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HOW DO STUDENTS NEGOTIATE THEIR AFFECT TOWARD MATHEMATICAL PROBLEM-SOLVING AND HOW DO THEY ASSIGN MEANING TO PROBLEMS AND PROBLEM-SOLVING? A QUALITATIVE META-SYNTHESIS.

By
RAFAH KHALIL AWDEH

A thesis
submitted in partial fulfillment of the requirements
for the degree of Master of Arts
to the Department of Education
of the Faculty of Arts and Sciences
at the American University of Beirut

Beirut, Lebanon
December 2021

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ACKNOWLEDGEMENTS

All the thanks and gratitude go to my mentor and advisor professor Murad Jurdak. I attribute the completion of this thesis to his professionalism in guiding me throughout this journey, with his unlimited support, encouragement, and belief in my abilities.
I would like to extend my gratitude to my husband, my mother and my daughter who had faith in me and never failed to be of great help and support.

ABSTRACT

OF THE THESIS OF

Rafah Khalil Awdeh for Master of Arts
Major: Math Education

Title: How Do Students Negotiate Their Affect Toward Mathematical Problem-Solving and How Do They Assign Meaning to Problems and Problem-Solving? A Qualitative Meta-Synthesis.

Qualitative studies have enriched the mathematics education field over the past 40 years across various topics. In spite of the fact that the number of these studies is increasing and the quality of qualitative research studies in mathematics education is changing, little is still known about how this qualitative research findings would contribute to our understanding of a particular topic within the field. Through the process of qualitative research meta-synthesis, our knowledge base can be widened to offer better and deeper understanding of attitudes, perceptions, and behaviors relevant to mathematics teaching and learning. This study is a qualitative research meta-synthesis that aims at synthesizing, analyzing, and integrating the findings of a collective body of qualitative research to understand the way students negotiate their affective field toward problem-solving, and the way students assign personal meanings to problems and to problem-solving. This study utilizes theory-generating approaches to qualitative meta-synthesis which is based on the grounded formal theory to analyze data extracted from twenty-one selected relevant qualitative studies. The grounded theory would provide a comprehensive framework that enables us to understand and explain how students negotiate their affective relationship with problem solving and the way they assign meaning to it by coding, categorizing, constant comparison, coming up with a theme, and generating a well-founded theory.

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CHAPTER 1

INTRODUCTION

Background

This section focuses on two themes. The first theme includes a discussion of the background of the affect as it relates to problem-solving. The second theme focuses on the background of negotiation as it relates to problem-solving.

Affect and Problem-Solving

At the end of the last century, there appeared a shift in the field of mathematics education. The interpretative research approach started to replace the normative approach. The intricacy of the educational process in mathematics made it vital for researchers to understand the humanistic aspect of mathematics learning instead of proving general rules about learning supported by the cause-effect paradigm (Di Martino, 2018).

With the humanistic aspect playing a significant role in the learning process, researchers could not ignore the affect if they were to consider the complexity of the mathematical learning experience. The growing interest in affect and the multitude of research on it came to light because of its significant role in students' mathematics learning. Researchers aimed to explore and understand the teaching/learning environments and conditions that would lead students to negotiate their affect toward their mathematical learning experiences positively. As a result of many researchers' calls to involve affect in mathematics education research, the theory of McLeod (1992) on affect came to existence (Di Martino, 2018).

McLeod's (1992) publication on affect presented itself as a manifesto in the field of mathematics education. Researchers adopted it to frame their work and explore students’ affective field during their mathematical learning journey. McLeod placed his theory on affect within Ernest Paul’s social-constructivist theory.

When social constructivism found its way to the field of mathematics education, mathematics ceased to be a merely cognitive process. Earnest Paul's contributions to mathematical education were instrumental in placing social constructivism at the heart of mathematical research. Piaget's stage theory and later Vygotsky’s social theory of mind inspired Paul to develop his social constructivist theory, which acknowledges that both the individual meaning-making and the social interaction or social processes are at the heart of the mathematical learning experience (Di Martino, 2018). McLeod’s theory of affect identified three main affective constructs: emotions, beliefs, and attitudes, which can be distinguished in terms of their stability, and they could describe the wide range of affective responses to mathematics (McLeod, 1992).

McLeod used the definition of Mandler (1984) for emotions. He viewed emotions as the most intense and the least stable among the three constructs; they develop due to cognitive analysis combined with psychological responses in reaction to momentary experiences. Emotions are transient, and they fluctuate with incidents and circumstances; they can be intense and can change rapidly. In contrast, beliefs and attitudes are more cognitive in nature and more stable than emotions; they develop over long periods, and they need time to change (McLeod, 1992). The relationship between beliefs, attitudes and emotions could be described so that the emotional state, through repetition, could become a trait that develops into deep beliefs and attitudes.

Beliefs and attitudes are not straightforward concepts that could be defined in a specific framework. The literature on beliefs is the richest among the three affective constructs in mathematics education. Despite this, its definition remains problematic.

Many researchers do not define belief, assuming that the readers know what beliefs are or formulate their definition of belief, which might contradict others (Furinghetti & Pehkonen, 2002).

The two categories of beliefs that are particularly relevant for mathematics education are beliefs about self and mathematics. Beliefs about mathematics develop gradually over time based on the experiences that the students encounter throughout their learning of mathematics journey; they involve beliefs about problem-solving, beliefs about self when solving a task, and beliefs about mathematics in general. Beliefs are also influenced by cultural factors such as the school and home mathematical culture that the student lives in (McLeod, 1992). The research on beliefs in mathematics education is interested in interpreting students' beliefs and studying their social and cultural origins, the students' awareness and degree of certainty concerning their beliefs, and the factors behind the changes in such beliefs.

Attitude is a multidimensional concept; its multiplicity of dimensions makes it harder for researchers to define it explicitly. Researchers mainly study attitude through questionnaires, and within the field of mathematics education, the attitude has cognitive, affective, and behavioral components (Martino & Zan, 2011). Attitude could be taken for the affective responses that involve positive or negative feelings that are moderately intense and reasonably stable, such as liking geometry or disliking problem solving, being bored by algebra, and curious about trigonometry (McLeod, 1992).

An attitude develops over time as an automatic result of positive and negative mathematical emotional experiences. These experiences start with strong negative or positive emotions and keep reoccurring in the same manner, ending with a negative or positive attitude respectively that is less emotionally intense but more stable (McLeod, 1992).

The way students' affect manifests itself during problem-solving is undeniable. When students face a problem, a wide range of emotions is triggered, and their beliefs about problem-solving and oneself influence the way students view and approach a problem (Schoenfeld, 1983). With that being said, researchers were very interested in exploring and understanding the nature of students' affective field and shifts during problem-solving.

Problem-solving has been a central theme in education for several decades. Educators and education decision-makers agree on the importance of problem-solving skills for academic and real-life success. The request by the National Council of Teachers of Mathematics (NCTM) in 2000 for problem-solving to become the focus of school mathematics was widely repeated in the field of mathematics education (NCTM, 2000). When students work on challenging problems, their affective responses are highly activated, and they are more explicit and more intense than they are when they work on routine problems. This phenomenon inspired us to explore and understand the topic through a qualitative meta-synthesis.

When it comes to problem-solving and to problems, they are as hard to define as attitude. Researchers introduced several definitions of "problem." The meaning assigned by students to "a problem" is also a challenge. The meaning that students usually assign to a problem is the student's definition of "a problem." Students' base their definitions on their personal mathematical experiences. Mtetwa and Garofalo (1989) recognized multiple unhealthy beliefs students have about mathematical problem-solving that determine their definition or the meaning they assign to problems.

They affirm that such faulty and stereotypical beliefs developed from textbook answers and classroom experiences.

Multiple definitions of "a problem" appeared in the literature. Bruner's (1961) definition of a problem stated that one needs to consider 'troubles,' 'puzzles,' and 'problems' when defining a problem. Kantowski (1977, p.163) defined a *problem* as "An individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him." Another definition by Mervis (1978) stated that a problem is "a question or condition that is difficult to deal with and has not been solved" (p. 27). Lester (1980) says that "A problem is a situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution" (Lester, 1980, p. 287). Buchanan (1987) defines mathematical problems as "non-routine problems that required more than ready-to-hand procedures or algorithms in the solution process" (p.402). McLeod (1988) defines *problems* as "those tasks where the solution or goal is not immediately attainable, and there is no obvious algorithm for the student to use" (p. 135).

Ohio Department of Education (1980, p.5) listed four elements that define a mathematical problem, among which "a person must desire a solution." This definition has an affective component to it, which does not appear in any previous definition. Relating the definition or the meaning of a problem to affect is of significant interest for us in our meta-synthesis because we believe that the meaning assigned to problems by students is highly influenced by their learning experiences and the emotional aspects that accompany those experiences.

Negotiation and Problem-Solving

When we mention negotiation, the first thing that comes to mind is a back-and-forth dialogue between two entities to resolve an issue or reach a conclusion. Negotiating oneself is an endeavor that individuals go through implicitly to grasp new ideas or concepts or change emotions, beliefs, and attitudes.

It is inevitable to discuss negotiating oneself without mentioning Piaget's cognitive equilibrium theory as a part of his contribution to radical constructivism (Piaget & Inhelder, 1969). According to Piaget, and in the process of constructing one's knowledge, humans seek equilibrium. A state of frustrating disequilibrium occurs when the cognition is subject to new knowledge that mismatches the individual's existing knowledge. When an individual enters a state of disequilibrium, the individual tries to manage it through assimilation or accommodation. To resolve the disequilibrium state, the individual assimilates the new knowledge to his old schema where it would fit. If the new information does not fit into any schema, the individual needs to modify his current schema to fit the new information. This process through which the individual is going involves a certain kind of internal or implicit negotiations between the individual and himself to reach cognitive equilibrium and construct his knowledge and formulate his own beliefs. On the other hand, social constructivism extended the theory of constructing knowledge outside of the personal cognition and psyche (Piaget & Inhelder, 1969).

The social constructivism theory was the one to reconcile the cognitive individual and the collective social aspects of learning. The theory acknowledges that both the individual meaning-making and the social interaction or social processes are at the heart of the mathematical learning experience. Social constructivism found its way to mathematics education in the eighties, influenced by Bruner, Bishop and Vygotsky; nevertheless, it was Paul Ernest who developed the social-constructivist theory (Di Martino,2018).

Social interactions and cultural influences precede individual knowledge in the process of knowledge construction; as Vygotsky mentioned, “Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first *between* people …, then *inside* the child” (Vygotsky 1978, p. 57).

Paul Ernest introduced two kinds of knowledge, objective, and subjective knowledge. According to Piaget's cognitive equilibrium model, subjective knowledge is the knowledge that the students build; it is built individually through assimilation and accommodation. With the influence of societal factors and interaction, a new form of knowledge is introduced, objective knowledge. Objective knowledge introduced by the society is internalized by the student and added to his subjective knowledge. For students to internalize this new knowledge, they pass through intersubjective scrutiny of ideas, reformulation, and acceptance of the new knowledge; a form of negotiation between students and themselves takes place. This negotiation process could lead to a change once the new knowledge is accepted (Ernest, 1991).

Based on the above, we can say that students enter their learning journey with a belief system about education and a vision of themselves already established (Laughran, 2006; Lortie, 1975; Wenger, 1998). Throughout the journey, each student’s meanings, and affective experiences cross paths with contextual social experiences to shape a unique path into their academic field (Wenger,1998). Contextual experiences include the learning environment they experience, the teaching practices they are subject to, the influence of significant others such as peers, educators, and other society members.

When discrepancies occur between the students' original vision and beliefs and a contextual element, they must negotiate or sort out the incongruity. This negotiation is an ongoing, context-related, and unique meaning-making process that could extend, redirect, or modify prior beliefs (Wenger,1998).

So, we define "negotiate" as whenever students experience misalignment between their assigned meaning to problem-solving, affect or beliefs toward problem-solving, based on previous learning experiences, and what their new or alternative learning experiences are subjecting them to, they need to negotiate and sort out their assigned meaning and beliefs to resolve the discrepancies.

Rationale

In the last fifteen years, a trend to utilize qualitative approaches to study different problem-solving aspects has emerged. Several primary qualitative studies have explored the way students negotiate their affective relationship with problem-solving and the personal meaning they assign to the latter. These studies, which were enacted in different countries, used a variety of classroom teaching/learning environments. These teaching/learning environments differed in their theoretical lenses for framing the research issues and interpreting their findings. Also, they differed in targeting students at the different educational levels, ranging from primary to university. The ultimate purpose of this study is to use qualitative meta-synthesis based on the grounded formal theory to integrate and interpret findings from a pool of qualitative studies published in the last twenty-four years. These studies addressed the way students negotiate their affective relationship with problem-solving and the personal meaning they assign to it.

An extensive literature research showed no presence for a qualitative meta-synthesis that addressed students' negotiation of their affective relationship with problem-solving and the personal meaning they assign to the latter. Our literature search identified twenty-one journal articles addressing the meaning students have of a problem and problem-solving and the way they feel towards problem-solving in a variety of classroom teaching/learning environments.

As for affect, research addressing classroom teaching/ learning environments varied in classroom settings, from students working individually or in groups to students collaborating on solving problems, sometimes followed by whole classroom discussions.

Few studies explored students' affect towards problem-solving in conventional classroom environments, where no intervention was employed and the delivery of knowledge was through direct instruction (Andersen, 2015; DiMartino, 2019; Higgins, 1997; Jader et al., 2016; Lerch, 2004; Martinez & Gonzalez, 2015; Moyer et al., 2018; Qaisar et al., 2015; Schindler & Bakker 2020; Wong, 2002; Yusof & Tall, 1999).

Several of the studies we identified targeted group work settings to understand more about students' affect towards problem-solving (Andersson et al. 2015; Bray & Tangney 2016; Higgins 1997; Hino, 2015; Ju & Kwon, 2007; Lynch & Star 2014; Perrenet & Taconis 2009; Qaisar et al. 2015; Satyam, 2020; Schindler & Bakker 2020; Yusof & Tall, 1999). Collaborative work differs from group work. In collaborative work, the students simultaneously solve a challenging problem that none of them could do on their own. On the other hand, group work responsibilities are divided among the group members, where this division makes the students do most of their work individually (Damon and Phelps,1989).

Multiple studies explored the students' affect towards problem-solving in teaching/ learning environments that adopt a reform curriculum setting, using collaborative work followed by whole classroom discussion (Andersson et al., 2015; Francisco, 2005; Higgins, 1997; Qaisar et al., 2015; Shindler & Bakker, 2020; Yusof & Tall, 1998). Bray & Tangney (2015) explored collaborative work with the assistance of technology. Other studies explored collaborative work preceded by instruction on problem-solving strategies and meta-cognitive models (Higgins, 1997; Lynch & Star, 2014).

As for the meaning that students usually assign to a problem and problem-solving, the way they perceive it, and how they make sense of it, the research explored that in conventional classroom settings (Di Martino, 2018; Higgins, 1997; Jablonka, 2005; Jurdak, 2006; Lerch, 2004; Stylianides & Stylianides, 2014; Wong et al., 2002), as well as in collaborative work setting, group work settings or authentic and real-life problem-solving settings (Francisco, 2005; Higgins, 1997; Hino 2015; Jurdak, 2006; Perrenet & Taconis, 2009; Stylianides & Stylianides, 2014).

This qualitative meta-synthesis aims at integrating and synthesizing the findings of the studies across a variety of classroom teaching/learning environments. The study intends to come up with a better understanding of how students negotiate their affective field with problem-solving and how they assign meaning to mathematical problems across alternative teaching and learning environments, leading to a deeper understanding of the issue.

Affect, Meaning, and Educational Levels

The studies on students' affective relationship with problem-solving and the meaning they assign to problems and problem-solving targeted different educational levels. Studies took place at either elementary, middle, high school, or university levels.

Among these studies, four targeted primary school students whose assigned meaning to problems and their affective disposition toward problem-solving were explored (DiMartino, 2018; Qaisar et al., 2015; Wong et al. 2002;). Qaisar et al. (2015) were interested in understanding the students’ affect towards problem-solving by exploring their emotional aspects and beliefs. On the other hand, Di Martino (2018) and Wong et al. (2002) explored what a problem means to students and how their assigned meaning changes as they progress in their grade levels from kindergarten to grade five.

Multiple studies explored our research questions at the middle school grade level (Francisco, 2005; Higgins, 1997; Hino, 2015; Jablonka, 2005; Moyer et al., 2018; Wong, 2002). Wong (2002) and Moyer et al. (2018) targeted students' affective field. They explored their emotions and beliefs towards problem-solving, whereas Jablonka (2005) and Francisco (2005) both investigated the meaning of learning from the students' point of view. The study by Hino (2015) tried to understand the way students view their affect and the meaning they assign to problems.

Seven of the studies identified for our qualitative meta-synthesis explored students' affect toward problem-solving and the meaning that the students assigned to problems and problem-solving at high school grade level (Andersson et al., 2015; Bray & Tangney, 2015; Jader et al., 2016; Jurdak, 2006; Lynch & Star, 2014; Moyer et al., 2018; Schindler& Bakker, 2020). The mentioned studies explored the students' affective field and emotional disposition towards problem-solving or their meaning of a problem and problem-solving, or both.

Researchers also showed interest in university-level students and attempted to figure out their emotions and beliefs, as well as the meaning they assign to problems and the way they view their affect in relationship to that meaning (Ju & Kwon, 2007;

Lerch, 2004; Martinez & Gonzalez 2016; Perrenet & Taconis, 2009; Satyam, 2020; Stylianides & Stylianides, 2014; Yusof & Tall, 1998). Ju and Kwon (2007), Lerch (2004), Martinez & Gonzalez (2016), Satyam (2020), and Yusof & Tall (1998) explored the beliefs of the students and their emotional disposition to problem-solving. Whereas Perrenet & Taconis (2009) and Stylianides & Stylianides (2014) explored students' affective field to problem-solving and the meaning they assign to problems.

The single study that followed students from grade one to university was the one done by Francisco (2005). However, in his study, he interviewed the students at the end of the intervention to explore how they perceive their mathematical knowledge and the meaning they assign to it. The fact that Francisco interviewed the students at the end of the intervention makes the study unfit to be considered one that explored the issue across grade levels. Francisco did not provide a baseline to conclude the changes that the students went through as they progressed in grade levels.

This qualitative meta-synthesis will synthesize and interpret the results of twenty-one studies across all educational levels from primary to university. The findings would be an essential factor in understanding the sequential development of students' assigned meaning, emotions, and beliefs as they grow.

Affect, Meaning, and Educational Systems

The studies chosen for this qualitative meta-synthesis took place globally, including America, Europe, Asia, and Australia. The fact that both students' learning experiences and national curriculums are embedded in culture and influenced by it made it essential to review these studies from the curriculum perspective.

Most of the studies in our qualitative meta-synthesis took place in the USA (Francisco, 2005; Higgins, 1997; Lerch, 2004; Lynch & Star, 2014; Moyer et al., 2018;

Perrenet & Taconis, 2009; Satyam, 2020; Stylianides & Stylianides, 2014). These studies explored either students' affect toward problem-solving, their assigned meaning to problems, or both.

In the United States, the curriculum frameworks vary according to state. Mathematics standards across the states emphasize learning mathematical content in real-world situations while still focusing on computational thinking and mathematical fluency to solve problems and promote a more profound understanding (Mullis et al., 2016).

