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DUAL-CHANNEL SUPPLY CHAIN

by
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ABSTRACT
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The rapid technological developments that have been witnessing over the past decade offer manufacturers the option to sell their products directly to customers through e-commerce. This paper analyzes a dual-channel supply chain. The first channel is the standard retail channel in which the retailer sells the manufacturer's non-customized product(s) to customers. Whereas in the second channel, the manufacturer is selling customized products directly through an online channel. The customized products consist of the standard product along with assembled multiple parts, rendering it a different version.

Two models are evaluated based on a constant and linearly price and lead-time dependent demand. The first model solves for the optimal ordering quantities for the basic product and the customizing features. As for the second model, the objective is to find the optimal pricing strategies for both, the basic product, and the customizing features, while holding the ordering policy constant.

A third model was solved numerically to find a local optimal solution having both the ordering and pricing policies as decision variables.

Finally, a sensitivity analysis study was conducted to the model's key input parameter to understand its behavior resulting in some interesting managerial insights.

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CHAPTER 1

INTRODUCTION

The success stories of many companies have been directly linked to their flexibility, responsiveness, and ability to meet the customers' continuously changing requirements. In the last decade, technological advancements have created a whole new sales model, e-commerce, which has offered manufacturers the option of selling their product directly to the customer without resorting to a retailer in their supply chain. IBM, Nike, Apple, and Dell are some of the companies that are adopting dual-channel supply chain models (Batrafi et al., 2016, p. 1). As a result, many manufacturers have made all or some of their products available through an online channel. This fact brought the customers' needs more to the attention of manufacturers and strengthened their relationship. The closeness between the manufacturers and the customers along with the technological improvements in computer-aided design or manufacturing have introduced the concept of mass customization (Batrafi et al., 2017, p. 1). Through mass customization, the customers can now customize or modify a certain basic product according to their requirements and convey it to the manufacturer. The manufacturer can then provide the customers with a customized product that meets their specific needs. From a business perspective, supply chain analysts now must decide whether to adopt an online channel or not and in case they decide to offer an online market, they should optimize over both channels. This topic resulted in many studies investigating dual-channel supply chains for different assumptions and parameters. Many papers, such as the work done by Batrafi et al. in 2016 and 2017, and the work done by Saha et. al in 2017, studied a manufacturer supply chain composed of a traditional channel selling standard products to the customers through retailers along with an online one offering customized versions of the basic product to the customers with specific requirements. This paper considers a similar model, but pushed further, looking into the customization products and the accessories needed to achieve them. It analyzes and optimizes a dual-channel supply chain modeled in terms of the basic product, the parts needed for customization, and the customized products.

The remainder of the paper is organized as follows. In Chapter 2 presents the literature that is most relevant to this work. The mathematical model is presented in Chapter 3. In Chapter 4, The model is evaluated based on a specific example and the optimal policies are derived. Sensitivity analysis and managerial insights are presented in Section 5. Finally, the paper is concluded in Chapter 6 and opportunities for future work are discussed.

CHAPTER 2

LITERATURE REVIEW

The recent supply chain literature has given a considerable attention to the dual-channel models. The major focus of the literature has been on pricing decisions and ordering policies. This work greatly relates to this stream of research, specifically the literature that is focused on dual-channel supply chain, channel structure, customized products, and inventory management. Previous research focusing on pricing decisions and ordering policies problems appear in the work of Batrafi et al. (2016). The authors studied a similar dual-channel supply chain structure. Their models have the selling prices, shipment batch sizes and the production quantities as decision variables. They concluded that the adoption of the online channel is highly beneficial to the manufacturer, but it cannibalizes the retailer's profit. They also discussed Yan and Pei's (2009) suggestion to improve the retail service to minimize the effect of the online channel over the retail one. Similarly, Saha et al. (2017) investigated the optimal pricing policies and the characteristics of a two-level dual-channel supply chain under price and delivery time-sensitive demand. They considered the whole products prices and delivery lead-time to be the decision variables. They concluded that when the manufacturer and the retailer are cooperating in taking the decisions, the delivery time is less compared to a non-cooperative scenario. They showed that an inconsistent pricing policy generates more channel profit than the consistent pricing policy (Saha et al., 2017, p. 22).

On the other hand, Mukhopadhyay and Setoputro (2004) investigated pricing and return policies for an e-commerce reverse logistics channel. They considered the price and the refund amount of the return policy to be their decision variables. They reached a closed optimal form for the price and the refund amount stated in terms of customer demand price sensitivity, the sensitivity of demand to the return policy and the return rate for given return policy. The trade-off in their model is between the increases in revenue and those of the costs. Price increases the first and the number of returned items the second (Mukhopadhyay & Setoputro, 2004, p. 17). However, a return policy was not introduced to this paper's model, but it could be a great development to this work. Another study that focused on the return policies is done by Batrafi et al. (2017). They examined the effect of adopting different return policies in a forward and reverse supply chain system, where unsatisfied customers may return their purchases for a refund. They ran their analysis on both a single channel and dual a channel supply chain and compared the profits. Their decision variables are the selling prices of products sold and returned, shipment batch sizes and production quantities. They focused on inventory decisions, return policy and refurbishing costs. The results favored a dual channel over a single one and revealed that the optimal prices are not affected by the adoption of the dual channel. They also showed that the more generous the return policy is, the higher the selling prices are (Batrafi et al., 2017).

More on pricing, Li et al. (2015) analyzed the pricing policies of a competitive dual-channel green supply chain. Their study consists of a standard manufacturer-retailer channel selling multiple non-green products and a direct channel selling only green products. They assumed that the marginal production cost will not be affected by adopting green products manufacturing process, but there are additional investment costs that need to be incurred. They concluded their analysis by stating that there is a threshold for greening cost above which the manufacturer does not open a green direct market (Li et al. 2015). Also, this paper does not address the issue of the environment, but it would also be a great development to this work given the current climate threats we are living in. A study that tackled the issue of the environment is done by Ji et al. (2016). They considered the impact of the supply chain on the environment. They studied a dual channel where the manufacturer is the leader, and the retailer is the follower and aimed at reducing its carbon emission by optimizing pricing policies and emission reduction decisions. The manufacturer can implement low-carbon technologies in his production process, while the retailer can choose to adopt low-carbon promotions. They concluded that the introduction of a low-carbon online market is only beneficial to the manufacturer when the consumers' low-carbon sensitivity is higher than a certain threshold. They also stated that increasing this sensitivity will lead the retailer to favorize the single-channel structure over the dual one. In addition, they showed that if the manufacturer and the retailer adopt low-carbon production process and promotions, the market coverage of low-carbon product will improve (Ji, Zhang, & Yang, 2016).

This work also relates to stream of research that focuses on introducing a direct online market. Chiang et al. (2003) studied the effect of a direct online market on the overall supply chain evaluation, in addition to the influence of the customer product acceptance through the direct or online market. They modelled a system of a retailer and a manufacturer, acting independently, then they further investigated the case of multiple independent retailers who are competing with prices, where the prices are the decision variables. They concluded that not having a direct online channel results in higher retailer prices, lower sales, lower profit, and lower overall channel efficiency. Thus, opening a direct channel, even if no sales happen over it, will force the retailer to decrease its selling prices, which in its turn, will increase demand and increase the total supply chain profit. When they pushed their model into a multiple independent retailers' stage, they realized that a manufacturer opening a direct channel may be misguided, which will force him to decrease his wholesale prices and benefit the retailers through additional profit (Chiang, Chhajed, & Hess, 2003).

Another stream of research that is greatly in correlation with this paper is the one focusing on product customization. Here is a brief review of the literature. Mikkola and Skjott-Larsen (2007) discussed the concepts of mass customization, modularization, and postponement in designing supply chains. Those concepts define different approaches to customization that would render the online channel serving customized products more powerful. Torres (2015) stated that mass customization of production would replace conventional mass production and mentioned three developing trends that contributed to the shift to the customization realm, which are: "Every customer is their own market", "Consumers are more expressive" and "Customization is the new loyalty". He furtherly mentioned the following four approaches to mass customization that would play a vital role in making it happen: Collaborative Customization, Adaptive Customization, Cosmetic Customization and Transparent Customization. He emphasized that the poor execution of customization, complex products and costly operations result in disappointment (Torres, 2015). Minguela-Rata and De Leeuw (2013) focused on a supply chain that supports customers with do-it-yourself repair kits. Their research used a web-based survey. In their model, a customer receives an incentive to repair his/her purchased item at home. They offered the spare parts through three different methods: Direct home deliveries, In-store pickups, and Post office pickups. They concluded that 80.79% of the respondents are willing to perform a "do it yourself repair", with 82.8% of them preferring spare parts direct deliveries, 6.62% opting for in-store pickups and 6.62% choosing post office pickups (Minguela-Rata & De Leeuw, 2013). This is important to the customization since it is possible to offer some do-it-yourself customization possibilities for the customers.

As seen from the literature and to the best of the author's knowledge, this paper is the first to consider separate demand functions for each customized product sold online. Also, it is the first to model and optimize the inventory of the basic product, each customized product as well as the inventory of the additional parts needed in the customization process.

CHAPTER 3

MODEL FRAMEWORK

In this chapter, the steps needed for constructing the model are discussed.

3.1. Description

The model considers a dual-channel supply chain consisting of a manufacturer and a retailer. The manufacturer and the retailer form an entity in the primary channel, which sells the standard product with the retailer being the sale point for the manufacturer. In the secondary channel, a customized product is sold directly to the customers through e-commerce. The manufacturer's production system allows him to mass-produce a standard product and to produce customized products as per customers' requirements. The customized products consist of a single standard product with multiple customization accessories installed on it.

The first part of this paper, which is presented in Section 4.1, intends to maximize the total profit of the whole dual-channel supply chain by optimizing the order quantities for the standard product, customizing accessories, customized products as well as the prices under constant demand. The second part, which is presented in Section 4.2, aims at maximizing the total profit by optimizing the selling prices of the standard product and the customized products. The demand, in this case, is modeled as a linear function of price & lead-time, and the ordering policy is a given constant. Finally, in Section 4.3, the total profit is maximized by optimizing over all the decision variables mentioned previously, where the demand is also a linear function of price & lead-time.

The manufacturer's production system can mass-produce a Basic Product (BP). The manufacturer can customize the BP and create customized products. Customized products, also referred to as features, are achieved by appending/attaching accessories, also referred to as spare parts (SP), to the BP. Let f be the number of customized products (features) that can be produced. The number of spare parts (used in features) available is m. Let $\mathbf{F}$ be an $(f \times n)$ matrix where $\mathbf{F}(i, j)$ denotes the number of spare parts of type i that are required to produce customized product j, for $i = 1, \ldots n$ and $j = 1, \ldots f$. The model assumes that the production of the BP and the customized products is happening over continuous and identically repetitive cycles.

