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MEAN-VARIANCE ASSORTMENT AND INVENTORY OPTIMIZATION FOR A NEWVENDOR

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AN ABSTRACT OF THE THESIS OF

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The single-period newsvendor model is a widely used application in Operations Management. Usually, fashion retailers encounter a problem in deciding the size of their orders before the start of the season. Most of the times they incur overage and underage costs while ordering too much or too little. The newsvendor model typically helps avoiding these costs by setting an order quantity that maximizes the expected profit of the retailer. Recent literature on the single-product case has shown, however, that the expected profit-maximizing (risk-neutral) newsvendor is prone to a high risk level reflected in a high profit variance. This literature also observes adopting a slightly smaller order quantity that the one utilized by the risk-neutral newsvendor caries significant variance reductions.

Motivated by the single-product observation on the high variance bared by the risk-neutral vendor, we consider the case of a fashion retailer managing an assortment of substitutable products under logit demand. We develop a model inspired by the classic mean-variance portfolio optimization problem in Finance, whereby the retailer sets the inventory levels of products in the assortment in a way that minimizes the profit variance while achieving a minimum targeted expected profit level. We develop useful analytical properties of this mean-variance assortment planning model. For example, we show that the ordered quantities are always below those of the risk-neutral newsvendor and that the expected profit target constraint is always binding. Numerical results indicate that our model is well-behaved in the sense that an optimal solution is reached quickly with a reasonable choice of the starting order quantities solution. In addition, we observe ample opportunities to reduce the profit variance involving small sacrifices in the expected profit. That is, multiple product management seems to allow better harnessing the risk-reward tradeoff than the single product one. Numerical results are also developed on the structure of the optimal assortment in the mean-variance setting. We observe some deviations from the common risk-neutral results. For example, for horizontally differentiated products, the optimal assortment among those having the same cardinality is not necessarily a popular set.

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CHAPTER I

INTRODUCTION

Operations management is concerned with ensuring high-efficiency production levels in various industries. Its adoption has been capable of providing a major transformation through a better understanding of business practices. Normally, retailers are concerned with different operation management decisions especially those related to pricing, inventory level, and assortments. The newsvendor model that is widely used and studied in order to meet the retailers' concerns. It is one of the most powerful models in OM.

The main concern that retailers care about is the order quantity. In case the retailer ordered more than the demand, a cost on lost-sales will be incurred. While, if the retailer ordered a quantity less than the desired one, sales would missed. The newsvendor model used to set ordering quantity to be sold in a single selling season with stochastic demand and without any opportunity to replenish inventory (Cachon and Kok 2007). The classical newsvendor's main objective is concerned with either maximizing the expected profit or minimizing the expected cost (Choi et al. 2008). Although most of the times the model is used to maximize the expected profit and balance it with the expected costs, however, the variance of the expected profit is not taken into account, thus leading to high returns but being exposed to risk (see Rubio-Herrero, et al. 2015).

The mean-variance framework developed by Markowitz (1952) has allowed investors to structure portfolios of financial securities taking into account the payoff (mean) and risk (variance of the profit) into account. In the (recent) retailing literature (e.g. van Ryzin and Mahajan 1999, Maddah and Bish 2007), the optimal assortment of products to offer (and their order quantities) in a certain category of substitutable products has been determined by looking at a single objective of maximizing expected profit in a newsvendor-type setting.

In this thesis, we seek to extend the results in this recent assortment planning literature by considering an additional objective of minimizing the variance of the assortment profit, in a manner similar to the classic Markowitz approach. The retailer takes the point of view of an investor optimizing a portfolio by investing in products having uncertain demand. The demand is assumed to follow a Normal distribution which is a good approximation to demand generated from Poisson arrivals. The mean-variance analysis of the single-product newsvendor model (e.g.: Choi et al. (2008)) indicates that the variance increases with the increase of the expected profits. This literature also observes that the variance of the profit can be significantly decreased if the order quantity is slightly decreased from its expected profit-maximizing level. In this thesis, we show that for multiple products, one can minimize the variance significantly and maintain a high percentage. We also analyze the structure of the optimal assortments in the mean-variance framework.

The remainder of this thesis is organized as follows. In Chapter II, we review the related literature. In Chapter III, we introduce background and assumptions for our model. In Chapter IV, we present the numerical results and our model. In Chapter V, we support our model with analytical results. Finally, in Chapter VI, we summarize our main findings and give suggestions for future work and research.

CHAPTER II

LITERATURE REVIEW

In this chapter, we review the literature related to our topic. In section 2.1, we review the work on portfolio optimization. In section 2.2, we go over the assortment planning based on the expected profits. In section 2.3, we review the different risk-related criteria utilized for in the single-product newsvendor problem. In section 2.4, we review the literature on the mean-variance single product newsvendor.

2.1 Portfolio Optimization

Portfolio optimization is used by investors as guidance for financial asset selection. It is mainly concerned with allocating competing resources. Most of these resources have an uncertain outcome however, this problem has been widely used in many decision-making areas (Pardalos et al. 1994), e.g.in insurance companies, governments budgeting tax revenues, and bond portfolios. Prior to 1952, the practice in diversified investments was very well established. However, Harry Markowitz realized the lack of a theory that covered the effect of diversification when the risks were correlated (Markowitz 1999). Back then, he contributed one of the most celebrated financial works related to portfolio optimization (Markovitz 1952). This theory is known as the Modern Portfolio Theory (MPT). His model has enabled investment in the least risky portfolio while meeting a guaranteed level of return through investment diversification. It also allows the construction of efficient frontiers through optimal portfolios that provide optimal trade-off between risk and reward (Masmoudi and Abdelaziz 2018). Markowitz model has been extended through several approaches. The most well-known one is the Capital Asset

Pricing Model CAPM by (Sharpe 1964). CAPM assumes that investors care about the mean-variance of their investment to choose efficient portfolios while being risk-averse and offers a framework for determining fair prices for risky assets. Then in (Ross 1976), the Arbitrage Pricing Theory considered that multiple factors can explain the return. User-specified confidence levels based on investor's and experts' opinions have been provided using (Black and Litterman 1992). Fernholz (2002) and Karatzas and Fernholz (2009) analyze portfolio behavior and equity market structure through theoretical and market applications. More advanced literature have been accomplished through the years (e.g. Doerner et al. 2004, Ortobelli et al. 2005, Balbs 2007, Sereda et al. 2010, and Still and Kondor 2010). These advancements were used in portfolio optimization by including constant and time-varying higher moments on the returns, and by utilizing sophisticated numerical search techniques such as metaheurisitcs and machine learning.

2.2 Assortment Planning Based on the Expected Profit

Van Ryzin and Mahajan (1999) highlight the importance of assortment planning in terms of a variety of product line structuring. In other words, this is related to the retailer in deciding the subset variants to be offered with the amount of inventory of each variant to be stocked. Under the Multinomial MNL logit choice (MNL) and horizontal product differentiation, they establish useful results on the structure of the optimal assortment, mainly that popular sets are optimal. The MNL is a consumer choice that is widely used due to its easy estimated parameters and similar product lines (Guadagni and Little 1983). It is applied in research related to inventory by (Hanson and Martin 1996 and Hopp and Xu 2005).

Maddah et al. (2007, 2014), study the pricing, inventory, and assortment and the interdependence among them in a newsvendor typesetting under logit choice similar to van Ryzin and Mahajan (1999). . (Cachon et al. 2005) shed the light on the consumer search and assortment decision by having a model similar to that of van Ryzin and Mahajan (1999). Maddah et al. (2011) review the recent works on pricing, variety (assortment), and inventory decisions for a product line of substitutable items. M-V Ghoneim and Maddah (2016) developed a model that is capable of optimizing assortment and pricing decisions. This optimization occurs under a classical deterministic consumer choice model targeting multiple complementary retail categories.

