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Covid-19 Shock: A Bayesian Approach

by
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ABSTRACT
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The coronavirus, that started in December 2019, became a pandemic that hit the world economy and had devastating consequences. The spread of the virus suggested preventive measures knowing that no vaccine was available. Therefore, city, district and then country-wide lockdowns were implemented. These variations are introduced as shocks to unemployment, and will be studied in a Vector Autoregressive Framework.

The shock to unemployment will be discussed using Bayesian inference. This approach has well-known advantages when studying heavily parameterized models like VARs, as it assigns prior probabilities to the model parameters. These prior beliefs can be updated whenever new information is available. This in turn can help in modelling and forecasting changes that occur following the shock.

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CHAPTER I

INTRODUCTION

The coronavirus, that started in December 2019, became a pandemic that hit the world economy and had devastating consequences. On March 11, 2020, The World Health Organization identified the outbreak as a pandemic. Greater attention was then provided to the infection and death rates given the limited health care capacities of each country. At first, most countries countered by requiring their population to implement forms of social distancing. However, the rapid spread of the virus suggested preventive measures knowing that no vaccine was available. In March 2020, a vast majority of countries registered a number of cases as the outbreak reached Europe and the U.S. Therefore, city, district and then country-wide lockdowns were implemented. Different countries took different measures. Ranging between 0 to a 100 (strictest), the stringency of the lockdown in each country varied. Starting March 2020, public events and gatherings were cancelled, schools and some work places closed, as well as international and domestic travel restrictions were imposed. The United States imposed lockdowns (<40) to gradually increase the stringency in subsequent months. While Italy urged hard lockdowns (>80) as Covid-19 cases soared and its health sector started suffering in February. France did not hesitate and quickly increased the strictness of its lockdown in March as a preventive measure against rising cases. The United Kingdom instituted soft lockdowns that became hard as of 2020Q2. Nevertheless, it’s important to note that Sweden alongside Japan, aimed for herd immunity and did not impose harsh lockdowns that interrupted economic activity, making both a good example to include in our study for future comparisons. The Republic of Korea tracked cases relentlessly accompanying lockdowns implementation in an effort to control and reduce the rapid spread of the virus. The different strategies adopted in each of these countries make them a good sample for comparison given the policies and laws introduced during that period to provide a social safety net for the public. On Marche 1st, 2020, the World Health Organization started recommending lockdowns for some countries, requiring others to close a number of sectors. However, on April 1st 2020, WHO required full lockdowns in most countries, including our sample. It’s important to note at this point that only Sweden did not impose a full lockdown, and instead only imposed minor lockdowns at some levels.

We summarize in the table below, the number of cases, mortality number and lockdown stringency index as of March 31st, 2020.

31-Mar-20	**Cases**	**Mortality**	**Lockdown Stringency**
Canada	820	17	72
France	4,272	346	88
Germany	5,546	88	77
Italy	5,230	801	92
Japan	149	4	40
Republic of Korea	107	6	76
New Zealand	70	1	97
Sweden	348	40	50
U. K	3,384	277	80
U. S	19,363	619	72

source: ourworldindata.org

Table 1: Covid-19 figures

The Covid-19 shock produced variations in many key macroeconomic variables that affected day-to-day life. In the U.S, unemployment rate soared to 14.8% in April 2020, increasing by 10 percentage points in a single month. This record high rate came as a shock after years of low levels of unemployment. Similarly, unemployment rate in Canada reached 13.7% as of April 2020, a record high not witnessed since the 1980s crisis. Most other macroeconomic variables also experienced changes, leading to an unstable economy and uncertainty about the future. Gross Domestic Product in many countries decreased substantially. GDP in Germany decreased 11.25% and private consumption decreased 11.65% in 2020Q2. While France witnessed a GDP decline of 18.78% in 2020Q2 accompanied by a 16.4% cut back in private consumption. These uncommon variations motivated the study for the future trajectory of these variables in a number of countries. These variations are introduced as shocks to the consumer price index (CPI) unemployment, and interest rate. The shock had visible effects on economies starting 2020 Q2, when country-wide lockdowns started to be implemented. Unemployment rate increased, prices decreased and consequently interest rates decreased. Table 1.1 summarizes the pre-lockdown average level of CPI, the average unemployment rates and the average interest rates, expressed in percentages for each country. Table 1.2 summarizes the average levels and percentages post-lockdown.

	Pre-Lockdown					
Country	CPI	σ(CPI)	UR	σ(UR)	R	σ(R)
France	93.91	7.71	8.2	0.81	1.59	1.78
Germany	93.40	8.11	3.1	2.41	1.59	1.78
Italy	92.01	9.07	8.4	1.89	1.59	1.78
Japan	98.38	1.95	2.5	0.93	0.25	0.25
Republic of Korea	89.10	12.72	3.6	0.35	3.35	1.62
New Zealand	90.87	11.46	4.5	0.92	4.26	2.35
Sweden	96.03	7.00	7.1	0.93	1.47	1.72
U.K	90.53	11.70	4.3	1.34	2.51	2.23
U.S	217.43	26.63	5.9	1.98	1.57	1.76

Table 2: Pre-Lockdown Stats

	Post-Lockdown		
Country	CPI	UR (%)	R
France	104.69	9.1	-0.30
Germany	106.25	4.5	-0.30
Italy	102.90	9.1	-0.30
Japan	101.80	3.1	-0.04
Republic of Korea	104.84	4.2	0.97
New Zealand	107.04	5.2	0.31
Sweden	106.87	9	-0.13
U.K	108.67	5.1	0.39
U.S	256.47	13.07	0.14

Table 3: Post-Lockdown Stats

Aiming to describe the future evolution of these variables, we will construct a Vector Autoregressive system and study the dynamic response of this system when one of its variables is shocked by an impulse. The lockdowns that occurred to amputate the rapid spread of the virus in many countries, will be identified as shocks and studied in a vector autoregressive context (VAR), the most popular time series model in macroeconomics. However, the shock registered huge data variations in the last few months, making the estimation of standard time series models like VAR a challenge. Should one discard the data from the pandemic? Or can one include them without distorting the parameter estimates? These questions are crucial when generating expectations about the future trajectory of our key macroeconomic series, as the latest data from the pandemic period can contaminate the time-series observations leading to weak and unreliable inferences. Having said that, our paper first develops a vector autoregressive model that captures consumer price index, unemployment rate, and interest rate. We will then look at the impulse responses for a one-standard deviation shock to unemployment. The first step consists in developing a standard VAR excluding the Covid-19 observations in order to estimate the parameters. However. disregarding the Covid-19 data is inappropriate for predictions and inferences, as it vastly underestimates uncertainty. In a second step we develop a VAR model including the data from the pandemic. It's important to note that the responses generated will not have any structural analysis as the impulse responses explode, and are only used as an interpretation for the estimated dynamics of the variables studied. Computing the impulse response by a standard procedure produces meaningless results. Recent data from the pandemic, despite being a tiny fraction of the series, can wildly influence parameter estimation. Therefore, we take a further step that consists in modeling the possible future trajectory of the residual variance by specializing our discussion to the case of Bayesian inference. This approach has well-known advantages when studying heavily parameterized models like VARs. and consists of assigning prior probabilities to the model parameters. The research will help in understanding and forecasting changes that occur following the shock. The idea is to include informative priors to shrink the unrestricted VAR in order to have a parsimonious model and hence minimize parameter uncertainty to improve our forecast accuracy. Historical data on the consumer price index, unemployment rate, and interest rate were retrieved from the Federal Reserve of St. Louise for the purpose of building our model. We also perform two exercises, first developing a Bayesian VAR without the Covid-19 observations; the predictions seem to be sharp with little volatility. A second exercise comprises of including the recent pandemic data in our Bayesian VAR. This estimation procedure stems the idea that economic fluctuations are volatile for months to come, leading to several recovery paths for each country following the Covid-19 crisis.

