AMERICAN UNIVERSITY OF BEIRUT

IPP WITH CREDIT FACILITY ON RAW MATERIAL

by
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AN ABSTRACT OF THE THESIS OF

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This paper develops two models that aim at studying the optimal production policy for a manufacturer. The inventory models presented in this paper are based on the classic Economic Production Quantity (EPQ) analysis. In the first base model, we determine the optimal production policy by accounting for the inventories of the raw material and the finished products. In this base model, the production policy consists of determining the number of batches of raw material to order for multiple identical production cycles, in addition to the optimal production quantity in each cycle. In the second model, we determine the optimal production policy with an additional fixed credit facility period for settling the procurement cost. A closed-form formula of the optimal number of production cycles is obtained in the first model and a thorough analysis is made to find that number for the second model. In addition to incorporating the effect of trade credit of raw material in the EPQ, our results could aid manufacturers in deciding on different supplier offers related to delays in payment.

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CHAPTER I
INTRODUCTION

Most production inventory models consider the economic ordering of raw material independently of the production of the finished products. However, quite often, raw materials should be procured and stocked in inventory prior to their manufacturing and transformation into finished products. To solve this problem, integrated procurement production (IPP) systems are introduced to build a link between procurement and production policies when raw materials are transformed into finished goods. (See Goyal, 1992, for a review of IPP models). In this paper, we propose an inventory model that develops economic order quantities (EOQ) for raw materials and economic production quantities (EPQ) for finished products with the objective of minimizing total inventory costs.

Another concept studied in this paper is credit facility that is not taken into consideration in the conventional inventory theory. In fact, the conventional inventory theory assumes that the manufacturer pays for an order as soon as the raw materials are received. Nevertheless, this assumption is not always applicable. In fact, in most business transactions, suppliers allow a certain credit period to settle the account. This leads to a win-win situation in which the supplier stimulates demand and the manufacturer accumulates benefits from interest on sales revenue during the credit period. Indeed, in the manufacturer's point of view, the longer the credit period the lower the inventory holding cost due to savings in the inventory financing cost. This delay period is important for the suppliers as well. In fact, it is considered a valuable promotional tool for the suppliers by which they can increase the demand on their materials without reducing their prices. The reader is referred to Haley and Higgins

(1973) and Goyal (1985) for early works and background of inventory management under trade credit and Maddah et al. (2004) for a review of related works.

A survey to determine and validate the reliance of manufacturers on credit facilities as a financing vehicle was conducted. A total number of twenty companies of varying sizes, operating in various industries and sectors, were questioned and inquired on their utilization of credit facilities to improve liquidity and cash flow. The companies surveyed had an upper bound of yearly revenues of 15 million dollars (USD) to the lower bound of 1 million dollars (USD) and an average of 6.8 million dollars (USD). The surveyed companies manufactured products from specialty construction products, to outfits, passing by porcelain tableware. Ninety percent, or 18 out of 20, confirmed the availability of credit facility as a financing method offered by their suppliers. The financing term differed from supplier to supplier depending on the contract size, specific project, credit limit, and other factors. Typical credit term ranged from as short a duration as 2 weeks to up to a lengthy duration of 180 days. Suppliers that failed to offer credit facilities served their customers by offering discounts on advanced payments.

The concept of credit period was extensively studied in the inventory management literature in many settings. This paper aims at studying the effect of credit facilities on the inventory of raw material in an IPP system. The rest of the paper is organized as follows. The literature for both IPP systems and inventory management under credit facility is discussed in Section 2. An EPQ model that takes into consideration raw materials is studied in Section 3. In this section, closed-form formulas of the optimal production quantity and of the number of batches of raw material that the manufacturer should order for multiple cycles are derived. A new model is discussed in Section 4 where credit facility is incorporated into the model of Section 3. In this section, a closed-form formula for the production quantity is given for a fixed number of raw material batches. Then, enumeration is used to find the number of batches that should be ordered for different cycles. Section 5 presents numerical results and managerial insights. Section 6 concludes this paper.

CHAPTER II
LITERATURE REVIEW

The traditional economic ordering quantity (EOQ) formula was first presented by Harris (1913) to determine the economic order size that balances fixed ordering cost and holding cost. Later, Taft (1918) introduced the economic production quantity EPQ formula to determine the economic manufacturing batch size under a similar cost trade-off. Since then, a large body of research was developed to extend these theories. Goyal (1977) seems to be the first to consider the procurement-production (IPP) system in which the inventory management of the raw materials and the finished product for a single product are integrated to minimize costs. Many papers extend Goyal's model to a multistage supply chain such as Korgaonker (1979), Adam and Ignall (1980). Other papers such as Park (1983) and Raafat (1985, 1988) use Goyal's integrated model but consider that the raw materials and finished products are deteriorating. More recent works on IPP include Priyan et al. (2015), Fauza et al. (2013) and Nasr et al. (2014), and references therein. However, to the best of our knowledge, no work in the literature considers the IPP system with credit facility on raw material ordering. Some assumptions in the classical inventory theory are sometimes not valid. For example, unlike many practical contexts, most inventory models assume that payment occur immediately upon receiving an order. Goyal (1985) is among the first works to assume that the supplier allows a defined period to settle the account after receiving an order. Goyal (1985) develops an economic order quantity model under the conditions of allowable delay in payments assuming that the unit selling price and the unit purchasing price are equal, and that the account is settled at the end of the credit period. Teng

(2002) revises Goyal's model by considering that the unit purchase cost and selling price are different. Many authors extended Goyal's model to account for deterioration such as Aggarwal and Jaggi (1995), Jamal et al. (2000), Sarker et al. (2000). Others assume that shortage is permitted to occur such as Jamal et al. (1997), Chen and Ouyang (2006), Chung and Huang (2009). Chung and Huang (2003) considers the economic production quantity (EPQ) inventory model for a manufacturer when the supplier offers a permissible delay in payments assuming that a part of the amount of the total cost will be paid at the end of the credit period and the other part of the amount would be paid by getting a loan from the bank. Along a similar avenue Teng and Chang (2009) investigate the optimal retailer's replenishment decisions with two levels of trade credit policy in the EPQ framework. Many other authors also considered the EPQ model under credit facilities such as Huang (2004), Huang (2007), Goswami et al. (2010), Mahata (2012). However, to the author's knowledge, none of them have studied EPQ models taking into consideration both raw materials costs and delay in payments. Our proposed research considers such an EPQ model.

CHAPTER III
BASE IPP MODEL

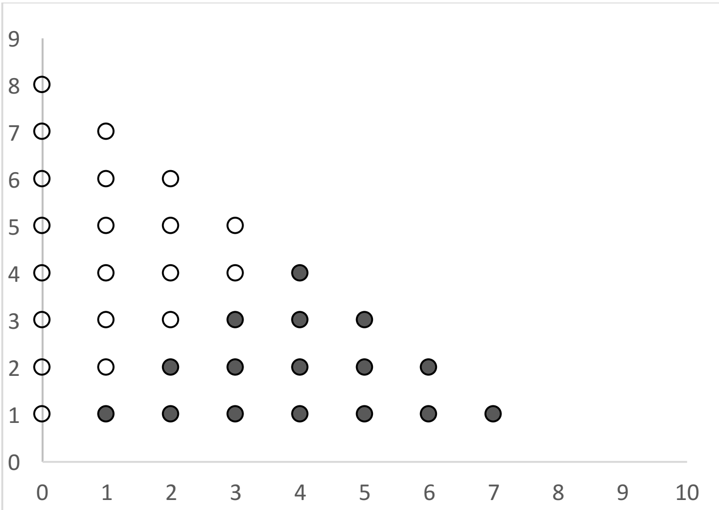

Figure 1: EPQ Model involving raw material and finished products

Consider the IPP model discussed in Section 1. Figure 1 shows the behavior of raw material and finished products in one cycle: raw material (empty circles) being transformed into finished products (filled circles) at a rate α then finished products being sold at a rate β , $\beta < \alpha$. (In this example $\beta = 1$, $\alpha = 2$)

The IPP model studied in this paper takes into consideration both the finished products and the raw material. In this model, following the literature (e.g. Goyal 1992) raw material is ordered in lots that cover n production cycles. The production rate and consumption rate are assumed to be deterministic and constant over time and the lead time is assumed to be zero. No shortage, on either raw materials or finished products, is allowed, and no quantity discount on raw material is also assumed. The objective of this model is to determine the optimal amount of raw material that should be ordered and the optimal number of production cycles in a raw material ordering cycle. Table 1 shows the notation we use for this model.

y	Production lot size
Z	Maximum inventory of the finished product
n	Number of production cycles in one raw material procurement cycle (the
α	Production rate of finished products or Depletion rate of raw material
β	Demand rate of finished products
t_1	time to accumulate Z units of finished products at the rate $(\alpha-\beta)$
t_2	time to consume Z units of finished products at the rate β units/unit time ,
t_0	time to produce y units of finished products at a rate α and consume them
h	unit holding cost ($/unit/year)
h_0	Unit holding cost of raw material ($/unit/year),
K_0	fixed raw material ordering (procurement) cost
K	fixed production (setup) cost