Our work is based on the assumptions and findings of Maddah and Bish (2007). By taking into account their costs assumptions of no salvage value and no additional holding or shortage costs, considering a “static substitution” not linked to stock-outs, and items having Normal demands. In the M-V analysis, we were able to find the variance for all the possible assortments by enumerating over all subsets. Then, we were able to find optimal assortments that aren’t considered as popular-sets, not having the highest profit margins. Then, we tried to optimize our optimal assortments. By having the optimal expected profits from the basic model, we minimized the variance for each assortment subjected to a profit constraint of high percentages and thus our variable was the order quantity.

2.3 Modeling Risk for a Single Product Newsvendor

Modeling risk has been widely developed and studied over the years and categorized into several models. Usually, these models can be mainly grouped under

Expected Utility Theory, Mean-risk optimization, Downside-risk, and Coherent measure of risk.

Starting with the expected utility theory, it was adopted by von Neumann and Morgenstern (2007). In such a model, the retailer aims to maximize his expected utility function. A newsvendor model under this function has been examined by Eckhoudt et al. (1995). Wang et al (2009) use the expected utility theory framework to analyze the classes of the utility function.

The mean-risk optimization approach is used under the Markowitz 1955 portfolio optimization. Usually, utility functions can be approximated by the M-V approach if the function is normally (Anvari 1987) or quadratic distributed (Chen and Federgruen 2000). Ohmura and Matsuo (2012a, 2012b) use the standard deviation as a risk measure.

The downside risk measure known as Value-at-Risk (VaR) by Charnes and Cooper (1959) is used in a way that calculates the probability of calculating certain events happening. Jammernegg and Kischka (2012) use Var and compares it with other risk preferences under a newsvendor problem without shortage costs.

The coherent measure of risk known as conditional value-at-risk (CVaR), is a model used to measure the profit falling under a certain level. Ahmed et al. (2007) show the existence of a newsvendor optimal solution using the CVaR maximization. Also, optimal prices and order quantities were provided using the CVaR approach by Chen et al. (2009) for different types of demand.

Retailers are mainly categorized under three risk preferences. It depends on their behavior and reaction to how they deal with their selling season while ordering their inventory. While being set up in a completely stochastic situation, these agents will act differently in the decision-making process. The risk parameters that they act upon are either too risky (risk-seeking), risk-neutral, or risk avoiders (risk-averse). According to Choi et al. (2008), a conservative newsvendor that does not enjoy profit uncertainty and his satisfaction increases with the increase of profit is a risk-averse newsvendor. The newsvendor that his satisfaction is based on the expected profit only and is neutral to profit uncertainty is known to be a risk-neutral newsvendor. While, the newsvendor that is a gambler type, who gets excited from profit uncertainty and his satisfaction increases with the profit and level of profit uncertainty is the risk-seeking type newsvendor. Ohmura (2015) reviewed the four approaches used in modeling risk-averse newsvendor models. These approaches are the mean-risk optimization, expected utility theory, downside risk, and coherent measure of risk. However, he faced difficulty while analyzing the risk-averse effect in the mean-risk approach. Also, in Choi, Li, Yan, and Chiu (2008), they try to capture the different risk preferences through building optimization models for individual decision-makers. They derive optimal order quantities for each preference and set their pricing contracts. Our work will be based on and M-V risk-averse optimization model.

The relationship between the expected profit and variance of the profit can be found by some specific distributions. One of the main inputs in the stochastic newsvendor problem is the demand probability distribution. Choi and Chiu (2012b) consider single-period inventory problems with a normally distributed demand. Perakis and Roels (2008) studied the newsvendor problem while having partial information about the demand distribution. They raise an important question regarding which distribution among the uniform, gamma, normal, or exponential to be used since each distribution leads to a different order quantity.

2.4 The Mean-Variance Single Product Newsvendor

Herrero et al. (2015) consider the single-period newsvendor while taking the price and stock quantity as the decision variables. An M-V analysis including a stochastic, price dependent demand is presented. In Chiu and Choi (2013), the importance of the M-V approach in conducting risk analysis has been discussed. The source of risk in the supply chain is classified into two sources. The first type being supply chain disruption risk emerging from natural and man-made problems, and the second type is the supply chain operational risk which we are mostly interested in that refers to variations that exist due to normal situations and demand uncertainties. Markowitz (1959) work has been widely adopted in the supply chain and extended in risk analysis based on its importance in this field of study by accounting for the mean and variance of the profit. Li et al. (2008) studied the M-V analysis of a single supplier and single retailer where the retailer controls the standard deviation of the profit and study both centralized and decentralized supply chain cases.

Chen and Federgruen (2000), conducted an M-V analysis for a quadratic utility function in a single period newsvendor model including the construction of an efficient frontier. Wu et al. (2009) applied the M-V approach on a risk-averse newsvendor setting by taking into consideration the stock-out cost while assuming a power distributed demand and a special case of uniformly distributed demand. In Choi and Chiu (2012a) the mean-variance and mean-downside risk of the newsvendor model are studied and then a fashion retailer’s inventory decision-making problem is modeled as a newsvendor problem. Agrawal and Seshadri (2000) conducted an M-V analysis on multiple retailers by using supply contracts. It has been analytically proven by Wu et al. (2009) that the optimal order quantity under an M-V can have a larger value than that of a risk-neutral case under a power demand distribution. Newsvendor problems under M-V frameworks have been studied for different risk scenario preferences while analytically exploring the optimal solution and efficient frontier by (Choi et al. 2008b).

All of the previous research in M-V analysis has been conducted on a single item model. Our work is to broaden the scope to multiple products.

CHAPTER III

MODEL AND ASSUMPTIONS

In the following, we present the base model and assumptions to be applied to Maddah and Bish (2007).

In a single-period multi-product newsvendor problem, the newsvendor composes a product line of fashionable products from the supplier from the set $\Omega = \{1, 2, ..., n\}$. Let S be the set of items with a unit ordering cost c_i per item $i \in S$ stocked by the retailer where $S \subseteq \Omega$. The normal random variable X_i is the item demand in S with mean $\lambda q_i(S)$ and a standard deviation $\sqrt{\lambda q_i(S)}$, where $q_i(S)$ is the probability of choosing Product i defined below. This can be seen as a reasonable approximation generated from a Poisson process with a customer arrival rate λ during the selling period. The Multinomial Logit Choice model is adopted with a utility of $i \in S \subseteq \Omega$ is $U_i = \alpha_i - p_i + \varepsilon_i$, where p_i represents the selling price (unit revenue), α_i is the mean reservation price (consumer choice), and ε_i are independent and identically distributed (i.i.d) Gumbel random variables. During the selling season, $p_i > c_i$.

There is no salvage value (zero salvage value) neither shortage penalty. The decision variable y_i is the amount of quantity to be made for the selling season known as the order quantity (inventory level).

For an item $i \in S$ to be bought, it will have a probability $q_i(S) = \Pr\{U_i = \max_{j \in S \cup \{0\}} U_j\}$ as a standard result of the MNL, while the probability of no-purchase is denoted by $q_0(S) = 1 - \sum_{j \in S} q_j(S)$.

It can be shown that $q_i(S)$ is given by

$$q_i(S) = \frac{e^{(\alpha_i - p_i)/\mu}}{v_o + \sum_{j \in S} e^{(\alpha_j - p_j)/\mu}}, \ i \in S, \qquad (1)$$

$$q_0(S) = \frac{v_o}{v_o + \sum_{j \in S} e^{(\alpha_j - p_j)/\mu}}, i \in S, \qquad (2)$$

where, $v_o = e^{u_o/\mu}$ (e.g. Maddah and Bish 2007).

u_o : The mean utility for the no-purchase option.

μ : The Gumbel random variable shape factor.

Following the literature (e.g. Chen and Federgruen 2000), the fashion retailer's expected profit under a newsvendor model is as follows

$$\Pi_i(y_i) = (p_i - c_i)y_i - p_i \int_0^{y_i} F(x)dx \ , (3)$$

where $F_i(.)$ is the C.D.F of the demand for Product i X_i.

