



AMERICAN UNIVERSITY OF BEIRUT

SIMULATING PATIENT FLOW AT THE LAUMC  
RADIOLOGY DEPARTMENT

by  
MARIO IBRAHIM YAZBECK

A thesis  
submitted in partial fulfillment of the requirements  
for the degree of Master in Engineering Management  
to the Department of Industrial Engineering and Management  
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at the American University of Beirut

Beirut, Lebanon  
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AMERICAN UNIVERSITY OF BEIRUT

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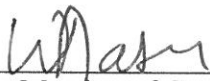
By  
MARIO IBRAHIM YAZBECK

Approved by:

\_\_\_\_\_  
[Dr. Bacel Maddah, Professor, Advisor]  
[Department of Industrial Engineering and Management]

  
\_\_\_\_\_  
Advisor

\_\_\_\_\_  
[Dr. Walid Nasr, Associate Professor]  
[Department of Industrial Engineering and Management]

  
\_\_\_\_\_  
Member of Committee

\_\_\_\_\_  
[Dr. Hussein Tarhini, Assistant Professor]  
[Department of Industrial Engineering and Management]

  
\_\_\_\_\_  
Member of Committee

Date of thesis defense: May 2, 2018

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## AN ABSTRACT OF THE THESIS OF

Mario Ibrahim Yazbeck for Masters of Engineering Management  
Major: Engineering Management

Title: Simulating Patient Flow at LAUMC Radiology Department

This thesis is on applying Operations Research (OR) techniques and Simulation to reduce patient waiting times and improve resource utilization at the Radiology Department of the Lebanese American University Medical Center (LAUMC) previously known as Rizk Hospital.

A study was conducted to collect data and analyze the actual behavior of the system, and extract key performance indicators (KPIs); for example, patient inter-arrival times, various service (procedure) times, patient waiting times, and equipment utilization. A simulation model was then developed in Arena to process the collected data. The model takes into account realistic factors such as patient no-shows and lateness, unscheduled patient “walk-ins”, and patients undergoing more than one radiology procedure per visit (such as X-Ray and mammography). The simulation model is used to find bottlenecks causing congestion in the system flow, and to suggest changes leading to improvements. Such changes include manipulating (adding/removing) resources (radiology machines or technicians) in order to reduce waiting time and ease the flow of patients in the system.

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# CHAPTER 1

## INTRODUCTION AND LITERATURE REVIEW

Over the past five decades, increasing health care costs have pushed researchers and healthcare professionals to look up ways to improve the efficiency of healthcare operations (Jun, S.H. Jacobson, and J.R. Swisher, 1999). Discreet event simulation is an Operations Research tool that assists decision makers in checking the efficacy of a system while accounting for uncertain/random behavior. Its flexibility allows to try out multiple *cost-free* alternatives or to even design new workflow methodologies that could improve the behavior of a whole system without altering its existing physical form. It can also assist in forecasting resource allocation (staffing) for the multiple, interacting activities and serve as a support for decision makers to achieve their objectives.

The application of discreet event simulation in healthcare is becoming an increasingly popular choice for researchers mainly because of the large number of successes in the field (Jun et al, 1999). Simulation tools, as well, are meeting the researchers half way by providing packages designed to meet healthcare simulation needs. Since the 1960's OR models have been successfully used to assist clinical decision-making, facility location and planning, resource allocation, evaluation of treatments, and organizational redesign. Simulation is one of the most commonly used OR approaches, and is widely regarded as the technique of choice in healthcare because of its power and flexibility (Davies, R and H. Davies. 1994).

Several excellent review articles have appeared that examine conducting a discrete-event simulation study in health care clinics, England and Roberts (1978) provide a thorough and comprehensive survey on the application of discrete-event simulation in 21

health care settings (including laboratory studies, emergency services, and the national health care system). Their detailed survey cites 92 discrete-event simulation models out of 1,200 models reviewed, including all published models through 1978. Klein et al. (1993) present a bibliography that includes operational decision making, medical decision making, and system dynamics planning models. Smith- Daniels et al. (1988) present a literature review pertaining to acquisition decisions (e.g., facility location, aggregate capacity, and facility sizing) and allocation decisions (e.g., inpatient admissions scheduling, surgical facility scheduling, and ambulatory care scheduling), including several operations research methodologies, such as heuristics, Markov chains, linear programming, and queuing theory; as well as discrete-event simulation. Jun et al. (1999) presents a survey of discrete-event simulation applications to clinic design and analysis.

This literature review cannot be complete without the citing of Dr. David Gaba who, amongst the multitude of articles and papers that I have reviewed, has contributed to about half of the editorials. He was the founding Editor-in-Chief of the *Simulation in Healthcare* community of practice that has released 38 articles, classified as editorials and written by 27 authors from inception, in 2006 till April 2016. Dr. David Gaba wrote 19 of them.

In 2007, as early successes of embedding simulation were seeing the light, Dr. Gaba and Dr Dan Raemer sought to establish practice standards and metrics to develop and encourage the use of simulation not as an optional extra, but as a crucial component to be integrated. Since then three essential components for effective simulation education were isolated training resources, trained educators and curricular institutionalization. Right now there are three additional journals with a strong focus on simulation in healthcare:

Advances in simulation, Clinical Simulation in Nursing as well as BMJ Simulation and

Technology-Enhanced Learning. This is a solid sign that the use of simulation in healthcare is escalating and proving to be very efficient by assisting doctors, Hospital's HR personnel, and management in taking decisions or choosing alternative scenarios.

This paper is on the simulation and modeling of the Radiology Department at the Lebanese American University Medical Center (LAUMC). Several papers tackled specifically radiology department simulation in chronological order like Ben Lev et al's "Patient Flow and Utilization of Resources in a Diagnostic Radiology Department: Analysis by Simulation Techniques" (1972), and "A simulation model of a diagnostic radiology department" by P. Ciaran O'Kane (1981). Klafehn also tried to reduce the waiting times of non-admission patients going through a radiology department in his paper "Impact points in patient flows through a radiology department provided through simulation" (1987). On a more recent note MA Centeno et al. studied the Radiology department at Jackson Memorial Hospital and suggested improvement scenarios in their paper "Project and process improvements in healthcare organizations: a simulation study of the radiology department at JMH" (2000) as well as Johnston et al.'s "Modeling radiology department operation using discrete event simulation" (2009).

Like this paper, the aforementioned studies were focused on reducing patient waiting time and optimizing resource utilization. This thesis has two distinctive features that were not handled in the above articles:

- Highly accurate operation service time distribution sets and patient inter-arrival rates mined from thousands of hospital records stretching over an overall duration of two quarters.
- Accounting for multiple successive operations undergone by one patient.

The remainder of this thesis is organized as follows:

Chapter 2 handles the introduction and motivation behind the thesis topic.

Chapter3 explains the workflow of the model and the system's various activities.

It also analyzes the patient inter-arrival (IA) time and operations' service time distributions. Chapter 4 will handle the interpretation of the simulated results and;

Chapter 5, the conclusion, will provide model assessments and recommended resource numbers, to optimize the system's operations and resource usage.

## CHAPTER 2

### MOTIVATION AND PROJECT OVERVIEW

In this chapter we will discuss the motivation behind taking up the Simulation of the Radiology Department at LAUMC project, as well as introducing the topic and laying out the thesis plan.

#### **2.1 Motivation**

Although the growth of OR and simulation has been relatively fast, its use in the healthcare sector has been quite limited and sometimes faced with a lot of resistance from practitioners. Over the last two decades, we have been gradually moving into an era of acceptance and open-mindedness towards new practices; this change in times has caused a rapid spurt in the use of simulation to assist/optimize healthcare procedures and even patient safety and wellbeing.

The LAUMC is committed to excellence in patient care, clinical outcomes, academics and research, part of their mission as a University Hospital, therefore, their request for assistance in reducing patient waiting times and increasing operations efficiency at their radiology department wasn't a surprise.

OR and simulation have proven to be the “game changers” in tackling challenging, large scale, industrial problems stretching from warfare, supply chain, construction and logistics, to most recently healthcare. Consequently, our LAUMC model will be resolved using a combination of OR and simulation to provide the hospital with accurate, reliable and cutting edge solutions to meet their department needs.



## 2.2 Introduction

The former Rizk Hospital, an iconic hospital of the Sassine, Achrafieh area, was recently acquired by the LAU and branded LAUMC (Lebanese American University Medical Center) to launch its new Medical Doctor program. The new management has since gone through a long list of challenges and improvements in order to advance the hospital and make it both student and patient ready.



*Figure 1- LAUMC Logo*

The radiology department there offers several medical imaging techniques to diagnose and treat diseases. Here is a list of the provided services at the LAUMC Radiology department with a brief description of each:

Table 1- LAUMC Radiology Procedures List (<http://www.radiologyinfo.org>)

Procedures	Description
BMD	<b>Bone mineral density</b> scan, is a special type of X-ray that measures bone mineral density. It provides information about bone strength or fragility and the risk of fractures or broken bones. The higher the density, generally, the lower the risk of fracture.
CT	<b>Computerized tomography</b> is a way of using X-rays to take pictures or images in very fine slices through the part of the body that the doctor has asked to be investigated.
INTXR	The <b>internal XR</b> is a special x-ray technique that makes it possible to see internal organs in motion.
MAMMO	A <b>mammography</b> is performed when a person, their doctor or another health professional discovers unusual signs or symptoms in one or both breasts, i.e. a lump, tenderness, nipple discharge or skin changes. The mammogram confirms whether the changes are benign (non-cancerous) and no treatment is needed, or whether the changes indicate breast cancer and further tests and treatment will be required.
MR	<b>Magnetic resonance imaging</b> is a medical imaging technique used in radiology to form pictures of the anatomy and the physiological processes of the body in both health and disease. MRI scanners use strong magnetic fields, radio waves, and field gradients to generate images of the organs in the body.
NM	<b>Nuclear medicine</b> scans use a special camera (gamma) to take pictures of tissues and organs in the body after a radioactive tracer is put in a vein in the arm and is absorbed by the tissues and organs. The radioactive tracer shows the activity and function of the tissues or organs.
PET	<b>Positron emission tomography</b> is a nuclear medicine exam that produces a three dimensional image of functional processes in the body. A PET scan uses a small amount of a radioactive drug to show differences between healthy and diseased issue. The diagnostic images produced by PET are used to evaluate a variety of diseases.
RF	<b>Radiography and Fluoroscopy</b> both use x-ray beams to acquire an image (radiography) or video (fluoroscopy) of internal bones and organs. Fractures and arthritis are commonly well imaged by radiography whereas fluoroscopy studies the upper gastrointestinal series to evaluate patients with suspected gastro-esophageal reflux and other problems such as swallowing difficulty.
US	<b>Ultrasound</b> is a type of imaging that uses high-frequency sound waves to look at organs and structures inside the body. Health care professionals use it to view the heart, blood vessels, kidneys, liver, and other organs. During pregnancy, doctors also use ultrasound to view and examine the fetus.
XR	<b>X-ray or radiography</b> uses a very small dose of ionizing radiation to produce pictures of the body's internal structures. X-rays are the oldest and most frequently used form of medical imaging. They are often used to help diagnosed fractured bones, look for injury or infection and to locate foreign objects in soft tissue. Some x-ray exams may use an iodine-based contrast material or barium to help improve the visibility of specific organs, blood vessels, tissues or bone.

