AMERICAN UNIVERSITY OF BEIRUT

IMPACT OF USING GEOGEBRA SOFTWARE ON THE LEVEL AND TYPE OF STUDENT ENGAGEMENT IN LEARNING GEOMETRY

by
LOUBNA ISSAM HAMZEH

A thesis
submitted in partial fulfillment of the requirements
for the degree of Master of Arts
of the Faculty of Arts and Sciences
at the American University of Beirut

Beirut, Lebanon
July 2020

AMERICAN UNIVERSITY OF BEIRUT

IMPACT OF USING GEOGEBRA SOFTWARE ON THE LEVEL AND TYPE OF STUDENT ENGAGEMENT IN LEARNING GEOMETRY

by

LOUBNA ISSAM HAMZEH

Approved by:

Dr. Hoda Baytiyeh, Associate Professor Advisor

Department of Education

p.p. Dr. Ghazi Ghaith,

Dr. Ghazi Gaith, Professor Member of Committee

Department of Education

p.p. Dr. Lina Khalil,

Dr. Lina Khalil, Assistant Professor Member of Committee

Department of Education

Date of thesis defense: July 28, 2020

AMERICAN UNIVERSITY OF BEIRUT

THESIS RELEASE FORM

Issam Hamzeh Loubna

(x) Master's Thesis ◯ Master's Project ◯ Doctoral Dissertation

[X] I authorize the American University of Beirut to: (a) reproduce hard or electronic copies of my thesis, dissertation, or project; (b) include such copies in the archives and digital repositories of the University; and (c) make freely available such copies to third parties for research or educational purposes.

[ ] I authorize the American University of Beirut, to: (a) reproduce hard or electronic copies of it; (b) include such copies in the archives and digital repositories of the University; and (c) make freely available such copies to third parties for research or educational purposes after:

One ---- year from the date of submission of my thesis, dissertation, or project.

Two ---- years from the date of submission of my thesis, dissertation, or project.

Three ---- years from the date of submission of my thesis, dissertation, or project.

July 28, 2020

Signature Date

ACKNOWLEDGMENTS

The fulfillment of this undertaking could not have been attainable without the contribution and support of a lot of people whose names may not all be set forth below. Their contributions are sincerely appreciated and gratefully acknowledged.

However, with boundless love and appreciation, I extend my heartfelt gratitude and appreciation to the people who believed in me, supported me, and continuously encouraged me to above and beyond. This thesis is dedicated to my parents, Issam Hamzeh and Soad Al Ghazal, my husband, Weam Massoud, and my two beloved daughters Celia and Eliana. Their prayers and confidence in me have helped me push through and accomplish more than I ever anticipated.

Above all, I would also like to express my sincere appreciation by acknowledging my advisor Professor Hoda Baytiyeh, whose expertise, consistent guidance, ample time spent, and consistent advice helped me bring this study into success.

I extend my profound gratitude to the committee members. I would like to thank Professor Ghazi Ghaith, for his continuous support, encouragement, and constructive comments and to thank Professor Lina Khalil, for her patient advice, valuable critiquing, and provisions.

My wholehearted thanks go to my sisters, and dear friends for their love, encouragement, and moral support. To my colleagues at the American University of Beirut, I say, "We meet to part, but more importantly we part to meet."

AN ABSTRACT OF THE THESIS OF

Loubna Hamzeh for Master of Arts
Major: Mathematics Education

Title: Impact of Using GeoGebra Software on the Level and Type of Student Engagement in Learning Geometry

We live in a mathematical world. In such a world, a lack of mathematical engagement closes many doors to productive futures. Research shows that the strategic use of instructional technology positively impacts student engagement in mathematics. Based on The Sociocultural Theory of Engagement, this mixed-method research aims to identify the effectiveness of using GeoGebra software for teaching Geometry lessons. The current quasi-experimental research addresses the impact of using GeoGebra on the level and type of the three dimensions of students' engagement "cognitive, behavioral, and affective" of grade eight students. Results show that implementing GeoGebra could be a catalyst for student engagement in geometry lessons. Overall, students have positive attitudes towards GeoGebra integration and feel that it has positive impacts on learner satisfaction, promotes engagement, and facilitates academic success. These findings are essential due to the technological shift that schools are currently facing.

Keywords: student engagement, Cognitive engagement, Emotional engagement, Behavioral engagement, mathematics, GeoGebra, Geometry

Table of Contents

Appendix

CHAPTER I

INTRODUCTION

The world is changing rapidly in parallel to revolutions of science and evolution of technology. As a result of this, researchers assign high priority on developing innovative research about the positive influence of instructional technology in 21^{st}-century mathematics classrooms (Bray & Tangney, 2016). Research reveals that instructional technology has a positive impact on students' perceptions of mathematics (Gupta, 2009), and on their attitudes towards mathematics education (Ingram et al., 2016). In a nutshell, research confirms that the use of instructional technology in mathematics classrooms enhances student engagement in mathematics (Bray & Tangney, 2016) and reinforces mathematics success (Litster et al., 2019).

Mathematics is the curricular villain that sieves students out of schools into professional careers and helps decision making in everyday life (Schoenfeld, 2002). Mounting evidence points to the relationship between low levels of student engagement and mathematics underachievement (Durksen et al., 2017). Student engagement in mathematics positively influences the standards of their mathematical achievement (Fung et al., 2018). To this end, researchers are increasingly drawing their attention to engagement in mathematics as a precursor of mathematical achievement (Fredricks, 2011).

Fredrick (2011) claims the motivation is what an individual does to generate the desire to be engaged in an activity, while engagement is a multi-component construct that encompasses three kinds of engagement, behavioral, cognitive, and affective. While definitions of engagement vary, most coalesce around the notions of cognitive engagement tap students' investment, including self-regulatory strategies (Fredrick, 2011). Behavioral engagement refers to the extent to which students participate and to the degree of effort applied. Furthermore, affective engagement includes the emotional dimensions of interest and student enjoyment.

Theoretical Framework

Watt and Goos, (2017) consider four theoretical foundations of engagement; "Expectancy-Value Theory" that deals with the perception of abilities, "Achievement Goal Theory" that aims at achievement goals, "Self-Determination" that concerns the innate psychological needs, and "The Sociocultural Theory." The Sociocultural Theory of Engagement shapes the three other theories and affirms social forces improve students' motivations, and thereby sculpt their engagement. Worthy of attention, is that this specific theory puts educators in a position of power and control of the classroom environment. This feature is crucial because education is about making use of what is on hand to create engaging learning environments. After an analysis of multiple dimensions of human experiences, Lerman proposed a unit labeled "person-in-practice-in-person," which certifies a "person-in-practice" involves more than individualized behavior, cognition, or affection.

On the contrary, participation in the practice of a classroom creates a new identity for the individual "practice-in-person" (Watt& Goos, 2017). This theory pertains to the social-cultural approach to learning, which helps educators understand why some students are not engaged and expedite the employment of strategies that promote their engagement. Accordingly, this theory sparks the interest of educators to adjust their classroom instructions to improve student engagement. In other words, the theory advocates instructional interventions that enable the students to work in pairs on their own using a step-by-step guide without the teacher's assistance.

GeoGebra Software

Markus Hohenwarter invented the freely available open-source GeoGebra software by 2002 to support mathematics educators by a facility that makes learning abstract concepts more meaningful. GeoGebra is an interactive geometry, algebra, statistics, and calculus application that support mathematics teaching (Hohenwarter et al., 2009). GeoGebra enables learners to gain stronger links between Geometry and algebra and to visualize abstract mathematical ideas, among all school cycles and throughout university studies. Research conveys GeoGebra software holds a positive impact on myriad dimensions of a mathematics classroom. Various research addressed the effects of GeoGebra mathematics software on enhancing students' mathematical achievement levels on the one hand (Zulnaidi et al., 2019), and on consolidating their conceptual and procedural knowledge on the other (Hutkemri et al., 2017). In the same fashion, considerable studies targeted the impact of GeoGebra on cultivating different mathematical aspects as analytical thinking (Jacinto & Carreira, 2017), abstract thinking (Redo et al., 2018), logical thinking, critical thinking (Aizikovitsh-Udi & Radakovic, 2012), reasoning (Granberg & Olsson, 2015), creative thinking, and problem-solving abilities (Olsson, 2017). Alternative research went far beyond to inquire about the potency of GeoGebra on intensifying discovery, experimentation, and communication (Zengin, 2019). Further inquiries revolved around the impact of GeoGebra on promoting generalization and visualization in teaching mathematics (Khalil et al., 2017).

Purpose of Research

The purpose of this mixed methods research is first to identify and understand the effectiveness of using GeoGebra software for teaching Geometry lessons on the level and type of the three dimensions of students' engagement "cognitive, behavioral, and affective"; case of grade eight students. Further, the secondary purpose is to elicit students' perceptions regarding their engagement in learning Geometry by the use of GeoGebra software.

The purpose was not to prove or disprove a hypothesis but rather to generate descriptions based upon an investigation of students' interactions with the technological tool and with each other and of the impact these might have on their engagement in learning mathematics, specifically, in learning "Geometry."

Rationale

Studies ascertain instructional technology elicits student engagement in mathematics (Litster et al., 2019). Further studies assert that instructional technology improves student attitudes towards mathematics (Mavridis et al., 2017), enhance their motivation, and generates engagement (Lam, & Tong, 2012). Researchers conducted many studies on GeoGebra software due to its importance in facilitating mathematics learning and its ease in delivering knowledge (Olsson, 2017; Hutkemri et al., 2017; Zengin, 2019).

On the flip side, missing pieces in the literature review of the GeoGebra software were inspected. Oddly, studies unveiled that by the day, none of the conducted research has addressed the impact of implementing GeoGebra on student's cognitive, behavioral, and emotional engagement. By that, the significance of "engagement" in mathematical learning processes calls for research on the impact of implementing GeoGebra in geometry classes on student engagement. This study extends the existing knowledge about the importance of exploiting "GeoGebra" in teaching and learning mathematics to fill the gap in the other literature.

The above descriptions do not represent the perceptions and practices of students across all schools and, accordingly, are of limited generalizability. Nevertheless, the study adds to the limited research in this area by providing detailed descriptions of students' use of GeoGebra application and its perceived benefits on their engagement in learning Geometry.

Research Questions

This study will seek answers to the below questions:

1. What is the relative effectiveness of using GeoGebra software on students' affective, behavioral, and cognitive engagement as compared to using conventional learning strategies for teaching Geometry in middle school classes?
2. What is the relation between the different strands of engagement; affective, behavioral, and cognitive engagement?
3. What are the students' perceptions of the role of GeoGebra in enhancing their engagement in learning Geometry?

Contribution to Educational Research and Practice

Findings from this study apprise teachers about students' engagement concerning using GeoGebra software for teaching geometry lessons. The results might introduce a tool for educators to strengthen up the level of students' engagement, specifically in Geometry.

Consistent with The Sociocultural Theory of Engagement, the role of the instructional environment in creating a socially and academically engaging context (Fredricks et al., 2011) may become more evident. In other words, the study reveals how GeoGebra integration redefines the role of the teacher as a facilitator for the sake of promoting student engagement. This study as well might serve as a reference to curriculum developers to design GeoGebra-Based Mathematics Curriculum.

CHAPTER II

LITERATURE REVIEW

Mathematics is labeled as the school villain due to its rigorous foundations, dynamic of reflective logic, complex structure, cumulative construction, formalized mathematical language, high essence of abstractness, and its role as a gatekeeper for many careers. Findings of the latest global research spark high levels of apprehension regarding students' motivation and engagement levels in mathematics. The results of various studies reflect the risk of a steady decrease in levels of student engagement in mathematics, along with insignificant levels of motivation. The reason behind raising the red flag in the field is due to the prominence of motivation and engagement in learning mathematics. Student motivation and engagement are proven factors in mathematics achievement and are as well as essential aspects of successful mathematics classrooms. Accordingly, researchers urged the need to delve into the issue of the efficacy of traditional approaches and strategies to mathematics teaching further. Recent results emphasize the need for a dramatic shift in the mathematic classroom practices to improve the status quo. This prevailing situation calls mathematics educators to implement classroom models that are conducive to stimulate student engagement and motivation. In the push to outfit classrooms with student engagement, researchers in mathematics set extensive guidelines that pave the way for educators to embrace instructional technology breakthroughs (Fredricks et al., 2011).

Motivation vs. Engagement

Despite the widespread confusion, motivation is different from engagement. Motivation and engagement are explicitly related and highly interdependent, yet researchers distinguish between the two (figure 1). In the main, researchers agree upon the fact that motivation underpins engagement. Generally speaking, the experience of being engaged outpaces motivation, as it embraces conjointly the "what," not only the "why" of the act.

Researchers deem motivation as the student's desire, intention, or willingness to act and to be actively involved in the learning process (Durksen et al., 2017). Motivation positively affects students' attention and increases their energy and effort levels. This focus of attention improves students' learning capacity, strengthens students' initiatives, and enhances persistence in carrying out academic activities (West & Uhlenberg, 1970).

While motivation encompasses private unobservable factors that energize epistemic curiosity and active search for new information, engagement comprises outward observable outcomes of this energy (Fielding-Wells et al., 2017). Researchers define engagement as the extent to which students get actively involved in the learning process and note that high levels of motivation are not necessarily related to high levels of engagement (Helme & Clarke, 2001).

Nevertheless, researchers took notice of several features that are common between engagement and motivation. As a start, student attention, which is a part of student motivation, is simultaneously part of the cognitive engagement as it represents a crucial segment of the psychological investment. In a similar vein, persistence coexists in behavioral engagement, cognitive engagement, and adaptive motivation. Along the same line, active student participation contributes to student motivation and, at the same time, influences behavioral engagement. However, underlying motivational processes that influence the ways students engage in mathematics are difficult to pinpoint. Therefore, teachers base their judgments on conspicuous students' behaviors and interactions as a sign of engagement (Skilling et al., 2016).

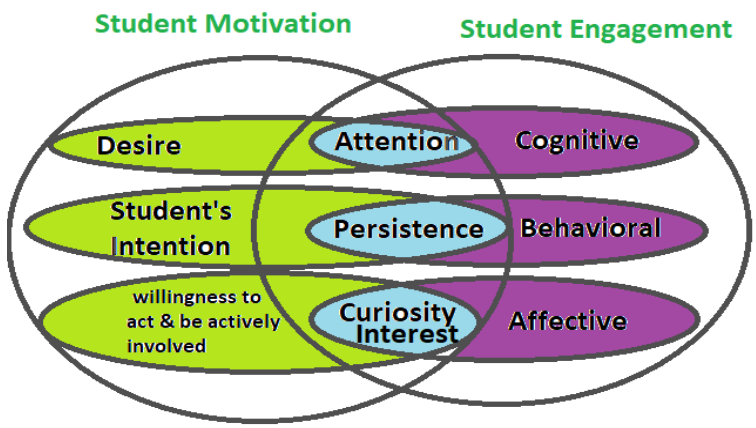


Figure 1: Motivation vs. Engagement

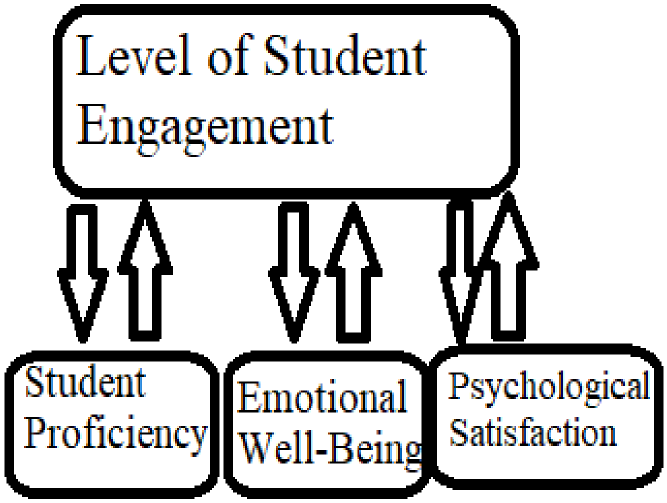


Figure 2: The correlation between levels of engagement and student proficiency, emotional well-being, and psychological satisfaction.

Student Engagement

There is an increasing interest in addressing the area of student engagement as educators continue to seek strategies that promote student engagement. Numerous studies emphasize the correlation between levels of engagement and student proficiency, emotional well-being, and psychological satisfaction (figure 2). However, improving engagement is a challenging task due to the complex construct of student engagement and the multiple factors that could influence it.

Types of Student Engagement

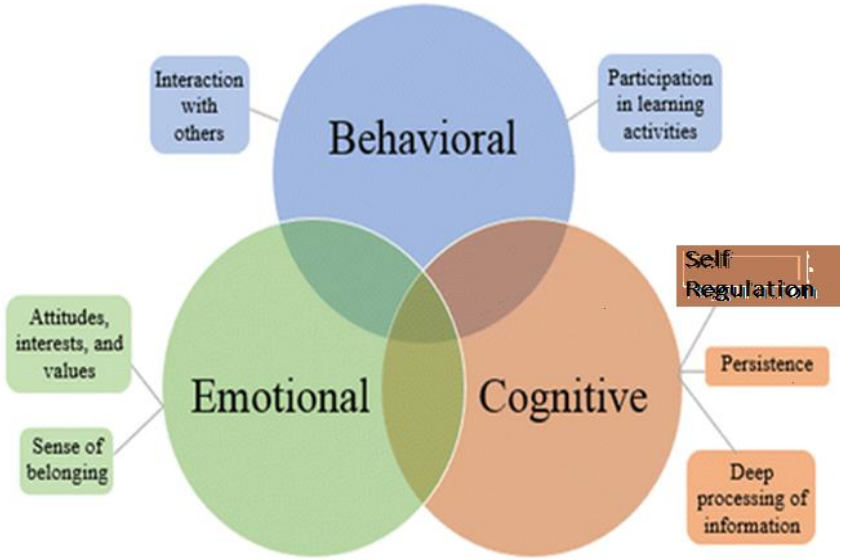


Figure 3: Types of student engagement

In an absolute sense, researchers differentiate between two broad kinds of engagement: productive and receptive engagement (Fung et al., 2018). Productive engagement focuses on the active role of the learner and the sustained activation of working and other executive processes. Contrarily, receptive engagement refers to activities that rely on passive observation, perception, activation of existing knowledge, and informal events. According to their research, Fung et al. (2018) note that students perform better when they get productively engaged as compared to being receptively engaged (figure 3).

In a more specific manner, Skilling et al. (2016, p. 9) designed the Engagement Spectrum to highlight the three main types of engagement: "behavioral," "emotional," and "cognitive." The engagement spectrum shows, in addition to the kinds of engagement, the different levels that form a nested hierarchy ranging from "disengaged," "variably engaged," to "substantially engaged."

Affective engagement. Affective engagement refers to student interest, perception, the value of learning and success, attitudes, and emotional reactions such as enjoyment, boredom, anxiety, and frustration. Researchers mainly rely on student self-report instruments in addition to observing student affective responses and behaviors to measure affective engagement. Given its extreme importance, a reliable means to measure student affective engagement involves capturing insights on students' intrinsic range of emotions while getting engaged in academic tasks (Fung et al., 2018).

Behavioral engagement. On the other hand, behavioral engagement refers to the active student participation, intense effort, communication including initiating and answering questions, interaction with others, involvement in voluntary activities, attentiveness, diligence, and persistence. Researchers inspect levels of student behavioral engagement through observation, in reliance on the displayed frequency of behavioral actions exhibited by students towards learning experiences (Fung et al., 2018).

Cognitive engagement. Cognitive engagement refers to the psychological investment that students apply to their learning (Skilling et al., 2016). More delicately, cognitively engaged students are learners who have a sense of control over their learning processes and on their academic outcomes (Helme & Clarke, 2001, p. 9).

According to Pierce et al. (2007), the four integral dimensions that constitute cognitive engagement are openness, perseverance, self-regulation, and mindfulness. The previously mentioned researchers specified openness as the shorthand for using a variety of creative problem-solving strategies. They defined determination as putting forth an effort to continue when encountering unfamiliar situations, openly expressing difficulties, acknowledging errors, and adaptively seeking help. On the other hand, active mental involvement in high cognition, concentration, attention, and efforts that students put to comprehend demanding tasks fall into the dimension of mechanical cognitive practices. In Brief, cognitively engaged students are those who persist in being mindful, thoughtful, and self-regulated. They are students who manage to set clear goals, organize, self-monitor, and self-evaluate their learning using regulating cognitive and metacognitive strategies at various points of the learning process.

From his side, Kong et al. (2003) proposed the cycle of three phases and the hierarchy of cognitive engagement. The lowest stage of cognitive engagement is the reliance on parents or on teachers to receive knowledge. The later stage is the surface phase that covers memorization, practicing, and handling tests whereas, the latest step in the deep phase, which implicates deep understanding, drawing summaries, and building connections to prior knowledge.

Similarly, Corno and Mandinach's (1983) model of cognitive engagement distinguishes four hypothetical forms of cognitive engagement: self- regulated learning, task focus, resource management, and recipience. Self-regulated strategies refer to the use of higher-order metacognitive components. These strategies constitute the highest form of cognitive engagement. A task focus is a process of metacognition, where students independently set their attention to the plan and execute a specific task and self-monitor of one's actions. Resource management relates to "help-seeking" and suggests students accessing different types of external sources to get educational objectives. The lowest form of cognitive engagement in this model is recipience, in which students respond inertly with little cognitive investment.

A significant body of research stresses the myriad positive effects of increasing the quality of cognitive engagement in classroom tasks. The benefits of cognitive engagement include an increase in learners' academic achievements, enhancement of their mathematical abilities, and the development of self-regulated learning (Helme & Clarke, 2001). Researchers identified intense levels of cognitive engagement as "the flow experience." Comprehensively, Helme & Clarke's (2001) study brought to light the fact that the flow experience or high-levels of cognitive engagement evolve through student-student interactions rather than teacher-student interactions.

More thoroughly, they affirmed as well that individual factors, task characteristics, and social rules that govern educational activities play a pivotal role in expressing cognitive engagement.

Measuring cognitive engagement. First and foremost, signs of behavioral and affective engagement are more in number and scope than for cognitive engagement and can be more obviously noticeable (Skilling et al., 2016).

The measurement and psychometric examination of any cognitive process involve self-reporting techniques. Nonetheless, self-reports on academic metacognitive experiences are unreliable, the thing that drives researchers to adopt combined measuring instruments. Some instruments that measure cognitive engagement along with self-reporting instruments are observation, interviews, transcribed video, and audio recording. The interdependence of cognitive aspects of engagement urges researchers to take account of a range of behaviors in an analysis of video record or the interpretation of classroom observation (Helme & Clarke, 2001). As a result of this, measuring cognitive engagement depends on the classroom situation, including specific linguistic and behavioral indicators.

Employing this, Helme & Clarke, (2001, p. 9) had set a model that helps researchers conduct a precise objective analysis of student behavior to spot cognitive engagement in different learning situations. In case students are assigned to work in parallel, their model advocates observers in classrooms to highlight practices that reflect cognitive engagement as verbalization of thinking, self-monitoring, concentration, resisting distractions and interruptions, gestures, and seeking information or feedback. When students are working in a small-group cooperative learning environment, the observer should inquire if students ask enough questions, complete their peer utterances, exchange ideas, give directions, explain information, or justify an argument in addition to gestures. When holding small group student-teacher interactions, the observer should track whether students evaluate comments, answer teacher's questions, give information, explain procedures and reasoning, address questions to the teacher, hold reflective self-questioning, and persist in completing tasks. In like manner, in a whole teacher-class interaction, the observer should seek to ask and answer questions, making evaluative comments, contributing ideas, and complete teacher utterances.

Theories of Engagement

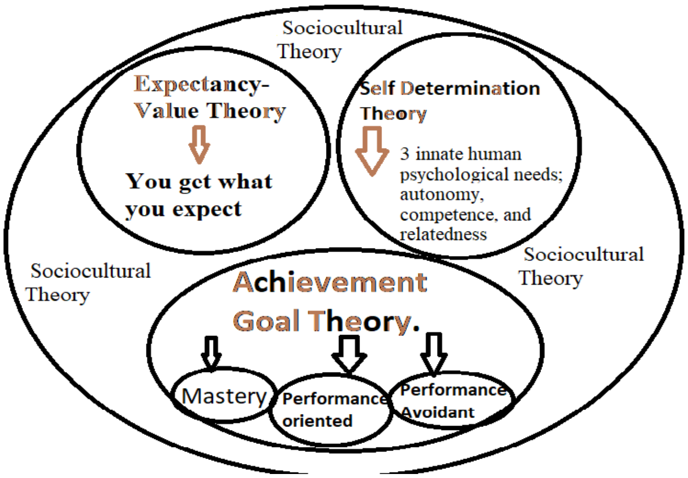


Figure 4: Sociocultural Theory of Engagement

Engagement scholars have used a wide range of theoretical frameworks to expound the engagement process (figure 4). The various theoretical lenses that researchers use reveal how engagement fits within more comprehensive psychological paradigms (Carmichael et al., 2017). Using empirical studies of engagement, Fredricks et al. (2011) and down the road, Watt & Goos, (2017) pinned down four predominant engagement theories: Expectancy-Value Theory, Achievement Goal Theory, Self-Determination Theory, and The Sociocultural Theory of Engagement.

Expectancy-Value Theory. The first theory is the "Expectancy-Value Theory" (EVT) that represents a framework to understand how young people's beliefs predict their educational choices. EVT as an explanatory framework enables researchers to realize that when teachers set a learning task, students question themselves and their capacities. Based on Fielding-Wells et al.'s (2017) work, when teachers ask students to perform a given task, they instantly query whether they have what it takes to succeed that task, whether they want to do it and the reason behind taking action towards the job. Eventually, student's responses to these questions shape the degree of energy, attention, and self-regulated work they direct towards the task.

Achievement Goal Theory. The second theory is the "Achievement Goal Theory," which divides individuals into three types. On the top, students who hold mastery goals put a high emphasis on improvement, understanding, and optimal self-development. Forasmuch, performance-oriented students are competent enough to put their highest focus on grades and on outperforming others. "Performance-avoidant" individuals are those who get motivated by the fear of demonstrating poor performance. With the three types of individuals noted in the Achievement Goal Theory, Helme & Clarke (2001) inferred that students who are more mastery-oriented use more cognitive strategies and engage in more metacognitive activities than the others.

Self-Determination Theory. The third theory is "Self-determination theory" that proposes a set of three basic innate human psychological needs; autonomy, competence, and relatedness (Carmichael et al., 2017). As an illustration of the theory, teachers continuously strive to create socio-emotional support for their students to stimulate their cognition.

The Sociocultural Theory of Engagement.

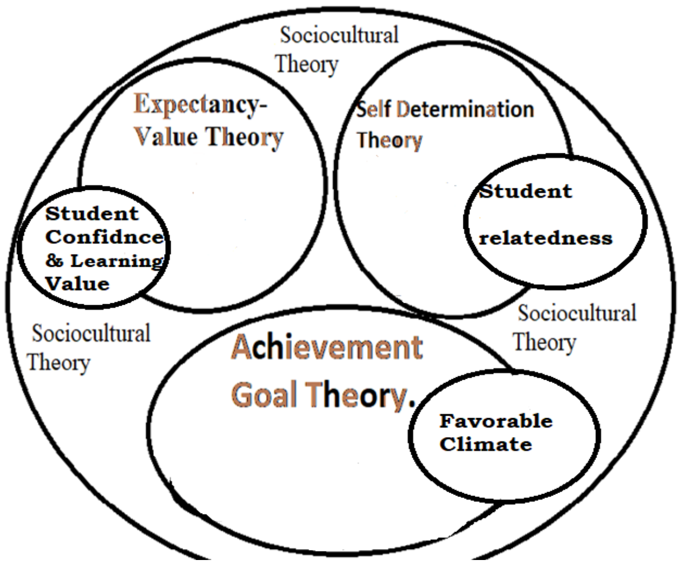


Figure 5: Commonalities of Sociocultural Theory

The preceding three theories of engagement are "inside-out" theories being primarily focused on individual factors, though contextual forces shape them. On the flip side, the notion of engagement from a sociocultural perspective looks from the "outside-in" perspective. The sociocultural view aims to understand how learning environments promote different engagement structures and lead to different student interactions.

Studies reveal that promoting student engagement depends on various indicators some-as student confidence, favorable climate, relatedness, and using advanced materials and connections (figure 5). This theoretical perspective corresponds with the Expectancy-Value, Achievement Goal Theory, and Self-Determination Theory. The student confidence and value of learning coincide in EVT and Contextual Influences Theory. The favorable climate is a common factor of AGT and Contextual Influences Theory. Also, the student confidence and relatedness play a crucial role in SDT and The Sociocultural Theory of Engagement. In that context, the last theory of engagement embeds within the three other theories of engagement and combines all the elements of human social behavior: affect, cognition, communication, and meaning (Watt & Goos, 2017).

Substantially, "The Sociocultural Theory of Engagement" proposes that the encircling environment shapes one's behavior and engagement (Guardino & Fullerton, 2010). The Sociocultural Theory of Engagement emanates from classroom organization, relationships, division of roles, responsibilities, and content or type of knowledge presented in the classroom. In greater detail, teacher instruction, pedagogy, assessments, assumptions, convictions, teacher-student, and student-student interaction and, eventually, privileged features impact the student's engagement (Durksen et al., 2017). Ultimately, pedagogy and classroom instruction is adequate factors in promoting engagement. In short, students become part of their classroom practices, and their classroom practices become part of them (Lerman, 2001). Sociocultural theories do not take the individual learner or the individual teacher as the unit of analysis but instead, take a "person-in-practice-in-person" unit or "student in-classroom-in-student." These theories state that a person develops a new identity in the context of different social-cultural experiences. Psychologists assure students may engage and behave divergently upon different sociocultural normalization that surrounds them.

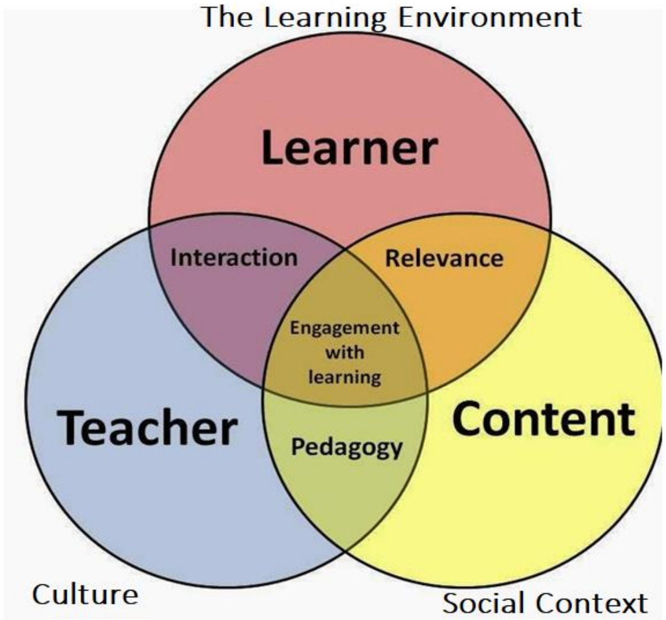


Figure 6: The learning environment

In light of this, Helme & Clarke (2001) assert that the interplay between "the individual," "the learning environment," "the academic tasks," and "the teacher instruction" constrain or promote different forms of student engagement (figure 6). Likewise, Geiger et al. (2017) affirm that student academic engagement gets shaped by classroom structure and culture, environment, and people who are around the classroom, affordances, and constraints, and by adopted approaches to teaching and learning. A wealth of empirical studies had been handled in MERJ special issue on "Theoretical Foundations of Engagement in Mathematics" to discuss thoroughly the sociocultural factors of engagement given their prominence in modern education (Sträßer, 2017).

The learning environment and culture. Stating things clearly, modifying one dimension of the learning environment could change students' behaviors and serve as a catalytic agent of student academic engagement. Muir (2014) revealed that classroom culture is the main reason for students' lack of academic engagement. From a similar perspective, Attard (2013) defines student engagement as the relation between the student and the classroom work.

Central to the cultural psychology approach is the notion of artifacts. The artifact is physical objects that gain meaning and function through a form of language. To this end, educators should be aware that the student's identity and student engagement get shaped by language as a mediating cultural artifact (Jorgensen, 2010).

Brown (2017) affirms engagement in learning is socially constructed on the ideological involvement of the teacher and the students in the learning encounters. The social context is the main foundation of engagement, given that it exhibits to students the significance of their work and directs their energy towards organizational goals. The social context exerts positive pressure on students to conform with a high level of the cognitive, behavioral, and environmental performance of their peers. In this context, learning needs to be viewed holistically through situated or sociocultural perspectives (Fredricks et al., 2011).

Academic tasks and teacher instruction. The instructional strategies that teachers use and choices they make to model thinking enhance or impair the subsequent classroom engagement (Fredricks et al., 2011). The Measures of Effective Teaching Project, held by Geiger et al., (2017), reflect that classroom structure, task characteristics, and the pedagogical practices regulate classroom engagement. Proceeding on the same track, Durksen et al., (2017) indicated that increasing positive engagement or reducing negative engagement hinges on the teacher's practices, instructional support, and active classroom organization. A case in point is utilizing interactive teaching activities that enhance students' engagement and contribute to keeping students busy with tasks (Guardino & Fullerton, 2010; Lazarides & Rubach, (2017).

Socio-Cultural Theorem in mathematics classrooms. Before diving into further details, mathematics in itself is a social construct and a social activity. In other words, student's self, student's mathematical identity, and student engagement in mathematics at multiple levels get constructed by interactional practices of school mathematics (Jorgensen, 2010). Subsequently, researchers shifted their focus from searching for ways to bring about mathematical understanding as a "decontextualized mental process" towards developing identities of students as speakers and actors of mathematics in school classrooms; the so-called "student-in-mathematics-classroom-in-student." Elements of this mathematical identity include ways in which the teacher, students' previous experiences, social relationships, division of roles in the classroom, and the artifacts in class frame mathematical activities (Lerman, 2001).

On his side, Litster et al., (2019) adopted in his study the definition of "affordances" as cues to the potential uses of an artifact by an agent in a mathematical environment. When children engage with a mathematics application; for example, the application becomes their environment, the interactive features become the artifacts, and the child becomes the agent. The features provide potential cues to help children understand possible actions and relationships between mathematical concepts and procedures in the application.

Accordingly, several factors, including mathematics classroom cultures and ineffective teaching instruction, have been identified as contributors to student disengagement in mathematics (Carmichael et al., 2017). Valsiner's zone theory in mathematics focuses on the influences within an environment of the teachers' actions concerning the promotion of students' learning and engagement (Valsiner, 1997). The ZPA relates to how adopting innovative instructional approaches advances student actions, thinking, learning, and engagement in mathematics (Geiger et al., 2017). Pedagogical approaches that involve active participation, mathematical investigations, and exploration with digital technologies appear to concurrently foster student engagement and student achievement in mathematics (Angelini et al., 2017).

Engagement in Mathematics

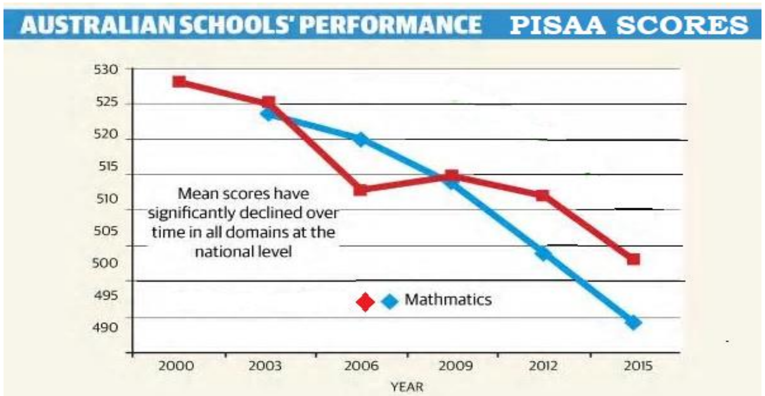


Figure 7: PISAA Scores

Mathematics is the critical filter that delimits access to a range of STEM occupations, enables discipline for various areas of intellectual inquiry, and profoundly affects national well-being (Lazarides & Rubach, 2017). Engagement in mathematics is of paramount importance since it is directly proportional to mathematics achievement. Brown (2017) confirms that the level of engagement with mathematics has a significant impact on student performance in the short term and on the quality of their learning in the long run. Regrettably, the latest PISA (figure 7) and TIMSS results in Austria sounded alarm bells among educators, policymakers, politicians, and in the public media as they apprised a critical issue of low engagement levels in mathematics (Geiger et al., 2017; Watt & Goos, 2017). Regarding this dilemma, researchers agreed that the reduced achievement levels on PISSA and TIMSS relate to the insignificant engagement levels in Mathematics. A wealth of studies concluded that mathematical engagement was the precursor to students' performance in those national and international assessments (Watt & Goos, 2017).

Significance of engagement in mathematics. Student engagement is deemed to be one of the critical solutions to improve students' academic performance in mathematics (Fung et al., 2018). The interrelated components of students' engagement jointly reinforce each other to benefit mathematical learning. Particularly, cognitive engagement has the most substantial impact on achievement in mathematics among the different dimensions of engagement. In the same vein, affective engagement plays a significant role in embellishing negative attitudes towards mathematics that are extremely difficult to change and persist into adult life. Besides, behavioral engagement spurs students to attend mathematics classes more regularly and to put in the required effort in mathematical tasks.

Risks of disengagement in mathematics. The appalling decline in universal mathematical performance during a critical period in students' lives demands the extensive focus of research on the issue of engagement in mathematics (Durksen et al., 2017). Based on empirical research, Attard (2013) believes that the growing concern regarding lowered student engagement with mathematics goes back for what it holds of adverse effects on limiting students' future opportunities and limiting their ability to understand life experiences through a mathematical perspective. Along those lines, warning signals have been raised regarding students' disengagement with mathematics, which contributes not only to low achievement levels but also to ongoing decay in numbers of students undertaking senior secondary and tertiary studies in mathematics (Angelini et al., 2017). Accordingly, Fredricks et al. (2011) confirm that student disengagement in mathematics is causing notable school drop-out rates and is causing an emerging shortage of students studying subjects of applied mathematics.

Moreover, statistical data apprises that proportions of students taking intermediate and higher mathematics and proceeding to mathematically-based subjects in science, engineering, and technology are becoming extremely low. In several studies, students report negative feelings towards mathematics and declare that they avoid the subject choice of mathematics. Students justify this stance on the pretext that they perceive mathematics as less attractive, less enjoyable, monotonous, stressful, disengaging, and meaningless. The worst part is that even high achievers at school report that they fail to enjoy or recognize any personal relevance of mathematics, and few of them voluntarily continue to study mathematics in depth (Attard, 2011).

History of student disengagement in mathematics. The reality of low levels of engagement in mathematics is by no means new. Formerly, authors such as Fredericks et al. (2004) previously tackled the increasing need for educators, researchers, and policymakers to address the problem of students' disengagement in mathematics. Beforehand, researchers designed extensive studies to inquire about the drop-in mathematics engagement during the middle and secondary years of schooling. Mounting evidence in that body of research revealed a significant correlation between low levels of student engagement and academic underachievement, low participation rates, and low retention rates at school (Martin & Marsh, 2006). Even before, Helme & Clarke (2001) were as well concerned by the deterioration in the perceived self-control and engagement over the transition from primary to secondary school. Concisely, disengagement with mathematics begins early in primary school but becomes more pronounced when students' progress towards middle and high school, affected by a complex decline in positive attitudes towards the subject.

Even way back to 1989, the National Council of Teachers of Mathematics (NCTM) produced "The Principles and Standards for School Mathematics" as a response to the abominable national assessment data, that as well reflected a severe decline of engagement levels in high school mathematics (Fielding-Wells et al., 2017). Grievously, students taking advanced mathematics at high and intermediate schools nearly halved from 1980 to 2004 (Norton, 2006). The American students' disengagement in learning mathematics as they progress towards middle school was associated with an international downwards trend in the proportion of students taking higher mathematics (Kong et al., 2003). The critical mathematics situation became universal, where at least two-thirds of American high school students (Kong et al., 2003), and more than three-quarters of Australian students reported disengagement in learning mathematics (Carmichael et al., 2017).

Factors affecting student engagement in mathematics. Students' lack of engagement with mathematics became one of the most critical issues that educators face in the twenty-first century (Brown, 2017). In search of possible reasons why students become disenchanted in mathematics, researchers were quick to mainly blame it on the traditional methods of teaching mathematics (Norton, 2006; Brown, 2017; Lazarides & Rubach, 2017). In further investigation, Attard (2013) links the decrease in engagement levels with inappropriate teaching strategies, irrelevant math curricula, un-interesting classroom tasks, teaching practices that mismatch students' expectations along with unsuitable cultural and technological conditions.

The international conservative agendas, such as national standard testing regimes, do not account for the great diversity of the nation, are working against mathematics engagement at a deep level (Jorgensen, 2010). The current testing regimes not only cost the national budget considerable amounts but as well take funds away from ensuring quality engaging teaching is possible.

Solution for student disengagement in mathematics. Accordingly, researchers and educators pledged in a concerted effort to understand factors that improve teaching practices and open access doors for all students into the field of mathematics (Jorgensen, 2010). On its side, by 2003, Austria started taking some measures that could improve that problematic situation, where the Australian Curriculum Board acknowledged the demand to investigate the notion and constructs of student engagement in the context of mathematics learning (Kong et al., 2003). In accordance to the low the PISSA and TIMSS results, a major educational institution in Austria some as "The Australian Association of Mathematics Teachers (AAMT)," "The federal government agency for scientific research in Australia," "The Australian Curriculum Board" and many others declared a state of public alert toward mathematics education. They publicized the urgent need for initiatives to improve students' engagement in the learning of mathematics (Attard, 2013; Angelini et al., 2017).

Consequently, teacher awareness and perceptions of student engagement during mathematics instruction, the academic circumstances, and the classroom environments are wholly essential to student engagement (Skilling et al., 2016). Against this background, researchers urge educators to approach mathematics lessons through a variety of pedagogies that seriously foster engagement (Jorgensen, 2010). In means, teachers should regularly infer ways to adjust and implement productive mathematical tasks that students find authentic and engaging, to arrest disengagement effectively, and to persuade student decision of enrolling in higher-level mathematics (Norton, 2006). Executing such appropriate teaching practices over an extended period, impels children to enjoy and value mathematics and eventually become mathematically engaged (Attard, 2011).

Technology-Mediated Approach to Mathematics

In an age of digital technology, millennials and all succeeding generations are generations of information communication technology, alleged as Digital natives. The reliance on fixed personal computers is quickly diminishing, making way for a variety of mobile inferences for learning within formal and informal settings (Kyriakides et al., 2016). The recent few years have witnessed rapid worldwide adoption of mobile instructional devices based on the explorative, productive, instructive, instructional role that they fulfill at homes, schools, educational institutions, and outdoors. Mobile instructional technologies and applications pave the way to reshape aspects of mathematics learning experiences. Worth noting that despite the increased interest on mobile devices as learning tools, research that concerns integrating them into mathematics teaching and learning is still relatively small due to the novelty of these technologies (Larkin et al., 2016).

Impact of instructional technology on educators. The National Council of Teachers of Mathematics states the importance of using technology to assist students in learning mathematics, especially Geometry. National policies as well, advocate strategic integration of ICT in teaching and learning since it increases student participation and gets everyone involved, particularly special needs students and those who are at risk of disengagement (Muir, 2014). Above that, educational technology creates the ideal environment for experimenting with mathematical ideas, and a collaborative environment for students to work and master classroom transitions between different learning situations (Larkin et al., 2016). Adding to the preceding, Attard (2012) exposed that the use of mathematical educational technology allows creating practical lessons that incorporate peer-peer and peer-teacher interaction while practicing new concepts, especially in middle school. Such environments provide a foundation that turns the abstract structure of mathematical ideas into concrete, and a means to move students between activities smoothly, which saves time and secures a smooth lesson flow. Besides, educational technology can as well bring real-life situations into the classroom, making mathematics more relevant to students' lives and more attractive.

Impact of instructional technology on student thinking. Technology has the potential to enhance student's learning of mathematics. Findings gained from various teaching interventions suggest that mobile instructional technology holds much promise as a tool for reforming mathematics education (Kyriakides et al., 2016). In a province-wide evaluation of educational research, results suggest that the adoption of instructional devices and software within and beyond the classroom confines can provide an educationally rich dynamic environment. Educators put particular emphasis on Mathematics applications since they increase students' autonomy, allowing the student to develop a deep holistic understanding and to overcome any misconceptions (Litster et al., 2019).

Furthermore, the instant feedback of mathematics applications encourages students to be more efficient by linking together multiple representations simultaneously and making immediate connections between different mathematical concepts. In conjunction with that, incorporating digital technologies promotes logical thinking and reinforces student mathematical resilient intents. Instructional technology as well supports the optimal development of valuable mathematical problem-solving skills such as strategic thinking, planning, multi-tasking, self-monitoring, communication, negotiation, group decision-making, pattern recognition, accuracy, speed of calculation, and data handling (Larkin et al., 2016). Quite so, instructional technology holds the potential to shift students' conceptual understanding with their dynamic visual affordances and to uphold every bit of student mathematical thinking.

Instructional technology and mathematics software help students broaden their fundamentally narrow viewpoint of mathematics as being primarily computation and arithmetic (Kyriakides et al., 2016; Larkin et al., 2016). Instructional technology has the power to enhance mathematical exploration, to provide dynamically linked representations of ideas, and to encourage general metacognitive abilities such as planning and checking (Pierce et al., 2007).

Impact of instructional technology on student engagement in mathematics. Creating learning environments that engage struggling students and disengaged learners while at the same time provide profound learning opportunities is one of the biggest challenges facing mathematics teachers. The research literature suggests that digital educational application have many potential benefits for mathematics teaching and learning including the capacity to motivate, engage, immerse students, increase in students' learning outcomes and improve students' attitudes and positive emotions towards mathematics (Bray and Tangney 2016; Kyriakides et al., 2016; Larkin et al., 2016; Litster et al., 2019). In a related move, Pierce et al. (2007) consider that implementing instruction technology alters the levels of student engagement in mathematics. In parallel, Skilling et al. (2016) agree that using interactive materials, instructional technology, and visual aids mediates students' engagement.

