AMERICAN UNIVERSITY OF BEIRUT

THE INTERRELATIONSHIPS AMONG CONCEPT IMAGE, IDENTIFICATION AND SITUATION OF COMPLEX NUMBERS FOR GRADE 12 LEBANESE STUDENTS

by
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"And I am sure that he who began a good work in you will bring it to completion at the day of Jesus Christ" Philippians 1:6.

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AN ABSTRACT OF THE THESIS OF

Julien Michel Jalkh for Master of Arts
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Title: The Interrelationships among Concept Image, Identification, and Situation of Complex Numbers for Grade 12 Lebanese Students

The purpose of this research study is to (1) investigate Lebanese students' concept image of complex numbers, (2) examine the impact of the situation in which a complex number is used on students' concept image of complex numbers and (3) examine the impact of students' concept image on their identification of complex numbers. A mixed design was used in this study. A qualitative data analysis of 93 Lebanese students' response to the open-ended concept image questionnaire and the semi-structured complex number justification test was done. The responses to the identification test provided the qualitative data. Analysis of the results revealed that the five situations differ significantly in their distributions of concept image, and the students' concept image of complex numbers is moderately associated to the situation. Second, the situation in which complex numbers are used impacts the students' evoked concept image. The impact of the situation on the formation of students' concept image of complex numbers tends to be affected by the social aspects of constructivism, particularly textbook discourse, as reflected by the definition of complex numbers. Third, the total identification means score differed significantly between the students' five concept image categories of complex numbers in the identification situation. Moreover, the students with the conceptual view as their concept image had the highest total identification mean score. Finally, when identifying examples of complex numbers, the students rely on their concept image and not their concept definition. The students who rely on the formal definition to construct their concept image might score better in their identification of complex numbers.

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CHAPTER I

INTRODUCTION

Constructivists consider learning not the result of teaching, but the result of what students do with the new information that is presented to them (Sternberg & Williams, 2010). This means that the student is an active learner who constructs knowledge from prior understandings and social experiences in a schema. If part of this schema is not mathematically accurate or in conflict with the correct mathematical concept, an alternative conception (aka misconception) is present. A construct that helped in the interpretation of learning difficulties and assisted students in several domains of mathematics education and psychology is the constructs of concept image and concept definition.

In this study, the term “concept image” is used to represent the cognitive schema, which contains the personal perception, description, processes, and images of the student (Tall & Vinner, 1981). The term “complex number identification” is used to represent the degree to which the learner can identify if a given number is a complex number. One way of measuring complex number identification is by the recognition of examples of the concept. The term “situation” is used to represent the context in which the student uses a certain concept.

A review of recent empirical studies revealed that the students’ concept image was explored for several mathematical concepts: continuous functions (Dahl, 2017; Jeong & Kim, 2013), convergence of sequences (Jeong & Kim, 2013; Li & Tall, 1993), tangent lines in calculus (Vincent, LaRue, Sealey, & Engelke, 2015), concept of the slope in space geometry (Moore-Russo, Conner, & Rugg, 2011), limits (Li & Tall, 1993), fractions (Zhang, Clements, & Ellerton, 2015), linear algebra (Wawro, Sweeney,

& Rabin, 2011), square roots (Eisenberg, 2003) and complex numbers (Nordlander & Nordlander, 2012). Since several topics are not covered or not covered in depth using the concept image and concept definition constructs, there is clearly a need to further investigate several topics. Studies exploring students' concept image help in understanding and addressing students' alternative conceptions, especially in advanced mathematical concepts that are more abstract in nature. The purpose of this research study is to (1) investigate students' concept image of complex numbers, (2) examine the impact of the situation in which a complex number is used on students' concept image of complex numbers and (3) examine the impact of students' concept image on their identification of complex numbers.

Background

The constructs of concept definition and concept image are old and well-known constructs within cognitive theories. Both constructs appeared with Vinner and Hershkowitz (1980), and were elaborated since then. Nowadays, these constructs are still being used in several studies to understand students' and teachers' concept image in different fields of mathematics education (Dahl, 2017; Karakok, LaRue, Sealey, & Engelke, 2015; Soto-Johnson & Dyben, 2015; Zhang, Clements, & Ellerton, 2015). The constructs of concept image and concept definition are even used beyond the psychological facet of mathematics education; these constructs were also used to interpret data about students' learning from a social constructivist perspective (Bingolbali & Monaghan, 2008; Saglam, Karaaslan & Ayas, 2011).

The constructs of concept image and concept definition were born because of an interest in the mathematics educators' community to understand how students understand, view, and assimilate mathematical concepts and specifically the abstract concepts (Tall & Vinner, 1981; Vinner & Hershkowitz, 1980; Vinner, 1983, 1991). Tall and Vinner (1981) considered that mathematical concepts are the building blocks of mathematical theories. These concepts can be defined in a formal way since mathematics is exact, but these same exact concepts are perceived in a unique way by every person. Thus each student will perceive the concept in his/her own way and construct his/her own cognitive schema. Since there is a difference between the formal definition and the cognitive process of constructing this schema, Tall and Vinner (1981) used two terms to differentiate between them. In this thesis, the cognitive schema, which contains the personal perception, description, processes, and images, is referred to as "concept image". When the student recalls part of this concept image the term "evoked concept image" is used. The words used to define or identify a memorized mathematical concept are defined as "concept definition". When the student uses his/her own words to identify a concept, this is referred to as "personal concept definition", which differs from a "formal concept definition". In the students' cognitive schema, there is one slot for the concept definition and one for the concept image. One of these slots or both of them might be empty. How the two slots are filled and interact with each other, differ from one student to another. If the concept is memorized by the student in a meaningless way, the concept image slot is empty. At the beginning of a chapter of an abstract concept, the student's two slots are empty, and then when the formal definition is introduced, the student fills the concept definition slot. Later on, after several examples of the concept, the student starts giving meaning to the concept and starts filling the concept image slot along with a personal description (Tall, 1991).

The constructs of concept image and concept definition were used in understanding how students learn concepts. Whenever students are faced with a cognitive task, some of them refer to the concept definition and give an answer. Other students might consult the concept image slot intuitively and then answer. A third group of students might consult one of the two slots first, and an interchange would happen between the concept definition slot and concept image slot in a way that is unique to every student (Vinner, 1991). Several studies explored students' concept image of different mathematical concepts (Dahl, 2017; Eisenberg, 2003; Jeong & Kim, 2013; Li & Tall, 1993; Moore-Russo, Conner, & Rugg, 2011; Vincent, LaRue, Sealey, & Engelke, 2015; Zhang, Clements, & Ellerton, 2015). To our knowledge, there has been one study that investigated students' concept image and number identification of complex numbers (Nordlander & Nordlander, 2012). In their study, Nordlander and Nordlander (2012) grouped the students' concept image into four categories and were able to identify the difficulties students have in the basic property of identifying a complex number.

In the Lebanese curriculum, the concept of complex numbers is introduced in the scientific sections in grade 11 and grade 12. A complex number can be represented in its algebraic form as x + yi, where x and y are real numbers and $i = \sqrt{-1}$. The real number x is called the real part of the complex number, and the real number y is called the imaginary part of a complex number. In their study, Conner, Ramussen, Zandieh, and Smith (2007), claimed that although the algebraic form may look like a sum of two numbers, accepting the set of complex numbers as a new set of numbers depends on conceptualizing x + iy as one entity. Complex numbers have different representations: algebraic, trigonometric, exponential, and geometric.

Few studies were conducted to understand the difficulties students face while learning the concept of complex numbers. One study described how the students' misconceptions decreased after two weeks and a half of an instructional unit on complex numbers. Results revealed that students' conception of complex numbers is mainly limited to the idea that $i^2 = -1$ (Conner et al., 2007). A more recent study (Nordlander & Nordlander, 2012) was conducted to understand students' concept image and number identification of complex numbers. First-year engineering students were asked to answer a questionnaire and to reflect on their view of the concept of complex numbers. The answers revealed four categories of concept images of complex numbers. The first category is when students consider a complex number as a created extension of a real number to solve tasks that are unsolvable with the set of real numbers. The second category is when students consider a complex number as a complex combination of two numbers. The third category is when students reduce the concept of complex number to a symbol which is i or i^2. These students tried to minimize abstraction by representing a complex number by a symbol. The last category of students considered complex numbers as a mystery that cannot be grasped cognitively. These students' related negative emotions to complex numbers. In their study, Nordlander and Nordlander (2012) also asked the first-year engineering students along with secondary school students to take a test to study the students' identification of complex numbers. Results also revealed that students have difficulties identifying a complex number and what numbers are considered complex numbers. A study by Panaoura, Elia, Gagatsis, and Giatilis (2006) explored students' processes and performance in dealing with inequalities and equations including complex numbers. Results reported that the students who used the geometric representation of complex numbers had a fragmental understanding of them and that most students faced difficulties in complex problem solving irrespective of the representation used. Few studies tackled the students' understanding of complex numbers, while one of them explored students' concept image of complex numbers. On the other hand, no study addressed the impact of the situation in which a complex number is used on the students' concept image of complex numbers. One study addressed the students' identification of complex numbers (Nordlander & Nordlander, 2012). However, no study addressed the impact of the students' concept image on their identification of complex numbers.