One of the main issues the new management is tackling is the long waiting times of patients for radiology procedures. With an average waiting time of more than forty five minutes for the X-RAY machine, something had to be done. The LAUMC got in touch with our research team in the hopes of improving its systems' performance using operations research and simulation.

Several visits were conducted to the department and with the assistance of Dr. Daniel Mahfoud the Chief of Radiology & Nuclear Medicine and Ms. Jessica Saad, Senior Radiology Coordinator. A preliminary simulation model of Radiology Department at the LAUMC was built which is described in chapter 3.

## CHAPTER 3

### THE MODEL

In this chapter, we will discuss the several steps required to build a successful simulation model and, how these steps were implemented at the LAUMC Radiology Department. In section 3.1, we discuss data gathering and input analysis while in section 3.2 we explain how our simulation model was built in Arena.

#### **3.1 Data Gathering and Input Analysis**

The most crucial and, at the same time, most challenging task when developing a simulation model is data gathering. We were lucky that the LAUMC had already installed time keeping software that stores when radiology patients arrive to the front desk start their service and when they complete it. Nevertheless all big data contains errors and needs cleaning and processing. Out of 32,700 records, stretching between spring 2014 and spring 2015, only about 7,000 were retained for study after confirming that the data they hold is accurate i.e. removing records where durations surpassed the normal range. Consolidating multiple entries also accounted for reducing our data size, i.e. if a patient had to undergo five XR procedures then he would have five records; we combined these five records into one by taking the minimum start time and maximum end time grouped by patient name, date of visit and procedure type.

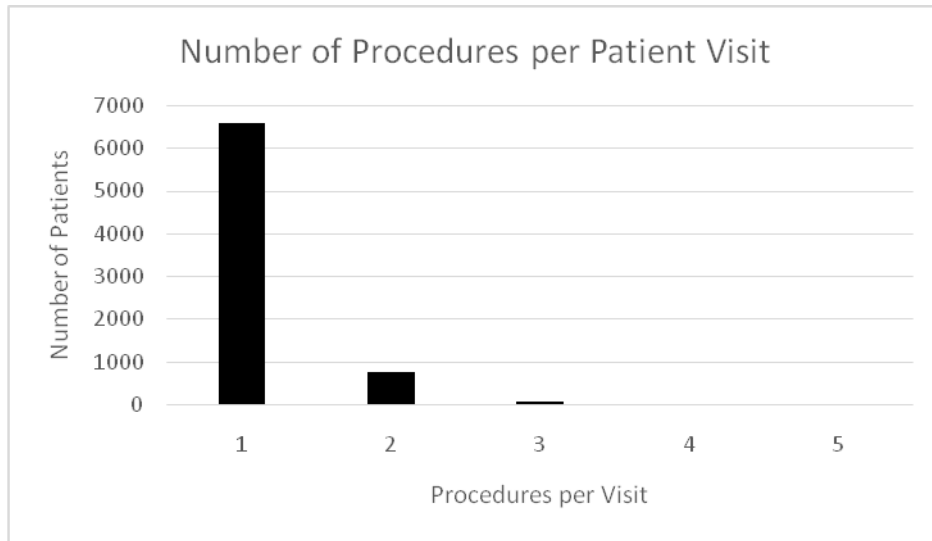


Figure 2- Number of Procedures per Patient Visit

From this data, several KPIs are extracted. Here is a brief list of each, to discuss in more detail over the following sections:

- Patient inter-arrival time
- Procedure service time
- Procedure demand

### 3.1.1 Patient Inter-Arrival Time

The time between arrivals is the kick-starter of every model. In the actual system, the cycle starts when a patient arrives to the hospital; in the model, however, it is the “Start” node that injects the patients into the system following a pre-set inter-arrival time that defines the rate at which these patients arrive to the hospital.

After studying the daily arrival numbers of patients and separating them into time frames to emphasize peak hours of congestion the below table was populated. Similar arrival numbers were grouped under the same time epoch.

Table 2- Rad. Patient Arrival Rate per Time Frame

<b>Time Frame</b>	<b>Rate (per hour)</b>	<b>Mean</b>	<b>Variance</b>
8am to 9am	7	7.17	7.53
9am to 10am	12	12.3	12.56
10am to 11am	16	15.86	15.2
11am to 12pm	12	12.03	12.25
12pm to 2 pm	9	8.66	8.67
2pm to 3 pm	8	7.73	7.974
3pm to 6pm	6	6.73	6.9

Having very close mean and variance values these averages can be safely considered as means for a Poisson distribution and inputted into Arena’s scheduler as arrival rates. Our arrival rates were also double checked using Arena Input analyzer to verify their fit as Poisson distributions; an example can be found in Appendix A where the distribution summary of the time epoch 10 to 11am is showcased. These rates were deduced from arrivals exclusive to the radiology department. However the daily radiology patient count amounts to 30% of the hospitals’ gross total; so in order to accommodate for the actual patient impact on the hospital’s cashiers (where all patients converge before being redirected to their point of interest), these rates were multiplied by a factor of 3.3.

Table 3- Gross Patient Arrival Rate per Time Frame

<b>Time Frame</b>	<b>Rate (per hour)</b>
8am to 9am	23
9am to 10am	40
10am to 11am	53
11am to 12pm	40
12pm to 2 pm	30
2pm to 3 pm	26
3pm to 6pm	20

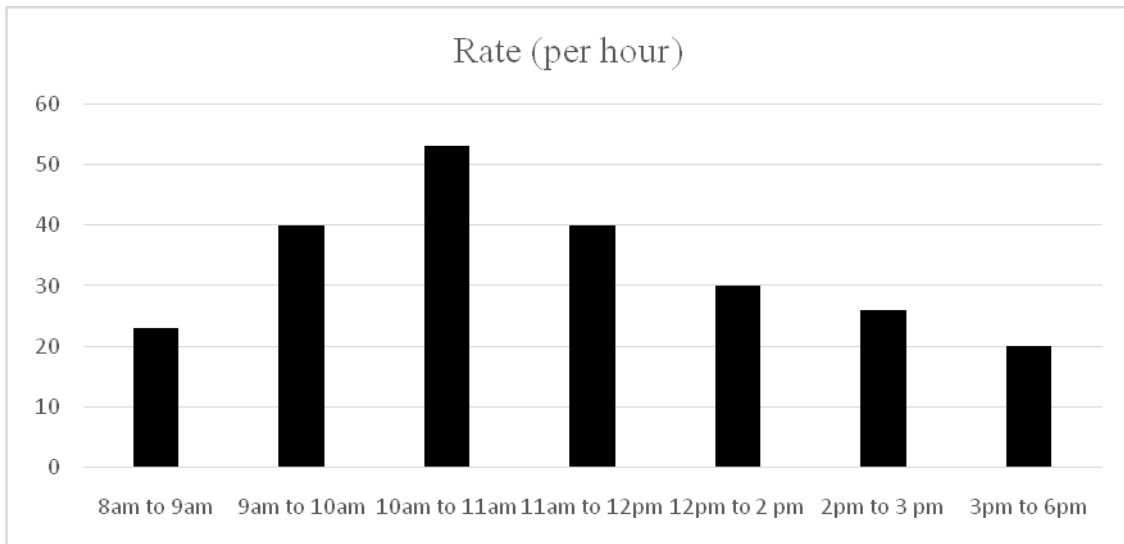


Figure 3- Gross Patient Arrival Rate per Time Frame Histogram

### 3.1.2 Procedure Service Time

PAT_FIRST_NAME	PAT_LAST_NAME	Registration Date	Registration Time	Procedure Start Time	Procedure End Time	Waiting Time	WORKPLACE_CODE	DEVICE_CODE
		6/22/2015	9:53:22	10:32:00	10:32:00	0:38:38	WRD-M1	XR
		6/23/2015	10:05:06	10:12:00	10:12:00	0:06:54	WRD-M1	XR
		6/5/2015	7:10:02	10:36:00	10:36:00	3:25:58	WRD-M1	XR

Figure 4- LAUMC Database Snippet

All radiology procedures were digitally imported from the LAUMC database, cleaned and processed the Arena Input Analyzer in order to finally find the below service time distributions. It is important to note that a triangular distribution of minimum 5, maximum 15 and most likely 10 minutes has been added to all service times as preparation time for the procedure.

Table 4- Procedure Service Time Distributions

Procedure	Service time Distribution	Mean	Std Deviation
BMD	1.5 + 42 * BETA(1.14, 1.54)	19.3	10.8
CT	2.5 + EXPO(14.9)	17.4	13.2
INTXR	TRIA(7, 72.4, 175)	84.8	39.5
MAMMO	7.5 + 55 * BETA(1.82, 2.5)	30.7	11.8
MR	7.5 + 54 * BETA(1.23, 1.65)	30.5	13.6
NM	6.5 + 57 * BETA(1.15, 1.11)	35.5	15.8
PET	9.5 + GAMM(9.53, 2.38)	32.2	14.1
RF	NORM(32.1, 12.9)	32.1	13.1
US	8.5 + WEIB(20.1, 1.28)	27.1	14
XR	0.5 + LOGN(3.88, 4.36)	4.24	3.32
Cashier	1.5 + GAMM(1.93, 2.08)	5.53	2.72

Unfortunately, some tasks, like the cashier’s for example did not have the time keeping system installed so data collection was done by hand, and in turn, inputted by hand to gather this task’s processing time distribution. Please find below a snippet of a sheet filled by hand by one of the cashiers (Loyal) at LAUMC:

LAYAL				
NAME OF PATIENT	DATE	COVENTION	H . A	H.S
[REDACTED]	1-07-2016	Perianal	10:10	10:10
[REDACTED]	"	Dallaa	10:10	10:12
[REDACTED]	"	Dallaa	10:12	10:14
[REDACTED]	"	Dallaa	10:16	10:16
[REDACTED]	"	Dallaa	10:16	10:20
[REDACTED]	"	Dallaa	10:20	10:23
[REDACTED]	"	Dallaa	10:23	10:26
[REDACTED]	"	Dallaa	10:26	10:29
[REDACTED]	"	Dallaa	10:30	10:37
[REDACTED]	"	Dallaa	10:32	10:35

Figure 5- Cashier Service time handwritten

A detailed summary of the Arena Input Analyzer files used to find the above procedure time distributions can be found in Appendix B of this document showcasing the validity of the aforementioned distributions. An example for XR can be found here below in table 5.



