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STRENGTHENING OF REINFORCED CONCRETE CORBELS USING PRE-TENSIONED ANCHOR RODS

by
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AN ABSTRACT OF THE THESIS OF

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Title: Strengthening of Reinforced Concrete Corbels Using Pre-tensioned Anchor Rods

Reinforced concrete (RC) corbels are defined as short cantilever members with span-to-depth ratio (*a/d*) less than one. They are typically used as supports for precast beams or indoor cranes. Strengthening those corbels might be needed due to aging, deterioration, or a change of the load demand on the structure. Several options are available for strengthening RC corbels including: RC jacketing, wrapping FRP sheets, steel plates, or adding pre-tensioned or passive anchor rods. This study aims at investigating the behavior of reinforced concrete (*RC*) corbels strengthened using pre-tensioned anchor rods. A total of ten as-built and strengthened corbels specimens were tested under monotonic loading up to failure. The test parameters included: (1) span to depth ratio of the corbel (*a/d*), (2) concrete compressive strength (f'_c), (3) number of pre-tensioned rods, (4) and location of pre-tensioned rods. The results showed that strengthened corbels had significant strength improvement with a delay in crack formation and propagation. The increase in corbel's strength was dependent on the aforementioned parameters. It was higher for corbels with higher concrete strength and lower span-to-depth ratio a/d. The results of the experimental tests conducted in this study in addition to ones available in the literature are compared against existing strength prediction models. The change in the failure mode with the increase in the pre-tensioning force is analyzed and correlated with the predicted governing mode. After comparing and analyzing the experimental results, modified strength prediction models are suggested for strengthening RC corbels with pre-tensioned anchor rods.

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ABBREVIATIONS

a : the distance of application of the load from the support

a/d : span to depth ratio

A_h : the area of the horizontal stirrups

A_p : the area of the pre-stressing rods

A_s : the area of the main steel reinforcement

A_{si} : the area of distributed reinforcement at spacing s in the direction of reinforcement crossing a strut at an angle α to the axis of a strut

b : the width of the corbel

b_s : the width of the concrete strut

d_p : the distance between the center of the rod and the extreme compression fiber

d : the depth of the corbel

f'_c : the concrete compressive strength

f_{cd} : the design compressive stress for the strut

$f_{0.1k}$: the yield strength of the pre-tensioning rods

f_t : the tensile strength of the reinforced concrete strut

f_{y1} : the main reinforcement yield stress

f_{y2} : the horizontal stirrups yield stress

f_u : the ultimate strength of the top pre-tensioning rods

jd : the moment arm of the corbel

jd_1 : the moment arm of the main reinforcement

jd_2 : the moment arm of the pre-tensioned rods.

H : the height of the corbel

M_n : the moment capacity of the corbel

n : modular ratio

P_e : the effective pre-tensioning force

P_i : the initial pre-tensioning force

P_{im} : the initial pre-tensioning force at mid-section

s : the secondary horizontal reinforcement spacing

v_c : the average shear stress at failure at the interface with the column

V_n : the Corbel load capacity

V_{shd} : the diagonal splitting strength of corbels

w : the width of the loading plate

α : the angle of the confining reinforcement relative to the strut.

β : a factor that reflects the shape of the strut and its confinement

δ : the angle of the strut

μ : the shear friction coefficient

ρ : the main reinforcement ratio

ρ_h : the secondary horizontal reinforcement ratio

CHAPTER I

INTRODUCTION

A. RC Corbels

Reinforced concrete (RC) corbels are defined as short cantilever members with span to depth ratio (*a/d*) less than one, they are typically used as supports for precast beams or indoor cranes. Strengthening those corbels might be needed sometimes due to aging, deterioration, or a change of the load demand on the structure. Several options are available for strengthening RC corbels including: RC jacketing, wrapping FRP sheets, steel plates, or adding pre-tensioned or passive anchor rods.

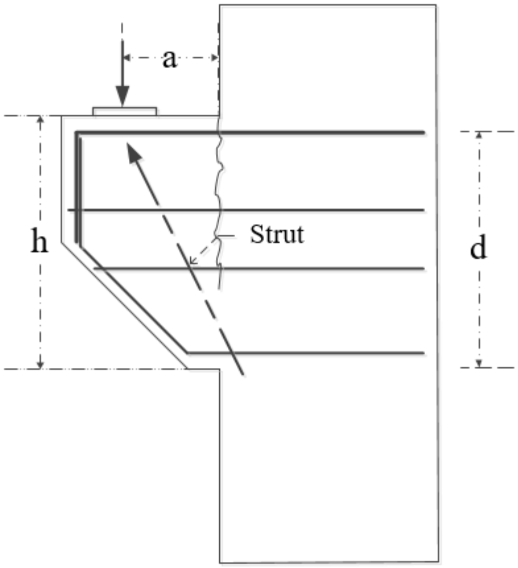


Fig. 1. RC Corbels

The RC corbels are considered short members of a low span to depth ratio and thus, their behavior is more complex than regular cantilevers. RC corbels strength generally increases with the decreasing load span to depth ratio (*a/d*), increasing concrete compressive strength (f'_c), reinforcement ratio (ρ), and cross-section [1]. RC corbels design is performed either by shear friction based methods or strut and tie methods. Shear friction based methods assume that the capacity of the corbels to be the smaller of its shear friction capacity and its flexural capacity measured at the interface of the supporting column. The conventional shear friction theory adopted by the ACI 318-14 code [2] assumes that the capacity of a concrete interface is equal to the horizontal forces developed by the clamping action of the steel bars multiplied by a friction coefficient. The shear friction reinforcement used is assumed to reach yield but the interface capacity is also limited by the ability of the concrete surface to withstand those shearing stresses without crushing. Another option is to design corbels based on a strut and tie model, where the corbel is modeled as a truss; the capacity in this case will be dependent on the strength of the ties, the struts, and the nodes. The failure limits that govern are the reinforcement yielding, concrete strut crushing or splitting, or nodal crushing.

B. Pre-Tensioned Corbels:

Limited research has been conducted on the behavior of pre-tensioned corbels, and their behavior is not very well understood. Based on the work of Chakrabarti and Kashou [3], Tan and Mansour [4], Godycka and Lachowicz [5] and Godycka [6], it is evident that adding pre-tensioned tendons or anchor rods can produce corbels of higher strength and less cracks at service load.

Chakrabarti and Kashou [3] examined pre-tensioned corbels without regular reinforcement. The used corbels were reinforced with high strength pre-tensioned anchoring rods of 15.8 mm diameter. Nine specimens were tested, where six of them had added wire meshes. It was concluded that the cracking load and the ultimate load capacity improves with the use of pre-tensioned rods. In addition the use of steel meshes decreases the cracks width in the corbels and increases their cracking load and ultimate load.

Tan and Mansour [4] examined partially pre-tensioned corbels (containing both pre-tensioned and non-pre-tensioned tendons). Twelve corbels specimens were tested. The varying parameters were f'_c, the *a/d* ratio, the ratio of the pre-stressed tendons out of the total tendons, and the pre-stressing stress in the tendons. The authors mainly concluded that the cracking load and the ultimate load of corbels increases with the increasing pre-stressing degree (ratio of the pre-stressing force out of the ultimate capacity of the tendons).

Godycka [6] examined 9 corbels specimens of a varying *a/d* ratio, the nine specimens were divided into three sets; one set having regular reinforcement, another set strengthened with passive #8 pre-tensioned rods of 396 MPa yield strength, and the last set strengthened with the same rods but with pre-tensioning force applied. The author mainly concluded that pre-tensioning increases the ultimate strength of corbels but this increase is more significant at higher *a/d* ratio, the higher the ratio the more the influence. The same conclusion is also made for the crack widths, where the decrease in crack width is more significant at a higher *a/d* ratio.

In a more recent work, Godycka and Lachowicz [5] examined six corbels of a varying *a/d* ratio and a varying stressing position. Those specimens were pre-tensioned with high strength #8 pre-tensioned rods, and they were compared with six other specimens reinforced with an assumed equivalent amount of regular reinforcement. They mainly concluded that the best location for pre-tensioning is to place the rods near the top of the corbel, and that the use of pre-tensioning significantly increases the cracking load of the corbels as compared to that of regular reinforcement, and it increases their load capacity at high *a/d* ratios.

Based on the aforementioned work, it is evident that adding a pre-tensioning force to RC corbels increases their load capacity and reduces their cracks at service load; in addition, similar to regular RC corbels, the strength of pre-tensioned corbels is higher for corbels of lower *a/d* ratio and higher f'_c.

C. Strengthened Corbels

Several Strengthening methods for RC corbels were studied in the literature. This includes external strengthening with CFRP [7, 8, 9, 10, 11] and GFRP materials [12], Ferrocement sheets [13], externally anchored steel plates or angles [14], and also bonded passive steel anchor rods [15, 16]. However, the available research is not enough to generalize design guidelines and strengthening systems for RC corbels of different geometric and material properties.

In this study, a proposed strengthening technique of RC corbels using pre-tensioned steel anchor rods is presented. In practical application for this strengthening method, horizontal holes have to be drilled in the corbels and extended to the supporting column. Anchorage can be provided by anchoring the rods in the supporting column using bonding materials, or by extending them to other side of the supporting column and anchoring them externally. After providing the needed anchorage, the pre-tensioning force is applied.

In this study, the proposed strengthening technique that is examined is strengthening RC corbels using pre-tensioned anchor rods. The experimental program consists of strengthened and as-built corbels specimens. The as-built corbels are designed and detailed in accordance with the shear friction based method of the ACI-14 318 [2]. The strengthened specimens have anchor rods added at the top, or at the top and the middle of the corbels. The strength of the corbels was examined as a function of the pre-tensioning force, the location of the pre-tensioning rods, the load span to depth ratio (a/d) and the concrete compressive strength (f'_c). Experimental results conducted in this study, in addition to existing experimental results available in the literature, are compared against existing strength prediction models. The change in the failure mode with the increase in the pre-tensioning force is monitored and correlated with the predicted governing mode. With the addition of the pre-tensioning force, the failure mode in some corbels changed from a shear tension failure mode to a shear compression failure mode. Afterwards, strength prediction methods are recommended for RC corbels strengthened with pre-tensioned anchor rods.