Rationale

Few studies were conducted to describe students' alternative conceptions of complex numbers (Conner et al., 2007; Panaoura et al., 2006). To our knowledge, one study explored students' concept image as well as their identification of complex numbers (Nordlander & Nordlander, 2012), and one study explored the impact of the situation on the students' concept image of the acid-base concept (Saglam et al., 2010). Moreover, no research studied the impact of the situation in which a complex number is used on the students' concept image of complex numbers, and no research was done to study the impact of the students' concept image of complex numbers on their identification of complex numbers. This research is an extension of the study conducted by Nordlander and Nordlander (2012) and it differs from it in four aspects: (1) This study is in a different sociocultural context, (2) this study is on a larger sample of high school students, Nordlander and Nordlander (2012) used two small samples on engineering and high school students, (3) this study investigates the impact of the situation in which a complex number is used on the students' concept image, and (4) this study investigates the impact of students' concept image of complex number on the students’ identification of complex numbers. This thesis will extend previous studies on students’ concept image in mathematics education and will be of benefit for the literature as well as the practitioners’ community.

Purpose

The study has three purposes. The first purpose is to identify the concept image of complex numbers held by grade 12 students in Lebanon who have studied complex numbers as part of their curriculum. The second purpose is to explore the impact of the situation in which the complex number is used on the concept image of grade 12 students in Lebanon who have studied complex numbers as part of their curriculum. Finally, the third purpose is to explore the impact of the students’ concept image on their identification of complex numbers.

Research Questions

In this study, the following questions were addressed:

1. What are the concept image of complex numbers held by students in Lebanon who have studied complex numbers as part of the Lebanese curriculum?
2. What is the impact of the situation in which the complex number is used on the concept image of grade 12 students who have studied complex numbers as part of the Lebanese curriculum?
3. What is the impact of the concept image of complex number of grade 12 students who have studied complex numbers as part of the Lebanese curriculum on their identification of complex numbers?

Significance

This study helps teachers better understand how teaching and learning of complex numbers are affected by students' concept image in different situations and hence reflect on their own methods of teaching. The results of this study can raise awareness among teachers and curriculum developers about the mediating role of situation in teaching and learning complex numbers as well as the impact of students' concept image of complex numbers on their number identification.

CHAPTER II

LITERATURE REVIEW

This chapter provides an overview of the construct of concept image and concept definition, their births, developments, and applications in different fields of mathematics education. In addition, this chapter offers a summary of the major studies in mathematics education using these constructs to understand students' learning and the major studies that use the impact of situation on the formation of students' concept image. This chapter also provides a historical background of how complex numbers came to exist in addition to an overview of some studies dealing with alternative conceptions in complex numbers.

The birth of theories known as the cognitive-developmental theories, was revolutionary in the field of educational psychology and contributed to defining the nature of mathematical proficiency. Piaget considered that the learner actively constructs his/her understanding by creating a schema, which is a cognitive framework that orders and helps the learner understand new knowledge (Sternberg & Williams, 2010). Moreover, whenever a student encounters a new concept that mismatches his or her preconceived concept, disequilibrium or cognitive dissonance happens. To reach a state of equilibrium again, the student needs to assimilate and then accommodate. In assimilation, the student attempts to add the new concept to the schema. In the second process, accommodation, the student creates a new schema to fit the new concept that could not fit in the already created schema (Sternberg & Williams, 2010). Piaget's theory led to several radical constructivist theories that focus on the individualistic aspects of learning where every student constructs his/her own image of the concept in a mental schema. However, with the development of social theories that consider learning as the result of the active interaction with the learner's social surrounding, these radical constructivist views were criticized by several theorists due to their neglect of the social aspects that affect learning (Ernest, 1994; Radford, 2008). Since radical constructivist theorists could not answer what the impact of social factors such as discourse, the role of peers and teachers, and other social factors have on the students' construction of learning, the social constructivist theory for learning was proposed (Ernest, 1994). Today several constructivist theories are presented in the literature that differ in how much emphasis individual aspects versus social aspects have on students' learning. Cobb, Wood and Yackel (1992) described mathematics learning as both an individual constructive process and a social process. Using this description Cobb et al. (1992) explained how students learn and construct the concept in their schema. One of the most used constructs that helped mathematics educators understand how students make sense of the mathematical concepts is the construct of concept image and concept definition. Below, an overview of the construct of concept image and concept definition will be presented to highlight the gaps in the literature and to point out the research needed.

Overview of Concept Image and Concept Definition

The constructs of concept image and concept definition first appeared with Vinner and Hershkowitz in 1980 in the proceedings of the fourth international conference for the psychology of mathematics education. The following year, the first journal article defining a concept image and concept definition was written by Tall and Vinner (1981). All the terminologies used in this construct were presented in this article. Despite the several studies conducted using this construct it is still the article most referred to concerning concept definition and concept image. Knowing that mathematical concepts are abstract, Tall, Vinner and Hershkowitz were interested in understanding how students view, adopt and internalize those mathematical concepts (Tall & Vinner, 1981; Vinner & Hershkowitz, 1980; Vinner, 1983, 1991). Several scholars built on the constructs and elaborated it so that today concept definition and concept image are an integral part of the terminology used to understand mathematical thinking and concepts in the field of cognitive mathematics education and psychology (Bingolbali & Monaghan, 2008; Li & Tall, 1993; Simon, 2017; Vinner, 1991).

According to Tall and Vinner (1981), mathematics is exact, which means that we can accurately define concepts as basic building blocks of mathematical theories. In reality, from a psychological perspective, these formal definitions are perceived uniquely by different human beings resulting in several cognitive schemas in the minds of each of those human beings. Since the formal definitions of mathematical concepts are in variance with the mental processes in which they are perceived, Tall and Vinner identified a distinction between the two.

Several concepts that we use are not formally defined; they are constructed by the accumulation of many experiences. Later on, these concepts are still developed without a formal definition. This definition is usually constructed in the cognitive schema of the person, and a name or symbol will be used to retrieve the data in the schema. The whole process of constructing, retrieving, and manipulating the schema is referred to as conception. This whole cognitive schema is called the “concept image” which includes all the descriptions, properties, processes, and images in the schema. It is constructed and matures following several experiences and stimuli over the years. For example, when the concept of subtraction is introduced at the early stages of mathematics education, the student learns this concept with natural numbers. The subtraction of natural numbers is always a number smaller than the initial one used.

Associating subtraction to the reduction of a number consciously or unconsciously is part of the concept image of that child. When this same child encounters subtraction of integers, more specifically negative numbers, he/she will face a problem because subtraction of negative numbers results in a greater number. Getting a greater number after subtracting two numbers will result in a conflict in this student's concept image. The concept image is not always coherent when it develops. Thus different parts of the schema are accessed through different stimuli resulting in a concept image that is not necessarily coherent as a whole. Tall and Vinner (1981) referred to the part of the concept image that is recalled in certain situations as the "evoked concept image". In different situations, evoked concept images that are contradicting might be recalled, but when conflicting concept images are evoked simultaneously, it is then that the student will experience confusion. For the same mathematical concept, a student might use two different processes to solve two different mathematical questions. For example, a student might factorize $2x + 2$ properly but choose a wrong method to factorize $2x^2 + 3x + 1$. These two problems elicit two different stimuli that would recall two different evoked concept images.

The concept definition is defined as the words or symbols memorized by the student to specify a certain mathematical concept. A student might use his own wordings to define a concept; this would be called a "personal concept definition" that might differ from a "formal concept definition" approved by the mathematical community. Therefore, we can consider the personal concept definition of a student as putting his/her own evoked concept image into words. The personal concept definition is part of the student's concept image. A student might learn and recall the formal concept definition at the beginning of the chapter and later on construct a concept image that is not directly related to the formal concept definition. For example, a student might learn in calculus that a function is a mapping from an element in set A onto an element in set B such that every element in set A corresponds to one and only one element in set B. Later, during the chapter, this student might consider a function as a machine that takes an input in set A and gives an output in set B, or this student might consider a function to be a formula or a graph. The student might not have a clear link in his cognitive structure between the formal concept definition and his/her evoked concept images. This phenomenon will leave the student with difficulties in the future when the need arises to use the formal concept definition when faced with a more advanced concept of the chapter. Conflicting images can exist in the schema of a student and be evoked at different instances without the student noticing the conflicting views, but a cognitive conflict will happen when the student experiences a conflict between two evoked images simultaneously in his/her cognitive schema (Tall & Vinner, 1981; Vinner, 1991).