3.2. Demand Functions

There are two options available to the customers. They can either buy a standard product from the traditional retail channel, or they can buy a customized product online. The customized products are the set of f features available to the customers. Each feature is composed of an assembly of a standard basic product and a certain combination of spare parts. This leads us to the two dimensional $f \times n$ array shown below. In this array, we see the list of products that can be chosen by customers labeled as features denoted by F_i in addition to the standard basic product. Each row dictates the quantity needed of each spare part denoted by $\mathbf{F}(i, j)$, along with the basic product, to assemble each feature. As a result, the demand for the spare parts can be determined from the demand for each feature and the spare part composition of it. This means that the demand of the spare parts is the product of the demand for the features and the number of spare parts used in each feature.

$D_{SP}(i) = \sum_{j=1}^{f} F_{ij} \times D_F(j)$.

Where:

- $D_{SP}(i)$ is the demand for spare part i
- F_{ij} is the quantity of spare part i used in feature F_j
- $D_F(j)$ is demand for feature F_j

Suppose that there exists a feature F_1 that needs 2 items of spare part 1, $F_{11} = 2$. Also, the demand for F_1 is 3 per year. Then, the demand for SP_1 is:

$$D_{SP}(1) = F_{11} \times D_F(1) = 2 \times 3 = 6 \; per \; year$$

Products Composition Array

$$\begin{bmatrix} \text{Product} & SP_1 & SP_2 & \dots & SPm & BP \\ Basic & 0 & 0 & \dots & 0 & 1 \\ Feature\ 1 & 1 & 0 & \dots & 0 & 1 \\ Feature\ 2 & 0 & 1 & \dots & 0 & 1 \\ Feature\ i & A & B & \dots & C & 1 \\ Feature\ f & X & Y & \dots & Z & 1 \end{bmatrix}$$

The demands for the basic product and the features are denoted as follows:

Products Demand Array

$$\begin{bmatrix} Product & \text{Demand} \\ Basic & Dretail \\ Feature\ 1 & D_F(1) \\ Feature\ 2 & D_F(2) \\ Featture\ i & D_F(i) \end{bmatrix}$$

Finally, after getting inspired by Batrafi et al. (2016)'s demand functions, the retailer's and each feature's demand functions are deterministically defined as linear functions of the selling prices of the products among other factors as follows:

$D_r = (1-\theta)a - \alpha_r \, P_{Bp} + \sum_{x=1}^{f} \beta_r \, l_F(x)$ (Demand for Basic Product Through Direct Channel)

$D_F(1) = \theta_F(1)a - \alpha_F(1)P_F(1) - \beta_F(1)l_F(1)$ (Demand for Feature 1 Through Online Channel)

$D_F(2) = \theta_F(2)a - \alpha_F(2)P_F(2) - \beta_F(2)l_F(2)$ (Demand for Feature 2 Through Online Channel)

$D_F(i) = \theta_F(i)a - \alpha_F(i)P_F(i) - \beta_F(i)l_F(i)$ (Demand for Feature x Through Online Channel)

$D_F(f) = \theta_F(f)a - \alpha_F(f)P_F(f) - \beta_F(f)l_F(f)$ (Demand for Feature f Through Online Channel)

Where:

- P_{BP} is the selling price of the standard product
- $P_F(i)$ is the selling price of the feature F_i
- a is the primary demand, potential demand when the product is free of charge
- $\theta = \sum_{i=1}^{f} \theta_F(i)$ and $1 - \theta$ represent the percentages of the demand going to the online channel and retail channel respectively
- α_r is the standard product demand's price sensitivity
- $\alpha_F(i)$ is the feature $F_i$demand's price sensitivity
- β's are the elasticity of customer demand with respect to the features' manufacturing and delivery lead-time

In these equations, D_r is the demand for the basic product through the retail channel, a is the market potential meaning the amount expected to be demanded if the products are offered for free. $(1 - \theta)$ is the direct channel fraction of the market if the prices are zero. This demand function decreases linearly when the basic product's price P_{BP} increases in a fraction of α_r . In addition, β_r is the factor by which the demand for the standard basic product increases when the lead-time for the features increases.

Whereas $D_F(1), D_F(2), \dots, D_F(f)$ are the features $F_1, F_2, \dots, F_f$ demand through the online channel. $\theta_F(1), \theta_F(2), \dots, \theta_F(f)$ are each feature's fraction of the market if the products are offered for free. Also, these demand functions are inversely linearly proportional with the respective price of each of the features with a fraction $\alpha_F(1), \alpha_F(2), \dots, \alpha_F(f)$ respectively. Finally, β_F(i) is the factor by which the demand for feature F_i decreases if its lead-time $L_F(i)$ increases.

To avoid the trivial case, we also assume that the selling prices are larger than their costs, i.e., $\boldsymbol{P(i) > C(i)}$ for all features $\boldsymbol{i}$.

3.2.1. Example

As an example, consider a manufacturer offering a laptop as a basic product that can be upgraded by adding either a high-performance graphics card (VGA), or a high-performance hard disk (SSD), or both. This model is as follows:

- One Basic Product
- Two Spare Parts
- Three Features

The products compositions are as follows:

Products Composition Array

$$\begin{bmatrix} \text{Product} & SP_1\ (VGA) & SP_2\ (SSD) & BP\ (Laptop) \\ Basic & 0 & 0 & 1 \\ Feature\ 1 & 1 & 0 & 1 \\ Feature\ 2 & 0 & 1 & 1 \\ Feature\ 3 & 1 & 1 & 1 \end{bmatrix}$$

The demands for the products are as follows:

Products Demand Array

$$\begin{bmatrix} Product & \text{Demand} \\ Basic & Dretail \\ Feature\ 1 & D_F(1) \\ Feature\ 2 & D_F(2) \\ Featture\ 3 & D_F(3) \end{bmatrix}$$

The demand for spare parts is determined according to the features' demand, since a single spare part 1 (SP_1) is used in feature 1 and in feature 3, the demand for spare part 1 can be written as follows:

$$D_{SP}(1) = D_F(1) + D_F(3)$$

Similarly, we can write the demand for spare part 2 (SP_2):

$$D_{SP}(2) = D_F(2) + D_F(3)$$

Moreover, the demand for basic product through both channels can be determined by:

$$D_{BP} = D_{Retail} + D_F(1) + D_F(2) + D_F(3)$$

3.3. Inventory Model of The Standard Channel

Consider a manufacturer that is mass producing a standard product and selling it to customers over its retail sales point. As mentioned earlier, the manufacturer and the retailer are considered to be a single entity. Thus, there are no costs incurred by transferring the produced products from the manufacturer's warehouse to the retail store. In addition, this channel has the following parameters:

- Order setup cost: K_{BP} $\$/setup$
- Holding cost: h_{BP} $\$/unit/year$
- Basic product cost: C_{BP} $\$/unit$
- Selling price: P_{BP} $\$/unit$ (decision variable)
- Production Quantity: Q_{BP} $unit/order$ (decision variable)
- Manufacturer Production Rate: R_m $unit/year$
- Standard Channel Demand: D_r $unit/year$

It is important to note that it is previously stated that the demand is considered either to be constant or a linear function. Also, the model is composed of a constant production rate, order setup cost, and holding cost. Consequently, the standard retail channel inventory system follows the economic production quantity model (EPQ) as shown in Figure 1.

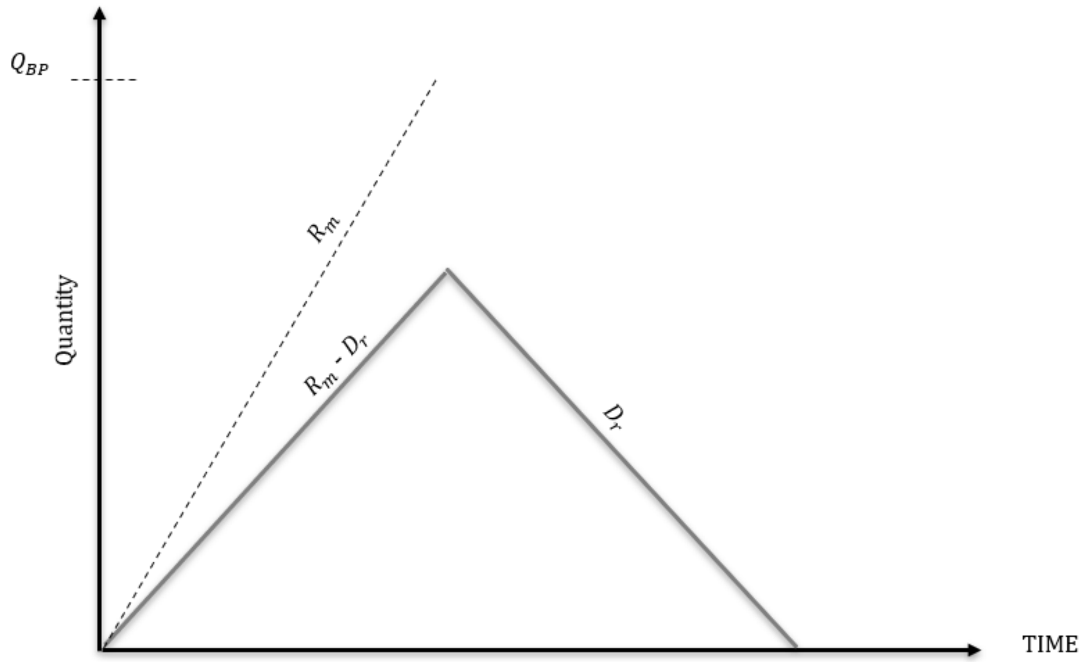

Figure 1: Standard Channel Inventory Model

3.4. Inventory Model of The Online Channel

Consider a manufacturer that is selling customers customized products through an online channel or e-commerce. The customization happens by adding spare parts to the standard product. This means, the customer can choose from a set of predefined features where spare parts are added to the basic product to achieve them. Consequently, the online channel inventory model is divided into three parts: The spare parts inventory model, the basic product used in the feature's creation inventory model and the assembled features inventory model.