Table 1: Notations

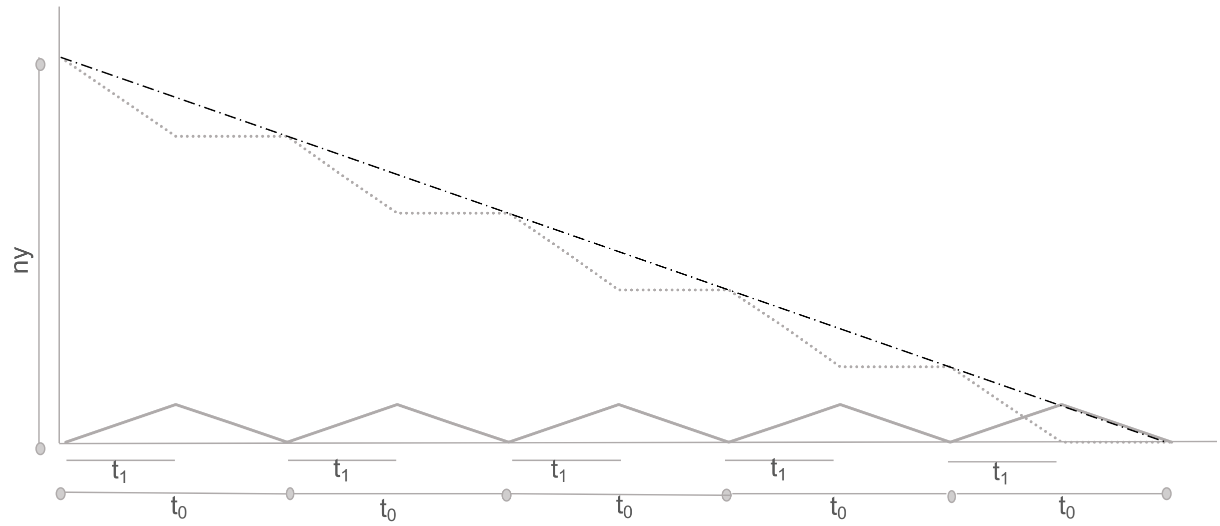

Figure 2: Behavior of Inventory level for finished products and raw material over time

Figure 2 shows a raw material ordering cycle with five production runs. Through a purchase, enough raw material is stored to produce finished products across multiple cycles, in this example, five in total. As finished products are manufactured and sold, raw material inventory decreases accordingly, eventually reaching 0 once the last batch of finished products are produced.

The total cost per ordering cycle is composed of the production cost, nK, the fixed cost of ordering raw material, K_0, and the holding costs of the finished products and of raw materials. Following, the standard EPQ model the holding cost of the finished product over n production cycles is given as $n(hZt_0 / 2)$, where $Z = y(1 - \beta/\alpha)$ is the maximum inventory level of the finished product. The following lemma gives the average inventory level of raw material.

Lemma 1. *The average inventory level of raw material is*

$$Z_0 = \frac{n\frac{yt_1}{2} + \frac{n(n-1)}{2}yt_0}{nt_0} = \frac{y\beta}{2\alpha} + \frac{(n-1)y}{2}$$

Proof. See Appendix 1.

Therefore, the total cost per cycle is

$$TC(y,n) = nK + h\frac{ny^2}{2\beta}\left(1-\frac{\beta}{\alpha}\right) + K_0 + h_0\left(\frac{n(n-1)}{2}\frac{y^2}{\beta} + n\frac{y^2}{2\alpha}\right)$$

With a cycle duration of nt_0, the annual ordering cost is then

$$TC_u(n,y) = \frac{TC_u(y,n)}{nt_0} = \frac{K\beta}{y} + h\frac{y}{2}\left(1-\frac{\beta}{\alpha}\right) + \frac{K_0\beta}{ny} + h_0\left(\frac{(n-1)}{2}y + \frac{y\beta}{2\alpha}\right) \quad (1)$$

For a given number of production runs per cycle, n, it can be easily shown that the annual cost, $TC_u(y,n)$ is convex in the production lot size y. Then, the optimal order quantity for a given n is obtained from the first order optimality condition,

$$\frac{dTC_u(y = y^*(n), n)}{dy} = 0$$

which implies that

$$y^*(n)=\sqrt{\frac{nK\beta+K_0\beta}{\frac{n}{2}\left[h\left(1-\frac{\beta}{\alpha}\right)+h_0\left((n-1)+\frac{\beta}{\alpha}\right)\right]}} \quad (2)$$

Next, we adopt a sequential optimization approach. The optimal order quantity, $y^*(n)$ in (2) is replaced in the annual cost in (1), to obtain the annual cost at optimal inventory level, $TC_u(n) = TC_u(n, y^*(n))$. Upon replacement,

$$TC_u(n)=\sqrt{\frac{2\left[h\left(1-\frac{\beta}{\alpha}\right)+h_0\left(\frac{\beta}{\alpha}+n-1\right)\right]\left[nK\beta+K_0\beta\right]}{n}} \quad (3)$$

Assuming that n is a continuous variable, the following theorem indicates that the annual cost $TC_u(n)$ is quasiconvex in n and gives an expression for the optimal n.

Theorem 1. *The annual cost $TC_u(n)$ is quasiconvex in n,* with a unique local minimum

$$n=\sqrt{\frac{K_0\beta\left(h\left(1-\frac{\beta}{\alpha}\right)+h_0\frac{\beta}{\alpha}\right)-h_0K_0\beta}{h_0K\beta}} \quad (4)$$

Proof. See Appendix 2.

Theorem 1 implies that the optimal (integer) number of shipments can be obtained by rounding the continuous solution up or down, whoever gives the least cost,

$$n = \arg\min_n \left(TC_u \left(\lfloor n \rfloor \right), TC_u \left(\lceil n \rceil \right) \right) \quad (5)$$

where $\lfloor x \rfloor$ ($\lceil x \rceil$) is the largest integer $\leq x$ ($\geq x$).

Once n is found from (5), the optimal production lot size is found (2) as $y^* = y^*(n)$. Some papers have discussed ways to find the optimal number of production cycles (e.g. Goyal 1992), Nasr (2014)). However, none of these papers have obtained the closed-form expression in (4) and (5). This is the first contribution of our proposed research.

CHAPTER IV
IPP MODEL WITH DELAY IN PAYMENT ON RAW MATERIAL ORDERS

Consider the same model as in Section 3. However, assume now that the manufacturer will pay the suppliers of raw material after a fixed time T (the credit facility period) of receiving an order. The objective of this model is similar to that in Section 3, to determine the optimal production lot size and the number of production runs per procurement cycle, in a way that minimizes the total cost.

Three general cases are studied in this model,

(i) Credit facility period longer than procurement cycle, $T > nt_0$.

(ii) Credit facility period shorter than procurement cycle, $T < nt_0$, and falls in a production phase, $T - \lfloor T / t_0 \rfloor < t_1$

(iii) Credit facility period shorter than the procurement cycle, $T < nt_0$, and falls in a consumption phase, $T - \lfloor T / t_0 \rfloor > t_1$

The same notations as Section 3 (Table 1) are used. We add the following.

- T : Delay in payment (credit facility) period
- j: Number of the production run in a procurement cycle where payment must be made; $j=T/t_0$
- i = Manufacturer cost of capital

- c_0 = Unit cost of raw material
- c_1 = added cost to transform one unit of raw material into finished products
- i_e = interest rate earned by the manufacturing on revenues during the credit period
- A_{rm} = area under the raw material inventory level between the beginning of the procurement cycle and the end of the credit period.
- A_{fp} = Similar area for finished products

  A' = area under the straight line representing the interest earned on the raw material between the beginning of the procurement cycle and the end of the credit period.