Concept Image and Concept Definition in Learning and Instruction

Definitions play an important role in the formation of students' concept image and are important for preventing personal concept definitions that are not accepted by the mathematical community. Technical contexts and using definitions build new ways and habits of thinking, which are different from daily life contexts. Students, at the beginning of their learning process might adopt the thinking habits of daily life and then grow in their habits of the technical contexts (Vinner, 1991).

In our cognitive structure, there are two slots: One for the definitions of concepts and one for concept images. One of these slots or even both of them might be empty. In fact, when the concept definition is memorized in a meaningless way, thus meaning is not associated with the concept name, the concept image slot is therefore considered empty. These two slots can be formed independently, yet they can still interact with one another. For example, a student might have a notion of the chapter of the coordinates system due to several exposures of orthonormal systems. This student might start creating a concept image of this concept as a point with two orthogonal equal vectors. Later, when the concept is introduced by the teacher, this student will notice that any point with two non-collinear vectors forms a coordinates system in a plane. As a result of this cognitive dissonance, three consequences might happen. (a) The student might accommodate and change his/her concept image of the coordinates system from a point with two orthogonal equal vectors to a point with any two non-collinear vectors. (b) the student might keep his/her concept image as it is and add the teacher's definition to the definition slot and will eventually forget it; so when the teacher asks the student to define an orthonormal system, he/she will define it as a point with two orthogonal equal vectors. (c) The student will add the teacher's definition to his/her definition slot. When asked about the definition, he/she will state it as the teacher defined it, and when he/she needs to draw a coordinates system, the student will refer to it as a point with two orthogonal equal vectors (Vinner, 1991).

Before a concept is introduced, the two slots are empty or the concept image slot might have a naïve image formed of the concept because of several experiences. If the two slots are empty at the beginning, one slot might be filled before the other depending on how the chapter is taught. When the formal definition is introduced, the student stores it in the concept definition slot with the concept image slot empty; then after several examples, the concept image slot starts forming with a personal concept definition of the concept which does not necessarily contain all the aspects of the concept. Cognitive theorists might have different views on how the cognitive system works when students are presented with a task to perform. Teachers' expectation of students' desirable process of task performance is not very realistic. They expect that whenever a student is faced with a cognitive task, he/she might refer to concept definition slot and then answer, while for other students an interplay between the concept definition slot and the concept image slot might happen and then the answer will finally come from the concept definition slot. A third group of students, when faced with a cognitive task, will refer to their concept image in an intuitive way, then from the concept image slot refer to the formal definition to give an appropriate answer. Vinner then claims that the student will address a cognitive task by only referring to the concept image intuitively and give an answer (Vinner, 1991).

Concept Image and Concept Definition in Mathematics Education

Since the construct of concept image and concept definition first appeared with Tall and Vinner (1981), many empirical studies were conducted to study students' concept image of several mathematical concepts.

The first research, conducted by Tall and Vinner (1981), studied the concept image of students concerning the concept of continuity of a function. A questionnaire was given to 41 level-A Mathematics students. They were given four explicit forms of functions along with their curve in addition to a fifth explicit function without its graph. The students were asked to specify which of these functions are continuous with justification if possible. The results revealed that most students justified their answers using their evoked concept images. For example, some students stated that a continuous function has a curve with no gaps, others said that a continuous function is drawn in one piece, or a continuous function has an explicit form that is written as one formula and not as a split function. Some students even considered that a continuous function has no angular points. These evoked concept images might be in contradiction with formal concept definitions in advanced cases, causing cognitive conflict with the student's concept image. Many students may possess a clear concept image but with a weak concept definition. Even if these students can solve most of the problems at their level, they will face difficulties at more advanced levels when they need to rely on their concept definition to solve their tasks. Relying on the concept image that conflicts with the formal definition will obstruct the student's formation of a formal concept definition. If the teacher is alert and conscious of the possible conflicts, he/she can discuss with the students and help them deal with this cognitive dissonance by guiding them towards a formal concept definition.

In their study, Li and Tall (1993) helped students construct different concept images of limits and sequences with the help of computer programs. A limit might be perceived as a process that never ends (informal paradigm). Using a computer program where we define a function as a sequence, the author tried to transit the concept definition of university students, from the informal paradigm to the formal paradigm. The use of computer programs permits the term of a sequence to be considered as a mental image or a process, which gives the advanced student the flexibility to alternate between manipulating the concept image as part of a more global mental map, and the process needed to complete a given task (Gray & Tall, 1991). Empirical data revealed that the limit concept is considered as a process, whereas the sequence is considered as a mental image for most students. At the end of this study, the authors concluded that problems will still remain despite the help of computer programs (Li & Tall, 1993).

Another research was conducted to study the mathematical concept images of engineering students (Maull & Berry, 2000). A questionnaire was developed and used with 200 mathematics and engineering students to address how they perceive their concept image and to study their level of understanding. The results obtained were useful for further in-depth research on certain concepts or modes of images and revealed that mathematics students have different concept image than engineering students. The results also focused on the preferred mode of representation used in the students' concept image. In general, algebraic, verbal, and visual are the three main types of representations. In mechanics questions, the engineering students mainly used the visual type of representation, which is the preferred type of most engineers, whereas in the mathematics questions, the students favored the option of verbal representation. The difference in the preferred type of representations between mechanics and mathematical questions might be due to the fact that since it is required to draw a diagram in most mechanics questions, the engineering students might find it more natural to use a visual approach to tackle a mechanics question. On the other hand, in mathematics questions, the visual representations are not a required step in solving the question.

A study on students' concept images of a slope in a three-dimensional system was conducted on graduate students in the USA (Moore-Russo, Conner, & Rugg, 2011). The students were working in a group, and they had to construct the definition of the slope in a three-dimensional system collectively. At the end of the study, the results revealed that argumentation impacted the students' concept image of the slope. In addition, the results revealed that social contexts and pedagogical structures affect the formation of students' concept image.

Two studies aimed at understanding university students' concept image of abstract fundamental ideas (Jeong & Kim, 2013; Wawro, Sweeney, & Rabin, 2011). The first study aimed to explore university students' concept image of the subspace concept and how these concept images are linked to the formal definition through individual, semi-structured interviews of eight first-year university students (Wawro, Sweeney, & Rabin, 2011). The study investigated the relationship between the geometric representations used in the students' concept image and the abstract algebraic language of the formal concept definition. The results revealed that the students' concept image were an accurate interpretation of many facets of the concept definition except for some very abstract algebraic concepts. Their concept definition aided them in understanding properties, as well as differentiating between correct and wrong identification of subspaces. The second study aimed to understand how students use proofs in sequences of continuous functions and their uniform convergence using their geometric concept image intuitively (Jeong & Kim, 2013). The research was conducted on three males and three females in their senior year in a university in South Korea. A questionnaire survey was done four times, and the six participants were divided into two groups by gender. Results showed that students have some misconceptions in the concept of convergence of continuous functions. Results also revealed that discussion about the concept helps them update and fix their concept images to be more consistent with the formal concept.

A recent study showed how a teaching intervention might affect fifth-grade students' concept images of unit fractions (Zhang, Clements, & Ellerton, 2015). Forty students in fifth grade solved a test and did interviews, and then they were divided into two groups. After the pre-test and interviews, the lesson was explained to one group using a multiple-embodiment approach, and then a post-test and interviews were administered to see the effect of the intervention on students' concept image. Results revealed that during the pre-test and first interviews, most of the students used the area-model embodiment to represent the concept of fractions as the textbook used. After the intervention, the students showed a better understanding of the concept of unit fractions; their concept images were more holistic. They used different embodiments to explain the concept of fractions during the interviews, and they also scored higher on the post-test.

The most recent two studies that explored students' concept image were conducted on university students for advanced mathematical concepts (Dahl, 2017; Vincent, LaRue, Sealey, & Engelke, 2015). In the first study, Vincent, LaRue, Sealey and Engelke (2015) investigated first-year calculus students' concept images of the concept of tangent lines. The authors, furthermore, explored the students' concept images of tangent line while applying Newton's method. Task-based interviews were administered to check the student's definition of tangent lines, the capacity of drawing a tangent line in different cases consistent with their personal definition, and if the students can apply Newton's method using tangent lines. Results showed that calculus students enrolled in their first year do not have a holistic concept image concerning tangent lines. As a result of the partially developed concept image, the students' are ready to modify their concept image when they are faced with tasks that are in conflict with their existing ones. Introducing Newton's method to the students while having a concept image of tangent lines that is not fully developed was problematic for them.

The latest study on students' concept images was conducted on first-year university students pursuing their studies in social sciences or humanities majors (Dahl, 2017). The purpose of the research was to explore how students refer to their concept image or their concept definition when faced with a task on the concept of limits, continuity, and asymptotes. Moreover, the study also investigated how much the students' concept images are coherent with all the dimensions of the formal concept definition. The six students majoring in non-STEM majors were interviewed and asked to solve four tasks. Results of the tasks showed that some students have partial understandings of the concepts of continuity, asymptotes, and limits. Other students were able to solve the tasks because of their coherent concept images, either by referring to concept definition or by relying on their concept image only.