3.4.1. Spare Parts Inventory Model

The first part is concerned with the inventory of the spare parts, which follows the economic order quantity model (EOQ) under the following parameters for each spare part SP_i as shown in Figure 2:

- Setup cost for the i^{th} SP: $\boldsymbol{K}_{SP}(i)\ \$/Order$.
- Holding cost: $\boldsymbol{h}_{SP}(i)\ \$/unit/year$
- Cost: $\boldsymbol{C}_{SP}(i)\ \$/unit$
- Order quantity: $\boldsymbol{Q}_{SP}(i)\ unit/order$ (decision variable)
- Demand for the i^{th} SP: $\boldsymbol{D}_{SP}(i)\ unit/year$

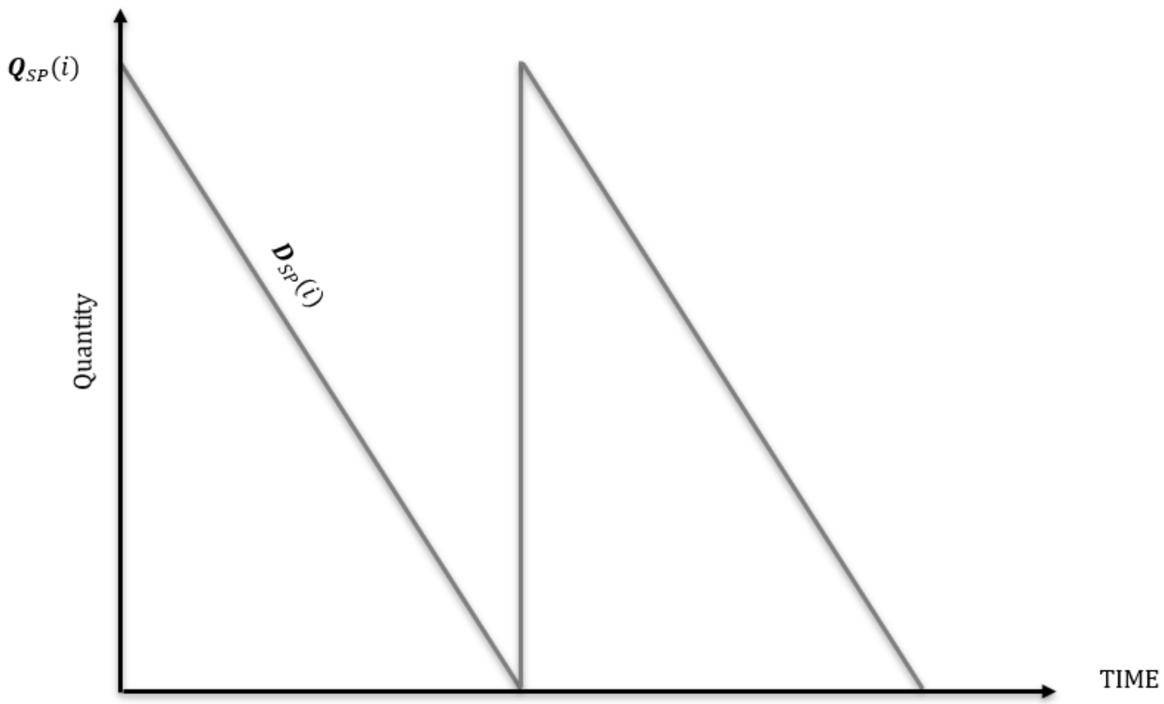

Figure 2: Spare Parts Inventory Model

3.4.2. Basic Product Used in The Customization Process Inventory Model

The second part is concerned with the inventory of the basic products used in building the features, which follows the EPQ as well under the following parameters as shown in Figure 3:

- Order set up cost: K_{BP} $\$/setup$
- Holding cost: h_{BP} $\$/unit/year$
- Cost: C_{BP} $\$/unit$
- Optimal order quantity: Q_{BP} $unit/order$ (Decision Variable)
- Manufacturer Production Rate: R_m $unit/year$
- Feature i Demand: $D_F(i)$ $unit/year$

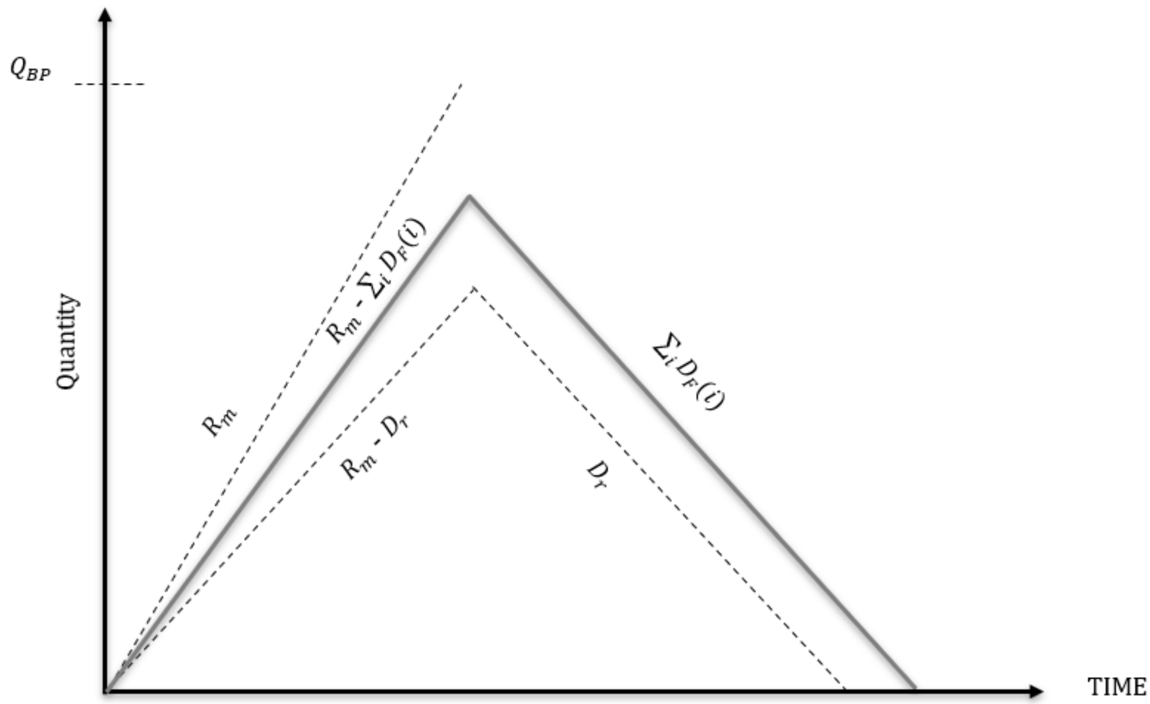

Figure 3: Basic Products Used in Features Inventory Mode along with the Standard Channel Inventory Model

3.4.3. Basic Product Used in The Customization Process Inventory Model

The third part is concerned with the finished and assembled customized products called features, their rate of assembly, cost of assembly, and inventory model. This part follows the EPQ inventory model under the following parameters as shown in *Figure 4:*

- Assembly set up cost: $\boldsymbol{K_F}(i)$ $\$/setup$
- Finished features holding cost: $\boldsymbol{h_F}(i)$ $\$/unit/year$
- Assembly cost: $\boldsymbol{C_F}(i)$ $\$/unit$
- Selling price: $\boldsymbol{P_F}(i)$ $\$/unit$ (Decision Variable)
- Features assembly rate: $\boldsymbol{R_F}(i)$ $unit/year$
- Optimal feature quantity: $\boldsymbol{Q_F}(i)$ $unit/order$ (Decision Variable)
- Feature i Demand: $\boldsymbol{D_F}(i)$ $unit/year$

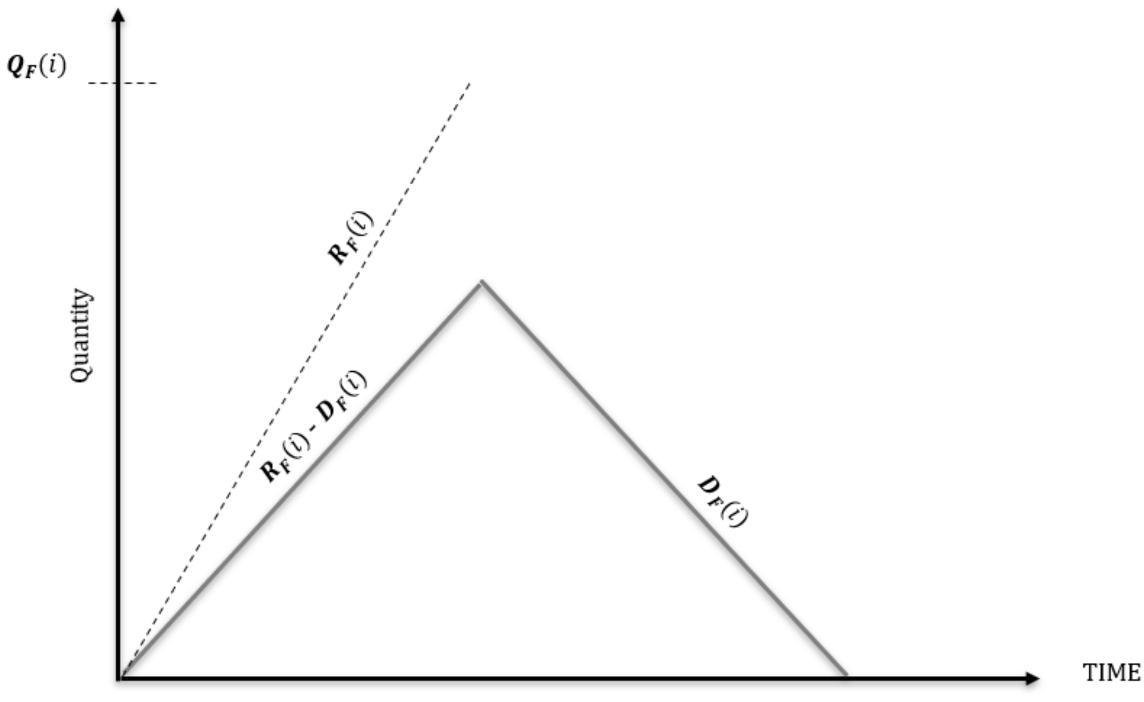

Figure 4: Features Inventory Model

3.5. Profit Functions

In this section, the profit functions for the standard channel as well as the online channel are defined.

