AMERICAN UNIVERSITY OF BEIRUT

RESERVE STOCK AND THE EFFECT OF SUPPLIER DIVERSIFICATION

by
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ABSTRACT

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Reserve stocks are used to protect supply chain operations from interruption effects caused by suppliers' interludes, delays, quality snags and many other reasons. A deliberate strategy proposed in this thesis is to diversify suppliers in order to smooth out the shortage risk induced from supply interruptions.

The model formulated is based on the work of Hansmann (1962) [7] who consider a reserve stock to meet demand in case one source of supply becomes unavailable. Hansmann's model is expanded to include two suppliers. Subsequently, sensitivity analysis is performed on parameters and variables such as demand rate, percentage distribution of stock between suppliers, expected interruption time, shortage cost and other elements in order to compare the results of a dual-sourcing strategy with that of a single-sourcing strategy.

Results indicated that diversification help mitigate shortage risk even when the additional supplier is not superior in terms of cost or reliability. In addition, results show that when having identical suppliers, minimum total cost is incurred when equal amounts are procured from each supplier. In particular, this research indicates that the scenario for the inclusion of a "dominated supplier" to the portfolio of suppliers may be worth taking into consideration. The dominated supplier is characterized by having higher interruption rate, higher mean of interruption time, and higher ordering costs

Finally, in this research, the model is extended to include three or more suppliers available for procurement having different characteristics. Results show that for a more diversified system, more benefits in reducing shortage and overall costs are expected.

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CHAPTER I

INTRODUCTION AND MOTIVATIONS

Managing the continuity of product supply requires the full understanding of all the interconnected risks throughout the full process. Most companies take into consideration risk mitigation strategies that help in reducing the effects of low-impact highly recurrent supply disruptions. However, many do ignore the importance of high-impact low-occurring risks where successful companies with reserve stock strategies tend to take the leap and take over the market with the failure of its competitors. The main challenge behind reserve stock strategies is to intelligently position and size reserves with a minimum cost impact (Chopra & Sodhi, 2004) [5]. Real world inventory situations involve various sources of uncertainties that are related to supply and consumption rates where these sources can be classified into either internal sources or external sources. Internal sources are those that are directly related to operations. For example, a certain machine stopped working in a factory, an electrical failure, lack of financial resources or any other reason. On the other hand, external sources are those that are out of control of operations. For example, the financial crisis that Lebanon is going through where people are not able to transfer money to other countries therefore some supplies cannot be procured. COVID-19 is considered as another external source that has led to interruption of some specified supplies. Thus, the need for reserve stock sustains its importance as a buffer against supply disruptions and tends to smoothen shortage costs by decreasing its direct effects. However, the reduction in shortage costs achieved by having a reserve supply is partially offset by the increase in carrying and holding costs. Therefore, a balanced strategy should be taken into consideration in order to find an optimum quantity in reserve that will imply minimum costs to the inventory system (Salameh & Schmidt, 1984) [19].

On the other hand, some tend to avoid having safety stock policies by implementing alternate strategies such as product substitution, demand management, holding a backup supplier or to accept shortage risk if carrying costs tend to be too costly to the business (Tomlin & Wang, 2011) [26]. Major factors affect the decision to be taken such as the size of the entity, the ability to hold reserve stock, and the type of supplies that are being held and many other factors.

In the process of selecting suppliers, one of the commonly used strategies is to select a unique supplier that provides the minimum cost per unit. The main element of interest behind such strategy depends on either economy of scale where products are procured from a single supplier to benefit from offered discounts or on capacities of suppliers when they are large enough compared to expected demand (Burke, Carrillo, & Vakharia, 2007) [4]. However, in case of any interruption, induced shortage costs outweigh the discounts offered causing a lot of burdens such as loss of sales, reduction in profit margins, and may lead to closure of business.

The two main motivations that this thesis takes into consideration are reserve stock and diversification. The advantages and benefits of each of them independently were studied by many researches threw literature. However, what is done in this thesis is that both concepts are combined into a model, and the combined model is analyzed in order to achieve minimization of total costs and optimization of operations.

What follows are some real life examples related to the two main motivations in order to make this research more concrete.

Before the coronavirus outbreak in December 2019, companies tend to assume that shortages are rare to happen and that they have incorporated into their operations preventive strategies that would overcome some short-lived supply interruptions. However, this pandemic exposed the risks in a way that was not experienced before where global companies that were depending on limited sources of suppliers in order to achieve reduction in costs faced huge interruptions in their supplies and faced gigantic shortage costs (O'Byrne,2020) [14]. Mainly, companies tend to procure, source and produce from the cheapest locations possible worldwide. Therefore, the country that has most of the basic elements that attract the interest of such companies is China. This is due to the fact that China includes the widest markets, with the broadest source of supplies, and the lowest production costs with the needed technological advancements. Therefore, China played a very important economic role that was created by the interest of these companies. After the pandemic, the concept of diversifying sources has come into serious consideration since operations with China was heavily interrupted and most global companies faced serious damages. However, shifting production to similar countries such as Bangladesh, Vietnam, India or etc. is not an optimum solution. The best solution as per Aydin, an expert in operations management and business analytics, in an interview with him done by John Hopkins Magazine is by diversifying a supply base that do not have high correlation with each other. Thus, if a certain region is undergoing some sort of disruptions, then the chain will continue with the other set of diversified suppliers. (Parsons, 2020) [13].

The incident of the Albuquerque production plant in New Mexico on March 17, 2000 serves as a good example regarding the importance of having risk mitigation strategies against supply disruption. A lightning struck a power line and caused a massive disruption in the electrical grid system starting a massive fire in the micro-ship production plant. The semiconductor plant, owned by Royal Philips Electronics, was forced to shut down for weeks in order to recover at full capacity. At time where mobile phone industry was booming all around the world, a disruption of production may lead to a major shift in the balance of power in the industry. The two major customers that were being served were "Nokia Corp." and "Telefon AB L.M. Ericsson". Directly after the incident, Nokia began transferring orders to other Philips plants as well as to other international suppliers. However, Ericsson had no other source of microchips and suffered from severe disruptions for months resulting in loss of sales of around 400 million dollars (Latour, 2001) [9] (Thomas, 2010) [25].

A similar incident happened with the American fast food chain KFC in the United Kingdom after supply interruption of chicken forced it to close hundreds of stores in different locations. It all started when the original supplier of chicken for KFC, Bidvest, lost tender against its rivals in 2017. Afterwards, the new contractors DHL and QSL faced problems in satisfying demand due to the centralized system of warehouses that they used and due to the massive demand for chicken that is spread along the whole country. Lacking a supply strategy, KFC faced gigantic reputational and shortage losses. In order to get things back to normal, KFC divided its contract between the two where Bidvest was re-awarded the supply of around 350 out of 900 restaurants in the north of the country and the remaining areas remained under the scope of DHL and QSL. Therefore, KFC used the concept of diversification in order to resolve the interruption issue that it faced. However, in March 2019, KFC decided to switch back to Bidvest who offered a lower tender price for the whole contract compared to its rivals. Even though Bidvest is considered a reliable supplier for KFC and is offering a lower tender price, the strategy that KFC is following remains vulnerable to shortages due to the various sources of interruptions. In this research, the effects of diversification will prove the advantages of sharing risks and optimizing operations while dividing procurement percentage between the reliable and un-reliable suppliers. (Topping, 2018) [27] (Bentham, 2018)[1].

Figure 1: Tweet #1: Customer's reaction against KFC's 2019 strategy

Figure 2: Tweet #2: Customer's reaction against KFC's 2019 strategy

Figure 3: Tweet #3: Customer's reaction against KFC's 2019 strategy

The rest of the thesis is organized as follows: Chapter 2 presents a brief review of the related literature. Chapters 3 and 4 introduce the model of Hansmann (1962) that this thesis depends on and describes the modifications that has been done on this model. Chapter 5 extends Hansmann's model to include the concept of diversification by having another supplier available for procurement. Chapter 6 includes sensitivity analysis and numerical examples on the model in order to optimize the decision variables that are taken into consideration and in order to study the effects of the variation of the initial parameters used on the results obtained. Chapter 7 shows 3D convexity plots for the total cost function with respect to the decision variables used. The final chapter includes the optimization of a 3-supplier system as a further step towards generalization.

CHAPTER II

LITERATURE REVIEW

In general, supply interruption can be either partial or complete. In our analysis, disruption is taken to be complete for one supplier while others continue normal operation thus resulting in a partial unmet demand. In addition, the probability of supply disruption is considered to be steady over time (stationary) with a negligible chance of having two suppliers interrupting at the same time. In addition, replenishment of reserve stock is taken to be instantaneous. Thus, when supply interruption for a certain supplier ends, all unsatisfied demand will be procured instantaneously.

Thus, an optimum solution is achieved based on minimizing a total cost that consists of three main components: Shortage, ordering and holding. What follows is a brief literature review on the proposed subject compared to our focus of study. Our review is along three main streams of study: supply disruption, reserve stock, and diversification of suppliers.

A. Reserve Stock

Starting with reserve stock, the base model we use in this thesis is proposed by Hasnmann (1962). This classical model calculates an optimum reserve stock for two production machines that work together to produce a common product. The objective is to minimize both idle time and carried supply that would incur holding and shortage costs (Salameh, 1981) [18]. On the other hand, a similar strategy is adopted in manufacturing systems where an optimum quantity of just in time buffer is calculated in order to avoid shortages caused by scheduled maintenance. (Salahmeh & Ghattas 2001)

[20]. Pal, Sana, & Chaudhuri (2013) [12] evaluate the optimal buffer inventory for stochastic demand during a preventive maintenance with an EPQ model. Unlike these works, our model focus on the effects of diversification and an optimum level of reserve that would minimize total cost. Maddah, Yassine, Salameh & Chatila, (2013) [11] propose adding ordering, deterioration and preventive replenishment costs to Hansmann's model. For the purpose of this thesis, the model of Hansmann will be also used and modified to include multiple supply sources that are interrupted independently of one another.

B. Supplier Interruption

The importance of supply diversification results from several international incidents that led to costly interruption of operations such as wars, terrorist attacks, financial crisis, weather conditions etc. Svensson (2000) [23] analyzes through case studies the vulnerability of supply chain operations related to manufacturing logistics. Through his study, supply disorders were assessed as either being qualitative or quantitative with either atomistic or holistic sources. On the other hand, Sheffi (2005) [21] emphasizes on the importance of having more than one supplier in order to share interruption risks. In addition, the dangers of having Just-in-time strategies have been thoroughly evaluated as a target for supply disruptions and shortage of supplies. Tang (2006) [24] emphasizes on the importance of developing models reflecting disruption-avoidance strategies where risks were classified into either operational (inherent uncertainties in supply chain models) or disruptive (natural events or man-made disasters).

C. Supplier Diversification

The basis of supply chain policies is choosing an effective sourcing combination to be protected against the various prevailing risks. The most common approaches are single sourcing, dual sourcing and multiple sourcing strategies (Burke, Carrillo, & Vakharia, 2007) [4]. While each strategy has its own pros and cons, this research will focus on how to distribute percentage procurement between suppliers for a dual sourcing strategy and compare its direct effects on the total cost of the system. Afterwards, the model is evaluated for three different suppliers and optimum results are evaluated. Without disruption risks, single sourcing strategies hold a lot of advantages to the system such as higher collaboration between buyers and suppliers, optimization of shared benefits, higher quality of products at a lower cost, and many other benefits (Larson & Kulchitsky, 1998) [10].

While companies became more aware of the risks associated with single sourcing strategies, multiple sourcing and holding reserves had been the focus of literature and were thoroughly evaluated to determine an optimum strategy that would minimize costs and increase safety of operations. Berger & Zeng (2006) [2] relies on a decision tree approach in order to determine the optimal level of supply taking into consideration disruption risks. An expected cost function was formulated capturing the relationship between the associated risks and its correlated trade-offs. In addition, the optimum level of suppliers is defined. However, our model focuses on reserve supply and the effects of diversification on the total cost of the system. Dada, Petruzzi & Schwarz (2007) [6] considers a newsvendor problem served by different types of suppliers that are defined by either being perfectly reliable or unreliable. They compare the importance of having a reliable supplier to the costs realized within. They concluded that cheapest suppliers are first selected; afterwards, the quantity of supply ordered from each selected supplier vary according to reliability. Yu, Zeng & Zhao (2009) [28] compare the impact of supply disruptions by having single and dual sourcing methodologies using expected profit functions in a two-stage supply chain with a non-stationary and price-sensitive demand. Schmitt and Snyder (2012) [22] take into consideration two types of risks: Disruptions and yield uncertainty. Their paper took into consideration an infinite time horizon were results focused on the increase in costs, underutilization of unreliable suppliers and distortion of ordered quantities that a single period model incur. Our research is different than these works in that is has a main focus on reserve stock whereby supply diversification is sought to reduce the impact of shortages.

D. Combined Model

The three concepts that our model aims to combine has been reflected through several researches. Diversification of suppliers, which is represented by order splitting, resulted in a reduction in the quantity of safety buffer that should be held. Similarly, this research stresses on the importance of diversification on total cost reduction. However, the approach used differs from the work of other papers since it depends on the work of Hansmann (1962) [7] where the focus is only on reserve stock. Kelle and Silver (1990) [8] approaches this analysis by considering the effects of order splitting on lead time. They compare the effects of depending on only one supplier with that of depending on multiple suppliers where each is characterized by variable lead times. Thus, rather than waiting for the unique supplier to replenish supplies, three suppliers with variable lead times will replenish the inventory system. This approach results in reduction in average stock level without increasing the probability of shortage under some stated conditions.

Ramasesh et al. (1991) [17] uses a similar approach where dual sourcing strategies are taken into consideration using identical order splitting. They determine that reduction in total cost is achieved when uncertainty in lead time is high and when ordering costs are low. Ryu and lee (2003) [16] study the effects of lead time reduction using a dual sourcing strategy and calculated the reduction in the total cost and the needed quantity using a modified model of Bookbinder and Çakanyildirim (1999) [3]. Ruiz-Torres and Mahmoodi (2006) [15] presents a decision model for procurement of suppliers where the model takes into consideration the increase in the output of suppliers when others fail. However, increase of output is not taken into consideration in this research where in case of supplier failure, the other suppliers will continue operations normally. Zhang, Lai and Wang (2019) [29] considers a dual sourcing problem with multiple products. They consider one reliable supplier with higher costs and the other which is less reliable with lower costs. Their main objective is to maximize profits with limited constraints using an extended newsvendor problem. A similar approach is used in this research where a dual-sourcing methodology is used for the same purpose of reducing total costs. However, reserve stock is considered based on Hansmann's work and an algorithm is proposed for the calculation of the optimum solution.

CHAPTER III

BASE MODEL (HANSMANN,1962)

The seed model that this research depends on considers two manufacturing machines working in series. The first machine produces a semi-finished product while the second machine completes it. It is clearly shown that the work of the second machine is directly dependent on the work of the first machine. Thus, if the production from the first machine interrupts, the second machine will also interrupt and the production line will experience idle time. A solution proposed by Hansmann is to hold reserve supply from the first machine in order to minimize idle time experienced. However, holding reserve will incur additional holding costs. Thus, a system cannot be overprotective by holding a huge quantity of reserve which will lead to ineffective costs. Thus, an optimum amount of reserve should be held to balance between both holding and shortage costs that would minimize reserves.

Hansmann's model considers an inventory having a reserve of level of supply S held as a buffer against supply interruptions. Supply interruptions are assumed to occur at random. The times between two supply interruptions are assumed to be independent and identically distributed random variables. Let λ be rate of supply interruptions Supply downtime is assumed to be relatively small compared to supply availability. In addition, during downtime, stock is consumed at a known demand rate per unit time. Let T be our random variable in our analysis where it represents supply interruption time of a supplier. At the end of supply interruption, stock replenishment is assumed to be instantaneous for model simplification.

Figure 4 shows a typical inventory profile over time.

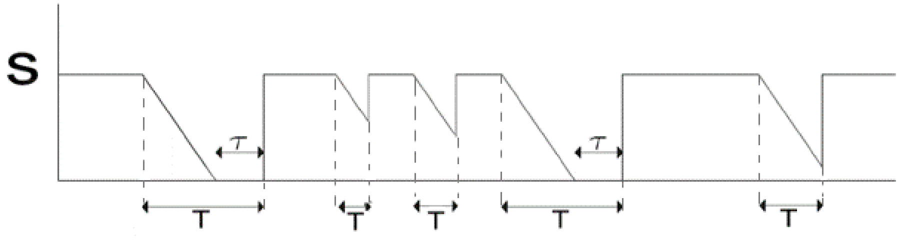

Figure 4: Reserve stock variation over time

The decision variable in the model is S, the reserve stock level

The model requires the following input parameters,

β: Demand rate (Units / year).

T: Supply interruption time (Consumption time from reserve supply) is a random variable with a known probability density function and a mean μ_T.

τ: Stock out time (Occurs when reserve supply is totally depleted).

h: Holding cost (\$ / unit / year).

$\gamma = E(\eta)$ where η represents time between interruptions.

$\lambda = \frac{1}{\gamma}$ (Rate of supply interruption).

π: Shortage cost per unit time (\$ / year).

c: Variable ordering cost (\$ / unit).

The model also makes the following assumptions:

- Drop in Reserve level is minimal (the average stock level is approximately S).
- Time between supply interruptions is relatively long.

- Shortage cost is assumed to be kept constant with respect to time (in general, shortage costs tend to increase with respect to stock out time).

shortage time (τ) is given by:

$$\tau = \begin{cases} 0 & T \leq \dfrac{S}{\beta} \\ T - \dfrac{S}{\beta} & T > \dfrac{S}{\beta} \end{cases}$$

The total cost in the base model is the sum of holding costs and shortage costs, where:

$$TC_U(S) = hS + \pi\,\lambda\,E\left(\tau \mid T > \frac{S}{\beta}\right)$$

Replacing $E\left(\tau \mid T > \frac{S}{\beta}\right)$ by $\int_{\frac{S}{\beta}}^{\infty} \left(t - \frac{S}{\beta}\right) f_T(t)\,dt$ results in the below equation:

$$TC_U(S) = hS + \pi\,\lambda \int_{\frac{S}{\beta}}^{\infty} \left(t - \frac{S}{\beta}\right) f_T(t)\,dt \tag{1}$$

A. First Order Optimality Condition

In order to determine the optimum reserve stock that would minimize cost, the total cost formula in (1) is differentiated with respect to S

Setting $\frac{\partial TC_U(S)}{\partial s}$ to 0, gives

$$h + \pi\lambda \frac{\partial}{\partial S} \int_{\frac{S}{\beta}}^{\infty} \left(t - \frac{S}{\beta}\right) f_T(t)dt = 0 \tag{2}$$

Leibniz rule is used to solve the derivative of the integral with an infinite upper limit,

$$\frac{\partial}{\partial S}\int_{\frac{S}{\beta}}^{\infty}\left(t-\frac{S}{\beta}\right)f_T(t)dt=\int_{\frac{S}{\beta}}^{\infty}\left(\left(-\frac{1}{\beta}\right)f_T(t)dt\right)$$

Replacing in (2) gives the following equation,

$$\int_{\frac{S}{\beta}}^{\infty} f_T(t)dt=\frac{h\beta}{\pi\lambda} \quad (3)$$

1. Physical Interpretation of the First Optimality Condition

The left hand side of Formula (3) represents the probability that T is greater than $\frac{S}{\beta}$

Therefore, $0 \leq P\left(T>\frac{S}{\beta}\right)=\frac{h\beta}{\lambda\pi}\leq 1$, or equivalently, $h\beta \leq \lambda\pi$

That is, an optimum level of reserve stock is only carried if annual holding costs incurred under a certain demand per unit time are less than annual shortage costs per unit time. Otherwise, it is more efficient to carry no reserve stock and incur a full shortage cost. This relation is important since it will be repeated in all the modifications performed in this research.

B. Example 1

Consider a model instance with the following input parameters,

h = 0.15 $ / unit / year

π = $ 45,000 / year

β = 18,000 units / year

γ = 1 year, λ= $1\ / year$

T (Supply interruption time) is assumed to follow a uniform distribution on (a,b) where a = 0 and b = $\frac{1}{12}$

Utilizing (3), the optimal buffer stock level is:

$S*$ = 1,410 units.

The expected annual cost is $TCU(S)$ = $ 3,637.5.

