AMERICAN UNIVERSITY OF BEIRUT

ASSORTMENT AND PRICING OPTIMIZATION IN AN OMNICHANNEL SETTING

by
SARA MOHAMAD DAHER

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SARA MOHAMAD DAHER

Approved by:

	Signature
Dr. Bacel Maddah, Professor Industrial Engineering and Management	Advisor
	Signature
Dr. Hussein Tarhini, Assistant Professor Industrial Engineering and Management	Member of Committee
	Signature
Dr. Maher Noueihed, Assistant Professor Industrial Engineering and Management	Member of Committee

Date of thesis defense: September 6, 2021

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ABSTRACT
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As firms are starting to move to selling through both online and offline channels, customers are getting the chance to explore products, touch and feel them, and identify diverse attributes before making a purchase. Such a setting in which a retailer uses several channels that interact together is referred to as an omnichannel setting. In this setting, the products offered in one channel affect the demand in other channels. In an omnichannel setting consisting of an online channel and an offline channel, we analyze how a firm can optimize assortment selection and set product prices so as to maximize profit. Our results include developing a model for a pure showcase decision problem, where all products are sold in the online channel, and a subset of these products is offered in the offline channel, which acts as a showroom only, with no sales. In this model, based on the multinomial logit demand, we incorporate how physical inspection of products modifies customer utility towards the product and affects the consumer demand model, and, accordingly, select the optimal assortment of products to showcase in the offline channel. We further extend the model to allow for price optimization and study the structure of the optimal pricing and assortment. We find that the optimal prices in the online channel are characterized by equal profit margins and that the optimal assortment in the showroom consists of "undervalued" having a valuation that improves upon physical inspection by the customer.

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CHAPTER 1

INTRODUCTION

In this chapter, we introduce our problem and present some motivations behind this research work. In section 1.1, we will introduce the topic of assortment and price optimization in an omnichannel setting and the reasoning that led to us pursuing this research topic. Then, in section 1.2, we will present the organization sequence of this thesis document.

1.1. Introduction

A growing number of firms are offering their products to customers not only through typical brick-and-mortar stores or online stores but rather through several channels at the same time. Such a setting is often referred to as an "omnichannel" setting (Leinbach Reyhle, 2021). As omnichannel retailing continues to grow in popularity, it is still a rather "undiscovered" aspect. As of a few years back, many retailers and researchers have entered the world of multichannel – or dual channel – retailing, which consists of using both an offline channel and an online channel to perform retailing activities. However, in such a setting, these two channels are considered rather independent, each having its own inventory, planning, profits, etc., even though the retailer managing both channels is ultimately the same entity. As the online world is growing even wider, channels have been increasing. For example, we can now have a channel for each social media platform, for a website, and for a mobile application, etc.

In recent years, studies have become more oriented on trying to consider these channels not as separate entities serving the same retailer but rather as interacting channels that could aid each other. This is where the omnichannel context arose, and as the world keeps evolving and retailers seek to increase their profits, a greater number of retailers are moving towards omnichannel retailing. As a simple definition, an omnichannel strategy is a form of retailing that enables consumers to shop through channels at any time and from any location by allowing real-time engagement, resulting in a special, full, and seamless shopping experience that breaks down the barriers between channels (Juaneda-Ayensa et al., 2016). Rather than working in parallel, communication channels and their supporting resources are designed and orchestrated to cooperate.

On another hand, assortment planning has been a topic of interest for researchers for over 20 years, and diverse studies have been conducted on this topic as it evolved over the years. Of the main researchers who began to embark on this journey in the operations management literature, are van Ryzin & Mahajan (1999) who discussed the relationship between inventory costs and variety benefits in retail assortment. Several other researchers took on the issue of assortment planning throughout the years. Some discussed assortment planning in relation to inventory decisions under different choice models, others looked at pricing, demand substitution, delivery times, etc. See Kök et al. (2008) and Maddah et al. (2011) for review of works on assortment planning. While a good amount of research has been conducted on assortment planning, we find that although omnichannel retailing has been growing more rapidly in recent years, there is not much literature discussing omnichannel retailing and assortment planning in a combined model. In fact, the domain of supply chain and inventory management in the omnichannel environment is scarce in the literature (Cai & Lo, 2020). Nonetheless, Rooderkerk & Kök (2020) began to explore this research area where they discussed omnichannel consumer behavior and the firm strategy. Then, they listed omnichannel assortment planning challenges: strategic challenges, tactical challenges and operational challenges. In the strategic challenges presented, they discuss the concepts of showrooming and webrooming, and the issue of channel coordination. When a customer performs the act of showrooming, it indicates that the shopper visits a store to check out a product but then purchases the product online from home. Webrooming, however, also known as reverse showrooming, is when consumers go online to research products, but then head to a brick-and-mortar store to complete their purchase. These phenomena are not particularly new to the retail world; however, it is only recently that retailers began to identify them and even try to capitalize on them (Bourg et al., 2021). Among the tactical challenges discussed by Rooderkerk & Kök (2020) are the coordination of assortment composition across channels and the assortment layout. As for operational challenges, topics covered include information provision, decision aids, inventory management and return management.

While Rooderkerk & Kök (2020) provide insight on omnichannel assortment planning in terms of concepts and theories, they do not provide mathematical models nor assortment structures for omnichannel assortment optimization. Thus, to the best of our knowledge, this topic has not been fully explored yet, and we therefore believe our research can offer a contribution towards its advancement. One important recent work on assortment planning in an omnichannel context is by Dzyabura and Jagabathula (2018) who investigate the problem of which assortment to offer in an offline channel acting mostly as a showroom for customers who visit both the offline and online channel. Our proposed work builds on the results of Dzyabura and Jagabathula (2018) by integrating a pricing decision aspect.

The main aim of our research is finding the optimal assortment of products displayed offline and the prices of products sold online to ensure the highest profit in an omnichannel setting composed of an offline (in-store) channel and an online channel. The model will be set up in a way similar to that proposed by Dzyabura and Jagabathula (2018), where the online channel contains the universal set of products while the aim is to select the best assortment of items to offer in the offline store, which will act solely as a showroom. In this sense, the offline assortment selected will be a subset of the full selection offered online. We will then explore price optimization on the offered products in a second model and investigate the effect of endogenizing prices on the structure of the optimal assortment obtained.

1.2. Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 presents a review of the related literature on both assortment and pricing planning, and on omnichannel decisions. Chapter 3 presents the general model formulation, assumptions made, and parameters defined. Chapters 4 and 5 offer analytical results and numerical results, respectively, on the structure of the optimal assortment and pricing. Finally, Chapter 6 presents the conclusion and recommendations for future extensions of this work.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we begin by viewing works related to the concept of omnichannel retailing in section 2.1, then move on to works on assortment planning with exogenous prices in section 2.2, followed by works on assortment pricing in section 2.3. We then follow-up with works on joint assortment and pricing optimization in a single channel in section 2.4, and finally, section 2.5 discusses works on assortment and pricing optimization in an omnichannel setting, which relate the most to our research.

2.1. Omnichannel Retailing

The concept of omnichannel is one of the most significant advancements in business strategy in recent years, with both practical and theoretical implications (Bell et al. 2014, Brynjolsson et al 2013, Piotrowicz & Cuthbertson 2014, Verhoef et al 2015b). The term omnichannel first appeared ten years ago (Rigby, 2011) but the concept remains unclear due to indistinct use of the concepts multi-, cross-, and omni-channel in the literature (Beck & Rygl, 2015; Klaus, 2013). While multi-channel refers to a retailer or company having a presence on multiple channels that work separately, in an omnichannel environment, the channels work together, such that customers can use digital channels for research and experience the physical store in a single transaction process. Because the channels are jointly managed, customers expect to have the same brand experience wherever and whenever they interact within the company (Piotrowicz & Cuthbertson, 2014). With the abundance of mobile technologies and social media, the omnichannel customer journey has become more complex; the simultaneous use of different communication channels by customers is facilitating the emergence of new behaviors such as showrooming and webrooming (Mosquera et. al, 2017). Some studies have taken to studying the impact of the buy-online pick-up-in-store (BOPS) in omnichannel setting on store operations (Gao & Su, 2017). Academics also explored omnichannel retailing through mathematical models that incorporate the omnichannel concept. Modak (2017) and Modak & Kelle (2018) developed a two-level omni-channel supply chain mathematical model under price and delivery time sensitive additive stochastic demands in which the retailer operates a direct online channel along with the traditional retail channel, and they analyzed the effects of delivery lead time of online marketing.