Table 5- XR Service Time Distribution Summary

<b>XR Service Time Distribution Summary</b>	
Distribution:	Lognormal
Expression:	0.5 + LOGN(3.88, 4.36)
Square Error:	0.000803
<b>Chi Square Test</b>	
Number of intervals	9
Degrees of freedom	6
Test Statistic	6.09
Corresponding p-value	0.426
<b>Data Summary</b>	
Number of Data Points	296
Min Data Value	1
Max Data Value	17
Sample Mean	4.24
Sample Std Dev	3.32
<b>Histogram Summary</b>	
Histogram Range	0.5 to 17.5
Number of Intervals	17

### 3.1.3 Procedure Probability

In this section we will be discussing the patients' repartition over the various radiology procedures. Please find below a table containing the patients' procedure distribution percentage per day.

Table 6- Patients' Daily Procedure Distribution

Procedures	Daily Patient %
BMD	1
CT	9
INTXR	1
MAMMO	6
MR	10
NM	2
PET	2
RF	1
US	14
XR	54

In the model we took into account the patients that perform more than one procedure at a time and that was mostly common with female patients performing their yearly check-ups.

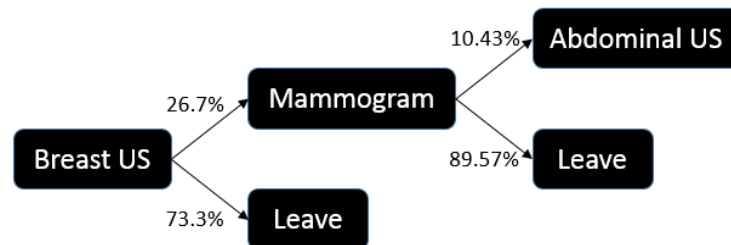


Figure 6- Multiple Procedure Patient Distributions

A detailed patient flow study of the whole system will be discussed in the following section.

## **3.2 Developing the Model**

In this chapter, we will be discussing the patient flow throughout the system, from entering to leaving the hospital. After covering the path and services the patients have to take to go through with their radiology procedures a detailed study of building the model in Arena will be given to highlight the degree of detail that was put into the development of this model in order to make it as realistic and life-like as possible.

### ***3.2.1 Patient Flow***

All patients have to first go through the cashier to check in and register; 5% of these arrivals are assumed to have “special cases” and are redirected to specialized insurance officers for clearance. Next stop is the radiology desk at the 1<sup>st</sup> floor where all the patients wait for the availability of their specialized radiology machine. The patients now go through with their procedures with the probabilities cited in section 3.1.3, with a specific service time which was also discussed in 3.1.2. After completing their service(s) the patients then proceed to exit the system. Please find below a diagram showing the patient routing around the radiology department.

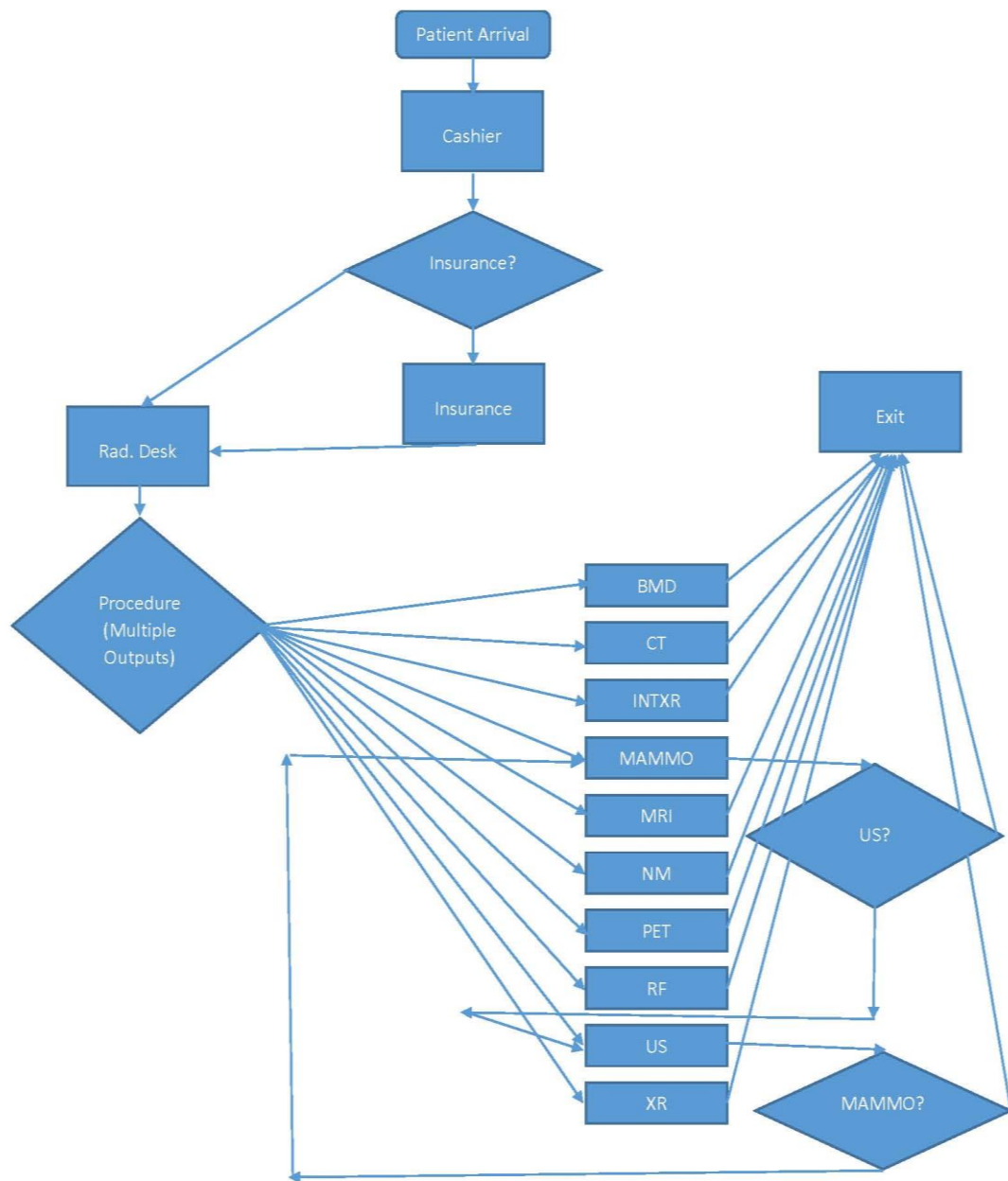


Figure 7- Patient Flow Diagram

### 3.2.2 Building the Model using Arena

Arena is one of the leading discrete event simulation engines that allows users to develop their own process flow in order to validate, improve and optimize its performance. AUB FEA were so kind to provide me with an Arena temporary license to develop my model once I took on this particular thesis topic.

In this chapter we will be covering the step by step development of the LAUMC Radiology Department Arena model in full detail before discussing findings and resource optimization techniques.

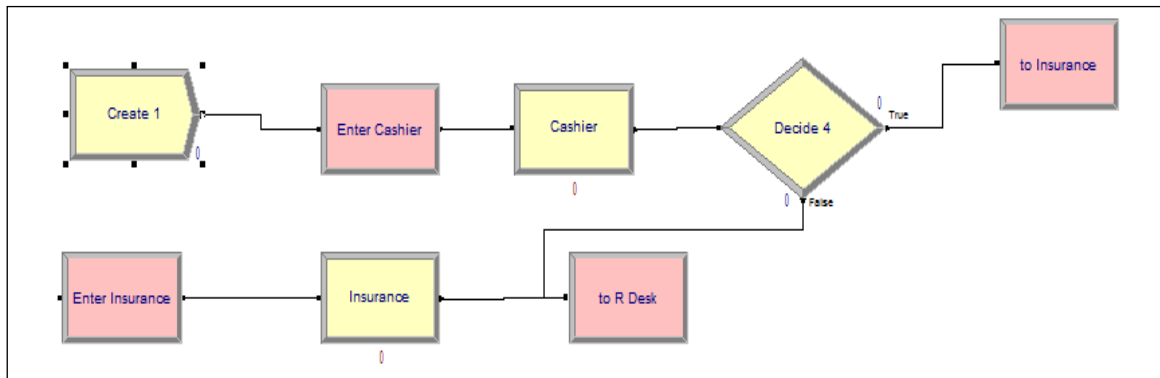


Figure 8- Model Snapshot 1- Patient arrival

The “Create” node is what injects the patients into the system. It is configured to inject with a certain rate that change throughout the day in order to mimic the hospital’s actual patient influx. This will assure that the model reflects the hospital’s rush hours and accurately portrays patient waiting times and machine utilizations. The Arena arrival rate schedule is captured below in figure 9.

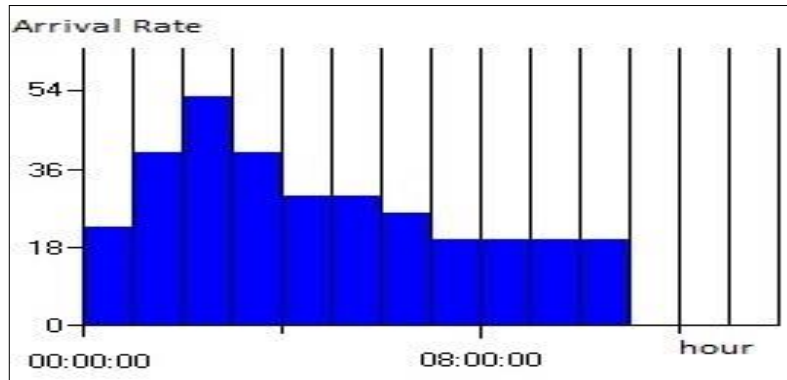


Figure 9 –Rate Schedule Arena arrival Snapshot

After being injected into the system and completing the cashiers and/or insurance processes the patients then proceed to the radiology desk where they wait for their procedures' availability (figure 10).

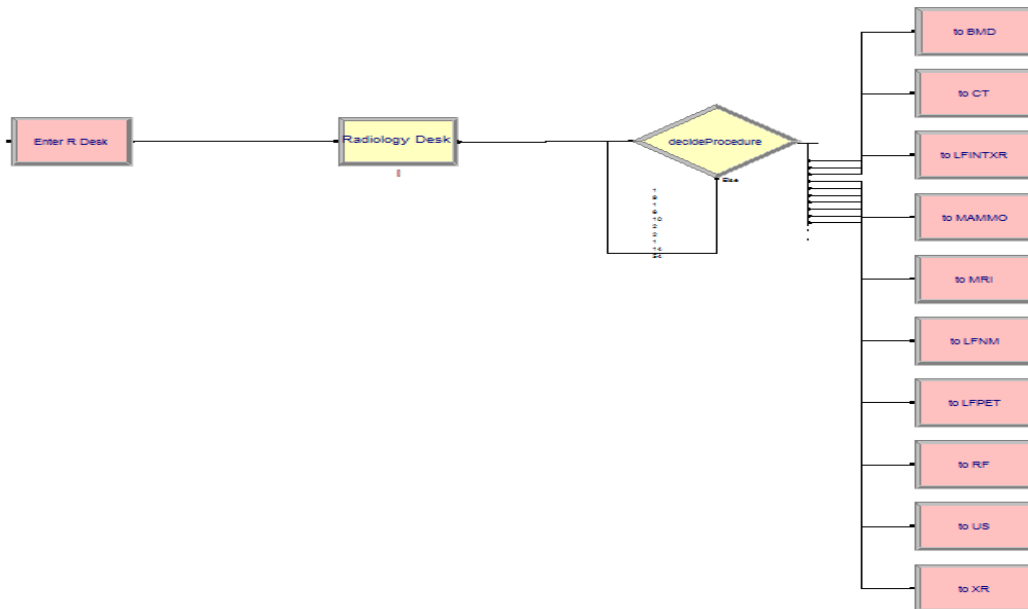


Figure 10- Model Snapshot 2- To procedures

Each process box, i.e. the BMD process has its own characteristics from service time to resource numbers and these are all found in the process properties box found by double clicking the process box.