CHAPTER IV

MODIFICATIONS ON HANSMANN'S MODEL

Hansmann's model (1962) [7] takes into consideration two types of costs, holding and shortage costs. However, ordering costs are not taken into consideration. Thus, with reference to Maddah et al. (2013) [11], ordering costs are derived based on the quantity ordered upon supply interruption,

$$\text{Ordering Quantity} = \begin{cases} \beta T, & T \leq \dfrac{S}{\beta} \\ S, & T > \dfrac{S}{\beta} \end{cases}$$

Therefore, the ordering cost per year is

$$Co \ = c\lambda(\int_{0}^{\frac{S}{\beta}} \beta t f_T(t)dt + \int_{\frac{S}{\beta}}^{\infty} S f_T(t)dt) \tag{4}$$

Total expected annual cost formula obtained in (1) and becomes

$$TC_u(S) = hS + \pi\,\lambda\, \int_{\frac{S}{\beta}}^{\infty} \left(t - \frac{S}{\beta}\right) f_T(t)\,dt + c\lambda(\int_{0}^{\frac{S}{\beta}} \beta t f_T(t)dt + \int_{\frac{S}{\beta}}^{\infty} S f_T(t)dt) \tag{5}$$

A. First Order Optimality Condition

Differentiation gives,

$$\frac{\partial TC_U(S)}{\partial s} = h - \frac{\pi\lambda}{\beta} \int_{\frac{S}{\beta}}^{\infty} f_T(t)dt + \ c\lambda(\int_{\frac{S}{\beta}}^{\infty} f_T(t)dt) \tag{6}$$

Equivalently, $\frac{\partial TC_U(S)}{\partial s} = h - \lambda\left(1 - F_T\left(\frac{S}{\beta}\right)\right)\left(\left(\frac{\pi}{\beta}\right) - c\right)$ (7)

Setting $\frac{\partial TC_U(S)}{\partial s} = 0$ gives,

$$\frac{h}{\lambda\left(\left(\frac{\pi}{\beta}\right) - c\right)} = \left(1 - F_T\left(\frac{S}{\beta}\right)\right) \qquad (7')$$

1. Physical Interpretation of the First Optimality Condition

$\left(1 - F_T\left(\frac{S}{\beta}\right)\right)$ Represents the probability for T greater than $\frac{S}{\beta}$,

$0 \leq \frac{h}{\lambda\left(\left(\frac{\pi}{\beta}\right) - c\right)} \leq 1$, or equivalently, $(h + c\lambda)\beta \leq \pi$

Therefore, a reserve stock is carried if average annual holding and ordering costs under a certain demand rate per unit time are less than average shortage costs incurred per unit time. Otherwise, it is more efficient to carry no reserve.

Simplifying (7) gives the following expression for the optimal reserve stock,

Differentiation gives,

$$S^* = \beta F_T^{-1}\left(1 - \left(\frac{h}{\left(\frac{\pi}{\beta} - c\right)\lambda}\right)\right) \qquad (8)$$

B. Convexity of TCu with respect to *S*

To check for whether the solution obtained is a global or a local solution, the second derivative is evaluated and is set equal to 0:

$$\frac{\partial TCu(S)^2}{\partial S^2} = \frac{\lambda}{\beta}\ f_T\left(\frac{s}{\beta}\right)(\frac{\pi}{\beta} - c) \tag{9}$$

Under the condition $\pi > c\beta$, TCU (S) is convex.

This is a non-restrictive condition stating that shortage costs should be greater than ordering costs in order for holding reserve to be a feasible solution.

C. Example 2

Consider the same input data as Example 1, with an additional unit cost

c = $1/unit.

Applying (8) gives the following optimal reserve stock,

S*= 1350 units.

The corresponding expected annual cost is $ 4,237.5.

CHAPTER V

DUAL SOURCING MODEL

The wide range of benefits that a dual sourcing strategy holds makes it one of the most used systems in supply chain structures. In this research, the modified model that was defined in the previous section is extended to include two suppliers. The objective of this is to quantify the benefits of diversifying suppliers by comparison with the single supplier model of Hansmann (1962) (as amended by Maddah et al. (2013), in the previous section) and to try to find strategy that would minimize total cost. Supply interruption is taken to be independent. Therefore, with two suppliers available, consumption rate of reserve is reduced, which will lead to decreasing shortages.

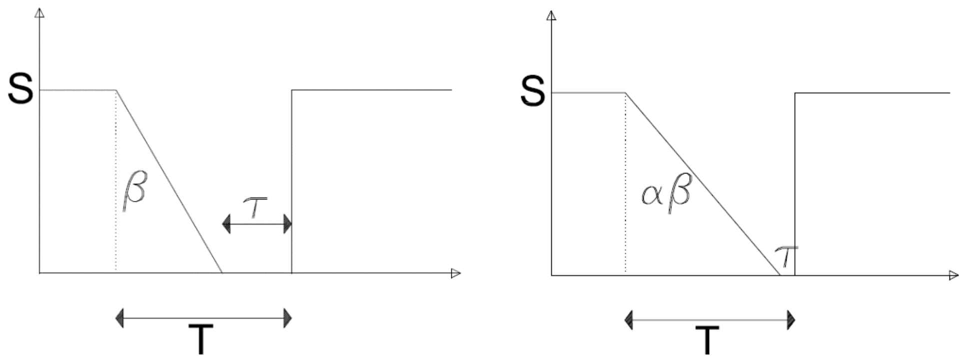

Figure 5: Order splitting effects on consumption of reserve

The decision variables of the dual sourcing model are the reserve stock S, and the proportion of reserve supplied from Supplier 1, $\alpha_1 \leq 1$.

The model has the following input parameters

Demand rate, $\beta = \alpha_1\beta + (1\text{-}\alpha_1)\,\beta$. When Supplier 2 (1) is interrupted, the demand rate is $\alpha_1\beta\,((1\text{-}\alpha_1)\beta)$.

$\gamma_i = E(\eta_i)$ where η_i represents time between interruptions, i = 1,2.

λ_i (Rate of disruption), i = 1,2.

c_i: Unit cost of stock received from Supplier i.

i: % of ordering cost that is equivalent to the holding cost / unit

The holding cost can be written as a weighted average of the holding costs of stocks received from both suppliers.

$$\bar{h} \cong \alpha_1 h_1 + \alpha_2 h_2$$
$$= i(\alpha_1 c_1 + \alpha_2 c_2)$$

A justification of this expression is given in the appendix.

A. Dual Sourcing Total Cost Formulation

The total cost for the dual-supply system is formulated as the sum of holding, shortage and ordering costs implied by each supplier.

$$\begin{aligned} E[TC_u(S,\alpha_1)] &= \bar{h}S + \pi\lambda_1 \int_{\frac{S}{\alpha_1\beta}}^{\infty} \left(t_1 - \frac{S}{\alpha_1\,\beta}\right) f_{T_1}(t)\,dt \\ &+ c_1\lambda_1 \left(\int_0^{\frac{S}{\alpha_1\,\beta}} \alpha_1\beta\; t_1 f_{T1}(t)dt + \int_{\frac{S}{\alpha_1\,\beta}}^{\infty} S f_{T1}(t)dt\right) \\ &+\pi\lambda_2 \int_{\frac{S}{\alpha_2\,\beta}}^{\infty} \left(t_2 - \frac{S}{\alpha_2\,\beta}\right) f_{T2}(t)\,dt \\ &+ c_2\lambda_2 \left(\int_0^{\frac{S}{\alpha_2\,\beta}} \alpha_2\beta\; t_2 f_{T2}(t)dt + \int_{\frac{S}{\alpha_2\,\beta}}^{\infty} S f_{T2}(t)dt\right) \end{aligned} \tag{10}$$

B. First Order Optimality Condition with respect to *S*

Differentiating with respect to S, and setting the derivative equal to 0, gives,

$$\bar{h} - \lambda_1\left(1 - F_{T1}\left(\frac{S}{\alpha_1\,\beta}\right)\right)\left(\left(\frac{\pi}{\alpha_1\,\beta}\right) - c_1\right) - \lambda_2\left(1 - F_{T2}\left(\frac{S}{\alpha_2\beta}\right)\right)\left(\left(\frac{\pi}{\alpha_2\,\beta}\right) - c_2\right) = 0 \tag{11}$$

or equivalently,

$$\lambda_1\left(1-F_{T1}\left(\frac{S}{\alpha_1\,\beta}\right)\right)\left(\left(\frac{\pi}{\alpha_1\,\beta}\right)-c_1\right)-\lambda_2\left(1-F_{T2}\left(\frac{S}{\alpha_2\beta}\right)\right)\left(\left(\frac{\pi}{\alpha_2\,\beta}\right)-c_2\right)=\bar{h} \tag{12}$$

1. Physical Interpretation of the First Optimality Condition

Note that in order for (11) to hold, the following inequality must be satisfied,

$$h\leq\lambda_1\left(\left(\frac{\pi}{\alpha_1\,\beta}\right)-c_1\right)+\lambda_2\left(\left(\frac{\pi}{\alpha_2\,\beta}\right)-c_2\right)$$

$$\pi\geq\frac{\beta(h+\lambda_1c_1+\lambda_2c_2)}{\left(\frac{\lambda_1}{\alpha_1}+\frac{\lambda_2}{(1-\alpha_1)}\right)} \tag{13}$$

Equation (13) conditions that shortage cost per unit time should be greater than ordering and holding cost incurred having a certain demand for a reasonable solution to be assessed. (This condition avoids the trivial case of not holding reserve).

C. Convexity with respect to *S*

In order to prove convexity, the second derivative is evaluated and shown in equation (14):

$$\frac{\partial^2}{\partial S^2}\mathrm{TC_U}(S,\alpha_1)$$

$$\left(\frac{\lambda_1}{\alpha_1\,\beta}\,\mathrm{f_{T1}}\left(\frac{S}{\alpha_1\,\beta}\right)\left(\left(\frac{\pi}{\alpha_1\beta}\right)-c_1\right)\right)+\left(\frac{\lambda_2}{\alpha_2\beta}\,\mathrm{f_{T2}}\left(\frac{S}{\alpha_2\,\beta}\right)\left(\left(\frac{\pi}{\alpha_2\,\beta}\right)-c_2\right)\right) \tag{14}$$

Under the reasonable conditions $\frac{\pi}{\alpha_1\,\beta}>c_1$ and $\frac{\pi}{\alpha_2\,\beta}>c_2$, second derivative is positive (TCu is convex) and an optimal solution for reserve stock exists.

D. First Order Optimality Condition with respect to α_1

The first derivative with respect to α_1 is evaluated and shown in equation (15):

$$\frac{\partial}{\partial \alpha_1}\mathrm{TC_U}(S,\alpha_1) = +\pi\lambda_1 \frac{S}{\alpha_1{}^2\beta}\int_{\frac{S}{\alpha_1\beta}}^{\infty} f_{T_1}(\mathrm{t})\,d\mathrm{t} + \mathrm{c}_1\lambda_1\left(\int_0^{\frac{S}{\alpha_1\beta}}\beta\,\mathrm{t}f_{\mathrm{T}1}(\mathrm{t})d\mathrm{t}\right)$$

$$-\pi\lambda_2\frac{S}{\alpha_2{}^2\ \beta}\int_{\frac{S}{\alpha_2\beta}}^{\infty} f_{\mathrm{T}2}(\mathrm{t})\,\mathrm{dt} - \mathrm{c}_2\lambda_2\left(\int_0^{\frac{S}{\alpha_2\beta}}\alpha_2\beta\,\mathrm{t}f_{\mathrm{T}2}(\mathrm{t})\mathrm{dt}\right) \qquad (15)$$

E. Convexity with respect to α_1

For the total cost formula to converge to a minimum, the function should be convex in both S and α_1. In order to prove convexity, the second derivative is evaluated and shown in equations (16) and (16'):

$$\frac{\partial^2}{\partial \alpha_1{}^2}\mathrm{TC_U}(S,\alpha_1) =$$

$$\pi\lambda_1\left(-\frac{2S}{\alpha_1{}^3\beta}\int_{\frac{S}{\alpha_1\beta}}^{\infty} f_{\mathrm{T}1}(\mathrm{t})d\mathrm{t} + \frac{S^2}{\alpha_1{}^4\beta^2}f_{\mathrm{T}1}\left(\frac{S}{\alpha_1\beta}\right)\right) - \mathrm{c}_1\lambda_1\frac{S^2}{\alpha_1{}^3\beta}f_{\mathrm{T}1}\left(\frac{S}{\alpha_1\beta}\right)$$

$$+\pi\lambda_2\left(-\frac{2S}{(1-\alpha_1)^3\beta}\int_{\frac{S}{(1-\alpha_1)\beta}}^{\infty} f_{\mathrm{T}2}(\mathrm{t})d\mathrm{t} + \frac{S^2}{(1-\alpha_1)^4\beta^2}f_{\mathrm{T}2}\left(\frac{S}{(1-\alpha_1)\beta}\right)\right) \qquad (16)$$

$$-\mathrm{c}_2\lambda_2\frac{S^2}{(1-\alpha_1)^3\beta}f_{\mathrm{T}2}\left(\frac{S}{\alpha_1\beta}\right)$$

$$= -2\pi\int_{\frac{S}{\alpha_1\beta}}^{\infty} f_{T1}(t)dt + f_{T1}\left(\frac{S}{\alpha_1\beta}\right)\left(\frac{\pi S}{\alpha_1\beta} - c_1\right) \qquad (16')$$

$$-2\pi\int_{\frac{S}{(1-\alpha_1)\beta}}^{\infty} f_{T2}(t)dt + f_{T2}\left(\frac{S}{(1-\alpha_1)\beta}\right)\left(\frac{\pi S}{(1-\alpha_1)\beta} - c_2\right)$$

The second derivative is positive under the following conditions:

1. $\frac{\pi S}{\alpha_1 \beta} > c_1$

2. $\frac{\pi S}{(1-\alpha_1)\,\beta} > c_2$

3. $f_{T1}\left(\frac{S}{\alpha_1 \beta}\right)\left(\frac{\pi S}{\alpha_1 \beta} - c_1\right) > 2\pi \int_{\frac{S}{\alpha_1 \beta}}^{\infty} f_{T1}(t)dt$

4. $f_{T2}\left(\frac{S}{(1-\alpha_1)\,\beta}\right)\left(\frac{\pi S}{(1-\alpha_1)\,\beta} - c_2\right) > 2\pi \int_{\frac{S}{(1-\alpha_1)\,\beta}}^{\infty} f_{T2}(t)dt$

As these conditions are too restrictive, we conclude that the expected cost is not generally convex in α_1. In fact, the total cost function is not even quasi-convex with respect to α_1 as we show later in this thesis via numerical examples. (Graphs drawn in the numerical section).

F. Generalization of the Total Cost Formula

A general form of the total cost formula can be written as shown in equation (18). This form includes multiple suppliers. Supplier j has a unit cost c_j and contributes a fraction α_j of the supply. Note that $\{\boldsymbol{\alpha} = \alpha_1 + \alpha_2 + \dots \alpha_n\}$

$$\mathrm{TCU}(S, \boldsymbol{\alpha_j}) = \bar{h}S + \sum_{j=1}^{n} \left\{ \pi\left(\lambda_j E\left(T_j - \frac{S}{\alpha_j \beta}\right)^{+}\right) + c_j \lambda_j \left(\left(\alpha_j \beta E\left(T_j \middle| T_j < \frac{S}{\alpha_j \beta}\right) \overline{F}_{T_j}\left(\frac{S}{\alpha_j \beta}\right)\right)\right\} \quad (17)$$

CHAPTER VI

SENSITIVITY ANALYSIS AND NUMERICAL EXAMPLES

A. Exponential Distribution for Supply Downtime

Supply downtime represents the time during which supply interruption takes place. In this model, interruption time is assumed to follow an exponential distribution for two main reasons. The first reason is that exponential distributions are used for reliability applications with constant failure rate. As for the second reason, exponential distributions result in simple and closed formulas for complex expressions.

The probability density function of the exponential downtime for supplier j is given by

$$f\{T_j\} = \mu_j e^{-\mu_j t} \ ,$$

where $\mu_j = (\frac{1}{E[Tj]})$.

Therefore, the total cost formula of the system using an exponential distribution for interruption time is given in equation (19). (Derivation of the total cost formula is in the Appendix)

$$\begin{aligned} \mathrm{TCU}(S,\alpha_1) &= hS + \pi\lambda_1 \frac{1}{\mu_1} e^{-\frac{\mu_1 S}{\alpha_1 \beta}} + \pi\lambda_2 \frac{1}{\mu_2} e^{-\frac{\mu_2 S}{\alpha_2 \beta}} \\ &+ c_1 \lambda_1 \alpha_1 \beta \left(-\frac{S}{\alpha_1 \beta} e^{-\frac{\mu_1 S}{\alpha_1 \beta}} - \frac{1}{\mu_1} e^{-\frac{\mu_1 S}{\alpha_1 \beta}} + \frac{1}{\mu_1} \right) \\ &+ c_2 \lambda_2 \alpha_2 \beta \left(-\frac{S}{\alpha_2 \beta} e^{-\frac{\mu_2 S}{\alpha_2 \beta}} - \frac{1}{\mu_2} e^{-\frac{\mu_2 S}{\alpha_2 \beta}} + \frac{1}{\mu_2} \right) \\ &+ c_1 \lambda_1 S e^{-\frac{\mu_1 S}{\alpha_1 \beta}} + c_2 \lambda_2 S e^{-\frac{\mu_2 S}{\alpha_2 \beta}} \end{aligned} \qquad (18)$$

B. Sensitivity Analysis and Results

The total cost formulated in this research depends on inputs that requires estimation that may be subject to inaccuracies. At this stage, sensitivity analysis is required in order to determine the effects of the variability of these inputs to the results obtained. For this purpose, three scenarios are investigated, the first scenario comprises two identical suppliers having the same input parameters while the second scenario consists of one reliable-costlier and one unreliable-low-cost supplier. All results are compared to the base model in order to define a minimization of cost strategy. As for the third scenario, a “dominated supplier” is added to the model that is costlier and less reliable than another one where the objective is to study the reduction of total cost incurred by sharing shortage costs.

1. Scenario I: Two Identical Suppliers

Scenario I takes into consideration two identical suppliers with independent and identical times between interruptions, denoted by T having rates $\lambda_1 = \lambda_2$, and unit costs $c_1 = c_2$. The main objective of this scenario is to study whether having a dual-sourcing strategy reduces costs compared to having a single supplier model. The base parameter values for the analysis in this section are shown in Table 1. Note that supply interruption times, T_1 and T_2, are assumed to be exponentially distributed with rates μ_1 and μ_2 through ought the numerical analysis.

In order to find the most probable optimum distribution between the available identical suppliers, the first order optimality condition in (15) gives,

$$\frac{\partial}{\partial \alpha_1} \mathrm{TC_U}(S, \alpha_1) = 0$$

$$\frac{\pi\lambda S}{\beta}\left(\frac{1}{\alpha_1{}^2}\int_{\frac{S}{\alpha_1\beta}}^{\infty} f_{\mathrm{T}}(\mathrm{t})d\mathrm{t} - \frac{1}{(1-\alpha_1)^2}\int_{\frac{S}{(1-\alpha_1)\beta}}^{\infty} f_{\mathrm{T}}(\mathrm{t})d\mathrm{t}\right) + \mathrm{c}\lambda\beta\left(\int_0^{\frac{S}{\alpha_1\beta}} tf_{\mathrm{T}}(\mathrm{t})d\mathrm{t} - \int_0^{\frac{S}{(1-\alpha_1)\beta}} tf_{\mathrm{T}}(\mathrm{t})d\mathrm{t}\right) = 0 \tag{19}$$

Note that setting $\alpha_1 = 1-\alpha_1$, or equivalently $\alpha_1 = \frac{1}{2}$, leads to satisfying the condition in (19). This is an intuitive result, indicating that with identical suppliers, it is optimal to split the supply equally among the suppliers.

This result can be generalized to n suppliers and the optimum supply split is given by $\alpha_1 = \alpha_2 = \ldots = \alpha_n = \frac{1}{n}$. Please see the appendix for a proof.