To further understand the omnichannel development and integration in modern markets, Cao (2019) used reports submitted by 91 US retailers to the New York Stock Exchange to analyze different stages of omnichannel development. According to Cao's proposed model, the integration between channels starts from a silo mode, when online and traditional are operated separately, through multi-channel with a minimal level of integration between channels. Then, the level of integration increases to multi-channel with moderate integration, and the final stage is full integration, the omnichannel. Omnichannel development is implemented through an evolutionary approach where the company is moving step by step, gradually changing its operations through integration. Studies conducted to investigate the impacts of channel integration on the customer experience confirm that channel integration significantly affects customers' channel preferences (Goraya et al, 2020). As customer preferences in an omnichannel setting continue to be studied, a quantitative approach was proposed to optimize the showcasing portfolio for a given retailer to maximize the exposure of the features that customers expect to experience from a visit to a showroom (Park et al., 2020).

2.2. Assortment Planning with Exogenous Prices

Assortment planning has been a topic of interest in operations research for over 20 years, and diverse studies have been conducted on this topic as it evolved over the years. Generally, one can say that assortment optimization refers to the problem of selecting a set of products to offer to a group of customers so that maximum revenue is realized when customers make purchases. Among early researchers who looked into assortment optimization are Smith & Agrawal (2000) who worked on developing a general probabilistic demand model for items in an assortment and captured the effect of substitution. Later on, Gaur & Honhon (2006) studied optimal assortment planning and inventory decisions for a retailer under a locational consumer choice model. Li (2007) then studied a single-period assortment optimization problem with varying cost parameters using a consumer choice process characterized by a multinomial logit model. Kök & Fisher (2007) attempted to estimate demand and optimize assortment under a model in which consumers might accept substitutes when their favorite product is unavailable, with a demand model similar to that in Smith & Agrawal (2000). Yücel et al. (2009) tackled the problem of product assortment and inventory planning under customer-driven demand substitution. They developed a flexible model to aid retailers in finding optimal assortments that allow them to maximize their expected profit. Honhon et al. (2010) also worked on determining the optimal assortment and inventory levels in a single-period problem with stockout-based substitution, while Rusmevichientong et al. (2010) looked into the assortment optimization problem subject to a capacity constraint. As more attention started being given to customer preferences, Miller et al. (2010) developed a methodology for choosing optimal retail assortments for infrequently purchased products and assessing the robustness of such assortments with regards to shifts in customer preferences.

2.3. Assortment Pricing

Another operations research topic that caught the attention of researchers over the years is pricing optimization. Pricing optimization can be considered as the practice of using data from customers and the market to find the most effective price point for a product or a service that will maximize sales or profitability. Tang & Yin (2007) tackled pricing optimization of products by developing a base model with deterministic demand to examine how a retailer should jointly determine the order quantity and the retail price of two substitutable products under the fixed and variable pricing strategies. Other researchers in the field are Dong et. al (2009) who studied dynamic pricing and inventory control of substitute products for a retailer who faces a long supply lead time and a short selling season. Most recently, Li et. al (2021) examined the retailer's customer returns policy strategy, pricing and ordering decisions in a supply chain selling seasonal products over two periods.

2.4. Joint Assortment and Pricing Optimization

While each of the two mentioned operations research topics had been predominantly studied separately, some researchers looked into integrating assortment and pricing optimization within their literatures. Maddah & Bish (2007) developed a multi-item newsvendor model with items having normal demands, under the additional complexities of pricing and assortment decisions. They used the multinomial logit choice model, considered the demand to be mixed multiplicative-additive, and used a static substitution case choice model. They derived a theorem that indicates that an optimal assortment has a popular set structure with the least cost, or the two most popular items with the least cost, and so on. Kök & Xu (2011) studied assortment planning and pricing for a product category with heterogeneous product types from two different brands. They modeled consumer choice using the nested multinomial logit framework with hierarchical structures. Katsifou et. al (2014) looked into joint product assortment, inventory and price optimization in an attempt to attract both loyal and non-loyal customers. Maddah et al. (2014) studied the structure of and the interdependence among the critical decisions on pricing, inventory and assortment of retailer's product line. They considered substitute retail products that are horizontally differentiated, adopted a multinomial logit choice model and a newsvendor-type inventory setting. They analyzed joint pricing and inventory decisions while using a multiplicative-additive demand model. Their study focused on popular-set assortments which had the highest consumer valuation. Ghoniem & Maddah (2015) examined a multi-period selling horizon where a retailer jointly optimizes assortment planning, pricing, and inventory decisions for a product line of substitutable products. They modeled the problem as a mixed-integer nonlinear program and used a deterministic demand model, generated by consumer's preferences. Building on their previous work, Ghoniem et al. (2016) investigated the joint optimization of assortment and pricing decisions for complementary retail categories constituting substitutable items and related by cross-selling considerations. Their mixed-integer nonlinear program maximized the retailer's profit by jointly optimizing assortment and pricing decisions for multiple categories under a classical deterministic maximum-surplus consumer choice model.

2.5. Omnichannel Assortment and Pricing Optimization

While omnichannel retailing has been growing more rapidly in recent years, there is not much literature discussing omnichannel retailing in an assortment planning context. However, Rooderkerk & Kök (2020) discuss omnichannel assortment optimization without providing mathematical models that can be used to advance the omni-channel assortment research development. Dzyabura and Jagabathula (2018) embarked on a study of a firm's showcase decision in which they determine the subset of products from the online channel to offer in the offline channel in such a way that maximizes aggregate sales or profits across both channels. They contribute to the literature by focusing on multi-attribute products that are sold by a single firm in a multichannel setting and studying assortment decisions under exogenous pricing. This study takes into account the fact that the assortment may change the product a customer would purchase because of the touch-and-feel information provided by the offline channel. This is due to the change in customer utility of a product before and after seeing it in a live setting. Their model captures the interaction between the online and offline channels, provides analytical results on the structure of the optimal offer sets, and they provide a scalable integer programming-based optimization algorithm to solve the firm's showcase decision. Their modeling contribution is to extend the standard utility model to allow the capture of the impact of physical evaluation through changing attribute part-worths. The model presented by Dzyabura and Jagabathula (2018) is for products that are considered expensive and include a high-involvement decision, and are differentiated along more than one dimension.

As for the literature discussing price optimization in an omnichannel setting, Gupta et al (2019) presented a price optimization problem for a retailer who had many offline stores but then added an online channel to improve its digital presence. Their paper takes an integrated approach to price-optimization, inventory control and e-fulfillment problem and develops a decision support model for such a retailer having multiple objectives of profit maximization and lost sales minimization. Another paper that looks into assortments in omnichannel environment is a recent paper by Zhang & Zheng (2020) that uses a two-dimensional open cylinder model to capture customers' spatial locations and preferences over product characteristics at the same time. They integrate three research streams: channel competition, pricing decisions and customization strategy. Their model allows both online and offline firms to determine their optimal product variety. This paper assumed one e-tailer (electronic retailer) and multiple retailers, and their study incorporated game theory by modeling several scenarios, producing each scenario's profit function for both e-tailer and retailer. This allowed the model to predict the best (most profitable) subgame scenario for each of the e-tailer and the retailers. However, among the limitations of this model is that it may not be suitable for certain goods and that product customization in their model is conducted only along a single dimension.

