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RE-ORDERING POLICIES FOR INVENTORY SYSTEMS WITH RECYCLABLE ITEMS AND STOCHASTIC DEMAND – OUTSOURCING VS. IN-HOUSE RECYCLING

by
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This thesis is dedicated to my family. For their endless love, support, and encouragement

AN ABSTRACT OF THE THESIS OF

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Title: Re-ordering Policies for Inventory Systems with Recyclable Items and Stochastic Demand – Outsourcing vs. In-house Recycling

Investing in recoverable items is an increasing trend in a variety of manufacturing industries. Such industries seek to balance their supply chain costs while reducing the amount of solid (non-biodegradable) waste. Our work develops mathematical models of inventory systems that rely on newly manufactured and recoverable items to satisfy the market demand. Specifically, we consider continuous (r, Q) re-ordering policies for single item inventory systems with stochastic demand and recycling. We solve for the re-ordering policy and safety stock for two models. The first model assumes that the recovery of items is outsourced to a supplier, where returns (collected used items) arrive in random quantities with every order. The second model assumes that product recovery is performed in-house at the manufacturer's facilities. The proposed mathematical frameworks consider an infinite time horizon where demand and the amount recovered are stochastic. The objective of this work is to focus on developing environmentally responsible inventory policies/models that could help in greening supply chains. A numerical study is presented to compare the proposed models and quantify the cost trade-off between recovering (recycling) items in-house and outsourcing them.

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ABBREVIATIONS

NOTATION	DEFINITION	UNIT
d	demand rate	unit /unit-time
d_e	effective demand rate	unit /unit-time
r_e	recovery rate	unit /unit-time
L	lead-time	unit-time
T	cycle time	unit-time
γ	proportion of demand collected	$0 \leq \gamma \leq 1$
θ	proportion of collected demand that is recoverable	$0 \leq \theta \leq 1$
r	reorder point	units
Q	order quantity	units
D_L	demand over lead-time	units
R_T	number of returned items during a cycle	units
$n(r)$	expected shortage per cycle	units
$z_{\alpha(r)}$	standard normal value satisfying a service level of $\alpha(r)$	-
$L(z_{\alpha(r)})$	expected number of lost sales as a fraction of the standard deviation σ_{D_L}	-
$cv(R_T)$	coefficient of variation of recovered items during a cycle	-
$cv(D_L)$	coefficient of variation of demand during lead time	-
$cv(R_L)$	coefficient of variation of recovered items during lead time	-
c_Q	purchase cost of newly manufactured item	$ /unit
c_{R_T}	purchase cost of a returned item	$ /unit
h	holding cost	$ /unit /unit-time

p	stockout cost	\$ /unit
K	ordering cost	\$ /cycle
c_1	collection cost	\$ /unit
c_2	cost of dispensing of the unusable items	\$ /unit
c_3	cost of recovery one item	\$ /unit
CC	collection cost per unit time	\$ /unit-time
DC	dispensing cost per unit time	\$ /unit-time
RC	recycling cost per unit time	\$ /unit-time
TRC	total cost of inhouse recovery per unit time	\$ /unit-time
PPC	purchase cost per cycle	\$ /cycle
HPC	holding cost per cycle	\$ /cycle
CPC	cost per cycle	\$ /cycle
EIL	expected inventory level	units
$CPUT$	cost per unit time	\$ /unit-time

CHAPTER 1
INTRODUCTION

1.1 Introduction

Population increase has been a widely discussed topic in the previous decades, where the environmental hazards that accompany its uncontrolled growth are well established in the literature (Ehrlich & Holdren, 1971). The world population is projected to reach 8.6, 9.8, and 11.2 billion by 2030, 2050, and 2100, respectively (Department of Economic and Social Affairs, 2017). This increase in population, accompanied by an increasing trend in market consumption, will result in a significant increase in waste generation. Existing supply chain systems have become a prime source of pollution, which calls for a serious re-evaluation of the manufacturing process. The challenge for such manufacturing systems is to reduce the number of items that end up in landfill sites while satisfying increases in market demand. Furthermore, the world's finite resources may not be able to keep up with the alarming demand increase (Bonney and Jaber, 2011). For example, global plastic production has reached 335 million tons in 2017, which is 4% higher compared to 2016 (Plastics Europe, 2017). The marine environment is severely affected by the plentiful plastic fragments from packaging material that ends up dumped in the seas (Zbyszewski and Corcoran, 2011). Furthermore, high-density plastics take more than two years to degrade in marine environments, while lower density ones disintegrate after 12 months (Tosin *et al.*, 2012). In a traditional landfill situation, plastics need more than 50 years to fully degrade, which is more than the useful lifetime of a landfill site (Agamuthu and Faizura, 2005), needless to mention the harmful chemicals, from dissolved plastics, that, potentially, affecting soil and water tables (Bonney and Jaber, 2011). This mismatch between the time to degrade and the rate of disposing plastics creates a new problem due to the limited number of landfill sites and the environmental hazards that a traditional landfill has on its surroundings.

Over the last few decades, governments, industries, and communities have been trying hard to reduce the amount of solid waste that goes to landfill sites. The implementation of recycling, remanufacturing, and reusing has been growing at different levels from households to large companies. According to the United States Environmental Protection Agency, the recycling rate improved from 6% of municipal solid waste in 1960 to slightly above 25% in 2015. The reported recycling and composting rate of some products are as follows: 29.9% for PET bottles and jars, 33.2% for glass containers, 39.8% for selected consumer electronics, and 54.9% for aluminum beer and soda cans (EPA, 2015). At the industrial level, several companies came up with solutions through implementing collection, recycling, and reuse of materials and products. This advance is not only encouraged by regulations from the government and increasing sense of responsibility towards the environment; companies have realized valuable commercial opportunities in collecting, recycling, and reusing materials and products. For example, John Deere & Co. invested $20 million in a returnable container program with its suppliers of assembly parts and Herman Miller Inc., claims to have saved over $600,000 in two years using returnable packaging material for steel shelves (Kroon and Vrijens, 1995). The majority of classical inventory models do not address recycling and do not account for the environmental consequences of the corresponding ordering policies. This limitation motivates developing inventory models that balance reductions in operational costs and increases in those of protecting the environment

(Bonney and Jaber, 2011). Such models created a new term known as the mixed manufacturing/remanufacturing systems, where returned assets could be reused or recovered and sold back to the market (Alinovi *et al.,* 2011). The models of this paper could be applied to any end-of-life product recovery activity, such as remanufacturing and repair. We capture the stochastic behavior of the recovery process and quantify the effects of this added uncertainty on the performance of the inventory system. Our models integrate two sources of uncertainty, the demand and recovery processes, and develop an iterative, but computationally efficient, approach that solves for the reordering policy. We refer to King *et al.* (2006) for a clear differentiation of end-of life product recovery strategies, which include repairing, reconditioning, remanufacturing, or recycling.

1.2 Organization of the thesis

The organization of the thesis is summarized in Figure 1. Chapter 2 provides a research background on the topics covered in this study on inventory systems with reverse logistics. It is divided into two sections covering the literature about deterministic and stochastic models. Chapter 3 considers the model formulation and assumptions. Chapters 4 and 5 provide a detailed description of the mathematical model formulation and solution procedure for two systems, (1) remanufacture in-house and (2) outsource remanufacturing. Chapter 6 presents an analysis and discussion on a numerical example that illustrates the efficiency of the proposed solution procedure in comparing in-house product recovery with outsourcing for different system parameters. Finally, Chapter 7 concludes the research work, highlights its limitation, and provides recommendations for future works.

