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TIME-TO-MARKET AND PRODUCT PERFORMANCE TRADE-OFF

by
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AN ABSTRACT OF THE THESIS OF

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New product development (NPD) is the engine of renewal in any organization. The profitability, and even the survival, of organizations depends on the number and quality of new products introduced into the market. However, the process of developing new products (and estimating the proper timing for their introduction into the market) remains a rich topic for research and investigation within the product development community as it lies at the intersection of the engineering and marketing disciplines. A unifying model that brings both engineering and marketing concerns together is necessary to bridge the gap between both disciplines. In this thesis, a mathematical model is introduced to study the trade-off between the product performance and the time-to-market for different product development (PD) scenarios. These scenarios vary in the three main aspects related to product characteristics: the product complexity, the product newness and the degree of supplier's involvement. The model aims to maximize the revenue of the firm over a limited marketing window. The optimal solution reveals interesting managerial insights regarding the time that must be spent on "system design phase" and the "detailed design phase" for each product development (PD) scenario.

CONTENTS

ILLUSTRATIONS

FIGURE PAGE

TABLES

CHAPTER I

INTRODUCTION

A. Background

Amid fierce competition that reigns most industries, firms are giving more attention to product development cycle-time, focusing on the fast go-to-market decision and the advantages of being first to market (Banu Goktan & Miles, 2001). Product quality and performance also provide these firms with the necessary competitive advantage. A quick product introduction into the market usually grants the firm a competitive advantage leading to a higher market share and longer product life, in addition to a favorable cash flow (Karlsson & Ahlstrom, 1999). However, this rushed introduction into the market which is associated with a compressed product development process, can result in inferior product quality or performance. Numerous examples can be listed in this context. For instance, GE's introduction of new fridge with a rotary type compressor led the company to recall millions of units sold and incur substantial expenses (Bayus, 1997). Likewise, Mercedes introduced the small segment A Class model car as a result of a development time reduced to half of the previous models. During test driving of the new car, the problem of turning over at moderate speed was witnessed; this led the company to recall all cars for redevelopment at considerable incurred costs (Langerak & Hultink, 2006). Chrysler as well rushed the Neon model to market without conducting sufficient road tests. The automobile manufacturer recalled the car twice after only one month of sales (Langerak & Hultink, 2006). In 2016, Samsung rushed Galaxy Note 7 to the market in order to outdo rivals, however after reports of exploding batteries and overheating, Samsung was obliged to recall more than 3 million items sold and halt the production of the smartphone model

(Mozur, 2017). In fact, a product recall is associated with significant incurred costs including litigation fees, defects repairing and the cost of sales loss (Ahsan & Gunawan, 2014).

On the other hand, as the product performance is considered an essential contributor to the product success in the market and a promoter of high profitability (Cohen, Eliashberg, & Tech-Hua, 1996), some firms tend to spend too much time on improving the quality of the product and end up missing the window of opportunity. Nike FuelBand for instance was launched when the market became saturated with fitness devices wrist band such as Fitbit, Apple watches, Garmins (Athul, 2018). Similarly, Microsoft Zune failed to compete with the iPod due to its late introduction into the market following a prolonged development process (Athul, 2018). The challenge in compressing the product development process time is not to cut corners, but to accomplish the development tasks without sacrificing the performance of the developed product (Karlsson & Ahlstro`m, 1999).

B. Motivation

The trade-off between maximizing the product performance and minimizing the product development time has been the subject of many studies. For instance, Cohen et al. (1996) introduced a mathematical model representing the new product development process and capturing the trade-off between the time to market and the new product quality. The model is based on a multistage structure of the product development process and characterized by a short marketing time (window of opportunity) due to a high degree of product obsolescence. The model suggests that for a given firm, the product performance improvement is attributed to the firm productivity level i.e. the size of the development team, the return of experience and the time spent on each stage of the process. The study considers both the costs and revenues of the product over its life cycle. The revenue is function of the product quality and the length of opportunity window and the cost is function of the total man-hours (development team size multiplied by the development time) and the wage rate. The analysis aimed to maximize the firm's profit. The resources and time allocation across the different stages is determined for different scenarios characterized by different factors such as the size of the potential market, the extent of the opportunity window, the level of rivalry and the firm productivity.

Similarly, Bayus (1997) formulated a mathematical model incorporating both the time to market and product performance levels to examine the existing trade-off. His approach was also based on the relationship between the cost incurred in compressing the new product development process and the revenues generated as a function of the performance level and the window of market opportunity. Bayus (1997) suggested that the cost of product development is function of both the product performance and the development time and could be represented by a U shaped curve (Figure 4 in chapter II section B-4). The generated revenue is also function of the product performance and development time. The trade-off is examined by formulating and solving a mathematical problem. The analysis of the model results depended on the level of competition, the development cost, the level of sales, and the extent of market window.

However, the existing literature does not investigate the impact of the product properties and the supply chain configuration on the product performance-time-to-market trade-off.

C. Research Objectives

The goal of the thesis is to thoroughly explore the product and process aspects that impact the product performance and the new process development time. The thesis aims also to formulate and solve a mathematical optimization model that captures the trade-off between the time spent on engineering design (both systems design and detailed design) and marketing decisions (both introduction timing and product performance) for different NPD scenarios. These scenarios differ in three important factors that are part of any NPD process. These factors are: product complexity, product newness and the level of supplier's involvement. The objective of the proposed model is to maximize the firm's revenue with respect to a time-limited market window.

D. Thesis Outline

The remainder of the thesis is structured in four chapters. Chapter 2 presents the literature review covering four major topics: (1) new product development (NPD) process, (2) the different product properties, (3) new product development teams and (4) marketing & demand. In Chapter 3 the mathematical model is formulated, solved and analyzed. In chapter 4 a direct numerical application to the model is provided based on data collected from existing literature. Finally, chapter 5 concludes the thesis with research limitations and suggestions for the future work.

CHAPTER II

LITERATURE REVIEW

The literature review section is divided into four major subsections: (1) new product development (NPD) process, (2) new product characteristics, (3) new product development (NPD) teams and (4) marketing & demand.

The first subsection provides an overview of the different new product development (NPD) process models studied in the literature. The second subsection details some of the main product characteristics including its architecture, modularity and complexity. The third part focuses on the product development organization and the characteristics of a successful team. Finally, the fourth subsection elaborates on the marketing issues and product demand.

A. Product Development Process

The product development (PD) process is the sequence of all the essential tasks that a firm must perform to develop, manufacture and sell a product (Ulrich and Eppinger, 2004). These tasks include concept development (where customer needs are identified and product attributes are specified), system design (where product architecture is defined along with subsystem identification and interfaces), detailed design (where details of all product modules and components, such as geometry and material, are specified), prototyping (where product validation and testing are performed), manufacturing (manufacturing design and planning for ramp-up and full production), and a whole chain of suppliers (supply chain design).

The two major models of new product development (NPD) process that are studied in the literature are discussed in the following section. The two models are: the traditional product development process (or the waterfall process) and the agile stage gate process.

1. ***Traditional Product Development Process***

As defined by Ulrich and Eppinger (2004), the traditional product development process or the waterfall process consists of the following six consecutive stages.

a. <u>Planning Phase</u>

The first phase of the process is the planning phase, mainly referred to as "phase zero", it is the phase during which the main opportunity is identified and the technology development and the market objectives are assessed (Ulrich & Eppinger, 2004). The output of this phase is the project's mission statement specifying the targeted customers and market, the business objectives and the main assumptions and constraints (Ulrich & Eppinger, 2004).

b. <u>Concept Development</u>

At this phase of the process, the needs of the target customers are clearly identified; diverse concepts are listed and evaluated, many alternatives are dropped and few are selected for further development and testing (Ulrich & Eppinger, 2004). The concept includes the definition of the main functions, features and main specifications of the developed product. An economic evaluation of the product is also carried out during this phase to justify the project (Ulrich & Eppinger, 2004).

c. System Design

At the system design stage, the architecture of the product is defined, the decomposition into subsystems is carried out, the primary design of the main components is performed, and the detail design responsibility is allocated to internal and external resources (Ulrich & Eppinger, 2004). The original plan for the manufacturing (production system and assembly) is also established during this phase. The main outputs of this phase include: the general layout of each pre-selected concept, the specification of each sub system of the product, and the process flow diagram of the production process (Ulrich & Eppinger, 2004).

d. Detail Design

During the detailed design stage, the essential specifications of the products (shape, dimensions, geometry, weight, material of construction) are established, the parts to be purchased from the suppliers are identified and the tools needed for the fabrication of each part are designed (Ulrich & Eppinger, 2004). At the end of this phase, the detailed drawings of the final design are completed (Ulrich & Eppinger, 2004).

e. Testing and Refinement

At the testing stage, preproduction models of the product are constructed and evaluated (Ulrich & Eppinger, 2004). Prototypes are built with production-intent parts, these parts have the same material and specification as the parts to be included in the final product, but not necessarily produced by the foreseen final production process. These prototypes are usually referred to as “Alpha prototype”, they are usually tested to check whether the product will work as expected and satisfy the main customers' needs. "Beta prototypes" on the other hand, are built with parts fabricated by the foreseen production process (Ulrich & Eppinger, 2004). The beta prototype is used to verify the performance and reliability of the final product and to identify required modifications to the final product or production process (Ulrich & Eppinger, 2004).

f. Production

At the production stage, the product is manufactured through the foreseen production process (Ulrich & Eppinger, 2004). During the ramp up phase, the production workforce is trained and any remaining flaws or problems in the production are resolved and any defects in the product are evaluated (Ulrich & Eppinger, 2004). A gradual transition takes place from the ramp up production to the ongoing production. During this gradual transition period, the product is released. After the release, a review is performed to evaluate the product commercially and technically and to improve the development process of future projects (Ulrich & Eppinger, 2004).

The stage gate model, manages the different stages of the traditional product development process discussed above while imposing gates between consecutive stages in order to improve the process' effectiveness and efficiency (Cooper R. G., 2017). The model is graphically represented in Figure 1. Each gate is considered a checkpoint for quality control process, allowing the idea/product to proceed from one stage to the other (Cooper R. G., 1990). At each gate a set of deliverables is specified and the idea/product progresses from one stage to the other only if the deliverables meet certain standards and criteria (Cooper R. G., 1990). In other words, a set of inputs are required at each gate, so that an output can be provided. The inputs are the deliverables that the project leader presents at each gate, these deliverables are evaluated based on defined criteria before a decision of go/ kill/ hold/ is taken as an output along with the approval of the action plan for the following stage (Cooper R. G., 1990). Senior managers evaluate the deliverables at each gate, acting as gatekeepers (Cooper R. G., 1990).

Following researches and studies conducted in the domain, the stage gate model was shown to be of great importance since it helps the product development team to give "quality" a special focus, (Cooper R. G., 1990).

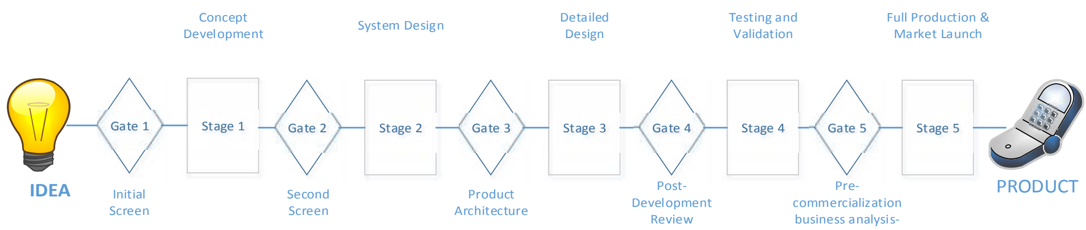

Figure 1 Stage Gate Model (adapted from Cooper (2001))

2. *Agile Stage-Gate Model*

Recently, a hybrid product development methodology has been developed. It is an integration, or a hybrid model combining both the "agile development methodology" and the traditional gate-stage system (Cooper & Sommer, 2016). The "agile" approach was devised in the 1990s to be applied mainly to the software development projects (Cooper R. G., 2017). The "agile development methodology" brings adaptability, responsiveness, agility and speed to the development project in order to develop - rapidly- a functioning product (Cooper R. G., 2017). This methodology incorporates numerous short "sprints" or iterations ranging from two to four weeks, at the end of which a "potentially releasable version" of the product (mainly a software) is ready to be demonstrated to customers (Cooper R. G., 2017). This methodology was created to cope with the new markets that are faster paced, more dynamic and less foreseeable (Cooper R. G., 2017).

There have been evidences that applying the hybrid "Agile-Stage-Gate" methodology to physical product development results in a management that is more adaptive to the changing customers' needs, a more efficient process, a reduced work load and rework per project, less unplanned iterations, fewer mistakes, and therefore a quicker product development process and a compressed time-to-market (Cooper R. G., 2017).

3. *Testing, Refinement and Iteration*

Even before the emergence of the "agile-stage gate model", many researchers considered the product development process an iterative, uncertain and creative process (Browning & Eppinger, 2002) (Martinez Leon, Farris, & Letens, 2013). Iterations, that mainly prevail in the development of complex systems such as aircrafts (Wynn & Eckert, 2016) help solve design problems, make ideas converge into real products and fix the incompatibilities between the components. The iteration by definition is a feedback loop followed to revisit the work previously performed (Safoutin & Smith, 1996) or a repetition with a goal of improvement. Iterations can be applied for a simple engineering task or it can extend to span over the release of successive generations of the same product (Safoutin & Smith, 1996).

In addition to the span of the loop, iterations differ in purpose. According to Safoutin and Smith (1997), iterations must influence the cost, quality, time or other aspects of the design; they are perceived by some as a usually unaddressed issue (Browning & Eppinger, 2002), a time consuming problem that creates additional cost or an important tool to improve the design (Safoutin & Smith, 1996). All iterations involve repetition in order to improve a specific aspect of the design (Safoutin & Smith, 1996). Browning and Eppinger (2002) agreed also that the quality of the product design improves with each iteration (Browning & Eppinger, 2002).

a. Iteration and the Product Development Process

According to Wynn and Eckert (2016), the research done so far is not conclusive on whether an iteration accelerates or delays the product development process. However, it is agreed that the product and the development process characteristics affect the duration and number of iterations in the product development process; for instance Wynn and Eckert (2016) state that, the overlapping design activities lead to fewer iterations during the testing and refinement. On the other hand, assigning the development work to different teams often leads to more iterations after discovering problems during the testing and integration stage of the product development process (Wynn & Eckert, 2016).

Furthermore, the product architecture (defined in next section) directly influences the level of iterations (Browning & Eppinger, 2002); for instance, adopting a standard design, reused components and a modular architecture lower the iteration level in a NPD process (Wynn & Eckert, 2016). According to Safoutin and Smith, iterations can be prevented by better application of existing information; in other words, a more elaborate study of a design before the progression in a prototype will result in less and more effective iterations (Safoutin & Smith, 1996). "If the problem is sufficiently understood at the beginning of the process, and design knowledge is optimally applied at exactly the time and place that it is needed, and there are no mistakes or oversights, iteration should be left without a role in the design transformation" (Safoutin & Smith, 1996).

b. Rework Duration

The estimation of the rework time could be assumed non-changing over consecutive attempts, however this assumption would render a product development model very simplistic (Maier, Wynn, & Biedermann, 2014). A more realistic assumption is to shorten the task duration with every iteration based on a predetermined learning curve (Browning & Eppinger, 2002) (Maier, Wynn, & Biedermann, 2014). In other words, the duration of a task is subject to learning effects reflecting the fact that it takes less time and effort to rework a task than to perform it for the first time (Maier, Wynn, & Biedermann, 2014).

In their developed model, Maier et al. (2014), assumed that the total duration of the task is linearly related to the number of iterations while other researchers assume that the duration of the iteration follows a stochastic model (Maier, Wynn, & Biedermann, 2014).

Certain product characteristics (defined in next section) directly influences the duration of an iteration since the latter is dependent on the amount of task rework and the sensitivity of the downstream tasks to this change. In particular, the duration of an iteration is related to the level of product modularity; a reduced level of modularity increases the time taken to revisit a design.

4. Measures of a Successful Development Process

According to Ulrich and Eppinger (2004), from the perspective of a for-profit investor, a successful product development process is measured based on the following three criteria:

- The development time: which is the time spent by the development team to complete the product development process. It reflects the responsiveness of the firm to competition and the speed to receive the economic returns of the development team efforts (Ulrich & Eppinger, 2004).
- The development cost: which is the money spent to develop the product. The development cost is considerable proportion of the investment needed to attain the targeted profit (Ulrich & Eppinger, 2004).
- The development capability: defined as the ability to learn from the current experience to develop future products (Ulrich & Eppinger, 2004).

B. Product Characteristics

1. Product Architecture

An essential aspect of the new product, is its architecture. According to Ulrich (1995), the product architecture is “the scheme by which the function of a product is allocated to physical components” (Ulrich K. , 1995). In other words, the product architecture is the arrangement by which the functional components of the product are assembled in physical groups, the architecture also defines the way these groups are interacting (Ulrich K. , 1995).

The product architecture is the main structural scheme of the product, comprising information on the number of components forming the product and the relationship/ interaction between the components (Fixson, 2004).

In his article “The Role of Product Architecture in the Manufacturing Firm” Ulrich, 1995 defines the product architecture by:

- The functional elements arrangement.
- The matching between the functional elements and the physical components.
- The interfaces between the interacting physical elements.

In terms of architecture, products are divided into two main categories: the modular and integral architecture. A modular product is known for its one to one mapping of functions to physical components and for the decoupled interfaces between the components (Ulrich K. , 1995). The decoupling means that the interfaces are well specified so that a change in one component does not require a modification in other components. In the modular architecture, each functional element is served by one physical group known as the functional building block or module; the interfaces and interactions between the different physical groups are clear and well delineated (Ulrich K. , 1995).

There are several types of product modularity that exist (Ulrich K. , 1995), these types are illustrated in Figure 2.

- Slot modular architecture: in this type of architecture, each interface has a unique type. The interfaces between the different modules are of different types (Ulrich K. , 1995).

- Bus modular architecture: in this type, there is a common element, the bus, to which the different elements are connected by the same type of interface (Ulrich K. , 1995).

• Sectional modular architecture: In this type, all interfaces are of the same type, however there is no “bus” or single component to which the other elements attach (Ulrich K. , 1995).

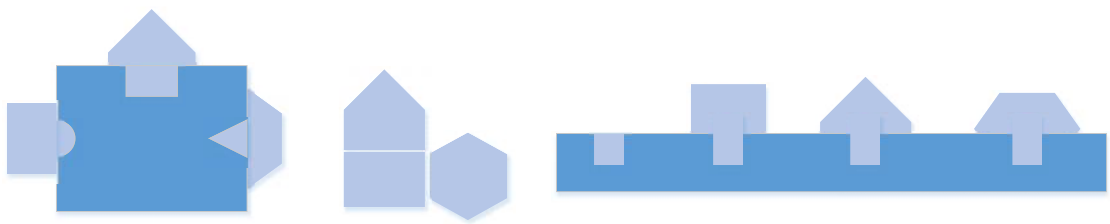

Figure 2 Different Types of Product Architecture (adapted from Ulrich (1995))

In the integral product on the other hand, the functions are performed by multiple physical groups, “functions are shared by physical elements” and unlike modular products, integral products do not show clear interfaces between the different physical groups (Ulrich K. , 1995).

The product architecture is defined during the system design phase of the product development process through the system engineering. The architecture impacts how the product is made, sold and assembled (Ulku & Schmidt, 2010). It is more of a result of an evolution rather than a thoughtful decision of the company (Ulrich K. , 1995).

a. <u>Modularity</u>

The modularity is a characteristic of both natural and artificial systems (Baldwin, 2015). Even though there is no universal measure for modularity, a high modular product can be recognized from its various, little and loosely coupled units (Baldwin, 2015) and the one to one mapping between the physical parts of the product and its designed functions (Ulrich K. , 1995) , whereas an integral product is characterized by a single large unit “in which everything depends on everything else” (Baldwin, 2015). The various definitions of modularity provided by different researchers and the characteristics of the modular products are listed in Table 1.

The modularity is often associated with the complexity of a product; designers tend to adopt the modular architecture when designing a complex product in order to simplify their design jobs (Brabazon & Matthews, 2003). Modularity is found to be an efficient way to manage complex products (Vickery, Koufteros, Droge, & Calantone, 2016) (Mikkola & Gassman, 2003).

In order to measure the degree of modularity of a product, Mikkola and Gassman (2003) introduced a mathematical function, the “modularization function”, that takes into account the number of standard components, the number of new to firm components, the degree of coupling and the degree of substitutability of a new to firm component.

The product modularity is the result of a learning and design process, called “modularization” in which, a complex product is broken down into smaller units (Baldwin, 2015). This process dates back to the years 1960s when IBM was the first to address the design and production problems of the earliest huge computers (Baldwin, 2015). IBM System/360 is considered the first modular system computer (Baldwin, 2015).

In her article “Modularity and Organizations”, Baldwin (2015) briefs the steps in the process of modularization which incorporates:

- Identifying the dependencies among the elements and figuring out ways to solve them (Baldwin, 2015).

- Eliminating the dependencies between the different building blocks, by following certain design rules (Baldwin, 2015).

A successful modularization contributes to an architecture characterized by a high level of interaction between the components within each building block (or subsystem) associated with a low level of interaction across the different subsystems (Baldwin, 2015).

b. Advantages of Modular Products

Even though it is known to enhance the sharing of knowledge among the team members (Mikkola & Gassman, 2003), a non-modular design is at a disadvantage due to the interdependencies between its elements, therefore any change in one of the components leads to changes in the other elements (Baldwin, 2015). On the other hand, modular products are associated with numerous advantages, listed in Table 1. In this thesis we are interested in the impact of modularity on two major aspects: (1) the product performance and (2) the NPD process management.

c. Correlation between Architecture and Product Performance

According to Ulrich (1995), two main components contribute to the total performance of a given product: (1) the local performance which is the performance of each component of the product and (2) the global performance which is a characteristic of the whole product (product size, weight, efficiency, power...). Ulrich (1995), claims that a higher local performance is associated with a modular product, whereas a higher global performance is associated with integral architectures (Ulrich K. , 1995).

Similarly, Holtta and Deweck (2005), distinguish between two types of performance: (1) the business performance and (2) the technical performance. The authors argue that technical constraints such as lightweight, speed and efficiency limit the advantage of modular products and favor the use of integral architecture. For instance, the lightweight version of the same product tends to have fewer parts, coupled interfaces and therefore an integral architecture (Holtta-Otto & de Weck, 2007). On the other hand, modular designs, characterized by an increased number of decoupled parts, tend to be less advantageous when mass, energy, volume occupancy and other technical aspects are concerned (Holtta-Otto & de Weck, 2007). However, modular architectures emerge when a business driven project is developed (Holtta-Otto & de Weck, 2007) since they favor business performance through promoting the mix and match (Baldwin, 2015) (Ulrich K. , 1995) (Mikkola & Gassman, 2003), re-configurability (Baldwin, 2015) , upgrade, add ons, adaptation (Ulrich K. , 1995) and more frequent product introduction into the market (Vickery, Koufteros, Droge, & Calantone, 2016)

Later, Danese and Filippini (2012), studied the mediating effect of the supplier’s involvement on the product modularity-performance relationship by examining 201 development projects from manufacturing plants in the mechanical, electronics and transportation industries. The authors hypothesized that the impact of product modularity on performance is at least partially attributed to the supplier’s participation through the creation of innovative ideas and its application to the outsourced modules. Their statistical analysis showed a significant positive impact of product modularity on product performance mediated by the degree of supplier’s involvement. The results of their study suggest that a NPD firm can improve its performance by following a product modularity strategy. The modularity paves the way for supplier's collaboration and integration within the NPD and leads to higher product and time performance.

d. Correlation between Architecture and Process Time

- System level design: According to Ulrich (1995), more efforts are invested in this phase when a modular product is being developed. This can be attributed to the time and effort spent to map functions to the different physical elements and to clearly and accurately define the interfaces between the modules and specify the relevant standards and protocols. In addition, the performance targets of each component is drawn in this phase of the process (Ulrich K. , 1995). For integral product on the other hand, less effort would be required; the efforts during this stage are allocated to setting clear performance targets.

- Detailed design: the detailed design of each component can be performed independently and in parallel for modular products (Ulrich K. , 1995). The communication between the developing teams are infrequent. Whereas for the integral product, the component designers create a core team, within which many channels of communications are formed (Ulrich K. , 1995).

- Testing and refinement: testing a modular product results in checking unanticipated interaction between the physical components of the product, it is usually considered a simple checking activity which results in fixing some bugs by modifying only few components (Ulrich K. , 1995). More time is expected to be spent on "testing and refinement” of integral products, since any error detected in this phase requires alteration to many components of the product (Ulrich K. , 1995).

PRODUCT MODULARITY				
Definition	**Characteristics of Modular Products**	**Correlation with Product Performance**	**Correlation with NPD Process Performance and NPD Time**	**Marketing advantages**
By definition the modularity is: the scheme by which functional properties are allocated to physical elements (Ulrich K. , 1995). the hierarchical structure of the product system, consisting of sub systems or sub-assemblies that can be designed independently yet work as a one coherent system (Vickery, Koufteros, Droge, & Calantone, 2016). the extent to which a complex product can be divided into simpler units, called modules (Baldwin, 2015).	A modular product is characterized by: The existence of various small assemblies of elements (modules) that are loosely coupled unlike the integral product that is characterized by a larger unit incorporating elements depending on each other (Baldwin, 2015). The one to one mapping between the physical elements and the functional components, known as the functional mapping (Ulrich K. , 1995) (Vickery, Koufteros, Droge, & Calantone, 2016). The clear and decoupled interfaces among the physical elements, or the interfaces standardization (Ulrich K. , 1995) (Vickery, Koufteros, Droge, & Calantone, 2016). The high level of decomposability of the product system (Baldwin, 2015) (Vickery, Koufteros, Droge, & Calantone, 2016) The existence of one or more physical element.in each module (Brabazon & Matthews, 2003) (Ethiraj & Levinthal, 2004). The high level of interdependence between the elements within each module associated with a low level of interdependence between the different the subsystems forming the whole product (Baldwin, 2015).	The local performance of a product is higher for a modular product, whereas the global performance is optimized with integral products. (Ulrich K. , 1995) (Holtta, Suk, & De Weck, 2005). A modular architecture is favored when flexibility and quick innovation are sought and judged more important than the product performance (Ethiraj & Levinthal, 2004).	The product modularity is correlated with: A reduced risk of mistakes in case any changes are made (Rebentisc, et al., 2016). A compressed New Product Development (NPD) lead time (Mikkola & Gassman, 2003) (Vickery, Koufteros, Droge, & Calantone, 2016) since the different modules can be worked on concurrently (Baldwin, 2015). An improved product introduction performance by decomposing and decreasing the size of a design problem and reducing the interdependence among the various components of a single product (Vickery, Koufteros, Droge, & Calantone, 2016). A reduced cognitive complexity of the system/ product (Baldwin, 2015) by transforming it into a decoupled, easy to understand system (Baldwin, 2015) and offering “the ability to decompose Higher chances of independent and concurrent design and supply chain (Ulku & Schmidt, 2010).	Modular products are associated with: The possibility of mix and match, and substitutability of components, without compromising the system integrity. (Baldwin, 2015) (Ulrich K. , 1995) (Mikkola & Gassman, 2003). Re-configurability for high variety products (Baldwin, 2015). Easier upgrades, adds-on and adaptation (Ulrich K. , 1995). Higher level of innovation (Vickery, Koufteros, Droge, & Calantone, 2016). More frequent product introduction into the market (Vickery, Koufteros, Droge, & Calantone, 2016) Quicker and less costly introduction of subsequent versions of the product at an increasing performance level (Mikkola & Gassman, 2003) Improved handling of “environmental perturbations” (Vickery, Koufteros, Droge, & Calantone, 2016) Possible standardization opportunities (Ulrich K. , 1995)

Table 1 Summary of Product Modularity

2. Product Complexity

The complexity of the engineering systems is becoming more significant due to the complexity of technologies and infrastructure supporting these systems. Unwisely managed complex systems lead to undesirable results in terms of development time and cost (Sinha & Suk Suh, 2018) (Rebentisc, et al., 2016) (Sinha & De Weck, 2013) (Griffin, 1997) (Tatikonda & Rosenthal, 2000).