An appropriate estimation of the inventory costs under delay in payments requires appropriate accounting of the financing cost. Assume the cost of capital of the manufacturer is at the rate i, the unit financing cost of raw material is then ic_0. To estimate the unit financing cost of the finished product, assume that, in addition to the raw material cost, a cost c_1 is needed to produce the finished product. The cost c_1 accounts for all operational costs beyond raw material, e.g., labor, utility, accessories, etc. Then, the unit financing cost of the finished product is $i(c_0 + c_1)$. With the fixed ordering cost not changing, and with an additional negative cost (profit) for earned interest on revenue during the credit period, the total cost per ordering cycle can now be written as,

$$TC(y,n) = nK + K_0 + h_{sf} n \frac{y^2}{2\beta}\left(1 - \frac{\beta}{\alpha}\right) + i\left(c_0 + c_1\right) A_{fp} + h_{s0}\left(\frac{n(n-1)y^2}{2\beta} + n\frac{y^2}{2\alpha}\right) + ic_0 A_{rm} - i_e c_0 A' \quad (6)$$

<u>Analysis of Case (i).</u>

In this case,

$A_{rm} = 0$

$A_{fp}=0$

$$A' = \frac{\beta\left(nt_0\right)^2}{2} + \beta nt_0\left(T - nt_0\right)$$

Then, the annual cost and optimal inventory level for a given number of production runs are given by

$$TCU_1(n,y) = \frac{K\beta}{y} + \frac{K_0\beta}{ny} + \left(h_{sf} + c_1\right)\left(\frac{1}{2}y\left(1 - \frac{\beta}{\alpha}\right)\right) + h_{s0}\left[\frac{y\beta}{2\alpha} + \frac{(n-1)y}{2}\right] - i_e c_0\left[\frac{2\beta T - \beta nt_0}{2}\right]$$

(7)

$$y_1(n) = \sqrt{\frac{K\beta + \frac{K_0\beta}{n}}{\frac{1}{2}\left(h_{sf} + ic_1\right)\left(1 - \frac{\beta}{\alpha}\right) + h_{s0}\left(\frac{\beta}{2\alpha} + \frac{n-1}{2}\right) + \frac{i_e c_0 n}{2}}} \qquad (8)$$

Analysis of Case (ii). We derive the following expressions for the inventory areas A_{rm} and A_{fp} using geometry.

$$A_{rm} = \sum_{i=1}^{j}(n-i)yt_0 + \sum_{i=1}^{j}\frac{yt_1}{2} - (n-j)y\left(jt_0 - T\right) - \frac{\left(t_1 + (j-1)t_0 - T^2\right)}{2t_1}y$$

$$A_{fp} = \sum_{i=1}^{j-1}\frac{Zt_0}{2} + \frac{1}{2}\frac{\left(T - (j-1)t_0\right)^2}{t_1}Z$$

$$A' = \frac{\beta T^2}{2}$$

With these expressions used to estimate the inventory holding cost, proceeding in a similar manner to the base model analysis in Section 3, we derive the following expression for the total annual cost and optimal order quantity for a given number of production runs,

$$TCU_2(y,n) = \frac{K\beta}{y} + \frac{K_0\beta}{ny} + \frac{1}{2}(h_{sf} + ic_1)y\left(1 - \frac{\beta}{\alpha}\right) + h_s\left[\frac{y\beta}{2\alpha} + \frac{(n-1)y}{2}\right] + ic_0\left[\frac{ny}{2} - \beta T + \frac{T^2\beta^2}{2ny}\right] - i_e c_0\left[\frac{\beta^2 T^2}{2ny}\right]$$

(9)

$$y_2(n) = \sqrt{\frac{K\beta + \frac{K_0\beta}{n} - \frac{i_e c_0 T^2\beta^2}{2n} + \frac{ic_0 T^2\beta^2}{2n}}{\frac{1}{2}(h_{sf} + c_1)\left(1 - \frac{\beta}{\alpha}\right) + h_{s0}\left(\frac{\beta}{2\alpha} + \frac{n-1}{2}\right) + ic_0\left(\frac{n}{2}\right)}} \qquad (10)$$

<u>Analysis of Case (iii).</u> Similar to the previous case, we derive the following expressions for the inventory areas A_{rm} and A_{fp} using geometry to estimate the inventory holding cost.

$$A_{rm} = \sum_{i=1}^{j}(n-i)yt_0 + \sum_{i=1}^{j}\frac{yt_1}{2} - (n-j)y(jt_0 - T)$$

$$A_{fp} = \sum_{j=1}^{j-1}\frac{Zt_0}{2} + \frac{Zt_1}{2} + Z\left[\frac{jt_0 - T}{t_0 - t_1}\right]\left[T - ((j-1)t_0 + t_1)\right] + \frac{1}{2}\left[T - ((j-1)t_0 + t_1)\right]\left[Z - \frac{jt_0 - T}{t_0 - t_1}Z\right]$$

Then, upon simplification, the annual cost and the optimal inventory level for a given number of production runs are found to have the same expressions as those in Case (ii) which are given in (9) and (10).

As we have seen, Cases (iii) and (ii) have the same expressions for the annual cost. That leaves us with only two cases to analyze, Case 1 ($T > nt_0$) and Case 2 ($T < nt_0$).

To find the optimal solution, the following algorithm is used.

Algorithm 1.

Step 1. Set $n = 1$ and $TCU^* = \sqrt{2(K+K_0)\beta[h(1-\beta/\alpha)+h_0(\beta/\alpha)]}$.

Step 2. Find $y_1(n)$ from (8). Set $y_1^*(n) = min(y_1(n), \beta T/n)$ and $TCU_1^*(n) = TCU_1(y_1^*(n), n)$, where $TCU_1(y_1^*(n), n)$ is obtained from (7).

Step 3. Find $y_2(n)$ from (10). Set $y_2^*(n) = max(y_2(n), \beta T/n)$ and $TCU_2^*(n) = TCU_2(y_2^*(n), n)$, $TCU_2(y_2^*(n), n)$ is obtained from (9).

Step 4. If $TCU_1^*(n) < TCU_2^*(n)$, set $TCU^*(n) = TCU_1^*(n)$ and $y^*(n) = y_1^*(n)$. Otherwise, set $TCU^*(n) = TCU_2^*(n)$ and $y^*(n) = y_2^*(n)$.

Step 5. If $TCU^*(n) < TCU^*$, set $n^* = n$, $y^* = y^*(n)$, $n = n + 1$, and go to Step 2. Otherwise, stop.

Step 1 of Algorithm 1 initiates the search for the optimal solution by setting the annual cost to a large enough value. This value is obtained by setting $n = 1$ in (3), the annual cost for the base case with no credit period. (Since credit facility reduces the cost, this is obviously an upper bound on the optimal cost.) For a given number of production batches n, Step 2 searches for the minimum cost from Case 1 ($T > nt_0$), which is equivalent to finding the minimum of $TCU_1(y, n)$ in (7) *for* $y \in (0, \beta T/n]$. The convexity of $TCU_1(y, n)$ in y implies that this minimum is achieved at $y = min(y_1(n), \beta T/n)$. A similar analysis is done in Step 3 for Case 2 ($T < nt_0$), which translates into finding the minimum cost for $y \in [\beta T/n, \infty)$, which is achieved at $y = \max(y_2(n), \beta T/n)$. Step 4 determines the optimal order size for a given n by comparing the solutions obtained in Steps 2 and 3. Finally, Step 5 updates the optimal solution, and sets the stopping criterion. The algorithm stops when the minimum cost at optimal inventory level, $TCU^*(n)$ starts increasing in n.

CHAPTER V
NUMERICAL RESULTS AND INSIGHTS

In this section, the results of our numerical study are presented. Consider the input data for the base case in Table 2.

Parameter	Base Value
K_0	\$2000/order
K	\$50/production run
c_0	\$50/unit
c_f	\$100/unit
i	10%/year
h_{s0}	\$0.5/unit/month
h_{sf}	\$1/unit/month
α	1,200 units/month
β	8,00 units/month
c_1	\$50/unit
i_e	3%/year
T	0.3 years

Table 2: Base parameter values

For the base model in Section 3, utilizing Theorem 1, the optimal (continuous) number of production runs is n =3.65 To get the optimal integer value of the number, we find from (3) that $TC_u(3)$ = \$4,585 and $TC_u(4)$ = \$4,579.66 which implies that the optimal number of production runs is n^* = 4. The optimal production lot size is given from (2) as $y^* = y^*(4)$ = 192 units. Finally, a plot of the annual cost at optimal inventory level $TC_u(n)$ in (3) is shown in Figure 7. This plot clearly indicates that $TC_u(n)$ is indeed quasiconvex as shown in Theorem 2.

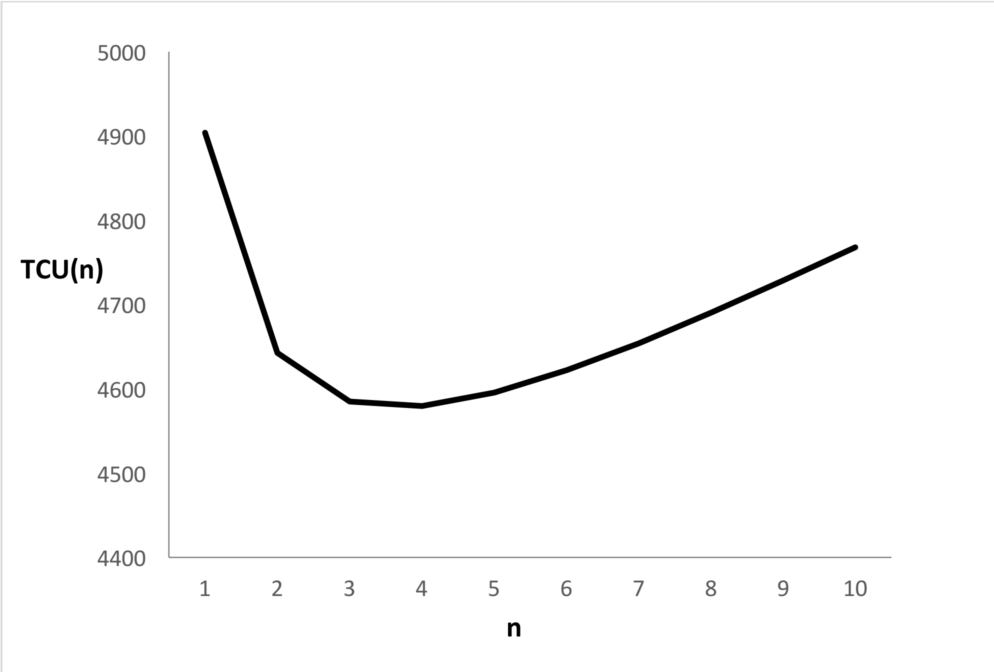

Figure 3: Cost at optimal inventory level vs number of production runs

Next we numerically analyze our main credit facility model in Section 4 using Algorithm 1.