Impact of Situation on Concept Image

From a social constructivist point of view, social processes and individual cognitive construction both affect the students' learning process of mathematics (Ernest, 1994; Cobb et al., 1992). Since the social processes impact the formation of students' concept image, this section will present the only paper to our knowledge on the impact of situation on the formation of students' concept image.

Saglam et al. (2011) studied the impact of situation on students' concept image of the acid-base concept. An open-ended questionnaire on equilibrium and acid-base concepts was given to 106 students. The students who answered the questionnaire accurately were chosen to participate in an interview to study the impact of situation on their conception of the concepts. The results revealed that different situations evoke a different concept image, an evoked concept image that works in a particular situation could be inadequate in a wider situation, and a concept image that is used in a particular situation might lead to a misconception in a different situation.

Complex Numbers

Numbers are divided into several sets, the most known set of numbers and the one that is first introduced to children is the set of natural numbers (0, 1, 2, 3, 4…). After the introduction of natural numbers, students face a problem in calculating '4 – 6' thus, the set of integers is introduced. Integers include the natural number as well as their opposites (…, -3, -2, -1, 0, 1, 2, 3…). Following the sets of integers, students face a problem in calculating seven divided by two, therefore students learn the set of rational numbers, which includes all the ratios of integers except zero in the denominator. Later on, students are confronted with the set of real numbers, which contains all the rational numbers in addition to the numbers that cannot be written as a ratio of two integers like π or $\sqrt{2}$. The set of real numbers can be represented on a numbered line where every point on the line corresponds to one real number and vice versa. Finally, students will again face a problem when trying to solve $x^2 + 1 = 0$. The equation has no solution if x belongs to the set of real numbers; consequently, the set of complex numbers is introduced. Complex numbers can be represented in a plane where every point on the plane corresponds to a complex number, and every complex number can be represented on a plane. If we use an orthonormal system, the Cartesian coordinates (x, y) of a point, refer to the image of the complex number that is written in its algebraic form as x + iy. The real number x is called the real part of the complex number, and the real number y is called the imaginary part of a complex number; thus the abscissa of the orthonormal system, also known as the complex plane, is called the real axis and the ordinate axis is called the imaginary axis. The algebraic representation of a complex number is written as a real part added by an imaginary part which is multiplied by the letter i, where $i^2 = -1$. For example, point (3,-4) on the complex plane refers to the complex number 3 - 4i.

Complex numbers are used in several domains of mathematics and science, like for example in quantum mechanics and electromagnetism in physics, or transformations in mathematics. With the use of complex numbers, scientists and mathematicians can solve any equation. For example, using real numbers, the equation "$x^2 + 4 = 0$" has no solution, but using the set of complex numbers, the solutions for the above equation are 2i and -2i. Historically, the use of complex numbers started to solve equations, but nowadays, their use has gone way beyond merely solving equations (Nemirovsky, Rasmussen, Sweeney & Wawro, 2012).

Historical development. In their study, Conner, Ramussen, Zandieh and Smith (2007) summarized different perspectives for the historical development of complex numbers. The first perspective is related to the process/objective view that is similar to how other number sets were developed. For example, there is no rational number squared which solves the equation $x^2 = 2$, thus the need for a new set which is the set of real numbers that contains all the rational numbers in addition to irrationals. The process of solving the equation led to a new object which is an irrational number. The square root of negative numbers led to the need for complex numbers (Sfard, 1991). Another perspective is that complex numbers started with defining $i^2 = -1$ and then added to it the two real parts, x and y, which led to the algebraic form of complex numbers x + iy. Although the algebraic form may look like a sum of two numbers, accepting the set of complex numbers as a new set of numbers depends on conceptualizing x + iy as one entity.

Alternative conceptions in complex numbers. Few studies were conducted targeting the difficulties students face while learning the concept of complex numbers. The first study described the changes in students' conceptions after two weeks and half of an instructional unit on complex numbers. The study also described some of the challenges students face while conceiving the complex plane. The results revealed that students' conception of complex numbers is mainly limited to the idea that $i^2 = -1$ (Conner et al., 2007).

Another more recent study was conducted to study university and high school students' concept images of complex numbers. Forty-seven Swedish 1st year engineering students were asked to answer a questionnaire to reflect on their view of the concept of complex numbers. The answers revealed four categories of concept images which are, (1) a mathematical artifice, (2) two-dimensional numbers or the two-dimensional view, (3) a symbolic extension of mathematics or the symbolic view, and (4) an ungraspable mystery or the mystery view. In addition, a test was administered to university and high school students to study their identification of complex numbers. The results of the test revealed that students have difficulties identifying a complex number and what numbers are considered complex numbers (Nordlander & Nordlander, 2012).

A third research studied students' processes and performance in questions containing inequalities and equations of complex numbers. These questions required transformations from a geometric to an algebraic representation and vice versa. In addition, this research studied students' complex problem-solving skills. Using a questionnaire, data were collected from a sample of 95 students in their final high school year in Greece (17–18 years old). The results revealed that the geometric approach was used more often, whereas the algebraic approach was used more consistently and systematically. The students who used the geometric approach had a fragmental understanding of complex numbers, and most students faced a problem in complex problem solving irrespective of the representation used (Panaoura et al., 2006).

Conclusion

As the above review of literature shows, the constructs of concept definition and concept image proved to be beneficial in exploring students' understanding of several mathematical concepts. Few studies tried to identify the alternative conceptions students construct when presented with the set of complex numbers. One study investigated students' concept images of complex numbers in different situations and the students' identification of complex numbers (Nordlander & Nordlander, 2012). Several studies tried to find the source and reason for students' incoherent concept images and conceptions of complex numbers (Conner et al., 2007; Nordlander & Nordlander, 2012; Panaoura et al., 2006). One study explored the impact of situation on the students' concept image (Saglam et al., 2011). After the review and analysis of the recent empirical studies of students' concept image and concept definition in the different fields of mathematics, there is clearly a need to further explore (1) students' concept image of complex numbers, (2) the impact of situation in which the concept of complex numbers is used on the students' concept image, and (3) the impact of the concept image of complex numbers on the students' identification of complex numbers. This thesis is an extension of the study conducted by Nordlander and Nordlander (2012). It will add to the literature by exploring students' concept image and identification of complex numbers while focusing on the impact of situation on the students' concept image, as well as the impact of concept image on the identification of complex numbers.

CHAPTER III

METHODOLOGY

This chapter presents an overview of the participants, instruments, research design, data collection procedures, and data analysis methods that were used in this thesis.

Participants

In Lebanese schools that follow the Lebanese curriculum, the concept of complex numbers is taught in grade twelve, life sciences (LS) and general sciences (GS) section classes. The sample of students chosen for this research was a purposeful sample of schools and students. The participants were selected from three private schools that follow the Lebanese curriculum in the North Mount Lebanon region. One school has French as the language of instruction in the mathematics classes, out of which two classes, one LS and one GS class, were purposefully chosen. The other two schools have English as the language of instruction in the mathematics classes. Out of the second school, a GS class was purposefully chosen, and out of the third school, an LS class was purposefully chosen as well. The total number of participants was 93 grade twelve students in the three schools stratified by the language of instruction (English, French) and the stream of specialization (GS, LS). Fifty-two point seven percent of the sample learned math in English, while 47.3 % learned math in French. Forty-seven point three persent of the sample was in the LS section, while 52.7 % of the sample was in the GS section.

Variables

In this study, the term “concept image of complex number” is used to represent part of the cognitive cognitive schema, which contains the student’s perception, description, processes, and images of the concept of complex numbers (Tall & Vinner, 1981). The term “complex number identification” is used to represent the degree to which the learner can identify if a given number is a complex number. One way of measuring complex number identification is by the recognition of examples of the concept. The term “complex number situation” is used to represent the context in which the student uses the concept of complex numbers.

Data Collection Tools

Two data collection tools were used in this study. The first data collection tool was a concept image questionnaire to identify students’ concept image in four different situations. The second data collection tool was a complex number identification and justification test to measure students’ identification of complex numbers and their justification for their identification of the numbers. The two data collection tools were translated to French and used in the schools that have French as their language of instruction in the mathematics classes. The translated data collection tools were approved by the IRB.

The concept image questionnaire consists of an open-ended questionnaire that allowed the students to express themselves on how they view complex numbers. The questionnaire is based on a framework adopted by Nordlander and Nordlander (2012) (see Appendix A). The concept image questionnaire includes four questions which correspond to four different situations of using complex numbers. A fifth situation is also present where the student justifies the identification of a complex number in the complex number identification and justification test. The definitions of the five situations are as follows:

1. *Intuitive situation:* This concept image is inferred from the student's response to the question 'what comes to your mind when asked about the concept of complex numbers? List the experiences (class, book, personal experiences, etc.) that guided you to give the above answer'.
2. *Application situation:* This concept image is inferred from the student's response to the question 'Can you give some real life applications for complex numbers?'.
3. *Learning difficulties situation:* This concept image is inferred from the student's response to the question 'what is the greatest difficulty you experienced regarding complex numbers? List the experiences (class, book, personal experiences, etc.) that guided you to give the above answer'.
4. *Concept map situation:* This concept image is inferred from the student's response to the question 'fill the mind map below' related to the word 'complex numbers'.
5. *Identification situation:* This concept image is inferred from the student's response to the question in the identification test 'write a justification why you chose yes or no'.