Table 1: Base parameter values for Scenario I

Parameters	Supplier	Notation	Value	Units
Financial Cost	-	i	15	%
Variable Ordering Cost	S_1	c_1	1	USD/unit
Variable Ordering Cost	S_2	c_2	1	USD/unit
Demand	-	β	15,000	Units/year
Shortage Cost	-	π	60,000	USD/year
Percentage of Supplies from	S_1	α_1	0.5	Unit-less
Rate of supply interruption time	S_1	μ_1	365/30	Days^{-1}
Rate of supply interruption time	S_2	μ_2	365/30	Days^{-1}
Mean time between interruptions	S_1	γ_1	1	years
Mean time between interruptions	S_2	γ_2	1	years

a. Optimization of the Decision Variables

The total cost function derived in this research showed convexity with respect to S and non-convexity with respect to α_1. Thus, in order to optimize the decision variables, a grid search analysis is used where α_1 is variated from its minimum value to its maximum value by a certain increment. At each value for α_1, optimum reserve stock, total cost incurred and expected shortage time are calculated and compared using either single or dual sourcing strategies. Table 2 shows the different values obtained.

Table 2: Scenario I optimization table

α_1	S_1 (Units)	TCu_1	S_{1-2} (Units)	TCu_{1-2}	τ_1 (days / year)	τ_{1-2} (days / year)
0	3,694	$ 1,971.81	3,694	$ 1,971.81	1.5	1.5
0.05			3,585	$ 1,946.78		1.402
0.1			3,474	$ 1,920.91		1.307
0.15			3,356	$ 1,894.19		1.214
0.2			3,236	$ 1,866.60		1.124
0.3			3,001	$ 1,810.07		0.94
0.4			2,842	$ 1,763.04		0.739
0.5			2,796	$ 1,744.78		0.643
0.6			2,841	$ 1,763.04		0.739
0.7			2,996	$ 1,810.06		0.94
0.8			3,236	$ 1,866.60		1.124
0.9			3,475	$ 1,920.91		1.307
1			3,692	$ 1,971.81		1.5

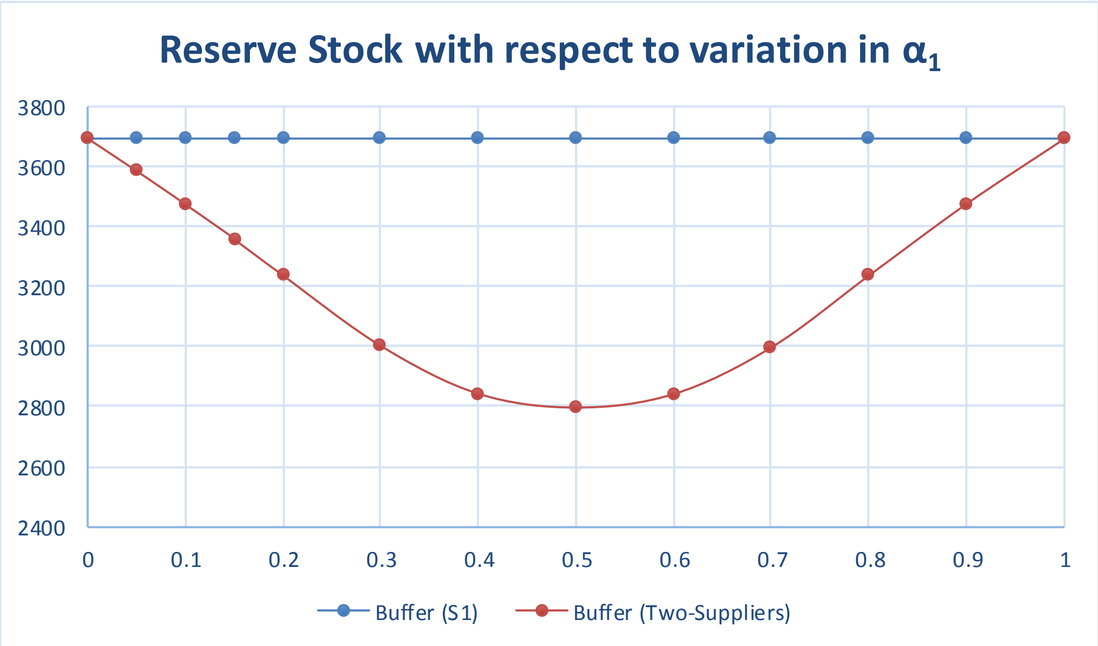

Figure 6: Reserve stock variation with respect to α_1

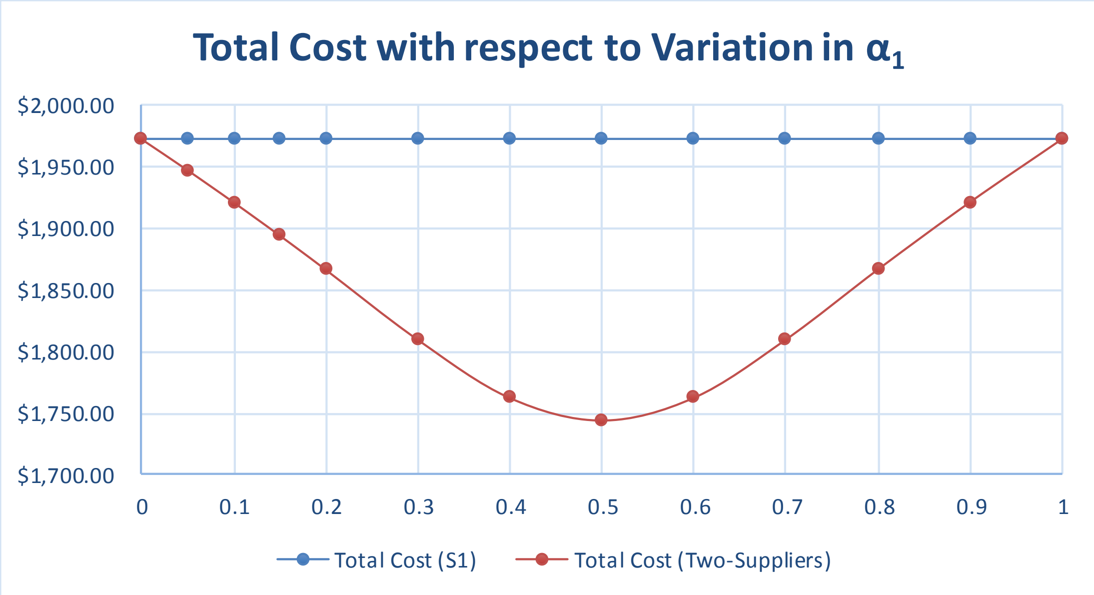

Figure 7: Total cost variation with respect to α_1

As shown in Figure 6 and Figure 7, optimum quantity of reserve and the minimum total cost for the dual-supplier system is achieved at α_1 = 0.5 with an optimal amount of reserve $S*$ = 2,796 units and corresponding average annual cost $TCu*$ = $1,744.79 compared to $S*$ = 3,694 units and $TCu*$ = $1,971.81 for single supplier strategy.

Table 3: Cost comparison

	One Supplier	Two-Suppliers	% Variation
Holding Cost	$554.24	$419.50	-24%
Shortage Cost	$246.26	$105.61	-57%
Ordering Cost	$1,171.31	$1,219.68	4%
Total Cost	$1,971.81	$1,744.79	-12%

Results show a decrease of around 24% in holding costs for reserve stock. In addition, Table 3 shows a 12 % decrease in total cost by having a replenishment strategy from two suppliers compared to only one supplier. Shortage costs have showed a total reduction of 57%. However, ordering costs increased by 4%, This is due to the fact that the dual-system embraces ordering more supplies compared to the single supplier model which is reflected in the reduction achieved in the expected shortage days per year (0.65

days / year for two suppliers at $\alpha_1 = 0.5$ compared to 1.5 days / year for a single supplier).

In conclusion, Table 4 represents a summary table regarding scenario where the total cost of the system is calculated with respect to the variation in α_1 and α_2.

Table 4: Table summary on scenario I

α_1	α_2	Reserve Stock	Total Cost per unit time
0.0	1.0	3,694.00	$1,971.81
1.0	0.0	3,694.00	$1,971.81
0.5	0.5	2,796.00	$1,744.78

b. Sensitivity Analysis on Shortage Cost / year

Table 5: Scenario I sensitivity analysis on π

π	TCu_1	TCu_{1-2}	τ_1 (Days / year)	T_{1-2} (Days / year)
$ 18,000	$ 1,471.01	$1,595.97	1.5	0.643
$ 20,000	$ 1,565.48	$1,612.09		
$ 30,000	$1,768.65	$1,666.44		
$ 40,000	$1,863.11	$1,700.44		
$ 50,000	$1,925.34	$1,725.25		
$ 60,000	$1,971.81	$1,744.78		
$ 70,000	$2,008.92	$1,760.91		
$ 80,000	$2,039.82	$1,774.63		
$ 90,000	$2,066.28	$1,786.58		
$ 100,000	$2,089.43	$1,797.16		

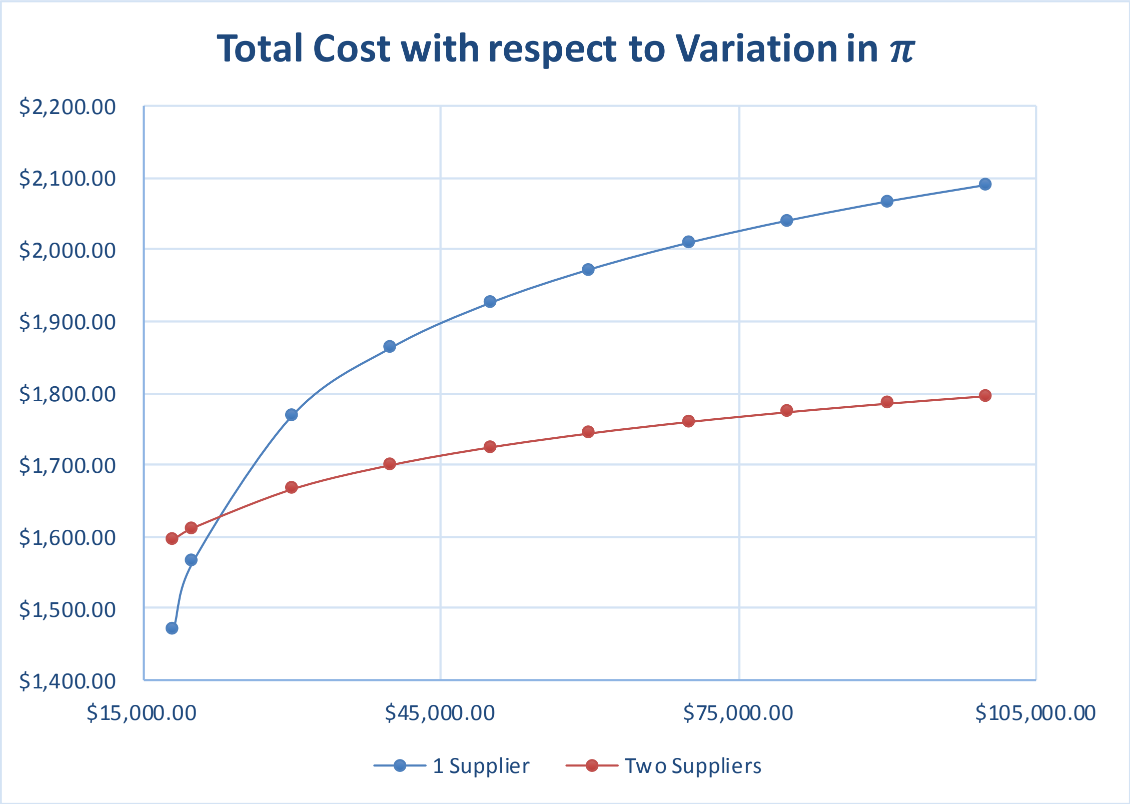

Figure 8: Total cost variation with respect to shortage cost α_1

Sensitivity analysis is done on shortage costs where values are variated from $18,000 to $100,000 per year. As shown in Table 5 and Figure 8, for values between $18,000 till $22,000, one supplier seems to incur lower costs compared to having two suppliers. There are two important insights that are shown in Figure 8. The first insight is that the blue line which represents the single sourcing strategy fluctuates much more with respect to the variation in shortage costs per unit time compared to the orange line which represents the dual sourcing strategy. Thus, using the concept of diversification, the system created is more stable and withstand variation in shortages. As for the second insight, as shortage costs per unit time increase, the difference between the two systems also increase which highlights on the benefits of diversification in minimizing the negative effects of shortage costs on operations.

Table 6: Shortage cost comparison

		One Supplier			Two Suppliers		
π	Type of Cost	S_1	Cu_1	TCu_1	S_{1-2}	Cu_{1-2}	TCu_{1-2}
$ 18,000	Holding	355	$53.26	$ 1,471.02	1,805	$270.71	$ 1,595.96
	Shortage		$1,109.27			$158.36	
	Ordering		$308.49			$1,166.89	
$ 20,000	Holding	984	$147.59	$ 1,565.48	1,911	$286.67	$ 1,612.09
	Shortage		$740.03			$148.07	
	Ordering		$677.86			$1,177.35	
$ 22,000	Holding	1,399	$209.81	$ 1,627.70	2,003	$300.42	$ 1,625.81
	Shortage		$581.48			$140.36	
	Ordering		$836.41			$1,185.03	
$ 25,000	Holding	1,834	$275.10	$ 1,693.66	2,118	$317.76	$ 1,643.20
	Shortage		$464.21			$132.23	
	Ordering		$954.35			$1,193.21	
$ 50,000	Holding	3,388	$508.16	$ 1,925.34	2,667	$400.05	$ 1,725.24
	Shortage		$263.28			$108.61	
	Ordering		$1,153.89			$1,216.58	
$ 70,000	Holding	3,938	$590.75	$ 2,008.92	2,904	$435.54	$ 1,760.91
	Shortage		$235.84			$103.59	
	Ordering		$1,182.34			$1,221.78	
$ 80,000	Holding	4,146	$621.87	$ 2,039.82	2,995	$449.25	$ 1,774.63
	Shortage		$227.78			$102.07	
	Ordering		$1,190.17			$1,223.31	
$ 100,000	Holding	4,473	$671.01	$ 2,089.43	3,144	$471.62	$ 1,797.16
	Shortage		$218.28			$100.17	
	Ordering		$1,200.13			$1,225.36	

Table 6 shows that the effects of ordering costs for $\pi \leq 20{,}000$ USD per year favored the option of having one supplier only. The reasoning behind this result is that as shortages per year has low effects on the total cost of the system, the single sourcing strategy is holding a certain amount of reserve and is letting for shortages to take place. While, the dual sourcing strategy holds a certain amount of reserve from both suppliers and is evading shortages as much as possible. (This is represented by the increase in the amount of reserve held in the dual sourcing system compared to that held in the single sourcing). Thus, the increase in holding and ordering costs outweighs the decrease in shortage costs which makes the single sourcing strategy seems to incur lower costs.

Taking into consideration high values for shortage costs per year, the effects of shortages highly impact the total cost of the system. Thus, the single sourcing strategy is holding higher amount of reserve in order to avoid shortages as much as possible. However, the dual sourcing system, by holding less amount of reserve, was able to reduce shortage much less than the single sourcing system. Thus, the benefits of diversification are shown in controlling shortage costs by holding fewer amount of reserve compared to single sourcing strategies.

c. Sensitivity analysis on demand rate

Table 7: Scenario I sensitivity analysis on β

Demand (Units / year)	One Supplier (USD)	Two Suppliers (USD)	Difference (USD)
5,000	$737.36	$618.26	($119.10)
10,000	$1,377.52	$1,191.05	($186.47)
15,000	$1,971.81	$1,744.78	($227.03)
20,000	$2,529.11	$2,284.90	($244.21)
25,000	$3,051.45	$2,813.83	($237.63)
40,000	$4,374.57	$4,343.86	($30.71)
50,000	$4,903.37	$5,319.89	$416.51

Sensitivity analysis is done on demand rate variation where the demand rate β is varied from 5,000 till 50,000 units per year. Table 7 shows that for demand values less than 40,000 units per year, the dual-sourcing strategy tends to be more beneficial compared to having only one supplier. As demand rate increases beyond 40,000 units per year, the single sourcing system seems to incur lower costs compared to the dual sourcing system. In addition, the maximum difference between both systems is achieved at a demand rate of 20,000 units per year.

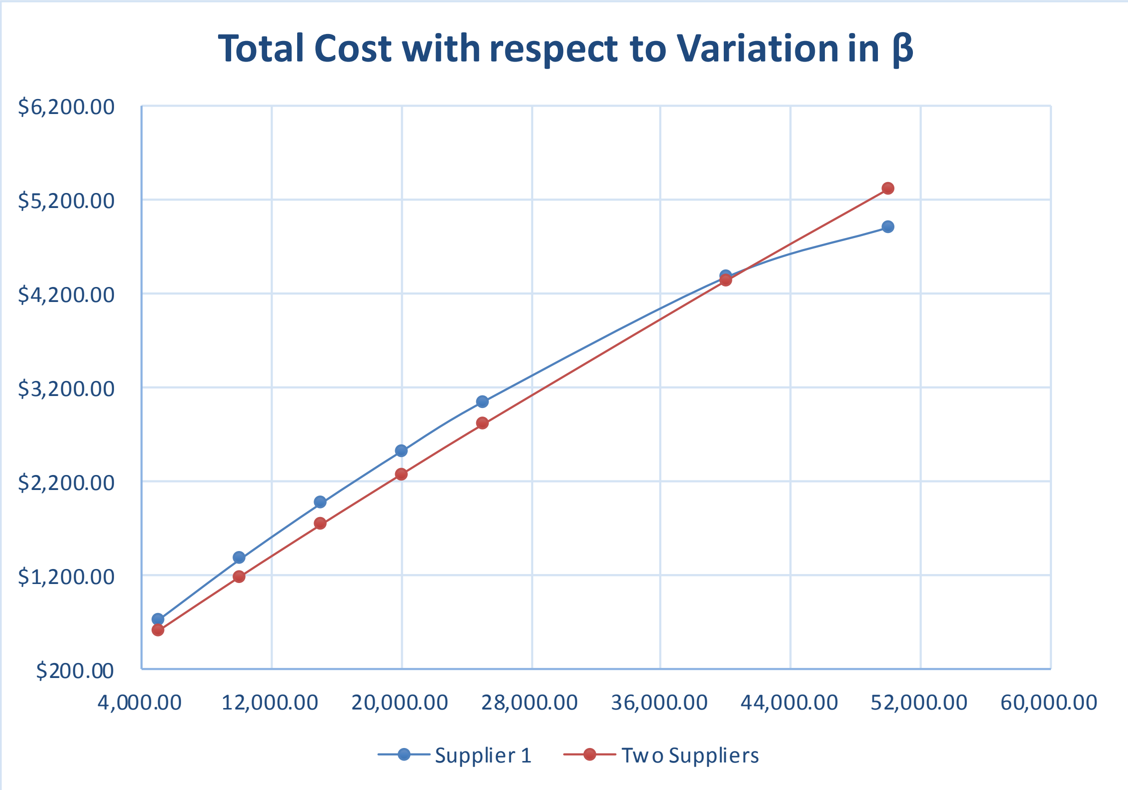

Figure 9: Total cost variation with respect to demand rate

Table 8: Total cost comparison with respect to variation in demand rate

		One Supplier			Two Suppliers		
β	Type of Cost	S_1	Cu_1	TCu_1	S_{1-2}	Cu_{1-2}	TCu_{1-2}
5,000	Holding	1,765	$ 264.8	$ 737.36	1,177	$ 176.50	$ 618.26
	Shortage		$ 67.20			$ 32.14	
	Ordering		$ 405.36			$ 409.62	
15,000	Holding	3,686	$ 552.83	$ 1,971.81	2,795	$ 419.30	$ 1,744.78
	Shortage		$ 248.14			$ 105.83	
	Ordering		$ 1,170.84			$ 1,219.65	
25,000	Holding	4,579	$ 686.80	$ 3,051.46	4,031	$ 604.65	$ 2,813.82
	Shortage		$ 531.19			$ 195.00	
	Ordering		$ 1,833.47			$ 2,014.17	
40,000	Holding	3,957	$ 593.54	$ 4,374.57	5,406	$ 810.86	$ 4,343.87
	Shortage		$ 1,480.06			$ 368.00	
	Ordering		$ 2,300.97			$ 3,165.01	
50,000	Holding	1,183	$ 177.44	$ 4,903.37	6,012	$ 901.77	$ 5,319.89
	Shortage		$ 3,698.02			$ 528.91	
	Ordering		$ 1,027.91			$ 3,889.21	

Table 8 shows the variation of total cost for either having a single or a dual supplier strategy with respect to variation in demand rate. It is shown that at β = 20,000 units per year, the difference between both strategies is maximized reaching $ 244.21. As demand rate increase, the difference between both systems decreases till demand reaches a value of around 40,000 units per year where both graphs coincide. Table 8 explains briefly the reaction of the system with respect to variation in demand rate. At low demand rates, the diversified system is holding less amount of reserve and is experiencing less shortages. Thus, the diversified system incurs lower holding and shortage costs, but higher ordering costs due to the decrease in idle time. However, for very high demand rate (50,000 units per year), the single sourcing system is not able to hold huge amount of reserve in order to avoid shortages (1182.93 compared to 6011.78). Thus, the single sourcing system is incurring huge shortage costs, a lot of idle time and is not ordering products. However, the diversified system is able to hold huge amount of reserve in order to avoid shortage as much as possible ($528.91 shortages in the dual sourcing compared to $3698.02 in the single sourcing). In addition, the diversified system is incurring high ordering costs ($3889.21 compared to only $1027.91). Thus, the diversified system is able to satisfy demand and is able to manage the continuity of operations. While the single sourcing system is not able to satisfy demand, thus it is facing huge idle and shortage time.