As the literature lacks extensive research in the area of assortment optimization in an omnichannel setting, our work will contribute to the literature by extending the work of Dzyabura & Jagabathula (2018) by endogenizing pricing decisions. In an omnichannel setting, our work will focus on joint assortment and pricing decisions of relatively expensive products, such as bags, with customizations along more than one dimension. The model we present allows for assortment optimization for the offline channel jointly with the prices of all offered products, assuming the online channel includes all these products.

CHAPTER 3

MODEL FORMULATION AND ASSUMPTIONS

In this chapter, we present the model used in this research work. In section 3.1, we introduce the problem statement indicating the objective behind this work. Then, section 3.2 presents the model formulation and assumptions used throughout.

3.1. Problem Statement

Our preliminary objective is to determine the assortment of products to "showcase" – i.e. to offer – in the offline channel to maximize the profit from the online channel. We focus on a firm selling products through an online channel and showrooming some of these products in the offline channel (a brick-and-mortar store). The products considered in the model are reasonably high-priced and infrequently purchased. Therefore, customers can benefit from an offline store visit before choosing a product to purchase. Such products can be furniture, designer bags, apparel, etc. In our "pure showcase" setting, the firm does not carry inventory in the offline stores and sells products only through the online channel. In this setting, the offline store acts only as a showroom while purchases are made through the online channel. This is similar to one of the models presented by Dzyabura & Jagabathula (2018).

3.2. Model Formulation and Assumptions

The products considered in the model are close substitutes but are differentiated along K pre-specified attributes. Attribute k, $k = 1, \ldots, K$, can take on a "level" l that belongs to a predefined set L_k. Example of levels can be small, medium, and large, for the size attribute, and black, blue, and red for the color attribute. The set L is the Cartesian product of all attribute sets L_k, i.e. $L = L_1 \times L_2 \times \ldots \times L_k$. A product is defined by the set of attributes it has $\mathbf{x} \in L$. Equivalently, a product can be represented by $\mathbf{x} = (x_1, x_2, \ldots, x_K)$, where $x_k \in L_k$, $k = 1, \ldots, K$.

Assuming that all customers visit the offline channel before shopping online, the model explores how exposure to a product in the offline channel impacts the purchase behavior of customers. To model this impact, we suppose that customers associate different utility part-worths with each feature of a product, depending on whether they were exposed to the feature in the offline channel or not. The difference between the online and offline part-worths for an attribute-level may be interpreted as being caused by the information gained by the customer from “touching and feeling” the particular attribute-level in the offline store (Dzyabura and Jagabathula, 2018). For example, a customer may think she likes the red color but changes her mind upon physical inspection and decides she prefers other non-tropical colors like black.

The utility of a product is considered to be of an additive nature in which the utility is decomposed into the sum of utilities of the product’s constituent attributes. For example, the utility for a large red bag is the sum of the utility for the base bag, the utility for the large level of the size attribute, and the utility for the red level of the color attribute.

Let w_{kl}^{on} be the online utility part-worth assigned to level $l \in L_k$, of attribute k, $k = 1, \ldots, K$. We define a utility part-worth w_{kl}^{off} associated with a customer who has seen the product in the offline channel and then her utility for this product changed from w_{kl}^{on} to w_{kl}^{off} accordingly. We can also say that w_{kl}^{off} is the customer utility part-worth if the customer was exposed to feature (k, l) in the offline store and w_{kl}^{on} is the customer utility part-worth if the customer was *not* exposed to feature (k, l) in the offline store. When $w_{kl}^{\text{off}} > w_{kl}^{\text{on}}$, we say that the level l of attribute k is undervalued by the customer who shops online. Otherwise, when $w_{kl}^{\text{off}} < w_{kl}^{\text{on}}$, we say that the level l of attribute k is overvalued by the customer who shops online. Dzyabura & Jagabathula (2018) rely heavily on this undervaluation/overvaluation of attributes in deriving their results on the structure of the optimal assortment, as explained in the sequel.

Let M be the assortment of products offered (for display only) in the online channel. The assortment M affects the sales of the online channel through the level of the attributes of its products. To capture this, let S_k^M be the levels of attribute k offered in assortment M. For example, if only red and black bags are offered in M, and the color attribute is numbered 1, then $S_1^M = \{\text{Red}, \text{Black}\}$. Then, for a product offered online, the part-worth of attribute k will depend on whether the level of this attribute is offered in M. Specifically, the utility, $U(\mathbf{x}, M)$, of a product $\mathbf{x} \in L$ is additive in the part-worth of attributes in $\mathbf{x}$, taking into account the effect of the offline assortment, and is linearly decreasing, in the price of $\mathbf{x}$, $p_{\mathbf{x}}$. That is,

$$U(\mathbf{x}, M) = \left[\sum_{k=1}^{K} u_k(\mathbf{x}, M)\right] - p_{\mathbf{x}} + \varepsilon_{\mathbf{x}} \qquad (1)$$

where $u_k(\mathbf{x}, M) = \sum_{l \in S_k^M} w_{kl}^{\text{off}} \theta_l(x_k) + \sum_{l \notin S_k^M} w_{kl}^{\text{off}} \theta_l(x_k)$, with $\theta_l(x_k) = 1$, if $x_k = l$, and $\theta_l(x_k) = 0$, otherwise, and $\varepsilon_{\mathbf{x}}$ are independent and identically distributed Gumbel random variables (iid) with mean 0 and shape factor 1. A customer chooses to buy Product $\mathbf{x}$ from the online channel if the utility of $\mathbf{x}$ is larger than that of all other products in the attribute space L and that of the no-purchase option, $U(0, M)$.

Assuming, without loss of generality, that $U(0, M) = \varepsilon_0$, where ε_0 is also i.i.d with $\varepsilon_{\mathbf{y}}$, $\mathbf{y} \in L$. It follows that the probability that a customer purchases product $\mathbf{x}$ from the online channel is: $q_{\mathbf{x}}(M) = P\{U(\mathbf{x}, M) = \max_{\mathbf{y}\in L\cup\{0\}} U(\mathbf{y}, M)\}$. Letting $\mathbf{p} = (p_1, p_2, \ldots, p_{|L|})$ denote the prices of all the products in the attributes space, and applying well-known results on the logit demand model (e.g. Maddah et al. 2014), it follows that:

$$q_{\mathbf{x}}(M, \mathbf{p}) = \frac{\exp\{[\sum_{k=1}^{K} u_k(\mathbf{x}, M)] - p_{\mathbf{x}}\}}{1 + \sum_{\mathbf{y}\in L} \exp\{[\sum_{k=1}^{K} u_k(\mathbf{y}, M)] - p_{\mathbf{y}}\}} \qquad (2)$$

The expected retailing profit, assuming a market size of 1, is then given by

$$\Pi(M, \mathbf{p}) = \sum_{\mathbf{x}\in L} (p_{\mathbf{x}} - c_x)\, q_{\mathbf{x}}(M, \mathbf{p}) \qquad (3)$$

Finally, the retailer's objective of selecting the assortment and pricing in a way that maximizes profit can be written as

$$\max_{M\subseteq L} \max_{\mathbf{p}} \Pi(M, \mathbf{p}) \qquad (4)$$

We base the numerical results in the sequel on the expected profit formulation in (3) and the profit maximization objective in (4).

CHAPTER 4

ANALYTICAL RESULTS

This chapter presents the analytical results obtained from this research. Section 4.1 provides the structure of the optimal assortment of products to offer offline, then section 4.2 offers the structure of the optimal pricing of products that are sold online. Section 4.3 then provides a sensitivity analysis to test the effects of utility part-worth and production cost, and, finally, section 4.4 presents the structure of the jointly optimal offline assortment and online pricing of products.