Chapter II

EXPERIMENTAL STUDY

A. Specimens:

The experimental program consisted of ten corbels tested in two phases. Phase 1 consisted of corbels having $f'_c = 20$ MPa, and Phase 2 of corbels having $f'_c = 27$ MPa. Table 1 shows the details of the tested specimens. The corbels were monolithically cast with a 21×70 cm column and have a 21×40 cm section at the connection with the column as shown in the Fig. 2.

The specimens were detailed in accordance with the ACI 318-14 requirements [2]. The as-built corbels were reinforced with minimum longitudinal reinforcement, and their corresponding minimum required horizontal stirrups. The strengthened specimens had horizontal pre-tensioned rods added either at the top, or at the middle and the top of the corbels cross section. The top pre-tensioned rods were placed at 7 cm from the top of the corbel. This location allowed the highest possible eccentricity with an acceptable proximity to the main tension reinforcement. A concrete cover of 3cm is used. To avoid spalling of concrete, the loading plates were positioned such that their edges are 5 cm away from the edge of the corbels in eight specimens. For the other two specimens S3-1B and S3-1B-A1 the edge distance was 15 cm. Bearing plates were also used to support the anchor rods' nuts, and were designed with enough thickness and bearing area to prevent the yielding of the steel plate or the bearing failure of concrete.

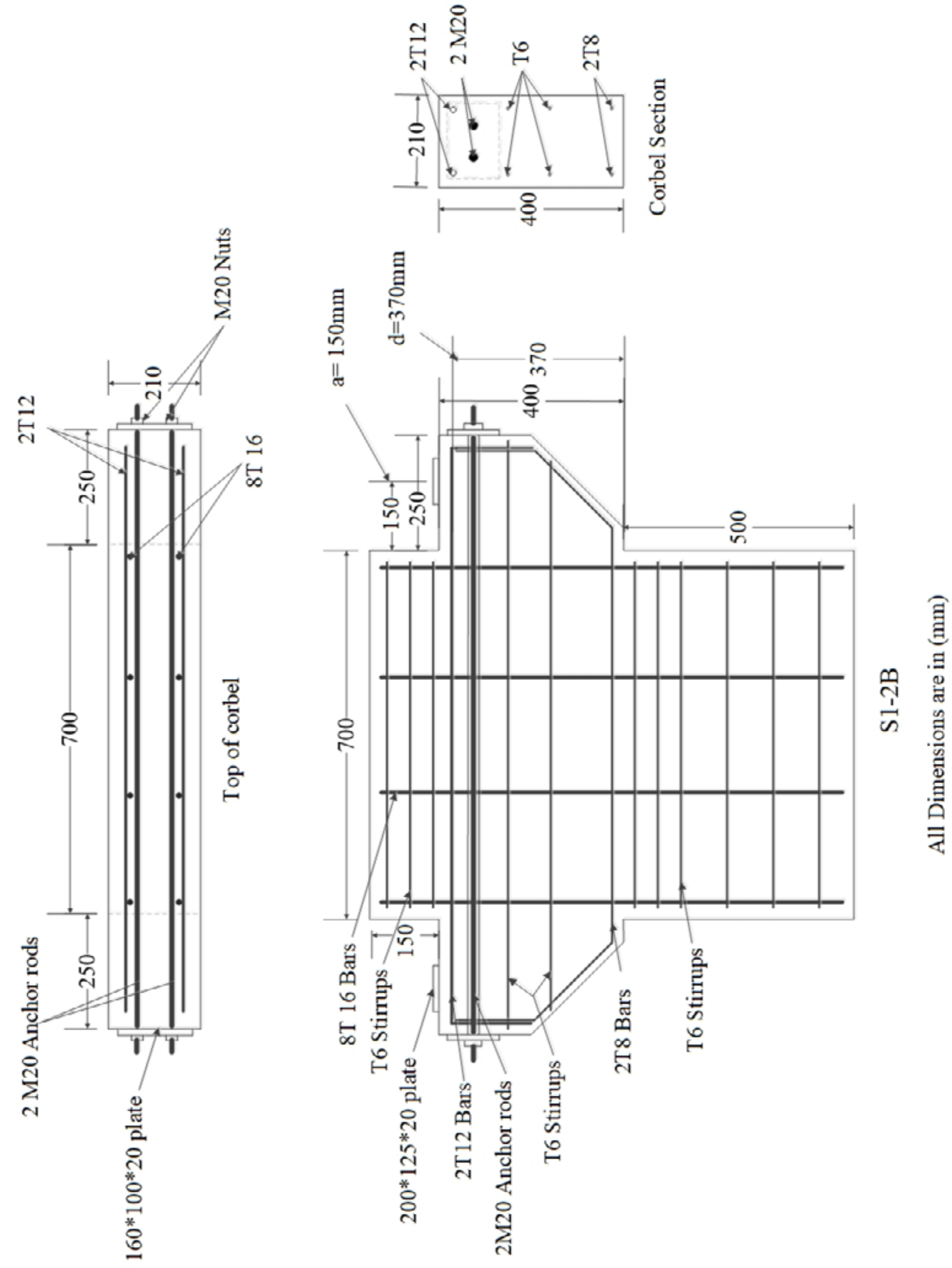


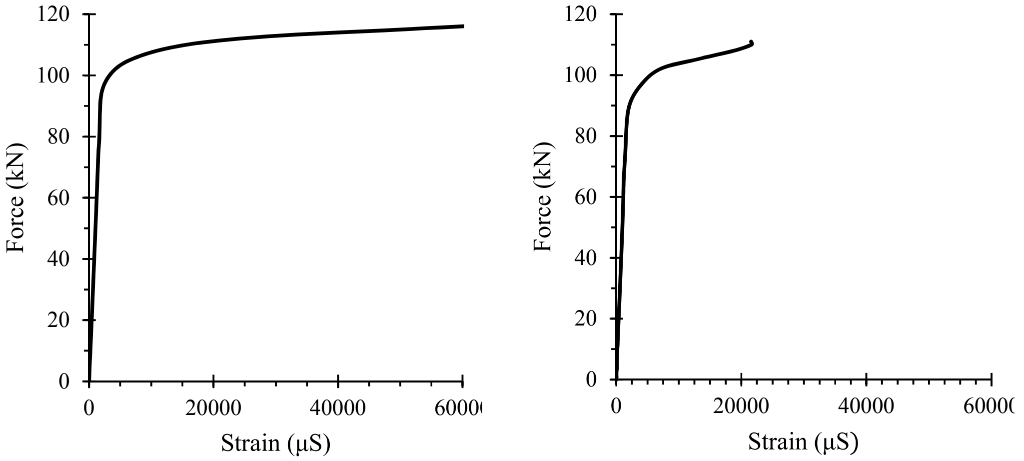
Fig. 2. Specimen S1-2B reinforcement

Table 1. Testing plan

Specimen	H (cm)	d (cm)	d_p (cm)	b (cm)	a (cm)	a/d	A_s (cm^2)	A_h (cm^2)	A_p (cm^2)	A_p mid (cm^2)	f'_c (MPa)	f_{y1} (MPa)	f_{y2} (MPa)	$f_{0.1k}$ (MPa)	$f_{0.1k}$ mid (MPa)	P_i (kN)	P_e assumed (kN)
Phase 1																	
C1-0B	40	37	-	21	15	0.41	2.26	1.13	0	0	20.2	486	311	-	-	-	-
S1-1B	40	37	33	21	15	0.41	2.26	1.13	2.35	0	20.2	486	311	403	-	81	91
S2-1B	40	37	33	21	15	0.41	2.26	1.13	4.7	0	20.2	486	311	403	-	162	182
C2-0B	40	37	-	21	25	0.68	2.26	1.13	0	0	20.2	486	311	-	-	-	-
S1-2B	40	37	33	21	25	0.68	2.26	1.13	2.35	0	20.2	486	311	403	-	81	91
S2-2B	40	37	33	21	25	0.68	2.26	1.13	4.7	0	20.2	486	311	403	-	162	182
Phase 2																	
C3-0B	40	37	-	21	15	0.41	2.26	1.13	0	0	27.1	447	396	-	-	-	-
S3-1B	40	37	33	21	15	0.41	2.26	1.13	3	0	27.1	447	396	458	-	81	140
S3-1B-A	40	37	33	21	15	0.41	2.26	1.13	2.35	1.5	27.1	447	396	412	458	121	150
S3-1B-A1	40	37	33	21	15	0.41	2.26	1.13	2.35	1.5	27.1	447	396	412	458	121	150

B. Materials:

T12 bars with yield strength f_y = 486 MPa and T6 stirrups with f_y = 311 MPa were used in Phase 1. For Phase 2, bars with f_y = 447 MPa and stirrups with f_y = 396 MPa were used. For all the specimens, M20 rods were used as top anchor rods except for specimen S3-1B, where M16 rods were used. For the midsection of specimens S3-1B-A and S3-1B-A1, M16 rods were used. In Phase 1, the pre-tensioned rods used were DIN975 4.6 B with $f_{0.1k}$ = 403 MPa and ultimate strength f_u = 460 MPa, and they were pre-tensioned to 75% of their ultimate strength. Fig. 3 shows typical force vs. strain curves for pre-tensioning rods. The pre-tensioned rods used in Phase 2 had a slightly higher strength than that of phase 1, but they were pre-tensioned to the same force. The plates used to support the pre-tensioned rods were of A36 steel having a yield strength of 250 MPa.



(a) (b)

Fig. 3. Force vs. strain in two M20 pre-tensioned rods

C. Testing Setup

All the specimens were tested on 100 tons MTS machine. As shown in Fig. 4, the load was applied to the bottom of the supporting column, with the top of the corbels positioned on the machine supports, transferring the load equally to the corbels on both sides. For the strengthened specimens, PVC pipes were installed before casting in order to insert the threaded rods afterwards; as shown in Fig. 5, temporary steel bars were inserted to keep the pipes straight during casting. The initial pre-tensioning force was applied using a torque meter, and it was applied directly before testing to limit steel relaxation. Strain gages were used for the reinforcing bars and the threaded rods. They were installed before casting, and placed 1cm away from the interface with the supporting column. Positioning the strain gages near the column face allows checking whether the main reinforcement reaches yield. The strain gages used for the pre-tensioned rods allow determining the accuracy of the pre-tensioning force applied and the increase the force in the rods upon loading. The testing of corbels was done at a rate of 0.2 mm per minute until failure, and LVDTs were positioned on both sides of the corbels near their interface with the column for determining the resulting deflections.