A study by Ingram et al. (2016) indicated that integrating mathematics instructional technology and applications reinforces not only the level of students' engagement but also its quality. In their study, students went beyond being engaged by the technology to become involved with mathematics both independently and cooperatively. Similarly, Kyriakides et al. (2016) reflect that mobile technology not only affected the engaged learning of mathematics and the level of engagement but also addressed the three dimensions of engagement. The technology device in the study increased the exploratory behavior, promoted higher levels of student control over their learning of mathematics, and at the same time, increased the students' affective and cognitive engagement. In a nutshell, a promising way to encourage students to understand mathematical concepts and become highly engaged at the same time is the use of information and communication technologies in the teaching and learning process (Zulnaidi et al., 2020).

GeoGebra as an Instructional Technology

GeoGebra is a free open mathematics software created by Markus Hohenwarter in 2001. GeoGebra is a simple but powerful tool that joins Geometry, algebra, statistics, and calculus for all levels of education at once. GeoGebra enables teachers to shorten distances between anchoring knowledge points and new mathematics concepts (Francisco & Barros, 2019). What sounds interesting about GeoGebra is that it supports student learning, besides being a teachers' instructional tool (Larkin et al., 2016). GeoGebra is handy in teaching and learning mathematics since it eases the visualization of complex abstract concepts and creates links between various mathematical knowledge (Zulnaidi et al., 2020). This dynamic software makes the concepts of mathematics dynamic, and the teaching of mathematics more explorative in which the students can find the relationship between the analytic and the visual representation of mathematical concepts instinctively.

Using ICT terms, GeoGebra is the Dynamic Mathematics Environment that combines both the Computer Algebra System and the Dynamic Geometry Environment in mathematics (Kaya & Öcala, 2018).

Research in GeoGebra

There is a rapid growth in research that investigates the contributions of GeoGebra software in learning and teaching mathematics. As a result, educationists became able to see the capabilities of GeoGebra utilization in discovery-based mathematics classes.

Impact of GeoGebra on student achievement.

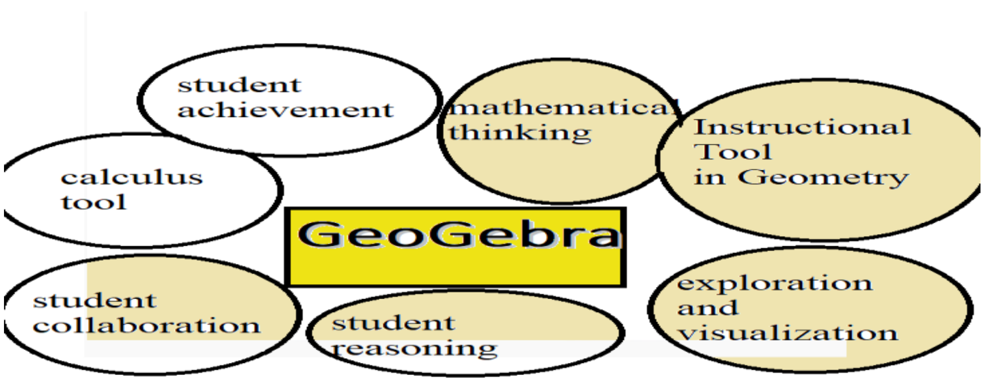


Figure 8: Impact of GeoGebra

GeoGebra is an instructional tool that could lessen mathematical misconceptions and positively reflect on students' performance (Zulnaidi et al., 2020). The results of the previous study align with prior research of Kaya & Öcala (2018), identifying that GeoGebra is a teaching aid that could strengthen students' mathematics achievement (figure 8).

Impact of GeoGebra on student mathematical thinking. GeoGebra increases students' interest in learning mathematics, and correspondingly enhances their cognitive abilities (Zulnaidi et al., 2020). Results from considerable research show that GeoGebra enhances five aspects of mathematical thinking: generalization, analytical thinking, logical thinking, abstract thinking, and representation (Khalil et al., 2017). GeoGebra includes game-strategic thinking that helps students in the construction of valid links between concepts during the argumentation phase of the proving process (Larkin et al., 2016). The use of GeoGebra amplifies critical mathematical thinking and creates a constructivist approach towards inquiry-based learning. This environment, in turn, empowers techno-mathematical fluency that enhances tackling complex mathematics problems (Jacinto & Carreira, 2017).

There is a shred of mounting evidence that shows dynamic visualizations of GeoGebra create means for the more in-depth analysis of mathematical concepts. GeoGebra deepens students' understanding of knowledge learned in classes through the display of mathematical concepts. (Aizikovitsh-Udi & Radakovic, 2012). GeoGebra enables students to grasp abstract mathematical concepts that require complex reasoning and critical thinking skills.

Impact of GeoGebra on student reasoning. GeoGebra visualizations positively affect students' exploration skills in problem-solving and their engagement in reasoning (Olsson, 2017). Those visualizations have the power to enhance investigating, conjecturing, and discovering geometric properties on a given construction. Accordingly, GeoGebra enables students to utilize an inductive, deductive, or abductive approach to reasoning using visual, numerical, algebraic, or mathematical rationale (Redo et al., 2018).

Impact of GeoGebra on exploration and visualization. In the last couple of years, researchers focused on dynamic learning environments that allow students to explore mathematics concepts visually and dynamically. Research assures that GeoGebra serves as a technological tool and dynamic representation that eases students' problems in calculating, constructing, modeling, and reflection (Zulnaidi et al., 2020).GeoGebra embellishes student self-exploration, which empowers the constructivist approach to learning (Shadaan & Leong, 2013) and offers students the opportunity to explore and visualize abstract concepts in a concrete manner (Shadaan & Leong, 2013).

The interactions with GeoGebra aid in creating explorative approaches to solve tasks and task designs that leave students with the responsibility to construct solutions (Olsson, 2017). GeoGebra improves student visualization skills and allows them to accurately build multiple representations in a short time, explore the illustrations dynamically, and receive immediate feedback on their actions (Chang & Bhagat, 2015). In other respects, GeoGebra provides multiple representations of concepts and develops mathematical connections (Baltaci et al., 2015).

Impact of GeoGebra on student collaboration. GeoGebra software has a significant potential to provide a mathematical learning environment that is rich in cooperation and communication (Baltaci et al., 2015). GeoGebra encourages interactive teacher-student interactional environment and student teamwork, where students assist one another in learning mathematical concepts (Shadaan & Leong, 2013). The dynamic software intensifies social interaction in the classroom and grants students the chance to help each other reach their full learning potential (Shadaan & Leong, 2013). Moreover, Olsson (2017) assures that GeoGebra could be used as a means of supporting collaboration and creative reasoning during a problem-solving process.

GeoGebra for calculus. GeoGebra is the best software amongst extensive software packages tailored for the study of calculus (Zulnaidi et al., 2020). Primarily, GeoGebra plays a vital role in some subject matters in mathematics that require reversible thinking such as Calculus since the software allows graphic visualization (Nobre et al., 2016).

GeoGebra as an Instructional Tool in Geometry

Geometry. Geometry is an essential branch of mathematics that is taught from elementary school until college. It contributes to logical and deductive reasoning about spatial objects and relationships. The traditional pencil-paper approach to geometry is deemed as ineffective. Such conventional methods of geometry result in difficulties in producing accurate geometrical representation and of deeply understanding geometry concepts (Jelatu et al., 2018). The underlying reason is that Geometry requires visualization ability, but many students cannot visualize three-dimensional objects in a two-dimensional perspective. Students should be provided with a supportive learning media where they get the opportunity to observe, explore, and reach out to abstract geometrical principles.

Impact of GeoGebra on learning Geometry. A review of the literature shows that using GeoGebra alleviates the teaching and learning Geometry in specific and enhances students' understanding of Geometry (Shadaan & Leong, 2013). GeoGebra bridges the gap between abstract and concrete. The concrete structure of mathematics supports the student to understand the topic, grasp the logic of the course, and let them know what is taught in the class. GeoGebra turns abstract concepts into concrete to make high spatial ability students and low spatial ability ones balanced in their understanding (Jelatu et al., 2018). To sum up, GeoGebra supports the development of professional skills in mathematics and encourages different task designs when solving geometric exercises (Redo et al., 2018).

CHAPTER III

METHODOLOGY

Based on The Sociocultural Theory of Engagement, this mixed-method research aims to identify the effectiveness of using GeoGebra software for teaching Geometry lessons on the level and type of the three dimensions of students' engagement "cognitive, behavioral, and affective" of grade eight students.

Design

The researcher used the quasi-experimental mixed-method design in the study. The study aims to estimate the causal impact of GeoGebra on the students' engagement without random assignment.

Participants

The sample consists of thirty-four eighth-graders in a private school located at Mount Lebanon Governate, where the medium of instruction is English, eighteen students in one section, and sixteen in the other without any random pre-selection processes. The study does not recruit students on a casual basis since the administration already allocates students in two sections. The school assembles sections upon proportional numbers of male and female and based on student academic abilities and social behavior. As much as possible, the administration counselor includes equal numbers of the high, medium, and low achieving students in each class. The first section will be designated as the control group A, and the second section will be designated as the experimental group B, respectively.

The 29 years old female teacher will be a participant researcher in this study. The teacher has eight years of experience in teaching mathematics and holds a bachelor's degree in pure mathematics, along with a teaching diploma in mathematics education for middle-secondary classes.

Setting

In response to coronavirus disease (COVID-19) pandemic, all the classes were set up online on the Zoom application. All the Zoom mathematics sessions were video recorded during the three research weeks.

The targeted mixed-gender school in the study is a private school in Aley that follows the Lebanese curriculum. This school lies in a rural area in the east of Khaldeh - Mount Lebanon Governate, where most students come from a middle socioeconomic background. Eighth graders in this school are not familiar with GeoGebra software, as the teacher has not offered any session using this software before the study. The school does not adopt the software, and no other math teacher used this software at the previous level. Accordingly, students did not have prior access to the GeoGebra software inside or outside the classroom for learning or any other purposes.

Measuring engagement requires self-reporting tools, which limits holding the study in cycles one (grades one till three) and two (grades four till six). Indeed, middle (grades seven till nine) and high school students (grades ten till twelve) express themselves more clearly. Besides, school rules for high school students would not encourage holding the study within. In preference, the study will be conducted in grade 8.

Procedure

After gaining official access to the school as a researcher, informing students about voluntary participation in the study, and receiving their parents' informed consent, the researcher labeled one section as the control group and the other section as the experimental group adopting a quasi-experimental design. The researcher offered the control group a conventional lecture approach and a GeoGebra teaching intervention in the experimental group. In the first place, both groups filled out a pre-Test "5-point Likert scale Survey" on engagement in Geometry classes, where scale ranges among strongly disagree, disagree, neutral, agree, and strongly agree (Fung et al., 2018). The researcher in this study used the National Survey of Student Engagement (NSSE survey) with permission from The College Student Report, National Survey of Student Engagement, and Copyright 2001-19 of "The Trustees of Indiana University." Each of the four Geometry instructional sessions lasted around 45 minutes and was conducted four times a week for three consecutive weeks; the researcher had to abide by the annual mathematics plan of grade eight.

Noteworthy, before holding the GeoGebra intervention in the experimental group, the participant researcher conducted a GeoGebra practice session to introduce the software. Students in the experimental group first watched instructional videos on their iPads (2 students – 1 iPad). Each session, the researcher provided students with a controller schematic sheet (appendix E, appendix F, appendix G) to assist in learning the basic operations of GeoGebra. Pointing out that the researcher conducted video-based observation to study frequency and interval of behavior's occurrence on the one hand, and used duration recording to review the time students spend engaging in practice on the other (Fung et al., 2018). Subsequently, after the intervention took place, students of both groups filled a post-Test "5-point Likert scale Survey" on engagement in Geometry classes. Eventually, the researcher conducted recorded semi-structured interviews on a group of five students in which probing mostly took place. Participation in the interview was voluntary. Five students decided to participate in the research interview and had the free will to withdraw from participating at any time with no penalization.

The treatment in the experimental group. After assigning the two sections as control and experimental groups, the control group received traditional instruction in learning four different Geometry lessons per week - 45 minutes each, over three consecutive weeks, abiding by the annual mathematics plan of grade eight. The same instructor taught the same Geometry lessons in both groups, asked the same questions, and did the same activities and applications in both sessions. In the control group's classroom, the lecturer explained those lessons and solved problems by getting help from presented models on the board. The only difference among the teaching activities between the experimental and control group was that the instructor did the Geometry activities employing GeoGebra as a treatment in the experimental group. Four main GeoGebra tasks were assigned during the GeoGebra consecutive sessions. A random example of a GeoGebra task (International GeoGebra Institute, 2013), for the grade eight-Special Parallelograms lesson (figure 9) might look like this:

- Create a parallelogram using GeoGebra.
- Change it by dragging the vertices of the figure.
- Explore how the values of the sides and angles are changing.
- Identify the discoveries of the sides of parallelograms.
- Identify the discoveries of the angles of parallelograms.
- Explore how the values and properties of the diagonals of the parallelogram are changing.
- Identify the discoveries of diagonals of parallelograms.
- Using GeoGebra, change the same parallelogram into a rectangle.
- Explore how the values of the sides and angles are changing.
- Identify the discoveries of the sides of a rectangle.
- Identify the discoveries of the angles of the rectangle.

- Explore how the values and properties of the diagonals of the rectangle are changing.
- Identify the discoveries of diagonals of a rectangle.
- Using GeoGebra, change the same rectangle into a square.
- Explore how the values of the sides and angles are changing.
- Identify the discoveries of the sides of a square.
- Identify the discoveries of the angles of the square.
- Explore how the values and properties of the diagonals of the square are changing.
- Identify the conclusion of the diagonals of a square.

The same activity was held in the control group from the presented models on the board. The only difference in the instruction between the two groups was the use of GeoGebra in the experimental group in the activities, controlling for everything else.

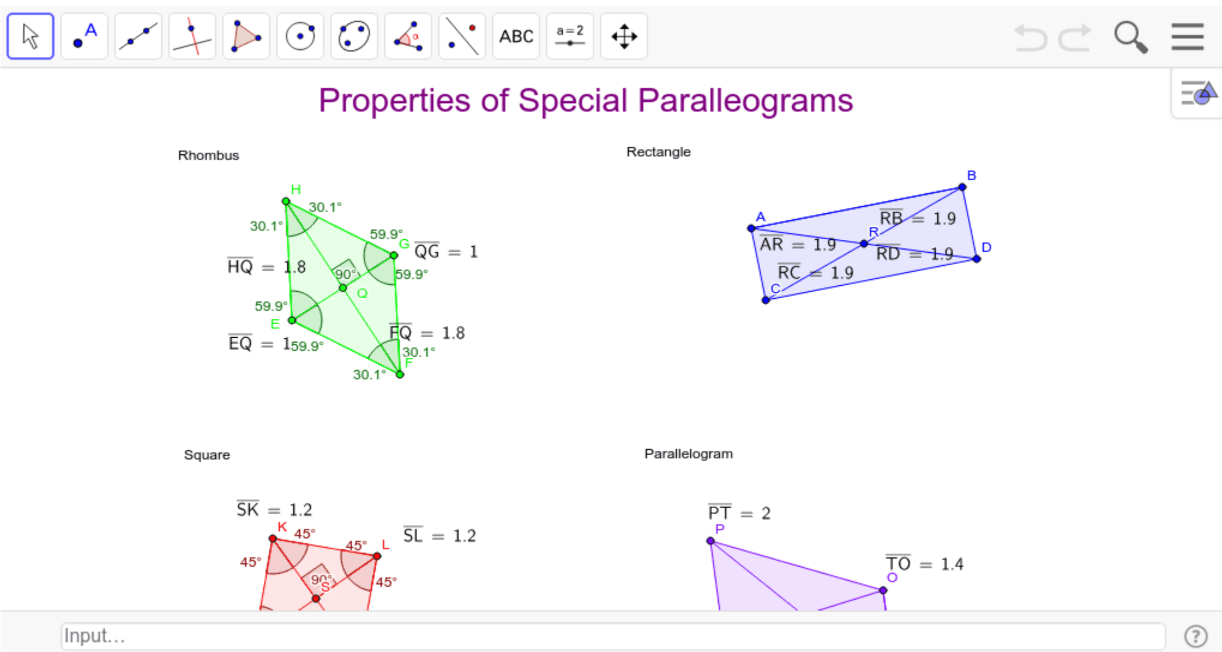


Figure 9: Sample GeoGebra Lesson

Instruments

Three instruments were used in the study to ensure triangulation; a 5-point Likert scale NSSE survey, video-based observation of the classroom, and recorded semi-structured interviews on focus groups of five students. The 5-point Likert scale NSSE surveys were used for quantitative purposes, while the video-based observation of both classrooms and the recorded semi-structured interviews for qualitative purposes. The pre-Test "5-point Likert scale NSSE Survey" on engagement in Geometry classes was the first instrument to be used. Throughout the study, a video-based observation was applied to both groups, along with duration recording. The four GeoGebra sessions were proceeded by a post-Test "5-point Likert scale NSSE Survey" on engagement in Geometry classes. Eventually, recorded semi-structured interviews on a case study of five students were held, allowing probing and asking students for meaning to ensure reliability when required.

Tools used for quantitative purposes. The pre-Test and post-Test "5-point Likert scale Survey" on engagement in Geometry classes consisted of three parts each.

Part of Survey on affective engagement. The affective engagement was measured by student mathematics interest, enjoyment, positivity, sense of value, confidence, desire for understanding, curiosity, and knowledge of ownership. Data for these variables were derived from student responses to items using a 5-point Likert scale (1= strongly disagree, 2= disagree, 3= neutral, 4= agree, 5= strongly agree), for example, "I am interested in Geometry-Math classes" (Fraenkel & Wallen, 2012).

Part of the survey on behavioral engagement. The behavioral engagement was first measured by student participation and communication of mathematical tasks using both instruments, the NSSE survey, and classroom observation. The research checked if students were reflecting, asking questions, representing data, comparing, explaining, justifying, agreeing about, cooperating with peers and teachers to validate ideas, keeping a record of information, voluntarily choosing optional Geometry activities. Behavioral data were derived from student responses to items using a 5-point Likert scale (1= strongly disagree, 2= disagree, 3= neutral, 4= agree,5= strongly agree), for example, "I work hard on Geometry tasks" (Fraenkel & Wallen, 2012).

Part of the survey on cognitive engagement. Student cognitive engagement was measured by implementing different cognitive strategies, mainly using the NSSE survey and the interview, with some reliance on classroom observation. The researcher inquired for practices as self-regulation, self-monitoring, personal investment, employment of consistent mathematical goals, and seeking help when needed. Measuring cognitive engagement data was derived as well from exerting mental effort. The researcher checked whether students pay attention in mathematics Geometry classes, avoid distractions when studying Geometry, focus and concentrate on Geometry tasks given by the teacher, preserve on solving a task- "time-wise," inquire and work independently. Data for this variable were derived from students' responses to items using a 5-point scale NSSE survey; for example, "I try to seek explanations and to link facts together when solving a complex Geometry problem." (Fraenkel & Wallen, 2012).

The video-based observation was applied to both groups, along with duration recording, to measure cognitive and behavioral engagement.

Tools used for qualitative purposes. Having the Zoom sessions recorded by the administration, the school allowed the researcher to review the zoom video after every learning intervention, without any concern about student privacy policy. Without any violation of the students' privacy, the administration offered the researcher limited access to the Zoom video recordings. Stating things out clearly, the school owns that data and not the researcher. The limited access prohibited the researcher from copying the videos or from using the data for different purposes. This video recording did not induce any changes in the virtual classroom setting, which assured students did not modify any aspect of their behavior. The fact that there was no observer effect undermined the reliability and integrity of the study.

Classroom observation for behavioral engagement. The researcher measured aspects of interaction in terms of checking the amount of talk and study the quality of participation. The researcher checked if students asked questions or only answered when asked, in means investigated if students initiated or only responded. The researcher studied the frequency of actions along with the interval of behavior's occurrence (Fung et al., 2018).

Classroom observation for cognitive engagement. The researchers interpreted students' metacognitive thinking and analyzed the type of strategies they used (Michalewicz, 2000). The more the students used cognitive strategies, the more engaged in the task they were. The measure of cognitive engagement required the researchers to check for self-regulating approach as well and for student personal cognitive investment using a duration recording method that studies the time spent on each task (Fraenkel & Wallen, 2012).

Recorded semi-structured interviews. The interviews offered participants the freedom to express their views and perceptions of GeoGebra in their terms. The researcher used the interview-group to gain an in-depth understanding of the students' perception regarding their engagement in learning Geometry by the use of GeoGebra software. The researcher conducted semi-structured interviews on a voluntary selected group of five students in which probing mostly took place (Fraenkel & Wallen, 2012). Those semi-structured interviews mainly measured, among other dimensions, the affective dimension of engagement. The best way to study emotions was through self-reporting measures. Students deeply reflected upon their feelings and desires in their own words.

Teaching Materials

The study focused on four Geometry lessons from the eighth-grade mathematics book – Al Ahlia Lebanese Curriculum Book. The teacher designed controller schematic sheets, along with illustrations, to assist students in understanding the GeoGebra tasks. The intervention required personal iPads or personal laptops with the GeoGebra software installed.

Independent Variables

The use of GeoGebra was the independent variable, where GeoGebra is an interactive mathematics software program for learning and teaching mathematics starting from primary school up to university level (Redo et al., 2018).

Dependent Variables

The three dependent variables were the multi-components of students' engagement

- Cognitive engagement
- Behavioral engagement
- Affective engagement

Case Delineation

In this paper, The Sociocultural Theory model has been adopted as a lens to examine a complete unit of mathematical inquiry as undertaken with a class of 13–14-year-old students. The mixed methods research took place in two eighth grade sections at "West Hill College," Mount Lebanon Governorate. Students of mixed gender mostly belonged to the middle socioeconomic status. Adopting a quasi-experimental method, the researcher, as a participant in the study, applied four conventional Geometry lectures in the first section labeled as the control group. The other section, marked as the experimental group, received the same four Geometry lectures using the GeoGebra software. In the first place, both groups filled out a pre-Test "5-point Likert scale

Survey" on engagement in Geometry classes (Fung et al., 2018). Researchers in this study used the NSSE survey with permission from The College Student Report, National Survey of Student Engagement, and Copyright 2001-19 of "The Trustees of Indiana University." The researchers asked students to indicate the extent of their agreement with each statement, on a five-point scale that goes from strongly agree to disagree strongly. Pointing out that the researcher conducted video-based observation to study frequency and interval of behavior's occurrence, and used duration recording to explore the time students spend engaging in an act (Fung et al., 2018). The bright spot was that all classes at Westhill College were equipped with surveillance cameras, which saved the illusion of the Hawthorne effect. The fact that students were used to having an observation camera set in their actual classes, guaranteed that they did not act differently on the Zoom camera. Researchers believe the social context establishes student actions, and therefore, since the classroom conditions stayed similar, the Hawthorne effect did not exist.

Subsequently, after the intervention took place, students of both groups filled a post-Test "5-point Likert scale Survey" on engagement in Geometry classes. Eventually, the researcher conducted recorded semi-structured interviews on a voluntary group of five students in which probing mostly took place.

Reliability

The researcher took several steps to increase the authenticity of the study and minimize bias. The researcher kept an audit trail throughout the research process. The purpose behind using three instruments in the research was to ensure triangulation. For the sake of increasing validity and reliability, data was collected from multiple perspectives using multiple tools (Babchuk, 2017; Fraenkel & Wallen, 2012). Further methodological triangulation occurred through combining the variety of multiple data sources for the study.

Also, the interview and the student surveys were piloted on different students, teachers, university classmates, and university professors to make sure that the survey questions were readable, understandable, clear, and target the purpose (Babchuk, 2017; Fraenkel & Wallen, 2012).

Ethical Consideration

In this study, respect for the research participants was prioritized. Full consents were obtained from participants, parents, and administration before the survey. Besides, participation in the study was voluntary. Students had the free will to participate in a follow-up interview, and to withdraw from the interview at any time, provisioned on the parental consent forms to agree to this. Interviews will be the typically fifteen-minute duration in the parental presence. The researcher ensured confidentiality of participation as all participants selected pseudonyms to protect their identity. Beyond a shadow of a doubt, the research participants were not subjected to any harm, and its procedure did not include any irrelevant components.

CHAPTER IV

RESULTS AND DATA ANALYSIS

Data Collection

In response to coronavirus disease (COVID-19) pandemic**,** schools across the globe implemented national school closure by March 2020. Halfway through the school year, schools had to shift away from classrooms towards remote learning undertaken on digital platforms. West Hill College responded to the unique and challenging circumstances of this academic year by taking measures that ensure student learning could proceed safely and effectively. The school settled into the new "normal," where all the classes were set up online on the Zoom application. All teachers scheduled daily lessons on the Zoom app and sent the meeting invitation email to the students. The teachers sometimes used Zoom Breakout Room Feature to either pre-assign or auto-assign students into groups for a short discussion on particular topics. Students were able to be presenters by sharing their screens, which allowed them to collaborate and participate continuously.

Due to scheduling constraints, the researcher spent an average of three weeks of four days a week in 45-minutes mathematics sessions, implementing the study. Time was divided equally among the two groups, "control" and "experimental." The researcher primarily worked with students individually or in small groups and collected artifacts from GeoGebra's work. Students' artifacts generally took the form of screenshots of work or recordings of student geometry problem-solving. The primarily quantitative component consisted of pre-post student surveys. Before holding the GeoGebra intervention in the experimental group, both groups filled out the pre-test “5-point Likert scale NSSE Survey”. The GeoGebra sessions were then proceeded by a post-Test “5-point Likert scale NSSE Survey” on engagement in Geometry classes in both groups. The qualitative part consisted of interviews and observations. The researcher conducted open-ended interviews with five students after the three weeks of the GeoGebra intervention. The interview was voluntary, and students could opt-out at any time by notifying the researcher. The researcher held zoom audio-meetings with five students from the experimental group who were willing to participate in the follow-up interview, in the presence of their parents. The interviews were limited to 10-15 minutes each, and the interview recordings were later transcribed. The tapes were destroyed following transcription, and student names changed to ensure student privacy. The interview questions were based on a scale for monitoring students' perception of learning geometry using GeoGebra software.

All the zoom mathematics sessions were video recorded during the three research weeks. Students who did not sign the parental consent for videotaping were asked to turn off their video. The dense and rich students' interactions were "winnowed" for the sections that pertained to the research questions above. The transcriptions were coded based on these research questions to facilitate analysis (Creswell, 1996).

Analysis

The researcher of this study analyzed the quantitative data from the survey using descriptive statistics to find evidential data. After arranging the available data from the NSSE survey, the researcher used SPSS tabular format to represent the data and derive the objective quantitative analysis.

The researcher of this study could not make decisions based solely on quantitative analysis. Therefore, the researcher attempted to derive qualitative data through an in-depth search of meaning. The data analysis of the video records and the interviews went hand in hand. The researcher conducted a multimodal detailed, thick description of the study and adopted a direct interpretation of the recorded videos and of the data that students self-report during the semi-structured interview. The researcher organized the data, broke them into codes, then categorized them to discover patterns and universal themes. Those descriptive, multi-dimensional categories provided a framework for in-depth inductive analysis. When the data did not seem plausible, the researcher referred to the students to confirm the tentative researcher interpretations. After that, the researcher assessed the accuracy of the emerging qualitative findings through the triangulation of data.

The researchers utilized both the qualitative and quantitative methods for triangulation. They used the three data sources to gain a complete understanding of the data and to decide if that data corroborates. The researchers triangulated the outcomes of the interview group with the video observation reports as well as the engagement survey results. This triangulation was done at various levels to focus on a conclusion based on multiple perspectives.

A multimodal analysis of the video-based observation was held using frequency and interval of behavior's occurrence on the one hand and using duration recording on the other. A thematic analysis of recorded interviews was conducted where student's talks were transcribed and evaluated using data coding and categorization.

Statistical analysis was held using SPSS to evaluate the surveys using the Likert rating survey scale. SPSS was conducted to check independent sample t-test results (*p*-value) and the level of error. Also, statistical analysis monitored the change in levels of engagement dimensions and determined the correlation between the three variables. Nevertheless, boxplots and additional graphs were used to compare the variables using an exploratory data analysis.

Quantitative Analysis – SPSS Results

The first data source of this quasi-experimental study was a survey, which was administered to eighth-graders of West Hill College. A total of thirty-three participants were consented to participate. The two sections were similar in their academic level and demography. One class was randomly chosen to receive the treatment (GeoGebra), and the other class, therefore, was set as the control group. Fifteen students from the control group and eighteen students from the experimental group returned the surveys. Respondents were asked to answer thirty-two multiple-choice questions based on a 5-Likert scale.

Once the components were obtained, the researcher calculated component scores for each student. This score allowed a comparison between the extents of engagement across each of the three strands of engagement. Descriptive statistics were generated on the overall pre- and post-tests scores. Frequency tables were calculated, and the data was proved to be normally distributed. Subsequently, the normally distributed data analyses were completed using a two-sample independent t-test to mean percentage of the experimental score on the pre-test score versus the control score, once in the pretest and once in the posttest. The researcher conducted two-sample t-tests using IPM SPSS statistics 19 to assess the mean difference of student engagement between the experimental and the control group. The engagement levels of each student were measured twice; before and after the GeoGebra intervention. The level of significance was placed at 0.05.

To establish the reliability and internal consistency, the researcher calculated Cronbach's alpha coefficients, which yielded adequate levels of inter-item reliability.

The two-sample independent t-test was used to test for a statistically significant difference between the treatment and the control samples in mean pretest scores. No statistically significant pretest difference was found. The two-sample t-test was then used to test for a statistically significant difference between mean posttest scores. A statistically significant posttest difference was found. The customary *p*-value of 5% was used to decide if the observed difference was statistically significant. Using $p < 0.05$ keeps the chance of a false positive small, where five percent strikes a balance between the chance of a false positive and false negative.

There was no significant difference between the GeoGebra classroom, and the control group pretest means. That is before the experiment, both groups of students started at the same level of engagement in geometry classes. There was a significant difference between the GeoGebra classroom and the control classroom posttests means. That is, after the GeoGebra intervention, the experimental students were identified to have higher levels of behavioral, cognitive, and affective engagement than the control. The experimental group gained virtually higher levels of engagement from the control group, on average, from pre to post.

Cronbach's Alpha of behavioral questions. A Cronbach's analysis was conducted on the "Behavioural Questions" subscale of the GeoGebra survey. It was found that the subscale's alpha level was (0.87), which indicates that the subscale has an adequate level of inter-item reliability. However, it was found that deleting any of the items would not significantly increase the alpha level. Worth noting, SPSS could indicate that the deletion of some items makes minor

Case Processing Summary

		N	%
Cases	Valid	33	100.0
	Excluded[a]	0	.0
	Total	33	100.0

a. Listwise deletion based on all variables in the procedure.

Table 1: Result of for the total cases

improvements to the Cronbach's Alpha. The researcher was inclined to leave all of the survey scale items because they provided valid variability within the final data.

Reliability Statistics	
Cronbach's Alpha	N of Items
.870	7

Table 2: Results for reliability statistics for "Behavioral Question."

Item-Total Statistics				
	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Behavioral Question1	20.03	33.843	.602	.857
Behavioral Question2	19.91	31.023	.805	.831
Behavioral Question3	21.24	30.814	.637	.853
Behavioral Question4	20.18	31.778	.608	.857
Behavioral Question5	20.64	31.176	.706	.843
Behavioral Question6	20.33	32.979	.615	.855
Behavioral Question7	19.85	33.508	.567	.861

Table 3: Results showing Cronbach's Alpha if item deleted among "Behavioral Questions

Cronbach's Alpha of Cognitive Questions. A Cronbach's analysis was conducted on the "Cognitive Questions" subscale of the GeoGebra survey. It was found that the subscale's alpha level was (0.84), which indicates that the subscale has an adequate level of inter-item reliability.

However, it was found that deleting any of the items would not significantly increase the alpha level.

Reliability Statistics

Cronbach's Alpha	N of Items
.849	17

Table 4: Results for reliability statistics for "Cognitive Questions."

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Cognitive Question1	62.18	76.216	.476	.841
Cognitive Question2	61.67	76.479	.529	.838
Cognitive Question3	61.97	74.780	.682	.831
Cognitive Question4	61.64	74.864	.636	.833
Cognitive Question5	61.67	79.604	.312	.849
Cognitive Question6	61.61	80.121	.399	.844
Cognitive Question7	62.45	72.881	.525	.839
Cognitive Question8	62.00	74.063	.561	.836
Cognitive Question9	61.42	81.877	.216	.853
Cognitive Question10	61.45	80.068	.381	.845
Cognitive Question11	61.61	75.559	.563	.836
Cognitive Question12	61.48	79.008	.418	.843
Cognitive Question13	61.88	74.610	.681	.831
Cognitive Question14	61.55	82.006	.292	.848
Cognitive Question15	61.52	79.008	.586	.838
Cognitive Question16	61.82	82.341	.242	.851
Cognitive Question17	61.79	79.860	.371	.845

Table 5: Results showing Cronbach's Alpha if item deleted among "Cognitive Questions."

Cronbach’s Alpha of affective questions. A Cronbach’s analysis was conducted on the “Affective Questions” subscale of the GeoGebra survey. It was found that the subscale’s alpha level was (0.78), which indicates that the subscale has an adequate level of inter-item reliability. However, it was found that deleting any of the items would not significantly increase the alpha level.

Reliability Statistics

Cronbach's Alpha	N of Items
.789	8

Table 6: Results for reliability statistics for “Affective Questions”

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Affective Question1	29.48	17.883	.602	.747
Affective Question2	28.58	22.064	.258	.797
Affective Question3	28.67	20.042	.561	.759
Affective Question4	28.79	20.922	.417	.777
Affective Question5	28.64	20.676	.534	.765
Affective Question6	28.82	18.591	.536	.759
Affective Question7	29.24	15.814	.699	.726
Affective Question8	29.21	19.360	.406	.783

Table 7: Results showing Cronbach's Alpha if item deleted among "Affective Questions"

Normality Test. If the significant value of the Shapiro-Wilk test is higher than 0.05, the data shall be normal. If it is below 0.05, the data significantly deviate from a normal distribution. According to Shapiro-Wilk's test, the results of the Pre-test, Post-test, and Post- Pre-Difference of the three variables, Behavioral, Cognitive, and Affective of the 33 students, were customarily distributed ($p > 0.05$). The Shapiro-Wilk results mean that the data set is well-modeled by a normal distribution and that the random variables underlying the data set were normally distributed. Histograms for each variable are shown below in the following page.

	Shapiro-Wilk		
	W	df	p
Pre Behavioral	.976	33	.665
Post Behavioral	.963	33	.316
Difference Behavioral	.982	33	.842
Pre Cognitive	.944	33	.087
Post Cognitive	.971	33	.508
Difference Cognitive	.964	33	.343
Pre Affective	.958	33	.227
Post Affective	.946	33	.100
Difference Affective	.943	33	.085
Pre Total Engagement	.984	33	.900
Post Total Engagement	.982	33	.833
Difference Total Engagement	.978	33	.738

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Table 8: Results of the Shapiro-Wilk test

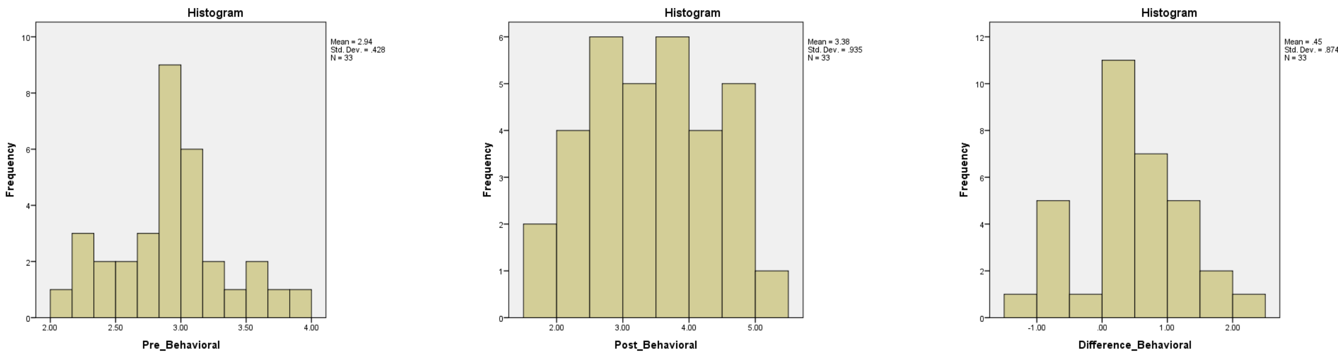


Figure 10: Behavioral Normality Histograms

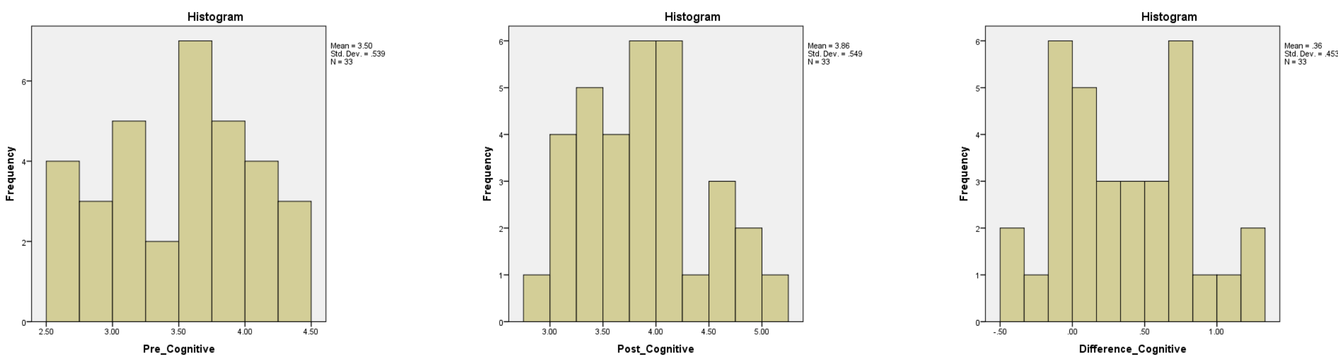


Figure 11: Cognitive Normality Histograms

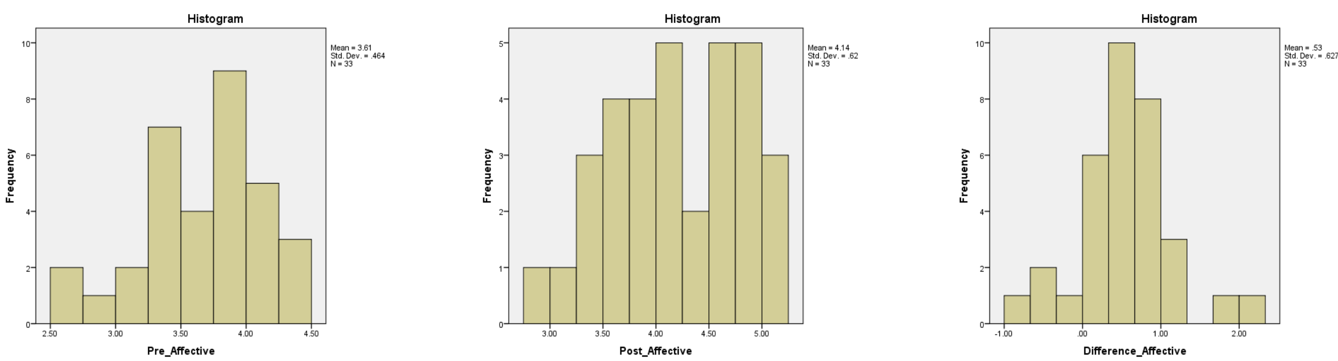


Figure 12: Affective Normality Histograms

Pursuant to the first research question, the researcher used Independent Samples t-Test to inquire about the relative effectiveness of using GeoGebra software on students' affective, behavioral, and cognitive engagement as compared to using conventional learning strategies for teaching Geometry in middle school classes.

Independent Samples t-Test. The Independent Samples t-Test, also called the two-sample t-Test, compared the means of the two independent groups (control – experimental) in order to determine whether there was statistical evidence that the associated student means are significantly different. The independent sample t-Test was valid in the study since the sample was of small size, and the population distribution was approximately normal. The researcher conducted the two-sample t-Test to compare the engagement levels of the two sections before (pre) and after (post) the GeoGebra teaching intervention.

Independent Samples t-Test - pretest. Levene's variances equality test does not assume the equality of variances ($F = 10.437$; $p < 0.05$). According to the t-Test for independent samples in the pretest, the difference between the two data means for the Behavioral engagement of the two groups, experimental and control, is not significant ($t\ (25.664) = 0.199$; p 0> .05)**.** The Levene's variances equality test assumes the equality of variances ($F = 3.834$; $p > 0.05$). According to the T-test for independent samples in the pretest, the difference between the two data means for the Cognitive engagement of the two groups, experimental and control, is not significant ($t\ (31) = 0.053$; $p > 0.05$). The Levene's variances equality test assumes the equality of variances ($F = 0.003$; $p < 0.05$). According to the T-test for independent samples in the pretest, the difference between the two data means for the Affective engagement of the two groups, experimental and control, is not significant ($t\ (31) = .346$; $p > 0.05$). Levene's variances equality test does not assume the equality of variances ($F = 5.783$; $p < 0.05$). According to the T-test for independent samples in the pretest, the difference between the two data means for the Behavioral engagement of the two groups, experimental and control, is not significant (*t* (29.544) = 0.244; $p > 0.05$). This data reveals that there is no significant difference in the mean scores between any of the engagement strands in the pretest.

There was no significant difference (*Sig. (2-tailed)* = 0.852; $p < 0.05$) between the means of behavioral engagement of the control group (*Mean* = 2.9238) and that of the experimental group (*Mean* = 2.9524) in the pretest. There was no significant difference (*Sig. (2-tailed)* = 0.96; $p < 0.05$) between the means of cognitive engagement of the control group *(Mean* = 3.5) and that of the experimental group (*Mean* = 3.4902) in the pretest. There was no significant difference (*Sig. (2-tailed)* = 0.731; $p < 0.05$) between the means of affective engagement of the control group (*Mean* = 3.575) and that of the experimental group (*Mean* = 3.6319) in the pretest. There was no significant difference (*Sig. (2-tailed)* = 0.816; $p < 0.05$) between the means of the total engagement of the control group (*Mean* = 3.3297) and that of the experimental group (*Mean* = 3.3614) in the pretest.

Group Statistics

Pretest	Group	N	Mean	Std. Deviation	Std. Error Mean
Behavioral Total	Experimental	18	2.9524	.53676	.12652
	Control	15	2.9238	.26378	.06811
Cognitive Total	Experimental	18	3.5000	.61472	.14489
	Control	15	3.4902	.45601	.11774
Affective Total	Experimental	18	3.6319	.47641	.11229
	Control	15	3.5750	.46243	.11940
Pretest Total	Experimental	18	3.3614	.44835	.10568
	Control	15	3.3297	.29452	.07604

Table 9: Pretest Means – Experimental vs. control

Independent Samples Test

Pretest		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Behavioral Total	Equal variances assumed	10.437	0.003	0.188	31	0.852	0.02857	0.15216	-0.28175	0.3389
	Equal variances not assumed			0.199	25.664	0.844	0.02857	0.14368	-0.26696	0.32411
Cognitive Total	Equal variances assumed	3.834	0.059	0.051	31	0.96	0.0098	0.19185	-0.38147	0.40108
	Equal variances not assumed			0.053	30.641	0.958	0.0098	0.1867	-0.37115	0.39076
Affective Total	Equal variances assumed	0.003	0.958	0.346	31	0.731	0.05694	0.16437	-0.27828	0.39217
	Equal variances not assumed			0.347	30.237	0.731	0.05694	0.16391	-0.27769	0.39158
Pretest Total	Equal variances assumed	5.783	0.022	0.235	31	0.816	0.03177	0.13513	-0.24384	0.30738
	Equal variances not assumed			0.244	29.544	0.809	0.03177	0.13019	-0.23429	0.29784

Table 10: Independent sample t-test - Pretest

Independent Sample t-Test – posttest. The Levene's variances equality test assumes the equality of variances (F =0.842; p > 0.05). According to the T-test for independent samples in the posttest, the difference between the two data means for the Behavioral engagement of the two groups, experimental and control, is significant (t (31) = 2.368; $p < 0.05$). The Levene's variances equality test assumes the equality of variances ($F = 1.741$; $p > 0.05$). According to the T-test for independent samples in the posttest, the difference between the two data means for the Cognitive engagement of the two groups, experimental and control, is not significant (t (31) = 0.373; $p > 0.05$). The Levene's variances equality test assumes the equality of variances ($F = 1.765$; p > 0.05). According to the T-test for independent samples in the posttest, the difference between the two data means for the Affective engagement of the two groups, experimental and control, is significant (t (31) = 2.502; $p < 0.05$). Levene's variances equality test does not assume the equality of variances ($F = 10.323$; $p < 0.05$). According to the T-test for independent samples in the posttest, the difference between the two data means for the Total engagement of the two groups, experimental and control, is significant (t (31) = 2.329; $p < 0.05$) and ($df = 18.249$; $p < 0.05$).