4.1. Structure of Optimal Assortment

The structure of the profit maximizing subset of attribute levels to offer offline is as follows

Theorem 1 Pure Showcase Profit Max Solution Structure (Dzyabura and Jagabathula, 2018). For given online prices, any optimal solution $\boldsymbol{S}^* = (S_1^*, \ \dots, S_k^*)$ *to the pure showcase profit maximizing problem satisfy*

$$\{l \in L_k^+ : r_{kl} > t_k^*\} \subseteq S_k^{*+} \subseteq \{l \in L_k^+ : r_{kl} \geq t_k^*\},$$

$$\{l \in L_k^- : r_{kl} < t_k^*\} \subseteq S_k^{*-} \subseteq \{l \in L_k^- : r_{kl} \leq t_k^*\},$$

where $t_k^* \coloneqq R_k(S_k^*) - \frac{R(\boldsymbol{S}^*)}{D(\boldsymbol{S}^*)}$ *where* $R(S)$ *is the expected profit function,* $D(S)$ *is the expected sales function,* r_{kl} *is the profit margin associated with level* l *of attribute* k*,* L_k^+ *comprises the set of undervalued attributes and* L_k^- *comprises the set of overvalued attributes,* S_k^+ *denotes* $S_k \cap L_k^+$ *and* S_k^- *denotes* $S_k \cap L_k^-$ *for any subset* $S_k \subseteq L_k$.

The theorem establishes that it is optimal to offer offline the most profitable undervalued attribute levels and the least profitable overvalued attribute levels. Because offering undervalued levels increases their attractiveness and offering overvalued levels decreases their attractiveness, the result provides the following intuitive suggestion: increase the attractiveness of the most profitable levels and decrease the attractiveness of the least profitable levels.

4.2. Structure of Optimal Pricing

Optimal pricing depends on the optimal profit margin, since price is known to be the cost plus the profit margin. Therefore, we explore the structure of the optimal profit margin, which is as follows

Theorem 2 Optimal Profit Margin (Li & Huh, 2011). If an offline assortment is given, and the utility partworths α for products are also given, then all products in the online assortment have the same profit margin m^* defined by

$$m^* = 1 + W\left(\sum_{x \in L}^{n} e^{\alpha_x - c_x - 1}\right), \text{ where } \alpha_{\boldsymbol{x}} = \sum_{k=1}^{K} U_k(\boldsymbol{x}, M).$$

where $W(z)$ refers to the Lambert W function which is defined by the solution to the equation $we^w = z$.

Utilizing Mathcad, we found that the following form of the W function provided more accurate results, $W(z) := root(\ln(w) + w - \ln(z),\ w,\ 0.00001,\ 100)$.
The behavior of the W function is shown in Figure 1.

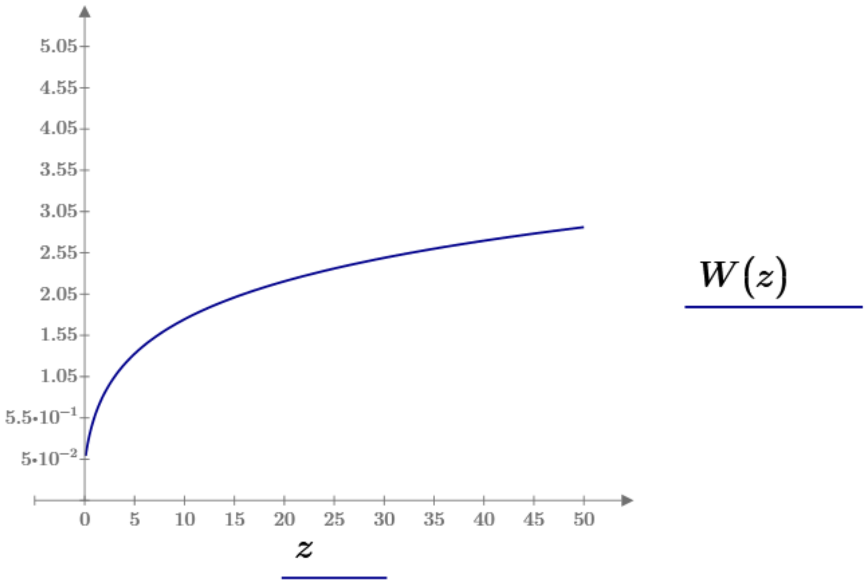

Figure 1 Plot of Lambert W Function

Thus, we can say that the optimal pricing of a product is

$$p_x^* = m^* + c_x.$$

4.3. Sensitivity Analysis

We perform a sensitivity analysis to test the effect of utility part-worth, α, and the production cost, c, on the profit margin and, consequently, on the optimal product pricing. We summarize our findings in the lemmas presented below.

Lemma 1: The optimal profit margin m_s^* is increasing in α_x.

This lemma is proved in the Appendix.

Lemma 2: The optimal profit margin m_s^* is decreasing in c_j.

This lemma is proved in the Appendix.

4.4. Heuristic for the Structure of the Jointly Optimal Assortment and Pricing

For the jointly optimal assortment and pricing, fixing the offline assortment at $\boldsymbol{S^*}$ should give product prices having equal profit margins, and fixing the profit margin m at $\boldsymbol{m^*}$ should give the structure presented by Dzyabura and Jagabathula (2018). Noting that fixing the profit margin m causes the concepts of most profitable and least profitable presented by Dzyabura and Jagabathula (2018) to become irrelevant, we conjecture that the optimal offline assortment structure follows that for equal profit margins in the pure showcase omnichannel setting, we include in the offline assortment only the most undervalued attribute levels.

To simplify the search for the optimal assortment, we propose a heuristic that exploits assortments with the most undervalued attribute levels. The heuristic starts with a single product assortment having the most undervaluation level, then adds a second product with the next highest undervaluation level, and then adds a third product with the third highest undervaluation level, and so on. In a space with L products, this heuristic exploits $|L|$ assortments, whereas full enumeration requires $2^L - 1$ assortments.

CHAPTER 5

NUMERICAL RESULTS

In this chapter, we begin by presenting the setting of Example 1 in Section 5.1, then move on to the estimation of the base parameters for this example in Section 5.1. Section 5.3 then presents some variants of Example 1, followed by Example 2 in Section 5.4. We then validate the analytical results obtained from Chapter 4 in Section 5.5, and finally, in Section 5.6, we present sensitivity plots related to the lemmas previously presented in Chapter 4.

5.1. Example 1 Setting

In the initial setting of the model, we present a numerical example in which we are offering one product, which is a designer bag.

These bags have two attributes ($K = 2$),

- Attribute 1: “color”, which has two levels ($|L_1| = 2$),
    - Level 1 : “black,”
    - Level 2: “red,”
- Attribute 2: “size”, which has 2 levels ($|L_2| = 2$),
    - Level 1: “small,”
    - Level 2: “large,”

Suppose that the online channel has all four possible products (universe of feasible products) – small black bag, large black bag, small red bag and large red bag – and our goal is to decide which item(s) to offer in the offline channel to maximize profits.

The offered products’ symbology follows the {k, l} form, shown in Table 1.

Table 1 Products for Example 1

Product	**Symbol**
Small Black	{1, 1}
Large Black	{1, 2}
Small Red	{2, 1}
Large Red	{2, 2}

The following cases, which amount to a total of 15 ($2^4 - 1$) assortments, were explored to determine the optimal assortment to offer offline.

