AMERICAN UNIVERSITY OF BEIRUT

CAPTURING THE EFFECTS OF OIL PRICE UNCERTAINTY IN CARBON INTEGRATION NETWORK DESIGN

by
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Carbon integration is a novel concept that targets the recovery and allocation of industrially emitted carbon dioxide, CO_2, streams into CO_2-using sinks, with the goal of attaining a source-to-sink allocation strategy that meets a desired carbon dioxide emission reduction target, and an ultimate aim of minimizing the cost of the network, while maximizing any revenue attained.

Enhanced Oil Recovery, EOR, is considered one of the most attractive CO_2 sink options. CO_2 streams that are delivered and injected into EOR sites are often classified as great revenue sources for CO_2 supplying entities. Since oil pricing heavily affects the revenue generated from sending captured CO_2 streams into EOR sites, and since oil prices continuously vary, this paper studies the effect of oil price fluctuations onto the design of carbon integration networks. Hence, oil pricing has been selected as the main uncertainty parameter, and has been fed into a Linearized Multi-Period Carbon Integration model using stochastic data. Since oil prices vary periodically, this model has been formulated over several time periods, in which the oil pricing parameters are allowed to change over time. Subsequently, the proposed model has been optimized using two different approaches: (1) the Binomial Lattice approach, which primarily utilizes average uncertainties as expected values, and (2) the Multi-Scenario approach, which provides the variables' values as well as the main objective function of the model as a solution basis after accounting for different scenarios.

The performance of both methods has been analyzed and compared using random selection of different scenarios which involves simulating each scenario individually. The results obtained demonstrate that each approach has its own advantages and disadvantages. Sometimes, decision makers may find the information extracted from the average values provided by the Binomial Lattice approach to be suitable; other times, a more detailed set of solutions may be desired through the Multi-scenario approach. Hence, the proposed methods may be utilized whenever reckoned as seemingly fitting for decision-making circumstances.

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CHAPTER I

INTRODUCTION

A. Motivation and Objective

Most greenhouse gas, GHG, emissions in industry are primarily a result of burning fossil fuels, or natural gas material, for energy production in the form of either heat or power. In 2010, such activities accounted for about 65% of all global CO_2 gas emissions [1]. Several industrial processes also produce CO_2 emissions through chemical reactions that do not involve combustion activities. Many countries are realizing an imperative need for the industrial sector to manage their carbon footprints by enforcing carbon dioxide reduction targets on GHG emissions [1]. However, meeting such targets introduces numerous challenges especially for energy intensive industries.

Carbon integration aims to identify CO_2 capture, recovery and allocation schemes in the form of carbon dioxide networks by employing cost effective and revenue generating source-to-sink allocation strategies [2]. Carbon dioxide can be utilized in many different ways through the chemical or biological conversion to other value added products [3]. An industrial zone, which usually consists of a cluster of processing facilities within geographic proximity, could ideally incorporate many economic options for carbon dioxide converting processes which are often referred to as CO_2-using sinks. Many of those processes are very useful in converting the majority of carbon dioxide emissions that result from industrial activities into value-added products. Introducing those conversion routes greatly facilitate industrial symbiosis, which ideally involves the reuse of a generated waste stream as useful material in other processes that could potentially exist within an industrial cluster [4].

There exist a plethora of studies that focus on carbon dioxide allocation into geological storage, such as the work of Middleton and Bielicki [5] and Tan et al. [6]. While carbon dioxide allocation into storage tanks incurs additional cost onto a given system, CO_2-using sinks allow for revenue generating opportunities like the Enhanced Oil Recovery, EOR, sink [7]. It should be emphasized that revenue generating sinks often require high purity carbon dioxide streams, which are easily attainable by incorporating treatment units that are capable of separating CO_2 gas from the remaining gaseous emission material. Treatment costs are easily attainable, and depend on the technology adopted [2]. Another main cost factor to consider are transmission costs, which can be accounted for based on the geographic distances between emission sources and CO_2 sinks within the city, as well as the initial pressure of a given emission source and the required pressure conditions at the sink [2].

Hence, assessing capture costs, pipeline costs, and compression costs, as well as any CO_2 revenue options at the sink are vital for determining economically attractive CO_2 connectivity decisions [2]. When accounting for revenue aspects, any financial returns that may be generated from CO_2 sink options can vary greatly. For instance, oil prices in the range of 15-20 US$/barrel EOR have been able to generate revenue of 10 to 16 US$/ton of CO_2 injected [8]. When oil prices were reported to be 90 US$/barrel in 2015, the revenue generated upon injecting one ton of CO_2 grew up to 70 US$/ton of CO_2 injected [9]. With the continuous oil price fluctuations, the revenue from such sink options become quite uncertain.

This paper mainly aims to incorporate the effect of such uncertainty factors into carbon integration network design. Carbon integration networks were first introduced by Al-Mohannadi and Linke using a deterministic Mixed Integer Non Linear Programming, MINLP, model [2]. Since oil prices have been found to directly affect the corresponding

CO_2 sink price, and Enhanced Oil Recovery sinks were reported to be one of the greatest revenue generating sinks by Al-Mohannadi and Linke, this paper presents a stochastic model that captures oil pricing uncertainty in the course of identifying optimal design strategies of carbon integration networks.

B. Literature Review

Generally speaking, investigating uncertainty elements is crucial where they have severe impact on the model, the network configuration, and the system as a whole. Not accounting for uncertainties is like observing and dealing with the system on one specific point in time, whereas in reality, the system experiences many different cases and scenarios.

Uncertainties occur in Carbon Capture and Utilization and Storage, CCUS, systems, where they may be associated with CO_2 emission sources (e.g. source operating lifetime), or CO_2 sinks (e.g. sink storage capacity), as well as in water networks, where they may be associated with water quality levels and flow rates. Also, uncertainties can happen in energy networks, supply chain networks, etc.

Tan [10] finds an optimal solution for a fuzzy mathematical nonlinear programming model of a water network which includes uncertainties in the flow rates, quality levels of the source streams, and quality tolerances of stream sinks. The fuzzy uncertain parameters have lower and upper bounds. This optimal solution aims to achieve the minimum freshwater consumption while avoiding the adverse process effects[11]. Tan [10] presents three different case studies from literature along with their optimal solutions. The first two case studies are for a single-plant network with and without topological constraints, and the third case study is for inter-plant water integration network.

One article presents the design and optimization of a Carbon Capture, Utilization, and Sequestration, CCUS, supply chain network that addresses the issues of selecting the sources, the alternate capture technologies and materials, utilization of CO_2 sites, and sequestration in different storage sites [7]. It is a cost-model whose aim is to minimize the overall network cost where the cost includes the dehydration, capture, compression, transportation, and injection costs as well as the revenues generated from utilizing the CO_2. Hasan et al. [7] discusses the computational methods of each of the aforementioned costs as well the revenues generated from selling high-purity CO_2 to the CO_2-EOR sites which are the Enhanced Oil Recovery sites. In doing so, a Mixed Integer Linear optimization, MILP, model is developed to select the optimum network. This work concludes that the selection of the right material and capture technology is a crucial element that directly affects the overall network cost [7]. Furthermore a reduction in the cost is obtained by diversifying the selected sources between both utilization and sequestration. It has been reported that the higher the flow rates from the sources are, the lower the cost is, particularly the capture and compression costs. One important fact is that the CCUS cost is directly related to the minimum CO_2 reduction target; the higher the target, the higher the cost. Finally, and most importantly, this work concludes that it is possible to reduce more than 50% of the current CO_2 emissions from the sources in the United States at reasonable costs by implementing the CCUS networks, thus enabling the continual use of fossil fuels [7].

He et al. [11] propose a Mixed Integer Linear Programming, MILP, model that accounts for the following uncertain parameters: the ending time of the operating life of each source, the upper limit of carbon dioxide storage capacity of each sink, and the carbon footprint of compensatory power to make up for CCS energy losses. In doing so, He et al. [11] assume that those uncertain parameters can be represented as uniformly distributed parameters, each having a minimum and a maximum value incorporated into their model. According to their work, this model is referred to as the worst-case model. Subsequently, He et al. [11] propose a robust MILP model that incorporates uncertainty parameters as probability distributions. Upon developing the deterministic MILP model, the robust MILP model, and the worst-case model, their results show that uncertainties associated with those parameters greatly affect the CCUS network configuration, as well as the corresponding operating conditions. Although the deterministic model is able to provide feasible and optimal network configurations, the solutions generated from the deterministic model may turn out to be infeasible in real life practice. Hence, capturing those uncertainty elements is able to provide more realistic solutions of CCUS systems [11].

King et al. [12] discuss the economics of a CCUS network in which anthropogenic CO_2 captured from a fossil power plant is processed into the Enhanced Oil Recovery, EOR, sinks. In their study they define producers as electricity generation plants, whereas consumers as owners of the oil reservoirs [12]. King et al. [12] also explain how the oil reservoirs have been selected for their study, as they illustrate which plants have been considered as their carbon dioxide sources. After discussing their pipeline network as well as the different costs that should be taken into account, King et al. [12] develop a cash flow model that integrates CO_2 capture at the power plants, CO_2 transport through the pipelines, and CO_2 processing into the EOR sinks. Their goal is to achieve a 20% internal rate of return for each part of the process. It has been shown that there is a huge uncertainty in various parameters in their model, for example, in the capital cost of capture and EOR, oil prices, amount of CO_2 needed for each barrel of oil, etc. Monte

Carlo simulations have been used to conclude that EOR operations are more likely to yield more than 20% internal rate of return due to the uncertainty in the oil prices, that have been mainly expected to be higher, and uncertainty in the CO_2 prices which have been expected to be lower than the values used in their reference case [12]. Monte Carlo simulation is an iterative approach where the sample inputs vary probabilistically and consequently the model outputs are computed. Subsequently, these results are statistically analyzed [13]. Since a complex Monte Carlo simulation is required, this approach has not been adopted in this work.

Ahmed and Sahinidis [14] utilize a two-stage approach in which their variables have been categorized into two different sets. First stage variables, known as the design variables, are identified as variables that should be decided before the realization of the uncertain parameters. Second stage variables, known as the control or operating variables, are identified as variables that can be decided after the uncertain parameters have been embedded into the scenario. The objective, in such a case, is to minimize both first stage costs, and the expected value of the random second stage recourse costs. The only limitation of this approach is the fact that it only accounts for the expected value of second stage costs, while ignoring any other variations that might occur due to the realization of the uncertainties [14]. Thus, in an attempt to resolve this issue, a deviation term, referred to as the 'robustness measure', has later been incorporated into their model. In their paper, an alternative formulation that handles all nonlinearities which result from the use of this robustness measure term has been then proposed [14].

Ahmed and Sahinidis [14] also present a motivating example to illustrate the effects of the real values of the uncertain parameters, that in turn can lead to huge variations in the real cost value. To account for this variability, a goal programming approach is utilized, which in turn aims to minimize the total cost, both first stage and recourse costs, in addition to a weighted variability contribution. Their study also discusses two different frameworks: (1) a robust optimization framework which accounts for a risk measure in the objective function, and (2) a restricted recourse framework that accounts for the same risk measure in the constraints. Many different applications are presented in their study, but their main difficulty is the nonlinearity involved [14]. Due to the nonlinearity introduced into the model as a result of using variance as a robustness measure, and due to the fact that the variance is a symmetric risk measure which penalizes both costs, higher and lower than the expected recourse cost, Ahmed and Sahinidis [14] use an upper partial mean, UPM, that is an asymmetric measure of recourse costs variability. Their work continues to illustrate the optimization robustness of their approach, using the restricted recourse framework explained earlier, for a chemical process planning problem under uncertainty. In this problem, a new variable is added, so as to account for the positive deviation in a linearized fashion. Additionally, they develop a heuristic for the restricted recourse formulation, and finalize the discussion of their study using several scenarios [14].

Liu and Sahinidis [15] introduce a two-stage stochastic programming approach for a chemical process planning problem under uncertainty. Due to the large size of the model, Liu and Sahinidis [15] develop a Benders-based decomposition approach. This decomposition algorithm divides the Mixed Integer Linear model into two: an integer and a continuous component. Another feature of the Benders decomposition model is its ability to break the problem into small components. Liu and Sahinidis [15] explain the approach with using both discrete parameters as well as continuous random parameters [15].

Bidhandi and Yusuff [16] present a two stage stochastic programming model for a supply chain network design problem under uncertainty. First-stage decisions are configuration decisions, while second-stage decisions are those associated with processing and transporting products and materials from suppliers to customers, under uncertainty. The uncertain parameters are considered to be operational costs, customer demand, and facility capacities. Each of those parameters are associated with a log-normal distribution. Due to the large number of scenarios and due to the difficulty in evaluating the expected values in the objective function, Bidhandi and Yusuff [16] utilize a Sample Average Approximation, SAA, technique in their study, together with a Monte Carlo simulation, to determine approximations of expected values in place of finding real values. The objective function in this case is to minimize the total cost while satisfying the customer demands [16]. Furthermore, to improve the computational time, the accelerated Benders' decomposition approach is used in which the integer master problems are replaced by linear problems. This modified algorithm along with the surrogate constraints leads to much better and improved results compared to the original SAA approach [16].

Another paper that discusses a two-stage approach to plan for a Carbon Capture and Storage, CCS, network under uncertainty which includes CO_2 capture, transportation, storage, sequestration, and utilization is presented by Han and Lee [17]. In their study, Han and Lee [17] account for uncertainties such as CO_2 emissions, operating costs, and product prices. While their model accounts for an objective function that can either maximize profit or minimize cost, their paper addresses the case of maximizing the profit while meeting a certain carbon dioxide reduction target. Furthermore, their study experiments the effect of different sizes and scales of all facilities used in the network as utilization facilities, capture facilities, etc., as well as the effect of several CO_2 reduction targets over a long-term horizon. Their MILP model is capable of deciding where, how much, and how to capture, transport, utilize, store, and sequester carbon dioxide [17].