In conclusion, even though the single sourcing system is incurring lower costs at huge demand; however, the system is not working at optimal conditions. If our objective function includes a profit margin that reflects the continuity of operations, the diversified system will result in huge profits compared to the single sourcing system. In addition, the two main objectives in this research as stated before are to minimize costs and to maximize operations. Thus, the diversified system is able to optimize operations by managing the continuity of operations even though the system incurs higher total costs.

d. Sensitivity Analysis on mean of interruption time

Table 9: Scenario I sensitivity analysis on µ

Supply Interruption (Days)	µ	One Supplier (USD)	Two Suppliers (USD)	Difference
2	182.5	$ 131.45	$ 116.32	($ 15.14)
4	91.25	$ 262.91	$ 232.64	($ 30.27)
7	52.14	$ 460.09	$ 407.12	($ 52.97)
14	26.07	$ 920.18	$ 814.23	($ 105.95)
30	12.17	$ 1,971.81	$ 1,744.78	($ 227.03)
40	9.125	$ 2,629.08	$ 2,326.38	($ 302.71)
60	6.08	$ 3,943.63	$ 3,489.57	($ 454.06)

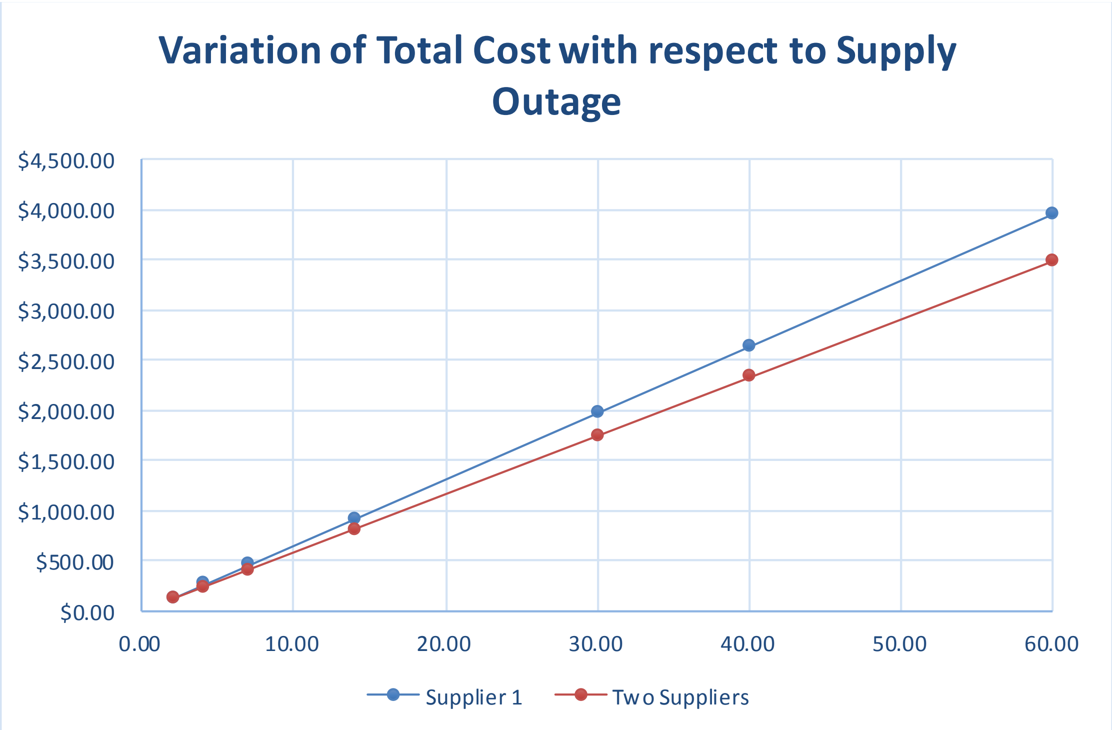

Figure 10: Total cost variation with respect to mean of interruption time

A direct relationship is realized between the increase in the mean of interruption time and the total cost incurred to the system. As the expected interruption time increase, the difference between the two systems increase reflecting the benefits of diversifying suppliers on the total costs incurred.

2. Scenario II: Two Non-Identical Suppliers

Scenario II represents a more realistic case where two different types of suppliers are available for procurement. The first supplier sells at lower costs but is less reliable while the second supplier is costlier but more reliable.

Table 10: Base parameter values for Scenario II

Parameters	Supplier	Notation	Value	Units
Financial Cost	-	i	15	%
Variable Ordering Cost	S_1	c_1	0.15	USD/unit
Variable Ordering Cost	S_2	c_2	0.25	USD/unit
Demand	-	β	18,000	Units/year
Shortage Cost	-	π	40,000	USD/year
Percentage of Supplies	S_1	α_1	0.3	Unit-less
Rate of supply interruption time	S_1	μ_1	365/30	Years^{-1}
Rate of supply interruption time	S_2	μ_2	365/20	Years^{1}
Mean time between interruptions	S_1	γ_1	1	years
Mean time between interruptions	S_2	γ_2	1.5	years

a. <u>Optimization of the decision variables</u>

Table 11: Scenario II optimization table

α_1	S_1 (Units)	TCu_1 (USD)	S_2 (Units)	TCu_2 (USD)	S_{1-2} (Units)	TCu_{1-2} (USD)	τ_1 (Days/year)	τ_2 (Days/year)	τ_{1-2} (Days/year)
0					3,506	$332.93			0.375
0.1					3,277	$320.85			0.325
0.2					3,079	$309.07			0.271
0.25					3,002	$304.17			0.239
0.3					3,002	$301.84			0.203
0.35					3,096	$303.32			0.181
0.4	6,709	$405.76	3,523	$332.94	3,239	$308.57	0.322	0.375	0.177
0.5					3,708	$326.27			0.206
0.6					4,282	$346.32			0.24
0.7					4,853	$364.58			0.268
0.8					5,457	$380.50			0.291
0.9					6,063	$394.19			0.311
1					6,672	$405.75			0.322

The same grid search analysis is used in this section in order to optimize the decision variable in use.

From Table 11, it is shown that minimum total cost is achieved at $\alpha_1 = 0.3$. This result is considered counter-intuitive since most traders tend to procure from suppliers who offers lower costs for the same product ordered. However, this example shown that for some initial parameters used, the effects of interruptions and of shortage costs highly affects the total costs incurred where more weight is given to the costlier but more reliable supplier that incurs less total costs compared to the cheaper supplier.

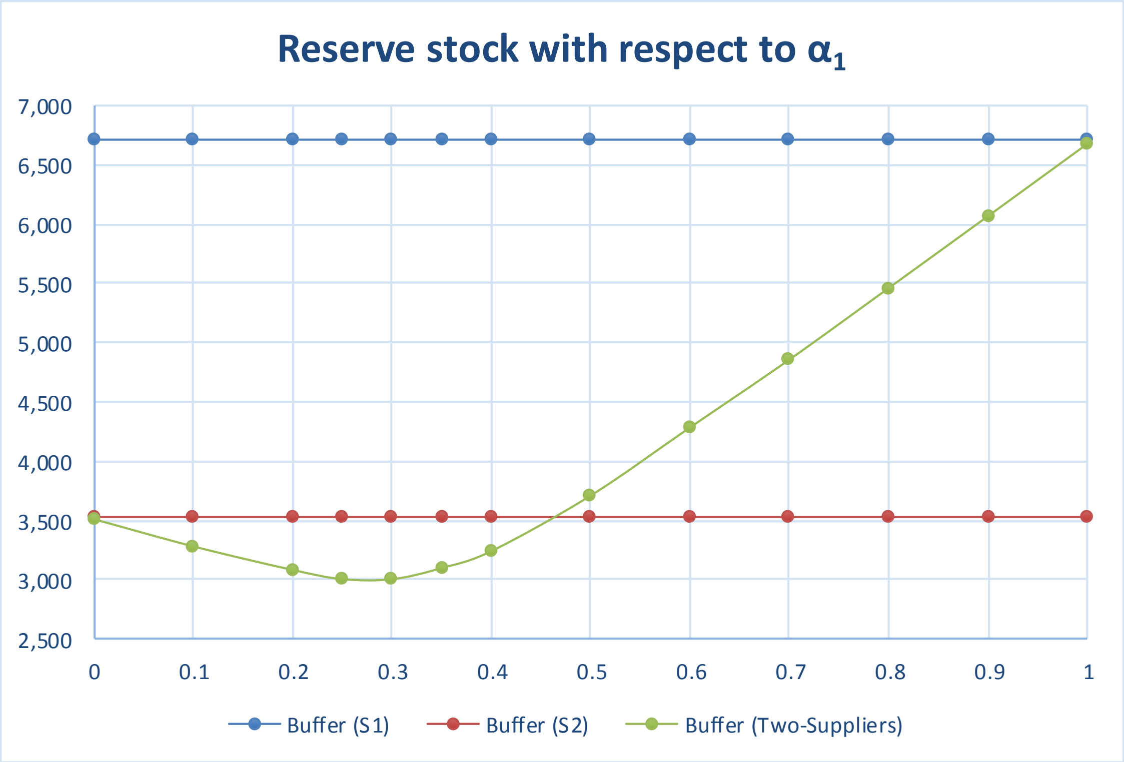

Figure 11: Reserve stock variation with respect to α_1

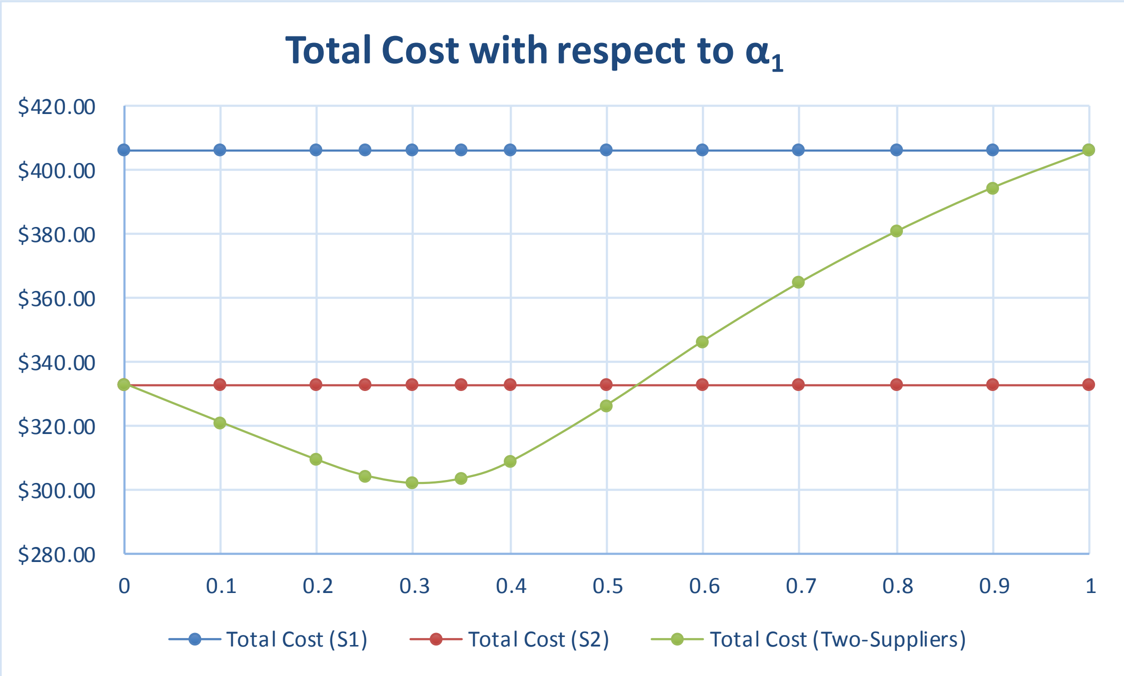

Figure 12: Total cost variation with respect to α_1

Figure 12 shows that minimum total cost is incurred at $\alpha_1 = 0.3$ where 30% of supplies should be procured from the cheaper supplier while 70% from the costlier one. In conclusion, Table 12 is a summary table where the total cost of the system is calculated with respect to the variation in α_1 and α_2.

Table 12: Table summary on scenario II

α_1	α_2	Reserve Stock	Total Cost per unit time
0.0	1.0	3,523.00	$332.94
1.0	0.0	6,709.00	$405.76
0.3	0.7	3,002.00	$301.84

As shown in Table 12, if the system depends only on the costlier but more reliable supplier (Supplier 2), then total cost incurred is equal to 332.94 USD. However, if the system depends on the cheaper but less reliable supplier (Supplier 1), total costs incurred are higher and equal to 405.76 USD. This example stress on the importance of reliability and shortage effects through the supplier selection process.

b. Sensitivity analysis on shortage cost per year

Table 13: Scenario 2 Sensitivity analysis on π

	Total Cost			**Shortage Time (days)**		
Shortage Cost	**TCu_1**	**TCu_2**	**$TCu_{1\text{-}2}$**	**τ_1**	**τ_2**	**$\tau_{1\text{-}2}$**
$ 8,000	$340.81	$247.25	$263.80	2.31	3.86	1.22
$ 10,000	$351.47	$263.97	$269.55	1.70	2.49	0.91
$ 20,000	$380.19	$302.28	$286.15	0.69	0.86	0.41
$ 30,000	$395.38	$320.70	$295.39	0.45	0.53	0.27
$ 40,000	$405.79	$332.93	$301.83	0.34	0.38	0.20
$ 50,000	$413.68	$342.13	$306.80	0.26	0.30	0.16
$ 60,000	$420.06	$349.47	$310.84	0.21	0.24	0.13
$ 70,000	$425.41	$355.59	$314.25	0.18	0.20	0.11
$ 80,000	$430.02	$360.85	$317.20	0.16	0.18	0.10
$ 90,000	$434.07	$365.45	$319.80	0.14	0.15	0.08
$ 100,000	$437.68	$369.54	$322.13	0.13	0.14	0.08

Sensitivity analysis is done on shortage cost per unit time where values varied from $8000 till $100,000 per year. As shown in Table 13, for shortage costs above 10,000 USD / year, the dual-sourcing strategy incurs lower costs compared to having either supplier 2 alone or the combined system.

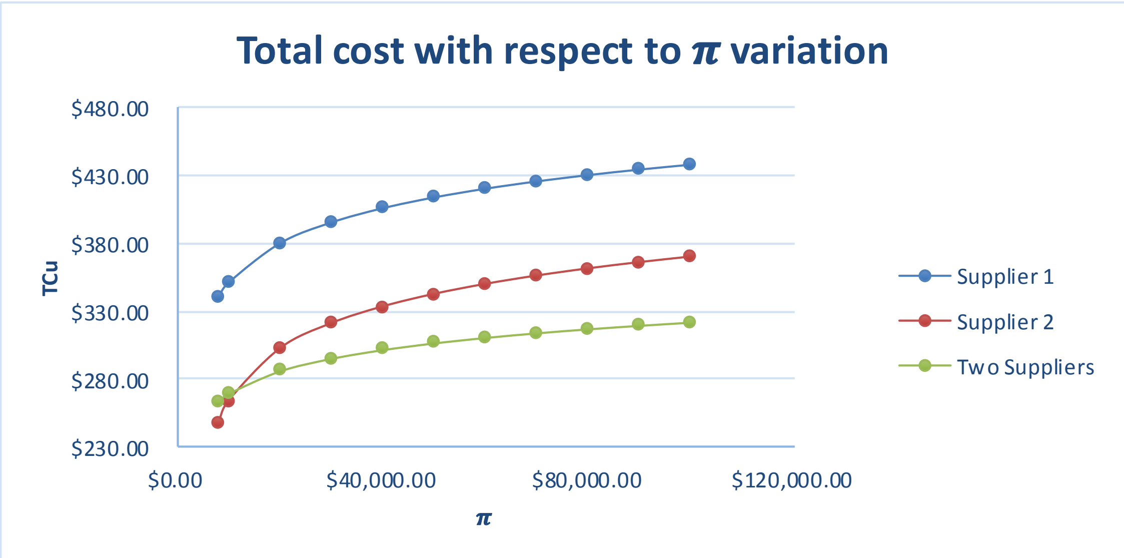

Figure 13: Total cost vs. shortage cost variation

As shown in Figure 13, for values of π between $8,000 / year and around $13,000 / year, ordering only from Supplier 2 incur lower costs compared to having a combination of two suppliers. Thus, at low shortage costs per year, the effects of ordering and holding costs seems to favor the single sourcing strategy. The reasoning behind this result is that shortage costs has low effects on the total cost of the system. However, as shortage costs per year increases, the dual supplier model becomes more efficient thus reducing the total cost of the system. In addition, depending only on Supplier 1 (cheaper supplier) results in higher total cost than depending either on supplier 2 alone or on a combination between both suppliers. Thus, supplier 1 is considered an out-of-comparison supplier incurring higher costs at all values of π. In addition, a common strategy followed by some organizations is shown in this graph where at low shortage values, depending on one supplier while experiencing shortages seems to be more efficient compared to having a reserve-diversified strategy due to the limited capabilities to diversify.

Table 14: Total cost division with respect to shortage cost variation

		Supplier 1			**Supplier 2**			**Two Suppliers**		
π		S_1	Cu_1	TCu_1	S_2	Cu_2	TCu_2	S_{1-2}	Cu_{1-2}	TCu_{1-2}
$20,000	Holding	5,572	$125.4	$380.1	2,695	$101.1	$302.3	2,562	$84.5	$286.2
	Shortage		$38			$47.5			$23	
	Ordering		$216.8			$153.7			$178.6	
$40,000	Holding	6,629	$149.2	$405.8	3,502	$131.3	$332.9	3,009	$99.3	$301.8
	Shortage		$37.2			$42			$22.4	
	Ordering		$219.4			$160			$180.1	
$60,000	Holding	7,300	$164.2	$420.1	3,930	$147.4	$349.5	3,265	$107.8	$310.8
	Shortage		$35.5			$40.8			$22.5	
	Ordering		$220.3			$161.3			$180.6	
$80,000	Holding	7,759	$174.6	$430	4,244	$159.2	$360.9	3,458	$114.1	$317.2
	Shortage		$34.7			$39.5			$22.3	
	Ordering		$220.7			$162.2			$180.9	

Table 14 shows a breakdown of total cost under the proposed strategies while varying shortage costs per year from \$ 20,000 per year till \$ 80,000 per year. It is shown that the dual-supplier system holds the minimum buffer supply. In addition, diversifying suppliers reduced total costs for all values taken into consideration (shortage cost reduced by around 35% to 40% compared to the single supplier strategy). However, procurement from two suppliers results in higher ordering costs than that of having Supplier 2 only due to the fact that the diversified system is experiencing less idle time (System is ordering more products).

c. Sensitivity analysis on demand rate

In Scenario I, high demand rates reflect lower total costs achieved using a single sourcing strategy. While taking into consideration un-identical suppliers, similar results are achieved as shown in Table 15 and Figure 14.