Group Statistics

Post test	Group	N	Mean	Std. Deviation	Std. Error Mean
Behavioral	Experimental	18	3.7143	.78399	.18479
	Control	15	2.9905	.97261	.25113
Cognitive	Experimental	18	3.8922	.47199	.11125
	Control	15	3.8196	.64417	.16633
Affective	Experimental	18	4.3611	.47722	.11248
	Control	15	3.8583	.67458	.17418
Total	Experimental	18	3.9892	.28709	.06767
	Control	15	3.5561	.67084	.17321

Table 11: Posttest Means – Experimental vs. control

There was a significant difference (*Sig. (2-tailed)* = 0.024; $p < 0.05$) between the means of behavioral engagement of the control group (*Mean* = 2.9905) and that of the experimental group (*Mean* = 3.7143) in the posttest. There was no significant difference (*Sig. (2-tailed)* = 0.712; $p < 0.05$) between the means of cognitive engagement of the control group (*Mean* = 3.8196) and that of the experimental group (*Mean* = 3.8922) in the posttest. There was a significant difference (*Sig. (2-tailed)* = 0.018; $p < 0.05$) between the means of affective engagement of the control group (*Mean* = 3.8583) and that of the experimental group (*Mean* = 4.3611) in the posttest. There was a significant difference (*Sig. (2-tailed)* = 0.019; $p < 0.05$) between the means of the total engagement of the control group (*Mean* = 3.5561) and that of the experimental group (*Mean* = 3.9892) in the posttest.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Behavioral	Equal variances assumed	0.842	0.366	2.368	31	0.024	0.72381	0.30563	0.10047	1.34715
	Equal variances not assumed			2.321	26.796	0.028	0.72381	0.31179	0.08385	1.36377
Cognitive	Equal variances assumed	1.741	0.197	0.373	31	0.712	0.07255	0.19452	-0.32417	0.46927
	Equal variances not assumed			0.363	25.179	0.72	0.07255	0.2001	-0.33942	0.48452
Affective	Equal variances assumed	1.765	0.194	2.502	31	0.018	0.50278	0.20095	0.09293	0.91262
	Equal variances not assumed			2.425	24.59	0.023	0.50278	0.20734	0.07539	0.93016
Total	Equal variances assumed	10.323	0.003	2.485	31	0.019	0.43305	0.17425	0.07765	0.78844
	Equal variances not assumed			2.329	18.249	0.032	0.43305	0.18596	0.04274	0.82335

Table 12: Independent sample t-Test - Posttest

Boxplots. In response to the first question, the researcher employed boxplots to summarize the set of data measured on an interval scale. In descriptive statistics, the researcher used the boxplots to depict the difference in engagement levels graphically. The boxplots include overall spread and the interquartile ranges. More specifically, the boxplots show the minimum engagement level, the first lower quartile, the median, the third upper quartile, and the maximum engagement level in each of the three types of engagement.

Behavioral Boxplot. The behavioral box plot shows how the distribution of gains is virtually higher for the experimental than that of the control group. The experimental box plot shows that the bottom of the box is the 40th percentile, the line near the middle of the box is the 80th percentile, and the top of the box is the 125th percentile (figure 13). The vertical distance from the bottom to the top of the box is the inter-quartile range.

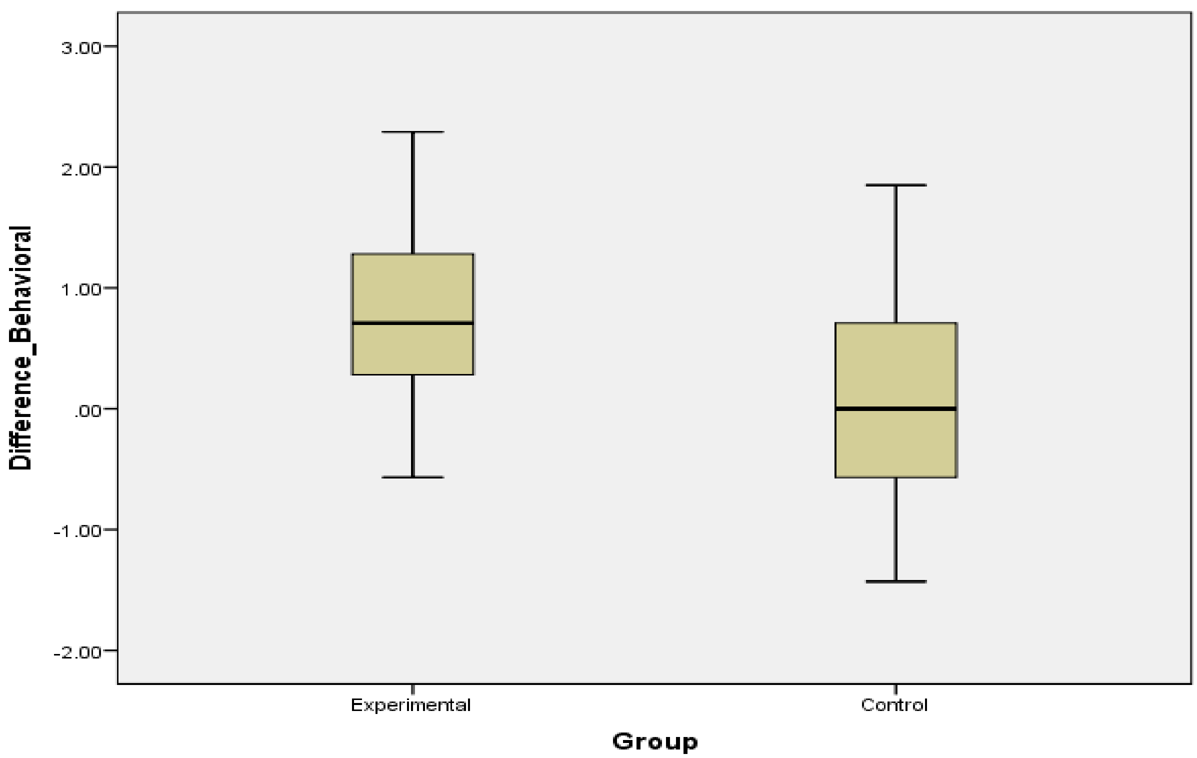


Figure 13: Difference-Behavioral boxplots

The so-called whiskers the vertical line atop the box and below the box show the range of the top and bottom 75 percent of scores. The researcher noticed that the control group has more variation in the below behavioral engagement than the treatment group. However, the treatment group has higher above average segments.

Cognitive Boxplot. The cognitive box plot shows how the distribution of gains is virtually higher for the experimental than that of the control group. The bottom of the experimental box lies by the 10th percentile. The middle line near the box is the 50th percentile, and the top is the 75th percentile. The vertical distance from the bottom to the top of the box is the inter-quartile range. The so-called whiskers the vertical line atop the box and below the box show the range of the top and bottom 65 percent of scores (figure 14).

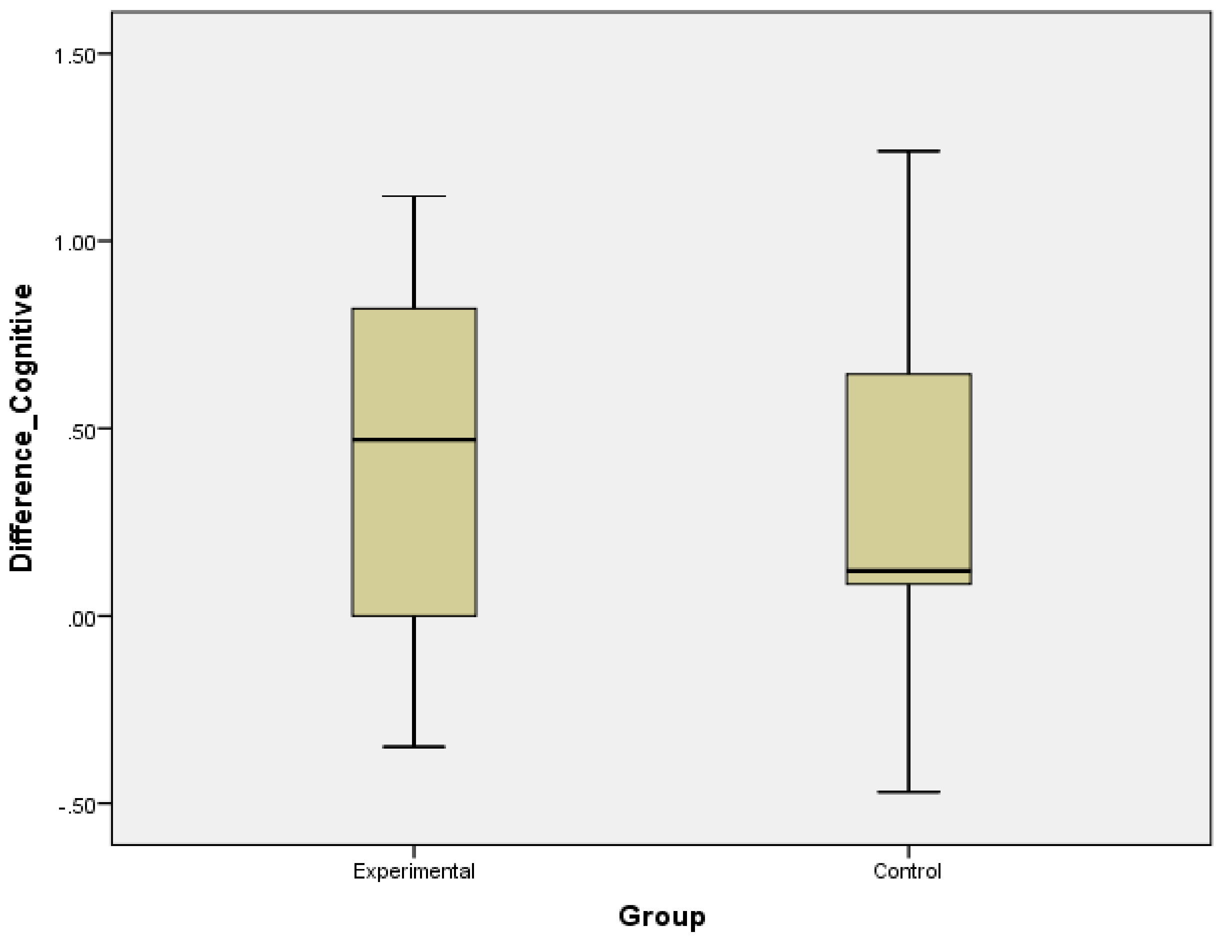


Figure 14: Difference-Cognitive boxplot

The researcher noticed that the treatment group has more variation in the below cognitive engagement and above average segments than the control group.

Affective Boxplot. The affective box plot shows how the distribution of gains is virtually higher for the experimental than that of the control group. The bottom of the experimental box plot is the 40th percentile. The middle line of the box is 80th percentile, and top of the box is 75th percentile. The vertical distance from the bottom to the top of the box is the inter-quartile range. The so-called whiskers the vertical line atop the box and below the box show the range of the top and bottom 40 percent of scores. The researcher noticed that the control group has more variation in the below affective engagement than the treatment group. However, the treatment group has higher above average segments (figure 15).

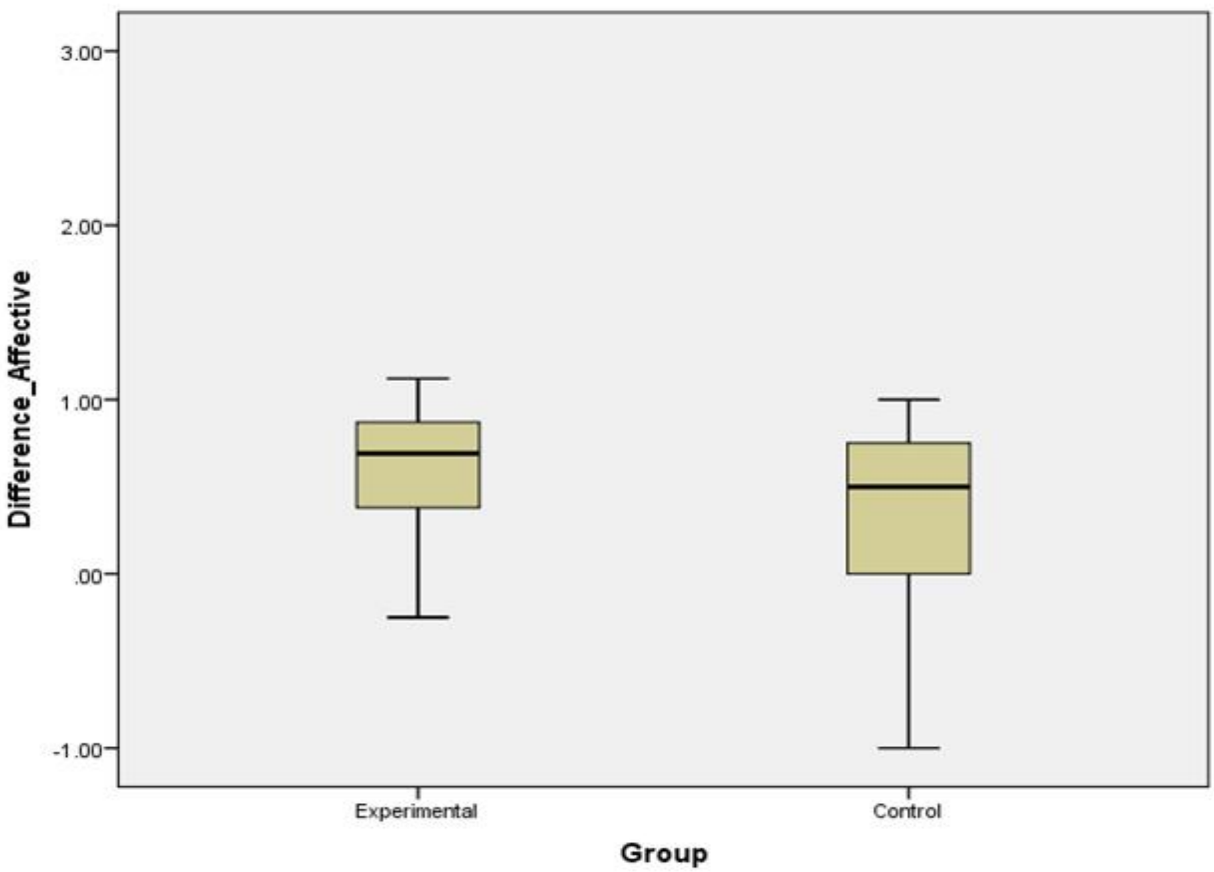


Figure 15: Difference-Affective boxplot

The results revealed no significant differences among the control group pre-posttests. The delayed posttest indicates that the control group who did not use GeoGebra did not show statistically significant improvement. In contrast, a significant difference was found in the experimental group's mean scores. The difference between the pretest and delayed posttest was statistically significant. The results indicate that the experimental group engagement levels were significantly better in the posttest than the pretest. Overall, the results suggest that GeoGebra had some positive effect on the students' engagement in geometry. Based on the results of the study, GeoGebra had a positive impact on behavioral and affective engagement specifically. However, it did not impact the students' cognitive engagement in geometry. On the other hand, cognitive engagement was found to be positively correlated with affective engagement. However, the correlation coefficient of the behavioral engagement with the affective and the cognitive engagement was not found to be significant.

Pearson's Correlation Test. On the ground of the second research question, the researcher conducted Pearson's Correlation test for the sake of investigating the relationships between the three dependent variables, "Cognitive engagement," "Affective engagement," and "Behavioral engagement." Pearson's correlation coefficient (*r*) is a computation of the strength of the association among every two dependent variables. The correlation was set significant at the 0.01 level (2-tailed). Saying that $p < 0.01$ means that confidence in the tested value is greater than 99%. Contrary to expectations, using Pearson's correlation coefficient, it was found that the "Behavioral Question" subscale was not significantly correlated with the "Cognitive Questions" subscale, r (32) = 0.32, p = 0.064. Furthermore, the "Behavioral Question" subscale was not significantly correlated with the "Affective Question" subscale, r (32) =0.26, p = 0.14. The study confirmed the expectations, as it was found that the "Cognitive Question" subscale was significantly correlated with the "Affective Question" subscale, r (32) = 0.58, $p < 0.01$. The calculation of Pearson's correlation coefficient and the subsequent significance testing reveals a moderate positive correlation between Affective and Cognitive engagement and an insignificant relation with behavioral engagement. The results mean that when the affective engagement increases, the cognitive engagement increases, and the other way around. Also, when affective engagement decreases, cognitive engagement decreases as well. However, the insignificant positive correlation reveals that a change in behavioral engagement has no impact on cognitive engagement neither on affective engagement.

Correlations

		Behavioral	Cognitive	Affective
Behavioral	Pearson Correlation	1	.326	.260
	Sig. (2-tailed)		.064	.143
	N	33	33	33
Cognitive	Pearson Correlation	.326	1	.588**
	Sig. (2-tailed)	.064		.000
	N	33	33	33
Affective	Pearson Correlation	.260	.588**	1
	Sig. (2-tailed)	.143	.000	
	N	33	33	33

**. Correlation is significant at the 0.01 level (2-tailed).

Table 13: Results of Pearson's correlation test between Behavioral, Cognitive and Affective Questions.

Qualitative Analysis

The researcher held the qualitative analysis in light of the three research questions. A fragment of the qualitative analysis met the first research question in studying the relative effectiveness of using GeoGebra software on students' affective, behavioral, and cognitive engagement as compared to using conventional learning strategies for teaching Geometry in middle school classes. Another split responded to the second question in studying the relation between the different strands of engagement; affective, behavioral, and cognitive engagement Besides, the qualitative analysis contributed to answering the last research question that discussed students' perceptions of the role of GeoGebra in enhancing their engagement in learning Geometry.

Qualitative Analysis – Video Data Observation

One of the research objectives focused on monitoring students' activities during experimental teaching phases in online learning classes. Altogether, the researcher collected twelve 45-minutes GeoGebra work sessions of the experimental group 8th graders (N = 34) by continuous videotaping of their GeoGebra-work phases. Subsequent qualitative analysis revealed nine categories characterizing the students' most relevant activities. Intra-observer objectivity and reliability scores confirmed the excellent fit of this categorization since the researcher did the observation and reviewed it at two different intervals of time and consistently got the same results. Concerning the qualitative analysis, the researcher viewed all videos to get a feel for how the different lessons played out. After this, the first video was scrutinized several times for evidence of the three strands of mathematical engagement. As incidents were observed, the lesson transcript was annotated to illustrate the proof warranting the identification of the particular strand observed. With each viewing, the annotations were refined, and decisions clarified. This researcher repeated the same process for all the subtitled videotapes.

Multimodal analysis. The researcher used multimodal analysis to seek "Audio meanings," "Oral meanings," "Written meanings," "Visual meanings," "Spatial meanings," "Tactile meanings," and "Gestural meanings" at once. The multimodal analysis includes verbal and non-verbal communication, including gestures, gazes, tone of voice, and a range of different modes to define what goes on in interactions (Jewitt, 2012). In brief, multimodality was the best choice to support the complex fine-grained analysis of all the artifacts and interactions. The multimodality analysis used involved the process of multimodal layering. Multimodal layering consists of three distinct but interconnected stages. The first stage is about linking the classroom socio-cultural environment and the GeoGebra, the second is about connecting prior geometry knowledge, and the third is about linking GeoGebra and with personal experiences (Jewitt, 2006).

The analytic dimensions of multimodality refer to the content of visual data that are of interest when analyzing situations. These contents include facial expressions, body postures, interactions, and context. The person's face and body convey the nonverbal information. The verbal interactions significantly affect the socio-cultural environment and the people within (Jewitt, 2006). These dimensions were the lenses that allowed the researcher to derive information from the video recording and to understand its situational dynamics.

Multimodality allowed the researcher not only to reconstruct the virtual classroom situation but also to analyze inner dynamics by establishing overall storylines. The analysis included breaking down situations by focusing on who did what, when, how, and where (Jewitt, 2012). The aim was to understand the student situational path that led to the chain of linked actions and interactions. The researcher first developed coding schemes through a phase of intensive, detailed labeling of information. The second phase included grouping connected codes into categories.

The multimodality approach to analysis opened up possibilities for recognizing, analyzing, and theorizing how students created meaning and context while using the GeoGebra software. The researcher explained student engagement in the iterative connection between GeoGebra's meaning potentials, the social and cultural environment in which the app was encountered, intentions, and knowledge that the students brought to that encounter. The researcher strived to connect student engagement with their expression of emotion and what it signifies in this social context, such as gestures, images, and objects notated in the scripts. Not only the researcher used a multimodal analysis to map the students' gaze, facial expression, tone of voice, but they also used multimodality to identify specific strands of engagement and to interrogate episodes of engagement as a result of the GeoGebra visual stimuli (Jewitt, 2005).

The researcher examined how the student's interaction was constrained and engendered relative to the use of the GeoGebra, their interactions with one another, and crucially their cognitive apprehension of tasks. The researcher emphasized how students interacted with their virtual classroom environment and engaged with the software. The analysis process was excerpted by getting immersed in the data by producing a rough multimodal descriptive overview along with a rough multimodal transcript.

The analysis process of the video-recordings. The first stage of this qualitative analysis comprised repeated viewing of the video data and the production of an overview of the students' activity in the video through video logging.

Time	What is happening?	Interaction with the software	Movement	Body	Talk	Teacher's interaction
9:05 AM	S2 suggests drawing the geometry figure with animation	Start the geometry construction	S2 holds the Ipad to start drawing	Nodding head with an A-ok hand sign	Here we go…	Ok, let us start
9:12 AM	S5 suggests taking a screen shot of his work to share it with the school group	Takes a screen shot	Holds Ipad up to show his GeoGebra representation	Moves back to change his sitting position	That was a good start for today	Good for you, keep up the good work
9:15	S3 asks his classmate S9 to share screen to help him construct his figure	S9 shares screen to demontrate step by step the geometry figure construction	S9 Points to the GeoGebra tools he used, S3 follows him		S3 That was helpful	Thanks S9 for his help

Table 14: Example of a transcript

The video logging included descriptive labeling of the videos according to its content. A first-stage dismantling of the video of each student's interaction was produced to gain an overview account focused on the use of modes. This process, labeled as "transcript," included recording movement, body action (including gaze), and the students' interaction with GeoGebra and with their peers and teacher (table 14).

A discussion was defined as the exchange, exploration, and possibly the debate of ideas. The demonstration was defined as showing something, illustrating, providing an example, or displaying a behavior. Finally, in-class discussion, student solving techniques 'in action' were assessed in addition to the frequency of the use of each method. The "Interaction category" with the GeoGebra recorded how the students interacted with the application, monitored their progress and responded to geometrical representation. The "Body action category" recorded all reciprocal influence, such as pointing, gesturing, or facial expressions. The researcher noted the students' speech along with the teacher's interaction with them. These first-stage transcripts provided an additional lens with which to identify and verify episodes of engagement.

The second analytical stage used these overview transcripts alongside the video to identify episodes when the engagement was prompted or supported by the use of GeoGebra. The researcher identified these episodes by attending to a range of multimodal aspects of the students' interaction with GeoGebra and geometry learning. Following Skilling et al., (2016) and Helme & Clarke, (2001) in the understanding of the signs of affective, behavioral, and cognitive engagement, modal indicators were considered relative to the students' experience of the GeoGebra software environment.

The third stage of the analysis focused on in-depth modal analysis and the interaction between modes. This stage involved further iterative viewing of the research questions. A fine grained multimodal transcription of each of the episodes included a detailed, time-stamped transcript of the incident. This transcription included a descriptive analysis focused on verbalization of thinking, participating in class discussions and bodily interaction through the modes of movement, body orientation, body posture, gaze, gesture, touch, and facial expression.

A grounded analysis of the data enabled the researcher to identify the different strands of engagement that were supported by the use of GeoGebra. The researcher explored these instances of engagement in consideration of varying communication and representation potentials of modes. The researcher noted how these modes prompted and supported engagement in the context of learning geometry.

Findings of the video analysis. The findings presented in this section provide insights on how GeoGebra can prompt and support learners' engagement in learning geometry using a multimodality analysis approach. Four aspects of the students' multimodal interaction with the GeoGebra were central to provide the organizational structure for this section: student's interpretation, student's action, student's perception, and student's computation. These were developed iteratively through intensive grounded analysis of the empirical data in conversation with multimodal theory.

Interpretation	**Action**	**Capture**	**Computational Thinking**
Intention	Speech - Audio Cues	Microphone	Decomposition
Attention	Gesture - Visual Cues	Video Camera	Pattern Recognition
Emotion	Facial Expression	Video Camera	Abstraction
Observation of behavior	Writing - Speech to Text	Tablet Haptic Deivice	Algorithm Design

Table 15: Four aspects of the students' multimodal interaction

Different modal resources as logical interaction, physical interaction, gaze, gesture, and others were combined to produce student's experience. The examples discussed in this section illustrate how students connected the GeoGebra visual stimuli and the online classroom environment, how the connection prompted their engagement, and how the engagement supported geometry learning. The below four examples illustrate a range of student interaction and provide an interpretative insight into students' engagement with the GeoGebra experiences.

Example 1: "Pythagorean Theorem Investigation."

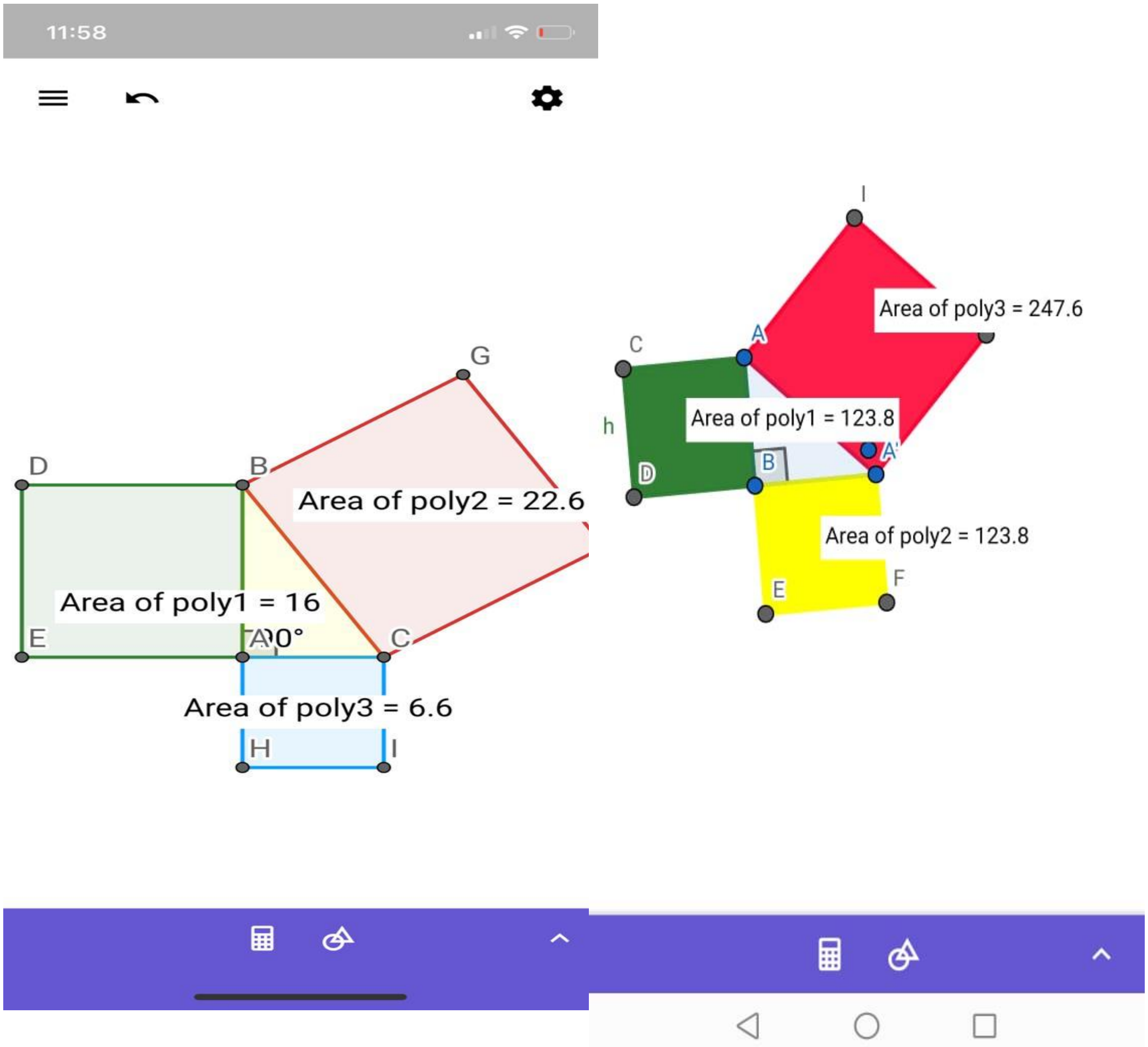


Figure 16: Sample of students' work on Pythagorean Investigation

The following activity was an investigation of the Pythagorean Theorem proof (figure 16). Pythagorean Theorem asserts that the square of the hypotenuse of any right triangle is equal to the squared sum of the two legs. Over the years, some mathematicians proved Pythagorean by considering the areas for the square surrounding the three sides of the right triangles. In this study, comparing all three areas, students reached that the two small areas of the squares sum up to the big area of the square lying by the hypotenuse. Being involved in the first activity, few students felt stressed out and found it hard to keep track of the teacher's instructions. This stress stemmed out from a lack of some basic necessary GeoGebra skills. This session introduced critical new techniques and aimed to build the students' fundamental skills in GeoGebra.

However, it was surprising how the other students were helping their struggling peers overcome obstacles. They were discussing work results and seeking formative feedback to help each other understand. Besides, students provided self-assessment of the quality of their work, flipped through the other's work, and rated them in comparison. They highlighted their weakness and strength in GeoGebra and commented on the work of one another. They gave their peers hints about what they could have done better and helped them comprehend the task. This example demonstrates the interplay between students' interpretation of the GeoGebra visual stimuli, the current online classroom context, and the students' engagement.

Few students were struggling by the beginning of the session, which made student S1 say: "I have just shared a video on the class group to show the procedure step by step and a detailed explanation. Check it so that everyone may follow up with us."

Student S2 answered: "Yes, she is right; you need to enter the settings first, then change the increment values."

Then afterward, S1, S2, S3, and S4 shared an image for their geometry figure done on the class group. Incidents like S1 and S2 explaining the task for others and when S1, S2, and S3 revealed the completion of the investigation task determine the presence of cognitive engagement, (Fung et al., 2018). Persistence in completing tasks was shown by S8, who said, "I cannot get it right, but I will try it all over again and again."

This investigation prompted students to cognitively link the activity with their self-monitoring progress and their reflective self-questions in which S5 assessed his job as messed up

"So I have messed up the whole thing. GeoGebra needs much practice". S10 kept on holding reflective self-questions. "Where am I now? How shall we proceed now?" (Skilling et al., 2016).

S5 then went, "May I share my work? Check this progress." Then after S5 shared her work, she proceeded, "What do you think? Is it correct?" Then S4 addressed S3, "You need to close the figure, or else your figure would be linear."

Again, student S6 said, "Check mine! This work does not seem very good!". So, student S7 replied, "You cannot put B= (b,0) where B= (a, b)." and S2 said, "As soon as you fix point "B," it would no longer stay linear! Just go ahead and give it a try, stick to the variables (a, b)."

S8: "The trick is to follow the procedures in order! Order of the points is basic in GeoGebra."

S9: "Tomorrow, we will get prepared for the coming task."

These conversations sum up both student participation (S5), student interaction (S4) at once, holding discussions (S6), exchanging ideas, and cooperation with other students (S2), which are all linked to the behavioral engagement (Fung et al., 2018).

Observation "looks-for"	**Category**	**Observed**	**Specify Details (frequency, time interval)**
Verbalization of thinking	Cognitive	11:33	S11: We can use identities to calculate the areas
Self-monitoring	Cognitive	11:50	S12: I need to work harder in the coming activity
Concentration	Cognitive	11:40	S3: Give me just one minute I need to focus
Resisting Distractions	Cognitive	11:10	S13: Please mute your audios
Seeking Help	Cognitive	11:12	S8, S9, S10: I need help
Seeking Information	Cognitive	11:28	S1, S2: How did you do it?

Seeking Feedback	Cognitive	11:25	S6: What do you think of my work?
Evaluate Comments	Cognitive	11:43	S14: No, you are wrong! The two lends are AB and AC.
Answer Teacher's Questions	Cognitive		Random students
Give Information	Cognitive		Random students
Explain Procedures	Cognitive	11:26	S3: You need to hold the point b and drag to get different examples at once
Reasoning	Cognitive	11:30	S1: The areas added together are equal to the area of the big square
Address Deep Questions	Cognitive	11:34	S4: How does it work? S5: Why is Pythagorean important in our daily life?
Hold Reflective Self-Questioning	Cognitive	11:27	S10: Where am I now? How will I proceed now?
Self-Monitoring Progress	Cognitive	11:11	S5: So, I have messed up the whole thing. I need to practice more
Persist in Completing Tasks	Cognitive	11:25	S8: I cannot get it right, but I will try it all over again and again.
Using Different Strategies	Cognitive	11:48	S2: I tried to draw on a different figure as well to make things clear
Ask Random Questions	Behavioral	all over the session	Students were continuously asking random questions.

Participating in class Discussions	Behavioral	11:13	S5: May I share my work? I want to show you all my progress.
Interacting with others	Behavioral	11:13	S4 addressing S3: You need to close the figure.
Hold Discussion	Behavioral	11:16	S6: "Check mine! This work does not seem very good!".
Complete Tasks	Behavioral	11:08	S1, S2, and S3 share an image of their geometry figure done.
Cooperate with Other Students	Behavioral	11:15	S4: "You need to close the figure…" S2: As soon as you fix point B…
Explain the task to Others	Behavioral	1st: 11:06	S1: I have just shared a video… S2: You need to enter the settings
Get Prepared for Tasks	Behavioral	11:22	S8: The trick is to follow the procedures in order!
Get Prepared for Tasks	Behavioral	11:30	S9: Tomorrow, I will prepare for the following task.
Seem Interested	Affective	11:46	S15: This was awesome!
Seem Curious	Affective	11:04	S11: I shall know more about GeoGebra
Hold Positive Attitude	Affective	11:50	S1: This worked out well!
Seem Stressed Out	Affective	11:01	S9: The activity seems hard to do.
Enjoyment	Affective	11:49	S14: This session was fun!
Finding Relevance and Value of Task	Affective	11:50	S1: Now GeoGebra made it easier to get the Pythagorean thing

Table 16: Observation Protocol – first video

This investigation triggered students' physical link with the GeoGebra activity. To do this, the students used to gaze and gesture, point to or look at GeoGebra tools. They expressed excitement seen in shifts in the students' embodied interaction, such as nodding the head up and down, placing their hands on their heads, making significant and quick hand gestures, and moving closer to the iPad. As affective engagement and interaction between the students unfolded, their action was directed toward linking the GeoGebra visual stimuli to the surrounding online classroom environment. This example demonstrates the interplay between students' interpretation of the GeoGebra software and the three strands of student engagement; cognitive, behavioral, and affective.

***Example 2: Pythagorean Spiral*.** The following activity revolves around the Spiral of Theodorus, which is a spiral composed of right triangles placed edge-to-edge. Students started by constructing an isosceles right triangle, with each leg of the triangle having a unit length (figure 17). The trick in this spiral is that each right triangle formed would have one of its legs as the hypotenuse of the prior triangle. The students repeated the same process until the nth triangle in the sequence was a right triangle with side lengths $\sqrt{n}$, with hypotenuse$\sqrt{n+1}$.

The teacher used the first several minutes of class to encourage students to get on task more efficiently. This time typically involved the discussion of the goal of the day and instructions for the GeoGebra activity. Student S9 started the session by saying, "Yesterday, I practiced GeoGebra, I can almost draw any polygon and even 3D shapes. We enjoy using it."

S12 replied, "This application is amazing. It is incredible what we became capable of doing using it."

S1 continued, "Yesterday, I kept playing with all those geometry tools! It is awesome!"

S5 said, "Same here!"

Though some students seemed challenged with the GeoGebra tools, by the time of the second activity, most students gained an overall competence in using GeoGebra. They covered most of the fundamental GeoGebra skills and felt that they had a better idea of what was going on in the GeoGebra task. Throughout the session, students were more capable and confident about various GeoGebra tools. As a case in point, student S6 said: "GeoGebra seemed challenging at first, but once I realized how useful it was, I understood it." They reflected on their new ways of thinking about GeoGebra and geometry. Students had to learn new strategies, new ways of thinking in solving geometry, and self-monitoring skills. As of interest was how students were already developing positive attitudes towards GeoGebra and evolving their ability to solve geometry. Students' positive attitudes affected their performance in the geometry class. For example, S13 said: "Using GeoGebra in each activity needs a sort of new way of thinking for me." Then S9 replied, "It rather requires a great deal of discipline and technical skills."

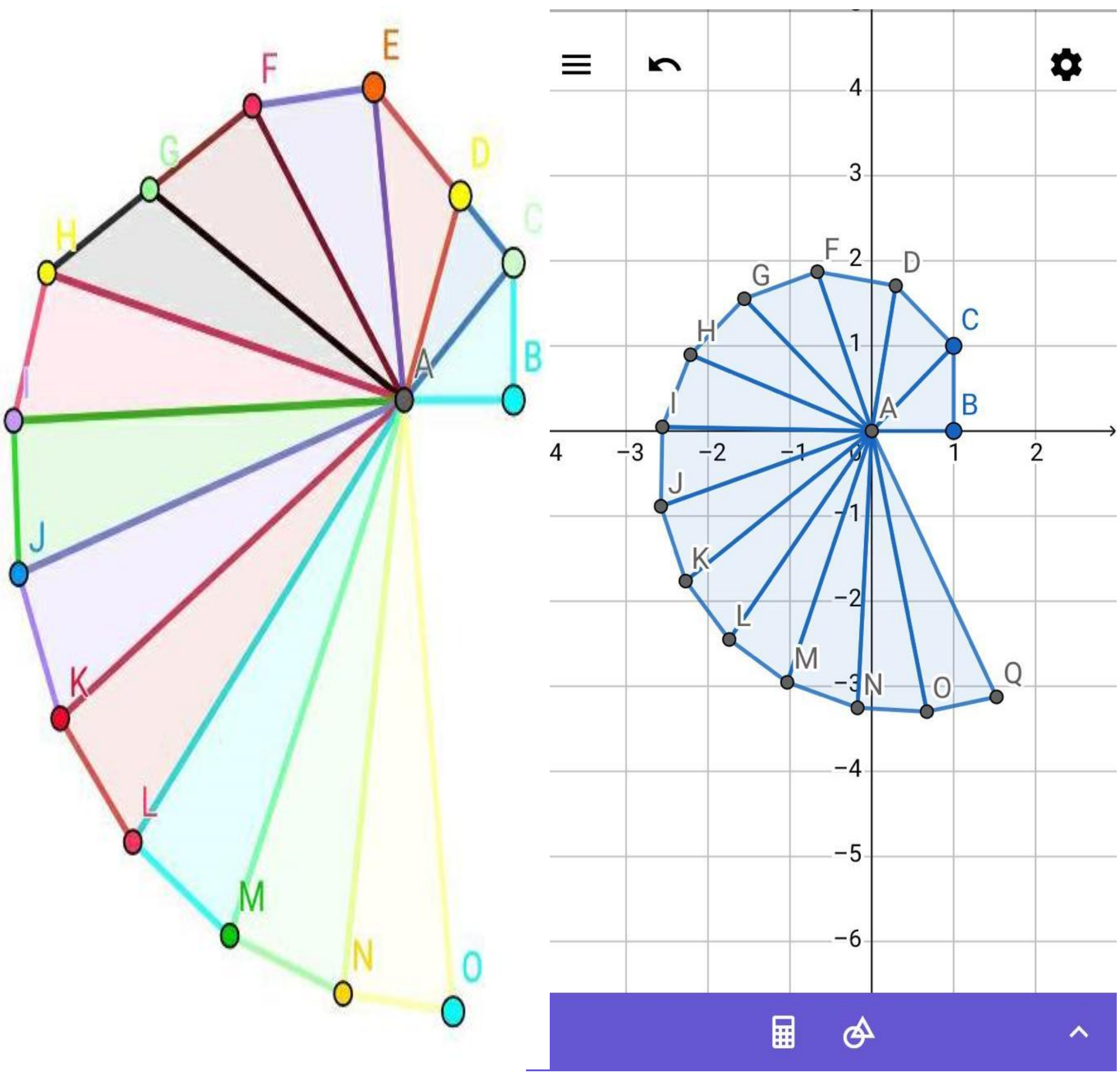


Figure 17: Sample of students' work on Pythagorean Spiral

Later in the activity, S5 said: "I am slow in drawing figures; GeoGebra will help me speed up solving geometry problems."

Generally, students worked the task independently and often expressed enthusiasm for GeoGebra. However, they did express frustrations with the device at times. They faced what they called technical issues, including errors, delays around device, and app updates. For example, S8 was unable to keep up with the class pace, and S15 said, "Every time this is getting more confusing! GeoGebra can be tricky! Even my saved drawings and work got deleted."

It was clear that students' attitudes towards geometry problems were less procedural or algorithmic than those expected in conventional mathematics lessons. A good example would be when S1 said, "There is no one way to do those tasks; you just need to try and check out if it was relevant, right?" Abstract geometry and mathematical reasoning as well were in reach of the students, as they accompanied each solution by a method. Rather than memorizing, GeoGebra helped students learn by doing. Students were navigating multiple strategies to solve geometry problems. These multiple strategies were encouraged by the teacher and supplemented by GeoGebra. As a demonstration, S3 said: "The is no more imagine this and imagine that now we can see it all in 3D GeoGebra." Whereas S1 said: "Pythagorean applies to all right triangles, but right isosceles shall have a different rule, right?" Students as well spent more time on tasks even in free choice lessons. The free choice of experiences took the form of playing geometry games on GeoGebra, as mentioned previously by S1, S12, and S5.

Students were collaborating and directing their learning as they all worked together to discuss concepts and find solutions to geometry problems. On the contrary, students who were struggling to learn new ideas in traditional geometry classrooms would quickly fall behind their peers. The self-paced learning of GeoGebra blended with peer collaboration replenished their knowledge. Whenever a student needed more time or extra help, they practiced `outside class with guided exercises at their own pace. They had the privilege of accessing the resources whenever they needed to and asked the teacher or their peers when extra help was needed. Most students were aware of the valuable benefits that the GeoGebra added up to their learning outcomes.

S4: "How do we construct the circle passing through A?"

S2: "Construct the perpendicular line to AB at B, B will be the center, then construct a circle passing through A."

S7: "Mark the point of intersection of the circle and the perpendicular line as C."

S4: "Okay, but what is the length of the hypotenuse of the second triangle?"

S8: "Repeat the construction till you get the triangle with a hypotenuse of length."

S11: "I suggest you clean up your figure by hiding all the perpendicular lines and circles. If you do so, you would get it."

S7: "This worked well for me."

S7 addressing the teacher: "Check mine! For the first time in long, I can do geometry!"

S15: "Seems like I am the only one who cannot get it right! Why can't I get right? You are going fast, and I cannot follow."

S12: "We can work on it after class, save your work for now, and help you fix it."

S15: "Okay, great."

Teacher: "If you need any help, get me involved."

S12: "Sure, we will."

A similar pattern of behaviors was observed in other instances to demonstrate the students' textual reflections and interpretation processes. For example, students looking at an image of the Pythagorean spiral used gaze, gesture, and movement to link the image to their immediate meaning construction with context, clues, and prior knowledge. The students then moved back and forth, looked around the software trying to accomplish the task, directing their gaze to find a particular GeoGebra tool to focus their attention on, and then gestured to identify these tools to their peers. In this interaction, movement, gaze, gesture, and speech in the virtual classroom were coordinated to bring visual stimuli of GeoGebra alive through interaction with the social-cultural interaction

(Fung et al., 2018). The students then reflected on the images and talked about the experience of using GeoGebra. Through their gaze direction, body orientation, and gesture, they prompted and supported the students' engagement.

Observation "looks-for"	**Category**	**Observed**	**Specify Details (frequency, time interval)**
Verbalization of thinking	Cognitive		Random students
Self-monitoring	Cognitive	9:22 AM	S5: I am slow in drawing figures, GeoGebra will help me speed up in solving geometry problems."
Concentration	Cognitive		Random students
Visualization of abstract ideas	Cognitive	9:40 AM	S3: The is no more imagine this and imagine that now we can see it all in 3D GeoGebra.
Seeking Help	Cognitive	9:20	S4: How do we construct the circle passing through A?
Seeking Information	Cognitive	9:24 AM	S4: Okay, but what is the length of the hypotenuse of the second triangle?
Seeking Feedback	Cognitive	9:31 AM	S7: Check mine!
Evaluate Comments	Cognitive	9:31 AM	S7: This worked well for me.
Answer Teacher's Questions	Cognitive		Random students
Give Information	Cognitive	9:21 AM	S2: Construct the perpendicular line to AB at B, B will be the center, then construct a circle passing through A.

Explain Procedures	Cognitive	9:22 AM	S7: Mark the point of intersection of the circle and the perpendicular line as C.
Reasoning	Cognitive	9:49 AM	S1: Pythagorean applies to all right triangles, but right isosceles shall have a different rule.
Address Deep Questions	Cognitive	9:46 AM	S1: There is no one way to do those tasks; you just need to try and check out if it was relevant, right?
Hold Reflective Self-Questioning	Cognitive	9:27 AM	S15: Why can't I get right? You are going fast, and I cannot follow.
Self-Monitoring Progress	Cognitive	9:15 AM	S9: GeoGebra requires a great deal of discipline and technical skills.
Persist in Completing Tasks	Cognitive	9:31 AM	S7: For the first time in so long, I can do geometry!
Organization in solving tasks	Cognitive	9:33 AM	S11: I suggest you clean up your figure by hiding all the perpendicular lines and circles. This suggestion would help you get it.
Using Different Strategies	Cognitive	9:14 AM	S13 Using GeoGebra in each activity needs a sort of new way of thinking for me.
Ask Random Questions	Behavioral		Random students
Participating in class Discussions	Behavioral		Random students
Interacting with others	Behavioral		Random students

Hold Discussion	Behavioral		Random students
Prepare Extra Work	Behavioral	9:00 AM	S9: Yesterday, I practiced GeoGebra; I can almost draw any polygon and even 3D shapes.
Complete Tasks	Behavioral		Random students
Cooperate with Other Students	Behavioral		Random students
Explain the task to Others	Behavioral		Random students
Exchange Ideas	Behavioral		Random students
Get involved in Voluntary Tasks	Behavioral	9:40 AM	S12: We can work on it after class, save your work for now and I will help you fix it."
Seem Interested	Affective	9:01 AM	S12: This application is fantastic. It is incredible what we became capable of doing using it.
Seem Anxious	Affective	9:07 AM	S15: Every time this is getting more confusing! GeoGebra can be tricky
Hold Positive Attitude	Affective	9:02 AM	S1: Yesterday, I kept playing with all those geometry tools! It is awesome! S5: Same here
Seem Stressed Out	Affective	9:39 AM	S15: It seems like I am the only one who cannot get it right!
Enjoyment	Affective	9:00 AM	S9: I enjoy using it.

