (in parenthesis) next to each test outcome. For the cases involving the smaller proof load level, the likelihood of having 100% positive tests for all the tested piles ranges from 0.47 to 0.71. This associated range of likelihood for the case of the higher proof load level is only 0.12 to 0.28. On the other hand, there is 51% to 72% chance that all the piles will fail for the case of the higher proof load level compared to 10% to 29% for the case involving the smaller proof load level. Since the decision regarding the optimal proof load level is based on the “expected” benefit, the likelihoods of successes and failures will play a significant role in the decision making framework, rendering the smaller proof load level an optimal decision alternative in the case analyzed.

The illustrative example that was analyzed in the previous section indicates that the proposed decision making framework is project specific and reflects the contribution of all the factors that affect the design process including: (1) the site-specific soil profile and properties (reflected in the mean values of  $r_{LB}$  and  $r_{mean}$  and in the coefficients of variation of  $r_{LB}$ ,  $r_{mean}$ , and  $r$ ), (2) site specific loads (reflected in the superstructure load  $s_{super}$ ), and (3) project-specific load testing parameters (reflected in the cost of the pile  $C_{pile}$  and cost of conducting a load test  $C_{r,proof}$ ). It is expected that the proof-load test program that results in the largest expected benefit to the project would depend on the above factors.

To illustrate the sensitivity of the decision to the superstructure load, cost of manufacturing and installing piles, and the cost of implementing the load test program,

the decision analysis of the illustrative example was repeated for several possible scenarios where the total super structure load was varied from 25 MN to 300 MN. For each load scenario, the cost of pile testing was taken as \$10/kN (base case used in example) and \$20/kN, and the cost of the piles (including installation) was taken as \$2000, \$5000 (base case) and \$8000 per pile. For each scenario, the number and level of proof load tests that would maximize the expected benefit were calculated. Results of all tests indicate that the optimum proof load level was 1.5 times the design load. The optimal number of proof load tests for each scenario is presented in Table 5.1.

Table 5. 1 - Sensitivity of Optimal Pile Load Test program to Input Parameters

Load(MN)	No. of Piles, $N_{required}$	Optimal Number of Tests for $C_{r,proof} = 10 \text{ \$/kN}$			Optimal Number of Tests for $C_{r,proof} = 20 \text{ \$/kN}$		
		$C_{Pile} = \$2000$	$C_{Pile} = \$5000$	$C_{Pile} = \$8000$	$C_{Pile} = \$2000$	$C_{Pile} = \$5000$	$C_{Pile} = \$8000$
25	49	0	1	2	0	1	1
50	97	1	2	3	1	1	2
100	195	2	3	5	1	2	3
150	292	3	5	7	2	3	3
300	584	5	7	10	3	5	5

Interestingly, results indicate that the optimal number of tests is highly dependent on the applied superstructure load, since high loads require a larger number of piles. For example, for the smallest superstructure load considered (25 MN), the required number of piles if no tests are to be conducted is 49 piles. The optimal number of proof load tests for this case is only one test for the base case considered. If the superstructure load is assumed to increase to 300 MN (584 piles required in this case), the optimal number of tests increases to 7 tests. The sensitivity of the optimal number of tests to the total number of piles in the site is expected since the benefits associated with reducing the number of piles will outweigh the costs of implementing the load test

program as the number of piles increase. It is interesting to note that the optimum number of piles is found to be around 1% to 2% of the total number of piles required for the case where no pile load tests ( $FS = 3.5$ ) are conducted. These percentages are in line with typical values that are currently being implemented in the pile design and testing industry for test programs involving static proof load tests.

Results in Table 5.1 also indicate that the optimal number of proof load tests depends on the cost of the pile and the cost of the proof load test. For the base case example, as the cost of piles increases from \$2000 to \$8000 per pile, the optimal number of tests increases from 1 to 3 for a superstructure load of 50 MN (97 piles in the site) and from 5 to 10 tests for the case of a load of 300 MN (584 piles in the site). These results are expected since they indicate that as the cost of manufacturing and installing a pile increases, the benefits associated with the cost savings due to reducing the number of piles increase, allowing for conducting more pile load tests. On the other hand, as the cost of conducting the pile load test increases, the optimal number of tests has to be reduced so as not to negatively affect the net benefit significantly. This is clearly illustrated in Table 5.1 where the optimal number of tests is found to decrease for cases involving test costs of  $20 \text{ \$/kN}$  compared to the base case where the test cost is  $10 \text{ \$/kN}$ .

## CHAPTER 6

### CONCLUSIONS AND FUTURE WORK

#### 6.1 Conclusions

A rational decision making framework that is based on reliability-based principles was presented in this thesis to address the current world-wide inconsistencies that are inherent in the design of proof-load test programs for piles. The proposed methodology will probably reduce the need for conducting unnecessarily costly pile load test programs in some cases and insufficient or deficient load test programs in others. In both cases, the proposed decision framework constitutes a tangible solution to the problem of depleting resources due to the lack of rational methodologies for designing pile load test programs. The methodology has been proven to be simple, realistic, and efficient in quantifying the value of different test program alternatives and could be used in the future as a basis for recommending international guidelines on the selection of efficient pile load test programs in civil engineering design and construction.

Several simplifying but realistic assumptions have been adopted in this study to simplify the mathematical complexities that are associated with the Bayesian updating and reliability calculations required. Another major simplifying assumption that was required to quantify the benefits of the proof-load test alternatives involved the assumption that all piles in the site are part of a group and that the superstructure load is transferred to the piles equally. In addition, it was assumed that the major design

decision following a pile load test program involves adding or reducing the number of piles in the site, without any change to the geometry.

An illustrative example whereby the proposed decision making framework was utilized to choose the number and level of proof load tests for steel pipe piles that were driven in a site with medium dense sand resulted in the following observations:

- The optimum proof load level that resulted in the maximum benefit to the project was 1.5 times the design load or 0.75 of the ultimate pile capacity, irrespective of the number of piles in the site, the cost of the pile, and the cost of the test.
- The optimum number of tests was found to be a function of the number of piles (superstructure load) and the costs of the pile construction and testing.
- As the number of piles in the site increases (due to large superstructure load), the optimal required number of proof load tests also increase. Interestingly, the optimum number of pile load tests is found to be around 1% to 2% of the total number of piles required for the case where no pile load tests are conducted. These percentages are realistic and in line with typical values adopted in the industry.
- Finally, the optimal number of pile load tests is found to increase as the cost of pile construction and installation increases and as the cost of implementing the pile test program decreases.

## **6.2 Recommendations for Future Work**

Future work should be mainly focused on two main areas: (1) relaxing most (if not all) of the assumptions and constraints that were made in the proposed decision

framework with regards to the inability to introduce changes in the geometry (length or diameter) of the pile in the current framework, and (2) developing the Bayesian methodology to allow it to update the within-site variability in the pile capacities in a given site.



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