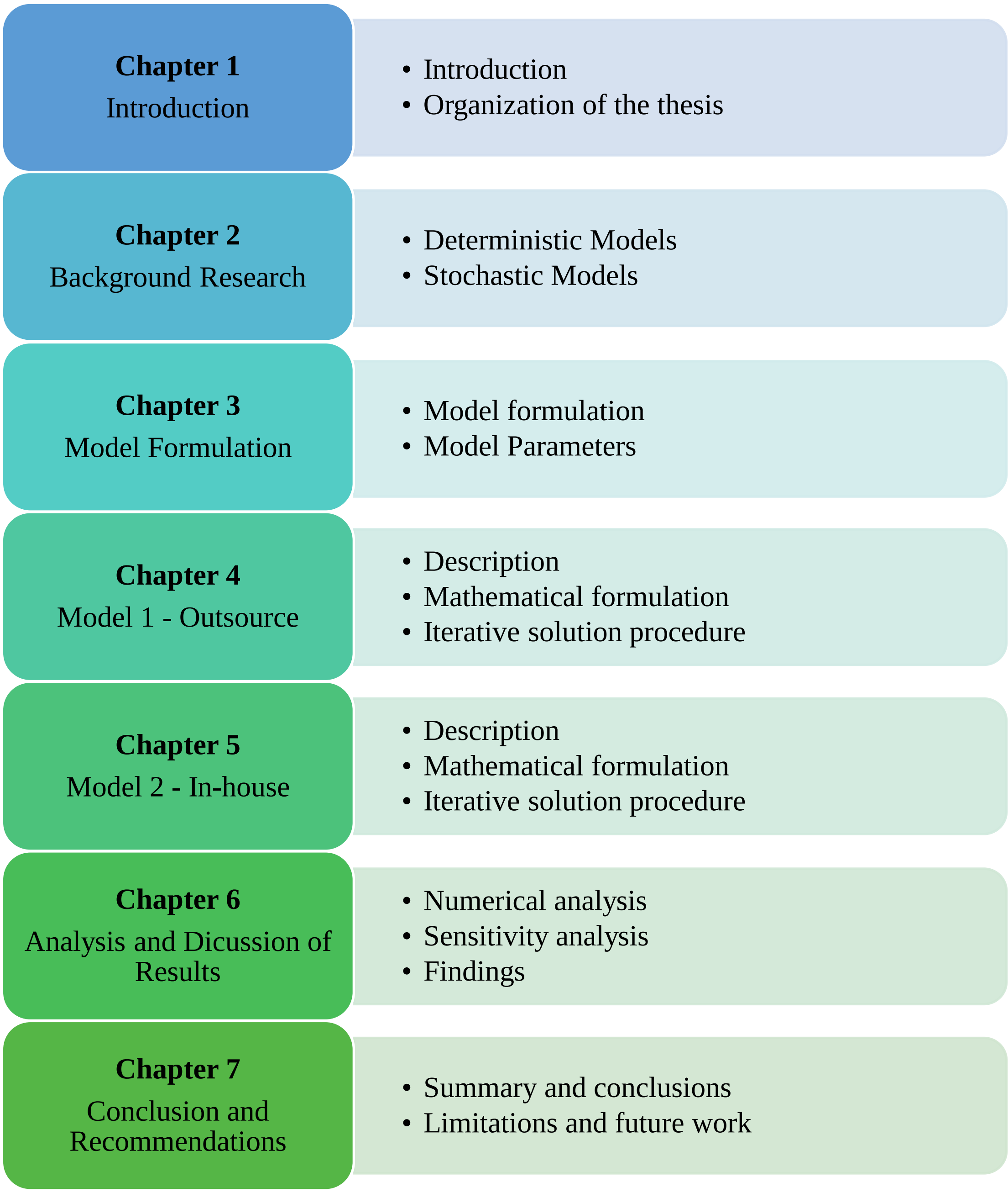

Figure 1 – Organization of the thesis

CHAPTER 2

BACKGROUND RESEARCH

We present a review of inventory systems with product recovery and re-use, by way of remanufacturing and/or recycling processes. Section 2.1 considers the related deterministic models, and Section 2.2 extends the review to account for the stochastic models. We conclude the chapter with a literature summary table, which serves to highlight our contribution to the literature.

2.1 Deterministic Models

Inventory management with reverse logistics can be traced back to Schrady (1967), who developed a deterministic EOQ-based model for a single item inventory system with no backorders. The model assumed that products could be returned for repair, with those unrepairable scrapped. Schrady (1967) noted that an item designated reparable (as opposed to consumable) is presumably more economical to repair, than to dispose of or replace. Another work along this line of research is that of Nahmiasj and Rivera (1979), which extended the work of Schrady (1967) by assuming a finite repair rate and limited storage. The resulting model also considered the interaction between procurement and repair. Mabini *et al.* (1992) presented a similar model to that of Schrady (1967), but allowed for backordering.

With the rise of environmental issues in the 1990s, this research line took a new turn with the work of Richter (1996), who developed an EOQ model with product collection and disposal. Richter (1996) considered a system of two shops. The first shop stocked newly produced and recovered items, while the second shop stocked collected/returned/used items. The model in Richter (1996) assumed that nonrecoverable items are disposed of at a cost and solved for the optimal disposal rate. The work in Teunter (2001) studied a deterministic EOQ model with recoverable (repaired/refurbished remanufactured/recovered) items with different holding costs. Teunter (2001) accounted for the disposal of used items by categorizing the stock as a manufacturing batch or a recovery batch and obtained a simple EOQ for each stock category over an infinite-time planning horizon. Koh *et al.* (2002) considered joint EPQ and EOQ models with stationary demand, where items are either newly purchased or recovered. Dobos and Richter (2004) examined a production and recycling system with a predetermined production-inventory policy and assumed that recovered items are as good as new. Their results showed that a bang-bang (recycle or produce all) not a mixed strategy is optimal. However, Dobos and Richter (2004) concluded that despite what their results showed, such pure strategies are, probably, not technologically feasible, where relying solely on recycled items entails buying back all sold and used items. In a follow-up paper, Dobos and Richter (2006) considered the quality of collected items and showed that for such an assumption, a mixed strategy of production and recycling is optimal. Singh and Saxena (2012) considered a similar model, which allowed for shortages and backordering. The authors assumed time-dependent rates and investigated coordinating the manufacturing and remanufacturing processes. Other related works include but are not limited to Hui Oh and Hwang (2006) and Matar *et al.* (2014). The latter being the closest in scope to this paper, where the authors discussed production/recycling/reuse of plastic bottles that are either sold to produce low-grade plastics or disposed of in landfills. Two novel ideas were brought forth in the work of Matar *et al.* (2014), which are (i) using biodegradable plastics to minimize the environmental impact of disposing of bottles and (ii) the rehabilitation of landfill sites.

2.2 Stochastic Models

Stochastic inventory models include Alinovi *et al.* (2011), who formulated a stochastic EOQ-based inventory control model for a mixed manufacturing/remanufacturing system. The authors utilized Monte-Carlo simulation to estimate the optimal return policy while accounting for the uncertainty in demand, returned quantity, and return delay. Fleischmann *et al.* (2002) proposed a basic inventory control model with stochastic returns. They adopted Poisson distributions to model the number demanded and returned. Shi *et al.* (2010) formulated a mathematical model to maximize the overall profit by optimizing the production and recycling processes, subject to uncertain demand and return rates. The authors adopted a Lagrangian relaxation and a sub-gradient heuristic. Hsueh (2011) investigated time-dependent inventory-control policies in a manufacturing/ remanufacturing system with normally distributed demand and return processes. For different points in time, closed-form solutions were obtained for the optimal production lot size, reorder point, and safety stock.

Benedito & Corominas (2013) integrated Markov decision processes with reverse logistics models to obtain the optimal manufacturing policy. The authors assumed that the quantity returned to be stochastic and dependent on sales. The system developed in Benedito & Corominas (2013) considered a company that recovers, produces, and sells the product. Serrato *et al*. (2007) considered a Markov decision model where reverse logistics was either performed internally or outsourced. The authors based their model on a reward function that accounted for the capacity and operating costs. They concluded that as the return fraction increased, the outsourcing threshold was more likely to be crossed, and thus internal reverse logistics would become more favorable

(Serrato, Ryan, & Gaytán, 2007). Our work also analyzes the outsourcing vs. in-house recycling decision while integrating the stochastic aspect of demand and return in a modified continuous (r, Q) inventory-control system. The closest to our work is that of Teunter (2002), who considered an inventory system with stochastic demand and return and a discounted cost. Unlike our work, Teunter (2002) resorted to simulation to determine the best possible (not optimal) values of the decision variables relating to the economic and manufacturing order quantity systems. Our work also considers stochastic demand return, but for a continuous (r, Q) inventory-control system. We present an iterative analytical approach to solve for the best possible cost per unit of time, which is a computationally efficient alternative to simulation.

The above-surveyed work shows that stochastic inventory models for reverse logistics systems are still not that many, despite the importance of the topic. This paper, therefore, contributes to this line of research by formulating models that build on an approach proposed in Silver *et al*. (2016), which calculated the reorder point and quantity for fast-moving items. They developed a procedure to find the optimal solution by iterating between two values, the EOQ and the reorder point. We implement a similar approach in our model to compute the ordering policy by integrating the remanufacturing/recycling process.