CHAPTER II

LITERATURE REVIEW

The health crises caused by the Covid-19 induced several countries to implement country-wide lockdowns in order to prevent the propagation of the pandemic. Key macroeconomic indicators were affected as a result. The pandemic emphasized the importance of countries' interconnections, as it not only rapidly moved across borders, but made economic indicators behave similarly. A majority of countries countered the pandemic by recommending their population to adhere to some form of social distancing, in order to reduce the infection rate and attenuate the tension on healthcare providers. However, the responses were widely heterogeneous. Italy implemented restrictive stay-at-home measures. Sweden and the U.K lessened the restrictions and aimed for herd immunity, but soon the U.K moved away from this policy. A minority of countries, like New Zealand and the Republic of Korea acted decisively and quickly, attempting to eradicate the virus before it spread. Both the public and the private sector lived in uncertainty. Businesses froze, attempting to climb out of the accumulated debt they've been offered to survive in the lockdown (Casado et al., 2020). The shock was the center of debates for many researchers. Sheldon (2020), compares the current labor market situation with previous employment crisis and presents the possible future trajectory of the unemployment rate in Switzerland on the basis of the same set of leading indicators previously used. These leading indicators for the unemployment rate point towards worsening situations in the labor market in the future. The strong increase, from16% to 40 % in long-term unemployment in Switzerland is daunting, as long-term unemployment can put a drag on recovery. Indicators suggest that unemployment responses vary amply across countries, relative on the characteristics of their labor market. Unemployment in some countries witnessed a strong response to the pandemic shock. Over 16.5 million people filled unemployment claims in the U.S as of April 4th 2020, with new claims reaching 6-7 million per week. (Coibion et al., n.d.), while fluctuations in other countries were less profound.

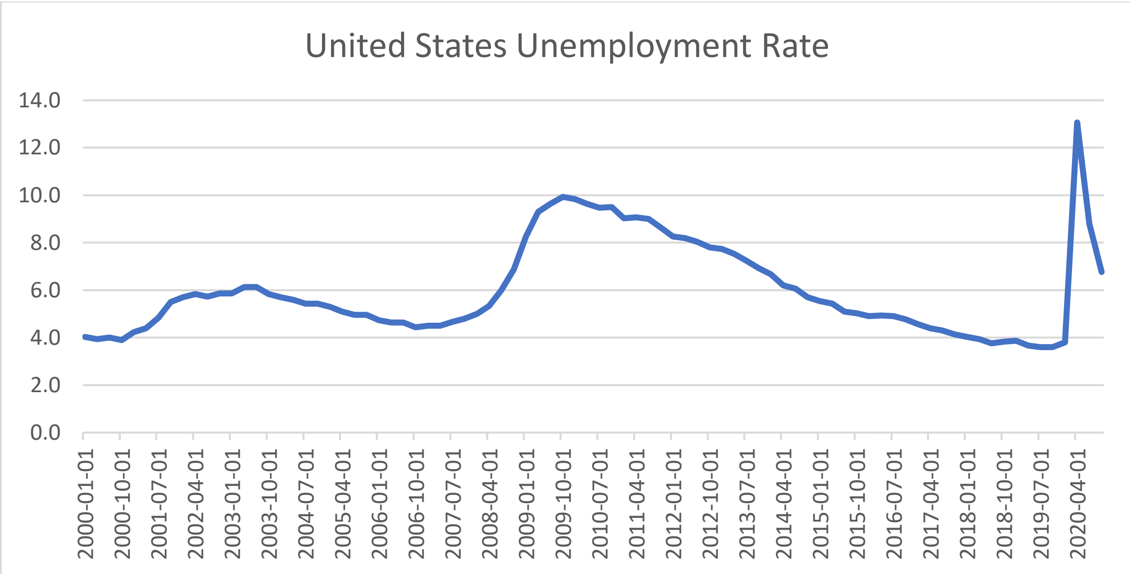


Figure 1 - US Unemployment Rate

The implications on unemployment vary widely across countries, this suggests that institutional differences can partially insolate a country's population from the effects of large exogenous shocks (Milani, n.d.). The outbreak was an unpredicted, unprecedented shock on the macroeconomy. Shocks generally induce a recession in an economy where an increase in the unemployment rate follows. However, in this case, the shock in the unemployment significantly contributed in amplifying the recession. How was economic activity affected by this major pandemic and what medium and long run effects can be identified presently? How should one model the shock? A growing number of literature model the dynamics of the pandemic and the economy to identify, quantify, and interpret the costs of different economic policies. Some researchers assessed the treatment effect on unemployment via difference-in-difference approach, lockdown v/s no-lockdown regions (Goodman-Bacon & Marcus, n.d.). Researchers also studied planned price changes to infer the relative importance of supply and demand forces during the pandemic period (Balleer et al., 2020). Whether for government purposes, central banks, or private institutions, studies were conducted in order to quantify the shock, or forecast its future implications. Except time varying volatility models are numerous. Quite a few, including VARs, were developed to capture the variations in macroeconomic variables in certain countries. Fabio Milani (2020) studies the interdependencies between the Covid-19 shocks and economies. (Milani, n.d.) estimates a Global VAR (GVAR) model to study the transmission of the pandemic shock both domestically and globally. Macroeconomic practitioners often analyze and interpret with multivariate time series models. The most popular of all are Vector Autoregression models (VAR). In economics, VAR models were made popular by Sims. The model consists of an extension of a univariate autoregressive model (AR). Generally, VARs provide useful description of the dynamic behavior of time series and for forecasting their future evolution, they are a natural tool for forecasting. Their joint generation mechanism is useful in economic analysis. Superior estimates of the future trajectory of variables and simultaneous interaction of equations can be provided when modelling with VARs. The vector autoregressive setup is such that present values of a group of variables can be partially explained by their past values. However, choosing the optimal number of lags to identify the true volatility model can be difficult. It is recommended to use selection criteria given that they reduce the mean squared error of the impulse response estimates, rather than selecting arbitrarily based on a researcher's preferences. Among the three information criterion, Akaike Information Criterion (AIC) is generally the best in determining the right volatility process. AIC outperforms its competitors in indicating the true model to work with as well as providing impulse response estimates that have the smallest mean squared errors (Kilian, 1990; Lutkepohl & Schlaak, n.d.).The flexibility of forecasting can be conditional on the potential future paths of the time series. In addition, Vector autoregressive models can be used for structural inference, where one can impose certain assumptions about the causal structure of the data studied and summarize the subsequent impacts of unanticipated shocks and innovations on the variables (Eichenbaum et al., 2020). We can summarize these impacts with impulse response functions. VAR models allow the researcher to treat all the variables as endogenous, this is helpful since unemployment increased as a result of social distancing. An important feature time series must possess is stationarity without time trends. By including deterministic polynomial terms, one can capture the trending behavior of the variables. The cointegration concept developed by Granger (1981) and Johansen (1987) and others proved that VAR models can also capture these stochastic trends.