Following their multi-period deterministic model, Han and Lee [17] formulate a multi-period stochastic model, combined with an inexact two-stage stochastic programming. Subsequently, they compare it to other approaches like Multi-scenario stochastic programming, and a two-stage stochastic programming [17]. The uncertainties in operating costs and product prices, which are the coefficients of the objective function, have been modeled using a Multi-scenario stochastic programming approach, where each uncertain parameter is associated with a finite set of scenarios, each with a given probability of occurrence. Nevertheless, the uncertainties in CO_2 emissions have been formulated using a two-stage stochastic model using the expected scenario approach. This defines the inexact two-stage stochastic programming approach that combines both the Multi-scenario as well as the two-stage stochastic programming approaches [17].

An alternative approach is presented by Wang et al. [18] which utilizes the concept of a bi-random variable and adopts an Equilibrium Chance-Constrained Programming, ECCP, to model a CCUS system under uncertainty. A 'bi-random' variable is a parameter that has dual random characteristics. In other words, any random variable will follow a probabilistic distribution whose characteristic values also follow a random distribution. The equilibrium chance concept is used to compare the degree of occurrence of two bi-random events. In their study, it is selected over the other chance measures, like the primitive chance and average chance, because it is presented as a real number that facilitates the comparison and decision making process [18]. Wang et al. [18] assume that every bi-random variable has a normal distribution, with characteristic values (the mean) being random as well. First, the uncertain optimization model whose objective function is to minimize the total cost while satisfying different constraints, such as those related to environmental and capacity limits, must be formulated. Following this, the random constraints that consist of the bi-random variables must be converted into their deterministic equivalents using the Equilibrium Chance-Constrained algorithm. After solving the deterministic model, various optimal solutions can be generated, each based on a specific value of the probability-violation level. The results of the case study presented by Wang et al. [18] show that for a low constraint violation level, the model is more restricted, leading to a higher system cost, and a larger amount of treated CO_2 [18]. Decision makers can then select the best solution, whilst taking into account the trade-off between the profitability and reliability of the system [18].

Tan et al. [19] present a continuous time Mixed Integer Non Linear Programming model for a Carbon Capture and Storage, CCS, network whose objective is to maximize the reduction of CO_2 emissions by matching m-CO_2 sources to n-CO_2 sinks [19]. The nonlinearity in the model is due to the presence of some bilinear terms. Due to its computational difficulties, this model is linearized into a Mixed Integer Linear Programming model, MILP, by eliminating these terms. Then, Tan et al. [19] present two case studies to illustrate the importance of their model. An important assumption in their model is that the sources have fixed flow rates and operating lives, and the sinks have an earliest time of availability and a maximum CO_2 storage capacity [19]. The result of these assumptions is the primary focus of the model on physical and temporal aspects of CCS systems.

Compernolle et al. [20] apply continuous time real-options models coupled with dynamic programming concepts so as to define the investment threshold levels. The real-options models are split into two: one that focuses on the investment in a CO_2 capture unit, while the other focuses on the investment in EOR [20]. From the first model, the critical price level of CO_2 for which the CO_2 producer becomes willing to invest in a CO_2 capture unit is determined. Similarly, from the second model, the critical price level of oil for which the oil producer becomes willing to invest in EOR is also determined, for a given CO_2 cost [20]. According to Compernolle et al. [20], CCUS is not economically feasible due to the high investment cost required. However, one way that allows pursuing this technology is to effectively use the CO_2, for instance, in Enhanced Oil Recovery [20]. Assuming that CO_2 exchange will take place from a CO_2 producer to an oil producer, attractive price ranges for such transactions can be identified [20]. Uncertainties in oil and CO_2 prices are also addressed using a sensitivity analysis approach. In their paper, the minimum oil price needed to process the trade in CO_2 depends on several factors, such as the CO_2 permit price, the lifetime of the oil field, the rate of oil extraction per ton of CO_2 injected, and the discount rate [20]. The sensitivity analysis shows that a longer oil field lifetime results in a lower minimum oil price. For high CO_2 permit prices, electricity producers that invest in the capture unit must be willing to pay a fee to oil producers to store the CO_2; thus CO_2 input could also become revenue to oil producers according to Compernolle et al. [20].

The impact of various other uncertainty elements on Carbon Capture, Utilization and Storage, CCUS, problems are profound. Many previous work investigated the impact of those effects, using a variety of different techniques. For instance, Tan et al. [13] propose a two-stage approach that relies on the P-graph framework in the first stage to specify the n-best networks that are optimal and sub-optimal solutions. Such alternative solutions are then considered in the second stage, in which a Monte Carlo simulation is utilized to test for the system's sensitivity to changes in the parameters. Two different case studies are considered by Tan et al. [13]: (1) a carbon-constrained energy planning problem, and (2) a carbon dioxide capture and storage planning problem between sources and sinks. After arbitrary optimal and near-optimal solutions have been obtained from both stages, the P-graph framework has then been used to test the robustness of such attained alternatives to parameter variations using Monte Carlo simulations.

Subsequently, the best network for implementation can then be selected. In the second case study, Tan et al. [13] assume a normal distribution for both the amount of carbon dioxide that can be captured from the sources and the total capacity of the sinks. Optimizing the model using the P-graph resulted in 71 networks. Some of those solutions are optimal and the others are sub-optimal networks. These networks fail if the excess storage capacity of at least one sink is negative. The Monte Carlo simulation shown in this specific case shows that there is a high probability of failure of two networks, one of which is optimal and the other is sub-optimal. Thus, it has been found that the use of Monte Carlo simulations are beneficial to study the system sensitivity to perturbations of such parameters [13]. Their work addresses two case studies, one of which is the carbon integration network with uncertainty in the availability of carbon dioxide in the sources and in the storage capacity of the sinks [13].

In contrast to the work presented by Tan et al. [13], and all other previous contributions that were discussed above, this work studies a different uncertainty variable, in the form of oil pricing, and investigates its effect on carbon integration network design problems.

CHAPTER II

BACKGROUND

Based on the literature review discussion presented above, it has been realized that only certain mathematical techniques can be utilized to model oil price uncertainty. For instance, Ross [21] concludes that the continuous-time geometric Brownian Motion Model, which is an extensively used approach that is often utilized for modeling stock prices of real assets, cannot be used to model stochastic oil prices. This is due to a key assumption that is required by this model, which states that the future pricings are independent of past prices and past price movements. Therefore, having a general understanding of such appropriate methods that can accurately capture and model oil pricing uncertainty is crucial. This chapter illustrates two approaches: (1) Risk-Neutral uncertainty model or models that use data, and (2) Chance Constrained Programming.

A. Risk-Neutral uncertainty model or models that use data

Ross [21] discusses many methods for modeling the oil price uncertainties, such as the Risk-Neutral model. This method can help identify whether a certain option is underpriced or overpriced with respect to a current price of the security itself [21]. This model assumes, being at any state i, the log ratio of the next state will be a random variable with a normal distribution with a mean μ_i, and a standard deviation s_i that are related according to Equation (1), where: r is the interest rate, and N is the number of trading days in a year (taken to be equal to 252 days).

$$\mu_i = {}^{r}/_{N} - {}^{s_i^2}/_{2} \tag{1}$$

If this type of model is to be adopted, a separate simulation would be required to find the expected worth of an option. Hence, to avoid simulation models, oil prices will not be modeled using the Risk-Neutral model in this paper. On the other hand, if the aim is to value an option, one could use a model that assumes that the future will tend to follow the past. In such a case, the model would then assume that currently being at any state i, the logarithm of the ratio of tomorrow's price to today's price is a random variable that is normally distributed using a mean $\bar{x}_i$ value and a standard deviation s_i value, which in turn would require a certain computation process so as to obtain those values based on given data.

Other methods also exist, which mainly rely on a bootstrap approach instead of a normality assumption, for which the future is dependent on the past [21]. This latter approach assumes that the best approximation for the log distribution ratio of a certain state is to randomly choose one of the data values. In both cases, a separate simulation is required to be able to determine the expected value of a future price.

B. Chance Constrained Programming

In addition to all aforementioned approaches, the Chance Constrained Programming is also an alternative modeling method, which in turn combines a mathematical programming model with chance constraints in the form of probability levels of attainment [22]. For instance, if α is a confidence level that is predetermined by the decision maker, then a constraint will be satisfied at least α times of all the possible cases [22]. A typical mathematical programming model often follows the following structure:

$$Max\ f(x) \tag{2}$$

$$s.t. Ax \leq b \tag{3}$$

Chance constrained models can be utilized in many applications, usually involving financial planning, portfolio selection models, blending applications, agent recruitment planning, etc. [22]. The objective function $f(x)$ can be a profit function that needs to be maximized. It consists of n variables x, and it includes the profit contribution rate constants. There are m constraints in Ax, each of which is limited by constant b [22]. Charnes and Cooper present three formulations of the chance constraint models [22]:

1. Maximize the expected value of a probabilistic function

$$Max\ E[Y] \ \text{(Where } Y = f(x)\text{)} \tag{4}$$

$$s.t.\ \Pr(Ax \leq b) \geq \alpha \tag{5}$$

Any of these coefficients, $Y, A, or\ b$ may be probabilistic. This formulation renders the maximization (or minimization) of a function and guarantees that a constraint is met at least α times. While the expected value of a function is often linear, chance constraints are usually nonlinear. This formulation is recommended when the target is to optimize the objective function while staying within the limits of the resources at a certain probability level [22].

2. Minimize the variance

$$Min\ Var\ [Y] \tag{6}$$

$$s.t.\ \Pr(Ax \leq b) \geq \alpha \tag{7}$$

This formulation is usually applied to identify the portfolio investments with the minimum variance while satisfying the set of chance constraints. It is often used to measure the risk associated with a certain activity, which is not the case in this paper.

3. Maximize the probability to satisfy a chance constraint set

$$Max\ \Pr(Y \geq target) \tag{8}$$

$$s.t.\ \Pr(Ax \leq b) \geq \alpha \tag{9}$$

This formulation is generally much more difficult to accomplish [22]. The structure of the chance constraints in the form of probability levels renders it a nonlinear set which requires a nonlinear programming solution. This nonlinearity limits the size of the model; as such, it can no longer yield a solution in case a large number of constraints or variables is involved [22]. As a result, this approach is not adopted in this work.

CHAPTER III

METHODOLOGY

The deterministic model presented by Al Mohannadi and Linke [2] that represents the carbon integration network has been utilized as the base model in this study. The base model combines a set of equations that mathematically describe the capture process of carbon dioxide sources, in both treated or untreated forms, followed by a series of pressurization stages (compression and pumping), then transportation of CO_2 streams to the various sinks into which they are allocated [2]. The same model has also been extended in later studies, so as to account for multi-period considerations [23], and natural gas monetization strategies [24].

The objective function of the base model aims to minimize the total cost of the network, which is a combination of the following individual costs: the treatment cost to treat the captured CO_2 streams, the capital and operating costs of the compressors and pumps needed in each source-sink connection, the transportation cost to transport the treated and untreated flows from the sources to the sinks, and the processing cost of these streams into the sinks [2]. The deterministic model includes a set of constraints that must be satisfied, for instance, the capacity's limit of each sink, the available amount of CO_2 in each source, etc. Since the model involves several integer and binary variables, it was classified as a Mixed Integer Non Linear Programming, MINLP, model according to Al Mohannadi and Linke [2]. Since most parts of the model were already linear to begin with, only two modifications have been implemented onto the MINLP model adopted from Al Mohannadi and Linke. Hence, the MINLP has been linearized into a Mixed Integer Linear Programming, MILP, by implementing an alternative linear equation that describes the pumping cost. In addition, the highly non-linear diameter computations which were previously utilized in the transportation cost using if statements have been replaced with a linear alternative. Following this, the linearized deterministic model has then been converted into a stochastic multi-period one.

As explained earlier, the oil prices that are never certain, directly affect the processing cost into the EOR sink, which in turn affects the total cost. This is why it is crucial to capture the uncertainty and fluctuations of the oil prices by which the goal of this paper is achieved which is to find the optimal carbon integration network under the uncertain oil prices that vary randomly with the objective to minimize the total cost. Dealing with uncertain parameters is more realistic because uncertainty is with no doubt one of the most controlling phenomena. Our model's main target is to investigate the effect of having stochastic parameters, particularly the oil price, on the design of the carbon integration network as well as on the total cost/revenue of the network.

In this paper, the variability, uncertainty, and oil price fluctuations are first modeled using a Binomial Lattice model, which was developed by Luenberger [25]. This model is classified as a discrete-time approximation of the continuous-time geometric Brownian motion model [26]. This approach overcomes the independence assumption of the continuous-time geometric Brownian motion model, this is why it is the first chosen approach in this paper. Second, the uncertain oil prices are modeled using a Multi-scenario approach which considers real scenarios that may occur and averages the result over a subset of scenarios. This approach is heavily used since it takes into account realistic scenarios that might take place.

This chapter illustrates the two approaches: (1) the Binomial Lattice approach, and (2) the Multi-scenario approach to model the uncertain oil prices.

