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MIXED-INTEGER OPTIMIZATION FOR VOLT/VAR CONTROL IN DISTRIBUTION NETWORKS

by
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AN ABSTRACT OF THE THESIS OF

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As the load demand persists to increase globally with the growing capacity of distributed generation, distribution systems witness various voltage violation problems. In this context, designing strategies to ensure voltage profile enhancement is a significant challenge in the current operation of distribution networks. Volt/VAr control (VVC) is a major function that is employed by distribution management systems to manage the voltage magnitudes throughout the distribution system. VVC also serves other secondary objectives such as the minimization of the real power loss in distribution networks. To address its objectives, VVC periodically adjusts capacitor switches, transformer taps, and the reactive power set-points of distributed generation. This thesis builds on solving the VVC problem using mixed-integer conic programming (MICP) in radial and meshed networks. To speed up computations and alleviate voltage violations in meshed networks, the thesis proposes solving the VVC problem using a discrete coordinate-descent algorithm, starting from a solution to the continuous relaxation of the VVC mixed-integer conic program. The optimality of such an approach is investigated by evaluating the gap relative to the MICP objective function value. Numerical results are reported on radial and meshed test distribution networks with up to 3146 nodes. The obtained results demonstrate the superior performance of discrete-coordinate descent (DCD), when initialized by solving a continuous relaxation of the MICP, against the DCD algorithm with the classical initialization from the current operating point. Although the DCD is a local search method, the proposed approach yields an acceptable VVC solution with improved computational performance in various distribution networks.

Keywords—Centralized control, distributed power generation, load flow control, reactive power control, voltage control.

CONTENTS

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CHAPTER I

INTRODUCTION

As the world population increases, the need for the optimal exploitation of the existing power systems becomes more apparent. In this context, numerous power grids have been transformed into smart grids after being subjected to fundamental improvement [1].

Under the influence of unprecedentedly increasing demand, the power distribution systems usually suffer from unpermitted voltage reduction at distant nodes [2]; in addition, the increased installed capacity of distributed generation and its intermittent nature contribute to voltage fluctuations. Therefore, efficient voltage control is studied as a major practical task which distribution networks should perform to ensure voltage profile enhancement [3]. Volt/VAr control (VVC) is a major function that is employed by distribution management systems to manage the voltage magnitudes and the reactive power flow throughout the distribution system [4, 5, 6]. Maintaining the voltage magnitudes within acceptable limits is a primary task of VVC, while power loss minimization is a secondary objective. Still, Volt/VAr optimization serves multiple operation modes in the advanced distribution management systems [6]. For instance, Volt/VAr optimization evaluates, during the operation planning stage of a grid, the settings of the voltage regulators and the reactive compensation elements that are tailored to execute the monitoring and control mode of the modern distribution systems [6]. In addition, Volt/VAr optimization plays a fundamental role in the regular execution of the distribution system demand response.

VVC periodically adjusts the transformer tap positions, the capacitor switch statuses, and the reactive power output of distributed generation by solving an optimization problem; the optimization problem is formulated such that the real power loss is minimized and the operational security constraints are guaranteed [4, 7, 8, 9]. Hence, VVC performs the relevant dispatching of reactive control devices as function of time, so that the required objectives are fulfilled.

In this thesis, the On-load Tap Changing mechanism is a subject of study as it permits the adjustment of the tap positions of an energized transformer. In contrary, the No-load Tap Changing mechanism may still exist in some power systems, where the adjustment of the tap positions of a transformer is not possible unless the transformer is de-energized. With the diverse controllers involved in VVC, the necessity of a centralized solution becomes explicit to accomplish a well-coordinated management in response to load changes and network alterations [10]. Subsequently, the centralized VVC, in advisory and closed loop modes, is most often implemented as a complementary part of SCADA-based modern distribution management systems [10]. It should be pointed out that the only difference between the advisory and closed loop modes relies in the way the Volt/VAr control addresses the control actions. Specifically, the Volt/VAr advisory mode tackles the control actions manually, while the closed loop control mode of VVC entails automatic execution.

Technically, VVC algorithms exist in two forms: rule-based and network-based [4, 10]. While rule-based VVC is simple to employ, the network-based VVC yields better solutions, using the grid's mathematical model and the real-time measurements from the SCADA system [10]. Network-based VVC solves an optimization problem to compute the optimal vector of transformer tap positions, discrete shunt controls, and distributed generation reactive powers. According to [4], the network-based VVC is a nondeterministic polynomial (NP) problem with discrete and continuous variables that are coupled through nonlinear constraints [4]. To solve this problem, several techniques have been reported in the literature. In general, these methods are either nondeterministic or deterministic. In what follows, the use of both methods is briefly reviewed. Then, the contributions of the thesis are described.

A. VVC using nondeterministic techniques

According to some field experts, it is always sufficient to seek a near optimal solution to such VVC problems using nondeterministic methods, including heuristics and meta-heuristics. For example, the authors in [11] adopted a particle swarm optimization technique to solve a mixed-integer nonlinear optimization problem, whose multi-objective function not only involves power loss but also includes the cost resulting from the switching operations associated with the tap changers and switched capacitors. The VVC solution was achieved in [8] with a new method that consolidates the primal-dual interior point method and the genetic algorithm (GA). Based on the 69- and 119-node test systems, the authors in [8] argued that the proposed method was useful in achieving faster convergence, better accuracy, and a more reliable VVC solution than the conventional GA. Using an improved harmony search algorithm with adaptive parameter selection, the reactive power coordinated optimization problem was solved in [12] for distribution networks involving distributed generators and switched-type controllers. However, the performance of the given algorithm, as is the case with many bio-inspired techniques, is still questionable when dealing with large-scale networks in real-time applications.

B. VVC using deterministic techniques

Several deterministic approaches have been investigated to solve the network-based VVC problem in distribution networks. Together with the branch-and-bound approach, a trust region sequential quadratic programming method was used in [3] to iteratively compute an approximate solution to the coordinated VVC problem. To accommodate the integration of distributed generators, the authors in [13] proposed a decentralized reactive power optimization method for transmission and distribution networks based on the generalized Benders decomposition, after employing the second-order conic programming relaxation technique. To further address the reactive power optimization problem in active distribution networks, a sensitivity-based relaxation and decomposition technique was utilized in [14] to formulate a mixed-integer second-order cone optimization problem. The authors in [14] supported the significance of their methodology through improved computational capabilities. In [15], a mixed-integer second-order cone programing model was formulated to solve the optimal operation problem of radial distribution networks with energy storage; this formulation was obtained after performing variable substitution, convexification of constraints, and an equivalent disjunctive formulation for the model of tap-changing transformers. The authors described in [16] a distributed second-order cone programming solver for VVC; in contrary with other second-order cone programming formulations, this algorithm was derived based on the alternating direction method of multipliers and can be implemented in a distributed manner.