Six of our studies took place in Asia (Hino, 2015; Jablonka, 2005; Ju & Kwon, 2007; Jurdak, 2006; Qaisar, 2015; Yusof & Tall, 1998). Ju Kwon (2007) explored students' affect during problem-solving in South Korea. Jablonka's (2005) study on how students attribute meaning to mathematical problem-solving compared Hong Kong's cultural teaching practices to USA's. Hino's (2015) study on students' affect toward problem-solving and the personal meaning they associate with problems took place in Japan. Jurdak (2006) explored students' views on situated problems that they were asked to solve and compare to real-world and school problems in Lebanon. Qaisar (2015) explored students' emotions and beliefs on mathematics in a Pakistani school, and Yusof & Tall (1998) had their study on university students' affect on problem-solving done in Malaysia.

It is noticeable that three Asian curriculums stress everyday problems to be targeted through mathematical learning. We find it imperative to mention that the three curriculums tackle the aesthetic aspect of mathematics, requiring students to appreciate the beauty of mathematics, the value of mathematical learning, and develop a positive personal attitude towards mathematics (Mullis et al., 2016).

Andersson et al. (2015), Jader et al. (2016), and Schindler & Bakker (2020) explored Swedish students’ beliefs on mathematical problem-solving. The Swedish curriculum highly stresses problem-solving and the formulation of mathematical questions in everyday situations and different subject areas and their application/modeling in different situations (Mullis et al., 2016).

Some of the studies on students' affect toward problem-solving and the meaning they assign to problems in our qualitative meta-synthesis took place in different European countries. Bray & Tangney's (2015) study took place in Ireland, Di Martino's (2015) study in Italy, and Jablonka's (2005) study in Germany. The curriculums in these countries stress the importance of problem-solving as a part of the learning process, involving everyday situations as a medium for practice and learning (Mullis et al., 2016).

Martinez & Gonzalez's (2016) study took place in Mexico, where the authors explored students' emotional experiences in their linear algebra course.

The selected studies for this qualitative meta-synthesis took place in a variety of cultural contexts. This variety in cultural contexts would enrich the analysis and interpretations of our qualitative meta-analysis. However, these countries belong to the Western developed world (the USA and Europe) or developed South-Eastern Asian countries (South Korea, Hong Kong, Japan). Therefore, the findings of this study are likely to be relevant to developed Western and South Asian countries.

Purpose

The purpose of this study is to integrate and interpret the findings of a selected pool of qualitative studies in terms of:

1. How students negotiate their meaning of a problem and problem-solving across alternative teaching and learning classroom environment, educational levels, and educational systems.
2. How students negotiate their affective relationship with problem-solving across alternative teaching and learning classroom environments, educational levels, and educational systems

Research Questions

1. How do students negotiate their meaning of a problem and problem-solving across alternative teaching and learning classroom environments, educational levels, and educational systems?
2. How do students negotiate their affective relationship with problem-solving across alternative teaching and learning classroom environments, educational levels, and educational systems?

Significance of the Study

Theories generated using primary qualitative research methods are generally not considered transferable outside the context in which they were developed because they originate from minimal samples. This qualitative meta-synthesis aims to integrate, synthesize, and interpret data from a collective body of qualitative research findings related to students' beliefs, emotions towards problem-solving, and meaning of a problem in different educational levels and in alternative classroom teaching/learning environments. Such a qualitative meta-synthesis would lead to a deeper understanding of students' beliefs and emotions towards word problems as well as their assigned meaning to problems and problem-solving and contribute to a more generalizable and transferable theory, thus helping us to move from knowledge generation to knowledge application.

The transferability of the results from this study might help educators build positive experiences for the students across various grade levels and alternative learning environments. These positive experiences would help those students negotiate their affective relationship with problem-solving and the meaning they assign to it in the direction that educators desire, a positive view towards mathematical problem-solving.

CHAPTER 2

REVIEW OF THE LITERATURE

Qualitative Meta-Synthesis

Qualitative meta-synthesis is a recent development in the field of qualitative research. It provides means to enhance the contribution of qualitative findings to the generation of more formalized knowledge. Human experiences share many common attributes; however, using a single lens to interpret these multiple experiences might not be feasible. A qualitative meta-synthesis contextualizes these experiences and brings them into the center, all under one lens; thus, making these experiences more meaningful and context-specific (Erwin et al., 2011).

Qualitative meta-synthesis uses rigorous qualitative methods to synthesize previous studies' findings to construct greater meanings and deeper understandings. In a qualitative meta-synthesis, the researcher uses the findings from a carefully selected pool of qualitative studies as his data instead of the studies' raw data. On top of that, the researcher engages heavily in a complex and deep analysis of the data to better understand the meaning within a specific context and generate a generalizable theory based on his new comprehensive findings (Erwin et al., 2011). The goal of qualitative meta-synthesis is to look for themes across studies that promote new insights into the body of qualitative literature while preserving the integrity of the original studies.

Theory-generating meta-synthesis research is placed within the qualitative research paradigm, and it is based on grounded theory methods (Corbin & Strauss, 2008). Being placed within this paradigmatic and methodological orientation, theory-generating meta-synthesis research is established on the assumption that theory can be inductively generated from qualitative data (Finfgeld- Connett, 2010).

As mentioned earlier in our paper, there are various approaches to qualitative meta-synthesis, such as meta-ethnography, meta-study, thematic synthesis, formal grounded theory, cross-study analysis, and meta-aggregation (Finfgeld-Connett, 2010). Some of these approaches have been influenced by an interpretive paradigm where they show a particular interest in placing qualitative insights within a larger discourse and in developing conceptual, theoretical frameworks to increase the understanding of how things connect and interact. Other approaches have found their theoretical base in critical realism, a paradigm supporting the acquisition of knowledge about the external world as it is, while recognizing that perception is a function of, and thus fundamentally marked by, the human mind. Pragmatism, a method that uses scientific logic to clarify the meaning of certain concepts or ideas through investigating their potential relationship with the real world, also found its way as a theoretical framework in qualitative meta-synthesis studies (Hannes & Lockwood, 2011).

The first qualitative meta-synthesis came from synthesizing four qualitative studies in sociology, it was done in the late 1960 s by Glaser and Strauss, the creators of grounded theory (Glaser & Strauss, 1967). Stern and Harris (1985) were the first who documented a meta-synthesis of qualitative studies in the field of nursing, where qualitative meta-synthesis is very popular currently (Zimmer, 2006). It took much longer for the qualitative meta-synthesis approach to reach the discipline of mathematics education. In the 2000s calls for qualitative meta-synthesis studies started to echo and very few studies started to appear (Yore & Lerman, 2008).

Students' Affective Relationship with Problem-Solving

More than seventy years after Polya's comments on the importance of affect in mathematical problem-solving in which he said "Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. …If he [the teacher] fills his allotted time with drilling his students in routine operations, he kills their interest… "(Pólya, 1945, p. V), substantial efforts have been made to better understand how the affective domain of students is related to problem-solving.

The interplay between cognition and emotion in the field of mathematics has been the concern of researchers for a while. Emotion is not simply a part of human thought and action due to anecdotal or phenomenal basis; there is now a growing body of evidence that emotional states interact significantly with various cognitive functions (McLeod & Adams,1989).

The research area investigating the interplay between cognitive and emotional aspects in mathematics education is known as affect, where the affective domain is divided into beliefs, attitudes, and emotions that could describe the wide range of affective responses to mathematics (McLeod, 1992). With the affective domain involving emotions, beliefs, and attitudes, these three factors influence the way students negotiate their relationship with mathematics in general and mathematical problem-solving in particular. (Evans et al., 2006; McLeod, 1992). Because affective factors heavily influenced problem-solving performance, research on problem-solving started to take affective issues into account.

When it comes to emotions in mathematics, research has shown of a multitude of emotional aspects that are involved in the process of mathematical problem-solving. Interest, motivation, anxiety, and fear are all emotions that students experience while solving problems. They acknowledge that mathematics is one of the school subjects that triggers the strongest negative emotions in students, which may cause students to develop an attitude of refusal towards it, thus hindering their thinking process (Buxton, 1981). According to researchers, students' struggle when problem-solving, even when they possess the necessary cognitive skills, is caused by their affect towards the subject. (Schoenfeld, 1983). A history of negative experiences with problem-solving that creates momentary negative emotions leads to a long-lasting beliefs and attitudes on problem-solving.

With all being said, it became a need that research provide the mathematical community with evidence-based data on how students negotiate their affective relationship with problem-solving and understand more about their affective responses to promote better beliefs and attitude towards mathematics. Recently, researchers have begun to conduct intervention studies in the mathematics classroom to understand the relationship between various classroom environments and students' affect toward problem-solving (Schukajlow et al., 2017). Thus, multiple qualitative studies tried to explore and understand this affect and the way it is shaped during problem-solving activities.

Research on Affective Relationship with Problem-Solving across Teaching /Learning Environments

Researchers have explored conventional and alternative classroom settings where group work, collaborative work, technology, and different teaching strategies were employed. Conventional classroom settings are when the student is a receiver of the knowledge through direct instruction and not a constructor (Jader et al., 2016; Lerch, 2004). In group work, students do most of their work independently while working within the group; they discuss and negotiate solutions during work. On the other hand, collaborative work happens when students work in groups interdependently, they share ideas and thoughts to perform a task or solve a challenging problem. (Andersson et al. 2015; Hino, 2015; Ju & Kwon, 2007; Moyer et al., 2018; Qaisar et al., 2015; Satyam, 2020; Shindler & Bakker, 2020; Stylianides & Stylianides, 2014; Yusof & Tall, 1998).

The use of heuristics and other metacognitive and problem-solving strategies is a well-known approach in mathematical problem solving, it can be used mostly while tackling unfamiliar problems that need unconventional ways to be solved. The effects of teaching and using these various strategies on students' affect has been the focus of research for a while (Higgins, 1997; Lynch & Star, 2014).

The popularity and advancement in technology made it easier for educators to design and incorporate more meaningful learning activities in a technology-enhanced classroom setting. Computers, mobile technology, and various mathematical software are at the disposal of educators and could be used to make the students' learning experiences more meaningful, practical, and joyful. Researchers investigated the impact of technology use on students' mathematical attitudes and beliefs in multiple studies. (Bray & Tangney, 2015)

The first part of this review will shed light on the studies that have explored students" affect toward mathematical problem solving in a variety of teaching/learning environments explored in a number of qualitative studies.

Conventional Classroom Environment

Jader et al. (2016) explored the beliefs of three secondary school Swedish students during problem-solving sessions. Jader found out that the students displayed a negative attitude towards problem-solving. Students' self-expectations and insecurities impacted their abilities to be creative thinkers and successful problems solvers. Students' belief of being good at solving only familiar problems strengthened their negative intrinsic motivation, which led to a feeling of insecurity towards the task. In contrast, students who displayed positive intrinsic motivation had less insecurity and were more persistent at solving the problem. Jader concluded that students' failure was highly influenced by the negative beliefs they carry about problem-solving.

Lerch (2004) explored how four Algebra I university students practice their control decisions while solving routine and non-routine mathematical word problems and how these decisions are influenced by their personal mathematical beliefs. Narratives written by the students and interviews revealed that none of the students have the control decision needed for problem-solving when working on non-routine problems. Lerch concluded that this lack of control was due to the personal belief systems of the students on non-routine problem-solving. Faulty beliefs, such as "formal mathematics has nothing to do with real life problem-solving", "mathematical problems should always be solved in less than ten minutes", and "only geniuses are capable of creating or discovering mathematics" influenced the way students approach problem-solving tasks, and their abilities to take control of the situation.

Martinez and Gonzalez (2016) interviewed twenty-seven university students to explore the range of emotions they go feel while working on problem-solving. Emotions such as disappointment, fear, and distress were explored. The authors found out that most of these negative emotions were caused by the goals that the students set for themselves, and the goal that the educational system and the culture set for them.

Group Work

Ju and Kwon (2007) explored how group work followed by classroom discussions would influence the beliefs of nineteen university students on mathematical problem-solving. Students worked in groups on problems borrowed from their real life during a differential equations course, where mathematical concepts will emerge and be discussed. A whole classroom discussion would follow the group work. Students would discuss the various ideas and justify their concepts with the teacher who orchestrated the discussion. The teacher would redirect the questions asked by one student to the other students to come up with ideas and construct arguments collectively. Analyzing the student's conversation during group work and whole class discussions, the authors noticed a change in the students' speech pattern. As the course progressed, the way students discussed their solutions changed from the third-person perspective mode to the first-person perspective mode. The third-person perspective reflects the students placing themselves as passive recipients and consumers of mathematics, reflecting the belief that mathematics is immutable and independent of what the students think. Whereas in a first-person perspective, students view themselves as active catalysts in producing their mathematical knowledge. Ju and Kwon concluded that this change in belief was due to the classroom practice that the students have experienced; they added that classrooms are communities of practice where norms and emerging students' beliefs are in a dialogical relationship. Altering the norms would lead to altering the beliefs.

Moyer et al. (2018) explored the effect of the reformed curriculum on the attitude, beliefs, and the emotional disposition of students towards mathematics in comparison to a traditional curriculum during the middle school years was explored by. The reformed curriculum takes a functional approach to algebra teaching in middle school. Concepts are taught through an exploration of the relationships between real-world quantities that arise in daily problem situations. On the other side, the traditional curriculum follows a structural approach by moving away from contextualized problems and focusing on students' procedural abilities. Forty-four grade twelve students were the study subjects, some of whom had studied the reformed curriculum during their middle school and the rest studied the traditional curriculum. The authors found a remarkable difference in the two groups' vision of mathematics. The reformed curriculum students were more independent learners, relied on themselves or their colleagues more than their teachers. On the other hand, the traditional curriculum students preferred structured instruction and individual work along with the teacher's assistance. The emotional disposition of the two groups did not vary significantly. Students from both groups expressed dislike for mathematics, but for different reasons. The authors argued that the reformed curriculum students' current dislike of mathematics could be because those students moved from the reformed curriculum back to the traditional lecture-based curriculum in high school. Moyer et al. concluded that the middle school learning experience provided by different kinds of curriculums would shape the attitudes and beliefs of students towards mathematics, making it vital to be wise in choosing the right curriculum that would provide the positive learning experience.

To understand what elicits significant positive emotions in students during their work on problem-solving, Satyam (2020) interviewed eleven university students attending a transition to proof mathematics course, where tenants of active learning and group work were used. The purpose, as indicated by Satyam, of understanding what elicits or triggers these positive emotions that he calls satisfying moments is that they could be utilized to build learning experiences around them. Positive learning experiences that elicit positive emotions would lead to a positive attitude towards mathematics. The author concluded that the most common satisfying moments happened when students succeeded at their task without struggle, overcame past challenges, worked with peers, and had the feeling that they understood the problem and were capable of doing it on their own.

A Swedish study explored the mathematical identity narratives of two high school students who labeled themselves as “math haters” and how these narratives were problem-context bound (Andersson et al., 2015). When the students could relate to the context of problem-solving on a human or personal level as they work in groups, they had a different narrative about their relationship with mathematical problem-solving. Both girls use words with positive connotations such as “good” and “useful” to describe their group work experience. The girls also talked about their experience with the task itself more actively using verbs such as “participated” and “talked about”. The girls expressed a positive attitude that was context-bound, but they also expressed their fear that that was temporary and unsustainable. The authors concluded that as the context changes, the narrative changes as well. According to the authors, experience with negative emotions would hinder the ability of the student to change their affective relationship with mathematics on a long-term basis, even if they can do it during a specific task.

Stylianides and Stylianides (2014) explored the impact of a short intervention on students' problem-solving common and counterproductive beliefs. Thirty-nine university students participated in the study, where they had to work in small groups followed by a whole classroom discussion on real-life problems that looked unsolvable, but they were. The problems were designed in a way that they were interesting, curiosity arousing, and fun. The role of the teacher was to assure the students at the beginning of the session that the problem was solvable and that solving it was within their capabilities if they persevered and thought well about it. The teacher provided scaffolding to a group only when it was needed. At the end of the study, the interviews indicated significant improvement in the students' problem-solving beliefs compared to those at the onset of the study. The authors concluded that the students' whole outlook on how to solve problems was changed, they stated that they started looking at problems more deeply and in more than one way. Students believed that persevering and working hard to solve a challenging problem would be satisfying and fulfilling.

Another study investigated the way students attend to their peers' multiple solutions during structured problem-solving sessions (Hino, 2015). During these sessions, the teacher presents the problem, the students work on it individually then in groups to compare solutions. A whole classroom discussion follows to summarize and highlight the major points. Twenty-four Japanese grade eight students were interviewed to express their thoughts about the learning practice after every session. Hino noted that comparing solutions had a positive impact on students' affect; they expressed developing interest and being impressed by their peers' novel ideas. Collecting information from peers' work empowered the students and created confidence that made them ready to try new and unfamiliar problem-solving. The author concluded that the students' attending to peers' different and multiple solutions was for the sake of self-development and having their exploratory questions answered instead of just getting the right solution for the problem; the exploratory questions supported their goals of sense-making and knowledge construction.

Collaborative Work

A case study in Sweden (Shindler & Bakker, 2020) investigated the affective fluidity and multiplicity as well as the change in affect of an eighteen-year-old female student during problem-solving and problem posing sessions. The sessions were inquiry-oriented collaborative problem-solving sessions taught by the first author of the article. During those sessions, the student first worked on the problems individually for a short time and then worked with a group for most of the time. At the end of each session, reflections took place. The student reflected using her own words on how the session went. The authors noted that the change in the student's affective field was dramatic. The student went from being an anxious mathematical problem solver to someone who enjoyed mathematical problem-solving. The student's affective field at the end of the study, compared to the beginning showed an apparent positive evolution. The student was ready to try new things and look into different directions; her attitudes shifted considering that mathematics was boring into saying that trying new things makes problem-solving more fun. The authors concluded that collaborative work and group dynamics provide students with a safe atmosphere, where students appreciate each other's ideas and approaches. They comfort each other and provide empathy and support, all this leading to a positive shift in their beliefs and attitudes towards problem-solving.

Qaisar et al. (2015) explored the beliefs of elementary students in two Pakistani schools on mathematical learning and understating and on the social context in which learning takes place. Collaborative work was a new context for the students who were used to direct instruction mode of learning. Students were not enthusiastic about working in groups and preferred to work independently before the intervention, as expressed during the interviews. Interviews after the intervention revealed a significant shift in students' attitudes toward collaborative work and mathematical learning. Collaborative work increased students' confidence in their abilities of problem-solving; they believed they could depend on peers for help. The intervention could create a shift in students' beliefs about who can and who cannot learn mathematics, the nature of learning mathematics, and the importance of the social context in learning mathematics.

Yusof & Tall (1998) explored the impact of collaborative problem-solving as a teaching approach on the mathematical beliefs and attitudes of 44 Malaysian university students. The conventional lecturing mode of learning was replaced by problem-solving with collaboration, where students would discuss, argue, conjecture, and come with a solution for the problem collaboratively. Students expressed their feelings towards mathematical learning before and after the intervention through written comments. The comments showed a significant shift in the way students think of mathematical learning, from boredom and frustration to enjoyment, satisfaction, and finding pleasure in problem-solving. The students' beliefs and attitudes changed from viewing mathematics as a body of procedures to learn into viewing mathematics as a process of thinking.