Case 1**:** Only one product $\boldsymbol{x}$ is offered offline:

- {1, 1}, {1, 2}, {2, 1} or {2, 2}

Case 2**:** For the case of two products offered offline, six assortment options are considered as to what to offer, as follows:

- {1, 1} and {2, 1}
- {1, 1} and {1, 2}
- {1, 1} and {2, 2}
- {1, 2} and {2, 1}
- {1, 2} and {2, 2}
- {2, 1} and {2, 2}

Case 3**:** For the case of three products offered offline, four assortment options are considered as to what to offer, and they are as follows:

- {1, 1}, {1, 2} and {2, 1}
- {1, 1}, {1, 2} and {2, 2}
- {1, 1}, {2, 1} and {2, 2}
- {1, 2}, {2, 1} and {2, 2}

Case 4**:** The last case would be to offer all 4 products offline:

- {1, 1}, {1, 2}, {2, 1} and {2, 2}

5.2. Estimation of Example 1 Base Parameters

To set the initial exogenous prices of the products, we first set a base price of $140 for the small black bag, which we considered as the base product. We assumed a hedonic price model in which the product price is “feature-based” and configurable. This allows us to obtain prices for customized products. We therefore assume a customization cost of $10 for change of color (from black to red) and $10 for change of size (from small to red). As the price of each product is of additive nature, one can then say that, for example, the price of a large black bag consists of the base price, $140, plus a customization cost of $10 for changing the size from small to large, making its price a total of $150.

Table 2 Preliminary Product Prices for Example 1

Product	Price ($)
Small Black Bag, {1, 1}	140
Large Black Bag, {1, 2}	150
Small Red Bag, {2, 1}	150
Large Red Bag, {2, 2}	160

To find an approximate value for the cost of the base product, the gross profit margin for the industry was obtained from Industry ratios (benchmarking): Gross margin (Ready Ratios, 2019) based on the financial statements of companies in the industry that were submitted to the U.S Securities and Exchange Commission (SEC). For the leather & leather products industry, in which the products in this example lie, an average value of 58% between the years 2014 and 2019 is adopted to find the cost of the production of the products.

Table 3 Production Costs for Example 1

Product	**Cost ($)**
Small Black Bag, {1, 1}	58.8
Large Black Bag, {1, 2}	63
Small Red Bag, {2, 1}	63
Large Red Bag, {2, 2}	67.2

As for product utility part-worth, we adopt a standard multi-attribute utility model in which utility of a product is the sum of the utilities of its attributes. We assume that when customers visit the offline store, they prefer black to exotic colors such as red, therefore black bags are assumed to be undervalued while red bags are assumed to be overvalued. Moreover, it was further assumed in this example that customers prefer simple small bags over large bags.

Table 4 Product Utility Part-worth for Example 1

Product, {k, l}	**Online Part-worth, w_{kl}^{on} ($)**	**Offline Part-worth, w_{kl}^{off} ($)**
Small Black Bag, {1, 1}	142	144
Large Black Bag, {1, 2}	152	154
Small Red Bag {2, 1}	151	150
Large Red Bag {2, 2}	161	160

5.3. Example 1 Variants

To further verify our numerical findings, we present several variants of the example in which we modified which products are undervalued or overvalued, or changed the profitability of products, etc. By changing some parameters, eight different scenarios were tested, and results were observed to verify our findings regarding the structure of the optimal assortment and pricing.

In the first four scenarios, we vary undervaluation and overvaluation of products without modifying profitability of products. Originally, products {1, 1} and {1, 2} were undervalued products with online utility part-worth values of 144 and 154, respectively, and offline utility part-worth values of 142 and 152, respectively. Products {2, 1} and {2, 2} were originally overvalued products with online utility part-worth values of 150 and 160, respectively, and offline utility part-worth values of 152 and 162, respectively. However, this no longer holds in the following four scenarios in which we change overvaluation and undervaluation of the products.

- In the first scenario, we explore results when we set products {1, 2} and {2, 2} to be undervalued with offline utility part-worth values of 154 and 163, respectively while products {1, 1} and {2, 1} to be overvalued with offline utility part-worth values of 140 and 150, respectively.
- In the second scenario, we explore results when we set products {1, 1} and {2, 1} to be undervalued with offline utility part-worth values of 144 and 152, respectively while products while products {1, 2} and {2, 2} to be overvalued with offline utility part-worth values of 150 and 160, respectively.
- In the third scenario, we explore results when all products are overvalued with offline utility part-worth values of 140, 151, 150 and 158 for products {1, 1}, {1, 2}, {2, 1} and {2, 2}, respectively.
- In the fourth scenario, we explore results when all products are undervalued with offline utility part-worth values of 144, 154, 152 and 164 for products {1, 1}, {1, 2}, {2, 1} and {2, 2}, respectively.

In the fifth to eighth variants, we do not modify undervaluation and overvaluation of products, but instead we change the profitability of products by changing their production costs.

- In the fifth scenario, we decrease the production cost of product {1, 1} from $58.8 to $50.
- In the sixth scenario, we increase the production cost of product {1, 2} from $63 to $70.
- In the seventh scenario, we decrease the production cost of product {2, 1} from $63 to $55.
- In the eighth scenario, we increase the production cost of product {2, 2} from $67.2 to $75.

5.4. Example 2 Setting and Parameters

Similar to example 1, products offered in this example are designer bags. These bags have two attributes (K = 2),

- Attribute 1: "size", which has three levels ($|L_2|$ = 3),
    - Level 1: "small,"
    - Level 2: "medium,"
    - Level 3: "large,"
- Attribute 2: "color", which has three levels ($|L_1|$ = 3),
    - Level 1 : "black,"
    - Level 2: "blue,"
    - Level 2: "red,"

The offered products' symbology follows the {k, l} form, as shown in Table 5.

Table 5 Products for Example 2

Product	**Symbol**
Small Black Bag	{1, 1}
Small Blue Bag	{1, 2}
Small Red Bag	{1, 3}
Medium Black Bag	{2, 1}
Medium Blue Bag	{2, 2}
Medium Red Bag	{2, 3}
Large Black Bag	{3, 1}
Large Blue Bag	{3, 2}
Large Red Bag	{3, 3}

To set the initial exogenous prices of the products, we first set a base price of $140 for the small black bag, which we considered as the base product. We assumed a hedonic price model in which the product price is "feature-based" and configurable. This allows us to obtain prices for customized products. We therefore assumed a customization cost of $5 for change of color from black to blue, $5 for change of color from black to red, $10 for change of size from small to medium, $15 for change of size from small to large.

Table 6 Preliminary Product Prices for Example 2

Product	**Price ($)**
Small Black Bag, {1, 1}	140
Small Blue Bag, {1, 2}	145
Small Red Bag, {1, 3}	145
Medium Black Bag, {2, 1}	150
Medium Blue Bag, {2, 2}	155
Medium Red Bag, {2, 3}	155
Large Black Bag, {3, 1}	155
Small Red Bag, {3, 2}	160
Large Red Bag, {3, 3}	160

Finding the cost for production was done the same way used for Example 1. The cost values can be seen in the table below.

Table 7 Production Costs for Example 2

Product	Cost ($)
Small Black Bag, {1, 1}	58.8
Small Blue Bag, {1, 2}	60.9
Small Red Bag, {1, 3}	60.9
Medium Black Bag, {2, 1}	63
Medium Blue Bag, {2, 2}	65.1
Medium Red Bag, {2, 3}	65.1
Large Black Bag, {3, 1}	65.1
Small Red Bag, {3, 2}	67.2
Large Red Bag, {3, 3}	67.2

As for product utility part-worth, we adopt a standard multi-attribute utility model in which utility of a product is the sum of the utilities of its attributes. We use the following product utility part-worth values.