A. The Binomial Lattice Approach

The Binomial Lattice model is derived from the continuous-time geometric Brownian motion model that is widely used to model the stock price behavior. This model takes into consideration the quality of the oil being extracted and sold; the oil might be treated, liquefied, or both before being sent to the consumers [26]. These mentioned factors explicitly affect the selling price of the oil, which in turn will affect the revenue generated from carbon dioxide streams injected into these EOR sites. Melki [26] states that to model this uncertainty and fluctuations of the oil price, the respective data must be collected and the volatility and the expected growth rate of the oil price must be estimated. These data and estimates will then be used to model the oil prices using a Binomial Lattice model. Furthermore, oil treatment issues will not be accounted for. Instead, it was assumed that consumers are held responsible for treating the oil received, based on its respective use at the sink [26].

Generally speaking, the Binomial Lattice, which is a discrete-time model, is capable of capturing oil prices periodically. This means that if the price at the beginning of the period is S, then the price at the beginning of the next period will have one of two values. It will either go up by a factor $u > 1$ with a probability $0 < p < 1$ to be equal to uS, or it will go down by a factor $d < 1$, with a probability $1 - p$ to be equal to dS. Having S, u and d all positive then the price can never have a negative value which allows the consideration of the logarithm of the price as a variable [25]. Luenberger [25] defined the parameters of the Binomial Lattice model as follows:

- The expected yearly growth rate:

$$gr = E[\ln(S_T/S_s)] \tag{10}$$

Where S_T is the price at the end of the whole period (1 year), and S_s is the initial stock price;

- The yearly variance:

$$\sigma^2 = var[\ln(S_T/S_s)] \tag{11}$$

- The probability:

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{gr}{\sigma}\right)\sqrt{\Delta t} \tag{12}$$

Where σ is the yearly standard deviation and Δt is the period length that is really small.

- The factor associated with price increase:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{13}$$

- The factor associated with price decrease:

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{14}$$

An option is defined as the right to buy (a call option) or to sell (a put option) an asset under specified price and specified period of time. The option premium is defined as the price of the option itself which may be only a fraction of the price of the asset. This premium cannot be returned in case the option holder does not want to exercise the option. The term exercise the option is usually used when the holder actually buys or sells the asset by obeying the terms of the option [25].

It is required to specify the details of the option, which are often description of the asset, stating whether it is a call or a put option, the exercise price or the strike price, the expiration date stating if it is an American or European option, and the premium price. The strike price is the price at which the asset will be bought or sold when the option is exercised. The expiration date is the period of time during which the option is valid. There are two styles of the expiration date; the American option states that the option can be exercised anytime during this period until the last day, whereas the European option states that the option can only be exercised on the last day which is exactly the expiration date [25].

The procedure to find the parameters of the Binomial Lattice model requires collecting several data such as the risk-free interest rate, the current oil price, the oil futures prices and the oil futures options prices on a specific date. The daily treasury yield curve rates for October 2017 have been obtained from the U.S. Department of the Treasury [27]. Table 1 summarizes the relevant information. Assuming a maturity of 3 months for future options prices, the U.S. Treasury interest rate at 3 months becomes $r = 1.07\%$. The market price of crude oil has been considered as a spot market which has then been used to obtain the future oil prices. The corresponding data is provided in Table 2. The current price is taken to be the price of the hydrocarbon that'll be delivered the next month due to the delay that occurs in delivering the oil which is almost a month. Thus, the current crude oil price according to October 2017, as shown in Table 2, is $\$49.25/barrel$.

Table 1: U.S Daily Treasury Yield Curve Rates of October 2017[27] (Source: www.ustreas.gov)

Date	1 Mo	3 Mo	6 Mo	1 Yr.	2 Yr.	3 Yr.	5 Yr.	7 Yr.	10 Yr.
10/02/17	0.95	1.01	1.22	1.31	1.49	1.63	1.94	2.17	2.34
10/03/17	1.01	1.07	1.21	1.32	1.47	1.62	1.92	2.15	2.33
10/04/17	1.00	1.08	1.21	1.33	1.47	1.62	1.92	2.15	2.33
10/05/17	1.02	1.07	1.21	1.35	1.49	1.63	1.94	2.17	2.35
10/06/17	1.03	1.07*	1.22	1.35	1.54	1.66	1.97	2.20	2.37

*The treasury yield interest rate at 3 months maturity

Table 2: Crude Oil Futures Prices [29] (Source: Wall Street Journal on 08/10/2017)

Month	Last
Crude Oil – Electronic Nov 2017	49.25*

Crude Oil – Electronic Dec 2017	49.60
Crude Oil – Electronic Jan 2018	49.84
Crude Oil – Electronic Feb 2018	50.09
Crude Oil – Electronic Mar 2018	50.22
Crude Oil – Electronic Apr 2018	50.38
Crude Oil – Electronic May 2018	50.37
Crude Oil – Electronic Jun 2018	50.39
Crude Oil – Electronic Jul 2018	50.34
Crude Oil – Electronic Aug 2018	50.31
Crude Oil – Electronic Sep 2018	50.27
Crude Oil – Electronic Oct 2018	50.28
Crude Oil – Electronic Nov 2018	52.31**

*The current crude oil future price on November 2017

**the crude oil future price 1 year from now, on November 2018

In addition to the above information, future crude oil prices are needed to estimate the volatility of the oil price. In order to be able to use Black-Scholes equation, the data collected must correspond to a European call option with an expiration date on January 2018 (3 months maturity). The strike price taken from Chicago Mercantile Exchange [28] which was found to satisfy the aforementioned criteria is $50/barrel and the value of the option is $6.82/barrel.

Following the above data collection process, the Black-Scholes equation has then been used to calculate the implied volatility of the oil price. The call option Black-Scholes formula is stated in Equation (15), where: C is the price of the option, S_0 is the current future price, $N(x)$ is the standard cumulative normal probability distribution, K is the strike price, r is the risk-free interest rate, T is the expiration time, and σ is the volatility of oil price [25]:

$$C = S_0 N(d_1) - K e^{-r(T)} N(d_2) \tag{15}$$

$$d_1 = \frac{\ln(S_0/K) + \left(r + \sigma^2/2\right) * T}{\sigma\sqrt{T}} \quad (16)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (17)$$

It should be noted that the Black-Scholes equation was found to yield a volatility $\sigma = 72.46\%$.

Moving on, the two factors that represent the increase and the decrease (per period) of the oil price have been calculated using Equations (18) and (19) respectively.

$$u = e^{\sigma\sqrt{\Delta t}} = 2.064 \quad (18)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = 0.485 \quad (19)$$

Each time period, $\Delta t,$ was taken as 1 year. The probabilities associated with an increase (u) or decrease (d) in the oil price have been defined as q_u and q_d, respectively. Those two values have been determined by setting the current value of futures price to zero, as shown in Equations (20), (21), and (22) below.

$$q_u S[u - (1 + r')] + q_d S[d - (1 + r')] = 0 \quad (20)$$

$$q_u = \frac{(1 + r') - d}{u - d} = 0.365 \quad (21)$$

$$q_d = 1 - q_u = 0.635 \quad (22)$$

In the above equations, r' is defined as the rate of increase of the different oil futures prices, which is not the same as the risk-free interest rate, *r* [25]. Using the future oil prices highlighted in Table 2, r' has been calculated to be 6.2%, according to Equation (23):

$$r' = \frac{Crude\ Oil\ Future\ Price_{Nov\ 2018}}{Crude\ Oil\ Future\ Price_{Nov\ 2017}} = \frac{52.31}{49.25} - 1 \quad (23)$$

Upon collecting all the necessary data which is required to execute the multi-period Binomial Lattice model, it is essential to note the following:

1. Due to the limited information available, it has been assumed that the same values for the parameters (u, d, q_u, q_d) hold on for all the time periods.
2. Starting by the current price $S_0 = \$49.25$ at the end of the 0th period, the oil price has a probability q_u to increase and become $u * S_0$ and a probability q_d to decrease and become $d * S_0$. Similarly, the Binomial Lattice tree goes on with each node giving two new nodes.
3. At the end of every time period, the expected value is calculated using Equation (24) and is considered to be the value for the processing cost parameter, CRS, into the EOR sink in the model:

$$CRSperiod = \sum_{i \in I} p_i * op_i \tag{24}$$

Where p_i is the probability of oil price increase/decrease in time period i, and op_i is the oil price relative to this probability at the specific node in time period i (the summation must be carried out over all the nodes at each time period separately).

It should be noted that the main multi-period Binomial Lattice model was implemented using AMPL. However, since the expected value of the CRS parameter that is associated with the EOR sink must first be calculated, a separate MATLAB code was used to perform those calculations. The AMPL model file that has been used to implement the multi-period Binomial Lattice approach is provided in Appendix A. This model has been implemented using "MATLAB version R2017, and optimized using the CPLEX solver via AMPL, on a laptop with Intel ® Core ™ i5-2410M, 2.30 GHz, 4.00 GB RAM, 64-bit Operating System". A flowchart that summarizes the main steps involved in the MATLAB code which has been utilized to iteratively calculate the expected CRS value for each time period is illustrated in Figure 1. The code starts by defining a Pascal matrix for n time periods, as well as a row vector to have the CRS values in after calculating these values using the for loops. The detailed MATLAB code is provided in Appendix B. All MATLAB codes have been implemented using "MATLAB version R2017 on a laptop with Intel ® Core ™ i5-2410M, 2.30 GHz, 4.00 GB RAM, 64-bit Operating System". Following the execution of the MATLAB code, the eight expected CRS values attained have then been exported into AMPL and have been used as input data for the EOR sink in each of the eight time periods in the carbon integration multi-period Binomial Lattice model.

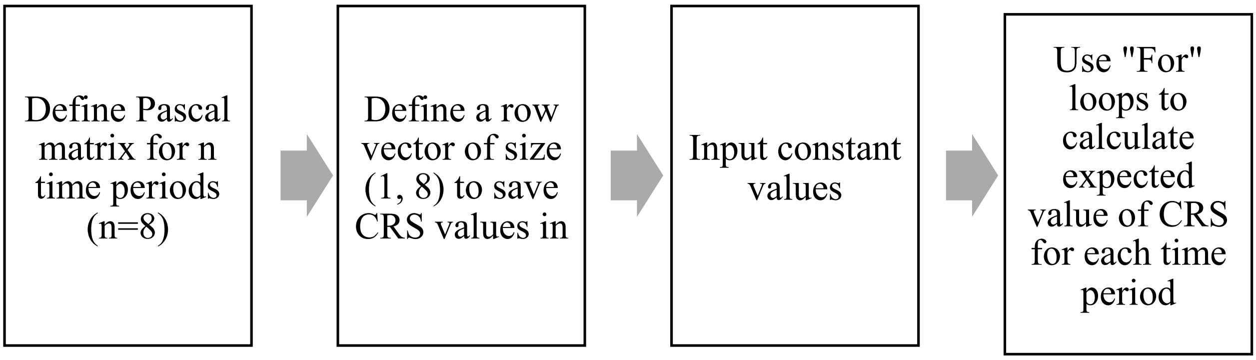

Figure 1: Steps in MATLAB code to calculate expected values of CRS for each time period for the Binomial Lattice approach

In summary, Figure 2 outlines the main sequence of steps that have been utilized to execute the Binomial Lattice approach. As it has been discussed above, the MATLAB code is first used to calculate the expected values of CRS parameters for each time period, then these values are used as input data into the optimization model to obtain the optimal solution for the Binomial Lattice multi-period approach.

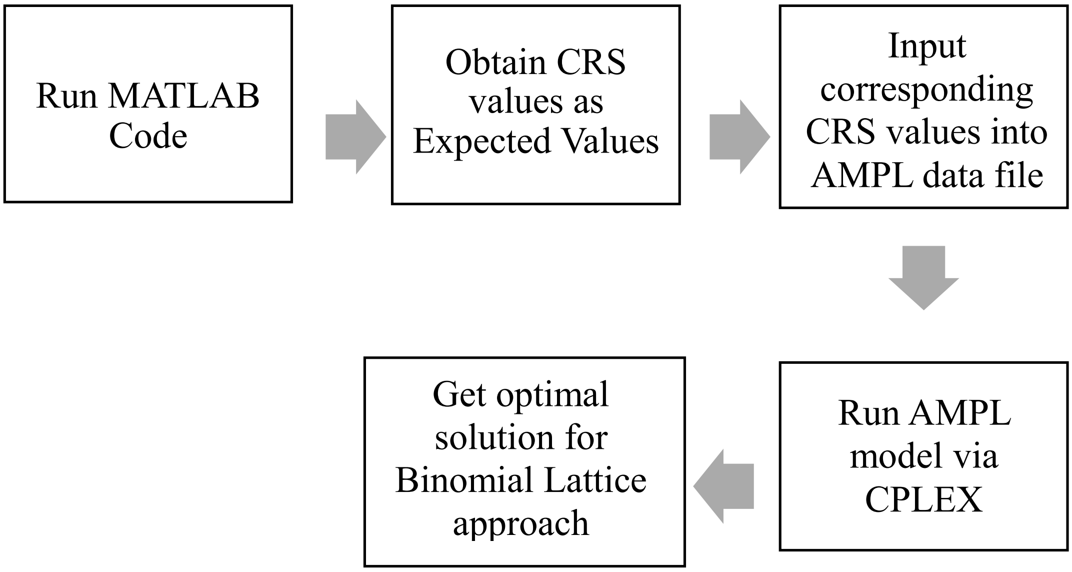

Figure 2: Sequence of steps utilized for the multi-period Binomial Lattice approach

B. The Multi-Scenario Approach

The second approach is Multi-scenario approach where each scenario consists of several time periods. This approach relies on oil prices either increasing or decreasing by the end of each time period, in each scenario. Using MATLAB version R2017, a matrix of all the possible scenarios is first created. Out of this matrix, a subset of random and unique scenarios are to be selected for uncertain oil prices that might occur over the time periods under study (8 different cases have been utilized in this study). This subset of scenarios, represented by binary numbers for either an increase (one) or a decrease (zero) in the oil price in each time period, is to be converted into another matrix to calculate the processing cost parameter, CRS, into the Enhanced Oil Recovery sink for each time period in every scenario to be used as data in the AMPL model. Then, this AMPL model is run to get the optimal solution for the Multi-scenario multi-period approach. This procedure is summarized in Figure 3. Moreover, the AMPL model, detailed in Appendix C, is a modified version that incorporates the multi-period Multi-scenario approach, which in turn relies on averaging the objective function (total cost) over all scenarios considered in the subset. This AMPL model has also been implemented and optimized using the CPLEX solver via AMPL, on a laptop with Intel ® Core ™ i5-2410M, 2.30 GHz, 4.00 GB RAM, 64-bit Operating System".