C. Thesis contributions

The need for a novel and more realistic model of the VVC problem persists to be an active research area [17]. Due to the wide application of mixed-integer conic programming in VVC, as reported in the literature, the significance of such a promising convexification technique is highlighted in the reduction of the problem complexity for distribution networks. Currently, the practical exploitation of convex optimization techniques gains substantial attention as seen in [18, 19] and the design of novel convex optimization algorithms represents an extensive research area as observed in [20, 21]. By taking advantage of the derived conic programming formulation of radial distribution load flow in [22], this thesis proceeds by examining the power of mixed-integer conic programming (MICP) for VVC, which was proposed in [4], in guaranteeing a lower bound of the optimal objective function value in multiple distribution networks. However, MICP is computationally intensive, which makes it not suited to real-time applications. This thesis proposes solving a continuous relaxation of the MICP, and then using this solution to initiate a search via discrete-coordinate descent (DCD). The continuous relaxation can be efficiently solved using state-of-the-art convex optimization software such as CPLEX [23], and the DCD method is used to restore feasibility (if lost due to rounding of the continuous MICP control variables) and to improve the objective function value.

Thesis Outline

The rest of the thesis is organized in 4 chapters:

Chapter 2: In this chapter, the conic programming and the mixed integer optimization for VVC are reviewed. The global optimality of the VVC solution, as provided by the mixed integer conic programming (MICP), is investigated in various radial networks; the reason behind the voltage violations that may arise in meshed networks is clarified.

Chapter 3: The discrete coordinate-descent algorithm (DCD) is illustrated as a viable Volt/VAr control algorithm. Its effectiveness is illustrated in the different network configurations.

Chapter 4: A cheap starting solution for DCD is proposed in this chapter. The more advantageous performance of DCD with such a starting point is described in terms of the achievable higher loss reduction, as compared to the DCD with the classical initialization method.

Chapter 5: The thesis concludes with a summary describing the obtained results in a comparative form between the four methods which are: the MICP formulation, the DCD initialized with the MICP solution, the DCD initialized with the solution of the MICP continuous relaxation, and the DCD initialized with the current operating point.

Table 1 shows the characteristics of the test networks, which are the modified Brazilian distribution system in addition to 1464- and 3146-node test systems; R describes a radial network while M represents a meshed one. The table shows for each network the load multiplying factor (LMF), the number of nodes n, the number of switched capacitors (CAP), the number of tap-changing transformers (TR), and the number of DG connections. The resolution per distribution generation VAr output is set to 10^{-3} pu [4] and the complete data files are available for download from [24].

Table 1. A summary of the test networks

Name	LMF	*n*	CAP	TR	DG
B_R	1	161	2	6	7
1k5_LR	0.33	1464	8	8	5
1k5_MR	0.5	1464	8	8	5
1k5_HR	1	1464	8	8	5
3k_LR	0.5	3146	13	15	10
3k_MR	0.8	3146	13	15	10
3k_HR	1	3146	13	15	10
B_M	1	160	2	6	7
1k5_LM	0.33	1464	8	8	5
1k5_MM	0.5	1464	8	8	5
1k5_HM	1	1464	8	8	5
3k_LM	0.5	3146	13	15	10
3k_MM	0.8	3146	13	15	10
3k_HM	1	3146	13	15	10

CHAPTER II

MIXED INTEGER CONIC PROGRAMMING (MICP)

This chapter implements the Volt/VAr control (VVC) optimization problem as a mixed integer conic program. The chapter starts by a brief description of conic programming and mixed integer optimization. Then, the VVC formulation is cast for radial networks as a mixed integer conic program; the mixed integer conic formulation is relevant for VVC in meshed networks, in the sense that it guarantees a tight lower bound on the VVC objective function value. The numerical results are discussed for radial and meshed test networks, and the chapter ends with the relevant conclusions.

A. Conic programming and mixed integer optimization

In the field of mathematical optimization, convex optimization gains great practical interest where the global minimum could be systematically attained using commercial software such as CPLEX [23]. According to [25], a convex set J is a set that contains the line segment between any two points in the set. Mathematically speaking,

$$\forall x_1, x_2 \in J, 0 \leq \alpha \leq 1 \Rightarrow \alpha x_1 + (1-\alpha) x_2 \in J \quad (1)$$

This is illustrated in Figure 1.

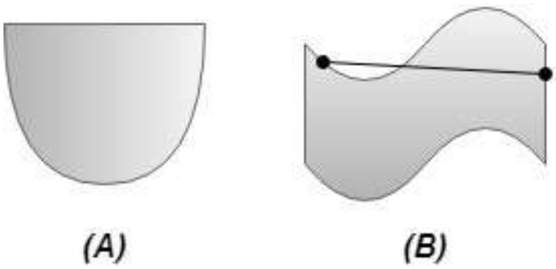

Figure 1. Convex set (A) and nonconvex set (B).

In accordance with [25], a function $f : J \rightarrow \mathbb{R}$ is called convex if

$$\forall x, y \in J, 0 \leq \alpha \leq 1 \Rightarrow f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha)f(y) \quad (2)$$

Consequently, a convex optimization problem seeks the minimization of a convex objective function over a convex set; the convexity of the problem guarantees that the local minimum must be a global minimum.

Although a wide range of distinct convex optimization problems exists, linear programming and the conic programming are still among the most common and easily implemented formulations. Still, the implementation of conic programming in VVC has always been technically attractive as previously illustrated in the literature review. Before defining the mathematical formulation of conic programming, it is required to explore the definition of the norm cone as shown in (3).

Norm cone: $\{(\boldsymbol{x}, m) \in \mathbb{R}^{n\times 1} \times \mathbb{R}^{+} / \|\boldsymbol{x}\| \leq m\}$ (3)

The norm cone is called second-order cone $\boldsymbol{C}$ whenever the used norm is the standard Euclidean norm $\|\boldsymbol{x}\|_2$.