Table 15: Scenario II sensitivity analysis on β

	Total Cost (USD)			**Average Shortage Time (Days / year)**		
β	**TCu_1**	**TCu_2**	**TCu_{1-2}**	**τ_1**	**τ_2**	**τ_{1-2}**
5,000	\$128.8	\$110.8	\$94.2	0.06	0.06	0.04
7,000	\$176	\$150.2	\$129	0.08	0.09	0.05
10,000	\$244.6	\$207	\$179.9	0.12	0.13	0.08
15,000	\$355.3	\$297.3	\$262.4	0.18	0.20	0.11
20,000	\$462.6	\$383.6	\$342.8	0.24	0.28	0.15
22,000	\$504.8	\$417.2	\$374.5	0.26	0.30	0.17
25,000	\$567.4	\$466.8	\$421.6	0.30	0.35	0.19
30,000	\$670	\$547.5	\$499.1	0.38	0.43	0.23
35,000	\$770.8	\$626	\$575.6	0.44	0.52	0.27
40,000	\$870	\$702.4	\$651.1	0.51	0.60	0.31
45,000	\$967.8	\$777	\$725.8	0.57	0.70	0.35
70,000	\$1,438.8	\$1,125.3	\$1,089.3	0.97	1.24	0.56
90,000	\$1,797.7	\$1,125.3	\$1,370.3	1.35	1.82	0.75
110,000	\$2,142.7	\$1,605.5	\$1,644.4	1.72	2.57	0.96
130,000	\$2,475	\$1,808.2	\$1,912.4	2.21	3.55	1.16

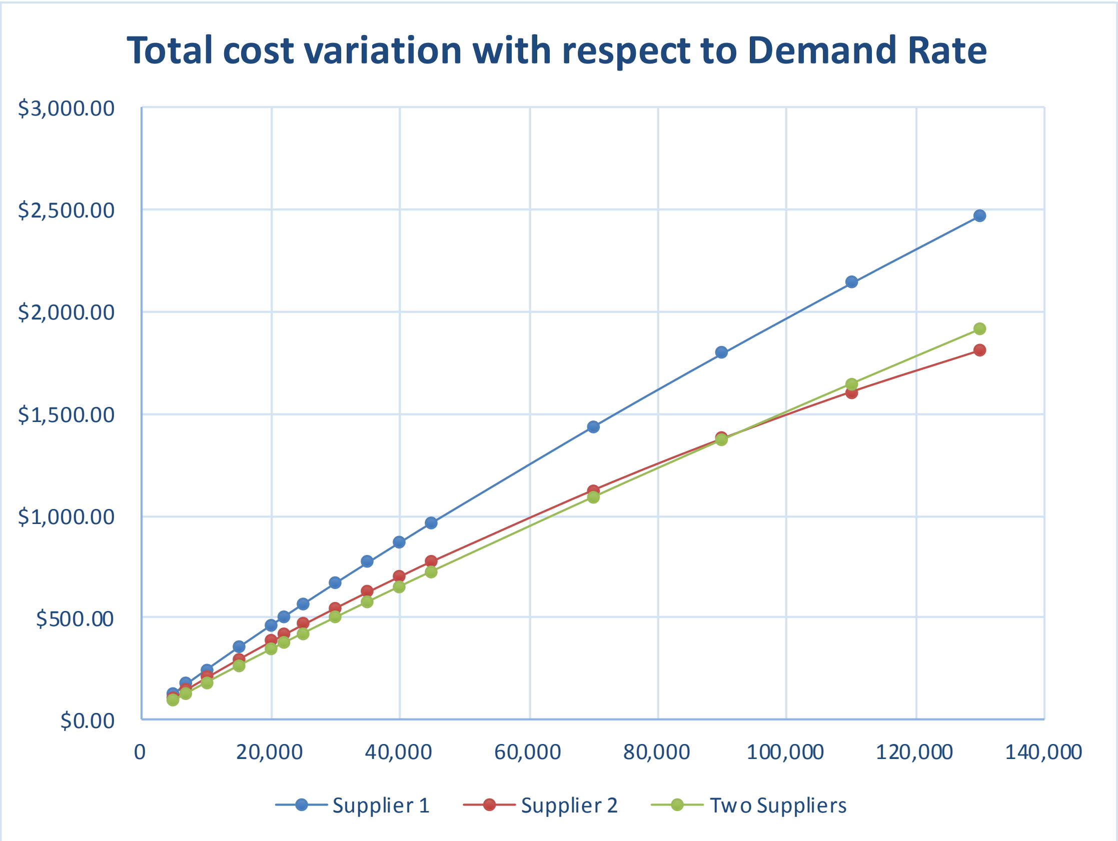

Figure 14: Total cost variation with respect to demand rate

Similar to Scenario I, the difference between both is maximized at around 45,000 units per year. This is due to the effect of the reduced shortage costs that has resulted from the reliance on more than one supplier. As demand rate increases, the dual supplier system remains on incurring lower total costs till demand reaches a value of 110,000 units per year where the single supplier model seems to incur lower costs.

Table 16: Total cost division with respect to demand variation

β	Type of Cost	Supplier 1 S_1	Supplier 1 Cu_1	Supplier 1 TCu_1	Supplier 2 S_2	Supplier 2 Cu_2	Supplier 2 TCu_2	Two Suppliers S_{1-2}	Two Suppliers Cu_{1-2}	Two Suppliers TCu_{1-2}
5,000	Holding	2,576	$58	$128.8	1,462	$54.9	$110.8	1,142	$37.7	$94.2
	Shortage		$9.3			$10.6			$6.1	
	Ordering		$61.5			$45.5			$50.4	
15,000	Holding	6,334	$142.5	$355.3	3,439	$129	$297.3	2,823	$93.2	$262.4
	Shortage		$29			$33.4			$18.6	
	Ordering		$183.9			$134.9			$150.6	
25,000	Holding	9,434	$212.3	$567.4	4,984	$186.9	$466.8	4,236	$139.8	$421.6
	Shortage		$50			$57.7			$31.6	
	Ordering		$305.1			$222.3			$250.3	
40,000	Holding	13,425	$302.1	$870	6,791	$254.7	$702.4	6,100	$201.3	$651.1
	Shortage		$83.1			$98.9			$51.3	
	Ordering		$484.8			$348.8			$398.6	
60,000	Holding	18,075	$406.7	$1,253	8,532	$319.9	$990.6	8,315	$274.4	$945.7
	Shortage		$126.3			$163.6			$77	
	Ordering		$720.8			$507			$594.3	
80,000	Holding	21,552	$484.9	$1,620	9,470	$355.1	$1,254.2	10,147	$334.8	$1,231
	Shortage		$186			$252.7			$109.1	
	Ordering		$949.1			$646.4			$786.8	
100,000	Holding	24,509	$551.5	$1,972	9,995	$374.8	$1,494.4	11,886	$392.2	$1,508
	Shortage		$250			$353.7			$138.7	
	Ordering		$1,170.4			$765.9			$977.3	
120,000	Holding	26,791	$602.8	$2,311	9,895	$371.1	$1,710.3	13,370	$441.2	$1,779
	Shortage		$326.1			$486.7			$173.8	
	Ordering		$1,381.6			$852.6			$1,164	

Table 16 shows that for values below 80,000 units per year, a dual-supplier strategy holds the lowest buffer supply. However, as demand rate increases, less reserve is held using supplier 2 only and less amount of reserve is ordered. Thus, at high demand rates, ordering costs are relatively high for a dual-supplier strategy and this is related to the lower shortage time incurred. Higher ordering costs might as well be viewed as continuation of work where business is gaining higher revenue compared to a single supplier strategy that is facing huge idle time. Therefore, the results achieved at high demand rate considers optimization of operations using diversification of suppliers even though total costs incurred are higher.

3. Scenario III: Inclusion of a Dominated Supplier

In Scenarios I and II, the effects of diversification showed that for given initial parameters, a dual-supplier system reduces total cost incurred and optimizes operations. This is due to the fact that demand is being partially met by more than one supplier and shortage costs are reduced. However, in this section, a dominated supplier is being introduced on top of a lower-cost and, on average, more reliable supplier. Specifically, for Supplier 1, the unit cost is $0.23, the average time between interruptions is 1.1 years, and the average down time is 26 days. These numbers are $0.25, 1 year, and 30 days for Supplier 2. The results obtained are compared to single sourcing strategies.

Table 17: Base parameter values for Scenario III

Parameters	Supplier	Notation	Value	Units
Financial Cost	-	i	15	%
Variable Ordering Cost	S_1	c_1	0.23	USD/unit
Variable Ordering Cost	S_2	c_2	0.25	USD/unit
Demand	-	β	18,000	Units/year
Shortage Cost	-	π	40,000	USD/year
Percentage of Supplies from	S_1	α_1	0.63	Unit-less
Rate of supply interruption time	S_1	μ_1	26	Days^{-1}
Rate of supply interruption time	S_2	μ_2	30	Days^{-1}
Mean time between interruptions	S_1	γ_1	1.1	years
Mean time between interruptions	S_2	γ_2	1	years

a. <u>Optimization of the Decision Variables</u>

Figure 15: Reserve stock variation with respect to α_1

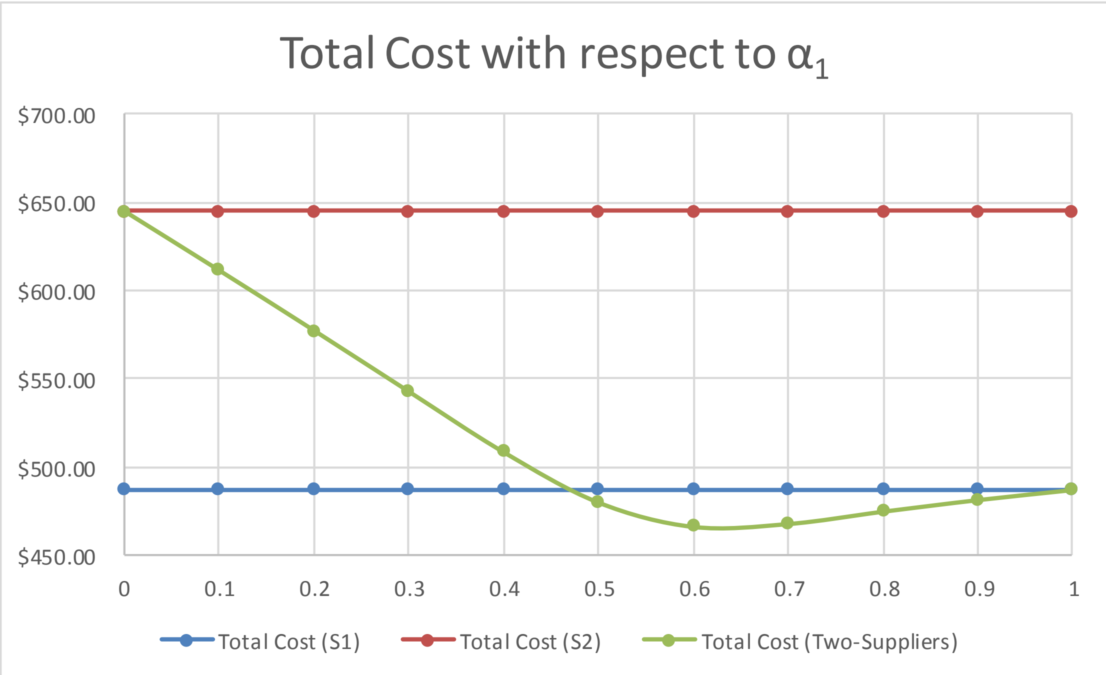

Figure 16: Total cost variation with respect to α_1

Figure 15 and Figure 16 shows the variation of the held reserve and the total cost of the system with respect to variation of procurement percentage from supplier 1. An optimum solution for the whole system is achieved at a value of α_1 = 0.63. This means that around 63% of supplies should be procured from the reliable supplier while the remaining 37% should be procured from the “dominated supplier”. Thus, the importance of diversification is realized where the inclusion of a “dominated supplier” reduced the total cost of the system by around 5% compared to having only a single supplier available for procurement. It should be noted that results obtained are case sensitive where they heavily depend on the initial given parameters.

Table 18: Scenario 3 sensitivity analysis on α_1

α_1	S_1 (Units)	TCu_1 (USD)	S_2 (Units)	TCu_2 (USD)	S_{1-2} (Units)	TCu_{1-2} (USD)	τ_1 (Days)	τ_2 (Days)	τ_{1-2} (Days)
0	5,085	$487.54	5,825	$645.20	5,862	$645.18	0.45	0.572	0.572
0.1					5,452	$611.73			0.503
0.2					5,017	$577.79			0.438
0.3					4,554	$543.31			0.375
0.4					4,094	$509.06			0.309
0.5					3,792	$480.55			0.247
0.6					3,702	$466.74			0.249
0.7					3,890	$468.33			0.308
0.8					4,287	$475.29			0.358
0.9					4,687	$481.83			0.404
1					5,072	$487.54			0.45

In conclusion, Table 18 represents an optimization procedure where the total cost of the system is calculated with respect to the variation in α_1. In addition, all results are compared by having either single or dual sourcing methodology.

Table 19: Table summary on scenario III

α_1	α_2	Reserve Stock	Total Cost per unit time
0.0	1.0	5,825.00	$645.20
1.0	0.0	5,085.00	$487.54
0.6	0.4	3,702.00	$466.74

Table 19 shows a summary table where if the system considered depends only on supplier 1 (the dominated supplier), highest total cost is achieved. However, if the system procures from supplier 2 (better supplier), then a lower total cost is resulted.

While taking into consideration the concept of diversification, minimum total cost achieved between the three systems under consideration.

b. Sensitivity analysis on shortage cost per year

In Scenarios I and II, both the single and the dual sourcing strategies were compared and results showed that diversification effects are amplified given high shortage costs per unit time. However, suppliers in scenarios I and II where in a complementary relation where the more reliable supplier is costlier than that of the less reliable. In this section, the main objective is to study the effects of diversification with respect to variation in shortage costs per unit time and to study whether diversification effects are also realized by having non-complementary suppliers (the inclusion of a dominated supplier).

Table 20: Scenario III Sensitivity analysis on π

	Total Cost (USD)			**Shortage Time (days)**		
π	**TCu_1**	**TCu_2**	**TCu_{1-2}**	**τ_1**	**τ_2**	**τ_{1-2}**
$ 8,000	$388.94	$516.65	$415.46	4.58	5.72	1.68
$ 10,000	$407.40	$541.73	$423.31	3.04	3.70	1.25
$ 20,000	$451.45	$599.22	$445.44	1.138	1.33	0.56
$ 30,000	$473.07	$626.83	$457.55	0.68	0.79	0.36
$ 40,000	$487.54	$645.19	$465.95	0.49	0.57	0.27
$ 50,000	$498.43	$658.97	$472.40	0.39	0.45	0.21
$ 60,000	$507.15	$669.99	$477.63	0.32	0.37	0.18
$ 70,000	$514.43	$679.17	$482.03	0.27	0.311	0.15
$ 80,000	$520.68	$687.05	$485.84	0.23	0.26	0.13
$ 90,000	$526.18	$693.95	$489.21	0.21	0.23	0.12
$ 100,000	$531.03	$700.09	$492.19	0.18	0.21	0.1

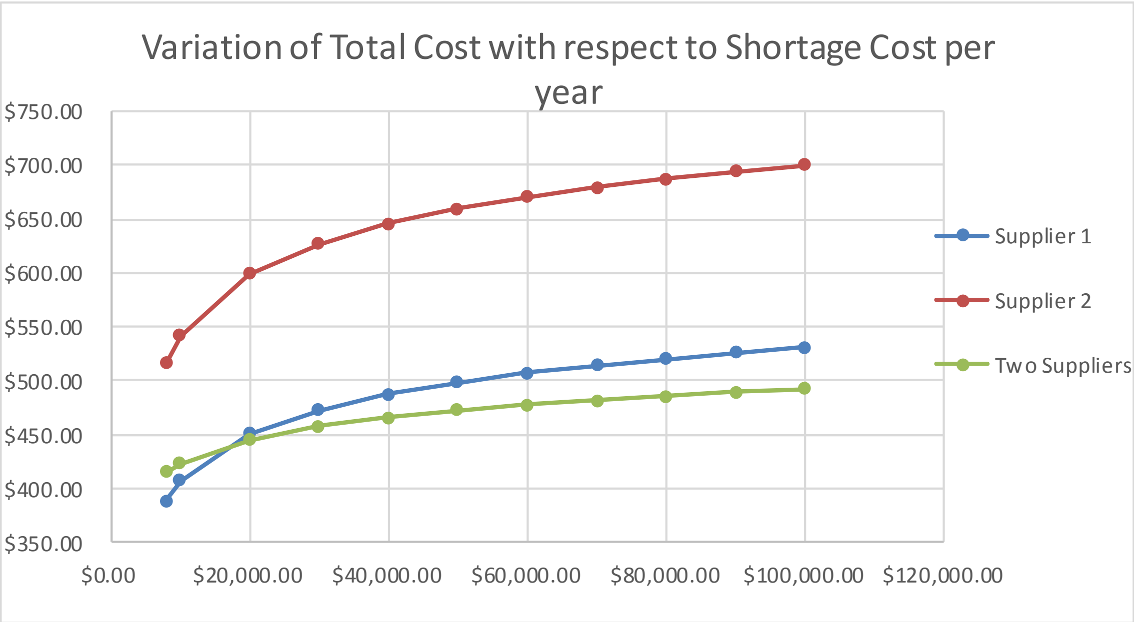

Figure 17: Total cost vs. shortage cost variation

Figure 17 shows the same graphical insights that were analyzed in Scenario I. However, the out of comparison supplier is the dominated supplier that incurs higher total costs for every value of shortage costs per year compared to either depending on the better supplier or on the combined system.

Table 21: Total cost division with respect to shortage cost variation

		Supplier 1			Supplier 2			Two Suppliers		
π		S_1	Cu_1	TCu_1	S_2	Cu_2	TCu_2	S_{1-2}	Cu_{1-2}	TCu_{1-2}
$20,000	Holding	4,011	$138.4	$451.5	4,601	$172.6	$599.2	3,176	$113.1	$445.4
	Shortage		$56.7			$73.3			$30.5	
	Ordering		$256.4			$353.4			$301.9	
$40,000	Holding	5,071	$174.9	$487.5	5,853	$219.5	$645.2	3,734	$133	$466
	Shortage		$49.6			$62.9			$29.2	
	Ordering		$263			$362.8			$303.8	
$60,000	Holding	5,623	$194	$507.2	6,495	$243.6	$670	4,047	$144.1	$477.6
	Shortage		$48.4			$61.1			$29.1	
	Ordering		$264.8			$365.3			$304.4	
$80,000	Holding	6,971	$208.1	$520.7	6,032	$261.4	$687.1	4,282	$152.5	$485.8
	Shortage		$46.9			$59.1			$28.6	
	Ordering		$265.7			$366.5			$304.7	

It is shown that for small values of shortage costs per year, a single supplier system incurs lower costs compared to that of a dual supplier system since holding and ordering costs outweigh the increase in costs resulted from supply interruptions. However, as shortage costs per unit time increase to around $18,000 per year, the dual supplier system becomes more efficient and incurs lower costs compared to having either supplier 1 or supplier 2 alone. Results obtained clarifies the main objectives of diversification where for high values of shortage costs, the inclusion of the dominated supplier attains lower total costs than depending only on the better supplier.

c. Sensitivity analysis on demand rate

Table 22: Scenario III sensitivity analysis on β

	Total Cost (USD)			**Shortage Time**		
β	**TCu_1**	**TCu_2**	**TCu_{1-2}**	**τ_1**	**τ_2**	**τ_{1-2}**
5,000	$152.2	$200.3	$139.6	0.13	0.14	0.07
7,000	$207	$272.9	$191.7	0.18	0.21	0.10
10,000	$286.5	$378.2	$268.2	0.27	0.30	0.15
15,000	$413.7	$547.1	$392.7	0.41	0.47	0.22
20,000	$535.9	$709.5	$514.3	0.57	0.66	0.30
22,000	$583.6	$773	$562.3	0.64	0.72	0.33
25,000	$654.1	$866.9	$633.8	0.72	0.84	0.38
30,000	$769	$1,019.9	$751.6	0.90	1.05	0.46
35,000	$880.9	$1,169	$867.8	1.09	1.27	0.55
40,000	$990	$1,314.6	$982.7	1.31	1.53	0.63
45,000	$1,096.5	$1,456.6	$1,096.4	1.52	1.79	0.72
70,000	$1,592.6	$2,117.7	$1,649.9	2.97	3.57	1.21
90,000	$1,944.7	$2,583.3	$2,077.3	4.61	5.91	1.69
110,000	$2,249	$2,975.2	$2,492.6	7.43	10.02	2.20

Table 22 shows that for high values of demand rate, the diversified system incurs higher total costs. However, this insight is explained before since the single sourcing system is not able to hold a huge amount of reserve in order to avoid shortages. Thus, the system is allowing for shortages while facing idle time. However, the diversified system is able to optimize operations and to manage the continuity of operations.