Due to the concavity of the expected profit, there exists an order quantity that maximizes the expected profit in (3).

$$y_i^0 = F_i^{-1}\left(\frac{p_i - c_i}{p_i}\right) \qquad (4)$$

Also, since we are mainly focused on the variance of the profit equation, its general form equation is given by:

$$\sigma_i^2(S, \mathbf{y}) = E[\Pi(y_i)]^2 - (E[\Pi(y_i)])^2$$

Under a newsvendor type-setting problem, the profit variance for Product *i* is given by (e.g. Choi et. al 2008)

$$\sigma_i^2[y_i] = p_i^{\,2}\left(2y_i n(y_i) - 2\int_0^{y_i} xF(x)dx - [n(y_i)]^2\right),$$

where,

$$n(y_i) = \max(y_i - x_i) = y_i - \mu_i + L(y_i) = \int_0^{y_i} F_i(x_i)dx$$

The function $n(y_i)$ is the amount of leftover remaining at the end of the retailing season and $L(y_i)$ is the loss function representing the lost sales generated from shortage quantity at the end of the season.

Under a Normal demand distribution multi-item newsvendor model we get the following equations for each of the order quantity, expected profit, and profit variance equations:

$$y_i^0(S) = \lambda q_i(S) + \Phi^{-1}(1 - \frac{c_i}{p_i})\sqrt{\lambda q_i(S)} \quad \text{for } i \in S \quad . \quad (5)$$

$$\begin{aligned}\Pi(S,\mathbf{y}) &= \sum_{i\in S}\Pi_i(y_i) \\ &= \sum_{i\in S}\{p_i\mu_i(y_i) - c_i y_i - p_i\sigma_i(y_i)[\phi(z(y_i)) - z(y_i)(1-\Phi(z(y_i))]\}\end{aligned} \quad (6)$$

where

$$\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2} ... \mathbf{y_n}),\ \mu_i = \lambda q_i(y),\ \sigma_i(y_i) = \sqrt{\lambda q_i(y_i)},\ z(y_i) = \frac{y_i - \mu_i}{\sigma_i},$$

ϕ is the standard normal pdf, and Φ is the standard normal C.D.F.

$$\begin{aligned}\sigma^2[S,\mathbf{y}] &= \sum_{i\in S}\sigma_i^{\,2}[y_i] \\ &= \sum_{i\in S} p_i^2\sigma_i^2\left[z(y_i)\phi(z(y_i)) + \varphi(z(y_i))(1+z(y_i))^2 - (\phi(z(y_i)) + z(y_i)\varphi(z(y_i)))^2\right]. \quad (7)\end{aligned}$$

Usually, retailers can have high expected profits using the maximizing profit optimal order quantity equation. At the same time, following this approach may be risky and costly.

A better way to deal with this problem is to understand more what profit level can make the retailer satisfied. Based on his profit tolerance level, we will be able to sacrifice the profit level below the maximum level but at the same time subjecting him to a huge reduction in his profit variance.

In this chapter, our main goal is to optimize the newsvendor multi-item problem by minimizing the retailers' risk exposure being subject to a constraint on the expected profit. Our model is as follows:

$$\begin{aligned}&\min_{S,\mathbf{y}} \ \sigma^2(S,\mathbf{y})\\&\text{subject to}\\&\Pi(S,\mathbf{y}) \geq \pi_0\end{aligned}$$

where π_0 is the target profit level which is defined by the retailer that cannot exceed $\Pi(S,\mathbf{y}^0)$ the maximum expected profit value obtained by using y^0.

CHAPTER IV

ANALYTICAL RESULTS

The following lemmas capture our main analytical resuls. Lemmas 1 and 2 are straightforward extension of results from the single-product case.

Lemma 1: *For a given assortment S, the total expected profit function* $\Pi(S, \mathbf{y})$ *is a concave function in y with a unique maximum* at $\mathbf{y}^0(S) = (y_1{}^0, \ldots, y_{|S|}{}^0)$, where $y_i^0 = F_i^{-1}(\frac{p_i - c_i}{p_i})$.

Proof. See Appendix.

Lemma 2: *The total variance* $\sigma^2(S, \mathbf{y})$ *is a monotone increasing function* y_i *for all i.*

Proof. See Appendix.

While Lemma 2 establishes the monotonicity of the variance function, the following lemma provides some new insights into the convexity of the assortment variance.

Lemma 3: *The total variance function* $\sigma^2(S, \mathbf{y})$ *is a monotone increasing function in* y_i *that changes from being convex to the right of 0 to becoming concave to the left of* ∞.

Proof. See Appendix.

Lemma 3 implies that the variance function starts convex and increasing in y_i (like a hockey stick), and ends concave and increasing (like an inverted hockey stick). We observe the risk neutral oreder quantity, y_i^0, is often at the boundary between the convex and concave regions. This is a useful observation, especially, as we show next, in Lemma 4, that the optimal mean-variance order quantity is such that y_i^*, is such that $y_i^* \le y_i^0$. As such, Lemma 4 suggests that our mean-variance problem involves a convex opitimzation.

Lemma 4. *For a given assortment S and a given profit level* π_0 *the optimal order quantity* y_i^* *for the mean-variance problem should be below the profit-maximizing (risk-neutral) order quantity* y_i^0 .

Proof. See Appendix.

When formulating the optimization problem, it is important to set the profit constraint to a level that the retailer is satisfied with. Mainly, our profit level is set to greater than or equal to a target value π_0. However, since we are minimizing the variance, we show in Lemma 5, that the expect profit constraint is always binding. Lemma 5 is useful as it can serve to simplify the solution of the problem. For example, in the single-product case, Lemma 5 directly gives the optimal solution.

Lemma 5: *The profit constraint is binding at optimality* $\Pi(S, \mathbf{y}) = \pi_0$.

Proof. See Appendix.

CHAPTER V

NUMERICAL RESULTS

In this section, we start in Section 5.1 by showing the numerical results obtained from the different scenarios built based on Maddah and Bish (2007). In Section 5.2 we present the optimization problem supported by the Mean-Variance analysis. In 5.3, we show the M-V behavior as a function of the order quantities. In 5.4, we show some numerical results on the level of a fixed profit level and possible order quantity relationship. In 5.5, we provide an explanation on the convexity of the profit variance function.

5.1 Risk-Reward Trade-Off in Assortment Planning

In this section, we will refer to an assortment of three items. Each item has a different mean reservation price. The prices and costs are exogenously set.

Consider two cases from Maddah and Bish (2007). The difference between the two cases is the consumer choice for item 1. We capture the total expected profits and total profit variance for each assortment in the two cases. We can see from Table 1 that assortment {1,2} has the highest expected profit while at the same time it captures the highest expected profit variance.

Case 1	S	Consumer Choice	Expected Profit	Variance of Profit	Std. Deviation of Profit
a	{1}	11	104.39	824.31	28.71
b	{2}	10	73.01	493.86	22.22
c	{3}	9	42.50	236.23	15.37
d	{1,2}		124.86	968.06	31.11
e	{1,3}		103.04	778.07	27.89
f	{2,3}		80.55	535.57	23.14
g	{1,2,3}		116.88	901.22	30.02

Table 1 - Expected Profit and Profit Variance calculation for Case 1.

$\alpha_1 = 11,\ \alpha_2 = 10,\ and\ \ \alpha_3 = 9$
$p_1 = 11.572,\ p_2 = 10.567,\ p_3 = 9.563$
$c_1 = 9,\ c_2 =\ 8,\ and\ c_3 =\ 7$
$v_0 = 1,\ \mu = 1,\ and\ \lambda = 100$

In Case 2, the single item {1} has the highest expected profit and highest profit variance. However, item {3} has the lowest expected profit and captures the lowest variance among the remaining assortments.