Collaborative Work with Technology

Bray and Tangney (2015) studied the effect of teaching mathematics based on the "Realistic Mathematics Education" model on fifty-four high school students' engagement and confidence in their mathematical learning process. The one yearlong qualitative study investigated the transformation that occurred to students in depth through interviews during and at the end of the study. Realistic Mathematics Education (RME) stems from the progressive philosophical view in education. In RME, learning mathematics starts with a problem that should have a meaningful context and relates to everyday life; the students identify the relevant mathematical concept in that problem, refine the problem into a mathematical one, solve it and interpret the solution in relevance to the original situation. In addition to the model the students used technology to work on the problems collaboratively with the teacher's scaffolding. Bray & Tangney found out that engaging in this kind of learning changed the outlook of some students on mathematics in a radical way. They said the process of guided discovery that the students went through made them have a sense of meaningfulness that positively impacted their engagement and confidence. The authors related this to the sense of autonomy and ownership the students experienced over their learning process. Bray & Tangney concluded that the real and personal understanding of the mathematical concepts that underline the problems students worked on created a feeling of confidence. The use of technology helped the students represent, relate, and manipulate things. The authors added that teamwork contributed positively to the students' affective engagement; they found enjoyment in mathematics and active participation in class where all students engaged in meaningful activities.

Collaborative Work with Teaching Strategies

Lynch and Star (2014) studied the effects of instruction using multiple strategies on the attitudes of six struggling high school students. The new curriculum material utilized in the intervention focused on the specific practice of comparing multiple strategies to solve word problems. Students will solve a word problem and then discuss a worked example presented to them. The worked example used two strategies that the students discussed and compared to their solutions in a whole classroom discussion. Lynch and Star concluded that learning multiple strategies made the students aware of the presence of more than one way to solve a problem, which alleviated their past difficulties, improved their understanding, reduced their problem-solving anxiety, and improved their attitudes toward mathematics and mathematical problem-solving.

Another study explored the effects of systematic instruction in problem solving on eighteen grade six and seven students, the study aimed to understand how problem-solving skills' teaching approach would influence students' attitudes and beliefs towards problem-solving (Higgins, 1997). After teaching the students five problem-solving strategies, students were presented with challenging word problems that they could solve in more than one way. The intervention took place over the course of the academic year. The students solved these problems in small groups, classroom discussions, and with the teacher's scaffolding when needed. The author found out that students who learned problem-solving strategies and had the chance to use them on daily basis developed confidence in their problem-solving abilities. The students admitted that they have transferred these skills to other classes and even to outside the school environment. The students also realized that problem-solving needed perseverance, so they were ready to spend hours trying to solve a problem with high levels of motivation. Higgins (1997) added that these students realized the usefulness of mathematics in general and appreciated the fact that it made them “use their brains.” The students’ opportunities to share and discuss their ideas and ways of thinking with their peers and teachers using a common language fostered their mathematical understanding and created a positive attitude towards mathematical problem-solving.

In summary, most of these studies have shown that group and collaborative work helps students alter their negative views towards problem-solving into positive ones. The fact that students can build their knowledge in collaboration with their peers creates various positive feelings such as motivation, engagement, and a feeling of control. These positive emotions could lead to the desired change in their attitudes and beliefs regarding mathematics and mathematical problem-solving.

The teaching of problem-solving strategies and other metacognitive strategies influences students’ attitudes towards problem-solving tasks positively, leading to more confidence and feeling in control. These strategies have been shown to provide students with a feeling of empowerment and competence over problem-solving tasks.

Research has also concluded that technology-enhanced learning positively affects students’ affective field towards solving non-familiar and challenging problems. The use of technology had a significant positive effect on students’ motivation, engagement, and joy. These positive context-bound feelings, upon recurrence, would lead to positive attitudes and beliefs towards mathematics and mathematical problem-solving.

The Meaning Students Assign to a Problem and Problem-Solving

Problem-solving has been a central theme in education for several decades. Mathematical problems and problem-solving have been defined by many researchers and many ways over the past 60 years. Mathematics educators and researchers have perceived mathematical problem solving and defined it as a heuristic process, a logic-based program, a means of inductive and deductive discovery, a framework for goal-oriented decision making, methodologies with multiple variables, and a model-eliciting activity (Yee & Bostic, 2014). However, the way students define problems and problem-solving is our area of concern.

The way students assign meaning to a problem or define a problem is a part of their epistemological view that emerges from their learning experiences. Their definition or meaning is related to their beliefs on problems, and problem-solving that are built based on their classroom experiences with problems (Callejo & Vila, 2009). Two kinds of conceptualization of a problem appear in students' narratives; students define and assign meaning to problems as an expert, or they provide a "grounded in practice" meaning (DiMartino, 2018).

Experts describe a problem as a situation in which the solver has gotten stuck, and the activation of creative and productive thinking is needed (Perkins, 2000). On the other hand, a grounded in practice definition views a problem as a task of a stereotypical nature that can be correctly solved using formulas and rules that comes from rote learning (Sidenvall et al., 2014).

An expert will have sophisticated view towards mathematics and mathematical problem solving that is consistent with the idea of mathematical problem solving as a sense-making activity, learning as constructing one's own knowledge rather than just receiving it, and teaching as a facilitating of the learning process instead of simply showing or telling students what they need to learn (Francisco, 2013). Whereas a student with the grounded in practice view will be searching for rules and will hold the belief that problems should be solvable by a distinct rule within a relatively short time and have only one answer (Jablonka, 2005).

Multiple studies have explored how students view mathematical problem-solving and the meaning they assign to problems. In this part of the review, we go over few of these studies and their findings.

Research on the Meaning Students Assign to a Problem and Problem-Solving Across Teaching/Learning Environments

Conventional Classroom

Di Martino (2018) explored how children change their vision of problems from kindergarten to upper elementary classes. He interviewed two hundred and eighty-four Italian students for the study; from kindergarten to grade five. The study showed that students from kindergarten to grade one, where the word “problem” is not usually mentioned in the classroom, have viewed, and referred to problems in an auspicious way; a problem for them was an everyday life problem that they wanted to solve. They expressed that thinking, trying different ways, collaborating with others, and asking for help are the ways to solve any problem. Students in grades three to five had a different vision of what a problem was. For these students, a problem is no longer an everyday problem that they want to solve, it is a text problem that needs to be solved. The problem should have a familiar context, one answer, and can be solved using a mathematical operation. The author concluded that the optimistic view of what a problem is that kindergarten students adopt changes into a more rigid and stereotypical view as they move to upper elementary classes, displaying a deterioration in their attitude towards problems and problem-solving. Di Martino added that the education experience that the students are going through at this stage is negatively influencing their attitude to problem-solving and the meaning they assign to problems.

One hundred grade eight students from the United States, Hong Kong, and Germany were interviewed to explore the way they attribute meaning to mathematics classroom activities (Jablonka, 2005). Jablonka discovered commonalities among the beliefs and attitudes of these students that she attributed to classroom practices. Most of the students studied math for the mere reason of passing a test, even though they thought that mathematics involved thinking and using the brain. Most of the students saw mathematical content as useless and could not be used in everyday situations. The goal of generating a product was dominant in students' responses to what learning is; they all focused on learning how to solve a question or find an answer. Jablonka concluded that learning as a process was not on the mind of the students, they saw the importance of learning mathematics and doing it right as a task with expectations and goals set by their teachers. From the students' perspective, the teacher provided the problem, the goals, the tasks and the way of solving them, whereas the students were supposed to be mentally engaged in finding the solution.

In Wong et al. (2002), 1216 students from grades 3,6,7, and 9 attempted solving three sets of mathematics problems. These were computational problems, word problems, and open-ended problems. Two students from each class were then interviewed individually on their strategies to solve the open-ended problems. The interviewers asked the students to explain their working procedures for solving the problems and to give their opinions on these problems. Wong concluded that the students were constrained by their faulty beliefs about mathematical problem-solving when they tackled the open-ended problems. He believed that these constraints were due to the years of exposure to traditional teaching classroom culture.

Group Work

Stylianides and Stylianides (2014) investigated if a short duration intervention would have a positively impact students' problem solving common and counterproductive beliefs and their assigned meaning to problem-solving. Thirty-nine university students participated in the study, where they had to work in small groups followed by a whole classroom discussion on real-life problems that looked unsolvable, but they were. The problems were designed in a way that they were interesting, curiosity arousing, and fun. The role of the teacher was to assure the students at the beginning of the session that the problem was solvable and that solving it was within their capabilities if they persevered and thought well about it. The teacher provided scaffolding to a group only when it was needed. The interviews at the end of the study indicated significant improvement in the way students view problem-solving beliefs compared to those at the onset of the study. The authors concluded that the students' whole outlook solving problems was changed; they started looking at problems more profoundly and in more than one way. Students believed that persevering and working hard to solve a challenging problem would be satisfying and fulfilling.

Collaborative Work

A study by Francisco (2005) investigated students' views on mathematical learning when they work collaboratively to explore patterns, make conjectures, and justify their reasoning during mathematical work. The students experienced a constructive approach that put the student at the center of the learning process. The longitudinal study followed five students from first grade through high school and university years. According to Francisco, the interviews at the end of the study revealed a positive, progressive view on mathematics and mathematics learning. The students agreed that acquiring mathematical knowledge should not be done through lecturing or showing and telling, it should be built up by the students themselves. The students believed that mathematical knowledge should be constructed through arguing and discussing ideas with peers, which they considered a source of motivation to do mathematics. Motivation for the students was not a task-related concept; it was more related to the whole learning process of collaborative work and discovery regardless of the task itself. Francisco added that the students exhibited sophisticated views about learning mathematics; they did not distinguish between learning and proofs, they viewed learning as a discovery process where they argue, negotiate and convince. They also viewed proofs as an integral part of knowledge acquisition through convincing arguments.

Collaborative Work with Authentic Problem-Solving

Perrenet and Taconis (2009) explore mathematical problem-solving beliefs from the students' perspective in a mixed study. First-year university students had to answer a questionnaire on precise and metacognitive processes in mathematical problem-solving, productive aspects of mathematical problem solving, and technical approaches to mathematical problem-solving. They answered the questionnaire at the beginning of the first year and at the end of the year. During that year, the students worked in pairs on authentic open-ended word problems related to their culture. The students’ responses to the questionnaires showed a significant shift in their views on the meaning of a problem and problem-solving, their views became more sophisticated due to the kind of problems they were exposed to according to the authors.

Real-Life Problem-Solving

In a theoretical and empirical study, Jurdak (2006) contrasted problem-solving of situated problems in school and the real world from the students' point of view. Three experiential word problems were given to 31 of grade twelve science students. While solving these tasks, the students were asked about their approach to the task's solution, their opinion of the tasks, and to compare them with school and real-life tasks. The students expressed that a real-life situation problem differs from the problems they solved in the study and from classroom problems. They mentioned that they would have more tools to use in real-life problems, such as the experience of others, logic, and their point of view. Jurdak concluded that students should be more involved in solving authentic situated problems to get more meaningful mathematical learning.

In summary, we can say that students' views about mathematical problem-solving and the meaning they assign to problems are highly influenced by the learning experience. Students' views could be naïve or sophisticated. Their epistemological beliefs on the nature of knowledge and how it should be acquired would vary depending on the learning approach and the social contexts in which this learning occurs.

Conclusion

Students, in general, display a range of negative emotions towards problem-solving; they lack the feelings of security and confidence to tackle word problems with flexibility and openness. Their beliefs and how they perceive and view the problem-solving process are mostly faulty, including their expectations that all problems should be solved in one way and in a short time and that mathematics is not for everyone.

Solving word problems represents an emotional challenge for students as much as it is a cognitive challenge.

Many studies have tried to research how classroom teaching/ learning environments would impact students' beliefs and emotions, hoping to build on these environments and reach the state desired by teachers and educators. Researchers also tried to understand the way students view, define, and relate to problems and problem-solving and what conditions would affect this relationship. Various contexts such as collaborative work, teaching problem-solving strategies, and the use of technology have shown to be promising in most situations. Research suggested that altering the learning environment will alter the learning experience. Thus, it might alter the students' affect towards problem-solving and the meaning they assign to problems and problem-solving. Researchers require a deeper understanding of the issue to reach an answer that would present a guide or a clear pathway for educators and decision-makers to follow.

CHAPTER 3

METHODOLOGY

Research Designs in a Qualitative Meta-Synthesis Study

This study is a qualitative meta-synthesis that synthesizes and interprets findings across a pool of selected qualitative studies to explore how students negotiate their affect toward problem-solving and the personal meanings they assign to problem-solving. Synthesizing a collective body of qualitative studies on a specific topic to determine common themes provides a richer and more thorough understanding that might not be possible to achieve from a single study (Finfgeld-Connett, 2018).

Before heading into our design's details, we find it imperative to distinguish among qualitative meta-synthesis, a literature review, secondary analysis, and meta-analysis. A researcher summarizes, critiques, or evaluates a specific area of interest in qualitative and quantitative studies in a literature review. Literature reviews do not generate new theories to the existing literature. On the other hand, the secondary analysis method uses the studies' raw data as their subject for analysis to come up with new interpretations based on a new look at data. A meta-analysis is a statistical procedure that merges a body of quantitative research, collects, aggregates, and analyzes data to a numerical value to infer a cause-and-effect relationship (Zimmer, 2006).

Several methods were adopted for carrying out qualitative meta-synthesis, such as meta-ethnography, meta-study, meta-summary, and theory generating meta-synthesis (Finfgeld-Connett, 2018). Meta-ethnography was the first method to be developed in qualitative meta-synthesis. Noblet and Hare developed meta-ethnography in the late eighties. They targeted the field of social sciences, aiming to synthesize findings from primary qualitative research in a systematic, cross-comparative, inductive, and interpretive way to create a more comprehensive and holistic interpretation of individual qualitative research findings (Noblet & Hare, 1988). Meta-study is another method to conduct qualitative meta-synthesis, which analyzes primary research results and reflects on the perspectives and processes involved in that primary research. Meta-study employs critical examination of the primary research's theory, method, and data to culminate in a synthesis that generates new knowledge (Paterson et al., 2001). Qualitative research synthesis is the third approach used to tackle qualitative meta-synthesis. An approach grounded in methodology and rigorous, the qualitative research approach extracts the meaning from primary research at a higher level by combining them into a new whole. Information from primary research is aggregated and interpreted to present a comprehensive view of the knowledge contained in multiple individual research (Major & Savin, 2010).

Sandelowski et al. (2007) developed the meta-summary approach to meta-synthesis. Meta-summary is an approach that is a quantitatively oriented aggregation approach to synthesis. However, it lends itself convenient and workable for qualitative research where the studies are survey-based, or the findings are summaries of the data in primary studies. Meta-summary sums up data of primary research and filters it to conclude. (Sandelowski et al., 2007).

Finfgeld-Connett (2010) developed the theory-generating qualitative meta-synthesis approach. The approach is based on grounded formal theory research methods and situated within the qualitative research paradigm (Corbin & Strauss, 2008). Theory-generating meta-synthesis analyzes and interprets the data from the findings of multiple heterogeneous primary research studies. Data from the findings are coded and categorized to extract a theme and a generalizable theory from the findings of the primary research studies. Results from theory-generating meta-synthesis go beyond summing and aggregating previous research findings; these results amount to a newly synthesized theory that could support decision-making and applicability of findings (Finfgeld-Connett, 2018).

Qualitative meta-synthesis emerged in health sciences and gained a growing interest in the broad field of education (Au, 2007; Cobb et al., 2009); nevertheless, it later found its way to the mathematics education field. Very few qualitative meta-synthesis studies are available in mathematics education. Their meta-synthesis approaches are based on the grounded formal theory or thematic and content analysis, where primary research findings represent the meta-synthesis subjects (Thomas & Berry, 2019; Thunder & Berry, 2016).

This study utilizes theory-generating approaches to qualitative meta-synthesis based on the grounded formal theory (Finfgeld-Connett, 2018). The grounded theory methodology has its roots in interpretivism, the philosophical view that revolves around the way people make sense and meaning of their reality, a look that aligns with the questions of this study. The grounded theory provides a comprehensive framework that enables the researcher to understand and explain what is happening by coding, categorizing, constant comparison, coming up with a theme, and generating a well-founded theory (McCann& Polacsek, 2018).

Elements of Qualitative Meta-synthesis

After identifying our research question, the qualitative meta-synthesis approach includes the following steps:

1. Carry on a comprehensive search for articles.
2. Select initial relevant articles.
3. Appraise the quality of the selected studies.
4. Code data from the findings of the selected articles. By reading and re-reading and identifying and labeling segments of a text containing an idea or concept (called code) that are relevant to the research questions.
5. Generate categories by grouping codes into categories that deal with similar ideas through the researcher's reflective analysis.
6. Generate themes by reflecting on the categories to generate more general themes.
7. Describe and interpret themes through labeling, describing, interpreting, and exemplifying the emerging themes.
8. Synthesize findings from successfully appraised studies.

The Initial Selection of the Articles

We started searching using the American University of Beirut digital library to search the following databases:

1. ERIC
2. Academic Search Ultimate
3. Educational Research Complete

The search was restricted to the following parameters:

1. The year 2000 onwards.
2. Peer-reviewed journal articles.
3. Mathematics education discipline.

The first step in choosing articles was to decide on the relevance of an article based on its title; the second step was reading the abstract. Most of the articles were quantitative studies; qualitative studies were rare; thus, we changed the search to studies published from 1990 onwards. Few articles were selected based on their abstracts before another round of search that targeted the databases of a few mathematical journals. We ran a search within the following journals due to their known credibility and relevance:

1. International Journal on Mathematics Education (ZDM)
2. Journal of Mathematical Behavior
3. Journal of Educational Studies in Mathematics

We used the search term “students’ affect in problem-solving,” which did not yield enough studies that meet the search criteria. Therefore, we added “emotions and problem solving,” “students’ attitudes and problem-solving,” and “students’ beliefs and problem-solving.” A total of 24 studies were selected that met the initial search criteria. The inclusion criteria were:

1. Qualitative or mixed studies (methodological parameter)
2. Published 1990 onwards (temporal parameter)
3. Targeted students (population parameter).

Appraisal of Selected Articles

A quality appraisal of the individual studies was performed to finalize the selection process. Another appraisal for the journals in which the articles were published was done. The initial inclusion and exclusion criteria were based on our meta-synthesis topic, population, temporal, and methodological parameters for our study, but every good quality qualitative research study should include basic elements. So, it is essential to check the quality of these elements, such as research problem and purpose, data collection techniques, data analysis, and report of findings, in addition to implications and conclusions (Thunder & Berry, 2016).

To appraise the quality of the journals from which our articles were extracted, we used the "Scientific Journals Ranking" (Scimago Journal & Country Rank, n.d.). SJR is a portal that ranks journals by their scientific influence based on the number of citations they receive from other journals and the importance of citing journals. Twenty-one out of the twenty-four articles we selected passed the SJR appraisal from the first quartile journals (Q1), the highest rank in the system. Our next step was to appraise the articles individually.

Individual appraisal of the twenty-one articles followed the recommendations of Finfgeld-Connett (2018). We used the "Critical Appraisal Skills Program Checklist" (CASP, 2018) for qualitative studies, which is usually used to help researchers examine research studies for quality, trustworthiness, and potential inclusion in a qualitative meta-synthesis. CASP requires a qualitative study to address the following issues:

Q1) Was there a clear statement of the aims of the research?

Q2) Is a qualitative methodology appropriate?

Q3) Was the research design appropriate to the aims of the research?

Q4) Was the recruitment strategy appropriate to the aims of the research?

Q5) Was the data collected in a way that addressed the research issue?

Q6) Has the relationship between researcher and participants been adequately considered?

Q7) Have ethical issues been taken into consideration?

Q8) Was the data analysis sufficiently rigorous?

Q9) Is there a clear statement of findings?