3.5.1. The Standard Channel Profit Function

As mentioned previously, in this model the manufacturer and the retailer are considered as a single entity and the standard channel's inventory model follows a typical EPQ inventory model. Thus, the profit function for the standard channel is the typical EPQ profit function as follows:

$$\Pi_{M,S} = P_{BP}\,D_r - \left[\frac{K_{BP}\,D_r}{Q_{BP}} + \left(\frac{h_{BP}\,Q_{BP}}{2}\right)\left(1 - \frac{D_r}{R_m}\right) + C_{BP}\,D_r\right]$$

3.5.2. The Online Channel Profit Function

As mentioned previously, in this channel the manufacturer is directly selling customized products to customers through e-commerce. In addition, there are three parts that constitute the online channel: the spare parts, the basic products used in the customization and the finalized features. Thus, the profit function for this channel has a multiple stream of costs, but a single stream as revenue as follows:

- $Revenue \;=\; \sum_{i=1}^{f} P_F(i)\,D_F(i)$
- $Cost\ of\ Spare\ Parts\ (EOQ) = \sum_{i=1}^{m}\left[\frac{K_{SP}(i)\,D_{SP}(i)}{Q_{SP}(i)} + \frac{h_{SP}(i)\,Q_{SP}(i)}{2} + C_{SP}(i)D_{SP}(i)\right]$
- $Cost\ of\ Basic\ Product\ (EPQ) = \frac{K_{BP}\,\sum_{i=1}^{f} D_F(i)}{Q_{BP}} + \left(\frac{h_{BP}\,Q_{BP}}{2}\right)\left(1 - \frac{\sum_{i=1}^{f} D_F(i)}{R_m}\right) + C_{BP}\sum_{i=1}^{f} D_F(i)$
- *Cost of Features Assembly (EPQ)* $= \sum_{i=1}^{f}\left[\frac{K_F(i)\,D_F(i)}{Q_F(i)} + \left(\frac{h_F(i)Q_F(i)}{2}\right)\left(1 - \frac{D_F(i)}{R_F(i)}\right) + C_F(i)D_F(i)\right]$
- *Online Channel Profit Function:* $\Pi_{M,C} = Total\ Revenue - Total\ Cost$

3.5.3. The Total Supply Chain Profit Function

The total supply chain profit function is the summation of the standard channel and the online channel profit functions as follows:

$$\Pi_M = \mathit{\Pi}_{M,S} + \mathit{\Pi}_{M,C}$$

CHAPTER 4

MODEL EVALUATION

In this chapter, the model is evaluated by finding the optimal strategies and solutions for the following scenarios. First, the optimal ordering policy is found when the demand is constant. Second, the optimal pricing policy is found when the ordering policy is predefined and constant. Finally, the optimal ordering and pricing policies are found when they are both variables. In the evaluation, the previously mentioned example is considered as the basis. The example is as follows:

- One Basic Product (Laptop)
- Two Spare Parts (High-Performance VGA & High-Performance Hard-Disk)
- Three Features

The products compositions are as follows:

Products Composition Array

$$\begin{bmatrix} \text{Product} & SP_1\ (VGA) & SP_2\ (SSD) & BP\ (Laptop) \\ Basic & 0 & 0 & 1 \\ Feature\ 1 & 1 & 0 & 1 \\ Feature\ 2 & 0 & 1 & 1 \\ Feature\ 3 & 1 & 1 & 1 \end{bmatrix}$$

The demands for the products are as follows:

Products Demand Array

$$\begin{bmatrix} Product & \text{Demand} \\ Basic & Dretail \\ Feature\ 1 & D_F(1) \\ Feature\ 2 & D_F(2) \\ Featture\ 3 & D_F(3) \end{bmatrix}$$

4.1. Optimal Ordering Policy Under Constant Demand

This section is aimed at finding the optimal ordering policy under constant demand. Thus, the decision variables are: $Q_{BP}, Q_F(1), Q_F(2), Q_F(3), Q_{SP}(1), Q_{SP}(2)$. The first step in the evaluation, we need to check if there exists an optimal solution by proving that the total supply chain profit function is jointly concave in the decision variables mentioned above. To check for joint concavity, the Hessian Matrix must be constructed and proven to be negative semi-definite. The Hessian Matrix is an $n \times n$ matrix where n is equal to the number of decision variables. For each column and each row, we will assign one of our decision variables in the same order of assignment. The matrix components are nothing but the differentiation of the total profit objective function with respect to the row decision variable first, and with respect to the column decision variable second. This means the first row will contain the following:

$\frac{\partial^2 \prod Total}{\partial^2 Q_{BP}}, \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F1}}, \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F2}}, \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F3}}, \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{SP1}}, \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{SP2}}.$

The hessian matrix is as follows:

$$\begin{bmatrix}
\frac{\partial^2 \prod Total}{\partial^2 Q_{BP}} & \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F1}} & \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F2}} & \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{F3}} & \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{SP1}} & \frac{\partial \prod Total}{\partial Q_{BP}\,\partial Q_{SP2}} \\
\frac{\partial \prod Total}{\partial Q_{F1}\,\partial Q_{BP}} & \frac{\partial^2 \prod Total}{\partial^2 Q_{F1}} & \frac{\partial \prod Total}{\partial Q_{F1}\,\partial Q_{F2}} & \frac{\partial \prod Total}{\partial Q_{F1}\,\partial Q_{F3}} & \frac{\partial \prod Total}{\partial Q_{F1}\,\partial Q_{SP1}} & \frac{\partial \prod Total}{\partial Q_{F1}\,\partial Q_{SP2}} \\
\frac{\partial \prod Total}{\partial Q_{F2}\,\partial Q_{BP}} & \frac{\partial \prod Total}{\partial Q_{F2}\,\partial Q_{F1}} & \frac{\partial^2 \prod Total}{\partial^2 Q_{F2}} & \frac{\partial \prod Total}{\partial Q_{F2}\,\partial Q_{F3}} & \frac{\partial \prod Total}{\partial Q_{F2}\,\partial Q_{SP1}} & \frac{\partial \prod Total}{\partial Q_{F2}\,\partial Q_{SP2}} \\
\frac{\partial \prod Total}{\partial Q_{F3}\,\partial Q_{BP}} & \frac{\partial \prod Total}{\partial Q_{F3}\,\partial Q_{F1}} & \frac{\partial \prod Total}{\partial Q_{F3}\,\partial Q_{F2}} & \frac{\partial^2 \prod Total}{\partial^2 Q_{F3}} & \frac{\partial \prod Total}{\partial Q_{F3}\,\partial Q_{SP1}} & \frac{\partial \prod Total}{\partial Q_{F3}\,\partial Q_{SP2}} \\
\frac{\partial \prod Total}{\partial Q_{SP1}\,\partial Q_{BP}} & \frac{\partial \prod Total}{\partial Q_{SP1}\,\partial Q_{F1}} & \frac{\partial \prod Total}{\partial Q_{SP1}\,\partial Q_{F2}} & \frac{\partial \prod Total}{\partial Q_{SP1}\,\partial Q_{F3}} & \frac{\partial^2 \prod Total}{\partial^2 Q_{SP1}} & \frac{\partial \prod Total}{\partial Q_{SP1}\,\partial Q_{SP2}} \\
\frac{\partial \prod Total}{\partial Q_{SP2}\,\partial Q_{BP}} & \frac{\partial \prod Total}{\partial Q_{SP2}\,\partial Q_{F1}} & \frac{\partial \prod Total}{\partial Q_{SP2}\,\partial Q_{F2}} & \frac{\partial \prod Total}{\partial Q_{SP2}\,\partial Q_{F3}} & \frac{\partial \prod Total}{\partial Q_{SP2}\,\partial Q_{SP1}} & \frac{\partial^2 \prod Total}{\partial^2 Q_{SP2}}
\end{bmatrix}$$

Theorem 1: The total profit function is concave in $Q_{BP}, Q_F(1), Q_F(2), Q_F(3), Q_{SP}(1), Q_{SP}(2)$.

Proof: In order to check if the total profit function is concave, the hessian matrix must be multiplied by a vector $Z = [Z_1 , Z_2 , Z_3, Z_4, Z_5, Z_6]$ and its transpose Z^t. If the result is negative, then the hessian matrix is negative semi-definite, and the total supply chain profit function is jointly concave in the decision variables. If, the hessian matrix positive semi-definite, and the total supply chain profit function in jointly convex in the decision variables.

$$ZHZ^t = {Z_1}^2\left(\frac{-2K_{BP}\, D_r}{Q^3{}_{BP}} - \frac{2K_{BP}\sum_{x=1}^{f} D_{Fx}}{Q^3{}_{BP}}\right) + {Z_2}^2\left(\frac{-2K_{SP}(1)\, D_{SP}(1)}{Q^3{}_{SP}(1)}\right) + {Z_3}^2\left(\frac{-2K_{SP}(2)\, D_{SP}(2)}{Q^3{}_{SP}(2)}\right)$$
$$+ {Z_4}^2\left(\frac{-2K_F(1)\, D_F(1)}{Q^3{}_F(1)}\right) + {Z_5}^2\left(\frac{-2K_F(2) D_F(2)}{Q^3{}_F(2)}\right) + {Z_6}^2\left(\frac{-2K_F(3)\, D_F(3)}{Q^3{}_F(3)}\right) \le 0$$

This proves that the total supply chain's profit function is jointly concave in $Q_{BP}, Q_F(1), Q_F(2), Q_F(3), Q_{SP}(1), Q_{SP}(2)$. Thus, there exists an optimal ordering policy that maximizes the total profit. There is also a pattern that concludes concavity regardless of the number of spare parts and features present in the model.

The general mathematical proof is as follows:

$$ZHZ^t = {Z_1}^2\left(\frac{-2K_{BP}\, D_r}{Q^3{}_{BP}} - \frac{2K_{BP}\sum_{x=1}^{f} D_{Fx}}{Q^3{}_{BP}}\right) + \sum_{i=1}^{m} {Z_{i+1}}^2\left(\frac{-2K_{SP}(i)\, D_{SP}(i)}{Q^3{}_{SP}(i)}\right) + \sum_{j=1}^{n} {Z_{j+m+1}}^2\left(\frac{-2K_F(j)\, D_F(j)}{Q^3{}_F(j)}\right) \le 0$$

The optimal ordering policy formulas are as follows:

$$Q_{bp} = \sqrt{\frac{2\,\left(-K_{BP}\ \times R_m\,(\sum_{i=1}^{3} D_F(i) + D_r)\right)}{h_{BP}(\,\sum_{i=1}^{3} D_F(i) + D_r\ - 2\,R_m)}}$$

$$Q_{SP}(i) = \sqrt{\frac{2\,D_{SP}(i)\,K_{SP}(i)}{h_{SP}(i)}}$$

$$Q_F(i) = \sqrt{\frac{-\,2\,D_F(i)\,K_F(i)\,R_F(i)}{h_F(i)\,(D_F(i) - R_F(i))}}$$

4.2. Optimal Pricing Policy Under Constant Ordering Policy

This section considers the previous example where demand is a linear function of price and lead-time and the ordering policy is fixed. Recalling the demand functions:

$$D_r = (1-\theta)a - \alpha_r \, P_{Bp} + \sum_{i=1}^{f} \beta_r \, l_F(i)$$

$$D_F(f) = \theta_F(f)a - \alpha_F(f)P_F(f) - \beta_F(f)l_F(f)$$

The same profit functions of the channels mentioned previously apply here but, the demand is no longer considered a constant. The demand functions of the spare parts as previously mentioned are as follows:

$$D_{BP} = Dr \; + \; D_F(1) + D_F(2) + D_F(3)$$

$$D_{SP}(1) = D_F(1) + D_F(3)$$

$$D_{SP}(2) = D_F(2) + D_F(3)$$

To check for concavity, the theorem previously used in section 4.1 applies here, but with the following decision variables: P_{BP}, $P_F(1)$, $P_F(2)$,and $P_F(3)$.