Case 2	S	Consumer Choice	Expected Profit	Profit of Variance	Std. Deviation of Profit
a	1	12	184.54	1383.23	37.19
b	2	10	73.01	493.86	22.22
c	3	9	42.50	236.23	15.37
d	1,2		180.84	1379.81	37.15
e	1,3		166.49	1248.88	35.34
f	2,3		80.59	535.75	23.15
g	1,2,3		164.56	1266.75	35.59

Table 2 - Expected Profit and Profit Variance calculation for Case 2.

$\alpha_1 = 12,\ \alpha_2 = 10,\ and\ \ \alpha_3 = 9$
$p_1 = 11.572,\ p_2 = 10.567,\ p_3 = 9.563$
$c_1 = 9,\ c_2 =\ 8,\ and\ c_3 =\ 7$
$v_0 = 1,\ \mu = 1,\ and\ \lambda = 100$

Figure 1 and Figure 2 clearly illustrates the results obtained in Table 1 and Table 2. We can notice that an assortment with high expected profit is associated with a very large variance.

Figure 1 – The M-V relationship in Case 1.

Figure 2 - The M-V relationship in Case 2.

5.2 Optimization Formulation of the M-V Problem

Usually, retailers can have high expected profits using the maximizing profit optimal order quantity equation. At the same time, following this approach may be risky and costly. His profit is prone to a lot of risks depending on his ordered quantity, demand, consumer search, price, and cost of each item in the assortment.

A better way to deal with this problem is to understand more what profit level can make the retailer satisfied if he targets a level below the maximum profit. Based on his profit tolerance level, we will be able to sacrifice the profit level below the maximum level but in the same time subjecting him to a huge reduction in his profit variance.

In this section, our main goal is to optimize the newsvendor multi-item problem by minimizing the retailers' risk exposure being subject to a constraint on the expected profit, as defined in Chapter 3.

In order to conduct a numerical approach using the above optimization problem, we use the following parameters:

$$v_0 = 1,\ \mu = 1,\ and\ \lambda = 100$$
$$\alpha_1 = 12.85,\ \alpha_2 = 10,\ and\ \alpha_3 = 9$$
$$p_1 = 11.572,\ p_2 = 10.567,\ p_3 = 9.563$$
$$c_1 = 9,\ c_2 = 8,\ and\ c_3 = 7$$

Based on these inputs, we can obtain 7 different assortments of different sizes ranging from one assortment of one item to an assortment composed of three items. The expected profit and variance of each of the possible outcomes are present in the table below.

S	Consumer Choice	Expected Profit	Variance of Profit	Std. Deviation of Profit
{1}	12.85	208.49	1886.67	43.44
{2}	10	73.01	493.86	22.22
{3}	9	30.58	235.58	15.35
{1,2}		196.04	1244.55	35.28
{1,3}		197.64	1240.64	35.22
{2,3}		82.13	596.17	24.42
{1,2,3}		192.35	1246.71	35.31

Table 3 – The M-V results of the formulation model.

We choose assortment {2,3} as a reference for our optimization problem. And apply a range of constraints on the profit level while trying to obtain the minimum variance possible on a broader. To have a wider look at what possible values we can get for that assortment, we applied a range of constraints to the profit by setting the profit threshold from a range of $\pi_0 = 0$ to $\pi_0 = 82.13$. This was done by applying percentage changes to the maximum expected profit $\Pi(y^0_{2,3}) = 82.13$.

The problem was optimized using Excel Solver, by having an objective to minimize the variance equation while having the profit level set at several % levels of the total maximum expected profit and the order quantities being our variables. It is also important to note that order quantities were set at most equal to the level of maximizing the expected profit ($y_2 \le y^0_2$ and $y_3 \le y^0_3$).

% of Total EP	y_2	y_3	Expected Profit	Variance of Profit	Std. Deviation of Profit	Coefficient of Variation
0	0.00	0.00	0.00	0.00	0.00	0.00
10	3.20	0.00	8.22	0.00	0.01	0.08
20	6.40	0.00	16.43	0.01	0.03	0.18
30	9.60	0.00	24.64	0.01	0.12	0.47
40	12.44	0.36	32.85	0.20	0.45	1.36
50	14.41	1.60	41.07	0.80	0.89	2.18
60	16.40	2.84	49.28	2.93	1.71	3.47
70	18.41	4.11	54.49	9.81	3.13	5.75
80	20.53	5.43	65.70	30.93	5.56	8.46
85	21.68	6.14	69.81	54.57	7.39	10.58
90	22.95	6.89	73.92	97.83	9.89	13.38
95	24.51	7.77	78.02	186.11	13.64	17.48
96	24.89	7.97	78.84	215.41	14.68	18.61
97	25.32	8.19	79.67	252.34	15.89	19.94
98	25.82	8.44	80.49	301.46	17.36	21.57
99	26.47	8.74	81.31	374.58	19.35	23.80
99.5	26.92	8.93	81.72	432.74	20.80	25.46
100	27.97	9.35	82.13	590.51	24.30	29.59

Table 4 – Construction of the efficient frontier for assortment {2,3} at optimal order quantities solutions for both items.

If we compare the profit variance obtained $100\%\ \Pi(y_{2,3}^{0})$ with that $80\%\ \Pi(y_{2,3}^{0})$, we can realize the huge tradeoff in the variance level. Decreasing the profit by only 20% was capable of decreasing the total variance by 94.7% and the profit sigma by 77.1%. Furthermore, achieving a $90\%\ \Pi(y_{2,3}^{0})$ is capable of dropping the total variance by 83.43% and the total profit sigma by 59.3%. Most retailers' can be satisfied with an $80-90\%$ of $\Pi(y_{2,3}^{0})$. This tradeoff can be clearly seen on the efficient frontier graph below.

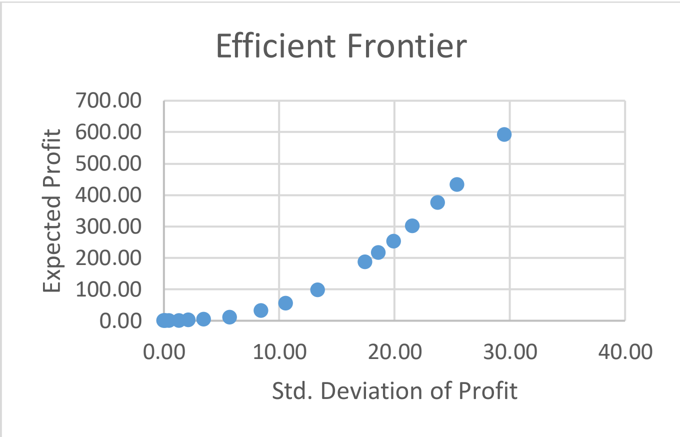

Figure 3 – The efficient frontier plot of assortment {2,3}.

This tradeoff result is valid on several assortment sizes. Taking assortment {1,2,3} and into consideration and applying the same optimization problem on a multi-profit level percentages with constraints on the order quantities $y_1 \le y_1^0, y_2 \le y_2^0$ and $y_3 \le y_3^0$ to it gives us the following results:

%Expected Profit	y_1	y_2	y_3	Expected Profit	Variance of Profit	Std. Deviation of Profit	Coefficient of Variation (%)
0	0.27	0.00	0.00	0.70	0.00	0.00	0.00
10	7.77	0.00	0.00	19.94	0.00	0.00	0.01
20	10.52	0.00	0.00	27.02	0.00	0.00	0.02
30	15.78	0.00	0.00	40.50	0.00	0.04	0.09
40	19.70	1.34	0.00	53.98	0.04	0.19	0.35
50	22.93	3.35	0.00	67.46	0.29	0.53	0.79

60	26.17	5.38	0.00	80.94	1.86	1.37	1.69
70	29.04	7.18	0.69	94.37	10.28	3.21	3.40
80	31.80	8.91	1.77	107.86	37.24	6.10	5.66
85	33.29	9.83	2.33	114.60	69.92	8.36	7.30
90	34.95	10.84	2.92	121.34	133.01	11.53	9.50
92.5	35.90	11.39	3.23	124.71	186.60	13.66	10.95
95	36.99	12.01	3.56	128.08	268.83	16.40	12.80
96	37.50	12.29	3.70	129.43	315.27	17.76	13.72
97	38.07	12.59	3.85	130.78	374.58	19.35	14.80
98	38.75	12.93	4.02	132.12	454.59	21.32	16.14
99	39.62	13.34	4.20	133.47	575.62	23.99	17.98
99.5	40.24	13.62	4.31	134.15	673.29	25.95	19.34
100	41.65	14.19	4.53	134.82	938.76	30.64	22.73

Table 5 – Construction of the efficient frontier for assortment {1,2,3} at optimal order quantities solutions for the three items.