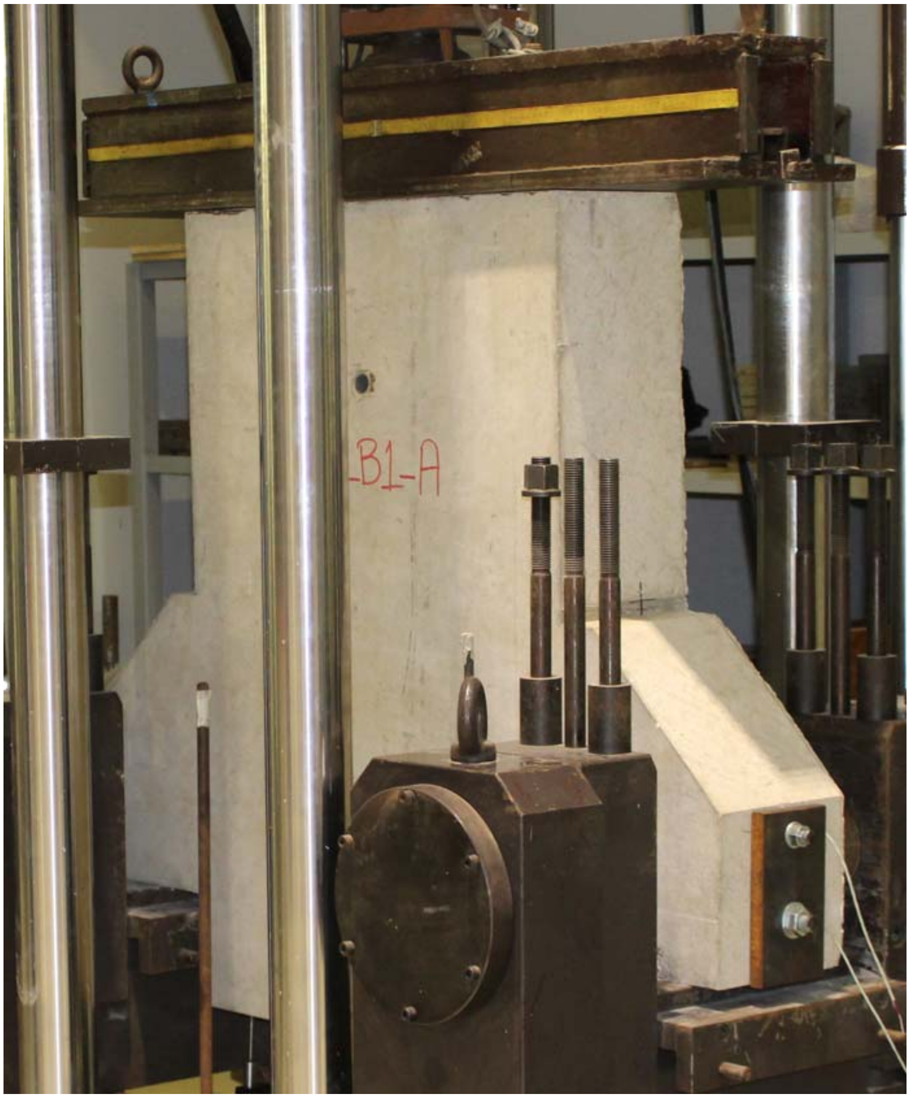


Fig. 4. Specimen S3-1B-A

Fig. 5. S1-1B before casting

D. Capacity Increase

As shown in Table 2, all strengthened specimens had a strength increase reaching up to 67% at 0.012% total reinforcement ratio. This increase indicates the effectiveness of the strengthening method. However, the contribution of the applied pre-tensioning force on the strength increase was not consistent, and it was affected by the *a/d* ratio and the *f'c*. The specimens in phase 2 had a strength increase higher than the added pre-tensioning force with a ratio up to 1.7. This indicates that the pre-tensioning is more effective for corbels of higher *f'c*. It can be noticed that the strengthened specimens had no significant difference in the strength increase. The three specimens had 80.8 KN pre-tensioning force applied on top, but only S3-1B-A and S3-1B-A1 had 40.4 KN pre-tensioning force applied on the middle. Thus, Specimen S3-1B was expected to have a lower strength then the other two. The fact that the 2 M16 rods have a total yield strength and ultimate strength higher that of a single M20 rod can explain the similarity in the strength. In relation to that, as shown in table 1, the assumed total effective pre-tensioning force at failure for S3-1B is not far from that of the two other strengthened specimens. It can be concluded based on those results, that adding pre-tensioning force at midsection had no advantage over adding pre-tensioning force to the top of those corbels.

Table 2. Strength increase in the strengthened specimens

Specimen	*a/d*	f'_c (Mpa)	Capacity V_n (kN)	Capacity Increase (kN)	**Capacity Increase (%)**	P_i Top (kN)	P_i Middle (kN)	$\boldsymbol{\Delta V_n / \Delta P_i}$
C1-0B	0.41	20.2	221.7	0	**-**	0	0	**-**
S1-1B	0.41	20.2	287.4	65.4	**29.6**	80.8	0	**0.8**
S1-2B	0.41	20.2	368.2	146.4	**66.0**	161.6	0	**0.9**
C2-0B	0.68	20.2	162.8	0	**-**	0	0	**-**
S2-1B	0.68	20.2	235.6	72.8	**44.7**	80.8	0	**0.9**
S2-2B	0.68	20.2	272.3	109.5	**67.3**	161.6	0	**0.7**
C3-0B	0.41	27.1	313.1	0	**-**	0	0	**-**
S3-1B	0.41	27.1	447.3	134.15	**42.8**	80.8	0	**1.7**
S3-1B-A	0.41	27.1	450.3	137.1	**43.8**	80.8	40.4	**1.1**
S3-1B-A1	0.41	27.1	445.5	132.4	**42.3**	80.8	40.4	**1.1**

As can be seen in Table 3, the tested corbels reached a max shear stress of $0.23f'_c$ at failure. This stress is slightly higher than the conservative shear crushing limit corresponding to ACI 318-14 [2] of $0.2f'_c$. Based on RC and pre-tensioned corbels experiments in the literature, this stress can be considered far less than the potential shear stresses that could be developed in corbels, especially at higher pre-tensioning forces and higher reinforcement ratios [5, 17].

Table 3: Shear stresses achieved in strengthened corbels

Specimen	Capacity (kN)	F_{pi} Top (kN)	F_{pi} Middle (kN)	f'_c (Mpa)	$\mathbf{v_c/f'_c}$
C1-0B	221.7	0	0	20.2	**0.14**
S1-1B	287.4	80.8	0	20.2	**0.18**
S1-2B	368.2	161.6	0	20.2	**0.23**
C2-0B	162.8	0	0	20.2	**0.10**
S2-1B	235.6	80.8	0	20.2	**0.15**
S2-2B	272.3	161.6	0	20.2	**0.17**

C3-0B	313.1	0	0	27.1	**0.15**
S3-1B	447.3	80.8	0	27.1	**0.21**
S3-1B-A	450.3	80.8	40.4	27.1	**0.21**
S3-1B-A1	445.5	80.8	40.4	27.1	**0.21**

In addition to the strength increase, pre-tensioning also increased the stiffness of the corbels as shown in Figs. 6, 7, and 8. This can be attributed to the delay in the cracks formation, which reduces the service load cracks of the corbels. On the other hand, although the peak load was achieved at higher displacement values, the failure tended to be more brittle with the increase in the pre-tensioning force.