The complexity of a product is mainly defined in the initial concept generation stage of the NPD process (Sinha & Suk Suh, 2018); after establishing the architecture each module in the product (ideally contributing to one function, and the different modules contribute to the overall performance of the product) becomes characterized by a certain level of structural complexity.

There is no consensus among academia and industry on the definition of a system complexity. For instance, in a study aiming to examine the impact of product newness and product complexity on the product development cycle time, Griffin (1997), defines the product complexity as the number of functions incorporated within a product. On the other hand, Ethiraj and Levinthal (2004), define a complex system as a system composed of a “large number of parts that interact in a non-simple manner”, these interactions make the performance unpredicted. Similarly, in their research on its modulating effect on modularity-new product performance relation, Vickery et al (2016) define the complexity as the number of components establishing a system. Furthermore, Novak and Eppinger (2001), state that three main characteristics are present in a complex product: (1) the number of components in a product that need to be designed and executed, since each component needs its own drawings, part number, its own testing and validation, (2) the extent of interaction between the components since a change in one part to be designed require modification in other parts of the product and (3) the level of product novelty.

The most extensive and recent complexity related work was done by Sinha and De Weck (2013) who developed a method to measure the structural complexity of a system. The authors distinguish between the complexity of components and that of the system. Whereas the complexity of the component is related to the understanding of that object, the system complexity depends on the heterogeneity, quantity and the connectivity between these elements. For a given system, there are three types of internal complexity: 1) the structural complexity, 2) the dynamic complexity and 3) the organizational complexity (Sinha & Suk Suh, 2018). The structural complexity is a form driven complexity, the dynamic complexity is a function driven complexity and depends on the functions performed by the product and the organizational complexity relates to the engineering firm structure; knowing that the organization structure usually mirrors the system's architecture, the organization's complexity is related to the structural complexity (Sinha & Suk Suh, 2018). According to authors, the structural complexity metric, closely related to the architecture of the system, is given by: $C = C1+C2C3$ where C1 is the sum of complexities of individual components (does not involve any architectural information unlike C2C3), C2 and C3 are the total number of pair-wise interfaces and the topological complexity respectively. C2 reflects the number and complexity of each pair wise interaction and C3 reflects the impact of architecture of the arrangement of interfaces; a distributed system is associated with a higher structural complexity unlike more centralized structure as shown in Figure 3

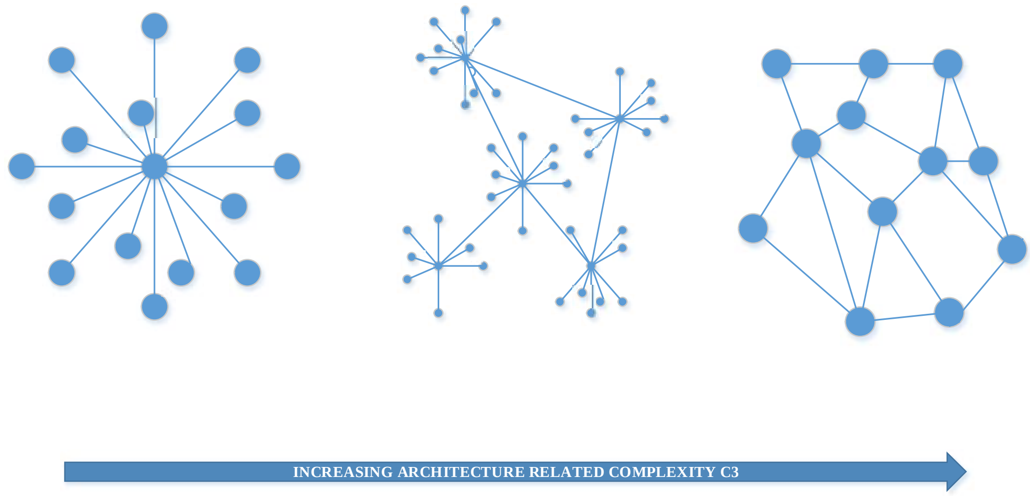

Figure 3 Increasing architecture driven complexity, adapted from (Sinha & De Weck, 2013)

3. Measure of a Successful Product

According to Ulrich and Eppinger (2004), from the perspective of a for-profit investor, a successful product is measured based on the following criteria:

- Product quality: also known as the product performance, measures the reliability, robustness of the product, and the extent to which the product satisfies the customers' needs.
- Product cost: the product cost includes the capital cost invested on equipment and tools as well as the costs of producing the different units of the product. The cost of the product determines the profit earned by the firm based on a certain sales volume or a particular price.
-

4. Time to Market and New Product Performance Trade-off

As previously mentioned in section A.4 of this chapter, a successful product development process is characterized by a short time to market. Knowing the impact of the time to market and the product performance on the ultimate success or failure of the product development firm, several researchers, including Bayus (1997) and Cohen et al. (1996), have investigated the trade-off between the time to market and the level of product performance.

Bayus (1997) investigated the optimal product introduction time and performance level for different competitive scenarios. Bayus' approach was based on the relationship between the time to market and the cost of the product development process (Bayus, 1997).

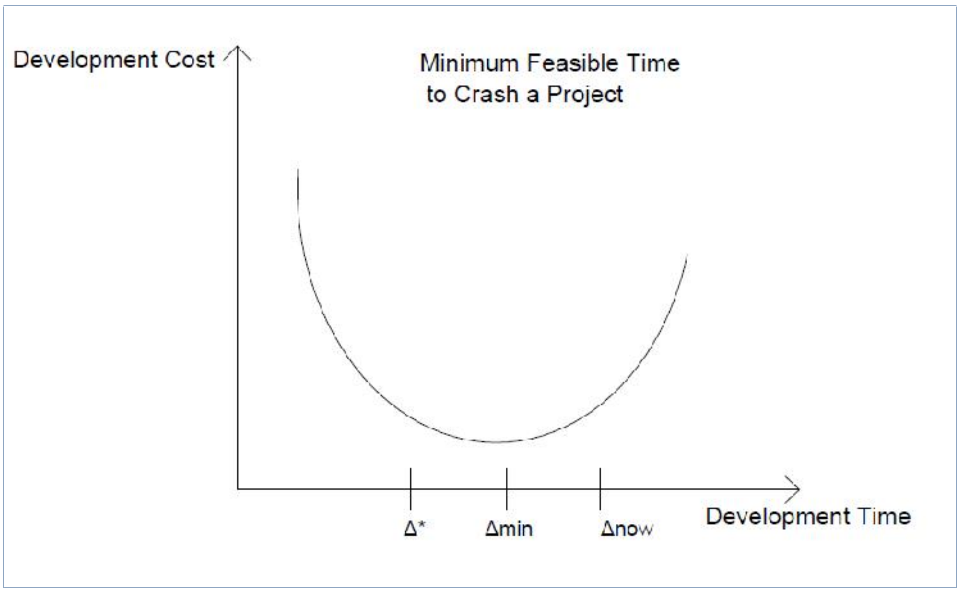

Figure 4 Development Time and Development Cost Trade-off (adapted from Bayus (1997))

As shown in Figure 4, the development time - cost trade-off follows a U shape curve. The development cost increases as the development time is reduced below a minimum time Δ_{min}. This is justified by the increased number of people involved in the project to compress the development time, lowering the overall productivity due to the additional required communication and training for new team members. This increase in cost is also attributed to the concurrent work that is more expensive than the sequential work (Bayus, 1997). For development times longer than Δ_{min} the development cost also increases as the development time extends, this is due to the lower know-how of the team members in addition to their decreased motivation. According to Bayus, the optimal development time Δ^* is not associated with the lowest development cost, yet with an extended marketing window. Product performance is associated as well with higher development cost. Therefore, each firm aims to balance “the benefits and costs” of rushing to market with various performance levels, taking into consideration the degree of the competition (Bayus, 1997).

Similarly, Cohen et al. (1996) introduced a mathematical model representing the new product development process and showing the trade-off between the time to market and the new product quality. This model is based on a multistage structure of the product development process (discussed in section A of this chapter) and characterized by a short marketing time (window of opportunity) due to a high degree of product obsolescence. Cohen et al. (1996) concluded that if the product quality improvements are additive (over the phases of the process), it will be optimal to spend most of the time at the most productive development phase of the multi stage process. In addition, they found that it is not always optimal to be faster, especially if there are high market potential and high margin for the new product. An optimized minimum break even time may risk a premature introduction to the market.

C. Product Development Teams

1. Types of Product Development Teams

The different people involved in the product development process are referred to by “project team”. The project team usually consists of a core team and an extended team. The core team, a small sized team, comprises a team leader, design, and marketing manufacturing specialists; whereas the extended team, consisting of a higher number of members that extends to include suppliers and other members of the financial, legal and sales sectors (Ulrich & Eppinger , 2004).

Firms are more relying on external teams to develop the different subsystems of the product (Ulku & Schmidt, 2010). Therefore, teams are not only in-house (within the firm) or they can also be external, when the development work is outsourced.

It is commonly presumed that a product architecture “mirrors” the team and the organization architecture (Ulku & Schmidt, 2010) (Baldwin, 2015). i.e a modular product matches decentralized organizations (when product development is outsourced and teams are outside the firm) whereas the integral product requires integrated organizations (in house teams) (Ulku & Schmidt, 2010).

It is also known that the product architecture affects the level of interaction and collaboration between the developing teams; an integral architecture requires extensive interaction between the development teams, whereas the development teams of modular products can work on the different subsystems relatively independently (Ulku & Schmidt, 2010)

However, numerous new empirical studies proved that the relationship between the organization structure and the product structure is more complicated than the simple one to one mapping (Ulku & Schmidt, 2010). In their paper, Ulku et al. (2010) studied the optimal mapping between the supply chain configuration and the product architecture (modular and integral) and explored the options of developing the product internally by the firm or in collaboration with the supplier for each product architecture type (modular and integral).

Ulku et al. (2010) developed a mathematical model to find the optimal mapping between the supply configurations to the product architecture. They considered a product composed of two main subsystems. The model assumes that a higher degree of modularity (m) leads to a lower performance. In addition, the model introduced a new parameter “H” reflecting the degree of inseparability of the product. Ulku et al. (2010) argue that some systems, due to their types and nature, are more separable than others; for instance, the natural interaction among the electromechanical systems is strong, therefore the degree of inseparability is low, unlike other systems such as a software. In addition, the degree of inseparability also depends on the newness of the design task, in other words, if the product development firm has previous experience in the structure of interdependencies, it is easier for the firm to decompose the system into subsystem and the opposite is true.

Regarding the costs of development process, Ulku et al. (2010) attribute the cost of the development process to various stages. First, the cost of technical collaboration that is assumed zero for fully modular products as there is no need for collaboration between the separate development teams working independently on clearly decoupled modules (m=1). Conversely, less modular (more integral) architecture needs more collaboration during the detailed design phase of the product development process. Second, the development cost incurred during the detailed design phase is function of the subsystem quality specified by the responsible product development team.

2. Outsourcing and Product Quality Improvement

Literature provides evidence that the supplier’s involvement in NPD contributes to more innovative products and improves the development time (Danese & Roberto, 2012). The supplier’s involvement in NPD can take place at the different stages of the process. In the early phase of concept generation, suppliers can provide valuable information on technology trends and contribute to value of engineering and more creative and innovative concepts (Danese & Roberto, 2012). During the detailed engineering phase, suppliers help identify risks, prevent potential problems and employ their expertise to design/produce their assigned components or modules. (Danese & Roberto, 2012). In the testing and integration phase, suppliers can perform additional tests that help identify potential future failures (Danese & Roberto, 2012)

Overall, an increased supplier’s involvement contributes to an improved product quality, better features and characteristics and less costly design (Petersen, Handfield, & Ragatz, 2005).

D. Demand and Utility Function of New Products

In general, the demand of each product is a function of its price and value relative to the customer. Customers’ needs are translated into system-level attributes that are important to customer value, called critical to value attributes (CTV) (Cook H. , 2006). In order to stay in business, a given firm should satisfy the needs of an adequate number of customers better than its rivals in order to provide the cash flow required to develop future products. This customer’s loop is represented in Figure 5.

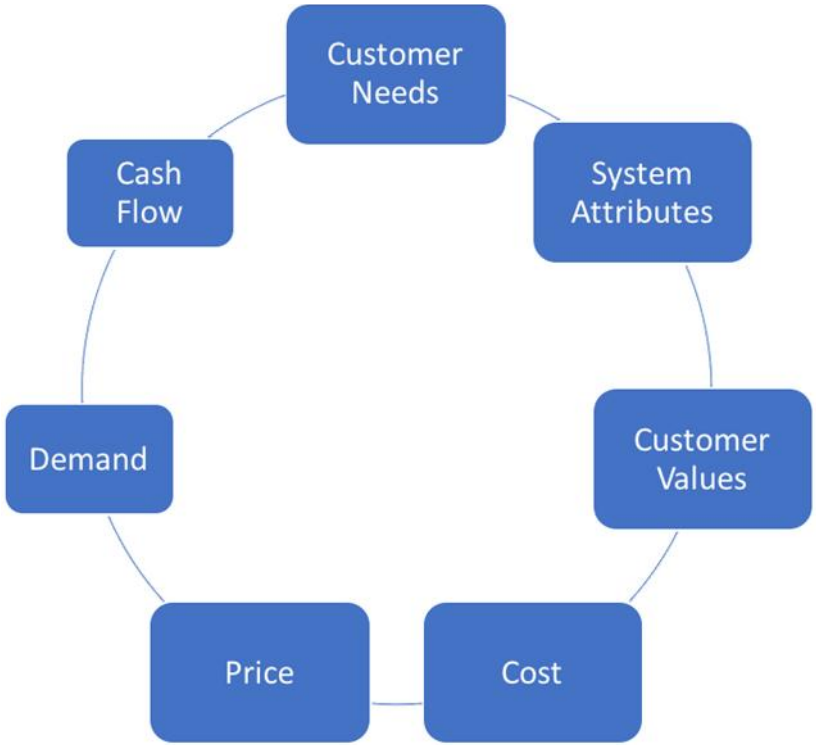

Figure 5 Structure of product planning (customer loop) (adapted from (Cook H. , 2006))

Therefore, the demand function can be represented by D= f (V, P), where P is the product price and the V is the value given by the utility function for a specific customer type (Cook H. E., 2005).

In the case of monopoly, a demand can be represented by a downward sloping curve with respect to the price, as shown in Figure 6. A demand curve can be expanded in terms of Taylor series about the demand, price and value of the reference state (D0, P0, V0) respectively. The linear approximation is given by $D = K\ (V\text{-}P)$, where the only the linear terms of the function are kept (Cook H. E., 2005).

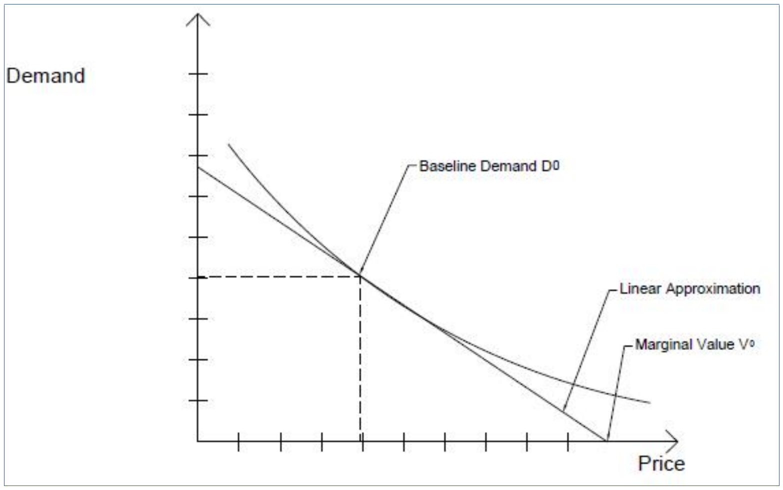

Figure 6 Demand Curve (adapted from Cook (2005))

An increase in the value of a product shifts the demand curve upwards, as shown in Figure 7, such that for the same price, the demand of the higher value product exceeds the demand of the lower value product; or for the same demand level the price of the lower value product is less than the price of the higher value product.

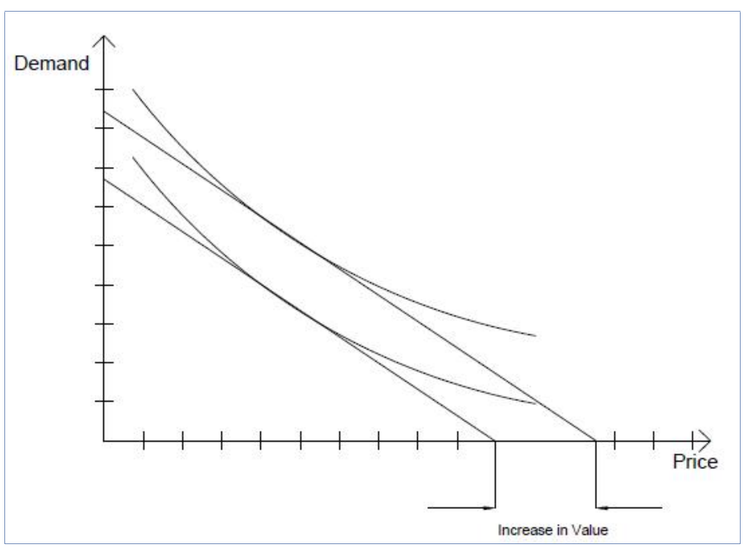

Figure 7 The Shift in Product Value (adapted from Cook (2005))

In a competitive market, the demand of a given product is function of its value and the value of its rivals with respect to the customer. The logit model is frequently used to forecast the demand of a certain product, given the utility of this product and the total utilities of the competitors. The logit function is given by $D_i = D_T \frac{e^{U_i}}{\sum_{j=1}^{n} e^{U_j}}$ where U_i is the utility of the product, D_i is the demand, U_j is the utility of the competitor and n is the total number of competitors (Cook H. , 2006). The logit model is built on the assumption that the customer's utility is the summation of two components, a deterministic component and a random component (Cohen, Eliashberg, & Tech-Hua, 1996). The probability that a chosen costumer buys form the firm is equal to the probability that this same firm's product produces higher utility than the competitors.

Knowing that the deterministic part is function of the product performance, Cohen et al. 1996 introduced the product quality into the utility function such that the demand takes the following form: $D_i = D_T \frac{e^{U(Q_i)}}{e^{U(Q_i)} + e^{U(Q_c)}}$ where Q_i and Q_c are the firm's product quality and that of the competitor respectively.

CHAPTER III

MODEL FORMULATION

This chapter is structured in three major sections. In the first section, the mathematical model description and assumptions are discussed and the various mathematical constructs required for capturing the time-to-market and product performance trade-off problem are formulated. In the second section the solution methodology is detailed, including the numerical values assumed, the simulation and the results obtained. In the last part the analysis of the results is discussed.

A. Mathematical Formulation

1. New Product Development Process

In our proposed mathematical formulation, the traditional stage gate model is followed with ideas adopted from the agile model; we consider that three major consecutive stages contribute to the New Product Development (NPD) time frame. These three stages are the system design, detailed design and testing and integration. At the end of the testing stage a decision to go to market, not to go to market or iterate i.e, rework is made. If the decision is to rework, the detailed design stage is revisited before proceeding again to testing and integration. The structure of the model, including the consecutive three stages of the NPD, the marketing stage and the time allocated for each stage (not to scale) is represented in Figure 8.

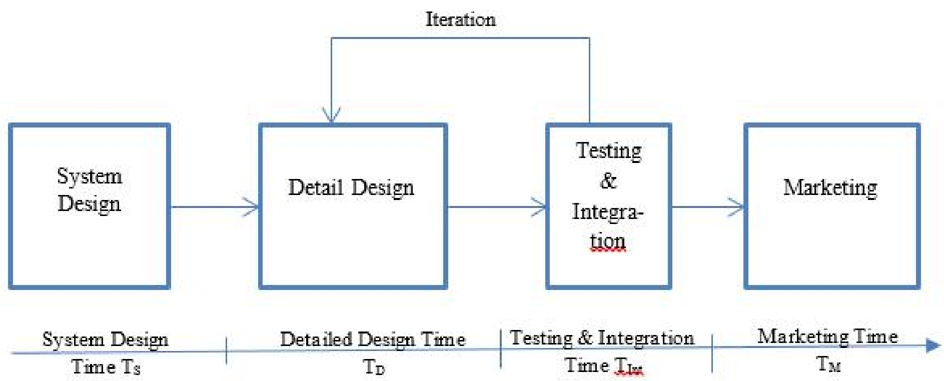

Figure 8 Product Development Process Model and Time Span

As previously discussed in Chapters I & II, the time reduction of the NPD process, contributes to a longer marketing window, the window of opportunity; however it is associated with low product quality and therefore a lower expected profit due to a reduced market share and/or lower product price (Cohen, Eliashberg, & Tech-Hua, 1996). On the other hand, an extended NPD process results in higher product performance at the expense of the marketing window. Therefore, there is a trade-off between the time spent on the system design, detailed design, testing and integration, i.e. the time to market on one hand and the quality/performance of the product on the other hand. The formulated mathematical model will capture this trade-off, taking into account the product characteristics (detailed in section B of Chapter II) that affect the duration of each stage of the NPD process and the new product performance level.

Similar to Cohen et al. (1996) our model is based on a multi-stage process spanning over a limited time frame, after which the product becomes obsolete, loses its value and its demand becomes insignificant. This assumption mainly applies to highly competitive markets and technological products such as computers, tablets, smart phones in addition to software industry and automobile industry that also exhibit these demand characteristics.

2. *System Design Phase*

In the system design phase of the NPD process, the system architects define the product architecture, identify the subsystems forming the product, specify the interfaces between the subsystems and establish the mapping of the physical components to the functional elements (Ulku & Schmidt, 2010); therefore, the product architecture emerges and the extent of product modularity is uncovered during this phase.

As argued by Baldwin (2015), the modularity of a product is defined through the process of "modularization" during which the degree of dependence and interaction between the different elements of the product is defined. As a result of this process, the different elements are clustered within relatively independent modules such that the interaction between the elements within the module is maximized and the interaction across the modules is minimized (Baldwin, 2015). In other words, the modularization entails identifying the highly interactive cluster of elements and grouping them in modules (Ko, 2013). It also requires recognizing the dependencies between the elements and trying to eliminate them by the adoption of certain design rules (Baldwin, 2015). Through the "systematic repetition of this process" the unnecessary interdependencies across the different clusters can be eliminated and the main modules emerge (Baldwin, 2015), therefore, a highly modular product architecture is the result of substantial effort and considerable time spent by the system engineers to identify the highly interactive cluster of elements.

It is understood that for a complex product - characterized by a large number of elements and a diverse number of functions assigned to the product- the process of modularization is more time consuming. Bashir and Thomson (1999) showed that the amount of effort required to establish the product architecture is higher when the product complexity increases. Fujimoto and Clark (1989) agreed that the level of product complexity impacts the product planning phase currently known as the system design time. In addition, if we consider the example given by Vickery et al. (2016) a toaster oven is considered much less complex than cars since it comprises less components and has fewer and less diverse interfaces between the components, in addition toaster oven encompasses fewer secondary functions that contribute to the main/ primary function of the product. Compared to automobiles, toaster ovens which are considered less complex, require less time spent on the system design phase to reach the same level of modularity as automobiles (same level of standardized interfaces, clustering of interactive elements within decomposable modules, same level of mapping between the modules and functional elements). Therefore, more time is required to eliminate and organize the interdependencies among components and modularize a more complex system.

Based on the above, our model reflects two major characteristics:

- A longer time spent by the system architect on the modularization process is associated with a higher level of product modularity (Al-Kindi & Yassine, 2009) (Weisera, Baasner, Hosch, Schlueter, & Ovtcharova, 2016).
- A higher level of product complexity requires a longer time consumed by the system architects on the modularization process to reach a preset level of modularity,

Therefore the modularity $m\ (T_s, \lambda)$ can be expressed in equation (1), where the system design time is designated by T_s ($0 \leq T_s \leq 1$), and the product complexity by λ ($\lambda \geq 0$). The plot of the function, Figure 9, shows the evolution of the modularity “m” with time spent on system design “T_s” for different products having different levels of complexity "λ". The graph reveals that, for the same product, a longer time spent of system design (higher T_s for the same level of complexity λ) results in a product architecture of higher modularity. On the other hand, when an equal amount of time is allocated on system design for different products having different levels of complexity "λ", a higher modularity will emerge for the product having the lowest level of complexity "λ".

$$m = T_S^{\lambda} \quad (1)$$

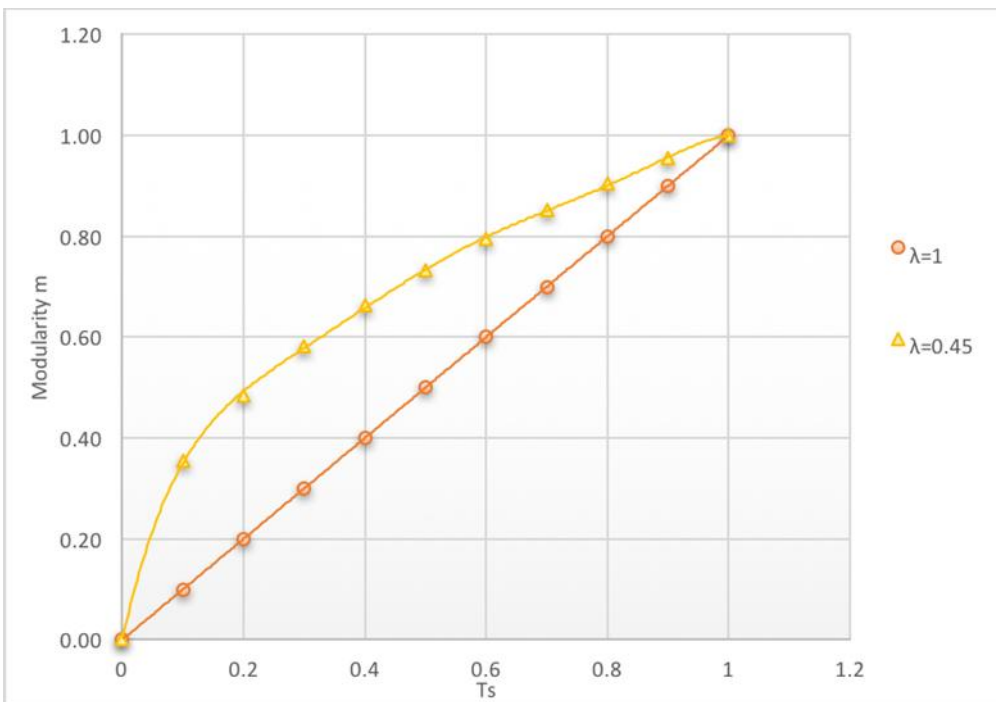

Figure 9 Plot of Modularity vs System Design Time

3. Detail Design Phase

During the detailed design phase of the NPD process, the design of different subsystems, defined in the previous phase (system design phase), is assigned to different teams working concurrently on the different subsystems. These activities can be performed in-house or they can be outsourced. Each team, in-house team or supplier, defines the subsystem geometry, material, tolerances and other specifications during this stage of the process. The teams first spend an amount of time T_D to deliver the detailed design at a certain quality level Q at the end of this phase. This detailed design time T_D is considered the maximum time consumed by any of the different design teams working on the different subsystems.