Commonly, VAR analysis begins by specifying and estimating reduced form models for the data generating process to afterward check their suitability.
If the VAR model, in its reduced form, passes the checking stage, one can use it for forecasting and structural analysis (Asteriou & Hall, 2011). Below is a simplified figure for the process:

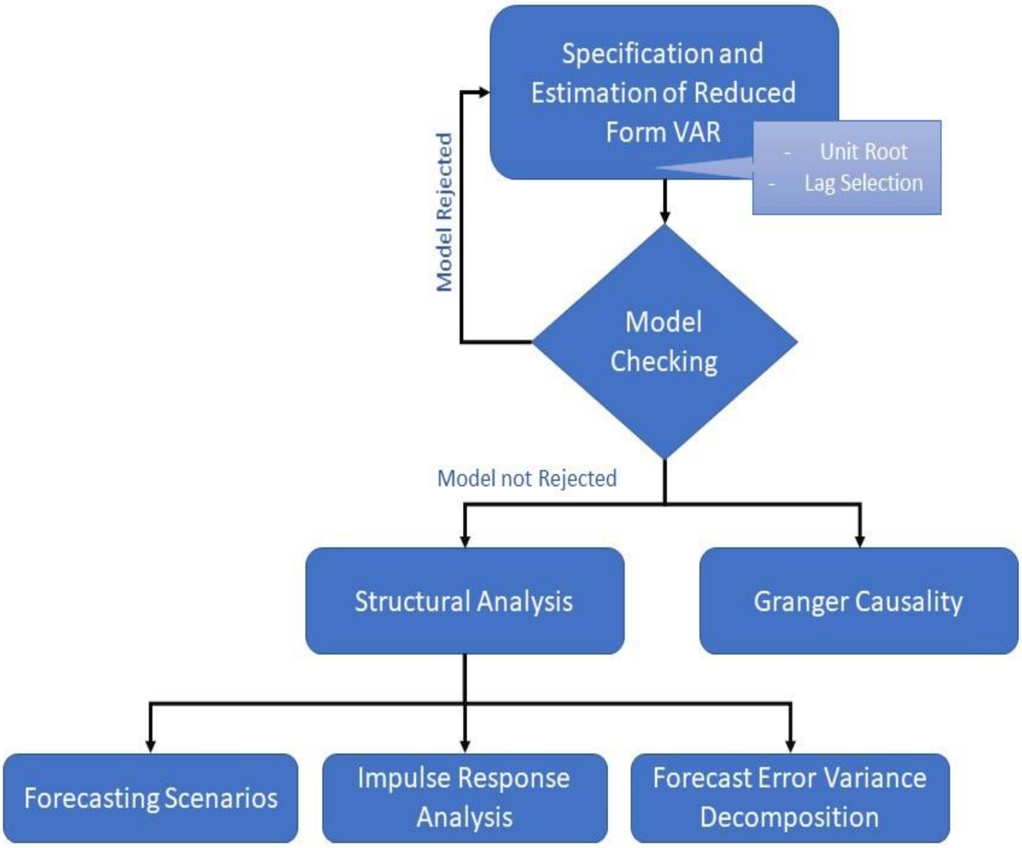


Figure 2 - Vector Autoregressive Model

Multivariate time series models, like VARs, usually consist of a large number of parameters, thus resulting in over-parameterization (Litterman, 1980). Due to the rapid increase in data availability, macroeconomists were able to operate with large data sets collected by governmental agencies and policy institutions. Consequently, worries about over-parameterization surfaced. Time-variation in the coefficients of a vector autoregressive model leads to the proliferation of the number of parameters as time-variation in the error covariance matrix increases concerns about over-parameterization. The common approach to resolve such issues was shrinkage, the most successful way in reducing over-parameterization. Therefore, many researchers started using Bayesian methods since prior information introduced shrinkage in a logical and consistent way. In addition, stationarity can pose great obstacles for time series analysis. It is essential for the researcher to ensure that all the components in the VAR are stationary in order to examine the statistical significance of the coefficients (Wooldridge, 2016). Some researcher would argue that the purpose of VAR estimation is only to observe the relationship between the variables. Having said that, from a Bayesian point of view, no special account of non-stationarity needs to be taken. The Bayesian approach is purely grounded on the likelihood function which takes a Gaussian shape irrespective of the presence non-stationary series (Sims et al., 1990; Uhlig, 1994). This constitutes a big difference between classical and Bayesian inference. In their paper "Inference in Linear Time Series Models with Some Unit Roots", Sims, Stock and Watson argue that the common practice of researchers attempting to ensure stationarity either by difference or cointegration is not always necessary. Whether the data are integrated or not is not the issue, but rather whether the estimated coefficients have a nonstandard distribution if the regressors were in fact integrated. It is often the case that their distribution is unaffected by non-stationarity, hence a Bayesian analysis finds no motive to use transformed models. Classical and Bayesian inference on unit roots differ significantly. From a Bayesian perspective, the researcher is allowed to identify the uncertainty by using weights, without taking a stand on stationarity. The unit root of a specified series is just one of several possibilities, and obtains posterior weight based on the data. Posterior probabilities are proportional to the joint probabilities of the prior and the likelihood. While the conditional likelihood function of the data may not be standard, the conditional likelihood function of the parameters is standard. In other words, non-stationarity is only present in the data, not in the parameters (Uhlig, 1994).

Applying Bayesian methods may be uncomfortable given the many choices in choosing a prior. However, only a small number of priors can be useful to use. Uhlig (1994) argues that if the prior belongs to the Normal-Wishart density, then the posterior will follow a Normal-Wishart density as well. A researcher will find it more natural to use Bayesian approach by including parameter uncertainty and use the observed data as given. Classical inference may not take into account the uncertainty underlying the coefficients when pretesting for unit roots. The Normal-Wishart family which includes the Minnesota prior, unites around a distaste towards explosive roots. They are reasonable when centered at the unit root, conditional that they are adjusted in reduced-form models by centralizing the prior weight for the coefficient toward zero as the largest root approaches unity from below. Uhlig also states that for persistence and medium run forecasting, the Bayesian approach takes uncertainty about the existence of unit roots into account. Predictive density tails can be subtle towards the prior treatment of explosive roots.

The selected prior in our research is the Minnesota prior. Introduced in 1980 by Litterman, the Minnesota prior assumes that each variable included in the model follows a random walk process, probably with a drift. Litterman states that it is a "reasonable approximation of the behavior of an economic variable" yet parsimonious. It is characterized by its first and second moment:

$$\mathrm{E}\ [\ (\beta_s)_{i\,j}\,|\Sigma\ ] = \begin{cases} 1 \ if\ i\ =\ j\ and\ s\ =\ 1 \\ 0 \qquad\qquad otherwise \end{cases}$$

$$\mathrm{Cov}\ (\ (\beta_s)_{i\,j}\,,(\beta_r)_{h\,m}\ |\ \Sigma) = \begin{cases} \lambda^2\,\frac{1}{s^2}\frac{\Sigma_{ih}}{\Psi_j\,/(d-h-1)}\ if\ m = j\ and\ r = s \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

And can be easily modeled to the form of:

$$\beta|\Sigma \sim N\ (b, \Sigma \otimes \Omega).$$

The variance of the Minnesota prior is notably lower for coefficients that are associated with distant lags and that the coefficients that are related to that same variable and same lag in a different equation are allowed to be correlated. It's key hyperparameter λ, is responsible in determining the overall tightness of the prior besides controlling the scale of all the variances and covariances. For $\lambda \rightarrow 0$ the posterior approaches the prior, whereas λ approaching infinity will make the posterior distribution closer the sample information, or the likelihood function (Kuschnig & Vashold, n.d.)