Table 8 Product Utility Part-worth in Example 2

Product	Online Part-worth, α^{on} ($)	Offline Part-worth, α^{off} ($)
Small Black Bag, {1, 1}	145	147
Small Blue Bag, {1, 2}	147	148
Small Red Bag, {1, 3}	147	147
Medium Black Bag, {2, 1}	154	157
Medium Blue Bag, {2, 2}	158	158
Medium Red Bag, {2, 3}	158	157
Large Black Bag, {3, 1}	159	159
Small Red Bag, {3, 2}	162	160
Large Red Bag, {3, 3}	162	159

5.5. Validation of Analytical Results

The numerical examples were used to validate the theorems presented in Chapter 4. The results obtained can be summarized into two categories: results for offline assortment optimization and results for the jointly optimal offline assortment and online pricing.

5.5.1. Results for Offline Assortment Optimization

To find the optimal offline assortment, we apply theorem 1 from Chapter 4 as follows. The first step in finding that optimal assortment is to group attribute levels into undervalued and overvalued. We then arrange undervalued attribute levels from most to least profitable, and overvalued attribute levels from least to most profitable. We then select attribute levels to include in the optimal assortment based on the size of assortment needed. For assortment size one, we select the most profitable undervalued attribute level; for size two, we select the most profitable undervalued attribute level and the least profitable overvalued attribute level. Then moving to size three, we add to the assortment either the second most profitable undervalued attribute level or the second least profitable overvalued attribute level, and then enumerate these two options to select the one with the higher profit. We then continue following this procedure until we reach the assortment of size n.

Thus, for Example 1, assortment size one would include product {1, 2}, assortment size 2 would include {1, 2} and {2, 1}, assortment size three would include {1, 2}, {2, 1} and either {1, 1} or {2, 2}. To verify the results achieved by following the heuristic, we also tried enumerating each of the 15 assortment combinations and selecting the assortment that provides the highest value for maximum profit. The results were consistent with the heuristic, as can be seen in the table below.

Table 9 Results for Assortment Optimization of Example 1

Items Offered Offline	Max Profit ($)
{1, 1}	81.3309
{1, 2}	**85.3326**
{2, 1}	81.1481
{2, 2}	80.6370
{1, 1} & {1, 2}	83.6454
{1, 1} & {2, 1}	81.1848
{1, 1} & {2, 2}	81.0354
{1, 2} & {2, 1}	**85.2896**
{1, 2} & {2, 2}	85.1402
{2, 1} & {2, 2}	80.0220
{1, 1}, {1, 2} & {2, 1}	83.5948
{1, 1}, {1, 2} & {2, 2}	83.5073
{1, 1}, {2, 1} & {2, 2}	80.8777
{1, 2}, {2, 1} & {2, 2}	**85.0911**
{1, 1}, {1, 2}, {2, 1} & {2, 2}	83.4538

To further verify the optimal assortment structure presented by, we present the optimal offline assortment for each of the model variants as well, and they can be summarized as follows:

Table 10 Summary of Results for Assortment Optimization of Variants of Example 1

Model	Assortment Optimization	
	Optimal Assortment	Max Profit ($)
Variant 1	{1, 1}, {2, 1}, {2, 2}	87.7776
Variant 2	{2, 1}	82.5925
Variant 3	{1, 1}	81.8040
Variant 4	{2, 2}	89.5558
Variant 5	{1, 1}, {2, 1}	88.3876
Variant 6	{1, 1}	80.5749
Variant 7	{1, 2}	85.6504
Variant 8	{1, 2}, {2, 2}	85.0233

For each of the eight variants presented in Table 10, we notice that the assortment structure holds as the optimal assortment includes the most profitable undervalued attribute level and the least profitable overvalued attribute level.

Further to these results, we also present the results obtained by Example 2. As the example is large with $2^9 - 1 = 511$ possible assortments, we will present the heuristic followed to obtain the optimal assortment and then showcase the results with the optimal assortment relative to each assortment size. Following the heuristic, the optimal assortment relative to each assortment size would be as follows:

- For assortment size 1, we consider the most profitable undervalued attribute levels, i.e. product {2, 1}.
- For assortment sizes 2 to 5, we take assortment of size 1 and add to it one of the three products which are at value {1, 3}, {1, 2} or {3, 1}
- For assortment size 6, we add to assortment of size 5 the least profitable overvalued attribute level, i.e. product {3, 2}.
- For assortment sizes 7 and 8, we consider adding to the previous assortment size either the second most profitable undervalued attribute level {1, 2} or the second least profitable overvalued attribute level {3, 3}.
- For assortment sizes 8 and 9, we consider add into the previous assortment either the third most profitable undervalued attribute level {1, 1} or the third least profitable overvalued attribute level {2, 3}.

As verification to the results of the heuristic followed, we enumerated all possible assortments and came to the conclusion that the steps followed by the heuristic provide valid results, thus eliminating the need to enumerate a very large number of assortment combinations.

Table 11 Optimal Assortments for Example 2

Assortment Size	Optimal Assortment
1	{2, 1}
2	{1, 3}, {2, 1} OR {2, 1}, {2, 2} OR {2, 1}, {3, 1}
3	{1, 3}, {2, 1}, {2, 2} OR {1, 3}, {2, 1}, {3, 1} OR {2, 1}, {2, 2}, {3, 1}
4	{1, 3}, {2, 1}, {2, 2}, {3, 1}
5	{1, 3}, {2, 1}, {2, 2}, {3, 1}, {3, 2}
6	{1, 3}, {2, 1}, {2, 2}, {3, 1}, {3, 2}, {3, 3}
7	{1, 2}, {1, 3}, {2, 1}, {2, 2}, {3, 1}, {3, 2}, {3, 3}
8	{1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}, {3, 2}, {3, 3}
9	{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}, {3, 2}, {3, 3}

5.5.2. Results for the Jointly Optimal Offline Assortment and Online Pricing

To find the optimal offline assortment, we present a heuristic to be followed. The first step in finding that optimal assortment is to arrange attribute levels from most to least undervalued. We then select attribute levels to include in the optimal assortment from most to least undervalued.

For Example 1, we arrange products from most to least undervalued as follows: {1, 2} > {1, 1} > {2, 1} > {2, 2}. Then, we note that assortment of size 1 would include product {1, 2}, that of size 2 would add to it product {1, 1}, that of size 3 would to it product {2, 1} and finally the assortment of size 4 would add to it product {2, 2}.
We then present the numerical results for the jointly optimal offline assortment and online pricing upon endogenizing prices and enumerating each of the assortment combinations to verify the assortment selection we made (i.e. verify that the assortment that provides the highest value for maximum profit is the one selected.)

Table 12 Results for Joint Assortment and Price Optimization of Example 1

Items Offered Offline	**Max Profit ($)**
{1, 1}	88.3303
{1, 2}	**88.3782**
{2, 1}	88.3283
{2, 2}	87.3602
{1, 1} & {1, 2}	**88.3804**
{1, 1} & {2, 1}	88.3284
{1, 1} & {2, 2}	87.3606
{1, 2} & {2, 1}	88.3785
{1, 2} & {2, 2}	87.4886
{2, 1} & {2, 2}	87.3552
{1, 1}, {1, 2} & {2, 1}	**88.3786**
{1, 1}, {1, 2} & {2, 2}	87.4889
{1, 1}, {2, 1} & {2, 2}	87.3556
{1, 2}, {2, 1} & {2, 2}	87.4842
{1, 1}, {1, 2}, {2, 1} & {2, 2}	87.4845

We present the results of the joint assortment and price optimization for each of the model variants as well, and they can be summarized as follows:

Table 13 Summary of Results for Joint Assortment and Price Optimization of Example 1 Variants

Model	**Joint Assortment and Price Optimization**	
	Optimal Assortment	**Max Profit ($)**
Variant 1	{1, 2} & {2, 2}	90.3054
Variant 2	{1, 1} & {2, 1}	88.3352
Variant 3	{1, 1}	88.3301
Variant 4	{1, 2}, {2, 1} & {2, 2}	91.2881
Variant 5	{1, 1} & {1, 2}	89.1363
Variant 6	{1, 1}, {1, 2} & {2, 1}	87.3384
Variant 7	{1, 1} & {1, 2}	90.6046
Variant 8	{1, 2}	85.6056

In each of the variants, we note that the optimal assortment follows the structure presented in the heuristic.