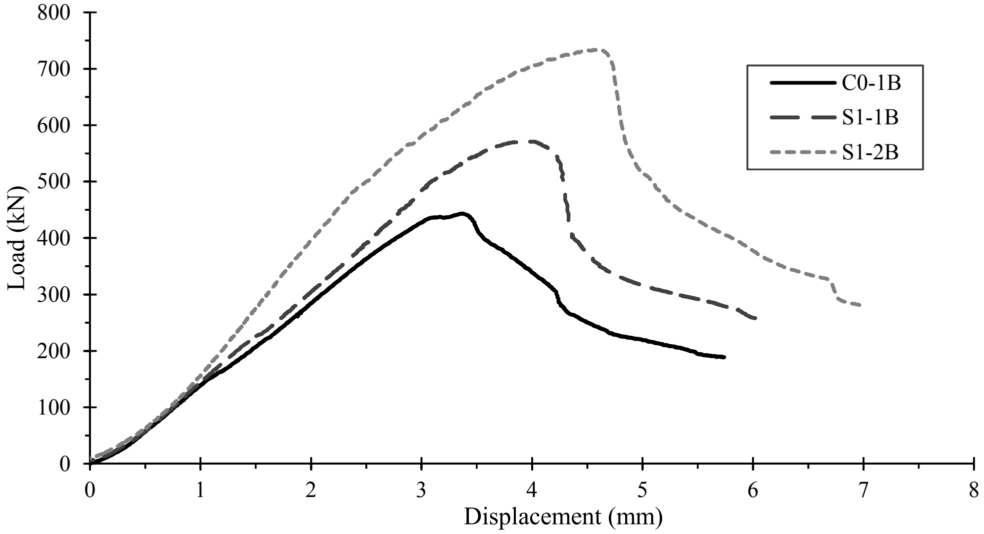


Fig. 6. Force vs. displacement for the short corbels specimens

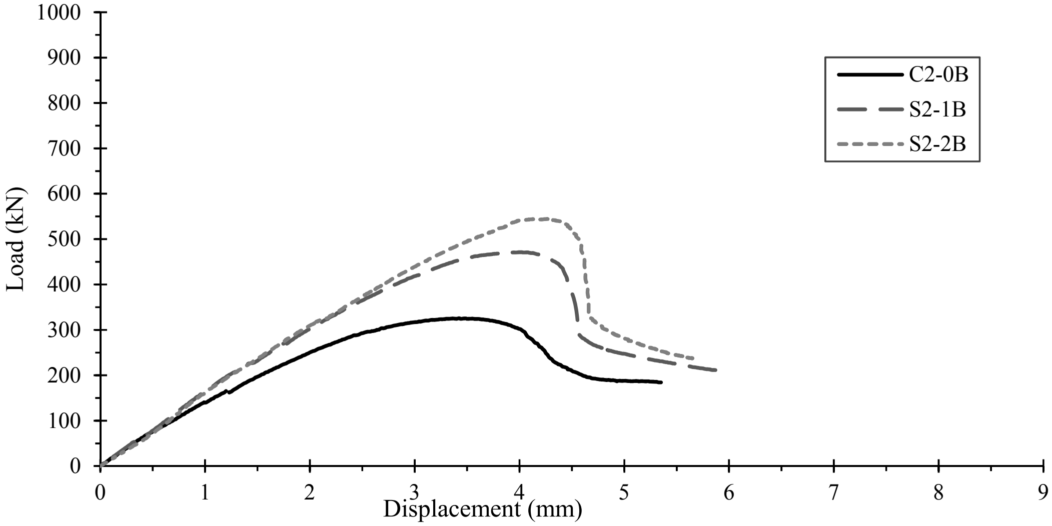


Fig. 7. Force vs. displacement for the long corbels specimens

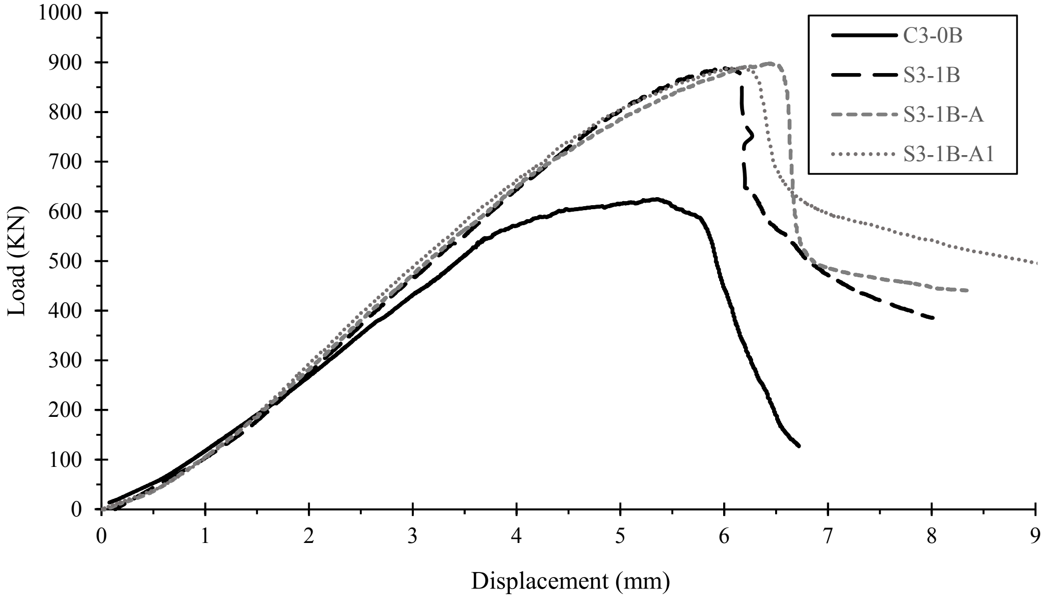


Fig. 8. Force vs. displacement for the 28 MPa corbels specimens

E. Cracks Formation and Failure Pattern

All the tested corbels had a flexural crack opening at the interface with the column; with the increase in the applied load single or multiple diagonal cracks opened near and below the loading plate. Cracks continued to widen, and the peak load was reached when the cracks connected with each other.

Table 4: failure and cracking patterns in the tested specimens

	Shape of the flexural crack at the column interface	Failure mode at the compression zone at the column interface
C1-0B	Short diagonal crack	compression
S1-1B	Short diagonal crack	compression
S1-2B	Short diagonal crack	compression
C2-0B	diagonal then straight at midsection	compression
S2-1B	diagonal then straight at midsection	splitting
S2-2B	Short diagonal crack	splitting
C3-0B	diagonal then straight at midsection	compression
S3-1B	Medium Length diagonal crack	compression
S3-1B-A	Short diagonal crack	compression
S3-1B-A1	Short diagonal crack	compression

Table 4 summarizes the observed cracking and failure modes in the tested specimen. The corbel specimens showed mainly two types of failure modes: shear compression failure mode, and a diagonal splitting failure mode. In the first type, a compression failure is observed at the compression zone in the column interface. In the second type, failure was observed with limited or no crushing at the compression zone in the column interface. As shown in Fig. 10, the as-built long corbel had the first type of failure mode; while the second type of failure mode can be clearly observed in the strengthened long corbels S2-1B and S2-2B. This type of failure should be avoided because it typically causes a premature failure in corbels. To avoid this type of failure, enough horizontal reinforcement should be provided at mid-section, and this will be discussed further in chapter 3.

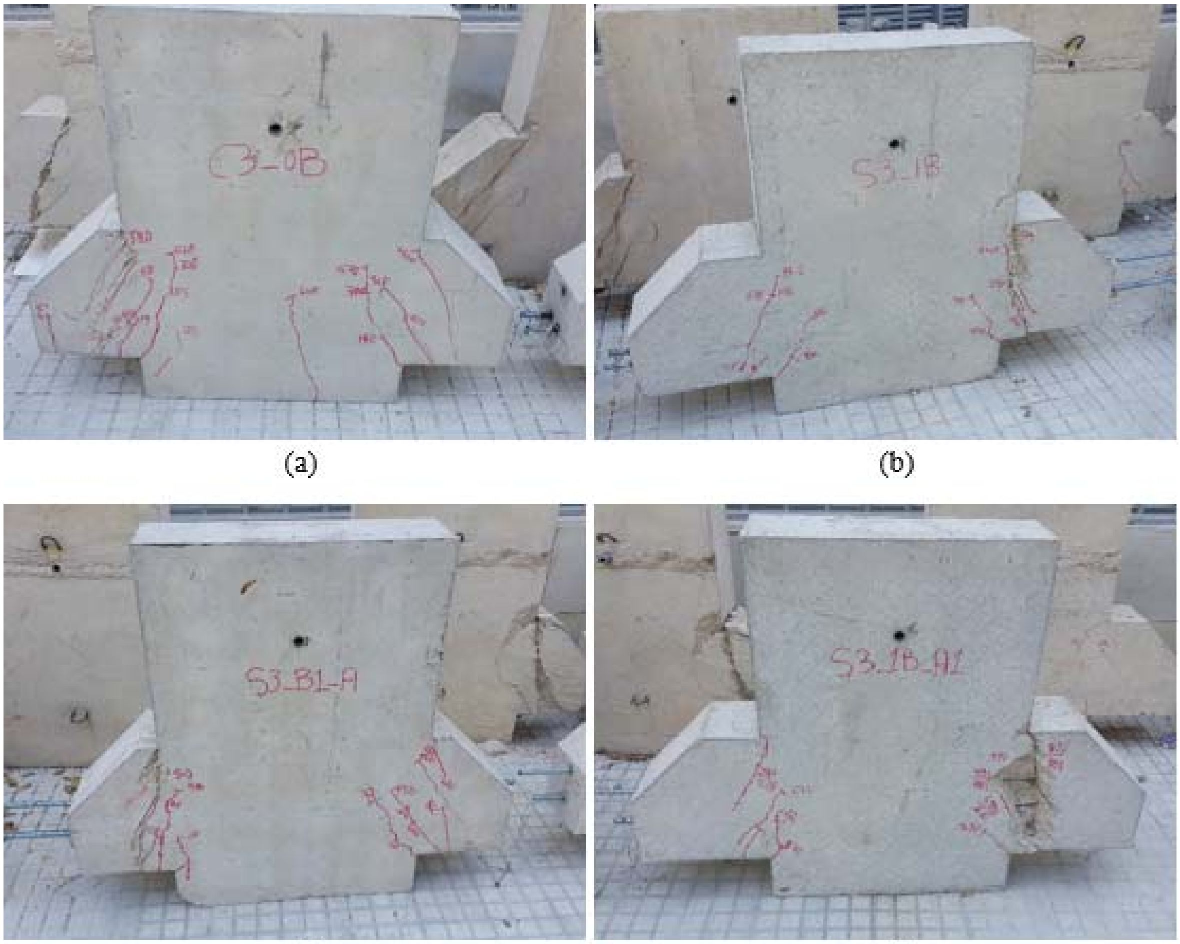
As shown in Figs. 9 and 11, the first type of failure mode was observed all the short corbel specimens, where the orientation of the diagonal cracks was not steep enough to cause a splitting failure. In Phase 2, specimen S3-1B had two M16 top rods pre-tensioned to the same force as that of the top rod in specimens S3-1B-A and S3-1B-A1. Comparing specimens S3-1B with specimens S3-1B-A and S3-1B-A1, no clear change in the failure and cracking pattern was observed with the addition of a pre-tensioned rod at the midsection. This suggests that mid-section rods could be more influential for longer corbels.

(a) (b) (c)

Fig. 9. Failure pattern in specimens: (a) C1-0B, (b) S1-1B, (c) S1-2B.

(a) (b) (c)

Fig. 10. Failure pattern in specimens: (a) C2-0B, (b) S2-1B, (c) S2-2B.



(a) (b)

(c) (d)

Fig. 11. Failure pattern in specimens: (a) C3-0B, (b) S3-1B, (c) S3-1B-A, (d) S3-1B-A1.

It was observed that in most specimens that the corbels on the two sides of the column showed some difference the cracking patterns. Some differences were also observed between the front and a back face of the specimen, an example is shown in Fig. 12. Variations in cracks and load capacities are expected in replicate reinforced concrete elements; they can be attributed to the non-homogeneous nature of concrete and the steel reinforcement; also due to expected minor geometric errors in placing reinforcement, loading, formwork, etc. In addition to those cracks, seven out of the ten specimens had cracks in the vertical plain, parallel to the reinforcement orientation as shown in Fig. 13; such cracks are caused by the high compression stresses near the interface of the column.

(a) (b)

Fig. 12: Specimen S3-1B after failure: (a) Front face, (b) Back face

(a) (b) (c)

Fig. 13: Vertical cracks after failure: (a) S1-2B, (b) S2-2B, (c) S3-1B-A

F. Strain Measurements

Based on the readings of the strain gages that were mounted on the main reinforcement and the pre-tensioned rods, in all corbel specimens the main reinforcement reached the yield stress near the interface with the column. However, the failure strain in the main rods tended to decrease as pre-tensioning force increased. As shown in Figs. 14 to 17, the addition of pre-tensioned rods delayed the cracks formation, and thus a significant capacity increase was achieved before major engagement of the main reinforcement in the force development.