n	$y_1(n)$	$y_1^*(n)$	$y_2(n)$	$y_2^*(n)$	$TCU_1^*(n)$	$TCU_2^*(n)$	$TCU^*(n)$
1	925	240	689	689	$6,933	$3,853	$3,852.89
2	537	120	373	373	$6,990	$3,581	$3,580.54
3	383	80	257	257	$7,120	$3,517	$3,517.44
4	299	60	198	198	$7,268	$3,509	**$3,508.98**
5	247	48	161	161	$7,424	$3,523	$3,522.58
6	211	40	136	136	$7,583	$3,547	$3,547.09
7	184	34	118	118	$7,745	$3,578	$3,577.69
8	164	30	105	105	$7,908	$3,612	$3,611.96
9	148	27	94	94	$8,071	$3,649	$3,648.54
10	135	24	86	86	$8,235	$3,687	$3,686.62

Table 3: Numerical example of IPP Model with Delay in Payment on Raw Material Orders

To validate the model and gain insights, a one-way sensitivity analysis was run for both models. First, K_0 is varied from $2,000 up to $5,000 with other parameters fixed at their base values in Table 1. Tables 4 & 5 respectively show the sensitivity analysis on K_0 for both first and second models. Figure 4 shows the effect of the change of K_0 on the number of raw materials that should be ordered for both models and figure 5 shows how that affects TCU* in both cases.

K_0	y	n~	n*	ny	TCU*
2000	192	3.63	4	768	4580
2500	213	4.08	4	852	5073
3000	232	4.47	4	928	5523
3500	202	4.83	5	1010	5933
4000	215	5.16	5	1075	6316
4500	228	5.47	5	1140	6677
5000	201	5.77	6	1206	7016

Table 4: Sensitivity analysis over K_0 for the first model

K_0	y*	n*	n*y*	TCU*
2000	198	4	792	3509
2500	218	4	872	3990
3000	192	5	960	4429
3500	206	5	1030	4832
4000	218	5	1090	5209
4500	194	6	1164	5564
5000	204	6	1224	5899

Table 5: Sensitivity analysis over K_0 for the second model

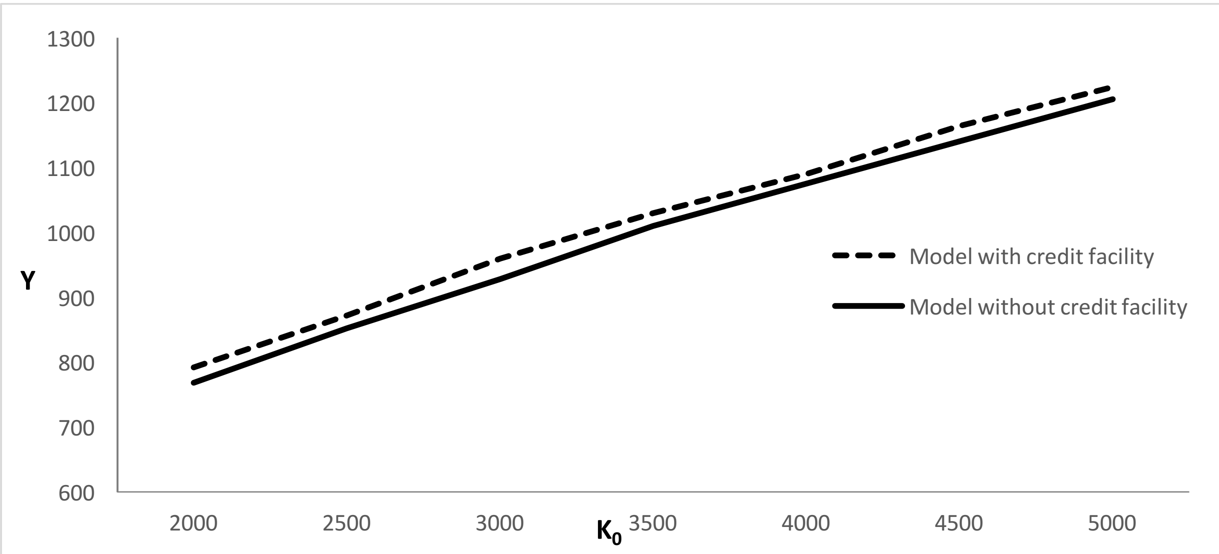

Figure 4: Relationship between n*y* and K_0 for both base model and credit facility model

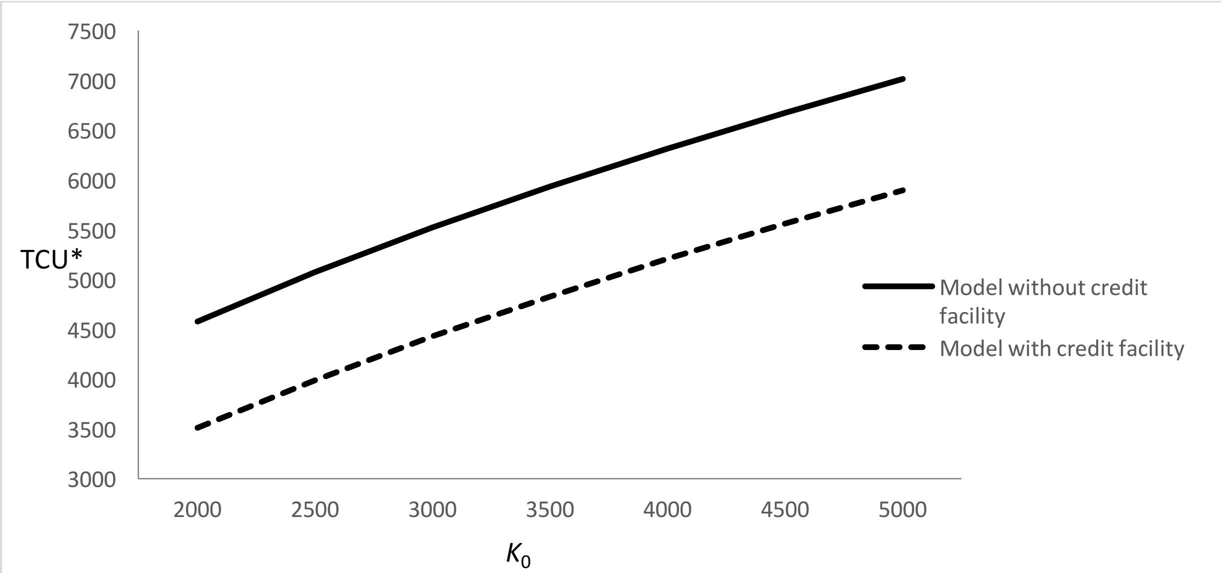

Figure 5: Relationship between K_0 and TCU* for both base model and credit facility model

As expected, when K_0 increases ,the number of raw materials that should be ordered increases and the Total cost increases for both cases.The higher K_0 is, the more raw material it is expected to be ordered at one shot in order to have the optimal situation. No matter how K_0 was varied, there was always one local minimum of *TCU* in *n*. In some situations, alternate optimal solutions existed at two consecutive values of *n*. Also, note that no matter how K_0 is varied the total cost for the model with credit facility is less than that for the one without it.

The next scenario analysis was done by increasing *K* from \$100 to \$700, leaving everything else constant. The results were shown in the tables below.