Table 1 Background Research Summary

Author	Stochastic Demand	Stochastic Returns	In-house	Supplier	Decision Variables	Solution Procedure
Schrady (1967)	-	-	X	-	(r, Q)	Closed form solution
Nahmiasj and Rivera (1979)	-	-	X	-	(r, Q)	Closed form solution
Mabini *et al.* (1992)	-	-	X	-	(r, Q)	Closed form solution
Richter (1996)	-	-	-	X	(r, Q)	Closed form solution
Teunter (2001)	-	-	X	-	Manufacturing and recovery batch size	Closed form solution
Koh *et al.* (2002)	-	-	X	-	Quantity of newly produced items, inventory level of recoverable items, number of orders	Search heuristic
Dobos and Richter (2004)	-	-	X	-	Marginal use and buyback rates, number and size of recycling lots, number and size of production lots	Closed form solution
Dobos and Richter (2006)	-	-	X	-	Marginal use and buyback rates, number and size of recycling lots, number and size of production lots	Closed form solution
Singh and Saxena (2012)	-	-	X	-	Acceptable returned quantity for used items, maximum inventory level from production and remanufacturing	Closed form solution
Hui Oh and Hwang (2006)	-	-	X	-	Number of production setups, number of raw material orders, production lot size, order size of raw material, cycle time	Closed form solution
Matar *et al.* (2014)	-	-	X	-	Cycle time	Closed form solution
Teunter (2002)	X	X	X	-	Economic order quantity for manufacturing and remanufacturing	Approximations and testing via simulation
Hsueh (2011)	X	X	X	-	Number of production activities and safety stock for every stage of the product life cycle (finite time horizon)	Closed form solution for every life every stage
Benedito & Corominas (2013)	X	X	X	-	Number of products to be manufactured	MOLP adapted to the approximated Markov model
Silver et al. (2016)	X	-	X	-	(r,Q)	Solution obtained via iterative algorithmic approach
Alinovi *et al.* (2011)	X	X	X	-	Size of the manufacturing purchasing order	Simulation
Fleischmann *et al.* (2002)	X	X	X	-	(s,Q)	Simulation
Shi *et al.* (2010)	X	X	X	-	Stocking, manufacturing and remanufacturing quantities	Lagrangian based Heuristic
This Work	X	X	X	X	(r,Q)	Solution obtained via iterative algorithmic approach

Table 1 presents a concise summary of the relevant literature and shows that the existing inventory models that accounted for the uncertainty in the demand or recovery processes, based their analysis on heuristics or simulations that solved for near-optimal solutions. Our work considers the (r, Q) policy over an infinite time horizon, and we show that the cost per unit time is convex (for outsourcing and in-house recycling models). The convexity of both models allows us to develop an iterative algorithm that solves for the solution parameters.

CHAPTER 3

MODEL FORMULATION AND PARAMETERS

3.1 Model Formulation

We consider a continuous review inventory system with recoverable (e.g., recyclable) items over an infinite time horizon, where a manufacturer procures new items (raw material) from an external supplier. Each mathematical model calculates the total cost, which is the performance measure, of the inventory system for a continuous re-ordering policy, denoting the re-order point by r, and the order quantity by Q. Two recycling models: The first model outsources recycling activities to a supplier (Model 1), while the second performs them in-house (Model 2). Model 1 accounts for the case where the manufacturer is offered an outsource option for the recycling process from the supplier. Model 2 investigates the option of investing in a recycling process at the manufacturer's end. The stochastic components of the inventory system include the demand process over the lead-time as well as the number of recovered items. The number of recovered items is related to the demand process through parameters γ and θ, which denotes the proportion of demand that is collected and the proportion of collected items that are recoverable.

We assume that recovered items and newly manufactured items have the same selling price, and accordingly the same holding cost (Teunter, 2001). Both models assume that there are never two or more outstanding orders. The stockout cost of both inventory systems is p (in $/unit).

3.2 Model Parameters

The following summarizes the notations used in the mathematical models:

Monetary Parameters:

c_Q: purchase cost of newly manufactured item ($ /unit)

c_{R_T}: purchase cost of a returned item ($ /unit)

h: holding cost ($ /unit / unit-time)

p: stockout cost ($ /unit)

K: ordering cost ($ / production cycle)

System Parameters:

d: demand rate (unit /unit-time)

d_e: effective demand rate (unit / unit-time)

r_e: recovery rate (unit / unit-time)

L: lead-time (unit-time)

T: cycle time - time to consume inventory (unit-time) –random variable

γ: proportion of demand collected, $0 \leq \gamma \leq 1$

θ: proportion of collected demand that is recoverable, $0 \leq \theta \leq 1$

r: reorder point (units) – decision variable

Q: order quantity (units) – decision variable

D_L: demand over lead-time (unit) – random variable

R_T: number of returned items during a cycle (unit) – random variable

$n(r)$: expected shortage per cycle (unit)

$cv(R_T)$: coefficient of variation of recovered items during a cycle

$cv(D_L)$: coefficient of variation of demand during lead time

$cv(R_T)$: coefficient of variation of recovered items during lead time

CHAPTER 4

MODEL 1: OUTSOURCE

4.1 Model 1 Mathematical Formulation

Figure 2 illustrates the behavior of the manufacturer's inventory when recovering activities are outsourced. The supplier requires a fixed lead-time, L, to deliver an order. An order of size Q and a random number of recovered items R_T are received at the beginning of each cycle/period. The supplier is responsible for the collection of used items from the market, disposing of those unrecoverable, and recovering and delivering those that are to the manufacturer.

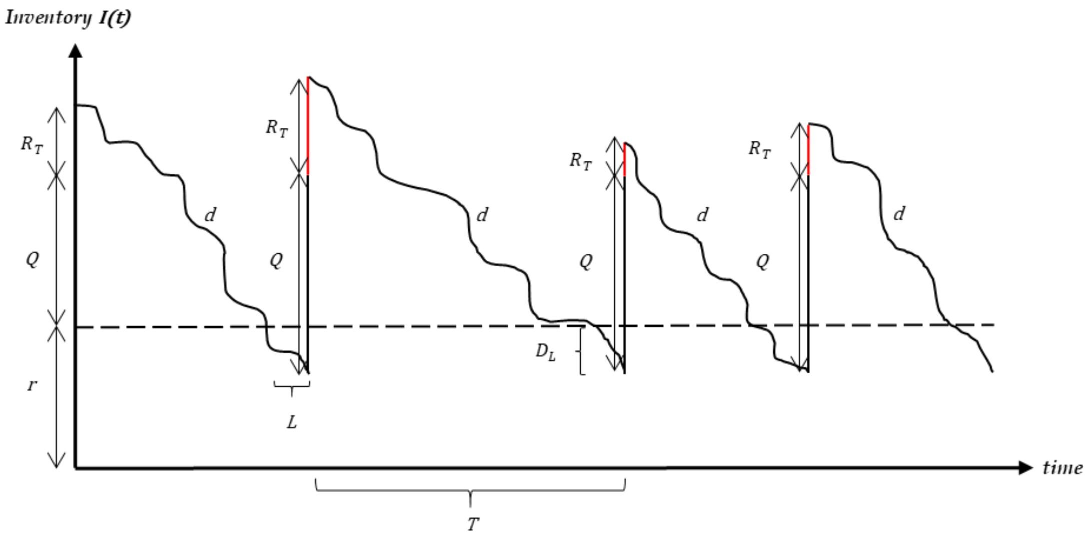

Figure 2 – Behavior of inventory for Model 1

We assume that the inventory system of Figure 2 follows a renewal process, where the time between replenishments, T, represents a renewal cycle. The demand process is assumed to be stochastic, where the time between arrival epochs follows a general distribution and is iid. Let X_i denote the time between the $(i-1)^{st}$ and the i^{th} demand arrival epochs. Therefore, $\mathrm{E}[X_i]$ is the expected inter-arrival time of a demand item and the demand rate, d, is expressed as follows,

$$d = \frac{1}{\mathrm{E}[X_i]} \tag{1}$$

, i.e., the manufacturer's inventory is consumed at a rate of d items per unit of time. Furthermore, the duration of the inventory renewal cycle, as denoted by T, is expressed as follows,

$$T = \sum_{i=1}^{Q+R_T} X_i\ , \tag{2}$$

where R_T is the number of recovered items received in a cycle of duration T, and Q is the quantity of ordered items received at the beginning of the cycle. Since $X_1, \dots, X_{Q+R_T}$ is a sequence of independent and identically distributed observations, then Wald's equation (Ross, 1996) is used to calculate the expected value of the cycle time,