Starting from the definition of the second-order cone, it is now possible to illustrate the mathematical model of any second-order cone program as shown in (4)-(6), where the problem parameters are: $\boldsymbol{l} \in \mathbb{R}^{1\times n}$, $\boldsymbol{E} \in \mathbb{R}^{t\times n}$, $\boldsymbol{F} \in \mathbb{R}^{t\times 1}$, and $\boldsymbol{x} \in \mathbb{R}^{n\times 1}$ is the vector of decision variables.

$$\min \ \boldsymbol{l}\boldsymbol{x} \quad (4)$$

subject to:

$$\boldsymbol{E}\boldsymbol{x} \geq \boldsymbol{F} \quad (5)$$

$$\boldsymbol{x} \in \boldsymbol{C} \quad (6)$$

If some of the elements of $\boldsymbol{x}$ are restricted to take integer values, then the optimization formulation becomes a mixed integer optimization problem.

B. Mixed integer conic optimization for VVC

In [22], the conic programming formulation of radial distribution load flow was proposed and tested on several radial networks. The major merit of such a modeling approach was manifested in [4], where MICP was presented as a suitable formulation for VVC in radial networks. In what follows, the description of such a VVC formulation for radial networks is provided based on [4].

In this section, n denotes the total number of nodes in a radial network; in addition, the number of capacitor bank settings and transformer tap positions (see Figure 2) are referred to as nc and nt respectively.

The following notation is employed for sets in the VVC formulation:

$N(i)$	Collection of nodes connected to node *i* by a branch.
LB	Collection of branches in a radial system.
SC	Collection of nodes where switchable shunt capacitors are located.
DG	Collection of nodes where distributed generators are connected.
TB	Collection of branches having tap-changing transformers.

As mathematical notations:

r^{min}, r^{max}	Lower and upper limits of the interval set in which a real quantity r varies.
b^{re}, b^{im}	Real and imaginary components of a complex quantity $\underline{b}$.
$\underline{b}^*$	Conjugate of the complex quantity $\underline{b}$.

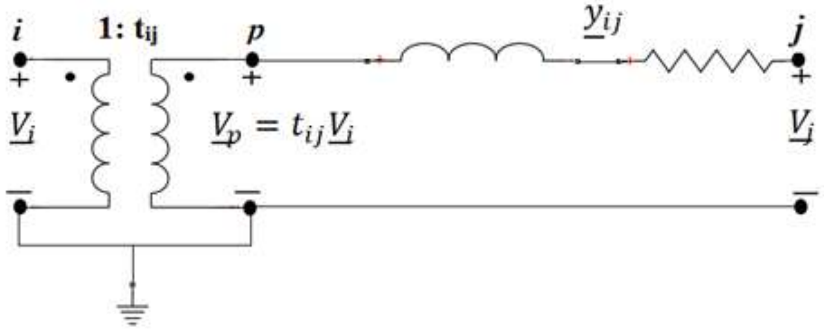

Figure 2. Tap-changing transformer.

From a power system modeling point of view, the following list of symbols is used to define the problem:

$\underline{V}_i$, $\underline{I}_i$	Voltage and injection current at node *i*.
V_{slack}	Voltage magnitude at the slack node.
θ_i	Voltage angle at node i.
$\underline{\boldsymbol{V}}, \underline{\boldsymbol{I}}$	Vectors of nodal voltages and injection currents.
$\underline{y}_{ij}$	Series admittance of branch *ij*.
$\underline{I}_{gi}$	Compensation current due to distributed generation at node *i*.
I_{ij}	Magnitude of current traversing branch *ij*.
p_{di}, q_{di}	Load real and reactive power at node *i*.
p_{gi} , q_{gi}	Supplied real and reactive power by distributed generation at node *i*.
q_{ci}	Produced reactive power by the switched capacitor at node *i*.
p_i , q_i	Injected real and reactive power at node *i*.
q_{0i}	Nominal reactive power produced (at 1 p.u voltage) by the switched capacitor at node *i*, with the possible set of values $\left\{q_{0i}^{(1)}, \dots \dots, q_{0i}^{(nc)}\right\}$.
t_{ij}	Transformer tap in branch *ij*, with the possible set of values $\left\{t_{ij}^{(1)}, \dots \dots, t_{ij}^{(nt)}\right\}$.

In the context of such a VVC formulation for radial networks, define $U_i = (V_i^2)/\sqrt{2}$ for every node i, and $C_{ij} = V_i\, V_j \cos(\theta_i - \theta_j)$ and $S_{ij} = V_i\, V_j \sin(\theta_i - \theta_j)$ for every branch ij.

The Volt/VAr optimization problem can be mathematically formulated so that the objective function is to minimize the network losses (7) while satisfying the power injection equality constraints (8)-(9), the conic constraints (10), the current magnitude constraints (12), the constraints (13-14) accounting for the tap-changing transformers and the discrete capacitor switch statuses together with the distributed generation VAr output (15)-(17). It is worth mentioning that the optimization problem maintains the voltage magnitudes (11) within their prescribed limits. The control variables in the VVC problem are the transformer tap positions, the capacitor switch statuses, and the reactive power outputs of the distributed generation.

$$\min \ \sum_{i=1}^{n} \sum_{j \in N(i)} [\sqrt{2}\, y_{ij}^{\text{re}} U_i - y_{ij}^{\text{re}} C_{ij}] \tag{7}$$

subject to:

$$\sum_{j \in N(i)} [\sqrt{2} y_{ij}^{re} U_i - y_{ij}^{re} C_{ij} - y_{ij}^{im} S_{ij}] = \ p_{gi} - p_{di} \tag{8}$$

$\forall i \in \{1, \dots \dots, n\}$

$$\sum_{j \in N(i)} \left[-\sqrt{2} y_{ij}^{\text{im}} U_i + y_{ij}^{\text{im}} C_{ij} - y_{ij}^{\text{re}} S_{ij}\right] = q_{gi} + q_{ci} - q_{di} \tag{9}$$