Theorem 2: The total profit function is concave in P_{BP}, $P_F(1)$, $P_F(2)$,and $P_F(3)$.

Proof: To check if the total profit function is concave, hessian matrix must be multiplied by a vector $Z = \left[Z_1\,, Z_2\,, Z_3, Z_4\right]$ and its transpose Z^t. If the result is negative, the function is jointly concave.

The hessian matrix is as follows:

$$\begin{bmatrix} \frac{\partial^2 \prod Total}{\partial^2 P_{BP}} & \frac{\partial \prod Total}{\partial P_{BP}\, \partial P_{F1}} & \frac{\partial \prod Total}{\partial P_{BP}\, \partial P_{F2}} & \frac{\partial \prod Total}{\partial P_{BP}\, \partial P_{F3}} \\ \frac{\partial \prod Total}{\partial P_{F1}\, \partial P_{BP}} & \frac{\partial^2 \prod Total}{\partial^2 P_{F1}} & \frac{\partial \prod Total}{\partial P_{F1}\, \partial P_{F2}} & \frac{\partial \prod Total}{\partial P_{F1}\, \partial P_{F3}} \\ \frac{\partial \prod Total}{\partial P_{F2}\, \partial P_{BP}} & \frac{\partial \prod Total}{\partial P_{F2}\, \partial P_{F1}} & \frac{\partial^2 \prod Total}{\partial^2 P_{F2}} & \frac{\partial \prod Total}{\partial P_{F2}\, \partial P_{F3}} \\ \frac{\partial \prod Total}{\partial P_{F3}\, \partial Q_{BP}} & \frac{\partial \prod Total}{\partial P_{F3}\, \partial P_{F1}} & \frac{\partial \prod Total}{\partial P_{F3}\, \partial Q_{F2}} & \frac{\partial^2 \prod Total}{\partial^2 P_{F3}} \end{bmatrix}$$

Multiplying the hessian matrix by the vector Z and its transpose Z^t results in the following:

$$ZH\mathrm{Z}^t = -2\left[{Z_1}^2\alpha_r + {Z_2}^2\alpha_F(1) + {Z_3}^2\alpha_F(2) + {Z_4}^2\alpha_F(3)\right] < 0$$

This proves that the total supply chain's profit function is jointly concave in P_{BP}, $P_F(1)$, $P_F(2)$,and $P_F(3)$. Thus, there exists an optimal pricing policy to maximize the total profit. There is also a pattern that concludes concavity regardless of the number of spare parts and features present in the model.

The general mathematical proof is as follows:

$$ZH\mathrm{Z}^t = -2\left[{Z_1}^2\alpha_r + \sum_{j=1}^{n} {Z_{j+1}}^2\alpha_F(\mathrm{j})\right] < 0$$

The optimal pricing policy formulas are as follows:

$$P_{bp} = \frac{C_{BP}}{2} + \frac{K_{BP}}{2\,Q_{BP}} - \frac{Q_{BP}\,h_{BP}}{4\,R_m} - \frac{a(\theta+1)}{2\,\alpha_r} + \frac{\beta_r\,(L_F(1) + L_F(2) + \ L_F(3))}{2\,\alpha_r}$$

$$P_F(1) = \frac{C_F(1) + C_{SP}(1) + C_{BP}}{2} + \frac{-\beta_F(1)L_F(1) + \theta_F(1)a}{2\,\alpha_F(1)} + \frac{K_F(1)}{2\,Q_F(1)} + \frac{K_{SP}(1)}{2\,Q_{SP}(1)} + \frac{K_{BP}}{2Q_{BP}} - \frac{Q_F(1)\,h_F(1)}{4\,R_F(1)} - \frac{Q_{BP}\,h_{BP}}{4\,R_m}$$

$$P_F(2) = \frac{C_F(2) + C_{SP}(2) + C_{BP}}{2} + \frac{-\beta_F(2)L_F(2) + \theta_F(2)a}{2\,\alpha_F(2)} + \frac{K_F(2)}{2\,Q_F(2)} + \frac{K_{SP}(2)}{2\,Q_{SP}(2)} + \frac{K_{BP}}{2Q_{BP}} - \frac{Q_F(2)\,h_F(2)}{4\,R_F(2)} - \frac{Q_{BP}\,h_{BP}}{4\,R_m}$$

$$P_F(3) = \frac{C_F(3) + C_{SP}(1) + C_{SP}(2) + C_{BP}}{2} + \frac{-\beta_F(2)L_F(3) + \theta_F(3)a}{2\,\alpha_F(3)} + \frac{K_F(3)}{2\,Q_F(3)} + \frac{K_{SP}(1)}{2\,Q_{SP}(1)} + \frac{K_{SP}(2)}{2\,Q_{SP}(2)} + \frac{K_{BP}}{2Q_{BP}} - \frac{Q_F(3)\,h_F(3)}{4\,R_F(3)} - \frac{Q_{BP}\,h_{BP}}{4\,R_m}$$

To generalize, regardless of the number of features and spare parts used, the optimal pricing policy formulas are the following:

$$P_{bp} = \frac{C_{BP}}{2} + \frac{K_{BP}}{2\,Q_{BP}} - \frac{Q_{BP}\,h_{BP}}{4\,R_m} - \frac{\theta\,a}{2\,\alpha_r} + \frac{a}{2\,\alpha_r} + \frac{\sum_{i=1}^{f} \beta_r\,L_F(i)}{2\,\alpha_r}$$

$$P_F(i) = \frac{C_F(i) + C_{BP} + \sum_{n(i)} C_{n(i)}}{2} + \frac{-\beta_F(i) L_F(i) + \theta_F(i) a}{2\,\alpha_F(i)} + \frac{K_F(i)}{2\,Q_F(i)} + \sum_{n(i)} \frac{K_{n(i)}}{2\,Q_{n(i)}} + \frac{K_{BP}}{2Q_{BP}} - \frac{Q_F(i)\,h_F(i)}{4\,R_F(i)} - \frac{Q_{BP}\,h_{BP}}{4\,R_m}$$

Where $n(i)$ is the list of spare parts used in feature F_i

4.3. Optimal Ordering & Pricing Polices

This section tackles the model by considering the prices and ordering quantities as decision variables. The model in this section is similar to the previous one, with the decision variables being the only difference. It is hard to prove the concavity of the profit function analytically. Thus, a numerical approach is adopted.

To evaluate the model numerically, the model parameters are defined in Table 1 after getting inspired from Batrafi et al. (2016)'s model parameters.

Model Parameter	*Value*	*Unit*	*Model Parameter*	*Value*	*Unit*
R_m	*18000*	*unit/cycle*	$h_F(2)$	*30*	*\$/unit/cycle*
$R_F(1)$	*18000*	*unit/cycle*	$h_F(3)$	*30*	*\$/unit/cycle*
$R_F(2)$	*18000*	*unit/cycle*	K_{BP}	1000	*\$/setup*
$R_F(3)$	*18000*	*unit/cycle*	$K_{SP}(1)$	300	*\$/setup*
a	*30000*	*unit/cycle*	$K_{SP}(2)$	300	*\$/setup*
α_r	*20*	$unit^2$*/cycle*	$K_F(1)$	800	*\$/setup*
$\alpha_F(1)$	*2*	$unit^2$*/cycle*	$K_F(2)$	800	*\$/setup*
$\alpha_F(2)$	*2*	$unit^2$*/cycle*	$K_F(3)$	800	*\$/setup*
$\alpha_F(3)$	*2*	$unit^2$*/cycle*	$\theta_F(1)$	10	*%*
C_{BP}	*150*	*\$/unit*	$\theta_F(2)$	10	*%*
$C_{SP}(1)$	*100*	*\$/unit*	$\theta_F(3)$	10	*%*
$C_{SP}(2)$	*75*	*\$/unit*	β_r	40	*customer/cycle*
$C_F(1)$	*25*	*\$/unit*	$\beta_F(1)$	50	*customer/cycle*
$C_F(2)$	*15*	*\$/unit*	$\beta_F(2)$	50	*customer/cycle*
$C_F(3)$	*40*	*\$/unit*	$\beta_F(3)$	60	*customer/cycle*
h_{BP}	*30*	*\$/unit/cycle*	$l_F(1)$	6	*Day*
$h_{SP}(1)$	*30*	*\$/unit/cycle*	$l_F(2)$	6	*Day*
$h_{SP}(2)$	*30*	*\$/unit/cycle*	$l_F(3)$	10	*Day*
$h_F(1)$	*30*	*\$/unit/cycle*			

Table 1: Model's Defined Input Parameter

After having the input parameters set, it is time to find the local optimal solutions that maximizes the total profit function in terms of the following decision variables: $Q_{BP}, Q_F(1), Q_F(2), Q_F(3), Q_{SP}(1), Q_{SP}(2), P_{BP}$, $P_F(1)$, $P_F(2)$ and $P_F(3)$.

To do so, two different numerical methodologies are adopted, and the results are compared. The two approaches are: Iterative Analysis & Built-in Local Minimum Function.

4.3.1. Iterative Analysis

The work in this paper has divided the model into two parts. First, the demand is considered a constant, and the optimization happened over the ordering policy leading to optimal quantity formulas. Then, the ordering police is considered a constant, and the optimization happened over the pricing policy leading to optimal prices formulas. This section uses an algorithm that is constructed using the previous results to find a local optimal solution. The algorithm takes random values for the prices as initial input values and uses them to compute values for the quantities using the optimal quantities formulas derived previously in section 4.1. Then, the values found for the quantities are inserted in the optimal prices formulas also derived previously in section 4.2 to find new values for the prices. These new prices values are inserted back in the optimal quantities formulas to find new values for the quantities. This process is repeated until all the quantities and prices values converge to a number between an iteration and the preceding one. After the convergence happens, these values are considered to be the local optimal ones achieving the highest profit possible. The local optimal values are presented in Table 2:

Decision Variable	*Value*	*Unit*
Q_{BP}	*796*	*Piece*
$Q_F(1)$	*247*	*Piece*
$Q_F(2)$	*251*	*Piece*
$Q_F(3)$	*216*	*Piece*
$Q_{SP}(1)$	*195*	*Piece*
$Q_{SP}(2)$	*197*	*Piece*
P_{BP}	*622*	*$*
$P_F(1)$	*815*	*$*
$P_F(2)$	*797*	*$*
$P_F(3)$	*786*	*$*
$Total\ Profit$	*5,934,941*	*$*

Table 2: Local Optimal Solution

4.3.2. Built-in Local Minimum Function

In this section, the built-in local minimum function in Python is used to find the local optimal solution. After feeding all the equations, parameters, objective function, and constraints to the program and running it, the results are found to be exactly equal to the ones found in section 4.3.1.