Although not bonded the pre-tensioned rods showed a substantial strain increase, which indicates an influential increase in the effective pre-tensioning force prior failure. The measured strain increase in the pre-tensioned rods ranged from 1300 to 4700 micro strain. The effective strain values at failure measured at least 2500 micro strain, indicating that the effective pre-tensioning force at failure reached a minimum of 95 kN, equivalent to about 89% of the rod capacity. Despite some expected inaccuracies in applying the pre-tensioning force by the torque meter, all the rods reached higher force at failure as compared to the initial assumed pre-tensioning force, as shown in Fig. 18. Accordingly, it is more accurate to include the effect of the strain increase in the capacity prediction equations by using an effective pre-tensioning force instead of the initial one. However, for design purposes, the increase in the pre-tensioning force could be regarded as a reserve strength. It was noticed that long corbel specimens had higher strain increase in the main reinforcement and in the pre-tensioned rods as compared to the short corbels, and this is expected due to the higher flexural stresses that occur in those corbels. However, more research is required to better understand the influence of various parameters on the strain increase in the pre-tensioned rods of corbels upon loading.

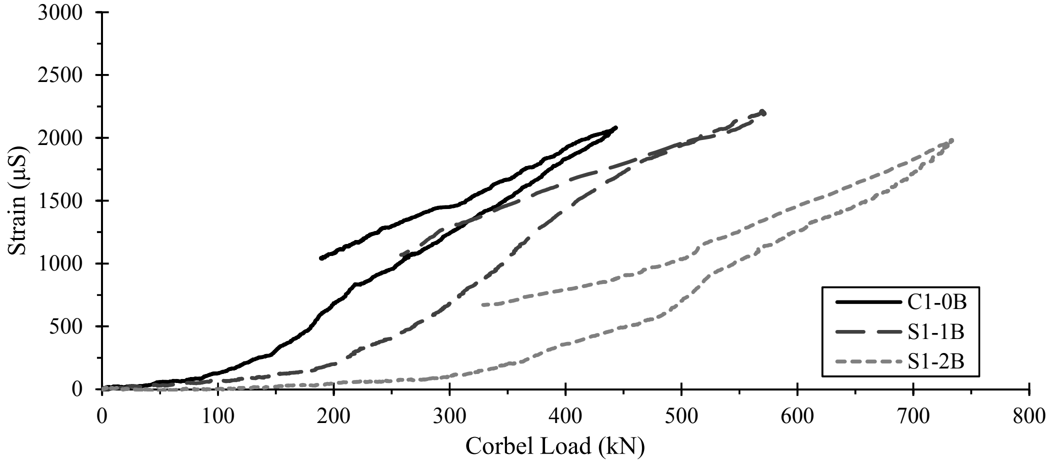


Fig. 14. Force vs. Main Reinforcement Strain on the Non-Failing Side for Corbels of Set 1

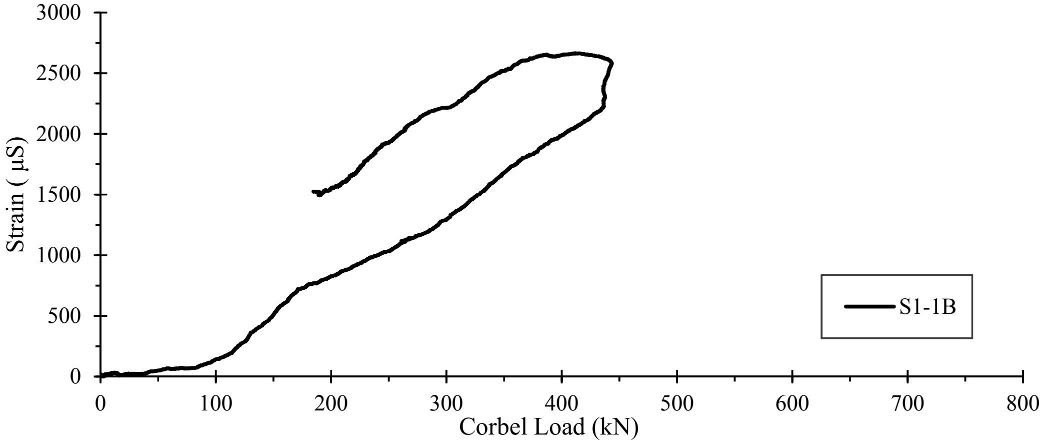


Fig. 15. Force vs. Main Reinforcement Strain on the Failing Side for Corbel S1-1B of Set 1

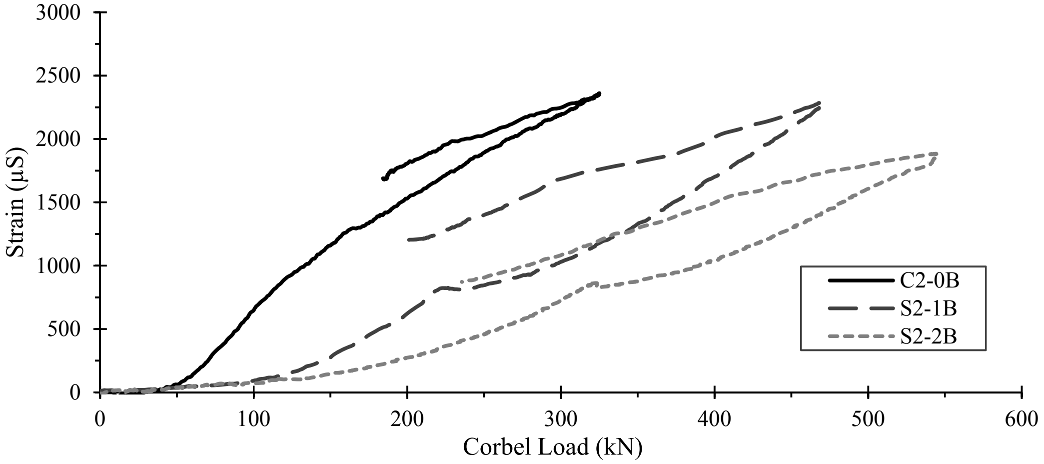


Fig. 16. Force vs. Main Reinforcement Strain on the Non-Failing Side for Corbels of Set 2

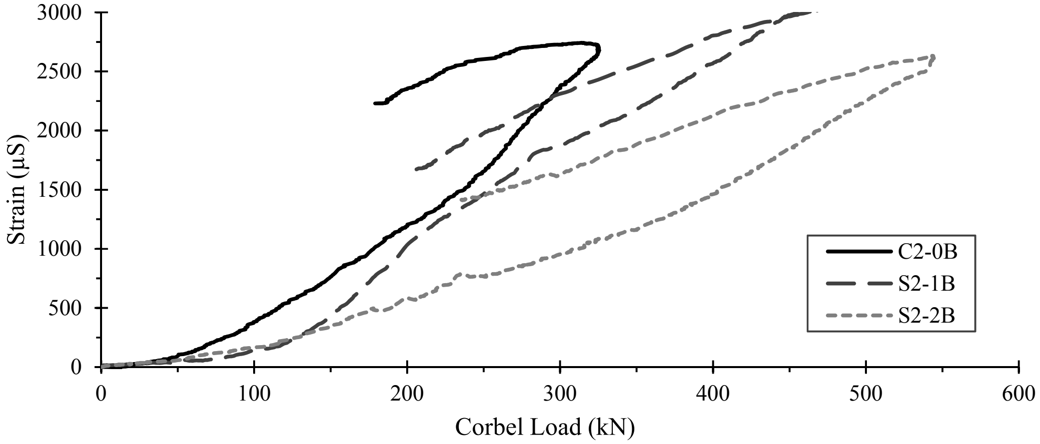


Fig. 17. Force vs. Main Reinforcement Strain on the Failing Side for Corbels of Set 2

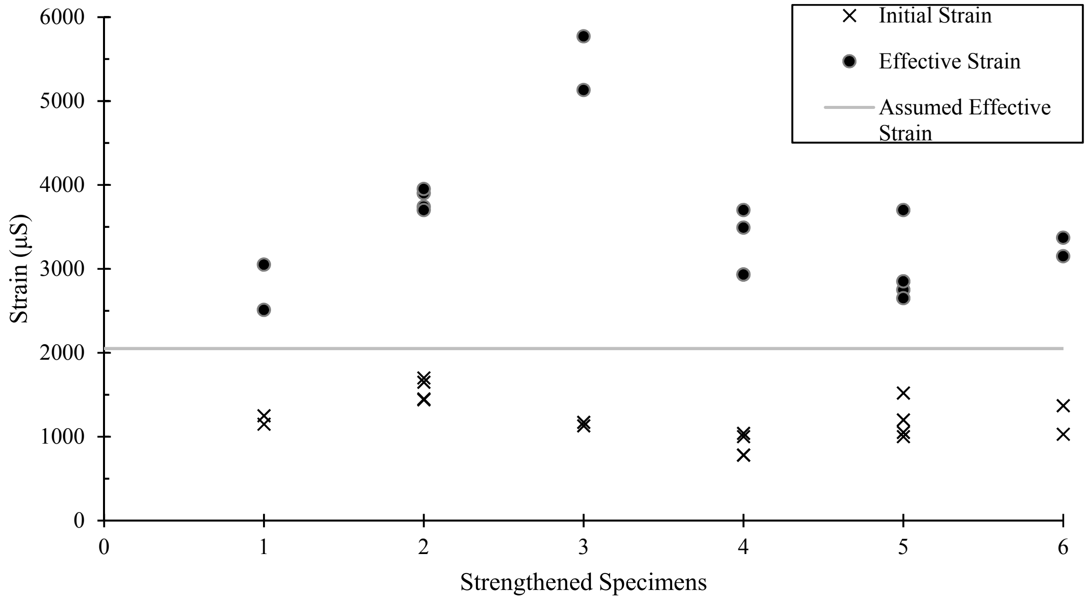


Fig. 18. Initial strain and effective strain in the pre-tensioned rods for the strengthened specimens

CHAPTER 3

STRENGTH PREDICTION USING EXISTING MODELS

Given that RC corbels are short members of a low span to depth ratio, their behavior in shear and bending is more complicated than other concrete members that have a stress-strain compatibility behavior.

A. RC Corbels ACI Shear Friction Design

ACI 318-14 [2] requires corbels to be designed based on either the shear friction theory, or based on the strut and tie model. The ACI shear friction based method [2] assumes the capacity of the corbels to be the smaller of its shear friction capacity and its flexural capacity measured at the interface with the supporting column. The conventional shear friction theory assumes that the capacity of a concrete interface is equal to the horizontal forces developed by the clamping action of the steel bars multiplied by a friction coefficient, and it is calculated as follows:

$$V_{shear} = A_v f_y \mu \tag{1}$$

where A_v is the area of the reinforcement crossing the shear interface, f_y is the yield stress of the reinforcement, and μ is the shear friction coefficient.