$\forall i \in \{1, \dots \dots, n\}$

$$2U_i U_j \geq C_{ij}^2 + S_{ij}^2 \ , \quad C_{ij} \geq 0 \ , \quad S_{ij} = -S_{ji} \qquad \forall ij \in \text{LB} \tag{10}$$

$$(V_i^{min})^2/\sqrt{2} \leq U_i \leq (V_i^{max})^2/\sqrt{2} \qquad \forall i \in \{1, \dots \dots, n\} \tag{11}$$

$$U_1 = (V_{slack})^2/\sqrt{2}$$

$$\sqrt{2} y_{ij}^2 \left(U_i + U_j\right) - 2 y_{ij}^2 C_{ij} \leq \left(I_{ij}^{max}\right)^{\mathbf{2}} \qquad \forall ij \in \text{LB} \tag{12}$$

$$U_p - \left(t_{ij}^{(k)}\right)^2 U_i \geq b_{ij}^{(k)} [\left(t_{ij}^{(1)}\right)^2 - \left(t_{ij}^{(k)}\right)^2] U_i^{max} \tag{13}$$

$$U_p - \left(t_{ij}^{(k)}\right)^2 U_i \leq b_{ij}^{(k)} [\left(t_{ij}^{(nt)}\right)^2 - \left(t_{ij}^{(k)}\right)^2] U_i^{max}$$

$\forall ij \in \text{TB}, \forall k \in \{1, \dots, nt\}$

$$\sum_{k=1}^{nt} b_{ij}^{(k)} = nt - 1 \ , \ b_{ij}^{(k)} \in \{0,1\} \tag{14}$$

$$q_{ci} - \sqrt{2} q_{0i}^{(k)} U_i \geq b_i^{(k)} \sqrt{2} (q_{0i}^{(1)} - q_{0i}^{(k)}) U_i^{max} \tag{15}$$

$$q_{ci} - \sqrt{2} q_{0i}^{(k)} U_i \leq b_i^{(k)} \sqrt{2} (q_{0i}^{(nc)} - q_{0i}^{(k)}) U_i^{max}$$

$\forall i \in \mathrm{SC}, \forall k \in \{1, \ldots., nc\}$

$$\sum_{k=1}^{nq} b_i^{(k)} = nc - 1 \ , \ b_i^{(k)} \in \{0,1\} \qquad (16)$$

$$q_{gi}^{min} \leq q_{gi} \leq q_{gi}^{max} \qquad \forall i \in \mathrm{DG} \qquad (17)$$

In accordance with (13), the transformer taps are discrete variables which obey the fact that $t_{ij}^{(k)} < t_{ij}^{(k+1)}$. The reactive power injection by the capacitor banks also changes in discrete steps (15) at 1 per-unit voltage, with $q_{0i}^{(k)} < q_{0i}^{(k+1)}$ [4]. If the voltage magnitude is different from 1 per-unit at the node to which the capacitor bank is connected, then the reactive power injection by the capacitor bank is proportional to the square of the nodal voltage magnitude.

The VVC problem, as modeled above for radial networks, falls in the category of mixed-integer conic optimization problems whose global minimum could be systematically searched for using commercial software such as CPLEX [23]. If the conic constraints (10) are binding at the optimal solution, then the solution to the mixed integer conic programming (MICP) formulation is feasible for the original VVC problem; this is likely to hold in practice and gives a globally optimal solution.

Even though the above formulation describes the VVC problem in radial networks only, the MICP formulation can also be employed to solve the VVC problem in meshed networks after relaxing the cycle constraints. Because cycle constraints are relaxed, the MICP solution will not in general be feasible for meshed networks. As a matter of fact, if the conic constraints are binding at the optimal solution, then the optimal objective value of the MICP solution serves as a lower bound for the original VVC problem. Thus, the MICP solution is not a global optimum for the meshed VVC problem unless such solution satisfies the cycle constraints with binding conic constraints.

In comparison with mixed-integer conic programming, mixed-integer linear programming solvers have a longer history of research and development; it is therefore useful to tightly approximate the conic constraints in (10) by a set of polyhedral constraints [4]. With this conversion, state-of-the-art optimization tools such as the CPLEX mixed-integer linear programming solver can be employed to solve the VVC problem. In what follows, the obtained results following the VVC MICP formulation are studied for the various radial and meshed test networks.

C. Results on radial and meshed networks using VVC MICP formulation

The execution of the different VVC algorithms in this thesis was carried out in Matlab on a Windows 10 virtual machine having Intel(R) Xeon(R) CPU E5-2695 v4 at 2.10GHz, 2100 MHz, 24 Cores, 24 Logical Processors and 42.0 GB of RAM. In this section, the CPLEX [23] solution of the mixed-integer conic VVC problem is shown, after the tight polyhedral approximation of the conic constraints. The default relative optimality gap tolerance used in CPLEX is 0.01%, and a time limit of 48 hours is employed.

In Table 2, the percentage loss reduction (LR (%)) is shown as obtained from MICP. As previously noted, the MICP generated a global solution to VVC for the radial test instances; this is validated by checking that the conic constraints are binding at the optimal solution. Regarding the meshed networks, the majority of the instances show slight voltage violations (which are marked with *) as expected. Because cycle constraints are not accounted for in MICP, a power flow based iterative procedure starting from the MICP control set points is generally required to recover a feasible solution in meshed networks; this feasible solution will satisfy the power flow equations that include cycle constraints, in addition to voltage magnitude limits. The feasibility recovery procedure is examined in the next chapter. However, in some meshed test instances (B_M and 3k5_LM), a single power flow execution shows that the MICP control settings do not give rise to voltage magnitude violations and therefore there is no need for feasibility recovery for these instances.

Table 2. The obtained percentage loss reduction and the corresponding execution time using the MICP formulation (instances having voltage violations are marked with *)

Name	LR (%)	Time (s)
B_R	11.92	170.7
1k5_LR	16.35	2389.7
1k5_MR	15.91	2161.8
1k5_HR	19.93	4195.2
3k_LR	12.5	63192.6
3k_MR	16.71	35338.5
3k_HR	20.05	61626.3
B_M	13.87	155.2
1k5_LM	14.42*	4704.2
1k5_MM	14.47*	4600.4
1k5_HM	18.16*	5969.8
3k_LM	11.78	172815.3
3k_MM	15.66*	172832.6
3k_HM	18.78*	172837.6

D. Chapter conclusion

To sum up, this chapter examines the performance of the mixed integer conic programming (MICP) solution for the VVC problem in different distribution network configurations. First, the chapter reviews the effectiveness of the convex property of conic problems in establishing a globally optimal solution. Then the chapter examines, through numerical results using CPLEX, the computational power of the MICP in solving the VVC problem for predefined radial and meshed networks. As VVC is a real-time control function, and it has to comply with the distribution management system's real-time computing requirements, the numerical results reveal that MICP, which has long computing time, could be practically used to give a tight lower bound against which the solution quality of heuristic techniques can be measured. The tight lower bound coincides with the global optimum whenever the conic constraints are binding and cycle constraints (in meshed networks) are satisfied; the globally optimal solution could be used as a benchmark for the solution set points of heuristic techniques. To speed up the VVC computational time in real-time applications, the next chapter studies the discrete coordinate-descent algorithm (DCD) as a viable network-based VVC algorithm.