K	n~	n*	y*	n*y*	TCU*
100	2.6	3	259	777	4,742
300	1.5	2	403	806	5,167
500	1.2	1	739	739	5,416
700	1.0	1	768	768	5,629

Table 6: Sensitivity analysis over K for the First Model

K	n*	TCU*	y*	n*y*
100	3	3670.41	266	798
300	2	4090.27	412	824
500	1	4350.83	757	757
700	1	4558.33	785	785

Table 7: Sensitivity analysis over K for the Second Model

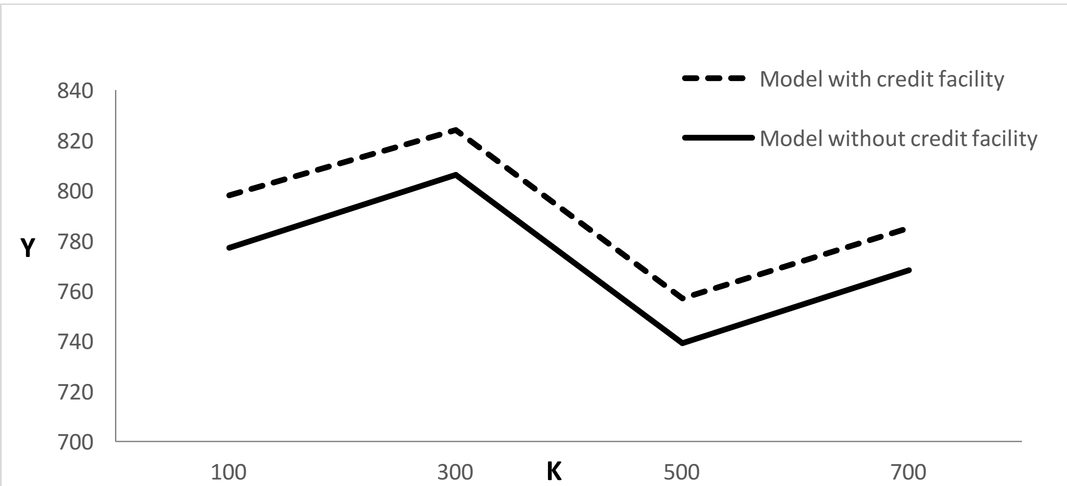

Figure 6: Relationship between n*y* and K for both base model and credit facility model

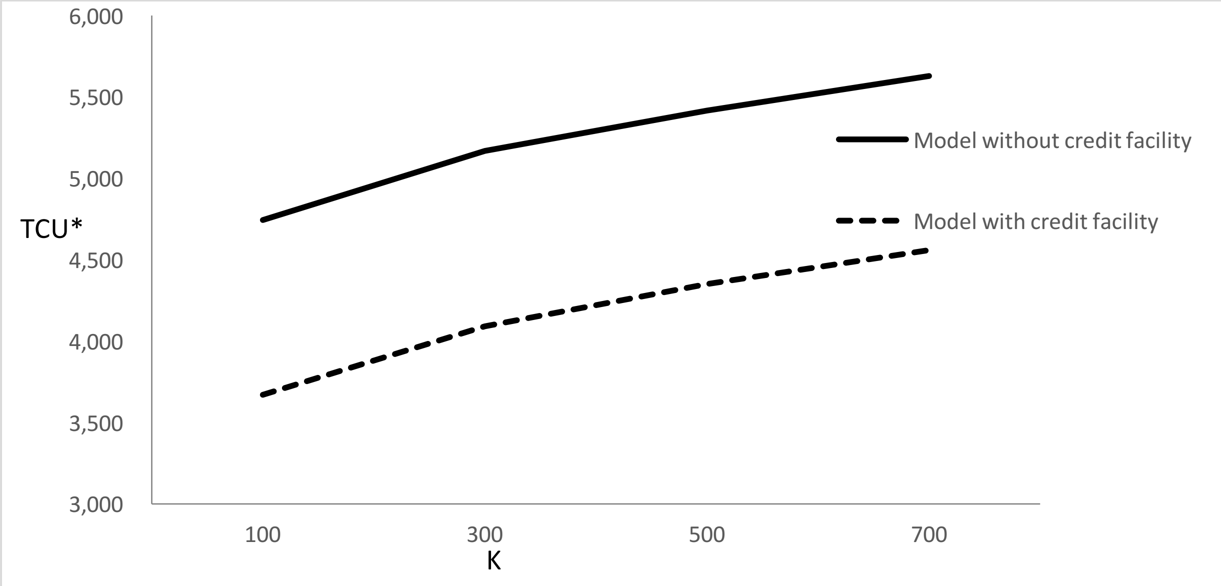

Figure 7: Relationship between K and TCU* for both base model and credit facility model

When K increases, the number of raw materials does not get directly affected but the Total cost increases for both cases. When K is increased, the optimal order size $y^*(n)$ increases. However, K and ny are not directly proportional. In fact, it is true that it is better to produce more finished products out of the ones ordered, however that is not directly correlated to the number of batches of raw materials that should be ordered. Also, note that no matter how K is varied the total cost for the model with credit facility is less than that for the one without it.

A similar sensitivity analysis is done for the storage cost of raw material, h_{s0}, that was varied from \$0.1 to \$0.9. The results are shown in the Tables 8 and 9 and Figure 8 & 9. When h_{s0} increases, the number of raw materials decreases and the Total cost increases for both cases.

In fact, n^*y^* is directly affected by h_{s0} because the higher the holding cost of the raw material, the less raw material should be ordered at one shot at the beginning of the cycle. In this analysis, we also note that no matter how K is varied the total cost for the model with credit facility is less than that for the one without it

hs_0	h_{sf}	y*	n~	n*	n*y*	TCU*
0.1	1	198	3.93	4	792	4436.52
0.3	1	195	3.79	4	780	4508.66
0.5	1	192	3.65	4	768	4579.67
0.7	1	189	3.52	4	756	4649.59
0.9	1	243	3.39	3	729	4716.5

Table 8: Sensitivity analysis over h_{s0} for the First Model

hs0	hsf	y*	n*	n*y*	TCU*
0.1	1	816	4	3264	3361.8
0.3	1	804	4	3216	3435.97
0.5	1	792	4	3168	3508.98
0.7	1	780	4	3120	3508.9
0.9	1	768	4	3072	3651.7

Table 9: Sensitivity analysis over h_{s0} for the Second Model

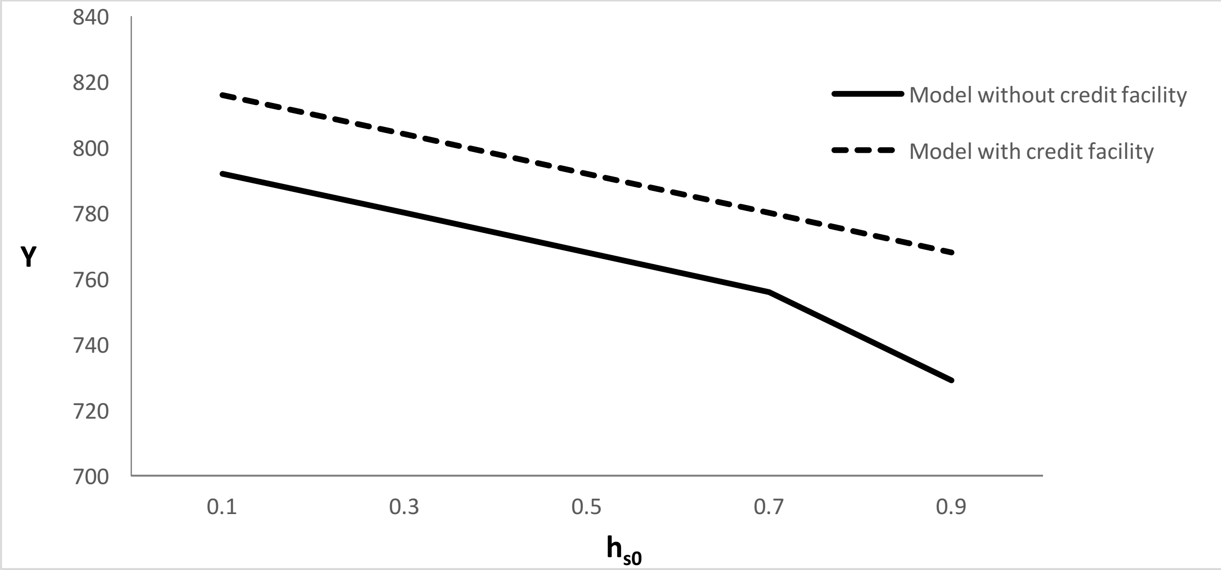

Figure 8: Relationship between n^*y^* and h_{s0} for both base model and credit facility model