The shear friction reinforcement used is assumed to reach yield but the interface capacity is also limited by the ability of the concrete to withstand those shearing stresses without crushing. In addition, research has shown [18] that the flexural stresses lower than the flexural capacity of the section do not affect the shear friction capacity of a concrete interface. Accordingly, using the ACI shear friction based method [2], RC corbels capacity is calculated as the smaller value of equations (2) and (3):

$$V_{shear} = \left(A_s f_{yd1} + A_h f_{yd2}\right)\mu < \left\{0.2f'_c bd; \left(480 + 0.08f'_c\right)bd; 1600bd\right\} \quad \text{(N, mm)} \tag{2}$$

$$V_f = \frac{M_n}{a} = \frac{A_s f_y jd}{a} \tag{3}$$

where f_{yd1} and f_{yd2} are the yield stresses of the main reinforcement and the horizontal stirrups respectively, and they are limited to 420 MPa.

B. RC Corbels Modified Shear Friction Equations

In addition to its influence on the max shear capacity, research has shown that the concrete compressive strength can also affect the shear friction capacity even when the shearing stresses do not reach the crushing stresses [19]. This influence is not taken into account in the conventional shear friction theory. Several researchers [19, 20, 21] have suggested the use modified shear friction equations for the design and strength prediction for RC corbels. Such equations take into account the effect of concrete compressive strength on the corbels capacity at low reinforcement ratios, and set different limits for the shear crushing capacity at high reinforcement ratios.

Using Mattock's modified shear friction equation [20], the capacity for monolithically cast RC corbels is also calculated as the smaller value of equations (3) and (4):

$$V_{shear} = 0.8\left(A_s f_{yd1} + A_h f_{yd2}\right) + 0.1f'_c bh < \min\{0.3f'_c bh, 17.4bh\} \quad \text{(N, mm)} \tag{4}$$

C. Influence of Eccentricity on Shear Friction Capacity in Corbels:

Mattock [18] tested the influence of eccentricity on the shear friction capacity of RC corbels at their interface with the supporting column. He examined specimens subjected to loads with varying eccentricities, and the failure was induced to occur at the interface with the column. The eccentricity had no significant influence in the mentioned tests. However, research has shown that the strength of corbels tends to decrease as the *a/d* ratio decreases even for low *a/d* ratios when flexural failures do not govern [4]. This is probably because failure in RC corbels does not typically occur at the interface with the supporting column, but usually through diagonal cracks developing at the area between the load location and the bottom of the interface with the supporting column, and the cracking pattern is affected by the *a/d*.

In practice, using ACI shear friction based method [2] for the design of corbels is considered conservative as long as the detailing requirements are satisfied. Using shear friction based designs for RC corbels, a minimum amount of horizontal stirrups is required, usually of an area half of that of the main reinforcement. In case not enough amount of stirrups are provided, corbels can fail in a premature diagonal failure. Such failure can prevent corbels from reaching the other assumed shear frictional or flexural capacities.

D. Strengthened Corbels Capacity Using ACI Shear Friction Design:

To account for the effect of the pre-tensioning force on the corbels capacity using the ACI shear friction based method [2], the pre-tensioning force can be assumed to cause a net compression force on the interface between the corbels and their supporting column. ACI 318-14 [2] allows shear friction interfaces with a permanent net compression to be designed by adding the net compression force to the assumed clamping force of shear friction steel reinforcement.

For the pre-tensioning rods, the effective pre-tensioning force at failure is used instead of the initial force in all the used corbels strength prediction equations. For the

DIN975 4.6 rods, the strain increase in the pre-tensioned rods at failure is expected to cause the pre-tensioning force in the rods to reach more than 85% of the ultimate capacity of the rods, accordingly the 85% value is used for the effective pre-tensioning force in the strength prediction equations. In addition, ACI 318-14 [2] limits yielding force of the reinforcement used in the shear friction calculations to 60 ksi, since achieving strains higher than 0.0021, due to the clamping action, is not guaranteed. However, such limit is not imposed on the flexural capacity.

Accordingly, the pre-tensioned corbels capacity can be calculated using the ACI method [2] as the smaller value of Equations (5) or (6).

$$V_n = \left(A_s f_{yd1} + A_h f_{yd2} + P_e\right)\mu < \min\left\{0.2f'_c bd;\left(3.5+0.08f'_c\right)bd;11.6bd\right\} \qquad \text{(N, mm)} \qquad (5)$$

$$V_n = \frac{M_n}{a} = \frac{A_s f_y jd_1 + P_e\, jd_2}{a} \qquad (6)$$

where P_e is the assumed effective pre-tensioning force at failure.

As shown in Table 5, the strengthened specimens in phase 1 had substantial strength increase but did not reach the capacity assumed by the ACI shear friction method [2]. This is possibly because with the addition of the pre-tensioned rods in phase 1, the existing minimum horizontal stirrups no longer satisfy the ACI detailing requirements [2].

For specimens in phase 2, the ACI method [2] was conservative, where those specimens had a higher concrete strength than specimens of phase 1.

E. AASHTO LRFD 2010 Shear Friction Design:

AASHTO's general shear friction design equation [22] includes a cohesion parameter added to the clamping shear friction force produced by the horizontal reinforcement. In addition to that, limits less conservative than the ACI 318 [2] values are imposed concrete shear crushing strength. However, for RC corbels design the cohesion contribution is taken as zero, and in addition, shear friction limits are more conservative. Accordingly, the design of corbels using AASHTO is very similar to design using ACI 318 [2] but with a more conservative upper limit, and the shear friction capacity for pre-tensioned corbels can be calculated similarly as follows:

$$V_{shear} = 1.4\left(A_s f_{y1} + A_h f_{y2} + P_e\right) < \min\{0.2f'_c bd, 5.62bd\} \qquad \text{(N, mm)} \qquad (7)$$

F. Mattock's Modified Shear Friction Equation:

For pre-tensioned corbels, the load capacity is taken similarly as the smaller value of equations (5) or (8):

$$V_n = 0.8\left(A_s f_{y1} + A_h f_{y2} + P_e\right) + 0.1f'_c bh < \min\{0.3f'_c bh, 17.4bh\} \qquad \text{(N, mm)} \qquad (8)$$

As a modification for this method, and in order to apply them to pre-tensioned corbels, similar assumptions are made as the ones used for the ACI method [2]; therefore, in the equations the pre-tensioning force was added to the yielding force of the passive regular reinforcement, and the same semi-empirical factors are used. This is based on work done by Mattock and Hawkins [17], which shows that net compression has similar influence on the shear friction capacity as that of the shear friction reinforcement.

G. Premature Diagonal Splitting Failure in Strengthened Corbels:

Based on the work of zsutty [23] the diagonal shear capacity of deep beams without stirrups can be approximated by equation (9). Mattock et al. [1] assessed the applicability of this equation for corbels without stirrups, and found a good correlation with test results of such corbels in the literature. To estimate the contribution of adding stirrups to the capacity of corbels Mattock et al. [1] used equation (10) for the design of their specimens.

$$V_{shd} = 8110\sqrt[3]{f'_{c}\rho\frac{d}{a}} \qquad \text{(MPa)} \qquad (9)$$

$$V = V_{shd} + A_{H}f_{y} \qquad (10)$$

A premature diagonal splitting failure might occur in regular RC corbels in case not enough horizontal stirrups are used. Such failure was observed in the two strengthened long corbels. Thus when strengthening corbels with top pre-tensioning rods, a possible premature failure should be taken into account, especially for long corbels specimens. To include the effect of the pre-tensioning force, the capacity can be calculated as follows:

$$V = V_{shd} + A_{H}f_{y} + P_{im} \qquad (11)$$

Where P_{im} is the initial pre-tensioning force at midsection.

H. Hagberg Strut and Tie Model

Another option used for the design of RC and pre-tensioned corbels is to use the strut and tie model. The corbel is modeled as a truss; the capacity in this case will be dependent on the strength of the struts, the ties, and the nodes.

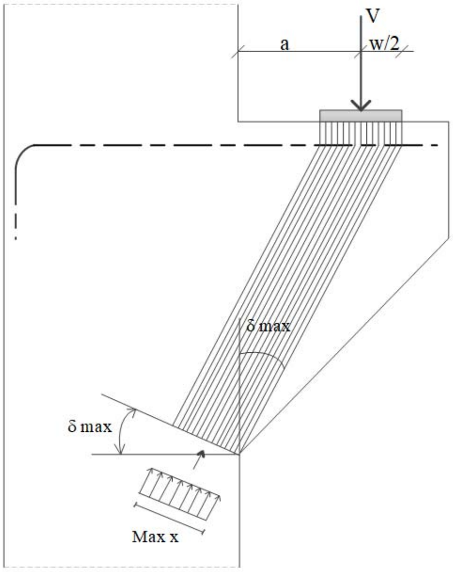


Fig. 19. Hagberg Strut and tie model [24].

Hagberg [24] suggested a simplified strut and tie model for RC corbels, the load capacity from the smaller value of equations (12) and (13) related to the strut and tie capacities respectively in:

$$V_n = \frac{bwf_{cd}}{1+\left(\frac{a}{d}+\frac{w}{2d}\right)^2} \tag{12}$$

$$V_n = \frac{\left(A_s f_y\right)}{tan\delta} \tag{13}$$

$$f_{cd} = \beta f'_c \tag{14}$$

where w is width of the loading plate, f_{cd} is the design concrete strength for the strut, δ is the angle of inclination of the strut, and β is a factor that reflects the shape of the strut and its confinement. Here, the value of β used is 0.75.

Based on ACI-14 318 [2] $\beta = 0.75$ is used for corbels satisfying the minimum required stirrups that act as strut confinement based on equation (15)

$$\sum \frac{A_{si}}{b_s s} \sin\alpha > 0.003 \tag{15}$$

For horizontal stirrups, A_{si}, is the area of distributed reinforcement at spacing s in the direction of reinforcement crossing a strut at an angle α to the axis of a strut, and b_s is the width of the strut. For struts not satisfying this requirement β of 0.6 is used. In addition, the code allows confined struts to have a higher f_{cd} if the confinement influence is documented by tests and analysis.