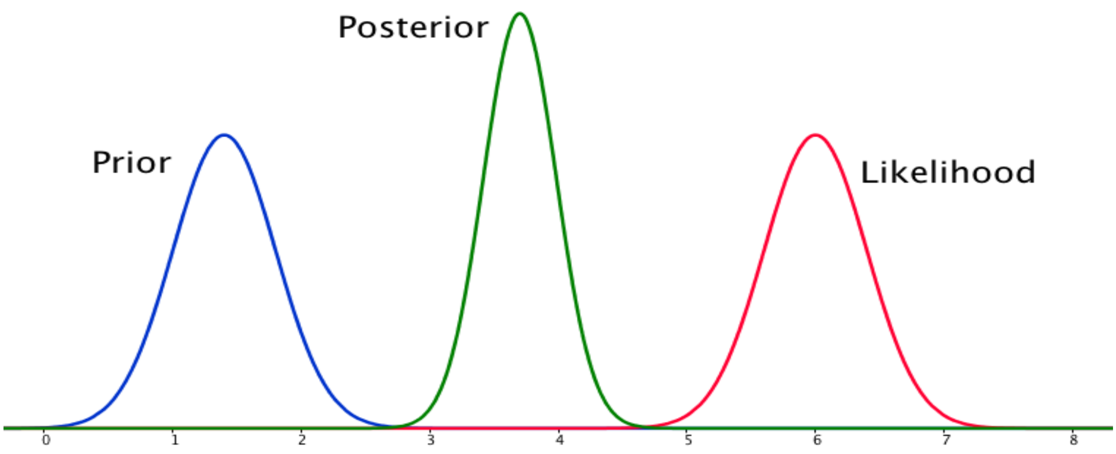


Figure 3 - Distribution

The frequentist approach to statistics treats parameters as fixed but unknown quantities where we can estimate these parameters using a sample from population. However, different samples will yield different estimates. The distribution of these estimates is known as a sampling distribution and quantifies the uncertainty about the estimates even though the parameter itself is considered fixed. The Bayesian approach is a different way of thinking. Our parameters are treated as random variables which can be described with a probability distribution (Giannone et al., n.d.). Probability is our degree of belief, absent data. This mathematical expression of our belief about the parameter included is called the prior distribution. Furthermore, we can investigate by conducting the experiment to produce another distribution, known as the likelihood function. Bayesian inference allows the researcher to update his prior beliefs about the parameter with the results obtained from the experiment. In other words, we can compute the posterior distribution by multiplying the prior with the likelihood. Additionally, if the posterior belongs to the same family as the prior, the prior is called conjugate. The posterior can closely resemble the prior when the sample size is small and the prior is informative. In contrast, the posterior will be closer to the likelihood as we increase sample size or use an uninformative prior, such as a flat prior.

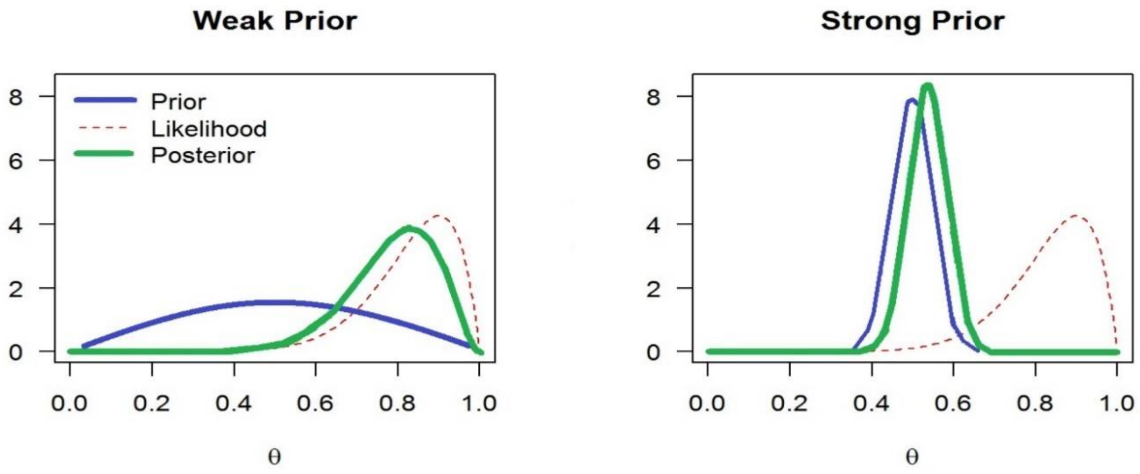


Figure 4 - Weak and Strong Priors

The consequences of the pandemic and its transmission mechanism differ by sector and firm size. Small and medium size enterprises (SMEs) played an important role in the decline of economic activity (Pedauga et al., 2021). Hence, policy makers looking to lessen the macroeconomic effect of the pandemic should be largely targeting SMEs as disruptions in this sector produced large reduction in demand (Balleer et al., 2020; Pedauga et al., 2021). The lockdowns following the pandemic do affect output. However, the current discrepancies differ substantially from the previous financial crisis in 2008. Throughout history, financial and other endogenous crises similarly affected the economy, applying a significant and persistent downward shift in the potential level of output. This is supported by the fact that following such recessions, no overshooting in growth rates was observed, pointing to long-lasting impact on the potential level of production. Therefore, exogenous shock like Covid-19 and endogenous shocks like financial crises have different costs, as exogenous shocks contracting economic activity are followed by a surge in growth rates bringing output back to its long-term trend. Moreover, the decisions on how early to intervene, and if so, to what degree was probably affected by various factors and trade-offs such as the potential containing contagion and a country's health system and capacity. A number of countries' reaction to the virus were intriguing to study, due to the diversity of interventions and policy decisions. The "health v/s wealth" took the center stage. One of the key issues within the "health versus wealth" talk about the implementation of Non-Pharmaceutical Interventions (NPIs) such as stay-at-home orders, is whether such approaches thrust relief to a point where they force net economic costs on society.

CHAPTER III

METHODOLOGY

To study the Covid-19 shock on our key macroeconomic variables, we first have to examine our variables behavior prior to the shock. Given that we have more than one dependent variable we introduce a set of linear dynamic equations where each equation of a dependent variable is specified as of lags of itself and other remaining variables in the system. Therefore, all three variables are endogenous and we have a set of three equations; we use a multivariate Vector Autoregressive (VAR) model to investigate the relationship between the consumer price index (CPI), the unemployment rate (UR), and the interest rate (R) and if there is any significant impact from shocks to the variables studied. VAR models are useful in describing the dynamic behavior of time series and for forecasting. Our VAR model equations:

$$CPI_t = \alpha_{10} + \beta_{11}.\ UR_{t-k} + \gamma_{12}.R_{t-k} + \lambda_{13}.CPI_{t-k} + \mu_{1t}$$

$$UR_t = \alpha_{20} + \beta_{21}.\ UR_{t-k} + \gamma_{22}.R_{t-k} + \lambda_{23}.CPI_{t-k} + \mu_{2t}$$

$$R_t = \alpha_{30} + \beta_{31}.\ UR_{t-k} + \gamma_{32}.C_{t-k} + \lambda_{33}.CPI_{t-k} + \mu_{3t}$$

Where α_{ij}, β_{ij}, γ_{ij}, and λ_{ij} are the coefficients to be estimated, alternatively we have:

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \mu_t \qquad \textbf{(1)}$$

$$\mu_{t\,\sim}\, N\,(0, \textstyle\sum)$$

Where $Y_t = (CPI_t, UR_t, R_t,)$ is a 3 x 1 vector of our endogenous variables, A_p are k x k matrices of the coefficients, and $\mu_t = (\mu_{1t}, \mu_{2t}, \mu_{3t})$ is a 3 x 1 white noise process.

Past data will help us identify the values at time t. However, the obtained coefficients in Vector Autoregressive models can be difficult to interpret given their atheoretical background. Therefore, impulse response functions can help by examining the response of the dependent variables in the VAR system to shocks in the error term. A shock in a residual in one equation causes change in the dependent variable of the equation, and because this variable is present in all other equations, the shock has an effect on all other dependent variables in the next period. Economies evolve and grow over time, providing non-stationary series that can constitute an obstacle for VAR inference. Examining how long and to what degree a residual shock in any of the equations has on the consumer price index, the unemployment rate or the interest rate present in the system can lead to unreliable results. Developing such VAR models is possible, if the stationarity condition is satisfied. Yet, other obstacles occur, should one include the recent Covid-19 data? Or disregard in order to estimate the parameters? The estimation of a Standard VAR model excluding the Covid-19 observations yields an impulse response function that does not generate proper predictions as it drastically underestimates uncertainty. Having said that, the standard estimation that includes the recent data contaminated by the Covid-19 shock will only lead to a spurious regression and unreliable results. Moreover, VAR models applied to bounded sample sizes of macroeconomic data like ours, can suffer from over-parameterization. Estimating with a high lag order can result in a very large number of coefficients relative to our data. This over-parametrization can lead to the loss of degrees of freedom and that affects our predications and inferences.