$$E[T] = \frac{Q + E[R_T]}{d}\,. \tag{3}$$

Throughout a cycle, the portion of items that are collected from consumers is γ. Since not all collected items can be used, θ is defined as the potion of collected items that can be recovered. Therefore, the rate at which the market generates usable recovered items is $r_e = \gamma\ \theta\ d$ items per unit time. Consequently, the expected number of items recovered over a cycle of duration T is expressed as follows,

$$E[R_T] = r_e\ E[T] = \gamma\ \theta\ d\ E[T] = \ \gamma\ \theta\ (Q + E[R_T]), \tag{4}$$

$$\Rightarrow E[R_T] = \frac{\gamma\ \theta}{1 - \gamma\ \theta}\ Q. \tag{5}$$

We consider the demand over lead time, D_L, as a random variable with a coefficient of variation, $cv(D_L)$. Accordingly,

$$E[D_L] = d\,L \tag{6}$$

$$Var[D_L] = cv(D_L)^2 E[D_L]^2 \tag{7}$$

To capture the variability of the number of recovered items delivered to the supplier during an inventory cycle and over the lead time, we define cv_{R_T} and cv_{R_L} to be the coefficient of variations of the number of recovered items during intervals of duration T and L, respectively. We related cv_{R_T} and $cv_{R_L}$by the following equation,

$$cv(R_T)^2 \times E[T] \approx cv(R_L)^2 \times L, \tag{8}$$

$$\Rightarrow cv(R_T) \approx \sqrt{cv(R_L)^2 \times \frac{L}{\frac{Q+E[R_T]}{E[d]}}} = cv(R_L)\sqrt{\frac{L\,E[d]}{Q+E[R_T]}}. \tag{9}$$

Notice that the approximation in Equations (8) and (9) is exact if the demand process is Poisson (time between arrivals follows an exponential distribution). Furthermore, the assumption in Equations (8) and (9) is accurate if the inter-arrival epochs are renewal, and the accuracy improves for long lead times where the error is $o(1)$ (Whitt, 1982; Nasr et al. 2018). Accordingly, Equation (9) relates the variability of the number of recovered items during a cycle and over the lead-time.

Therefore, the variance of the number of recovered items over the cycle time is

$$Var[R_T] = \mathrm{cv}(R_T)^2 \times E[R_T]^2. \tag{10}$$

We define the safety stock, SS, as the lowest inventory level realized by the system, i.e., the inventory just before the order is received. The safety stock is a random variable and is expressed as follows,

$$SS = r - D_L. \tag{11}$$

4.2 Performance Measures/System Costs – Model 1

In this section, we calculate the system cost per cycle (CPC) of Model 1, which includes the purchasing, holding, ordering, and shortage costs. The manufacturer purchases newly manufactured items for c_Q each. The supplier provides the recovered items for a discounted price of c_{R_T} for each. Therefore, the expected purchase cost per cycle is calculated as follows

$$PPC_1 = c_Q\, Q + c_{R_T}\, E[R_T] \tag{12}$$

Recovered items delivered to the manufacturer, along with the purchased items, are assumed to be of the same quality. Consequently, purchased and recovered items have the same holding cost, h. The expected inventory level, EIL_1, held per cycle is calculated as,

$$EIL_1 = \frac{Q + E[R_T]}{2} + \mathrm{E}[SS]\,, \tag{13}$$

which is the average value of the ordered quantity $(Q + E[R_T])/2$ plus the expected safety stock $E[SS]$. Accordingly, the expected holding cost per cycle $HPC_1$is,

$$HPC_1 = h\left(\frac{Q + R_T}{2} + SS\right)\sum_{i=1}^{Q+R_T} X_i. \tag{14}$$

Let K be the fixed ordering cost per cycle, and p be the stockout cost per unit incurred by the manufacturer. A shortage is present when the demand over the lead-time is more than the reorder point inventory. Let $f_{D_L}(x)$ be the density function of the number of demanded items over the lead-time. Therefore, the expected shortage per cycle in this model is

$$n(r) = \int_r^{\infty}(x-r)f_{D_L}(x)dx = \int_r^{\infty}(x-r)f_{D_L}(x)dx = \sigma_{D_L} \times L\big(z_{\alpha(r)}\big), \tag{15}$$

where $z_{\alpha(r)}$ is the standard normal value satisfying a service level of $\alpha(r)$ and $L(z_{\alpha(r)})$ is the corresponding standard loss function, i.e., the expected number of lost sales as a fraction of the standard deviation σ_{D_L}. The loss function is the expected quantity by which demand exceeds a determined threshold value. This threshold value corresponds to the reorder point, r. If the demand exceeds r, a shortage cost per item, p, is incurred.

4.3 Computing the Re-Ordering Policy – Model 1

The cost per cycle equation is expressed as a function of the holding, ordering, and shortage costs as follows,

$$CPC_1 = c_Q\, Q + c_{R_T}\, E[R_T] + h\left(\frac{Q+R_T}{2} + SS\right) \sum_{i=1}^{Q+R_T} X_i + K + p \times n(r). \quad (16)$$

The expected cost per cycle is then calculated by the following equation,

$$E[CPC_1] = c_Q\, Q + c_{R_T}\, E[R_T] + h \times \left(\left(\frac{Q}{2} + E[SS]\right) \frac{Q + E[R_T]}{d} + \frac{E\left[{R_T}^2\right]}{2d} + Q\, \frac{E[R_T]}{2d}\right) + K + p \times n(r) \quad (17)$$

Since the behavior of the inventory system of Model 1 is a renewal process with a cycle time of duration *T*, the cost per unit time of Model 1 ($CPUT_1$) is calculated as the expected cost per cycle $E[CPUT_1]$ divided by the expected cycle time, $E[T]$,

$$E[CPUT_1] = \frac{E[CPC_1]}{E[T]}. \quad (18)$$

This results in the following equation,

$$E[CPUT_1] = \frac{\left(c_Q\, Q + c_{R_T}\, E[R_T] \right) d}{Q + E[R_T]} + h \times \frac{d}{Q + E[R_T]} \times \left(\left(\frac{Q}{2} + E[SS] \right) \times \frac{Q + E[R_T]}{d} + \frac{E\left[{R_T}^2\right]}{2\,d} + Q \times \frac{E[R_T]}{2\,d} \right) + K \times \frac{d}{Q + E[R_T]} + p \times \frac{d}{Q + E[R_T]} \times n(r) \tag{19}$$

The cost function of Equation (19) can be shown to be convex as a function of the order quantity Q. To prove convexity, the Hessian for $E[CPUT_1]$ is calculated as,

$$H(Q,r)_1 = \begin{bmatrix} \frac{2(1-\theta)(p\,n(r)+K)d}{Q^3} & \frac{(1-\theta)\,p\,\left(1-F(r)\right)\,d}{Q^2} \\ \frac{p\,d\,\left(1-F(r)\right)\left(1+\frac{\theta}{1-\theta}\right)}{\left(Q+Q\frac{\theta}{1-\theta}\right)^2} & \frac{p\,d\,f(r)}{Q+Q\frac{\theta}{1-\theta}} \end{bmatrix}. \tag{20}$$

The resulting semidefinite for $E[CPUT_1]$,

$$\frac{(1-\theta)\left(2\,z_1^2\,d\,(p\,n(r)+K) + 2\,z_1\,z_2\,Q\,d\,p\,\left(1-F(s)\right) + z_2^2\,Q^2\,d\,p\,f(s)\right)}{Q^3}, \tag{21}$$

is positive for all positive values of Q and r, and, hence, completes the proof of convexity of $E[CPUT_1]$. Thus, the equation for the order quantity Q is obtained by taking the derivative of the expected total cost per unit time, as expressed in Equation (19), with respect to Q,

$$Q^* = \frac{(1-\theta)\sqrt{2\,h\,d\,(p\,n(r)+K)}}{h}. \tag{22}$$

Furthermore, the equation for the reorder point s is calculated by considering the derivative of $E[CPUT_1]$, Equation (19), with respect to r,

$$F(r^*) = 1 - \frac{(Q + E[R_T])\,h}{p\,d}. \tag{23}$$

Accordingly, Equations (22) and (23) are solved iteratively to calculate Q^* and r^*.

4.4 Iterative Procedure – Model 1

Silver *et al.* (2016) discussed the simultaneous determination of Q and r. We describe a similar iterative procedure to find the ordering policy. Let Q_i and r_i denote the order size and reorder point for the *i^th^* iteration, respectively.