As agreed on by scholars such as Bayus (1997), Griffin (1993) and Ulrich (1993), the performance of the new product is positively related to the time spent on NPD process. Moreover, in their model, Cohen et al. (1996) assume that the performance of the product improves as more time is allocated to the design phase.

In addition to the amount of time spent on the detailed design phase, the final quality of the product is also directly affected by its architecture defined during the system design phase (Ulku & Schmidt, 2010). In our model, a higher level of modularity is assumed to positively impact the level of product performance. This hypothesis was validated by Danese and Filippini (2012) who confirmed the significant positive impact of product modularity on product performance mediated by the degree of supplier's involvement.

Furthermore, the supplier's involvement in NPD is closely related to the level of product performance. Researchers agree the product performance will be improved when the level of supplier's involvement increases (Petersen, Handfield, & Ragatz, 2005). A supplier helps the firm achieve product distinctiveness, innovativeness (Clark & Fujimoto, 1989) and improve the development time (Danese & Roberto, 2012).

Finally, the degree of product newness influences the time spent on the detail design to reach the desired level of performance. The product newness is defined by how much of the new product has to be redesigned, compared to the product issued from the previous generation (Griffin, 1997). It reflects the familiarity of the firm with the design task. Firms do not always start from scratch to design a new product (Griffin, 1997). Incremental modifications or improvements are more common in NPD rather than new to firm or new to the world projects (Griffin, 1997), nevertheless each firms develop products ranging over a wide range of newness level. Higher degrees of newness are found to extend the NPD time (Griffin, 1997) (Karlsson & Ahlstrom, 1999) (Duhamel & Santi, 2012); if the engineers are more familiar with the product, a quicker time is expected to develop the product (Griffin, 1997). Therefore, we can assume that as the level of newness-from a firm perspective- increases, more time is required to reach the targeted product performance level; if the product is designed from scratch, it takes more time for the detailed design to be accomplished when compared to a new generation of products that imitates a considerable part of the previous design or other concurrent models of the product.

Building upon the above assumptions and literature, we assume a simple function for generating performance reflecting the following major characteristics:

- The product performance improvement can be represented as a continuum $Q(T_d)$ such that a longer time spent on the detailed design phase is associated with a higher product performance.
- Compared to modular products, integral products require more time invested by the design team to reach a preset performance level.

- A higher degree of supplier's involvement contributes to higher performance level.

A higher level of product newness delays the achievement of the product performance target during the detailed design phase.

Therefore the product performance $Q\ (T_D, \mathrm{m}, \mathrm{\mu}, \mathrm{\alpha})$ can be expressed in equation (2), where T_D is the time is spent on detailed design ($0 \leq T_D \leq 1$), m the level of product modularity defined in the system design phase ($0 \leq \mathrm{m} \leq 1$) , μ the product newness ($0 \leq \mu \leq 1$) and $\mathrm{\alpha}$ the degree of supplier's involvement ($0 \leq \mathrm{\alpha} \leq 1$)

$$Q = T_D e^{-\frac{\mu}{\alpha + m}} \qquad (2)$$

The plot of equation (2) is shown in Figures 10, 11 and 12. Figure 10 reveals the evolution of the product performance "Q" with the time spent on the detailed design "T_D" to develop products characterized by different levels of modularity "m" and the same level of newness "μ" and supplier's involvement "α". The figure shows that a longer time spent on detailed design (higher T_D) results in an improved product performance. When an equal amount of time is allocated to detailed design of different products having different levels of modularity "m", a higher performance will result in the product with the highest level of modularity "m".

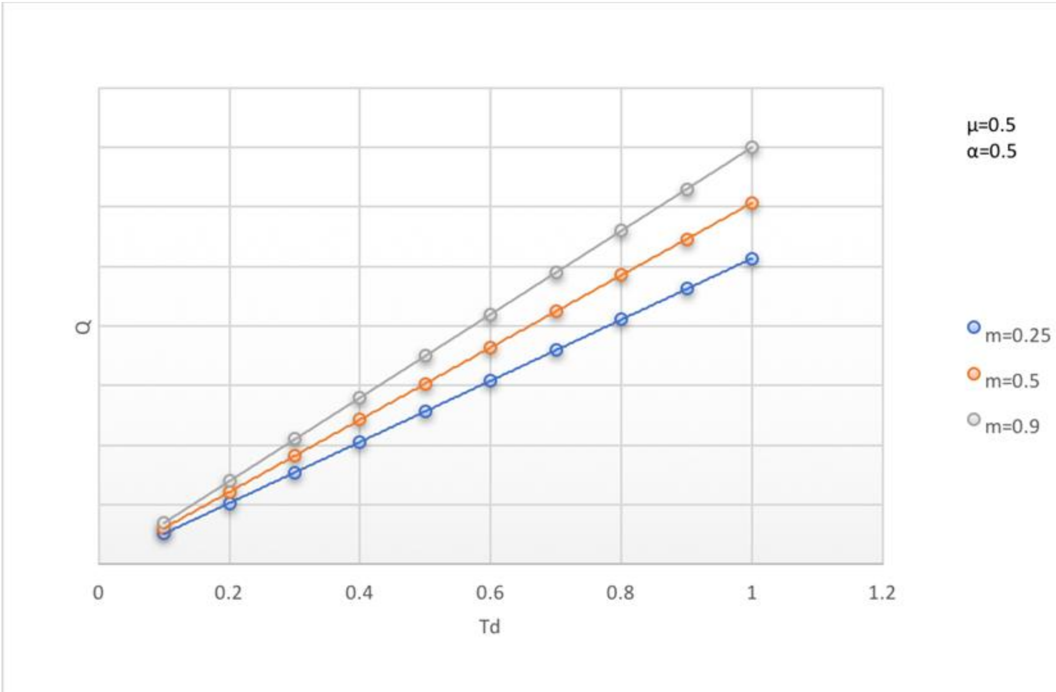

Figure 10: Plot of Product Performance vs Detailed Design Time TD for fixed product newness μ=0.5, α=0.5 and different levels of modularity

Similarly, Figure 11 reveals the evolution of the product performance "Q" with time spent on the detailed design "T_D"for products characterized by different levels of newness "μ". The firms are assumed to develop products of the same level of modularity and involving suppliers to the same extent. The plot shows that a longer time spent on detailed design (higher T_D) improves the product performance. And, when an equal amount of time is allocated to different products having distinguished levels of newness "μ", a higher product performance will result in the product having the lowest newness level "μ".

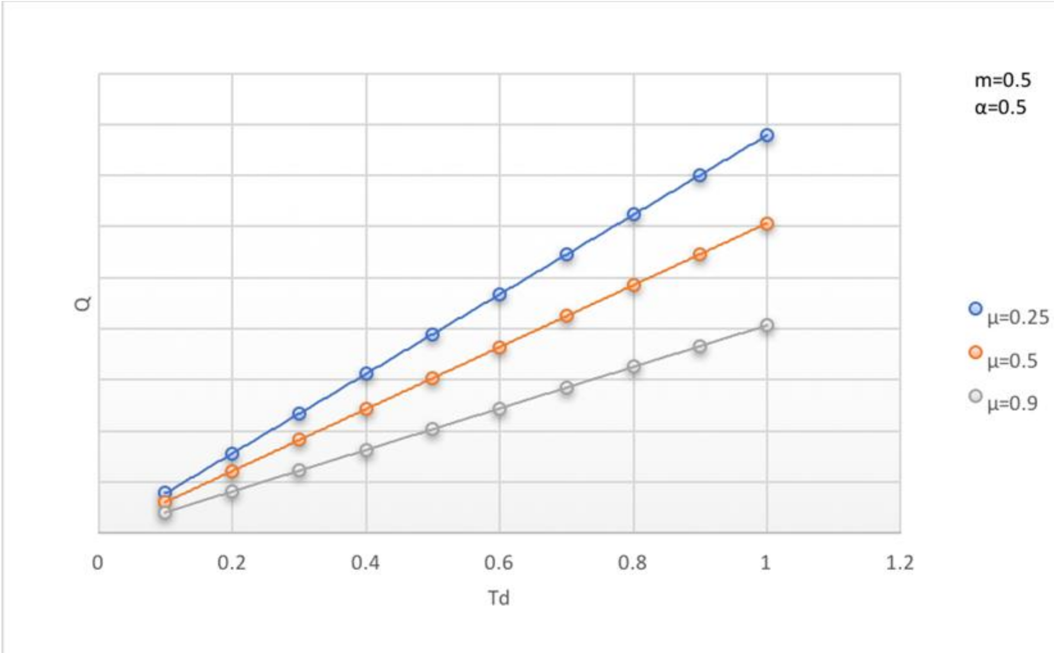

Figure 11 Plot of the product performance for different levels of newness μ, and fixed levels of supplier's involvement and modularity

Figure 12 shows the plot of the product performance evolution "Q" with the time spent on the detailed design "T_D" to develop products characterized by different degrees of supplier's involvement and the same level of newness"μ" and modularity "m". For the same level of product newness and modularity, a longer time spent on detailed design (higher T_D) results in an improved product performance. When an equal amount of time is allocated to detailed design of different products having different levels of supplier's involvement, a higher performance will result in the product with the highest level supplier's involvement"α".

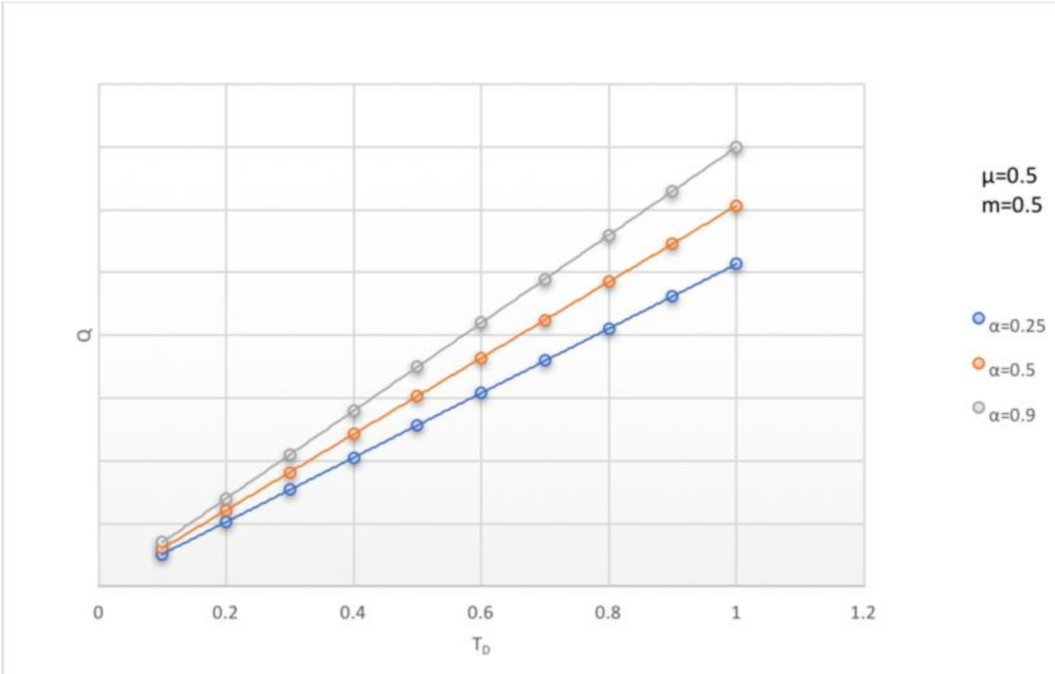

Figure 12 Plot of the product performance with detailed design time for different levels of supplier's involvement and fixed levels of newness and modularity

4. Testing and Integration

During the testing and integration stage of the NPD process, the different subsystems designed by the different design teams are tested to check their compatibility when brought together to function as one system.

There is a probability that the product fails the testing and integration, in this case the detailed design is rechecked to fix errors. In our model we assume that the extra amount of time spent on detailed design (iteration) does not contribute to any improvement in the product quality, however this time is spent to fix the bugs, errors and incompatibilities and allows the product to be launched into the market; we consider that the product quality is only determined by the original amount of time spent on the detailed design phase (T_D). In addition, we consider that the system design is not revisited in case the product fails the testing and integration only the detail design is rechecked.

The failure of the design at this stage is correlated with its architecture previously defined in the system design phase. As the modularity of the product increases, the concurrently designed and clearly defined subsystems will be more likely to fit together to form a functioning product, unlike the integral product that will need further modifications and adjustments of the different modules in order to fit together and contribute to a well-functioning product. Therefore, a higher level of modularity is associated with a lower probability of failure in testing and integration and a reduced number of iterations (Browning & Eppinger, 2002) (Wynn & Eckert, 2016).

Furthermore, the duration of the testing and integration phase is also closely related to the extent of supplier's involvement. If the detailed design of a large fraction of components, or chunks of components, is outsourced to an external supplier, the outsourced parts will be less likely to fit together to form a functioning product. Additional effort and collaboration between the firm and the suppliers will be required to further adjust and modify the parts in order to produce a functioning product. On the other hand, if the detail design is performed in house, a lower probability of failure in testing and integration is expected along with a reduced number of iterations, and in case of failure, less collaboration effort will be required to review and update the design since the whole job is carried out internally.

Therefore, we assume that the time spent on testing and integration phase is given in equation (3), where "α" is the fraction of supplier's involvement $(0 \leq \alpha \leq 1)$ and "m" is the level of product modularity $(0 \leq m \leq 1)$.

$$T_{INT} = \alpha\,(1 - m)T_D \qquad (3)$$

5. Marketing

The revenues are only realized during the “market window” of the process. The demand function adopted in our model is the logit model. Similarly to Cohen et al. (1996) and Bayus (1997), a log utility function is used for performance. $U_i = U(\mathrm{Q}_i) = \ln(Q_i)$. Therefore, the demand rate given in section D of Chapter II ($D_i = D_T \frac{e^{U_i}}{\sum_{j=1}^{n} e^{U_j}}$) takes the form given in equation (4), where Q is the product performance level, Q_C is the competitive products performance level and M is the product category demand rate.

$$D'(Q) = M\,\frac{Q}{Q + Q_C} \qquad (4)$$

The demand rate of an item of quality Q is the product of the demand rate of the item’s category and the market share of the firm (Cohen, Eliashberg, & Tech-Hua, 1996) (Bayus, 1997). The latter is function of the product’s quality compared to that of its competitors (Cohen, Eliashberg, & Tech-Hua, 1996). Similar to Bayus (1997) our model does not consider the advertising expenses and the advantageous return of other expenditures across competitors. However, unlike some researchers like Bayus (1997) who assumed in their mathematical model that the sales following the product launch start low and increase to reach a peak after a certain period of introduction to market then decline at the end of the marketing period, we consider that the market demand and sales rate are constant throughout the marketing phase. We consider also that the price is an exogenous factor that the firm cannot control, in our model the firm is considered a price taker and not a price setter due to the high level of competition.

By placing the equations (1) through (4) of the mathematical formulation together, the revenue is calculated as follows:

$$R\left(D^{'}, p,\ T_M\right) = D^{'}. \qquad (5)$$

The marketing window T_M is the total time spent on the whole project minus the duration consumed by the system design, detailed design and iterations.

Therefore $T_M = T_T - T_S - T_D - T_{INT} = T_T - T_S - T_D - \alpha\,(1-m)T_D$

6. *Optimization*

In our model, we normalize to 1 the total duration of the product development process and the market window (product life cycle including the time spent on system design, detailed design, marketing window).

The model can be now written as an optimization problem expressed in equation (6). The target is to maximize the firm revenue, and solve for the main decision variables, i.e the duration spent on each stage of the development process, in particular the system design stage (or modularity) and the detailed design stage.

$$\text{Max } R = M.\frac{Q}{Q+Q_c}\left[\ T_T - T_S - T_D - \alpha\,(1-m)T_D\right]p = M.\frac{Q}{Q+Q_c}\left[1 - \ T_S - T_D - \alpha\,(1-m)T_D\right]p$$

$$R = M.p.\frac{T_D e^{\frac{\mu}{\alpha+m}}}{T_D e^{\frac{\mu}{\alpha+m}} + Q_c}\left[1 - m^{\frac{1}{\lambda}} - T_D - \alpha\,(1-m)T_D\right] \qquad (6)$$

Subject to:

Time limitation constraint: $T_T - T_S - T_D - \alpha\,(1-m)T_D - T_M = 0$ (7)

System design time: $0 \leq T_S \leq 1$ (8)

Detailed design time: $0 \leq T_D \leq 1$ (9)

B. Model Solution

The objective of this analysis is to develop a managerial insight on the optimal firm's time allocation that maximizes the revenue depending on the characteristics of the product. As it is difficult to obtain a generic solution to every decision variable as a function of the various parameters, we opted for the simulation as it is the best alternative that fulfils the objective of this analysis. We use the the symbolic and numeric computing software Maple to calculate the optimal solution for each set of parameters and then we analyse the results to gain managerial insights and determine the optimal policy for the different NPD scenarios.

1. Input Ranges

In order to perform the simulation analysis of the model, the different values for the different input parameters are assessed. The parameters and their respective numerical ranges are listed in Table 2.

Parameter	Definition	Range
m	Modularity	0-1
λ	Complexity	0-1
μ	Newness	0-1
M	Product market share	0-1
p	Product price	>0

Parameter	Definition	Range
Qc	Competitor's product quality	0.5-1

Table 2 List of the different parameters used in the mathematical model and their numerical ranges

Qc and Q, given in equation (2) have the same unit and fall within the same range of values. The lower limit of Q is 0, reflecting a very poor quality of the product. Whereas the upper limit is calculated when TD tends to 1, μ (the newness) tends to 0 and α (the supplier's involvement) and m (the modularity) tend to 1. Therefore the upper limit is calculated to be 1.

$$\lim_{T_D \to 1, \mu \to 0, \alpha \to 1, m \to 1} Q = T_D e^{-\frac{\mu}{\alpha + m}} = 1$$

Even though the values of Qc range between 0 and 1, the NPD process represented by our model takes place in a competitive atmosphere; therefore our model is evaluated only for the upper range of Qc otherwise the optimization would not be meaningful. If the competition is very low, in case of monopoly, and the absence of obsolescence, the optimization of the NPD process would be meaningless.

The ranges listed in Table 2 cover most product development processes.

2. Simulation Results

The results of the simulation are shown in Table 3.

NPD Scenario	1	2	3	4
M	1	1	1	1
Qc	1	1	1	1

NPD Scenario	1	2	3	4
complexity (λ)	0.1	0.1	0.9	0.9
supplier’s involvement (α)	0.1	0.1	0.1	0.1
newness (μ)	0.1	0.9	0.1	0.9
T_d^*	0.406	0.433	0.372	0.283
m^*	0.614	0.734	0.120	0.424
T_s^*	0.008	0.045	0.094	0.386
T_{int}	0.016	0.012	0.033	0.016
$T_{marketing}$	0.571	0.510	0.501	0.315

NPD Scenario	5	6	7	8
M	1	1	1	1
Qc	1	1	1	1
complexity (λ)	0.1	0.1	0.9	0.9
supplier's involvement (α)	0.9	0.9	0.9	0.9
newness (μ)	0.1	0.9	0.1	0.9
T_d^*	0.329	0.349	0.238	0.250
m^*	0.681	0.711	0.000	0.048
T_s^*	0.021	0.033	0.000	0.028
T_{int}	0.095	0.091	0.214	0.214
$T_{marketing}$	0.555	0.527	0.548	0.508

Table 3 Simulation results for Qc=1

Each NPD scenario is characterized by a specific level of product newness μ, degree of complexity λ, supplier's involvement α, competition level Qc, level of product market share M and product price p.

P.S: A multiplicative constant does not impact the optimal solution, therefore the market share M and the product price p do not affect the optimal solution in terms of time allocation between system design time TS and detailed design TD, however, their values impact the calculation of the firm's revenue. And as our objective is not to calculate the revenue, yet to find the optimal allocation of time for managerial conclusions, we can drop M and p from further consideration in the analysis.

The NPD scenarios (1 through 8) represented in Table 3 are characterized by same competition level Qc yet different degrees of newness μ, supplier's involvement α, and complexity λ. Two numerical values for each of the parameters are considered to simulate its low and high level. These values are 0.1 and 0.9.

For instance the NPD project represented by the first scenario is characterized by a low level of complexity (λ=0.1), low level of supplier's involvement (α=0.1) and low newness (μ=0.1). Whereas the second scenario reflects an NPD characterized by a low level of complexity (λ=0.1), low level of supplier's involvement (α=0.1) and a high level of newness (μ=0.9). The third one reflects an NPD characterized by a high level of complexity (λ=0.9), low level of supplier's involvement (α=0.1) and a low level of newness (μ=0.1). Similarly for the other scenarios, we vary the level of complexity, supplier's involvement and newness.

For each given scenario, the optimal solution is obtained using Maple which solves for the optimal TD* and modularity m*, then the values of Ts* Tint are recalculated based on function $Ts = m^{\frac{1}{\lambda}}$ and $Tint = \alpha.(1 - m).Td$.

3. Illustrative Example

Maple results are detailed for scenario 2 as an illustrative example.

Scenario	2
Qc	1
complexity (λ)	0.1
supplier's involvement (α)	0.1
newness (μ)	0.9

Table 4 Input of scenario 2

Replacing the values in Equation (6) the revenue will acquire the following form:

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right)$$

The revenue plot as a function of m and T_D is show in Figure 13.

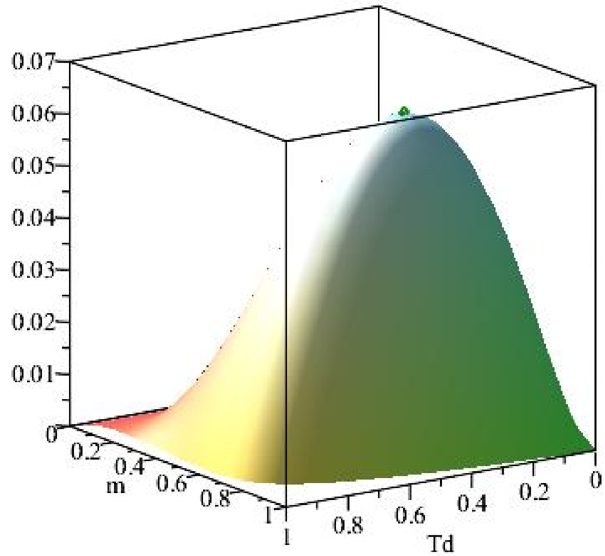

Figure 13 Plot of the revenue function (scenario 2)

The optimal solution is given by Td *= 0.433 and m*= 0.734. Ts and Tint are then calculated

	Scenario 2
T_d^*	0.433
m^*	0.734
T_s $(Ts = m^{\frac{1}{\lambda}})$	0.045
T_{int} $(Tint = \alpha.(1-m).Td)$	0.012

Table 5 Optimal solution of scenario 2

This same exercise is repeated for each of the eight scenarios.

4. Simulation results for Qc=0.5

The same calculation is also repeated for the lower range of completion, i.e for Qc =0.5. The results are summarized in Table 6.

NPD Scenario	1	2	3	4
M	1	1	1	1
Qc	0.5	0.5	0.5	0.5
complexity (λ)	0.1	0.1	0.9	0.9
supplier's involvement (α)	0.1	0.1	0.1	0.1
newness (μ)	0.1	0.9	0.1	0.9
T_d^*	0.363	0.409	0.345	0.279
m^*	0.607	0.730	0.112	0.420
T_s^*	0.007	0.043	0.088	0.381
T_{int}	0.014	0.011	0.031	0.016
$T_{marketing}$	0.616	0.537	0.537	0.324
NPD Scenario	5	6	7	8
M	1	1	1	1
Qc	0.5	0.5	0.5	0.5
complexity (λ)	0.1	0.1	0.9	0.85
supplier's involvement (α)	0.9	0.9	0.9	0.9
newness (μ)	0.1	0.9	0.1	0.9
T_d^*	0.296	0.323	0.220	0.240
m^*	0.673	0.706	0.000	0.041
T_s^*	0.019	0.031	0.000	0.023

NPD Scenario	5	6	7	8
T_{int}	0.087	0.086	0.198	0.207
$T_{marketing}$	0.597	0.560	0.582	0.529

Table 6 Simulation results for Qc=0.5

The differences in TD and TS between the two cases (Qc =1 and Qc=0.5) are minor (the differences are in the order of 10%) showing the relatively low sensitivity of the model to the differences in competitors quality. Therefore the analysis that follows is applicable for both very high and average competition.

5. *Optimality proof*

In Appendix 1 we prove that the optimal solution generated by Maple for each scenario and represented by a point S (m*, Td*) is a strong local maximum. The proof entails evaluating the gradient of the objective function and the determinants of the first and second order leading principals of the Hessian matrix at each point S (m*, Td*) for each NPD scenarios.

C. Optimization Analysis

In this section, we assess the impact of each of the input parameters on the optimal solution for our hypothesized product development process as formulated in our proposed model. The main objective here is to be able to draw some generic conclusions and insights regarding the management of the process based on the estimates of the different parameters which represent the product properties.

The following analysis is structured in three different sections evaluating the impact of complexity, newness and supplier's involvement separately.

1. Impact of Complexity

Figure 14 shows the optimal allocation of time between system design and detailed design for the low complexity level scenarios (i.e scenario 1,2, 5 and 6 in Table 3). The optimal solutions of all cases fall within the quadrant characterized by a high amount of time and effort spent on detailed design rather than the system design. This quadrant is referred to as the “modular focus” quadrant. The plot indicates that when the level of product complexity is low, it is optimal to spend more time on detailed design and very little time on system design. This low complexity level is coupled with an implicitly high level of modularity achieved through minor effort put into the modularization process. The high modularity leads to high performance level along with the long time spent of detailed design and optimizes the revenues.

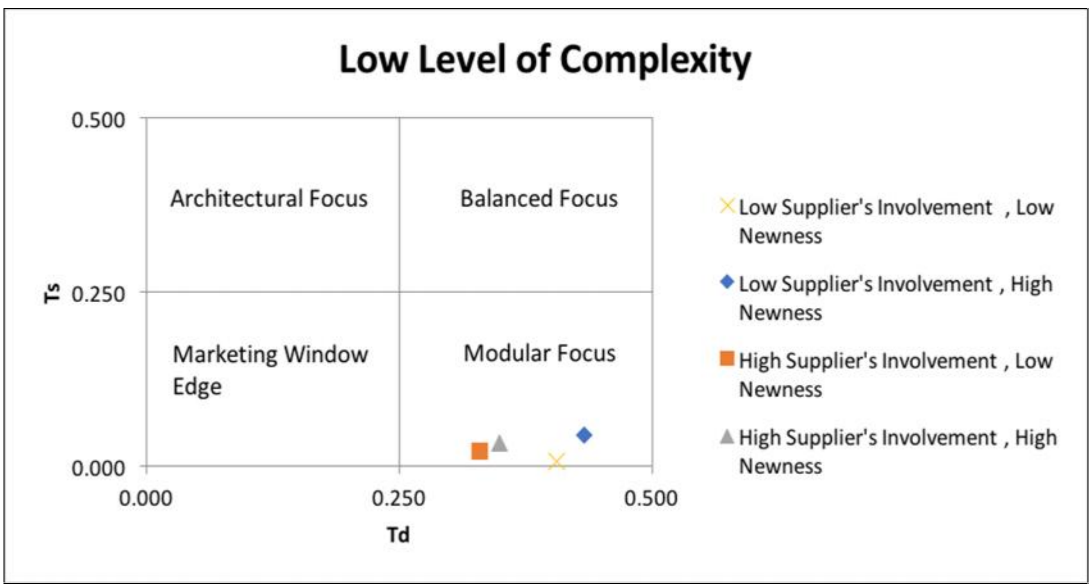

Figure 14 Optimal solutions for low complexity scenarios

The impact of the drastic increase in product complexity is shown in the Figure 15.