CHAPTER 5

SENSITIVITY ANALYSIS & MANAGERIAL INSIGHTS

This section discusses the sensitivity analysis study that has been performed over various model's key input parameters to understand the behavior of the supply chain model presented in this paper under different scenarios and draw some managerial insights from the results. Since multiple features exist in the online channel, the parameters of features F_3 are considered as representation for the online channel parameters. This means feature F_3's effect and behavior are considered as representatives for the online channel behavior, and they are used as a basis to draw the managerial insights for the online channel.

The sensitivity analysis study is conducted on the model presented in section 4.3 and its solution, and the studied input parameters are: the basic product manufacturing rate R_m, feature F_3 assembly rate $R_F(3)$, online channel customer acceptance $\theta_F(3)$ and feature F_3 lead-time $L_F(3)$.

Before proceeding with the sensitivity analysis an interesting realization must be discussed regarding the problem's initial solution. The feature F_3 consists of the basic product and two spare parts added to it, SP_1 & SP_2, and it is optimally priced at $P_F(3) = \$786$. Whereas the feature F_1 consists of the basic product and one spare part added to it, SP_1, and it is optimally priced at $P_F(1) = \$815 > P_F(3) = \$786$, which is also true for feature F_2 & $P_F(2)$. At first, this sounds counterintuitive since F_3 is made of the same basic product and a spare part that is used in making F_1 & F_2, but with one more additional spare part, thus it should have a higher price. However, the high 10-day lead-time of feature F_3 forces the manufacturer to price it at a lower price to attract customers. This decision might not be sound from a marketing perspective, and the manufacturer might still price it at a higher price than the other two features settling for a near optimal solution. In addition, if an optimization over the assortments is to be made, feature F_3 will be out of assortment.

5.1. Effect of Varying the Standard & Online Channels' Production Rates

The production rates play a crucial role in the supply chain and greatly affect its behavior. This section investigates the effect of the basic product production rate R_m, and feature F_3 assembly rate $R_F(3)$ on the supply chain.

5.1.1. Effect of Varying the Basic Product's Production Rate

The results show that as the manufacturer' standard product production rate increases from 6,500 unit/year to 20,500 unit/year, the supply-chain's total profit decreases from $5,958,549 to $5,933,952, the selling prices for all products slightly increase by $5 and the basic product quantity decreases from 3520 to 772 units as shown in Figure 5 and Figure 6.

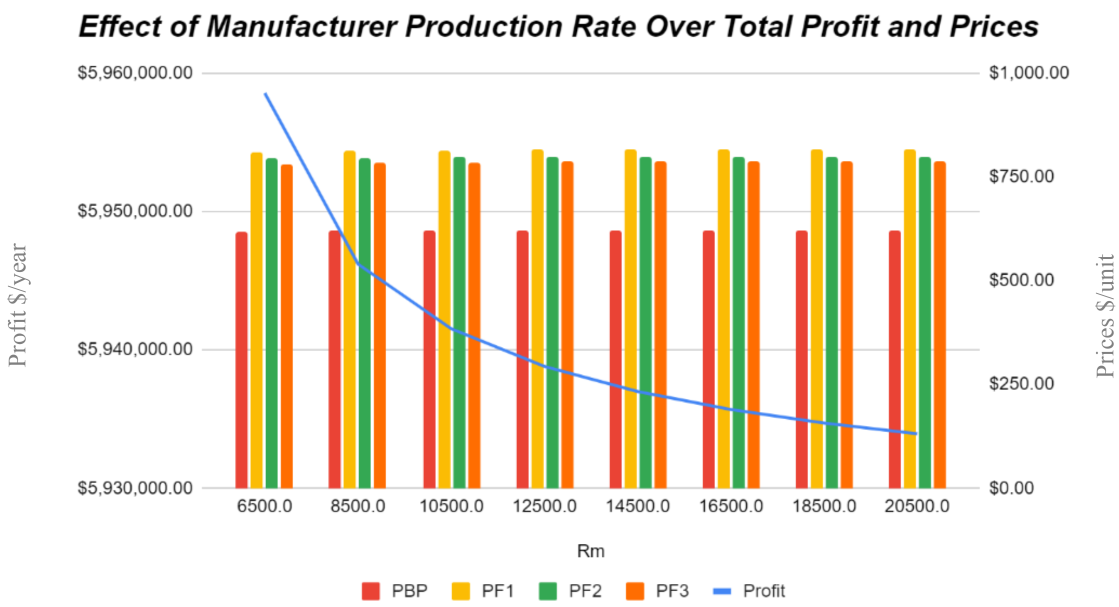

Figure 5: Effect of the Basic Product Production Rate Over the Total Profit & Prices

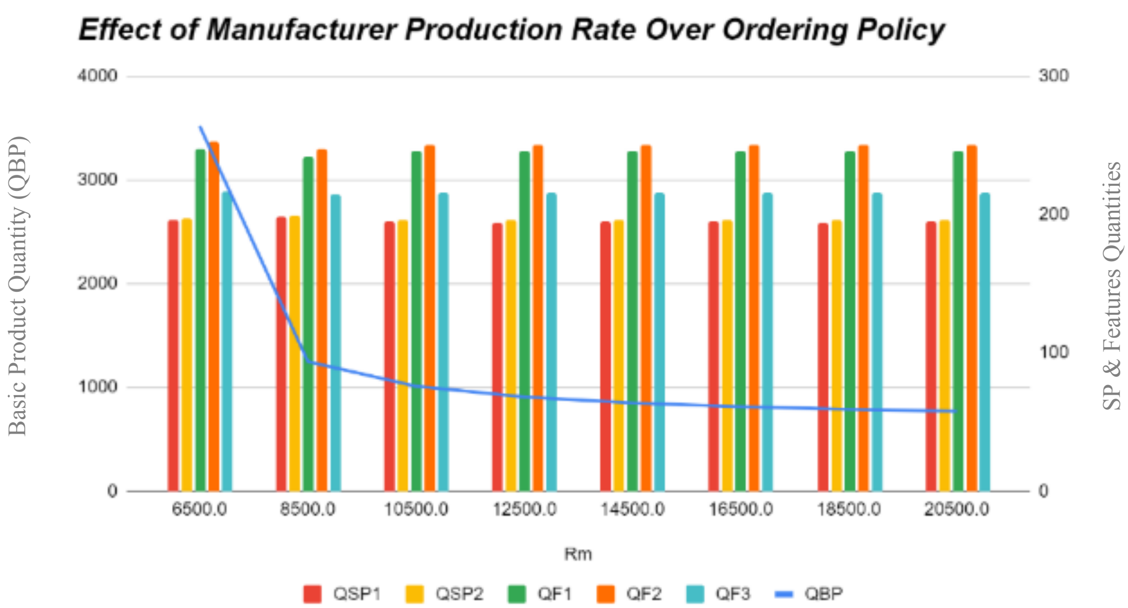

Figure 6: Effect of the Standard Product Production Rate Over the Ordering Policy

5.1.2. Effect of Varying Feature 3's Production Rate

The results show that as the manufacturer' F_3 production rate increases from 6,500 unit/year to 20,500 unit/year, the supply-chain's total profit slightly decreases from $5,935,211 to $5,934,924, the selling prices and ordering policy for all products remained the same except for feature 3, where the quantity produced decreases by approximately 10 units as shown in Figure 7 and Figure 8.

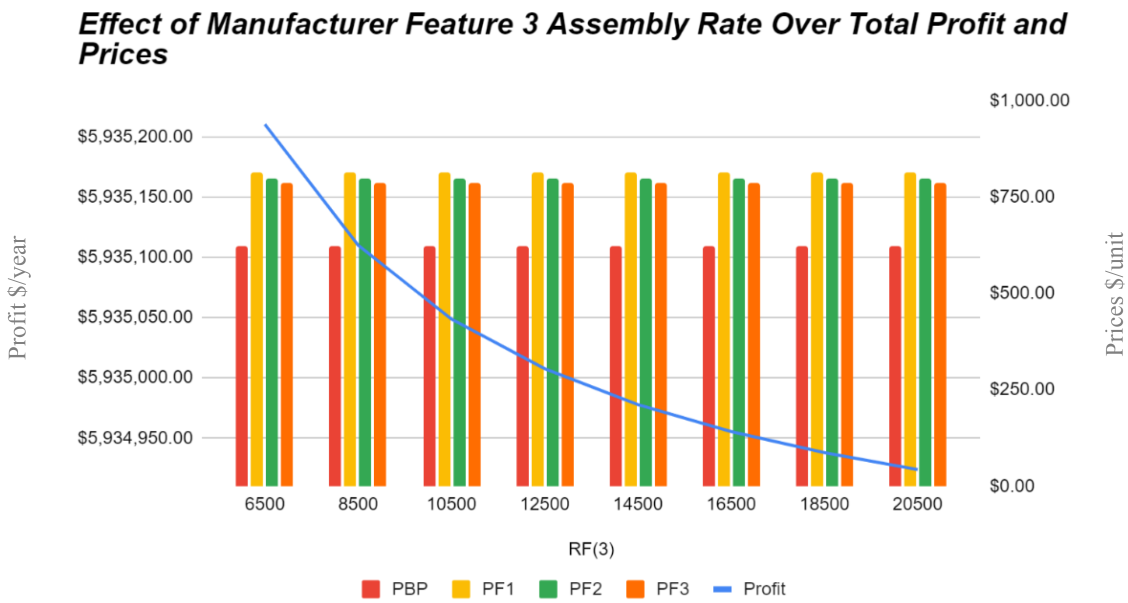

Figure 7: Effect of Feature 3 Assembly Rate Over the Total Profit & Prices

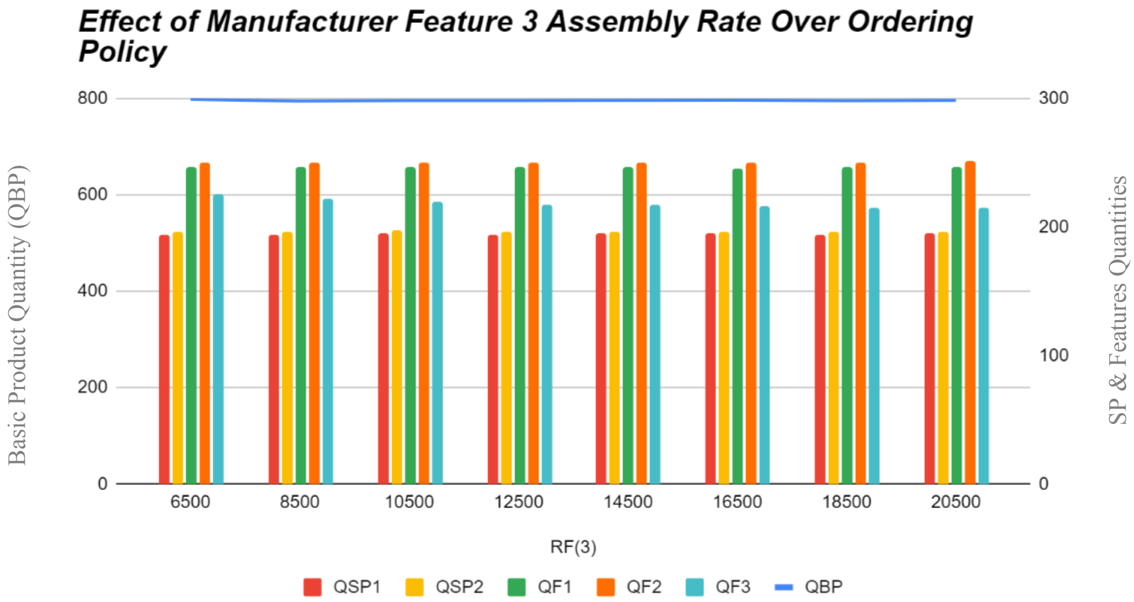

Figure 8: Effect of Feature 3 Assembly Rate Over the Ordering Policy

5.1.3. Managerial Insights

These results suggest that it is not beneficial for the manufacturer to increase the production rates, as more products will be stored, and higher holding costs will be paid. It is advised that the rates of manufacturing be close to the demands. Furthermore, the standard product manufacturing rate has more impact over the supply-chain than the features. When the former changes from 6,500 unit/year to 20500 unit/year, the total supply-chain profit drops by 0.41%, as for the latter, the profit drops by 0.0048%.