CHAPTER III

DISCRETE COORDINATE DESCENT (DCD)

This chapter explores the discrete coordinate descent (DCD) algorithm as a practical tool to tackle VVC in real-life implementations. The chapter starts by an introduction covering the DCD as a VVC industry standard. Then, the flowchart of the VVC solution by DCD is illustrated. Finally, the chapter investigates the DCD numerical results on recovering feasible solutions for meshed networks, starting from the MICP solutions.

A. Introduction to DCD

When applying the network-based VVC formulation in real-time applications, reliable performance and rapid execution are the most desirable attributes of any proposed algorithm. The practical complexity of such a problem appears mainly in having a nonlinear objective function and a diverse set of control variables that are coupled by power flow equality constraints. Other soft operational constraints exist; however, such constraints may be more flexible to relax based on the expert's knowledge and the load variation curve [26]. As a result, the class of multistep discrete programming search methods has been endorsed as a possible tool to tackle VVC in real-life implementations [4, 10, 26]. Amongst different discrete programming methods, the multistep discrete programming method searches for the best solution in the neighborhood of an initial starting point.

The discrete coordinate-descent algorithm (DCD) has been illustrated as a viable VVC algorithm. During the minimization process, the DCD algorithm gradually approaches a local minimum by iterative update of an individual control variable in a discrete variable step. While the discrete movement is simply one reasonable increment or decrement step in the chosen control variable, the DCD algorithm takes over the preferred direction, which guarantees the most admissible decrease in the objective function value throughout the iterative search.

B. Flowchart of DCD algorithm for VVC

In terms of its operation, and with respect to all permissible directions of all the control variables, the DCD algorithm iteratively evaluates the value of the objective function under concern. Consequently, the DCD algorithm selects in each iteration the search direction that achieves the most favorable reduction in the objective function value. The control variable corresponding to this system wise reduction is then updated in the associated direction. During each iteration, the algorithm repeats the same evaluation over all search directions; when no additional decrease in the objective function is achievable, the algorithm ends. Figure 3 shows the flowchart of DCD for VVC. In the flowchart, DCD_iteration symbolizes the DCD iteration number, SD denotes the index of the search direction, BSD represents the index of the best search direction, and IOF/ COF represent the initial/current objective function values.

To solve the VVC using DCD, two main conditions should be met. In the first place, all the control variables of the problem should be discrete; this condition is fulfilled by discretizing the range from which the distributed generation reactive power can be supplied. Secondly, the mathematical problem is reformulated to minimize an objective function $Objfct$ (18) accounting for power loss and the penalized operational violations. Specifically, f_V penalizes violated voltage limit constraints, and f_I penalizes violated current limit constraints.

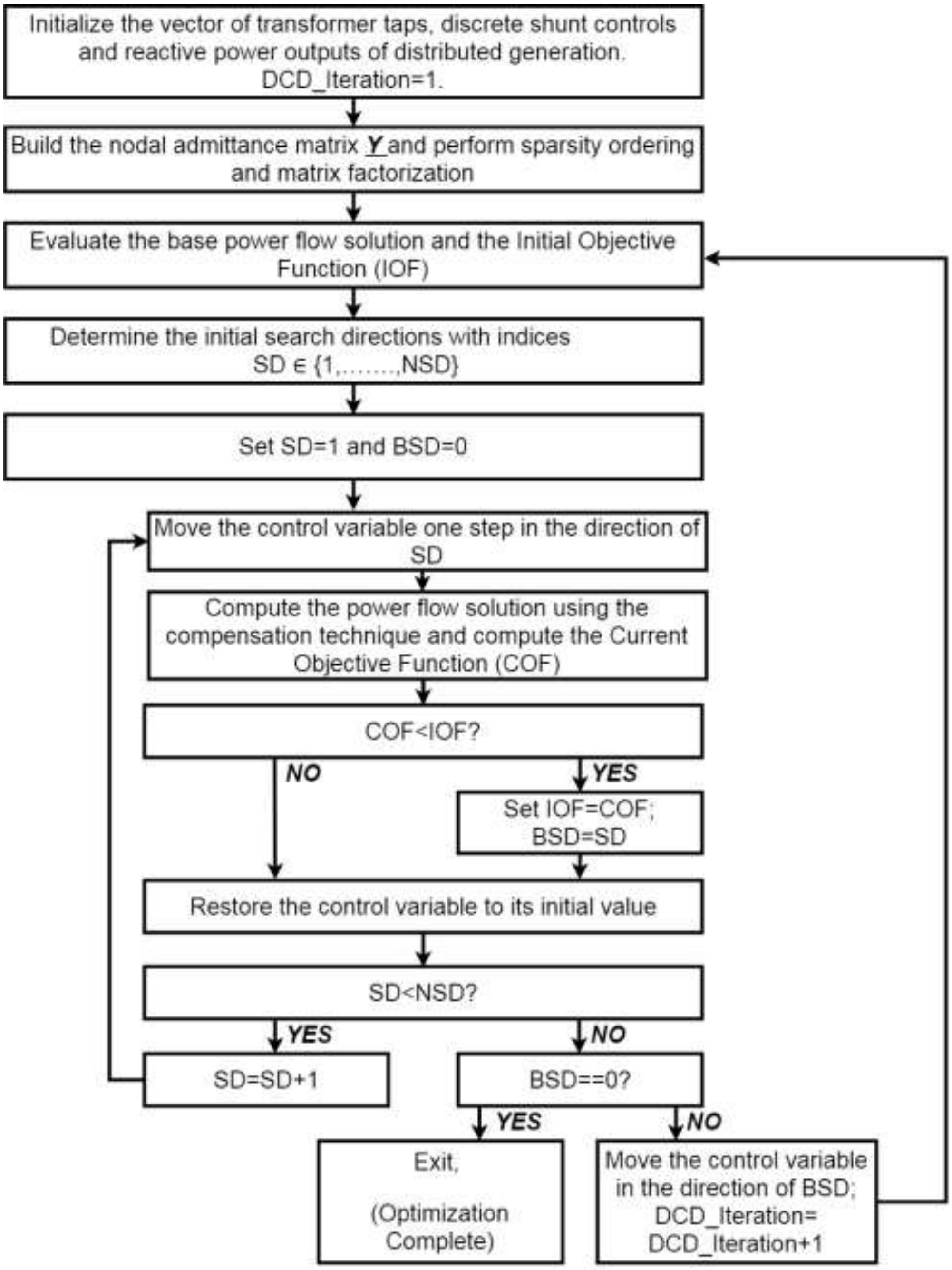

Figure 3. Flowchart of DCD algorithm for VVC.