Therefore, we take a further step in order to model the possible future trajectory of the residual variance by conditioning our discussion to the case of Bayesian inference. This approach has well-known advantages when studying heavily parameterized models like VARs. and consists of assigning prior probabilities to the model parameters. The research will help in understanding and forecasting changes that occur following the shock. The idea is to include informative priors to shrink the unrestricted VAR in order to have a parsimonious model and hence minimize parameter uncertainty to improve our forecast accuracy. In sum, BVARs contrast with standard VAR by treating the model parameters as random variables with prior distribution rather than fixed values. The multivariate normal assumption of μ_t gives a multivariate normal distribution of Y. In BVAR the coefficient matrix β is random, therefore we identify it as:

$$\beta \sim N\ (\beta_0, \textstyle\sum_0).$$

The vector β_0, our prior mean, can be assigned any value and the matrix. $\sum_0$ is our variance and measures our uncertainty about our prior beliefs. Therefore, Bayesian VAR forms prior distributions for β and ∑.

The coefficient matrix is obtained after modifying our standard VAR to Bayesian VAR. and stacking our observations to have:

$$Y_t = X_t\ \beta + E_t \text{ in matrix form} \qquad \textbf{(2)}$$

where $Y = (Y_1, \ldots, Y_t)$ and $E = (\mu_1, \ldots, \mu_t)$ are T x M, $X = (Y_{t-1}, \ldots, Y_{t-p})$ is K x K and $\beta = vec\ (B_1, B_2, B_3, C)$. The unknown parameters of the model are β and ∑.

In principle, Bayesian estimation follows that, given the probability density function (pdf) of our data conditional on our parameters, our likelihood function corresponds to:

$$L\ (Y \mid \beta, \textstyle\sum) \propto |\textstyle\sum|^{-T/2} \exp \left\{ -\frac{1}{2} \textstyle\sum_t (Y_t - X_t\ \beta)' \textstyle\sum^{-1} (Yt - X_t\ \beta) \right\} \quad \textbf{(3)}$$

Given the joint prior distribution on our parameters p (β, ∑), and according to Bayes rule, the joint posterior distribution of our model parameters conditional on our data is derived

$$p\ (\beta, \sum \mid Y) = \frac{p\ (\beta, \sum)\ L\ (Y \mid \beta, \sum)}{p(Y)}$$

$$p\ (\beta, \sum \mid Y) \propto p\ (\beta, \sum)\ L\ (Y \mid \beta, \sum)$$

In sum, our posterior is obtained by multiplying the prior by the likelihood function. The choice of a prior distribution is a fundamental step that can cause problems. Neither a flat prior nor an assertive prior can optimize the fit of the model, this makes the selection of the informativeness of the prior distribution for the model parameters an essential issue. As the number of variables increases relative to the number of observations in the VAR model, the important role of prior probabilities increases. The informativeness of the prior is set by treating it as an additional parameter based on the hierarchal modeling interpretation. Linear regression models as equation (2) are vast. Proposed by Litterman (1986), the Litterman/Minnesota prior, which we will be using is based on the assumption that ∑ is known. Its idea is that recent lags yield more valid information than distant ones, and a variable's own lag explains its variation better than the lags of other variables. The Minnesota prior which replaces ∑ by a known estimate facilitates the process, as it simplifies computations making analytical posteriors and predictive results available. For a prior mean, the Minnesota prior implies setting most or all of its elements to Zero.

Litterman assumes that:

$$p\ (\beta_g) = N\ (\bar{\beta}_g, \bar{\Omega}_g)$$

where $\bar{\beta}_g$ and $\bar{\Omega}_g$ are the prior mean and variance-covariance matrix of β_g respectively. The residual variance-covariance matrix $\sum$ is assumed fixed and diagonal $\sigma_{gg}{}^2 I_T$. We can rewrite:

$$Y_g = X_g\ \beta + E_g$$

Where Y and E are T x 1 vectors and X is stacked version of X_t of equation (2). The assumed independence of the error terms is such that the likelihood is just a product of independent normal densities

$$L\ (Y \mid \beta, \sum) \propto |\sigma_{g,g}|^{-T/2} \exp \left\{- \frac{1}{2\sigma g,g2}(Y_g - X\ \beta_g)'(Y_g - X\ \beta_g)\right\}$$

Hence the posterior distribution of the parameters becomes:

$$p\ (\beta_g \mid Y) = p\ (\beta_g)\ L\ (Y \mid \beta, \sigma_{g,g}{}^2)$$

and is proportional to:

$$|\ \sigma_{g,g}{}^2|^{-T/2}\ |\ \bar{\Omega}\ _g\ |^{-T/2} \exp \left\{- \frac{1}{2} \left[(\beta_g - \bar{\beta}_g)'\ \bar{\Omega}_g{}^{-1}(\beta_g - \bar{\beta}_g) + \frac{1}{\sigma^2} (Y_g - X\ \beta_g)'\ (Y_g - X\ \beta_g)\right]\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left[\left(\frac{1}{\sigma^2}\left(Y_g'Y_g - 2Y'X\,\beta_g + \beta_g'\,X'\,X\,\beta_g\right) + \beta_g'\bar{\Omega}_g^{-1}\,\beta_g - 2\bar{\beta}_g'\,\bar{\Omega}_g^{-1}\,\beta_g + \bar{\beta}_g'\,\bar{\Omega}_g^{-1}\,\bar{\beta}_g\right)\right]\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left[\left(\beta_g'\left(\frac{1}{\sigma^2}X'X + \bar{\Omega}_g^{-1}\right)\beta_g - 2\left(\frac{1}{\sigma^2}X'Y + \bar{\Omega}_g^{-1}\,\bar{\beta}_g\right)'\,\beta_g\right)\right]\right\}$$

Where $|\sigma_{g,g}{}^2|^{-T/2}$ and $|\bar{\Omega}_g|^{-T/2}$ are constants in the first proportionality, whereas $Y_g'Y_g$ and $\beta_g'\bar{\Omega}_g^{-1}\,\beta_g$ are constants in the second. We complete the square in the last exponential, we have:

$$p\,(\beta_g\,|\,Y) \propto \exp\{-\frac{1}{2}[\,(\beta_g' - \tilde{\Omega}_g\tilde{\beta}_g)'\,\tilde{\Omega}_g^{-1}\,(\beta_g' - \tilde{\Omega}_g\tilde{\beta}_g)]\}$$

with

$$\tilde{\beta}_g = \tilde{\Omega}_g\,(\,\bar{\Omega}_g^{-1}\,\beta_g\, + \sigma_{g,g}^{-2}\,X'Y_g)$$

and

$$\tilde{\Omega}_g = (\,\bar{\Omega}_g^{-1} + \sigma_{g,g}^{-2}\,X'X)^{-1}$$

Meaning, $p\,(\beta_g\,|\,Y) = N\,(\tilde{\beta}_g, \tilde{\Omega}_g)$. Litterman assigns values to these hyperparameters on the assumption that most macroeconomic series follow a random walk process.

Of course, different priors can be chosen, reshaping our vector autoregressive model and its forecast results. However, this Bayesian VAR focuses on the Minnesota prior following a Monte Carlo integration. It’s important to note that prior information and prior selection do affect the model, however they yield similar results when forecasting. The BVAR forecast is an unadjusted produce of a statistical process that tends to pick a point as close as probable to the future value of the variable studied. Furthermore, a central point about economic forecasts is the timing of the release. The forecasting procedure is too volatile and sensitive when presented with new information. Hence, forecasts published at different dates are thus based on slightly different information, housing different results.