Q10) How valuable is the research?

Table 1 shows that 21 of the 24 journal articles met the CASP and SJR criteria and hence qualified for our meta-synthesis.

Table 1

SJR and CAPS Appraisal

The study appraised	**CASP**										**SJR**
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	
Andersson, A. et l. (2015).	+	+	+	+	+	0	+	+	+	+	Q1
Bray, A., & Tangney, B. (2015).	+	+	+	+	+	0	+	+	+	+	Q1
Di Martino, P. (2018).	+	+	+	+	+	0	+	+	+	+	Q1
Francisco, J. M. (2005)	+	+	+	+	+	0	+	+	+	+	Q1
Higgins, K. M. (1997).	+	+	+	+	+	0	+	+	+	+	Q1
Hino, K. (2015).	+	+	+	+	+	0	+	+	+	+	Q1
Jablonka, E. (2005).	+	+	+	+	+	0	+	+	+	+	Q1
Jäder, J. et al. (2016).	+	+	+	+	+	0	+	+	+	+	Q1
Ju, M.-K., & Kwon, O. N. (2007).	+	+	+	+	+	0	+	+	+	+	Q1
Jurdak, E. M. (2006)	+	+	+	+	+	0	+	+	+	+	Q1
Lerch, C. M. (2004).	+	+	+	+	+	0	+	+	+	+	Q1
Lynch, K., & Star, J. R. (2014).	+	+	+	+	+	0	+	+	+	+	Q1
Martínez-Sierra, G., & García-González, M. D. S. (2015).	+	+	+	+	+	0	+	+	+	+	Q1
Moyer, J. et al. (2018).	+	+	+	+	+	0	+	+	+	+	Q1
Perrenet, J. & Taconis, R. (2009)	+	+	+	+	+	0	+	+	+	+	Q1

Qaisar, S. et al. (2015)	+	+	+	+	+	0	+	+	+	+	Q1
Satyam, V. R. (2020).	+	+	+	+	+	0	+	+	+	+	Q1
Schindler, M., & Bakker, A. (2020).	+	+	+	+	+	0	+	+	+	+	Q1
Stylianides, A. J., & Stylianides, G. J. (2014).	+	+	+	+	+	0	+	+	+	+	Q1
Wong, N. et al. (2002)	+	+	+	+	+	0	+	+	+	+	Q1
Yusof, Y. B. M. & Tall, D. (1998).	+	+	+	+	+	0	+	+	+	+	Q1

Note. SJR quartile and Critical Appraisal Skills Programme (CASP) questions scoring: Yes =+, Can't Tell = 0, No = -

Selected Studies Based on Appraisal

The quality appraisal of the articles yielded 21 studies that qualified for our meta-synthesis as shown in Table 2. The articles addressed our research questions on how students negotiate their affect with problem-solving and the meaning they assign to problems.

Table 2

Post Appraisal Selected Articles

Articles that addressed students' affect toward problem solving	Articles that addressed meaning students assign to problem solving	Articles that addressed both questions
Andersson, A., Valero, P., & Meaney, T. (2015). Bray, A., & Tangney, B. (2015). Jäder, J., Sidenvall, J., & Sumpter, L. (2016). Ju, M.-K., & Kwon, O. N. (2007). Lynch, K., & Star, J. R. (2014). Martínez-Sierra, G., & García-González, M. D. S. (2015).	Jablonka, E. (2005). Francisco, J. M. (2005). Jurdak, E. M. (2006) Stylianides, A. J., & Stylianides, G. J. (2014).	Di Martino, P. (2018). Higgins, K. M. (1997). Hino, K. (2015). Lerch, C. M. (2004). Perrenet, J. & Taconis, R. (2009) Wong, N. et al. (2002)

Moyer, J. C., Robison, V., & Cai, J. (2018). Qaisar, S., Dilshad, M., & Butt, I. H. (2015). Satyam, V. R. (2020). Schindler, M., & Bakker, A. (2020). Yusof, Y. B. M. & Tall, D. (1998).

The individual appraisal of the articles helped our research questions emerge. Examining the articles revealed trends, commonalities, and differences in the studies; these factors led to refining the research questions. The articles selected varied in the teaching/learning environments that the students experienced, in the educational levels they targeted, and varied in educational systems that framed their studies. Therefore, we categorized our articles across three domains that match our research question: classroom environments (Table 3), educational levels (Table 4), and educational systems (Table 5).

Table 3

Studies across Classroom Environments

	Conventional classroom environment	**Group work**	**Collaborative work**	**Technology with collaborative work / real-life PS**	**Collaborative work with teaching strategies**
Affect	Andersson, A., Valero, P., & Meaney, T. (2015). Di Martino, P. (2018). Higgins, K. M. (1997). Jäder, J., Sidenvall, J., & Sumpter, L. (2016). Lerch, C. M. (2004). Martínez-Sierra, G., & García-	Ju, M.-K., & Kwon, O. N. (2007). Andersson, A. et al. (2015). Satyam, V. R. (2020). Hino, K. (2015).	Qaisar, S., Dilshad, M., & Butt, I. H. (2015). Schindler, M., & Bakker, A. (2020). Yusof, Y. B. M. & Tall, D. (1998). Francisco, J. M. (2005).	Bray, A., & Tangney, B. (2015).	Lynch, K., & Star, J. R. (2014). Higgins, K. M. (1997).

	González, M. D. S. (2015). Moyer, J. C. et al. (2018). Qaisar, S., Dilshad, M., & Butt, I. H. (2015). Schindler, M., & Bakker, A. (2020). Wong, N. et al. (2002) Yusof, Y. B. M. & Tall, D. (1998).				
Meaning	Jablonka, E. (2005). Di Martino, P. (2018). Wong, N. et al. (2002) Stylianides, A. J., & Stylianides, G. J. (2014). Lerch, C. M. (2004).	Hino, K. (2015). Stylianides, A. J., & Stylianides, G. J. (2014).	Francisco, J. M. (2005).	Jurdak, E. M. (2006). Perrenet, J. & Taconis, R. (2009)	Higgins, K. M. (1997

Table 4

Studies across Grade Levels

	Primary level	**Middle school**	**High school**	**University**
Affect	Higgins, K. M. (1997). Qaisar, S., Dilshad, M., & Butt, I. H. (2015).	Moyer, J. C., Robison, V., & Cai, J. (2018). Hino, K. (2015).	Andersson, A. et al. (2015). Bray, A., & Tangney, B. (2015). Jäder, J. et al. (2016). Lynch, K., & Star, J. R. (2014). Schindler, M., & Bakker, A. (2020). Francisco, J. M. (2005	Ju, M.-K., & Kwon, O. N. (2007). Lerch, C. M. (2004). Martínez-Sierra, G., & García-González, M. D. S. (2015). Satyam, V. R. (2020). Yusof, Y. B. M. & Tall, D. (1998). Stylianides, A. J., & Stylianides, G. J. (2014).

Meaning	Di Martino, P. (2018).	Hino, K. (2015). Jablonka, E. (2005). Wong, N. et al. (2002)	Francisco, J. M. (2005). Jurdak, E. M. (2006).	Stylianides, A. J., & Stylianides, G. J. (2014). Perrenet, J. & Taconis, R. (2009)

Table 5

Studies across Educational Systems

	U. S. A and south American educational system	European educational systems	Asian and Australian educational systems
Affect	Higgins, K. M. (1997). Lerch, C. M. (2004). Lynch, K., & Star, J. R. (2014). Martínez-Sierra, G., & García-González, M. D. S. (2015). Moyer, J. C., Robison, V., & Cai, J. (2018). Satyam, V. R. (2020). Francisco, J. M. (2005). Stylianides, A. J., & Stylianides, G. J. (2014).	Andersson, A., Valero, P., & Meaney, T. (2015). Bray, A., & Tangney, B. (2015). Jäder, J., Sidenvall, J., & Sumpter, L. (2016). Schindler, M., & Bakker, A. (2020).	Ju, M.-K., & Kwon, O. N. (2007). Qaisar, S., Dilshad, M., & Butt, I. H. (2015). Yusof, Y. B. M. & Tall, D. (1998). Hino, K. (2015).
Meaning	Francisco, J. M. (2005). Stylianides, A. J., & Stylianides, G. J. (2014).	Di Martino, P. (2018). Perrenet, J. & Taconis, R. (2009)	Jablonka, E. (2005). Hino, K. (2015). Jurdak, E. M. (2006). Wong, N. et al. (2002)

Data Analysis

To analyze our data, we used the grounded theory. The grounded theory came into existence with Glaser and Strauss's book "*The Discovery of Grounded Theory: Strategies for Qualitative Research"* (1967). Their groundbreaking work explained how theory could be generated from data inductively and iteratively. When using the grounded theory, data are collected, then coded and categorized in a continuous iterative process that moves toward saturation and results in themes that constitute a theory grounded in the data.

This qualitative meta-synthesis uses the grounded theory to analyze data extracted from the twenty-one selected articles. The articles' findings represented the raw data for analysis; they were our source of information; all article's findings represented a subject for our meta-synthesis.

As mentioned earlier, our analysis started with coding; we identified and labeled segments of a text containing ideas or concepts (called code) relevant to the research questions by reading and re-reading the text, using constant comparison as we moved across subsequent texts. Then the codes were grouped into categories that dealt with similar ideas through the researcher's reflective analysis, and finally, by reflecting on the categories, more general themes emerged. Themes were interpreted to generate a theory. An example showing how theme one emerged from the data is provided in the appendix of this thesis.

Each of our twenty-one articles was coded individually. Coding filtered data and classified them to give us a grip for making comparisons with other data segments; we studied each article's findings and began to separate, sort, and synthesize these data through qualitative coding.

After coding all our articles separately, we used constant comparison to answer each research question by conducting a qualitative analysis of the corresponding set of articles, as shown in Table 6.

Table 6

Set of Articles Analyzed

Research question	***Set of articles***
Meaning of problem solving	
1) How do students negotiate their meaning of problem solving?	Articles on students' meaning of problem solving

1a) How do students negotiate their meaning of problem-solving across alternative teaching and learning classroom environments?	Articles on students' meaning across alternative classroom environment
1b) How do students negotiate their meaning of problem-solving across educational levels?	Articles on students' meaning across educational levels
1c) How do students negotiate their meaning of problem-solving across educational systems?	Articles on students' meaning across educational systems
Affect towards problem solving	
2) How do students negotiate their affective relationship with problem-solving?	Articles on students' affective relationship with problem-solving
2a) How do students negotiate their affective relationship with problem-solving across alternative teaching and learning classroom environments?	Articles on affect across alternative classroom environment
2b) How do students negotiate their affective relationship with problem-solving across educational levels?	Articles on affect across educational levels
2c) How do students negotiate their affective relationship with problem-solving across educational systems?	Articles on affect across educational systems

The resulting themes from these analyses were discussed, compared, and contrasted to answer the research questions of the study.

CHAPTER 4

RESULTS

This section has three parts. The first part is on students' negotiation of the meaning of a problem, the second part is on student's negotiation of the meaning of problem solving, and the third part is on students' negotiation of their emotions towards problem-solving

Students' Negotiation of the Meaning of a Problem

Upon analyzing the findings of the qualitative studies to answer our question on how students negotiate the meaning of a problem, three themes have emerged.

1. Students' negotiation of the meaning of a problem across learning environments.
2. Students' negotiation of the meaning of a problem across grade levels.
3. Students' negotiation of the meaning of a problem across educational systems.

Theme Related to Students' Negotiation of the Meaning of a Problem across Learning Environments

Our analysis of journal articles that dealt with students' view of a problem suggests that when students experience alternative learning environments characterized by individual/collaborative work, as contrasted to a status quo whole-class learning environment, they negotiate their meaning of a problem. The students display a shift in the way they view a problem. In status quo learning environments students view a problem as a closed, narrow-focused narrative posed by a textbook or a teacher in a familiar context and asking for a numerical answer; whereas in alternative learning environments, students start to view the problem as an open and broad-focused narrative that may be non-routine, in a non-familiar context, that may require thinking and analyzing the data to resolve the issue posed in the narrative.

Status Quo Learning Environments

Students experiencing status quo learning environments displayed a closed and narrow-focused view of a problem in mathematics. The attributes for a task to be considered a problem include a textbook or a teacher posing this problem. It is posed in a familiar context and asks for a numerical answer. In addition to the fact that the students characterize what tasks they would view as problems, they also exclude any task that does not meet their criteria. For example, proofs in geometry, tasks posed in non-familiar contexts or have non-routine content, issues that they identify while learning mathematics are not viewed as problems. Similarly, tasks that do not include all the information needed for a solution or have extra information, tasks that ask for non-numerical answers, and tasks that do not require calculation are not considered problems.

The quotes in Table 4.1 (status quo studies) that illustrates the students' meaning of a problem in status quo learning environments, come from five qualitative studies.

Status Quo Learning Environments Studies. DiMartino (2019) collected interview data from 284 students from kindergarten, grades one, three, and five in the forms of oral and written interviews. Teachers asked students to define and explain what a "problem" means, requiring them to provide examples of a problem and suggestions on how they would solve a problem.

In Wong et al. (2002), 1216 students from grades 3,6,7 and 9 attempted solving three sets of mathematics problems. These were computational problems, word problems, and open-ended problems. Two students from each class were then interviewed individually on the strategies they used to solve the open-ended problems. The interviewers asked the students to explain their working procedures for solving the problems and give their opinions regarding these problems.

Table 7

Student's Negotiation of the Meaning of a Problem across Learning

Article	Quotes
	Status-quo Studies
DiMartino (2019)	"For me, a problem is a ***text with some data".*** (p.303). "A problem is a text, with ***a familiar context and a final question***." (p.301).
Wong et al. (2002)	"Some even suggested that those [problems]that needed explanations ***were not mathematics, "as they do not involve numbers"*** (p. 33). "They did not take our non-routine problems as mathematics, since they ***were not calculable, did not involve numbers, and involved decision-making***." (p. 31).
Jablonka (2005)	A student: "actually, all ***problems are equations.***" (p.375)
Stylianides & Stylianides (2014)	"The Problem ***included information that had nothing to do with numbers*** or mathematics it did not include an equation, it read like ***a riddle."*** (p. 21). "***Did not follow the typical format*** in which the givens are offered first, and a question follows." (p.26)
Lerch (2004)	"I don't understand the way the problem is set up… If they just came right out and said it, I think it ***would be much easier than putting it in a whole paragraph form***." (p.32)
	Collaborative work studies
Francisco (2005)	"Brian emphasized the importance of coverage of ideas in-depth … which help get to the "***the full meaning of the problem***" (p.62). "I understand a lot better the whole ***concept behind each problem***." (p.56).
Stylianides & Stylianides (2014)	"Beth said that the problem gave [her***] a new sense of looking at word problems*** in terms of ***what kind of information is necessary*** to have in a problem or how this information can be presented." (p. 24), "She [the student] added that the problem was challenging and, although initially she thought it had nothing to do with mathematics, ***after some thinking she realized it was indeed a mathematical problem***." (p. 26).
Higgins (1997)	"7 of these students informed me that their teachers did give them impossible problems to work on. They claimed that it was all right because ***the problems made them think hard and that one learns by working on such problems."*** (p.19),
Perrenet & Taconis (2009)	"At school the problems were of a type where a precise answer was possible. ***In the real world a precise answer is almost impossible"*** (p.191),

The open-ended problems had some problems with irrelevant information, allow more than one solution, allow multiple methods or different interpretations, and need judgment. The open-ended problems were very different from what the students used to encounter in their daily status quo classroom environment.

In Jablonka (2014), the qualitative study followed ten consecutive mathematics lessons in six classrooms; the lessons were videotaped in a status quo classroom environment and documented for data collection. In addition to videotaping the lessons, some of the students provided feedback and comments after each lesson. One hundred nine students from six classrooms took part in the interviews about how they engage in mathematics at school, the meaning they assign to specific mathematical topics and mathematics in general, and their feelings during mathematics lessons.

Stylianides and Stylianides (2014), and to help expand the students' view about the nature or importance of different kinds of referents in mathematical problems, presented 39 students with an unfamiliar problem to solve. The problem was supposed to appear to the students as unsolvable. However, the problem was solvable and within their capabilities if they try hard. The problem included few numbers that seemed to offer insufficient information for its solution. The students needed to consider other kinds of data than numbers to be able to solve the problem. The researchers asked the students to describe their initial reaction upon reading the problem and explain whether the problem was different from other problems they encountered in their mathematics classes.

In the study by Lerch (2004), the researchers assigned both routine (textbook) and non-routine (recreational mathematics) problems for four students to solve. The students were supposed to talk out loud and tape their comments on their thinking process as they solved these problems. The students worked on these problems outside of class, with no restrictions on with whom they worked. They used their existing mathematical knowledge to solve the problems using whichever methods they preferred and spending no more than twenty minutes on any problem.

Alternative Learning Environments

As students experience alternative classroom environments characterized by collaborative work or collaborative work with heuristics or with authentic problem solving, they adopt a new view to the meaning of a problem and develop a new sense of looking at a word problem. Students accept that a problem is like problems they face in the real world. It could be non-routine, unfamiliar, and not necessarily like what they are used to encounter, occurring in a non-familiar context. A problem could be challenging, with the information presented in different ways and with many possible answers, like real-world problems. A problem has a meaning behind it, which requires understanding. They view that thinking is required when facing a problem to understand the concept behind it, and this thinking leads to learning and resolving the problem

The quotes in Table 4.1 (collaborative work learning environments studies) that illustrate the students' meaning of a problem in collaborative work learning environments, come from five qualitative studies.

Collaborative Work Learning Environment. Collaborative work took place across different classroom environments; these environments are explained below providing the context of each of the studies.

Collaborative Work. Francisco's (2005) study explored the students' views on mathematics learning from different angles of a group of five high school students who were a part of a 12year long longitudinal study. The students were interviewed at the end of the study to talk about their mathematical experiences. These students learned and experienced mathematics under research conditions consistent with a constructivist approach to mathematical learning in which collaborative work played a significant role.

Stylianides & Stylianides (2014), and after they presented their students with the unfamiliar "Problem "mentioned earlier, they were prompted to think about it differently. The teacher assured the students that the Problem was solvable and encouraged them to think about all the information the Problem offered. The teacher asked to work on the "Problem" individually and then in small groups. Members of the same group worked collaboratively and shared responsibility for completing the assigned task. Each small group member was expected to explain to the rest of the class the solution reached by the small group during a whole classroom discussion. Then the students were asked to write down and describe their experience with working on the Problem.

Collaborative Work with Heuristics. Higgins (1997) had two groups of students, the heuristics, and the non-heuristics. Teachers of the heuristic students attended a 3-week training session. The training involved immersing the teachers in mathematical problem-solving and providing them with specific training on the nature of the problem-solving teaching, which they were to apply with their students. The heuristic teachers taught five problem-solving skills at the beginning of the school year to their students: guess and check, look for a pattern, make a systematic list, make a drawing or model, and eliminate possibilities. The teachers taught the skills using direct instruction. Students worked on solving routine and non-routine problems individually and collaboratively.

The non-heuristic teachers had not received any formal training in mathematical problem-solving and did not use an instructional approach to teach problem solving. The non-heuristics students experienced few problem-solving strategies that were scattered throughout the textbooks with little emphasis on problem-solving strategies instruction, thus experiencing a status quo learning environment in comparison to the heuristic group. To assess these students' problem-solving processes, the author interviewed both groups of students individually at the end of the intervention. The students talked about memories of the mathematic classes that were taught, what problem-solving means to them, their attitudes toward different types of problems in mathematics, and their ability to solve nonroutine mathematical problems.