$$Objfct = \sum_{i=1}^{n}\sum_{j=1}^{n}\{V_i^{re}Y_{ij}^{re}V_j^{re} + V_i^{im}Y_{ij}^{re}V_j^{im}\} + f_V\sum_{i=1}^{n} max(0, V_i - V_i^{max}) + f_V\sum_{i=1}^{n} max\left(0, V_i^{min} - V_i\right) \quad (18)$$

$$+f_I \sum_{ij \in BR}^{n} max(0, I_{ij} - I_{ij}^{max})$$

To ensure a time efficient computation when solving multiple power flows in DCD, the current injection (CI) method is employed; the result of the CI method is the nodal voltages and the network power loss. The vector of nodal voltages is initialized as a starting point of the method; the present vector of nodal voltages is then used to evaluate the next vector of nodal injection currents via (19). By applying sparsity ordering and matrix factorization, the newly estimated nodal voltages are then computed using the nodal matrix equation (20). As shown in (20), $\underline{\boldsymbol{Y}}$ is the nodal equations coefficient matrix. Following the most recently computed nodal voltages, an iterative update of the injection currents repeats itself in a loop mode until a convergence test is satisfied.

$$\underline{I_i} = \frac{p_i - jq_i}{\underline{V_i^*}} \quad (19)$$

$$\underline{\boldsymbol{I}} = \underline{\boldsymbol{Y}}.\underline{\boldsymbol{V}} \quad (20)$$

Due to the presence of switchable shunt capacitors, the elements of the $\underline{\boldsymbol{Y}}$ matrix change, requiring re-factorization of the matrix in the DCD execution. To use the same factorization in the power flow computations, switchable capacitors in the VVC can be handled using the compensation method [4]. By referring to [4], it is useful to start with the initial $\underline{\boldsymbol{Y}}$ matrix, to which corresponds an injected current $\underline{I_i}$ for every node *i*, and where the initial setting of the shunt capacitor is denoted by q_{0i}^{old}. Then, any update in the switch status of shunt capacitors at node *i* (from q_{0i}^{old} to q_{0i}) is dealt with using the updated injected current $\underline{I_i'}$ that includes the compensation term as follows:

$$\underline{I_i'} = \underline{I_i} + j[q_{0i}^{old} - q_{0i}]\,\underline{V_i} \quad (21)$$

Similarly, the compensation technique is adopted to handle tap changes in the DCD implementation. The $\underline{\boldsymbol{Y}}$ matrix is formulated at the initialization stage by having the tap-changing transformer between nodes *i* and *j* with tap t_{ij}^{old}. At this level, the injected currents at nodes *i* and *j* are respectively $\underline{I}_i$ and $\underline{I}_j$. Thereafter, when the tap position between nodes *i* and *j* becomes t_{ij}, the injected currents at both nodes *i* and *j* are updated to $\underline{I}_i'$ and $\underline{I}_j'$. In this way, there is no need to update the $\underline{\boldsymbol{Y}}$ matrix continuously in the power flow computations. On the contrary, any updates in the tap position between two nodes *i* and *j* are dealt with using the updated injected currents $\underline{I}_i'$ and $\underline{I}_j'$ which account for the compensation terms as follows [4]:

$$\underline{I}_i' = \underline{I}_i + \left[\left(t_{ij}^{old}\right)^2 - (t_{ij})^2\right]\left(\underline{y}_{ij}\underline{V}_i\right) - [t_{ij}^{old} - t_{ij}](\underline{y}_{ij}\underline{V}_j) \quad (22)$$

$$\underline{I}_j' = \underline{I}_j - [t_{ij}^{old} - t_{ij}](\underline{y}_{ij}\ \underline{V}_i) \quad (23)$$

In addition, the impact of distributed generation at node *i* is handled using the compensation current:

$$\underline{I}_{gi} = \frac{p_{gi} - jq_{gi}}{\underline{V}_i^*} \quad (24)$$

C. Results on feasibility recovery via DCD

As previously illustrated in chapter 2, the MICP solution of the VVC problem was not feasible for the majority of the meshed network instances. This follows the fact of employing a necessary relaxation of the cycle constraints in meshed networks in order to make use of the VVC MICP formulation. In this section, DCD is used to recover a feasible solution for the original VVC problem in the meshed networks. The

DCD feasibility recovery process, starting from the MICP solution, will achieve a feasible solution satisfying power flow equations and voltage magnitude limits.

The results of the execution of DCD starting from the MICP control set points (referred to by MICP+DCD) are shown in Table 3. In Table 3, the percentage loss reduction (LR (%)) is as obtained from MICP and MICP+DCD. To study the optimality of MICP+DCD, as benchmarked relative to MICP, the GAP (%) (via (25)) is illustrated in column 4 relative to the MICP objective function value. The significance of MICP+DCD is confirmed through achieving reliably feasible solutions, where voltage violations (in instances marked with *) are no more observed.

$$\text{GAP (\%)} = \frac{(real\ power\ loss\ using\ MICP{+}DCD)-(real\ power\ loss\ using\ MICP)}{real\ power\ loss\ using\ MICP{+}DCD} \tag{25}$$

Table 3. The DCD feasibility recovery, starting from the MICP solution, in the meshed test networks

Name	MICP	MICP+DCD	
	LR (%)	LR (%)	GAP (%)
B_M	13.87	MICP is Feasible	
1k5_LM	14.42*	14.19	0.2684
1k5_MM	14.47*	14.40	0.0811
1k5_HM	18.16*	18.01	0.1810
3k_LM	11.78	MICP is Feasible	
3k_MM	15.66*	15.51	0.1772
3k_HM	18.78*	18.60	0.2174