We nonetheless present the results obtained for Example 2, in which, following the heuristic leads to arranging products as follows:

{2, 1} > {1, 1} > {1, 2} > {1, 3} > {2, 2} > {3, 1} > {2 ,3} > {3, 2} > {3, 3}.

Thus, we start assortment of size 1 with the most undervalued attribute level given by product {2, 1}, then include {1, 1} in assortment size 2, and move forward in the same way, including the next most undervalued attribute level as the assortment size increases. To verify this structure, we enumerated all possible assortment combinations (511 combination) and summarized the optimal assortment for each assortment size as follows:

Table 14 Jointly Optimal Assortments for Example 2

Assortment Size	**Optimal Assortment**	**Max Profit ($)**
1	{2, 1}	91.4444
2	{1, 1}, {2, 1}	91.4448
3	{1, 1}, {1, 2}, {2, 1}	91.4449
4	{1, 1}, {1, 2}, {1, 3}, {2, 1}	91.4449
5	{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 3}	91.4449
6	{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}	91.4449
7	{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}, {3, 2}	91.4148
8	{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}, {3, 2}, {3, 3}	91.0867

5.5.3. Discussion of Results

Upon looking into the results of the assortment optimization examples presented, we note that the optimal assortment structure presented by Dzyabura and Jagabathula (2018) holds, as the optimal offline assortment includes the most profitable undervalued attribute level and the least profitable overvalued attribute level. This was shown to be true for the original model of Example 1, the eight variants of Example 1 that were presented and for Example 2.

Moreover, we examined the structure of the optimal offline assortment upon endogenizing prices and found that the optimal structure follows that the optimal assortment would include the most undervalued attribute levels. We also notice a change in the size of the offline assortment of products to offer between the assortment optimization and the joint assortment and pricing optimization. However, no particular pattern was deduced at this point, and further investigation in that matter could be adopted in future extensions of this work.

Upon looking into the maximum profit margin achieved by each of the two optimization problems, we find that jointly optimizing the assortment structure and product prices achieves a higher profit margin than that achieved by Dzyabura and Jagabathula (2018) for the optimal assortment structure. What is interesting about the increase is its magnitude, as we notice an average of 6% increase in profit achieved when jointly optimizing pricing and assortment structure.

Table 15 Summary of Results of Optimization Problems for Example 1

Items Offered Offline	**Max Profit ($)**		**% Increase**
	Assortment Optimization	**Joint Assortment and Price Optimization**	
{1, 1}	81.3309	88.3303	8.61%
{1, 2}	**85.3326**	88.3782	3.57%
{2, 1}	81.1483	88.3283	8.85%
{2, 2}	80.6370	87.3602	8.34%
{1, 1} & {1, 2}	83.6454	**88.3804**	5.66%
{1, 1} & {2, 1}	81.1848	88.3284	8.80%
{1, 1} & {2, 2}	81.0354	87.3606	7.81%
{1, 2} & {2, 1}	85.2896	88.3785	3.62%
{1, 2} & {2, 2}	85.1402	87.4886	2.76%
{2, 1} & {2, 2}	80.0220	87.3552	9.16%
{1, 1}, {1, 2} & {2, 1}	83.5948	88.3786	5.72%

Items Offered Offline	Max Profit ($)		% Increase
	Assortment Optimization	Joint Assortment and Price Optimization	
{1, 1}, {1, 2} & {2, 2}	83.5073	87.4889	4.77%
{1, 1}, {2, 1} & {2, 2}	80.8777	87.3556	8.01%
{1, 2}, {2, 1} & {2, 2}	85.0911	87.4842	2.81%
{1, 1}, {1, 2}, {2, 1} & {2, 2}	83.4538	87.4845	4.83%

5.6. Sensitivity Plots

To validate the results of the sensitivity analysis presented in Chapter 4, we plotted the profit margin of product {1, 1} with respect to its utility part-worth, as well as plotted the profit margin of product {2, 2} with respect to its production cost. The lemmas previously presented were validated graphically as the plots show that the profit margin is increasing with the increase in the utility part-worth and decreasing with the increase in production cost.