Collaborative Work with Authentic Problem Solving. Perrenet & Taconis (2009) investigated mathematical problem-solving beliefs and behavior from the students' perspective in a mixed study. First-year university students had to answer a questionnaire on precise and metacognitive processes in mathematical problem-solving, productive aspects of mathematical problem solving, technical approach to mathematical problem-solving at the start of the first year. They answered the same questionnaire at the end of their bachelor's program. The students then compared their answers to the two questionnaires and noted any shift in their views. The students explained the shift in their views, providing reasons about what might have caused it. The curriculum used during those university years reflected the local mathematical culture and included a series of authentic mathematical modeling projects that the students worked on in pairs; the open-ended problems were posed in a non-mathematical language.

Theme Related to Students' Negotiation of the Meaning of a Problem across Grade Levels

As students move across grade levels and get exposed to formal mathematics education, they negotiate their assigned meaning to a problem, from an issue that they can resolve and that they need to resolve, into a textual and mathematical structure that has specific characteristics.

Kindergarten and elementary

Kindergarten children view a problem as an issue that needs to be solved, and they want to resolve it, even if they are unable to do that initially. The problem is a daily real-life situation such as having a fever or a broken washing machine with no clean clothes, losing a tooth and cannot find it, or not being able to open a doorknob.

Once the term "problem" is formally introduced at the elementary grade level in the mathematics curriculum, we notice a shift in the way students define a problem. The problem stops to be an issue that they want to resolve, it becomes as something that they must solve. For these students, a problem is a text with data in a familiar context and has a final question. The question posed by a problem has a unique answer. Elementary students do not consider non-numerical problems as mathematical problems. The non-routine problems that need explanation, do not involve numbers, are nor calculable and involve decision making and thinking are excluded from their definition of a problem. The quotes in Table 4.2 (kindergarten and elementary levels studies) that illustrates the students' meaning of a problem at kindergarten and elementary levels, come from six qualitative studies.

Table 8

Students' *Negotiation of the Meaning of a Problem across Grade Levels*

Article	Quotes
	Kindergarten and elementary levels studies
DiMartino (2019)	"the majority of the students characterize a problem as ***something that can be solved and they want to solve***, but—at least initially—they are not able to solve." (p. 300). "A problem is when I ***cannot open the do***or because I cannot reach the doorknob." (p. 300). "The washing machine is broken, and I have ***got no clean clothes***", (p.300), "I ***have fever.***" (p.300),
DiMartino (2019)	"A ***problem is a text, with a familiar context and a final question***. The question has a ***unique numerical answer***." (p.301) "a problem is no longer characterized as something that students want to solve but as ***something that must be solved***" (p.301)
Wong et al. (2002)	"Some [students]even suggested that those [problems]that needed explanations were not mathematics, "***as they do not involve numbers"*** (p. 33). "they did not take our non-routine problems as mathematics, since they were ***not calculable***, did not involve numbers, ***and involved decision-making***." (p. 31).
	Middle School level studies
Jablonka (2015)	A student's comment "actually, all ***problems are equations***." (p.375).
Wong et al. (2002)	"I feel these problems are like ***composition rather than mathematics***" (p.35). be "some felt that ***these non-routine problems were more challenging*** while others "don't like these problems as ***they do not follow a fixed rule*** [for solution]" (p.34)
Higgins (1997)	"Three of the non-heuristic students also said that problem ***solving involved words and numbers, not just numbers"*** (p.16).
	High school level study
Wong et al. (2002)	"These problems involve real life situations [rather than mathematics]***, not asking for a definite answer***" (p.33). "This question ***looks like logical reasoning more than mathematics***…logical reasoning involves words more, and ***for math, all are numbers***." (p.31)
	University level studies
Stylianides & Stylianides (2014)	"It [the problem] differs from other problems because the ***last clue has nothing to do with numbers and math."*** (p. 22), "When I first saw it [the problem***], I thought it was a joke" (p.25***). "The Problem included information that had nothing to do with numbers or mathematics it ***did not include an equation, it read like a riddle." (p. 21).***
Lerch (2004)	"I don't understand the ***way the problem is set up***… If they just came right out and said it, I think it would be ***much easier than putting it in a whole paragraph form." (p.32)***

Middle school

At middle school, students view all mathematical problems as equation. They view challenging non-routine problems that don not follow fixed rules for solving as composition rather than mathematical problems. They do not like these problems as well. The middle school students consider that a problem is what involves words and numbers and not just numbers. The quotes in Table 4.2 (middle school level studies) that illustrates the students' meaning of a problem at middle school level, come from three qualitative studies.

High School

When students move to high school, the meaning they assign to a problem is not much different; a problem still has specific characteristics; otherwise, it is not a part of mathematics. A "problem" to high school students should involve numbers rather than real-life situations and ask for a specific answer. An open-ended problem that does not entail numbers is not a mathematical problem; the students see that it belongs to logical reasoning labeling rather than mathematics. The open-ended problem has more words, whereas it should have numbers instead. The quotes in Table 4.2 (high school level study) that illustrates the students' meaning of a problem at high school level, come from one qualitative study.

University Level

Even at the university level, the shift continues. Students still hold a similar view of their colleagues on what a problem is. An unfamiliar problem that is different from problems students have seen before and does not have enough numbers and does not include an equation is not considered a problem. The students see it as a riddle or a joke by the students. The students would not welcome a problem written in the form of a paragraph; they would prefer a problem to be set or asked directly and clearly. The quotes in Table 4.2 (university level studies) that illustrates the students' meaning of a problem at university level, come from six two studies.

Theme Related to Students' Negotiation of the Meaning of a Problem across Educational Systems

Students across American, Hong Kong, and Italian educational systems adopt similar views on the meaning of a problem. A problem should be "mathematical" in nature, based on a text with data and numbers to be combined to reach a solution.

Italy

Italian students viewed a problem as a text with a familiar context, numerical data, and a final question. The given data is to help answer the problem through some mathematical operations.

Table 9

Student's Negotiation of the Meaning of a Problem across Educational Systems

Article	Quotes
	Italy study
DiMartino (2019)	"For me, a problem is a ***text with some data***. These data have to be used with some operations to answer the written questions" (p.303). "A problem ***is a text, with a familiar context and a final question***." (p.301).
	Hong Kong study
Wong et al. (2002)	"Some even suggested that those ***[problems]that needed explanations were not mathematics,*** "as they ***do not involve numbers***" (p. 33). "They did not take our non-routine problems as mathematics, since ***they were not calculable, did not involve numbers, and involved decision-makin***g." (p. 31). "Most [routine] ***problems in textbooks involve calculations*** and these [non-routine] problems involve thinking" (p.32)
	USA studies
Stylianides & Stylianides (2014)	When I first saw it, ***I thought it was a joke***" (p.25)., "Blond hair and addresses [problem***] has nothing to do with Math"*** (p.22). "According to the students, the Problem included information that had nothing to do with numbers or mathematics it ***did not include an equation, it read like a riddle***." (p. 21). "She noted that the Problem was ***not a typical mathematics problem***, …and ***did not follow the typical format*** in which the givens are offered first, and a question follows." (p.26)
Lerch (2004)	"I don't understand the way ***the problem is set up…*** If they just came right out and said it, I think it ***would be much easier than putting it in a whole paragraph form***." (p.32)

Hong Kong

The Chinese students view the problem as a mathematical entity that should involve numbers and calculation. A non-routine problem that does not involve numbers and requires thinking will not fit their criteria for what a problem is. Most problems in textbooks involve calculations and do not involve much thinking or decision making. The quotes in Table 4.3 that illustrates the students' views of a problem in Italy, China and USA studies, come from four qualitative studies that addressed this issue.

The USA

USA students shared similar views on the meaning of a problem; for them, a problem involves numerical mathematical data, follows a typical format, and has all the numerical information necessary to solve it. A problem is supposed to be set up clearly rather than a paragraph or a story that looks like a riddle or a joke.

Students' Negotiation of the Meaning of Problem-Solving

Upon analyzing the findings of the qualitative studies to answer our question on how students negotiate the meaning of problem solving, three themes have emerged.

1. Students' negotiation of the meaning of problem-solving across learning environments.
2. Students' negotiation of the meaning of problem-solving across grade levels.
3. Students' negotiation of the meaning of problem-solving across educational systems.

Theme Related to Students' Negotiation of the Meaning of Problem-Solving across Classroom Learning Environments

The analysis of the studies regarding the view of students on problem-solving as a process revealed that as students experience alternative learning environments, characterized by collaborative work, multiple solutions with scaffolding, multiple strategies and authentic problem-solving in contrast to a status quo learning environment, they negotiate and shift their view on the process of problem-solving. They shift their view from a text and data-based view where solving a problem is a matter of using rules into a view where problem solving entails cooperation in work, discussion of solutions, thinking and perseverance

Status-Quo Classroom Environment

Students in a status quo learning environment revealed a closed and narrow-focused view on the process of problem-solving in mathematics. For the process of problem-solving to be carried, there are specific characteristics and criteria to be met. These characteristics affirm that a problem is solved in one correct way, using standard recipes and applications such as sets of rules, formulas, and algorithms. The information given in the problem must be sufficient for solving the problem, entailing words and numbers. The numbers are combined using some mathematical operations such as addition and subtraction to reach a single solution for the problem. A problem must be solved using a step-by-step procedure in one try, and this is done in a short time frame. Any problem whose elements do not satisfy these assigned characteristics will be deemed unsolvable.

The quotes in Table 4.4 (status quo studies) that illustrates the students' views of a problem-solving in a status quo learning environment, come from nine qualitative studies that addressed this issue. The paragraphs below provide the context of each of the studies that are quoted in Table 4.4.

The contexts of the studies by DiMartino (2019), Higgins (1997), Jablonka (2014), Lerch (2004), Perrenet & Taconis (2009), Stylianides & Stylianides (2014), and Wong et al. (2002) were discussed earlier, in these studies the students were interviewed in a status quo or pre-intervention classroom environment.

Table 10

Students' Negotiation of the Meaning of Problem-Solving across Status Quo Learning Environments

Article	Quotes
	Status quo learning environments studies
DiMartino (2019)	"A mathematical problem can always be solved, ***and it has only one solutio***n, which should be reached ***in a single correct way***" (p.302).
Hino (2015)	"During individual problem solving, ***students usually used one or two methods*** to get the answer." (p.133).
Wong et al. (2002)	"***Each mathematics [problem] has only one solution"*** (p.33). "Every problem in the mathematics classroom ***has a unique answer***, has only one way of tackling and ***can be solved within minute***s" (p.41), "***Jot down the numbers*** in the question. Then read through the question to see how I can ***tackle it, e.g., +, −,*** and thus I get the answer" (p. 36),
Higgins (1997)	"The most common responses from the non-heuristic students were "just solving a problem" or "***finding an answer when you add, subtract, multiply, or divide the two numbers"*** (p.16). "I would only ***work on a problem for 2-3 minutes*** because your brain swells if you concentrate too long." (p.18)
Jablonka (2005)	"Classical algebraic text problems for setting up equations are introduced together with ***"steps" for solving these problems***." (p. 376), "...students recognize mathematics by the ***use of terminology*** and content and perceive mathematics as ***a set of rules*** …. when dealing with "word problems …the students appreciate this form of representation and ***otherwise feel lost.***" (p.377).
Jurdak (2006)	"***Different from school tasks"*** (p. 292) "In classroom math, the teacher ***asked questions that were applications of already memorized formulas"*** (p.293) "In class we have ***to stick to one way of solving a problem"*** (p.293)
Lerch (2004)	"The students ***applied general arithmetic procedures***, supporting the belief that there is a ***step-by-step procedure*** to solve any problem situation" (p.31). "Students are well conditioned to view problems as ***solvable in a short time frame***." (p.31)
Perrenet & Taconis (2009)	"The mathematics problems I met at school came up within the context of a ***certain technique ... or they were solved using standard recipes"*** (p.189), "At school most of the times only one method existed***; problems were right in one try; solutions were always short"*** (p.191) "At school it was simply ***the application of rules***." (p.191).
Stylianides & Stylianides (2014)	"One third of the students wrote that they did not know how to solve the Problem, whereas the rest deemed the ***Problem unsolvable***" (p.21) "Any math problem I have ***encountered has usually given me enough information to solve the problem.*** This problem, I feel, leaves out critical information to solving the problem." (p.22).

Jurdak (2006) contrasted theoretically and empirically problem-solving of situated problems in school and the real world. Three potentially experiential problem tasks were given to 31 last year high school science students. While solving the task, students were asked by the investigator about their approach to the solution of the task. After completing the task, students were asked their opinion of the tasks and to compare them with school and real-world tasks.

In Hino (2015), ten consecutive mathematics lessons in two classrooms were observed and recorded. The lessons were sessions of ‘structured problem-solving”. The teacher reviews the previous lesson then presents the problem for the day. The students work on the problem individually or in groups, and the author called this “individual problem-solving”. A whole classroom discussion of the various solution methods follows where the major points are highlighted and summarized, the author denotes this step by “collective problem-solving”. Some of the students were interviewed at the end of the lessons to express their opinions about the lessons, their goals, and their idea of a good lesson.

Alternative Learning Environments

Students who experienced alternative classroom environments such as collaborative problem-solving, collective problem-solving with scaffolding, collaborative work with multiple strategies, and authentic problem-solving negotiated their past views and outlook on problem-solving as a process and adopted more open views. They developed an alternative view of problem-solving as a collective endeavor where cooperation and arguing with colleagues play a significant role in problem-solving. They viewed problem-solving as an exercise that prepared them for real-world problems and required deep thinking and using past knowledge, enough time, and perseverance. These students became interested in multiple solutions to a problem; they became open to different and new solutions posed by colleagues that would enable them to solve problems they could not solve before and viewed as impossible. The way to reach a solution and answer a problem became more important than the answer itself. In contrast to problem-solving in school, real-world problems allow choice of methods and need argumentation and not simple applications of rules. In real-life problem-solving environment, students would use mental calculation and logic, solve their problems quickly and easily, consider factors other than calculation that might interfere in the solution like getting the advice of others who have experienced similar problems, and weighing the solution in terms of their own affordances.

The quotes in Table 4.5 (collaborative work studies and real-life problem-solving study) that illustrate the students' meaning of a problem-solving in collaborative work learning environments, come from six qualitative studies.

Collaborative Work. The contexts of the two studies by DiMartino (2019) and Stylianides & Stylianides (2014) were explained earlier, in both studies the students collaborated on solving problems and were interviewed after they experienced the collaborative work environment.

Collective Problem Solving/ Scaffolding. In the study by Hino (2015) whose context was discussed earlier, the students reflected on their experience of collective problem-solving and attending to multiple solutions that were presented by their colleagues.

Collaborative Work/Heuristics. Higgins (1997) heuristic students, as explained in the study context earlier, were taught heuristics as skills and tools to use in problem-solving through direct instruction and worked on solving problems individually and collaboratively. The heuristic students were interviewed to assess their problem-solving processes.

Collaborative Work/Authentic Problem Solving. In the study by Perrenet & Taconis (2009) that was mentioned earlier, the students reflected on their change of view towards the process of problem-solving after experiencing an environment of authentic problem-solving through their university years, where they worked on solving authentic modeling problems in pairs.

Table 11

Students' Negotiation of the Meaning of Problem-Solving across Alternative Learning Environments

Study	Quote
	Collaborative work learning environments studies
Francisco (2005)	"Romina said ***that through arguing*** the students "learn more" and ***become better prepared to handle real world problems"*** (p.61), "***Thinking deeply about a problem …*** and using past knowledge is a way of solving a new problem" (p.58) "If you're stuck on a problem and you don't know the answer, ***if you have two or three people together, it's much easier to come up with an answer***." (p.63)
Stylianides & Stylianides (2014)	"The answer requires figuring out exactly what the problem is asking ***and thinking about what you know about math*** to obtain the answer" (p. 26) " I didn't believe this was a real problem to begin with but ***once discussing it with my group*** and coming up with possible solutions ***I realized it could be solved***" (p. 23).
Hino (2015)	"Students ***become interested in different solutions*** " (p. 132), "I noticed that there ***was another way of solving that question***". (p. 132) "Students … commented that they learned the solution methods by ***listening to classmates discuss concepts that they could not grasp themselves during individual problem solving***" (p. 131) "The commenters usually mentioned that ***they saw ways to solve problems that they had not been able to solve previously.***" (p. 133)
Higgins (1997)	"They claimed that it was all right because the ***problems made them think hard and that one learns by working on such problems."*** (p.19), "The length of time they said they ***would work on a problem*** before they would believe it was impossible ... ranged from ***4 hours to 1 week***" (p.20).
Perrenet & Taconis (2009)	"And besides that, ***it is more the way to the answer that is more important than the answer itself.***" (p.191), "Now ***there is a choice of methods*** and some of those take lots more time than others" (p.191), "At school it was simply the application of rules; ***now a totally new argument can be needed for a solution"***, (p.191).

	Real-life problem-solving study
Jurdak (2006)	"In real life I could get and ***give the answers directly without showing a way"*** (p.296) "In real life ***other factors interfere***: the services the phone companies provided, the promotion they are making, and ***the experience of others*** who used these offers to tell us about their experience with these offers" (p.296) "In real life, ***factors other than calculations influenced a decision" (p.296)*** ***"In real life, the final decision depends on the person's point of view***" (p.296) "In real life, we ***use mental calculations and logic"(p.296)***

Real Life Problem-Solving. In real life problem-solving (Jurdak, 2006), decision making is a complex activity that occurs within the larger social context, and which results in a decision constrained by the acceptable social and personal rules and using all available mathematical and non-mathematical tools. In real-life problem-solving, they would use mental calculation and logic and solve their problems quickly and easily, considering that factors other than calculation might affect the solution like getting the advice of others who have experienced similar problems, and weighing the solution in terms of their own affordances.

Theme Related to Students' Negotiation of the Meaning of Problem-Solving across Grade Levels

As students progress across grade levels and get exposed to formal mathematical education, they negotiate and shift their view on the solvability of a problem from an open view into a narrow one. The students shift their view from being ready to think, analyze, and try various ways to solve a problem into prescribing conditions for problem-solving, such as a problem can be solved in one single way with a single correct answer in a short time.

Kindergarten and Elementary

Children believe that every problem they face at the kindergarten level could be solved, except for death. These kindergarten children have various tools to solve a problem; they think and analyze and reach multiple solutions for one problem.

At the elementary level, students still believe that all problems are solvable, but they set rules and conditions for solving these problems. Students believe that a problem is solved by combining the given numbers, using some operations in a single correct way, and reaching a unique correct answer. To solve a problem, elementary students would read the question, figure out what is required to answer and the operations they need to use. They use these operations to combine the numbers and get their unique solution within few minutes. The quotes in Table 4.6 that illustrates the students' negotiation of the meaning of problem-solving at kindergarten and elementary grade levels, come from three qualitative studies that addressed this issue.