D. Chapter conclusion

This chapter explores the practical value of the discrete coordinate-descent algorithm (DCD) in accomplishing the most admissible decrease in the VVC objective function value throughout an iterative search. In the first place, the chapter describes the prominence of DCD in the reliable and rapid execution of the network-based VVC problem in real-time applications. The practical integration of DCD in VVC optimization problems is explained next through a flowchart associated with the required clarifications. Then, the power of DCD in recovering feasible solutions for meshed MICP solutions is revealed through numerical results. All in all, DCD is manifested as a practical tool to recover feasible solutions for MICP solutions. The feasibility recovery could be useful even for radial MICP solutions, where the conic constraints are not binding at the optimal solution; however, this case was not observed in the studied radial test instances. Building on the findings of the current chapter, a cheap starting solution is proposed in chapter 4 for the DCD algorithm applied to VVC optimization problems.

CHAPTER IV

CHEAP STARTING SOLUTION FOR DCD

Since the previous chapters reveal that MICP has long computing time, this chapter proposes solving the VVC problem using a discrete coordinate-descent algorithm, starting from a solution to the continuous relaxation of the VVC mixed-integer conic program. In this chapter, the local search behavior of DCD is studied by investigating the effect of the starting solution point on the DCD final result. The effectiveness of the proposed cheap starting point, which is the solution of the MICP continuous relaxation, is demonstrated numerically against the classical initialization method. At the end of this chapter, a conclusion is derived from this comparison in terms of reliability and computational time.

A. DCD: A local search method

In many real-life challenging optimization problems, it is appropriate to reach a local optimum within acceptable time, rather than to spend extensive search and expensive resources until a global optimum is guaranteed. A local search algorithm, such as DCD, is an optimization algorithm that yields a locally optimal solution by conducting iterative transitions from one solution to another in the existing search space. The search process is launched by initializing the local search method with a starting point.

During the optimization process, the local search method iteratively updates the current solution by taking over the most preferred direction in its neighborhood, which eventually establishes the most admissible change in the objective function value.

Subsequently, the starting solution point has an effect on the final results obtained using DCD, as is the case with any other local search method. For this reason, it is desirable to initialize the DCD with a point close to the optimum. In fact, the execution of any local search method stops in one of two possible events: either the time limit is exceeded or no additional change is experienced in the objective function value.

As modeled in (7)-(17), the VVC mixed-integer conic program accounts for the tap-changing transformers, the reactive power injections by the capacitor banks, and the reactive power outputs of the distributed generation. While the VAr outputs of the distributed generation vary as continuous variables within their prescribed limits (17) in MICP, the transformer tap positions and the capacitor switch statuses are treated as discrete control variables (13)-(15). Hence, such discrete variables are coupled with binary variables (14)-(16) to guarantee that every transformer tap t_{ij} (from the set $\left\{t_{ij}^{(1)}, \dots \dots, t_{ij}^{(nt)}\right\}$), and every nominal switched capacitor reactive power q_{0i} (from the set $\left\{q_{0i}^{(1)}, \dots \dots, q_{0i}^{(nc)}\right\}$), takes one value at the end of execution of the MICP program. Given that MICP requires long computing time, and that the local search behavior of DCD is desirable in practice, this chapter proposes solving the VVC problem using DCD starting from a solution to the continuous relaxation of the VVC mixed-integer conic program.

Whenever the binary constraints of the variables $b_{ij}^{(k)}$ and $b_{i}^{(k)}$ in (14) and (16) are dropped, such binary variables are seen as continuous ones. Thus, a continuous relaxation of the VVC mixed-integer conic program arises. Following the previously described polyhedral approximation of the conic constraints in (10), the CPLEX linear programming solver is consequently employed to solve the VVC continuous relaxation, and thus to generate a continuous solution of the discrete control variables in a very reasonable computing time, in comparison with the original time intensive MICP formulation.

In what follows, the proposed DCD initialization methodology, based on such solution to the continuous relaxation of the VVC mixed-integer conic program, is investigated against the classical initialization method. While the proposed methodology rounds the control set-points to their nearest discrete values, the classical initialization method is based on the current operating point (COP as provided in the data files [24]).

B. Results for DCD starting from: MICP relaxation and current operating point

This section compares the numerical results for the execution of DCD starting from: (i) the current operating point (COP+DCD) and (ii) the solution to the continuous relaxation of the VVC mixed-integer conic program (CP+DCD). The results are reported for both radial and meshed networks.

To conduct the required comparison, the percentage loss reduction (LR (%)) is given in Table 4 as obtained from COP+DCD and CP+DCD. The optimality of COP+DCD and CP+DCD, as benchmarked relative to MICP, is illustrated in columns 3 and 5 respectively through the gap (GAP (%)) relative to the MICP objective function value. The superior performance of CP+DCD as compared to COP+DCD is confirmed through achieving a maximum GAP < 0.45% for CP+DCD, while the GAP for COP+DCD reaches around 5.2% for the 1464-node network at peak load.

Moreover, Table 5 shows the required number of the coordinate-descent iterations (CD iterations) and the corresponding time (CD time (s)) for COP+DCD and CP+DCD. CP+DCD attains a VVC solution with a much smaller number of CD iterations (ranging between 2 and 23) and consequently a much shorter time (maximum

CD time < 14 s); this is because the CP (i.e. the MICP relaxation) initializes DCD with a point close to the optimum, thus reducing the number of steps in the coordinate decent search.