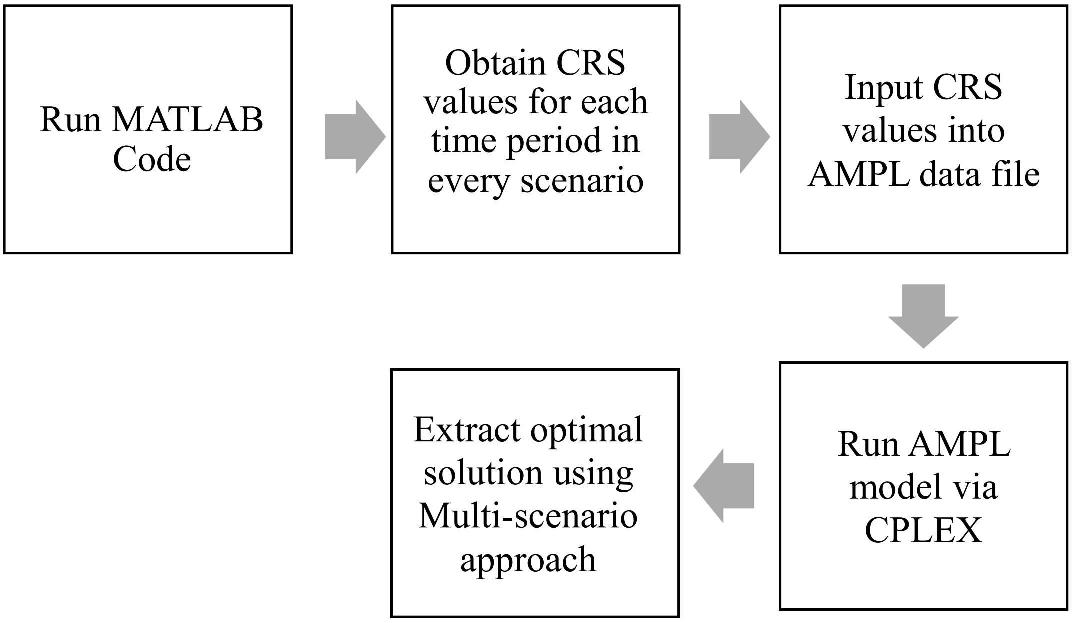

Figure 3: Sequence of steps utilized for the multi-period Multi-scenario approach

In order to run the formulated AMPL multi-period Multi-scenario model for a subset of random and unique scenarios as previously explained, the CRS values associated with the EOR sink must be calculated for each time period of these scenarios. The MATLAB code that is utilized to perform those calculations is summarized in Appendix D.

CHAPTER IV

RESULTS AND DISCUSSION

This chapter illustrates the main objective of this paper, describes the carbon integration network studied, the implementation of both approaches, and the detailed procedure to analyze their performance. It further analyzes the results obtained.

A. The Detailed Procedure

The presented optimization-based model is primarily useful for determining which CO_2 sources are the best to capture, and to which sinks they should be allocated, under uncertain sink revenue conditions. The model takes into account the treatment, compression and pumping of the streams needed to satisfy the sinks requirements. In order to illustrate the benefits of the proposed model, the same case study which was presented by Al-Mohannadi and Linke [2], has been revisited, for the purpose of illustrating the two different approaches which have been detailed above (1) the multi-period Binomial Lattice approach and (2) the multi-period Multi-scenario approach, for estimating the total cost of an optimum carbon integration network. The case study involves five different plants within an industrial cluster: namely an ammonia plant, an iron and steel plant, a refinery, a power plant, and a fuel additive plant. In total, there are four carbon dioxide source streams which have been considered: one stream from the ammonia plant, one stream from the steel plant, one stream from the power plant, and one stream from the refinery. Moreover, a total of six carbon dioxide utilizing sinks have been considered: (1) an algae sink, (2) a greenhouse sink, (3) a methanol sink, (4) a urea sink, (5) an EOR sink, and (6) a storage sink. All details pertaining to flowrate and composition data associated with the outlined source and sink streams are outlined in Al-Mohannadi and Linke [2].

The purpose of this section is to analyze and compare the accuracy of each described approach. Eight different time periods (assuming 1 year each) have been considered when conducting this analysis, and the net carbon reduction target, NCRT, has been set to 3%, over eight time periods. Depending on which specific scenario will be occurring, the actual total network cost can be calculated and compared to the estimated optimal total network cost, which was attained using the AMPL model. Hence, a MATLAB code is initially used to generate a subset of 100 random scenarios. Following this, two different MATLAB codes are used to calculate the actual total cost of each scenario after the realization of uncertain oil prices of this specific scenario. In this step, the first MATLAB code corresponds to the Binomial Lattice approach, while the second MATLAB code corresponds to the Multi-scenario approach. Once all the above steps have been completed, a total of 100 simulations have been executed, one at a time using each MATLAB code separately, and the actual total costs attained from each simulation have been extracted and reported. The cost information was found imperative at this stage, to be able to compare it with the estimated total network cost attained using each approach, respectively. Finally, the two approaches are compared.

To provide a better understanding of the procedure followed, the sequence of steps outlined above is demonstrated using an illustrative flowchart, Figure 4, which is shown below.

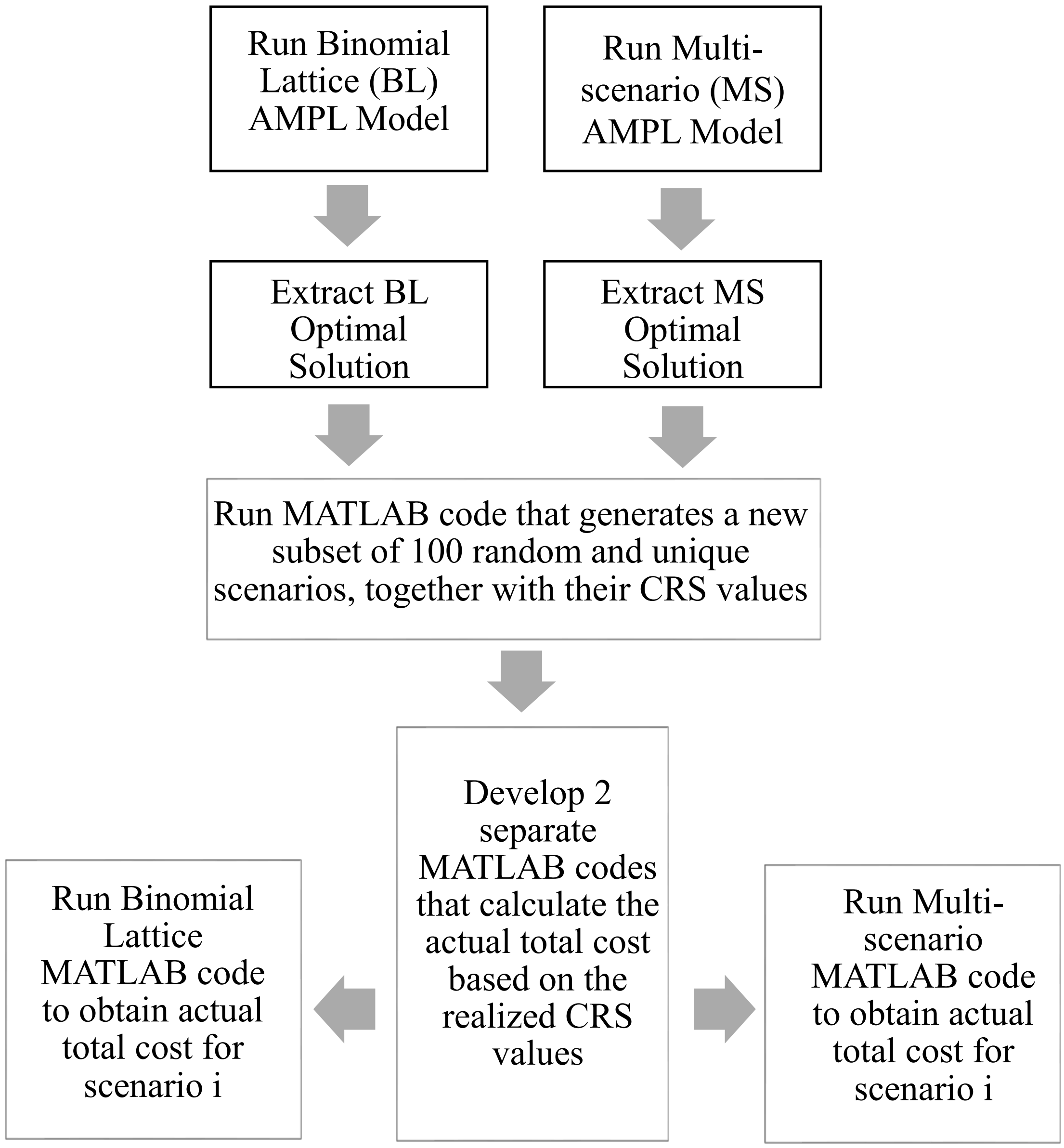

Figure 4: Summary flowchart

First, an optimal network solution is attained independently for each of the two approaches, using an AMPL model that calls the CPLEX solver. Then, 100 specific scenarios are generated using a MATLAB code along with their CRS values. Following the generation of those specific scenarios, each scenario is implemented separately using two independent MATLAB codes that calculate the actual total network cost; the first code follows the Binomial Lattice approach while the second follows the Multi-scenario approach. The two MATLAB codes which have been used to obtain the total actual cost for each scenario are discussed in sections B and C below.

B. Binomial Lattice MATLAB Code Description

The MATLAB code which was developed to calculate the actual total cost for each realized scenario using the Binomial Lattice approach starts off by inputting the fixed data values that were calculated optimally using the AMPL Binomial Lattice model. The $x_i's$ values are constant numbers obtained from the optimal solution:

$$Ctreatment = x_1 \tag{25}$$

$$CCAP = x_2 \tag{26}$$

$$COP = x_3 \tag{27}$$

$$POP = x_4 \tag{28}$$

$$Ctransportation = x_5 \tag{29}$$

Then, to calculate the pump capital cost, $PCAP$, for all source-sink connections over the eight time periods, four matrices are created, where each matrix is for a source and has all $PCAP$ for every sink, in which each row refers to a sink. The $PCAP$ value will simply be the summation of all elements of these matrices. These values y_{ij} for each matrix are also obtained from the optimal solution of the Binomial Lattice AMPL model. The matrix for source 1 is presented below, and in similar way, the matrices of the remaining sources are done:

$$S_1 = \begin{pmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \end{pmatrix} \tag{30}$$

Then, the elements are summed up along the columns:

$$S_1C = sum(S_1, 1) \tag{31}$$

Same steps are repeated for the remaining sources. Then, the resulting elements are added, which are the $PCAP$ value for each source:

$$PCAP = S_1C + S_2C + S_3C + S_4C \tag{32}$$

Next, to calculate the processing cost into the sinks, $Csink$, a matrix CS (k, Pd), where k is the number of sinks and Pd is the number of time periods is created and filled with zeros. Similarly, another matrix $FCO_2(k, Pd)$ is defined and filled with its corresponding values of the flow of CO_2 coming from all sources to each sink k in every time period Pd from the optimal solution of the AMPL model. Also, a third matrix $CRS\ (k, Pd)$ is defined and filled with its corresponding parameters of processing costs except for the Enhanced Oil Recovery sink whose parameters are randomly selected by the MATLAB code explained before. The last step is to fill in the CS matrix by using for loops (k=6, Pd=8)

$$CS\ (k, Pd) = 0 \tag{33}$$

$$FCO_2(6,8) = \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} & e_{18} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & e_{27} & e_{28} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} & e_{37} & e_{38} \\ e_{41} & e_{42} & e_{43} & e_{44} & e_{45} & e_{46} & e_{47} & e_{48} \\ e_{51} & e_{52} & e_{53} & e_{54} & e_{55} & e_{56} & e_{57} & e_{58} \\ e_{61} & e_{62} & e_{63} & e_{64} & e_{65} & e_{66} & e_{67} & e_{68} \end{pmatrix} \tag{34}$$

$$CRS(6,8) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} & r_{18} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} & r_{28} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} & r_{37} & r_{38} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} & r_{47} & r_{48} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{56} & r_{57} & r_{58} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & r_{66} & r_{67} & r_{68} \end{pmatrix} \tag{35}$$

$$for\ k = 1{:}\,6 \tag{36}$$

$$for\ Pd = 1{:}\,8 \tag{37}$$

$$CS(k, Pd) = FCO_2(k, Pd) * CRS(k, Pd) * 365 \tag{38}$$

$$end$$

end

$$CSC = sum(CS, 1) \tag{39}$$

$$CSR = sum(CSC, 2) \tag{40}$$

The total actual cost for a specified realized scenario with the Binomial Lattice approach is:

$$Tot_{cost} = Ctreatment + CCAP + COP + POP + Ctransportation + PCAP + CSR \tag{41}$$