Finding Relevance and Value of Task	Affective	9:11 AM	S6: GeoGebra seemed challenging at first, but once I realized how useful it was, I understood it.

Table 17: Observation Protocol – second video

Example 3: Investigation of the Inscribed Angle Conjecture. This GeoGebra activity aims at investigating the relationship between the inscribed angle, the central angle, and the arc they intercept. Each student had to draw a circle and an inscribed angle within. Then they had to draw the central angle intercepted within the same arc. Using the "measure angle" GeoGebra tool when they drag points to change the measure of the angles, they would comprehend that in a circle, the measure of an inscribed angle is half the measure of the central angle with the same intercepted arc. This activity led students to comprehend two corollaries. The first result is that the inscribed angle is half the measure of its intercepted arc (figure 18).

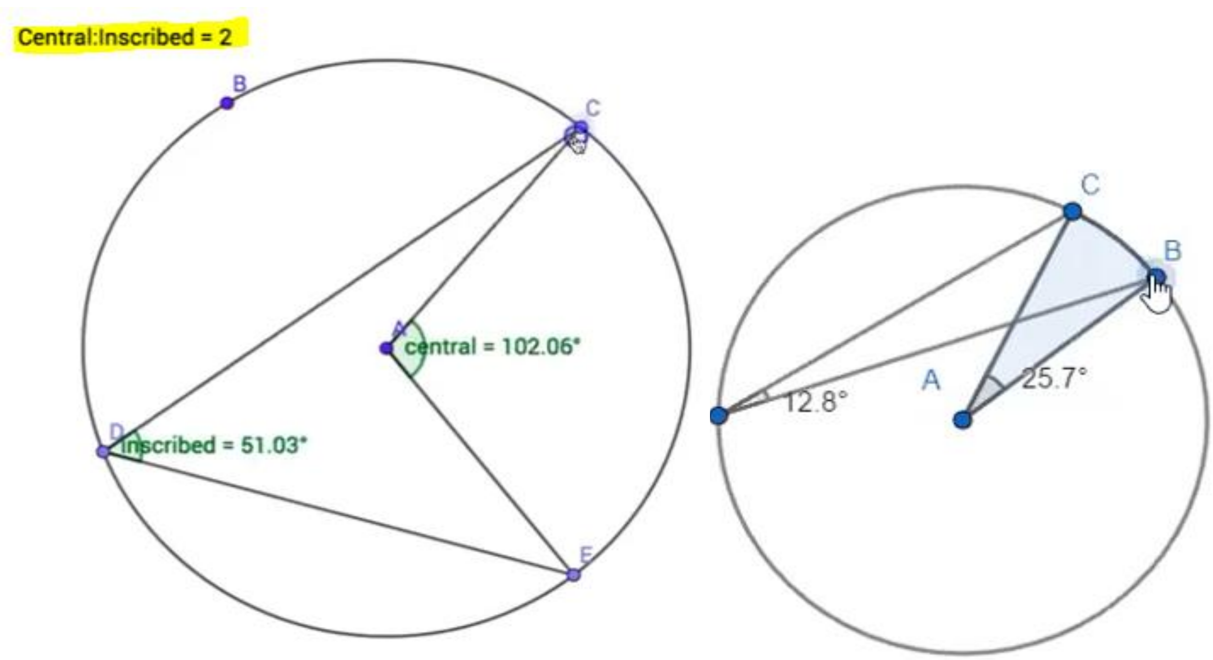


Figure 18: Sample of students' work on Inscribed Angle Conjecture

The second result is that any angle inscribed in a semi-circle is a right angle. Some would state it as if A, B, C are distinct points on a circle where the line AB is the diameter, then angle ACB is right (Thale's).

Using GeoGebra in the current learning activity created room for growth in terms of achieving the kind of active student engagement. The teacher was introducing new geometry concepts, new relations, and new theorems. However, GeoGebra provided an efficient, comfortable, and enjoyable pathway for students. Students by that session were becoming more proficient with the use of GeoGebra and better equipped with its transformative power. Circle geometry in GeoGebra has numerous benefits since it offers a wide choice of content and tools to help make concepts more evident and observable. The dynamic geometry of GeoGebra comes alive with circle geometry, and circle geometry was what the session was all about. Students were able to represent and visually display the circle theorems efficiently. Using the "move tool," students were able to get many examples at once only by dragging point "D." Using the same tool, they were able to form many circles by an eye blink only by dragging point "B." They used the "relation tool" in GeoGebra to demonstrate the relation between the inscribed angle and the measure of the central angle. No matter how students changed the circle and how they changed the vertex of the inscribed angle, still, the inscribed angle always measured half of the central angle. These intelligent GeoGebra tools illustrated concepts with multiple and diverse examples. Such examples provided the broadest range of students with supplement to their raw information construction, especially so for students who learn and think differently where one example would be unclear. The presence of multiple cases helped students understand and meet new concepts through a variety of ways and approaches. In other words, GeoGebra provided a feasible, scalable solution for tailoring instruction to learners' needs.

The specific modes used in the GeoGebra played a role in shaping students' interaction and their potential for engagement. The GeoGebra visual aids prompted students to reflect on and interpret the geometry concepts through intense participation and involvement. Students not only were interacting with their peers and teachers but also with the GeoGebra itself as a visual stimulus. The urge to necessarily keep eyes on the GeoGebra software constrained students' movement and their use of gestures. For the most part, patterns of activity revolved around the digital device.

The session was self-paced, and students were keen to complete the task. The teacher explained that she wanted students talking more, working together, and communicating their understanding with less direct instruction. Rich classroom discourse was happening at the group, whole class, and student-to-teacher level. This strategy offered students a way to express their ideas, reasoning, and thinking. This rich discourse was central for students to acquire an understanding of the new concepts. At the heart of it, dialogue among peers facilitated creating meanings and helped students take ownership of their learning. The search for answers to this GeoGebra investigation stimulated curiosity questions that elicited insights for other students to find meaning and value in the task. Students worked independently on the software, and then after that, they excitedly debated, explained, and compared their solutions until they agreed on the solution. The collaborative discourse and its external influence helped students generate a greater sense of purpose and excitement. The teacher answered a question with more well-framed questions. Not only that, but she also helped them gain entry into the language of math. Students' deep engagement with the content they were learning was translated through actions in the virtual classroom. Such a learning approach relates to self-regulation, task engagement, and persistence as each student had his personalized way of dealing with each specific content. There were three types of students in the virtual class: students who finished the task quickly and started working on exercises, students who sought support from peers and teachers, and some who did not complete the job before the whole class went over.

S1: "What if the angle was exterior? What would be its measure?"

S13: "What do you mean by exterior?"

S2: "The teacher mentioned this before. The vertex is outside the circle."

Teacher: "How many types of exterior angles do we have?"

S6: "One type."

S7: "I think more than one because the leg of the angle can be either a secant or a tangent.'

S12: "So we have two types of exterior angles? One formed from two tangents, one formed from two secants?"

S2: "Three, three, cause an exterior angle could be formed of one secant and one tangent."

Teacher: "What do you think? Who is right?"

S11: "I will draw to check."

S6: "I did the first two types; I am trying to draw the third."

S8: "I cannot draw an exterior angle with one secant and one tangent."

S10: "I did it! Yes, everyone can, check mine." (Then shares screen)

S2 (shares screen): "And mine."

S4: "Then we have three types of exterior angles, but what about the measure? You did not answer S1 yet."

S1: "The measure is getting smaller as we move further from the center of the circle."

S2: "Just like a rubber band."

S16: "Ha-ha, I liked the example."

Teacher addressing S16: "What do you think? When we stretch the rubber-band, does the angle gets smaller or bigger?"

S16: "Definitely smaller."

S2: "Then, my example is correct."

Teacher: "You said the angle gets smaller, then what would be the measure of any exterior angle?"

S2: "We subtract its two intercepted arcs."

Teacher: "Check the answer to your figures. Is it true?"

S1: "No, it is even smaller than that."

S4: "We subtract, and then we divide by two since we have two intercepted arcs."

S16: "What? How? I do not get it!"

S13 (shares screen) "Look, the first intercepted arc is equal to its central angle = 112 degrees, and the second intercepted arc is equal to its central angle=62 degrees. Now compare the difference of these two angles with the exterior angle."

S16: "The exterior angle is 25."

S2: "The difference between the two arcs is 50 degrees. The exterior angle= 25 degrees is half of the difference."

S9: "Nice, so now we have the rule for exterior angles! Nevertheless, what if the angle was interior?"

S15: "When I dragged the angle vertex from outside the circle into the inside, the angle seemed larger."

S3: "Getting larger means it is not subtraction, then the operation shall be the addition."

S2: "I will check if the two intercepted arcs sum up to the angle."

S1: "It is the same as the exterior, but instead of half the difference of the intercepted arcs, it is half the sum of the intercepted arcs."

Teacher: "How can you explain that?"

S16: "As we said about the rubber band, this time, we shorten the rubber band rather than stretch it."

Again, the question of S9 yielded to an interactive discourse that enhanced students' critical literacy and comprehension skills. The researcher observed collaborative discussion, an increase in attention, and practicing of higher-level critical thinking skills. The classroom environment elucidates that students were behaviorally and cognitively engaged in the learning process (Fung et al., 2018).

Student enjoyment in using GeoGebra contributed to students' interest in learning geometry. These high levels of interest promoted students' desire to focus their attention and to learn geometry with excitement. Students were captivated by GeoGebra and felt that the software was worth further exploration. This absorption reflected affective reactions, perceived value, and cognitive functioning intertwine. Students were driven by enthusiasm rather than compliance.

For example, sometimes, students used language to explicitly refer to emotion as "happy" or "fun." Other times they used a gesture or movement associated with passion as a clenched fist raised in the air to indicate triumph or joy. Even changes in movement and pace or the presence of a facial expression suggested an intensity of feeling like a big smile. The observed interactions implicitly linked GeoGebra’s role with student affective engagement.

To give an instance, S8 addressed the teacher: "Whenever I want to have fun, I go back to the previous geometry exercises and do them using GeoGebra. I will send you the figures that I did yesterday on GeoGebra."

S2: "I am doing it for fun, but rather doing all my assignments using the software. It helps me finish my assignments faster."

S13: "I am doing much creative stuff on GeoGebra."

S1: "GeoGebra became my game. When the electricity goes out, and I get bored, I play with GeoGebra."

S4: "I enjoy doing GeoGebra in class, but I do not use it outside class."

S14: "Neither do I."

S10: "Even when I fail to solve a geometry problem using GeoGebra, I turn my figure into an art. I turn geometry figure into something awesome. Still grappling with the skills."

Teacher: "Okay, this is good. Nevertheless, try to practice more in geometry figures."

S8: "I bought the book Be Smart, and I choose random geometry exercises to solve using GeoGebra."

S15: “I am not using GeoGebra that often.”

S16: "Whenever I do not understand a geometry concept, I go back to the solved exercises and practice them uses GeoGebra. Never understood geometry as now."

Observation "looks-for"	**Category**	**Observed**	**Specify Details (frequency, time interval)**
Verbalization of thinking	Cognitive	10:11 AM	S12: "So we have two types of exterior angles? One formed from two tangents, one formed from two secants?"
Self-monitoring	Cognitive	10:26 AM	S2: "I will check if the two intercepted arcs sum up to the angle."

Concentration	Cognitive	10:25 AM	S3: "Getting larger mean not subtraction, then it shall be the addition."
Resisting Distractions	Cognitive	10:27AM	S1: "It is the same as the exterior, but instead of half the difference of the intercepted arcs, it is half the sum of the intercepted arcs."
Gestures	Cognitive		Random students
Seeking Help	Cognitive		Random students
Seeking Information	Cognitive	10:08AM	S13: "What do you mean by exterior?"
Seeking Feedback	Cognitive	10:24 AM	S15: "I dragged the vertex of the angle from outside the circle into the inside of it. The angle seemed to get larger."
Evaluate Comments	Cognitive	10:12AM	S11: "I will draw to check."
Answer Teacher's Questions	Cognitive	10:10 AM	S7: "I think more than one because the leg of the angle can be either a secant or a tangent.'
Give Information	Cognitive	10:09AM	S2: "The teacher mentioned this before. The vertex is outside the circle."
Explain Procedures	Cognitive	10:15 AM	S1: "The measure is getting smaller as we mover further from the center of the circle."
Reasoning	Cognitive	10:22 AM	S2: "The difference between the two arcs is 50 degrees. The exterior angle= 25 degrees is half of the difference."

Address Deep Questions	Cognitive	10:08 AM	S1: "What if the angle was exterior? What would be its measure?"
Hold Reflective Self-Questioning	Cognitive	10:23 AM	S9: "Nice, so now we have the rule for exterior angles! Nevertheless, what if the angle was interior?"
Self-Monitoring Progress	Cognitive	10:14 AM	S10: "I did it! Yes, everyone can, check mine."
Persist in Completing Tasks	Cognitive	10:13 AM	S6: "I did the first two types; I am trying to draw the third."
Organization in solving tasks	Cognitive	10:12 AM	S2: "Three, three, cause an exterior angle could be formed of one secant and one tangent."
Using Different Strategies	Cognitive	10:22 AM	S13 (shares screen) "Look, the first intercepted arc is equal to its central angle = 112 degrees, and the second intercepted arc is equal to its central angle=62 degrees. Now compare the difference of these two angles with the exterior angle."
Ask Random Questions	Behavioral		Random students
Participating in class Discussions	Behavioral	10:15 AM	S4: "Then we have three types of exterior angles, but what about the measure?
Interacting with others	Behavioral	10:17 AM	S16: "Definitely smaller."
Hold Discussion	Behavioral	10:21 AM	S4: "We subtract, then we divide by two since we have two intercepted arcs."

Prepare Extra Work	Behavioral	10:46 AM	S16: "Every time I do not understand a geometry concept, I go back to the solved exercises and practice them using GeoGebra. Never understood geometry as now."
Complete Tasks	Behavioral	10:18 AM	S2: "Then, my example is correct."
Cooperate with Other Students	Behavioral	10:19 AM	S2: "We subtract its two intercepted arcs."
Explain the task to Others	Behavioral	10:20 AM	S1: "No, it is even smaller than that."
Exchange Ideas	Behavioral	11:16 AM	S2: "Just like a rubber band."
Get Prepared for Tasks	Behavioral	10:01 AM	S5: "GeoGebra is ready."
Get involved in Voluntary Tasks	Behavioral	10:43 AM	S8: "I bought the book Be Smart, and I choose random geometry exercises to solve using GeoGebra."
Seem Interested	Affective	10:46 AM	S16: "Every time I do not understand a geometry concept, I go back to the solved exercises and practice them using GeoGebra. I never understood geometry as I do now."
Seem Anxious	Affective	10:20 AM	S16: "What? How? I do not get it!"
Seem Curious	Affective	10:42 AM	S13: "I am doing much creative stuff on GeoGebra."

Hold Positive Attitude	Affective	10:44 AM	S1: "GeoGebra became my game. When the electricity goes out, and I get bored, I play with GeoGebra."
Seem Stressed Out	Affective	10:25 AM	S4: "I do not get this."
Enjoyment	Affective	10:17 AM	S16: "Ha-ha, I liked the example."
Finding Relevance and Value of Task	Affective	10:45 AM	S2: "I am doing it for fun, but rather doing all my assignments using the software. It helps finish assignments faster."

Table 18: Observation Protocol – third video

Example 4: Circle Geometry Investigations. This task tickled students' minds and sparked deep conversation through the below three thought-provoking investigations (figure 19).

1. Let us investigate how many circles can one draw through two given points.
2. Let us investigate how many points are needed to draw a circle.
3. Let us find out how many circles we can draw through three collinear points.

A circle is an essential shape in the field of geometry. What students might not see is how the distinct properties of a circle make it useful in various ways. To help students understand how this unique shape can be used to solve problems and understand its relation to the world around, it is essential to understand all of its properties. This activity allowed students to explore, predict, prove, and confirm those properties. For a better investigation of the circle properties, students illustrated interactive demonstrations using GeoGebra. Students continued to build their understanding of geometry as they were exploring the circle properties.

While some of the properties were associated with line segments related to circles, others were associated with arcs and angles. This specific activity was designed to help students discover some features that are associated with points related to circles. The teacher started the session by: "Using GeoGebra, we will investigate powerful facts about circles today."

S8 replies: "Exciting!

Teacher: "You need to scroll down the GeoGebra tools until you find the circle tool section."

S8: "I have tried them all. I know how to use them."

S2: "Me too…"

Teacher: "Using GeoGebra let us think about the first question, how many circles can we draw through two points?"

S1: "I think that would be two."

S5: "No, I think one. We can draw a one-line segment through any two points, so here the same thing goes."

S7: "This has nothing to do with line segments."

S12 (shares screen): "I have already drawn two circles passing through these two points."

Teacher: "How did you do it?"

S12: "Ahh, I am not sure. I just tried several circles, and this worked."

S4: "So the answer is two circles?"

Teacher: "What do you think? Is it two?"

S13: "You mean, it can be three?"

Teacher: "You need to check if you can draw three circles through two points."

S2: "Infinity. We can draw infinite circles."

Teacher: "Why did you say so?"

S2: "Because we have infinite radii in the circle."

Teacher: "How can this be related?"

S2: "The two points are on the circle so they can be endpoints of diameters, radii, or chords. If these are infinite, we can draw infinite circles."

S13: "I am still cannot draw three circles. How come we will draw infinite?"

Teacher: "What is common between the two circles that you have drawn drew?"

S6: "The two points."

Teacher: "I mean, other than the points?"

S7: "The two circles have two different centers."

S3: "But they have the same radii as S2 said."

Teacher: "Check the centers. Where do these centers lie?"

S1: "I think I have reached something. What if we draw on GeoGebra, a triangle from the center, and the two given points?"

S14: "Wait, guys… We need to talk about triangles now?"

S1: "I said that all the points when connected on GeoGebra form a triangle. Does this help us here?"

S11: "I still cannot reach anything here. We drew the triangle but cannot see the point. How does this relate to circles?"

S4: "I am still trying to draw the infinite circles; I did three of them, and I almost finished the fourth."

S15: "How?"

S4: "Using the GeoGebra circle tool, start from point A and drag till point B, the center will show up later."

S9: "This is smart."

S4: “Ha-ha, GeoGebra is smart not me.”

S13 addressing S4: "Finally I did the circles on GeoGebra. Thanks."

S16: "I did the circles on GeoGebra as well."

S1: "Interesting! Check all the centers lie on the same line and the form triangles."

S14: "If you draw all the triangles, the GeoGebra figure will be all messed out."

S2: "You do not need to draw IA=IB for "I" any radius, then the line S2 mentioned is the perpendicular bisector."

Teacher: "How did you know this?"

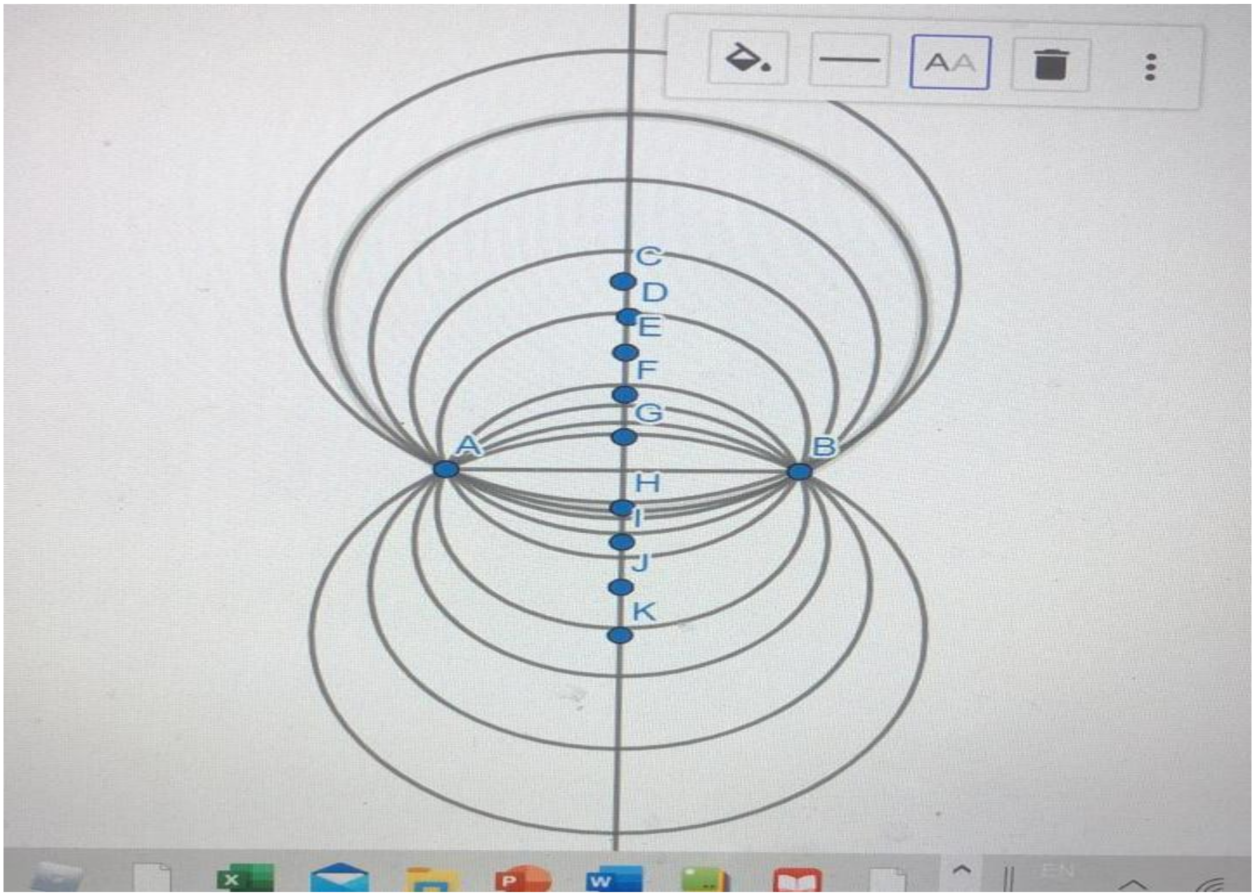


Figure 19: Sample of student work (Circle Properties)

S2 (figure 19): "GeoGebra made it very clear, even the measures show that OA and OB are equidistant. We know equidistant means perpendicular bisector."

S8: "This is awesome. We did not see that coming because some still did not draw the GeoGebra infinite circles."

S1: "Share your screen, and I will help you do it."

Students defined the problem by determining what exactly was required to solve the task. Then they set the goal and started brainstorming possible solutions like the answer is "one circle," "two circles," "infinite circles," or "three circles." Later through discussion, they ruled out the distinct poor options by examining results. At the end of the session, they were able to identify the best solution and to put it into practice through productive discourse. After getting an answer to the first question, they used the same strategy and a similar process as they proceeded to the second question. Generally, students solved the third question by just a simple conclusion. Impressively, students were pedantic and gave attention to the small details of a problem to reach the final solution. They built upon each other's' ideas to collaboratively reach the solution. The solving process also included decision-making as they evaluated multiple proposed solutions and evaluated all potential options.

GeoGebra stimulated not only crucial organizational skills in solving tasks but also enhanced creative problem-solving techniques. The problem-solving process reflected firm, well-informed, innovative decisions. The software helped students gain essential skills as they learned to generate compelling, creative, and quick problem-solving. They acquired knowledge of how to evaluate possibilities and reach a clear-cut solution accurately. The problem-solving process included peer collaboration, collective thinking, and bridging of knowledge. Students got engaged in solving the question in a series in collaborative steps. They were organizing the information received from their peers while developing novel ideas to share, which created an integral practice of collaborative and listening skills. Every student as an individual contributor in solving the task exercised his curiosity questions. The practical discourse facilitated the flow of diverse ideas and built an extensive network of information. According to Halpin et al. (2017), such cohesion in the learning environment inspires a collaborative culture and fosters an intuitive notion of student engagement. Worth noting, the teacher played a fundamental role by inviting student participation, hearing all voices, encouraging silent students, and answering each question by another question.

With a clear articulation of the task in hand, students used both convergent and divergent thinking to bring together disparate ideas, determine the best answer, and then generate new ones. Students' reasoning was characterized by logic, creativity, curiosity, flexibility, and originality. The process involved a bit of reproductive thinking where students used some of their previous geometry knowledge; however, it mostly relied on generating intuitive ideas. In other words, the core of the problem-solving was dependent upon divergent and convergent thinking with a balance of reproductive-generating thinking. Such mental processes are especially helpful for students who continuously struggle with new geometry tasks and changes. Previous studies reveal that such mental processes are influenced mainly by student engagement (Fung et al., 2018).

Student engagement in this session took a wide variety of forms, including participation, productive discourse, and cognition. Being attentive to details, students were curious and showed a degree of interest, optimism, and passion for the material in hand. Besides, the use of GeoGebra promoted positive emotions that made the classroom environment conducive to learning geometry. Students used gestures to stay focused on the task; they were raising hands and sometimes doing hand signals for their peers to stop talking as a sign of distraction resistance. The overall socio-cultural environment in the virtual session reduced the feelings of geometry confusion and disconnection, thereby increasing engagement. A few illustrative examples include the following:

S11: "I prefer GeoGebra more the paper-pencil drawing."

S2: "I liked it and found helpful from the very beginning."

S12: "It is entertaining! Like a math game."

S13: "GeoGebra makes geometry construction easy and enhances our creativity."

S15: "Very interesting and easy to study math and to draw a figure."

Observation "looks-for"	**Category**	**Observed**	**Specify Details (frequency, time interval)**
Verbalization of thinking	Cognitive	11:07 AM	S1: "I think that would be two."
Self-monitoring	Cognitive	11:14 AM	S1: "I think I have reached something."
Concentration	Cognitive	11:13 AM	S2: "The two points are on the circle so they can be endpoints of diameters, radii, or chords. If these are infinite, we can draw infinite circles."
Resisting Distractions	Cognitive	11:15 AM	S14: "Wait, guys… We need to talk about triangles now?"
Gestures	Cognitive		Random students
Seeking Help	Cognitive	11:13 AM	S13: "I am still cannot draw three circles. How come we will draw infinite?"
Seeking Information	Cognitive	11:11 AM	S4: "So the answer is two circles?"
Seeking Feedback	Cognitive	11:15 AM	S1: "Does this help us here?"
Evaluate Comments	Cognitive	11:16 AM	S11: "I drew the triangle but cannot see the point. How does this relate to circles?"

Answer Teacher's Questions	Cognitive	11:14 AM	S7: "The two circles have two different centers."
Give Information	Cognitive	11:09 AM	S12: "I have already drawn two circles passing through these two points."
Explain Procedures	Cognitive	11:12 AM	S2: "Because we have infinite radii in the circle."
Reasoning	Cognitive	11:11 AM	S2: "Infinity. We can draw infinite circles."
Address Deep Questions	Cognitive	11:14 AM	S1: "What if I draw a triangle from the center and the two given points?"
Hold Reflective Self-Questioning	Cognitive	11:15 AM	S1: "I said that all the points when connected on GeoGebra form a triangle. Does this help us here?"
Self-Monitoring Progress	Cognitive	11:16 AM	S11: "I still cannot reach anything here."
Persist in Completing Tasks	Cognitive	11:16 AM	S4: "I am still trying to draw the infinite circles; I did three of them, and I am almost done with the fourth."
Organization in solving tasks	Cognitive	11:18 AM	S14: "If you draw all the triangles, the GeoGebra figure will be all messed out."
Using Different Strategies	Cognitive	11:10 AM	S12: "I just tried several circles, and this worked."
Ask Random Questions	Behavioral		Random students

Participating in class Discussions	Behavioral	11:14 AM	S3: "But they have the same radii as S2 said."
Interacting with others	Behavioral	11:07 AM	S5: "No, I think one. Through any two points, one can draw a one-line segment, so here the same thing goes."
Hold Discussion	Behavioral		Random students
Prepare Extra Work	Behavioral	11:06 AM	S8: "I have tried them all; I know how to use them."
Complete Tasks	Behavioral	11:17 AM	S16: "I did the circles on GeoGebra as well."
Cooperate with Other Students	Behavioral	11:16 AM	S13 addressing S4: "Finally I did the circles. Thanks."
Explain the task to Others	Behavioral	11:17 AM	S1: "Check all the centers lie on the same line and the form triangles."
Exchange Ideas	Behavioral	11:08 AM	S7: "This has nothing to do with line segments."
Get involved in Voluntary Tasks	Behavioral	11:19 AM	S1: "Share your screen, and I will help you do it."
Seem Interested	Affective	11:05 AM	S8 "Exciting!"
Seem Anxious	Affective	11:16 AM	S11: "I still cannot reach anything here. We drew the triangle but cannot see the point. How does this relate to circles?"
Seem Curious	Affective	11:18 AM	S1: "Interesting!"
Hold Positive Attitude	Affective	11:17 AM	S4: “Ha-ha, GeoGebra is smart not me.”

Enjoyment	Affective	11:19 AM	S8: "This is awesome. Did not see that coming."
Finding Relevance and Value of Task	Affective	11:18 AM	S2: "GeoGebra made it very clear, even the measures show that OA and OB are equidistant."

Table 19: Observation Protocol – fourth video

Creation of Artifacts. As students explored GeoGebra, they made multimodal digital artifacts, images, and video recordings on their phones or iPads. A universal analysis conveys that engaging with the modal affordances of GeoGebra shaped the students' engagement in significant ways.

The interaction with GeoGebra empowered students' interest in geometry, as students were reengaging with the subject over time. GeoGebra triggered a self-sustaining student interest for geometry that was becoming by every session enduring and more meaningful. The individual interest in GeoGebra reflected well-developed personal preferences to enjoy and value geometry. The teacher supported students' pursuit of investment as their connection to GeoGebra became more stable and generalizable. She cultivated this interest with interventions to expand their knowledge of geometry and to solidify its perceived value. She provided activities that use structural features as problems and challenges to catch students' attention and arousal and eventually to stimulate their engagement. Students created a variety of unique complex artifacts (figure 20).

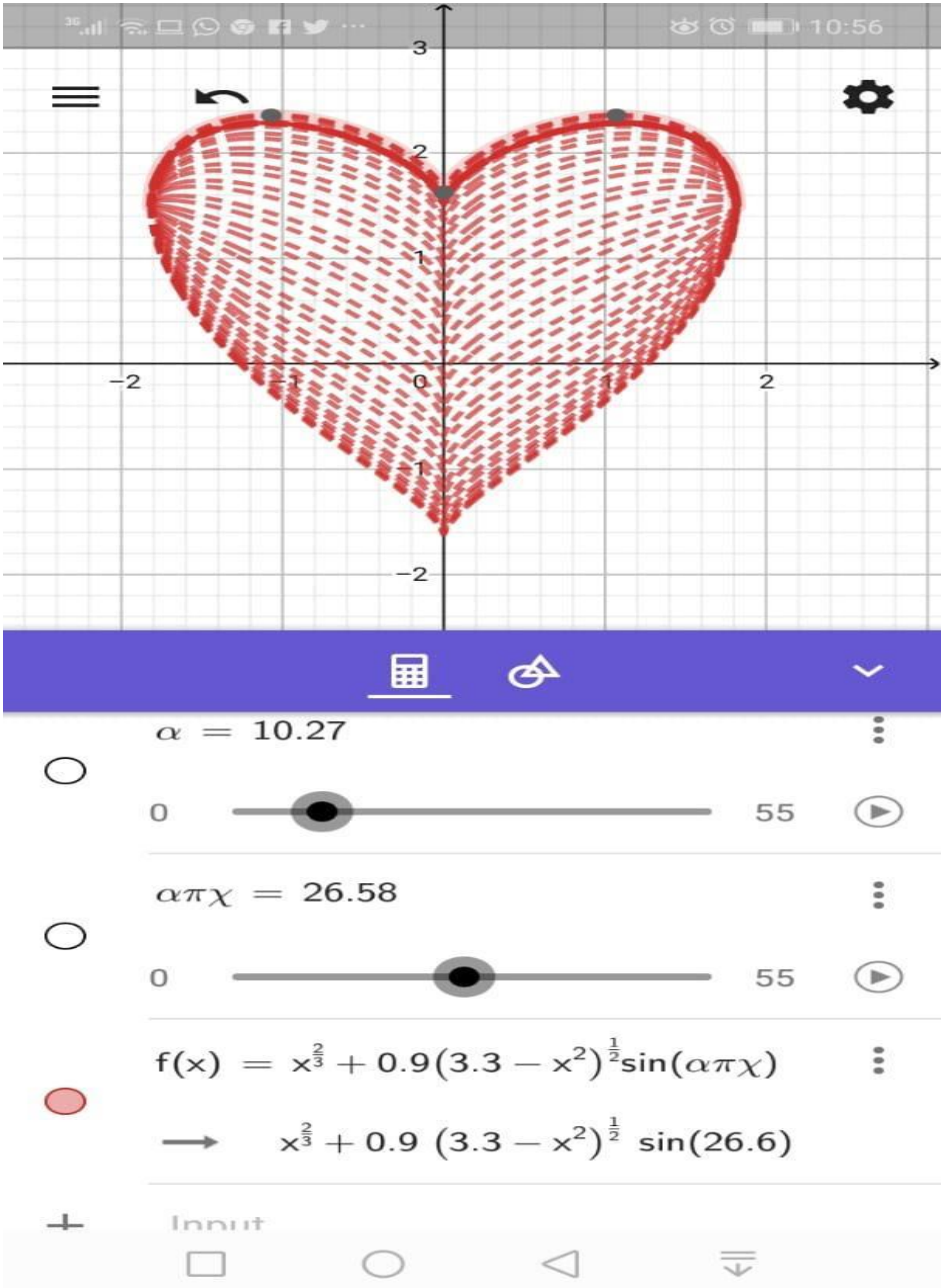


Figure 20: Sample of students' artifacts

The teacher harnessed students' deep connection to GeoGebra by asking them to generate these connections for themselves. To do so, the teacher infused the opportunity of sharing students' extracurricular GeoGebra work on the class group. This infusion aimed to understand students' perceptions of the value of using GeoGebra on their terms, and whether GeoGebra sparked connections between geometry and the real-life. This sparkle of joy grabbed high-achieving students' attention and helped remove pressures that were exacerbated by students who used to perform poorly in geometry. The GeoGebra artifacts impacted students with far-reaching positive benefits. They reached students at varying levels and helped students who tended to struggle the most in geometry connect to their unique potentials rather than one-size-fits-all. The GeoGebra visual stimulates, and catchy features to geometry enhanced students' engagement in these artifacts.

Notably, the modal qualities of both students' handling and manipulation of GeoGebra when creating artifacts, indicates a degree of engagement in the process. The students' interaction formed an act of paying tribute to GeoGebra experiences. The students even shared voice recordings linked to their GeoGebra video recordings and images. For example, after sharing her artifact (figure 24), S5 stated, "It took me two hours to finish it. I did it. Yesterday, but it was late, so I could not send it." The image and audio recording can be understood as both affective and behavioral engagement (Fung et al., 2018). This exhibition of interest and of doing extra work itself was a kind of multimodal interaction with the software (Jewitt, 2012).

Students' engagement sometimes extended beyond the creation of artifacts. This transfer made them suggest GeoGebra use in art sessions. Students weaved GeoGebra into everything they did as they found this approach to geometry not only intensifies engagement but also promotes creativity, transformational learning, and integrative thinking.

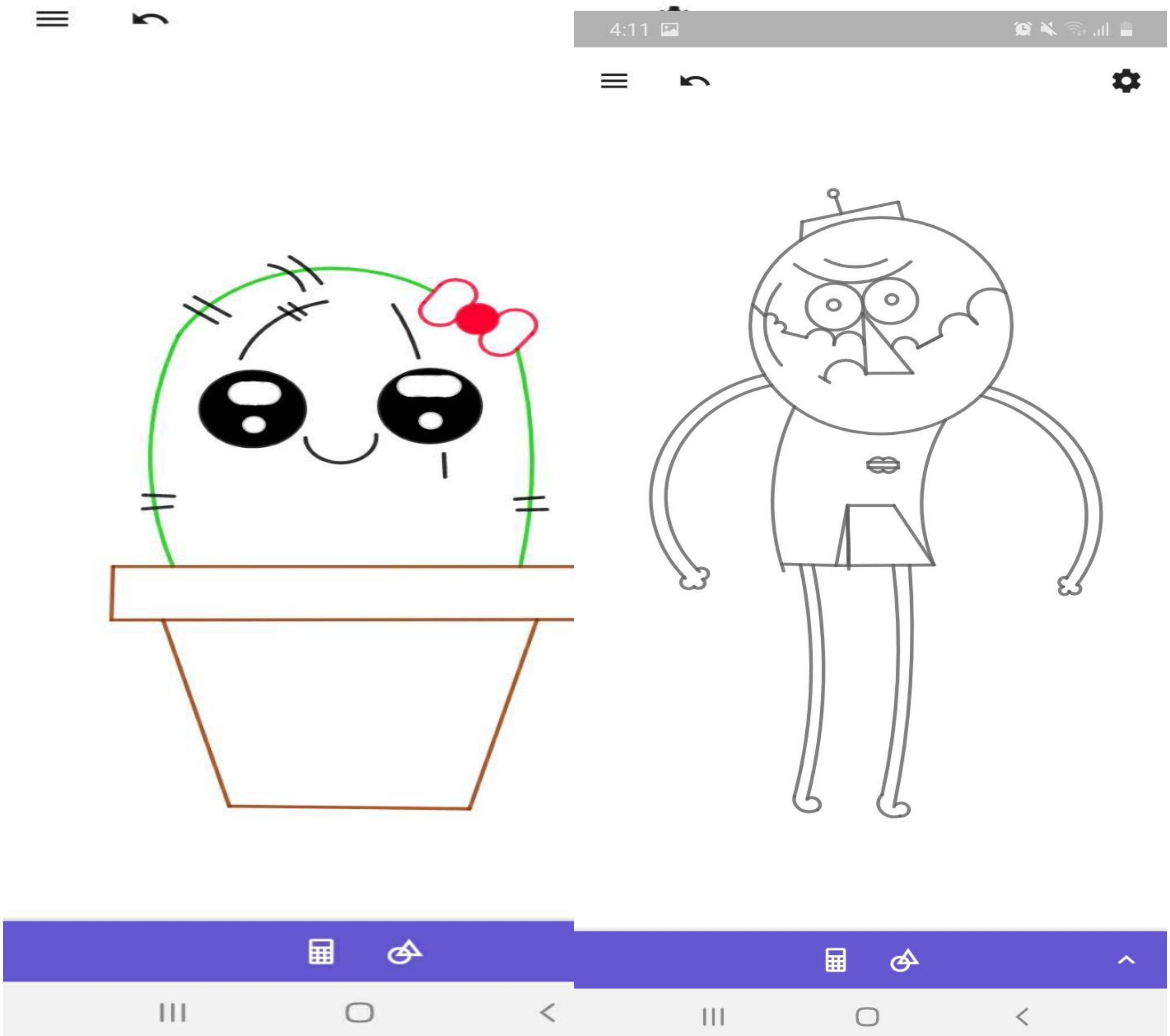


Figure 21: Sample of students' artifacts

Students created connections between different subjects and found creative confidence in their expression. Previous research shows that bringing art concepts into the math classroom improves engagement levels and helps students construct a deeper understanding of math concepts ("Benefits of Incorporating Art Concepts in Math," 2018). The students' integrative thinking required peer collaboration in addition to the commitment to planning.

With the aid of GeoGebra, students combined the creation of an art project with basic mathematical concepts. Some of these mathematics concepts were: "balance," "proportions," "variety," "geometrical lines," "space," "polygons," "circles," "quadrilaterals," "special parallelograms," and "congruent triangles." Multimodality analysis revealed that students' interaction with GeoGebra sparked creativity, increased their mathematical skills, and increased their engagement.

GeoGebra activated students' dormant imaginations and creativity, which are crucial to mathematics and problem-solving skills. Since mathematics is an abstraction, there is a sense in it which is entirely in the realm of imagination, and progress in math gets only stimulated by imaginers (Frabasilio, 2019). GeoGebra interweaved students' creativity with mathematics and assisted them in connecting threads to create, illustrate, and share geometrical artistic figures.

Moreover, GeoGebra bridged the gap between school geometry and real-life mathematics through computational skills. Student S6 used GeoGebra geometry tools to create the school logo (figure 22). He was so excited that he contacted the school administration to adopt his innovative logo. Students' ability to establish a connection between GeoGebra and real life is crucial with regards to the recognition and development of conceptual learning (Benson-O'Connor, 2019).

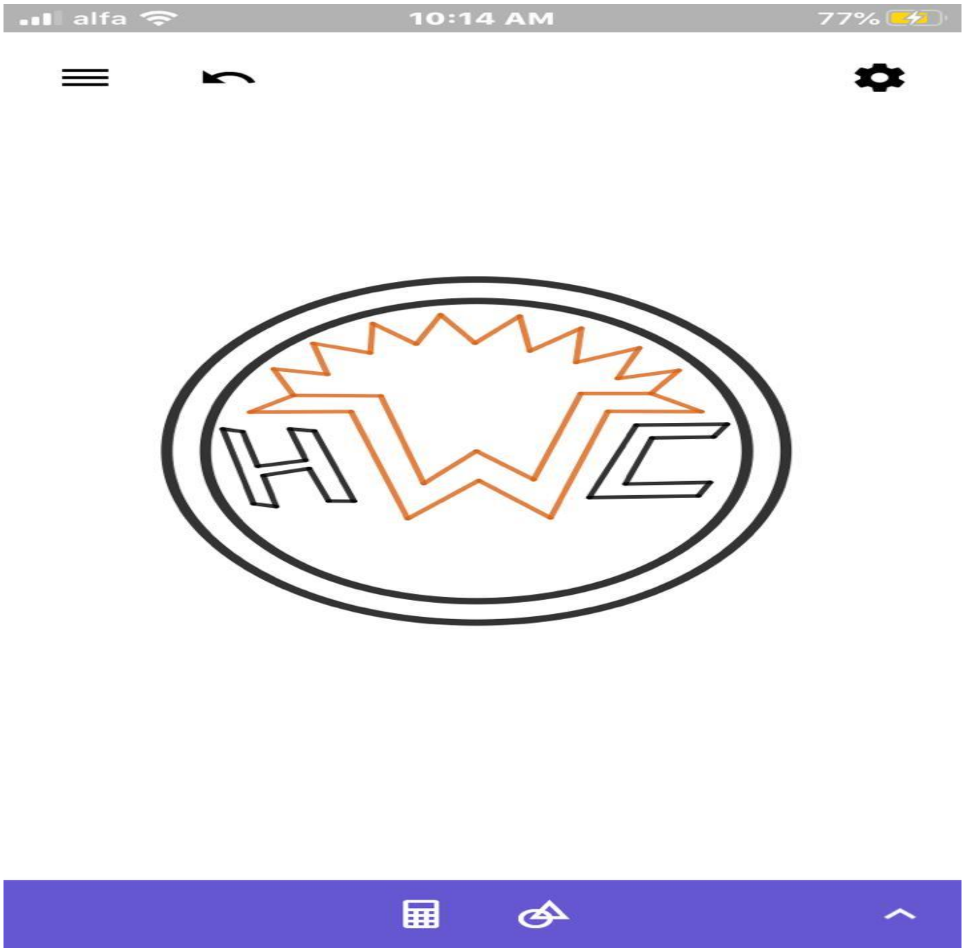


Figure 22: Sample of students' artifacts (School Logo)

In these episodes, students used GeoGebra logically to organize, analyze, and represent the geometrical figures through abstractions. The constructed knowledge of GeoGebra first became internalized as a social construction, then modified and later externalized throughout these artifacts. The creation of digital artifacts made an active contribution to the process of students' cognitive, behavioral, and affective engagement. The digital artifacts that students created sometimes acted as conceptual props to support making sense of experiences between previous lessons. Rendering artifacts led the students to make connections between GeoGebra, new geometry lessons, and previous ones. One pair of students used GeoGebra to demonstrate special parallelograms (figure 24). Students took this lesson by the beginning of the school year, and math can be an intense subject with many intricate practices that can be forgotten when not used regularly. Looking back over such previously learned information ensured the mind-refreshing benefits of GeoGebra. A different student did an artifact using the midsegment theorem (figure 23). At the same time, the rest stuck to the new lessons by creating Pythagorean artifact (figure 25) along with some animated arcs and circles (figure 26).

In several instances, the observer perceived interplay between students' visual and a digital audio artifact with GeoGebra in ways that prompted student engagement. In one episode, after sharing the artifacts on the classroom group, while watching the videos and checking the figures, one of the students leaned over the iPad. Another student held the iPad close to his body so that his whole arm was underneath the device. Students were discovering, creating, and receipting from their peers. They developed a great deal of GeoGebra skills by explaining ideas to one another and by participating in collaborative activities. They were giving and receiving feedback and evaluating their learning. This feedback facilitated GeoGebra skills development and eased, transferring GeoGebra knowledge across various categories, disciplines, and fields. As the students watched their friends' recordings, they were sitting still and paying attention, each with a slight smile on their faces. Their gaze shifted between the iPad, the teacher, and the screen that is being shared. Most students held the iPad in a cradling gesture while watching the shared screen. However, the way they kept the iPad was not observed at all the points during the student's interaction with the iPad. This multimodal analysis of students' artifact creation processes indicated that the students' decision to create artifacts was highly linked to their engagement. Four key aspects of the process illustrate how multimodality can provide insight into real opportunities for engagement. These four aspects are: Students connected art and geometry (figure 21), connected GeoGebra to geometry (figure 23), connected GeoGebra to real-life connected (figure 22), and connected geometry to real life (figure 25).

GeoGebra enhanced students' skills in connecting mathematical concepts to real life. The Realistic Mathematics Education theorem talks about how most people can only relate mathematics to numbers, algorithms, and calculations (Fesakis et al., 2018). Having the students in this study connects their geometry lessons to real life, gives meaning and relevance to mathematics—Student S17 related midsegment theorem (figure 23) to real life. The midsegment connecting two sides of a triangle is parallel to the third and are half as long. It is well-known to be used by geologists in real-life to find distances of sinkholes. It seemed thought-provoking to create a real-life artifact that uses midsegment as part of its structure. The creation of artifacts supported the students in making corresponding links between the common in the previous and present geometry sessions and fostered opportunities for students' engagement.