Table 4. The optimality of COP+DCD and CP+DCD as benchmarked relative to MICP

Name	Configuration	COP+DCD		CP+DCD	
		LR (%)	GAP (%)	LR (%)	GAP (%)
B_R	Radial	11.88	0.0529	11.89	0.0367
1k5_LR	Radial	14.11	2.6091	16.32	0.0368
1k5_MR	Radial	14.07	2.1374	15.88	0.0326
1k5_HR	Radial	17.67	2.7524	19.92	0.0232
3k_LR	Radial	11.12	1.5504	12.26	0.2749
3k_MR	Radial	14.92	2.1079	16.54	0.2025
3k_HR	Radial	17.4	3.2189	19.91	0.1789
B_M	Meshed	13.37	0.5728	13.78	0.0965
1k5_LM	Meshed	12.14	2.5958	14.40	0.0258
1k5_MM	Meshed	11.83	2.9948	14.28	0.2117
1k5_HM	Meshed	13.67	5.2000	18.07	0.1164
3k_LM	Meshed	8.81	3.2527	11.53	0.2786
3k_MM	Meshed	11.85	4.3170	15.54	0.1469
3k_HM	Meshed	14.56	4.9372	18.42	0.4409

Table 5. The computational performance of COP+DCD and CP+DCD in terms of the number of coordinate-descent iterations and the execution time

Name	Configuration	COP+DCD		CP+DCD	
		CD iterations	CD time (s)	CD iterations	CD time (s)
B_R	Radial	54	1.1	5	0.2
1k5_LR	Radial	109	11.8	20	2.0
1k5_MR	Radial	102	15.5	14	2.4
1k5_HR	Radial	136	24.0	23	4.4
3k_LR	Radial	92	34.6	11	4.4
3k_MR	Radial	109	110.1	8	8.2
3k_HR	Radial	104	53.6	21	11.6
B_M	Meshed	58	1.1	2	0.2
1k5_LM	Meshed	85	6.6	16	1.3
1k5_MM	Meshed	60	7.8	12	1.7
1k5_HM	Meshed	72	14.6	18	3.6
3k_LM	Meshed	73	32.6	14	6.3
3k_MM	Meshed	80	41.8	6	11.4
3k_HM	Meshed	79	61.4	19	13.6

C. Chapter conclusion

The current chapter proposes the MICP relaxation as a cheap starting solution for the discrete coordinate descent (DCD) algorithm in VVC optimization problems. After highlighting the local search behavior of DCD and consequently the effect of the starting solution point on the DCD final solution, the chapter illustrates that initiating DCD with the solution to a continuous relaxation of the MICP, is a practically good initiative to yield an acceptable VVC solution in a very reasonable computing time for different network configurations. The superior performance of CP+DCD against COP+DCD is evident through numerical results showing that not only do the CP+DCD solutions have a much smaller gap relative to the MICP solutions, but that also such solutions are attained with a much shorter time. The overall conclusion of the thesis work is clarified in chapter 5.

CHAPTER V

CONCLUSION

This chapter seeks to shed light on the overall conclusions, which are reached by the end of this thesis work. The conclusions are derived for radial and meshed networks after performing comparisons, per each network type, between the MICP solution (as seen in chapter 2), DCD initialized with the MICP solution (MICP+DCD as observed in chapter 3), DCD initialized with MICP relaxation (CP+DCD as examined in chapter 4), and DCD initialized with the current operating point (COP+DCD as also illustrated in chapter 4).

A. Radial networks

Regarding the seven radial networks which are tested in this thesis, the MICP solution not only represents the tight lower bound against which the quality of the attained solution by heuristic techniques could be measured, but also is feasible to the original VVC problem. Thus, the MICP solution practically coincides with the global optimum; this is demonstrated by having binding conic constraints at the MICP solution. Table 2 shows that the MICP control settings do not give rise to voltage magnitude violations. Hence, the feasibility recovery via DCD (denoted by MICP+DCD) is not required.

Since DCD is an endorsed tool to tackle VVC in real-life implementations, the MICP solution could be used as a benchmark for the solution set points of CP+DCD and COP+DCD. While CP+DCD and COP+DCD gain the advantage of requiring shorter computing time against the MICP approach, CP+DCD appears much more practical than the classical technique COP+DCD in industrial applications. As compared to the COP+DCD solutions, not only do the CP+DCD solutions have a much smaller gap relative to the MICP solutions (see Table 4), but also such solutions are attained with a much shorter time (see Table 5); this is because the MICP relaxation initializes DCD with a point close to the optimum, thus reducing the number of steps in the coordinate descent search. In Table 6, the remarkable difference in the execution time between CP and MICP is shown for the radial networks.

Table 6. The execution time of MICP and CP (s) for the radial networks

Name	MICP	CP
B_R	170.7	1.5
1k5_LR	2389.7	37.3
1k5_MR	2161.8	54.3
1k5_HR	4195.2	39.5
3k_LR	63192.6	159.8
3k_MR	35338.5	207.0
3k_HR	61626.3	134.9

B. Meshed networks

With respect to the seven meshed networks which are employed in this research, the generated MICP solution is generally not feasible to the original VVC problem (see Table 2); the solution serves to be the tight lower bound against which the quality of the achievable solution by heuristic techniques could be measured. This observation is attributed to relaxing the cycle constraints in the MICP formulation of the meshed VVC problem. Hence, the DCD initialized with the MICP solution (referred to by MICP+DCD) is carried out to yield a feasible solution (see Table 3), whose degree of optimality is gauged against the MICP lower bound; such a feasible solution satisfies power flow and voltage constraints.

Therefore, only a tight lower bound of the VVC actual solution is established by the MICP formulation in the meshed networks; nevertheless, the MICP formulation requires extensive computations especially for such meshed networks (see Table 2). Yet, a local search method, as DCD, represents the system wise resolution technique in practice. As a result, CP+DCD and COP+DCD are studied as practical VVC implementation tools in the meshed networks. The optimality of both tracks, CP+DCD and COP+DCD, is benchmarked relative to the VVC tight lower bound; this bound is attained by the MICP formulation as discussed before. By taking advantage of the computing performance of the VVC industry standard, the DCD algorithm, CP+DCD and COP+DCD both solve the VVC problem in a time-efficient manner. However, the obtained solutions via CP+DCD are more reasonable than those acquired by the classical technique COP+DCD in terms of the gap (relative to the VVC lower bound which the MICP formulation sets up) and the required computing time (see Tables 4, 5); this improvement is achievable because the MICP relaxation launches DCD with a starting point in the neighborhood of the optimum. In Table 7, the remarkable difference in the execution time between CP and MICP is shown.

Table 7. The execution time of MICP and CP (s) for the meshed networks

Name	MICP	CP
B_M	155.2	1.2
1k5_LM	4704.2	37.2
1k5_MM	4600.4	40.6
1k5_HM	5969.8	47.9
3k_LM	172815.3	162.6
3k_MM	172832.6	264.9
3k_HM	172837.6	151.8

A future research direction is to consider the uncertainty of the distributed generation real power output (representing renewable sources) in the VVC solution.

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