5.2. Effect of Varying Customers' Online Channel Acceptance

In this section, the effect of customers' online channel acceptance is examined. As mentioned previously, since we have multiple features to sell online and each feature has its own specific contribution to the online channel acceptance, the effect of varying the parameter associated to feature F_3, which is $\theta_F(3)$, will be examined and considered as a representation.

The results show that as the $\theta_F(3)$ increases from 0.1 to 0.45, the supply-chain total profit greatly increases from $5,934,942 to $20,503,359, the quantity of feature F_3 assembled increases from 216 units to 694 units, the quantity of spare part SP_1 ordered increases from 195 units to 374 units, the quantity of spare part SP_2 ordered increases from 197 to 375, the basic product produced quantity Q_{bp} only slightly increases by 5 units, the price of feature F_3 increases from $786 to $3,409 and the price of the basic product decreases from $622 to $360 as shown in Figure 9 and Figure 10.

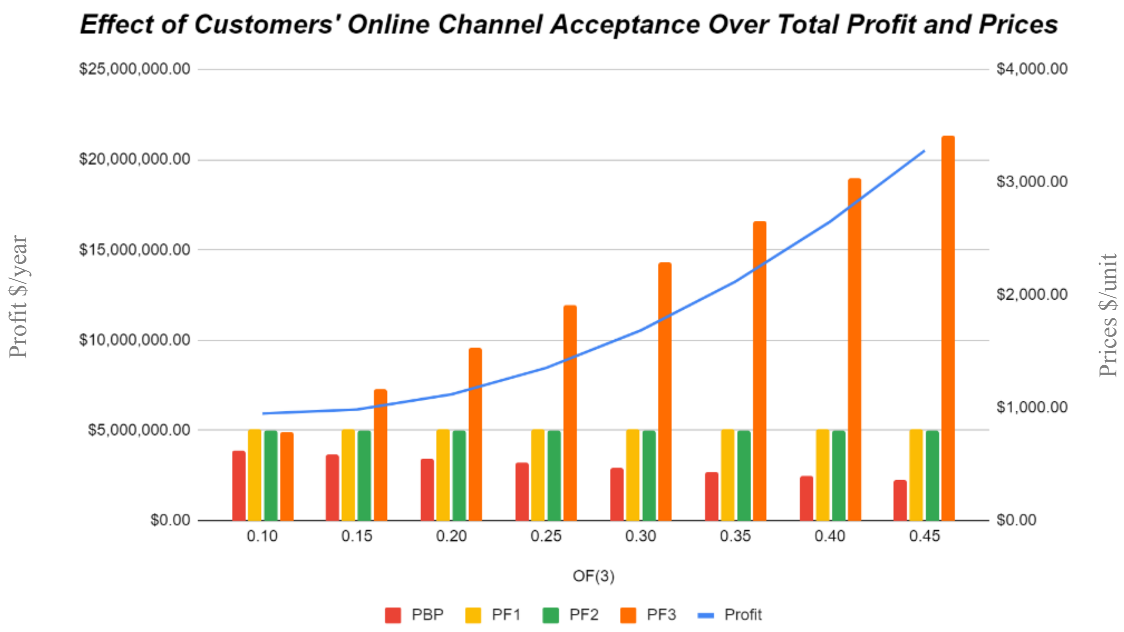

Figure 9: Effect of Customers' Online Channel Acceptance Over Total Profit & Prices

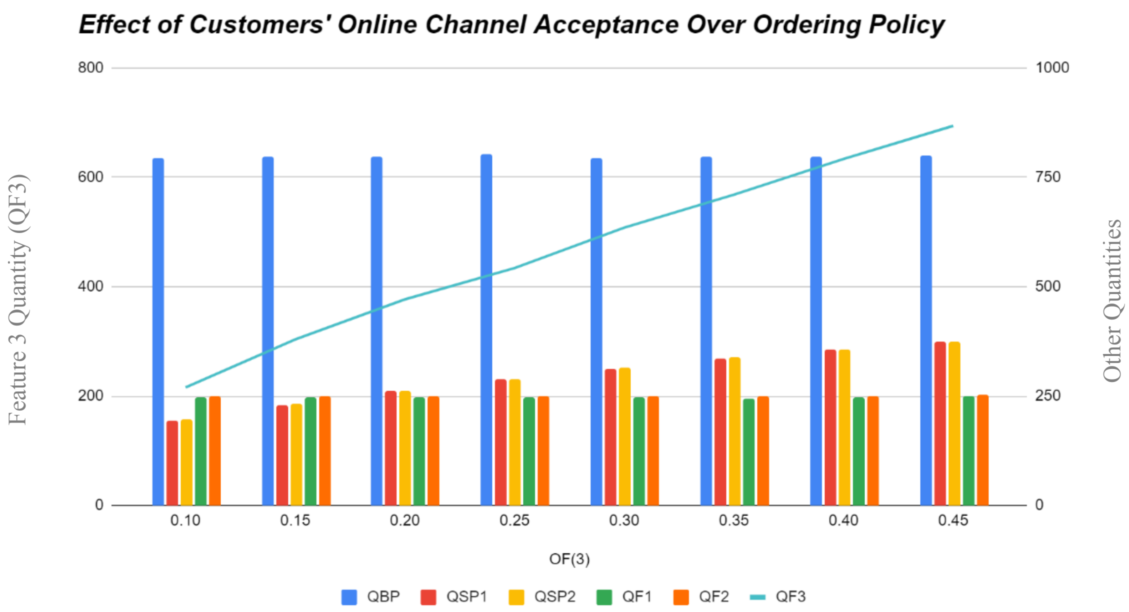

Figure 10: Effect of Customers' Online Channel Acceptance Over the Ordering Policy

5.2.1. Managerial Insights

These calculations suggest that when the online channel acceptance increase, the online channel products' demand increases and the standard channel products' demand decreases. As a result, the prices for the online channel products increase and the prices for the standard channel product decrease. Moreover, the quantities of the online channel products produced increase to cope with the customers increasing demand, which directly translates into increasing the quantities ordered and stored of the spare parts that are used in assembling them. Furthermore, even though the basic product is also used in the online products, over which the demand increases, its produced quantity only slightly increases as its demand generated by the standard channel is cannibalized by the online demand and decreases. Finally, the total supply-chain profit greatly increases by approximately 246% as the online acceptance increases. Thus, the manufacturer must incentivize the customers to buy the features sold online by adopting different strategies such as offering return policies advertising the online products.

5.3. Effect of Varying the Quoted Lead-Time

The quoted lead-time is the time needed by the manufacturer to prepare a customized product and deliver it right after the order initiation. Such factor is very important to the supply-chain as it can greatly affect the demand. In this section, effect of the quoted lead time of feature 3, which is $L_F(3)$, over the total supply-chain is examined.

The results show that as the lead time for feature 3 $L_F(3)$ increases from 1 to 10 days, the supply-chain total profit decreases from \$6,026,811 to \$5,934,941. The quantity assembled of the feature F_3 decreases from 250 to 216 units, the quantity of spare part SP_1 ordered decreases from 209 units to 195 units, the quantity of spare part SP_2 ordered decreases from 211 to 194, the basic product produced quantity Q_{bp} only slightly decreases by 5 units, the price of feature F_3 decreases from \$920 to \$786 and the price of the basic product increases from \$613 to \$622 as shown in Figures 11 & 12.