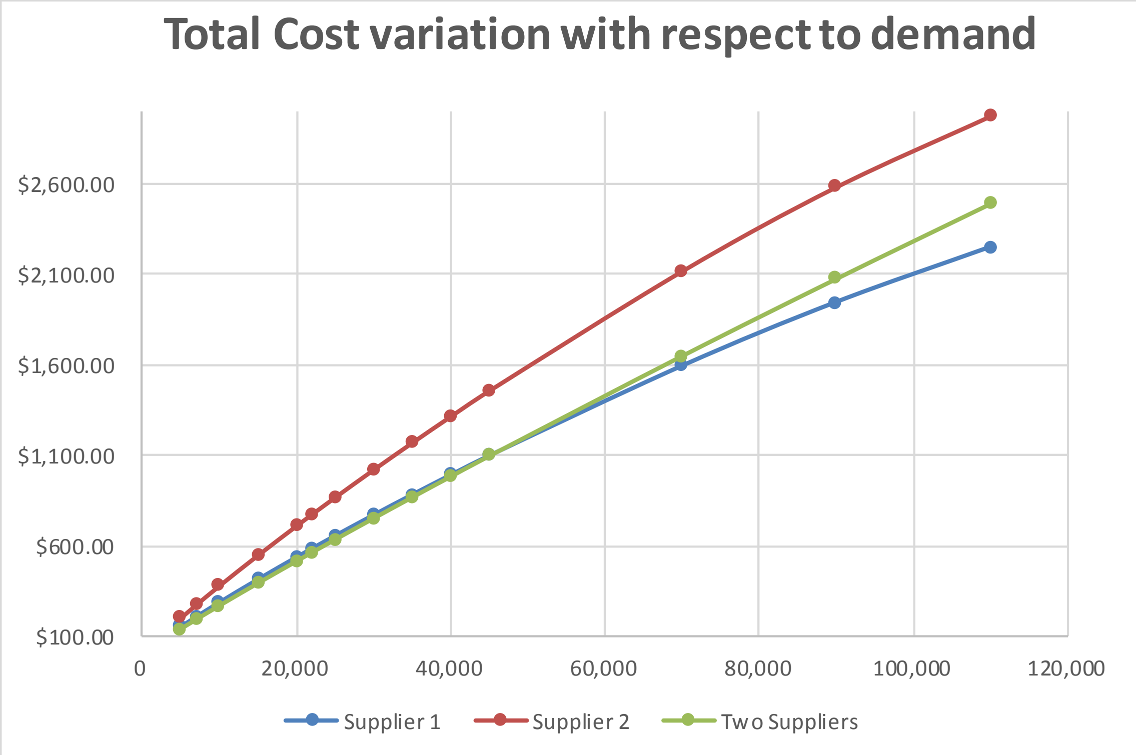

Figure 18: Total cost variation with respect to demand rate

Figure 18 and Table 23 show that for demand values below 45,000 units per year, having a dual-supplier strategy incurs lower costs to the total system compared to having a single supplier strategy. Similar to Scenarios I and II, the difference between both is maximized at around 20,000 units per year. This is due to the effect of the reduced shortage costs that results from the reliance on more than one supplier. As the value of demand increases, the diversified system keeps to be advantageous till it reaches a value of 45,000 units per year where having two suppliers incurs higher costs and optimizes operations by managing the continuity of operations.

Table 23: Total cost division with respect to demand rate variation

β	Type of Cost	Supplier 1 S_1	Supplier 1 Cu_1	Supplier 1 TCu_1	Supplier 2 S_2	Supplier 2 Cu_2	Supplier 2 TCu_2	Two Suppliers $S_{1\text{-}2}$	Two Suppliers $Cu_{1\text{-}2}$	Two Suppliers $TCu_{1\text{-}2}$
5,000	Holding	1,896	$65.4	$152.2	2,192	$82.2	$200.3	1,319	$47	$139.6
	Shortage		$12.6			$15.9			$7.9	
	Ordering		$74.1			$102.2			$84.8	
15,000	Holding	4,431	$152.9	$413.7	5,118	$191.9	$547.1	3,232	$115.1	$392.7
	Shortage		$40.9			$51.8			$24.1	
	Ordering		$219.9			$303.1			$253.5	
25,000	Holding	6,376	$220	$654.1	7,354	$275.8	$866.9	4,810	$171.3	$633.8
	Shortage		$72.2			$91.8			$41.6	
	Ordering		$362			$499.4			$420.9	
40,000	Holding	8,514	$293.7	$990	9,780	$366.8	$1,315	6,845	$243.7	$982.7
	Shortage		$130.5			$167.9			$69.2	
	Ordering		$565.8			$780			$669.8	
70,000	Holding	10,815	$373.1	$1,593	12,246	$459.2	$2,117	10,098	$359.6	$1,650
	Shortage		$296			$391.3			$132.9	
	Ordering		$923.4			$1,267.2			$1,158	
110,000	Holding	9,811	$338.5	$2,249	9,916	$371.9	$2,975	13,220	$470.7	$2,493
	Shortage		$740.6			$1,097.9			$241.2	
	Ordering		$1,170			$1,505.2			$1,781	

For consumption rate below 45,000 units per year, the diversified system sustains lower holding and shortage costs but higher ordering costs. As demand rate increases, the effects of ordering costs make the single supplier system seem to experience less total costs. As discussed earlier, the increase in ordering costs results from lower interruption rate of supplies experienced; thus, the system experiences lower shortage time and higher revenues due to continuity of operations.

CHAPTER VII

CONVEXITY PLOTS OF THE TOTAL COST FUNCTION

A. Convexity Plots

The total cost of the system depends on two decision variables. The first variable is how much to stock as reserve supply and the second decision variable is how much to procure from each available supplier. Thus, the optimization problem calculates values for S and α_1 that would minimize the total cost of the system. However, a main condition for the derivation of one optimum solution is the convexity nature of the total cost formula in order to make sure that the value obtained is a global optimum solution.

1. Convexity with respect to reserve supply S and α_1

Regarding reserve stock, the total cost function is convex with respect to S and the solution obtained is a global optimum solution (proved earlier). As for the percentage procured from each supplier, the total cost function is plotted for various values and results are shown in 3D plots in Figures 16 till 21.

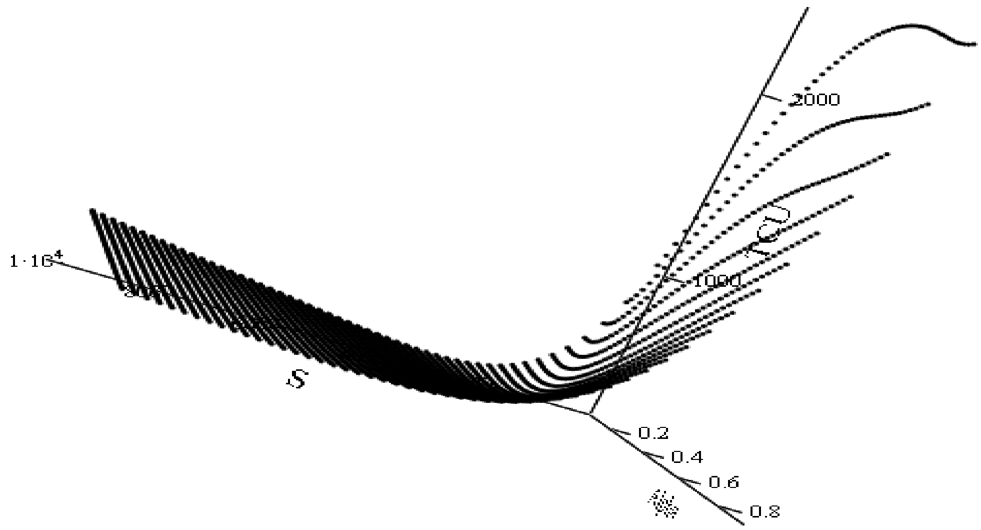

Figure 19: Variation of TCu with respect to S and α_1 (1)

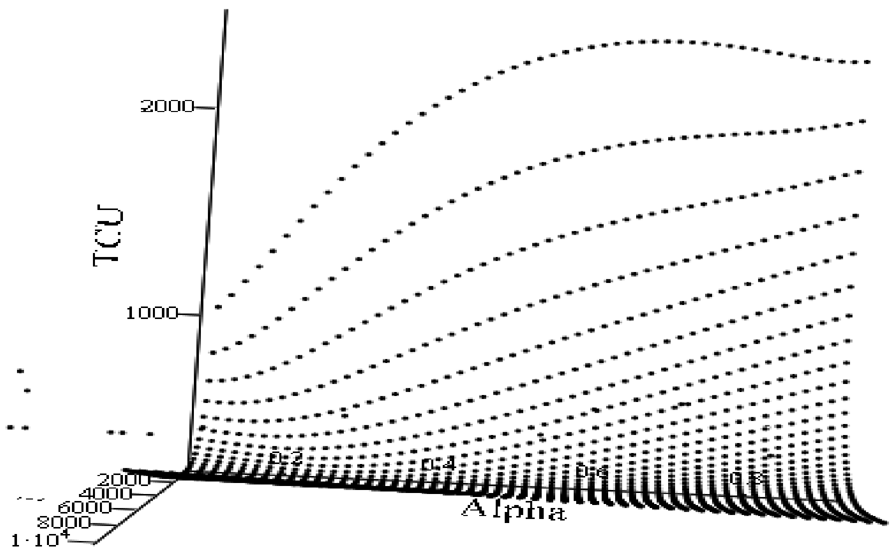

Figure 20: Variation of TCu with respect to S and α_1 (2)

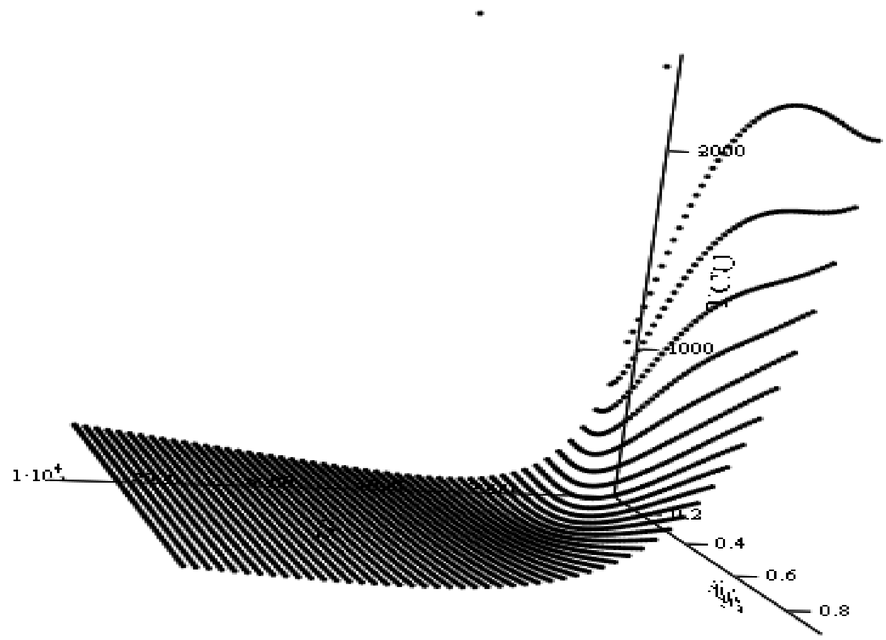

Figure 21: Variation of TCu with respect to S and α_1 (3)

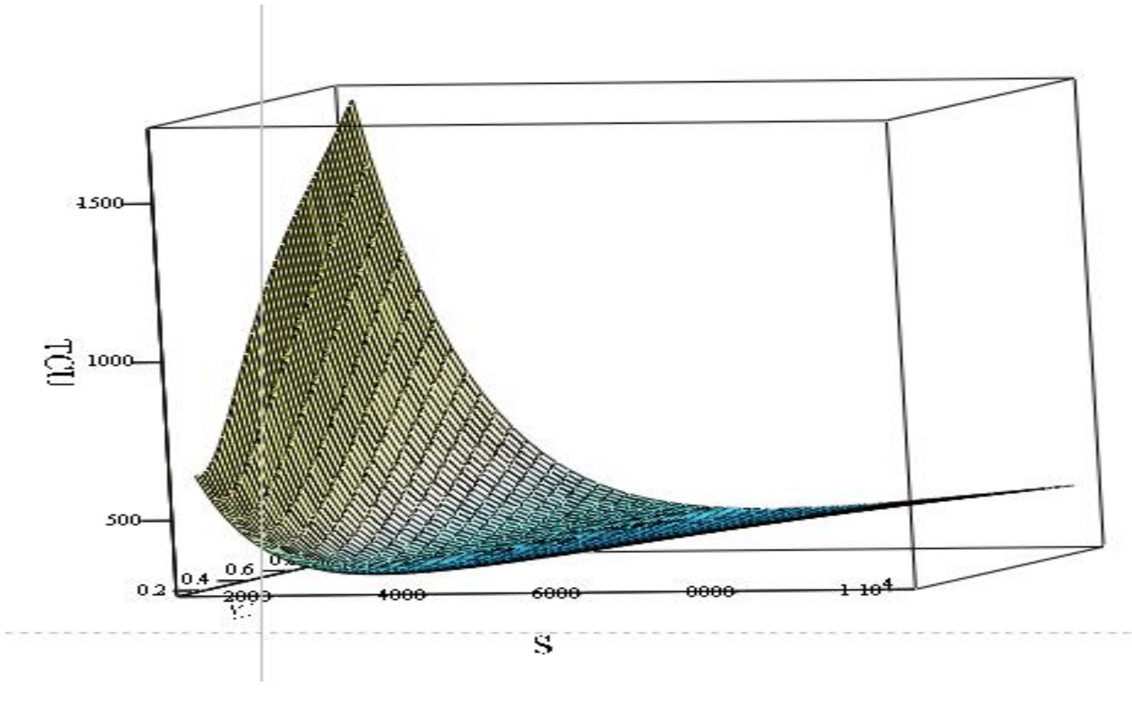

Figure 22: Variation of TCu with respect to S and α_1 (4)

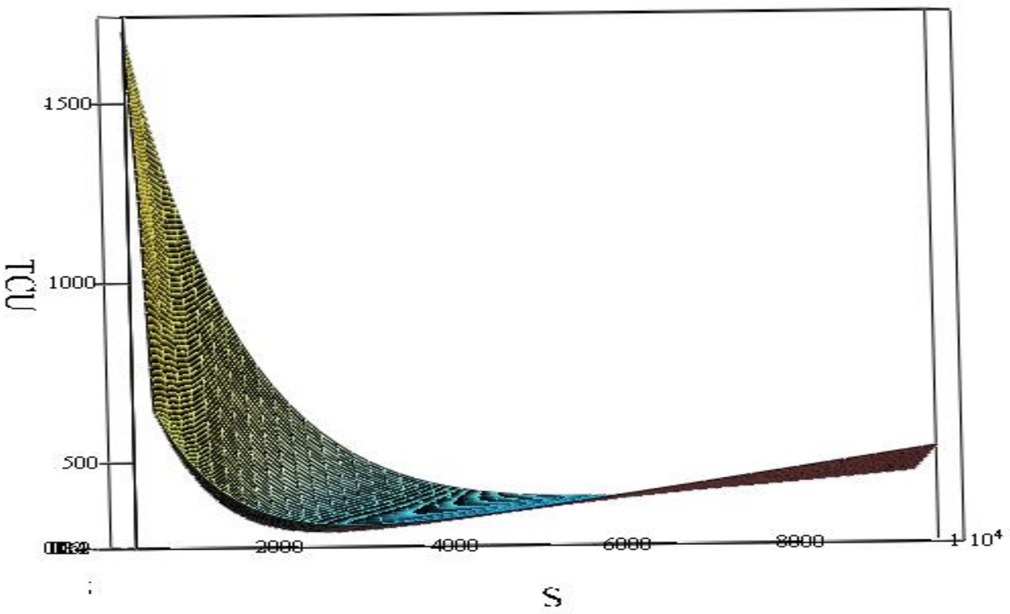

Figure 23: Variation of TCu with respect to S and α_1 (5)

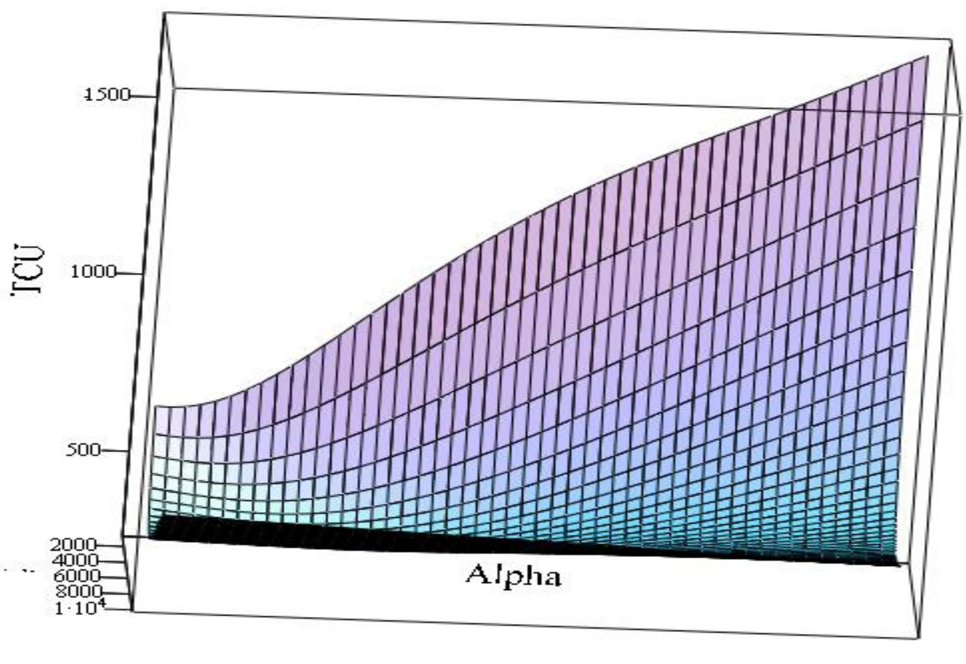

Figure 24: Variation of TCu with respect to S and α_1 (6)

As shown in Figure 19 till Figure 24, the total cost function is convex with respect to S but not with respect to α_1.