Step 1: Calculate Q_i using the basic EOQ re-ordering quantity,

$$Q_i = \sqrt{\frac{2Kd}{h}}. \tag{24}$$

Step 2: Calculate $F(r_i)$ according to Equation (23),

$$F(r_i) = 1 - \frac{h(Q_i + E[R_T])}{p\ d}.$$

Step 3: Calculate the relevant $z_{\alpha(r_i)}$ from $F(r_i)$, the inverse of the cumulative distribution of the standard Normal.

Step 4: Calculate r_i using the value $z_{\alpha(r_i)}$ (as calculated in Step 3), corresponding to a safety level $\alpha(r_i)$,

$$r_i = \sigma_{D_L} z_{\alpha(r_i)} + E[D_L]\,. \tag{25}$$

Step 5: Calculate $n(r_i)$ using Equation (15),

$$n(r_i) = \sigma_{D_L} \times L\big(z_{\alpha(r_i)}\big).$$

Step 6: Calculate Q_{i+1} according to Equation (22),

$$Q_{i+1} = \frac{(1-\theta)\sqrt{2\ h\ d\ (p\ n(r_i) + K)}}{h}.$$

Step 7: If $|Q_{i+1} - Q_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_i). Otherwise, set $Q_i = Q_{i+1}$ and repeat **Step 2** to **Step 4**.

Step 8: If $|r_{i+1} - r_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_{i+1}).

Otherwise set $r_i = r_{i+1}$ and repeat **Step 5** to **Step 6.**

Step 9: Set $Q_i = Q_{i+1}$, and restart algorithm from **Step 2**.

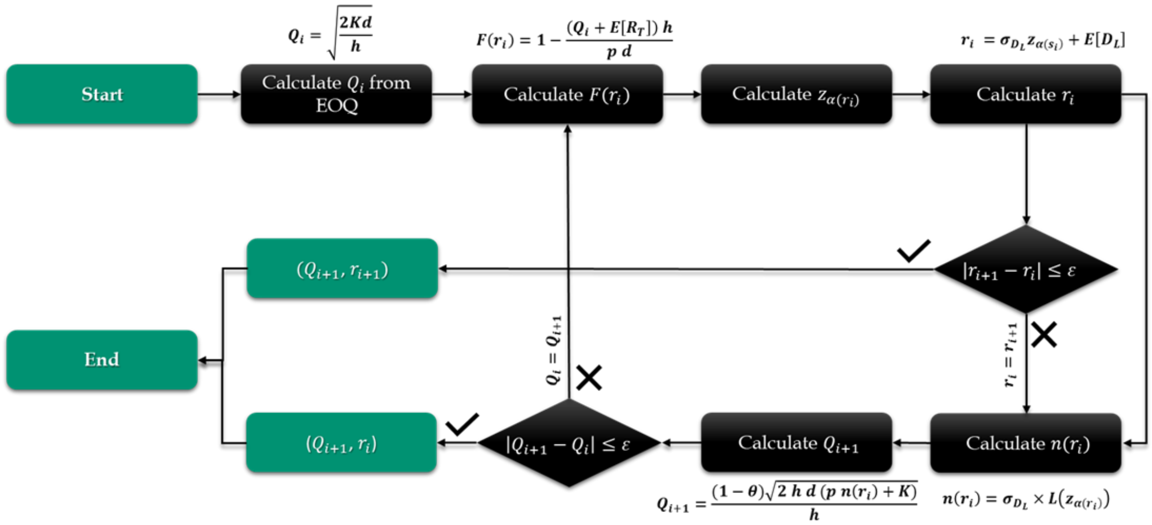

Figure 3 – Model 1 Solution Algorithm

CHAPTER 5

MODEL 2: IN-HOUSE

5.1 Model 2 Mathematical Formulation

Figure 4 illustrates the stochastic behavior of the manufacturer's inventory when returned items are recovered in-house (Model 2). A period defines the time between replenishments. The supplier requires a fixed lead time L to deliver an order/replenishment. The model assumes that the recovered items are added to the inventory as they arrive throughout the cycle at a rate of r_e items per unit time. Accordingly, the inventory of returned items is consumed by the demand process at a rate of d and partially replenished at a rate of r_e. Thus, we define the effective demand rate as the difference between the rates of demand, d, and manufacturer recovers collected used items is r_e, respectively. This results in a lower effective demand rate,

$$d_e = d - r_e. \tag{26}$$

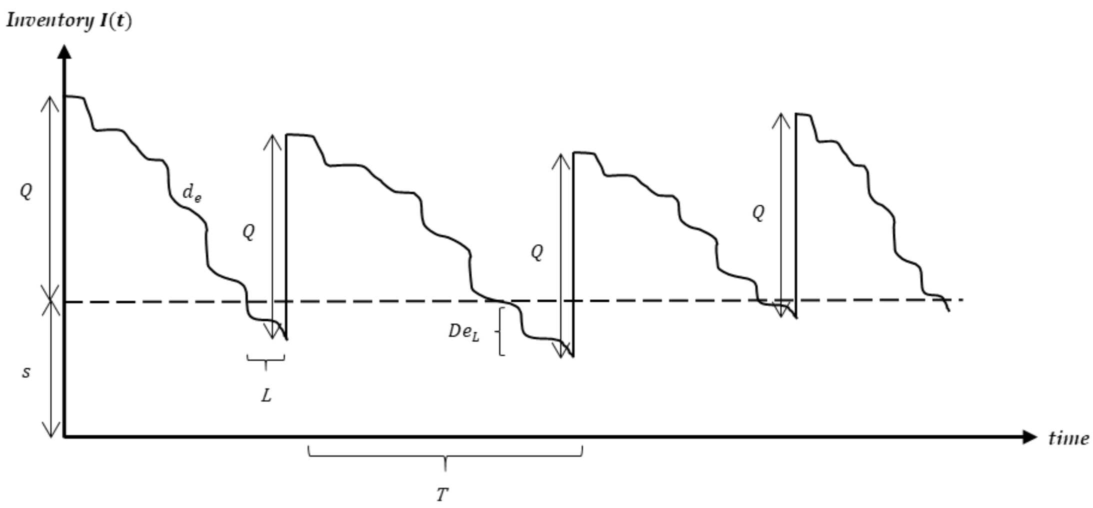

Figure 4 – Behavior of inventory for Model 2

The number of effectively demanded items over the lead time is a random variable, De_L, with an expected value and variance of,

$$E[De_L] = E[D_L] - E[R_L], \tag{27}$$

$$Var[De_L] = cv(D_L)^2 \times E[D_L]^2 + cv(R_L)^2 \times E[R_L]^2, \tag{28}$$

where $cv(D_L)$ and $cv(R_L)$ are the coefficients of variation of the number of demanded items and returned items over the lead time respectively. The cycle time as illustrated in Figure 4 becomes,

$$T = Q/d_e \, . \tag{29}$$

Similar to Model 1, the manufacturer in this model adopts a safety stock policy to decrease the number of random stock outs $n(r)$ to accommodate for the variation over the lead-time,

$$SS = r - D_{Le}. \tag{30}$$

5.2 Performance Measures/System Costs – Model 2

In this section, we calculate the system cost per cycle (CPC) of Model 2, which includes the purchasing, holding, ordering, and shortage costs. The recycling cost is discussed in detail in section 3.2.2. Like Model 1, the manufacturer purchases newly manufactured items for c_Q each. Therefore, the expected purchase cost per cycle is calculated as follows

$$PPC_2 = c_Q \, Q \tag{31}$$

Since the quality and price of the recovered items are assumed in the literature to be "as-new" (e.g., Richter, 1996; Teunter, 2001), we use the same holding cost assumptions adopted in Model 1. Consequently, purchased and recovered items have the same holding cost, h. The expected inventory level, EIL_2, held per cycle is calculated as follows,

$$EIL_2 = \frac{Q}{2} + E[SS] \quad (32)$$

which is the average value of the ordered quantity $Q/2$ plus the expected safety stock $E[SS]$. Accordingly, the holding cost per cycle HPC_2 is

$$HPC_2 = h\left(\frac{Q}{2} + SS\right)\sum_{i=1}^{Q}(X_i - Z_i). \quad (33)$$

The manufacturer incurs a fixed cost when it places an order. The shortage cost in this model is calculated by a similar expression to Model 1, but utilizes the standard deviation of the effective demanded items when calculating the expected number of shortages per cycle,

$$n(r) = \sigma_{De_L} \times L\left(z_{\alpha(r)}\right). \quad (34)$$

5.3 In-house Recycling Cost – Model 2

In addition to the inventory-related costs (holding, ordering, and shortage costs), Model 2 incurs additional costs due to the in-house recycling process. In this section, we describe the in-house recovery process, which consists of three components, and define its corresponding costs. The first sub-process is the collection process, where we assume γ to be the collection rate of items, $(0 \leq \gamma \leq 1)$. Let c_1 be the collection cost of one item ($ / item). The collection cost per unit time is

$$CC_2 = c_1\, \gamma\, d. \quad (35)$$

The second sub-process is for dispensing of items that do not pass inspection, which involves screening the screening and testing the collected items and labeling them as unusable. The second sub-process is for dispensing of items that do not pass inspection, which involves screening the screening and testing the collected items and labeling them as unusable; i.e., not repairable/recoverable.