I. Pre-Tensioned Corbels Strut and Tie Models

To apply Hagberg strut and tie method to pre-tensioned corbels, the strut capacity is calculated using equation (13) similar to RC corbels; the tie capacity is assumed to be equal to the effective pre-tensioned rod force at failure added to the yielding force of the passive regular reinforcement. The load capacity based on Hagberg model is the smaller value of equations (13) and (16) related to the strut and tie capacities respectively:

$$V_n = \frac{\left(A_s f_y + P_e\right)}{tan\delta} \tag{16}$$

In order to better account for the effect of adding pre-tensioning rods at midsection on the corbels capacity, we need to account to their effect on increasing the strut capacity. Siao [25] and Kassem [26] used bottle shaped truss model to help account for this effect as shown in Figs. 20 and 21.

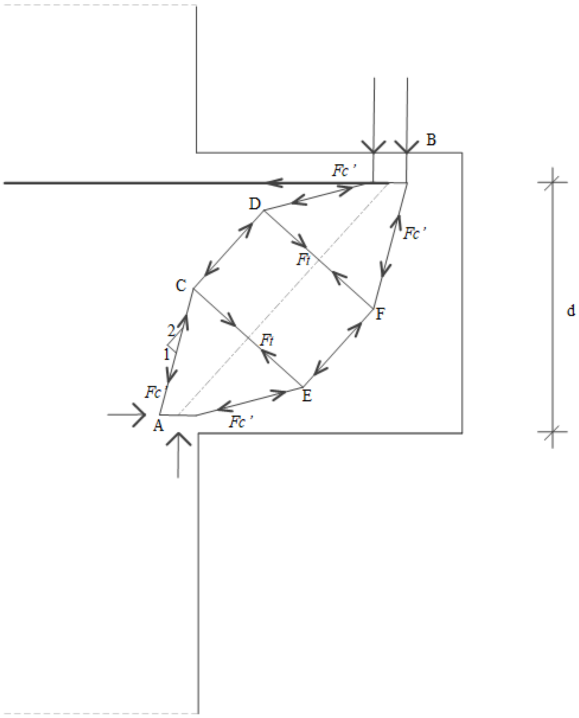


Fig. 20. Siao strut and tie model [24].

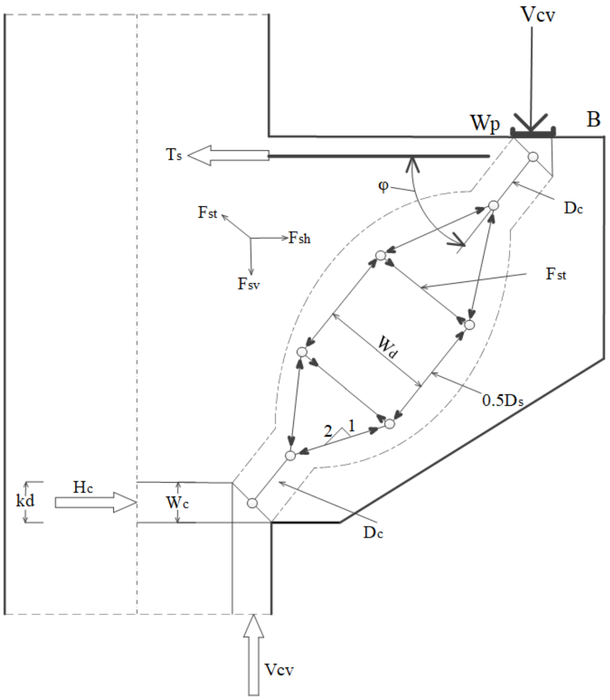


Fig. 21. Kassem strut and tie model [25].

Using Siao strut and tie model [25] the strut capacity is calculated based on equations (17) and (18):

$$V_n = 1.8\ f_t\ b\ d \tag{17}$$

where:

$$f_t = 0.52\sqrt{f'_c}\left[1 + n\left(\rho_h sin^2\delta\right)\right] \quad \text{(N, mm)} \tag{18}$$

f_t is the tensile strength of the reinforced concrete strut and it has two components: The splitting strength of concrete and the added strength provided by the stirrups.

To account for the effect of the added pre-tensioning force at midsection, equation (18) is modified into equation (19) as follows:

$$f_t = 0.52\sqrt{f'_c}[1+n(\rho_h sin^2\theta)] + \frac{P_{i\_mid} sin^2\theta}{bd} \qquad \text{(N, mm)} \qquad (19)$$

J. Comparison between the Strength Prediction Models

Table 5 presents a comparison between the strength predictions of the various equations for the tested specimens in this study. As shown in Figs. 22, 23, and 24, Mattocks equation provided better prediction for the strength of pre-tensioned corbels in the literature than the ACI [2] and ASHTOO [22]. ACI and ASHTOO were more conservative, especially for higher strength corbels. On the other hand, as shown in Table 5, Mattocks equation produced un-conservative strength predictions for the corbels tested in this study. To apply shear friction based method to strengthening RC corbels, pre-tensioned anchor rods must be added at midsection of the corbels. The area should be at least half of that of the area of the top pre-tensioned rods; this can guarantee that the strengthened corbels would reach their assumed shear frictional or flexural strength. In case not using midsection rods, the equation 11 could be used to produce conservative results. Strut and tie models by Hagberg and Siao showed conservative strength predictions for pre-tensioned corbels in the literature as shown in Figs. 25 and 26; however, this was not the case for all the specimens tested in this study (Table 5). Such variance could be attributed to the simplified nature of those two strut and tie models.

Table 5. Test Results vs. Strength Prediction

Specimen	**Actual Strength**	**Bending**	**ACI 318**	**Ratio**	**AASHTO**	**Ratio**	**Mattock**	**Ratio**	**Hagberg**	**Ratio**	**Siao**	**Ratio**
			[2]		[22]		[20]		[24]		[25]	
C1-0B	221.7	248.2	203.0	0.92	203.0	0.92	248.2	1.22	242.9	1.1	242.9	1.10
S1-1B	287.1	414.8	313.9	1.09	313.9	1.09	358.9	1.25	358.9	1.34	327.8	1.14
S1-2B	368.1	549.9	313.9	0.85	313.9	0.85	432.2	1.17	348.4	1.04	327.8	0.89
C2-0B	162.8	152.1	152.1	0.93	155.9	0.96	152.1	0.93	165.2	1.01	165.2	1.01
S2-1B	235.5	257.0	257.0	1.09	266.5	1.13	257.0	1.09	261.9	1.11	261.9	1.11
S2-2B	272.3	347.4	313.9	1.15	313.9	1.15	347.4	1.28	291.3	1.07	328.5	1.21
C3-0B	313.1	234.8	204.1	0.65	204.1	0.65	234.8	0.75	223.4	0.71	223.4	0.71
S3-1B	447.3	423.2	341.5	0.76	341.5	0.76	422.8	0.95	375.8	0.84	375.8	0.84
S3-1B-A	450.3	411.5	396.4	0.88	396.4	0.88	411.5	0.91	415.2	0.92	397.7	0.88
S3-1B-A1	445.6	411.5	396.4	0.89	396.4	0.89	411.5	0.92	415.2	0.93	397.7	0.89
Av. Ratio	-	-	-	**0.93**	-	**0.93**	-	**1.06**	-	**1.01**	-	**0.98**
COV	-	-	-	**0.16**	-	**0.16**	-	**0.18**	-	**0.17**	-	**0.16**

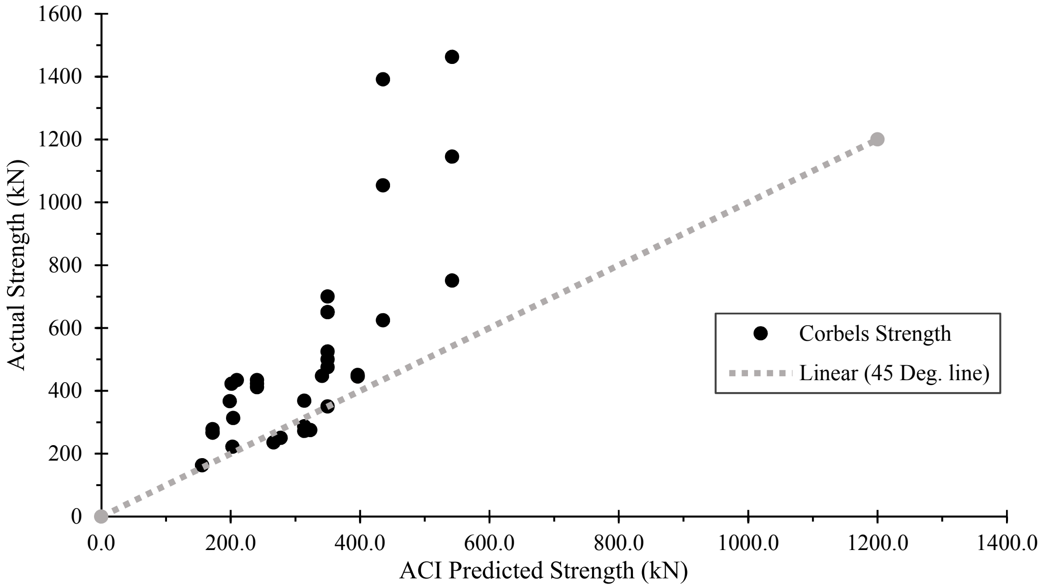


Fig. 22. Actual vs. ACI [2] Predicted Strength for pre-tensioned corbels experiments in the literature.

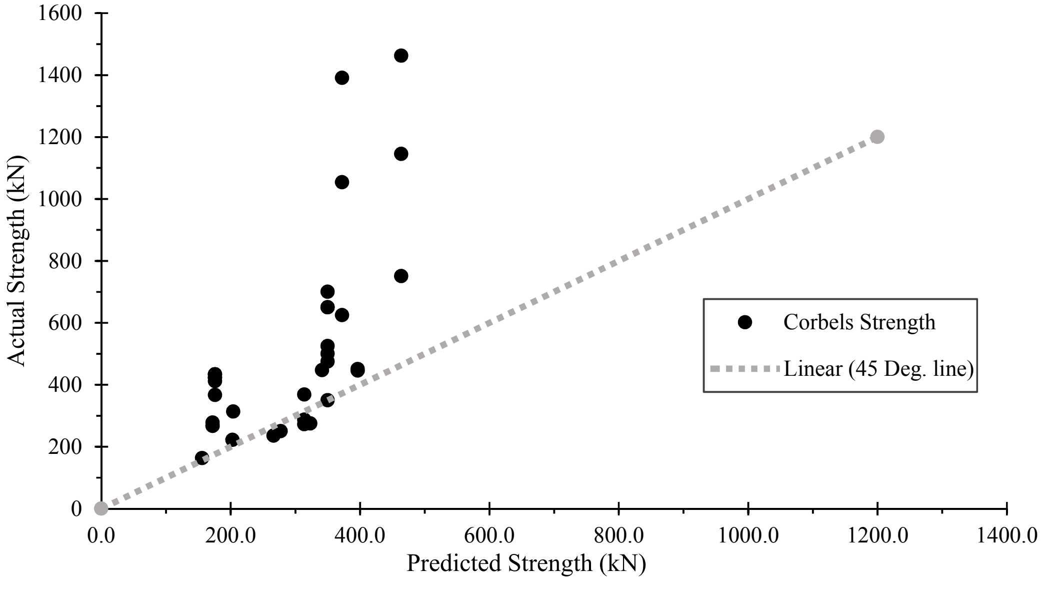


Fig. 23. Actual vs. AASHTO [22] Predicted Strength for pre-tensioned corbels experiments in the literature.