C. Multi-scenario MATLAB Code Description

The MATLAB code which was developed to calculate the actual total cost for each realized scenario using the Multi-scenario approach starts off by inputting the fixed data values calculated optimally by AMPL Multi-scenario model. The $t_i's$ values are constant numbers from the optimal solution:

$$Ctreat = t_1 \tag{42}$$

$$CCAPT = t_2 \tag{43}$$

$$COPT = t_3 \tag{44}$$

$$PCAPT = t_4 \tag{45}$$

$$POPT = t_5 \tag{46}$$

$$Ctransportation = t_6 \tag{47}$$

Now, to calculate $Csink$ a matrix of size k rows and Pd columns is defined and filled with zeros:

$$CS(6,8) = 0 \tag{48}$$

Then, another matrix for FCO_2T is defined and its values are obtained from the optimal solution for the AMPL model which are the average flows of CO_2 coming to sink k in every time period Pd over all scenarios, each row refers to a sink k and each column refers to a time period Pd:

$$FCO_2T(6,8) = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} & s_{38} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} & s_{47} & s_{48} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} & s_{57} & s_{58} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} & s_{67} & s_{68} \end{pmatrix} \quad (49)$$

Now, a third matrix CRS of size k*Pd is defined and its values are obtained from the parameters of processing cost into sink k, except for the Enhanced Oil Recovery sink, whose values are those selected randomly by the MATLAB code explained earlier:

$$CRS(6,8) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} & r_{18} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} & r_{28} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} & r_{37} & r_{38} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} & r_{47} & r_{48} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{56} & r_{57} & r_{58} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & r_{66} & r_{67} & r_{68} \end{pmatrix} \quad (50)$$

Now, the CS matrix is filled by using for loops:

$$for\ k = 1{:}6 \quad (51)$$

$$for\ Pd = 1{:}8 \quad (52)$$

$$CS(k, Pd) = FCO_2T(k, Pd) * CRS(k, Pd) * 365 \quad (53)$$

$$end$$

$$end$$

Then, the total $Csink$ cost is calculated by summing up the individual elements of the CS matrix:

$$CSC = sum(CS, 1) \quad (54)$$

$$CSR = sum(CSC, 2) \tag{55}$$

Hence, the total actual cost (Tot_{cost}) for a specified realized scenario using the Multi-scenario approach may be obtained according to the following:

$$Tot_{cost} = Ctreat + CCAPT + COPT + PCAPT + POPT + Ctransportation + CSR \tag{56}$$

D. Analyzing the performance of the two approaches

The results of optimizing both approaches with a net carbon reduction target, NCRT, of 3% are presented in Table 3. Moreover, the resulting values of the flow of CO_2 coming from all sources to each sink in every period in the Binomial approach are presented in Tables 4 and 5, while those of the Multi-scenario approach are presented in Tables 6 and 7.

Table 3: Optimal Solution of both approaches

	Binomial Lattice	Multi-scenario
Objective Function	-372,266,106	-943,331,624
Net capture over all periods	21,850	15,627

Table 4: Flow of CO_2 coming from all sources to each sink in every period (periods 1 to 4) (Binomial)

	Period 1	Period 2	Period 3	Period 4
Algae	0	0	0	0
EOR	2,914	2,914	2,914	2,914
Greenhouse	0	0	0	0
Methanol	0	0	0	0
Storage	0	0	0	0
Urea	0	0	0	0

Table 5: Flow of CO_2 coming from all sources to each sink in every period (periods 5 to 8) (Binomial)

	Period 5	Period 6	Period 7	Period 8
Algae	0	0	0	0
EOR	2,914	2,914	2,914	2,914
Greenhouse	0	0	0	0
Methanol	0	0	0	0
Storage	0	0	0	0
Urea	0	0	0	0

Table 6: Flow of CO_2 coming from all sources to each sink in every period (periods 1 to 4) (Multi-scenario)

	Period 1	Period 2	Period 3	Period 4
Algae	0	0	0	0
EOR	2,082	2,232	1,829	1,889
Greenhouse	0	0	0	0
Methanol	5	198	102	324
Storage	0	0	0	0
Urea	0	0	0	0

Table 7: Flow of CO_2 coming from all sources to each sink in every period (periods 5 to 8) (Multi-scenario)

	Period 5	Period 6	Period 7	Period 8
Algae	0	0	0	0
EOR	1,683	1,884	1,696	1,732
Greenhouse	0	0	0	0
Methanol	169	324	247	382
Storage	0	0	0	0
Urea	0	0	0	0

The last step involved performing 100 simulations on both MATLAB codes, one scenario at a time, and the total actual cost is reported. All terms of the objective function take the optimal values obtained by AMPL model, except for the processing cost into the sinks which depends on the scenario chosen because it is based on the uncertain oil prices. First, 100 simulations of the Binomial Lattice approach are documented, followed by those of the Multi-scenario approach, which are outlined in Tables 8 and 9, respectively. It should be emphasized that the individual scenarios reported in Tables 8 and 9 represent specific cases that might occur, and are not solutions of the network problem itself. The

AMPL model has been primarily utilized to extract the optimal network solution upfront, but then, a single scenario will actually occur in reality, which would ultimately yield a different network cost compared to that attained from the AMPL model. It should be noted that the oil prices change very similarly in both approaches, and because a specific scenario is carried out individually each time, the exact oil price is associated with the respective scenario being carried out. The two different approaches have been compared in order to identify which of the two provides a better and a closer estimate of the network cost with that of the optimal case obtained from AMPL model.

Table 8: Results attained for 100 scenarios implemented using the Binomial Lattice approach

Scenario Number	Scenario	Total Cost ($)
1	0 1 0 0 0 0 0 0	54,129,000
2	1 1 0 1 1 0 1 0	-1,852,400,000
3	1 0 0 0 0 0 1 0	-35,427,088
4	1 0 1 1 1 0 0 1	-1,328,700,000
5	1 0 0 1 0 1 0 1	-214,113,568
6	1 1 0 1 0 1 0 0	-975,445,606
7	1 0 0 0 1 0 0 1	-73,823,409
8	1 1 1 1 0 0 0 0	-2,407,700,000
9	1 1 0 0 0 1 0 0	-427,367,373
10	1 0 1 1 0 1 1 0	-1,329,000,000
11	1 1 0 0 1 0 1 0	-633,601,352
12	0 1 0 0 0 1 1 0	8,819,088
13	0 1 1 1 1 1 0 1	-3,055,800,000
14	0 1 1 1 0 0 1 0	-550,958,855
15	1 1 0 1 1 0 1 1	-2,581,200,000
16	0 1 0 0 1 0 0 0	18,392,000
17	1 0 0 0 1 1 0 0	-133,598,291
18	1 1 1 0 1 0 1 0	-2,204,700,000
19	0 1 1 1 0 1 0 1	-893,228,553
20	0 0 1 1 0 1 1 0	-174,228,193
21	0 0 0 1 1 1 1 0	-114,559,672
22	0 0 0 0 1 0 0 1	120,923,582
23	0 1 0 1 1 1 0 1	-639,557,568
24	1 0 0 0 1 0 1 1	-133,704,652
25	0 0 0 0 0 1 0 0	127,730,686
26	1 0 1 0 0 0 0 1	-196,883,086
27	1 1 1 0 0 0 1 1	-1,327,900,000
28	1 0 0 0 1 1 1 0	-256,870,690
29	0 0 1 0 1 1 0 0	-10,751,336
30	0 0 1 1 1 0 1 0	-257,189,773
31	1 1 1 1 1 0 1 0	-7,392,500,000
32	1 1 0 0 1 0 0 0	-510,328,953
33	0 0 1 0 1 0 0 1	49,023,546
34	1 1 0 0 1 0 0 1	-550,639,772
35	0 1 1 1 0 1 1 0	-1,246,200,000
36	1 0 0 1 0 0 0 0	-104,561,738

37	1	0	1	1	0	0	1	1	-804,948,923
38	0	1	0	1	0	0	0	0	-21,812,880
39	1	1	0	0	0	0	0	1	-367,592,491
40	1	0	1	0	1	0	1	1	-634,133,157
41	1	1	1	0	0	0	0	1	-1,073,700,000
42	1	0	0	1	0	0	1	1	-173,909,110
43	1	1	1	1	1	0	0	0	-5,161,100,000
44	0	1	0	0	1	1	1	1	-345,363,042
45	1	0	1	1	0	0	1	0	-633,707,713
46	0	0	1	1	0	0	1	0	-10,751,336
47	0	1	1	0	1	0	0	1	-297,181,509
48	1	0	0	1	0	1	1	1	-468,316,358
49	1	0	0	1	1	1	0	0	-551,065,216
50	0	0	1	1	0	1	0	1	-91,266,613
51	1	1	1	1	1	1	0	0	-10,491,000,000
52	0	0	0	1	0	0	0	1	111,457,453
53	1	0	1	0	0	0	1	0	-216,453,510
54	0	1	1	0	0	1	0	1	-214,219,929
55	1	0	1	0	0	0	0	0	-187,416,957
56	0	1	1	0	0	1	0	0	-173,909,110
57	0	1	1	0	0	0	1	0	-133,704,652
58	0	1	0	1	1	0	0	0	-174,015,471
59	1	1	1	0	1	1	0	0	-2,932,700,000
60	0	0	0	1	0	0	0	0	113,691,034
61	1	0	1	0	0	1	0	0	-256,657,968
62	0	1	1	1	1	0	0	0	-1,074,700,000
63	0	0	1	0	0	1	0	1	68,487,609
64	1	0	0	0	0	0	0	1	-30,853,565
65	1	0	0	1	0	1	1	0	-297,075,148
66	1	1	0	1	1	1	1	0	-4,811,900,000
67	0	1	0	0	1	1	0	0	-50,849,000
68	0	1	1	1	0	0	1	1	-722,200,065
69	0	1	0	0	1	0	1	1	-50,956,000
70	0	0	0	0	1	0	0	0	123,157,163
71	0	1	0	0	0	1	0	1	28,389,512
72	1	1	0	0	0	0	1	0	-387,162,915
73	1	0	1	1	1	1	1	1	-7,742,700,000
74	1	0	1	0	0	1	1	0	-379,930,367
75	1	1	1	1	0	1	1	0	-5,891,500,000
76	0	0	0	1	1	0	0	0	77,953,738
77	0	0	1	0	1	1	1	1	-305,264,945
78	0	1	1	0	1	1	0	1	-722,412,787
79	1	0	0	0	0	1	0	0	-44,893,217
80	0	0	0	0	0	0	0	0	131,453,321
81	1	1	0	1	0	0	1	0	-804,417,118
82	1	0	0	0	0	0	0	0	-28,619,984
83	1	0	1	0	1	1	0	1	-805,161,645
84	1	0	0	0	1	1	1	1	-428,111,900
85	1	0	1	1	0	1	0	1	-975,977,411
86	0	0	0	0	0	1	0	1	125,497,105
87	0	0	0	1	1	1	0	0	8,712,727
88	0	0	1	1	0	1	1	1	-345,469,403
89	1	0	0	1	1	0	0	1	-297,075,148
90	1	0	0	0	1	1	0	1	-173,909,110
91	1	0	0	1	1	0	1	0	-380,036,728
92	1	0	1	1	0	1	1	1	-2,057,800,000
93	1	1	1	0	0	1	1	1	-2,580,800,000
94	1	0	0	1	1	0	1	1	-551,277,938

95	0 1 0 1 1 0 1 1	-468,529,080
96	1 0 1 0 0 1 0 1	-296,968,787
97	0 0 1 0 0 0 1 0	87,419,867
98	1 1 0 1 0 1 0 1	-1,146,700,000
99	0 1 0 1 0 1 1 0	-214,326,290
100	0 1 0 0 0 0 1 0	47,322,000

** 1 represents an increase in the oil price while 0 represents a decrease in the oil price, for each time period.*

Table 9: Results attained for 100 scenarios implemented using the Multi-scenario approach