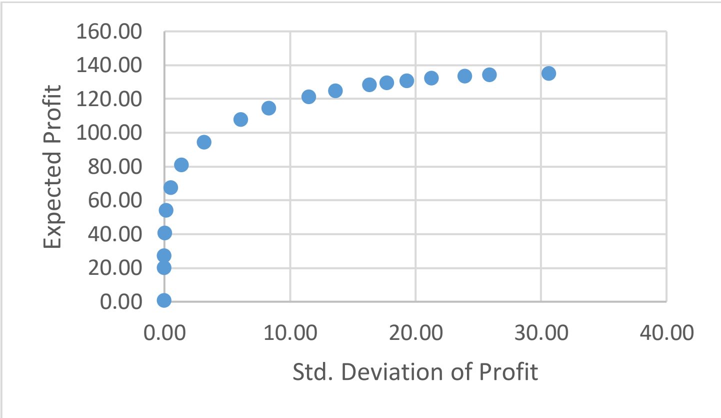

Figure 4 – The efficient frontier of {1,2,3}.

For this size of assortment, we can realize that a 90% of $\Pi(y^0_{1,2,3})$ leads to a variance drop by 85.8% and a sigma drop of 62.37%.

Not to mention the low values in the coefficient of variation on both cases which shows consistency in the findings we have.

To understand more the constraints set at the level of the order quantities, we can take a look beyond the optimization problem of assortment {2,3}. Setting order quantity values beyond y^0_2 and y^0_3 lowers the total profit and increases the total variance. At high order quantities, the profit can reach 0 and may become negative in the sense of only paying for the costs more than receiving returns. The variance also can reach a plateau that can be reached at high order quantity values. In the table below, we can see that when $y_2 \geq y_2^0$ and $y_3 \geq y_3^0$, the total profit decreases from its total maximum value and the variance keeps on increasing. At higher order quantities and specifically at $y_2 \geq 43.54$ and $y_3 = 13.03$, the $\Pi(y_{2,3}) = 0$ and beyond that it becomes negative while $\sigma^2(y_{2,3})$ reaches a plateau of 4354.5.

y_2	y_3	Expected Profit	Variance
0.01	0.00	0.00	0.00
3.20	0.00	8.22	0.00
9.60	0.00	24.64	0.01
14.41	1.60	41.07	0.80
18.41	4.11	54.49	9.81
21.68	6.14	69.81	54.57
25.32	8.19	79.67	252.34
26.47	8.74	81.31	374.58
27.97	9.35	82.13	590.51
33.85	12.29	65.70	2255.30
36.85	12.61	49.28	3095.10
39.27	12.79	32.85	3613.80
41.46	12.92	16.43	3922.30
43.54	13.03	0.00	4095.00
45.00	13.50	-13.58	4232.10
47.00	14.00	-31.96	4354.50

Table 6 – The total expected profit for order quantities exceeding the optimal solutions.

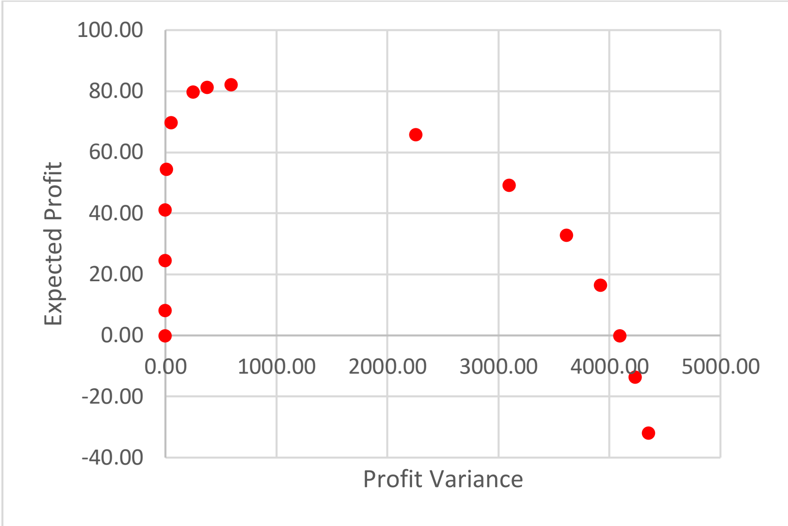

Figure 5 – The plot of the expected profit and the variance when the order quantities exceed the optimal solutions.

5.3 The Behavior of the Expected Profit and the Variance as a Function of the Order Quantities

Order quantities are our main variables in the optimization problem. They have a huge impact on both the expected profit and the variance of the profit. Since we are mainly concerned with minimizing the variance, it is important to know how the order quantities of a certain assortment affect both components.

It is important to mention that we are concerned with the order quantity range $y_i \in [0, y_i^0]$.

In assortment {2,3}, we plot the total expected profit as a function of the order quantities. We can realize that the expected profit increases in the range $y_2 \in [0, y_2^0]$ and $y_3 \in [0, y_3^0]$ until it reaches its maximum y_2^0 and y_3^0.

Also, the total variance as a function of order quantities has a similar behavior on the range of $y_2 \in [0, y_2^0]$ and $y_3 \in [0, y_3^0]$. We can realize that the total variance increases in both order quantities. The behavior resembles a hockey stick shape, where the variance starts from 0 at 0 order quantities, and at relatively higher order quantity values the variance then instantly increases. In the graph below, we can see how the variance responds to both y_2 and y_3 having the blue trend as a representation for item 2 and the orange trend represents item 3.

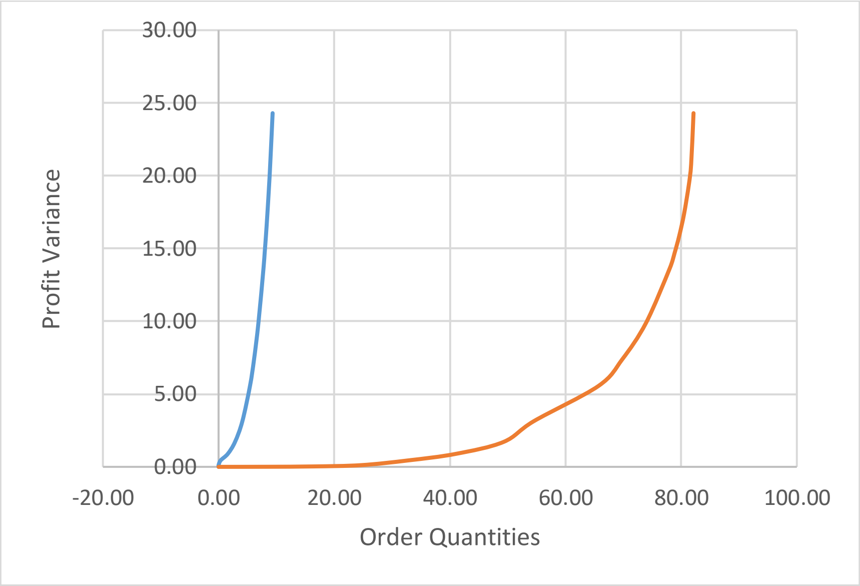

Figure 6 – The total variance of assortment {2,3} as a function of the order quantities y_2 and y_3.