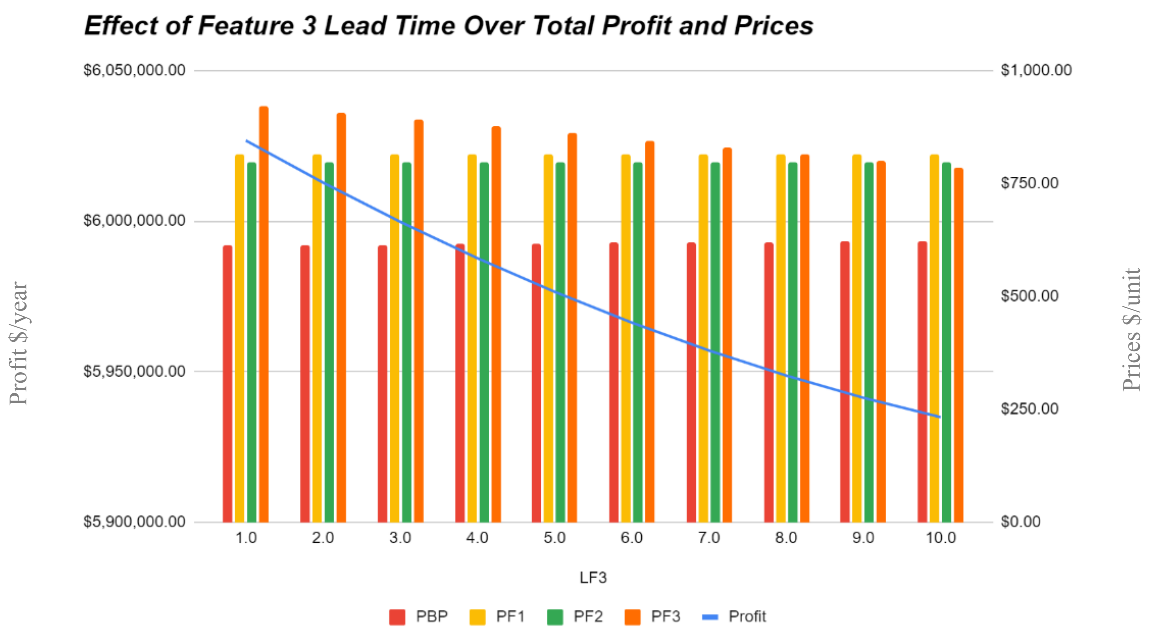

Figure 11: Effect of Feature 3 Lead Time Over the Total Profit & Prices

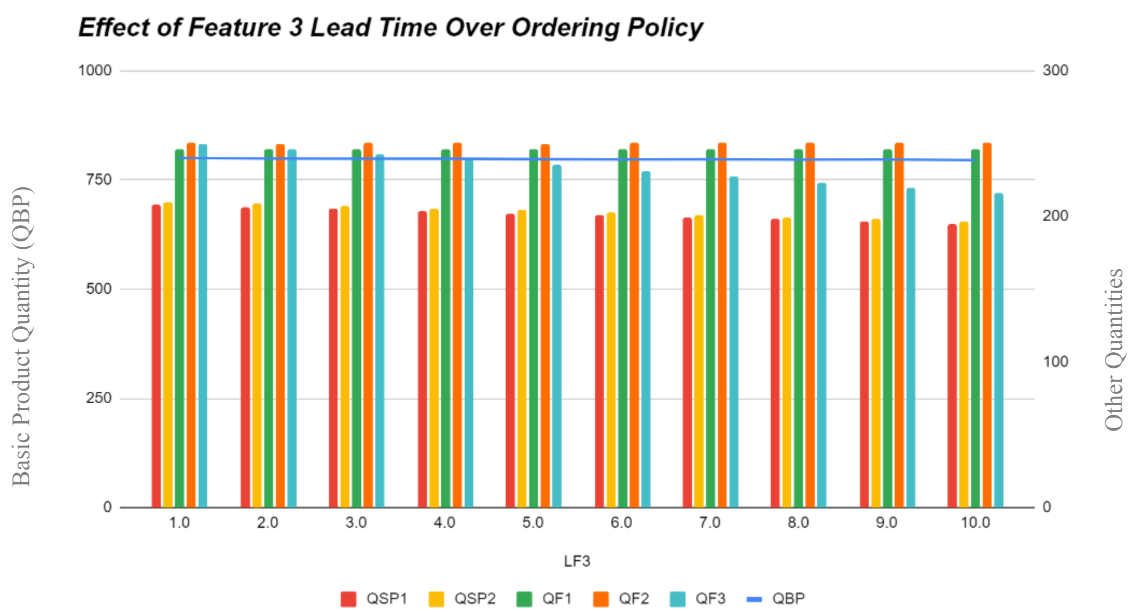

Figure 12: Effect of Feature 3 Lead Time Over the Ordering Policy

5.3.1. Managerial Insights

These data suggest that as $L_F(3)$ increases, demand channels from feature F_3 to the basic product leading to a decrease in the supply-chain's total profit. Consequently, less quantity of feature F_3 should be produced and lower quantities of spare parts SP_1 & SP_2 should be ordered. This leads to a decrease in the total supply chain profit because customers will favorize the basic product, which has a lower or even zero lead-time. To improve this situation, the manufacturer should hire a third-party logistics service to enhance the delivery related lead-time.

CHAPTER 6

CONCLUSION

This paper discusses a dual-channel supply chain model where a standard basic product is sold through traditional retail channel and customized products are sold online directly by the manufacturer. The features are achieved by adding a different combination of spare parts to the standard basic product. The manufacturer and the retailer are assumed to be a single entity. This means the manufacturer's finished goods inventory is also the retailer's inventory. The standard retail channel inventory follows the EPQ model. The online channel's inventory is divided into three parts: the basic products used in the features also following the EPQ inventory model, the spare parts used in customizing the basic products following the EOQ inventory model and the final customized goods or the features also following the EPQ inventory model.

The first part of this paper considers the demand to be a constant and the ordering quantities to be the decision variables. The second part considers the demand to be a linear equation of price and quoted lead time and the pricing strategy to be the decision variable, where the quantities are constant. The last part also considers the demand to be a linear equation of price and quoted delivery lead time but considers the ordering quantities and the product prices to be the decision variables.

The first two parts are solved mathematically. As for the last part, a numerical example is developed and solved. Furthermore, a sensitivity analysis study is performed to evaluate the effect of varying the following input parameters over the ordering policy, pricing policy and total supply chain profit: Manufacturer production rate, retailer production rate, customer online acceptance and quoted lead time.

This paper has some limitations that could be addressed in future works. It assumes that the manufacturer and the retailer are the same entity. It would be interesting to separate these two and track the profit cannibalization that could occur from introducing the customized features and evaluate the status of each independently. In addition, the demand cannibalization resulting from the products prices is a great development to this work. Also, considering the demand cannibalization between features resulting from their respective quoted lead-time is of a great value. Furthermore, this paper models the demand in a deterministic way. It would be interesting to model the demand stochastically. Also, in this paper the basic product is only sold through the retailer. Offering it through the online channel would also be valuable.

This work could also be improved by considering multiple retailers or a three-level supply channel. Finally, considering return polices with reverse logistics for returned products would also be an interesting and a major development.

APPENDIX

Manufacturer Standard Product Production Rate Sensitivity Analysis

R_m	*Profit*	$Q_{BP}(3)$	$Q_{SP}(1)$	$Q_{SP}(2)$	$Q_F(1)$	$Q_F(2)$	$Q_F(3)$	P_{BP}	$P_F(1)$	$P_F(2)$	$P_F(3)$
6500.0	5277354.7	1987.2	164.1	167.2	214.0	221.6	169.7	620.0	938.2	913.1	939.5
8500.0	5268343.5	1112.6	163.2	166.3	214.4	221.6	168.9	621.5	939.7	914.6	941.1
10500.0	5264366.2	934.2	163.3	166.6	214.1	220.9	168.8	621.9	940.0	915.0	941.5
12500.0	5261993.5	852.1	163.4	166.6	214.0	220.9	168.7	622.1	940.2	915.2	941.7
14500.0	5260397.4	806.4	163.5	166.8	212.8	219.6	167.1	622.2	940.4	915.3	941.9
16500.0	5259245.7	775.3	163.8	166.6	213.9	220.4	168.6	622.3	940.5	915.4	941.9
18500.0	5258373.0	752.7	163.3	166.3	213.8	220.6	168.6	622.4	940.5	915.5	942.0
20500.0	5257688.2	734.9	163.6	166.5	213.7	220.5	168.7	622.4	940.6	915.5	942.0

Manufacturer Feature 3 Production Rate Sensitivity Analysis

$R_F(3)$	*Profit*	$Q_{BP}(3)$	$Q_{SP}(1)$	$Q_{SP}(2)$	$Q_F(1)$	$Q_F(2)$	$Q_F(3)$	P_{BP}	$P_F(1)$	$P_F(2)$	$P_F(3)$
6500.0	5258700.6	757.0	163.1	166.4	213.4	220.1	172.7	622.3	940.5	915.5	941.8
8500.0	5258652.4	757.6	163.4	166.5	213.8	220.6	171.4	622.3	940.5	915.5	941.9
10500.0	5258622.8	756.8	163.4	166.3	213.6	220.2	170.1	622.4	940.5	915.5	941.9
12500.0	5258602.8	757.1	163.4	166.5	213.8	220.7	169.6	622.3	940.6	915.5	941.9
14500.0	5258588.3	756.8	163.5	166.5	213.5	220.6	169.3	622.4	940.5	915.5	942.0
16500.0	5258577.4	757.8	163.4	166.5	213.8	220.5	168.6	622.4	940.5	915.4	942.0
18500.0	5258568.8	756.6	163.5	166.6	213.6	220.5	168.4	622.3	940.5	915.4	941.9
20500.0	5258562.0	757.6	163.3	166.5	213.6	220.6	168.1	622.3	940.6	915.4	941.9

Online Channel Customer Acceptance Sensitivity Analysis

$\theta_F(3)$	*Profit*	$Q_{BP}(3)$	$Q_{SP}(1)$	$Q_{SP}(2)$	$Q_F(1)$	$Q_F(2)$	$Q_F(3)$	P_{BP}	$P_F(1)$	$P_F(2)$	$P_F(3)$
0.1	5258570.7	758.7	163.9	166.9	213.8	220.6	168.6	622.3	940.6	915.5	941.9
0.2	5248657.6	758.1	204.8	207.2	214.4	221.2	269.7	584.9	940.4	915.4	1315.7
0.2	5858758.0	758.2	238.7	240.9	214.1	221.0	344.4	547.4	940.3	915.2	1690.2
0.3	7088283.2	758.3	268.0	269.8	214.6	220.9	412.6	509.8	940.3	915.0	2064.7
0.3	8937012.2	760.3	296.0	298.0	212.8	220.8	481.0	472.3	940.1	915.1	2439.5
0.4	11404830.5	757.1	319.2	320.2	214.7	220.9	546.9	434.8	940.1	915.1	2814.3
0.4	14491667.7	757.5	339.6	340.5	215.0	222.6	613.2	397.3	940.1	915.1	3189.4
0.5	18197481.6	757.0	367.4	364.5	214.0	221.5	683.5	359.9	939.8	914.8	3564.0

Feature 3 Lead Time Sensitivity Analysis

$L_F(3)$	*Profit*	$Q_{BP}(3)$	$Q_{SP}(1)$	$Q_{SP}(2)$	$Q_F(1)$	$Q_F(2)$	$Q_F(3)$	P_{BP}	$P_F(1)$	$P_F(2)$	$P_F(3)$
1.0	5266370.0	761.5	180.1	182.8	213.2	220.1	207.7	613.3	940.4	915.4	1076.4
2.0	5261727.4	761.2	178.2	181.4	213.3	220.0	203.5	614.3	940.5	915.4	1061.4
3.0	5258028.7	759.2	175.7	178.8	213.4	220.2	200.7	615.4	940.5	915.4	1046.5
4.0	5255273.3	760.7	174.3	177.2	214.1	220.9	197.0	616.4	940.4	915.4	1031.5
5.0	5253461.8	759.1	172.4	175.2	213.6	220.6	192.1	617.3	940.5	915.4	1016.6
6.0	5252594.3	759.9	170.7	173.7	214.0	220.6	187.6	618.4	940.5	915.4	1001.6
7.0	5252670.6	758.2	170.2	173.1	212.5	219.3	181.8	619.3	940.5	915.4	986.7
8.0	5253692.7	758.0	167.1	170.1	213.7	220.3	178.3	620.3	940.5	915.4	971.8
9.0	5255659.2	757.4	165.2	168.3	213.7	220.6	173.6	621.3	940.5	915.5	956.9
10.0	5258570.7	758.7	163.9	166.9	213.8	220.6	168.6	622.3	940.6	915.5	941.9

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