The complex number identification and justification test measures the students' identification of complex numbers by asking them to respond whether each of the eight given numbers is a complex number. The identification test also has a justification part that allowed the students to justify their responses to the identification of each number. The identification test is also based on a framework developed by Nordlander and Nordlander (2012) (see Appendix B).

Research Design

The research design used in this study is a mixed design. A qualitative data analysis of the students' response to the open-ended concept image questionnaire (see

Appendix A) and the semi-structured complex justification test (see Appendix B) was done. The responses to the identification test provided the quantitative data.

This study is designed to explore Lebanese students' concept image of complex numbers, the impact of the situation in which the complex number is used on the concept image held by the students, as well as the impact of students' concept image of complex numbers on their identification of complex numbers.

Data Collection Procedures

Before starting with the data collection, approval from the Institutional Review Board (IRB) was obtained. After the chapter of complex number was explained in class, and after collecting the parental permissions and students' assent, the questionnaire and identification test were distributed to the participants. They were held during class time, and their duration was 30 minutes for the questionnaire and 15 minutes for the test. For each situation, the student's answer was coded into one or more of five categories. Four categories developed by Nordlander and Nordlander (2012) and the fifth category was developed by the researcher due to the presence of several students' responses that expressed emotional feelings towards complex numbers that cannot be categorized in any of the other four categories. The five categories are as follows:

1. A mathematical artifice: This category views complex numbers as a human creation, which is an extension of real numbers to solve tasks that were not solvable with the set of real numbers.
2. The two-dimensional view: This category views complex numbers as a complex combination of two numbers.

3. The symbolic extension of mathematics view: This category considers that it is the symbols (i) or (i^2) that make a number a complex number.
4. The mystery view: This category is the student's response to complex numbers as an ungraspable mystery that is very abstract.
5. The emotional view: This category is when a student associates complex numbers with an emotion that is mainly negative.
6. N/A: This category was used if no answer was presented, or the answer provided by the students could not be categorized into one of the preceding five concept image categories.

In the identification situation, the N/A concept image category was divided into two subcategories due to the presence of several students who justified their complex number identification using the personal concept definition or conceptual understanding of properties of complex numbers that are found in textbooks and accepted by the mathematical community. The two subcategories are:

1. The conceptual view: This category views complex numbers as personal concept definitions similar to how they are defined in the math textbooks with references to conceptual understandings of properties.
2. N/A: This category was used if no answer was presented, or the answer provided by the students could not be categorized into one of the preceding six concept image categories.

Data Analysis Procedures

The written responses of the questionnaire and the justification part of the identification test were analyzed by qualitative data analysis methods. The keywords in each of the five concept image categories were identified. These keywords were used as codes to classify students in one or more of the five concept image categories for each of the five situations described above. The categorization for a sample of twenty-one students was cross-checked by another expert judge, and the inter-rater degree of agreement was 0.81. For the test, each item was given a score of 1 (yes) and a score of 0 (no or missing answer). Each student had a total identification score that was between 0 and 8. The total identification score was the addition of the eight scores obtained for each number. For each number justification, the student's answer was coded into one of six categories mentioned above. In addition, after coding each justification, each student's responses were coded into one category of concept image for the identification situation. Descriptive statistics, chi-squared, contingency coefficient, cross-tabulation, table of means, one-way ANOVA, and Scheffe were used to answer the research questions.

Students' responses for each of the five situations were analyzed and categorized into the five concept images. Illustrative quotations from students' responses were used to exemplify students' meaning of concept image. To answer the third research question, the same was done for students' responses to the identification and justification test, but the responses were analyzed and categorized into six concept images.

This was followed by a cross-tabulation of concept image by situation that was done, chi-squared and contingency coefficient were calculated and reported. In addition, a table of means was done to represent the total identification score mean of each concept image category. Finally, a one-way ANOVA and Scheffe were done to compare the total identification score means of each concept image category in the identification situation.

CHAPTER IV

RESULTS

The purposes of the study were: (1) to identify the concept image of complex numbers held by 93 students in Lebanon who have studied complex numbers as part of their Lebanese curriculum, (2) to explore the impact of the situation in which the complex number is used on students' concept image, and (3) to explore the impact of students' concept image on their identification of complex numbers. To answer the research questions of this study, a concept image questionnaire consisting four questions was given to the students to identify their concept image (see Appendix A) and after filling the questionnaire, the participants had to answer a complex number identification and justification test (see Appendix B).

This chapter contains the results of the concept image questionnaire and the complex number identification and justification test. The results for each question of the questionnaire in addition to the justifications used to answer the identification test are presented separately to explore students' concept images. Afterwards, the outcomes of the identification test will also be presented. It is important to note that both instruments were given in English for students who study mathematics in English and in French for students who study mathematics in French. Hence, in this thesis, all the results of the French speaking students are translated and presented in English.

Concept Images of Complex Numbers

This section presents the results of the first research question on what are the concept image held by 93 students that studied complex numbers as part of the Lebanese curriculum. The concept image questionnaire includes four questions that correspond to four different situations of using a complex number in addition to the situation where the student justifies the identification of a complex number.

A cross-tabulation (see Table 1) of situation and concept image category was done. The first inference suggested by Table 1 is that its total distribution indicates the most frequent concept image category is the non-identifiable category (N/A) (33.4%), followed by the symbolic extension of mathematics view (24.1 %), mystery view (17.4 %), two-dimensional view (14.9 %), emotional view (5.7 %) and a mathematical artifice view (4.5 %). In the subsequent section, the students' concept image category mode of each situation will be presented.

Impact of Situation on Students' Concept Image of Complex Numbers

This section presents the results of the second research question on the impact of the situation in which the complex number is used on the students' concept image of it.

In addition to the cross-tabulation (see Table 1) of situation and concept image category that was done, chi-squared and contingency coefficient were calculated. The overall impact of situation on the concept image is first presented, followed by the impact of each situation with examples from students' responses, and finally, the impact of the five situations are compared.

Overall impact of situation on concept image. Table 1 suggests two additional inferences. First, the five situations differ significantly in their distributions of concept image (χ^2 is significant at 0.05). Second, the degree of association between the concept image and the situation is moderate, as indicated by the contingency coefficient (0.59). The details of these overall effects are presented in the subsequent sections.

Table 1
Cross-tabulation of concept image by situation

Situation	mathematical artifice		two-dimensional		symbolic extension of mathematics		mystery view		The emotional view		N/A		Total	
	#	%	#	%	#	%	#	%	#	%	#	%	#	%
Intuitive	11	8.9	18	14.5	36	29	31	25	6	4.8	22	17.7	124	100
Application	8	8.5	1	1.1	0	0	15	16	1	1.1	69	73.4	94	100
Learning difficulties	1	1.1	2	2.1	1	1.1	19	20	5	5.3	67	70.5	95	100
Concept map	5	3.3	35	23.2	55	36.4	28	18.5	20	13.2	8	5.3	151	100
Identification	0	0	27	29	42	45.2	4	4.3	0	0	20	21.5	93	100
Total	25	4.5	83	14.9	134	24.1	97	17.4	32	5.7	186	33.4	557	100

Note. Chi-squared = 297.6 significant at the $p < 0.05$ level

Intuitive concept image situation. The purpose of the first question of the concept image questionnaire was to identify students' intuitive concept image of complex numbers. The results revealed that the highest frequency of concept image, 29% of the responses, was the symbolic extension of mathematics view. This shows that the most common intuitive concept image is that a complex number is a number that contains the imaginary number *i*, which is equal to $\sqrt{-1}$. For example, student 92 answered, '*what comes to my mind is imaginary numbers and i without knowing what i is*'. Student 38 answered '*le nombre complexe trouvera des racines en posent i^2=-1 ce qui donnera l'esprit que rien n'est imposible' [the complex number can help us find roots by considering i^2 = -1 which will lead us to consider that nothing is impossible]*.

The second highest frequency of concept image, 25% of the responses, in the intuitive situation was the mystery view category. This result shows that 25% of the students relate to complex numbers as an abstract, ungraspable mystery. For example, student 45 answered, '*I think about numbers I cannot see or count*'. Student 7 answered, '*Quelque chose incomprehensible et quelque chose fictif' [Something incomprehensible and something fictional]*.