Table 12

Students' Negotiation of the Meaning of Problem- Solving across Grade Levels

Article	Quotes
	Kindergarten level study
DiMartino (2019)	"The only problem recognized as ***not being solvable by all children is death."*** (p.298). "Three other ways of solving problems…in Group 1: ***thinking/reasoning***, trying in ***various ways***, ***not giving up."*** (p.300) "Their naturalness in ***analyzing in depth a solution***, criticizing it, and finding multiple solutions to a single problem" (p.300)
	Elementary level studies
DiMartino (2019)	"A mathematical problem can always be solved, and it ***has only one solution,*** which should be ***reached in a single correct way***" (p.302) "Highlighting ***data and finding keywords in the text,*** therefore finding the right ***arithmetical operation***." (p. 303).
Wong et al. (2002)	"Read through questions, pick out the numbers, ***try out with +, −,*** and see what the problem is asking for." (p. 35) "Every problem in the mathematics classroom ***has a unique answer***, has only ***one way of tackling*** and can ***be solved within minutes***" (p.41)
	Middle school level studies
Jablonka (2015)	"Classical algebraic text problems for setting up equations are introduced together with **"steps" for solving these problems**." (p. 376)

Wong et al. (2002)	"***Jot down the numbers in the question***. Then read through the question to see how I can tackle it, e.g., +, −, and thus I get the answer" (p. 36), "There is ***definitely one answer***, very definite. One won't be asked of possibilities" (p.33).
Higgins (1997)	""just solving a problem" or "***finding an answer when you add, subtract, multiply, or divide the two numbers***" (p.16). "I would only ***work on a problem for 2-3 minutes*** because your brain swells if you concentrate too long." (p.18)
	High school level studies
Wong et al. (2002)	"Look at what conditions I have in hand, ***what the given information*** is, and what is being asked" (p.35), "We see that not only is finding the answer stressed in most cases, but also ***obtaining the answer in a few minutes"*** (p.33)
Jurdak (2006)	"***Different from school tasks***" (p. 292) "Classroom math helped in ***reaching answers quickly***" (p.293) "In classroom math, the teacher ***asked questions that were applications of already memorized formulas"*** *(p.293)* "In class we have ***to stick to one way of solving a problem***" (p.293)
	University level studies
Stylianides & Stylianides (2014)	"Any math problem I have encountered has usually given me ***enough information to solve the problem.*** This problem, I feel, leaves out critical information to solving the problem." (p.22). "When I first looked at this problem, I saw ***there were no numbers***" (p.25) "I realized with a lot of these problems it took me ***a lot longer than 5 to 10*** minutes to solve them." (p.26),
Lerch (2004)	"The students applied ***general arithmetic procedures***, supporting the belief that there is a ***step-by-step procedure*** to solve any problem situation...they applied an ***inappropriate algorithm…. students*** are well conditioned to view ***problems as solvable in a short time frame."*** (p.31)
Perrenet & Taconis (2009)	"The mathematics problems I met at school came up within the context of a certain technique ... or they were ***solved using standard recipes***" (p.189), "At school most of the times ***only one method existed***; problems were right in one try; solutions were always short, … it was simply the ***application of rules***." (p.191)

Middle School

Middle school students see mathematics and problem-solving as a set of rules, they appreciate using this form of representation, otherwise they feel lost. They usually use one or two methods to solve a problem through the usage of arithmetic operations on the given numbers they have. To solve a problem means to read the question, figure out how to tackle it, use operations to find the one and single definite answer. A problem for them should be solved within minutes. The quotes in Table 4.6 that illustrates the students' negotiation of the meaning of problem-solving at middle school grade levels, come from three qualitative studies that addressed this issue.

High School

The high school students share the same view of elementary and middle school students that a problem is solved looking at the given information and what is being asked, then their concern is to find the answer in few minutes. see that not only is finding the answer stressed in most cases, but also obtaining the answer in a few minutes. They get lost when they had to solve non-routine problems. The quotes in Table 4.6 that illustrates the students' negotiation of the meaning of problem-solving at high school grade levels, come from a qualitative study that addressed this issue.

University Level

The same view on problem-solving continues at university level. A word problem is deemed unsolvable if it does not have enough information to solve it, and the information should be numbers. Students have the conviction that a problem should be solved within a specific short period of time, not more than ten minutes. To solve a problem the students will apply general arithmetic step-by-step procedures, they could also apply inappropriate algorithm to solve a problem which they view as should be solvable in a short time frame. for university students problems are solved using standard recipes and application of rules, following one method and getting an answer in one try. The quotes in Table 4.6 that illustrates the students' negotiation of the meaning of problem solving at university levels, come from three qualitative studies that addressed this issue.

Theme Related to Students' Negotiation of the Meaning of Problem-Solving across Educational Systems.

Students across American, Hong Kong, Italian, German, Lebanese, and Netherland educational systems adopt similar views on the meaning of problem-solving. Students have set conditions for problem-solving, such as a problem can be solved quickly, using one single way, and getting a unique answer.

Italy

Italian students have rules for problem-solving; they assign specific conditions that a problem should meet for it to be solvable and the way it could be solved. They think that all problems are solvable, and this is done by combining the given numbers, using some operations in a single correct way, and reaching a unique correct answer. Highlighting the text, finding the right data, and the correct operation are the used approaches to problem-solving. The quotes in Table 4.7 that illustrates the students' negotiation of the meaning of problem-solving in Italy, China, and USA studies, come from eight qualitative studies that addressed this issue.

Germany

For German students, solving a problem entails rules and formulas and specific algebraic techniques and strategies that they should follow in a step-by-step manner; otherwise, they will be lost.

Japan

Japanese students will use one or two methods only to solve a problem.

Table 13

Students' Negotiation of the Meaning of Problem-Solving across Educational Systems.

Article	Quotes
	Italy
DiMartino (2019)	"a mathematical problem can always be solved, and it ***has only one solution***, which should be reached in ***a single correct way***" (p.302). "***Numerical data and arithmetic operations*** are considered necessary in order to solve mathematical problems." (p.304)
	Japan
Hino (2015)	"During individual problem solving, students usually ***used one or two methods*** to get the answer." (p.133).
	Hong Kong

Wong et al. (2002)	"Every problem in the mathematics classroom ***has a unique answer***, has ***only one way of tackling*** and can be solved within minutes" (p.41), "***Jot down the numbers*** in the question. Then read through the question to see how I can tackle it, e.g.**, +, −,** and thus I get the answer" (p. 36)
	USA/Germany/Japan
Jablonka (2005)	"Classical algebraic text problems for setting up equations are introduced together ***with "steps" for solving these problems***." (p. 376),
	Lebanon
Jurdak (2006)	"***Different from school tasks"*** (p. 292) "In classroom math, the teacher ***asked questions that were applications of already memorized formulas" (p. 293)*** "In class we have ***to stick to one way of solving a problem*"** (p.293)
	USA
Higgins (1997)	"Just solving a problem" or "***finding an answer when you add, subtract, multiply***, or divide the two numbers" (p.16). "I would only ***work on a problem for 2-3 minutes*** because your brain swells if you concentrate too long." (p.18)
Lerch (2004)	"The students applied general arithmetic procedures, supporting the belief that there is a ***step-by-step procedure to solve any problem*** situation." (p.31). "Students are well conditioned to view problems as ***solvable in a short time frame***." (p.31)
Perrenet & Taconis (2009)	"The mathematics problems I met at school came up within the context of a certain technique ... or they were solved ***using standard recipes"*** (p.189), "At school most of the times ***only one method existed***; problems were right in one try; solutions were always short… at school it was simply ***the application of rules***." (p.191).
Stylianides & Stylianides (2014)	"One third of the students wrote that they did not know how to solve the Problem, whereas the rest ***deemed the Problem unsolvable***" (p.21) "Any math problem I have encountered has usually ***give me enough information to solve*** the problem. ***This problem, I feel, leaves out critical information*** to solving the problem." (p.22).

Hong Kong

Hong Kong students view problem-solving as a process in which they will use one method to reach a single answer, and this is done within a short period of time. To solve a problem, they would figure out what information is given, what they are required to do, and which arithmetic operations they will use to reach a solution.

The USA

American students need enough information to solve the problem. The presence of numerical data in a problem is a critical condition for it to be solved. They solve the problem using one method, with one final answer and a short period of time. To solve a problem, American students will use general arithmetic procedures or algorithms in a step-by-step manner.

Netherlands

Students in the Netherlands view problem-solving as a procedure where using specific techniques or standard recipes will solve a problem. They think that there is only one method to solve a problem, where one applies rules and gets the correct answer.

Lebanon

Lebanese students view problem-solving in the school context as an activity within the school community, which results in a written solution using mostly mathematical tools and constrained by school rules, norms, and expectation. Classroom problem-solving involves applying formulas that they have memorized and sticking to one way of solving the problem on hand quickly.

Students' Negotiation of their Emotions towards Problem-Solving

Upon analyzing the findings of the qualitative studies to answer our question on how students negotiate their emotions towards problem-solving, three themes have emerged.

4. Students' negotiation of their emotions towards problem-solving across learning environments.
5. Students' negotiation of their emotions towards problem-solving across grade levels.
6. Students' negotiation of their emotions towards problem-solving across educational systems.

Definition of Emotion

Emotion is an affective construct that develops as a psychological response in reaction to momentary experiences. Emotions are usually transient, and they vary with incidents and circumstances; they can be intense and change rapidly (McLeod, 1992). A significant characteristic of emotions is their valence. Most emotional constructs are characterized by positive or negative valence. For example, enjoyment and love are positive emotions, whereas boredom and dislike in mathematics are negative emotions (Scherer et al., 2013).

Theme Related to Students' Negotiation of their Emotions towards Problem-Solving across Grade Levels

As students move across grade levels and get exposed to formal mathematics education, they gradually negotiate their emotions towards problem-solving from ambivalent positive (love word problems) and negative (despair) emotions into a wide range of negative emotions towards the whole discipline of mathematics such as dislike, boredom, discomfort, anxiety, confusion, frustration, and stress.

Elementary School

Once students are introduced to formal mathematical problem-solving in grade three, some students express their love for problem-solving and show their interest in it as a tool that enhances thinking, whereas other grade three students express their hate for problem-solving and the whole discipline of mathematics. Finally, when students reach grade six, the shift in emotions is characterized by a collective dislike of problem-solving and viewing mathematics as a boring subject. The quotes in Table 4.8 illustrate how kindergarten and elementary students negotiate their emotions toward problem-solving from three qualitative studies that researched the issue.

Table 14

Students' Negotiation of their Emotions towards Problem-Solving across Grade Levels

Article	Quotes
	Elementary School studies
DI Martino 2019	"When I have to solve a problem, I hold my head in my hands ***and I despair***." (p.304)
Wong 2002	"Some students even ***loved mathematics*** because it could provoke thinking." (p.32) "***I don't love math*** since I don't like thinking" (p.32) "I always find difficulty but the more I need to think the more ***I find it interesting."*** (p.32) "***Love word problems*** because writing enhances thinking" (p.34)
Qaisar et al. 2015	"Three of the students said that they ***do not like mathematics*** and believe that it is a difficult ***and boring*** subject" (p.76)
	Middle school studies
Higgins 1997	"Three non-heuristic students ***did not like to do these types of problems*** [non-routine] (p. 19) "They were unsuccessful in solving…this lack of success led to their ***dislike of the problems***" (p.19)
Wong 2002	"While younger students (mainly from ***Grade 3) loved word problem***s, students from upper grade levels ***[grade 6]*** found it ***"a waste of time."*** (p.34)
	High school studies
Andersen 2015	"Petra classified herself I am a ***math hater***" (p.149) "Malin described her previous experiences of mathematics education with the word Bobehagligt^ ***(unpleasant)"*** (p.150) "It has become so associated ***with difficulties and anxiety***" (p.150)
Jader et al. 2017	"On two occasions she [Leila] also indicates an ***insecurity*** *r*egarding her own ability." (p.768) "All three students indicate beliefs of ***insecurity"*** (p. 773)
Moyer et al. 2018	"We found that of the students who expressed ***distaste for mathematics….all of the non-CMP students did so*** because they thought it was difficult." (p. 128)
Schindler & Bakker 2020	"It kind of affects me when I cannot solve it***. I don't feel very confident and strong***" (p.9) "Anna's utterances reflect a low self-efficacy ("I can't do this") and ***helplessness/sadnes***s ("crying"). (p.9)
	University studies
Yusof & Tall 1999	"The students were initially ***very confused*** "(p.70) "***I did not enjoy most of the math courses*** – too dependent on the lecturers." (p.78)
Martinez & Gonzalez 2016	"***Solving problems...***triggers ***disappointment emotions*** when not achieved" (p. 97) "I spend a lot of time in the same problem, and then ***I get frustrated"*** (p.98) "This [problem solving on blackboard] ***stresses me*** and makes me wish to skip class" (p.99)
Lerch 2004	"Three of the four student participants found their ***experience working with the recreation problems uncomfortable***." (p.29) "Steve indicated that they were ***very confusing*** to him." (p.29)

Middle School

Students at middle school express their dislike for problem-solving in general and non-routine problem-solving in particular. They disliked problem-solving of non-routine problems because they considered it a waste of time. The quotes in Table 4.8 illustrate how middle school students negotiate their emotions toward problem-solving from two qualitative studies that addressed the issue.

High School

Students at high school grade levels describe themselves as math haters. They associate mathematics with anxiety, difficulties, and unpleasant feeling. They experience feelings of sadness and helplessness when working on problem-solving and a feeling of insecurity. They lack the feeling of confidence and strength when it comes to mathematics in general. The quotes in Table 4.8 illustrate how high school students negotiate their emotions toward problem-solving from four qualitative studies that addressed the issue**.**

University Level

When students reach university level, their emotions towards problem-solving and mathematics continue to be negatively valanced, expressing unenjoyment in mathematics courses and confusion regarding non-routine problem-solving. When working on non-routine problem-solving, frustration, stress, and discomfort are the most experienced emotions by university students. Disappointment is what they feel when they fail to solve a problem. The quotes in Table 4.8 illustrate how high university students negotiate their emotions toward problem-solving from four qualitative studies that addressed the issue.

Theme Related to Students' Negotiation of their Emotions towards Problem-Solving across Learning Environments

Our analysis of journal articles that dealt with students' emotions towards problem-solving indicates that when students experience alternative learning environments characterized by collaborative work, compared to a status quo whole-class learning environment, they negotiate their emotions towards problem-solving by experiencing a shift in their emotions. While elementary students experience ambivalent positive and negative emotions in status quo learning environments, other students experience mostly negative emotions toward problem-solving such as dislike, boredom, sadness, helplessness, anxiety, insecurity, frustration, and stress. In contrast, alternative learning environments trigger in students a wide range of positive emotions such as enjoyment, interest, fun, excitement, and engagement. Students feel more confident, secure, and satisfied during collaborative problem-solving.

Status Quo Learning Environments

While elementary students' emotions swing between positive and negative emotions in status quo learning environments, other students experience negative emotions toward problem-solving. Dislike, boredom, sadness, helplessness, anxiety, insecurity, frustration, and stress are among the students' emotional experiences. While working on problem-solving, students in status quo environments lack confidence, feel uncomfortable and confused, and see the time they spend on problem-solving as wasted. In addition, feelings of disappointment and frustration arise when the students fail at their problem-solving tasks.

The quotes in Table 4.9 (status quo studies) that illustrates the students' negotiation of emotions towards problem-solving in status quo learning environments, come from eleven qualitative studies.

Status Quo Learning Environments Studies. DiMartino (2019) collected interview data from 284 students from kindergarten, grades one, three, and five in oral and written interviews. Teachers asked students to mention what they did not like about problems.

In Wong et al. (2002), 1216 students from grades 3,6,7, and 9 attempted solving three sets of mathematics problems. These were computational problems, word problems, and open-ended problems. Two students from each class were then interviewed individually to give their opinions regarding their conceptions of the problem-solving process they have experienced.

Table 15

Students' Negotiation of their Emotions towards Problem-Solving in a Status-Quo Learning Environment

Article	Illustrative Quotes
	Status quo studies
DI Martino 2018	"When I have to solve a problem, I hold my head in my hands ***and I despair***." (p.304)
Wong 2002	"While younger students (mainly from ***Grade 3) loved word problem***s, students from upper grade levels ***[grade 6]*** found it ***"a waste of time."*** (p.34)
Qaisar et al. 2015	"Three of the students said that they ***do not like mathematics*** and believe that it is a difficult ***and boring*** subject" (p.76)
Higgins 1997	"Three non-heuristic students ***did not like to do these types of problems*** [non-routine] (p. 19) "They were unsuccessful in solving…this lack of success led to their ***dislike of the problems***" (p.19)
Andersen 2015	"Petra classified herself I am a ***math hater***" (p.149) "Malin described her previous experiences of mathematics education with the word Bobehagligt^ ***(unpleasant)"*** (p.150) "It has become so associated ***with difficulties and anxiety***" (p.150)
Jader et al. 2017	"On two occasions she [Leila] also indicates an ***insecurity*** *r*egarding her own ability." (p.768) "All three students indicate beliefs of ***insecurity"*** (p. 773)
Moyer et al. 2018	"We found that of the students who expressed ***distaste for mathematics…. all of the non-CMP students did so*** because they thought it was difficult." (p. 128)

Schindler & Bakker 2020	"It kind of affects me when I cannot solve it***. I don't feel very confident and strong***" (p.9) "Anna's utterances reflect a low self-efficacy ("I can't do this") and ***helplessness/sadnes***s ("crying"). (p.9)
Yusof & Tall 1999	"The students were initially ***very confused*** "(p.70) "***I did not enjoy most of the math courses*** – too dependent on the lecturers." (p.78)
Martinez & Gonzalez 2015	"***Solving problems...***triggers ***disappointment emotions*** when not achieved" (p. 97) "I spend a lot of time in the same problem, and then ***I get frustrated"*** (p.98) "This [problem solving on blackboard] ***stresses me*** and makes me wish to skip class" (p.99)
Lerch 2004	"Three of the four student participants found their ***experience working with the recreation problems uncomfortable***." (p.29) "Steve indicated that they were ***very confusing*** to him." (p.29)

Qaisar et al. (2015) collected data from two elementary schools; they interviewed students in a status quo setting (pre-intervention) and post collaborative work intervention, where students collaborated on various mathematical tasks. The purpose of the study was to look for changes in attitudes, beliefs, and emotions regarding mathematics.

Higgins (1997) had two groups of middle school students, the heuristics, and the non-heuristics. The non-heuristic students had not received any training in mathematical problem-solving, thus experiencing a status quo learning environment compared to the heuristic group. The students talked about memories of their mathematic classes and expressed their opinions on different types of problems in mathematics.

Andersen (2015) interviewed two 15-year-old girls Malin and Petra, who labeled themselves as "math-anxious" and a "math hater." Andersen interviewed the girls before and after collaborative problem-solving to express their thoughts on mathematics learning and problem-solving tasks.

Jader et al. (2017) explored the expectations, motivational beliefs, and feelings of security in three high school students through interviews after working on a task of non-routine problem-solving.

Moyer et al. (2018) interviewed twenty-six grade twelve students from ten high schools. These students had been taught using a traditional curriculum. The interviewers explored the student's attitudes and emotional disposition towards mathematics.

Schindler and Bakker (2020) explored the evolvement in the affective field of one high school student. The interviews took place pre- and post-collaborative work intervention to detect any change in students' emotions towards mathematics.

Yusof and Tall (1999) asked forty-four university students to write few sentences describing their feeling towards mathematics before and after learning through collaborative problem-solving. The students' narratives were compared to discover the effect of collaborative problem-solving on the students' attitudes, beliefs, and emotions to mathematics.

Martinez and Gonzalez (2016) interviewed twenty-seven university students in a status quo environment to explore their emotional experiences regarding problem-solving. Emotions such as disappointment, fear emotions, and distress were explored.

In the study by Lerch (2004), the researchers assigned both routine (textbook) and non-routine (recreational mathematics) problems for four students to solve. The students talked out loud, tape their comments while working on these problems, and wrote reflective essays about themselves and their experience with mathematics.