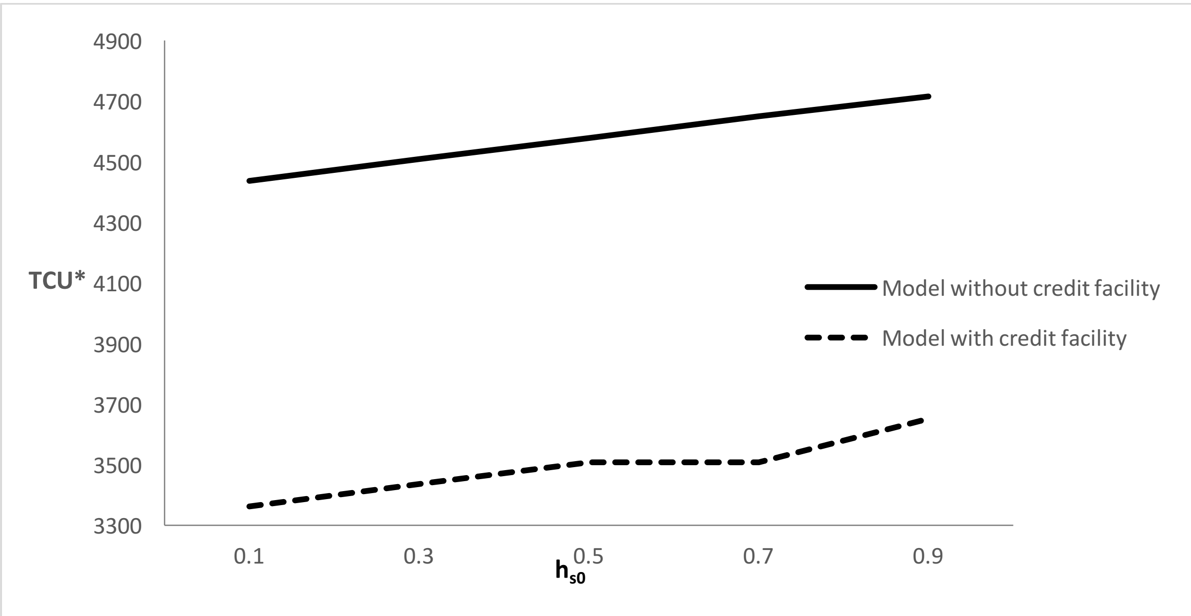

Figure 9: Relationship between h_{s0} and TCU* for both base model and credit facility model

As for h_{sf}, it was varied from 0.6 to 1.4 leaving everything else constant. The results are shown in Tables 10 and 11 and Figures 10 & 11. When h_{sf} increases, the number of raw materials does not get directly affected but the Total cost increases for both cases. n*y* is not directly affected by h_{sf} because the raw material can be ordered, manufactured and sold without staying in the inventory for long while at the finished product phase. Similarly to the previous analysis, note that no matter how h_{sf} is varied the total cost for the model with credit facility is less than that for the one without it.

h_{s0}	h_{sf}	n~	n*	TCU*	y*	n*y*
0.5	0.6	3.52	4	4566.84	193	772
0.5	0.8	3.58	4	4573.26	192	768
0.5	1	3.65	4	4579.67	192	768
0.5	1.2	3.72	4	4586.07	192	768

Table 10: Sensitivity analysis over h_{sf} for the First Model

hs0	hsf	n*	y*	n*y*	TCU*
0.5	0.6	4	198	792	3495.8
0.5	0.8	4	198	792	3502.4
0.5	1	4	198	792	3508.98
0.5	1.2	4	197	788	3515

Table 11: Sensitivity analysis over h_{sf} for the Second Model

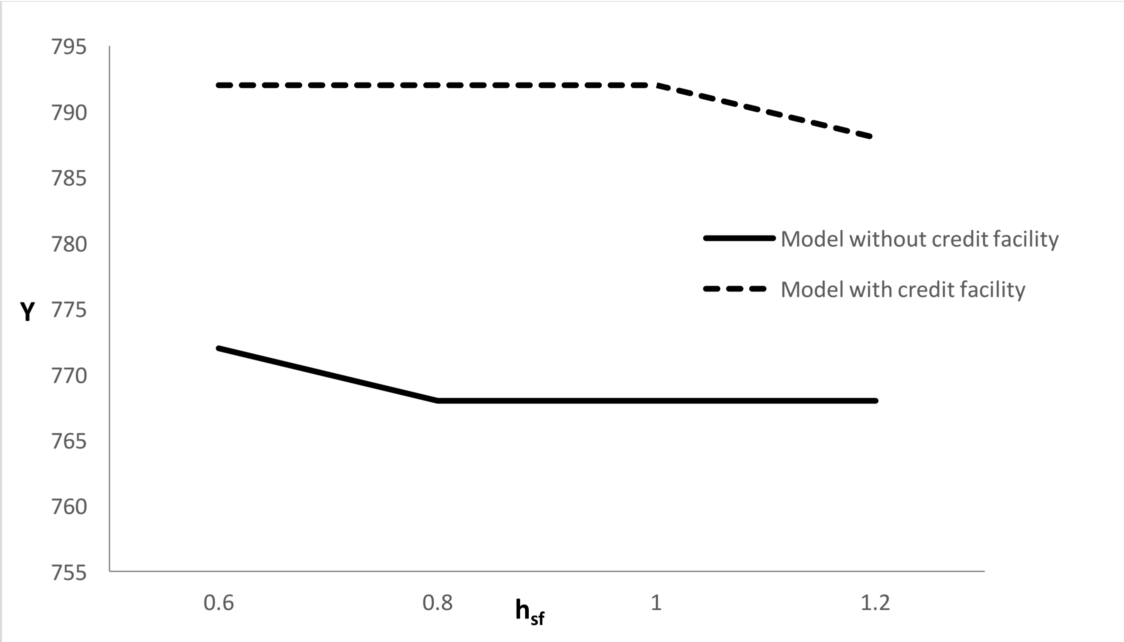

Figure 10: Relationship between n*y* and h_{sf} for both base model and credit facility model

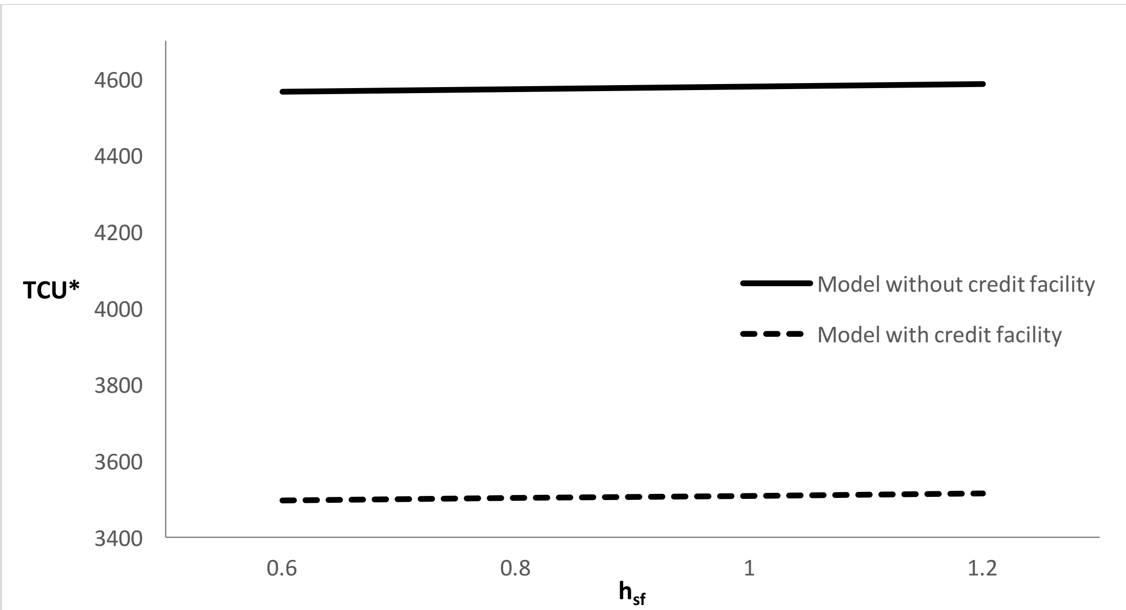

Figure 11: Relationship between h_{sf} and TCU* for both base model and credit facility model

Then sensitivity analysis with respect to the financing cost *i* is shown next. This was varied from 3% to 11%. The results are shown in Tables 12 and 13 and Figures 12 & 13. As expected, when i increases, the number of raw materials decreases but the Total cost increases for both cases.n*y* is directly affected by i because the financing cost can be seen as an opportunity cost and the higher the opportunity cost on ordering the raw material , the less raw material should be ordered.Note that no matter how i is varied the total cost for the model with credit facility is less than that for the one without it.

i	**y***	**n~**	**n***	**n*y***	**TCU***
0.03	319	3.65	4	1276	2761.64
0.05	260	3.65	4	1040	3382.31
0.07	225	3.65	4	900	3905.55
0.09	202	3.65	4	808	4366.54
0.11	184	3.65	4	736	4783.3

Table 12: Sensitivity analysis over i for the First Model

i	**y***	**n***	**n*y***	**TCU***
0.03	319	4	1276	2401.64
0.05	262	4	1048	2809.87
0.07	229	4	916	3128.95
0.09	206	4	824	3392.43
0.11	190	4	760	3617.37