For the two cases characterized by high level of supplier's involvement, the optimal solution is shifted to the left with minor modification in Ts. This is attributed to the fact that a high level of supplier's involvement coupled with a low level of modularity (resulting from an increased complexity) require a long testing and integration effort, irrespective of the newness level. Therefore, in this case, it is optimal to maximize the marketing window by spending less time on detailed design in order to balance the extra time needed for testing in integration. It is more advantageous to put less effort into the modularization phase to save system design time rather than benefiting from the positive impact of a higher modularity on product performance.

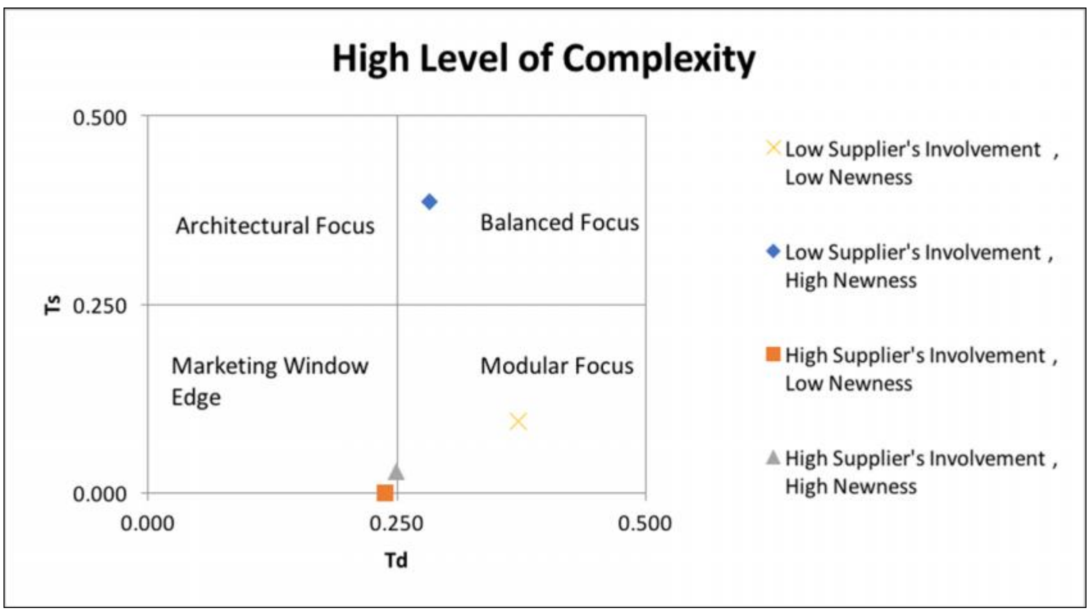

Figure 15 Optimal solutions for high complexity scenarios

However, for a low level of supplier's involvement, associated with a low level of newness, a radical increase in the product complexity contributes to a slight increase in the “modularization” time with minor change in the detail design time. In this scenario, the testing and integration time is minimum, due to low supplier’s involvement. The higher resulting modularity, coupled with the low level of newness improve the product performance level and therefore optimizes the revenues.

Finally, the impact of a drastic increase in product complexity on a low level of supplier’s involvement and high newness scenario leads to a higher system design time and lower detailed design time; the solution is shifted to the “balanced focus” zone. The high level of newness added to the low supplier’s involvement will make it hard for the company to improve performance; this could only be compensated by an increase in modularity level. This extra time spent on modularization should also be traded off with a slightly less time spent on detailed design to maintain a reasonable marketing window and allow the company to sell the product and generate revenue.

2. Impact of Newness

This same analysis is carried out from a different perspective to investigate the impact of a radical change in product newness on the optimal solution and time allocation to different phases on NPD.

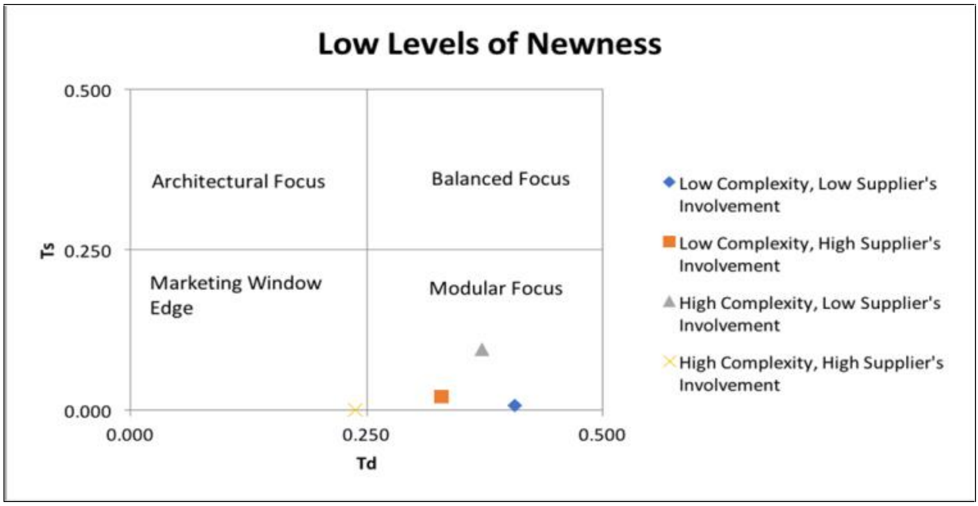

Figure 16 Optimal solutions for low newness scenarios

For the low level of newness, the majority of scenarios fall within the modular focus zone, where the time spent on the detailed design of each module considerably exceeds the time spent on system design. Except for the scenario where both complexity and supplier's involvement are high, where the solution falls within the "marketing edge" zone, and both T_D and T_S are relatively low. This case is associated with high testing and integration time (low modularity and high supplier's involvement), therefore in order to gain some marketing window time, some T_D duration must be saved.

In case of drastic increase in newness level, the low complexity scenarios remain in the "modular focus" zone, with little time spent on system design.
The scenario of high complexity and high supplier's involvement was subject to a slight shift to the right towards a higher modular focus. This is due to the negative impact of the newness on the product performance, an increase in T_D tries to recuperate some lost performance level due to the change in newness levels.

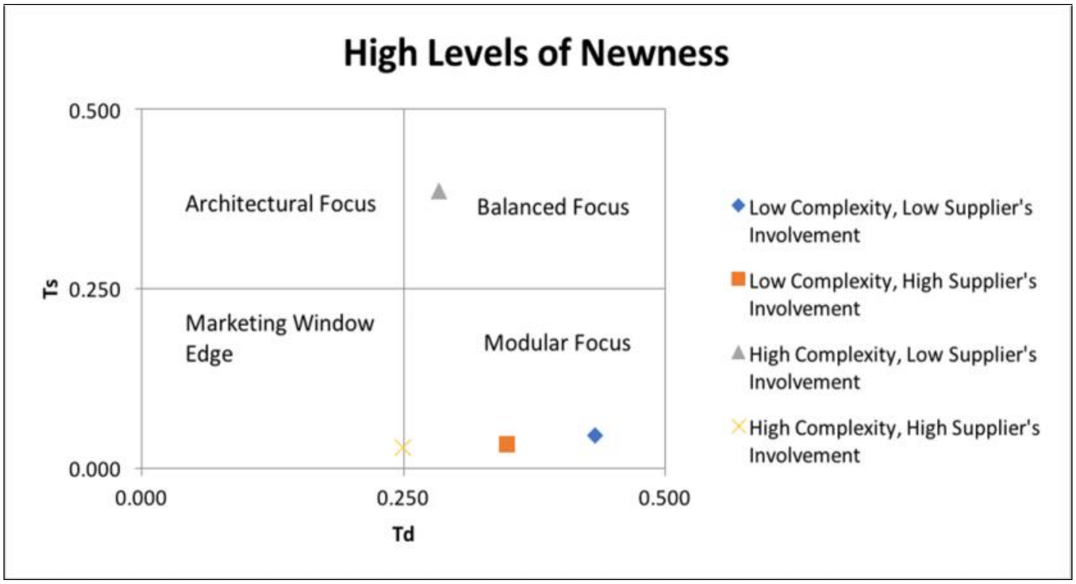

Figure 17 Optimal solutions for high newness scenarios

The optimal solution of the "high complexity, low supplier's involvement" case is shifted from the "modular focus" zone to the "balanced focus" zone following the dramatic increase in newness level. The detailed design time is reduced and the system design time is increased. The system design time is increased to improve the modularity and therefore the product performance and balance the negative impact of the high newness, low supplier's involvement and the implicitly low modularity on the product quality. T_D is slightly reduced to maintain an optimum marketing window.

3. Impact of Supplier's Involvement

Finally, the same analysis is carried out from a different perspective to investigate the impact of a radical change in supplier's involvement on the optimal solution and time allocation to different phases on NPD.

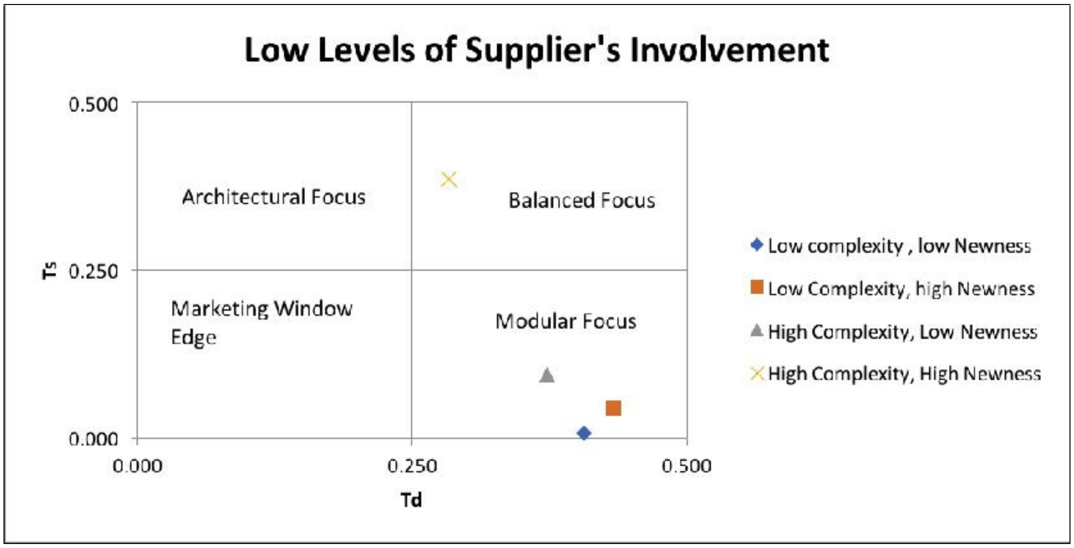

Figure 18 Optimal solutions for low supplier's involvement scenarios

The solution of low complexity scenarios fall within the “modular focus” category where the company is supposed to spend a higher amount of time on the detailed design phase compared to the system design. When a high complexity is coupled with low newness and supplier’s involvement, a slightly higher Ts is required to increase the modularity and therefore enhance the product performance. Tint in this case is very low since the supplier’s involvement is minor.

The fourth scenario characterized by a high complexity, high newness and low supplier’s involvement, all contributing to a low performance, the only way to contribute to some performance is to spend more time on system design and increase the modularity in order to improve the product performance. That’s why the solution falls in the “balanced focus” zone.

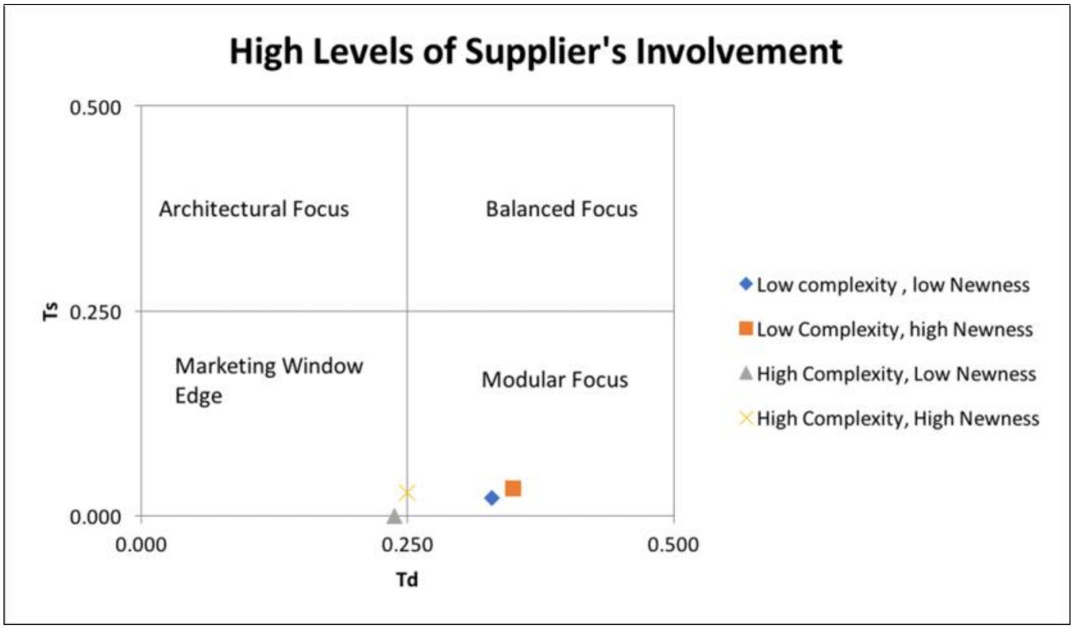

Figure 19 Optimal solutions for high supplier's involvement scenarios

The drastic change in supplier's involvement only impacts the high complexity scenarios by reducing the time spent on T_D and T_S.

For low newness, high complexity (low modularity), the increase in supplier's involvement leads to an increase in testing and integration time, therefore the marketing window is optimized by reducing the time spent on both the detailed design and system design. Some level of product performance is maintained due to the low newness levels and high supplier's involvement.

Similarly, for high complexity (low modularity) and high newness (deteriorating the performance level) a high supplier's involvement increases the testing and integration time, therefore the system design time should be reduced to gain some marketing window edge. The detailed design T_D cannot be compromised since that will affect the product performance level. Therefore, when developing a complex and innovative product, a firm is advised to involve more external suppliers to design, develop a higher fraction of its parts. This will help the firm optimize the marketing window, maximize the product performance and generate high revenues.

CHAPTER IV

CASE STUDY

A. Introduction

In this chapter, numerical values are applied to the introduced mathematical model. This numerical application aims to evaluate how practical it is to find/ match numerical values to the theoretical parameters included in the model and to test the reliability of its output by comparing it to real data.

The empirical data is obtained from Clark and Fujimoto (1989), (1991) and Novak and Eppinger (2001).

B. Empirical Data Source

Between 1985 and 1988, Fujimoto and Clark collected data from 29 car production projects in Europe, Japan and the United States. The data were gathered from 20 car manufacturers worldwide in different countries (8 in japan, 3 in the US and 9 in Europe). These companies accounted for around 70% of the global car manufacturing in 1986 (Clark & Fujimoto, 1991).

Extensive data were collected on three distinctive levels: the project performance (lead time, engineering hours and design quality), the project content and the organization/company. Where a “project” is defined as a major new car development including both a model renewal and the development of completely new model.

Through their research Clark and Fujimoto aimed to analyze the competition that exists in the world of auto industry and examine the drives behind the productivity and quality difference between the main Japanese, European and American players. In terms of quality, the European companies tend to rank first in design quality, while Japanese ranked first in the conformance quality (absence of defects). Regarding the lead time and productivity, the authors noticed the striking Japanese advantage compared to the European and American competitors. These differences are closely related to the size of the developed vehicle, its price, new parts ratio and body types. More importantly, the authors argued that the Japanese advantage is attributed to the extent of supplier's involvement in addition to managerial and organizational strengths. Clark and Fujimoto did detail the internal project management scheme and manufacturing strength through which Japanese manufacturers contributed to higher productivity and product quality.

1. Definition

In the world of auto industry, the attraction and satisfaction of customers depend mainly on three outcomes of the product development process (Clark & Fujimoto, 1991):

- The total product quality (TPQ)
- The lead time
- The productivity (engineering hours)

A rich base of data was collected for the above three dimensions, however our analysis focuses on the "lead time" only. By definition, the lead time is the duration spanning between the concept generation and the product introduction into the market.

It is the time needed to define, design, produce and introduce the product. Unlike engineering hours, the lead time is not the summation of each activity's duration since some activities are performed in parallel. In addition to the "concept to market" lead time, data were collected for each phase of the development process. Framed broadly, the product development process in the auto industry involves four different stages: concept generation, product planning, product engineering and process engineering (Clark & Fujimoto, 1991).

The concept generation which is the first stage of the process, focuses on creating the main concept of the car and the distinctive features that recognize it from competitors. This stage requires input from competitors as well as customers by identifying their needs. The generated concept is then translated into more concrete assumptions at the product planning phase, it incorporates the choice of component specification, style, layout in addition to the targeted performance. This phase bridges the concept generation with the design phase.

The product engineering phase starts following the approval of the product planning by the management. Detailed engineering drawings are elaborated at this stage in addition to the creation of a full scale prototype. Finally the process engineering stage focuses on the effective commercial production of the car model.

2. Raw Data

Table 7 summarizes the information collected by Fujimoto and Clark from 1987 till 1989 on 24 projects (out of 29 in total) at nineteen main car developers worldwide. These data will set the base for the calculations and analysis of the following sections.

	Variables \ Strategic- regional groups	Japanese volume producer	U.S volume producer	European Volume producer	Overall
	Number of projects	11	5	8	24
	Year of introduction	1981-1985	1984-1987	1980-1987	1980-1987
	Overall Lead time (months)	43.3	60.3	66.5	54.6
	Planning Lead time (months)	13.6	19.3	20.6	17.1
	Engineering Lead time (months)	32.2	39.7	45.9	38.3
Product Content / Innovation	Average price (1987 US dollars)	8,783	13,702	23,785	14,808
	Vehicle size (# of projects)				
	Micromini	3	0	0	3
	Small	7	1	4	12
	medium to large	1	4	4	9
	Average number of body types	2.4	1.8	1.5	2
Project scope	off the shelf parts	18%	45%	29%	27%
	Supplier involvement (%of parts cost)				
	Supplier proprietary (SP)	8%	3%	7%	7%

	Variables \ Strategic- regional groups	Japanese volume producer	U.S volume producer	European Volume producer	Overall
	Black Box (BB)	62%	16%	39%	44%
	Detail-controlled (DC)	30%	81%	54%	49%
	Supplier-engineering ratio	51%	14%	34%	37%
Organization capability	Die lead time (months)	13.8	25	28	21
	prototype lead time (months)	6.2	12.4	10.9	9.1

Table 7 Raw Data extracted from Clark and Fujimoto (1989)

C. Numerical Values for the Mathematical Model

Among the rich base of information collected by the researchers and summarized in Table 7, three parameters are shared with our introduced mathematical model, therefore their respective numerical values will be used in our application.

1. Newness (μ)

The value of newness is determined from the value of the “Off the shelf parts”. The latter is defined as the fraction of components that is common to previous or other models. Newness, the fraction of newly designed parts, is therefore calculated as 1- “off the shelf part”.

2. Supplier’s Involvement (α)

The data collected by Clark and Fujimoto on the parts procured from suppliers are broken down into three different categories, distinguishing the levels of supplier's engineering involvement:

- The supplier's proprietary part is the fraction of the elements or parts that are completely developed by suppliers.

- Black box parts are the parts whose basic design is carried out by the main car developer however, the detailed design is done by the supplier.

- Detail controlled part: the elements that are completely developed by the car producer.

Further, a ratio called "the supplier's engineering ratio" is calculated. It is defined as the fraction of the total engineering hours accounted by suppliers. This value is calculated assuming that 100% of engineering work is done by suppliers for proprietary parts and 70% of engineering work is done in black box parts and 0 % in detail controlled. This supplier's engineering ratio value is used for the parameter "α" in our model.

3. *Lead Time (T_S and T_D)*

As defined previously, the "lead time" refers to the duration spanning between the concept generation and the product introduction into the market; it includes four stages, concept generation, product planning, product engineering and process engineering

In their paper "Lead Time in Automobile Product Development Explaining the Japanese Advantage" Clark and Fujimoto grouped the first two stages (concept generation and product planning) of the product development process under a wider category which is the "planning phase" and the last two (product engineering and process engineering) under the "engineering phase".

4. Planning vs Engineering

Planning activities lay the ground for engineering work by determining the structure and function of the new product. The output of the planning phase is intangible and the problem solving at this phase is mainly analytical, within the head of the engineers. The output of each cycle within this stage is usually a document, therefore it does not require significant organizational coordination (Clark & Fujimoto, 1989). The result of the planning phase is a decomposition of the product into different cluster of components, each of which can be handled separately during the engineering phase (Clark & Fujimoto, 1989). The planning also determines the interfaces (Clark & Fujimoto, 1989). The planning lead time extends from the beginning of concept generation to the end of product planning (Clark & Fujimoto, 1989).

On the other hand the "Engineering phase" follows the decomposition of the product into smaller parts in the planning phase, the problem solving for each component (or cluster of components) goes independently (Clark & Fujimoto, 1989).

According to the above definition provided by Clark and Fujimoto (1991) we can conclude that the planning time is equivalent to the system design time of our model (T_s) and the engineering time is equivalent to the detailed engineering time added to the testing and integration ($T_d + T_{int}$) since the engineering phase includes testing and prototyping.

T_s = Planning time = Concept Generation + Product Planning

$T_d + T_{int}$ = Engineering time = Product Engineering + Process Engineering

Therefore the solution of our mathematical model will result in the calculation of the optimal Ts*and Td* and Tint*, these optimal values will be compared to the planning time and engineering time respectively.

5. Complexity

The value of complexity was adopted from Novak and Eppinger (2001) that aimed to study the effect of complexity level on the outsourcing decision.

According to Novak and Eppinger (2001), the product complexity that is driven by a number of factors including the targeted level of performance, architecture and adopted technology, is defined by three main criteria: 1) the number of elements within a product 2) the extent of interaction between the elements and 3) the level of product novelty. These three criteria are evaluated for the auto industry by collecting data through on site interviews with CEOs, project engineers, project managers. The complexity of each sub-system within the car is evaluated separately i.e, the complexity of the suspension, brakes, transmission, engine, steering, body, electrical systems is evaluated on a scale of 0 to 1and the mean complexity of the whole product is then calculated.

The mean complexity of the cars is determined to be 0.42. This value will be used in our analysis and calculations.

D. Numerical Solution by Maple

Table 8 summarizes the data provided by Clark and Fujimoto and needed to solve the mathematical model for each of the Japanese, European and American scenarios.

The data related to product content and organization capability are deleted since they are not included in our mathematical model. Furthermore, in the regression analysis done by the authors, they were proved to have minor effect on lead time.

	Strategic- regional groups / variables	Japanese volume producer	U.S volume producer	European Volume producer
Lead time	Overall Lead time (months)	43.3	60.3	66.5
	Planning Lead time (months)	13.6	19.3	20.6
	Engineering Lead time (months)	32.2	39.7	45.9
	T_{total} (2* Overall lead time)	**86.6**	**120.6**	**133**
	Average price (1987 US dollars)	8,783	13,702	23,785
Project scope	off the shelf parts	18%	45%	29%
	newness (μ)	**82%**	**55%**	**71%**
	Supplier involvement (% of parts cost)			
	Supplier proprietary (SP)	8%	3%	7%
	Black Box (BB)	62%	16%	39%
	Detail-controlled (DC)	30%	81%	54%
	Supplier-engineering ratio (α)	51%	14%	34%

Table 8 Data extracted from Clark and Fujimoto (1989) needed for the solution of the mathematical model

The bold values are calculated values, which are not originally present in the set of data collected in Clark and Fujimoto (1989).

Starting with the Ttotal which is by definition the concept to market lead time plus the market window, it is the time at which the car model becomes obsolete due to the introduction of a competing model by rivals or a newer model by the same car developer. According to Clark and Fujimoto, car developers forecast for a period that is at least twice the concept to market lead time. For instance if the lead time is six years the company shall forecast 6-12 years ahead. Therefore Ttotal is assumed to be twice the duration of the overall lead time.

As discussed previously the newness is calculated as “1- off the shelf parts”.

The optimization problem is solved using Maple, the symbolic and numeric computing software.

1. *Japanese Model*

Starting with the Japanese car development scenario, the numerical values substituted in the mathematical model are summarized in Table 9.

Parameter	Definition	Japanese Numerical Values
M	Product market share	Assumed 1 (value has no impact on the optimal solution)
Q	Developed product quality	Calculated : $Q = Td.e^{-\frac{\mu}{\alpha + m}}$
T_d^*	Detailed Engineering Time	Optimal value to be calculated by Maple
μ	Product newness	0.82

Parameter	Definition	Japanese Numerical Values
α	Supplier's engineering involvement percentage	0.51
Q_C	Competitors product quality	Assumed 1 (value has low sensitivity on the optimal solution)
m^*	Product modularity	Optimal value to be calculated using Maple
λ	Product complexity	0.42 Source: Novak and Eppinger (2001)
T_s^*	System design time	Calculated based on $Ts = m^{\frac{1}{\lambda}}$
T_{int}	Testing and integration time	Calculated based on $Tint = \alpha.(1 - m).Td$
R	Total product revenue	$R = \frac{M\,Q}{Q + Qc}.p.(1 - Td - Ts - Tint)$
p	Product price	8,783

Table 9 Japanese model numerical values

After substituting the numerical values, the total product revenue function takes the following form:

$$R = 8783 \left(\frac{Td \cdot e^{-\frac{0.82}{0.51 + m}}}{Td \cdot e^{-\frac{0.82}{0.51 + m}} + 1} \right) \cdot \left(-m^{2.380952381} + 1 - Td - (0.51 - 0.51\, m) \cdot Td \right).$$

The revenue function R=f(Td,m) is plotted by Maple as shown in Figure 20.

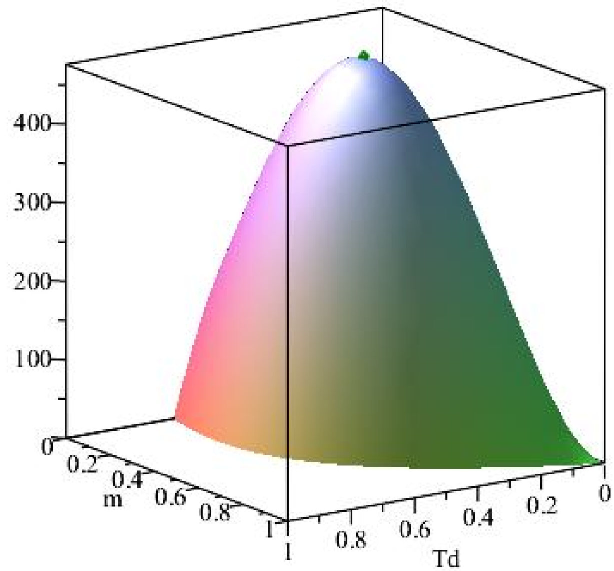

Figure 20 Plot of the Japanese revenue model (Mapel)

The optimization problem is solved by Maple as well; by maximizing R subject to two constraints 0≤Td≤1 and 0≤m≤1. The optimal solution written in terms of Td* and Ts* is Td* = 0.322 and Ts* = 0.096.