The project builder can then input all the variables to make this process as distinct and accurate as it can be in order to properly and accurately portray the model at hand.

Please find below a snapshot of the process properties for BMD (figure 11).

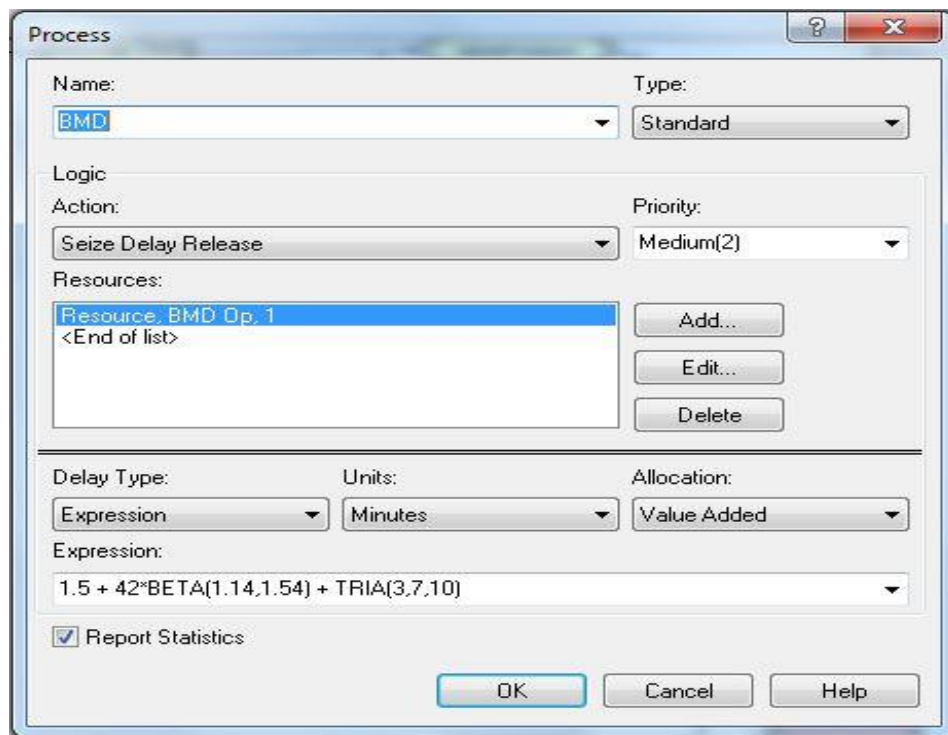


Figure 11- Arena's Process Properties

Figure 10 below highlights the multiple procedure paradigm incorporated in Arena. This was thoroughly discussed in section 3.1.3 and below is a representation of that particular process flow using Arena (figure 12).

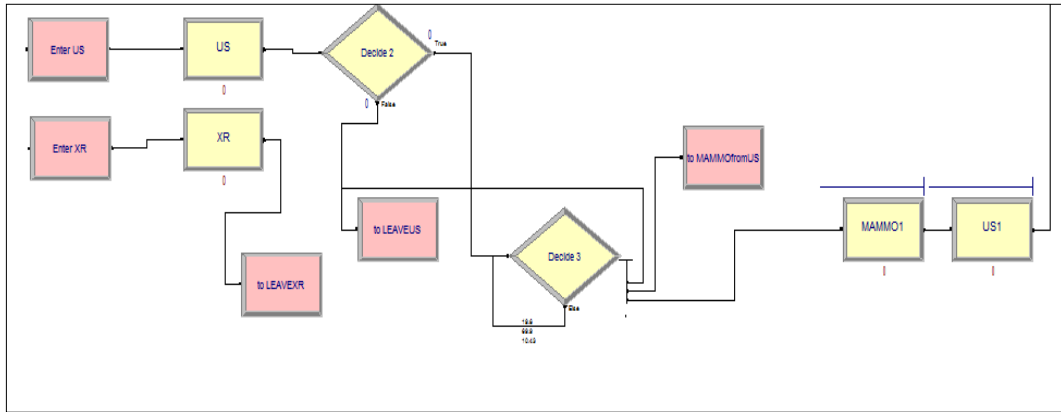


Figure 12- Model Snapshot 3- Multiple Procedures

After completing their required examinations the patients then proceed to exit the system. Different “Exit” nodes can be found before the final “Entity Dispose” node (“Dispose 1” in Figure 13) in order to make the animation unique for each distinct procedure.



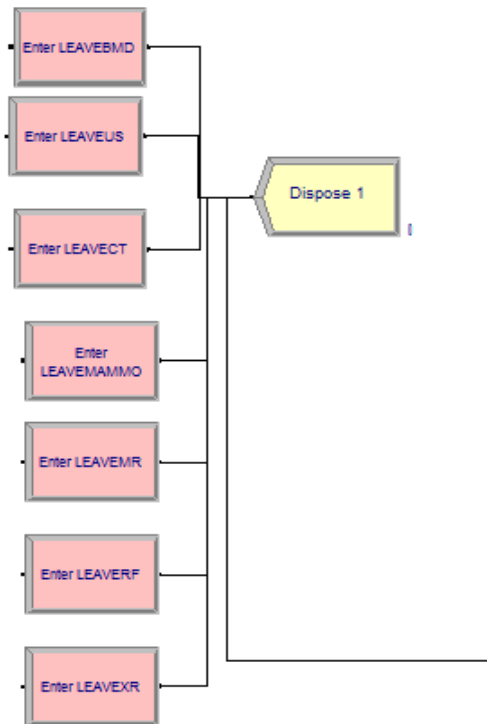


Figure 13- Model Snapshot 4- Patient exit

Now, that the model is complete, we will be running it, discussing its results and outcomes in order to assess its performance and recommend fixes if the need arises.

Details can be found in the next chapter.

# CHAPTER 4

## SIMULATION RUN RESULTS

### **4.1 Running the Model**

The model was run for the duration of a full work day, 10 hours (from 8am to 6pm), terminating, with 1000 iterations to make sure that the results hold up to our precision standards. Arena then gathers all the result data for example waiting times, total time spent in the system... to machine utilization percentage and stores them in a report form with their average, half width, minimum and maximum values. While studying these statistics we can assess the overall system performance and recommend fixes where congestion occurs.

The below result sets were found by running the model exactly as is it at the LAUMC Radiology Department. For more info about inter-arrival times, resource numbers and service times please review chapter 3.

#### ***4.1.1 As is Simulation Run Inputs and Results***

Before each run we will be citing the resource capacities and arrival rates of patients for these are the factors that will be changing most frequently between runs.

The procedure service times, for sure, are fix throughout all the experiments found below.

*Table 7- Resource Capacities As Is*

<b>Resources</b>	<b>Number</b>
Cashiers	4
Insurance Officers	6
Radiology Desk Clerks	2
BMD	1
CT	1
INTXR	1
MAMMO	2
MR	2
NM	1
PET	1
RF	1
US	2
XR	1

*Table 8- Arrival Rates As Is*

<b>Time Frame</b>	<b>Radiology Rate (per hour)</b>	<b>System Rate (per hour) *3.3</b>
<b>8am to 9am</b>	7	23
<b>9am to 10am</b>	12	40
<b>10am to 11am</b>	16	53
<b>11am to 12pm</b>	12	40
<b>12pm to 2 pm</b>	9	30
<b>2pm to 3 pm</b>	8	26
<b>3pm to 6pm</b>	6	20

After running the simulator we took a closer look at the several procedure waiting times and resource utilization figures to assess the actual system's performance. Please find below snippets of these figures followed by a brief explanation and evaluation of the system.

Waiting Time	Average
BMD.Queue	0.2278
Cashier.Queue	6.1184
CT.Queue	7.2839
Insurance.Queue	0.00
INTXR.Queue	2.6700
MAMMO.Queue	1.4624
MAMMO1.Queue	0.5781
MR.Queue	1.7416
NM.Queue	2.1644
PET.Queue	1.8247
Radiology Desk.Queue	0.00211451
RF.Queue	0.4100
US.Queue	3.4310
US1.Queue	1.1912
XR.Queue	43.7271

Figure 14- Procedure Waiting Time As Is

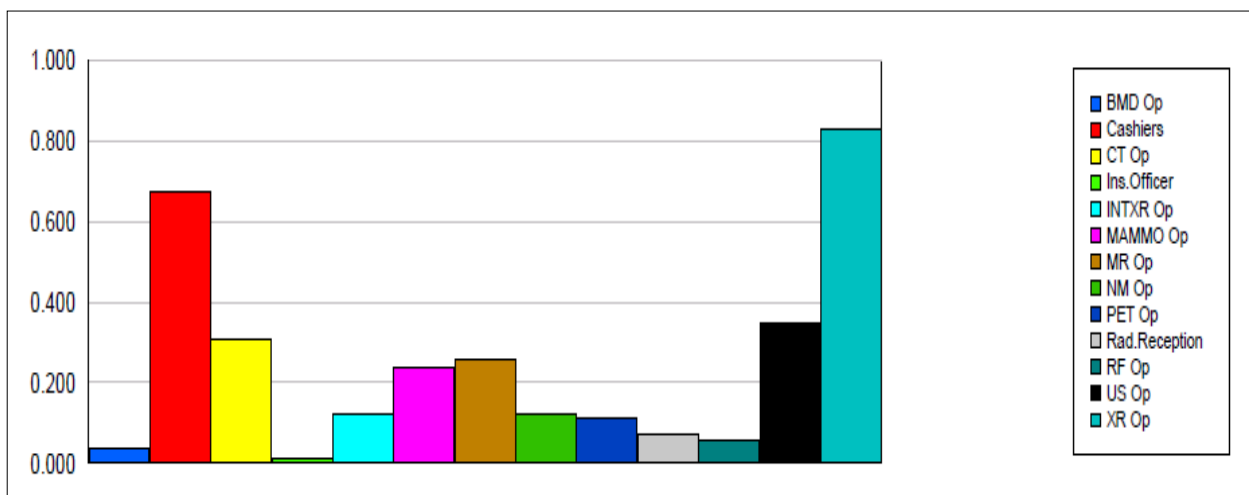


Figure 15- Resource Utilization As Is

The total number of patients that entered the hospital as a whole is 316; 95 of these went to radiology. It is clear to say that utilizations and waiting times are pretty reasonable for all procedures except XR. The average waiting for XR is a striking 43 minute average and the utilization of the XR machine is of 83%.