The BVAR including our three key macroeconomic variables is estimated for 9 countries: France, Germany, Italy, Japan, Republic of Korea, New Zealand, Sweden, U.K and the U.S. This application includes the Covid-19 observations and studies the dynamic response of our variables to a positive shock to CPI, unemployment, and the interest rate. As mentioned, with Vector Autoregressive models, the parameters themselves are seldom of direct interest. The great number of VAR coefficients represents a challenge in analyzing and interpreting. Nevertheless, the table below presents the posterior mean and standard deviation of the 3 Bayesian VAR coefficients for each country. It is important to highlight the mechanism through which the pandemic lead to unprecedented variations in our economies. As social distancing was key for countering the pandemic, the lockdown shock translated into an unemployment shock. Therefore, a shock to unemployment propagated to several other key macroeconomic variables. The lockdown shock, translated to a shock in the unemployment rate to subsequently induce discrepancies in prices, interest rates, consumption, trade, productivity and broadly speaking, output.

The posterior distributions of our coefficients following the MATLAB code by Gary Coop and Dimitri Korobilis are summarized in the table below.

	Posterior Distribution					
	Consumer Price Index		Unemployment Rate		Interest Rate	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
France	105.38	0.45	6.6	0.21	-0.06	0.32
Germany	106.74	0.44	4.64	0.17	-0.49	0.32
Italy	102.88	0.33	8.25	0.27	-0.1	0.29
Japan	101.63	0.55	2.82	0.16	-0.06	0.03
Rep. of Korea	105.08	0.51	4.09	0.2	0.59	0.39
New Zealand	107.18	0.48	4.01	0.29	-0.15	0.38
Sweden	107.05	0.63	8.69	0.32	-0.49	0.39
U.K	109.12	0.38	4.12	0.15	0.23	0.33
U.S	254.32	3.1	16.16	2.7	-0.35	0.66

Table 4: Posterior Distribution

In order to see the future evolution and the dynamics of our variables, we study the impulse response of each of our equations. The shock in each country provides different trajectories for each of our variables. However, it can be seen that the unemployment shock in most countries lead to a negative response for the consumer price index (CPI), a positive response for unemployment rate and a negative response for interest rates given a shock to unemployment. It’s important to note however, that using different priors changes the results and posterior distributions of our parameters, but the direction of change remains the same. It will be recalled that the lockdown was indeed an unemployment shock and hence we represent the impulse response function for the consumer price index, the unemployment rate and the interest rate following a shock to unemployment.