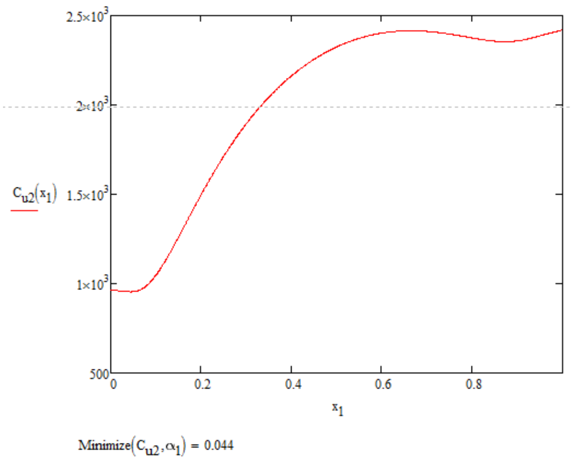

Figure 25: Variation of TCu with respect to α_1 at S = 500 units

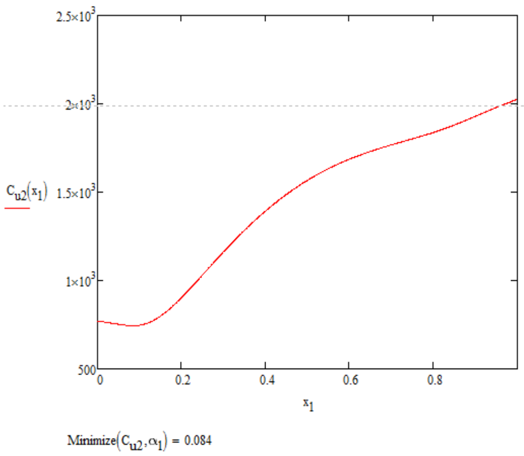

Figure 27: Variation of TCu with respect to α_1 at S = 700 units

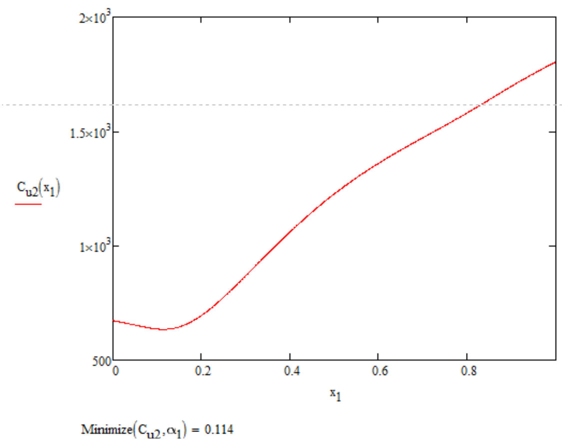

Figure 28: Variation of TCu with respect to α_1 at S = 1000 units

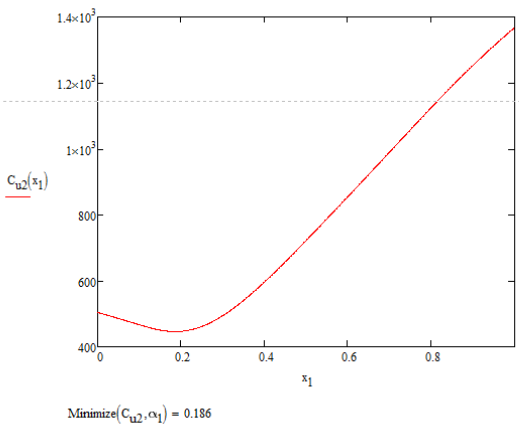

Figure 26: Variation of TCu with respect to α_1 at S = 1500 units

Figure 25 till Figure 28 show the variation of the total cost function with respect to α_1 for given values of S. As S increases from 500 till 1,500 units, the function changes from being non-convex to convex. Note that Figure 25 indicates that the total cost function is not even quasi-convex in α_1. Therefore, for a certain supply split among the two suppliers, given α_1, an optimum reserve stock for the system can be found with a simple line search and is considered a global solution for the problem. However, the solution obtained for α_1 might be either a local or a global solution. Thus, an algorithm must be followed in order to be able to determine the optimum values of α_1 and S.

B. Proposed Algorithm for an Optimal Solution

It is shown that the total cost formula is convex with respect to S and quasi-convex with respect to α_1. The following algorithm is proposed in order to find the optimum solution.

1. Given initial parameters, start with an initial assumed value for α_1.
2. TCu is convex with respect to S, find the value of S.
3. Replace the calculated optimum value for S to find α_1.
4. Repeat steps 2 and 3 till the final solution converge.

Using this proposed algorithm, one can find the optimum percentage of distribution of supply between suppliers using a minimum number of iterations.

1. Example of the Proposed Algorithm

Table 24: Base parameter values for the proposed algorithm

Parameters	Notation	Value	Units
Holding Cost	h	0.03	USD/unit/unit time
Variable Ordering Cost (S_1)	c_1	0.15	USD/unit
Variable Ordering Cost (S_2)	c_2	0.25	USD/unit
Demand	β	18,000	Units/year
Shortage Cost	π	40,000	USD/year
Percentage of Supplies from (S_1)	α_1	0.3	Unit-less
Rate of supply interruption time (S_1)	μ_1	365/30	Years^{-1}
Rate of supply interruption time (S_2)	μ_2	365/20	Years^{-1}
Mean time between interruptions (S_1)	γ_1	1	years
Mean time between interruptions (S_2)	γ_2	1.5	years

Table 25: Algorithm example for different initial values of α_1

Trial 0		Trial 1		Trial 2		Trial 3		Trial 4	
α_0	S_0	α_1	S_1	α_2	S_2	α_3	S_3	α_4	S_4
0.05	3406	0.321	3041	0.307	3021				
0.1	3298	0.317	3035	0.307	3021				
0.15	3186	0.313	3029	0.307	3021				
0.2	3076	0.309	3023	0.307	3021				
0.25	3004	0.306	3020	0.307	3021				
0.3	3013	0.306	3020	0.307	3021				
0.35	3102	0.31	3025	0.307	3021				
0.4	3255	0.316	3033	0.307	3021				
0.45	3461	0.323	3045	0.308	3022	0.307	3021		
0.5	3710	0.332	3062	0.308	3022	0.307	3021		
0.55	3990	0.344	3087	0.309	3023	0.307	3021		
0.6	4286	0.36	3128	0.311	3026	0.307	3021		
0.65	4586	0.382	3193	0.313	3029	0.307	3021		
0.7	4887	0.411	3296	0.317	3035	0.307	3021		
0.75	5187	0.447	3447	0.323	3045	0.308	3022	0.307	3021
0.8	5486	0.49	3657	0.33	3058	0.308	3022	0.307	3021
0.85	5786	0.537	3915	0.341	3081	0.309	3023	0.307	3021
0.9	6086	0.587	4208	0.355	3114	0.31	3025	0.307	3021
0.95	6388	0.639	4520	0.376	3174	0.313	3029	0.307	3021

Table 25 shows several iterations done using this algorithm, each starting from a different initial assumed value for α_1.

C. Optimum supplier split

Table 26: Base parameter values for supplier split

Parameters	Supplier	Notation	Value	Units
Financial Cost	-	i	15	%
Variable Ordering Cost	S_1	c_1	0.15	USD/unit
Variable Ordering Cost	S_2	c_2	0.25	USD/unit
Demand	-	β	18,000	Units/year
Shortage Cost	-	π	40,000	USD/year
Percentage of Supplies	S_1	α_1	0.3	Unit-less
Rate of supply interruption time (S_1)	S_1	μ_1	365/30	Years^{-1}
Rate of supply interruption time (S_2)	S_2	μ_2	365/20	Years^{1}
Mean time between interruptions (S_1)	S_1	γ_1	1	years
Mean time between interruptions (S_2)	S_2	γ_2	1.5	years

In this section, the main purpose is to study the variation of the optimum percentage of diversification between suppliers with respect to changes in input parameters. Using the assumed initial parameters given in Table 26 where there exist one less reliable / cheaper supplier and one more reliable / costlier supplier. The optimum value of α_1 is calculated using the proposed algorithm and the results are shown in Table 27 and Table 28.

Table 27: Supplier diversification with respect to variation in input parameters

Parameter	Parameter Value	Optimum Stock	Optimum α_1	Total Cost
c_1	0.15	3,021	0.307	$301.799
	0.18	2,997	0.268	$318.443
	0.2	3,025	0.23	$328.153
	0.22	3,508	0	$332.938
β	14,000	2,482	0.316	$238.937
	18,000	3,021	0.307	$301.799
	22,000	3,520	0.298	$363.415
	30,000	4,416	0.28	$483.843
	40,000	5,365	0.257	$628.311
π	20,000	2,503	0.266	$285.522
	30,000	2,816	0.293	$295.356
	40,000	3,021	0.307	$301.799
	50,000	3,172	0.315	$306.606
	60,000	3,292	0.32	$310.445
μ_1	365/25	2,908	0.397	$281.532
	365/30	3,002	0.307	$301.799
	365/35	3,095	0.24	$315.65
	365/40	3,154	0.187	$325.232
λ_1	1/0.8	3,016	0.261	$318.186
	1	3,021	0.307	$301.799
	1/1.2	3,035	0.34	$288.895
	1/1.4	3,050	0.369	$279.029

The following results can be summarized:

As ordering costs of supplier 1 increase, the dependence is more on supplier 2. The same result is achieved if ordering cost of supplier 2 increase.

As the demand rate increases, the dependence is shifted to the more reliable supplier since this will decrease shortage costs during shortage time because supplier 2 is more reliable and experiences less shortage time.

As shortage cost per unit time increase, results seem to be counter intuitive. The increase in shortage costs per year lead to holding more reserves, this can be seen in the increase in holding and ordering costs and in the reduction in shortage costs incurred as given in Table 27. Thus, the system tends to hold more reserve and depend more on the cheaper supplier (supplier 1).

The increase in the expected shortage time is directly proportional to the quantity of reserve ordered from the reliable supplier. As expected shortage time increases, more supplies are order from the more reliable supplier. Similarly, the increase in the rate of interruption is indirectly proportional to the quantity of reserve ordered from the reliable supplier. As rate of interruption decreases, more supplies are ordered from the less reliable supplier thus incurring lower costs to the system

Table 28: Division of costs with respect to variation in input parameters

Parameter	Parameter Value	Holding Cost	Shortage Cost	Ordering Cost	Total Cost
c_1	0.15	\$99.376	\$21.835	\$180.587	\$301.798
	0.18	\$103.954	\$24.723	\$189.766	\$318.443
	0.2	\$108.219	\$27.671	\$192.263	\$328.153
	0.22	\$131.55	\$41.691	\$159.693	\$332.934
β	14000	\$81.31	\$16.463	\$141.164	\$238.937
	18000	\$99.376	\$21.835	\$180.587	\$301.798
	22000	\$116.266	\$27.601	\$219.549	\$363.416
	30000	\$147.053	\$40.503	\$295.922	\$483.478
	40000	\$180.505	\$59.938	\$387.867	\$628.31
π	20000	\$83.876	\$25.863	\$175.784	\$285.523
	30000	\$93.224	\$23.038	\$179.094	\$295.356
	40000	\$99.376	\$21.835	\$180.587	\$301.798
	50000	\$103.962	\$21.244	\$181.4	\$306.606
	60000	\$107.648	\$20.895	\$181.901	\$310.444
μ_1	365/25	\$91.733	\$18.197	\$171.602	\$281.532
	365/30	\$98.751	\$22.513	\$180.545	\$301.809
	365/35	\$104.921	\$25.708	\$185.022	\$315.651
	365/40	\$109.428	\$29.456	\$186.348	\$325.232
λ_1	1/0.8	\$101.292	\$24.982	\$191.912	\$318.186
	1	\$99.376	\$21.835	\$180.587	\$301.798
	1/1.2	\$98.334	\$20.366	\$170.195	\$288.895
	1/1.4	\$97.493	\$19.671	\$161.226	\$278.39

CHAPTER VIII

THREE SUPPLIERS MODEL

The three cases studied in this research covered a comparison between a single supplier and a dual supplier model. In this section, an extension to the model is evaluated by adding a third supplier. Results should confirm with that obtained in the previous sections where the concept of risk sharing reduces the total cost of the system and makes the combined portfolio more beneficial in managing shortage costs.

A. Three Identical Suppliers

Initial analysis is done on three identical suppliers where for given values of α_1, α_2, and α_3 $(1-\alpha_1- \alpha_2)$; the optimum value of total cost is calculated and plotted below:

Table 29: Base parameter values for three suppliers (1)

Parameters	Notation	Value	Units
Financial Cost	i	15	%
Variable Ordering Cost (S_1)	c_1	1	USD/unit
Variable Ordering Cost (S_2)	c_2	1	USD/unit
Variable Ordering Cost (S_3)	c_3	1	USD/unit
Demand	β	15,000	Units/year
Shortage Cost	π	60,000	USD/year
Percentage of Supplies from (S_1)	α_1	Var.	Unit-less
Percentage of Supplies from (S_2)	α_2	Var.	Unit-less
Percentage of Supplies from (S_3)	α_3	Var.	Unit-less
Expected Down time (S_1)	μ_1	365/30	Years^{-1}
Expected Down time (S_2)	μ_2	365/30	Years^{-1}
Expected Down time (S_3)	μ_3	365/30	Years^{-1}
Time between interruptions (S_1)	γ_1	1	years
Time between interruptions (S_2)	γ_2	1	years
Time between interruptions (S_3)	γ_3	1	years

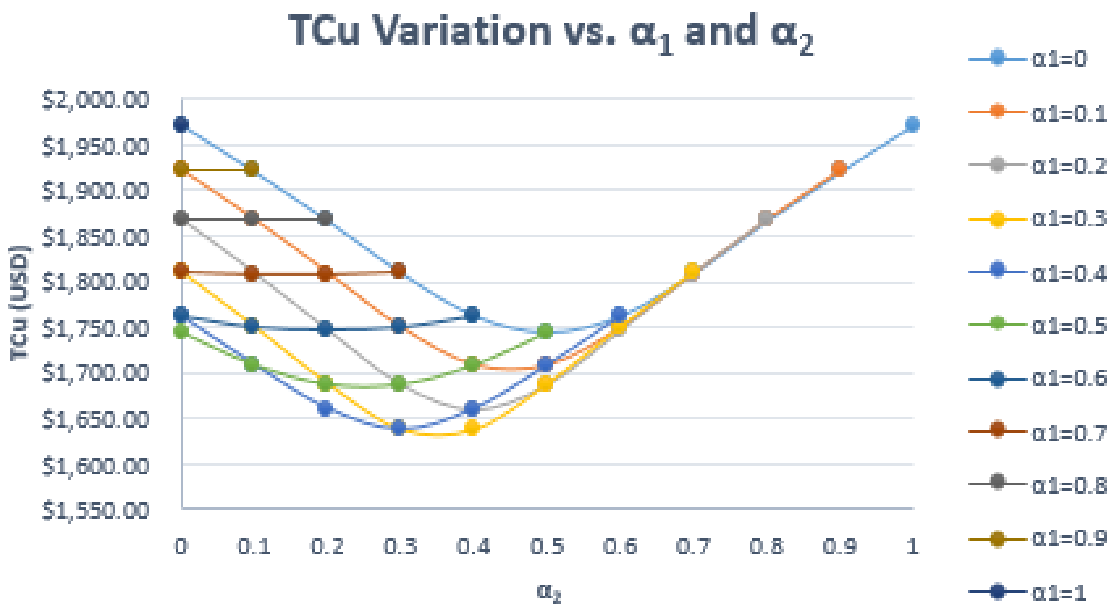

Figure 29: Variation of TCu w.r.t α_1 and α_2 and α_3

Table 30: Variation of TCu w.r.t. α_1, α_2, and α_3

	α_1										
α_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$1,972	$1,921	$1,867	$1,810	$1,763	$1,745	$1,763	$1,810	$1,867	$1,921	$1,972
0.1	$1,921	$1,867	$1,809	$1,750	$1,709	$1,709	$1,750	$1,809	$1,867	$1,921	
0.2	$1,867	$1,809	$1,747	$1,688	$1,661	$1,688	$1,747	$1,809	$1,867		
0.3	$1,810	$1,750	$1,688	$1,638	$1,638	$1,688	$1,750	$1,810			
0.4	$1,763	$1,709	$1,661	$1,638	$1,661	$1,709	$1,763				
0.5	$1,745	$1,709	$1,688	$1,688	$1,709	$1,745					
0.6	$1,763	$1,750	$1,747	$1,750	$1,763						
0.7	$1,810	$1,809	$1,809	$1,810							
0.8	$1,867	$1,867	$1,867								
0.9	$1,921	$1,921									
1	$1,972										

Figure 29 and Table 30 show the variation of total cost with respect to α_1, α_2, and α_3 $(1-\alpha_1-\alpha_2)$. Results show that for value of $\alpha_1 = 0$, the optimum value of total cost is at $\alpha_2 = \alpha_3 = 0.5$ (confirmed with results of previous sections). As values of α_1

increase, minimum cost of the system is achieved at $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$ with a TCu of 1,627 USD. Therefore, supplies should be equally procured from the three different suppliers available.

Table 31: Base parameter values for three suppliers (2)

Parameters	Notation	Value	Units
Financial Cost	i	15	%
Variable Ordering Cost (S_1)	c_1	1	USD
Variable Ordering Cost (S_2)	c_2	1	USD
Variable Ordering Cost (S_3)	c_3	1	USD
Demand	β	40,000	Units
Shortage Cost	π	60,000	USD
Percentage of Supplies from (S_1)	α_1	Var.	Unit-less
Percentage of Supplies from (S_2)	α_2	Var.	Unit-less
Percentage of Supplies from (S_3)	α_3	Var.	Unit-less
Expected Down time (S_1)	μ_1	365/30	Years^{-1}
Expected Down time (S_2)	μ_2	365/30	Years^{-1}
Expected Down time (S_3)	μ_3	365/30	Years^{-1}
Time between interruptions (S_1)	γ_1	1	years
Time between interruptions (S_2)	γ_2	1	years
Time between interruptions (S_3)	γ_3	1	years

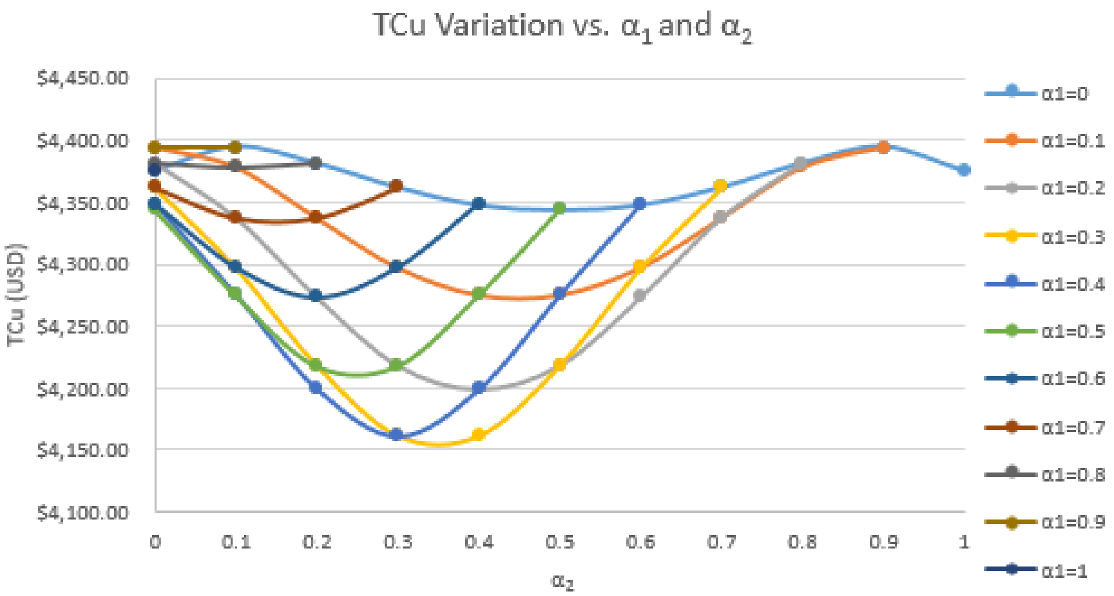

Figure 30: Variation of TCu w.r.t α_1 and α_2 and α_3

In this second case, the minimum cost is also achieved at $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$ with a TCu of 4,150 USD. However, for $\alpha_1 = 0$, the total cost not a quasi-convex and search methods cannot guarantee finding a global optimum solution.

B. Three Non-Identical Suppliers

In this scenario, the total cost is evaluated on three un-identical suppliers where for given values of α_1, α_2, and α_3 $(1-\alpha_1-\alpha_2)$; the optimum value of total cost is calculated and plotted below:

Table 32: Base parameter values for three suppliers (3)

Parameters	Notation	Value	Units
Financial Cost	i	15	%
Variable Ordering Cost (S_1)	c_1	1	USD
Variable Ordering Cost (S_2)	c_2	1.1	USD
Variable Ordering Cost (S_3)	c_3	0.9	USD
Demand	β	15,000	Units
Shortage Cost	π	60,000	USD
Percentage of Supplies from (S_1)	α_1	Var.	Unit-less
Percentage of Supplies from (S_2)	α_2	Var.	Unit-less
Expected Down time (S_1)	μ_1	365/30	Years^{-1}
Expected Down time (S_2)	μ_2	365/25	Years^{-1}
Expected Down time (S_3)	μ_3	365/35	Years^{-1}
Time between interruptions (S_1)	γ_1	1	years
Time between interruptions (S_2)	γ_2	1.1	years
Time between interruptions (S_3)	γ_3	0.9	years

From Table 32, supplier 1 is moderately costly and reliable, supplier 2 is the most expensive but the most reliable, and supplier 3 is the cheapest but the least reliable.