For the aforementioned inputs and system state, the best way to move forward is to suggest the addition of another XR machine in order to share the load and decrease patient waiting times.

#### ***4.1.2 As is Simulation Run Inputs and Results plus 1 XR Machine***

*Table 9- Resource Capacities +1 XR*

<b>Resources</b>	<b>Number</b>
Cashiers	4
Insurance Officers	6
Radiology Desk Clerks	2
BMD	1
CT	1
INTXR	1
MAMMO	2
MR	2
NM	1
PET	1
RF	1
US	2
XR	2

After adding another XR machine, the model was run with the same actual arrival rates and the updated results can be found below.

Waiting Time	Average
BMD.Queue	0.2448
Cashier.Queue	5.7904
CT.Queue	7.2908
Insurance.Queue	0.00
INTXR.Queue	2.3757
MAMMO.Queue	1.4124
MAMMO1.Queue	0.5720
MR.Queue	1.9098
NM.Queue	1.8108
PET.Queue	1.6626
Radiology Desk.Queue	0.00216761
RF.Queue	0.5345
US.Queue	3.1982
US1.Queue	1.3340
XR.Queue	2.0048

Figure 16- Procedure Waiting Time +IXR

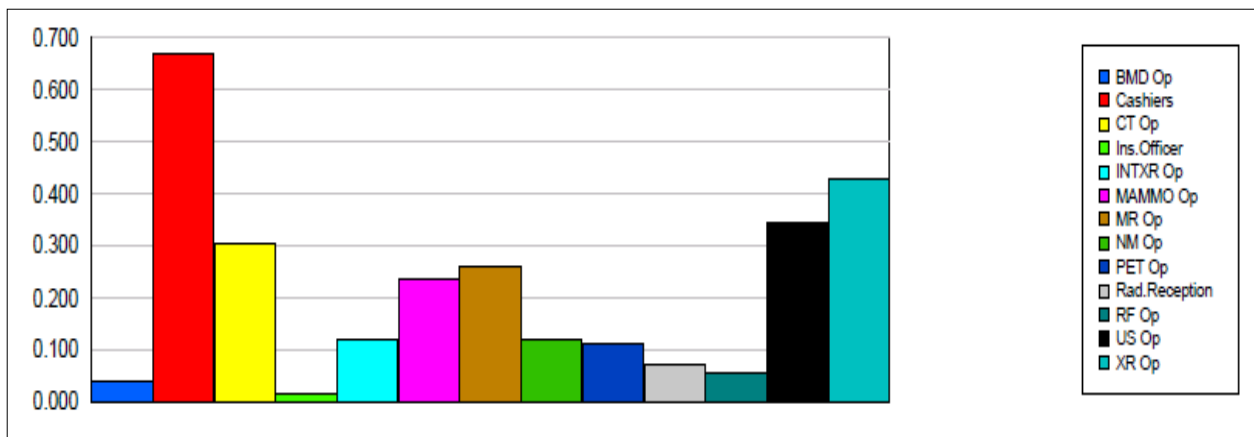


Figure 17- Resource Utilization +IXR

The XR procedure waiting time dropped from a 43 minute average to just 2 with the addition of just one machine, as well as the overall machine utilization moving down from 83% to just 43. With the waiting times of all procedures showing averages of less than 10 minutes, we can say that the system is running optimally.

Results were also double checked using Arena Process Analyzer which allows the user to conduct several different scenarios like in this case adding an additional XR machine and

then monitoring its potential effects on the system. Below are two figures highlighting first the scenario and desired response selection and then a histogram for easy visualization of the changes at hand.

S	Scenario Properties			Control	Responses		
	Name	Program File	Reps	XR Op	XR.Queue.WaitingTime	XR.Queue.NumberInQueue	XR Op.Utilization
1	1 x XR	25 : LAUMC.	1000	1.0000	47.273	3.870	0.831
2	2 x XR	25 : LAUMC.	1000	2.0000	2.471	0.204	0.429

Figure 18- Scenario Selection

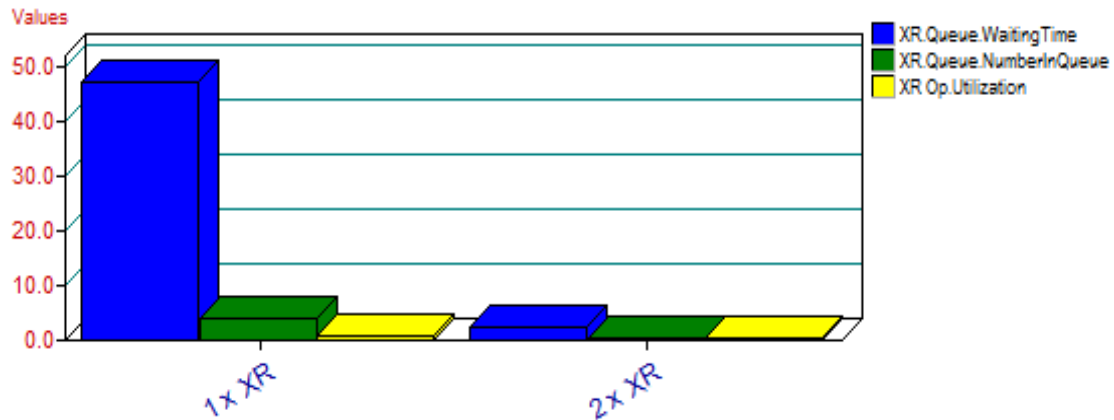


Figure 19- Comparison Histogram

#### 4.2 System Stress Test, the Hospital's Projected Growth

After making sure that the radiology department's resource numbers can handle the load as it currently is, the hospital wanted us to foresee, with the same analogy, the expected resource/machine numbers to go along with their planned growth in the upcoming 3 to 5 years. We were to predict and manage resource numbers to handle loads of +25, +50 and +100% to the actual, present arrival rates. Please find below a table showing the old and projected arrival rates, by sections, which will be implemented in our next simulation experiments.

Table 10- Resource Capacities +1 XR

Time Frame	Act. Rate (per hour)	Rate + 25%	Rate + 50%	Rate + 100%
8am to 9am	23	29	34	46
9am to 10am	40	50	60	80
10am to 11am	53	66	80	106
11am to 12pm	40	50	60	80
12pm to 2 pm	30	37	45	60
2pm to 3 pm	26	33	39	52
3pm to 6pm	20	25	30	40

**4.2.1 Actual Arrival Rates + 25%**

After applying a 25% increase to the actual arrivals and using the updated resource numbers found in section 4.1.1 (table 8); the following run results were found.

Waiting Time	Average
BMD.Queue	0.3867
Cashier.Queue	23.2226
CT.Queue	9.6739
Insurance.Queue	0.00
INTXR.Queue	4.4134
MAMMO.Queue	24.3914
MAMMO1.Queue	9.9532
MR.Queue	31.9123
NM.Queue	2.6191
PET.Queue	2.2253
Radiology Desk.Queue	0.00270318
RF.Queue	0.8432
US.Queue	4.8930
US1.Queue	2.2152
XR.Queue	2.8434

Figure 20- Procedure Waiting Time +25%



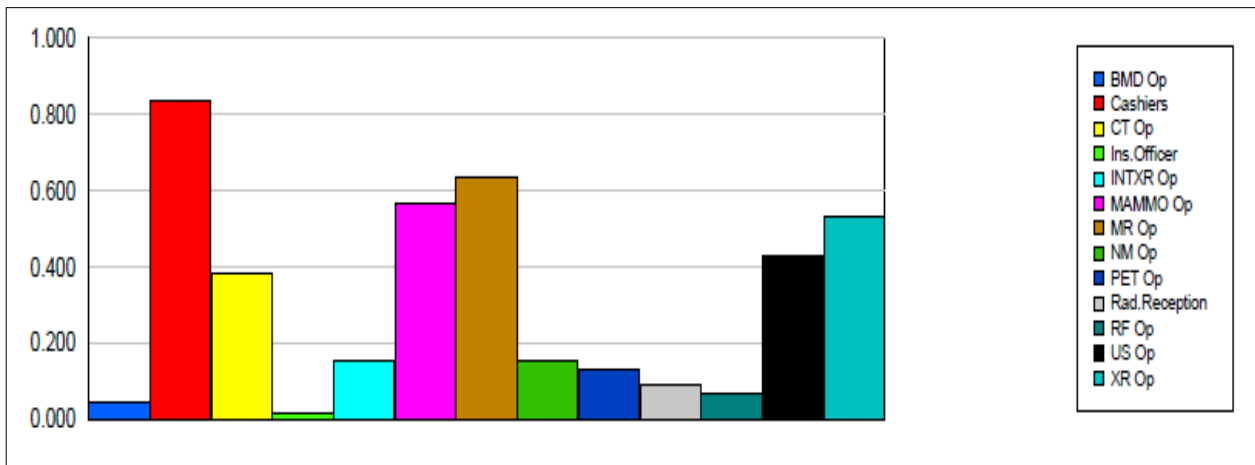


Figure 21- Resource Utilization +25%

The total number of patients that entered the hospital as a whole is 394; 118 of these went to radiology. MAMMO and MR waiting times have increased; 24mins for MAMMO and 32 for MR, but with their machine utilization being of 56 and 63%, it is safe to say that, at this stage, adding another machine to either is not that necessary. Cashiers on the other hand have an average waiting time of 23mins and a utilization of 83%, we should probably add another cashier to the resource pool before moving on to the next stage of +50% arrival rates. Please find below the optimal resource quantities table for the +25% arrival rates scenario.

Table 11- Opt. Resource Capacities Rate +25%

Resources	Number
Cashiers	5
Insurance Officers	6
Radiology Desk Clerks	2
BMD	1
CT	1
INTXR	1
MAMMO	2
MR	2
NM	1
PET	1
RF	1
US	2
XR	2

**4.2.2 Actual Arrival Rates + 50%**

After applying a 50% increase to the actual arrivals and using the updated resource numbers found in section 4.2.1 (table 11); the following run results were found.