2. U.S Modeltt

The same procedure is followed to solve the optimization problem taking into account the US car development data.

Parameter	Definition	U.S Projects Numerical Values
M	Product market share	Assumed 1 (value has low sensitivity on the optimal solution)
Q	Developed product quality	Calculated : $Q = Td . e^{-\frac{\mu}{\alpha + m}}$

Parameter	Definition	U.S Projects Numerical Values
T_d^*	Detailed Engineering Time	Optimal value to be calculated by Maple
μ	Product newness	0.55
α	Supplier's engineering involvement percentage	0.14
Q_C	Competitors product quality	Assumed 1 (value has low sensitivity on the optimal solution)
m^*	Product modularity	Optimal value to be calculated using Maple
λ	Product complexity	0.42
T_s	System design time	Calculated based on $Ts = m^{\frac{1}{\lambda}}$
T_{int}	Testing and integration time	Calculated based on $Tint = \alpha.(1 - m).Td$
R	Total product revenue	$R = \frac{M\,Q}{Q + Qc}.p.(1 - Td - Ts - Tint)$
p	Product price	13,702

Table 10 US model numerical values

$$R = 13702 \left(\frac{Td \cdot e^{-\frac{0.55}{0.14 + m}}}{Td \cdot e^{-\frac{0.55}{0.14 + m}} + 1} \right) \cdot \left(-m^{2.380952381} + 1 - Td - (0.14 - 0.14\,m) \cdot Td \right)$$

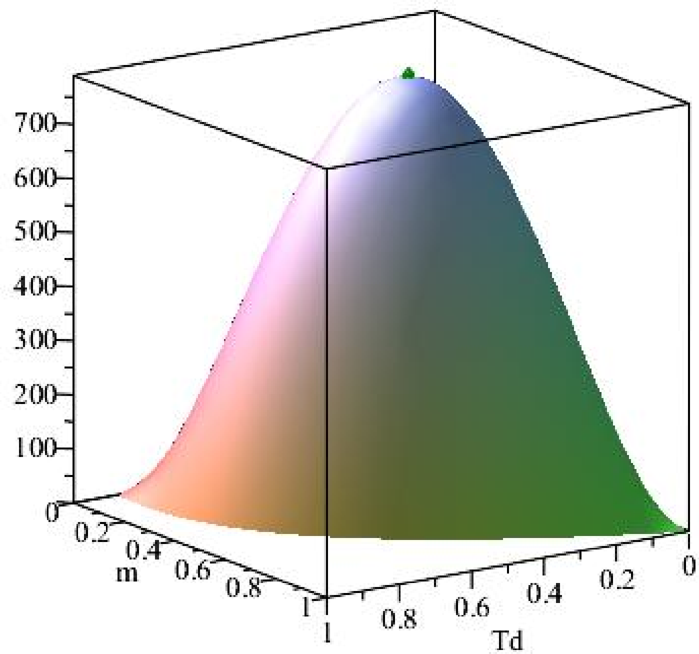

Figure 21 Plot of the U.S revenue model (Mapel)

The results of the optimization problem are Td* = 0.374 and Ts* = 0.133.

3. European Model

Similarly, the problem is optimized taking into account the European car developers scenario

Parameter	Definition	Europe Projects Numerical Values
M	Product market share	Assumed 1 (value has low sensitivity on the optimal solution)
Q	Developed product quality	Calculated : $Q = Td.e^{-\frac{\mu}{\alpha + m}}$
T_d^*	Detailed Engineering Time	Optimal value to be calculated by Maple

Parameter	Definition	Europe Projects Numerical Values
μ	Product newness	0.71
α	Supplier's engineering involvement percentage	0.34
Q_C	Competitors product quality	Assumed 1 (value has low sensitivity on the optimal solution)
m^*	Product modularity	Optimal value to be calculated using Maple
λ	Product complexity	0.42
T_s	System design time	Calculated based on $Ts = m^{\frac{1}{\lambda}}$
T_{int}	Testing and integration time	Calculated based on $Tint = \alpha.(1 - m).Td$
R	Total product revenue	$R = \frac{MQ}{Q + Qc}.p.(1 - Td - Ts - Tint)$
p	Product price	23,785

Table 11 European model numerical values

$$R = 23785 \left(\frac{Td \cdot e^{-\frac{0.71}{0.34 + m}}}{Td \cdot e^{-\frac{0.71}{0.34 + m}} + 1} \right) \cdot \left(-m^{2.380952381} + 1 - Td - (0.34 - 0.34\,m) \cdot Td \right)$$

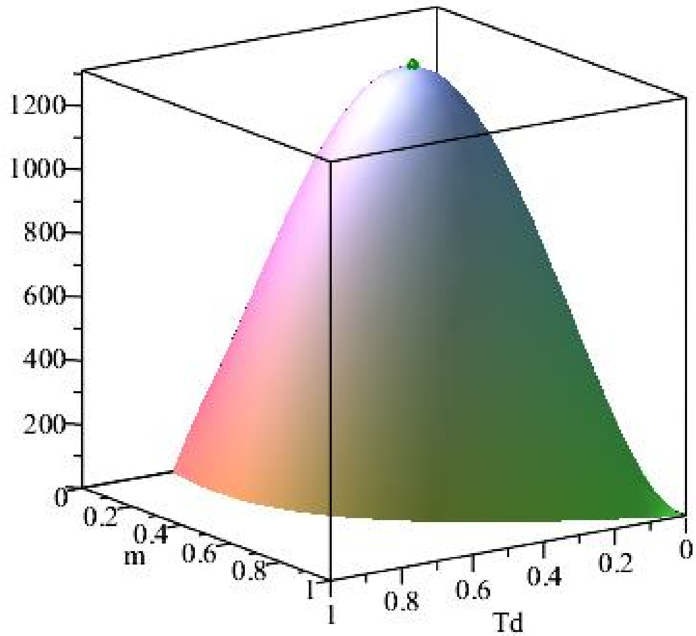

Figure 22 Plot of the European revenue model (Mapel)

The results of the optimization problem are Td* = 0.346 and Ts* = 0.111.

E. Summary of Results

The optimal solutions calculated in the previous section for the three car producers are summarized in Table 12, in addition the values for testing and integration time T_{int} are calculated.

	Japanese volume producer	U.S volume producer	European Volume producer
T_d^*	0.322	0.375	0.346
m^*	0.373	0.429	0.397
T_s^* ($Ts = m^{\frac{1}{\lambda}}$)	0.096	0.133	0.111

	Japanese volume producer	U.S volume producer	European Volume producer
T_{int} ($Tint = \alpha.(1-m).Td$)	0.104	0.030	0.072
T_d+T_{int}	0.426	0.405	0.418

Table 12 Summary of optimal solutions for the different car producers (years 1980s)

The values for Ts and Td are recalculated in units of months taking into account the Ttotal calculated previously.

	Japanese volume producer	U.S volume producer	European Volume producer
T_{total} (months)	86.6	120.6	133
m^*	0.373	0.429	0.397
$T_s{}^*$ ($Ts = m^{\frac{1}{\lambda}}$)	0.096	0.133	0.111
T_s^*(months)	8	16.1	14.7
T_d^*	0.322	0.375	0.346
T_d^* (months)	28	45	46
T_{int} ($Tint = \alpha.(1-m).Td$)	0.104	0.030	0.072
T_{int}(months)	9	3.7	9.5

	Japanese volume producer	U.S volume producer	European Volume producer
T_d+T_{int}	0.426	0.405	0.418
T_d+T_{int} (months)	37	48.8	55.6

Table 13 Summary of optimal solutions calculated in unit of "months", for years 1980s

The calculated values are compared to the empirical data collected by Clark and Fujimoto in Table 14.

	Japanese volume producer		American volume producer		European Volume producer	
	Calculated	Empirical	Calculated	Empirical	Calculated	Empirical
T_s (months)	8	13.6	16.1	19.3	14.7	20.6
T_d+T_{int} (months)	37	32.2	48.8	39.7	55.6	45.9
$T_{lead\ time}$	45	43.3	64.9	60.3	70.3	66.5

Table 14 Comparison between calculated and empirical values (for years 1980s)

F. Analysis and Discussion

The data collected in the 1980s on the automotive are updated by (Ellison, Clark, Fujimoto, & Hyun, 1995). These updated data are summarized in Table 15.

	Strategic- regional groups / variables	Japanese volume producer	U.S volume producer	European Volume producer
Lead time	Overall Lead time (months)	51	52	59

	Strategic- regional groups / variables	Japanese volume producer	U.S volume producer	European Volume producer
	Planning Lead time (months)	19	17	22
	Engineering Lead time (months)	28	33	32
	T_{total} (2* Overall lead time)	102	104	118
Project scope	off the shelf parts	28%	32%	32%
	newness (μ)	72%	68%	68%
	Supplier involvement (% of parts cost)			
	Supplier proprietary (SP)	6%	12%	12%
	Black Box (BB)	55%	30%	24%
	Detail-controlled (DC)	39%	58%	64%
	Supplier-engineering ratio (α)	45%	33%	29%

Table 15 Automotive industry data for the years 1990s, extracted from (Ellison, Clark, Fujimoto, & Hyun, 1995)

The same optimization exercise is performed based on these updated values and assuming that the complexity of automobiles has also increased from 1980s through 1990s from 0.42 to 0.5 due to the incorporation of more advanced mechanical systems and the integration of electronic features to improve safety, fuel efficiency and advanced diagnostics. The calculated optimal solutions are summarized in Table 16.

	Japanese volume producer	U.S volume producer	European Volume producer
T_d^*	0.323	0.339	0.344
m^*	0.325	0.354	0.369
T_s^* ($Ts = m^{\frac{1}{\lambda}}$)	0.105	0.126	0.136
T_{int} ($Tint = \alpha.(1 - m).Td$)	0.097	0.072	0.063
T_d+T_{int}	0.419	0.411	0.407

Table 16 Calculated optimal lead times for the different car producers (years 1990s)

The values for Ts and Td are recalculated in units of months in Table 17 taking into account the Ttotal calculated previously.

	Japanese volume producer	U.S volume producer	European Volume producer
T_{total} (months)	102	104	118
m^*	0.325	0.354	0.369
T_s^* ($Ts = m^{\frac{1}{\lambda}}$)	0.105	0.126	0.136
T_s^*(months)	11	13.1	16.0
T_d^*	0.323	0.339	0.344

	Japanese volume producer	U.S volume producer	European Volume producer
T_d^* (months)	33	35.3	40.6
T_{int} ($Tint = \alpha.(1 - m).Td$)	0.097	0.072	0.063
T_{int}(months)	10	7.5	7.4
T_d+T_{int}	0.419	0.411	0.407
T_d+T_{int} (months)	43	42.8	48.0

Table 17 Summary of optimal solutions calculated in unit of "months", for years 1990s

Table 18 shows a comparison between the calculated values and empirical values for the years 1990s.

	Japanese volume producer		American volume producer		European Volume producer	
	Calculated	Empirical	Calculated	Empirical	Calculated	Empirical
T_s (months)	11	19	13.1	17	16.0	22
T_d+T_{int} (months)	43	31	42.8	36	48.0	36
$T_{lead\ time}$	54	51	55.9	52	64	59

Table 18 Comparison between calculated and empirical values (for years 1990s)

As observed in Table 14 and Table 18, the output of the model and the empirical data are not exactly equal, however the values are comparable. In fact, the purpose of the model is not to get an accurate solution for the exact time that a firm must allocate to each phase of the development process; on the contrary the model aims to generate general managerial insights that help companies find the best division of time between the different project phases; in addition, there is always a difference between the theoretical values that a model generates and the real values that industries end up spending due to unanticipated events they face during the execution of their works. In addition, the introduced mathematical model is a general model that can be applied to all industries, it is not tailored for the automobile industry that has its own characteristics and specificities.

Comparing the information in Table 14 and Table 18 that summarize the output of the model and the empirical data for the years 1990s and 1980s respectively, we can observe that the model succeeded to forecast the change in time allocation between planning and engineering for the American and European car producers. The model output anticipated a decrease in system design time from 16 to 13 months (3 months difference) whereas a decrease from 19 to 17 months was observed (2 months difference). In engineering time, the model also anticipated a decrease in time from 48 to 42.8 months (around 6 months difference) and in reality the decrease was from 39 to 33 months (6 months difference).

This reduction in overall lead time may be attributed to the increased level the supplier’s involvement from 14% to 33% which led the American producers to save time in planning time and the overall engineering time.

For the European car developers, the model calculated one month increase in the system design time, from 14.7 to 16 months. In reality, the planning time for the European developers has increased by one month from 21 to 22 months. Regarding the engineering time, the calculated reduction in this phase was 8 months from 56 to 48 however the observed reduction was by 14 months from 46 months to 32 months.

For a lower new parts ratio and a lower supplier's engineering involvement along with the increase in the level of cars complexity, European car developers had to spend more time planning the product and grouping the highly interactive elements within modules and defining the interfaces between the subsystems. With these clearly defined subsystems the European car developers were able to spend less time in detail design and end up optimizing the marketing window.

For the Japanese scenario on the other hand, the model did not anticipate the change in time allocation. What is particularly observed in the Japanese scenario is that, unlike European and American developers, where a reduction of the overall lead time is noticed, the Japanese model data show a more prolonged overall lead time in 1990s compared to 1980s. Therefore we can hypothesize that in general, Japanese firms did not optimize their performance by adopting the best strategy (optimal supplier's involvement) and the applying the optimal allocation of time to maximize the revenues.

CHAPTER V

CONCLUSION

In this paper a mathematical model is formulated and solved to examine the trade-off between the time spent on the different stages of the NPD process and the product performance for different NPD scenarios. These scenarios differ in three important factors that are part of any NPD process. These factors are: product complexity, product newness and the level of supplier's involvement.

Firms in any product development industry can refer to the simulation output to generate some general ideas on the optimal time allocation between the different stages of their NPD process based on their product characteristics and their outsourcing strategy. Developers can also run the model to observe the optimal shift and change in the time allocation for the different process stages should the characteristics of the product change based on the changing customers' needs. This model can also help firms forecast the budget for the engineering discipline of the NPD process.

The presented case study proved that it is practical to run this model by assigning numerical values to the various parameters in order to draw important managerial insights. The case study also showed that the output of the model was in line with the collected data for two out of three scenarios, i.e European and American automobile developers but not the Japanese developers.

In future works, the model could be further developed and tailored to a specific industry so that it reflects its specificities and particularities. For instance some industries spend a considerable amount of time on the manufacturing stage whiles others don’t (software), some other industries also invest a substantial amount of time in testing and prototyping, such as the firms developing products for the environmental field.

Finally, as the model targets revenues only, the costs can be further incorporated, including engineering man power and collaboration costs in order to evaluate and optimize the net profit.

REFERENCES

Ahsan, K., & Gunawan, I. (2014). Analysis of Product Recalls: Identification of Recall Initiators and Causes of Recall. *Operations and Supply Chain Management*, 97-106.

Al-Kindi, M., & Yassine, A. A. (2009). The Impact of Engineering Design on Outsourcing Decisions. In K. E. Carettas (Ed.), *Outsourcing, Team Work and Business Management* (pp. 115-134). New York: Nova Science Publishers.

Athul, V. (2018, June 20). *Quora*. Retrieved from www.quora.com/What-are-some-of-the-most-epic-product-fails

Baldwin, C. (2015). Modularity and Organizations. *International Encyclopedia of the Social & Behavioral Sciences*, 718-723.

Banu Goktan, A., & Miles, G. (2001). Innovation Speed and Radicalness: Are they Inversly Related. *Management Decision*, 533-547.

Bayus, B. L. (1997). Speed-to-Market and New Product Performance Trade-offs. *Elsevier Science*, 485-497.

Brabazon, T., & Matthews, R. (2003). Product Architecture, Modularity and Product Design : A Complex Perspective. *Journal of American Academy of Business, Cambridge*, 1-20.

Browning, T. R., & Eppinger, S. D. (2002). Modeling Impacts of Process Architecture on Cost and Schedule Risk in Product Development. *IEEE Transactions on Engineering Management*, 428-442.

Calantone, J., Chan, K., & Cui, A. S. (2006). Decomposing Product Innovativeness and Its Effects on New Product Success. *Product Development and Management Association*, 408-421.

Clark, K. B., & Fujimoto, T. (1989). Lead Time in Automobile Product Development Explaining the Japanese Advantage. *Journal of Engineering and Technology Management*, 25-58.

Clark, K. B., & Fujimoto, T. (1991). *Product Development Performance: Strategy, Organization and Management in the World of Auto Industry.* Massachussets: Harvard Business School Press.

Cohen, M. A., Eliashberg, J., & Tech-Hua, H. (1996). New Product Development: The Performance and Time-To-Market Tradeoff. *Management Science*, 173-186.

Cook, H. (2006). The Role of Demand Modeling in Product Planning. In K. E. Lewis, W. Chen, & L. C. Schmidt, *Decision Making in Engineering Design.* New York: ASME.

Cook, H. E. (2005). *Design for Six Sigma as Strategic Experimentation* . Milwaukee: Quality Press.

Cooper, R. G. (1990). Stage-Gate Systems: A New Tool for Managing New Products. *Business Horizons*, 44-54.

Cooper, R. G. (2017). Idea-to-Launch Gating Systems. *Reseach- Technology Management*, 48-52.

Cooper, R. G., & Sommer, A. F. (2016). From Experience: The Agile- Stage-Gate Hybrid Model: A Promising New Approach and a New Research Opportunity. *Product Development and Mmanagement Association*, 513-526.

Danese, P., & Roberto, F. (2012). Direct and Mediated Effects of Product Modularity on Development Time and Product Performance. *IEEE Transactions on Engineering Management*, 260-271.

Duhamel, F., & Santi, M. (2012). Degree of innovativeness and new product preformance. *Technology analysis and strategic management*, 253-266.

Ellison, D. J., Clark, K. B., Fujimoto, T., & Hyun, Y.-s. (1995). Product Development Performance in the Auto Industry: 1990s Update. *Annual Sponsors Meeting* (pp. 1-37). Toronto: International Motor Vehicle Program.

Ethiraj, S., & Levinthal, D. (2004). Modularity and Innovation in Complex Systems. *Management Science*, 159-173.

Fixson, S. K. (2004). Product architecture assessment: a tool to link product, process, and supply chain design decisions. *Journal of Operations Management*, 345-369.

Griffin, A. (1997). The Effect of Project and Process Characteristics on Product Development Cycle Time. *Journal of Marketing Research*, 24-35.

Griffin, A. (1997). The effect of Project and Process Characteristics on Product Development Time. *Journal of Marketing Research*, 24-35.

Holtta, K., Suk, E. S., & De Weck, O. (2005). Tradeoff Between Modularity and Performance for Engineered Systems and Products. *International Conference of Engineering Design*, 1-14.

Holtta-Otto, K., & de Weck, O. (2007). Degree of Modularity in Engineering Systems and Products with Technical and Business Constraints. *Concurrent Engineering: Research and Applications*, 113-126.

Jensen, P. A., & Bard, J. F. (2003). Operations Research Models and Methods. In P. A. Jensen, & J. F. Bard, *Operations Research Models and Methods* (pp. 329-367). Austin: John Wiley & Sons, Inc.

Karlsson, C., & Ahlstrom, P. (1999). Technological Level and Product Development Cycle Time. *Elsevier*, 352-362.

Ko, Y. T. (2013). Optimizing Product Architecture for Complex Design. *Concurrent Engineering, Research and Applications*, 87-102.

Langerak, F., & Hultink, E. J. (2006). The Impact of Product Innovativeness on the Link between Development Speed and New Product Profitability. *Product Development and Management Association*, 203-214.

Maier, J. F., Wynn, D. C., & Biedermann, W. (2014). Simulating Progressive Iteration, Rework and Change Propagation to Prioritise Design Tasks. *Research Engineering Design*, 283-307.

Martinez Leon, H. C., Farris, J. A., & Letens, G. (2013). Improving Product Development Performance Through Iterations Front Loading. *IEEE Transactions on Engineering Management*, 552-565.

Mikkola, J., & Gassman, O. (2003). Managing Modularity of Product Architectures: Towards an Integrated Theory. *IEEE Transactions on Engineering Management*, 204-2018.

Mozur, P. (2017, January 22). Galaxy Note 7 Fires Caused by Battery and Design Flaws, Samsung Says. *The New York Times*.

Novak, S., & Eppinger, S. D. (2001). Sourcing by Design: Product Complexity and the Supply Chain. *Management Science*, 189-204.

Petersen, K. J., Handfield, R. B., & Ragatz, G. L. (2005). Supplier integration into new product development: coordinating product, process and supply chain design. *Journal of Operations Management*, 371-388.

Rebentisc, E., Schuh, G., Riesener, M., Breunig, S., Pott, A., & Sinha, K. (2016). Assessment of Changes in Technical Systems and their Effects on Cost and Duration Based on Structural Complexity. *5th CIRP Global Web Conference Research and Innovation for Future Production*, 35-40.

Safoutin, M., & Smith, R. (1996). The Iterative Component of Design. *IEMC 96 Proceedings: International Conference on Engineering and Technology Management. Managing Virtual Enterprises: A Convergence of Communications, Computing, and Energy Technologies* , (pp. 564-569).

Sinha, K., & De Weck, O. (2013). A network-based Structural Complexity Metric for Engineered Complex Systems. *Systems Conference.* Orlando, FL, USA: IEEE.

Sinha, K., & Suk Suh, E. (2018). Pareto-optimization of Complex System Architecture for Structural Complexity and Modularity. *Research Engineering Design*, 123-141.

Tatikonda, M., & Rosenthal, S. (2000). Technology Novelty, Project Complexity, and Product Development Project Execution Success: A Deeper Look at Task Uncertainty in Product Innovation. *IEEE Transactions on Engineering Management*, 74-87.

Ulku, S., & Schmidt, G. (2010). Matching Product Architecture and Supply Chain Configuration. *Production and Operations Management*, 16-31.

Ulrich, K. (1995). The role of product architecture in the manufacturing firm. *Reseach policy*, 419-440.

Ulrich, K. T., & Eppinger, S. D. (2004). *Product Design and Development.* New York: McGraw Hill.

Vickery, S., Koufteros, X., Droge, C., & Calantone, R. (2016). Product Modularity, Process Modularity, and New Product Introduction Performance: Does Complexity Matter? *Production and Operations Management*, 751-770.

Weisera, A.-K., Baasner, B., Hosch, M., Schlueter, M., & Ovtcharova, J. (2016). Complexity Assessment of Modular Product Families. *Procedia*, 595-600.

Wynn, D. C., & Eckert, C. C. (2016). Perspectives on iteration in design and development. *Research Engineering Design*, 153-184.

APPENDIX I

Theorem: if $f(x)$ is a continuous function in n variables and if the second order partial derivatives of $f(x)$ exist and are continuous, therefore:

- The gradient of this function $\nabla f(x)$ is the first order partial derivative of $f(x)$ with respect to each component of x (Jensen & Bard, 2003).

$$\nabla f(x) = (\frac{\partial f(x)}{\partial x_i})$$

- x^* is a stationary point of $f(x)$ if $\nabla f(x^*) = \left(\frac{\partial f(x)}{\partial x_i}\right) = 0$.

- The Hessian matrix $H(x)$ is the nxn matrix of second order partial derivatives of $f(x)$ with respect to each component of x (Jensen & Bard, 2003).

$$\mathrm{H}(x) = \left(\frac{\partial^2 f(x)}{\partial x_i \partial x_j}\right) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial^2 x_1} & \cdots & \frac{\partial^2 f(x)}{\partial x_n \partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(x)}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2 f(x)}{\partial^2 x_n} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \cdots & h_{1n} \\ h_{21} & h_{22} & h_{2n} \\ \vdots & & \vdots \\ h_{n1} & \ddots & h_{nn} \end{bmatrix}$$

The **definiteness** of the function $f(x)$ can be determined by computing the determinants of the submatrices of the Hessian (Jensen & Bard, 2003). The ith leading principal submatrix $H_i(x)$ is determined by taking the first “i” rows and columns of the original Hessian matrix $\mathrm{H}(x)$(Jensen & Bard, 2003).

Let H_i be the determinant of the leading principal $H_i(x)$. Therefore:

The determinant of the first leading principal is $H_1 = \frac{\partial^2 f(x)}{\partial^2 x_1} = h_{11}$

The determinant of the leading principal of the second order is

$$H_2 = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = h_{11}.h_{22}\text{-}h_{12}.h_{21} = h_{11}.h_{22}\text{-}{h_{12}}^2$$

And so on until H_n is determined.

The Hessian matrix H is **negative definite** if and only if $\boldsymbol{H_1 < 0}$ and the other subsequent leading principal determinants alternate in sign

$\boldsymbol{H_2 > 0, H_3 < 0, H_4 > 0}$ ...

There is a relationship between the optimality of a stationary point x^*, (where $\nabla f(x^*) = 0$) and the Hessian matrix evaluated at this stationary point $\mathrm{H}(x^*)$(Jensen & Bard, 2003).

If $\mathrm{H}(x^*)$ is **negative definite,** therefore $x^*$is a strong local maximum (Jensen & Bard, 2003).

<u>Application</u>: $R(Td, m)$ is the nonlinear, two variable, objective function which is twice differentiable.

The gradient of $R(Td, m)$ is given by:

$$\nabla R\,(Td, m) = \begin{bmatrix} \dfrac{\partial R}{\partial Td} \\ \dfrac{\partial R}{\partial m} \end{bmatrix}$$

The Hessian matrix of $R(Td, m)$ is given by:

$$Hessian\;\; H\,(Td, m) = \begin{bmatrix} \dfrac{\partial^2 R}{\partial^2 Td} & \dfrac{\partial^2 R}{\partial m \cdot \partial Td} \\ \dfrac{\partial^2 R}{\partial Td \cdot \partial m} & \dfrac{\partial^2 R}{\partial^2 m} \end{bmatrix}$$

The determinant of the first order leading principal is :

$$H1 = \frac{\partial^2 R}{\partial^2 Td}$$

The determinant of the second order leading principal is :

$$H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$$

In order to prove that the solutions provided by Maple in Section B-2 and B-4 of the Chapter III are strong local maxima, we need to show that each solution (Td^*, m^*) is a stationary point and that the Hessian Matrix is **negative definite** at this stationary point.

Therefore if

$$\nabla R\left(Td^*, m^*\right) = \begin{bmatrix} \frac{\partial R}{\partial Td} \\ \frac{\partial R}{\partial m} \end{bmatrix} = 0$$

And

$$H1 = \frac{\partial^2 R}{\partial^2 Td} < 0$$

And

$$H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2 > 0$$

then, the solution $\left(Td^*, m^*\right)$ is a local optimum.