Table 33: Variation of TCu w.r.t. α_1, α_2, and α_3

	α_1										
α_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$2,382	$2,296	$2,206	$2,111	$2,015	$1,937	$1,907	$1,934	$1,986	$2,039	$2,089
0.1	$2,283	$2,192	$2,097	$1,997	$1,900	$1,841	$1,849	$1,901	$1,956	$2,010	
0.2	$2,179	$2,083	$1,982	$1,877	$1,787	$1,765	$1,811	$1,870	$1,927		
0.3	$2,069	$1,967	$1,860	$1,752	$1,688	$1,718	$1,780	$1,839			
0.4	$1,953	$1,846	$1,737	$1,644	$1,637	$1,691	$1,751				
0.5	$1,838	$1,737	$1,650	$1,610	$1,632	$1,676					
0.6	$1,749	$1,678	$1,640	$1,629	$1,641						
0.7	$1,713	$1,684	$1,667	$1,655							
0.8	$1,727	$1,711	$1,695								
0.9	$1,753	$1,737									
1	$1,777										

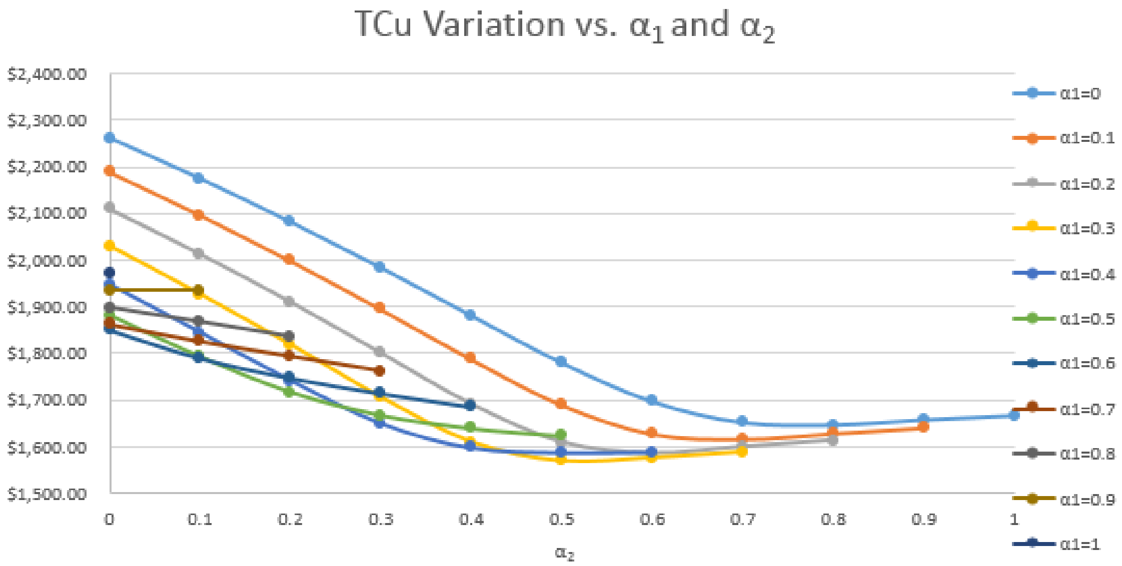

Figure 31: Variation of TCu w.r.t α_1 and α_2 and α_3

For the case of non-identical suppliers and for specific initial assumed parameters, the minimum cost is achieved at α_1=0.3, α_2=0.5, and α_3=0.2 with a TCu of 1,569 USD. Thus, the effects of diversification affect the system of three suppliers where the total cost of the system is minimized when all suppliers share risk depending on the input data of each. In addition, the highest weight is given to the costlier but most reliable supplier. Thus, this example shows the importance of taking into consideration minimization of shortage costs while taking into consideration that all results are sensitive to initial parameters used.

C. Inclusion of a dominated Supplier

In this scenario, the total cost is evaluated on three non-identical suppliers with a dominated supplier (Supplier 2) having higher ordering costs and higher interruption rate/time. For given values of α_1, α_2, and α_3 ($1-\alpha_1-\alpha_2$); the optimum value of total cost is calculated and plotted below:

Table 34: Base parameter values for three suppliers (4)

Parameters	Notation	Value	Units
Financial Cost	i	15	%
Variable Ordering Cost (S_1)	c_1	0.24	USD
Variable Ordering Cost (S_2)	c_2	0.25	USD
Variable Ordering Cost (S_3)	c_3	0.23	USD
Demand	β	18,000	Units
Shortage Cost	π	40,000	USD
Percentage of Supplies from (S_1)	α_1	Var.	Unit-less
Percentage of Supplies from (S_2)	α_2	Var.	Unit-less
Expected Down time (S_1)	μ_1	365/31	Years^{-1}
Expected Down time (S_2)	μ_2	365/32	Years^{-1}
Expected Down time (S_3)	μ_3	365/30	Years^{-1}
Time between interruptions (S_1)	γ_1	1	years
Time between interruptions (S_2)	γ_2	0.9	years
Time between interruptions (S_3)	γ_3	1.1	years

Table 34 shows supplier 1 which is moderately costly and reliable, supplier 2 which is the most expensive and the least reliable (a dominated supplier), and supplier 3 which is the cheapest but the most reliable supplier (best between suppliers).

Table 35: Variation of TCu w.r.t. α_1, α_2, and α_3

	α_1										
α_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$563	$549	$535	$520	$508	$509	$527	$556	$585	$614	$642
0.1	$557	$543	$528	$512	$503	$513	$540	$570	$600	$629	
0.2	$551	$535	$519	$503	$501	$522	$554	$584	$614		
0.3	$544	$528	$512	$500	$510	$538	$568	$599			
0.4	$540	$529	$521	$521	$536	$559	$586				
0.5	$553	$551	$554	$560	$571	$588					
0.6	$583	$588	$594	$601	$609						
0.7	$622	$628	$635	$641							
0.8	$661	$668	$674								
0.9	$700	$707									
1	$738										

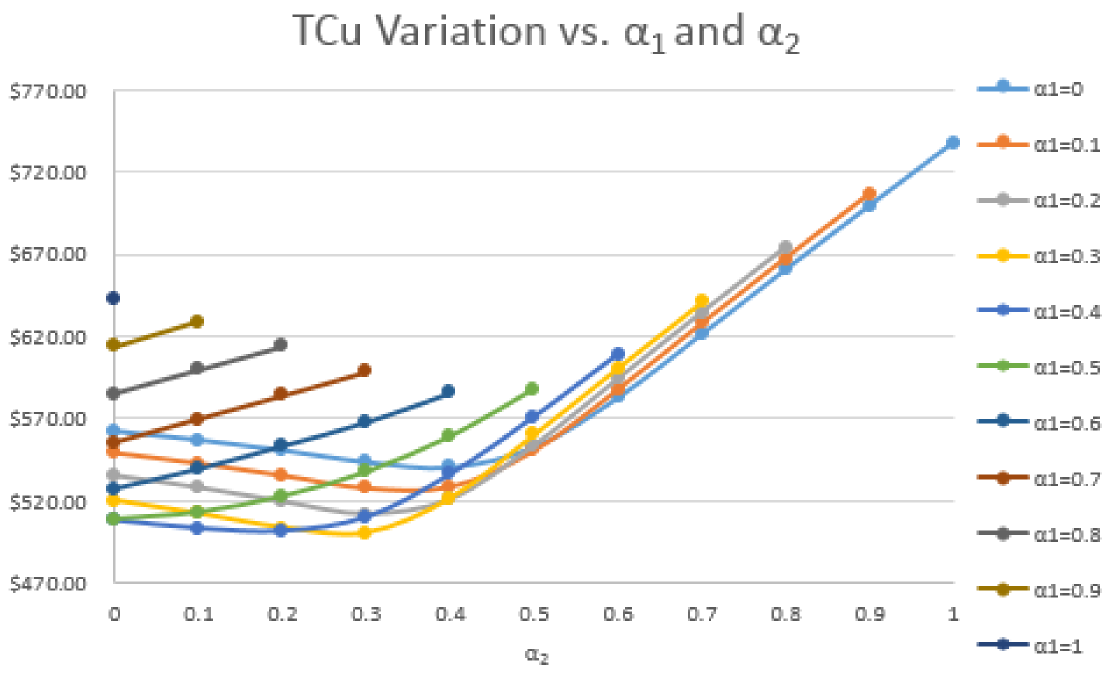

Figure 32: Variation of TCu w.r.t α_1 and α_2 and α_3

For the case of un-identical suppliers with a dominated supplier, and for specific initial assumed parameters, the minimum cost is also achieved at α_1=0.3, α_2=0.3, and α_3=0.4 with a TCu of 500.29 USD. Thus, procuring from a dominated supplier by around 30% will reduce the total cost of the system. In this example, it is shown that equal weights are given for the moderately costly and reliable supplier and the dominated supplier. This result stresses on the benefits of diversification where including a dominated supplier into the portfolio of suppliers reduces total costs due to the decrease in shortage time experienced.

CHAPTER IX

SUMMARY AND CONCLUSION

Supply chain disruptions can be limited by identifying the different types of risks that can be faced and develop strategies in order to ease interruption effects. In this research, a strategy of sizing a reserve safety stock through a dual-supply system has proved to be beneficial in controlling interruption risks. The analysis presented showed the importance of diversifying suppliers compared to a single-sourcing strategy. In addition, sensitivity analysis has been performed in order to measure the effects of their variability on the results obtained. Moreover, an optimum solution for reserve stock is mathematically calculated while an algorithm is proposed in order to find the percentage of procurement from each supplier available. It should be noted that the results obtained are directly affected by the initial assumed input. In addition, the numbers used are an example of a realization that might exist in real-life situations.

In conclusion, the main objective of this research is to focus on the importance of diversification and its effects on the total cost achieved. The following insights are achieved:

- If the available suppliers are identical in all their parameters, the minimum total cost is achieved if half of supplies are obtained from each. Thus, procurement should not be dependent on only one supplier.
- If there exist non-identical suppliers (one less reliable / cheaper supplier and one more reliable / costlier supplier), and taking into consideration assumed initial parameters, the minimum cost is achieved when procurement is

diversified where more reserve is bought from the more reliable supplier due to the less shortage costs achieved.

- If there exist a "dominated supplier" (a supplier that has a unit ordering costs and higher interruption rate), it has been shown that it is better to include this supplier in the portfolio of suppliers thus incurring lower costs due to the effects of diversification and sharing of risks.
- The total cost has been checked for convexity with respect to S and α_1. It has been shown that the total cost function is convex with respect to S while this total cost function is not even a quasi-convex with respect to α_1. depending on the initial assumed parameters.
- As an extension to the model, a third supplier is added and the results show that diversification also play a role in the modified model where the minimum total cost is achieved if procurement is done through all three suppliers.

Potential areas for future research might include the following ideas:

- In this research, instantaneous replenishment is taken into consideration. The variation in replenishment strategies might incur additional interruptions and might be analyzed further.
- Fixed ordering costs are disregarded in this model. This might be a factor that may be in the favor of single sourcing strategies.
- This model is extended to include three suppliers available for procurement. However, multiple sourcing strategies might reflect additional reductions in costs due to diversification effects.

- Demand rate in this research is taken as an average. However, randomness in demand rate might be taken into consideration in order to analyze its direct effects on the model.
- Product substitution might be taken into consideration as one of the mitigation strategies available to minimize interruption costs. Thus, a combined model can be formulated by having diversified suppliers and products.

APPENDIX

A. Holding Cost Formulation

Given:

$hS = ic_1x_1 + ic_2x_2$

$S = x_1 + x_2$

$\alpha_1 = \dfrac{x_1}{x_1 + x_2}$

$= \dfrac{x_1}{S}$

Similarly, $\alpha_2 = \frac{x_2}{S}$

Replace x_1 by $\alpha_1 S$ and x_2 by $\alpha_2 S$

$hS = ic_1\alpha_1 S + ic_2\alpha_2 S$

$= \mathrm{S}(\mathrm{ic}_1\alpha_1 + \mathrm{ic}_2\alpha_2)$

B. General Formulation for optimum *αi*

It was shown that the total cost function is not convex with respect to α_1. Therefore, an optimum solution cannot be determined that would optimize the objective function considered. However, a general form can be calculated by setting the first derivative to 0 and considering a most probable optimum solution:

$$\frac{\partial}{\partial \alpha_1}\mathrm{TC_U}(S, \alpha_i) = 0$$

$$\frac{\pi\lambda S}{\beta}\left(\frac{1}{\alpha_\mathrm{i}{}^2}\int_{\frac{S}{\alpha_\mathrm{i}\,\beta}}^{\infty} f_\mathrm{T}(\mathrm{t})d\mathrm{t} - \frac{1}{(1-\sum_{j\neq i}^{n}\alpha_\mathrm{j})^2}\int_{\frac{S}{(1-\sum_{j\neq i}^{n}\alpha_\mathrm{j})\beta}}^{\infty} f_\mathrm{T}(\mathrm{t})d\mathrm{t}\right) \tag{21}$$

$$+\ \mathrm{c}\lambda\beta\left(\int_0^{\frac{S}{\alpha_\mathrm{i}\,\beta}} t f_\mathrm{T}(\mathrm{t})d\mathrm{t} - \int_0^{\frac{S}{(1-\sum_{j\neq i}^{n}\alpha_\mathrm{j})\,\beta}} t f_\mathrm{T}(\mathrm{t})d\mathrm{t}\right) = 0$$

For the first derivative to be equated to 0:

$\alpha_i = (1 - \sum_{j \neq i}^{n} \alpha_j)$

$\alpha_i + \sum_{j \neq i}^{n} \alpha_j = 1,$

since suppliers are identical, $\alpha^* = \frac{1}{n}$.

C. Leibniz Integral Rule Basic Model

$$\frac{\partial}{\partial s}\int_{\frac{s}{\beta}}^{\infty}\left(t - \frac{s}{\beta}\right) f_T(t)dt$$

$$= \int_{\frac{s}{\beta}}^{\infty}\left(-\frac{1}{\beta}\right) f_T(t)dt$$

D. Leibniz Integral for Ordering Costs

$$c\lambda(\frac{\partial}{\partial S}\int_{0}^{\frac{S}{\beta}} \beta t f_T(t)dt + \frac{\partial}{\partial S}\int_{\frac{S}{\beta}}^{\infty} S f_T(t)dt)$$

$$= c\lambda(\beta\frac{\partial}{\partial S}\int_{0}^{\frac{S}{\beta}} tf(t)dt + \frac{\partial}{\partial S}\int_{\frac{S}{\beta}}^{\infty} Sf(t)dt)$$

$$= c\lambda(\frac{S}{\beta}f\left(\frac{S}{\beta}\right) + \int_{\frac{S}{\beta}}^{\infty} f(t)dt - \frac{1}{\beta}f(\frac{S}{\beta}))$$

E. Exponential Total Cost Formula

$$\mathrm{TCU(S)} = h\,S + \sum_{j=1}^{2}\left(\pi\left(\lambda_j E\left(T_j - \frac{S}{\alpha_j\beta}\right)^{+}\right) + c_j\lambda_j\left(\alpha_j\beta E\left(T_j \middle| T_j < \frac{S}{\alpha_j\beta}\right) + S\overline{F}_{T_j}\left(\frac{S}{\alpha_j\beta}\right)\right)\right)$$

For j=1,

$$E\left(T_1 - \frac{S}{\alpha_1\beta}\right)^{+} = \int_{\frac{S}{\alpha_1\beta}}^{\infty} t_1 f_{T_1}(t_1)dt_1 - \frac{S}{\alpha_1\beta}\int_{\frac{S}{\alpha_1\beta}}^{\infty} f_{T_1}(t_1)dt_1$$

$$= \mu_1 \int_{\frac{S}{\alpha_1\beta}}^{\infty} t_1 e^{-\mu_1 t_1}\, dt_1 - \frac{S\mu_1}{\alpha_1\beta}\int_{\frac{S}{\alpha_1\beta}}^{\infty} e^{-\mu_1 t_1}\, dt_1$$

The integral $\mu_1 \int_{\frac{S}{\alpha_1\beta}}^{\infty} t_1 e^{-\mu_1 t_1}\, dt_1$ can be estimated via integration by part, Let u = t1

du = dt1

dv = $e^{-\mu_1 t_1}$

v = $-\mu_1 e^{-\mu_1 t_1}$

$$uv - \int vdu$$

$$= \mu_1(-t_1 \frac{1}{\mu_1} e^{-\mu_1 t_1} - \int_{\frac{S}{\alpha_1 \beta}}^{\infty} -\frac{1}{\mu_1} e^{-\mu_1 t_1} dt_1)$$

$$= -t_1 e^{-\mu_1 t_1} - \mu_1 e^{-\mu_1 t_1}$$

For t1 varying from $\frac{S}{\alpha_1 \beta}$ to ∞

$$\mu_1 \int_{\frac{S}{\alpha_1 \beta}}^{\infty} t_1 e^{-\mu_1 t_1} dt_1 = \frac{S}{\alpha_1 \beta} e^{-\frac{S\mu_1}{\alpha_1 \beta}} + \mu_1 e^{-\frac{S\mu_1}{\alpha_1 \beta}}$$

The exponential distribution complementary CDF is $\int_{\frac{S}{\alpha_1 \beta}}^{\infty} f_{T_1}(t_1) dt_1$

$$= e^{-\frac{S\mu_1}{\alpha_1 \beta}}$$

$$E\left(T_1 - \frac{S}{\alpha_1 \beta}\right)^+ = \frac{S}{\alpha_1 \beta} e^{-\frac{S\mu_1}{\alpha_1 \beta}} + \mu_1 e^{-\frac{S\mu_1}{\alpha_1 \beta}} - \frac{S}{\alpha_1 \beta} e^{-\frac{S\mu_1}{\alpha_1 \beta}}$$

$$\pi\lambda_1 E\left(T_1 - \frac{S}{\alpha_1 \beta}\right)^+ = \pi\lambda_1 \mu_1 e^{-\frac{S\mu_1}{\alpha_1 \beta}}$$

Similarly,

$$\pi\lambda_2 E\left(T_2 - \frac{S}{\alpha_2 \beta}\right)^+ = \pi\lambda_2 \frac{1}{\mu_2} e^{-\frac{S\mu_2}{\alpha_2 \beta}}$$

2) $E(T_1 | T_1 < \frac{S}{\alpha_1 \beta}) = \mu_1 \int_0^{\frac{S}{\alpha_1 \beta}} t_1 e^{-\mu_1 t_1} dt_1$

Let u = t1

du = dt1

$\mathrm{dv} = e^{-\mu_1 t_1}$

$\mathrm{v} = -\frac{1}{\mu_1} e^{-\mu_1 t_1}$

$uv - \int vdu$

$= \mu_1(-t_1 \frac{1}{\mu_1} e^{-\mu_1 t_1} - \int_0^{\frac{S}{\alpha_1 \beta}} -\frac{1}{\mu_1} e^{-\mu_1 t_1} dt_1)$

$= -t_1 e^{-\mu_1 t_1} - \frac{1}{\mu_1} e^{-\mu_1 t_1}$

For t1 varying from 0 to $\frac{S}{\alpha_1 \beta}$

$\mu_1 \int_0^{\frac{S}{\alpha_1 \beta}} t_1 e^{-\lambda_1 t_1} dt_1 = -\frac{S}{\alpha_1 \beta} e^{-\frac{S\mu_1}{\alpha_1 \beta}} - \frac{1}{\mu_1} e^{-\frac{S\mu_1}{\alpha_1 \beta}} + \frac{1}{\mu_1}$

Similarly,

$E(T_2 | T_2 < S/(\alpha_2 \beta)) = c_2 \lambda_2 \alpha_2 \beta \left(-\frac{S}{\alpha_2 \beta} e^{-\frac{S\mu_2}{\alpha_2 \beta}} - \frac{1}{\mu_2} e^{-\frac{S\mu_2}{\alpha_2 \beta}} + \frac{1}{\mu_2} \right)$

3) Finally, $\overline{F}\left(\frac{S}{\alpha_1 \beta}\right) = e^{-\frac{S\mu_1}{\alpha_1 \beta}}$

Similarly $\overline{F}\left(\frac{S}{\alpha_2 \beta}\right) = e^{-\frac{S\mu_2}{\alpha_2 \beta}}$

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