Figure 5 - Impulse Response Function for France

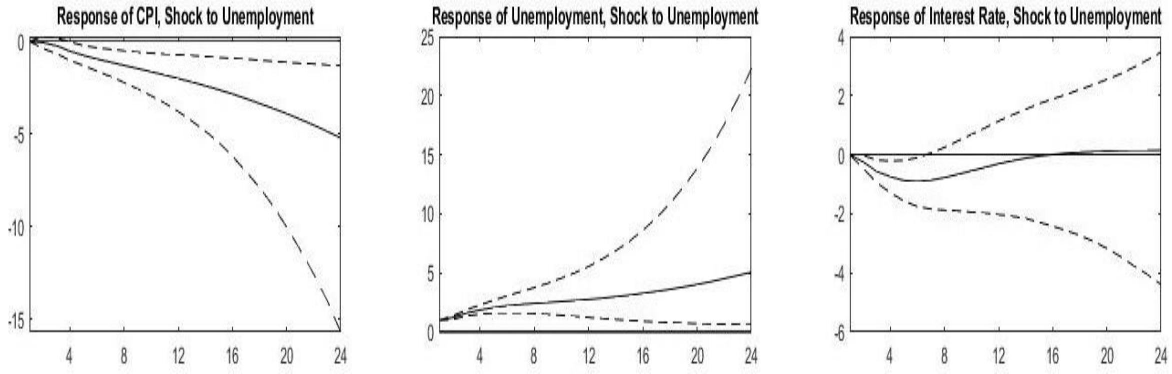


Figure 6 - Impulse Response Function for Germany

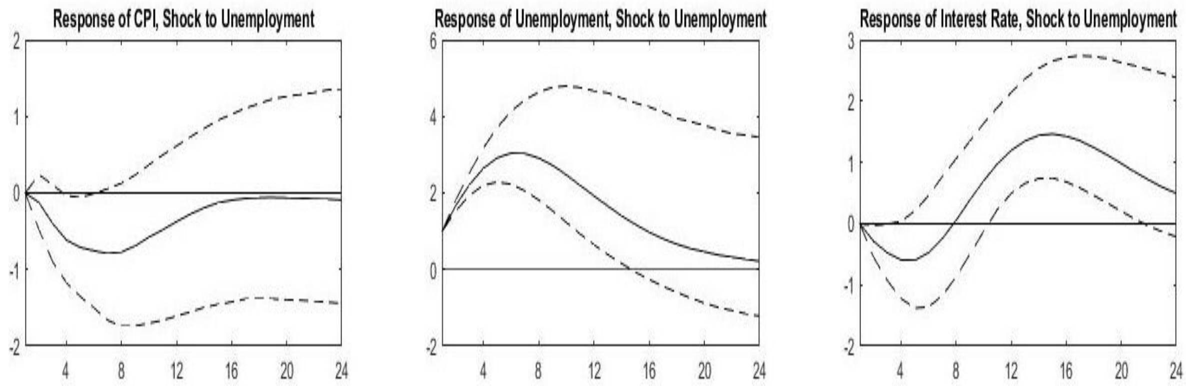


Figure 7 - Impulse Response Function for Italy

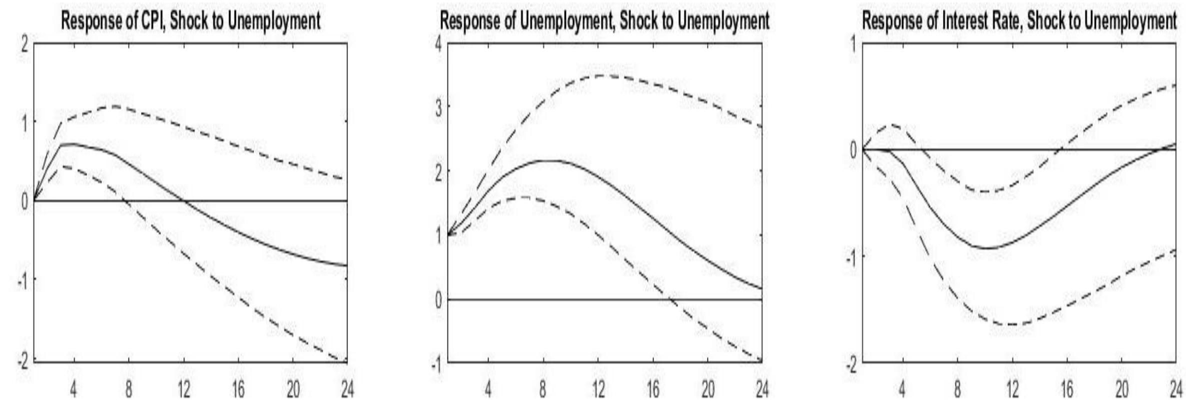


Figure 8 - Impulse Response Function for Japan

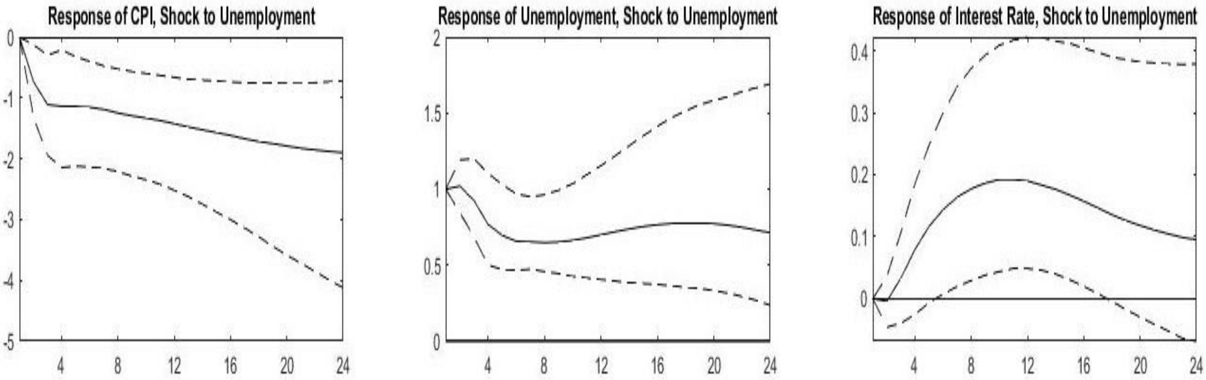


Figure 9 - Impulse Response Function for Rep. of Korea

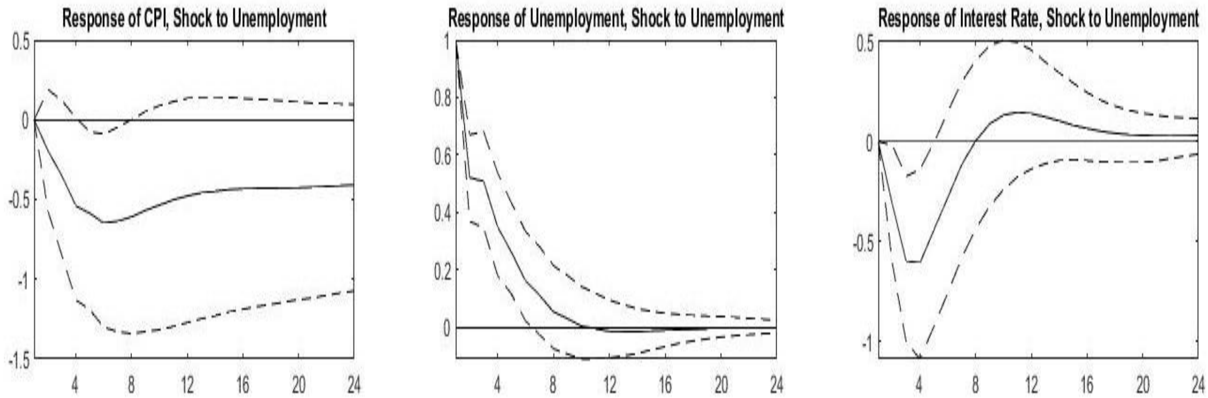


Figure 10 - Impulse Response Function for New Zealand

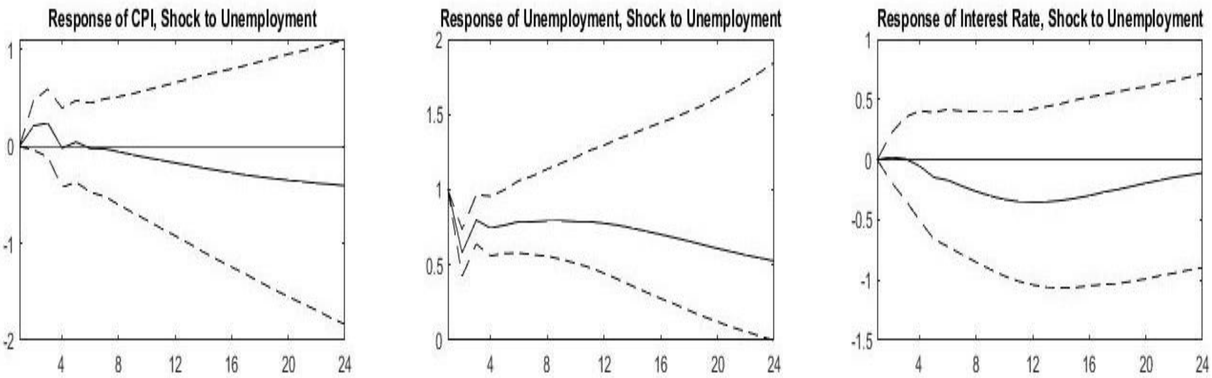


Figure 11 - Impulse Response Function for Sweden

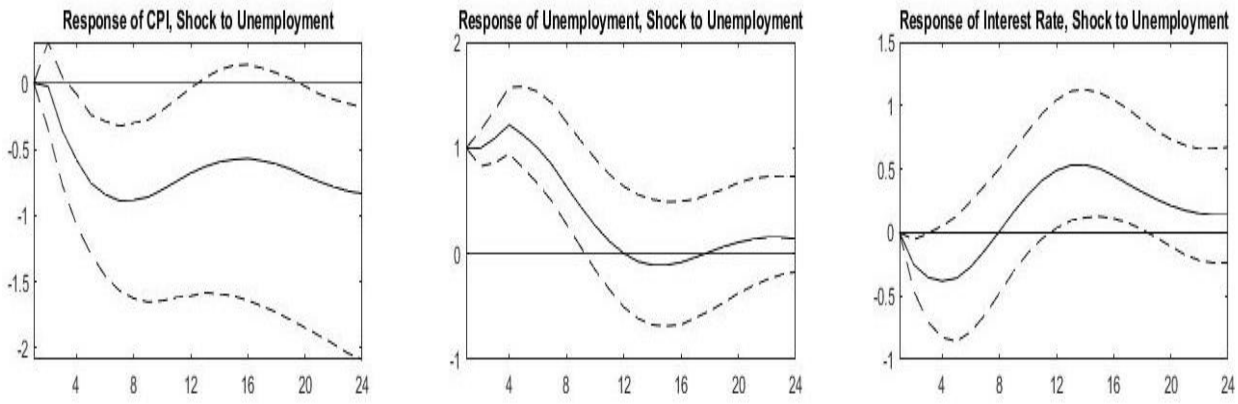


Figure 12 - Impulse Response Function for the U. K

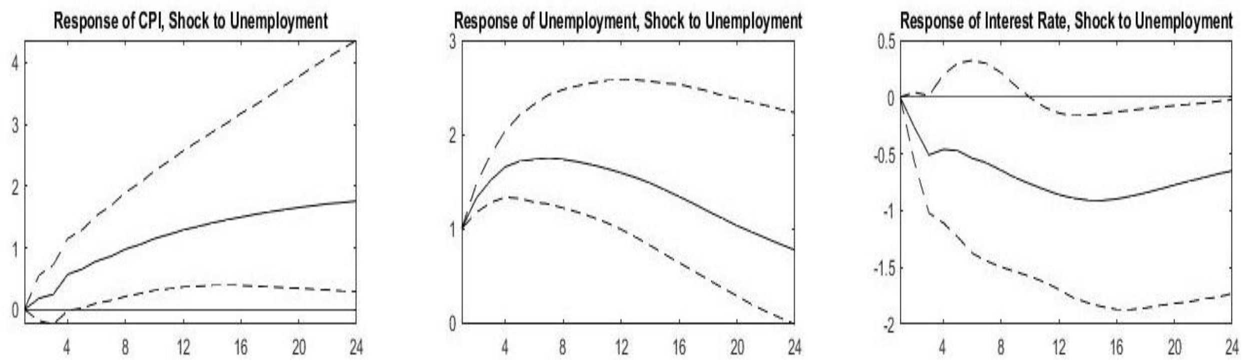
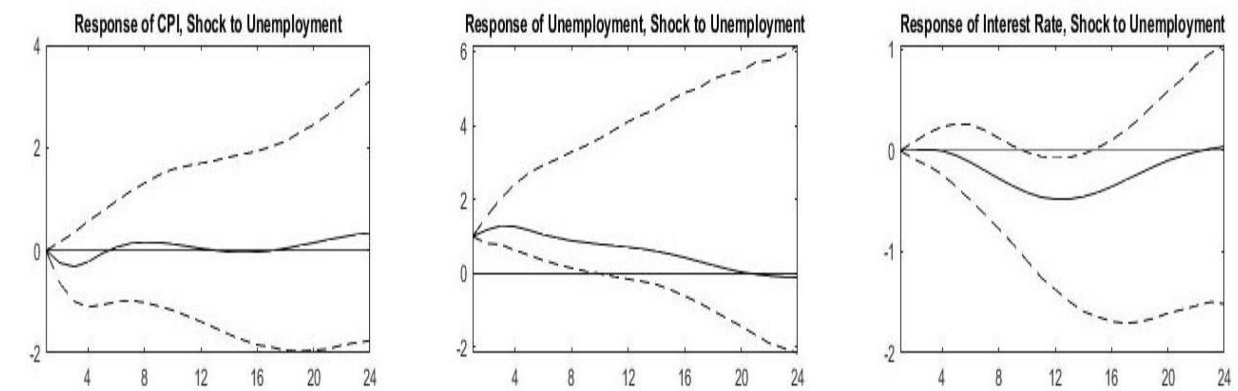


Figure 13 - Impulse Response Function for the U. S



CHAPTER IV

DISCUSSION

In this section we report our estimation results and analysis on the estimated Impulse Response Function. As an initial step, we have modeled a standard vector autoregressive model that includes our three key macroeconomics variables: consumer price index, unemployment rate, and interest rate (yield on 3-month treasury bill). Our data, that begins in 2000 Q1, included the latest Covid-19. The standard VAR model generates invalid impulse response functions as it undermines future projection based on uncertainty by working with a flat-prior, and including the latest data will result in an exploding impulse response. After specifying the proper VAR model for each country, we proceed by estimating a Bayesian vector autoregressive model that includes the Covid-19 observations. The reason is to generate the standard trajectory of our impulse response function given the recent lockdown shock in order to update prior expectations given the latest innovation. Our prior expectations are theoretically grounded on the Litterman/Minnesota prior. Including our residual shock, the BVAR model estimated gives solid and more reasonable results for the future projection of our key macroeconomic variables. The model we have estimated incorporates highly volatile residual shock, not only in the time of writing this research but also for the subsequent months through the mechanisms we have presented in our analysis. All three variables were subject to a shock in unemployment. The impulse response functions generated show how the consumer price index is negatively affected in most of our countries, while it was positively influenced in Italy, New Zealand and the U.K. According to economic theory, unemployment and inflation are negatively correlated, that is, when unemployment increases, inflation decreases. Throughout history this phenomenon was known as the Philips curve. It clearly stated the inverse relationship between the two series. However, Italy, New Zealand, and the U.K do not necessarily contradict standard results, as the relationship between the two variables has broken down over the years. The shock in unemployment also resulted in decreasing interest rates. Lower interest rates can stimulate borrowing and consequently spending among consumers; the increase in demand can lead to lower levels of unemployment, as businesses facing higher demand, will have an incentive to hire more workers. Nevertheless, a high degree of uncertainty is centered on the future evolution of unemployment rate, as several factors should be taken into account; remote work, labor automation and contemporary skill sets are a few examples that Covid-19 encouraged and motivated both the private and public sector to take into account for future operations. Historically, employment and unemployment rates were reverse images of one another, as workers move out from employment to unemployment, especially during recessions. Severe recessions occasionally lead to the “discouraged worker” phenomenon in which unemployed individuals stop looking for a job. Many of the unemployed are not actively looking for a new job (Coibion et al., n.d.) reclassifying them as “out of the labor force”, making the unemployment rate misleadingly decline. Moreover, the consumer price index is affected by the high levels of unemployment and also by low consumption rates. Consumption during the initial stages of the pandemic drastically decreased. The longer the pandemic period, the more firms will exit the market, as investment and productivity are pro-cyclical. It’s important to note that unemployment rate lags the business cycle by a quarter, interest rates tend to be pro-cyclical, and the yield curve on 3-month treasury bills is downward sloping in this recession. In addition, a number of econometric studies confirm that the robustness of the Okun relationship associating unemployment and the size of the output gap is steadily decreasing. The reason for this decreasing strength is not yet clear but might be due to the declining trend in production jobs, that normally react strongly to the fluctuations in the business cycle.