Collaborative Work Learning Environments

As students experience alternative classroom environments characterized by collaborative work or collaborative work with heuristics or technology, they negotiate their feelings towards problem-solving and experience a shift in the desired direction. Students express their liking of mathematics, describing it as an interesting, challenging, and exciting subject. Students experience a wide range of positive emotions such as fun, enjoyment, satisfaction, and appreciation while working on problem-solving in groups. Collaborative work seems to increase the students' engagement and confidence and reduce their anxiety. It also creates feelings of safety and curiosity towards mathematics and a sense of ownership of their knowledge.

The quotes in Table 4.10 (collaborative work learning environments studies) that illustrate the students' negotiation of their emotions towards problem-solving and mathematics in collaborative work learning environments, come from eleven qualitative studies.

Collaborative Learning Environments Studies. Collaborative work took place across different classroom environments; these environments are explained below providing the context of each of the studies.

Collaborative Work. The context in which the studies by Andersen (2015), Qaisar et al. (2015), Schindler and Bakker (2020), and Yusof and Tall (1999) took place were mentioned earlier.

Ju and Kwon (2007) interviewed nineteen university students after an inquiry-oriented course where students actively collaborated with peers to explore contextualized problems and construct mathematics through interaction. The purpose of the study was to explore if the intervention will have any impact on their attitude and emotional disposition towards mathematical learning.

Satyam (2020) explored the effect of one course on eleven university students' attitudes, beliefs, and emotions; the students had to describe the most satisfying moments they have experienced during the course. The course was a divergence for students, from computation to non-algorithmic problem solving and arguments.

Mathematical activity in the course was like that of "reform mathematics" in terms of problem-solving, explanation, writing, and collaboration with peers.

Perrenet and Taconis (2009) investigated mathematical problem-solving beliefs and of first-year university students. The students answered a questionnaire at the beginning and the end of their bachelor's program and compared their answers to the two questionnaires, justifying the affective shift that they went through. The intervention included a series of mathematical modeling projects that the students worked on collaboratively.

Table 16

Students' Negotiation of their Emotions towards Problem-Solving in Alternative Learning Environments

Article	Illustrative Quotes
	Collaborative work studies
Qaisar et al. 2015	"Mathematics is an ***interesting subject"*** (p.76) "Do you like mathematics?" … "I ***like it*** very much" (p.76)
Yusof & Tall 1999	"I try to connect the ideas together and talk about them with my friends … it is ***much more satisfying*** than rote-learning" (p.79)
Schindler & Bakker 2020	"The students related group work ***to enjoyment and to a feeling of safety***" (p.14) "Anna's and the group's increased interest and engagement …went along with positive ***emotions, such as fun, enjoyment, and excitement"*** (p.17***)***
Andersen 2015	"This is new and ***interesting*** for me." (p.151) "Both Petra and Malin did ***engage in learning*** during this group work." (p.152)
Ju & Kwon 2007	"I experienced what it really meant to do math, and ***I regained*** some of my lost ***confidence***" (p.271) "The students came ***to appreciate*** mathematics as emergent through co-***engagement*** with peers who have different kinds of expertise." (p.274)
Satyam 2020	"The act of engaging in the mathematics with other people was what was ***satisfying*** to Shelby" (p.11) "For Jordan, the act of explaining how to do a problem to classmates such that they understood ***was satisfying"*** (p.13) "I can explain it to the people in my group and when actually it makes sense to them, then ***I feel kind of good***" (p.13)
Perrenet & Taconis 2009	'University mathematics is much ***more interesting and exciting***.' (p.190)
	Collaborative work with heuristics and multiple solutions studies
Higgins 1997	The heuristic students liked them [non-routine problems] because they made you think; they were ***challenging, interesting, and fun***." (p.19)

Hino 2015	"The students commented that the discussion was important because they were ***relieved*** or ***encouraged*** to find they shared the same thinking or answer as their classmates" (p. 131) "They also ***developed an interest*** in finding similar ideas and solutions as their classmates "(p.131)
Lynch & Star 2014	"***I liked the multiple different ways***" (p.14) "Two students more explicitly noted ***reduced anxiety*** about mathematics" (p.14)
	Collaborative work with technology study
Bray & Tangney 2016	"Positive affective ***engagement*** was generated ***by interest…curiosity*** and a ***sense of ownership*** within the student cohort" (p. 189) "Basically, ***more exciting*** and involving ways for the people." (p.191)

Collaborative Work with Heuristics and Multiple Solutions. Higgins (1997) interviewed his group of heuristic students to talk about memories of the mathematic classes. The heuristic teachers taught five problem-solving skills at the beginning of the school year to their students using direct instruction. Students worked on solving routine and non-routine problems individually and collaboratively.

Hino (2015) observed and recorded ten consecutive mathematics lessons of structured problem-solving in two classrooms. First, the students worked on the problem individually or in groups, then discussed the various solution method as a whole classroom. Finally, the author interviewed some students at the end of the lessons to express their opinions about the lessons and communicate their idea of a good lesson.

Lynch and Star (2014) interviewed six struggling students to explore their experiences in an Algebra I course, mainly using multiple strategies in problem-solving and how it affected their emotional stance towards mathematical problem-solving.

Collaborative Work with Technology. Bray and Tangney (2016) conducted focus-group interviews, of four to six students each, after an intervention that entailed the use of digital technology, social constructivist pedagogies, and contextual problem-solving scenarios. The interviews explored the impact of the intervention on the students' engagement, confidence, and attitude towards mathematics.

Theme Related to Students' Negotiation of their Emotions towards Problem-Solving across Educational Systems

Our data analysis indicates that students across Italian, Swedish, Pakistani, Mexican, Malaysian, and American educational systems share the same emotional disposition towards problem-solving; they display negative emotions entailing dislike, boredom, sadness, helplessness, frustration, and stress. In addition, these students express feelings of anxiety, insecurity, and lack of confidence when they perform problem-solving tasks. On the other hand, Hong Kong elementary students express their love of mathematical problem-solving and mathematics.

Italy

Italian students express their despair when they are presented with mathematical problems to solve. Table 4.11 quotes that illustrate how students negotiate their emotions towards problem-solving come from eleven qualitative studies.

Table 17

Students' Negotiation of their Emotions towards Problem-Solving across Educational Systems

Article	Illustrative Quotes
	Italy
DI Martino 2019	"When I have to solve a problem, I hold my head in my hands ***and I despair***." (p.304)
	Pakistan
Qaisar et al. 2015	"Three of the students said that they ***do not like mathematics*** and believe that it is a difficult ***and boring*** subject" (p.76)
	Sweden
Andersen 2015	"Petra classified herself I am a ***math hater***" (p.149) "Malin described her previous experiences of mathematics education with the word Bobehagligt" ***(unpleasant)"*** (p.150) "It has become so associated ***with difficulties and anxiety***" (p.150)
Jader et al. 2017	"On two occasions she [Leila] also indicates an ***insecurity*** *r*egarding her own ability." (p.768) "All three students indicate beliefs of ***insecurity"*** (p. 773)
Schindler & Bakker 2020	"It kind of affects me when I cannot solve it***. I don't feel very confident and strong***" (p.9)

	"Anna's utterances reflect a low self-efficacy ("I can't do this") and ***helplessness/sadnes***s ("crying"). (p.9)
	Malaysia
Yusof & Tall 1999	"The students were initially ***very confused*** "(p.70) "***I did not enjoy most of the math courses*** – too dependent on the lecturers." (p.78)
	Mexico
Martinez & Gonzalez 2016	"***Solving problems...***triggers ***disappointment emotions*** when not achieved" (p. 97) "I spend a lot of time in the same problem, and then ***I get frustrated"*** (p.98) "This [problem solving on blackboard] ***stresses me*** and makes me wish to skip class" (p.99)
	USA
Lerch 2004	"Three of the four student participants found their ***experience working with the recreation problems uncomfortable***." (p.29) "Steve indicated that they were ***very confusing*** to him." (p.29)
Higgins 1997	"Three non-heuristic students ***did not like to do these types of problems*** [non-routine] (p. 19) "They were unsuccessful in solving…this lack of success led to their ***dislike of the problems***" (p.19)
Moyer et al. 2018	"We found that of the students who expressed ***distaste for mathematics…. all of the non-CMP students did so*** because they thought it was difficult." (p. 128)
	Hong Kong
Wong 2002	"While younger students (mainly from ***Grade 3) loved word problem***s, students from upper grade levels ***[grade 6]*** found it ***"a waste of time."*** (p.34) "Some students even ***loved mathematics*** because it could provoke thinking." (p.32) ***"Love word problems*** because writing enhances thinking" (p.34)

Pakistan

Students in Pakistan do not like mathematics and think of it as a boring subject.

Sweden

Swedish students display feelings of insecurity, lack of confidence, and anxiety when working on problem-solving. They hate math and describe their experience with it as unpleasant. Helplessness and sadness are shown during problem-solving when the students fail to solve a problem.

Malaysia

Malaysian students experience confusion while working on non-routine problem-solving; they express a lack of enjoyment in mathematics.

Mexico

Mexican students are stressed by problem-solving to the extent that they like to skip classes; they experience frustration when working on problems for long times. Disappointment is what they feel when they fail to solve a problem.

The USA

Students in the USA dislike mathematics and non-routine problems; they experience confusion and discomfort when dealing with such problems.

Hong Kong

Elementary Hong Kong students love mathematics and working on mathematical problem-solving; they think it provokes thinking. As they move to upper-grade levels, they stop liking problem-solving and start to see it as a waste of time.

Table 18

Emerging Themes from the Qualitative Research Meta-Synthesis

Students' negotiation of the meaning of a problem	
Theme one Across learning environments	As students experience alternative learning environments, characterized by individual/collaborative work, as contrasted to a status quo whole-class learning environment, they display a shift in the way they view a problem. they shift from viewing a problem as a closed, narrow-focused narrative posed by a textbook or a teacher in a familiar context and asking for a numerical answer; into viewing a problem as an open and broad-focused narrative that may be non-routine, in a non-familiar context, that may require thinking and analyzing the data to resolve the issue posed in the narrative.
Theme two across grade levels	As students progress across grade levels and get exposed to formal mathematics education, they negotiate their assigned meaning to a problem, from an issue that they can resolve and that they need to resolve, into a textual and mathematical structure that has specific characteristics.
Theme three across educational systems	Students across American, Chinese, and Italian educational systems adopt similar views on the meaning of a problem. A problem should be "mathematical" in nature, based on a text with data and numbers to be combined to reach a solution.
Students' negotiation of the meaning of problem-solving	
Theme four across learning environments	As students experience alternative learning environments, characterized by collaborative work in contrast to a status quo learning environment, they negotiate and shift their view on the process of problem solving. They shift their view from a text and data-based view where solving a problem is a matter of using rules into a view where problem-solving entails cooperation in work, discussion of solutions, thinking and perseverance.

Theme five across grade levels	As students progress across grade levels and get exposed to formal mathematical education, they negotiate and shift their view on the solvability of a problem from an open view into a narrow one. The students shift their view from being ready to think, analyze, and try various ways to solve a problem into prescribing conditions for problem-solving, such as a problem can be solved in one single way with a single correct answer in a short time.
Theme six across educational systems	Students across American, Chinese, Italian, German, and Netherland educational systems adopt similar views on the meaning of problem-solving. Students have set conditions for problem-solving, such as a problem can be solved quickly, using one single way, and getting a unique answer.
Students' negotiation of their emotions towards problem-solving	
Theme seven across grade levels	As students move across grade levels and get exposed to formal mathematics education, they gradually negotiate their emotions towards problem-solving from ambivalent positive (love word problems) and negative (despair) emotions into a wide range of negative emotions towards the whole discipline of mathematics such as dislike, boredom, discomfort, anxiety, confusion, frustration, and stress.
Theme eight across learning environments	As students experience alternative learning environments characterized by collaborative work in contrast to a status quo learning environment, they experience a shift in their emotions. While elementary students experience ambivalent positive and negative emotions in status quo learning environments, other students experience mostly negative emotions toward problem-solving. In contrast, alternative learning environments trigger in students a wide range of positive emotions such as enjoyment, interest, fun, excitement, and engagement. Students feel more confident, secure, and satisfied during collaborative problem-solving
Theme nine across educational systems	Students across Italian, Swedish, Pakistani, Mexican, Malaysian, and American educational systems share the same emotional disposition towards problem-solving; they display negative emotions entailing dislike, boredom, sadness, helplessness, frustration, and stress. In addition, these students express feelings of anxiety, insecurity, and lack of confidence when they perform problem-solving tasks. On the other hand, Hong Kong elementary students express their love of mathematical problem-solving and mathematics.

Students' Negotiation of the Meaning and Emotions across Learning Environments

Our meta-synthesis of the qualitative studies has shown that students negotiate the meaning of a problem and the process of problem-solving as well as their emotions towards problem solving differently in alternative learning environments versus a status-quo learning environment.

As students experience alternative learning environments, characterized by individual/collaborative work, as contrasted to a status quo whole-class learning environment:

1. They display a shift in the way they **view a problem**. they shift from viewing a problem as a closed, narrow-focused narrative posed by a textbook or a teacher in a familiar context and asking for a numerical answer; into viewing a problem as an open and broad-focused narrative that may be non-routine, in a non-familiar context, that may require thinking and analyzing the data to resolve the issue posed in the narrative.
2. They negotiate the way they view the **process of problem-solving** and shift their view from a text and data-based view where solving a problem is a matter of using rules into a view where problem-solving entails cooperation in work, discussion of solutions, thinking and perseverance.
3. They experience a shift in their **emotions toward problem-solving**. While elementary students experience ambivalent positive and negative emotions in status quo learning environments, other students experience mostly negative emotions toward problem-solving. In contrast, alternative learning environments trigger in students a wide range of positive emotions such as enjoyment, interest, fun, excitement, and engagement. Students feel more confident, secure, and satisfied during collaborative problem-solving

Students' Negotiation of the Meaning and Emotions across Grade Level

Our meta-synthesis of the qualitative studies has shown that the way students negotiate their view of a problem and the process of problem-solving as well as their emotions towards problem-solving change as they get exposed to formal education and move across grade levels.

As students progress across grade levels and get exposed to formal mathematics education:

1. They negotiate their **assigned meaning to a problem**, from an issue that they can resolve and that they need to resolve, into a textual and mathematical structure that has specific characteristics.
2. They negotiate and shift their **view on the solvability of a problem** from an open view into a narrow one. The students shift their view from being ready to think, analyze, and try various ways to solve a problem into prescribing conditions for problem-solving, such as a problem can be solved in one single way with a single correct answer in a short time.
3. They gradually negotiate their **emotions towards problem-solving** from ambivalent positive (love word problems) and negative (despair) emotions into a wide range of negative emotions towards the whole discipline of mathematics such as dislike, boredom, discomfort, anxiety, confusion, frustration, and stress.

Students' Negotiation of the Meaning and Emotions across Educational systems

Our meta-synthesis has shown that students across various educational system share widely the same views on the meaning of a problem, the process of problem-solving and their emotional disposition towards problem-solving, except for Hong Kong elementary students.

1. Students across American, Chinese, and Italian educational systems adopt similar **views on the meaning of a problem**. A problem should be "mathematical" in nature, based on a text with data and numbers to be combined to reach a solution.
2. Students across American, Chinese, Italian, German, and Netherland educational systems adopt similar **views on the meaning of problem-solving**. Students

have set conditions for problem-solving, such as a problem can be solved quickly, using one single way, and getting a unique answer.

3. Students across Italian, Swedish, Pakistani, Mexican, Malaysian, and American educational systems share the same **emotional disposition towards problem-solving**; they display negative emotions entailing dislike, boredom, sadness, helplessness, frustration, and stress. In addition, these students express feelings of anxiety, insecurity, and lack of confidence when they perform problem-solving tasks. On the other hand, Hong Kong elementary students express their love of mathematical problem-solving and mathematics.

CHAPTER 5

CONCLUSION AND DISCUSSION

The qualitative meta-synthesis in this thesis was designed to understand (1) the way students negotiate their meaning of a problem and the process of problem-solving (2) the way students negotiate their emotions towards problem-solving from their perspective. To achieve such an understanding, we analyzed and synthesized the finding of twenty one relevant qualitative studies reported in high-impact mathematics education journals between the years 1997 and 2020. This qualitative meta-synthesis utilized the theory-generating approach to qualitative meta-synthesis (Finfgeld-Connett, 2010). The theory-generating approach is based on the grounded formal theory. The theory employs coding, categorizing, constant comparison, coming up with a theme, and generating a well-founded theory to provide a comprehensive framework for understanding and explaining how students negotiate their affective relationship with problem-solving and how they assign meaning to it.

Our discussion is divided into two major parts, which correspond to the two research questions in this study. The first part discusses the theoretical interpretation of the findings on how students negotiate their assigned meaning to a problem and the process of problem-solving. The second part discusses the theoretical interpretation of how students negotiate their emotions towards problem-solving.

Part I: Theoretical Interpretation of Students' Negotiation of their Meaning of a Problem and Problem-Solving across Grade Levels, educational Systems and Classroom Environments

Our meta-synthesis shows that once students are introduced to formal mathematics education in status quo mathematics classroom settings, where direct instruction is the primary mode of delivering knowledge, they assign a rigid and stereotypical meaning to what a problem is. This rigid meaning appears across all grade levels and educational systems that we have encountered in our qualitative studies. However, as the students experience alternative learning environments characterized by individual and collaborative work, they display across all grade levels and educational systems that we have encountered in our qualitative study a shift in how they view a problem. They shift from viewing a problem as a closed, narrow-focused narrative posed by a textbook or a teacher in a familiar context and asking for a numerical answer; into viewing a problem as an open and broad-focused narrative that may be non-routine, in a non-familiar context, that may require thinking and analyzing the data to resolve the issue posed in the narrative.

As for the meaning students assign to the process of problem-solving, students in a status quo learning environment across all grade levels and educational systems revealed a narrow-focused view. For these students, the process of problem-solving entails standard recipes to solve problems in one way, using certain mathematical operations to obtain a single solution for a mathematical problem. Once these students experience alternative learning classroom environments such as collaborative problem solving, collaborative work with multiple strategies, and authentic problem solving, they display across all grade levels and educational systems that we have encountered in our qualitative studies a more open view on the process of problem-solving. They view problem-solving as an experience that prepares them for real-world problems; it requires deep thinking and the usage of past knowledge, enough time, and perseverance. Multiple solution strategies and multiple answers become a part of the problem-solving process.

Theoretical Perspective Used by Authors

The authors of our articles analyzed and interpreted their findings using different theoretical frameworks. The frameworks that appeared the most in our studies were the cognitive and the social-constructivist theoretical frameworks.

An Overview of Cognitive Theory

Schoenfeld (1985) introduced four types of knowledge that supported problem-solving: resources, strategies, control, and individual belief system. Some of our authors utilized the latter to interpret their findings. Schoenfeld (2016) theorized that faulty beliefs often have impairing effects on students, and they are formed based on their classroom experience "students abstract their beliefs about formal mathematics, their sense of their discipline, in large measure from their experiences in the classroom. Students' beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences." (Schoenfeld, 2016, p.27)

Schoenfeld (2016) believes that the kind of problem-solving students practice in their classroom focuses on artificial problems, where problems are solved fast, with specific rules, and in short periods, thus, contradicting the practical usefulness of mathematics. After recurring experiences with this problem-solving, students would give up trying to make sense of mathematics. They would view a problem as an exercise of little meaning and that there is always a rule to follow in problem-solving.