5.4 The Relationship between Order Quantities at a Fixed Expected Profit Level

Fixing the Expected Profit at a certain target value being below the total maximum value can be achieved by varying the order quantities. Order quantities behave in a way that allows us to vary them and still be able to achieve the same profit value in the case of multi-item assortments. Among the numerous results we get from this approach, it is important to note that each result yields a different profit variance value that varies according to the obtained order quantities. From an optimization point of view, only one solution leads to minimizing the variance while achieving the expected profit value.

Considering the assortment {2,3}, when achieving the profit value $80\%\ \Pi(y^0_{2,3})$, we can get multiple order quantity solutions with one that minimizes the variance.

Let us consider two conditions, where in the first one, y_2 is already known while y_3 is to be determined. In this sense, we will be solving an equation with one unknown being y_3.

In this table, we set y_2 values and solve them to obtain y_3 by satisfying the profit constraint and the order quantities constraints $y_i \in [0, y_i^0]$. Based on that we obtain the following results

Known	Unknown	Solve
y_2	**y_3**	**Variance**
19	7.3449	56.69
20	6.014	33.064
21	4.95	32.3137
22	4.0211	43.48
23	3.1867	65.36
24	2.45	99.7
25	1.833	149.8
26	1.355	219.679
27	1.04689	313.692
28	0.937	435.772

Table 7 – Obtaining the order quantity of item 3 as a function of the order quantity of item 2.

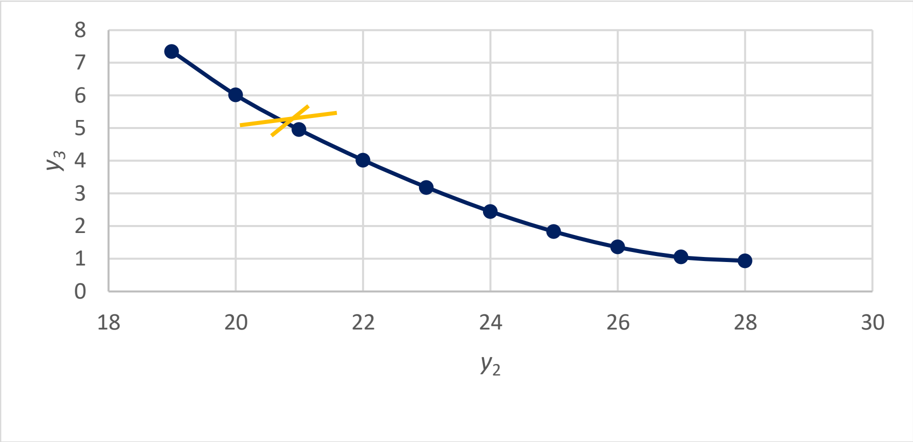

Figure 7 – The relationship between both order quantities of items 2 and 3 at an 80% profit target.

We can also consider the other way around, by having y_2 an unknown parameter, y_3 known, the same target profit level and order quantity constraints and we thus solve for y_2.

In the two different scenarios, there is only one solution that leads to the minimum variance value. That optimal solution is represented on both graphs as a yellow cross and it is achieved at $y_2 = 20.53$ and $y_3 = 5.43$.

Based on the given results, we can realize that when $y_2 > y_2^*$ the second-order quantity will be $y_3 < y_3^*$. The opposite is also true, that if $y_2 < y_2^*$ then $y_3 > y_3^*$. This comes in a way to balance the order quantities and be able to yield to the desired profit target. We can better understand this problem by looking into the 3-D plot below of the total expected profit on the vertical axis versus the order quantities of items 2 and 3 on the horizontal axes. The constraint is crossing the plane at the desired profit level $\Pi(y_{2,3}) = 65.7$. On that intersection we will have multiple solutions in that area, however only one will yield to the minimum variance optimal solution.

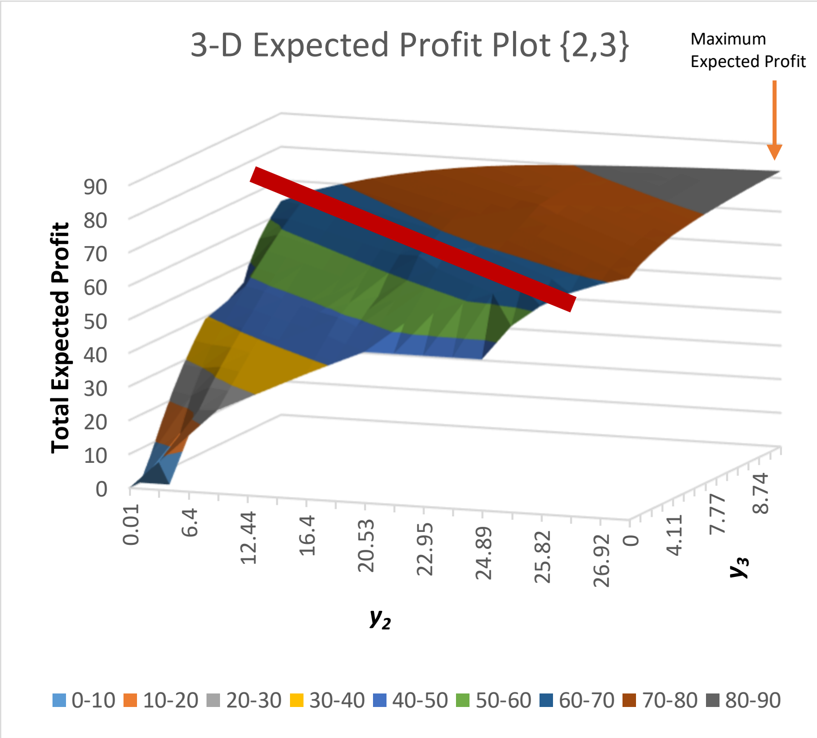

Figure 8 – A 3-D plot of the total expected profit of assortment {2,3} as a function of y_2 and y_3.

5.5 The Convexity of the Total Variance Equation

The profit variance function is a variance equation in terms of the order quantity. When plotting both components on a graph and exceeding the maximizing order quantity, we can be able to realize that the variance increases from 0 until reaching a plateau. By looking at item 2 in the assortment {2,3}, we can plot the graph and realize the S-shaped variance curved structure.

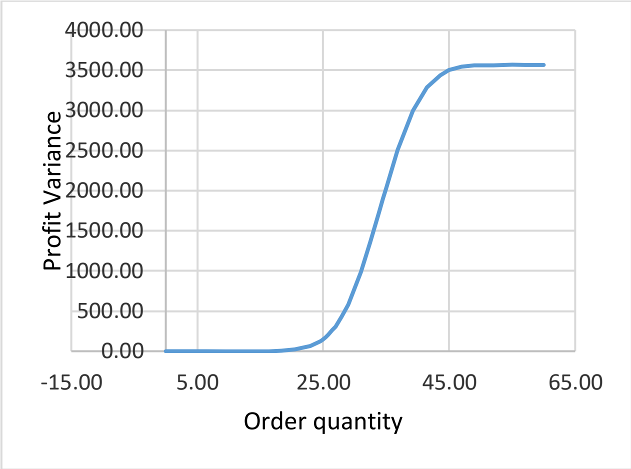

Figure 9 – The profit variance convexity function of {2} as a function of y_2.