Scenario 1- Chapter III-Section B-2

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc}.\left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td\right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.148881257894039443, [Td = 0.406084020182239, m = 0.613665303497815]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (9)$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) + \left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot(-1.1+0.1\,m) \tag{10}$$

$\frac{\partial^2 R}{\partial^2 Td}$ $\xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{3}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot(-1.1+0.1\,m) \tag{11}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{12}$$

$\frac{\partial R}{\partial m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right) \tag{13}$$

$\frac{\partial^2 R}{\partial^2 m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.1 \left(Td \cdot \left(-\frac{2\, \mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, \mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1} - \frac{0.02 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right.$$

$$+ \frac{0.02 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1 \right)^{3}}$$

$$\left. - \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(-\frac{2\, \mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, \mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right) \cdot \left(-m^{10.} + 1 - Td \right.$$

$$\left. - (0.1 - 0.1\, m) \cdot Td \right) + 2 \left(\frac{0.1 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1} \right.$$

$$\left. - \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right) \cdot \left(-10.\, m^{9.} + 0.1\, Td \right)$$

$$- 90. \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot m^{8.} \tag{14}$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \tag{15}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot \left(-m^{10.}+1-Td-(0.1-0.1\,m) \cdot Td\right) \quad \textbf{(16)}$$

$$+\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}\right) \cdot (-1.1+0.1\,m)$$

$$\frac{\partial^2 R}{\partial m \cdot \partial Td} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)}-\frac{0.2\,e^{-\frac{0.1}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right. \quad \textbf{(17)}$$

$$+\frac{0.2\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}$$

$$\left.-\frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2 (0.1+m)^2}\right) \cdot \left(-m^{10.}+1-Td-(0.1-0.1\,m) \cdot Td\right)$$

$$+\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot \left(-10.\,m^{9.}+0.1\,Td\right)$$

$$+\left(\frac{0.1\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot (\,$$

$$-1.1+0.1\,m)+\frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}$$

$$\frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2 : \textbf{(11)}\cdot\textbf{(14)}-\textbf{(17)}^2$$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.1 \tag{18}$$

$$-0.1\,m)\cdot Td\big)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right)\cdot(-1.1$$

$$+0.1\,m)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}$$

$$-\frac{0.02\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}+\frac{0.02\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.1-0.1\,m)\cdot Td\big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\Bigg)\cdot\big(-10.\,m^{9.}+0.1\,Td\big)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)}$$

$$- \frac{0.2\, e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1\right)^2}$$

$$+ \frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1\right)^3}$$

$$\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1\right)^2 (0.1+m)^2} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td\right)$$

$$+ \left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} - \frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1\right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.1\, Td\right)$$

$$+ \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1\right)^2} \right) \cdot \Big($$

$$\left. -1.1 + 0.1\, m\Big) + \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right)^2$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

eval(**(10)**, $\{m = 0.61366, Td = 0.40608\}$)

$$5.5090\ 10^{-6} \tag{19}$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial m}$

eval(**(13)**, $\{m = 0.61366, Td = 0.40608\}$)

$$2.95700\ 10^{-6} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\textbf{(11)}, \{m = 0.61366, Td = 0.40608\})$

$$-1.334587657 \quad \textbf{(21)}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\textbf{(18)}, \{m = 0.61366, Td = 0.40608\})$

$$0.7159290912 \quad \textbf{(22)}$$

Scenario 2- Chapter III-Section B-2

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0654402508914487396, [Td = 0.433060516433687, m = 0.734018580510200]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{3}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \qquad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right) \qquad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1}-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right. \tag{14}$$

$$+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1\right)^{3}}$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td\right.$$

$$\left.-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1}\right.$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right)$$

$$-90.\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1}\right)\cdot m^{8.}$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{15}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right) \cdot \left(-m^{10.}+1-Td-(0.1-0.1\, m) \cdot Td\right) + \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}\right) \cdot (-1.1+0.1\, m) \tag{16}$$

$\frac{\partial^2 R}{\partial m \cdot \partial Td}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.9\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)}-\frac{1.8\, e^{-\frac{0.9}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}} + \frac{1.8\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{3}} - \frac{0.9\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}(0.1+m)^{2}}\right) \cdot \left(-m^{10.}+1-Td-(0.1-0.1\, m) \cdot Td\right)$$
$$+\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right) \cdot \left(-10.\, m^{9.}+0.1\, Td\right)$$
$$+\left(\frac{0.9\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{0.9\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right) \cdot (-1.1+0.1\, m) + \frac{0.1\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1} \tag{17}$$

$\frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$ **(11)·(14)−(17)^2**

$$\Bigg(\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^3}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.1 \tag{18}$$

$$-0.1\,m)\cdot Td\big)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\right)\cdot\big(-1.1$$

$$+0.1\,m\big)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}$$

$$-\frac{1.62\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}+\frac{1.62\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^3}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.1-0.1\,m)\cdot Td\big)+2\Bigg(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\Bigg)\cdot\big(-10.\,m^{9.}+0.1\,Td\big)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)}$$

$$
-\frac{1.8\, e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}
$$

$$
+\frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^3}
$$

$$
-\frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2 (0.1+m)^2} \Bigg) \cdot \left(-m^{10.}+1-Td-(0.1-0.1\, m) \cdot Td\right)
$$

$$
+\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\right) \cdot \left(-10.\, m^{9.}+0.1\, Td\right)
$$

$$
+\left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\right) \cdot (
$$

$$
-1.1+0.1\, m)+\frac{0.1 \left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1} \Bigg)^2
$$

Evaluating $\nabla R\,(Td^*, m^*),\ \frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)}, \{m=0.734018580510200, Td=0.433060516433687\})$

$$
-8.5\ 10^{-9} \qquad \mathbf{(19)}
$$

Evaluating $\nabla R\,(Td^*, m^*),\ \frac{\partial R}{\partial m}$

$eval(\mathbf{(13)}, \{m=0.734018580510200, Td=0.433060516433687\})$

$$
3.679\ 10^{-8} \qquad \mathbf{(20)}
$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)}, \{m=0.734018580510200, Td=0.433060516433687\})$

$$-0.6083313304 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\textbf{(18)}, \{m = 0.734018580510200, Td = 0.433060516433687\})$

$$0.7328922664 \tag{22}$$

$M := 1$

$$M := 1 \quad \textbf{(1)}$$

$Qc := 1$

$$Qc := 1 \quad \textbf{(2)}$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad \textbf{(3)}$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad \textbf{(4)}$$

$\mu := 0.1$

$$\mu := 0.1 \quad \textbf{(5)}$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad \textbf{(6)}$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad \textbf{(7)}$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0955954998619407831,\ [Td = 0.372225465412315,\ m = 0.119537950593424]] \quad \textbf{(8)}$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad \textbf{(9)}$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot(-1.1+0.1\,m) \tag{10}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{3}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot(-1.1+0.1\,m) \tag{11}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{12}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right) \tag{13}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$
\begin{aligned}
&\left(\frac{0.1 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} - \frac{0.02 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right.\\
&+ \frac{0.02 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^{3}}\\
&\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right) \cdot \left(-m^{1.111111111} + 1 \right.\\
&\left. - Td - (0.1 - 0.1\, m) \cdot Td \right) + 2 \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right.\\
&\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^{2}} \right) \cdot \left(-1.111111111\, m^{0.111111111} + 0.1\, Td \right)\\
&- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}}
\end{aligned} \tag{14}
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \tag{15}
$$

$$
\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}
$$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot \left(-m^{1.111111111}+1-Td-(0.1-0.1\,m) \cdot Td\right)+\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}\right) \cdot (-1.1+0.1\,m) \tag{16}$$

$$\frac{\partial^2 R}{\partial m \cdot \partial Td} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)}-\frac{0.2\, e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}+\frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}-\frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2 (0.1+m)^2}\right) \cdot \left(-m^{1.111111111}+1-Td-(0.1-0.1\,m) \cdot Td\right)+\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot \left(-1.111111111\, m^{0.111111111}+0.1\, Td\right)+\left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right) \cdot (-1.1+0.1\,m) \tag{17}$$

$$+\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}$$

(11)·(14)−(17)^2 $\frac{^2R}{\partial^2 Td}\cdot\frac{\partial^2R}{\partial^2 m}-\left(\frac{\partial^2R}{\partial m\cdot\partial Td}\right)^2$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}\Bigg)\cdot\Big(-m^{1.111111111}+1-Td \qquad \textbf{(18)}$$

$$-(0.1-0.1\,m)\cdot Td\Big)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\right)\cdot\Big(-1.1$$

$$+0.1\,m\Big)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}$$

$$-\frac{0.02\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}+\frac{0.02\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^3}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\Bigg)\cdot\Big(-m^{1.111111111}+1$$

$$-Td-(0.1-0.1\,m)\cdot Td\Big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+1}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+1\right)^2}\Bigg)\cdot\Big(-1.111111111\,m^{0.111111111}+0.1\,Td\Big)$$

$$
- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}} \Bigg)
$$

$$
- \left(\left(\frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)} - \frac{0.2\, e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^2} \right.\right.
$$

$$
+ \frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^3}
$$

$$
\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^2 (0.1+m)^2} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)
$$

$$
+ \left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} - \frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^2} \right) \cdot \left(-1.111111111\, m^{0.111111111} \right.
$$

$$
\left. + 0.1\, Td \right) + \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right.
$$

$$
\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 1 \right)^2} \right) \cdot \left(-1.1 + 0.1\, m \right)
$$

$$
\left. + \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 1} \right)^2
$$

Evaluating $\nabla R\ (Td^{*}, m^{*}),\ \frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)}, \{Td = 0.372225465412315, m = 0.119537950593424\})$

$$0. \tag{19}$$

Evaluating $\nabla R\ (Td^{*}, m^{*}),\ \frac{\partial R}{\partial m}$

$eval(\mathbf{(13)}, \{Td = 0.372225465412315, m = 0.119537950593424\})$

$$-1.\ 10^{-10} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)}, \{Td = 0.372225465412315, m = 0.119537950593424\})$

$$-1.116408301 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\mathbf{(18)}, \{Td = 0.372225465412315, m = 0.119537950593424\})$

$$2.007878172 \tag{22}$$

Scenario 4- Chapter III-Section B-2

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0152406258151719185, [Td = 0.283300066132247, m = 0.424082220344776]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot(-1.1+0.1\,m) \tag{10}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{3}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot(-1.1+0.1\,m) \tag{11}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{12}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right) \tag{13}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3} + \frac{0.9\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} - \frac{1.62 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 1 \right)^2} \right. \tag{14}$$

$$+ \frac{1.62 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 1 \right)^3}$$

$$\left. - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3} + \frac{0.9\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 1 \right)^2} \right) \cdot \left(-m^{1.111111111} + 1 \right.$$

$$\left. - Td - (0.1 - 0.1\, m) \cdot Td \right) + 2 \left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right.$$

$$\left. - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 1 \right)^2} \right) \cdot \left(-1.111111111\, m^{0.111111111} + 0.1\, Td \right)$$

$$- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}}$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \tag{15}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}\right)\cdot(-1.1+0.1\,m) \tag{16}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)}-\frac{1.8\,e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{3}}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}(0.1+m)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right)+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+1\right)^{2}}\right)\cdot(-1.1+0.1\,m) \tag{17}$$

$$+\frac{0.1\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}$$

(11)·(14)−(17)² $\frac{{}^2R}{\partial^2 Td} \cdot \frac{\partial^2 R}{{}_2\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$$\Bigg(\Bigg(\left(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}+\frac{2\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^3}\right) \cdot \left(-m^{1.111111111}+1-Td\right. \tag{18}$$

$$\left.-(0.1-0.1\,m) \cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}-\frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\right) \cdot \left(-1.1\right.$$

$$\left.+0.1\,m\right)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td \cdot \left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}$$

$$-\frac{1.62\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}+\frac{1.62\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^3}$$

$$-\frac{0.9\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td \cdot \left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\Bigg) \cdot \left(-m^{1.111111111}+1\right.$$

$$\left.-Td-(0.1-0.1\,m) \cdot Td\right)+2\left(\frac{0.9\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.9}{0.1+m}}+1}\right.$$

$$\left.-\frac{0.9\left(Td \cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}}+1\right)^2}\right) \cdot \left(-1.111111111\,m^{0.111111111}+0.1\,Td\right)$$

$$- 0.1234567900 \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}} \Bigg)$$

$$- \Bigg(\Bigg(\frac{0.9\, \mathrm{e}^{-\frac{0.9}{0.1 + m}}}{(0.1 + m)^2 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)} - \frac{1.8\, \mathrm{e}^{-\frac{0.9}{0.1 + m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.1 + m}}}{(0.1 + m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)^2}$$

$$+ \frac{1.8 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} \right) \mathrm{e}^{-\frac{0.9}{0.1 + m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.1 + m}}}{(0.1 + m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)^3}$$

$$- \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} \right) \mathrm{e}^{-\frac{0.9}{0.1 + m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)^2 (0.1 + m)^2} \Bigg) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$+ \left(\frac{\mathrm{e}^{-\frac{0.9}{0.1 + m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1} - \frac{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} \right) \mathrm{e}^{-\frac{0.9}{0.1 + m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)^2} \right) \cdot \Big(-1.111111111\, m^{0.111111111}$$

$$+ 0.1\, Td \Big) + \left(\frac{0.9 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.1 + m}}}{(0.1 + m)^2} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1} \right.$$

$$\left. - \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.1 + m}}}{(0.1 + m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1 \right)^2} \right) \cdot (-1.1 + 0.1\, m)$$

$$+ \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} \right)}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.1 + m}} + 1} \Bigg)^2$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial Td}$

eval(**(10)**, $\{Td = 0.283300066132247,\ m = 0.424082220344776\}$)

$$-5.2\ 10^{-10} \tag{19}$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial m}$

eval(**(13)**, $\{Td = 0.283300066132247,\ m = 0.424082220344776\}$)

$$-4.4\ 10^{-10} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

eval(**(11)**, $\{Td = 0.283300066132247,\ m = 0.424082220344776\}$)

$$-0.3614028727 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

eval(**(18)**, $\{Td = 0.283300066132247,\ m = 0.424082220344776\}$)

$$0.09991106626 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.130980560676227320, [Td = 0.329209015998375, m = 0.680577174818062]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^3}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} - \frac{0.02 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^{2}} \right. \qquad \textbf{(14)}$$

$$+ \frac{0.02 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^{3}}$$

$$\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^{2}} \right) \cdot \left(-m^{10.} + 1 - Td \right.$$

$$\left. - (0.9 - 0.9\, m) \cdot Td \right) + 2 \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right.$$

$$\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^{2}} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td \right)$$

$$- 90. \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot m^{8.}$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right) \qquad \textbf{(15)}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \tag{16}$$

$$+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m)$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)}-\frac{0.2\,e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right. \tag{17}$$

$$+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^3}$$

$$\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2(0.9+m)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$+\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)$$

$$+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot($$

$$-1.9+0.9\,m)+\frac{0.9\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}$$

(11)·(14)−(17)² $$\frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$$

$$\Bigg(\Bigg(\Bigg(-\frac{2\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{2}}+\frac{2\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{3}}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.9 \tag{18}$$

$$-0.9\,m)\cdot Td\big)+2\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\mathrm{e}^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot\big(-1.9$$

$$+0.9\,m\big)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1}$$

$$-\frac{0.02\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{2}}+\frac{0.02\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{3}}$$

$$-\frac{0.1\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1}$$

$$-\frac{0.1\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\Bigg)\cdot\big(-10.\,m^{9.}+0.9\,Td\big)$$

$$-90.\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot\mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)}$$

$$- \frac{0.2\, e^{-\frac{0.1}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2}$$

$$+ \frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^3}$$

$$- \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2 (0.9+m)^2} \Bigg) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} - \frac{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td \right)$$

$$+ \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2} \right) \cdot ($$

$$-1.9 + 0.9\, m) + \frac{0.9 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \Bigg)^2$$

Evaluating $\nabla R\, (Td^*, m^*),\ \frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)},\ \{Td = 0.329209015998375,\ m = 0.680577174818062\})$

$$-2.208\ 10^{-7} \qquad \mathbf{(19)}$$

Evaluating $\nabla R\, (Td^*, m^*),\ \frac{\partial R}{\partial m}$

$eval(\mathbf{(13)},\ \{Td = 0.329209015998375,\ m = 0.680577174818062\})$

$$7.3647\ 10^{-8} \qquad \mathbf{(20)}$$

Evaluating $H\,1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)},\ \{Td = 0.329209015998375,\ m = 0.680577174818062\})$

$$-1.846483486 \quad \textbf{(21)}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\textbf{(18)}, \{Td = 0.329209015998375, m = 0.680577174818062\})$

$$1.775267083 \quad \textbf{(22)}$$

Scenario 6- Chapter III-Section B-2

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,\mathrm{e}^{-\frac{\mu}{\alpha+m}}}{Td.\,\mathrm{e}^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0877292351113289348, [Td = 0.348906218019577, m = 0.711347364424344]] \quad (8)$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 1} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (9)$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \qquad \textbf{(10)}$$

$$+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m)$$

$\frac{\partial^2 R}{\partial^2 Td}$ $\xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{3}}\right)\cdot\left(-m^{10.}+1-Td-(0.9\right. \qquad \textbf{(11)}$$

$$\left.-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot(-1.9+0.9\,m)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \qquad \textbf{(12)}$$

$\frac{\partial R}{\partial m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1\right. \qquad \textbf{(13)}$$

$$\left.-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)$$

$\frac{\partial^2 R}{\partial^2 m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right. \tag{14}$$

$$+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{3}}$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td\right.$$

$$\left.-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right.$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)$$

$$-90.\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot m^{8.}$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \tag{15}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \qquad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)}-\frac{1.8\,e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right. \qquad \textbf{(17)}$$

$$+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{3}}$$

$$\left.-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}(0.9+m)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$+\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)$$

$$+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot($$

$$-1.9+0.9\,m)+\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}$$

(11)·(14)−(17)^2 $$\frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^{2}$$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^3}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.9 \qquad \textbf{(18)}$$

$$-0.9\,m)\cdot Td\big)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\cdot\big(-1.9$$

$$+0.9\,m\big)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}$$

$$-\frac{1.62\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}+\frac{1.62\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^3}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\Bigg)\cdot\big(-10.\,m^{9.}+0.9\,Td\big)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)}$$

$$- \frac{1.8\, e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2}$$

$$+ \frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^3}$$

$$\left. - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2 (0.9+m)^2} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} - \frac{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td \right)$$

$$+ \left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2} \right) \cdot \Bigg($$

$$\left. -1.9 + 0.9\, m) + \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} \right)^2$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

eval(**(10)**, $\{Td = 0.348906218019577, m = 0.711347364424344\}$)

$$-6.7\ 10^{-9} \tag{19}$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial m}$

eval(**(13)**, $\{Td = 0.348906218019577, m = 0.711347364424344\}$)

$$-4.7\ 10^{-10} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

eval(**(11)**, $\{Td = 0.348906218019577, m = 0.711347364424344\}$)

$$-1.201501060 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\textbf{(18)}, \{Td = 0.348906218019577, m = 0.711347364424344\})$

$$1.221995708 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0961819471497491751, [Td = 0.237847000280800, m = 2.94012934694215\ 10^{-6}]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \tag{10}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(e^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{3}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot(-1.9+0.9\,m) \tag{11}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right) \tag{12}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right) \tag{13}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\Bigg(\frac{0.1 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3} + \frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} - \frac{0.02 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2}$$

$$+ \frac{0.02 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^3}$$

$$- \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3} + \frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2} \Bigg) \cdot \left(-m^{1.111111111} + 1 \right.$$

$$\left. - Td - (0.9 - 0.9\, m) \cdot Td \right) + 2 \Bigg(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1}$$

$$- \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 1 \right)^2} \Bigg) \cdot \left(-1.111111111\, m^{0.111111111} + 0.9\, Td \right)$$

$$- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}} \qquad \textbf{(14)}$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 1} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right) \qquad \textbf{(15)}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \tag{16}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)}-\frac{0.2\,e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{3}}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}(0.9+m)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right)+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+1}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+1\right)^{2}}\right)\cdot(-1.9+0.9\,m) \tag{17}$$

$$+\frac{0.9\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1}$$

(11)·(14)−(17)^2 $\quad \frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\left(\left(-\frac{2\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^2}+\frac{2\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^3}\right)\cdot\left(-m^{1.111111111}+1-Td\right. \tag{18}$$

$$\left.-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1}-\frac{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\mathrm{e}^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-1.9\right.$$

$$\left.+0.9\,m\right)\Bigg)\left(\left(\frac{0.1\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3}+\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1}\right.\right.$$

$$-\frac{0.02\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^2}+\frac{0.02\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^3}$$

$$\left.-\frac{0.1\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3}+\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{1.111111111}+1\right.$$

$$\left.-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{0.1\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1}\right.$$

$$\left.-\frac{0.1\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+1\right)^2}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right)$$

$$- 0.1234567900 \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}} \Bigg)$$

$$- \Bigg(\Bigg(\frac{0.1\, \mathrm{e}^{-\frac{0.1}{0.9 + m}}}{(0.9 + m)^{2} \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)} - \frac{0.2\, \mathrm{e}^{-\frac{0.1}{0.9 + m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.9 + m}}}{(0.9 + m)^{2}} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)^{2}}$$

$$+ \frac{0.2 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} \right) \mathrm{e}^{-\frac{0.1}{0.9 + m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.9 + m}}}{(0.9 + m)^{2}} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)^{3}}$$

$$- \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} \right) \mathrm{e}^{-\frac{0.1}{0.9 + m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)^{2} (0.9 + m)^{2}} \Bigg) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{\mathrm{e}^{-\frac{0.1}{0.9 + m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1} - \frac{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} \right) \mathrm{e}^{-\frac{0.1}{0.9 + m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)^{2}} \right) \cdot \Big(-1.111111111\, m^{0.111111111}$$

$$+ 0.9\, Td \Big) + \Bigg(\frac{0.1 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.9 + m}}}{(0.9 + m)^{2}} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1}$$

$$- \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.9 + m}}}{(0.9 + m)^{2}} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1 \right)^{2}} \Bigg) \cdot (-1.9 + 0.9\, m)$$

$$+ \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.9 + m}} + 1} \Bigg)^{2}$$

Evaluating $\nabla R\,(Td^{*}, m^{*})$, $\dfrac{\partial R}{\partial Td}$

$eval(\mathbf{(10)}, \{Td = 0.237847000280800, m = 2.94012934694215\ 10^{-6}\})$

$$-2.7\ 10^{-9} \tag{19}$$

Evaluating $\nabla R\,(Td^{*}, m^{*})$, $\dfrac{\partial R}{\partial m}$

$eval(\mathbf{(13)}, \{Td = 0.237847000280800, m = 2.94012934694215\ 10^{-6}\})$

$$-4.44998\ 10^{-7} \tag{20}$$

Evaluating $H1 = \dfrac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)}, \{Td = 0.237847000280800, m = 2.94012934694215\ 10^{-6}\})$

$$-2.803667533 \tag{21}$$

Evaluating $H2 = \dfrac{\partial^2 R}{\partial^2 Td} \cdot \dfrac{\partial^2 R}{\partial^2 m} - \left(\dfrac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\mathbf{(18)}, \{Td = 0.237847000280800, m = 2.94012934694215\ 10^{-6}\})$

$$5017.584529 \tag{22}$$

Scenario 8- Chapter III-Section B-2

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 1$

$$Qc := 1 \quad (2)$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc}.\left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td\right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0443803632171954968,\ [Td = 0.250348404332145,\ m = 0.0213216157747247]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (9)$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^3}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\begin{aligned}
&\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right. \\
&\quad+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{3}} \\
&\quad\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1\right. \\
&\quad\left.-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right. \\
&\quad\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right) \\
&\quad-0.1234567900\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\frac{1}{m^{0.888888889}}
\end{aligned}\tag{14}$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+1}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)\tag{15}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)}-\frac{1.8\,e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{3}}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}(0.9+m)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right)+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+1}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+1\right)^{2}}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(17)}$$

$$+\frac{0.9\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)}{Td\bullet e^{-\frac{0.9}{0.9+m}}+1}$$

(11)·(14)−(17)² $\quad \frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{{}_2\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^2}+\frac{2\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^3}\Bigg)\bullet(-m^{1.111111111}+1-Td \tag{18}$$

$$-(0.9-0.9\,m)\bullet Td)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\bullet e^{-\frac{0.9}{0.9+m}}+1}-\frac{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\bullet(-1.9$$

$$+0.9\,m)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td\bullet\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{Td\bullet e^{-\frac{0.9}{0.9+m}}+1}$$

$$-\frac{1.62\left(Td\bullet\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^2}+\frac{1.62\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)\left(Td\bullet\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^3}$$

$$-\frac{0.9\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)\left(Td\bullet\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^2}\Bigg)\bullet(-m^{1.111111111}+1$$

$$-Td-(0.9-0.9\,m)\bullet Td)+2\left(\frac{0.9\left(Td\bullet\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\bullet e^{-\frac{0.9}{0.9+m}}+1}\right.$$

$$\left.-\frac{0.9\left(Td\bullet e^{-\frac{0.9}{0.9+m}}\right)\left(Td\bullet\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\bullet e^{-\frac{0.9}{0.9+m}}+1\right)^2}\right)\bullet(-1.111111111\,m^{0.111111111}+0.9\,Td)$$

$$
- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} \right) \cdot \frac{1}{m^{0.888888889}} \Bigg)
$$

$$
- \Bigg(\Bigg(\frac{0.9\, e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)} - \frac{1.8\, e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2}
$$

$$
+ \frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^3}
$$

$$
- \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2 (0.9+m)^2} \Bigg) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)
$$

$$
+ \left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} - \frac{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2} \right) \cdot \Big(-1.111111111\, m^{0.111111111}
$$

$$
+ 0.9\, Td \Big) + \Bigg(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1}
$$

$$
- \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 1 \right)^2} \Bigg) \cdot (-1.9 + 0.9\, m)
$$

$$
+ \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 1} \Bigg)^2
$$

Evaluating $\nabla R\left(Td^{*}, m^{*}\right), \dfrac{\partial R}{\partial Td}$

$eval\left(\mathbf{(10)}, \left\{Td = 0.250348404332145, m = 0.0213216157747247\right\}\right)$

$$2.\ 10^{-10} \tag{19}$$

Evaluating $\nabla R\left(Td^{*}, m^{*}\right), \dfrac{\partial R}{\partial m}$

$eval\left(\mathbf{(13)}, \left\{Td = 0.250348404332145, m = 0.0213216157747247\right\}\right)$

$$-2.\ 10^{-11} \tag{20}$$

Evaluating $H1 = \dfrac{\partial^2 R}{\partial^2 Td},$

$eval\left(\mathbf{(11)}, \left\{Td = 0.250348404332145, m = 0.0213216157747247\right\}\right)$

$$-1.294234425 \tag{21}$$

Evaluating $H2 = \dfrac{\partial^2 R}{\partial^2 Td} \cdot \dfrac{\partial^2 R}{\partial^2 m} - \left(\dfrac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval\left(\mathbf{(18)}, \left\{Td = 0.250348404332145, m = 0.0213216157747247\right\}\right)$

$$0.5918674243 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td\right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.238120854544137867, [Td = 0.363268192901429, m = 0.606995830070805]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \quad \textbf{(10)}$$

$\frac{\partial^2 R}{\partial^2 Td}$ $\xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^3}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right)\cdot(-1.1+0.1\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \quad \textbf{(12)}$$

$\frac{\partial R}{\partial m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right) \quad \textbf{(13)}$$