Scenario Number	Scenario	Total Cost ($)
1	0 1 0 0 0 0 0 0	2,033,200
2	1 1 0 1 1 0 1 0	-1,182,600,000
3	1 0 0 0 0 0 1 0	-61,079,000
4	1 0 1 1 1 0 0 1	-846,430,000
5	1 0 0 1 0 1 0 1	-171,470,000
6	1 1 0 1 0 1 0 0	-671,690,000
7	1 0 0 0 1 0 0 1	-84,065,000
8	1 1 1 1 0 0 0 0	-1,568,000,000
9	1 1 0 0 0 1 0 0	-330,249,806
10	1 0 1 1 0 1 1 0	-848,230,000
11	1 1 0 0 1 0 1 0	-450,410,000
12	0 1 0 0 0 1 1 0	-25,094,000
13	0 1 1 1 1 1 0 1	-1,896,600,000
14	0 1 1 1 0 0 1 0	-371,290,000
15	1 1 0 1 1 0 1 1	-1,615,800,000
16	0 1 0 0 1 0 0 0	-19,316,000
17	1 0 0 0 1 1 0 0	-121,450,000
18	1 1 1 0 1 0 1 0	-1,403,800,000
19	0 1 1 1 0 1 0 1	-583,630,000
20	0 0 1 1 0 1 1 0	-129,930,000
21	0 0 0 1 1 1 1 0	-91,649,000
22	0 0 0 0 1 0 0 1	49,768,000
23	0 1 0 1 1 1 0 1	-420,890,000
24	1 0 0 0 1 0 1 1	-119,420,000
25	0 0 0 0 0 1 0 0	53,737,000
26	1 0 1 0 0 0 0 1	-162,140,000
27	1 1 1 0 0 0 1 1	-884,070,000
28	1 0 0 0 1 1 1 0	-193,700,000
29	0 0 1 0 1 1 0 0	-31,616,000
30	0 0 1 1 1 0 1 0	-177,840,000
31	1 1 1 1 1 0 1 0	-4,520,600,000
32	1 1 0 0 1 0 0 0	-378,160,000
33	0 0 1 0 1 0 0 1	5,764,300
34	1 1 0 0 1 0 0 1	-402,120,000
35	0 1 1 1 0 1 1 0	-789,110,000
36	1 0 0 1 0 0 0 0	-104,500,000
37	1 0 1 1 0 0 1 1	-532,200,000
38	0 1 0 1 0 0 0 0	-45,377,000
39	1 1 0 0 0 0 0 1	-292,870,000
40	1 0 1 0 1 0 1 1	-421,470,000
41	1 1 1 0 0 0 0 1	-733,990,000
42	1 0 0 1 0 0 1 1	-145,480,000
43	1 1 1 1 1 0 0 0	-3,212,700,000
44	0 1 0 0 1 1 1 1	-236,360,000

45	1	0	1	1	0	0	1	0	-430,410,000
46	0	0	1	1	0	0	1	0	-31,689,000
47	0	1	1	0	1	0	0	1	-212,270,000
48	1	0	0	1	0	1	1	1	-321,550,000
49	1	0	0	1	1	1	0	0	-378,220,000
50	0	0	1	1	0	1	0	1	-81,639,000
51	1	1	1	1	1	1	0	0	-6,523,400,000
52	0	0	0	1	0	0	0	1	43,632,000
53	1	0	1	0	0	0	1	0	-173,530,000
54	0	1	1	0	0	1	0	1	-164,360,000
55	1	0	1	0	0	0	0	0	-156,510,000
56	0	1	1	0	0	1	0	0	-140,400,000
57	0	1	1	0	0	0	1	0	-114,410,000
58	0	1	0	1	1	0	0	0	-136,290,000
59	1	1	1	0	1	1	0	0	-1,874,400,000
60	0	0	0	1	0	0	0	0	44,959,000
61	1	0	1	0	0	1	0	0	-199,520,000
62	0	1	1	1	1	0	0	0	-685,510,000
63	0	0	1	0	0	1	0	1	17,004,000
64	1	0	0	0	0	0	0	1	-58,417,000
65	1	0	0	1	0	1	1	0	-219,760,000
66	1	1	0	1	1	1	1	0	-2,961,100,000
67	0	1	0	0	1	1	0	0	-62,323,000
68	0	1	1	1	0	0	1	1	-473,070,000
69	0	1	0	0	1	0	1	1	-60,296,000
70	0	0	0	0	1	0	0	0	51,095,000
71	0	1	0	0	0	1	0	1	-13,703,000
72	1	1	0	0	0	0	1	0	-304,260,000
73	1	0	1	1	1	1	1	1	-4,674,000,000
74	1	0	1	0	0	1	1	0	-271,770,000
75	1	1	1	1	0	1	1	0	-3,653,700,000
76	0	0	0	1	1	0	0	0	23,610,000
77	0	0	1	0	1	1	1	1	-205,660,000
78	0	1	1	0	1	1	0	1	-472,900,000
79	1	0	0	0	0	1	0	0	-67,198,000
80	0	0	0	0	0	0	0	0	56,052,000
81	1	1	0	1	0	0	1	0	-561,140,000
82	1	0	0	0	0	0	0	0	-57,089,000
83	1	0	1	0	1	1	0	1	-532,030,000
84	1	0	0	0	1	1	1	1	-295,490,000
85	1	0	1	1	0	1	0	1	-642,750,000
86	0	0	0	0	0	1	0	1	52,409,000
87	0	0	0	1	1	1	0	0	-19,397,000
88	0	0	1	1	0	1	1	1	-231,720,000
89	1	0	0	1	1	0	0	1	-219,380,000
90	1	0	0	0	1	1	0	1	-145,410,000
91	1	0	0	1	1	0	1	0	-267,670,000
92	1	0	1	1	0	1	1	1	-1,281,400,000
93	1	1	1	0	0	1	1	1	-1,633,300,000
94	1	0	0	1	1	0	1	1	-369,460,000
95	0	1	0	1	1	0	1	1	-310,330,000
96	1	0	1	0	0	1	0	1	-223,480,000
97	0	0	1	0	0	0	1	0	28,750,000
98	1	1	0	1	0	1	0	1	-773,480,000
99	0	1	0	1	0	1	1	0	-160,640,000
100	0	1	0	0	0	0	1	0	-1,956,600

From the results attained, it has been observed that both approaches can be effective for decision-making, however, while both capture uncertainty using a multi-period base model, the two approaches differ in several aspects, especially in terms of the amount of random data it utilizes in the execution process. Hence, any decision maker should be aware of those differences. First off, it has been observed that the Binomial Lattice approach is able to provide an optimal multi-period carbon network solution that is based on average estimates of the CRS values, without taking into account the differences in network connectivity that could probably occur over the different time periods. On the other hand, the Multi-scenario approach can provide multi-period carbon network solutions, while accounting for network connectivity differences that may result due to the uncertain oil prices (which can either increase or decrease in each time period).

When comparing the CRS execution procedure of both methods, the Binomial model, involves a MATLAB code that calculates expected CRS values for every time period; thus only 8 CRS values have been utilized in this study as input values into the AMPL model for extracting optimal network solutions. On the other hand, the MATLAB code that was used to calculate the CRS values for the Multi-scenario approach results in 100 different scenarios, each of which consisting of 8 time periods; thus, in total 800 CRS values have been imported into the AMPL model for extracting optimal network solutions, averaged over all 100 scenarios. Thus, the Binomial approach uses expected CRS values that are calculated using the probabilities of the uncertain oil prices going up or down together with the new price predictions each case. On the contrary, the Multi-scenario approach utilizes a more rigorous approach for calculating CRS values, using data from various scenarios that might occur. Hence, the Multi-scenario approach calculates more exact CRS values given the current state of the oil price (i.e. whether it increased or decreased in this specific period of this specific scenario under study), then uses all attained CRS values to obtain the optimal network solution.

Analyzing the different results that are outlined in Tables 8 and 9 associated with every scenario, one can realize several differences. The AMPL model of the Binomial Lattice approach estimates a revenue of $ 372,266,106 as an optimal solution. Going through the individual scenarios, one can spot a case that generates a revenue of $ 10,491,000,000, differing by almost $ 10 Billion, compared to the optimal solution that was extracted through AMPL. Moreover, another odd case is reported, where the network is predicted to be highly un-profitable, with a total reported cost of around $ 131 Million. When analyzing the scenarios generated using the Multi-scenario approach, more realistic network cost estimates have been reported. The AMPL model reports an optimal solution with a suggested revenue of $ 943,331,624, which is obviously higher than the revenue suggested by the optimal AMPL solution obtained using the Binomial Lattice approach. This is most likely due to the fact that the Multi-scenario approach is able to generate different scenarios that might occur not only average values as it is the case with the Binomial approach. The most profitable scenario was reported to result in a revenue of $ 6,523,400,000 which differs by almost $ 5.5 Billion when compared to the estimated Multi-scenario solution generated by AMPL. Hence, almost half of the difference is reported using the Multi-scenario approach, compared to the Binomial Lattice case. Moreover, the most expensive scenario was reported at a total actual cost of $ 56,052,000, which is much more realistic than the most expensive case reported by the Binomial Lattice solutions.

CHAPTER V

CONCLUSION

Two different stochastic linearized multi-period models that can be utilized to generate optimal carbon integration networks have been proposed in this paper: (1) the Binomial Lattice model and (2) the Multi-scenario model. Ultimately, the goal was to be able to determine the best CO_2 source-to-sink allocations, under uncertain oil price conditions. Both approaches have been reported to be effective and easy to implement. However, several differences between the two methods have been reported. As for which approach to recommend, it has been found that it completely depends on the desired quality of information that could help achieve a viable and informed decision. In some cases, the use of average estimates provided by Binomial Lattice approach, may prove to be enough, while in other cases when a more detailed analysis is required, where the Multi-scenario approach might prove to be more rigorous in terms of the quality of network solutions that can be extracted under uncertain conditions. When it comes to decision-making activities, having a specified list of criteria before making any real decisions would certainly be useful, and it should be emphasized that exact same solutions may or may not be attainable in real situations, depending on the circumstances.

NOMENCLATURE

μ_i	The mean of a normally distributed random variable (log ratio of the next state)
s_i	Standard deviation of a normally distributed random variable
r	Interest rate (%)
N	Number of trading days in a year (taken to be equal to 252 days)
$\bar{x}_i$	The mean of a normally distributed random variable (logarithm of the ratio of tomorrow's price to today's price)
α	Confidence level (dimensionless)
$f(x)$	Objective function in chance constrained programming model
x	The decision variables in chance constrained programming model
A	Coefficients of the decision variables in chance constrained programming model
b	Constants in chance constrained programming model
$E[Y]$	The expected value of a probabilistic function
Y	The objective function f(x)
Pr	Probability (%)
$Var(Y)$	Variance of Y
u	The factor for the price increase (dimensionless)
p	Probability
d	The factor for the price decrease (dimensionless)
gr	The expected yearly growth rate
S_T	The price at the end of the whole period ($)
S_s	The initial stock price ($)
Δt	The period length (years)
C	The price of the option ($)
S_0	The current future price ($/barrel)
$N(x)$	The standard cumulative normal probability distribution
K	The strike price ($)
T	The expiration time (years)
σ	The volatility of the oil price ($/barrel)

q_u	The probability to have an increase in the oil price
q_d	The probability to have a decrease in the oil price
r'	The rate of increase of the different oil futures prices
p_i	The probability of oil price increase/ decrease in time period I
op_i	The oil price at a specific node in time period I ($/barrel)
Pd	Time period
$Ctreatment$	Positive, treatment and separation cost of CO_2 from source s to satisfy sink k's requirement over all periods ($)
$CCAP$	Positive, total capital cost of the compressors for all s-k connections over all periods ($)
COP	Positive, total operating cost of the compressors over all periods ($)
POP	Positive, total operating cost of the pump over all periods ($)
$Ctransportation$	Positive, total transportation cost over all periods ($)
S_1	The matrix for source 1
y_{ij}	The inputs for each matrix for each source
S_1C	The summation of the matrix S_1 along the columns
S_2C	The summation of the matrix S_2 along the columns
S_3C	The summation of the matrix S_3 along the columns
S_4C	The summation of the matrix S_4 along the columns
$PCAP$	Total pumping capital cost ($)
CS	The matrix that has CRS values
FCO_2	Flow of CO_2 coming from all sources to each sink k in every time period $ton\ CO_2/day$
CRS	Processing costs of CO_2 streams into the sinks ($)
CSC	The summation of CS elements along the columns
CSR	The summation of CSC elements along the rows
Tot_{cost}	The total cost ($)
$Ctreat$	Positive, Average treatment and separation cost of CO_2 from source s to satisfy sink k's requirement, over all periods and scenarios ($)
$CCAPT$	Positive, Average capital cost of the compressors, for all periods, all scenarios ($)

$COPT$	Positive, Average operating cost of the compressors, over all periods and scenarios ($)
$PCAPT$	Positive, Average capital cost of the pumps used in all periods and scenarios ($)
$POPT$	Positive, Average operating cost of the pumps over all periods and scenarios ($)
$FCO_2T[k, Pd]$	Positive, Average flow of CO_2 to sink k coming from all sources and treatment units in every time period, over all scenarios (ton CO_2/day)
s_{ij}	The inputs for the FCO2T matrix
r_{ij}	The inputs for the CRS matrix

APPENDICES

<u>Appendix A:</u> Linearized Optimization-based Model for the Binomial Lattice Approach:

The objective function, of the linearized optimization-based model for the Binomial Lattice, is to minimize the total cost subject to constraints that should be satisfied. This model is a modified version of the base model presented by Al-Mohannadi and Linke [2]. The total cost is a combination of several individual costs: treatment, compression, pumping, transportation, and processing costs. The constraints are mainly placed to avoid any violations on the limits of CO_2 available in the sources, and to satisfy the requirements and capacities of the sinks, the quality and the pressure of CO_2 processed into each sink, the merging of treated and untreated flows, and the net carbon reduction target to be achieved.

$$Min\ total\ cost = Min\ Ctreatment + CCAP + COP + \sum_S \sum_K PCAP[s,k] + POP + Ctransportation + Csink \quad (A.1)$$