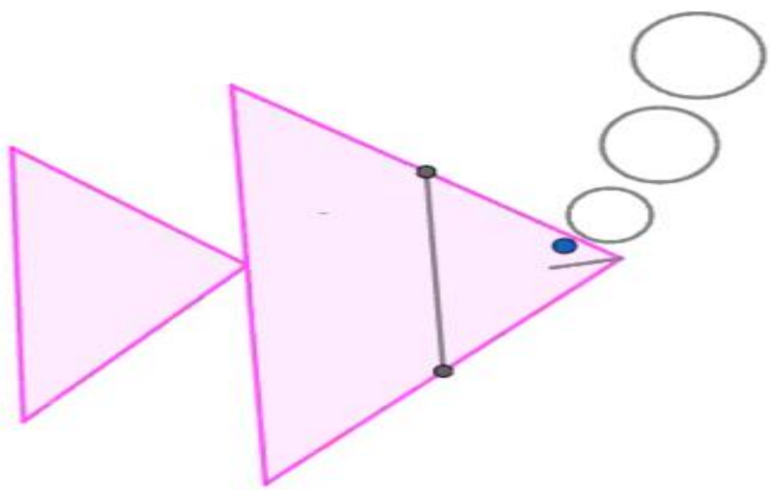


Figure 23: Sample of students' artifacts (Mid-segment)

GeoGebra is in itself an interactive geometry system that helps students create polygons, visualize transformations, and format constructions. Students have shown positive attitudes towards geometry when using the software. They believed dynamic GeoGebra was most beneficial in their geometry sessions since they took ownership of their learning. They enjoyed creating the figures and constructions quickly and being able to adjust and animate them. For example, S2 shared the audio with her artifact, saying: "I feel that GeoGebra is driving geometry ideas beyond the classroom for all of us. We never thought of doing geometry at home just for fun. With GeoGebra visual aids, only very few students would still be unable to understand geometry concepts." The modal affordance and process of making such a voice recording elicit particular forms of verbal expressions, which shape how engagement gets enacted. The method of creating an artifact gave the students' imagination of the realities of geometry more significance than the conventional lessons made available to them. The process of making a voice recording that emphasized a sharp distinction between the previous and the present geometry classes shaped the students' modal construction of the emotions associated with either type of geometry lectures.

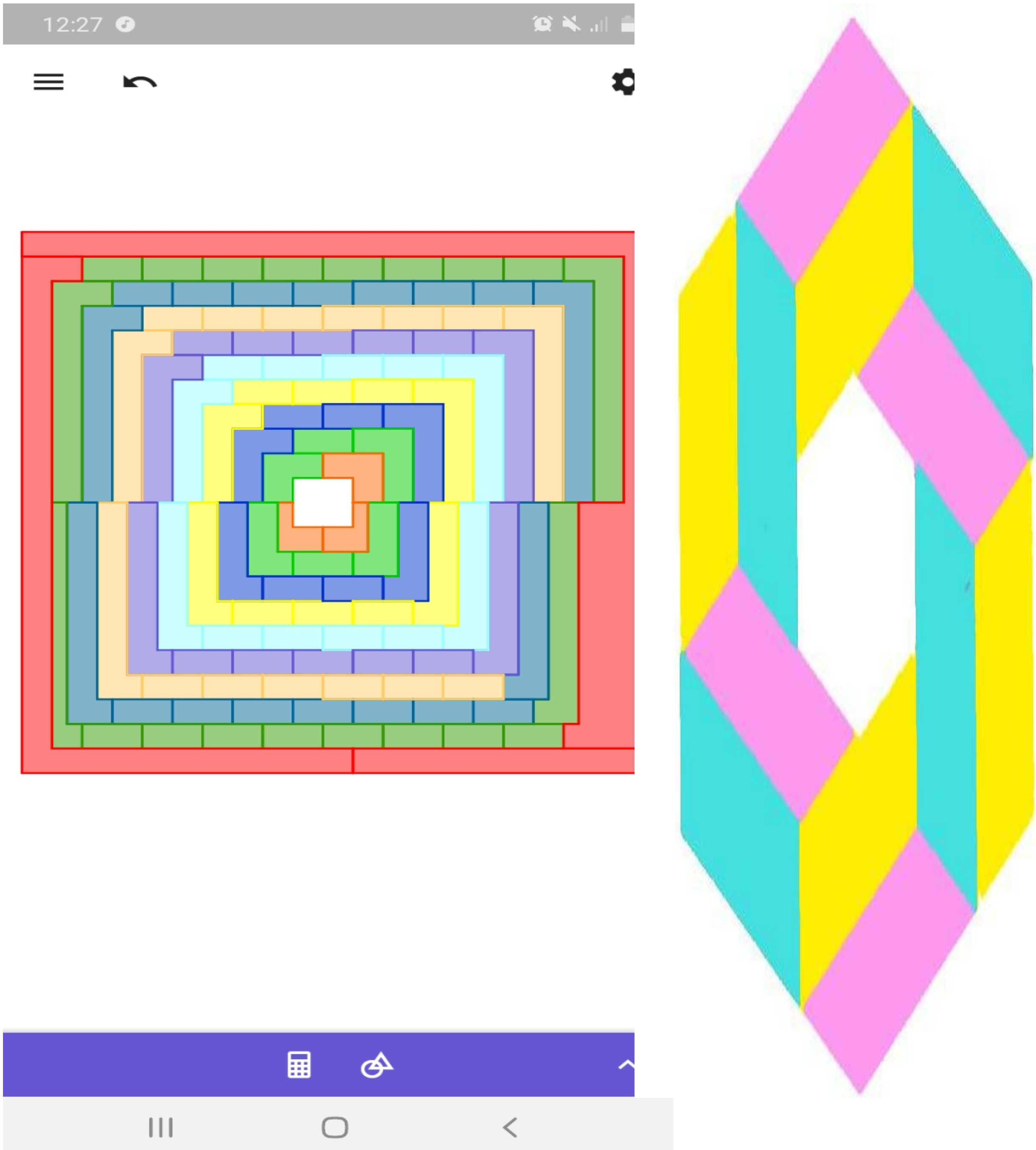


Figure 24: Sample of students' artifacts (Special Parallelograms)

Recordings were associated with a particular tone and pace, correct assertions, and descriptions of experiences as offered by the students in this clip.

The artifacts reflected a significant level of mathematical creativity. Through GeoGebra experiences, every student was able to show blended intellectual ability with positive emotions. Students developed expertise in using the software, which rewired their brain for creativity and helped students tap into original thinking. Students tried hard, made many mistakes, and then tried different strategies until they created what seemed an artifact of value. Through this experience, students had the opportunity to be in charge and to make their own choices. GeoGebra nourished creative thinking and turned the virtual classroom into an open space of free thinkers. The software acted like a new thinking tool that tweaked paths to facilitate new processes in doing geometry. Students learned how to think creatively, to revise their work, draw conclusions until they come up with different unique artifacts. The virtual class's socio-cultural environment was open for difference, and students learned from that diversity and creativity.

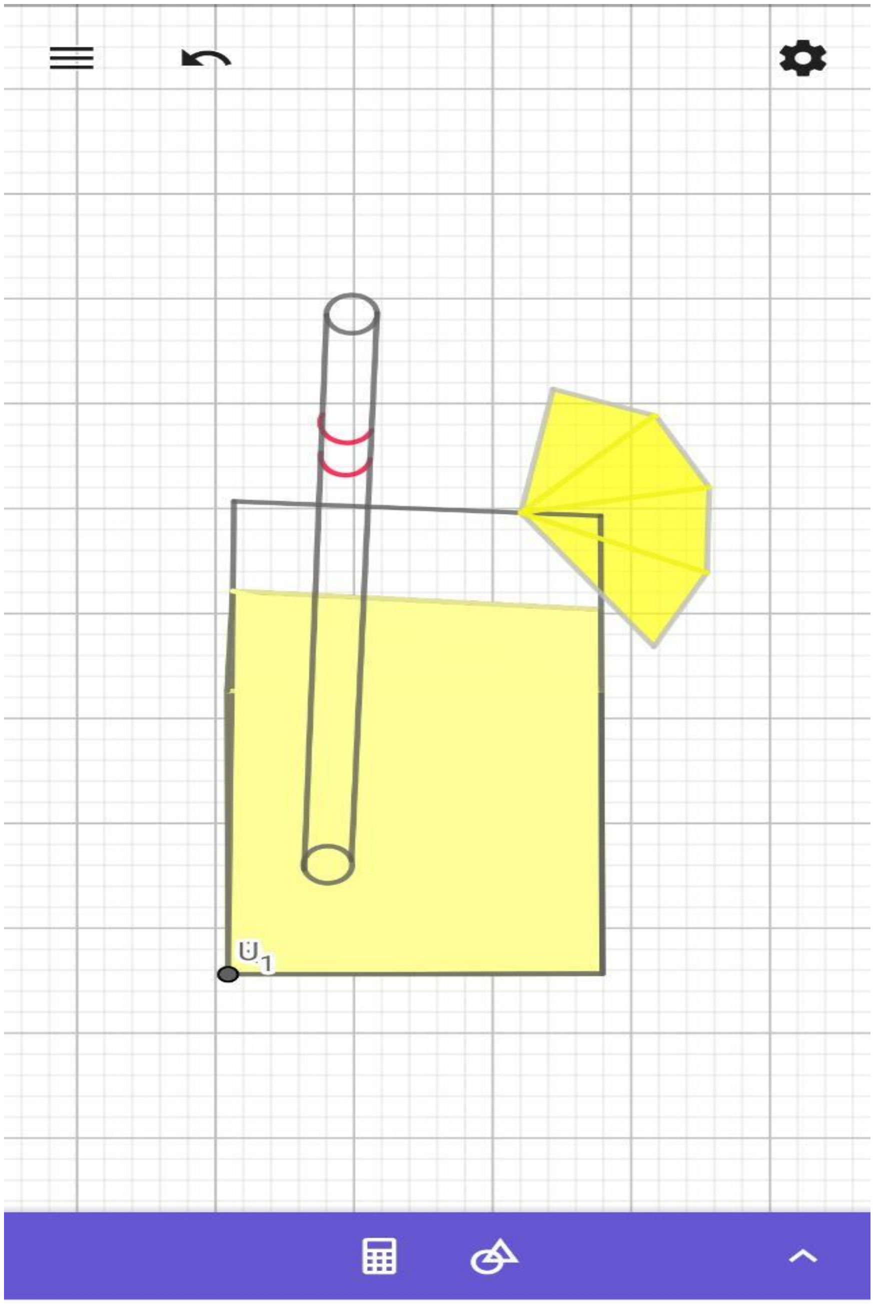
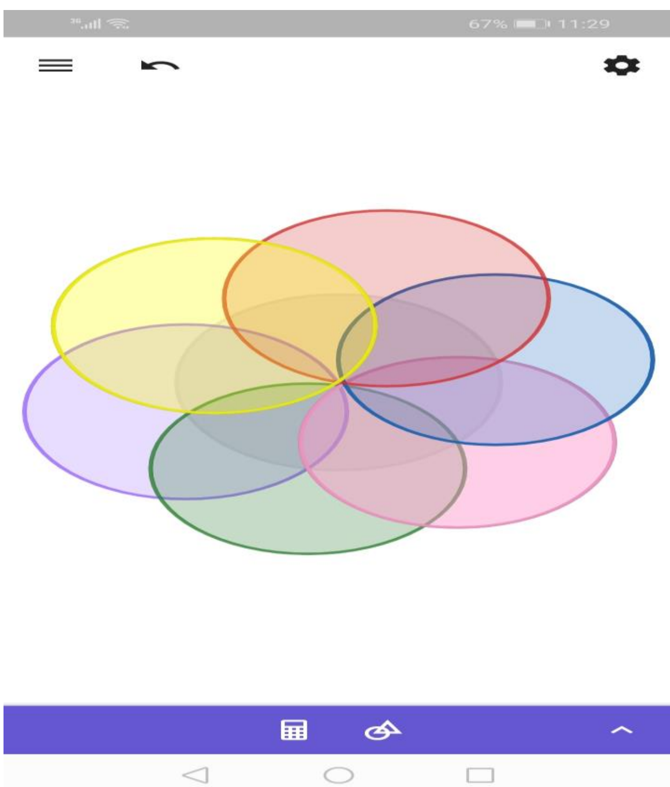


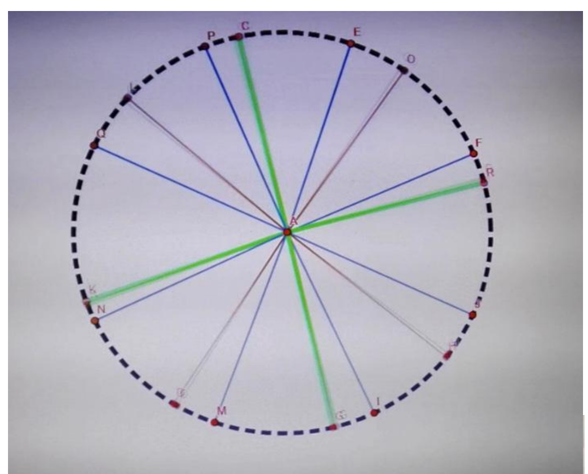
Figure 25: Sample of students' artifacts (Pythagorean Spiral)

Not only have GeoGebra intensified students' engagement and creative thinking, but they also promoted their argumentative skills. Students needed to structure arguments to explain and defend the logic of their thinking in creating such artifacts. The student S14 designed his Pythagorean spiral in an unusual creative decoration.

With an abundance of students’ unique design styles, this specific artifact had innovative details that added excitement. When the teacher asked where he got the idea of the spiral in a lemonade glass, he replied: "Pythagorean spiral is beautiful. Over the internet, I found amazing examples of it, and I thought, why not do it in a lemonade glass?

Students were comparing, discussing, and highlighting important details about their artifacts based upon their attributes. Collectively the students' multimodal comparisons of their work aided the creation of a collection of artifacts, supported engagement, and helped them establish a distinction between regular geometry sessions and GeoGebra intervention.





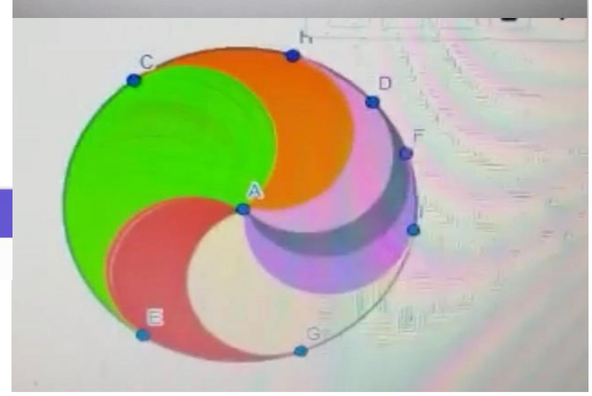


Figure 26: Sample of students' artifacts (Circle)

The examples discussed in this article suggest that creating related artifacts was a part of student engagement rather than merely reflecting or representing engagement.

As an example, S5 addressed S1: "I did like your animated arc wheel, but with a different version."

The pace is understood in terms of the tempo of the students' activity. During some episodes of engagement, changes in rhythm were observed through the students' use of multimodal resources in the interaction, including movement, body position, gaze, gesture, manipulation, and speech. An analysis of the pace of multimodal interaction points to a link between changes in the rhythm of students' interaction with GeoGebra and their socio-cultural classroom environment.

Observing the students getting entangled with some geometrical artifacts revealed a slowdown in pace. The students would gaze on some of the artifacts for nearly a minute, which is a long time relative to the rhythm of their interaction with other figures they did in the application. They would look carefully at the image, pointing at features and manipulating the image (e.g., zooming in) to focus on particular details, notably accompanies with amazement's facial expressions. Some students said addressing students S9 as a comment on figure 24: "Wow, very nice. I have been trying this for a whole week now, and you say it took you only a couple of hours!"

On the other hand, being able to finish a well-constructed artifact resulted in students excitedly interrupting one another and quickening their physical movement, making exaggerated gestures like raising thumbs or jumping up and down. In such examples, the accelerated pace in response to an emotive stimulus was a material manifestation of excitement. The abrupt change in speed, along with the pointing gesture and facial expression, involving widened eyes, suggest that the artifact was emotive to the student.

Slowing down and focusing attention can be useful for the development of the contextual understanding necessary for geometry as learners have more time to investigate a particular part of the artifact and thereby build more in-depth knowledge of what they are exploring. Besides, they have more time to reflect on what they have learned on GeoGebra so far and make links crucial to building a credible interpretation of the GeoGebra artifacts and geometry experiences.

Analysis of the Interview Data

The semi-structured interview contained twelve questions, all of which were asked to each of the five eighth-graders volunteer students. The questions and the following probes aimed to ascertain their perception of GeoGebra's impact on their engagement in mathematics. The interview involved asking key participants to respond to a variety of open-ended questions about the initiative. The researcher designed an in-depth interview to elicit detailed information, and students had the opportunity to elaborate in their own words. The topics and questions were organized thematically and in the order of the three types of student engagement. To obtain a certain degree of comparability between different types of engagement, the researcher structured the interviews by gathering questions holding similar engagement issues in GeoGebra. In other words, the structure of the interview generally enables categorizations. The first category of questions addressed cognitive engagement, the second category aimed at affective engagement, and the third focused on behavioral engagement. The researcher guided the interview conversations with the goal in mind of gathering information about students' perception of the software with their own words, phrases, and sentences. Every interview flowed differently from the other since participants were asked to raise points they believed as necessary. What every student said, shaped how the interview proceeded.

Participants in the interview signed a "participant consent form." When constructing the semi-structured interview, the researcher sought the opinions of two expert researchers. The preparation of the semi-structured interview involved brainstorming of various ideas, paring them down, and then cutting questions that seemed redundant. The process required the guidance, suggestions, and feedback of two researchers. The interviewer created audio recordings of the conducted interviews in order to explore student perception in-depth. It can be argued that the limitation of teacher interviews reduced their reliability at times.

After conducting the five interviews, the researcher used thematic qualitative analysis to analyze the interview data, specifically, a three-stage coding process since the purpose was broad and explorative. The breakdown of the interview transcripts was transferred to the computer, and a sixteen-page interview breakdown was obtained (Cohen, 2008). The texts were read with line by line reading technique and analyzed by content analysis. Codes generated from data obtained from manual encodings were listed. The researcher examined whether the codes have common points that could create universal themes.

The integrity of interviews. The integrity of the interviews was assessed to ensure that the researcher generally used the same interview procedures for each student (Creswell, 2009). The researcher audiotaped all interviews and created a checklist that detailed four specific interview objectives: “interviewer's use of an open-ended statement to begin the interview,” “follow-up questioning for vague responses,” “questioning to elicit additional responses and determine if the student was finished responding,” and “effectively terminating the interview when the student was finished responding.”

Credibility. For credibility, among the methods mentioned by Creswell (1996), the researcher sought participant approval on codes by sharing of interview breakdowns with participants. Besides, the researcher ensured the appliance of the rich and detailed description to ensure that the research context captures the reader. Also, the researcher reviewed the coding and categorization process twice in different instances. The researcher presented the codes, categories, sub-themes, and themes without changing the data. After reviewing the interview breakdown, the compatibility of the two encoders was 23 codes out of the 26 codes, with a compatibility percentage of 88.5%.

Coding. The researcher converted the interview audio recordings to text using a professional and intelligent audio-to-text transcription software, "Amber-Script." Afterward, on "ATLAS.ti 8," interviews were transcribed and coded. The researcher wrote the interview records belonging to the students without changing, and then they sent them to the participants for confirmation. For confidentiality, the audio-recordings were immediately destroyed after transcriptions were verified. The first stage in the interview analysis methodology was coding. Coding involved the examination, comparison, and conceptualization. The researcher read and reread the interview texts to get a sense of what they looked like and then went through the data word-by-word to code as much as possible. The researcher examined the data for similarities and differences, and initial conceptual categories or phenomena were identified. In the coding stage of data analysis for the current study, preliminary responses were identified by examining similarities in responses (Creswell, 1996). The initial examination of the data revealed a considerable number of homogeneous responses by the eighth graders. Within the respondent groups, participants often provided very similar responses.

The researcher highlighted the meaningful codes and derived codes for each case separately. After reading and coding the five interview texts, the researcher compared the codes on the condition that they match in several cases. However, when codes did not match, new additional codes were added, and the list was updated. The researcher followed this process until the list ended with an updated code list of all the" 26 codes" for the five cases. The researcher coded the same data twice for verification purposes and checked the number of agreements over both verifications.

Categorization of data. After creating the codes, the researcher put them into categories that frame them. For the sake of eliminating redundancy and creating a manageable working data set for the next stage of data analysis, comparable responses were collapsed into a single category for each data set. The researcher identified some first response categories for each data set and coded the data to reflect these categories. Categories were derived for only those responses where there was a distinct similarity in theme. Comparing categories promoted the comprehensibility of information and enabled the generation of causal relations and interdependencies (Creswell, 2009).

The researcher framed the codes in hierarchical frames to help organize the generated codes based on how they relate to one another. Therefore, all the positive emotions that emerged after the GeoGebra intervention, such as feelings of enjoyment, excitement, curiosity, and interest, were organized in one category. On the other side, and following the same hierarchical frame, codes that express how GeoGebra can lead to an improvement in attention, concentration, and learning skills were all put into one category, and the same strategy applies to all the other categories. This hierarchical way of framing the codes into categories allowed the presentation and inclusion of all the codes with different attributes under specific categories (Cohen, 2008). The generated categories were flexible categories that include all the codes. Eventually, the researcher captured all responses in some categories.

Creating themes. The last stage of the coding process involved selecting the core categories and relating them to one another. Categories were further refined and integrated into themes. For the sake of coming up with themes, the researcher searched for commonalities by making systematic comparisons across the categories. The researcher had to cut and sort categories based on the relationship between them to identify general response themes from the data set. For example, all emotional reactions linked to GeoGebra were included under one category and elicited in this case under a theme labeled "Affective Engagement." Generating the other three themes, relying on keywords, and looking at what most of the respondents expressed regarding their perspective on the impact of GeoGebra. With the above process, general themes for each of the data sets were identified (Cohen, 2008).

This constant comparison method of the qualitative analysis revealed four themes regarding students' perspectives on GeoGebra. Respondents had different perspectives on the impact of GeoGebra. Most of the respondents had positive, optimistic feelings towards, for example, the respondent S1 stated: "GeoGebra expands my imagination, helps me get better at graphics, discover new shapes and explore the beauty of drawing in math." In the same manner, S2 stated: "GeoGebra made us creative, and from my point of view it made me think or solve in different ways and helped improve my thinking ability." In this way, the final four themes, "Cognitive Engagement," "Affective Engagement," "Behavioral Engagement," and "Drawback" were created from clusters rather than individual responses. The respondent S3 stated that: "There are plenty of advantages of using GeoGebra:

- Making the geometry sessions more fun, easy and enjoyable
- Simplifies work efficiently by using only the app to draw great geometrical shapes
- It makes it easier to understand geometry lessons and engage in them."

However, other participants revealed slight drawbacks to using the software, for example, the respondent S1 stated that: "I have not been suffering from any drawbacks ever since I have started using GeoGebra, there might be a few problems in moving the figures, but it is no big deal, you can manage to work it out." The respondent S4 stated that: "Extensive use of GeoGebra might hurt our eyes. Also, it was hard at first to use GeoGebra, especially that I missed the first class, but it is a fantastic software that sure helps overcome mathematics struggle and frustration.

Interviews Findings. The below tables reveal the frequency (repetition) (f) and percentages (%) of the analyzed codes calculated according to the categorizations, and themes. The researcher interpreted the relation between different themes according to the data revealed.

<table>
<tr><th>Theme</th><th>Sub-theme</th><th>Code</th><th>Frequency</th><th>Percentage of the sub-theme</th><th>Percentage of theme</th></tr>
<tr><td rowspan="7">Cognitive Engagement</td><td rowspan="4">In-depth strategies</td><td>Concentration</td><td>5</td><td rowspan="4">6.90%</td><td rowspan="7">47.37%</td></tr>
<tr><td>Self-monitoring & self-control</td><td>4</td></tr>
<tr><td>Resisting distraction</td><td>3</td></tr>
<tr><td>Find relation among concepts</td><td>2</td></tr>
<tr><td rowspan="3">High Cognitive Practice</td><td>Persist in completing tasks</td><td>2</td><td rowspan="3">12.50%</td></tr>
<tr><td>Using different strategies</td><td>6</td></tr>
<tr><td>Visualization of abstract thinking</td><td>17</td></tr>
</table>

	Strategical learning support	Improve thinking skills	9	31%	
		Enhance discovery	7		
		Enhance learning ability	6		
		Enhance understanding	16		
		Enhance creativity	12		
		Expand imagination	4		
		Improve transformation	9		
Affective Engagement	**The value given to GeoGebra**	Find value and relevance	14	6.96%	39.85%
	The value given to mathematics	Explore the beauty of mathematics	2	1%	
	Positive emotions	Enjoyment	17	30.80%	
		Help overcome frustration	2		
		Hold a positive attitude	13		
		Seem curious	4		
		Seem excited	11		
		Seem interested	15		
Behavioral Engagement	**Peer Interaction**	Enhance communication	3	5%	9.02%

		Enhance cooperation	3		
		Increase participation	3		
	Voluntary activities	Get involved in voluntary tasks	7	2.40%	
Disadvantages	**In Task**	Hard task	2	1%	3.76%
	Other Drawbacks	Getting distracted	3	1.50%	
Total			201	100%	100%

Table 20: The theme, sub-theme, and codes of 8th-grade students' engagement in the mathematics course

Analyzing Table 20 reveals that the percentages of "Cognitive Engagement" and "Affective Engagement" themes are relatively close to one another among the total of the two hundred and one codes about eighth-graders engagement in mathematics. The two themes summed up to one hundred eighty codes, while the "Behavioral Engagement" theme has a relatively low percentage of nine percent only. The lowest percentage of all four themes was for the "Drawback" theme with a percentage of two and a half percent.

	Student Interview Gr=100	
	Absolute	Table-relative
Affective Engagement Gr=53; GS=8	53	39.85%
Behavioral Engagement Gr=12; GS=4	12	9.02%
Cognitive Engagement Gr=63; GS=13	63	47.37%
Drawback Gr=5; GS=1	5	3.76%
Totals	133	100.00%

Table 21: The Absolute and Relative Frequency Table of the Themes

Using "ATLAS.ti 8" table 21 shows the absolute and relative frequencies of each of the four different themes in the study. ATLAS.ti 8 counted eight codes in fifty-three quotations for affective engagement theme by a total frequency of 39.85%. It counted four codes in twelve quotations for the behavioral engagement theme by a total frequency of 9.02%. Besides, it counted thirteen codes in sixty-three quotations for the cognitive engagement theme by a total frequency of 47.37%. Lastly, it counted one code in five quotations for the drawback theme by a total frequency of 3.76%. Strikingly, the highest frequency was for the cognitive engagement theme, and the lowest was for the drawbacks.

Behavioral engagement. Some of the interview questions aimed at determining the level of behavioral engagement in the students' mathematics course (figure 27). Some questions determined the level of active student involvement in learning through participation, performing mathematics tasks, interacting, and communicating with peers. A sample of these questions would be "How does GeoGebra affect classroom discourse?" or "Do you use GeoGebra in your free time?

Why and why not? How often?" or "Do you think GeoGebra affects students' active participation? If yes, explain in what ways?"

Sub-themes for behavioral engagement included peer interaction and voluntary activities. Peer interaction was a multifaceted concept that included pieces of behavioral engagement, such as communication, cooperation, and participation (Fung et al., 2018). The second sub-theme for behavioral engagement was that "voluntary activities" included undertaking one's own free will and giving free time to naturally and spontaneously seek non-formal learning. According to the interview respondent, GeoGebra provided many features that promoted active learning and collaborative mathematics problem-solving and discussions. In addition to that, the software encouraged students to get involved in voluntary learning tasks (figure 12).

The codes included in the behavioral "peer interactions" sub-theme drew on the idea of active student involvement by participating and contributing to discussions and communicating their thoughts about a mathematical task. These behavioral interactions got manifested when students reflected, asked questions, represented, compared, explained, justified, agreed about, and validated ideas with peers and teachers. On the other side, the codes included in voluntary activities considered student conduct of and participation in optional math-related activities that helped students further explore math. For the most part, such voluntary participation in active learning tasks resulted in compelling mathematics teaching (Fung et al., 2018).

The first theme obtained from some questions of the research interview was "Behavioral Engagement," and sub-themes defined as "Peer Interaction, and Voluntary actions." While the theme of behavioral engagement generally had a percentage of 9.02%, the sub-themes had a share of 5% and 2.4%, respectively. Under the theme identified as behavioral engagement, students agreed mostly on the positive impact of GeoGebra on enhancing peer interaction in the learning process of mathematics.

Analyzing the students' answers to the interview question determined that they agreed on the impact of GeoGebra on "Peer Interaction," such as the software "Enhances communication" (S1, S3, S4), "Enhances cooperation" (S2, S4), and "Increase Participation" (S2. S5).

Giving an instance of enhancing communication.”

S1: "GeoGebra creates sort of a bond between students."

S2: "Now that we are using GeoGebra, I have been seeing much change in the participation of other students, the app is making them eager to answer and solve exercises."

S3: "GeoGebra has many various and essential primary advantages such as it makes it easier and faster for students to cope with the teacher discussion during classes.

For greater clarity of cooperation enhancement:

S2: "Students are working together to complete joint tasks on GeoGebra."

S4: "GeoGebra is fun because you can work on it alone or with friends in groups."

Taking two illustrations of the increase in participation:

S2: "The students get excited to use the GeoGebra app and are participating more and having new ideas."

S5: "GeoGebra makes it easier for students to understand the lessons, which results to let them participate more since they understand the lesson more clearly."

Analyzing students' answers to the interview question determined that the respondents believed GeoGebra triggers "Voluntary actions" (S1, S2, S4, S5).

S1: "I sometimes use GeoGebra for entertainment in my free time outside math classes."

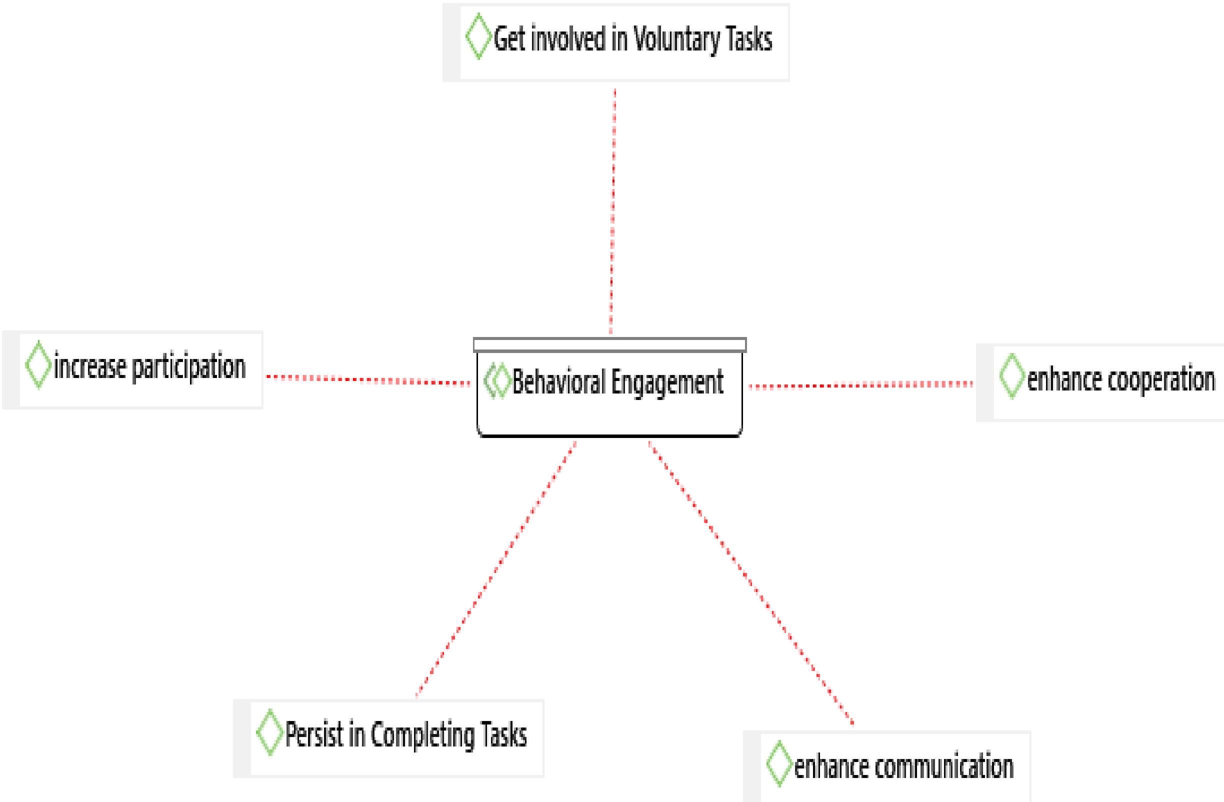


Figure 27: Codes of the theme "Behavioral Engagement."

S2: "I just use the GeoGebra whenever I feel bored."

S4: "I Practice GeoGebra in the free time because this will help us to get used to it."

S5: "I am using GeoGebra for 1 to 1:30 hours during my free time because it is fun, and I prefer it more than many games."

In the research, students sometimes discussed both dimensions simultaneously in their opinions regarding their behavioral engagement in a mathematics course. Several students described the impact of the software on enhancing peer interaction and emphasizing voluntary actions at once.

Cognitive engagement. Some questions in the interview addressed the cognitive engagement of the students in the mathematics course (figure 28). The students' mental investment in mathematics and self-regulated metacognition learning got determined through questions such as "Do you feel that the use of GeoGebra might distract you? Why and why not"? Alternatively, "What is GeoGebra's impact on learning geometry?" or "What are the advantages and drawbacks of using GeoGebra in a geometry session?"

Sub-themes for cognitive engagement included in-depth strategies, practice, and strategical learning support. In-depth strategies included pieces of cognitive engagement such as being attentive, self-regulating learning, using self-monitoring & self-control strategies, detailed thinking to find relationships among ideas, and finally, ways to resist distraction and stay concentrated (Attard, 2012). High cognitive practices required perceptions of personal responsibility for learning and a measure of self-regulation and personal investment (Skilling et al., 2016). These cognitive practices included persistence in completing tasks, using different mathematical strategies, and validating ideas with visualization of abstract thinking. The third sub-theme for cognitive engagement described how the software enhanced the core complex cognitive functions and the mental process of acquiring knowledge and understanding. According to the interview respondent, GeoGebra helped improve their thinking capacities, as discovery, learning ability, enhance understanding, creativity, imagination, transformation, among others (Attard, 2012).

The codes included in the in-depth cognitive strategies were mental processes consciously implemented to regulate thought and mathematics content. These in-depth cognitive strategies helped achieve goals such as solving mathematical problems through explicitly directing attentional focus on accomplishing mathematical tasks (Fielding-Wells et al., 2017). The codes included in the cognitive practices, on the other hand, were the crux of cognitive learning that evolved from the study of internal processes in the human mind associated with information processing. Furthermore, the codes of the strategical learning support that respondents gained from GeoGebra were all crucial cognitive prerequisites for learning mathematics. For the most part, those codes connect to the mental representation of the information present and to unleash the potential of existing ideas at once (Skilling et al., 2016).

The researcher identified the first theme obtained from some questions of the research interview as "Cognitive Engagement," and defined sub-themes as "In-depth Strategies, Practice, and Strategical Learning Support." While the theme of cognitive engagement generally has a percentage of 47.37%, the sub-themes have a share of 6.90%, 12.50%, and 31%. Findings determined that students agreed mostly on the positive impact of GeoGebra on supporting their strategical learning in the theme identified as cognitive engagement.

An in-depth analysis of the students' answers to the interview question, reveals that students used "In-depth Strategies" such as "Concentration" (S1, S3, S4), "Self-monitoring & self-control" (S2, S5), "Resisting distraction" (S3), "finding relations among different concepts" (S1, S2). By way of illustration of student concentration:

> Student S1: "If I would say anything about GeoGebra, it helps us focus on our lessons more. It is an advantage to know the properties of this software and how to use it for fast drawing. The whole thing would make it easier to focus on the lesson while using the app."
>
> Student S3 said, "GeoGebra makes it easier to concentrate on geometry tasks, understand geometry lessons, and engage in them."
>
> Student S4: "GeoGebra drags me to pay more attention and makes me curious about how to draw figures. It makes students concentrate on it, way more than on what is up."

From the other hand, to cite instances that reveal self-monitoring & self-control:

> S2: "I am working on a schedule to solve geometry tasks using GeoGebra."

S5: "I tried to assess my understanding of the Pythagorean lesson, and I found that GeoGebra was the reason I understood it."

As a model of the students who was resisting distractions:

S3: "GeoGebra helps me handle classroom distractions."

S4: "When I use GeoGebra, I avoid talking to others and put full focus on the figures and animations in hand."

As exemplary for the students who were finding relations among different concepts:

S1: "I am using GeoGebra quite often because I have been finding interest in it, and I have been relating different mathematics ideas through it. This thing I like the most!"

S4: "I am not that good in doing mathematics; I would love to be and especially in geometry. Nevertheless, this app GeoGebra shows how everything is related to math and how rules in math connect."

The interview respondents used "High Cognitive Practice," such as "Persistence in completing tasks" (S1, S3), "Using different strategies" (S1, S2 S3), "Visualization of abstract thinking" (S1, S2, S3, S4, S5). For case in persistence in completing tasks:

S1: "GeoGebra helps me gain a positive attitude towards the mathematical task and motivates me to do it right."

S3: "I change my strategy again and again until I do it right. It takes only a few clicks on GeoGebra. GeoGebra is fast and easy."

As in the present cases, students were using different strategies:

S1: "GeoGebra would make it easier to picture and imagine figures besides when being introduced to a new lesson, we draw different figures and use different strategies to solve, it would mean a great deal in teaching us how to solve mathematics problems..."

S2: "GeoGebra made us creative, and from my point of view, it made me think or solve in different ways, and my thinking ability improved."

S3: "GeoGebra perfectly affects geometry classroom. It taught the students how to think in different ways and draw figures correctly since some students find difficulties drawing."

On the other side, the visualization of abstract thinking includes instances such as:

S1: "GeoGebra makes it easier to picture and imagine figures besides."

S2: "The GeoGebra helps us solve by seeing instead of imagining and makes understanding math easier."

S3: "Do you know how hard it was to draw geometry figures without GeoGebra! It is exhausting, but this app makes the students' life easier when coming to draw a geometrical shape."

S4: "GeoGebra emphasizes my interest in geometry lessons by making me more curious to draw much more hard and complex geometrical figures. It pushes us to search for new ways to draw on different figures. It makes students look more forward to studying new geometry lessons using this app since animation, and the 3D helps understand the geometry problems."

S5: "The goal of using GeoGebra is to make geometric concepts understanding easier for the student by graphical construction, manipulation, and visualization of figures."

The interview respondents used "Strategically learning support" such as "Improve thinking skills" (S2, S3, S4, S5), "Enhance discovery" (S2, S3, S5), "Enhance learning ability" (S2, S3, S4), "Enhance understanding" (S1, S2, S3, S4, S5), "Enhance creativity" (S3, S5), "Expand imagination" (S1, S5), "Improve transformation" (S5). For an illustration of improving thinking skills:

S2: "GeoGebra empowers me a lot to think mentally."

S3: "GeoGebra enhances our critical thinking skills since sometimes, let us say we cannot draw a specific shape; it pushes us to think logically."

S4: "In my opinion, this app affected my thinking abilities a lot in a positive way."

As an indication of enhancing discovery:

S2: "GeoGebra made me discover new things and shapes in mathematics."

S3: "GeoGebra makes students discover hidden creative talents to do some amazing mathematics artworks using it."

S5: "GeoGebra helps us discover more about mathematics."

On an indicative basis of enhancing learning ability

S2: "GeoGebra enhances my learning capability by making me understand geometry lessons more."

S3: "GeoGebra enhances my learning skills and abilities a lot in a positive way."

S4: "GeoGebra makes it more fun by engaging our physical, critical thinking and learning abilities and skills."

Amongst others, employing examples for enhancing understanding

S1: "GeoGebra makes geometry easier and makes students understand geometry lessons more, especially when studying online."

S2: "With the help of GeoGebra, we understand geometry lessons more."

S3: "GeoGebra makes it easier to understand geometry lessons and engage in them."

S4: "GeoGebra made me understand geometry lessons that I never thought I would understand and master them."

S5: "From before, I did struggle in math, but now, thanks to GeoGebra, I tend to understand lessons more easily and enjoyably.

Enhancing creativity was made apparent when

S3 said: "GeoGebra pushes us to think creatively by using different methods to do a curve."

S5 said: "This app forces me to engage my creativity and critical-thinking to perform lots of hard, easy, amazing, or even simple geometrical shapes."

Giving an instance of expanding imagination:

S1: "GeoGebra expands my imagination."

S5: "GeoGebra would help me imagine the problems more easily."

Improving transformation can be seen through phrases like:

S2: "I am using GeoGebra in arts, not only mathematics."

S3: "The app affects students thinking in different subjects, not only math."

The research showed that students discussed more than one dimension simultaneously in their opinions regarding their cognitive engagement in a mathematics course.

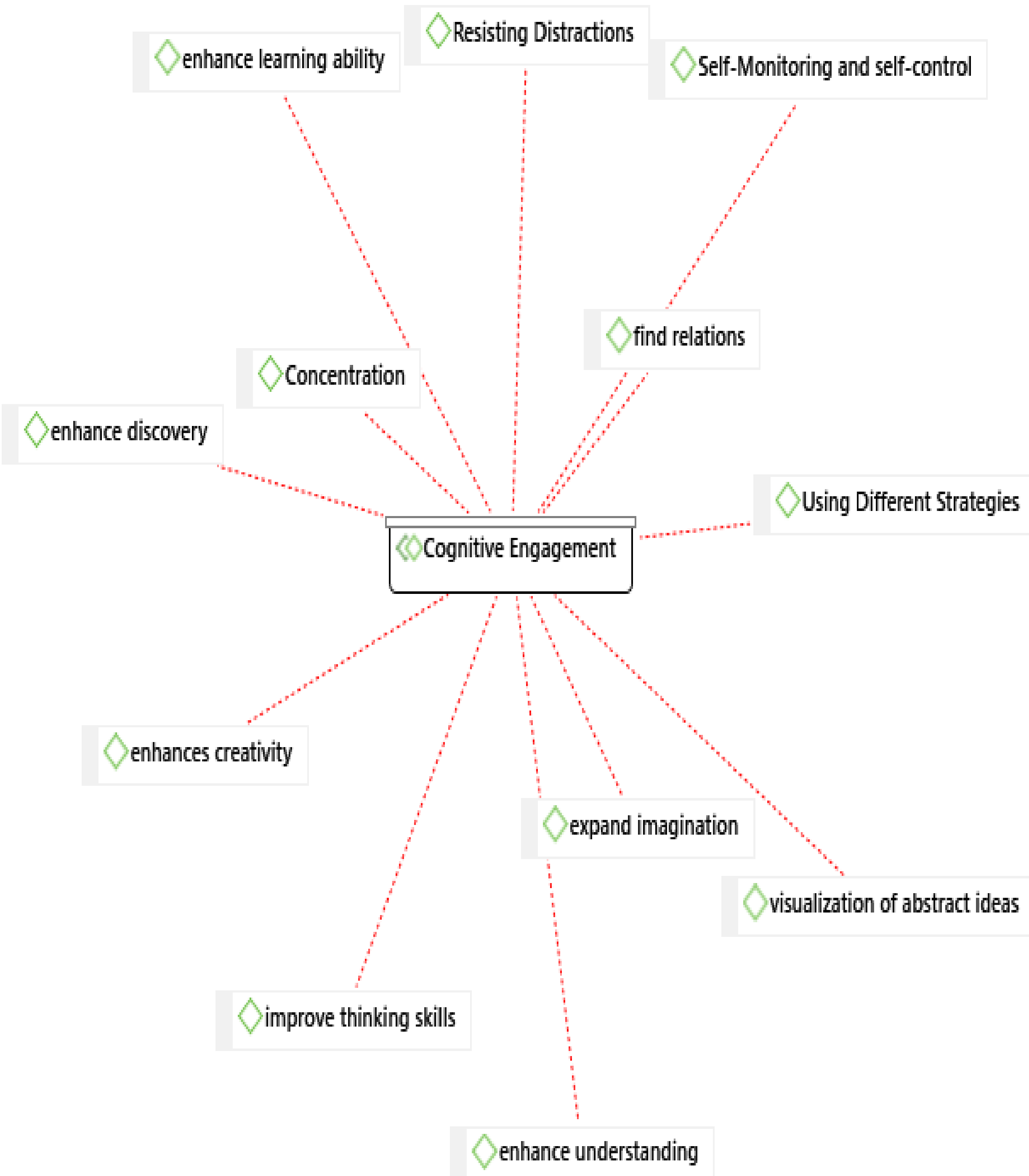


Figure 28: Codes of the theme "Cognitive Engagement."

Students use both in-depth and algorithm practices together. Other times they used cognitive practices along with strategically learning support in mathematics.

Affective engagement. The predictive value of using GeoGebra on the students' affective engagement in mathematics geometry course was investigated within a barrage of questions from the interview (figure 29). The researcher sought students' emotional reactions, personal commitment, and sentimental values of using GeoGebra for mathematics geometry tasks through questions such as "Do you think that GeoGebra makes learning geometry more fun? Why and why not?" or "Explain how GeoGebra impacts your interest in geometry lessons?" or "Do you struggle in geometry? Does the use of GeoGebra help overcome struggle and frustration? Why and why not?"