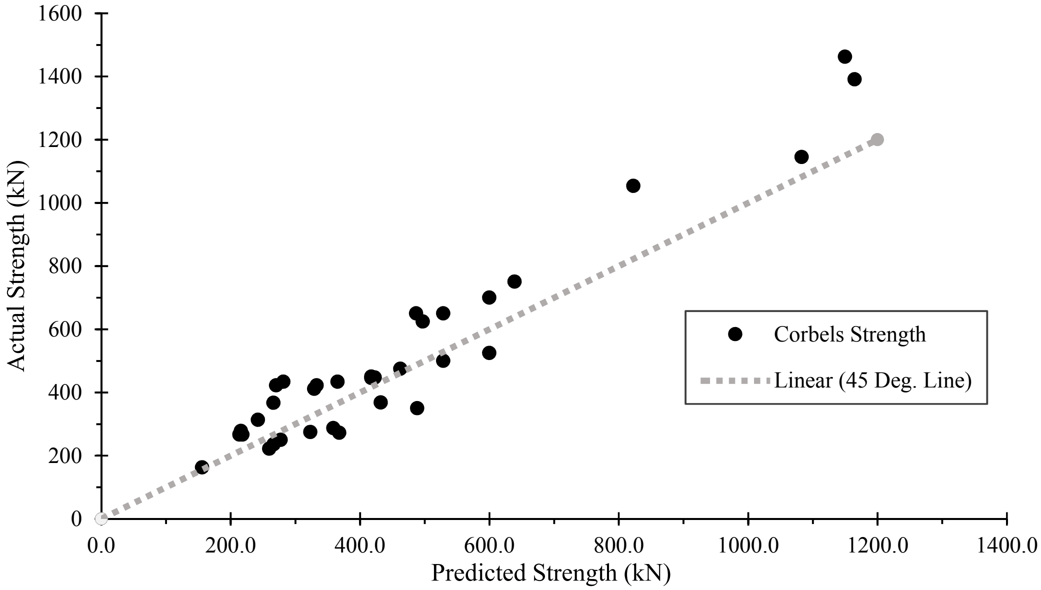


Fig. 24. Actual vs. Mattock [20] Predicted Strength for pre-tensioned corbels experiments in the literature.

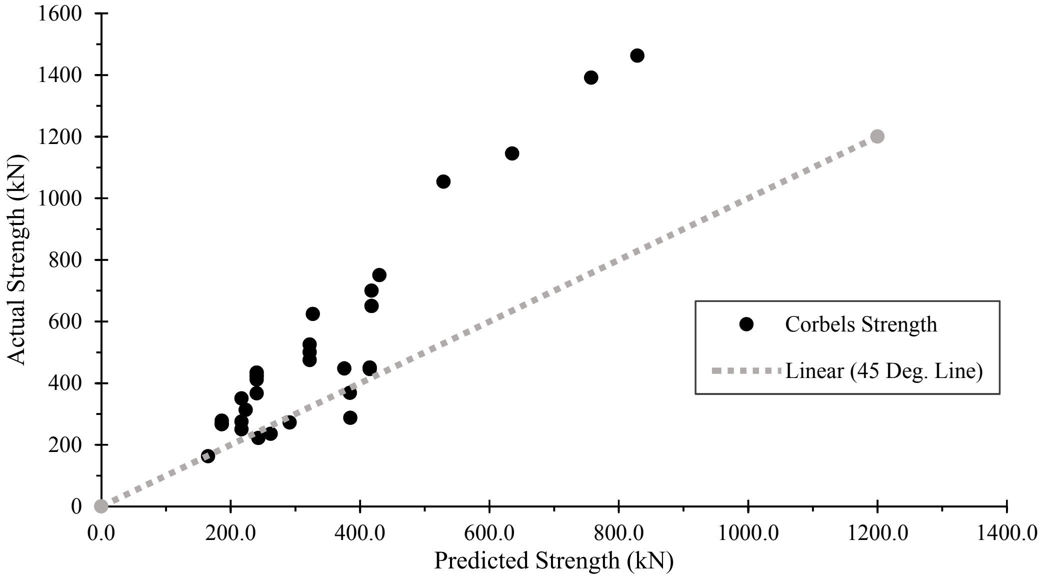


Fig. 25. Actual vs. Hagberg [24] predicted strength for pre-tensioned corbels experiments in the literature.

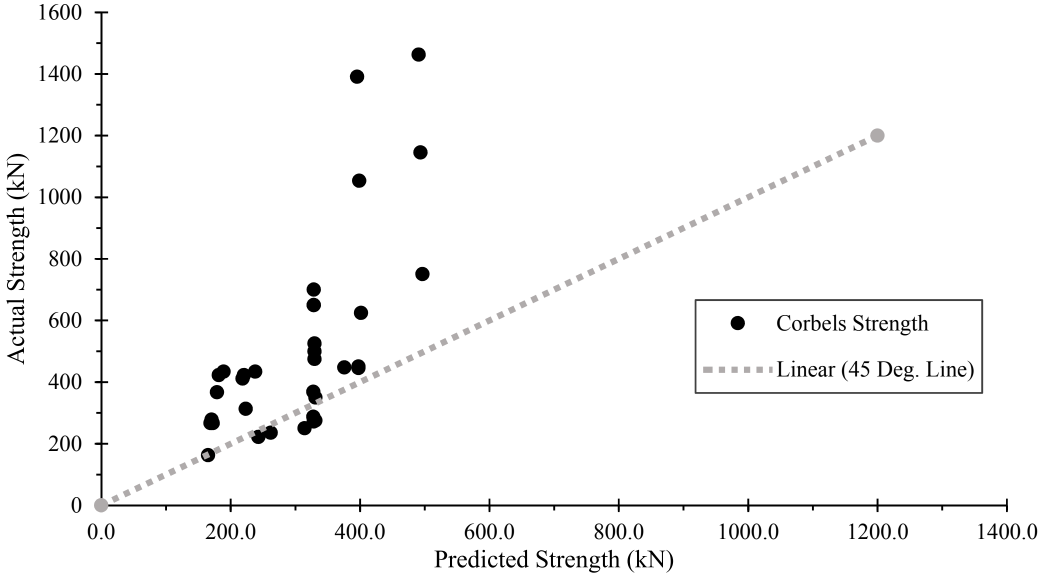


Fig. 26. Actual vs. Siao [25] predicted strength for pre-tensioned corbels experiments in the literature.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Strengthening RC corbels using pre-tensioned anchor rods can be used as a reliable strengthening method. It helps achieve significant strength improvements in these RC corbels' capacities, and improves their serviceability by reducing cracks at service loads. In the tested specimens a strength increase up to 67% was achieved compared to the as-built corbels, and this increase ranged between 70% and 170% of the added pre-tensioning force. This indicates the effectiveness of the strengthening method, and the potential to achieve higher strength increase for higher pre-tensioning forces. This strengthening method was found to be more efficient the higher the f'_c of corbels and the lower their reinforcement ratios. The efficiency of the applied force tended to decrease with the increasing pre-tensioning force. Analyzing experimental results showed that strength prediction using *Mattock's shear friction based method* has a good correlation with the results of pre-tensioned corbels experiments. However, when using this method for strength prediction of strengthened corbels, pre-tensioned rods might be required to be added at the mid-section. The area of those rods should be at least half of that of the added top rods to respect the detailing requirements. Strengthened corbels with a/d ratio of 0.68 tested in this study failed to reach the theoretical shear friction capacity based on ACI 318-14 and Mattock, indicating the importance of midsection reinforcement for long corbels. To calculate the capacity of corbels pre-tensioned from top only, modified zsutty's equation could be used to produce conservative results, especially for corbels with high a/d ratio. Other option for strength prediction of corbels is to use the strut and tie method. Their advantage is that they can allow more flexibility in designing the strengthening system. In addition, they can allow assessing the contribution of the added rods on strength increase in the struts, the ties, and the nodes. However, simplified strut and tie models might not be able to approximate the actual strength increase in strengthened corbels at various parametric conditions.

CHAPTER V

RECOMENDATIONS

More research is still needed on using pre-tensioned anchor rods for strengthening RC corbels, where the number of pre-tensioned corbels tested in the literature is still limited. Testing the influence of various parameters on the strength of pre-tensioned corbels can help recommend more reliable shear friction and concrete cohesion parameters for shear friction equations; also to examine the necessity of adding rods at the midsection of corbels of various a/d ratios. In absence of mid-section rods, more research is required to better predict the shear splitting capacity of those strengthened corbels. In case of using strut and tie methods for designing the strengthening system, more research is needed to improve existing models or to develop better representative ones. In relation to this, the effect of adding pre-tensioning force on ties, struts and nodes capacities can be further examined. The use of bonded pre-tensioned rods can be also examined. Bonding the pre-tensioned rods can have an influence on the strain increase in those rods, thus having a higher effective pre-tensioning force at failure. It can also influence the cracking pattern in corbels due to presence of dowel action. The use of inclined pre-tensioning rods can also be also examined. Urban et al. [15, 16] tested the use of inclined bonded passive anchor rods and they were found to be effective. Such rods, whether passive or pre-tensioned, can be applied for corbels with sloped exterior edge. They provide direct confinement for the inclined concrete strut and create additional paths for load transfer from the corbel to the supporting column. Better understanding for corbels failure mechanisms can also help in designing various strengthening system using frp or other strong materials.

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