Table 13: Sensitivity analysis over i for the Second Model

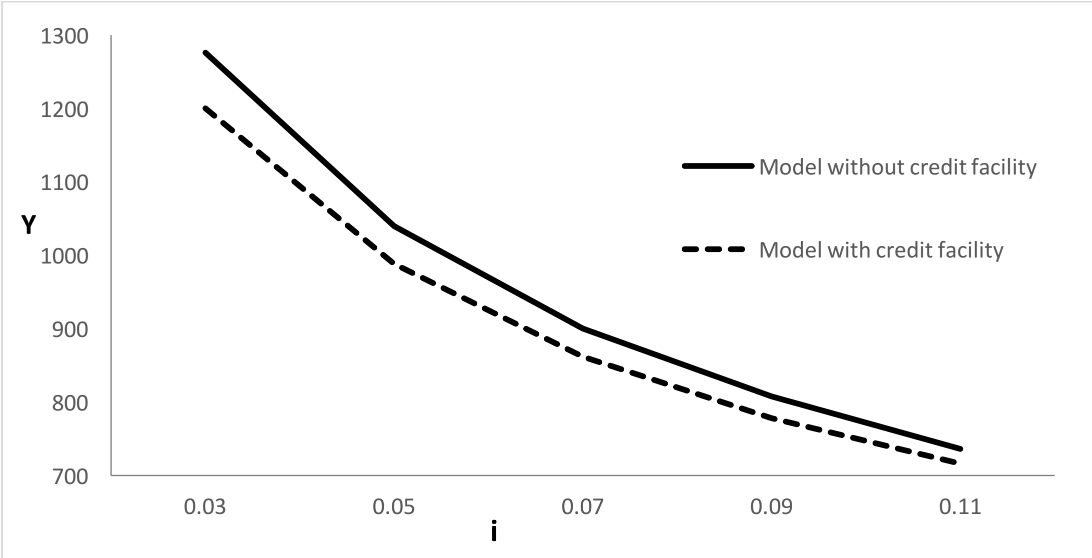

Figure 12: Relationship between n*y* and i for both base model and credit facility model

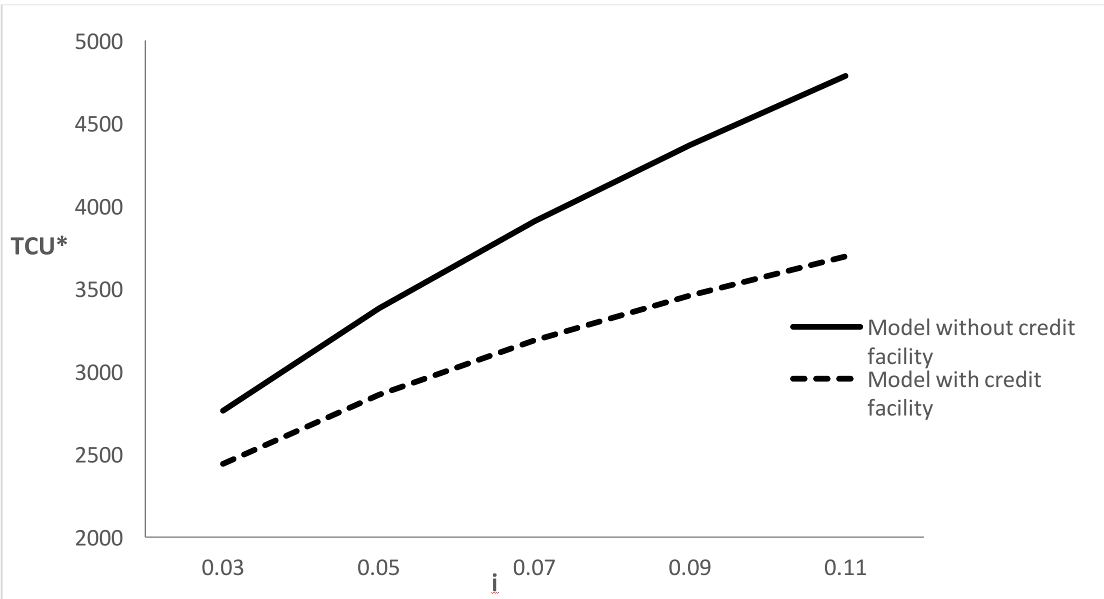

Figure 13: Relationship between i and TCU* for both base model and credit facility model

Next, the earned interest, i_e, is varied from 3% to 11%, leaving everything else constant. The results are shown in Table 14 and Figures 14 & 15. When i_e increases, the number of raw materials that should be ordered decreases and the Total cost decreases for both cases.

n^*y^* is directly affected by i_e because when i_e increases, the order size n^*y^* decreases in an attempt to shorten the cycle duration and allow the manufacturer to benefit from more interest on the generated revenue.

i_e	y*	n*	n*y*	TCU*
0.03	198	4	792	3508.98
0.05	196	4	784	3472.4
0.07	194	4	776	3435.53
0.09	193	4	772	3398.36
0.11	191	4	764	3360.89

Table 14: Sensitivity analysis over i_e for the Second Model

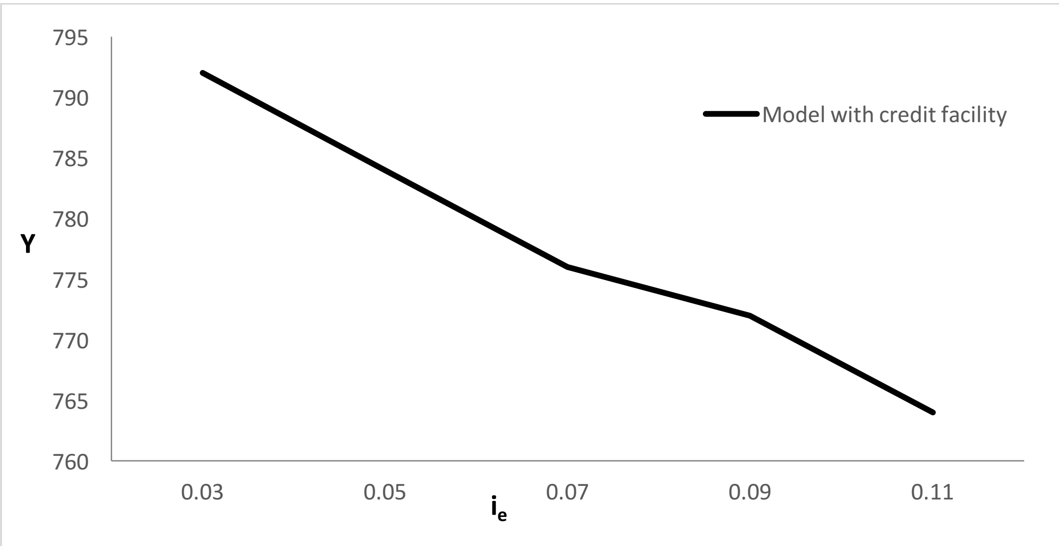

Figure 14: Relationship between n^*y^* and i_e for both base model and credit facility model

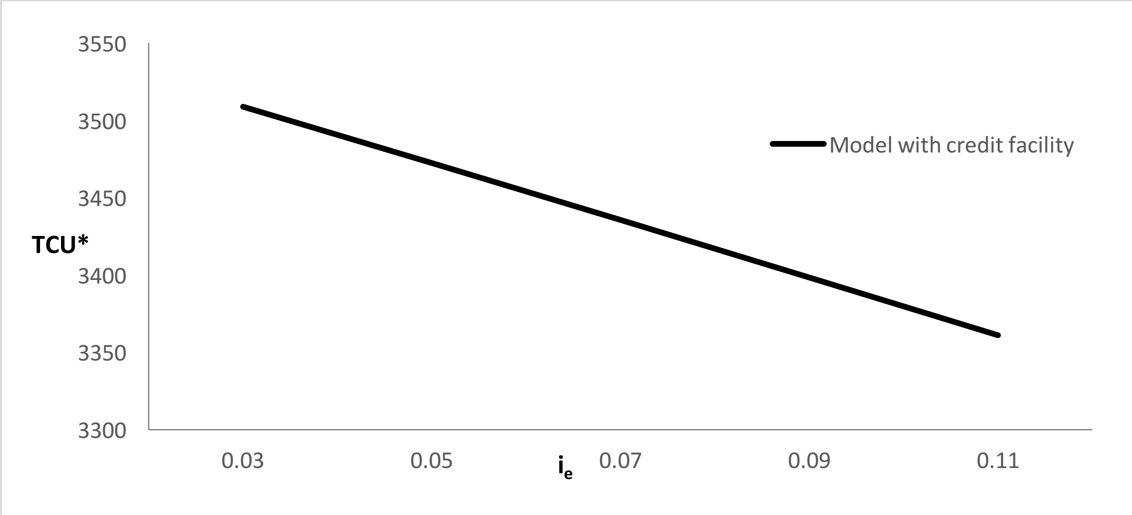

Figure 15: Relationship between i_e and TCU* for both base model and credit facility model

The last thing that we varied is the credit facility period, *T*, which was changed from 0 to 0.7 years. Results are shown in Table 15 and Figures 14 & 15. As we can see, the higher the credit facility period, more raw materials are ordered at a significantly less cost. This shows that ignoring credit facility in the analysis of the IPP model can lead to sub-optimal results with higher perceived costs.