Estimating a standard VAR model including the Covid-19 observations will yield highly flown estimates of β and $\sum$, and forecasts become inaccurate. Therefore, it will be preferable to examine the parameters using a BVAR. Bayesian methods perform better than their non-Bayesian counterparts in terms of forecasting and accuracy. As we have discussed, Bayesian methods appear to offer rational answers to overcome the complications of overparameterization and overfitting when estimating vector autoregressive models. The macroeconomic indicators we worked with behaved similarly worldwide, with or without lockdowns.

Off course, a variety of prior can be used when developing vector autoregressive models yielding distinct results. It's important to note that a more informed opinion about the variables would be expressed with a tighter prior.

CHAPTER V

CONCLUSION

The rapid spread of Covid-19 and its identification as a pandemic introduced a shock to economic activity. Lockdowns were implemented following the widespread of the virus. As a result, industrial and business activity halted and unemployment rates increased drastically. The wild macroeconomic discrepancies witnessed during the Covid-19 pandemic introduced new challenges for estimating macro-econometric models. VAR models have great capacity to fit the data yet their large number of parameters constitute a drawback, as VAR models overfit the data and lead to unreliable results. In this paper, we proposed analyzing the large changes using a Bayesian approach to Vector Autoregressive models (BVAR). By assigning prior probabilities to our model parameters about the long run dynamics of our data, Bayesian inference produces sharper results. Moreover, Classical and Bayesian inference can differ substantially regarding unit roots. Economists are interested in forecasting; unit roots are important as they yield explosive responses using the Classical approach. However, to a Bayesian, unit roots are just one of several possibilities and receives posterior wight according to the data, as the Bayesian approach is completely grounded on the likelihood function, which partakes the same Gaussian shape irrespective of the existence of nonstationary.

However, there are a number of reasons why modelling the effects of the pandemic remains a challenge. First, there still is a good deal of uncertainty around the volatility of our key macroeconomic variables. The recent resurgence of the virus provides a bedrock for prolonged lockdowns causing large variability in the macroeconomy. In contrast, the mass vaccination campaigns can lead to a fast recovery, bringing new developments to the shock volatility. Finally, it’s important to note that Bayesian vector autoregressive models are useful in modelling extreme observations. By providing probabilities to our statistical problems, Bayes provides us with a set of tools to update our beliefs when new data is presented. This allows the researcher to use posteriors from previous analysis as priors in new BVARs.

The Covid-19 pandemic introduced a distinct global shock to our interconnected world, simultaneously impacting and disrupting supply and demand. On the supply side, the pandemic reduced labor hours, increasing unemployment rate following social distancing rules and business shutdowns. On the demand side, investments, especially of small and medium size enterprises, and consumption dropped significantly, affecting economics prospects. The uncertainty that dominated the magnitude and the duration of the virus lead to extreme volatility in the macroeconomy. This research highlights the importance of a comprehensive and a harmonized cross-country policy response to Covid-19. Measures taken by the authorities should firstly control the spread of the virus, this includes a swift deployment of medical resources and mass vaccination campaigns. Thereafter, monetary and fiscal authorities should intervene to reinstate the smooth functioning of financial markets as well as actions to support firms and households.

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