Subject to

$$Qp[s,Pd] \geq LS[s] * w[s,Pd] \qquad \forall\, S, Pd \quad (A.2)$$

$$Qp[s,Pd] \leq R[s] * w[s,Pd] \qquad \forall\, S, Pd \quad (A.3)$$

$$Qp[s,Pd] = \sum_K \sum_T TF[s,k,t,Pd] * ET[t] + \sum_K U[s,k,Pd] \qquad \forall\, S, Pd \quad (A.4)$$

$$QQp[s,k,Pd] = \sum_T TF[s,k,t,Pd] * ET[t] + U[s,k,Pd] \qquad \forall\, S, K, Pd \quad (A.5)$$

$$R[s] * yr[s] \geq \sum_K \sum_T TF[s,k,t,Pd] * yu[s] + \sum_K U[s,k,Pd] * yu[s] \qquad \forall\, S, Pd \quad (A.6)$$

$$Fp[k,Pd] = \sum_S \sum_T TF[s,k,t,Pd] * ET[t] + \sum_S U[s,k,Pd] \qquad \forall\, K, Pd \quad (A.7)$$

$$Fp[k,Pd] * Zmin[k] \leq \sum_S \sum_T TF[s,k,t,Pd] * ET[t] * y[s,t] + \sum_S U[s,k,Pd] * yu[s] \qquad \forall\, K, Pd \quad (A.8)$$

$$FCO_2p[k,Pd] = \sum_S \sum_T TF[s,k,t,Pd] * ET[t] * y[s,t] + \sum_S U[s,k,Pd] * yu[s] \qquad \forall\, K, Pd \quad (A.9)$$

$$yu[s] = yr[s] \qquad \forall\, S \quad (A.10)$$

$$Fp[k,Pd] \leq Gmax[k] \qquad \forall\, K, Pd \quad (A.11)$$

$$Lp[s,k] * x[s,k] \leq \sum_T TF[s,k,t,Pd] * ET[t] + U[s,k,Pd] \qquad \forall\, S, K, Pd \quad (A.12)$$

$$Mp[s,k] * x[s,k] \geq \sum_T TF[s,k,t,Pd] * ET[t] + U[s,k,Pd] \qquad \forall\, S, K, Pd \quad (A.13)$$

$$PPUMP[s,k,Pd] = \frac{1000*0.01}{24*36} * \frac{QQp[s,k,Pd]*delppump[s,k]}{630*0.7} \qquad \forall\, S, K, Pd \quad (A.14)$$

$$PCOMP[s,k,Pd] = QQp[s,k,Pd] * SPP[s,k] * \frac{1000}{24} \qquad \forall\, S, K, Pd \quad (A.15)$$

$$Ctreatmentp[Pd] = \sum_S \sum_K \sum_T y[s,t] * TF[s,k,t,Pd] * ET[t] * CPT[s,t] * days \qquad \forall\, Pd \quad (A.16)$$

$$Ctreatment = \sum_{Pd} Ctreatmentp[Pd] \quad (A.17)$$

$$CCAPi[s,k] = 158902 * CRF * \left(\frac{PCOMP[s,k,Pd]}{224}\right)^1 \qquad \forall\, S, K, Pd \quad (A.18)$$

$$CCAP = \sum_S \sum_K CCAPi[s,k] \quad (A.19)$$

$$COPi[s,k,Pd] = 0.8 * PCOMP[s,k,Pd] * elec * days * 24 \qquad \forall\, S,K,Pd \tag{A.20}$$
$$COPp[Pd] = \sum_S \sum_K COPi[s,k,Pd] \qquad \forall\, Pd \tag{A.21}$$
$$COP = \sum_{Pd} COPp[Pd] \tag{A.22}$$
$$PCAP[s,k] = CRF * \left(\frac{1.11*10^6*PPUMP[s,k,Pd]}{1000} + zp[s,k,Pd]*0.07*10^6\right) \quad \forall\, S,K,Pd \tag{A.23}$$
$$PPUMP[s,k,Pd] \le 10000000 * zp[s,k,Pd] \qquad \forall\, S,K,Pd \tag{A.24}$$
$$POPi[s,k,Pd] = 0.8 * PPUMP[s,k,Pd] * elec * days * 24 \qquad \forall\, S,K,Pd \tag{A.25}$$
$$POPp[Pd] = \sum_S \sum_K POPi[s,k,Pd] \qquad \forall\, Pd \tag{A.26}$$
$$POP = \sum_{Pd} POPp[Pd] \tag{A.27}$$
$$P[s,k] = 4 * \frac{del[s,k]}{2} \qquad \forall\, S,K \tag{A.28}$$
$$QQp[s,k,Pd] \le ratio[s,k] * 2 * P[s,k] \qquad \forall\, S,K,Pd \tag{A.29}$$
$$del[s,k] \le 10000 * d[s,k] \qquad \forall\, S,K \tag{A.30}$$
$$Cpipe[s,k] = CRF * (96904 * d[s,k] + 95230 * 2 * del[s,k]) \qquad \forall\, S,K \tag{A.31}$$
$$Ctransportation = \sum_S \sum_K H[s,k] * Cpipe[s,k] \tag{A.32}$$
$$Csinki[k,Pd] = FCO_2p[k,Pd] * CRS[k,Pd] * days \qquad \forall\, K,Pd \tag{A.33}$$
$$Csinkp[Pd] = \sum_K Csinki[k,Pd] \qquad \forall\, Pd \tag{A.34}$$
$$Csink = \sum_{Pd} Csinkp[Pd] \tag{A.35}$$
$$NETCAPTUREp[Pd] = \sum_K FCO_2p[k,Pd] * (1 - EK[k]) - \sum_S \sum_K \sum_T ET[t] * TF[s,k,t,Pd] * y[s,t] * gamma[t] - \sum_S \sum_K (PPUMP[s,k,Pd] + PCOMP[s,k,Pd]) * EP * \frac{24}{1000} \qquad \forall\, Pd \tag{A.36}$$
$$NETCAPTURE = \sum_{Pd} NETCAPTUREp[Pd] \tag{A.37}$$
$$NETCAPTURE \ge NCRT \tag{A.38}$$

Appendix B: MATLAB Code for obtaining the CRS values for the Binomial Lattice

$$A(8,9) = 0 \tag{B.1}$$

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 & 0 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 & 0 & 0 \\ 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 & 0 \\ 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \end{pmatrix} \tag{B.2}$$

$$CRSperiod(1,8) = 0 \tag{B.3}$$

$$S_0 = 49.25 \tag{B.4}$$

$$u = 2.064 \tag{B.5}$$

$$d = 0.485 \tag{B.6}$$

$$q_u = 0.365 \tag{B.7}$$

$$q_d = 0.635 \tag{B.8}$$

$$for\ i = 1{:}\,size(A,1) \tag{B.9}$$

$$z = 0 \tag{B.10}$$

$$for\ j = 1{:}\,size(A,2) \tag{B.11}$$

$$z = z + A(i,j) * (q_u * u)^{(i-j+1)} * (q_d * d)^{(j-1)} * S_0 \tag{B.12}$$

$$end$$

$$CRSperiod(1,i) = -z \tag{B.13}$$

$$end$$

$$CRSperiod \tag{B.14}$$

<u>Appendix C</u>: Linearized Optimization-based Model for the Multi-Scenario Approach

The objective function, of the linearized optimization-based model for the Multi-scenario approach, is to minimize the total cost subject to constraints that should be satisfied. The model below is a modified version of the one presented by Al-Mohannadi and Linke [2]. This version of the model has been modified to incorporate the presence of different scenarios, unlike the model presented in Appendix A. The parameters, variables, and constraints have all been modified to account for each scenario that might happen.

$$Min\ total\ cost = Ctreat + CCAPT + COPT + PCAPT + POPT + Ctransportation + \sum_c \frac{Csink[i]}{c} \tag{C.1}$$

Subject to

$$Qp[s,i,Pd] \geq LS[s] * w[s,i,Pd] \quad \forall S,c,Pd \tag{C.2}$$

$$Qp[s,i,Pd] \leq R[s] * w[s,i,Pd] \quad \forall S,c,Pd \tag{C.3}$$

$$Qp[s,i,Pd] = \sum_K \sum_T TF[s,k,t,i,Pd] * ET[t] + \sum_K U[s,k,i,Pd] \quad \forall S,c,Pd \tag{C.4}$$

$$QQp[s,k,i,Pd] = \sum_T TF[s,k,t,i,Pd] * ET[t] + U[s,k,i,Pd] \quad \forall S,K,c,Pd \tag{C.5}$$

$$R[s] * yr[s] \geq \sum_K \sum_T TF[s,k,t,i,Pd] * yu[s] + \sum_K U[s,k,i,Pd] * yu[s] \quad \forall S,c,Pd \tag{C.6}$$

$$Fp[k,i,Pd] = \sum_S \sum_T TF[s,k,t,i,Pd] * ET[t] + \sum_S U[s,k,i,Pd] \quad \forall K,c,Pd \tag{C.7}$$

$$Fp[k,i,Pd] * Zmin[k] \leq \sum_S \sum_T TF[s,k,t,i,Pd] * ET[t] * y[s,t] + \sum_S U[s,k,i,Pd] * yu[s] \quad \forall K,c,Pd \tag{C.8}$$

$$FCO_2p[k,i,Pd] = \sum_S \sum_T TF[s,k,t,i,Pd] * ET[t] * y[s,t] + \sum_S U[s,k,i,Pd] * yu[s] \quad \forall K,c,Pd \tag{C.9}$$

$$FCO_2T\ [k,Pd] = \sum_c \frac{FCO_2p[k,i,Pd]}{c} \quad \forall K,Pd \tag{C.10}$$

$$yu[s] = yr[s] \quad \forall S \tag{C.11}$$

$$Fp[k,i,Pd] \leq Gmax[k] \quad \forall K,c,Pd \tag{C.12}$$

$$Lp[s,k] * x[s,k] \leq \sum_T TF[s,k,t,i,Pd] * ET[t] + U[s,k,i,Pd] \quad \forall S,K,c,Pd \tag{C.13}$$

$$Mp[s,k] * x[s,k] \geq \sum_T TF[s,k,t,i,Pd] * ET[t] + U[s,k,i,Pd] \quad \forall S,K,c,Pd \tag{C.14}$$

$$PPUMP[s,k,i,Pd] = \frac{1000*0.01}{24*36} * \frac{QQp[s,k,i,Pd]*delppump[s,k]}{630*0.7} \quad \forall S,K,c,Pd \tag{C.15}$$

$$PCOMP[s,k,i,Pd] = QQp[s,k,i,Pd] * SPP[s,k] * \frac{1000}{24} \quad \forall S,K,c,Pd \tag{C.16}$$

$$Ctreatmentp[i,Pd] = \sum_S \sum_K \sum_T y[s,t] * TF[s,k,t,i,Pd] * ET[t] * CPT[s,t] * days \quad \forall c,Pd \tag{C.17}$$

$$Ctreatment[i] = \sum_{Pd} Ctreatmentp[i,Pd] \quad \forall c \tag{C.18}$$

$$Ctreat = \sum_c \frac{Ctreatment[i]}{c} \tag{C.19}$$

$$CCAPi[s,k,i,Pd] = 158902 * CRF * \left(\frac{PCOMP[s,k,i,Pd]}{224}\right)^1 \quad \forall S,K,c,Pd \tag{C.20}$$

$$CCAPp[i,Pd] = \sum_S \sum_K CCAPi[s,k,i,Pd] \quad \forall c,Pd \tag{C.21}$$

$CCAP[i] \quad \forall\, c, Pd$ (C.22)

$CCAPT = \sum_c \frac{CCAP[i]}{c}$ (C.23)

$COPi[s,k,i,Pd] = 0.8 * PCOMP[s,k,i,Pd] * elec * days * 24 \quad \forall\, S, K, c, Pd$ (C.24)

$COPp[i,Pd] = \sum_S \sum_K COPi[s,k,i,Pd] \quad \forall\, c, Pd$ (C.25)

$COP[i] = \sum_{Pd} COPp[i,Pd] \quad \forall\, c$ (C.26)

$COPT = \sum_c \frac{COP[i]}{c}$ (C.27)

$PCAPp[s,k,i,Pd] = CRF * \left(\frac{1.11*10^6*PPUMP[s,k,i,Pd]}{1000} + zp[s,k,i,Pd] * 0.07 * 10^6\right) \forall\, S, K, c, Pd$ (C.28)

$PPUMP[s,k,i,Pd] \leq 10000000 * zp[s,k,i,Pd] \quad \forall\, S, K, c, Pd$ (C.29)

$PCAPi[i,Pd] = \sum_S \sum_K PCAPp[s,k,i,Pd] \quad \forall\, c, Pd$ (C.30)

$PCAP[i] \geq PCAPi[i,Pd] \quad \forall c, Pd$ (C.31)

$PCAPT = \sum_c \frac{PCAP[i]}{c}$ (C.32)

$POPi[s,k,i,Pd] = 0.8 * PPUMP[s,k,i,Pd] * elec * days * 24 \quad \forall\, S, K, c, Pd$ (C.33)

$POPp[i,Pd] = \sum_S \sum_K POPi[s,k,i,Pd] \quad \forall\, c, Pd$ (C.34)

$POP[i] = \sum_{Pd} POPp[i,Pd] \quad \forall\, c$ (C.35)

$POPT = \sum_c \frac{POP[i]}{c}$ (C.36)

$P[s,k] = 4 * \frac{del[s,k]}{2} \quad \forall\, S, K$ (C.37)

$QQp[s,k,i,Pd] \leq ratio[s,k] * 2 * P[s,k] \quad \forall\, S, K, c, Pd$ (C.38)

$del[s,k] \leq 10000 * d[s,k] \quad \forall\, S, K$ (C.39)

$Cpipe[s,k] = CRF * (96904 * d[s,k] + 95230 * 2 * del[s,k]) \quad \forall\, S, K$ (C.40)

$Ctransportation = \sum_S \sum_K H[s,k] * Cpipe[s,k]$ (C.41)

$Csinki[k,i,Pd] = FCO_2p[k,i,Pd] * CRS[k,i,Pd] * days \quad \forall\, K, c, Pd$ (C.42)

$Csinkp[i,Pd] = \sum_K Csinki[k,i,Pd] \quad \forall\, c, Pd$ (C.43)