$\frac{\partial^2 R}{\partial^2 m}$ $\xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.1 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} - \frac{0.02 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^{2}} \right.$$

$$+ \frac{0.02 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^{3}}$$

$$\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}} + \frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^{2}} \right) \cdot \left(-m^{10.} + 1 - Td \right.$$

$$\left. - (0.1 - 0.1\, m) \cdot Td \right) + 2 \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} \right.$$

$$\left. - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^{2}} \right) \cdot \left(-10.\, m^{9.} + 0.1\, Td \right)$$

$$- 90. \left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot m^{8.} \tag{14}$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \tag{15}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)}-\frac{0.2\,e^{-\frac{0.1}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}} \right. \qquad \textbf{(17)}$$

$$+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}$$

$$\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}(0.1+m)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$+\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right)$$

$$+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot($$

$$-1.1+0.1\,m)+\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}$$

$$\textbf{(11)}\cdot\textbf{(14)}-\textbf{(17)}^{2}\quad \frac{{}^{2}R}{\partial^{2}Td}\cdot\frac{\partial^{2}R}{\partial^{2}m}-\left(\frac{\partial^{2}R}{\partial m\cdot\partial Td}\right)^{2}$$

$$\Bigg(\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.1 \qquad \textbf{(18)}$$

$$-0.1\,m)\cdot Td\big)+2\Bigg(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\Bigg)\cdot(-1.1$$

$$+0.1\,m)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}$$

$$-\frac{0.02\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}+\frac{0.02\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.1-0.1\,m)\cdot Td\big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\Bigg)\cdot\big(-10.\,m^{9.}+0.1\,Td\big)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)}$$

$$- \frac{0.2\, e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2}$$

$$+ \frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^3}$$

$$- \left. \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2 (0.1+m)^2} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} - \frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.1\, Td \right)$$

$$+ \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2} \right) \cdot \Bigg($$

$$\left. -1.1 + 0.1\, m) + \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.1+m}} \right)}{Td \cdot e^{-\frac{0.1}{0.1+m}} + 0.5} \right)^2$$

Evaluating $\nabla R\ (Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

$eval((\mathbf{10}), \{Td = 0.363268192901429, m = 0.606995830070805\})$

$$-2.392\ 10^{-7} \tag{19}$$

Evaluating $\nabla R\ (Td^*, m^*)$, $\frac{\partial R}{\partial m}$

$eval((\mathbf{13}), \{Td = 0.363268192901429, m = 0.606995830070805\})$

$$-2.\ 10^{-11} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval((\mathbf{11}), \{Td = 0.363268192901429, m = 0.606995830070805\})$

$$-2.213075850 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\mathbf{(18)}, \{Td = 0.363268192901429, m = 0.606995830070805\})$

$$1.613721307 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\, e^{-\frac{\mu}{\alpha+m}}}{Td.\, e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad (6)$$

right hand side $\longrightarrow$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad (7)$$

optimization assistant $\longrightarrow$

$$[0.116342974525714238, [Td = 0.409252261156762, m = 0.730151015244792]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td}$ differentiate w.r.t. Td $\longrightarrow$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{3}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \qquad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right) \qquad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$
\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right. \tag{14}
$$

$$
+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{3}}
$$

$$
\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td\right.
$$

$$
\left.-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right.
$$

$$
\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right)
$$

$$
-90.\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot m^{8.}
$$

$$
\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)
$$

$$
\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{15}
$$

$$
\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}
$$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)}-\frac{1.8\,e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{3}}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}(0.1+m)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$
$$+\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.1\,Td\right)$$
$$+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot(-1.1+0.1\,m)+\frac{0.1\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5} \qquad \textbf{(17)}$$

(11)·(14)−(17)^2 $\quad \frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\Biggl(\Biggl(\Biggl(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^3}\Biggr)\cdot\bigl(-m^{10.}+1-Td-(0.1 \qquad \textbf{(18)}$$

$$-0.1\,m)\cdot Td\bigr)+2\Biggl(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\Biggr)\cdot(-1.1$$

$$+0.1\,m)\Biggr)\Biggl(\Biggl(\frac{0.9\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}$$

$$-\frac{1.62\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}+\frac{1.62\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^3}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\Biggr)\cdot\bigl(-m^{10.}+1-Td$$

$$-(0.1-0.1\,m)\cdot Td\bigr)+2\Biggl(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\Biggr)\cdot\bigl(-10.\,m^{9.}+0.1\,Td\bigr)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot m^{8.}\Biggr)-\Biggl(\Biggl(\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)}$$

$$- \frac{1.8\, e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2} + \frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^3}$$

$$- \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2 (0.1+m)^2} \Bigg) \cdot \left(-m^{10.} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} - \frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.1\, Td \right)$$

$$+ \left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2} \right) \cdot ($$

$$-1.1 + 0.1\, m) + \frac{0.1 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \Bigg)^2$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

eval(**(10)**, $\{Td = 0.409252261156762,\ m = 0.730151015244792\}$)

$$-2.5\ 10^{-9} \tag{19}$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial m}$

eval(**(13)**, $\{Td = 0.409252261156762,\ m = 0.730151015244792\}$)

$$-1.8\ 10^{-9} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

eval(**(11)**, $\{Td = 0.409252261156762,\ m = 0.730151015244792\}$)

$$-1.088080451 \quad \textbf{(21)}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\textbf{(18)}, \{Td = 0.409252261156762, m = 0.730151015244792\})$

$$2.123624350 \quad \textbf{(22)}$$

$M := 1$

$$M := 1 \quad \textbf{(1)}$$

$Qc := 0.5$

$$Qc := 0.5 \quad \textbf{(2)}$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad \textbf{(3)}$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad \textbf{(4)}$$

$\mu := 0.1$

$$\mu := 0.1 \quad \textbf{(5)}$$

$$R = \frac{M.Td.\, e^{-\frac{\mu}{\alpha + m}}}{Td.\, e^{-\frac{\mu}{\alpha + m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1 - m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.1 + m}}}{Td \cdot e^{-\frac{0.1}{0.1 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad \textbf{(6)}$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1 + m}}}{Td \cdot e^{-\frac{0.1}{0.1 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad \textbf{(7)}$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.161475948727308444, [Td = 0.344799772561704, m = 0.111840028332239]] \quad \textbf{(8)}$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1 + m}}}{Td \cdot e^{-\frac{0.1}{0.1 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.1 + m}}}{Td \cdot e^{-\frac{0.1}{0.1 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right) \quad \textbf{(9)}$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot(-1.1+0.1\,m) \qquad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right) \qquad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right) \qquad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1 \left(Td \cdot \left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3} + \frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5} - \frac{0.02 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)^2}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2} \right. \tag{14}$$

$$+ \frac{0.02 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)^2}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5 \right)^3}$$

$$\left. - \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^3} + \frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^4} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2} \right) \cdot \left(-m^{1.111111111} + 1 \right.$$

$$\left. - Td - (0.1 - 0.1\,m) \cdot Td \right) + 2 \left(\frac{0.1 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5} \right.$$

$$\left. - \frac{0.1 \left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5 \right)^2} \right) \cdot \left(-1.111111111\,m^{0.111111111} + 0.1\,Td \right)$$

$$- 0.1234567900 \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot \frac{1}{m^{0.888888889}}$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}}}{Td \cdot \mathrm{e}^{-\frac{0.1}{0.1+m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td \right) \tag{15}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right) \cdot \left(-m^{1.111111111}+1-Td-(0.1-0.1\, m) \cdot Td\right)+\left(\frac{Td \cdot e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right) \cdot (-1.1+0.1\, m) \tag{16}$$

$\frac{\partial^2 R}{\partial m \cdot \partial Td} \xrightarrow{\text{differentiate w.r.t. m}}$

$$\left(\frac{0.1\, e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2 \left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)}-\frac{0.2\, e^{-\frac{0.1}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}+\frac{0.2\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^3}-\frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2 (0.1+m)^2}\right) \cdot \left(-m^{1.111111111}+1-Td-(0.1-0.1\, m) \cdot Td\right)+\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right) e^{-\frac{0.1}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right) \cdot \left(-1.111111111\, m^{0.111111111}+0.1\, Td\right)+\left(\frac{0.1\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right) \cdot (-1.1+0.1\, m) \tag{17}$$

$$+\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}$$

(11)·(14)−(17)² $\frac{{}^{2}R}{\partial^{2}Td}\cdot\frac{\partial^{2}R}{\partial^{2}m}-\left(\frac{\partial^{2}R}{\partial m\cdot\partial Td}\right)^{2}$

$$\left(\left(\left(-\frac{2\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(e^{-\frac{0.1}{0.1+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}\right)\cdot\left(-m^{1.111111111}+1-Td\right.\right.\right. \tag{18}$$

$$\left.-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-1.1\right.$$

$$\left.\left.+0.1\,m\right)\right)\left(\left(\left(\frac{0.1\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right.\right.\right.$$

$$-\frac{0.02\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}+\frac{0.02\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{3}}$$

$$\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1\right.$$

$$\left.-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right.$$

$$\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right)$$

$$
-0.1234567900\left(\frac{Td\cdot e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)\cdot\frac{1}{m^{0.888888889}}\Bigg)
$$

$$
-\left(\left(\frac{0.1\,e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)}-\frac{0.2\,e^{-\frac{0.1}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right.\right.
$$

$$
+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^3}
$$

$$
\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2(0.1+m)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)
$$

$$
+\left(\frac{e^{-\frac{0.1}{0.1+m}}}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)e^{-\frac{0.1}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right)\cdot\left(-1.111111111\,m^{0.111111111}\right.
$$

$$
\left.+0.1\,Td\right)+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right.
$$

$$
\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5\right)^2}\right)\cdot(-1.1+0.1\,m)
$$

$$
\left.+\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.1+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.1+m}}+0.5}\right)^2
$$

Evaluating $\nabla R\ (Td^*, m^*),\ \frac{\partial R}{\partial Td}$

$eval(\textbf{(10)}, \{Td = 0.344799772561704, m = 0.111840028332239\})$

$$1.\ 10^{-10} \tag{19}$$

Evaluating $\nabla R\ (Td^*, m^*),\ \frac{\partial R}{\partial m}$

$eval(\textbf{(13)}, \{Td = 0.344799772561704, m = 0.111840028332239\})$

$$0. \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\textbf{(11)}, \{Td = 0.344799772561704, m = 0.111840028332239\})$

$$-1.899469822 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\textbf{(18)}, \{Td = 0.344799772561704, m = 0.111840028332239\})$

$$5.581902386 \tag{22}$$

Scenario 4- Chapter III-Section B-4

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad (3)$$

$\alpha := 0.1$

$$\alpha := 0.1 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc}.\left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td\right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0290930394404118851, [Td = 0.278600131569556, m = 0.419981718318083]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5}\right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\,m) \cdot Td\right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(e^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^3}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\right)\cdot(-1.1+0.1\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right.$$

$$+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{3}}$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1\right.$$

$$\left.-Td-(0.1-0.1\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right.$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right)$$

$$-0.1234567900\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\frac{1}{m^{0.888888889}} \tag{14}$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right) \tag{15}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}\right)\cdot(-1.1+0.1\,m) \tag{16}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)}-\frac{1.8\,e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{3}}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}(0.1+m)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.1-0.1\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)e^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.1\,Td\right)+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.1+m}}+0.5\right)^{2}}\right)\cdot(-1.1+0.1\,m) \tag{17}$$

$$+\frac{0.1\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)}{Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}$$

(11)·(14)−(17)^2 $\frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\Bigg(\bigg(-\frac{2\left(\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^2}+\frac{2\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(\mathrm{e}^{-\frac{0.9}{0.1+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^3}\bigg)\cdot\big(-m^{1.111111111}+1-Td \qquad \textbf{(18)}$$

$$-(0.1-0.1\,m)\cdot Td\big)+2\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}-\frac{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\mathrm{e}^{-\frac{0.9}{0.1+m}}}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\right)\cdot\big(-1.1$$

$$+0.1\,m)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}$$

$$-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^2}+\frac{1.62\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^3}$$

$$-\frac{0.9\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^3}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^4}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\Bigg)\cdot\big(-m^{1.111111111}+1$$

$$-Td-(0.1-0.1\,m)\cdot Td\big)+2\Bigg(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5}$$

$$-\frac{0.9\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.9}{0.1+m}}+0.5\right)^2}\Bigg)\cdot\big(-1.111111111\,m^{0.111111111}+0.1\,Td\big)$$

$$- 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right) \cdot \frac{1}{m^{0.888888889}} \Bigg)$$

$$- \Bigg(\Bigg(\frac{0.9\, e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)} - \frac{1.8\, e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2}$$

$$+ \frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^3}$$

$$- \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2 (0.1+m)^2} \Bigg) \cdot \left(-m^{1.111111111} + 1 - Td - (0.1 - 0.1\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.9}{0.1+m}}}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} - \frac{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) e^{-\frac{0.9}{0.1+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2} \right) \cdot \left(-1.111111111\, m^{0.111111111} \right.$$

$$\left. + 0.1\, Td \right) + \left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \right.$$

$$\left. - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.1+m}}}{(0.1+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5 \right)^2} \right) \cdot (-1.1 + 0.1\, m)$$

$$+ \frac{0.1 \left(Td \cdot e^{-\frac{0.9}{0.1+m}} \right)}{Td \cdot e^{-\frac{0.9}{0.1+m}} + 0.5} \Bigg)^2$$

Evaluating $\nabla R\ (Td^*, m^*),\ \dfrac{\partial R}{\partial Td}$

$eval(\mathbf{(10)},\ \{Td = 0.278600131569556,\ m = 0.419981718318083\})$

$$2.4\ 10^{-10} \tag{19}$$

Evaluating $\nabla R\ (Td^*, m^*),\ \dfrac{\partial R}{\partial m}$

$eval(\mathbf{(13)},\ \{Td = 0.278600131569556,\ m = 0.419981718318083\})$

$$1.\ 10^{-11} \tag{20}$$

Evaluating $H1 = \dfrac{\partial^2 R}{\partial^2 Td},$

$eval(\mathbf{(11)},\ \{Td = 0.278600131569556,\ m = 0.419981718318083\})$

$$-0.6823024853 \tag{21}$$

Evaluating $H2 = \dfrac{\partial^2 R}{\partial^2 Td} \cdot \dfrac{\partial^2 R}{\partial^2 m} - \left(\dfrac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\mathbf{(18)},\ \{Td = 0.278600131569556,\ m = 0.419981718318083\})$

$$0.3533654970 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc}.\left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td\right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.213504886137366012,\ [Td = 0.296443386322520,\ m = 0.672758218170760]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5}\right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td\right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \qquad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^3}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m) \qquad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \qquad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right) \qquad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$
\begin{aligned}
&\left(\frac{0.1\left(Td \cdot \left(\frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}} - \frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} - \frac{0.02\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5\right)^{2}} \right. \\
&\quad + \frac{0.02\left(Td \cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5\right)^{3}} \\
&\quad \left. - \frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td \cdot \left(\frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}} - \frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5\right)^{2}} \right) \cdot \left(-m^{10.} + 1 - Td\right. \\
&\quad \left. - (0.9 - 0.9\, m) \cdot Td\right) + 2 \left(\frac{0.1\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right. \\
&\quad \left. - \frac{0.1\left(Td \cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5\right)^{2}} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td\right) \\
&\quad - 90. \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot m^{8.}
\end{aligned} \tag{14}
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td\right)
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td\right) \tag{15}
$$

$$
\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}
$$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \tag{16}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)}-\frac{0.2\,e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{3}}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}(0.9+m)^{2}}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot(-1.9+0.9\,m)+\frac{0.9\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5} \tag{17}$$

(11)·(14)−(17)^2 $\quad \frac{{}^{2}R}{\partial^{2}Td}\cdot\frac{\partial^{2}R}{\partial^{2}m}-\left(\frac{\partial^{2}R}{\partial m\cdot\partial Td}\right)^{2}$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(e^{-\frac{0.1}{0.9+m}}\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{3}}\Bigg)\cdot\big(-m^{10.}+1-Td-(0.9 \tag{18}$$

$$-0.9\,m)\cdot Td\big)+2\Bigg(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\Bigg)\cdot(-1.9$$

$$+0.9\,m)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}}-\frac{2\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}$$

$$-\frac{0.02\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}+\frac{0.02\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{3}}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{4}}-\frac{2\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{3}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\Bigg)\cdot\big(-m^{10.}+1-Td$$

$$-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\Bigg)\cdot\big(-10.\,m^{9.}+0.9\,Td\big)$$

$$-90.\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot m^{8.}\Bigg)-\Bigg(\Bigg(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)}$$

$$- \frac{0.2\, e^{-\frac{0.1}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2}$$

$$+ \frac{0.2 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^3}$$

$$- \left. \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2 (0.9+m)^2} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} - \frac{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) e^{-\frac{0.1}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td \right)$$

$$+ \left(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2} \right) \cdot \Big($$

$$\left. -1.9 + 0.9\, m \Big) + \frac{0.9 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right)^2$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)}, \{Td = 0.296443386322520, m = 0.672758218170760\})$

$$-1.200\ 10^{-7} \tag{19}$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial m}$

$eval(\mathbf{(13)}, \{Td = 0.296443386322520, m = 0.672758218170760\})$

$$-1.1656\ 10^{-8} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)}, \{Td = 0.296443386322520, m = 0.672758218170760\})$

$$-3.122075996 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td} \right)^2$

$eval(\textbf{(18)}, \{Td = 0.296443386322520, m = 0.672758218170760\})$

$$4.145972731 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.1$

$$\lambda := 0.1 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.9$

$$\mu := 0.9 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.151106454357680664, [Td = 0.323454366387852, m = 0.705671151303974]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$
\begin{aligned}
&\Bigg(\frac{0.9 \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}} + \frac{0.9\, e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} - \frac{1.62 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^{2}} \\
&\quad + \frac{1.62 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}} \right) \right)^{2}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^{3}} \\
&\quad - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(-\frac{2\, e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}} + \frac{0.9\, e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^{2}} \Bigg) \cdot \big(-m^{10.} + 1 - Td \\
&\quad - (0.9 - 0.9\, m) \cdot Td \big) + 2 \Bigg(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \\
&\quad - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^{2}} \Bigg) \cdot \big(-10.\, m^{9.} + 0.9\, Td \big) \\
&\quad - 90. \left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot m^{8.}
\end{aligned}
\qquad \textbf{(14)}
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \big(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \big)
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \big(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \big) \qquad \textbf{(15)}
$$

$$
\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}
$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)}-\frac{1.8\,e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2(0.9+m)^2}\right)\cdot\left(-m^{10.}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m)+\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5} \quad \textbf{(17)}$$

(11)·(14)−(17)^2 $\frac{{}^2R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\left(\left(\left(-\frac{2\left(\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}+\frac{2\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{3}}\right)\cdot\left(-m^{10.}+1-Td-(0.9\right.\right.\right. \qquad \textbf{(18)}$$

$$\left.-0.9\,m)\cdot Td\right)+2\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\mathrm{e}^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-1.9\right.$$

$$\left.\left.+0.9\,m\right)\right)\left(\left(\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5}\right.\right.\right.$$

$$-\frac{1.62\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}+\frac{1.62\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{3}}$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{10.}+1-Td\right.$$

$$\left.-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5}\right.$$

$$\left.-\frac{0.9\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-10.\,m^{9.}+0.9\,Td\right)$$

$$\left.-90.\left(\frac{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot m^{8.}\right)-\left(\left(\frac{0.9\,\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot\mathrm{e}^{-\frac{0.9}{0.9+m}}+0.5\right)}\right.\right.$$

$$- \frac{1.8\, e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2}$$

$$+ \frac{1.8 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^3}$$

$$\left. - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2 (0.9+m)^2} \right) \cdot \left(-m^{10.} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} - \frac{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) e^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2} \right) \cdot \left(-10.\, m^{9.} + 0.9\, Td \right)$$

$$+ \left(\frac{0.9 \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} - \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2} \right) \cdot \Bigg($$

$$\left. -1.9 + 0.9\, m) + \frac{0.9 \left(Td \cdot e^{-\frac{0.9}{0.9+m}} \right)}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right)^2$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)}, \{Td = 0.323454366387852, m = 0.705671151303974\})$

$$0. \qquad \mathbf{(19)}$$

Evaluating $\nabla R\,(Td^*, m^*)$, $\frac{\partial R}{\partial m}$

$eval(\mathbf{(13)}, \{Td = 0.323454366387852, m = 0.705671151303974\})$

$$1.1\ 10^{-10} \qquad \mathbf{(20)}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td}$,

$eval(\mathbf{(11)}, \{Td = 0.323454366387852, m = 0.705671151303974\})$

$$-2.109495914 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval($**(18)**$, \{Td = 0.323454366387852, m = 0.705671151303974\})$

$$3.263109162 \tag{22}$$

$M := 1$

$$M := 1 \quad (1)$$

$Qc := 0.5$

$$Qc := 0.5 \quad (2)$$

$\lambda := 0.9$

$$\lambda := 0.9 \quad (3)$$

$\alpha := 0.9$

$$\alpha := 0.9 \quad (4)$$

$\mu := 0.1$

$$\mu := 0.1 \quad (5)$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha + m}}}{Td.\,e^{-\frac{\mu}{\alpha + m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1 - m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.1}{0.9 + m}}}{Td \cdot e^{-\frac{0.1}{0.9 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (6)$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9 + m}}}{Td \cdot e^{-\frac{0.1}{0.9 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (7)$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.164415361875960231, \; [Td = 0.219890759601015, \; m = 1.45049382370336\; 10^{-6}]] \quad (8)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9 + m}}}{Td \cdot e^{-\frac{0.1}{0.9 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.1}{0.9 + m}}}{Td \cdot e^{-\frac{0.1}{0.9 + m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \quad (9)$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(e^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^3}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$
\begin{aligned}
&\Bigg(\frac{0.1 \left(Td \cdot \left(\frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4} - \frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} - \frac{0.02 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2} \\
&\quad + \frac{0.02 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)^2}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^3} \\
&\quad - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{0.1\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4} - \frac{2\, e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2} \Bigg) \cdot \left(-m^{1.111111111} + 1 \right. \\
&\quad \left. - Td - (0.9 - 0.9\, m) \cdot Td \right) + 2 \Bigg(\frac{0.1 \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \\
&\quad - \frac{0.1 \left(Td \cdot e^{-\frac{0.1}{0.9+m}} \right) \left(Td \cdot \left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5 \right)^2} \Bigg) \cdot \left(-1.111111111\, m^{0.111111111} + 0.9\, Td \right) \\
&\quad - 0.1234567900 \left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot \frac{1}{m^{0.888888889}}
\end{aligned}
\tag{14}
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)
$$

$$
\left(\frac{Td \cdot e^{-\frac{0.1}{0.9+m}}}{Td \cdot e^{-\frac{0.1}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.111111111} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right) \tag{15}
$$

$$
\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}
$$

$$\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \qquad \textbf{(16)}$$

$$\frac{\partial^{2}R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)}-\frac{0.2\,e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{3}}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}(0.9+m)^{2}}\right)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-1.111111111\,m^{0.111111111}+0.9\,Td\right)+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^{2}}\right)\cdot(-1.9+0.9\,m) \qquad \textbf{(17)}$$

$$
+\frac{0.9\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5}
$$

(11)·(14)−(17)^2 $\frac{{}^2R}{\partial^2 Td}\,{}_2\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$
\Bigg(\Bigg(-\frac{2\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(\mathrm{e}^{-\frac{0.1}{0.9+m}}\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^3}\Bigg)\cdot\big(-m^{1.111111111}+1-Td \qquad \textbf{(18)}
$$

$$
-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\mathrm{e}^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-1.9
$$

$$
+0.9\,m\big)\Bigg)\Bigg(\Bigg(\frac{0.1\left(Td\cdot\left(\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4}-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5}
$$

$$
-\frac{0.02\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^2}+\frac{0.02\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^3}
$$

$$
-\frac{0.1\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{0.1\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^4}-\frac{2\,\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^3}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-m^{1.111111111}+1
$$

$$
-Td-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{0.1\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5}
$$

$$
-\frac{0.1\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{\mathrm{e}^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot \mathrm{e}^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-1.111111111\,m^{0.111111111}+0.9\,Td\big)
$$

$$-0.1234567900\left(\frac{Td\cdot e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right)\cdot\frac{1}{m^{0.888888889}}\Bigg)$$

$$-\Bigg(\Bigg(\frac{0.1\,e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)}-\frac{0.2\,e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}$$

$$+\frac{0.2\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^3}$$

$$-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2(0.9+m)^2}\Bigg)\cdot\left(-m^{1.111111111}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$+\left(\frac{e^{-\frac{0.1}{0.9+m}}}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)e^{-\frac{0.1}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-1.111111111\,m^{0.111111111}\right.$$

$$\left.+0.9\,Td\right)+\left(\frac{0.1\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\right.$$

$$\left.-\frac{0.1\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.1}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m)$$

$$+\frac{0.9\left(Td\cdot e^{-\frac{0.1}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.1}{0.9+m}}+0.5}\Bigg)^2$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)},\ \{Td = 0.219890759601015,\ m = 1.45049382370336\ 10^{-6}\})$

$$-2.\ 10^{-10} \tag{19}$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial m}$

$eval(\mathbf{(13)},\ \{Td = 0.219890759601015,\ m = 1.45049382370336\ 10^{-6}\})$

$$1.411\ 10^{-8} \tag{20}$$

Evaluating $H\,1 = \frac{\partial^2 R}{\partial^2 Td},$

$eval(\mathbf{(11)},\ \{Td = 0.219890759601015,\ m = 1.45049382370336\ 10^{-6}\})$

$$-4.880236875 \tag{21}$$

Evaluating $H\,2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\mathbf{(18)},\ \{Td = 0.219890759601015,\ m = 1.45049382370336\ 10^{-6}\})$

$$26338.21223 \tag{22}$$

Scenario 8- Chapter III-Section B-4

$M := 1$

$$M := 1 \tag{1}$$

$Qc := 0.5$

$$Qc := 0.5 \tag{2}$$

$\lambda := 0.85$

$$\lambda := 0.85 \tag{3}$$

$\alpha := 0.9$

$$\alpha := 0.9 \tag{4}$$

$\mu := 0.9$

$$\mu := 0.9 \tag{5}$$

$$R = \frac{M.Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}} + Qc} . \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha.(1-m).Td \right)$$

$$R = \left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.176470588} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \tag{6}$$

$\xrightarrow{\text{right hand side}}$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.176470588} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \tag{7}$$

$\xrightarrow{\text{optimization assistant}}$

$$[0.0824590703996496560, [Td = 0.239965213189667, m = 0.0412058618324981]] \tag{8}$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.176470588} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right)$$

$$\left(\frac{Td \cdot e^{-\frac{0.9}{0.9+m}}}{Td \cdot e^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \left(-m^{1.176470588} + 1 - Td - (0.9 - 0.9\,m) \cdot Td \right) \tag{9}$$