Waiting Time	Average
BMD.Queue	0.4454
Cashier.Queue	18.1009
CT.Queue	13.7434
Insurance.Queue	0.00
INTXR.Queue	5.6952
MAMMO.Queue	37.9347
MAMMO1.Queue	18.6341
MR.Queue	48.8399
NM.Queue	3.7045
PET.Queue	3.3533
Radiology Desk.Queue	0.00498039
RF.Queue	0.8193
US.Queue	9.2227
US1.Queue	3.8559
XR.Queue	6.0309

Figure 22- Procedure Waiting Time +50%

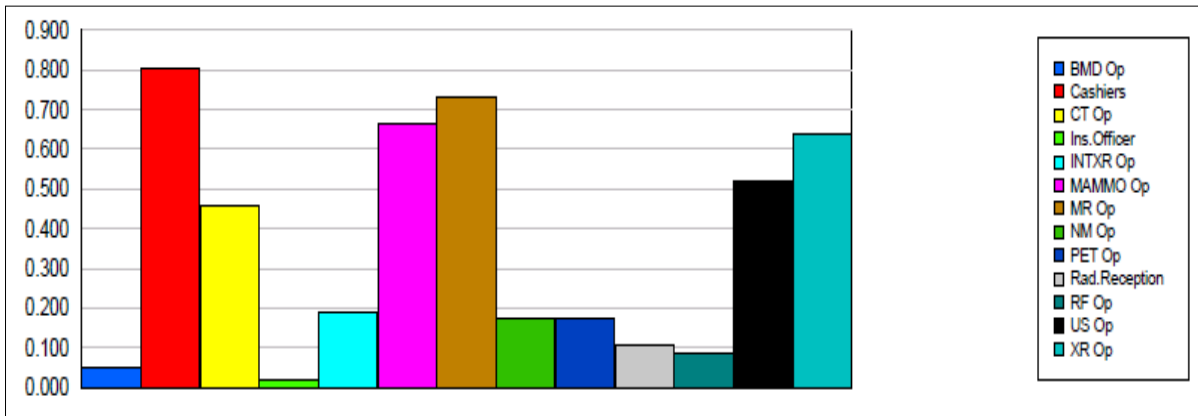


Figure 23- Resource Utilization +50%

The total number of patients that entered the hospital as a whole is 473; 142 of these went to radiology. The cashier waiting times are still too high, 18 minute average, with a utilization of 80%. At least one cashier should be added, in order to loosen up the system and increase inflow to the hospital, we will be adding 2 since cashiers are at the very start of the system. On the other hand, MR and MAMMO waiting times are no longer tolerable, 48 minutes for MR and 38 for MAMMO, we will be adding a machine of each before running our next exercise, the +100% arrival rate scenario. Please find below the optimal resource quantities table for the +50% arrival rates scenario.

Table 12- Opt. Resource Capacities Rate +50%

<b>Resources</b>	<b>Number</b>
Cashiers	7
Insurance Officers	6
Radiology Desk Clerks	2
BMD	1
CT	1
INTXR	1
MAMMO	3
MR	3
NM	1
PET	1
RF	1
US	2
XR	2

#### 4.2.3 Actual Arrival Rates + 100%

After applying a 100% increase to the actual arrivals and using the updated resource numbers found in section 4.2.2 (table 12); the following run results were found.

<b>Waiting Time</b>	<b>Average</b>
BMD.Queue	0.5663
Cashier.Queue	12.0326
CT.Queue	24.8680
Insurance.Queue	0.00
INTXR.Queue	9.3376
MAMMO.Queue	0.9191
MAMMO1.Queue	0.4807
MR.Queue	1.3886
NM.Queue	6.2878
PET.Queue	4.7371
Radiology Desk.Queue	0.01083735
RF.Queue	1.5933
US.Queue	21.2291
US1.Queue	12.2654
XR.Queue	29.2804

Figure 24- Procedure Waiting Time +100%

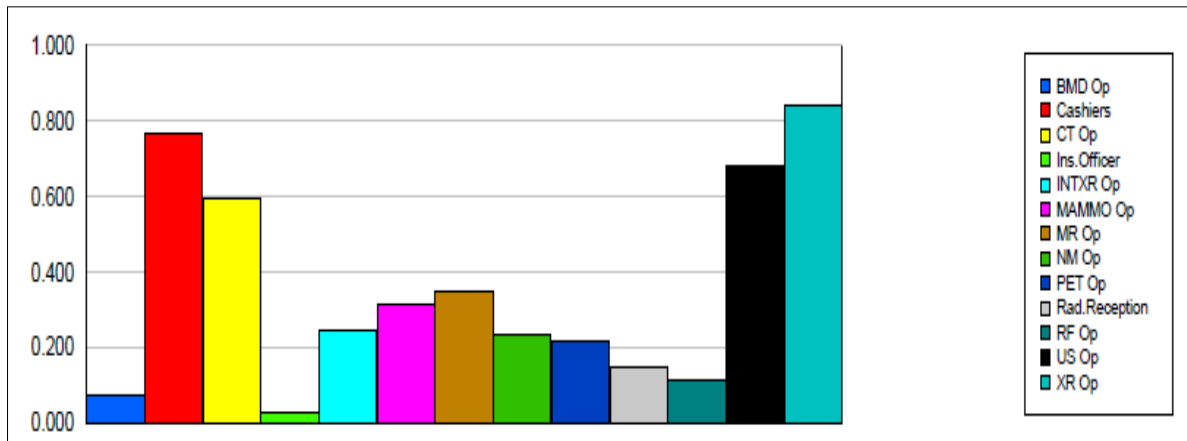


Figure 25- Resource Utilization +100%

The total number of patients that entered the hospital as a whole is 631; 190 of these went to radiology. System seems to be working fine. A peak of utilization can be found for the XR machines again with 84% and a average waiting time of 29 mins. Other high waiting times to be addressed are for CT and US, 25 and 21 minutes respectively. Optimally, one machine should be added for each of these above cited procedures, one for XR, one for CT and another for US. Please find below the optimal resource quantities table for the +100% arrival rates scenario.

Table 13- Opt. Resource Capacities Rate +100%

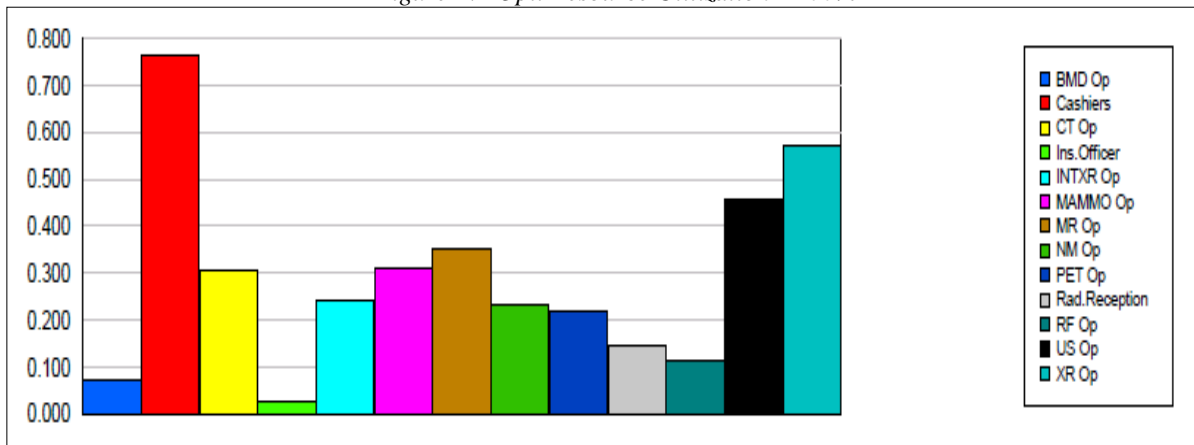
Resources	Number
Cashiers	7
Insurance Officers	6
Radiology Desk Clerks	2
BMD	1
CT	2
INTXR	1
MAMMO	3
MR	3
NM	1
PET	1
RF	1
US	3
XR	3

Results for this final stage of +100% arrival rates running optimally can be found below for validation.

Waiting Time	Average
BMD.Queue	0.7222
Cashier.Queue	11.9033
CT.Queue	1.9113
Insurance.Queue	0.00
INTXR.Queue	9.5295
MAMMO.Queue	1.0171
MAMMO1.Queue	0.4717
MR.Queue	1.6815
NM.Queue	5.8802
PET.Queue	4.6844
Radiology Desk.Queue	0.01116900
RF.Queue	1.6757
US.Queue	3.2661
US1.Queue	1.6853
XR.Queue	2.6367

Figure 26- Opt. Procedure Waiting Time +100%

Figure 27- Opt. Resource Utilization +100%



The total number of patients that entered the hospital as a whole is 638; 191 of these went to radiology. We can clearly notice an increase in the numbers of patients out of the system on a daily basis, 7 more patients to the hospital as a whole and 1 for radiology. Waiting times are all around or less than 10 minutes which is quite the norm for acceptable

waiting times. Cashier utilizations are still quite high, one more cashier could be added to share the load and limit overworking the staff, but going back to the service's waiting time, it is not that crucial.

## CHAPTER 5

### CONCLUSION AND FUTURE WORK

#### 5.1 Conclusion

After running the simulator with the various arrival rates and adapting the resource numbers to handle the overall system load, we were able to summarize our findings in the table below (table 14).

*Table 14- Optimal Resource Numbers Summary*

<b>Resource type/Arrival Rate</b>	<b>Original</b>	<b>Opt. Original</b>	<b>+25%</b>	<b>+50%</b>	<b>+100%</b>
Cashiers	4	4	5	7	7
Insurance Officers	6	6	6	6	6
Radiology Desk Clerks	2	2	2	2	2
BMD	1	1	1	1	1
CT	1	1	1	1	2
INTXR	1	1	1	1	1
MAMMO	2	2	2	3	3
MR	2	2	2	3	3
NM	1	1	1	1	1
PET	1	1	1	1	1
RF	1	1	1	1	1
US	2	2	2	2	3
XR	1	2	2	2	3

The resources affected with the increasing patient rate were surely the resources with the highest procedure utilization percentage. The cashiers' numbers nearly doubled, from 4 to 7 in total, for they handle not only arrivals to the radiology department, but the hospital as a whole. The XR machine numbers tripled (from 1 to 3) and an additional machine was added for MR and another for MAMMO all in order to accommodate for the projected 100% arrival rate increase.



When presenting the work to the Radiology crew at the LAUMC, Dr. Daniel Mahfoud, Head of Radiology, and his Senior Coordinator Ms. Jessica Saad were taken aback by the accuracy of the simulation runs; especially for the actual system's run results. Their data and findings reflect congestion on the XR machines with an average waiting time of 45 minutes. Our system also reflected this congestion and with a very accurate average waiting time of 43 minutes (figure 13). Dr. Daniel validated our findings and is going to recommend the use of simulation in other departments at the LAUMC. We are definitely looking forward to many more successful implementations.

## **5.2 Future Work**

Our future work, in the radiology department could be a purely financial study. We could at first set a dollar price to the average minute of waiting time on the machines in order to highlight the importance of introducing new resources and minimizing waiting times. Then, with the hospital's approval, we could acquire the machine prices and procedure pricing schemes, to estimate the time the hospital needs to recover from its expenses (break-even).

On the other hand, why not venture on to other departments. We could control the load on the Radiology department by checking its dependency on other departments (inpatient influx) and develop an optimization model for scheduling patients.