Sub-themes for affective engagement included giving value to GeoGebra, giving value to mathematics, and a whole bunch of positive emotions towards using the software. Giving value to GeoGebra included pieces of affective engagement that focused mainly on positive value development. Giving value to mathematics required finding meaning, relevance, and purpose in their classroom mathematics learning. (Brown, 2017). In this sub-theme, students expressed their pleasure by describing mathematics as beautiful. This sub-theme was a symbol of the intrinsic worth of mathematics and the value of its abstraction. The third sub-theme for affective engagement described six intense primary emotions that the interview respondents experienced while using the software and expressed. According to the respondents, GeoGebra enhanced positive and hopeful emotions that encouraged them to use GeoGebra more often. Students were able to identify and track their emotions. They described feelings of pleasure, sense of enjoyment, and contentment in using the software. These positive emotions made geometry tasks more comfortable to handle, helped build inner strength, and students' resilience (Fung et al., 2018). The most common positive emotions tracked in this sub-theme were enjoyment and amusement, positive attitude, curiosity, excitement, and coping math frustration.

The codes that represent giving value to GeoGebra revealed that the software made difficult geometry tasks more comfortable to handle, broadened student awareness, and made students experience more options for geometry problem-solving. This added value of the software acted as a buffer between students and stressful geometry tasks, allowing them to reduce their frustration and cope more effectively with mathematics as a core subject. The added value to the software eventually flourished and deepened the value of mathematic. Those codes summarized the role of mathematics in providing an effective way of building mental discipline and mental rigor. Besides, those codes explained the overall growing importance and the crucial role of mathematics. On the other hand, the codes included in the affective emotions were internal physiological reactions that unconsciously regulated students' experiences in mathematics. The outward affective expressions of the internal emotions revealed that GeoGebra created for the respondents a stimulus of positive reactions towards mathematical tasks (Fung et al., 2018).

The second theme obtained from some questions of the research was "Affective Engagement," and sub-themes as "giving value to mathematics," "giving value to mathematics," and "positive emotions towards using the software.". While the theme of affective engagement generally has a 39.85% percentage, the sub-themes have a share of 6.96%, 1%, and 30.8%, respectively. Students agreed mostly on all the positive emotions that GeoGebra boots in the theme identified as affective engagement.

Students' answers to the interview question determined that the respondents "value GeoGebra" (S3, S4, S5), as they regarded GeoGebra as relevant to the welfare of students since it saves them time and effort at once.

In an exemplary fashion for students who value of GeoGebra:

S3: "GeoGebra makes the student's life easier, especially when it comes to drawing geometrical shapes."

S4: "GeoGebra will help us very much, and at the same time, maybe it will help the teacher explain the lesson in less time."

S5: "The impact of this app is so good and helpful; it is much easier than using conventional learning of geometry."

In the same manner, respondents revealed that they "value mathematics" after exploring the beauty of mathematics by using GeoGebra (S1, S3, S5). For greater certainty that students value mathematics:

S1: "GeoGebra helps us explore the beauty of drawing in math."

S3: "This app is essential because it simplifies work and helps us solve easily by drawing great geometrical shapes."

S5: "GeoGebra is making me love the math sessions and wait for them."

The interview respondents expressed "positive emotions towards using the software" (S3) some as "enjoyment and amusement" (S1, S2, S3, S4), "positive attitude" (S1, S2, S3, S4, S5), "curiosity" (S3, S4), "excitement" (S2, S3), and "coping with math frustration" (S2). For clarity of students' enjoyment and amusement:

S1: "Sometimes I use GeoGebra for entertainment outside math classes. Everyone has been finding interest in it!"

S2: "GeoGebra is making students having fun in the geometry sessions."

S3: "One of the advantages is that GeoGebra makes students have fun."

S4: "Actually, every time I feel bored, I just use the GeoGebra app without hesitating."

S5: "As soon as I tried and started to do GeoGebra, I felt challenged to do things in a fun and enjoyable way!"

For the sake of the argument that students had a positive attitude towards using the software:

S1: "Using GeoGebra by itself is a great deal!"

S2: "This app is just great!"

S3: "In my point of view, this app has no drawbacks."

S4: "There are plenty of advantages of using this app."

S5: "I do not see any drawbacks for GeoGebra. It is fun and productive because one may do many geometry drawings."

For orienting students' curiosity:

S3: "GeoGebra app makes me curious on how to draw figures using it."

S4: "GeoGebra improves my interest in geometry lessons by making me more curious to draw much more hard and complex geometrical figures."

For ease of reference to students' excitement:

S2: "When we downloaded the GeoGebra app, I felt excited by the geometry lessons."

S3: "GeoGebra helps the student discover their hidden creative talents to do some amazing artworks using it."

Coping with math frustration was exemplifies as follows:

S2: “With the help of GeoGebra, we overcome our struggle and frustration in learning geometry.”

S5: “The use of GeoGebra helps overcome my struggle in math.”

In the research, students discussed more than one emotional dimension at the same time in their opinions regarding their affective engagement in a mathematics course. Students found the value of GeoGebra, mathematics, and tracked positive emotions towards the software at once.

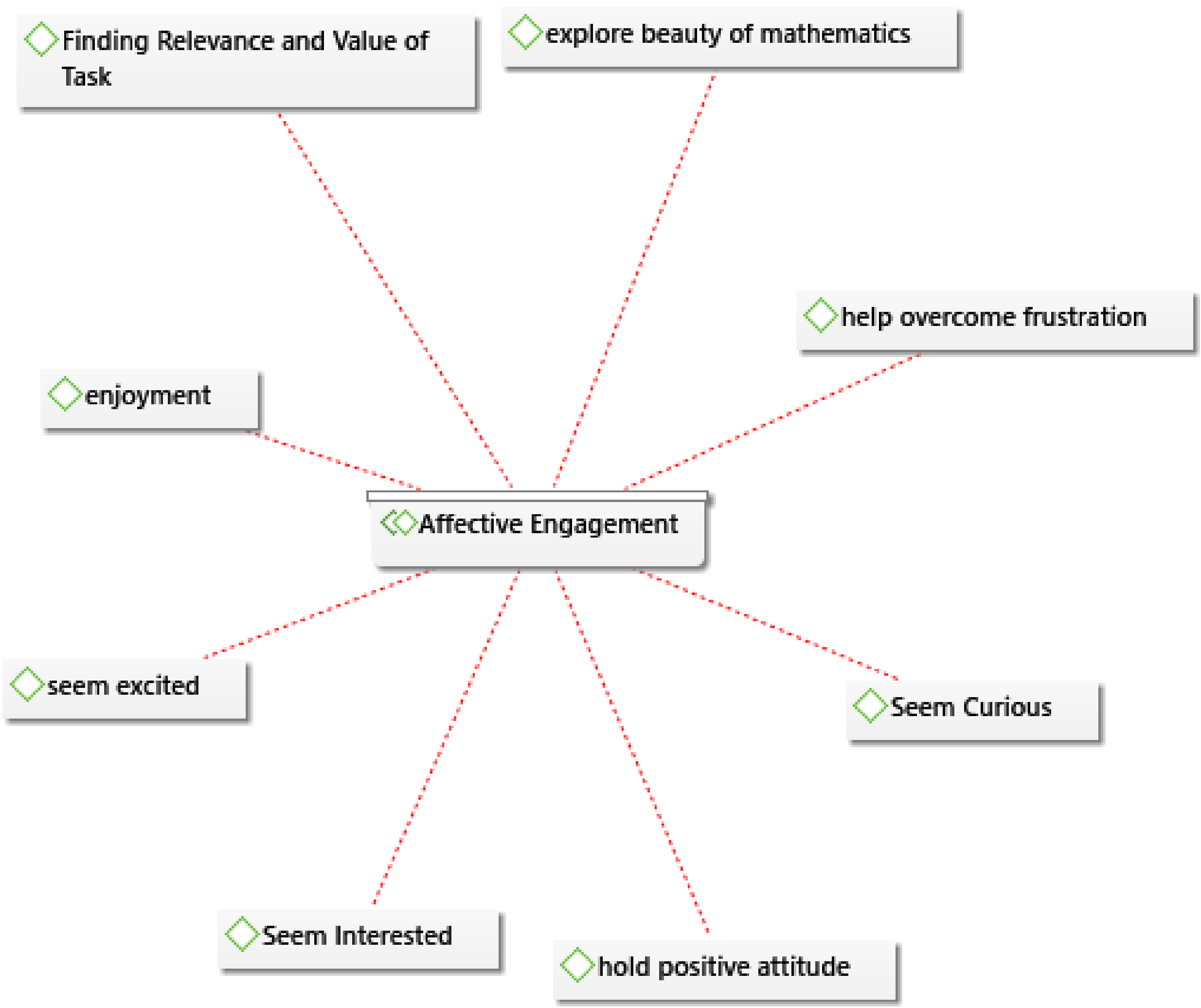


Figure 29: Codes of the theme "Affective Engagement."

Some questions in the interview determined whether students report the disadvantages of GeoGebra in a mathematics course. The unremarkable disadvantages got revealed through questions such as "What are the drawbacks of using GeoGebra in Geometry classrooms?". These disadvantages were held under sub-theme "Drawback" and include objectionable features or hindrances that render GeoGebra as less desirable. According to the interview respondent, GeoGebra has an unremarkable downside. The codes included disadvantages that were in the students' perspective unworthy to be noticed among the too many advantages.

The researcher labeled the theme obtained from some questions of the research interview "Disadvantages" and its sub-themes "Drawbacks in the task" and "Other drawbacks." While the theme of “Disadvantages” generally had a percentage of 3.76%, the sub-themes had a share of 1.00% and 1.50%, respectively. Students agreed mostly on those negative impacts of GeoGebra on their mathematics learning that are imperceptible. Examining students' answers to the interview question determined that they mentioned: "Drawback in the task" such as "Hard task" (S1, S3, S4) or "Getting distracted" (S2, S4, S5).

For instance, students who reported a hard task:

> S1: "There might be very few problems in moving the figures, but it is no big deal, you can manage to work it out."
>
> S3: "It was hard at first, especially which I missed some classes, but it is an amazing app, and it sure does."

For illustration for getting distracted:

> S2: "The drawbacks are for our distraction, which I think is not a big deal since we were using the phone to play games at that time."
>
> S4: "Both could work since students might go away and get distracted during class and draw, but on the other hand, if the class is under control, nothing like this would happen."
>
> S5: "It may slightly distract me from playing, like by drawing shapes for fun."

	○ enjoyment Gr=17		○ explore beauty of mathematics Gr=2		○ Finding Relevance and Value of Task Gr=14		○ help overcome frustration Gr=2		○ hold positive attitude Gr=13		○ Seem Curious Gr=4		○ seem excited Gr=11		○ Seem Interested Gr=15	
	count	coefficient	count	coefficient	count	coefficient	count	coefficient	count	coefficient	count	coefficient	count	coefficient	count	coefficient
○ Concentration Gr=5	0	0.00	0	0.00	1	0.06	0	0.00	1	0.06	1	0.13	0	0.00	1	0.05
○ enhance discovery Gr=7	1	0.04	0	0.00	0	0.00	0	0.00	0	0.00	1	0.10	2	0.13	2	0.10
○ enhance learning ability Gr=6	1	0.05	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	1	0.06	1	0.05
○ enhance understanding Gr=16	4	0.14	0	0.00	2	0.07	1	0.06	2	0.07	0	0.00	0	0.00	0	0.00
○ enhances creativity Gr=12	3	0.12	0	0.00	1	0.04	0	0.00	0	0.00	0	0.00	1	0.05	2	0.08
○ expand imagination Gr=4	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00
○ find relations Gr=2	1	0.06	0	0.00	0	0.00	0	0.00	1	0.07	0	0.00	0	0.00	1	0.06
○ improve thinking skills Gr=9	0	0.00	0	0.00	1	0.05	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00
○ Persist in Completing Tasks Gr=2	0	0.00	0	0.00	1	0.07	0	0.00	1	0.07	0	0.00	0	0.00	1	0.06
○ Resisting Distractions Gr=3	0	0.00	0	0.00	1	0.06	0	0.00	1	0.07	1	0.17	0	0.00	1	0.06
○ Self-Monitoring and self-control Gr=4	0	0.00	0	0.00	1	0.06	0	0.00	1	0.06	0	0.00	0	0.00	1	0.06
○ Using Different Strategies Gr=6	0	0.00	0	0.00	1	0.05	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00
○ visualization of abstract ideas Gr=17	1	0.03	0	0.00	6	0.24	0	0.00	2	0.07	1	0.05	2	0.08	3	0.10

Table 22: The "ATLAS.ti 8" code Co-occurrence table of Cognitive-Affective Engagement

Using the "ATLAS.ti 8" code Co-occurrence analysis table of Cognitive-Affective Engagement, the researcher sought a qualitative correlation between the cognitive and affective engagement codes. The code "Exploring the beauty of mathematics" that was part of the sub-theme "Value given to Mathematics" in affective engagement was the only code that had no relation to any of the cognitive engagement codes. With a very slight correlation, the code "Help overcome frustration" that was part of the sub-theme "Positive Emotions" in affective engagement theme correlated with frequency one and coefficient 0.06 to the code "Enhance understanding" from the theme cognitive engagement. The same goes to the code "Seem curious" that was part of the sub-theme "Positive Emotions" that correlated with frequency one and coefficient 0.17 to the code "Resisting Distraction" and with frequency one and coefficient 0.05 to the code "Visualization of abstract ideas" from the theme cognitive engagement. Similarly, a low frequency of 1 and low coefficient rate of 0.05 correlated the code "Seem Excited" that was part of the sub-theme "positive emotions" in affective engagement theme with the code "Enhances Creativity," and low frequency of 2 and a coefficient of 0.08 correlated it with the code "Visualization of Abstract Ideas" from the theme cognitive engagement. On the other side, there was the correlation between the codes "Enjoyment," "Seem Interested" and "Hold Positive Attitude" from the "Positive Emotions" sub-theme with various codes of the cognitive engagement theme with frequencies 9, 9, 8 respectively and coefficients 0.35, 0.42, 0.41 respectively.

The code "Expand Imagination" was in no way correlated to any of the affective engagement theme codes. Also, the "Improve Thinking Skills" and "Using Different Strategies" were insignificantly correlated by frequency one and coefficient 0.05 to the code "Finding Relevance and Value" of the affective engagement theme. The codes "Enhance Understanding," "Enhance Creativity," and "Visualization of Abstract Ideas" were mostly correlated by frequencies 9, 7, 15, and coefficients 0.34, 0.29, 0.57, respectively, with various affective engagement codes.

Significantly and remarkably, the code "Visualization of Abstract Ideas" from the theme cognitive engagement and the code "Finding Relevance and Value of Task" from the affective theme engagement were highly correlated by coefficient 0.24 and frequency 6. These numbers reveal that there was a correlation between cognitive and affective engagement.

Using the "ATLAS.ti 8" code Co-occurrence analysis table of Cognitive-Behavioral Engagement, the researcher sought a qualitative correlation between the cognitive and behavioral engagement codes. The two codes "Enhance Communication" and "Enhance Cooperation" that were part of the sub-theme "Peer Interaction" in behavioral engagement were not related to any of the cognitive engagement codes. The other two codes of the theme behavioral engagement "Increase Participation" from the sub-theme "Peer-Interaction" and "Get Involved in Voluntary Tasks" from the sub-theme "Voluntary Activities" were unremarkably related by frequencies 3, 3 and by coefficients 0.22, 0.24 respectively.

	○ enhance communication Gr=3		○ enhance cooperation Gr=3		○ Get involved in Voluntary Tasks Gr=7		○ increase participation Gr=3	
	count	coefficient	count	coefficient	count	coefficient	count	coefficient
○ Concentration Gr=5	0	0.00	0	0.00	0	0.00	0	0.00
○ enhance discovery Gr=7	0	0.00	0	0.00	1	0.08	1	0.11
○ enhance learning ability	0	0.00	0	0.00	1	0.08	0	0.00
○ enhance understanding	0	0.00	0	0.00	0	0.00	1	0.06
○ enhances creativity	0	0.00	0	0.00	1	0.06	1	0.07
○ expand imagination Gr=4	0	0.00	0	0.00	0	0.00	0	0.00
○ find relations Gr=2	0	0.00	0	0.00	0	0.00	0	0.00
○ improve thinking skills	0	0.00	0	0.00	0	0.00	0	0.00
○ Persist in Completing Tasks Gr=2	0	0.00	0	0.00	0	0.00	0	0.00
○ Resisting Distractions	0	0.00	0	0.00	0	0.00	0	0.00
○ Self-Monitoring and self-control Gr=4	0	0.00	0	0.00	0	0.00	0	0.00
○ Using Different Strategies	0	0.00	0	0.00	0	0.00	0	0.00
○ visualization of abstract ideas Gr=17	0	0.00	0	0.00	0	0.00	0	0.00

Table 23: The "ATLAS.ti 8" code Co-occurrence table of Cognitive-Behavioral Engagement

Nine codes related to the cognitive engagement theme were in no way related to any of the affective engagement themes. The codes labeled as “Expand Imagination,” “Improve Thinking Skills,” from the sub-theme “Strategical Learning support” had zero correlation frequency and zero correlation coefficient with the behavioral engagement theme. The same applied to the codes “Persist in Completing Tasks,” “Using Different Strategies,” and “Visualization of Abstract Ideas” from the sub-theme “High Cognitive Practices.” The codes “Resisting Distraction,” “Self-Monitoring and Self-Distraction,” and “Find Relation” from the sub-theme “In-depth strategies” from the cognitive engagement theme as well had zero correlation frequency and zero correlation coefficient with the behavioral engagement theme. Only the three codes “Enhance Discovery,” “Enhance Learning Ability” and “Enhance Creativity” from the cognitive engagement theme-related by frequency one and coefficients 0.08, 0.08, 0.06 respectively to the code “Get Involved in Voluntary Tasks” and by frequency one and coefficients 0.11, 0.06, 0.07 respectively to the code “Increase Participation” from the behavioral engagement theme. These numbers show that the two themes of cognitive engagement and behavior engagement were slightly unremarkably correlated.

	○ enhance communication Gr=3		○ enhance cooperation Gr=3		○ Get involved in Voluntary Tasks Gr=7		○ increase participation Gr=3	
	count	coefficient	count	coefficient	count	coefficient	count	coefficient
○ enjoyment Gr=17	0	0.00	0	0.00	3	0.14	0	0.00
○ explore beauty of mathematics Gr=2	0	0.00	0	0.00	0	0.00	0	0.00
○ Finding Relevance and Value of Task	1	0.06	1	0.06	0	0.00	0	0.00
○ help overcome frustration	0	0.00	0	0.00	0	0.00	0	0.00
○ hold positive attitude	1	0.07	1	0.07	1	0.05	0	0.00
○ Seem Curious Gr=4	0	0.00	0	0.00	0	0.00	0	0.00
○ seem excited Gr=11	0	0.00	0	0.00	1	0.06	0	0.00
○ Seem Interested Gr=15	0	0.00	0	0.00	3	0.16	0	0.00

Table 24: The "ATLAS.ti 8" code Co-occurrence table of Affective-Behavioral Engagement

Using the "ATLAS.ti 8" code Co-occurrence analysis table of Behavioral-Affective Engagement, the researcher sought a qualitative correlation between the affective and behavioral engagement codes. The code "Increase Participation" that was part of the sub-theme "Peer Interaction" in behavioral engagement is not related to any of the affective engagement codes. The other two codes of the theme behavioral engagement "Enhance Communication" and "Enhance Cooperation" from the sub-theme "Peer-Interaction" were related by frequency one and by coefficient 0.06 to the code "Finding Relevance and Value of Task" from the affective theme engagement. Simultaneously, these two codes were correlated by frequency one and coefficient 0.07 to the code "Hold Positive attitude" from the affective engagement theme. The only code from the theme behavioral engagement that was significantly related to the affective engagement theme is the code "Get Involved in Voluntary Tasks" by frequency 8 and 0.41 coefficient.

The two codes "Explore Beauty of Mathematics" from "Giving Value to Mathematics" and "Seem Curious" from the sub-theme "Positive Emotions" were not correlated to the behavioral engagement theme. The code "Enjoyment" from the sub-theme "Positive Emotions" of affective engagement theme was only related to the code "Get Involved in Voluntary Tasks" from the behavioral engagement theme by frequency 3 and 0.14 coefficient. In the same manner, the two codes "Seem Excited" and "Seem Interested" from the sub-theme "Positive Emotions" of affective engagement were slightly correlated to only one code "Get Involved in Voluntary Tasks" from the behavioral engagement theme by frequencies 1, 3 respectively and coefficient 0.06, 0.14 respectively. The code "Hold Positive Attitude" forms the sub-theme "Positive Emotions" of the affective theme was mostly correlated with behavioral engagement theme by a frequency three and by a 0.19 coefficient. These numbers showed that the two themes, affective and behavioral engagement, were partially correlated.

CHAPTER V

DISCUSSION OF RESEARCH FINDINGS

Methodological Triangulation of Data

Triangulation is an approach of comparative data analysis using differing methodologies. It is used to check and establish validity in the study by analyzing the research questions on different aspects (Creswell, 2009). It was not only about cross-checking data but rather to obtain a more comprehensive understanding of the impact of the software from different perspectives.

In specific, Methodological Triangulation was used since the study involved multiple qualitative and quantitative methods at once. The researcher did not rely solely on the quantitative findings since the qualitative component was useful in exploring in-depth scenarios. The researcher compared the areas of agreement and the areas of divergence from the surveys, interviews, and observation results.

Methodological Triangulation of this mixed-method research was insightful as it permitted cross-validation and facilitated exploration of the GeoGebra impact on student engagement (figure 30). With extensive analysis completed separately for the quantitative and qualitative parts of the study, the researcher combined the three measures to ensure that technique's fundamental biases have been overcome. When combined, the qualitative and quantitative methods supplemented each other.

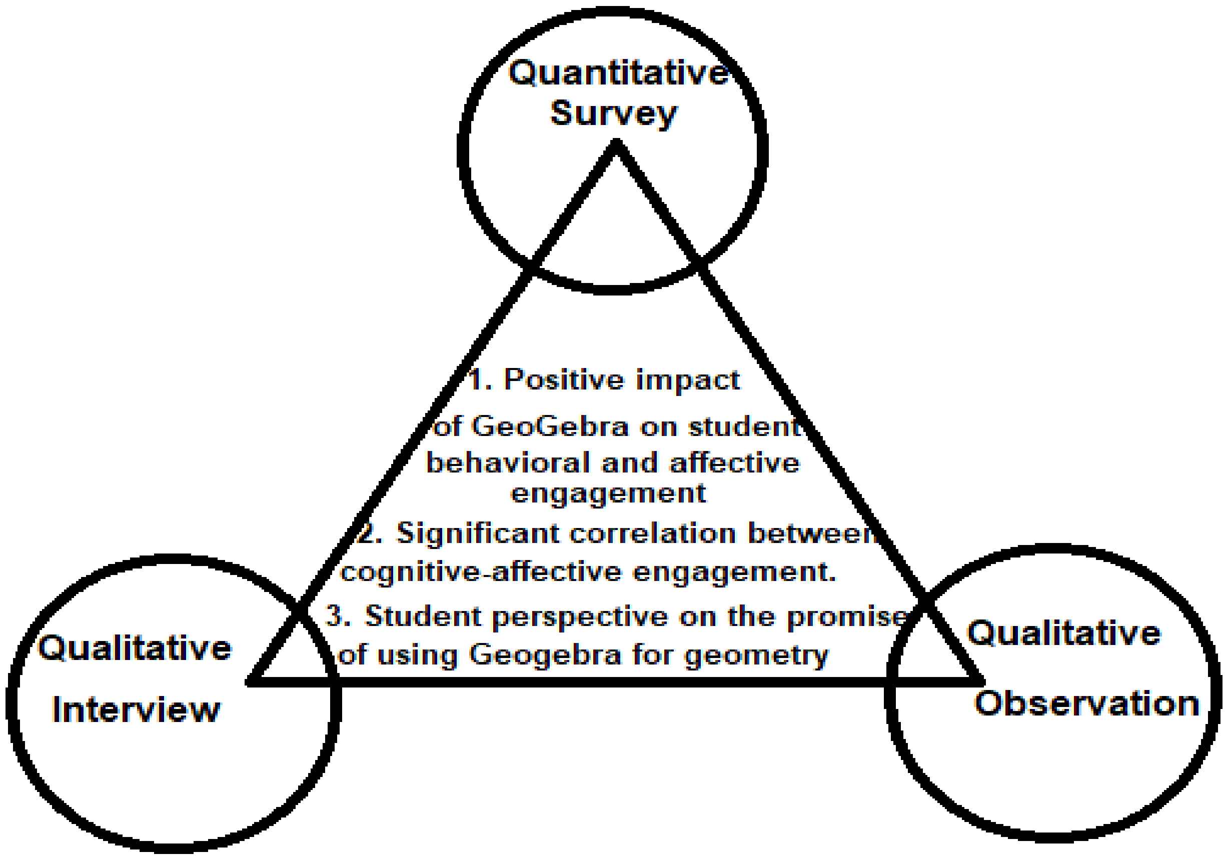


Figure 30: Triangulation of Methods

Not only did the video observations show moments of joy and intense concentration among the learners, but they also demonstrated the importance of engagement in shaping interpretations and in enabling insightful comparisons to be drawn between the students' work. Learning such GeoGebra activities was essential for providing the basis for understanding and identifying geometrical concepts. GeoGebra helped students develop a new layer of meaning and supported their learning by drawing attention to what was emotionally resonant. However, we do not argue that all GeoGebra artifacts reflected the same level of engagement; instead, we say that GeoGebra evoked appropriate engagement levels that supported geometry learning.

The results presented above from the video observation can be summarized in two points. First, GeoGebra positively impacted the different types of student engagement in learning geometry. Second, there existed a positive correlation between the depth of understanding, the discussion prompt, and the students' comfort in geometry classes. Students' emotional engagement was found to increase cognitive engagement and behavioral engagement. Nevertheless, the present analysis of the video data did not warrant reliable conclusions regarding the relationships between the multifaceted structures of student engagement. The distinctive patterns of relationships were observed, which encourage further inquiry into the unique aspects of using GeoGebra in geometry sessions.

On the other hand, the in-depth interviews revealed that students mostly agreed that the GeoGebra software mostly impacted their visualization ability of abstract thinking and enhanced their understanding of mathematics geometry lessons. This finding is in line with the research of Zulnaidi et al., (2020), who revealed that GeoGebra is handy in teaching and learning mathematics since it eases the visualization of complex abstract concepts and creates links between various mathematical knowledge. It also matches Jelatu et al. (2018) 's findings, who revealed that GeoGebra turns abstract concepts into concrete to make high spatial ability students and low spatial ability ones balanced in their understanding. Based on the students' perspectives in the theme of cognitive engagement in a mathematics lesson, the least impact of GeoGebra was on the sub-theme "In-Depth strategies." This situation is also mentioned by Aizikovitsh-Udi & Radakovic (2012), who believed there is a shred of mounting evidence that shows dynamic visualizations of GeoGebra create means for the more in-depth analysis of mathematical concepts. GeoGebra primarily impacted student's visualization abilities that minimally impacted the in-depth strategies. The most significant share under the theme of affective engagement was in the interview group's positive emotions after the GeoGebra intervention. Students enjoyed the positive experiences they had with the software. Attard (2011) expressed how executing such appropriate teaching practices, impels children to enjoy and value mathematics, and eventually become mathematically engaged. The lowest share of affective engagement in mathematics was the sub-theme of "Valuing Mathematics." In parallel with the present research, Attard (2011) stated that the worst of all is that even high achievers at school report that they fail to recognize any personal relevance of mathematics, and few of them continue to study mathematics in depth. Two of the seventy-eight codes identified in the theme of emotional engagement were in the sub-theme of value given to the mathematics course. The answers of the students determined that very few said that GeoGebra helped them value mathematics. Analyzing students' opinions given under the theme of value given to the mathematics, students had positive feelings towards learning mathematics with GeoGebra, but they have still had not found the actual value of mathematics itself.

The most significant share under the theme of behavioral engagement was for the "Peer Interaction" of the interview group after the GeoGebra intervention. The lowest share of behavioral engagement in mathematics was for the sub-theme of "voluntary Activities." In parallel with the present research, a similar situation was determined in a study carried out by Muir, (2014), who stated that such educational software increases student participation and gets everyone involved.

The analysis of the interview data reflected in addition to the positive impact of the GeoGebra on student engagement in geometry sessions, a correlation between the three strands: cognitive, affective, and behavioral. The three identified “themes” were found positively correlated to each other but with varying percentages. The behavioral engagement was not correlated to cognitive engagement and partially correlated with affective engagement. At the same time, the affective engagement was found correlated to cognitive engagement.

In summary, the emerging themes uncovered by the interview seemed to align with the data generated by the observations. The way students explained their GeoGebra work in the interview and observation, reflected cognitive, behavioral, and affective engagement. The emotional involvement in GeoGebra tasks increased allure and enhanced students' cognitive productivity. A great value of pleasure and passion accompanied geometry exploration with the aid of GeoGebra, reinforcing various critical thinking skills. These skills helped students rule the tasks as they found themselves adding different solving strategies and building upon the ideas of one another. These findings match the findings of (Redo et al., 2018) that affirms GeoGebra supports the development of professional skills in mathematics and encourages different task designs when solving geometric exercises. A great variety of the students' emotions were positive, pushing students to invest in creating solution plans and sharing them with their teacher and peers. Students' optimism about the GeoGebra intervention influenced their participation. The productive discourse, actions, and interactions fabulously debunked extraordinary levels of engagement. GeoGebra tasks were littered with the promise of good feelings, which shifted students' understanding of deep levels and nurtured students' sense of control over their behaviors and goals. While students' behavioral engagement was associated with ethical conduct, their cognitive engagement was delivered through exerting the necessary mental effort to master geometry concepts.

The interview and the observation revealed that students perceived GeoGebra activities as meaningful and worthy of their time and effort. Ensuring that GeoGebra activities were meaningful, students connected them to their previous knowledge and highlighted the assigned activities' value in personally relevant ways. Modeling among peers helped demonstrate why the math GeoGebra task was worth pursuing and how it might be used in real life. Modeling elicited the notion of competence with ongoing personal evaluation, which positively impacted subsequent engagement. It also provided peer coping models that included feedback to help struggling students' progress.

These qualitative findings coincided with the quantitative results of the survey from one side and diverged from another. The statistical analysis of the empirical data revealed convincing evidence of a clear positive impact of GeoGebra on student affective and behavioral engagement. Contrary to the qualitative findings, the quantitative data reflected that GeoGebra did not impact students' cognitive engagement. The quantitative results as well reflected a correlation between some strands of engagement only. This relation in both the qualitative and quantitative data was most evident and significant about emotional-cognitive engagement and was insignificant with cognitive-behavioral and affective-behavioral engagement. The SPSS quantitative did not show any significant association between the emotional-behavioral engagement, neither between cognitive-behavioral engagement, which partially collides with the previous qualitative arguments.

The comparison of the experimental group to the control group, which was not exposed to GeoGebra at all, elucidated significant differences in student engagement levels. The students' engagement and perception of the experimental group's geometry stated higher engagement levels at the post-test compared to the control group. The experimental group students mentioned more knowledge, more geometry-specific skills, and three strands of engagement than the control group students. Thus, as a result of the socio-cultural instructional development that the teacher had undergone by implementing GeoGebra activities, experimental students reported higher engagement outcomes at the post-test than at the pre-test.

Taking into account all the results obtained concerning the three research questions, one may conclude that the difference between experimental and control group students' engagement has been noticeable. The mere fact that they perceived GeoGebra as relevant and efficient provided an extra stimulus for experimental students to think about their learning. This fact was the starting point towards a modification in their engagement levels in geometry (figure 31).

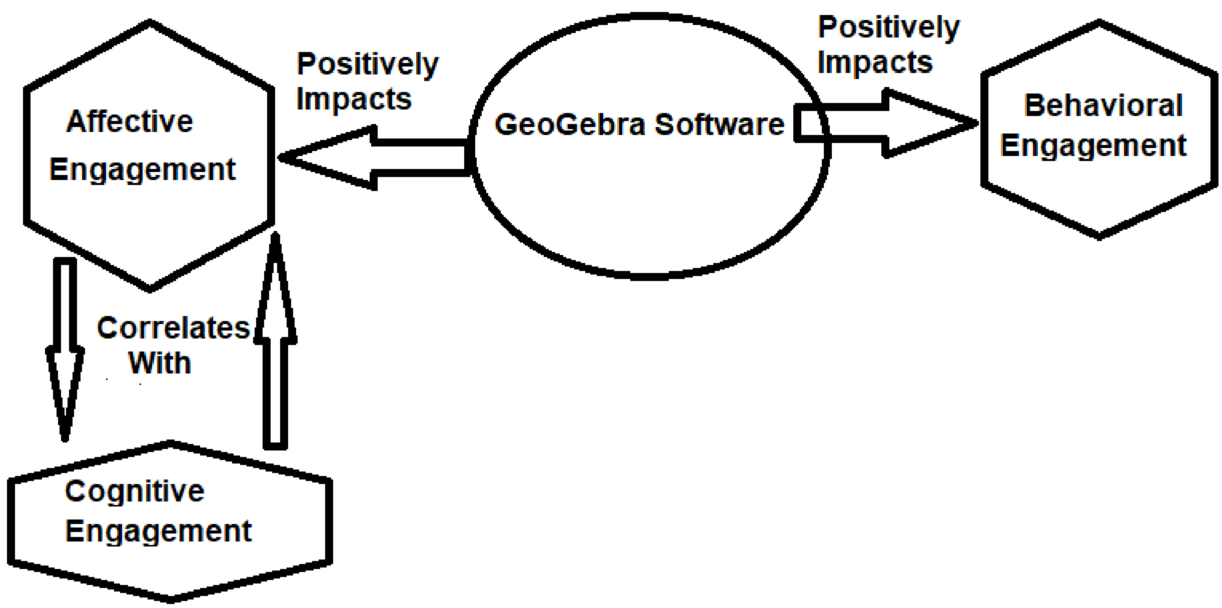


Figure 31: Sum-up the findings

CONCLUSION

The mixed-method study departed from the question of whether GeoGebra contributed to enhancing student engagement in geometry. An empirical investigation in the eighth grade of a private school was conducted to respond to the three research questions. In light of the empirical evidence, the research question was answered with an emphatic "yes": "The research findings strongly suggest that GeoGebra positively impacted eighth-graders’ engagement in learning geometry. GeoGebra had a significant positive impact on students' affective and behavioral engagement only. However, GeoGebra did not have a significant impact on student cognitive engagement. On the other hand, affective engagement was highly correlated to cognitive engagement yet was not correlated to behavioral engagement. Further investigations would contribute to uncovering the patterns of relationships among engagement facets, considering each engagement facet as a separate dimension.

In addition to the previous, qualitative results contributed to understanding students' perceptions and the role of GeoGebra augmentation beyond deepening enjoyment and developing contextual knowledge. The qualitative analysis reflected how various strands of engagement could be triggered. Implementing GeoGebra had different implications for learners' engagement, depending on the student and his unique interests.

The findings demonstrate how engagement unfolds via multimodal interaction between the person and his socio-cultural environment. In particular, certain qualities of the interaction were found to be indicative of engagement, including changes in working pace and linking GeoGebra to individual interests.

Limitations

A drawback of this quasi-experimental study might be the lack of a rigorous research design that would allow the drawing of robust conclusions and generalizations. The exploratory nature of the investigation, the qualitative methodology that was used in the study, the small scale of the study, its short duration, and its limited geographical nature mean that the drawing of generalizations to other cases would not be entitled.

Another limitation is that the collected data did not preclude any definitive claims of causality. It might be possible that those higher-performing students were more engaged in their learning as a result of their higher learning capacity, or that a third undefined variable caused higher levels of learning engagement. Therefore, further and more in-depth investigation into the use of this mathematics software is warranted and timely. Future research could adopt a longitudinal, mixed-methods research design to compare the relationship between GeoGebra and students' engagement in mathematics in different geographical areas.

Notwithstanding this straightforward conclusion, the limited scope of this study has to be reiterated. One of the study limitations is that the teacher was the co-investigator researcher, considering that the high-quality teacher-student relationship was another critical factor in determining student engagement. The teacher in this study adopted the constructivist approach in teaching where all sessions are student-centered and inquiry-based. This approach was another factor for triggering students' engagement.

Interestingly, the teacher relinquished control in favor of welcoming students' opinions and ideas in the flow of the activity. She also gave students all the time they needed to understand and absorb GeoGebra activities. Previous research suggests that student-created content is a crucial aspect in designing digital environments that effectively support engagement in mathematics (Holden, 2016). This study indicates that the types of material students are invited to create would impact on the nature of their engagement with the task. This matches the findings of Angelini et al., (2017), who stated that pedagogical approaches that involve active participation, mathematical investigations, and exploration with digital technologies appear to foster student engagement in mathematics concurrently.

Collaborative learning was another facilitator of engagement in GeoGebra activities. Students worked effectively with each other and experienced a sense of connection to each other during the exercises, which amplified their engagement levels. Finally, students' engagement was more likely full and thorough since they pursued the events because they wanted to learn and understand. There were no grades involved in any of the online tasks, which assured that the students had a mastery orientation mindset. They framed success in the GeoGebra activities in terms of learning rather than performing, emphasizing individual progress, and reduced social comparison.

Besides, some educators might argue that an excess of engagement may stand in geometrical interpretation and genuine experiences of geometry. One could observe how learners could become over-enthusiastic in making connections leading to instances in which learners constructed irrelevant links. Such situations highlight the importance of designing digital environments that support students in building relationships with the geometry and distinguish between appropriate and inappropriate links. Furthermore, it highlights the teacher's vital role in mediating the learning experience and following up peer or teacher-led feedback engagement with the material, ensuring that learners have an opportunity to probe further the links they have constructed and their subsequent emotional responses. Connecting extracurricular learning experiences and curriculum-specific topics are central to education in extending and reinforcing in-school work.

Recommendations

The time is ripe to consider the contribution of GeoGebra to geometry learning. Undoubtedly, the teacher's preparation of tasks and incentivizing may contribute to a more engaged learning experience. To get a better place, teachers need to develop a particularly useful instruction framework to embrace GeoGebra. The software shall be used through interventions that trigger and maintain students' engagement. As a case in point, teacher preparation could trigger students' engagement in curricular, co-curricular, and extracurricular efforts. One promising route is to implement core GeoGebra investigation sessions that emphasize engagement-triggering structural features. The adoption of incentivizing design in the geometry courses necessarily highlighted that GeoGebra triggered connections between different strands of engagement and in-depth information processing. Implementing GeoGebra interventions require careful attention to the details and getting down into the weeds of creating instructional opportunities and exploration pieces that sustain students' engagement. As an illustration, the teacher should effectively implement and facilitate meaningful conversations to improve students' comprehension proficiency. Also, modifying the structure of the questions throughout the discussion seems to play a significant role in student engagement. The findings go hand in hand with the findings of Geiger et al., (2017) that affirm the figure of sociocultural engagement theory that assures the task characteristics, and the pedagogical practices regulate classroom engagement. Proceeding on the same track, Durksen et al., (2017) indicated previously in their study that increasing positive engagement or reducing negative engagement hinges on the teacher's practices, instructional support, and active classroom organization

This research contributed to the literature with the definition of the impact of GeoGebra on student engagement. Although the level of student engagement is open for debate, this may be considered as a starting point for researchers seeking to conduct a study in this area. It can also give some clues for teachers and textbook writers to prepare examples for classroom practices in geometry to support student engagement in mathematics.

The next step is hopefully revising existing geometry curricula to go beyond performance standards and consider multiple indicators of students' mathematical achievement that include student engagement. Expanding the geometry curricula to include GeoGebra activities would set a new norm for the central role of students' engagement in mathematics. The educational system must respond to the students' ever-changing needs by integrating the software to sustain their developing engagement in the subject. The study is one step toward creating a future generation of students with a love of geometry. Whether a change in the instruction design of GeoGebra activities might lead to different results would be worthwhile to explore in future research.

It should be noted that the present study was conducted online and revealed similar results to those of previous studies conducted face-to-face. As a result of the unexpected circumstances, this research unintentionally also informed about students' engagement with online teaching. This study could be developed further, as it would be in order to further look into students' perceptions of online teaching.

References

Aizikovitsh-Udi, E., & Radakovic, N. (2012). Teaching probability by using GeoGebra dynamic tool and implementing critical thinking skills. *Procedia-Social and Behavioral Sciences, 46,* 4943-4947. DOI: 10.1016/j.sbspro.2012.06.364

Angelini, M., Prescott, A., Schuck, S., Rai, T., Lee, C., & Coupland, M. (2017). Maths Inside": A Project to Raise Interest in Mathematics. *Forty years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia, 1*(1), 181-188. Melbourne: MERGA Retrieved from Eric Search Database. (ERIC Number: ED589474)

Attard, C. (2011). "My favorite subject is math. For some reason, no-one really agrees with me": student perspectives of mathematics teaching and learning in the upper primary classroom. *Mathematics Education Research Journal, 23*(3), 363. DOI: 10.1007/s13394-011-00205

Attard, C. (2012). Engagement with Mathematics: What Does It Mean and What Does It Look Like? Australian Primary Mathematics Classroom, 17(1), 9-13.

Attard, C. (2013). "If I had to pick any subject, it wouldn't be math": foundations for engagement with mathematics during the middle years. *Mathematics Education Research Journal, 25*(4), 569-587. DOI:10.1007/s13394-013-0081-8

Babchuk, W. A. (2017). Book review: Qualitative research: A guide to design and implementation, by SB Merriam and EJ Tisdell. *Los Angeles, CA: SAGE Publications.* DOI:10.1177/0741713616671930

Baltacı, S., Yıldız, A., & Kösa, T. (2015). The Potential of GeoGebra Dynamic Mathematics Software in Teaching Analytic Geometry: The Opinion of Pre-service Mathematics Teachers. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 11*(1), 6. 483. DOI: 10.16949/turcomat.32803.

Bray, A., & Tangney, B. (2016). Enhancing student engagement through the affordances of mobile technology: A 21st-century learning perspective on realistic mathematics education. *Mathematics Education Research Journal, 28*(1), 173-197. DOI:10.1007/s13394-015-0158-7

Benefits of Incorporating Art Concepts in Math (2018, Feb 19). In-*School Specialty*. *"21st Century Learning"*. Retrieved from https://blog.schoolspecialty.com/

Benson-O'Connor, C. D., McDaniel, C., & Carr, J. (2019). Bringing math to life: Provide students opportunities to connect their lives to math. *Networks: An Online Journal for Teacher Research, 21*(2) DOI:10.4148/2470-6353.1299

Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. *Mathematics Education Research Journal*, *29*(2), 183-199. DOI:10.1007/s13394-017-0198-2

Carmichael, C., Callingham, R., & Anderson, J. (2017). Introduction to MERJ special issue, "theoretical foundations of engagement in mathematics: Empirical studies from the field." *Mathematics Education Research Journal, 29*(2), 131. DOI:10.1007/s13394-017-0207-5

Carmichael, C., Muir, T., & Callingham, R. (2017). The impact of within-school autonomy on students' goal orientations and engagement with mathematics. *Mathematics Education Research Journal, 29*(2), 219-236. DOI:10.1007/s13394-017-0200-z

Chang, Chun-Yen & Bhagat, Kaushal. (2015). Incorporating GeoGebra into Geometry learning A lesson from India. *Eurasia Journal of Mathematics, Science, and Technology Education. 11*(1). 77-86.

Cohen, E. (2008). Book review: Creswell, J. W., & Plano Clark, V. L. (2006). designing and conducting mixed methods research. *Thousand Oaks, CA: Sage. Los Angeles, CA: SAGE Publications.* DOI:10.1177/1049731508318695

Collins, Justin A., Valentine, Jerry. (2010) Higher-order thinking in the high-stakes accountability era linking student engagement and test performance. *Networked Digital Library of Theses and Dissertations,62,* 1-447. DOI:10.32469/10355/6769

Cordero, J. M., Cristóbal, V., & Santín, D. (2018). Causal inference on education policies: A survey of empirical studies using PISA, TIMSS, and PIRLS. *Journal of Economic Surveys, 32*(3), 878-915. DOI:10.1111/joes.12217

Creswell, J. W. (1996). Research design: Qualitative and quantitative approaches. *The Library Quarterly, 66*(2), 225-226. DOI:10.1086/602876

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approach* (3rd ed.). Los Angeles: SAGE.

Durksen, T. L., Way, J., Bobis, J., Anderson, J., Skilling, K., & Martin, A. J. (2017). Motivation and engagement in math: A qualitative framework for teacher-student interactions. *Mathematics Education Research Journal, 29*(2), 163-181. DOI:10.1007/s13394-017 0199-1

Fesakis, G., Karta, P., & Kozas, K. (2018). Designing math trails for enhanced by mobile learning realistic mathematics education in primary education. *International Journal of Engineering Pedagogy (iJEP), 8*(2), 49-63. DOI:10.3991/ijep. v8i2.8131

Fielding-Wells, J., O'Brien, M., & Makar, K. (2017). Using expectancy-value theory to explore aspects of motivation and engagement in inquiry-based learning in primary mathematics. *Mathematics Education Research Journal*, *29*(2), 237-254. DOI:10.1007/s13394-017 0201-y

Frabasilio, A. M. (2019). Extreme word problems: Interweave student imagination with math. *Mathematics Teaching in Middle School, 24*(4), 234-240. DOI:10.5951/mathteacmiddscho.24.4.0234

Francisco Regis Vieira Alves, & Barros, F. E. (2019). plane and space figurate numbers: Visualization with the GeoGebra's help. *Acta Didactica Napocensia, 12*(1), 57-74. DOI:10.24193/adn.12.1.4

Franklin, J.R. & Wallen, N.E. (2012). *How to design and how to evaluate research in education* (8th edition.). Boston: McGraw-Hill Education.

Fredricks, J. A. (2011). Engagement in school and out of school contexts: A multidimensional view of engagement. *Theory into Practice, 50*(4), 327-335. DOI:10.1080/00405841.2011.607401

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: The Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59-109. DOI:10.3102/00346543074001059

Fung, F., Tan, C. Y., & Chen, G. (2018). Student engagement and mathematics achievement: Unraveling main and interactive effects. *Psychology in the Schools, 55*(7), 815-831. DOI:10.1002/pits.22139

Geiger, V., Anderson, J., & Hurrell, D. (2017). A case study of effective practice in math teaching and learning informed by valsiner's zone theory. *Mathematics Education Research Journal, 29*(2), 143. DOI:10.1007/s13394-017-0191-9

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *The Journal of Mathematical Behavior,* 37, 48-62.