Let c_2 be the cost of dispensing of the unusable items ($ / item). The dispensing cost per unit time is

$$DC_2 = c_2\,(1-\theta)\,\gamma\,d. \tag{36}$$

The third sub-process is the recycling process where the cost to recycle one item is c_3. The recycling cost per unit time is

$$RC_2 = c_3\,\gamma\,\theta\,d. \tag{37}$$

We summarize the notation for the in-house recycling process,

c_1 : unit collection cost ($ / item)

c_2 : unit cost disposal cost ($ / item)

c_3 : cost of recovery one item ($ / item)

γ : proportion of demand collected ($0 \le \gamma \le 1$).

The total cost per unit time of the in-house recovery process is denoted by TRC_2 and calculated as follows,

$$TRC_2 = c_1\,\gamma\,d + c_2\,(1-\theta)\;\gamma\,d + c_3\,\gamma\,\theta\,d \tag{38}$$

5.4 Computing the Re-ordering Policy – Model 2

The cost per cycle equation for Model 2 (CPC_2) is now expressed as follows,

$$CPC_2 = c_Q\,Q + h\left(\frac{Q}{2} + SS\right)\sum_{i=1}^{Q}(X_i - Z_i) + K + p\,n(r) \tag{39}$$

The expected cost per cycle is then calculated as,

$$\begin{aligned} E[CPC_2] &= h \times E\left[\left(\frac{Q}{2} + SS\right)\sum_{i=1}^{Q}(X_i - Z_i)\right] + K + p\,n(r) \\ &= h \times \left(\left(\frac{Q}{2} + E[SS]\right)\frac{Q}{d_e}\right) + K + p\,n(r). \end{aligned} \tag{40}$$

Since the inventory system of Model 2 is also a renewal process, the cost per cycle is divided by the cycle time $E[T]$ to obtain the cost per unit time for Model 2, $(CPUT_2)$. This results in the following equation for Model 2,

$$E[CPUT_2] = \frac{\left(c_Q\, Q + \frac{h\, Q\, \left(\frac{Q}{2} + E[SS]\right)}{d_e} + K + p\, n(r) \right) d_e}{Q} + TRC_2. \tag{41}$$

To prove convexity, the Hessian for $E[CPUT_2]$ is calculated as,

$$H(Q,r)_2 = \begin{bmatrix} \frac{2\, d_e\, (p\, n(r)\, + K)}{Q^3} & \frac{p\left(1 - F(r)\right)\, d_e}{Q^2} \\ \frac{p\left(1 - F(r)\right)\, d_e}{Q^2} & \frac{p\, d_e f(r)}{Q} \end{bmatrix}. \tag{42}$$

The resulting semidefinite for $E[CPUT_2]$,

$$\frac{z_1^2\, d_e\left(2\, K + p\, n(r)\right) + 2\, z_1\, z_2\, Q\, p\, d_e\left(1 - F(r)\right) + z_2^2\, p\, d_e\, Q^2\, f(r)}{Q^3}, \tag{43}$$

which is positive for all positive values of Q and s then $CPUT_2$ is convex. Thus, the equation for the order quantity Q is calculated by taking the derivative of the expected total cost per unit time with respect to Q,

$$Q^* = \frac{\sqrt{2\, h\, d_e\, (p\, n(r) + K)}}{h}. \tag{44}$$

The equation for the reorder point s is calculated by deriving the expected total cost per unit time with respect to

$$F(r^*) = 1 - \frac{Q\, h}{p\, d_e} \tag{45}$$

Accordingly, Equations (44) and (45) are solved iteratively to calculate Q^* and r^*.

5.5 Iterative Procedure – Model 2

Step 1: Calculate Q_i using the basic EOQ re-ordering quantity:

$$Q_i = \sqrt{\frac{2Kd_e}{h}} \quad (46)$$

Step 2: Calculate $F(s_i)$ according to Equation (45),

$$F(r_i) = 1 - \frac{Q_i\, h}{p\, d_e}.$$

Step 3: Calculate the relevant $z_{\alpha(r_i)}$ from $F(r_i)$, the inverse of the cumulative distribution of the standard Normal.

Step 4: Calculate r_i using the value $z_{\alpha(r_i)}$ (as calculated in Step 3), corresponding to a safety level $\alpha(r_i)$,

$$r_i = \sigma_{De_L} \times z_\alpha + E[De_L]. \quad (47)$$

Step 5: Calculate $n(r_i)$ using Equation (34)

$$n(r_i) = \sigma_{De_L} \times L(z_{\alpha(r)})$$

Step 6: Calculate Q_{i+1} according to Equation (44),

$$Q_{i+1} = \frac{\sqrt{2\, h\, d_e\, (p\, n(r_i) + K)}}{h}.$$

Step 7: If $|Q_{i+1} - Q_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_i). Otherwise, set $Q_i = Q_{i+1}$ and repeat **Step 2** to **Step 4**.

Step 8: If $|r_{i+1} - r_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_{i+1}). Otherwise set $r_i = r_{i+1}$ and repeat **Step 5** to **Step 6**

Step 9: Set $Q_i = Q_{i+1}$, and restart algorithm from **Step 2**.

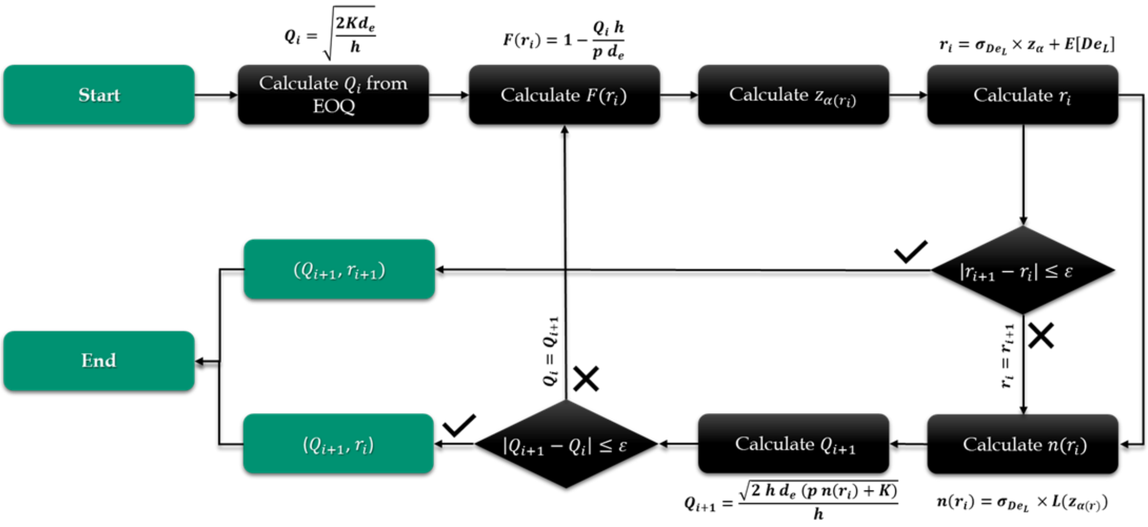

Figure 5 – Model 2 Solution Algorithm

CHAPTER 6

ANALYSIS AND DISCUSSION OF RESULTS

6.1 Numerical Analysis

This section provides numerical examples and sensitivity analysis to illustrate the behavior and the flexibility of our mathematical framework in comparing in-house recycling vs. outsourcing. Consider the following base case for both models with input parameters shown in Table 2, where we assume that items (plastic bottles) are recovered via recycling.

Table 2 Input Parameters for Base Case

Parameter	Value	Unit
d	50,000	bottles/month
γ	0.5	-
θ	0.5	-
L	0.2	months
cv_{D_L}	0.3	-
cv_{R_L}	0.1	-
c_Q	0.1	$/bottle
c_{R_T}	0.08	$/bottle
c_1	0.002	$/bottle
c_2	0.001	$/bottle
c_3	0.002	$/bottle
h	0.02	$/bottle/month
p	0.25	$/bottle
K	30	$

The percent savings of Model 2 (in-house recycling) compared to Model 1 (outsourcing the recycling process) is denoted by Δ, and is calculated as follows,

$$\Delta = \frac{E[CPUT_1] - E[CPUT_2]}{E[CPUT_1]} \times 100.$$

Define Δ as a measure of the cost-efficiency of choosing to recycle in-house instead of outsourcing the recycling process. The ordering policy is calculated in Table 3, along with the expected cost per unit time.