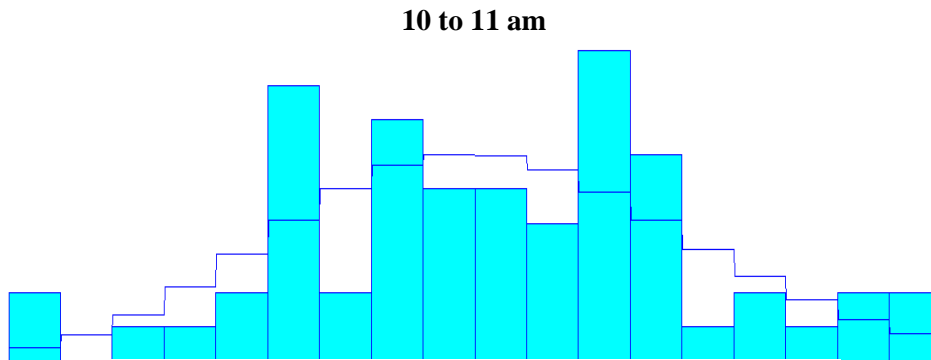
Dr. Mahfoud also recommended the simulation of the Internal Medicine center, maybe simulating patient flow there and advising on the best time between appointments could be a challenging call for a future project collaboration.

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# APPENDIX A:

## ARRIVAL RATES DISTRIBUTION



<b>Distribution Summary</b>	<b>10 to 11 am</b>
<b>Distribution:</b>	Poisson
<b>Expression:</b>	POIS(15.9)
<b>Square Error:</b>	0.017709

<b>Chi Square Test</b>	
<b>Number of intervals</b>	6
<b>Degrees of freedom</b>	4
<b>Test Statistic</b>	7.02
<b>Corresponding p-value</b>	0.147

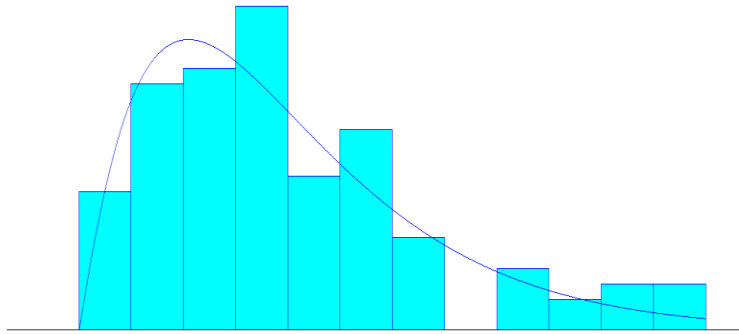
<b>Data Summary</b>	
<b>Number of Data Points</b>	60
<b>Min Data Value</b>	7
<b>Max Data Value</b>	24
<b>Sample Mean</b>	15.9
<b>Sample Std Dev</b>	3.9

<b>Histogram Summary</b>	
<b>Histogram Range</b>	= 6.5 to 24.5
<b>Number of Intervals</b>	18

## APPENDIX B:

### SERVICE TIME DISTRIBUTION BY TYPE OF PROCEDURE

**Cashier Service Time Distribution**



**Distribution Summary**

Distribution:	Gamma
<b>Distribution Summary</b>	
Expression:	1.5 + GAMM(1.93, 2.08)
Square Error:	0.00741

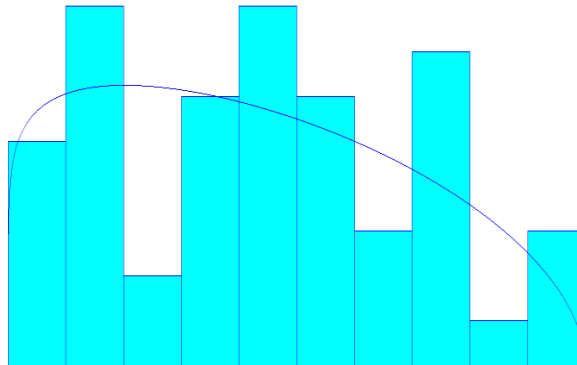
<b>Chi Square Test</b>	
Number of intervals	6
Degrees of freedom	3
Test Statistic	3.38
Corresponding p-value	0.355

<b>Data Summary</b>	
Number of Data Points	104
Min Data Value	2
Max Data Value	13
Sample Mean	5.53
Sample Std Dev	2.72

<b>Histogram Summary</b>	
Histogram Range	= 1.5 to 13.5
Number of Intervals	12

### BMD Service Time Distribution

---



#### Distribution Summary

Distribution:	Beta
Expression:	$1.5 + 42 * \text{BETA}(1.14, 1.54)$
Square Error:	0.018414

#### Chi Square Test

Number of intervals	8
Degrees of freedom	5
Test Statistic	5.32
Corresponding p-value	0.393

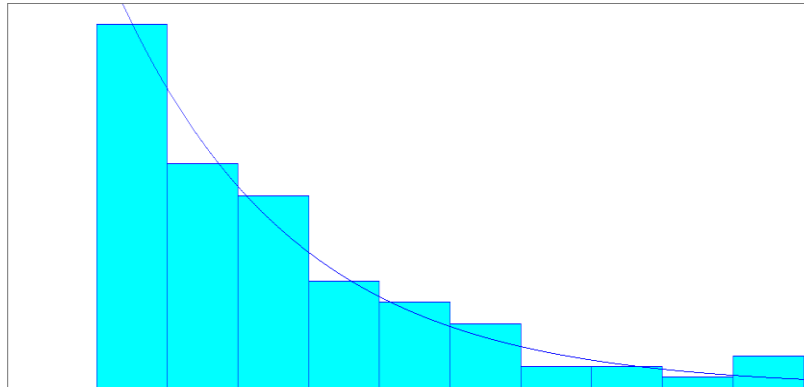
#### Data Summary

Number of Data Points	49
Min Data Value	2
Max Data Value	43
Sample Mean	19.3
Sample Std Dev	10.8

#### Histogram Summary

Histogram Range	1.5 to 43.5
Number of Intervals	10

### CT Service Time Distribution



#### Distribution Summary

Distribution:	Exponential
Expression:	$2.5 + \text{EXPO}(14.9)$
Square Error:	0.001919

#### Chi Square Test

Number of intervals	5
Degrees of freedom	3
Test Statistic	1.63
Corresponding p-value	0.659

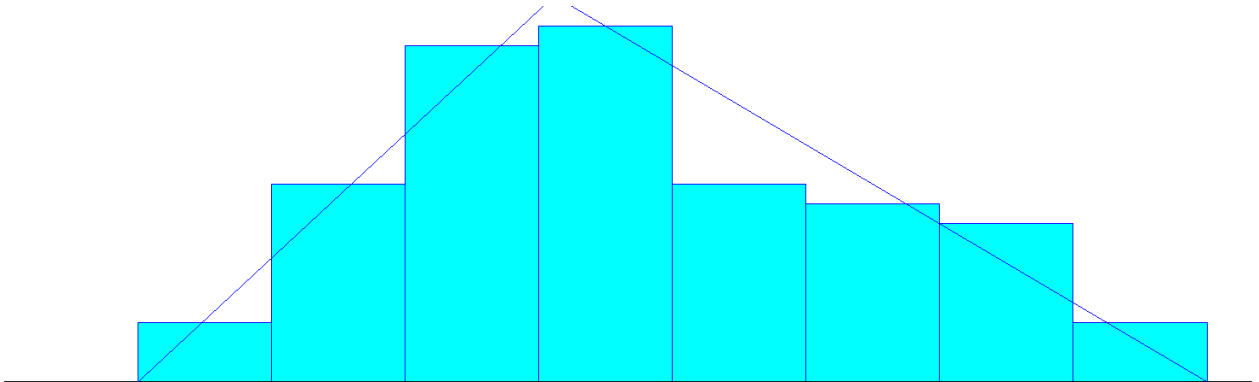
#### Data Summary

Number of Data Points	105
Min Data Value	3
Max Data Value	61
Sample Mean	17.4
Sample Std Dev	13.2

#### Histogram Summary

Histogram Range	2.5 to 61.5
Number of Intervals	10

### INTXR Service Time Distribution



#### Distribution Summary

Distribution:	Triangular
Expression:	TRIA(7, 72.4, 175)
Square Error:	0.003949

#### Chi Square Test

Number of intervals	6
Degrees of freedom	4
Test Statistic	2.5
Corresponding p-value	0.649

#### Kolmogorov-Smirnov Test

Test Statistic	0.0906
Corresponding p-value	> 0.15

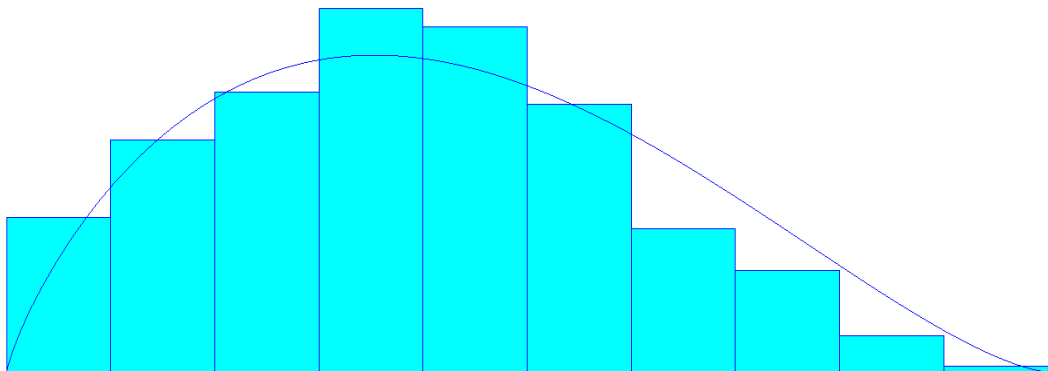
#### Data Summary

Number of Data Points	78
Min Data Value	7
Max Data Value	175
Sample Mean	84.8
Sample Std Dev	39.5

#### Histogram Summary

Histogram Range	7 to 175
Number of Intervals	8

### MAMMO Service Time Distribution



#### Distribution Summary

Distribution:	Beta
Expression:	$7.5 + 55 * \text{BETA}(1.82, 2.5)$
Square Error:	0.00215

#### Chi Square Test

Number of intervals	8
Degrees of freedom	5
Test Statistic	5.75
Corresponding p-value	0.346