$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(10)}$$

$$\frac{\partial^2 R}{\partial^2 Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(11)}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right) \quad \textbf{(12)}$$

$$\frac{\partial R}{\partial m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-1.176470588\,m^{0.176470588}+0.9\,Td\right) \quad \textbf{(13)}$$

$$\frac{\partial^2 R}{\partial^2 m}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\begin{aligned}&\left(\frac{0.9\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{1.62\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right.\\&+\frac{1.62\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)^{2}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^{3}}\\&\left.-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{3}}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{4}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-m^{1.176470588}+1\right.\\&\left.-Td-(0.9-0.9\,m)\cdot Td\right)+2\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right.\\&\left.-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^{2}}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^{2}}\right)\cdot\left(-1.176470588\,m^{0.176470588}+0.9\,Td\right)\\&-0.2076124564\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\frac{1}{m^{0.823529412}}\end{aligned}\tag{14}$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)\tag{15}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(16)}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)}-\frac{1.8\,e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{1.8\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2(0.9+m)^2}\right)\cdot\left(-m^{1.176470588}+1-Td-(0.9-0.9\,m)\cdot Td\right)+\left(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot\left(-1.176470588\,m^{0.176470588}+0.9\,Td\right)+\left(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\right)\cdot(-1.9+0.9\,m) \quad \textbf{(17)}$$

$$+\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}$$

(11)·(14)−(17)² $\frac{\partial^2 R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$\Bigg(\Bigg(-\frac{2\left(e^{-\frac{0.9}{0.9+m}}\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{2\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(e^{-\frac{0.9}{0.9+m}}\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}\Bigg)\cdot\big(-m^{1.176470588}+1-Td \qquad \textbf{(18)}$$

$$-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{e^{-\frac{0.9}{0.9+m}}}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}-\frac{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)e^{-\frac{0.9}{0.9+m}}}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-1.9$$

$$+0.9\,m\big)\Bigg)\Bigg(\Bigg(\frac{0.9\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}$$

$$-\frac{1.62\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}+\frac{1.62\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)^2}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^3}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(-\frac{2\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^3}+\frac{0.9\,e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^4}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-m^{1.176470588}+1$$

$$-Td-(0.9-0.9\,m)\cdot Td\big)+2\Bigg(\frac{0.9\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5}$$

$$-\frac{0.9\left(Td\cdot e^{-\frac{0.9}{0.9+m}}\right)\left(Td\cdot\left(\frac{e^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{0.9}{0.9+m}}+0.5\right)^2}\Bigg)\cdot\big(-1.176470588\,m^{0.176470588}+0.9\,Td\big)$$

$$\left. - 0.2076124564 \left(\frac{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5} \right) \cdot \frac{1}{m^{0.823529412}} \right)$$

$$- \left(\left(\frac{0.9\, \mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)} - \frac{1.8\, \mathrm{e}^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2} \right. \right.$$

$$+ \frac{1.8 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} \right) \mathrm{e}^{-\frac{0.9}{0.9+m}} \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)^3}$$

$$\left. - \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} \right) \mathrm{e}^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2 (0.9+m)^2} \right) \cdot \left(-m^{1.176470588} + 1 - Td - (0.9 - 0.9\, m) \cdot Td \right)$$

$$+ \left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5} - \frac{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} \right) \mathrm{e}^{-\frac{0.9}{0.9+m}}}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2} \right) \cdot \left(-1.176470588\, m^{0.176470588} \right.$$

$$\left. + 0.9\, Td \right) + \left(\frac{0.9 \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5} \right.$$

$$\left. - \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} \right) \left(Td \cdot \left(\frac{\mathrm{e}^{-\frac{0.9}{0.9+m}}}{(0.9+m)^2} \right) \right)}{\left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5 \right)^2} \right) \cdot (-1.9 + 0.9\, m)$$

$$\left. + \frac{0.9 \left(Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} \right)}{Td \cdot \mathrm{e}^{-\frac{0.9}{0.9+m}} + 0.5} \right)^2$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial Td}$

$eval(\mathbf{(10)},\ \{Td = 0.239965213189667,\ m = 0.041205861832498\})$

$$1.12\ 10^{-8} \tag{19}$$

Evaluating $\nabla R\,(Td^{*}, m^{*}),\ \frac{\partial R}{\partial m}$

$eval(\mathbf{(13)},\ \{Td = 0.239965213189667,\ m = 0.041205861832498\})$

$$-5.94\ 10^{-9} \tag{20}$$

Evaluating $H1 = \frac{\partial^2 R}{\partial^2 Td},$

$eval(\mathbf{(11)},\ \{Td = 0.239965213189667,\ m = 0.041205861832498\})$

$$-2.417977343 \tag{21}$$

Evaluating $H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2$

$eval(\mathbf{(18)},\ \{Td = 0.239965213189667,\ m = 0.041205861832498\})$

$$1.593954798 \tag{22}$$

In the following section, the gradient of the function and the determinants of the first and second order principal leading matrices are determined for each scenario provided in sections B-2 and B-4 of the Chapter III. Then these gradients and determinants are evaluated for each of the sixteen numerical solutions generated in sections B-2 and B-4 of the Chapter III. The results show that for all

scenarios: $\nabla R\left(Td^*, m^*\right) = \begin{bmatrix} \dfrac{\partial R}{\partial Td} \\ \dfrac{\partial R}{\partial m} \end{bmatrix} = 0$

and

$$H1 = \frac{\partial^2 R}{\partial^2 Td} < 0$$

and

$$H2 = \frac{\partial^2 R}{\partial^2 Td} \cdot \frac{\partial^2 R}{\partial^2 m} - \left(\frac{\partial^2 R}{\partial m \cdot \partial Td}\right)^2 > 0$$

Therefore all solutions generated by Maple are local maxima.

The general form of the objective function $R(T_d, m)$ gradient and the determinants of the first and second order leading principals of the Hessian matrix are presented in this section. Unlike the numerical applications provided in the previous section, the general forms of the Hessian matrix and in particular the determinant of the second order leading principal submatrix prove that it is really hard to make a statement on the optimality of the solution. As mentioned by Jensen and Bard (2003), it almost impossible to predict the convexity of a nonlinear objective function and the optimality of its stationary point by looking at the general form of the Hessian matrix and therefore the i^{th} order leading principal submatrices (Jensen & Bard, 2003).

$$\frac{Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}}+Qc}.\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha.(1-m).Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \quad \textbf{(1)}$$

$$\frac{\partial R}{\partial Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}-\frac{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}\right)e^{-\frac{\mu}{\alpha+m}}}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \quad \textbf{(2)}$$

$$+\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot(-1-\alpha\cdot(1-m))$$

$$\xrightarrow{\text{simplify symbolic}}$$

$$\left(\frac{e^{-\frac{\mu}{\alpha+m}}Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot(\quad \textbf{(3)}$$

$$-1-\alpha\cdot(1-m))$$

$$\frac{\partial^2 R}{\partial^2 Td} \xrightarrow{\text{differentiate w.r.t. Td}}$$

$$-2\left(\frac{\left(e^{-\frac{\mu}{\alpha+m}}\right)^2 Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^3}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \quad \textbf{(4)}$$

$$+\left(\frac{e^{-\frac{\mu}{\alpha+m}}Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot(-1-\alpha\cdot(1-m))+\left(\frac{e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right.$$

$$\left.-\frac{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}\right)e^{-\frac{\mu}{\alpha+m}}}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot(-1-\alpha\cdot(1-m))$$

$$\xrightarrow{\text{simplify symbolic}}$$

$$-2\left(\frac{e^{-\frac{2\mu}{\alpha+m}}Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^3}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \tag{5}$$

$$+2\left(\frac{e^{-\frac{\mu}{\alpha+m}}Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot(-1-\alpha\cdot(1-m))$$

$$subs\left(\left[-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m).Td=Tm,\ e^{-\frac{\mu}{\alpha+m}}=\rho,\ e^{-\frac{2\mu}{\alpha+m}}=\rho^2,\ Td\cdot e^{-\frac{\mu}{\alpha+m}}=Q\right],\mathbf{(5)}\right)$$

$$-2\left(\frac{\rho^2\,Qc}{(Q+Qc)^3}\right)\cdot Tm+2\left(\frac{\rho\,Qc}{(Q+Qc)^2}\right)\cdot(-1-\alpha\cdot(1-m)) \tag{6}$$

$$subs\left(\left[\left(\frac{\rho^2\,Qc}{(Q+Qc)^3}\right)=\frac{\rho^2\,Qc}{Qt^3},\ \left(\frac{\rho\,Qc}{(Q+Qc)^2}\right)=\frac{\rho\,Qc}{Qt^2}\right],\mathbf{(6)}\right)$$

$$-2\left(\frac{\rho^2\,Qc}{Qt^3}\right)\cdot Tm+2\left(\frac{\rho\,Qc}{Qt^2}\right)\cdot(-1-\alpha\cdot(1-m)) \tag{7}$$

substituting for $\frac{\partial R}{\partial Td}=0$

$$subs\left(\left[-1-\alpha\cdot(1-m)=-\frac{\rho\,Qc\cdot Tm}{Qt\cdot Q}\right],\mathbf{(7)}\right)$$

$$-2\left(\frac{\rho^2\,Qc}{Qt^3}\right)\cdot Tm+2\left(\frac{\rho\,Qc}{Qt^2}\right)\cdot\left(-\frac{\rho\,Qc\,Tm}{Qt\,Q}\right) \tag{8}$$

$$\frac{\partial^2 R}{\partial^2 Td}=-2\left(\frac{\rho^2\,Qc}{Qt^2\cdot Q}.Tm\right)$$

$$-2\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm \tag{9}$$

Final form for $\frac{\partial^2 R}{\partial^2 Td}$

$$\frac{Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}}+Qc}.\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha.(1-m).Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \tag{10}$$

$$\frac{\partial R}{\partial m} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right)}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} - \frac{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} \right) \left(Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^2} \right) \cdot \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha \cdot (1 - m) \cdot Td \right) + \left(\frac{Td \cdot e^{-\frac{\mu}{\alpha+m}}}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} \right) \cdot \left(-\frac{m^{\frac{1}{\lambda}}}{\lambda\, m} + \alpha \cdot Td \right) \quad \textbf{(11)}$$

$$\xrightarrow{\text{simplify symbolic}}$$

$$\left(\frac{\left(Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right) Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^2} \right) \cdot \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha \cdot (1 - m) \cdot Td \right) + \left(\frac{Td \cdot e^{-\frac{\mu}{\alpha+m}}}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} \right) \cdot \left(\frac{-m^{\frac{1-\lambda}{\lambda}} + (\alpha \cdot Td)\, \lambda}{\lambda} \right) \quad \textbf{(12)}$$

$$\frac{\partial^2 R}{\partial^2 m} \xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{\left(Td \cdot \left(-\frac{2\,\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^3} + \frac{\mu^2\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^4} \right) \right) Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^2} - \frac{2 \left(Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right)^2 Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^3} \right) \cdot \left(-m^{\frac{1}{\lambda}} + 1 - Td - \alpha \cdot (1 - m) \cdot Td \right) + \left(\frac{\left(Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right) Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^2} \right) \cdot \left(-\frac{m^{\frac{1}{\lambda}}}{\lambda\, m} + \alpha \cdot Td \right) + \left(\frac{Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right)}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} - \frac{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} \right) \left(Td \cdot \left(\frac{\mu\, e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc \right)^2} \right) \quad \textbf{(13)}$$

$$\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)-\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot\left(\frac{m^{\frac{1-\lambda}{\lambda}}\,(1-\lambda)}{\lambda^2\,m}\right)$$

simplify symbolic →

$$-\left(\frac{1}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^3}\left(Qc\left(\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)\left(Td\cdot\left(\frac{\mu\,e^{-\frac{\mu}{\alpha+m}}\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}\right)\right)\right.\right.\right.$$

$$\left.\left.\left.+2\left(Td\cdot\left(\frac{\mu\,e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2}\right)\right)^2\right)\right)\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right)$$

$$+2\left(\frac{\left(Td\cdot\left(\frac{\mu\,e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2}\right)\right)Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)+\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)$$

$$\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right) \qquad \textbf{(14)}$$

$$subs\left(\left[-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m).Td=Tm,\ e^{-\frac{\mu}{\alpha+m}}=\rho,\ e^{-\frac{2\mu}{\alpha+m}}=\rho^2,\ Td\cdot e^{-\frac{\mu}{\alpha+m}}=Q\right],\textbf{(14)}\right)$$

$$-\left(\frac{Qc\left((Q+Qc)\left(Td\cdot\left(\frac{\mu\,\rho\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}\right)\right)+2\left(Td\cdot\left(\frac{\mu\,\rho}{(\alpha+m)^2}\right)\right)^2\right)}{(Q+Qc)^3}\right)\cdot Tm \qquad \textbf{(15)}$$

$$+2\left(\frac{\left(Td\cdot\left(\frac{\mu\,\rho}{(\alpha+m)^2}\right)\right)Qc}{(Q+Qc)^2}\right)\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)+\left(\frac{Q}{Q+Qc}\right)$$

$$\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right)$$

$$subs\left(\left[Td\cdot\left(\frac{\mu\,\rho}{(\alpha+m)^2}\right)=\frac{\mu\cdot Q}{(\alpha+m)^2},\ (Q+Qc)=Qt,\ (Q+Qc)^2=Qt^2,\ (Q+Qc)^3=Qt^3\right],\textbf{(15)}\right)$$

$$-\left(\frac{Qc\left(Qt\left(Td\cdot\left(\frac{\mu\,\rho\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}\right)\right)+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm+2\left(\frac{(\mu\cdot Q)\;Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right) \qquad \mathbf{(16)}$$

$$subs\left(\left[\left(Td\cdot\left(\frac{\mu\,\rho\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}\right)\right)=Q\cdot\mu\cdot\frac{(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}\right],\mathbf{(16)}\right)$$

$$-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm+2\left(\frac{(\mu\cdot Q)\;Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right) \qquad \mathbf{(17)}$$

Substituting for $\frac{\partial R}{\partial m}$=0

$$subs\left(\left[\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}=-\frac{\mu\cdot Qc}{(\alpha+m)^2\,Qt}\right)\right],\mathbf{(17)}\right)$$

$$-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm+2\left(\frac{(\mu\cdot Q)\;Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot\left(-\frac{\mu\,Qc}{(\alpha+m)^2\,Qt}\right)+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right) \qquad \mathbf{(18)}$$

Final form for $\frac{\partial^2 R}{\partial^2 m}$

$$-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right).Tm-2\cdot\frac{\mu^2.Q\cdot Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right)$$

$$-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm-\frac{2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}(-1+\lambda)}{\lambda^2}\right) \tag{19}$$

$$\frac{Td.\,e^{-\frac{\mu}{\alpha+m}}}{Td.\,e^{-\frac{\mu}{\alpha+m}}+Qc}.\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha.(1-m).Td\right)$$

$$\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right) \tag{20}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{differentiate w.r.t. Td}}$$

$$\left(\frac{e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}-\frac{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}\right)e^{-\frac{\mu}{\alpha+m}}}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot(-1-\alpha\cdot(1-m)) \tag{21}$$

$$\frac{\partial R}{\partial Td}\xrightarrow{\text{simplify symbolic}}$$

$$\left(\frac{e^{-\frac{\mu}{\alpha+m}}\,Qc}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1-Td-\alpha\cdot(1-m)\cdot Td\right)+\left(\frac{Td\cdot e^{-\frac{\mu}{\alpha+m}}}{Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc}\right)\cdot(-1-\alpha\cdot(1-m)) \tag{22}$$

$$\frac{\partial^2 R}{\partial m\cdot\partial Td}\xrightarrow{\text{differentiate w.r.t. m}}$$

$$\left(\frac{\mu\,e^{-\frac{\mu}{\alpha+m}}\,Qc}{(\alpha+m)^2\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^2}-\frac{2\,e^{-\frac{\mu}{\alpha+m}}\,Qc\left(Td\cdot\left(\frac{\mu\,e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2}\right)\right)}{\left(Td\cdot e^{-\frac{\mu}{\alpha+m}}+Qc\right)^3}\right)\cdot\left(-m^{\frac{1}{\lambda}}+1\right. \tag{23}$$

$$- Td - \alpha \cdot (1-m) \cdot Td\Big) + \left(\frac{e^{-\frac{\mu}{\alpha+m}} Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)^2} \right) \cdot \left(- \frac{m^{\frac{1}{\lambda}}}{\lambda m} + \alpha \cdot Td \right)$$

$$+ \left(\frac{Td \cdot \left(\frac{\mu e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right)}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} - \frac{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}}\right) \left(Td \cdot \left(\frac{\mu e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right)}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)^2} \right) \cdot (-1 - \alpha \cdot (1$$

$$- m)) + \left(\frac{Td \cdot e^{-\frac{\mu}{\alpha+m}}}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} \right) \cdot \alpha$$

$$\frac{\partial^2 R}{\partial m \cdot \partial Td} \xrightarrow{\text{simplify symbolic}}$$

$$-2 \left(\frac{Qc\, e^{-\frac{\mu}{\alpha+m}} \left((\alpha+m)^2 \left(Td \cdot \left(\frac{\mu e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right) - \frac{\mu \left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)}{2} \right)}{(\alpha+m)^2 \left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)^3} \right) \cdot \Big(-m^{\frac{1}{\lambda}} \tag{24}$$

$$+ 1 - Td - \alpha \cdot (1-m) \cdot Td\Big) + \left(\frac{e^{-\frac{\mu}{\alpha+m}} Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)^2} \right) \cdot \left(\frac{-m^{\frac{1-\lambda}{\lambda}} + (\alpha \cdot Td)\, \lambda}{\lambda} \right)$$

$$+ \left(\frac{\left(Td \cdot \left(\frac{\mu e^{-\frac{\mu}{\alpha+m}}}{(\alpha+m)^2} \right) \right) Qc}{\left(Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc\right)^2} \right) \cdot (-1 - \alpha \cdot (1-m)) + \left(\frac{Td \cdot e^{-\frac{\mu}{\alpha+m}}}{Td \cdot e^{-\frac{\mu}{\alpha+m}} + Qc} \right) \cdot \alpha$$

$$subs\left(\left[-m^{\frac{1}{\lambda}} + 1 - Td - \alpha \cdot (1-m).Td = Tm,\ e^{-\frac{\mu}{\alpha+m}} = \rho,\ e^{-\frac{2\mu}{\alpha+m}} = \rho^2,\ Td \cdot e^{-\frac{\mu}{\alpha+m}} = Q \right], \textbf{(24)} \right)$$

$$-2 \left(\frac{Qc\, \rho \left((\alpha+m)^2 \left(Td \cdot \left(\frac{\mu \rho}{(\alpha+m)^2} \right) \right) - \frac{\mu (Q+Qc)}{2} \right)}{(\alpha+m)^2 (Q+Qc)^3} \right) \cdot Tm + \left(\frac{\rho\, Qc}{(Q+Qc)^2} \right) \tag{25}$$

$$\cdot \left(\frac{-m^{\frac{1-\lambda}{\lambda}} + (\alpha \cdot Td)\, \lambda}{\lambda} \right) + \left(\frac{\left(Td \cdot \left(\frac{\mu \rho}{(\alpha+m)^2} \right) \right) Qc}{(Q+Qc)^2} \right) \cdot (-1 - \alpha \cdot (1-m))$$

$$+\left(\frac{Q}{Q+Qc}\right)\cdot\alpha$$

$$subs\left(\left[Td\cdot\left(\frac{\mu\,\rho}{(\alpha+m)^2}\right)=\frac{\mu\cdot Q}{(\alpha+m)^2},\ (Q+Qc)=Qt,\ (Q+Qc)^2=Qt^2,\ (Q+Qc)^3=Qt^3\right],\ \mathbf{(25)}\right)$$

$$-2\left(\frac{Qc\,\rho\left(\mu\cdot Q-\frac{\mu\,Qt}{2}\right)}{(\alpha+m)^2\,Qt^3}\right)\cdot Tm+\left(\frac{\rho\,Qc}{Qt^2}\right)\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right) \qquad \mathbf{(26)}$$

$$+\left(\frac{(\mu\cdot Q)\,Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot(-1-\alpha\cdot(1-m))+\left(\frac{Q}{Qt}\right)\cdot\alpha$$

$$subs\left(\left[\left(\frac{\rho\,Qc}{Qt^2}\right)=\left(\frac{Q}{Qt\cdot Tm}\right)\cdot(1+\alpha.(1-m))\right],\ \mathbf{(26)}\right)$$

$$-2\left(\frac{Qc\,\rho\left(\mu\cdot Q-\frac{\mu\,Qt}{2}\right)}{(\alpha+m)^2\,Qt^3}\right)\cdot Tm+\left(\frac{Q}{Qt\cdot Tm}\right)\cdot(1+\alpha\cdot(1-m)) \qquad \mathbf{(27)}$$

$$\cdot\left(\frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}\right)+\left(\frac{(\mu\cdot Q)\,Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot(-1-\alpha\cdot(1-m))+\left(\frac{Q}{Qt}\right)\cdot\alpha$$

$$subs\left(\left[-1-\alpha\cdot(1-m)=-\frac{\rho\,Qc\cdot Tm}{Qt\cdot Q},\ \frac{-m^{\frac{1-\lambda}{\lambda}}+(\alpha\cdot Td)\,\lambda}{\lambda}=-\frac{\mu\cdot Qc}{(\alpha+m)^2\,Qt},\ 1+\alpha.(1-m)=\frac{\rho\,Qc\cdot Tm}{Qt\cdot Q}\right],\ \mathbf{(27)}\right)$$

$$-2\left(\frac{Qc\,\rho\left(\mu\cdot Q-\frac{\mu\,Qt}{2}\right)}{(\alpha+m)^2\,Qt^3}\right)\cdot Tm+\left(\frac{Q}{Qt\cdot Tm}\right)\cdot\left(\frac{\rho\,Qc\,Tm}{Qt\,Q}\right)\cdot\left(-\frac{\mu\,Qc}{(\alpha+m)^2\,Qt}\right) \qquad \mathbf{(28)}$$

$$+\left(\frac{(\mu\cdot Q)\,Qc}{(\alpha+m)^2\,Qt^2}\right)\cdot\left(-\frac{\rho\,Qc\,Tm}{Qt\,Q}\right)+\left(\frac{Q}{Qt}\right)\cdot\alpha$$

$$-2\,\frac{Qc\,\rho\left(\mu\cdot Q-\frac{\mu\,Qt}{2}\right)}{(\alpha+m)^2\,Qt^3}.Tm+\left(-\frac{\mu\cdot\rho\,Qc^2}{(\alpha+m)^2\cdot Qt^3}\right)+\left(-\frac{\rho\ Tm\ \cdot\mu\cdot Qc^2}{(\alpha+m)^2\,Qt^3}\right)+\left(\frac{Q}{Qt}\right).\alpha$$

$$-2\left(\frac{Qc\,\rho\left(\mu\cdot Q-\frac{\mu\,Qt}{2}\right)}{(\alpha+m)^2\,Qt^3}\right)\cdot Tm-\frac{\mu\,\rho\,Qc^2}{(\alpha+m)^2\,Qt^3}-\frac{\rho\,Tm\,\mu\,Qc^2}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha \qquad \mathbf{(29)}$$

$$-\frac{Qc\,\rho\,\mu\,[Tm\,Q+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha \qquad \textbf{(30)}$$

Final form for $\frac{\partial^2 R}{\partial m \cdot \partial Td}$

$$-2\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm \tag{1}$$

Final form for $\frac{\partial^2 R}{\partial^2 Td}$ ==> H1 which is always <0

$$-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm-\frac{2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right) \tag{2}$$

Final form for $\frac{\partial^2 R}{\partial^2 m}$

$$-\frac{Qc\,\rho\,\mu\,[Tm\,Q+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right).\alpha$$

$$-\frac{Qc\,\rho\,\mu\,[Tm\,Q+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha \tag{3}$$

Final form for $\frac{\partial^2 R}{\partial m\cdot\partial Td}$

(1)·(2)-(3)^2 : $\frac{\partial^2 R}{\partial^2 Td}\cdot\frac{\partial^2 R}{\partial^2 m}-\left(\frac{\partial^2 R}{\partial m\cdot\partial Td}\right)^2$

$$-2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(-\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm-\frac{2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(-1+\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,[Tm\,Q+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{4}$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right).Tm+\frac{2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,[Tm\,Q+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\left(\frac{Qc\left(\frac{Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)}{(\alpha+m)^4}+\frac{2\,(\mu\cdot Q)^2}{(\alpha+m)^4}\right)}{Qt^3}\right)\cdot Tm+\frac{2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,[Q\,Tm+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{5}$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\left(\frac{Tm\cdot Qc\,(Qt\cdot\,Q\cdot\,\mu\,(2\,\alpha+2\,m-\mu)+2\,\cdot\mu^2.Q^2)+2\,\cdot\mu^2.Q\cdot\,Qc^2}{(\alpha+m)^4\cdot Qt^3}\right)+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,[Q\,Tm+Qc]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\frac{Tm\,Qc\,(Qt\,Q\,\mu\,(2\,\alpha+2\,m-\mu)+2\,\mu^2\cdot Q^2)+2\,(\mu^2\cdot Q)\,Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\lambda}{\lambda}}\,(1-\lambda)}{\lambda^2}\right)\right)-\left(\tag{6}$$

$$-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\Bigg)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\frac{\left(Tm\;Qc\cdot Qt\;Q\cdot\mu\left(2\,\alpha+2\,m-\mu\right)+2\cdot Tm\cdot\;Qc\cdot\mu^2\cdot Q^2\right)+2\cdot\mu^2.Q\cdot\;Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\Bigg(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\Bigg)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\frac{Tm\;Qc\;Qt\;Q\;\mu\;\left(2\,\alpha+2\,m-\mu\right)+2\,\left(Tm\;Qc\;\mu^2\right)\cdot Q^2+2\,\left(\mu^2\cdot Q\right)\;Qc^2}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{7}$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\frac{Qc\;\mu\;Q\;\left(Tm\;\mu\;Q+\mu\;Qc(2-Tm)\;+2\;Tm\;Qt\;\alpha+2\;Tm\;Qt\;m\right)}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\Bigg(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\Bigg)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2\,Q}\right)\cdot Tm\right)\left(\frac{Qc\;\mu\;Q\;\left(Tm\;\mu\;Q+\mu\;Qc(2-Tm)\;+2\;Tm\;Qt\;\alpha+2\;Tm\;Qt\;m\right)}{(\alpha+m)^4\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{8}$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2}\right)\cdot Tm\right)\left(\frac{Qc\;\mu\cdot\left(Tm\;\mu\;Q+\mu\;Qc(2-Tm)\;+2\;Tm\;Qt\cdot(\alpha+m)\right)}{(\alpha+m)^4\,Qt^3}+\left(\frac{1}{Qt}\right)\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\Bigg(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\Bigg)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2}\right)\cdot Tm\right)\left(\frac{Qc\;\mu\;\left(Tm\;\mu\;Q+\mu\;Qc(2-Tm)\;+2\;Tm\;Qt\;(\alpha+m)\right)}{(\alpha+m)^4\,Qt^3}+\frac{1}{Qt}\cdot\left(\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{\lambda^2}\right)\right)-\left(-\frac{Qc\,\rho\,\mu\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{9}$$

substitue $\mu\;Qc$ by X

$$2\cdot\frac{\rho^2\,Qc}{Qt^2}.Tm\cdot\left(\frac{X\left(Tm\;\mu\;Q+X\cdot(2-Tm)\;+2\,Tm\,Qt\,(\alpha+m)\right)}{(\alpha+m)^4\,Qt^3}+\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{Qt\cdot\lambda^2}\right)-\left(-\frac{\rho\,X\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2$$

$$2\left(\left(\frac{\rho^2\,Qc}{Qt^2}\right)\cdot Tm\right)\left(\frac{X\left(Tm\;\mu\;Q+X\;(2-Tm)\;+2\;Tm\;Qt\;(\alpha+m)\right)}{(\alpha+m)^4\,Qt^3}+\frac{m^{\frac{1-2\,\lambda}{\lambda}}\;(1-\lambda)}{Qt\;\lambda^2}\right)-\left(-\frac{\rho\,X\,\left[Q\,Tm+Qc\right]}{(\alpha+m)^2\,Qt^3}+\left(\frac{Q}{Qt}\right)\cdot\alpha\right)^2 \tag{10}$$