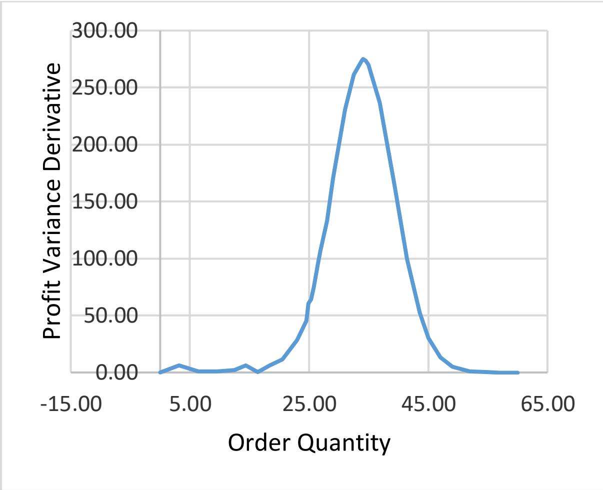

Figure 10 – The first derivative of the profit variance of {2}.

Figure 11 – The second derivative profit variance of {2}.

$\sigma^2(y_2)$ is a monotone increasing function of y_2. At y_2 = 20.53, the variance starts to rapidly increase from almost flat positive values slightly above 0 until reaching y_2 =41.45 where it then slows down reaching its maximum and stable value of 3565.5 thus forming another plateau when reaching high order quantities for this item (item 2).

The inflection point $\sigma^2(y_2)$ is at y_2 =34.06, where the function shifts from being convex to becoming concave. This point is the peak in $\sigma'^2(y_2)$. In addition, $\sigma'^2(y_2)$ has two inflection points before and after y_2 =34.06 when the function moves from 0 till the peak is reached and vice versa.

The peak point in $\sigma'^2(y_2)$ is a root at y_2 =34.06 in $\sigma''^2(y_2)$, the first inflection point in $\sigma'^2(y_2)$ at y_2 =28.969 is a maximum point in $\sigma''^2(y_2)$ at y_2 =28.969 and a minimum point in $\sigma''^2(y_2)$ for the second inflection point in $\sigma'^2(y_2)$ at y_2 =39.27. In addition, $\sigma''^2(y_2)$ has an inflection before y_2 =28.969, after y_2 =39.27, and between them at y_2 =34.06.

Thus, the extreme points will be roots in $\sigma'''^2(y_2)$ at y_2 =28.969 and y_2 =39.27. Also, the inflection points will have extreme points at y_2 =25.822, y_2 =34.06, and y_2 =43.539.

The two roots presented in $\sigma'''^2(y_2)$ represent the hockey stick behavior in $\sigma^2(y_2)$ at the lower and upper parts of the function. At these points, the graph changes its behavior twice from going from concave to convex and then from convex to concave.

Observation 1: *Applying the Assortment Diversification to Lower the Variance and Maintain the Profit Level:*

It is important to show the impact of diversification in a way to achieve a certain profit target while using more than one item. We can do so by having either identical items or different items. By different items, we mean items that have different consumer choices for this particular section.

Let us start with a single item and try to obtain its maximum profit level using assortments of larger sizes.

Consider item 1, with $\Pi(y_1^0) =$ 73.014, y_1^0 =32, and $\sigma^2(y_1^0) = 493.86$. Let us consider an additional identical item with item 1 and increase the assortment size from one item to two items. Obtaining the same profit using 1 items can be done with having y_1 =14.214 for each of the two items and a total variance of $\sigma^2(y_{1,2}^0)$ =10.36. Adding a third item to increase the assortment to 3 identical items with y_2 =9.516 for each item and a total $\sigma^2(y_{1,2,3}^0)$ =8.274. Considering a fourth identical item, we can achieve the same profit with order quantities of y_i =7.14 for each of the four items and a total variance $\sigma^2(y_{1,2,3,4}^0)$ =10.97.

These results highlight the importance of reaching the targeted profit through diversification of items where a significant variance drop was obtained. In the same time, it is important to balance the number of items in the assortment and avoid extra unnecessary items that can deviate the assortment from its optimal size and lead to an increased variance. In this case, an assortment of three items had the lowest variance amongst the other assortment sizes for identical items.

We can realize the impact of diversification not only on the level of this profit level, however, but this can also be applied at different target profit levels. For example, a profit of 62 which makes around 85% of the maximum profit, is best achieved through two identical items, since having a single item or three or four items attain higher variance levels ($\sigma^2(y_1^*) = 25.646$, $\sigma^2(y_{1,2}^*)$=2.43, $\sigma^2(y_{1,2,3}^*)$=2.751, and $\sigma^2(y_{1,2,3,4}^*)$=4.487). Thus, since we are optimizing on the variance values, two assortments best serve our needed profit in the lowest variance.

Lower profits can be also achieved through an assortment of size 1. For this particular case, a profit of 35 can be best attained using only one item and any additional item will imply a higher variance.

In other words, the higher the profit, the larger the assortment size, the lower the variance, depending on the profit level because sometimes large assortments can add on the optimal minimum variance value and make it an over diversified assortment. This is known as the thinning of demand. However, optimality should always be obtained on variance and the best assortment should be chosen accordingly.

We can see in the following table different profit levels of item 2 with the corresponding variance for each profit level achieved through 4 different assortments of size 1, 2, 3, and 4 constituted of multiple identical items. Each assortment variance is obtained in the table for its corresponding profit level and assortment size.

∏	73.01	72	71	69.363	67	65.7	64	62	60	58.41	52	45	40	35	30
{1}	493.86	259.84	190.06	124.90	73.28	55.50	38.87	25.65	16.90	12.09	2.97	0.56	0.15	0.04	0.01
{1,2}	10.36	9.11	8.02	6.85	4.76	4.02	3.19	2.43	1.84	1.48	0.58	0.20	0.09	0.04	0.02
{1,2,3}	8.27	7.50	6.80	5.79	4.58	4.02	3.38	2.75	2.24	1.89	0.95	0.43	0.24	0.13	0.07
{1,2,3,4}	10.97	10.12	9.35	8.20	6.77	6.10	5.30	4.49	3.80	3.32	1.91	1.03	0.65	0.41	0.25

Table 8 – The optimal profit solutions at different profit levels and multiple assortment sizes using identical items.

We can realize from the results that the assortment of size 4 is always undesirable for the given profit levels due to over-diversification that adds up on the variance due to the addition of an extra undesirable item to the assortment. And the best assortment size decreases as the profit level decreases. The assortment size does not remain the same through different profit targets.

Let us now consider 4 different items, with consumer choice α_1 =11, α_2 =10, α_3 =9, and $\alpha_4 = 8$ respectively whereas the prices and costs are the same. Sixteen different assortments can be obtained by combining these 4 items together with the possibility of having single, two, three, and four items assortment sizes. Let us pick assortment {1,2} with the highest profit amongst the other 15 assortments of $\Pi(y^0_{1,2})$ =136.7 and a variance $\sigma^2(y^0_{1,2})$ =925.643.

The objective is to achieve different profit levels of that assortment and see its impact on the variance. This was done by decreasing the profit level of this assortment and obtaining lower variance levels for the same assortment. This can be seen in the {1,2} row in the table. In addition, those same attained profit levels were also targeted through different assortment sizes constituted of item1, items 1 and 2 (the base case in this example), items 1, 2, and 3, and items 1, 2, 3, and 4.

A profit range from $\Pi(y_{1,2}) = 90$ (66% of $\Pi(y^0_{1,2})$) till $\Pi(y_{1,2})$ =132 (97% of $\Pi(y^0_{1,2})$) was achieved and studied under different assortment sizes based on minimum variance value. The following results in the table below were obtained. This shows that the best assortment is {1,2} for a profit range (115 to 132) and assortment item {1} for profit range of (90 to 115).

Additional items are unnecessary and adds on variance. The dashed cell is the case of being unable to achieve the profit with this assortment.

	132	130	125	120	115	110	105	100	90
{1}	-	827	200	85.9	38.416	17.045	7.34	3.04	0.45
{1,2}	324.43	233.3	127.71	72.691	41.771	23.914	13.539	7.544	2.204
{1,2,3}	497.32	338.5	192.92	116.77	72.6	45.5	28	17.7	6.6
{1,2,3,4}	962	514.89	273.18	166.8	105.9	68.656	44.941	30.03	14.75

Table 9 – The diversification effect on the profit variance using different items.