Guardino, C. A., & Fullerton, E. (2010). Changing behaviors by changing the classroom environment. *TEACHING Exceptional Children, 42*(6), 8-13. DOI:10.1177/004005991004200601

Gupta, M. L. (2009). Using emerging technologies to promote student engagement and learning in agricultural math. *The International Journal of Learning: Annual Review, 16*(10), 497508. DOI:10.18848/1447-9494/CGP/v16i10/46658

Halpin, P. F., von Davier, A. A., Hao, J., & Liu, L. (2017). Measuring student engagement during collaboration. *Journal of Educational Measurement, 54*(1), 70-84. DOI:10.1111/jedm.12133

Helme, S., & Clarke, D. (2001). Identifying cognitive engagement in the mathematics classroom. *Mathematics Education Research Journal, 13*(2), 133-153. DOI:10.1007/BF03217103

Hohenwarter, M., Jarvis, D., & Lavicza, Z. (2009). Linking geometry, algebra, and mathematics teachers: GeoGebra software and the establishment of the international GeoGebra institute. *International Journal for Technology in Mathematics Education, 16*(2), 83.

Holden, J. I. (2016). Mobile inquiry-as-play in mathematics teacher education. *On the Horizon, 24*(1), 71-81. DOI:10.1108/OTH-08-2015-0046

Hutkemri Z., Sharifah Norul, & Akmar Z. (2017). Mathematics; findings on mathematics reported by investigators at the University of Malaya (the effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics). *Computers, Networks & Communications, 13*(6), 2155-2180. DOI:1923796678.2017. 19441568

Ingram, N., Williamson-Leadley, S., & Pratt, K. (2016). Showing and telling: Using tablet technology to engage students in mathematics. *Mathematics Education Research Journal*, *28*(1), 123-147. DOI:10.1007/s13394-015-0162-y

International GeoGebra Institute. (n.d.). (2013). *GeoGebra.* Retrieved from https://www.geogebra.org/m/DmVNbn2V#material/vju3uhns

Jacinto, H., & Carreira, S. (2017). Mathematical problem solving with technology: The techno mathematical fluency of a student-with-GeoGebra. *International Journal of Science & Mathematics Education*, *15*(6), 1115-1136. DOI:10.1007/s10763-016-9728-8

Jelatu, S., Sariyasa, S., Ardana, I. M. (2018). Effect of GeoGebra-aided REACT strategy on the understanding of geometry concepts. *International Journal of Instruction, 11*(4), 325 336. DOI:10.12973/iji.2018.11421a

Jewitt, C. (2012). *Technology, literacy, learning: A multimodal approach* Taylor and Francis. DOI:10.4324/9780203964101

Jewitt, C. (2006;2005;2012;). *Technology, literacy, and learning: A multimodal approach*. London; New York; Routledge. DOI:10.4324/9780203964101

Jorgensen, R. (2010). Shifting practices of school mathematics: Engaging learners. *Mathematics Education Research Journal*, *22*(1), 1-2. DOI:10.1007/BF03217555

Khalil, M., Sultana, N., & Khalil, U. (2017). Exploration of mathematical thinking & its development through GeoGebra. *Journal of Educational Research, 20*(1), 83.

Kaya, A., & Öcala, M. (2018). Meta-Analysis for the Effect of GeoGebra on Students' Academic Achievements in Math. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 12* (2), 31-59. DOI: 10.17522/balikesirnef.505918

Kong, Q., Wong, N., & Lam, C. (2003). Student engagement in mathematics: Development of instrument and validation of construct. *Mathematics Education Research Journal, 15*(1), 4-21. DOI:10.1007/BF03217366

Kovács, Z., Recio, T., & Vélez, M. P. (2018). Using automated reasoning tools in GeoGebra in the teaching and learning of proving in geometry. *International Journal for Technology in Mathematics Education*, *25*(2), 33-50. DOI:10.1564/tme_v25.2.03

Kyriakides, A. O., Meletiou-Mavrotheris, M., & Prodromou, T. (2016). Mobile technology in service of students' learning of math: The example of game applicationA.L.E.X. in the context of a primary school in Cyprus. *Mathematics Education Research Journal, 28*(1), 53-78. DOI:10.1007/s13394-015-0163-x

Lam, P., & Tong, A. (2012). Digital devices in the classroom - hesitations of teachers-to-be. *Electronic Journal of E-Learning, 10*(4), 387.

Larkin, K., Larkin, K., Calder, N., & Calder, N. (2016). Mathematics education and mobile technologies. *Mathematics Education Research Journal, 28*(1), 1-7. DOI:10.1007/s13394 015-0167-6

Lazarides, R., & Rubach, C. (2017). Instructional characteristics in mathematics classrooms: Relationships to achievement goal orientation and student engagement. *Mathematics Education Research Journal, 29*(2), 201-217. DOI:10.1007/s13394-017-0196-4

Lerman, S. 2001, "Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics," *Educational Studies in Mathematics*, vol. 46, no. 1/3, pp. 87-113.

Litster, K., Moyer-Packenham, P. S., & Reeder, R. (2019). Base-10 blocks: A study of iPad virtual manipulative affordances across primary-grade levels. *Mathematics Education Research Journal, 31*(3), 349-365. DOI:10.1007/s13394-019-00257-2

Martin, A.J., & Marsh, H.W. (2006). Academic resilience and its psychological and educational correlates: A construct validity approach, *Psychology in the Schools, 43* (3), 267-281.

Mavridis, A., Katmada, A., & Tsiatsos, T. (2017). Impact of flexible online games on students' attitudes towards math. *The Educational Technology Research and Development, 65*(6), 1451-1470. DOI:10.1007/s11423-017-9522-5

Michalewicz, Z., Fogel, D. B., & SpringerLink (Online service). (2004;2000;). *How to solve it? Modern heuristics* (2nd, rev. and extended ed.). Berlin: Springer. DOI:10.1007/978-3-662 04131-4

Muir, T. (2014). Google, Mathletics, and Khan academy: Students' self-initiated use of online mathematical resources. *Mathematics Education Research Journal*, *26*(4), 833-852. DOI:10.1007/s13394-014-0128-5

Nobre, C. N., MEIRELES, M. R. G., JUNIOR, N. V., RESENDE, M. N. d., COSTA, L. E. d., ROCHA, R. C. d., . . . Department of Computer Science, Federal University of São João del Rey. (2016). The use of GeoGebra software as a calculus teaching and learning tool. *Informatics in Education - an International Journal, 15*(2), 253-267. DOI:10.15388/infedu.2016.13

Norton, S. (2006). Pedagogies for engagement of girls in the learning of proportional reasoning through technology practice. *Mathematics Education Research Journal, 18*(3), 69-99. DOI:10.1007/BF03217443

Olsson J., (2017). GeoGebra, Enhancing Creative Mathematical Reasoning. *UMEA Sverige University*. Doi: 978-91-7601-697-8/1650-8858

Olsson, J. (2017). The Contribution of Reasoning to the Utilization of Feedback from Software When Solving Mathematical Problems. *International Journal of Science and Maths Education, 10*(5), 1-21.

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education, 48*(2), 285-300. DOI: 10.1016/j.compedu.2005.01.006

Redo, T., & Richard P., & Velez, M. (2018). Designing Tasks Supported by GeoGebra Automated Reasoning Tools for the Development of Mathematical Skills. *International Journal of Technology in Mathematics Education, 26(*3), DOI: 10.1564/tme_v26.2.05

Schoenfeld, A. H. (2002). Making mathematics work for all children: The Issues of standards, testing, and equity. *The Educational Researcher, 31*(1), 13-25. DOI:10.3102/0013189X031001013

Shadaan, Praveen & Leong, Kwan Eu (2013). Effectiveness of Using GeoGebra on Students' Understanding in Learning Circles. *Malaysian Online Journal of Educational Technology, 1*(4), 1-11. Retrieved from Eric Search Database. (ERIC Number: EJ1086434)

Skilling, K., Bobis, J., Martin, A. J., Anderson, J., & Way, J. (2016). What secondary teachers think and do about students' engagement in mathematics. *Mathematics Education Research Journal, 28*(4), 545-566. DOI:10.1007/s13394-016-0179-x

Sträßer, R. (2017). Learners' engagement in mathematics—theories, frameworks, and results. *Mathematics Education Research Journal, 29*(2), 255-259. DOI:10.1007/s13394 0170208-4

Watt, H. M. G., & Goos, M. (2017). Theoretical foundations of engagement in mathematics. *Mathematics Education Research Journal,29*(2), 133-142.doi:10.1007/s13394-010206-6

West, S., & Uhlenberg, D. (1970). Measuring motivation. *Theory into Practice, 9*(1), 47-55. DOI:10.1080/00405847009542256

Zengin, Y. (2017). The Potential of GeoGebra Software for Providing Mathematical Communication in the Light of Pre-service Teachers' Views. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *11*(1), 101-127.

Zulnaidi, H., Oktavika, E., & Hidayat, R. (2019). Effect of use of GeoGebra on the achievement of high school mathematics students. *Education and Information Technologies*, 1-22. DOI:10.1007/s10639-019-09899-y

Appendix A

National Survey of Student Engagement – Indiana University Research Center

Behavioral Engagement

1. During the school year, about how often have you done the following in mathematics geometry sessions?

   (5=very often, 4=often, 3=sometimes, 2=rarely, 5=never)

   - Asked question or contributed to course discussions in other ways
   - Prepared extra drafts of a paper or assignment before turning it in
   - Come to class without completing assignments
   - Prepared for exams by discussing or by working through course material with other students
   - Explained course material to your peers
   - Discussed course topics, or ideas or concepts with a friend outside class

2. About how many hours do you spend in a typical 7-day week preparing for class (studying, reading, writing, doing homework or lab work, analyzing data, rehearsing)?

   (5=21hours-25 hours, 4=16hours-20 hours, 3=11hours-15 hours, 2=6hours-10 hours, 1=0hours-5 hours)

NSSE
national survey of
student engagement

Cognitive Engagement

1. During the school year, about how often have you done the following in mathematics geometry session?

   (5=very often, 4=often, 3=sometimes, 2=rarely, 5=never)

   - Asked another student to help you understand course material
   - Combined ideas from different lessons when completing assignments
   - Examined the strengths and weaknesses of your views on a topic or issue
   - Connected ideas from your courses to prior experiences and knowledge
   - Reviewed your notes after class
   - Reached conclusions based on your analysis of information (numbers, graphs, statistics, and others)
   - Used information to examine a real-world problem or issue
   - Evaluated what others have concluded
   - Summarized what you learned in class from the geometry course materials

2. During this current school year, how much has your geometry coursework emphasized the following?

   (5=very often, 4=often, 3=sometimes, 2=rarely, 5=never)

NSSE
national survey of
student engagement

- Memorizing course material
- Applying facts, theories, or methods to practical problems or new situations
- Analyzing an idea, experience, or line of reasoning in depth by examining its parts
- Evaluating a point of view, decision, or information source
- Forming a new idea or understanding from various pieces of information

3. How much has experience at this course contributed to your knowledge, skills, and personal development in the following areas?

   (5=very much, 4=much, 3= quite a bit, 2= some, 1=none)

- Writing mathematical language clearly and effectively
- Speaking mathematical language clearly and effectively
- Thinking critically and analytically
- Solving complex real-world problems

Affective Engagement

1. During the school year, about how often have you used the information to examine a real-world problem or issue in mathematics geometry sessions?

   (5=very often, 4=often, 3=sometimes, 2=rarely, 5=never)

NSSE
national survey of
student engagement

2. During this current school year, to what extent have the geometry mathematics courses challenged you to do your best?

(5=very much, 4=much, 3= quite a bit, 2= some, 1=none)

3. How much does this course emphasize the following?

(5=very much, 4=much, 3= quite a bit, 2= some, 1=none)

- Provide support to help students succeed academically
- Using learning support services (tutoring services, revision sheets, extra summaries)
- Provide opportunities for all to be involved
- Provide support for your overall well-being
- Helping you manage your non-academic responsibilities

Appendix B

Student Engagement in Mathematics Interview Questions

General

1. What is the primary goal of using GeoGebra in geometry classrooms?
2. What are the advantages and drawbacks of using GeoGebra in geometry classrooms?

Cognitive Engagement

3. What do you think might be the impact of using GeoGebra on your thinking ability?
4. In what ways would the use of GeoGebra enhance your learning capability?
5. In what ways would the use of GeoGebra empower you to control your learning of geometry?
6. Do you feel that the use of GeoGebra could distract your learning in the classroom? Why and why not?

Affective Engagement

7. Do you struggle in learning Geometry? Does the use of GeoGebra help overcome your struggle and frustration?
8. Explain whether using GeoGebra would enhance your interest in geometry lessons?
9. Do you think GeoGebra makes learning geometry more fun and productive? Explain how.

Behavioral Engagement

10. How does GeoGebra affect geometry classroom discourse?

11. Do you think using GeoGebra enhances students' active participation? Explain how.

12. How often are you using GeoGebra in your free time? Why and why not?

Appendix C

Student Indicator of Engagement Form

Teacher: ____________________ **Observer: ____________________**

Date: _______________________ **School: ______________________**

Subject: _____________________ **Grade Level: __________________**

Lesson Explained: _____________ **Observation Start Time: _________**

Observation End Time: ________

Observation "looks-for"	**Category**	**Observed**	**Specify Details (frequency, time interval)**
Verbalization of thinking	Cognitive	☐	
Self-monitoring	Cognitive	☐	
Concentration	Cognitive	☐	
Resisting Distractions	Cognitive	☐	
Gestures	Cognitive	☐	

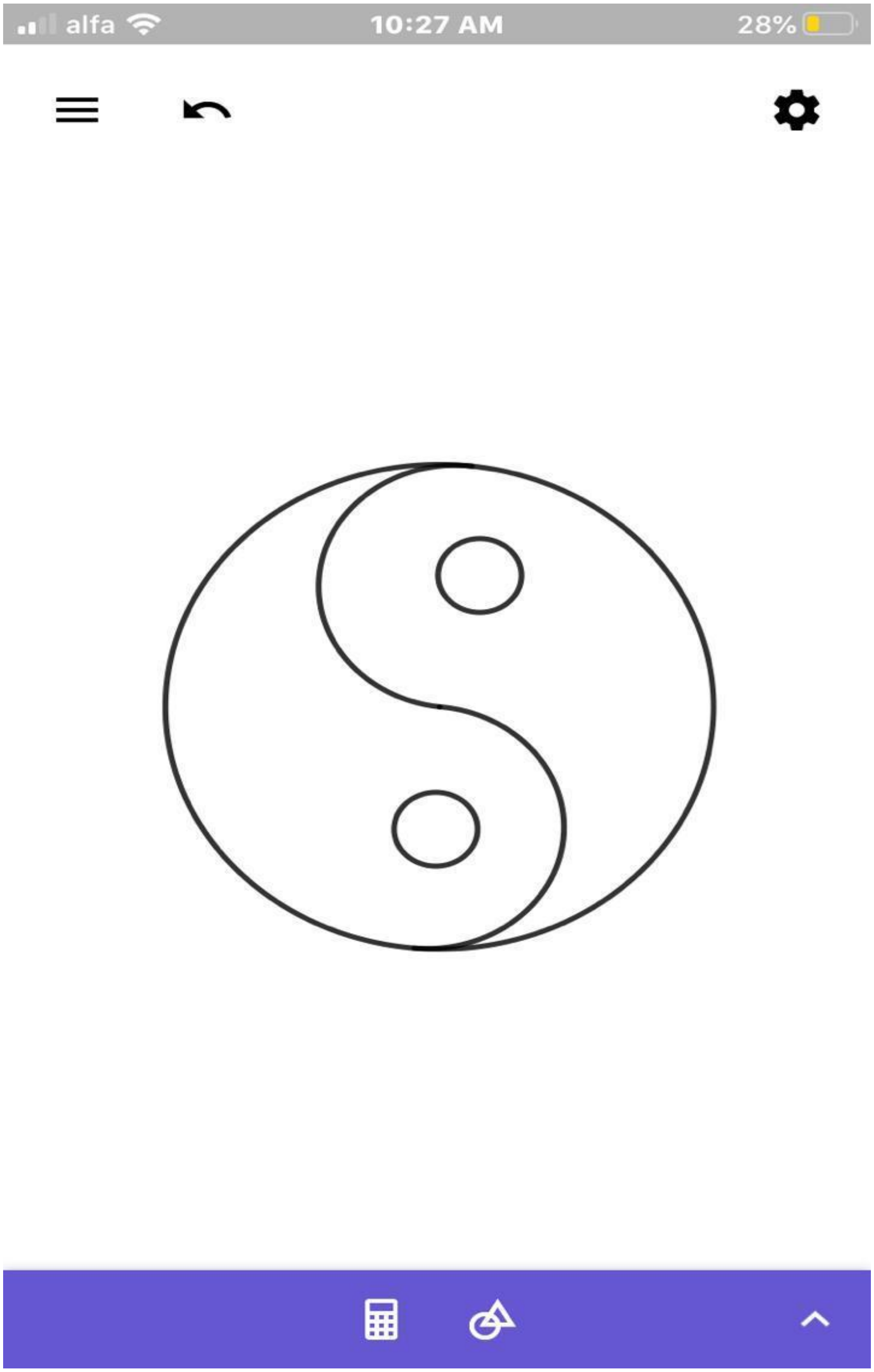
Seeking Help	Cognitive		
Seeking Information	Cognitive		
Seeking Feedback	Cognitive		
Evaluate Comments	Cognitive		
Answer Teacher's Questions	Cognitive		
Give Information	Cognitive		
Explain Procedures	Cognitive		
Reasoning	Cognitive		
Address Deep Questions	Cognitive		
Hold Reflective Self-Questioning	Cognitive		
Self-Monitoring Progress	Cognitive		

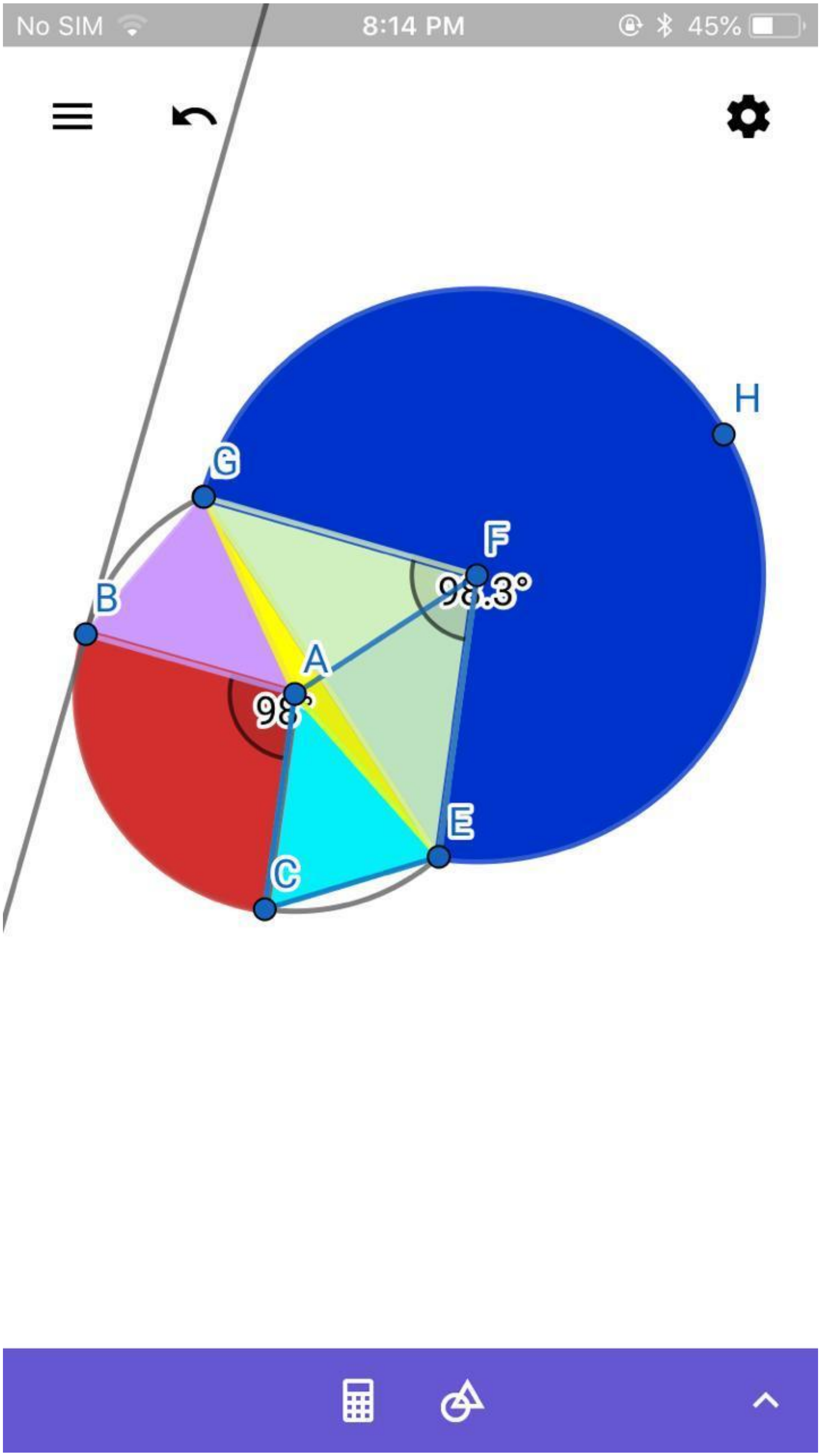
Persist in Completing Tasks	Cognitive	☐	
Organization in solving tasks	Cognitive	☐	
Using Different Strategies	Cognitive	☐	
Ask Random Questions	Behavioral	☐	
Participating in class Discussions	Behavioral	☐	
Interacting with others	Behavioral	☐	
Hold Discussion	Behavioral	☐	
Prepare Extra Work	Behavioral	☐	
Complete Tasks	Behavioral	☐	
Cooperate with Other Students	Behavioral	☐	
Explain the task to Others	Behavioral	☐	

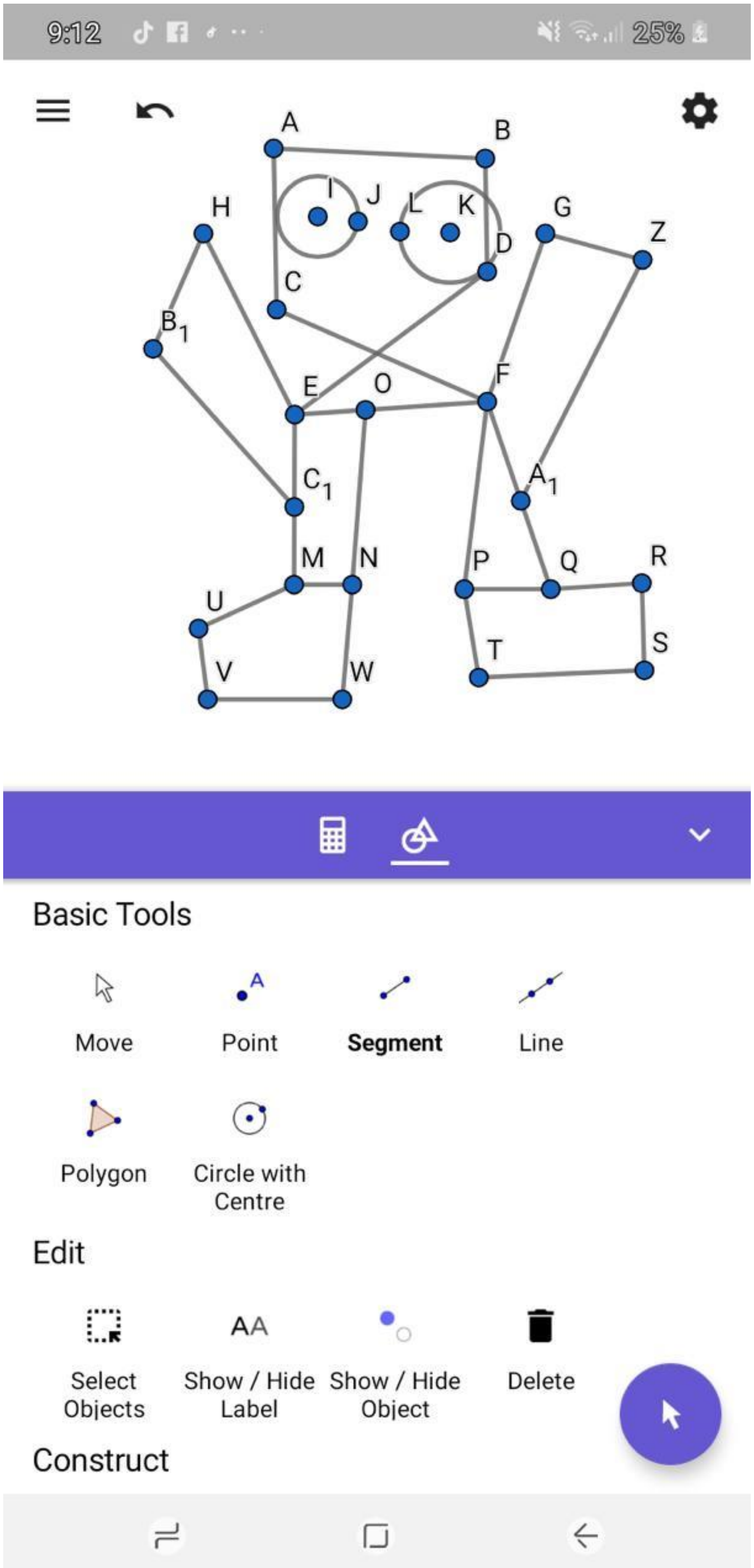
Exchange Ideas	Behavioral	☐	
Get Prepared for Tasks	Behavioral	☐	
Get involved in Voluntary Tasks	Behavioral	☐	
Seem Interested	Affective	☐	
Seem Anxious	Affective	☐	
Seem Curious	Affective	☐	
Hold Positive Attitude	Affective	☐	
Seem Stressed Out	Affective	☐	
Seem Bored	Affective	☐	
Enjoyment	Affective	☐	
Finding Relevance and Value of Task	Affective	☐	

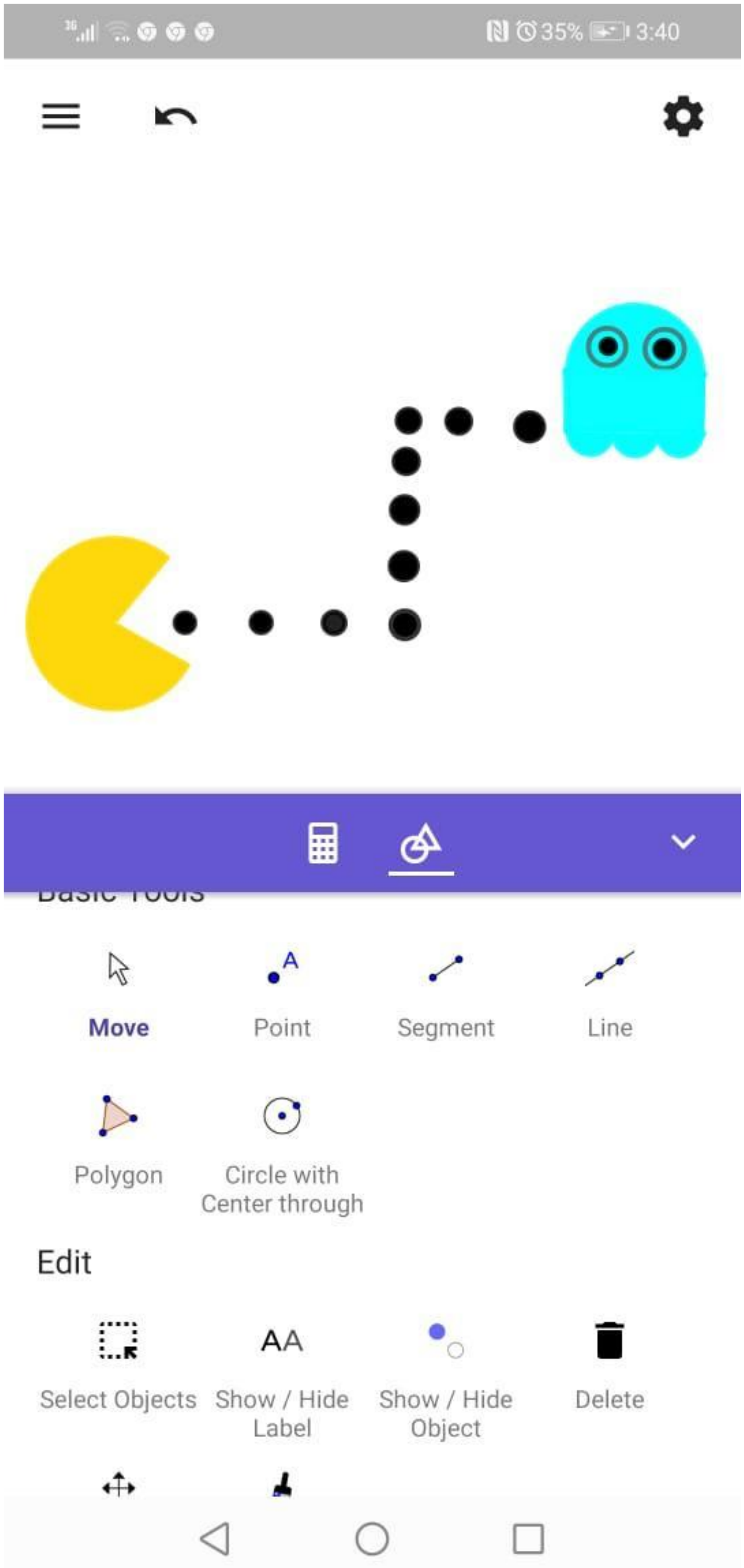
Appendix D

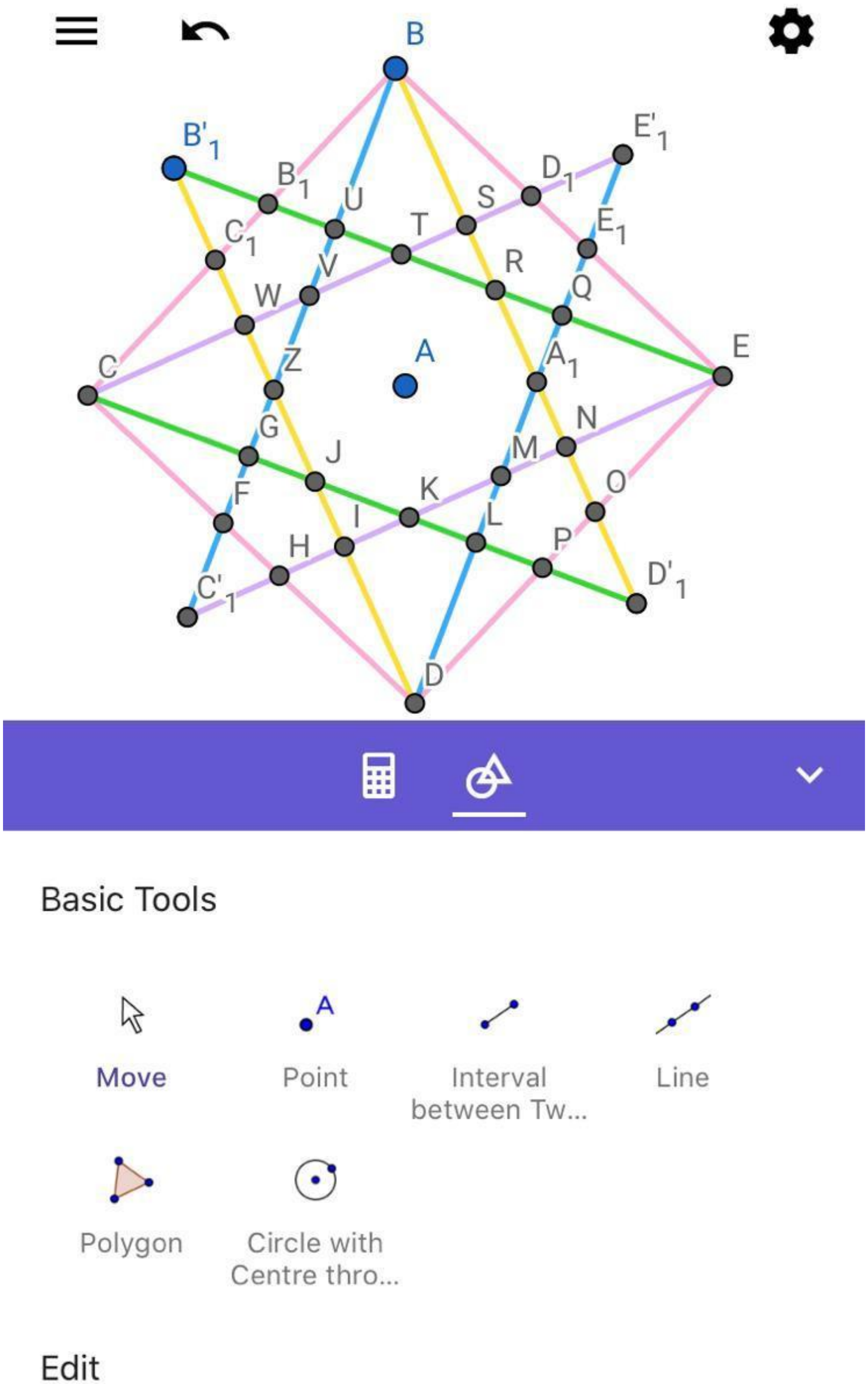
Students' GeoGebra Artifacts

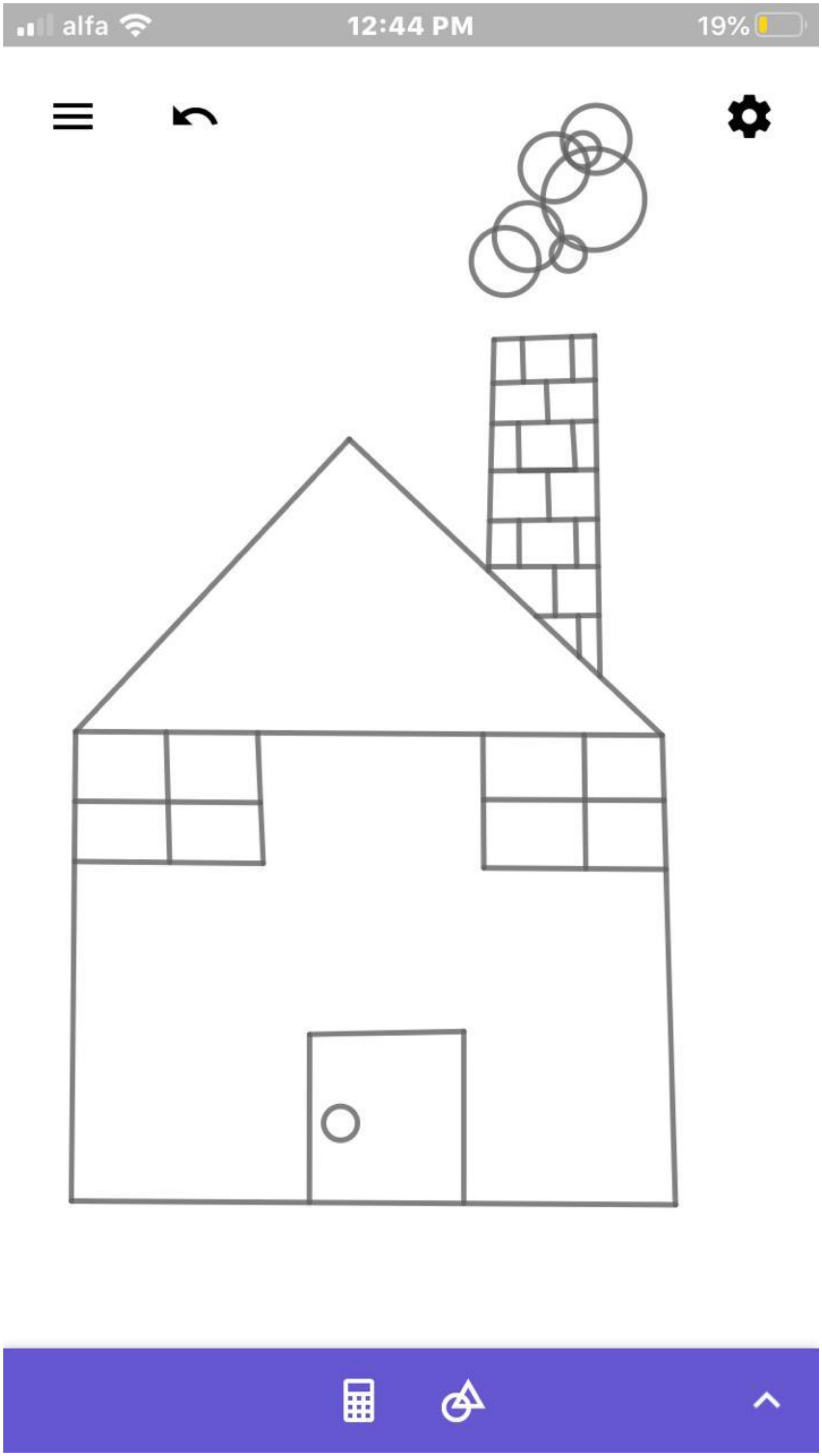


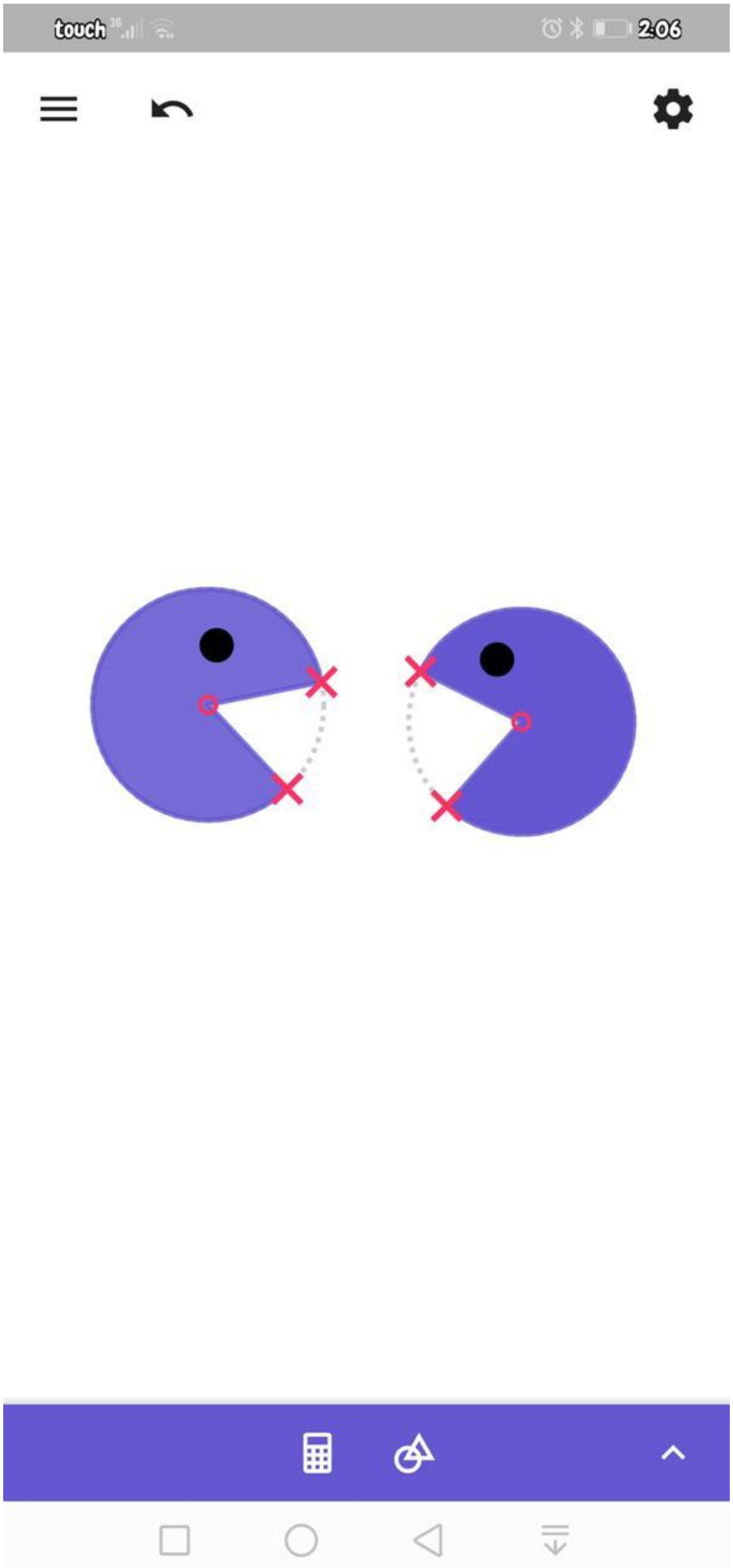

No SIM
8:14 PM
45%
H
G
F
B
A
E
C

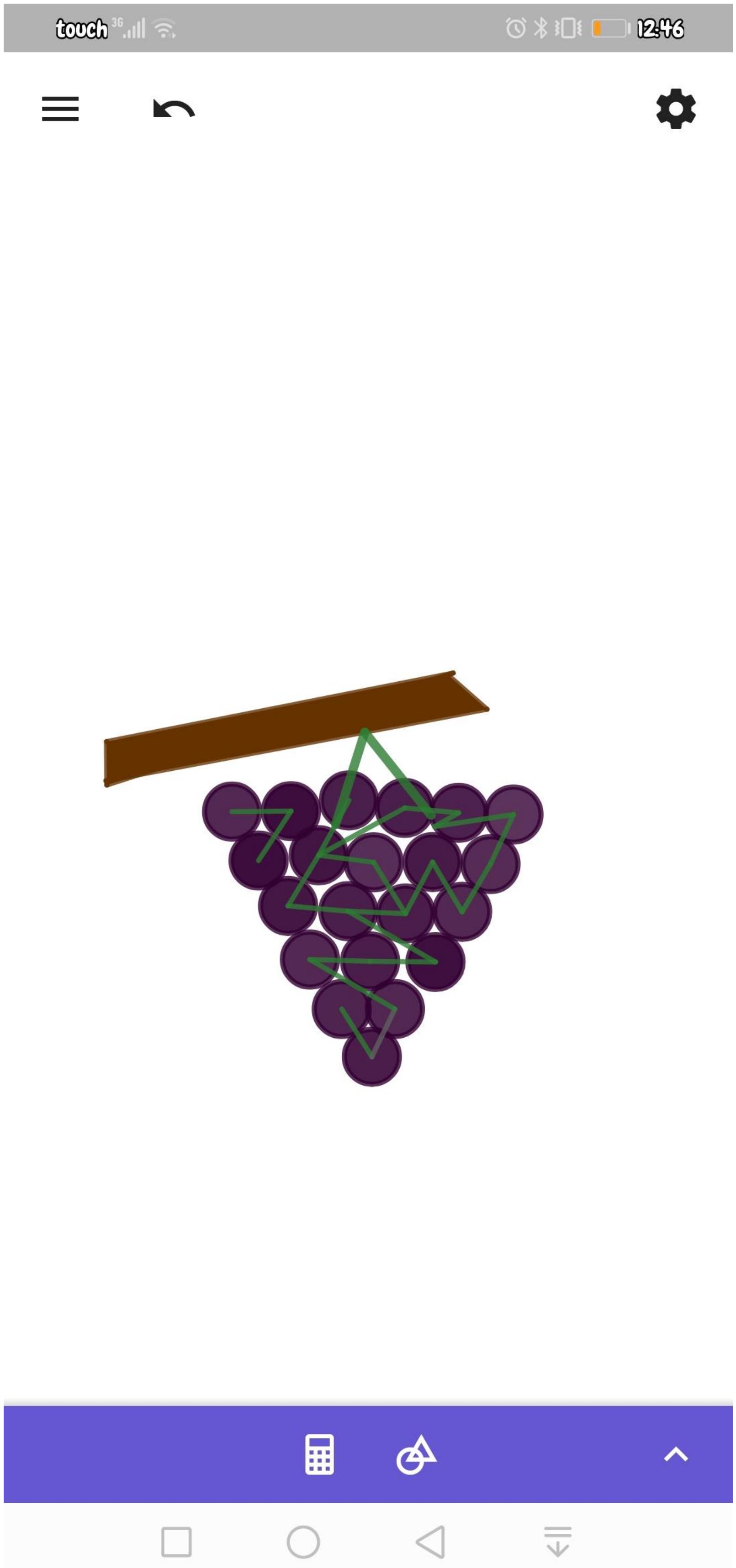

9:12
25%
A
B
I
J
L
K
H
G
Z
D
C
B1
E
O
F
C1
A1
M
N
P
Q
R
U
T
S
V
W
Basic Tools
Move
Point
Segment
Line
Polygon
Circle with Centre
Edit
Select Objects
Show / Hide Label
Show / Hide Object
Delete
Construct