Application concept image situation. When students were asked if they could give real-life applications of complex numbers, 73.4 % of them answered 'no', 'none', or gave irrelevant examples. For example, student 9 answered *'pas d'application reeles pour ces nombres' [No applications in real life for complex numbers]*. Student 51 answered, *'A chapter we use in the math program that we use to pass a certain class.'* 16 % of the responses were real-life applications related to mysterious or abstracts ideas. For example, student number 48 answered, *'its like solving a problem in the imaginary world hence you are solving it but theoretically and not practically. It resembles real life but has a twist. It is unique.'* Student number 54 answered *'might be used in philosophy to understand the origin of God'*.

Learning difficulties concept image situation. In the third question of the concept image questionnaire, the students were asked about the greatest difficulty they faced in complex numbers. The results revealed that the highest frequency, 70.5 % of the students, had not answered, or their answer could not have fit in any of the five concept image categories. For example, student 65 answered *'locus of a variable point in the complex plane and n^{th} roots of complex numbers'*. Student 25 answered *'la plus grande difficulté concernant les nombres complexes est purement le développement de grand* *nombre complex' [The greatest difficulty concerning complex numbers is only the expansion of long expressions of complex numbers].*

Concept map situation. In the last question of the concept image questionnaire, the students were asked to fill the concept map related to the word 'complex numbers'. Many answers were obtained and categorized in more than one concept image category. The highest frequency of concept image in this situation was the symbolic extension view. 36.4 % of the responses in the concept map were answers like *'i'*, '$\sqrt{-1}$', or '$i^2 = -1$'.

The second highest frequency of concept image, 23.2% of the responses, in the concept map situation was the two-dimensional view. 23.2% of the responses were answers like *'x +iy'*, *'two parts'*, or *'real part and imaginary part'*.

Thirteen point two percent of the responses in the concept map situation were categorized as the emotional view and related complex numbers to emotions such as disgust, and fear.

Identification concept image situation. In the complex number identification and justification test, eight numbers were presented, and the students answered if each of the eight numbers is a complex number or not; in addition the students were able to justify their answers. In the identification concept image situation, the highest frequency of concept image obtained was the symbolic extension of mathematics view. Forty-five point two percent of the students considered that if the number contains (i) then it is a complex number. For example student 38 answered *'-2.5 est un nombre complexe parce que* $-2.5 = 2.5i^2$*' [-2.5 is a complex number since* $-2.5 = 2.5i^2$*]* and *'cos* (π) + *sin*(π)

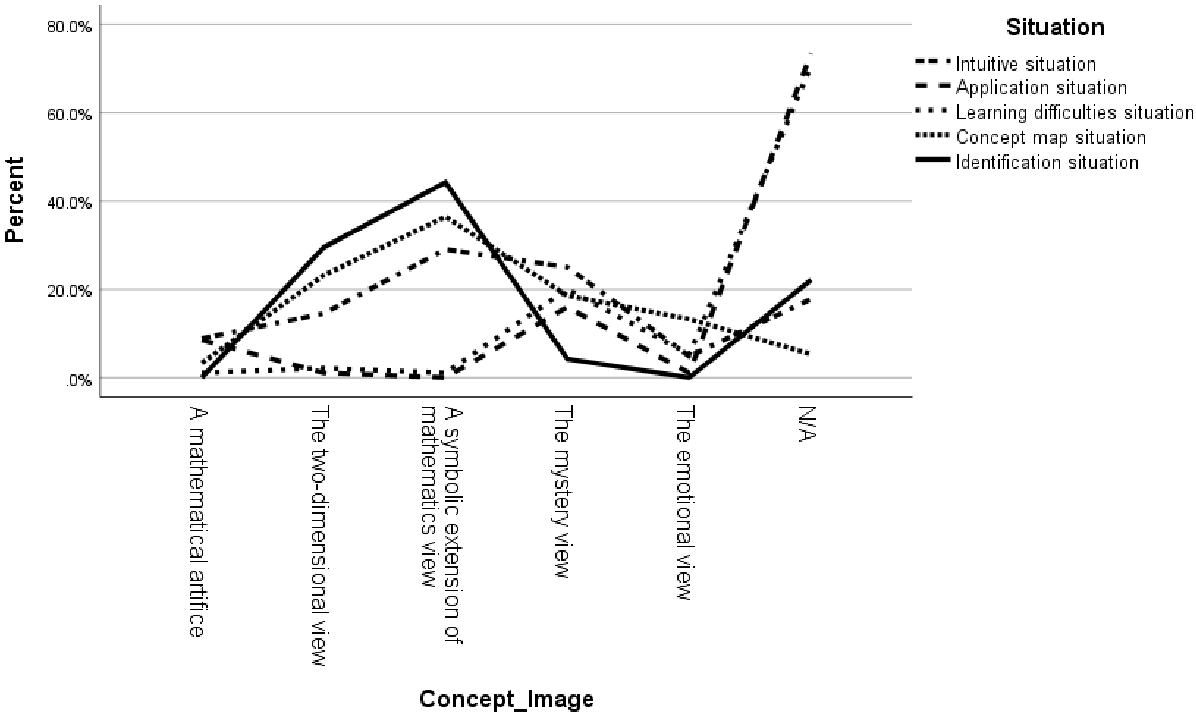
n'est pas un nombre complexe parce qu'il n y a pas d'imaginaire' [cos (π) + sin(π) is not a complex number because it does not have the imaginary number]. The second highest frequency of concept image obtained was the two-dimensional view. 29 % of the students considered that if the number is made up of two parts, then it is a complex number. For example, student 58 answered, *'-2.5 is not a complex number since there is no imaginary part, only a real part'*.

Comparison of the five concept image situations. Figure 1 shows the line graphs of the percentage of concept image categories for each situation. A comparison of the graphs of the five situations reveals the concept image of the symbolic extension of mathematics was the mode of three situations (intuitive, concept map, concept identification). Whereas the category of non-identifiable concept image (N/A) was the mode of the remaining two situations (application and difficulty).

Conclusion. At the end of this section on the impact of situation on the students' concept image of complex numbers, three conclusions can be drawn. (1) The students' concept image of complex numbers is moderately associated with the situation in which it is used. (2) The distribution of students' concept images is different depending on different situations. The mode of students' concept image of the three situations (intuitive, concept map, concept identification) was the symbolic extension of mathematics view. Whereas the mode of students' concept image of the other two situations (application and difficulty) was non-identifiable. (3) Even though the mode of students' concept image of the three situations (intuitive, concept map, concept identification) was the symbolic extension of mathematics view, the second-highest frequency of concept image differed between the different situations. For the intuitive situation, the second-highest frequency of concept image was the mystery view, whereas for the concept map and identification situations, the second highest frequency of concept image was the two-dimensional view.

Figure 1

Line graph of the percentage of frequency of concept image in different situations



Impact of Students' Concept Image of Complex Numbers on its Identification

This section presents the results of the third research question on the impact of students' concept image of complex numbers on its identification. The complex number identification and justification test had eight numbers, all being complex numbers, where students had to answer if each number is a complex number or not. In addition, students had to justify their answer for each number. Each correct identification was counted as one point, and each student had a total identification score that varied between zero and eight. For each number justification, the student's answer was coded into one of six categories (a mathematical artifice, the two-dimensional view, a symbolic extension of mathematics view, the mystery view, the emotional view, the conceptual view). In addition, after coding each justification, each student's responses were coded into one of the six categories mentioned above of the concept image for the identification situation.

A table of means (see Table 2) was done to represent the total identification mean score of each concept image category. The impact of students' concept image of complex numbers on the total identification mean score is first presented, and then a one-way ANOVA table (see Table 3) and Sheffe (see Table 4) were done to compare the total identification mean score of each concept image category in the identification situation.

Table 2

Total identification means score by concept image

Concept Image of Complex Numbers	Mean	N	Std. Deviation
The two-dimensional view	6.04	27	2.244
A symbolic extension of mathematics view	4.26	42	1.483
The mystery view	2.75	4	.957
The conceptual view	7.83	6	.408
N/A	3.79	14	2.082
Total	4.87	93	2.138

Table 2 shows that the students who had the conceptual view as their concept image of complex numbers in the identification situation had the highest total identification mean score (7.83 out of 8). These students justified the identification of complex numbers with answers like '*all numbers are complex numbers*' or '$e^{i\pi}$ *is the exponential form of a complex number*'. The students who considered that a complex number consists of two parts had the second-highest total identification mean score (6.04 out of 8). The students who had the most frequent concept image category in the identification situation considered complex numbers as a symbolic extension of mathematics and had a total identification mean score of 4.26 out of 8. Finally, the students with the lowest total identification mean score (2.75 out of 8) had the mystery view as their concept image of complex numbers.

Table 3
ANOVA of total identification scores as the dependent variable

Total Identification Score	Sum of Squares	df	Mean Square	F	Sig.
Between Concept Image categories	139.429	4	34.857	10.915	.000
Within Concept Image Category	281.022	88	3.193		
Total	420.452	92			

Table 3 shows that the total identification mean scores are significantly different between the five concept image categories. Post-hoc Scheffe (see Table 4) shows that the total identification mean difference between the concept image categories: (1) the two-dimensional view and the symbolic extension of mathematics view, (2) the two-dimensional view and the mystery view, (3) The two-dimensional view and the N/A category, (4) the symbolic extension of mathematics view and conceptual view, (5) The mystery view and the conceptual view, and (6) the conceptual view and the N/A category of concept image, are significant.