From the three different tables, we can realize that a lot of variances can be saved by choosing the best assortment size for the required profit level. Adding items to the assortment is an important diversification tool that saves up a lot of variances while achieving the same profit. Achieving higher profits are best tackled with larger assortment sizes. On the contrary, adding items can be harmful if not carefully managed. Over diversification implies adding additional items to the assortment that give the same profit but a higher variance from the optimal case.

Observation 2: *The Overall Optimal Assortment is a Popular Set*

Popular sets are defined by items or assortments having the highest consumer choice among their similar size assortments and lead to the highest expected profit. In order to find out in our case, after applying the M-V optimization if popular sets are optimal assortments, let us consider the case of 4 different items mentioned in the previous section. The consumer choice for each is of the four items can be shown as follows: α_1=11, α_2=10, α_3=9, and $\alpha_4 = 8$ respectively whereas the prices and costs are the same. Based on the above consumer choices, we can obtain that the popular sets are {1}, {1,2}, {1,2,3}, and {1,2,3,4}.

The optimization has been applied by taking the highest profit among all of the 15 possible assortments and setting each assortment to match a % profit constraint of that maximum expected profit. After applying the profit maximization approach, we get that assortment {1,2} has the highest profit amongst the other 14 assortments with $\Pi(y^{0}_{1,2}) = 136.7$ and a variance $\sigma^2(y^{0}_{1,2})$ =925.643.

The next stage was applying the optimization problem on all assortments by a profit constraint and order quantities being less than the optimal maximizing profit levels. The following results were then obtained. The blank cells represent a no solution on that profit level with its corresponding assortment.

%Π	96.56	87.7	80.46	73.15	65.83	58.5	51.2
∴ Π	**132**	**120**	**110**	**100**	**90**	**80**	**70**
1	-	85.9	17.04	3.045	0.45	0.054	0.005348
2	-	-	-	-	-	-	145.919
3	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-
1,2	324.43	72.691	23.914	7.544	2.204	0.59	0.1432
1,3	-	132.1215	37.83	10.838	2.879	0.362	0.03944
1,4	-	146.9	36.61	6.13	0.96	0.122	0.01254
2,3	-	-	-	-	-	270.437	56
2,4	-	-	-	-	-	-	15.75
3,4	-	-	-	-	-	-	-
1,2,3	446.0278	116.77	45.5	17.7	6.65	1.65	0.413
1,2,4	485.13	112.25	36.05	11.45	3.4	0.92	0.228
1,3,4	-	220.0977	58.45	16.67	4.49	0.66	0.075
2,3,4	-	-	-	-	-	309.66	63.784
1,2,3,4	846.26	166.38	68.65	24.53	9.24	2.47	0.633

Table 10: The overall optimal assortment using popular sets.

By looking at $\Pi(y_{1,2}) = 80.46$, we can notice that {1,2,3} which is a popular set has a higher variance than assortment {1,2,4} $\sigma^2(y_{1,2,3}) > \sigma^2(y_{1,2,4})$ in a sense that makes both assortments achieve the same profit level while the unpopular assortment {1,2,4} act better than {1,2,3} from a variance minimization point of view. The same can be realized at different profit levels among different assortment sizes. Taking into consideration {1,2}, it is dominated by {1,4} at $\Pi(y_{1,2}) = 70,\ 80, 90, \text{and } 100$.

However, when looking at the overall table, we can see that none of the non-popular sets are overall optimal at a certain profit level. By taking $\Pi(y_{1,2}) = 120 \text{ and } 132$ we can notice that {1,2} is the overall optimal assortment yielding to the least variance. Looking at a lower profit value $\Pi(y_{1,2}) \in [70,110]$ we can realize that the overall optimal assortment is {1} which is the single-sized assortment popular set. Based on this we can realize that popular assortments are the optimal assortments.

CHAPTER VI

CONCLUSION AND FUTURE WORK

The purpose of this thesis is to provide the retailer with a tool that enables them to meet their uncertain demand not only effectively but also efficiently. It is important that they will be able to enjoy high profits while being prone to much lower risks.

This model has been developed based on understanding the retailers' needs and meeting their struggles with uncertainty in terms of fulfilling the market demand. We were able to (i) provide the retailers with insights on the profit variance for each assortment they may have, (ii) provide an optimization formulation that is applied on multi-item assortments, (iii) show the variance tradeoffs with a little profit sacrifice by understanding its underlying risk, and (iv) numerically analyze the optimal assortment over a range of input parameters. In addition, diversification has shown its significant impact by reaching the needed profit while offering a low-profit variance in return.

Based on our analysis, we can see that it is very important to capture the full image of the profit not only estimating the expect but to go beyond that and capture the assortment's variance. The efficient frontier can help retailers choose a profit level that is in-line with their risk appetite.

A direction for future work is to provide more analytical results on the convexity of the variance as a function of the order quantities. Also, analytical results to support our observations on the structure of the optimal assortment are needed. Finally, pricing can be added to this model as an additional decision variable.

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APPENDIX

Proof of Lemma 1. The Expected Profit function $\Pi_i(y_i)$ for the single item is a concave function with a unique maximum at y_i^0 value. (Chen and Federgruen 2000).

In the multi-item case, the expected profit is the sum of the individual items profit functions, and is separable in the sense that the expected profit of Item *i* is a function of y_i only, it follows that the total expected profit is concave with a unique maximum at $\mathbf{y}^0$. (Turken et al. 2011).

Proof of Lemma 2. This also follow from the separability of the variance function, which implies (utilizing result from Choi et al. 2008) that

$$\frac{\partial^2\sigma^2(S,\mathbf{y})}{\partial y_i} = \frac{\partial^2\sigma_i^2(y_i)}{\partial y_i} = \sum_{i\in S} 2p_i^{\,2}\left(1-F_i(y_i)\right)n(y_i) \geq 0.$$

Proof of Lemma 3. At $y_i = 0$, it follows from the expression in the proof of Lemma 2 that $\lim_{y_i\to 0}\frac{\partial\sigma_i^2(y_i)}{\partial y_i} = 0,$ since $n(0) = 0$. (Recall that $n(y_i) = \int_0^{y_i} F_i(x_i)dx$). By computing the second derivative one can show that $\frac{\partial^2\sigma_i^2(y_i)}{\partial^2 y_i} > 0,$ for y_i around 0. This implies that the function is locally convex around 0.

As $y_i \to \infty$, $\lim_{y_i\to\infty}\frac{\partial\sigma_i^2(y_i)}{\partial y_i} = 0.$ We can also show from the second derivative that $\frac{\partial^2\sigma_i^2(y_i)}{\partial^2 y_i} < 0$ for y_i around ∞. Hence, the function is locally concave around ∞.

Proof of Lemma 4. Assume by contradiction that there exists a solution $y_i^* > y_i^0$, then since the expected profit from the assortment, $\Pi(S,\mathbf{y})$, is concave in y_i, with a maximum at y_i^0 and the assortment variance $\sigma^2(S, \mathbf{y})$ is increasing in y_i, then decreasing y_i^* to $y_i^* - \delta,\ \delta > 0$, will decrease $\sigma^2(S, \mathbf{y})$ and increase $\Pi(S, \mathbf{y})$ (implying the profitability constraint remains feasible). This contradicts the optimality of y_i^*.

Proof of Lemma 5: Again by contradiction assume that an optimal solution has an expected profit greater than the target profit level, $\Pi(S, \mathbf{y}) > \pi_0$. Then, since $y_i^* < y_i^0$, and the variance is decreasing in y_i, so decreasing y_i below y_i^* till $\Pi(S, \mathbf{y})$ reaches π_0 will decrease the variance while not violating the profitability constraint.