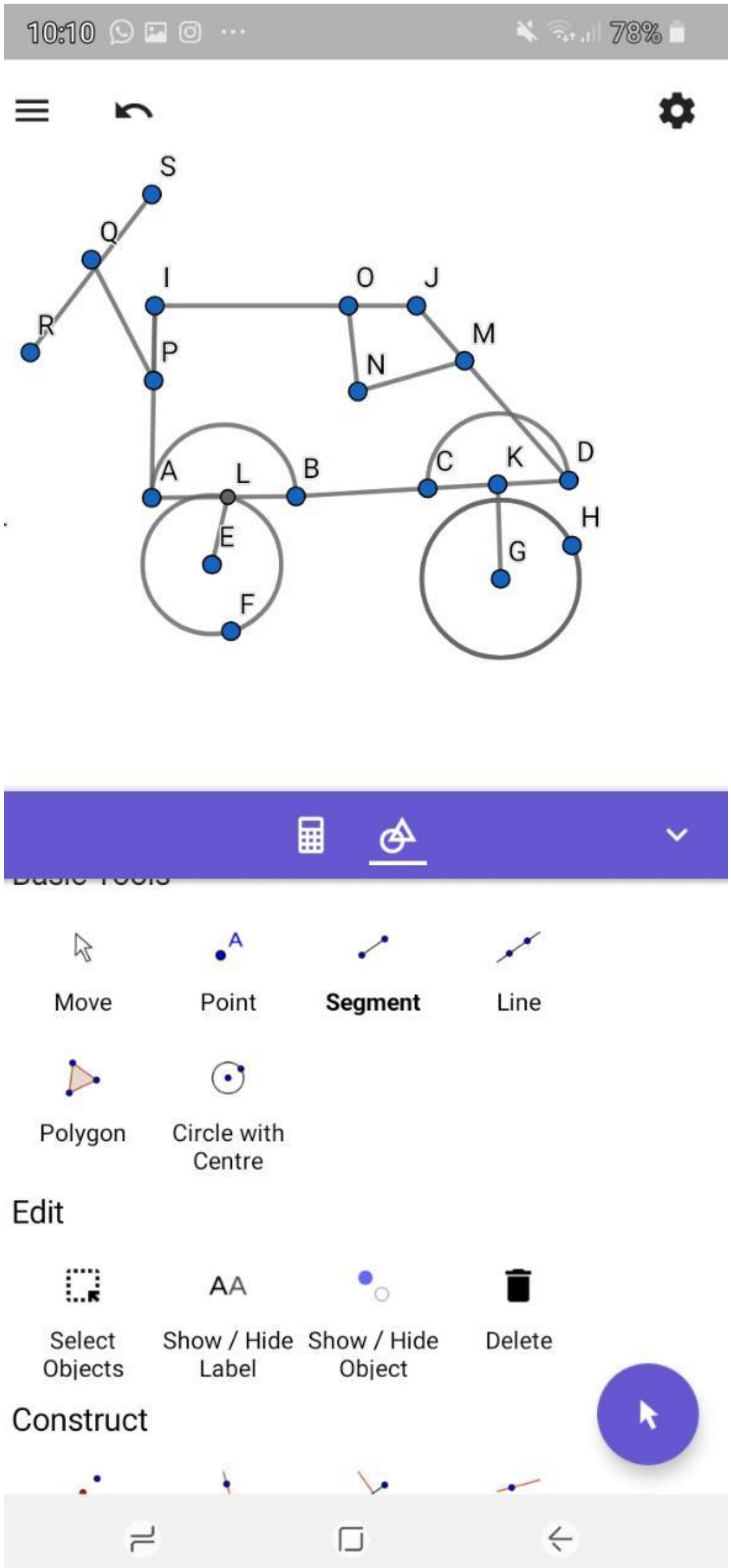

35% 3:40
Basic Tools
Move
Point
Segment
Line
Polygon
Circle with Center through
Edit
Select Objects
Show / Hide Label
Show / Hide Object
Delete

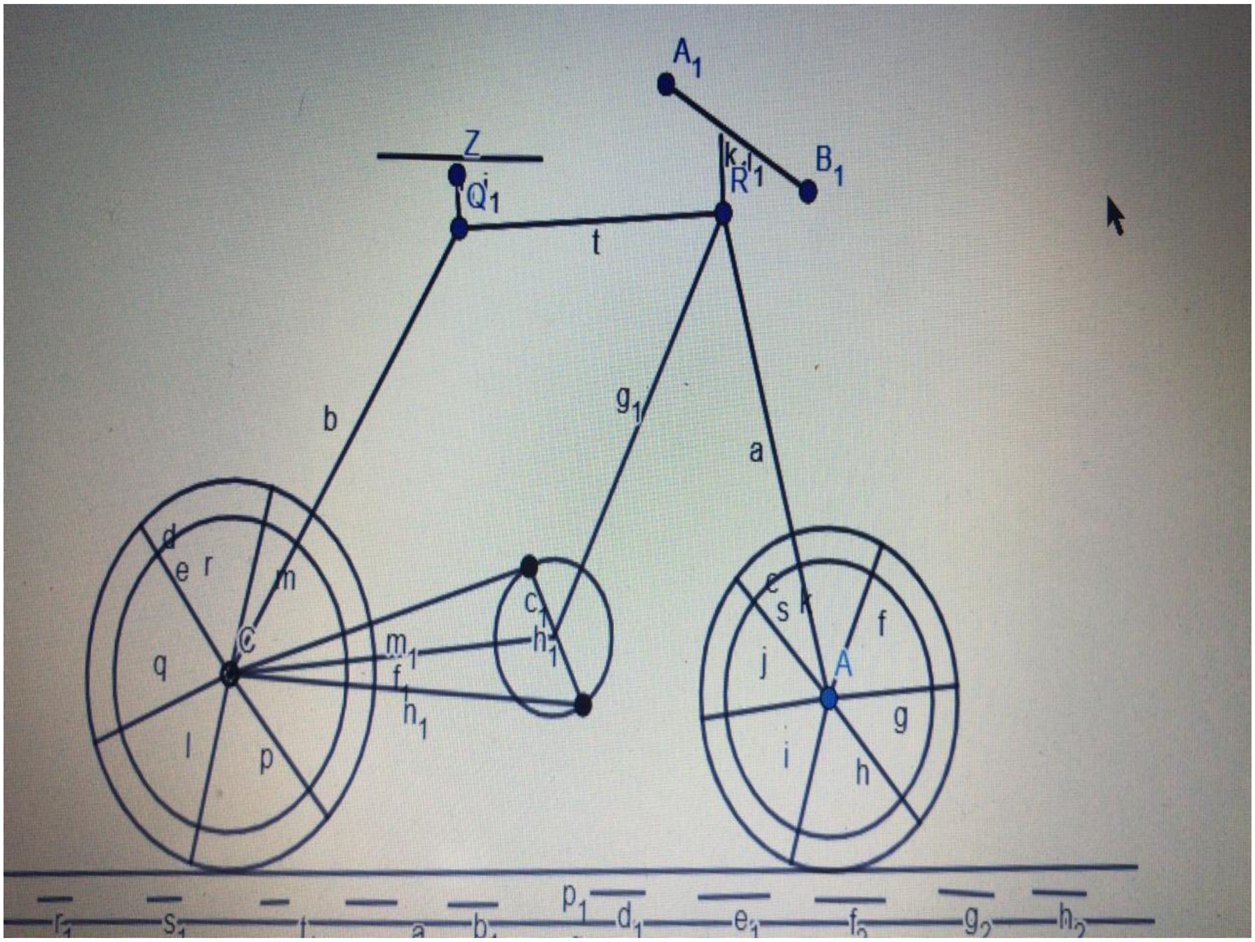

B
B'1
B1
U
T
S
D1
E'1
E1
C1
R
Q
W
V
A
C
Z
A1
E
G
J
M
N
F
K
O
I
L
H
P
C'1
D'1
D
Basic Tools
Move
A
Point
Interval between Tw...
Line
Polygon
Circle with Centre thro...
Edit


alfa
12:44 PM
19%

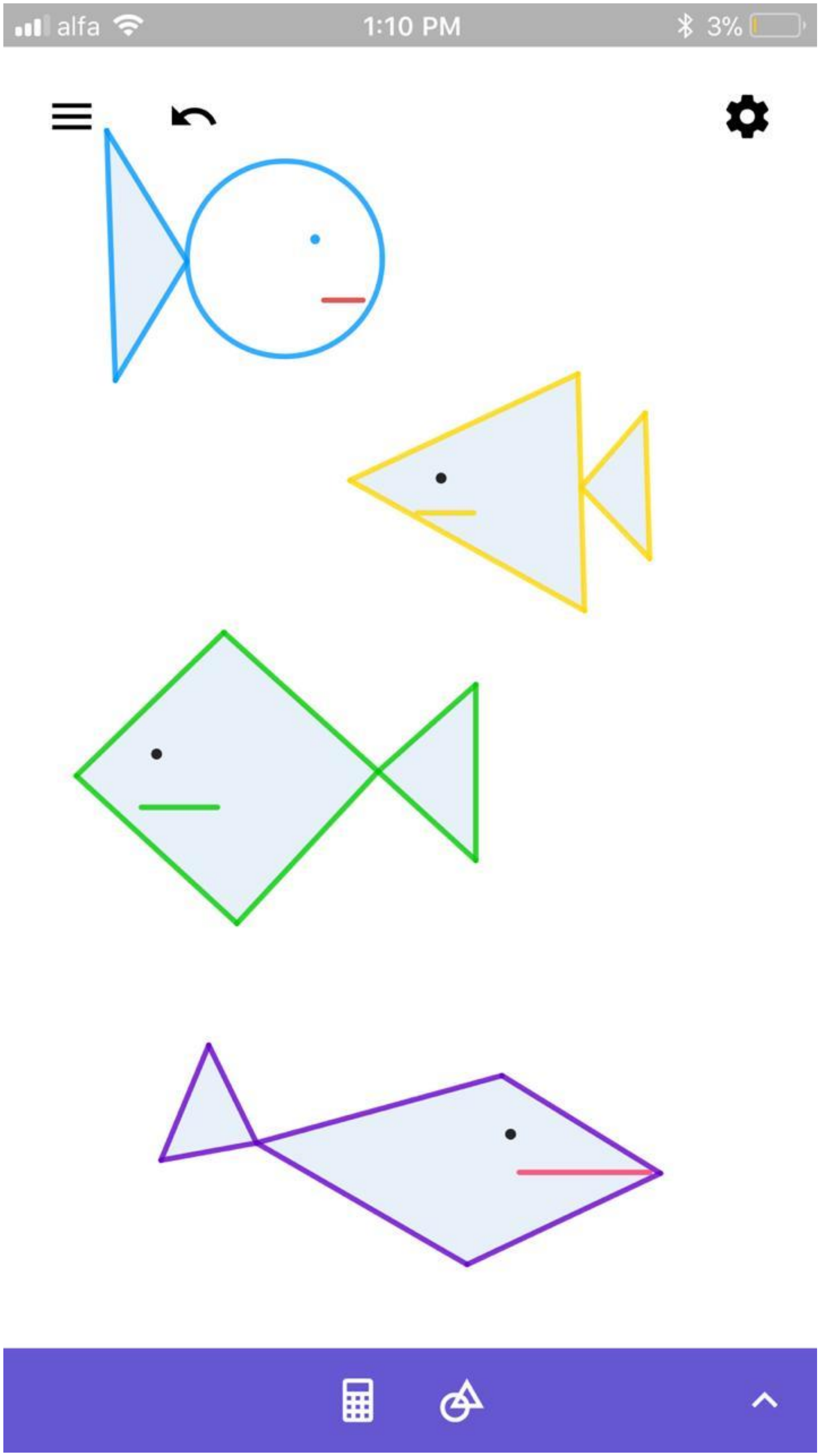

touch
2:06

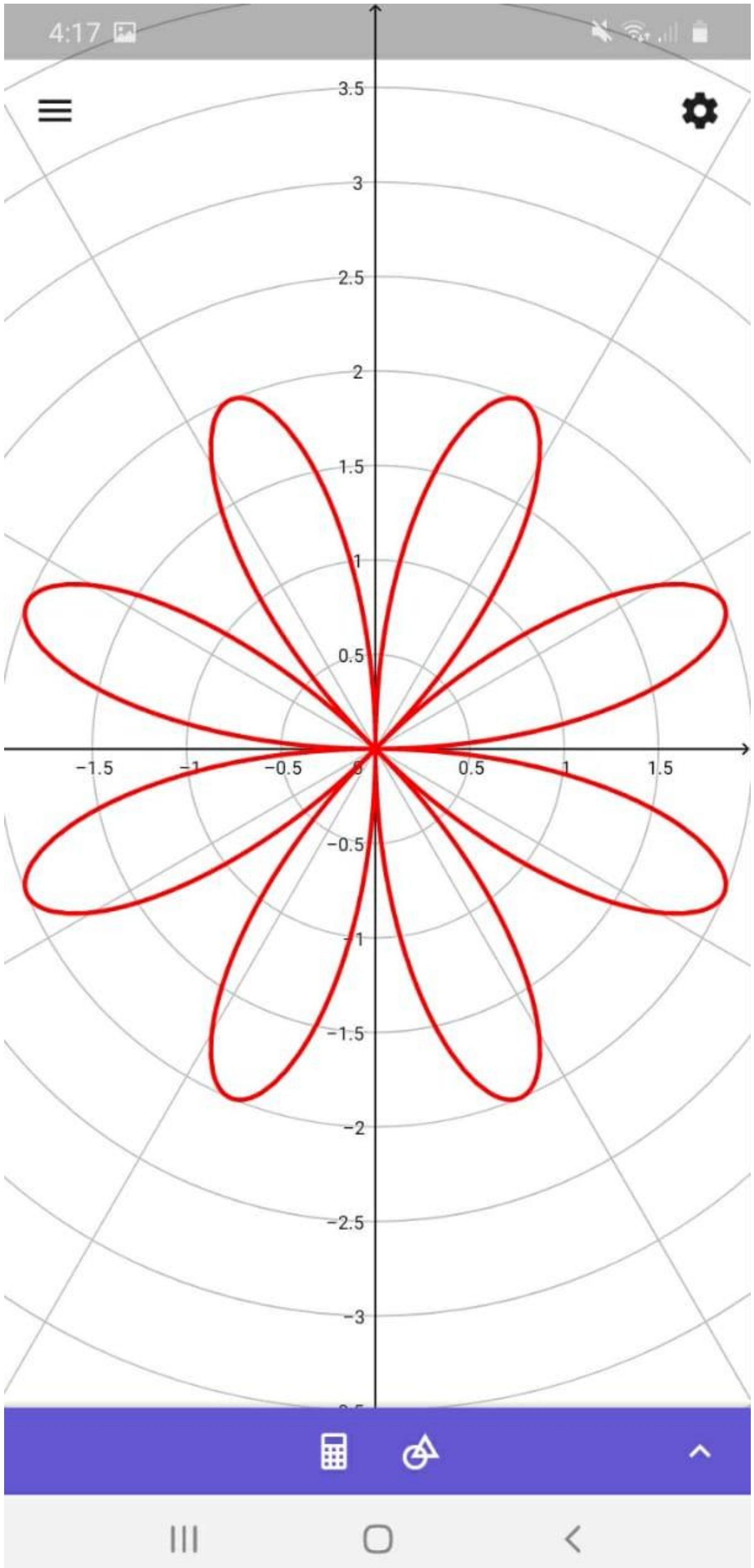

touch
12:46

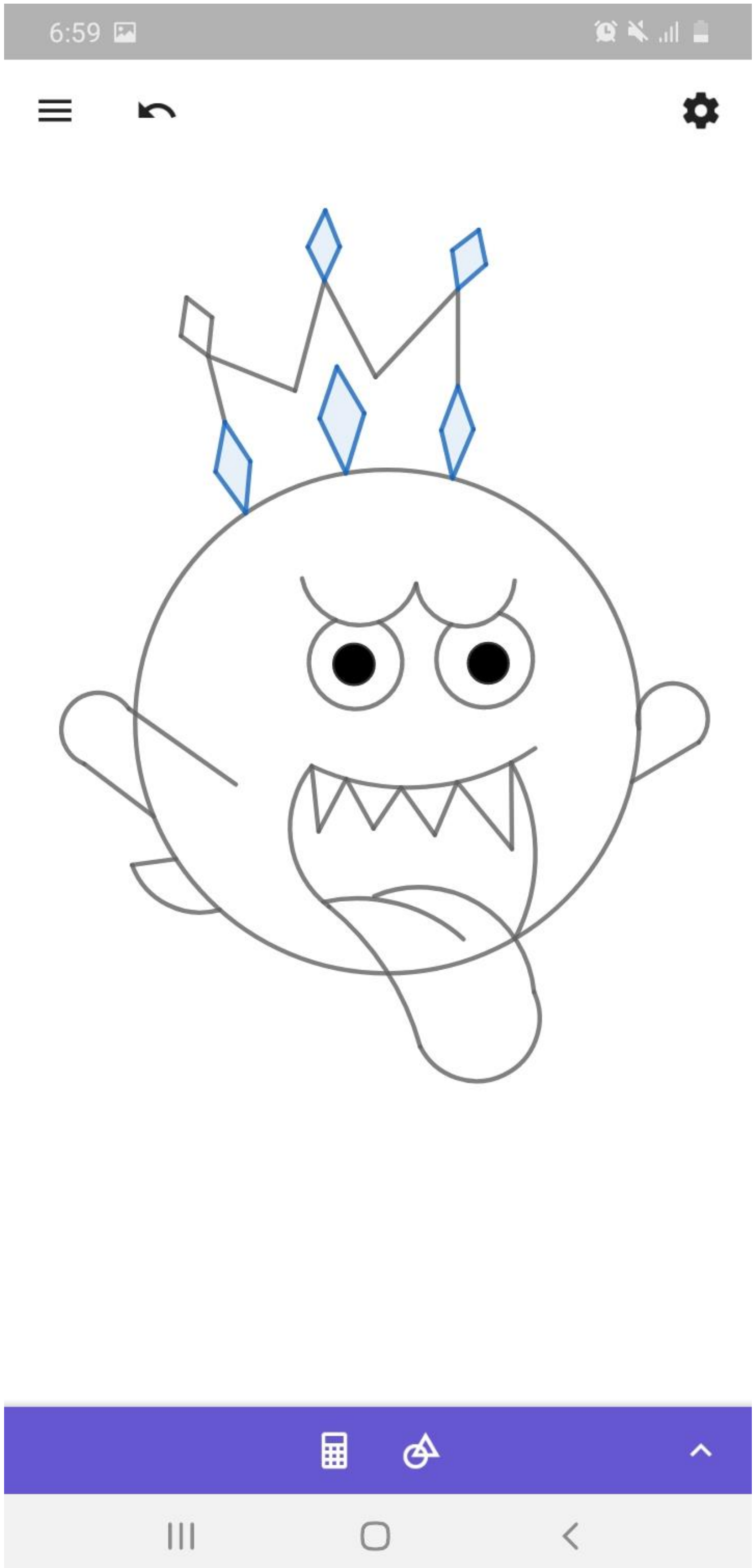

10:10
78%
S
Q
I
O
J
R
P
M
N
A
L
B
C
K
D
E
H
G
F
Move
Point
Segment
Line
Polygon
Circle with Centre
Edit
Select Objects
Show / Hide Label
Show / Hide Object
Delete
Construct

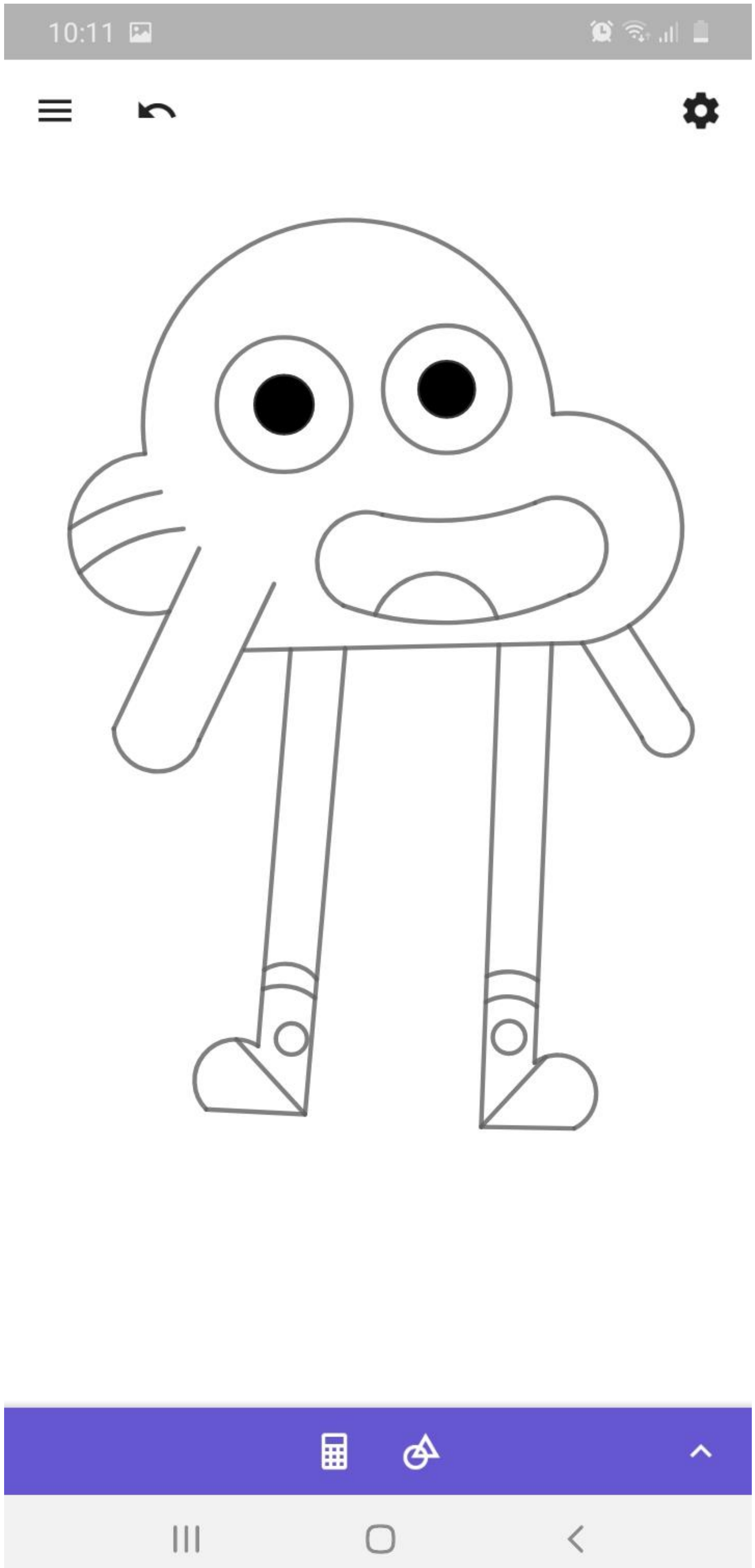

A1
Z
B1
Q1
R
k1
t
b
g1
a
d
e
r
m
C
c1
h1
m1
f1
q
l
p
c
s
k
f
j
A
g
i
h
p1
r1
s1
d1
e1
g2
h2

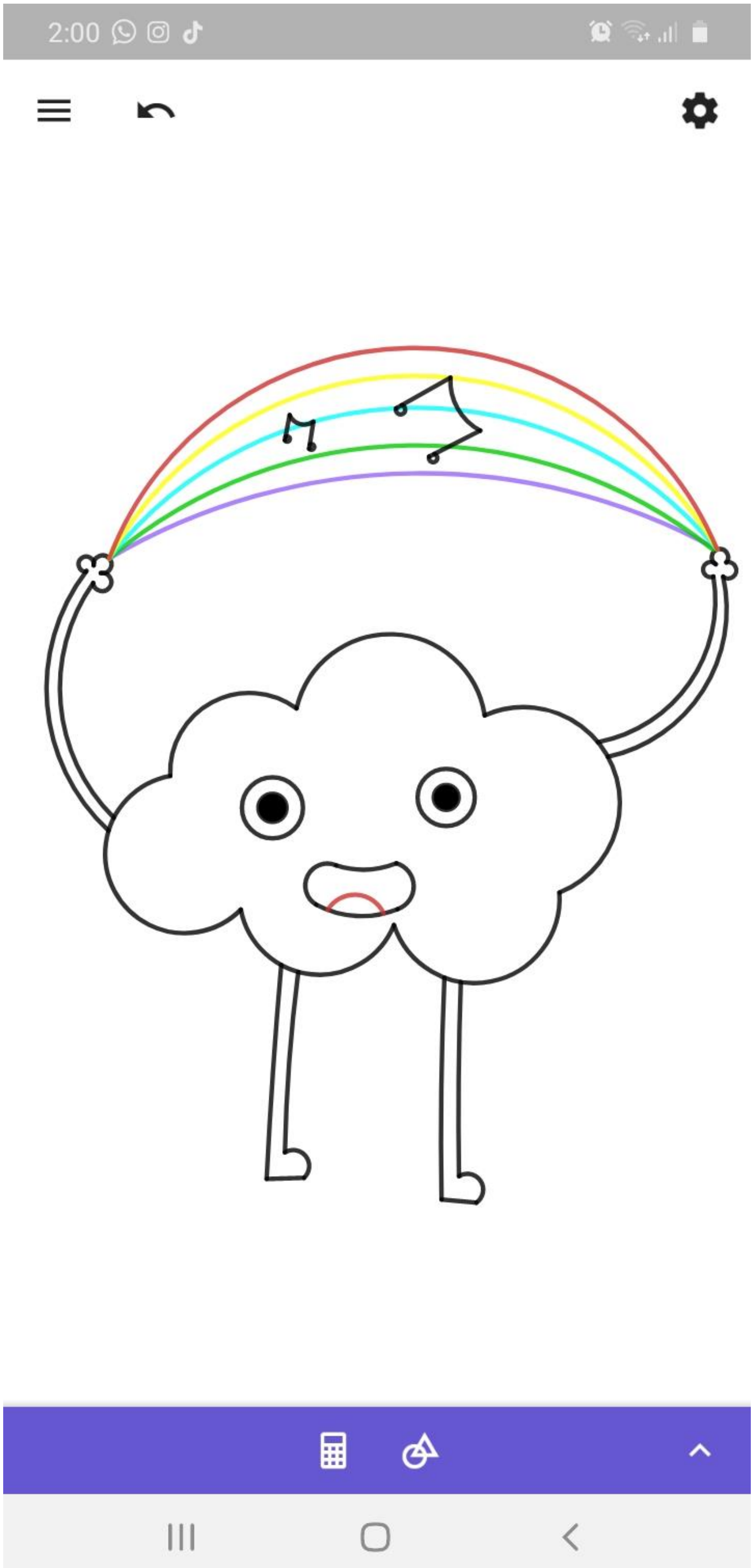

47%
3:23 pm


alfa
1:10 PM
3%


4:17
3.5
3
2.5
2
1.5
1
0.5
−1.5
−1
−0.5
0
0.5
1
1.5
−0.5
−1
−1.5
−2
−2.5
−3


6:59

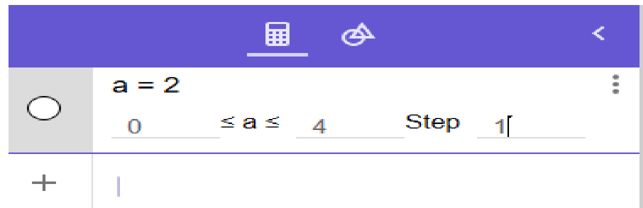

10:11


2:00

Appendix E

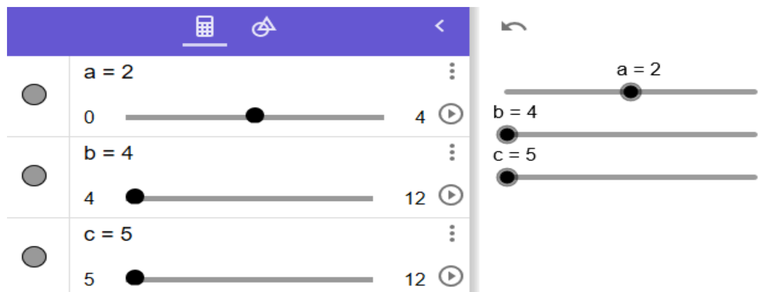
First GeoGebra Lesson- Investigation of Pythagorean Theorem

Follow the below instructions

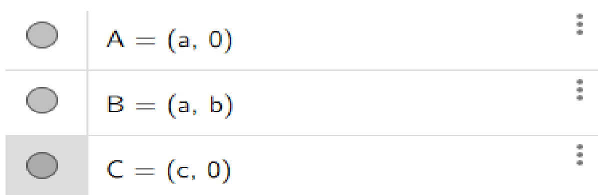


1. Input

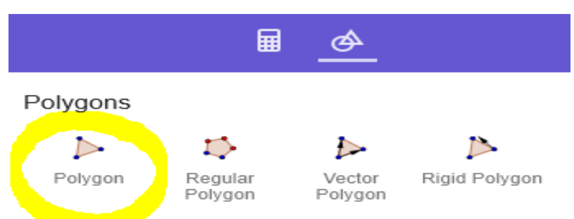
- a= 2
- set the slider (between 0 and 4) with (step or increment = 1), press enter



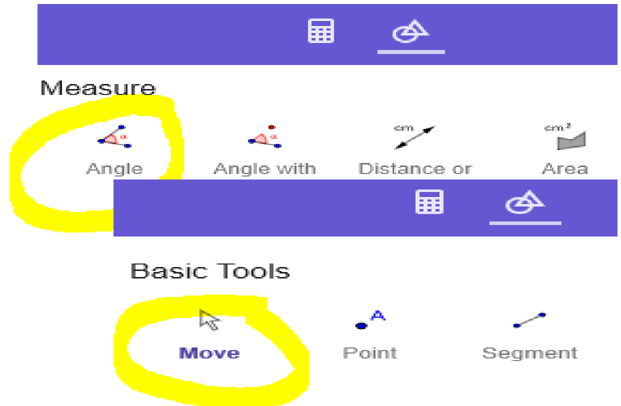
- b= 3
- set the slider (between 4 and 12) with (step or increment = 1), press enter
- c= 5
- set the slider (between 5 and 12) with (step or increment = 1), press enter
- (a, 0)
- (a, b)
- (c, 0)



2. Choose the Polygon Tool, then press on points A, B, C, A.

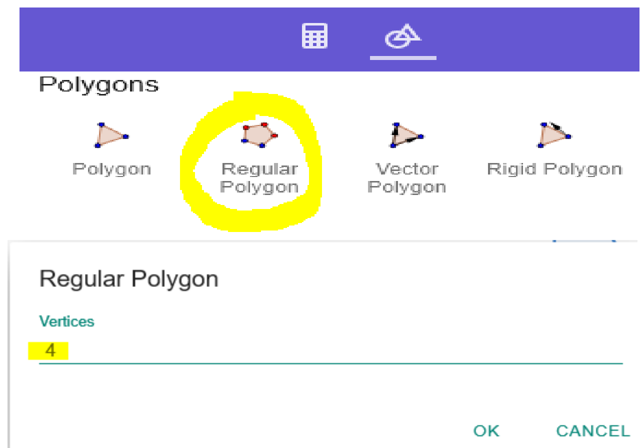


3. Choose the Measure angle tool, then press on points B, then A, then C.



4. Choose the Move Basic Tool, then organize the figure.

5. Choose Regular Polygon Tool.



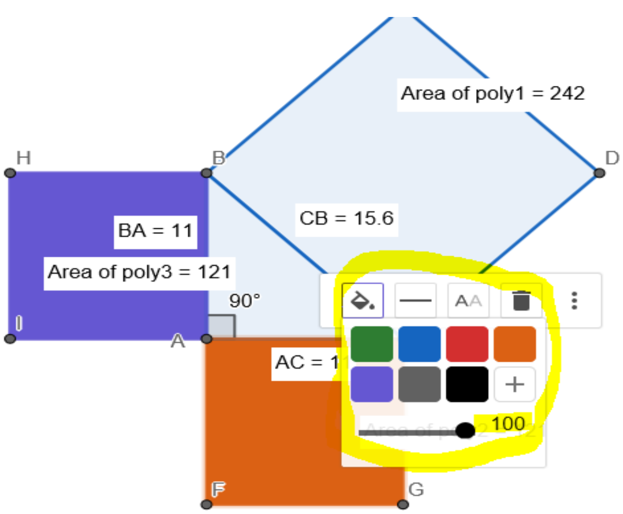
6. Press on point B, then press on point A, then set vertices as 4.

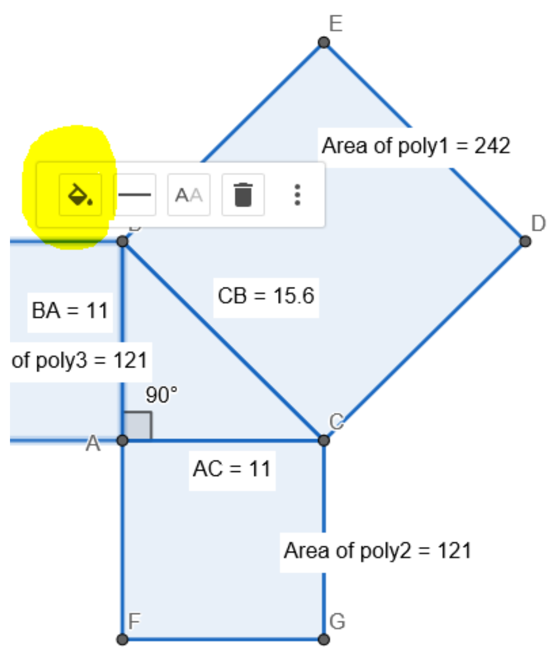
7. Press on point A, then press on point C, then set vertices as 4.

8. Press on point C, then press on point B, then set vertices as 4.

9. Press on the Move Basic Tool again.

10. Press on square ABED and color it purple.





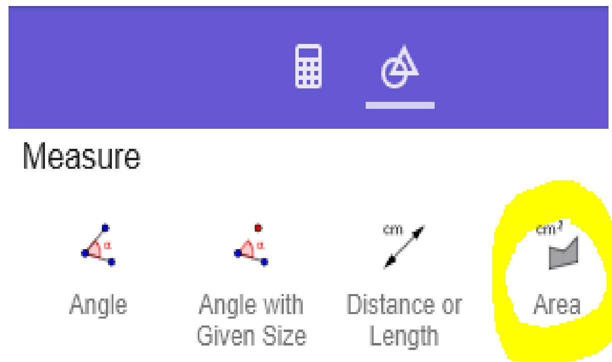
11. Press on square BCIH and color it orange.

12. Press on square ACFG and color it green.

13. Choose the Area Tool.

14. Measure the area of the purple square by pressing on it.

15. Measure the area of the orange square by pressing on it.



16. Measure the area of the green square.

17. Right-click. Remove axis.

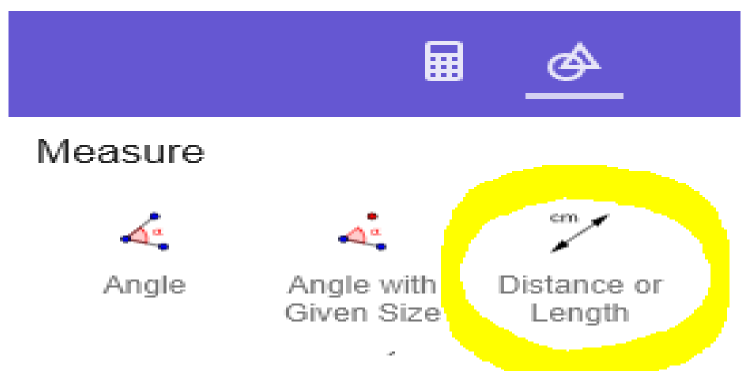
18. Move the sliders. Find the sum of “the orange area + the purple area.”

19. What do we notice?

20. Is this always true?

21. Write the equation that shows this relationship.

22. Choose the measure Distance or Length Tool.



23. Measure the length of side AB.

24. Measure the length of side BC.

25. Measure the Length of side CA.

26. Rewrite the previous equation in the using sides AB, BC, and CA.

Appendix F

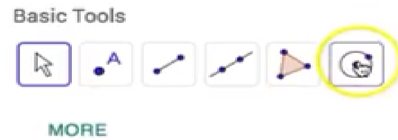
Second GeoGebra Lesson- Pythagorean Spiral

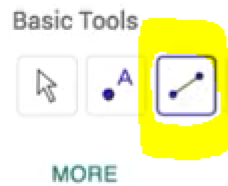
1. Draw a line segment, AB, of unit length
2. Construct a perpendicular line to AB at B
3. With B as the center, construct a circle passing through A.
4. Mark the point of intersection of the circle and the perpendicular line as C
5. Use the Polygon Tool to construct the triangle ABC. What is the length of the hypotenuse AC?
6. Change the color of the triangle ABC
7. Repeat the steps of construction (2-5) on point C. What is the length of the hypotenuse of the second triangle?
8. Repeat the construction till everyone gets the triangle with a hypotenuse of length. Clean up the construction by hiding the perpendicular lines and circles.

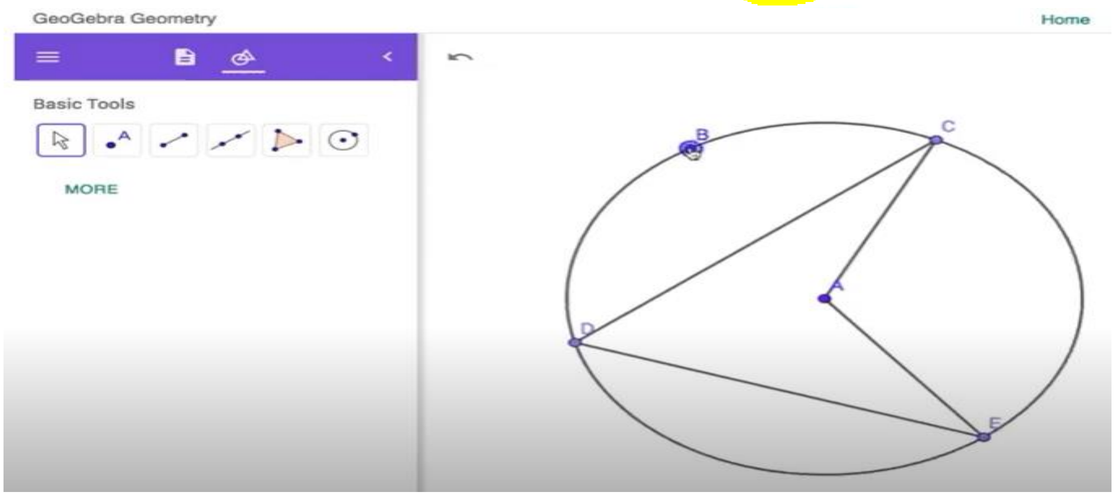
Appendix G

Third GeoGebra Lesson - Investigation of Arcs

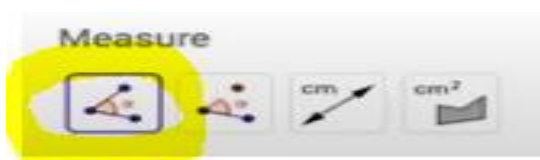
1. Press the Circle Tool C (A, r)
2. Press the Segment Tool to draw a chord [CD] inside the circle
3. Using the Segment Tool draw another chord [DE]
4. Then draw the radii [EA] and [AC]
5. Use the Move Tool on points C, D, E and B





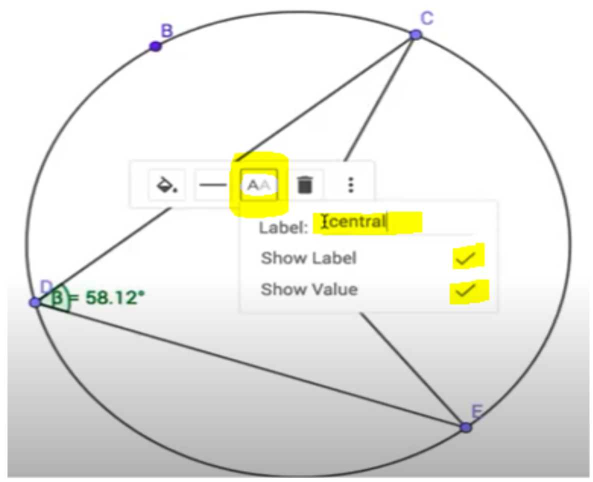


6. Press the Angle Measure Tool
7. Measure angle (EAC) and angle (EDC)

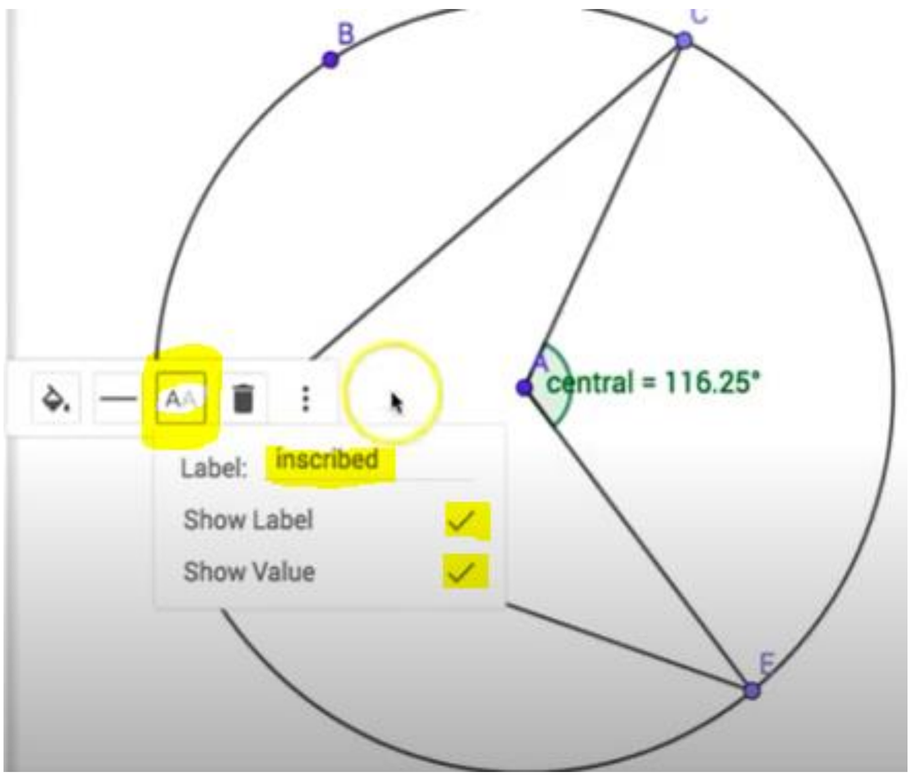


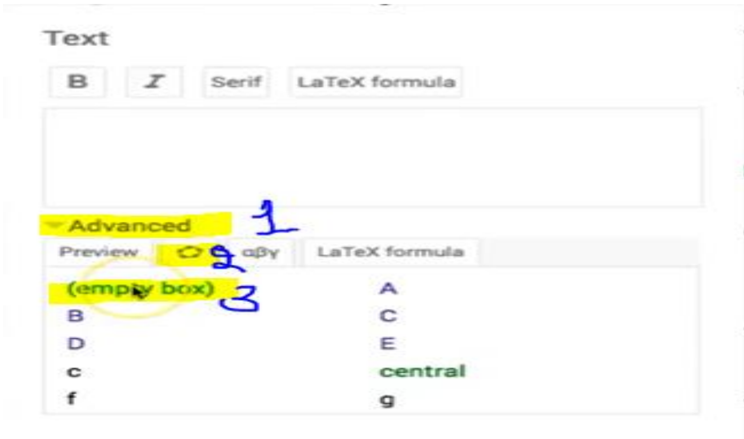
8. Press on point A by a left-clicks to get a toolbar, choose from it the Alphabetical Tool.

9. Label the angle (EAC) as "Central."
10. Press on point D by a left-click to get a toolbar, choose from it the Alphabetical Tool.



11. Label the angle (CDE) as "Inscribed."

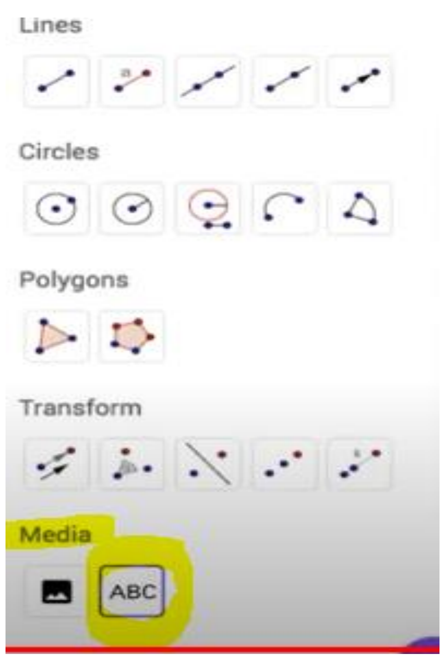




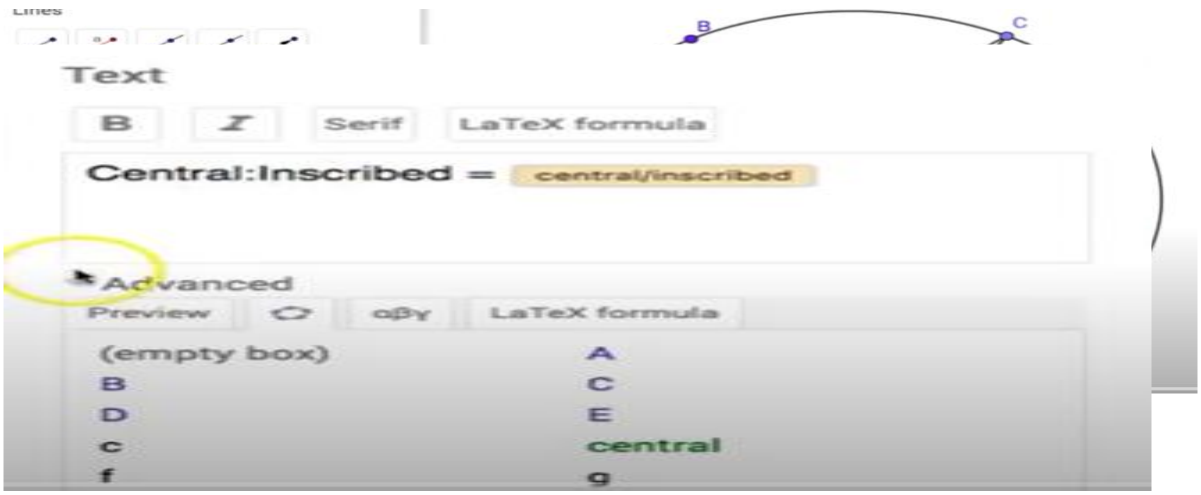
12. Press the Media Tool, then choose the ABC tab

13. Choose advanced in the box.

14. Press on the GeoGebra symbol in the advanced bar, then choose the (empty box)



15. Write in the empty box (Central/inscribed)



16. Write before the empty box: Central / inscribed=

17. What can we conclude?