Table 4

Scheffe with total identification score as the dependent variable

(I) Concept Image in Identification Test	(J) Concept Image in Identification Test	Mean Difference (I-J)	Sig.
The two-dimensional view	A symbolic extension of mathematics view	1.775*	.005
	The mystery view	3.287*	.025
	The conceptual view	-1.796	.300
	N/A	2.251*	.008
A symbolic extension of mathematics view	The mystery view	1.512	.626
	The conceptual view	-3.571*	.001
	N/A	.476	.945
The mystery view	The conceptual view	-5.083*	.001
	N/A	-1.036	.902
The conceptual view	N/A	4.048*	.001

Note. * the mean difference is significant at the 0.05 level

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CHAPTER V

DISCUSSION

The purposes of the study were first to identify the concept image of complex numbers held by 93 students in Lebanon who have studied complex numbers as part of their Lebanese curriculum. The second purpose is to explore the impact of the situation in which the complex number is used on students' concept image. The third purpose is to explore the impact of students' concept image on their identification of complex numbers. The research questions of this thesis are:

1. What are the concept image of complex numbers held by 93 students in Lebanon who have studied complex numbers as part of the Lebanese curriculum?
2. What is the impact of the situation in which the complex number is used on the concept image of grade 12 students who have studied complex numbers as part of the Lebanese curriculum?
3. What is the impact of the concept image of complex number of grade 12 students who have studied complex numbers as part of the Lebanese curriculum on their identification of complex numbers?

To answer these research questions, the students answered a concept image questionnaire and a complex number identification and justification test. The answers were analyzed, and the results were presented in the previous chapter. This chapter presents the discussion of the major findings, the limitations, recommendations for further research, implications for practice, and conclusion.

Concept Image of Complex Numbers

This section presents the concept image of complex numbers held by 93 students in Lebanon who have studied complex numbers as part of their Lebanese curriculum. In addition, a discussion of the results in relation to the study of Nordlander and Nordlander (2012) will be presented.

The total distribution of students' concept image indicates that the mode concept image category is the non-identifiable category (N/A), and the second most frequent concept image category is the symbolic extension of mathematics view. Since the students' concept image distribution differed according to the situation, a comparison of the five situations revealed that the concept image of symbolic extension of mathematics was the mode of three situations (intuitive, concept map, and concept identification), and the category of non-identifiable concept image (N/A) was the mode of the remaining two situations (application and difficulty). The second highest frequency of concept image differed between the three situations (intuitive, concept map, concept identification) even though the mode of the students' concept image of the three situations was the symbolic extension of mathematics view. For the concept map and identification situations, the second-highest frequency of concept image was the two-dimensional view, whereas for the intuitive situation, the second-highest frequency of concept image was the mystery view.

This study was an extension of the study conducted by Nordlander and Nordlander (2012). Comparing the results obtained by this study with the results obtained by Nordlander and Nordlander (2012), we can find several similarities as well as several differences of students' concept image. In their study, the mode of students' concept image in the intuitive situation was the two-dimensional view, and the second-highest frequency of concept image was the mystery view. The reason for the difference between the two studies concerning the mode concept image category in the intuitive situation is probably due to the different cultures where the studies were conducted. Nordalnder and Nordlander (2012) studied first-year engineering university students' concept image of complex numbers in Sweden, whereas in our study, the sample was high school students studying complex numbers as part of the Lebanese curriculum. The two curricula are different, thus affecting the formation of students' intuitive concept image differently. For the application and learning difficulties situations, the results of students' concept image in both studies were similar. The majority of the students in both studies did not answer the questions and had no idea how or where complex numbers are used in real life and, as a result, did not have a clear idea of what the difficulties of complex numbers beyond the classroom exercises are. Concerning the concept map situation, the mode for students' concept image categories in both studies was the symbolic extension of mathematics view. The comparison of the results shows that despite the difference in the results of students' concept image frequencies for the identification situation due to the different socio-cultural context of the two studies, the three other situations of the two studies have the same modes for the students' concept image categories.

To conclude, despite the different social contexts of the two studies, similarities in the results can be attributed to the fact that the concept of complex numbers is theoretical, abstract, and isolated from mathematics used in daily life. Since the concept of complex numbers is of abstract nature, the students' concept image in the application, learning difficulties, and concept map situations are not affected by the social context of the students.

Impact of Situation of Complex Number Use on Student Concept Image

This section presents the answers of the impact of situation in which the complex number is used on the concept image of grade 12 students. In addition, a discussion of the answers in terms of theory and relevant research findings will be presented.

The results revealed that the five situations differ significantly in their distributions of concept image, and the students' concept image of complex numbers is moderately associated to the situation.

Since the construct of concept image is rooted in the constructivist theories, we will discuss the second research question using a constructivist framework. The constructivist theories vary in their forms and interpretation of how learning happens. One dimension of how these constructivist theories vary is how much emphasis is placed on each of the two main aspects that affect learning i.e., the individual aspects and the social aspects (Ernest, 1994). The framework used in the discussion of the impact of situation on evoked students' concept image of complex numbers is the social constructivist framework as described by Cobb et al. (1992). From a social constructivist point of view, the learning of mathematics that happens by the students' construction of their mental schema is impacted socially and culturally (Cobb et al., 1992). The results obtained in chapter four are in line with the social constructivist theory. The results revealed that the students' concept image of complex numbers is moderately associated to the situation in which it is used and since the distribution of students' concept image is different depending on different situations, the students' concept image of complex numbers depend on the situation in which it is evoked. Since the students' construction of their concept image is impacted by the situation in which it is used, we still need to discuss the reasons for the situation affecting the students' concept image of complex numbers. Cobb et al. (1992) argued that learning mathematics does not happen only in an individualistic constructivist way. Students do not construct their schema in an organic way independent of social factors. Individualistic and social factors contributed to the formation of students' concept image of complex numbers.

Concerning the social factors that contributed to the formation of students' concept image, in the math school books of the Lebanese curriculum used in class by the sample of students in this study, the concept of complex numbers is introduced and defined as follows: "a complex number is any number of the form x + iy, where x and y are real numbers and $i^2 = -1$" (Attieh et al., 2007). Since discourse is one of the main social factors affecting the students' learning (Ernest, 1994), and the formation of concept image is affected by the social context in which students learn (Bingolbali & Monaghan, 2008), it follows that the manner in which the concept of complex numbers is introduced in the textbook has a major impact on the formation of students' concept image of complex numbers.

Discussing the individualistic factors that contributed to the formation of students' concept image, before the concept of complex numbers is introduced, the student's schema of numbers contained all real numbers. When complex numbers are introduced, the real imaginary parts of complex numbers are composed of real numbers, but the imaginary number (i) is a new number that was not part of their schema which will create a cognitive dissonance in their process of learning which includes assimilation and accommodation to reach equilibrium (Sternberg & Williams, 2010). The social and individualistic factors that are the definition of complex numbers presented in the Lebanese textbooks and the students' cognitive dissonance contributed to obtaining the concept image of symbolic extension of mathematics as the mode of three situations (intuitive, concept map, concept identification). This result aligns with the result obtained by Connor et al. (2007) that at the end of the chapter, students' conception of complex numbers is limited to the symbol (i) such that $i^2 = -1$. The students studying the concept of complex numbers as part of the Lebanese curriculum learn the definition of the algebraic form of a complex number, operations on complex numbers, the geometric representation of complex numbers, the modulus and argument of complex numbers, the trigonometric form and the exponential form of complex numbers, and finally De Moivre's formula. In the Lebanese textbook, all the required concepts are reinforced with abstract exercises, and the assessment is similar to the given exercises. In all the learning processes of complex numbers as part of the Lebanese curriculum, the concept is presented in a very abstract way. Due to this social factor, it is reasonable to have the category of non-identifiable concept image N/A as the mode of the two situations (application and difficulty), and the mystery view as the second-highest frequency of concept image in these two situations. The abstract nature of the concept, as well as the abstract exercises in the textbook, contributed to these frequencies of concept image categories. For the concept map and identification situations, the second-highest frequency of concept image was the two-dimensional view. In the math textbooks used, the definition of a complex number is that it is a number of the form x + iy. It is due to this social factor that the reason behind the second-highest frequency of concept image in the concept map and identification situation being the two-dimensional view is the textbook definition, which is very similar to that view. A lot of students' concept image are derived directly from the formal concept definition (Vinner, 1991).