Table 3 Base Case Results

Example	Model 1			Model 2			Δ
(#)	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	%
1	10,057	16,074	$ 5279.7	11,806	13,391	$ 4191.4	20.61%

The cost-efficiency of in-house recycling is Δ= 20.61%, where the results in Table 3 show that Model 1 (outsourcing) results in a lower order quantity yet a higher reorder point. A breakdown of the savings of Model 2 over Model 1 by cost type shows a saving of 40% in holding cost, a saving of 15% in ordering cost, and 21% saving in purchase cost.

6.2 Sensitivity Analysis

We perform a one-way sensitivity analysis by varying the demand rate, recovered proportion, lead-time, coefficients of variation of the demanded new and recovered items over the lead-time, purchase costs, holding cost, shortage cost, and ordering cost.

Table 4 Sensitivity Analysis - Demand

Example	Variable	Model 1			Model 2			Comparison
(#)	d	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
1	1,000	1,322	265	\$134.35	1,531	209	\$108.57	19.19%
2	10,000	4,298	3,012	\$1,112.85	5,005	2,472	\$887.01	20.29%
3	100,000	14,693	32,916	\$10,430.25	17,348	27,557	\$8,273.11	20.68%
4	1,000,000	58,090	350,046	\$102,351.24	71,237	296,164	\$81,098.01	20.76%

Table 4 presents the results of the sensitivity analysis for d where the demand rate is varied from 1,000 to 1,000,000 in multiples of 10. Model 2 results in better cost savings compared to all the demand values considered. As the demand rate increases, the percent saving of using Model 2 is maintained at around 20% (Δ ranges between 19.19% and 20.76% in Table 4). The overall saving value Δ is mainly attributed to the savings in holding, order, and purchase costs that Model 2 provides. For example, at d = 1,000,000, the savings reached 45% in holding cost, 18% in ordering cost, and 21% in purchase cost.

Table 5 Sensitivity Analysis - Recycling Proportion

Example	Variable	Model 1			Model 2			Comparison
(#)	$\gamma\theta$	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
6	0.05	12,739	16,074	\$5,479.65	13,105	15,539	\$5,230.37	4.55%
7	0.125	11,733	16,074	\$5,404.67	12,634	14,736	\$4,841.14	10.43%
8	0.25	10,057	16,074	\$5,279.71	11,806	13,391	\$4,191.43	20.61%
9	0.375	8,381	16,074	\$5,154.79	10,722	11,756	\$3,540.09	31.32%
10	0.5	6,705	16,074	\$5,029.90	9,929	10,643	\$2,886.42	42.61%

Next, we conduct a one-way sensitivity analysis on the recycling proportion $(\gamma\ \theta)$. We increase $(\gamma\ \theta)$ from 0.05 to 0.5, by increasing θ and fixing γ. These recycling parameters are dependent on societal recycling behaviors and can improve by implementing governmental regulations. As $(\gamma\ \theta)$ increases, Model 2 becomes more cost-efficient, as Table 5 indicates. The order quantity Q^* decreases as the number of recovered items in both models increases, which is mainly due to the continuous utilization of recovered items in supplying the demand in Model 2. However, the reorder point r^* does not vary in Model 1 since it is not directly dependent on $(\gamma\,\theta)$, Equation (23). Furthermore, by comparing the total expected cost behavior of Model 1 and Model 2, one notices that as $\gamma\,\theta$ increases, the cost per unit time decreases at a faster rate in Model 2. Looking at the behavior of cost components, one can see that the holding and order costs in Model 1 are insensitive to variations in $\gamma\,\theta$. The main parameter behind the decrease in the total expected cost in Model 1 is the purchase cost since the manufacturer acquires more recovered items for a lower unit cost. On the other hand, the holding, order, and purchase costs decrease in Model 2, resulting in 42.61% cost savings. By analyzing the reorder point, one notices that decreases in Model 2 have to do with more items are being recovered by the manufacturer throughout the inventory cycle.

Table 6 Sensitivity Analysis - Lead-time

Example	Variable	Model 1			Model 2			Comparison
(#)	L	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
11	0.4	11,026	31,915	$5,562.46	13,165	26,499	$4,330.77	22.14%
12	0.6	12,097	47,517	$5,843.12	14,694	39,310	$4,467.58	23.54%
13	0.8	13,273	62,874	$6,121.66	16,393	51,823	$4,601.82	24.83%
14	1	14,553	77,985	$6,398.08	18,262	64,036	$4,733.46	26.02%

We conducted a one-way sensitivity on the lead-time by increasing L from 0.2 months (base case) to 1 month in increments of 0.2-month. The results presented in Table 6 indicate that although the total cost per unit time for both models increases with lead-time, Model 2 becomes more efficient. Specifically, the holding and shortage costs in both models increase with longer lead-times. However, order cost decreases while the order quantity Q^* and reorder point r^* increase in both models since the manufacturer prefers to hold more inventory to decrease the probability of stockouts in longer lead-times. By comparing the reorder point of Model 2 to the reorder point in Model 1, at L of 1 month, one notices that it is lower by 18%, which indicates that Model 2 is more resilient to longer lead-times.

Table 7 Sensitivity Analysis - Holding Cost

Example	Variable	Model 1			Model 2			Comparison
(#)	h	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
15	0.04	7,432	15,569	$5,649.26	8,800	12,858	$4,403.82	22.05%
16	0.08	5,621	14,995	$6,309.38	6,744	12,244	$4,756.53	24.61%
17	0.16	4,409	14,314	$7,501.20	5,406	11,495	$5,341.61	28.79%
18	0.2	4,123	14,060	$8,062.19	5,105	11,210	$5,600.48	30.53%

We also conducted a one-way sensitivity analysis for holding cost h, where we increase the base case h from $0.02/bottle/month to $0.2 in multiples of two. The savings of Model 2 increases to reach 30.53% with higher holding costs, as presented in Table 7. The detailed results showed that as we increase h to $ 0.2, Model 2 resulted in 55% less holding cost than Model 1. At that h, the holding cost in Model 2 constitutes 22% of the total cost compared to 34% in Model 1, which indicates that Model 2 is less sensitive to increases in holding costs. Furthermore, the number of ordered items Q^* and reorder point r^* decrease in both models since the solution equations are inversely proportional to h. This behavior would mimic the natural response of the manufacturer to adapt to any increase in the holding cost from its side.

Table 8 Sensitivity analysis - $cv(D_L)$

Example	Variable	Model 1			Model 2			Comparison
(#)	$cv(D_L)$	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
19	0.1	9,465	12,050	$5,223.47	11,000	9,548	$4,098.46	21.54%
20	0.2	9,756	14,074	$5,251.71	11,392	11,475	$4,144.84	21.08%
21	0.4	10,369	18,047	$5,307.51	12,238	15,281	$4,237.88	20.15%
22	0.5	10,692	19,994	$5,335.07	12,689	17,141	$4,284.10	19.70%
23	0.75	11,549	24,747	$5,402.96	13,898	21,653	$4,398.51	18.59%
24	1	12,477	29,330	$5,469.39	15,224	25,962	$4,511.21	17.52%
25	1.25	13,479	33,743	$5,534.35	16,668	30,064	$4,622.15	16.48%
26	1.5	14,553	37,985	$5,597.83	18,230	33,959	$4,731.30	15.48%

Results show that $E[CPUT_2]$ is more sensitive to changes in $cv(D_L)$ than $E[CPUT_1]$. As it increases from 0.1 to 1.5, $E[CPUT_2]$ increases by about 15% compared to $E[CPUT_1]$, which increases by 7%. The uncertainty/variability in the number of demanded items over the lead-time is captured by $cv(D_L)$, the coefficient of variation of the demand counts over lead time. As variability increases, the manufacturer is interested in holding inventory to decrease the probability of stockouts. Both models recommend that the manufacturer makes larger orders (higher Q^*) at a higher reorder point r^*. The results of a detailed analysis showed, as expected, that increases in holding and shortage costs negatively affect the total cost. As we further analyze the savings at the holding cost level, we observe that Model 2 provides a 57% saving in holding cost for a $cv(D_L)$ of 0.1. At a $cv(D_L)$ value of 1.25, Model 1 outperforms Model 2 in terms of holding cost by 2% savings in holding costs, reaching 9% for a $cv(D_L)$ of 1.5. From a managerial perspective, holding more inventory (as proposed by Model 1) is a better solution in a market with highly variable demand process.