#### Data Summary

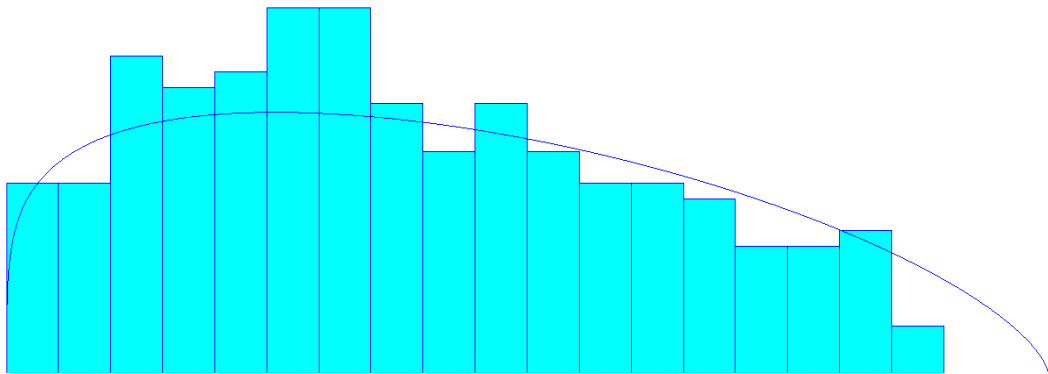
Number of Data Points	324
Min Data Value	8
Max Data Value	62
Sample Mean	30.7
Sample Std Dev	11.8

#### Histogram Summary

Histogram Range	7.5 to 62.5
Number of Intervals	10



### MR Service Time Distribution



#### Distribution Summary

Distribution:	Beta
Expression:	$7.5 + 54 * \text{BETA}(1.23, 1.65)$
Square Error:	0.001616

#### Chi Square Test

Number of intervals	15
Degrees of freedom	12
Test Statistic	5.48
Corresponding p-value	> 0.75

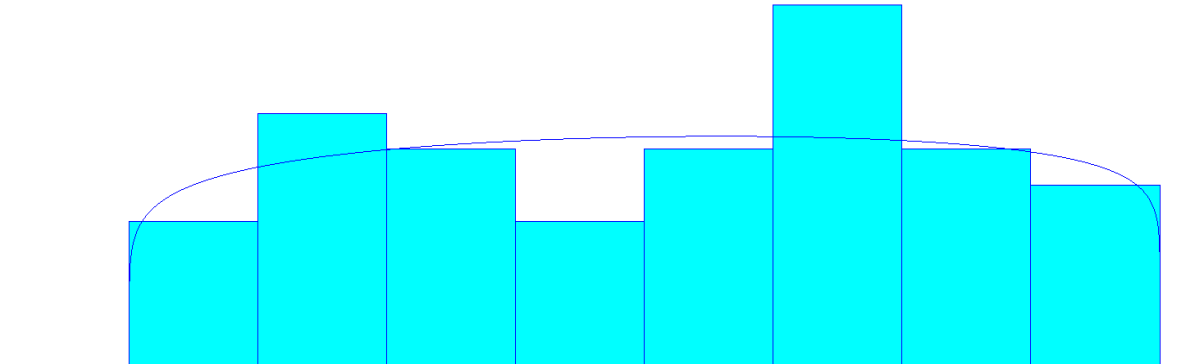
#### Data Summary

Number of Data Points	252
Min Data Value	8
Max Data Value	61
Sample Mean	30.5
Sample Std Dev	13.6

#### Histogram Summary

Histogram Range	7.5 to 61.5
Number of Intervals	20

### NM Service Time Distribution



#### Distribution Summary

Distribution:		Beta
Expression:	$6.5 + 57 * \text{BETA}(1.15, 1.11)$	
Square Error:		0.009446

#### Chi Square Test

Number of intervals	7
Degrees of freedom	4
Test Statistic	3.18
Corresponding p-value	0.531

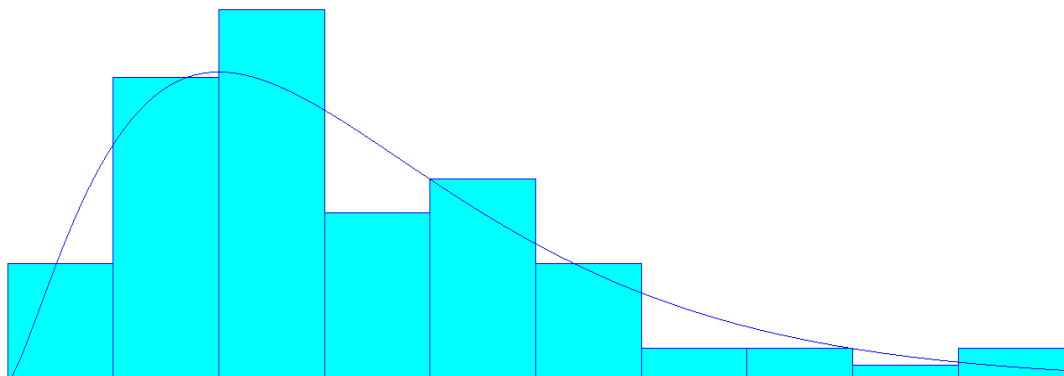
#### Data Summary

Number of Data Points	48
Min Data Value	7
Max Data Value	63
Sample Mean	35.5
Sample Std Dev	15.8

#### Histogram Summary

Histogram Range	6.5 to 63.5
Number of Intervals	8

### PET Service Time Distribution



#### Distribution Summary

Distribution:	Gamma
Expression:	$9.5 + \text{GAMM}(9.53, 2.38)$
Square Error:	0.006353

#### Chi Square Test

Number of intervals	5
Degrees of freedom	2
Test Statistic	2.74
Corresponding p-value	0.257

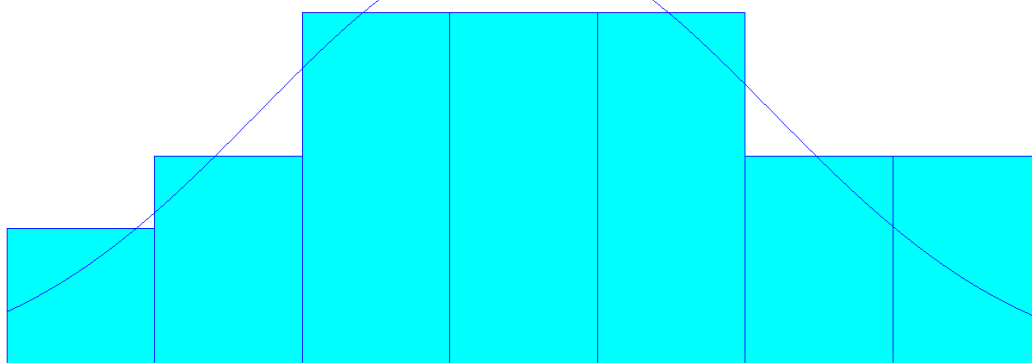
#### Data Summary

Number of Data Points	83
Min Data Value	10
Max Data Value	75
Sample Mean	32.2
Sample Std Dev	14.1

#### Histogram Summary

Histogram Range	9.5 to 75.5
Number of Intervals	10

### RF Service Time Distribution



#### Distribution Summary

Distribution:	Normal
Expression:	NORM(32.1, 12.9)
Square Error:	0.003998

#### Chi Square Test

Number of intervals	4
Degrees of freedom	1
Test Statistic	0.367
Corresponding p-value	0.562

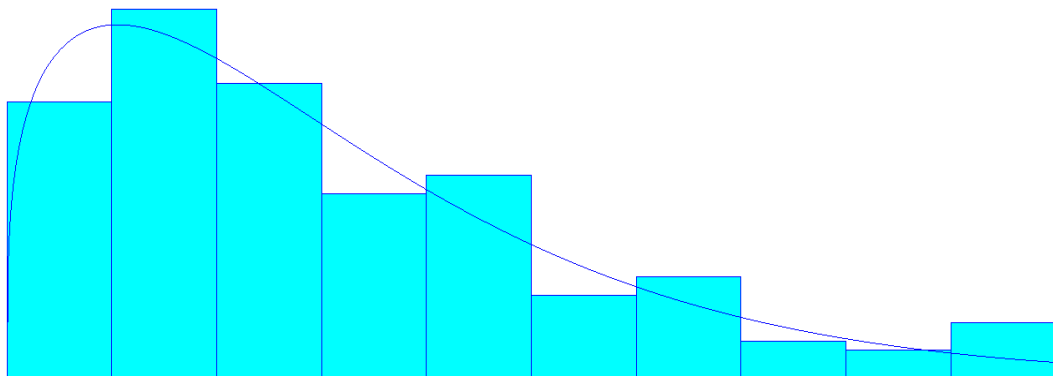
#### Data Summary

Number of Data Points	26
Min Data Value	7
Max Data Value	58
Sample Mean	32.1
Sample Std Dev	13.1

#### Histogram Summary

Histogram Range	6.5 to 58.5
Number of Intervals	7

### US Service Time Distribution



#### Distribution Summary

Distribution:	Weibull
Expression:	8.5 + WEIB(20.1, 1.28)
Square Error:	0.003076

#### Chi Square Test

Number of intervals	7
Degrees of freedom	4
Test Statistic	5.56
Corresponding p-value	0.239

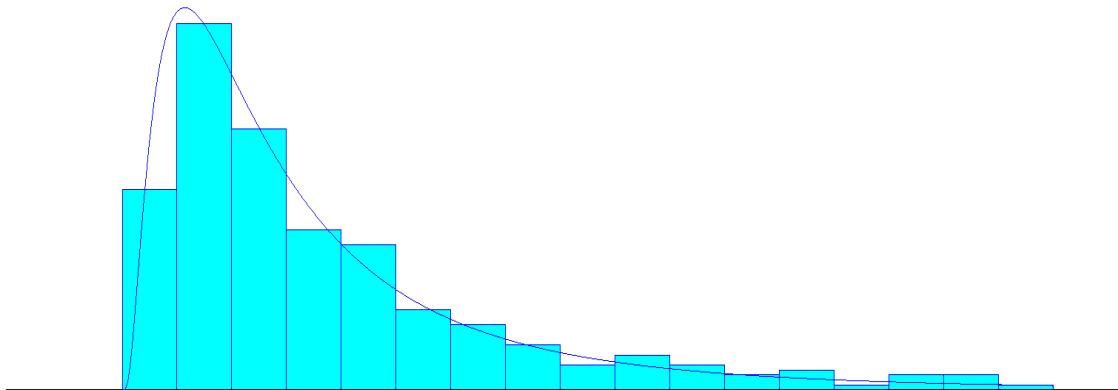
#### Data Summary

Number of Data Points	177
Min Data Value	9
Max Data Value	67
Sample Mean	27.1
Sample Std Dev	14

#### Histogram Summary

Histogram Range	8.5 to 67.5
Number of Intervals	10

### XR Service Time Distribution



#### Distribution Summary

Distribution:	Lognormal
Expression:	$0.5 + \text{LOGN}(3.88, 4.36)$
Square Error:	0.000803

#### Chi Square Test

Number of intervals	9
Degrees of freedom	6
Test Statistic	6.09
Corresponding p-value	0.426

#### Data Summary

Number of Data Points	296
Min Data Value	1
Max Data Value	17
Sample Mean	4.24
Sample Std Dev	3.32

#### Histogram Summary

Histogram Range	0.5 to 17.5
Number of Intervals	17