The results of a study on the students' concept image of the slope in space geometry revealed that the social context affected the formation of students' concept image (Moore-Russo et al., 2011). In addition, Jeong and Kim (2013) studied students' concept image of sequences and continuous functions. The results revealed that the formation of the students' concept image was affected by the discussion between the students. While studying the fifth-grade students' concept image of unit fractions, the results of the pre-test before the intervention revealed that the students' concept image of unit fractions were similar to the representation used by the textbook (Ahang et al., 2015). All these results show the impact of the social factors on the formation of the students' concept image. In relation to the studies on the impact of the situation on the students' concept image, the results of the study conducted by Saglam et al. (2010) on students' conception of the acid-base concept were similar to the results of this thesis. The study revealed that the situation impacts the students' evoked concept image of the acid-base concept, and the definition used for the acid-base concept contributes to the formation of these concept images.

To conclude the discussion of the second research question, it can be said that the situation in which complex numbers are used impacts the students' evoked concept image. The impact of the situation on the formation of students' concept image of complex numbers tends to be affected by the social aspects of constructivism, particularly textbook discourse as reflected by definition and discussion of complex numbers. Moreover, the individual aspects of constructivism, particularly cognitive dissonance resulting from the abstract nature of the chapter may have affected the students' concept image of complex numbers in different situations.

Impact of Students' Concept Image of Complex Numbers on its Identification

This section presents a discussion of the impact of students' concept image, in the identification situation, on their identification of complex numbers. The results will be discussed in terms of theory and relevant research findings.

The results revealed that the total identification means score differed significantly between the students' five concept image categories of complex numbers in the identification situation. Furthermore, the students with the conceptual view as their concept image had the highest total identification mean score, followed by the students with the two-dimensional view, the symbolic view, the N/A category, and the mystery view.

Vinner (1991) described the role of definition in the teaching and learning of mathematics using the constructs of concept image and concept definition. He then described a 'desirable theory and practice', which states that every student has two slots in his/her cognitive structure, one for the concept image and one for the concept definition. For a particular concept, the two slots might initially be empty or contain some information. After the concept is learned, the two slots are filled. Different parts of the concept image slot might be evoked in different situations. When abstract concepts are taught, like for example, the concept of complex numbers, the concept definition slot is first filled, then after several examples, the student constructs his/her concept image slot. When students are faced with a cognitive task, teachers expect that the student's process of task performance, despite its difference between students, is through an interplay between the concept definition slot and the concept image slot, and the answer will finally come from the concept definition slot. Vinner (1991) argues that the student's desirable process of task performances expected by teachers is not very realistic. He then claims that whenever students are faced with a task, they will refer to the concept image slot and then answer, which means that the students' answers to any task come from the concept image slot. Our results showed that the total identification mean scores differed significantly between the students' five concept image categories of complex numbers. Moreover, since all students were introduced to the same definition of complex numbers, it can be said that the results of this study align with the theory proposed by Vinner (1991) that when students are presented with a cognitive task, their concept image guides their answer rather than their concept definition. Furthermore, the students who had the conceptual view and the two-dimensional view as their concept images had the highest two total identification mean scores and this is because these two concept image categories were the closest to the formal definition used in their textbooks. The students who had the symbolic extension of mathematics view as their concept image in the identification situation had a total identification mean score of 4.26 out of 8, this is probably due to the fact that four out of the eight items in the complex number identification test had the imaginary number (i) visible in their given form (i, i^i, $e^{i\pi}$, $\cos(\pi)+i\sin(\pi)$).

In their study, Wawro et al. (2011) explored students' concept image of the abstract concept of subspace. Their results revealed that the students who had relied on the formal definition in constructing their concept image scored better in identifying examples and non-examples of subspaces. Similarly, in this thesis, the results revealed that students who had their concept image of complex numbers similar to the formal definition used in their textbooks (conceptual view and two-dimensional view) had the highest total identification mean score. Another study by Tsamir et al. (2015) explored early-years teachers' concept definition and concept image of basic geometric figures (triangles, circles, cylinders). The teachers were asked to give definitions for the geometric figures and to identify examples and non-examples of these geometric figures. The results revealed that teachers relied on their concept image rather than their concept definition in the identification of examples and non-examples of circles and cylinders. Similarly, our results showed that the students relied on their concept image of complex numbers in their identification of examples of complex numbers.

To conclude, the answer to the third research question states that when identifying examples of complex numbers, the students rely on their concept image and not their concept definition. In addition, the students who rely on the formal definition to construct their concept image might score better in their identification of complex numbers.

Limitations

This study presents some limitations. First, the study was conducted on 93 students studying in Lebanese schools following the Lebanese curriculum. The sample was purposefully chosen from three schools in the Mount Lebanon region with similar socio-economic status. Given the small sample size the generalization of the results to Lebanese students' concept image of complex numbers might be limited. The question arises if we would have gotten similar results if a bigger and more diverse sample would have been chosen to better represent the Lebanese students' concept image of complex numbers and the impact of situation on their concept image. Second, the data collection tools to explore students' concept image i.e. a concept image questionnaire and a complex number identification and justification test might have been a limitation because of the difficulty of identifying the students' concept image. The question arises if semi structured interviews would have been more helpful tools to help students express more and the researcher better understand the students' concept image of complex numbers and the reason for their construction.

Recommendation for Further Research

This study paves the way for further research opportunities. The first recommendation that stems out of this study is that further studies should be conducted to explore teachers' concept image of complex numbers. Since the social aspects of constructivism, particularly textbook discourse as reflected by definition and discussion of complex numbers impacts the formation of students' concept image of complex numbers, it is reasonable to explore teachers' concept image of complex numbers which will affect the students' concept image. Moreover, it may be helpful to explore the impact of classroom discourse on students' concept image of complex numbers, more specifically, the impact of the language of instruction on the students' concept image of complex numbers. Finally, it may be helpful to explore students' concept image of different abstract mathematical concepts since the results might be helpful for practitioners to improve their teaching methodologies, as well as for curriculum designers to improve their design of the scope and sequence of these concepts.

Implication for Practice

This thesis has implications for teachers and curriculum designers. The results helps the teacher be aware of different possible evoked concept images of complex numbers students might have in different situations. This will help teachers have more diverse methodologies to address different students' misconceptions of complex numbers. The results of this study as well, help curriculum designers better design the breadth and depth of the chapter of complex numbers. Real life examples in the course and applications will help students have a concept image of complex numbers that is more consistent with the formal definition and properties of the chapter.

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Appendix A

Concept Image Questionnaire

Instruction: There are 4 questions to answer, you have **30 minutes** to answer the following questions. The answer for every question should fit in the space assigned to them.

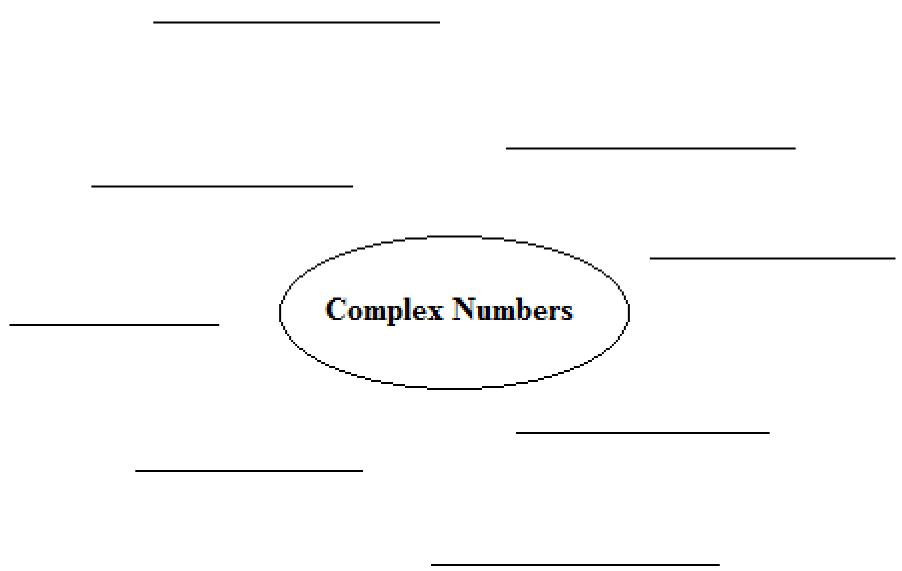
There are no right or wrong answers.

1. **What comes to your mind when asked about the concept of complex numbers? List the experiences (class, book, personal experiences, etc.) that guided you to give the above answer.**

2. **Can you give some real life applications for complex numbers?**

3. **What is the greatest difficulty you experienced regarding complex numbers? List the experiences (class, book, personal experiences, etc.) that guided you to give the above answer.**

4. **Fill the mind map below:**



Appendix B

Complex Numbers Identification and Justification Test

Instruction: Answer by yes or no if the following numbers are complex numbers.

You have **15 minutes** to answer all the questions.

Number	Yes	No	Write a justification why you chose 'yes' or 'no'
i			
i^i			
-2.5			
$e^{i\pi}$			
$5\cos(\pi)$			
$\cos(\pi)+i\sin(\pi)$			
$\cos(\pi)+\sin(\pi)$			
0			