Table 9 Sensitivity Analysis - Order Cost

Example	Variable	Model 1			Model 2			Comparison
(#)	K	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
27	10	6,147	16,668	$5,187.35	7,289	13,990	$4,113.07	20.71%
28	20	8,359	16,302	$5,239.00	9,843	13,622	$4,156.80	20.66%
29	40	11,488	15,905	$5,314.53	13,461	13,220	$4,221.11	20.57%
30	50	12,749	15,771	$5,345.47	14,919	13,083	$4,247.54	20.54%
31	60	13,889	15,659	$5,373.62	16,237	12,969	$4,271.61	20.51%
32	70	14,937	15,562	$5,399.64	17,449	12,870	$4,293.88	20.48%
33	80	15,913	15,477	$5,423.95	18,576	12,783	$4,314.69	20.45%

As we increase the order cost K from 10 dollars to 80 dollars in steps of 10, both models propose that the manufacturer increases the order quantity Q^* and lowers the reorder point r^*, which leads to a higher total cost per unit time in both models. Even though the efficiency of Model 2 decreases slightly with the increase of K, Model 2 remains more efficient than Model 1 in our analysis. The slight change in Δ indicates that both models are influenced equally by the variation of K.

Table 10 Sensitivity Analysis - Newly Manufactured Items Purchase Cost

Example	Variable	Model 1			Model 2			Comparison
(#)	c_Q	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
34	0.15	10,057	16,074	$7,154.72	11,806	13,991	$6,066.44	15.21%
35	0.2	10,057	16,074	$9,029.72	11,806	13,991	$7,941.44	12.05%
36	0.25	10,057	16,074	$10,904.72	11,806	13,991	$9,816.44	9.98%
37	0.3	10,057	16,074	$12,779.72	11,806	13,991	$11,691.44	8.52%

We also performed a one-way sensitivity analysis for the purchase cost of newly manufactured items where we varied c_Q from $0.15/unit to $0.3/unit. The results show that contrary to the total costs ($E[CPUT_1]$ and $E[CPUT_2]$ increased by about 79% and 93%, respectively), $Q*$ and $r*$ values are insensitive to changes in c_Q for both models. The difference in expected total costs between Model 1 and 2 decreased from 15.21% to 8.52%. The results suggest that there is a c_Q value for which both models become indifferent. The only parameter that is influenced by this variation is the purchase cost in both models. One could justify the effect of c_Q by analyzing the source of bottles in both models, where, in Model 1, it is the sum of the purchase cost of newly manufactured items and the recovered items provided by the supplier. On the other hand, Model 2 depends on purchasing newly manufactured items and recycling collected items.

Table 11 Sensitivity Analysis - Recovered Items Purchase Cost

Example	Variable	Model 1			Model 2			Comparison
(#)	c_{R_T}	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
38	0.02	10,057	16,074	\$4529.71	11,806	13,991	\$4191.43	7.47%
39	0.04	10,057	16,074	\$4779.72	11,806	13,991	\$4191.43	12.31%
40	0.06	10,057	16,074	\$5029.72	11,806	13,991	\$4191.43	16.67%
41	0.1	10,057	16,074	\$5529.72	11,806	13,991	\$4191.43	24.20%

Next, we conduct a sensitivity analysis concerning the purchase cost of recovered items c_{R_T}. As c_{R_T} increases from \$0.02/unit to \$0.1/unit, the efficiency of Model 2 increases from 7.47% to 24.2%, indicating that adopting in-house recycling is favored when the purchase cost of recovered items high. This situation appears when the supplier has a high cost of collection and recycling and could not provide the recovered item to the manufacturer at a competitive price.

The results of the one-way sensitivity analysis show the coefficient of variation, $cv(R_L)$, does not affect the number of recovered items over the lead-time. As we increase $cv(R_L)$ from 0.1 to 0.5 in steps of 0.1, the solution given by Model 1 remains the same, and thus the total cost is not affected. This is mainly due to the outsourcing assumption in Model 1, where it decreases the effect of variability on the manufacturer's inventory and incurs it on the suppliers. Furthermore, another justification is that the base case adopts a short lead-time, which decreases the effect of $cv(R_L)$ on the models. The results show a slight increase (by about 1%) in the total cost per unit time in Model 2. Varying the shortage cost from \$0.15/unit to \$0.35/unit in steps of 0.05 produced similar results. The results showed a decrease in the shortage cost component of the total cost per unit time, where both models recommend the use of a higher reorder point r^* to reduce the cost of probable stockouts.

6.3 Summary of Findings

The numerical examples quantify the improvement of in-house recycling over outsourcing by Δ. We summarize the findings of our numerical results as they related to Δ.

1- Fast moving items: The cost-efficiency of recycling in-house, as calculated by Δ, remained almost unchanged as the demand rate increased from 1,000 (Δ = 19.19%) to 1,000,000 (Δ = 20.76%). This finding shows that the cost efficiency of in-house recycling is not sensitive to an increase in the demand rate.

2- High recycling proportion: The numerical results of Table 5 show that for high recycling proportions, in-house recycling is more financially lucrative (cost efficiency improved from 4.55% to 42.61% when the proportion recovered increased from 0.05 to 0.5).

3- Long lead-time: When the system is subjected to longer lead-times by the supplier, our results illustrate that the cost-efficiency of in-house recycling improves (Δ increased from 22.14% to 26.02% as lead time increased from 0.4 to 1-time unit).

4- High holding cost: Higher holding costs also favor in-house recycling (Δ increased from 22.05% to 30.53% as holding cost increased from 0.04 to 0.2).

5- Low demand variability over lead-time: For highly variable demand, the cost-efficiency of in-house recycling is reduced (Δ decreased from 21.54% to 15.48% as the coefficient of variation of demand over lead time increased from 0.1 to 1.5).

6- Ordering cost: The cost-efficiency of recycling in-house is not sensitive to changes in the ordering cost (Δ decreased from 20.71% to 20.45% as the ordering cost increased from 10 to 80).

CHAPTER 7

CONCLUTION AND RECOMMENDATIONS

The well-established environmental benefits of product recovery (e.g., recycling, repair, remanufacturing) motivated this work, especially in an era where industries are seeking a balance between environmentally friendly processes and managing an economically efficient supply chain system. Furthermore, the effects of uncertainty on the operation of inventory systems, in a wide range of industries, are well established in practice and thoroughly investigated in the inventory-control literature. Calculating the re-ordering levels to manage and offset the effects of uncertainty becomes a key challenge for supply chain managers. Accordingly, this work addressed the operational side of a stochastic inventory system recyclable items and aimed at providing economical re-ordering policies. The first model investigates outsourcing the recycling process to an external supplier while the second model considers in-house recycling. Both models consider demand and the number of recovered items as the stochastic components of the system and have a continuous re-ordering policy with deterministic lead-times.

Although this work is motivated by the plastic-bottle industry, it can be extended to other industries with recyclable items (Abdulrahman *et al.*, 2015; Ordoobadi, 2009). This work provides a quantitative economic tool that accounts for stochastic demand that could be used by practitioners to make outsourcing decisions and setting inventory re-ordering levels. Numerical and simulation analysis helped in illustrating the behavior of the two mathematical models and compare their economic performance for different system parameters. We adopted percent savings as a measure to compare the results from both models, which serves to quantify the monetary advantages/disadvantages of recycling in-house vs. outsourcing. The numerical results illustrate that in-house recycling becomes significantly more favorable when the proportion of recovered items is high. The numerical study also indicates that the profitability of in-house recycling improves for higher holding costs and for longer lead times, whereas this profitability decreases when the variability of the demand over lead-time increases.

A limitation of this work is that it did not investigate the operational complexity or feasibility of recycling in-house. Future work would address this limitation and account for investing machinery, workers, transportation, among other operational aspects of in-house manufacturing. Another extension worth exploring is analyzing the stochastic nature of demand and recovery (repair/refurbish/remanufacture/recycle). Using real data to determine the distributions of demand and the returns (collected used items) will surely help in better positioning the models of this paper or those viewed as extensions. With respect to the solution convergence of the proposed heuristic to an optimal point, further research is required. The cost functions of both models will have to tested to find if they are not necessarily convex, but could admit a single local minimum.

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