$Csink[i] = \sum_{Pd} Csinkp[i,Pd] \quad \forall\, c$ (C.44)

$NETCAPTUREp[i,Pd] = \sum_K FCO_2p[k,i,Pd] * (1 - EK[k]) - \sum_S \sum_K \sum_T ET[t] * TF[s,k,t,i,Pd] * y[s,t] * gamma[t] - \sum_S \sum_K (PPUMP[s,k,i,Pd] + PCOMP[s,k,i,Pd]) * EP * \frac{24}{1000} \quad \forall\, c, Pd$ (C.45)

$NETCAPTURE[i] = \sum_{Pd} NETCAPTUREp[i,Pd] \quad \forall\, c$ (C.46)

$NETCAPTURE[i] \geq NCRT \quad \forall\, c$ (C.47)

Appendix D: MATLAB Code for obtaining the CRS values for the Multi-Scenario Approach

$function\ [OP] = scenario(n)$ (D.1)

$OP(2^n, n) = 0$ (D.2)

$for\ i = 1{:}\, 2^n - 1$ (D.3)

$binary = de2bi(i)$ (D.4)

$binary = fliplr(binary)$ (D.5)

$sizebinary = size(binary, 2)$ (D.6)

$difference = n - sizebinary$ (D.7)

$OP(i,:) = [zeros(1, difference)\ binary]$ (D.8)

end

$c = 100\ (c\ is\ the\ number\ of\ scenarios\ taken)$ (D.9)

$random = randperm(2^n, c)$ (D.10)

$B(c,n) = 0\ \ (n\ is\ the\ number\ of\ time\ periods)$ (D.11)

$for\ i = 1{:}\, c$ (D.12)

$B(i,:) = OP(random(i),:)$ (D.13)

end

B (D.14)

$CRS(c, n+1) = 0$ (D.15)

$for\ i = 1{:}c$ (D.16)

$for\ j = 2{:}n+1$ (D.17)

$CRS(i,1) = 49.25$ (D.18)

$u = 2.064$ (D.19)

$d = 0.485$ (D.20)

$if\ B(i, j-1) = 1$ (D.21)

$CRS(i,j) = u * CRS(i, j-1)$ (D.22)

$else$

$CRS(i,j) = d * CRS(i, j-1)$ (D.23)

end

end

end

$CRS\ (to\ display\ CRS\ matrix)$ (D.24)

end

Nomenclature (Appendices)

Sets:

Set S	Sources
Set K	Sinks
Set T	Treatment Techniques
Set Pd	Time Periods

Parameters:

$y[s,t]$	Composition of the treated flow from source s treated in t (%)
$yu[s]$	Composition of untreated flow (%)
$yr[s]$	Composition of the raw source flow from source s (%)
$ET[t]$	Treatment unit carbon removal efficiency (%)
$CPT[s,t]$	Cost parameter for carbon removal, ($\$/ton\ CO_2$)
CRF	Capital recovery factor, to annualize capital costs (dimensionless)
$elec$	Cost of electricity, ($\$/KWh$)
$days$	Number of days per year
$H[s,k]$	Distance from source s to sink k, ($mile$)
$TS[s]$	Temperature of source s, (K)
$v[s,k]$	Outlet velocity of flow from source s to sink k, (m/s)
$M[s]$	Molecular mass of source s (kg/kmol)
$PS[s]$	Pressure of source s (kPa)
$PK[k]$	Pressure of sink k (kPa)
$deltap[s,k]$	Pressure difference between source s and sink k (kPa)
$deltapipe[s,k]$	Pressure drop parameter within a pipe from source s to sink k (kPa)

$delppump[s,k]$	Pressure difference per stream, used for pumps (kPa)
pi	Constant =3.14
$CRS[k, Pd]$	Cost parameter of processing CO_2 into sink k in period Pd, negative value reflects a revenue ($\$/ton\ CO_2$)
$LS[s]$	Lower bound of flow available in source s (MTPD)
$R[s]$	Raw source flow in a given source s (MTPD)
$Zmin[k]$	Minimum composition required at sink k (%)
$Gmax[k]$	Maximum flow capacity requirement of sink k (MTPD)
$Lp[s,k]$	Lower flow limit of a pipe from source s to sink k having merged treated and untreated streams (MTPD)
$Mp[s,k]$	Upper flow limit of a pipe from source s to sink k having merged treated and untreated streams (MTPD)
$NCRT$	Net Carbon Reduction Target (%)
$EK[k]$	Sink efficiency factor (dimensionless)
$gamma[t]$	Amount of CO_2 emitted from treatment unit energy use 0.0338 $ton\ CO_2/ton\ CO_2$ processed out of treatment unit
EP	Carbon footprint parameter associated with power use (0.366 $kg\ \ CO_2/KWh$)
$SPP[s,k]$	Specific power for each connection, (KWh/kg)
$ratio[s,k]$	Ratio parameter for the diameter (dimensionless)
c	Number of scenarios
$CRS[k,i,Pd]$	Cost parameter of processing CO_2 into sink k in period Pd, in scenario i, negative value reflects a revenue ($\$/ton\ CO_2$)
S_1	The matrix for source 1
y_{ij}	The inputs for each matrix for each source
S_1R	The summation of the matrix S_1 along the rows
S_1C	The summation of the matrix S_1R along the columns
S_2C	The summation of the matrix S_2R along the columns
S_3C	The summation of the matrix S_3R along the columns
S_4C	The summation of the matrix S_4R along the columns
e_{ij}	The inputs for the FCO_2 matrix
r_{ij}	The inputs for the CRS matrix
s_{ij}	The inputs for the FCO2T matrix
CS	The matrix that has CRS values
CSC	The summation of CS elements along the columns
CSR	The summation of CSC elements along the rows
Tot_{cost}	The total cost ($)

<u>Variables:</u>

$w[s, Pd]$ (dimensionless)	Binary, 1 if flow is taken from source s, in period Pd
$PPUMP[s,k,Pd]$	Positive, pumping power ($KW\ d/ton\ CO_2$), in period Pd
$PCOMP[s,k,Pd]$	Positive, compression power from source s to sink k ($KW\ d/ton\ CO_2$), in period Pd
$TF[s,k,T,Pd]$	Positive, flow to be treated sent from source s to sink k per period Pd (MTPD)
$U[s,k,Pd]$	Positive, untreated flow to be sent from source s to sink k per period Pd (MTPD)

$x[s,k]$	Binary, relates to merging the treated and untreated streams or not (dimensionless)
$NETCAPTUREp[Pd]$	Positive, the net capture of CO_2 in each time period Pd
$NETCAPTURE$ $(ton\ CO_2/day)$	Positive, the net capture of CO_2 over all time periods
$Ctreatmentp[Pd]$	Positive, treatment and separation cost of CO_2 from source s to satisfy sink k's requirement in period Pd ($/period)
$Ctreatment$	Positive, treatment and separation cost of CO_2 from source s to satisfy sink k's requirement over all periods ($)
$CCAPi[s,k]$	Positive, individual capital cost of the compressor used in every s-k connection that satisfies all periods ($/period)
$CCAP$	Positive, total capital cost of the compressors for all s-k connections over all periods ($)
$COPi[s,k,Pd]$	Positive, individual operating cost of the compressor used in every s-k connection, for every period Pd ($/period)
$COPp[Pd]$	Positive, total operating cost of compressors per period Pd ($/period)
COP	Positive, total operating cost of the compressors over all periods ($)
$POPi[s,k,Pd]$	Positive, individual operating cost of the pump used in every s-k connection, for every period Pd ($/period)
$PCAP[s,k]$	Positive, individual capital cost of the pump used in every s-k connection that satisfies all periods ($)
$POPp[Pd]$	Positive, total operating cost of pumps per period Pd ($/period)
POP	Positive, total operating cost of the pump over all periods ($)
$Cpipe[s,k]$	Positive, cost parameter of the pipe from s to k ($\$/mile$)
$Ctransportation$	Positive, total transportation cost over all periods ($)
$Csinki[k,Pd]$	Processing cost of CO_2 in each sink in period Pd ($/ton CO_2/period)
$Csinkp[Pd]$	Total processing cost of CO_2 in all sinks k, per period Pd ($/ton CO_2/period)
$Csink$	Total processing cost of CO_2 in all sinks k, over all periods ($)
$Qp[s,Pd]$	Positive, the total of treated and untreated flows from every s, in every period Pd (MTPD)
$QQp[s,k,Pd]$	Positive, the total of treated and untreated flows from s to k, in every period Pd (MTPD)
$zp[s,k,Pd]$	Binary, for if else pumping cost, in every period Pd (dimensionless)
$Fp[k,Pd]$	Positive, flow into sink k coming from all sources and treatment units, in every period Pd (MTPD)
$FCO_2p[k,Pd]$	Positive, flow of CO_2 to the sink k coming from all sources and treatment units, in every period Pd (ton CO_2/day)
$del[s,k]$	Positive variable to find the diameter needed for the flow from s to k (dimensionless)
$P[s,k]$	Positive integer variable that approximates the diameter needed for every s-k connection (dimensionless)
$d[s,k]$	Binary variable that's related to del, it's 1 if there is a pipe between s and k (dimensionless)
$w[s,i,Pd]$	Binary, 1 if flow is taken from source s, in period Pd, and scenario i (dimensionless)

$PPUMP[s,k,i,Pd]$	Positive, pumping power ($KW\ d/ton\ CO_2$), in period Pd and scenario i
$PCOMP[s,k,i,Pd]$	Positive, compression power from source s to sink k ($KW\ d/ton\ CO_2$), in period Pd and scenario i
$TF[s,k,T,i,Pd]$	Positive, flow to be treated sent from source s to sink k per period Pd and scenario i (MTPD)
$U[s,k,i,Pd]$	Positive, untreated flow to be sent from source s to sink k per period Pd and scenario i (MTPD)
$NETCAPTUREp[i,Pd]$	Positive, the net capture of CO_2 in each time period Pd and scenario i
$NETCAPTURE[i]$	Positive, the net capture of CO_2 over all time periods for each scenario ($ton\ CO_2/day$)
$Ctreatmentp[i,Pd]$	Positive, treatment and separation cost of CO_2 from source s to satisfy sink k's requirement in period Pd and scenario i ($/period)
$Ctreatment[i]$	Positive, treatment and separation cost of CO_2 from source s to satisfy sink k's requirement over all periods for each scenario i ($)
$Ctreat$	Positive, Average treatment and separation cost of CO_2 from source s to satisfy sink k's requirement, over all periods and scenarios ($)
$CCAPi[s,k,i,Pd]$	Positive, individual capital cost of the compressor used in every s-k connection for every period Pd and scenario i ($/period)
$CCAPp[i,Pd]$	Positive, total capital cost of the compressors, for every period Pd, and scenario i, for all connections ($/period)
$CCAP[i]$	Positive, the highest compressor capital cost over all periods in each scenario i ($)
$CCAPT$	Positive, Average capital cost of the compressors, for all periods, all scenarios ($)
$COPi[s,k,i,Pd]$	Positive, individual operating cost of the compressor used in every s-k connection, for every period Pd and scenario i ($/period)
$COPp[i,Pd]$	Positive, total operating cost of the compressors, for each period Pd and scenario i ($/period)
$COP[i]$	Positive, total operating cost of the compressors over all periods for each scenario i ($)
$COPT$	Positive, Average operating cost of the compressors, over all periods and scenarios ($)
$PCAPp[s,k,i,Pd]$	Positive, individual capital cost of the pump used in every s-k connection, for every period Pd and scenario I ($/period)
$PCAPi[i,Pd]$	Positive, the capital cost of the pumps used for all s-k connections in each scenario and every time period ($/period)
$PCAP[i]$	Positive, the highest pump capital cost over all time periods in every scenario ($)
$PCAPT$	Positive, Average capital cost of the pumps used in all periods and scenarios ($)
$POPi[s,k,i,Pd]$	Positive, individual operating cost of the pump used in every s-k connection, for every period Pd and scenario i ($/period)
$POPp[i,Pd]$	Positive, total operating cost of the pumps, for every period Pd and scenario i ($/period)
$POP[i]$	Positive, total operating cost of the pump over all periods for each scenario i ($)

$POPT$	Positive, Average operating cost of the pumps over all periods and scenarios ($)
$Cpipe[s,k]$	Positive, cost parameter of the pipe from s to k ($\$/mile$)
$Ctransportation$	Positive, total transportation cost over all periods ($)
$Csinki[k,i,Pd]$	Processing cost of CO_2 in each sink in period Pd and scenario i ($/period)
$Csinkp[i,Pd]$	Total processing cost of CO_2 in all sinks k, for every period Pd and scenario i ($/period)
$Csink[i]$	Total processing cost of CO_2 in all sinks k, over all periods for each scenario i ($)
$Qp[s,i,Pd]$	Positive, the total of treated and untreated flows from every s, in every period Pd and scenario i (MTPD)
$QQp[s,k,i,Pd]$	Positive, the total of treated and untreated flows from s to k, in every period Pd and scenario i (MTPD)
$zp[s,k,i,Pd]$	Binary, for if else pumping cost, in every period Pd and scenario i (dimensionless)
$Fp[k,i,Pd]$	Positive, flow into sink k coming from all sources and treatment units, in every period Pd and scenario i (MTPD)
$FCO_2p[k,i,Pd]$	Positive, flow of CO_2 to the sink k coming from all sources and treatment units, in every period Pd and scenario i (ton CO_2/day)
$FCO_2T[k,Pd]$	Positive, Average flow of CO_2 to sink k coming from all sources and treatment units in every time period, over all scenarios (ton CO_2/day)

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