T	y*	n*	n*y*	TCU*
0.0000001	192	4	768	4579.67
0.1	193	4	772	4194.21
0.3	198	4	792	3508.98
0.5	207	4	828	2930.52
0.7	220	4	880	2445.3

Table 15: Sensitivity analysis over T for the Second Model

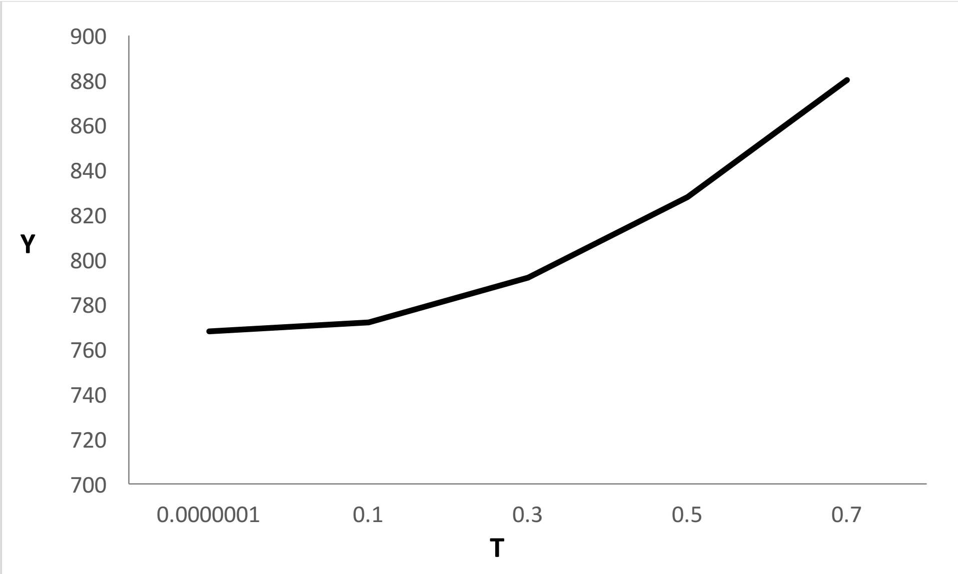

Figure 16: Relationship between ny and T for the Second Model

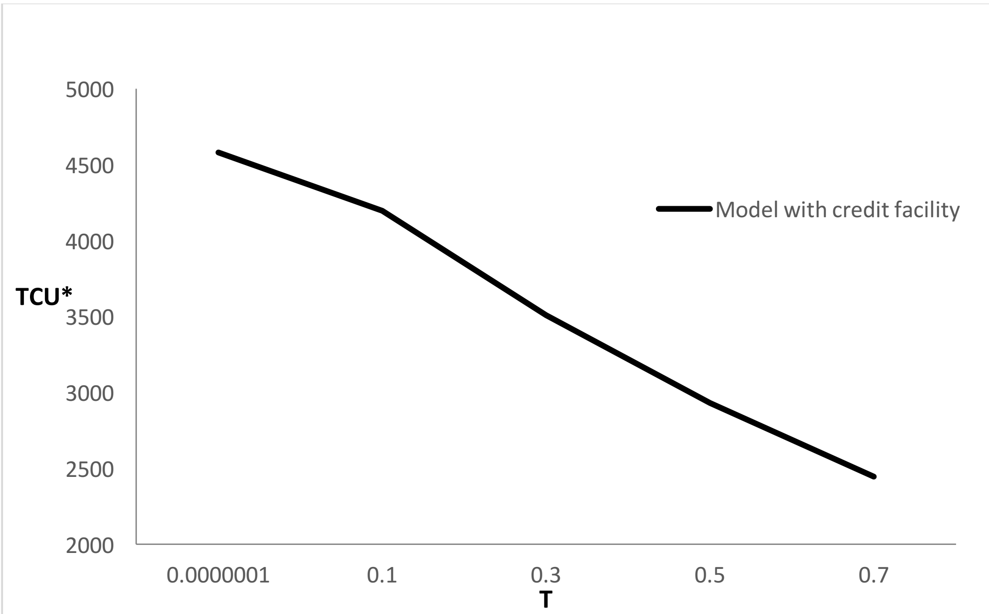

Figure 17: Relationship between TCU* and T for the Second Model

CHAPTER VI
CONCLUSION

In conclusion, unifying the economic lot size problem of the raw materials and the finished products is essential, also including the credit facility while studying the optimal production policy will have a huge impact on the cost and the ordering policy. To make the system even more optimal, the manufacturer should order raw materials for n production runs; this paper presents a closed formula for n in the case of an integrated procurement production system that orders raw materials according to the economic order quantity model and transforms them into finished products using the economic production quantity.

In the future, researchers can base their work on our models by taking different assumptions; they can take into consideration shortage on raw materials and finished products, they can consider the demand to be stochastic, consider there is quantity discount on raw material or they can take into account the time value of money. Considering multiple raw materials with different trade credit terms could also be a useful extension.

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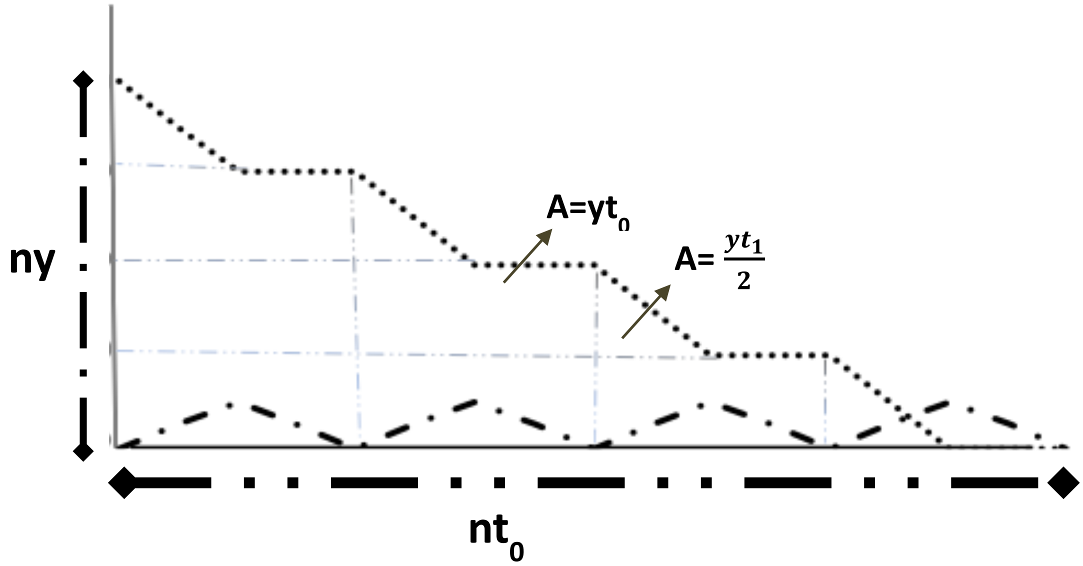
APPENDICES

Appendix 1

Area of raw material in model 1



To find the number of Raw material in inventory at different values of T, we found the areas under the curve that turned out to be in function of triangles and rectangles.

For T<nt$_0$, T falls in the production phase;

$$A_{rm} = \sum_{i=1}^{j}(n-i)\,yt_0 + \sum_{i=1}^{j}\frac{yt_1}{2} - (n-j)\,y\,(jt_0 - T) - \frac{\left(t_1 + (j-1)t_0 - T^2\right)}{2t_1}\,y$$

For T<nt$_0$, T falls in the consumption phase;

$$A_{rm} = \sum_{i=1}^{j}(n-i)\,yt_0 + \sum_{i=1}^{j}\frac{yt_1}{2} - (n-j)\,y\,(jt_0 - T)$$

For T>nt$_0$, $A_{rm} = \frac{yt_1}{2} + \frac{n(n-1)}{2}yt_0$; This is the area under the whole curve so can be used for the first model that doesn't take credit facility into consideration.

Appendix 2
Convexity of *TCU(n,*y*(n)) in model 1

$$TCU(n,y^*(n)) = \sqrt{2 * \frac{[n*K*\beta + K_0*\beta]*\left[h*\left(1-\frac{\beta}{\alpha}\right)+h_0*\left(\frac{\beta}{\alpha}\right)+h_0*(n-1)\right]}{n}}$$

$$N = [n * K * \beta + K_0 * \beta]\left[h * \left(1 - \frac{\beta}{\alpha}\right) + h_0 * \left(\frac{\beta}{\alpha} + (n-1)\right)\right]$$

$$\frac{dN}{dn} = K*\beta * \left[h\left(1 - \frac{\beta}{\alpha}\right) + h_0\left(\frac{\beta}{\alpha} + (n-1)\right)\right] + h_0[n * K * \beta + K_0 * \beta]$$

$$\frac{d^2N}{dn^2} = K * \beta * h_0 + K * \beta * h_0 = 2\,K * \beta * h_0 > 0 \rightarrow N \text{ is convex}$$

and n is linear → *$TC_u(n)$ is quasiconvex (*Refer to Avriel, M.(2003) that talked about the ratio of a convex function by a linear function in his book *Nonlinear Programming Analysis and Methods)*