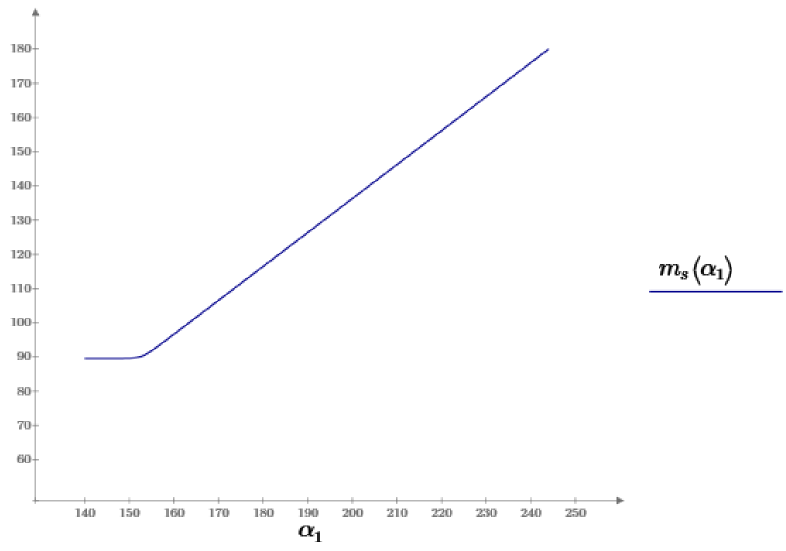

Figure 2 Sensitivity of Profit Margin w.r.t Product Utility Part-worth

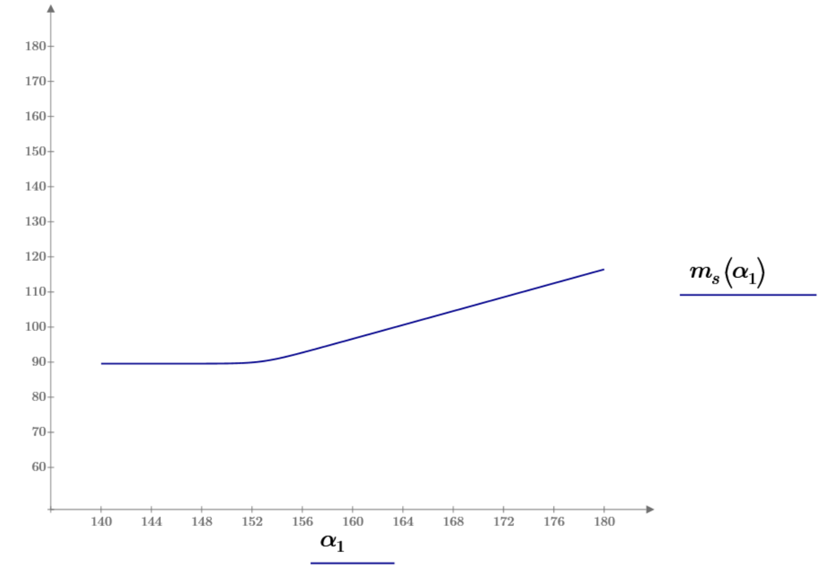

Figure 3 Close-Up of Profit Margin w.r.t Product Utility Part-worth

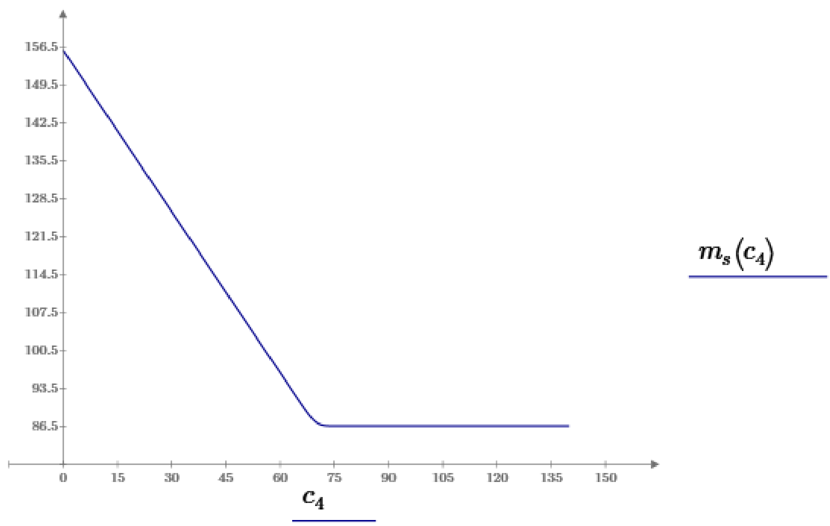

Figure 4 Sensitivity of Profit Margin w.r.t Product Cost

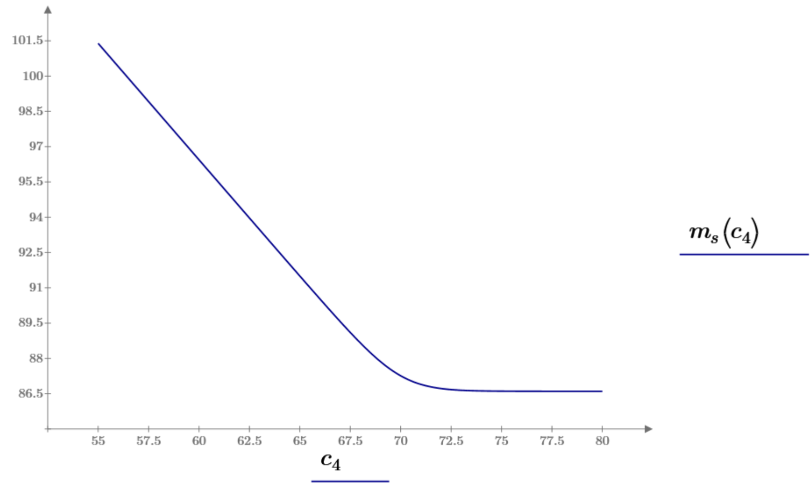

Figure 5 Close-up of Sensitivity of Profit Margin w.r.t Product Cost

To further comment on the plots presented, we notice that each of the two plots could be divided into two sections, one section is linearly increasing or decreasing, and the second section is relatively constant. The relatively constant section is due to the fact that when the product utility partworth (cost) is above (below) a certain threshold, this product no longer contributes to the profit margin, indicating that it no longer contributes to the assortment. As for the linearly increasing or decreasing sections in these plots, their linearity is due to the behavior of the Lambert W function as $z \to +\infty$ (Belkić, 2018). As Belkić (2018) proves, the Lambert W function becomes asymptotic as $z \to +\infty$ according to the following equation, $W(z)_{z\to+\infty} \approx \ln\left(\frac{z}{\ln(z)}\right)$, and this behavior can be seen in the plot of $W(z)$ as $z \to +\infty$ presented below.

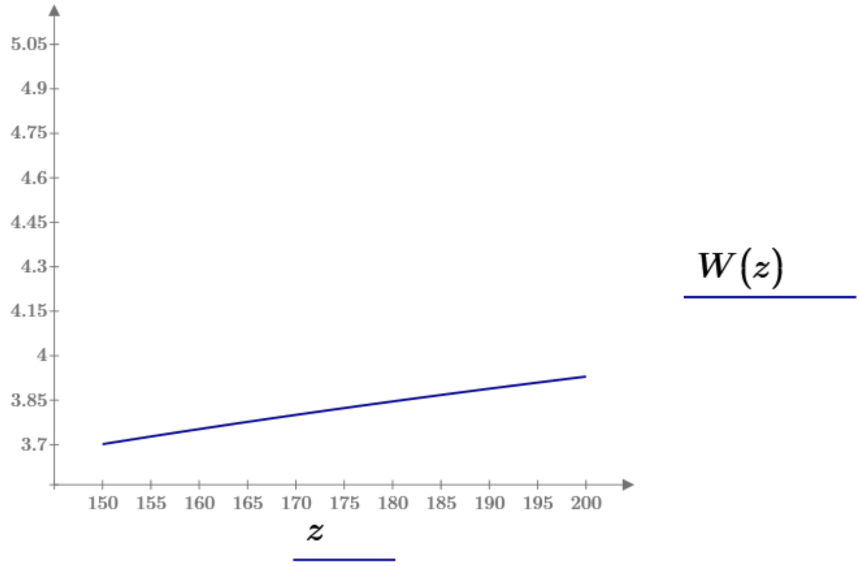

Figure 6 Plot of $W(z)$ as $z \to +\infty$

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

In this work, we focus on a retailer's showcase decision in an omnichannel setting. The setting consisted of two channels: an online channel which includes the universe of feasible products and where purchases are made, and an offline channel used purely to showcase some of the products. The goal of this work is to select an optimal subset of products to showcase in an offline channel and identify the optimal prices of products sold in the online channel. We initially validated the findings of Dzyabura and Jagabathula (2018) who presented an optimal offline assortment of products to offer under exogenous pricing, and, with the help of motivating numerical examples, validated the structure of the optimal assortment they presented. We then built on their work by incorporating price optimization in the model and achieved a jointly optimal solution structure for the offline assortment and the product prices. We demonstrated how this jointly optimal solution provides higher profit results than those obtained for the offline assortment optimization under exogenous product pricing. Our observations regarding the structure of the jointly optimal solution indicate that the offline assortment includes the products with the highest undervaluation, and the prices at which products are sold online have equal profit margins.

A limitation of this work is that it did not investigate different operations structures. Future work could address this limitation by incorporating inventory and returns policy. Another direction we believe is worth exploring is adopting a more elaborate consumer choice model, such as the nested logit model which can accommodate differential levels of interdependence between subsets of alternative options in a choice set. Finally, we also suggest exploring the general showcase decision problem, where one can explore the model in which a retailer can sell in both online and offline channels.

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APPENDIX

Proofs for Section 4.3

Proof of Lemma 1:

Note that $m^* = 1 + W(\sum_{x \in L}^{n} e^{\alpha_x - c_x - 1})$

Let $u(\alpha_x) = e^{\alpha_x - c_x - 1} + \sum_{y \neq x}^{n} e^{\alpha_y - c_y - 1}$

Then $\frac{\partial m^*(\alpha_x)}{\partial \alpha_x} = \frac{\partial W(u)}{\partial u} \times \frac{\partial u(\alpha_x)}{\partial(\alpha)} = \frac{W(u)}{u(1+W(u))} \times e^{\alpha_x - c_x - 1} > 0,$

which indicates that m^* is increasing in α_x.

Proof of Lemma 2:

Note that $m^* = 1 + W(\sum_{x \in L}^{n} e^{\alpha_x - c_x - 1})$

Let $u(c_x) = e^{\alpha_x - c_x - 1} + \sum_{y \neq x}^{n} e^{\alpha_y - c_y - 1}$

Then $\frac{\partial m^*(c_x)}{\partial c_x} = \frac{\partial W(u)}{\partial u} \times \frac{\partial u(c_x)}{\partial(c)} = \frac{W(u)}{u(1+W(u))} \times -e^{\alpha_x - c_x - 1} < 0,$

which indicates that m^* is decreasing in c_x.