Holding such views on problem-solving will lead students not to attempt problems for which they have no prescribed method to solve or may cut back on the time they are ready to devote to solving a problem.

In addition to the kind of problems students experience, Schoenfeld (2016), building on the social constructivist theory, states that the community of practice (classroom) that students belong to would influence how they think about mathematics and mathematical problem-solving. He argues that students develop how they see and use mathematics from their experiences with mathematics in a practically social setup, their classroom. Schoenfeld (2016) mentions that “mathematics *is* an act of sense-making that is socially constructed and socially transmitted” (Schoenfeld 2016, p.7). This argument puts his explanation of students’ beliefs and points of view in the realm of social constructivism and agrees with Vygotsky’s (1978) stance that meanings develop in the society in which the child exists and are transmitted by social interaction to the child.

An Overview of the Social Constructivist Theory

Ernest (1991) provided a theory on how the negotiation process takes place in the learner's cognition, influenced by social interaction during the learning process. "Social constructivism links subjective and objective knowledge in a cycle in which each contributes to the renewal of the other. In this cycle, the path followed by new mathematical knowledge is from subjective knowledge (the personal creation of an individual), via publication to objective knowledge (by intersubjective scrutiny, reformulation, and acceptance). Objective knowledge is internalized and reconstructed by individuals during the learning of mathematics to become the individuals' subjective knowledge. Using this knowledge, individuals create and publish new mathematical knowledge, thereby completing the cycle." (Ernest 1991, p. 43). The child's objective knowledge is built through social interaction with other individuals through the process of negotiation.

Authors' Theoretical Interpretations in Status-quo learning environments

Most of the authors (DiMartino, 2019; Higgins, 1997; Jablonka, 2014; Lerch, 2004; Perrenet & Taconis 2009; Stylianides & Stylianides, 2014; Wong, 2002) of the status quo studies analyzed their findings and interpreted them within a cognitive framework, consistent with Schoenfeld's (2016) claim that students develop their beliefs and views on mathematical problem-solving and mathematics from their classroom experience. They believe that this rigidity in the meaning they assign to a problem and the problem-solving process is acquired through years of exposure to direct instruction teaching where no place for creativity is available.

Wong (2002) claims that the extended exposure to the mathematics classroom culture has shaped students' views on the meaning of a problem and problem-solving. DiMartino (2019), on the other hand, believes that the exposition to mathematical problems in primary school has a negative effect on students' vision of problems and the way they are supposed to be solved; the problems posed allow neither multiple approaches and solutions nor different situations where students could create meaning.

Jablonka (2014) views the examination-oriented classroom culture, and the focus on the product instead of the process by teachers, as the main reason behind the way students view problems and problem-solving and assign meaning to it. Lerch (2004) believes that the classroom learning experience does not provide students with various resources and strategies from which to choose and that the student's knowledge base is not being expanded to include various strategies that are not dependent upon specific types of problems. These classroom learning experiences do not challenge student's faulty beliefs on the meaning of a problem and problem-solving.

Stylianides & Stylianides (2014) believe that there are various reasons behind the students' faulty epistemological beliefs. The type of problems presented to during their learning journey does not promote healthy epistemological beliefs on problem-solving. The difficulties they face while solving these problems and the role of the teacher that is not helping the students overcome these difficulties leads the students to have a rigid view of what a problem is and how it could be solved. Higgins (1997) interpreted his findings using Schoenfeld's (2016) cognitive theory, stating that students' work affects how they think about a particular subject and their beliefs about its nature, therefore the classroom environment with type of problems presented to students and the way these problems are solved affected the way students view what a problem is and how to solve it.

Perrenet & Taconis (2009) believe that school instills mathematical beliefs in students that do not align well with professional mathematical beliefs. Perrenet & Taconis (2009) claimed that the enculturation through learning in the status-quo environment, where the classroom presents a picture of mathematical problem-solving different from reality, leads to faulty beliefs on mathematical problem-solving.

Authors' Theoretical Interpretation in Alternative Learning Environment

Five of our authors analyzed and interpreted their findings on the effect of an alternative learning environment on how students negotiate the meaning of a problem and problem-solving within a cognitive framework, a constructivist framework, or both frameworks (Francisco 2005; Higgins 1997; Hino 2015; Perrenet & Taconis 2009; Stylianides & Stylianides 2014).

Hino (2015) adopts the standpoint of social constructivism. She states that comparing multiple solutions environments, which required active student participation and social interactions, led to a significant change. Students could develop a new, more sophisticated view and meaning on what problem-solving is.

Higgins (1997) interpreted his findings using Schoenfeld's (2016) version of the social constructivism theory. He states that students' work affects how they think about a particular subject and their beliefs about its nature. When the student's learning environment was altered by exploring exciting and challenging problems, they had the opportunity to share and discuss their thinking with their classmates using a common language that they developed through their heuristic's instruction. This collaboration with the type of problems used helped them develop a more sophisticated meaning on problem-solving than the students in a status-quo classroom setting.

Stylianides and Stylianides (2014) based their interpretations on Schoenfeld's (2016) theory that the appropriate classroom learning experience will challenge students' faulty beliefs. Therefore, when the students are presented with challenging problems and allowed to work collaboratively on these problems with the teacher's support, they will be capable of developing healthy epistemological beliefs on problem-solving, thus leading the students to have a sophisticated view of what a problem is. In addition to the cognitive framework, Stylianides & Stylianides mentions that using small groups in a class where students can socially interact and develop the solutions collaboratively can influence their opportunities to contribute to and benefit from the problem-solving experience, thus building on the tenets of social constructivism.

Schoenfeld's (2016) theory on the effect of classroom experience on students' learning and beliefs was also adopted by Perrenet & Taconis (2009). The authors claimed that the enculturation leading to faulty beliefs on mathematical problem-solving could be altered by changing the classroom environment. When students worked in pairs on challenging authentic problems, they could develop a more advanced view of the meaning of a problem.

Francisco (2005) framed his interpretation within the social constructivism theory. He claimed that a constructivist approach to learning, where students work collaboratively to build their knowledge, will lead to sophisticated epistemological beliefs about learning mathematics and the meaning of a problem and problem-solving. On the other hand, the activity theory (Leont'ev, 1981) appeared in the study on situated and real-life problem-solving by Jurdak (2006).

"A central assertion of activity theory is that our knowledge of the world is mediated by our interaction with it, and thus human behavior and thinking occur within meaningful contexts as people conduct purposeful goal-directed activities. This theory strongly advocates socially organized human activity as the major unit of analysis in psychological studies rather than mind or behavior." (Jurdak, 2006, p.286).

In Jurdak's study, the students expressed that a real-life situation problem differs from the problems they solved in the study and from classroom problems. They mentioned that they would use completely different methods in tackling each type of problem. Jurdak interpreted the result using a socio-cultural activity system. Problem-solving in the school context is an activity within the school community in which the tools, norms, and rules differ from the tools, rules, and norms of a real-life larger and more complex community. With their different cultural rules and norms, these two different social contexts make problem-solving in real life, where decision-making is required, a different activity from situated problem-solving.

Theoretical Interpretation of the Meta-synthesis Finding on Student's Meaning of a Problem and Problem-Solving

The themes that emerged based on our qualitative meta-synthesis have shown a sophisticated epistemological shift in students' assigned meanings to a problem and the process of problem-solving. The shift emerged because of cognitive negotiation that occurred once students experienced alternative learning environments.

The cognitive and the social-constructivist theoretical frameworks seem to provide a plausible interpretation of our findings. Consistent with Schoenfeld's (2016) cognitive theory, students' long experience in their status quo classroom environment influences their beliefs about problem-solving. The classroom environment and the kind of problems posed to students by the teachers and the textbooks create a set of faulty epistemological beliefs about the nature of mathematical problem-solving (Schoenfeld, 2016), leading to a narrow and focused view on the meaning of a problem and the way they solve it.

Schoenfeld believes that a change in the learning experience needs to happen to change these faulty beliefs and views. Our findings have shown that once these students experience an alternative learning environment where they need to collaborate and interact with colleagues; and where the problems they work on are more challenging and authentic, a negotiation process takes place.

Ernest's (1991) theory on the kinds of knowledge in social constructivism explains the negotiation process in the learner's cognition once their learning environment is altered. Students come to the classroom with their subjective knowledge and views on problem-solving based on their previous learning experience. The new collaborative environment that the students experience, and the new objective knowledge mediated by their social interaction with colleagues provide them with negotiations material. This new objective knowledge is reformulated and accepted by the students to become their subjective knowledge. In the studies that we have analyzed, the negotiation process that occurred because of social interactions led to a more sophisticated, open, and professional-like view on the meaning that students assigned to a problem and problem-solving.

Part II: Theoretical Interpretation of Students 'Negotiation of their Emotions towards Problem-Solving across Grade Levels, educational Systems and Classroom Environments

The emerging themes from our qualitative meta-synthesis have shown that elementary students experience ambivalent positive and negative emotions in status quo learning environments towards problem-solving. As students get introduced to formal mathematical education in grade one, they experience negative emotions such as dislike, boredom, sadness, helplessness, anxiety, insecurity, and frustration. This range of negative emotions appeared in our findings across all grade levels and different educational systems.

Once students experience alternative learning environments characterized by collaborative work, authentic problem solving, or real mathematics education across all grade levels and educational systems that we have encountered in our meta-synthesis, they experience a shift in their emotions. In contrast, alternative learning environments trigger a wide range of positive emotions such as enjoyment, interest, fun, excitement, and engagement. Students feel more confident, secure, and satisfied in these alternative learning environments

Theoretical Perspective Used by Authors

Multiple sociocultural theoretical and social constructivist frameworks appeared in the analysis and interpretations of our articles' findings. Cobb's holistic approach, McLeod's theory on affect, Lave and Wenger's sociocultural theory, and RME were utilized by the authors in the interpretations of their findings. Schoenfeld's (2016) and Hannula's (2006) cognitive theories were also used by some authors.

An Overview of Socio-cultural Theories

Sociocultural perspectives believe that students' beliefs and views about mathematics are socially constructed through years of social interactions at schools situated within larger societal and cultural contexts as the communities of practice. Students acquire their beliefs and views from these contextualized learning experiences rather than the mathematics discipline (Cobb et al., 1989; McLeod, 2002). Cobb et al. (1989) developed a holistic cognitive/constructivist approach to their theory on beliefs and emotions; they placed them in a cognitive/sociocultural context. Emotions are acts based on the way students cognitively appraise a specific social situation, such as their learning experience, in specific classroom culture. Social and cultural norms mediate students' appraisal of their experience. Therefore, to understand emotions, we need to understand the way students are appraising their experiences within their sociocultural domain and norms (Cobb et al., 1989).

Lave and Wenger (1991) and Wenger (1999) introduced and developed a sociocultural theory based on situated learning and communities of practice, asserting that learning is a social activity in which the learner participates in communities of practitioners. Through engaging with and contributing to their communities of practice, the learner moves to fully participate in the sociocultural practices of a community in a process that produces meaningful learning. (Wenger, 1999)

An Overview of social Constructivist and Cognitive Theories

Freudenthal's (1983) RME is a theory under the social constructivist umbrella, where the students construct their knowledge working on problems in a society-related context. It is an approach to teaching that aims at making mathematics more meaningful for students. This happens by introducing the students to problems within contexts relevant to their life experiences and compatible with their knowledge, where the role of the teacher is to scaffold students re-invent the mathematics they encounter.

McLeod (1992) developed a theoretical framework for affect. McLeod located his concept of affect within a socio/cognitive-constructivist perspective. Affect, according to McLeod, included three constructs: attitudes, beliefs, and emotions. McLeod's three constructs varied in duration and stability and in the degree to which cognition plays a role in each of them. He considered attitude and beliefs as trait-like in nature and emotions as state-like. Emotion is the affective construct that originates as a response to brief experiences; they are usually transient and affected by tangible events. With overtime recurrence of experiences, these emotions that they trigger could turn into beliefs that are hard to change. McLeod believes that the social context in which the educational process occurs influences the relationship between the three affective factors and mathematical learning. He also believes that cognitive processes and affective factors are highly related. "When students are involved in instruction in problem-solving, their affective states are influenced by a variety of factors. Some of the factors that seem particularly important to mathematics include the different kinds of cognitive processes that the problems require, the setting within which the instruction takes place, and the belief systems that the student holds." (McLeod, 1989, p.31)

Building on McLeod's (1992) theory on affect and Schoenfeld's (1992) cognitive theory, Hannula (2006) developed a framework relating emotions to cognition and social practices. He also believes that affect could be placed in either a cognitive or a social frame, "Affect in mathematics education can be studied as an element of social practice or as an aspect of the individual's thinking and learning." (Hannula, 2006, p.215) Hannula believes that cognition and emotions are related in a way that each one of these factors affect the other. The cognitive beliefs that one holds about mathematics will influence the way one feels about it. Hannula affirms the importance of the social context in which the learning occurs by stating that mathematics is not an individual venture but is created through communication between subjective experiences and interaction among mathematical communities. (Hannula, 2006)

Authors' Theoretical Interpretations in Status-quo learning environments

Three of our authors (DiMartino, 2019; Qaisar et al. 2015; Yusof & Tall, 1998) analyzed and interpreted their findings in the status-quo learning environment, where students expressed negative emotions toward problem-solving, using Schoenfeld's (2016) argument that students develop how they see and use mathematics based on their experiences with it in a specific social context.

Basing his interpretation on Schoenfeld's (2016) version of social-constructivism, DiMartino (2019) states that the way students experience mathematical problem-solving in primary school not only leads to a rigid view on students' meaning of problems and problem-solving, but it also influences their emotional disposition towards problem-solving negatively. Qaisar et al. (2015) also utilized Schoenfeld's

(2016) theory to state that the classroom context, where students acquire their learning experiences, plays a significant role in developing students' beliefs and feelings towards mathematics and mathematical problem-solving. He states that teacher-centered classrooms where rote learning is emphasized affect how students feel about mathematics and mathematical problem-solving.

Yusof & Tall (1998) to interpret the findings of their study within a similar framework, Schoenfeld (2016). They state that traditional learning, based on lecture style teaching where students are taught that it is best to follow the rules and procedures, affects their attitude and emotions towards mathematical problem-solving. Yusof and Tall conclude that the learning environment that the student experiences would affect their attitude towards problem-solving and thus how they express their emotions about it.

Two authors (Jader et al., 2016; Schindler and Bakker, 2020) used Hannula's theoretical framework to interpret their findings. Using Hannula’s (2002) interplay aspect between cognitive beliefs and emotions, Jader et al. (2016) state that negative cognitive belief about oneself and mathematical problem-solving abilities will lead to experiencing negative emotions during problem-solving activities.

Schindler and Bakker (2020) state that their study was inspired by the research of Hannula (2002) and Cobb et al. (1989) about affect during problem-solving. Schindler and Bakker claim that a student's beliefs about mathematical problem-solving affect the wide range of emotions that they experience during the task. These beliefs are a result of the student's experience with traditional problem-solving over their years of education.

Believing in the intense interplay among beliefs about mathematics, beliefs about self, and emotions, Moyer et al. (2018) framed their interpretation within McLeod's (1992) affective theory, which states that beliefs provide the context for the development of emotions during mathematical problem-solving. They believe that the way students are exposed to problem-solving in a traditional classroom environment influenced their beliefs and, ultimately, their emotional disposition towards mathematics and mathematical problem-solving.

Andersen et al. (2015) interpreted his finding using a socio-cultural theoretical framework influenced by Sfard & Prusak’s (2005) theory on the relation between identity and culture. He stated that the task context and the social context in which learning occurs affect how students feel about mathematics and mathematical problem-solving. When students can not relate to the task's context or the problem is detached from societal or critical issues that concern the student, negative feelings towards mathematics and mathematical problem-solving will arise.

Authors' Theoretical Interpretation in Alternative Learning Environment

Multiple theoretical frameworks appeared in our meta-synthesis to analyze and interpret the findings on students’ emotional disposition during problem-solving in an alternative learning environment. Multiple authors used the social constructivist framework. (Higgins, 1997; Hino, 2015; Qaisar et al., 2015; Satyam, 2020; Schindler and Bakker 2020; Yusof &Tall, 1999;) Whereas some of the authors of the alternative learning environment studies framed their interpretation within the socio-cultural theory (Andersen et al., 2015; Ju &Kwon, 2007; Perrenet & Taconis 2009)

Using the social constructivist theory and building on Cobb et al.’s (1989) holistic framework, Hino (2015) states that the collaborative work environment, in which students actively engage in social interactions while working on their tasks, affected the way they view problem-solving and how they feel about it.

Schoenfeld's (2016) version of the social constructivism theory, which we elaborated on in the previous part of this chapter, was utilized by Higgins (1997), Qaisar et al. (2015), Satyam (2020), and Yusof & Tall (1999) to interpret their findings on the way students in an alternative learning feel towards problem-solving.

Using Schoenfeld's (2016) theory, Higgins (1997) states that when the student's learning environment involved exploring challenging problems in a collaborative work setting, they had the chance to share ideas and discuss multiple strategies and solutions with their colleagues. These social interactions and approaches to mathematical problem-solving led to a positive emotional disposition towards mathematical problem-solving.

Satyam (2020) used McLeod's (1989) theory on affect but framed his findings within Schoenfeld's (2016) version of the social constructivism theory. The author stated that the most emotionally satisfying moments occurred when the students worked collaboratively on quality tasks that allowed conceptual understanding, made sense of concepts, and were group worthy. The collaborative social environment, together with the quality of tasks presented to the students, created what he called satisfying moments, which he defines as "significant positive feelings." (Satyam, 2020, p. 5)

Qaisar et al. (2015) used Schoenfeld's (1985) point that one's belief system is dynamic and changeable. They stated that when the students worked in groups on their mathematical problem-solving tasks, they significantly changed their beliefs and emotions about mathematics and mathematical problem-solving; they expressed feelings of encouragement and enjoyment.

Yusof & Tall (1999) framed their interpretation within Schoenfeld's (2016) version f the social-constructivist theory and concluded that by working collaboratively on problem-solving instead of a lecture-based classroom, the students displayed a significant shift in how they view mathematics and mathematical problem-solving. The fact that the students could construct their knowledge through formulating, modifying, refining, reviewing problems and solutions through social interactions with their colleagues created a wide range of positive emotions such as satisfaction and enjoyment.

Schindler and Bakker (2020), whose study was inspired by the research of Hannula (2002) and Cobb et al. (1989) on affect, interpreted their finding on alternative learning environment effect on emotions in a social constructivist approach. The authors concluded that when students worked in groups on problem-solving tasks, a significant shift in their emotional disposition towards mathematics and mathematical problem solving occurred. They stated that the emotional support and encouragement that the group members provided for each other created a safe atmosphere in which the students showed interest in each other's ideas. The collaborative work evolved the affective field of the students positively.

Freudenthal's (1983) RME theory appeared in Bray & Tangney's (2016) study. The authors interpreted their findings within the RME framework. They stated that when the students work collaboratively on open-ended tasks that allow guided discovery, problem-solving, investigation, sense-making, and different solutions in a meaningful context, they display positive affective engagement characterized by a sense of ownership, involvement, interest, and excitement.

Andersen et al. (2015) interpreted their finding using the sociocultural theoretical framework of Sfard & Prusak's (2005). The authors stated that when the task context is relatable to the students and to their societal concerns, they change the way they look at mathematics and mathematical problem-solving. They add that the social context in which learning takes place also significantly impacts the students' emotions. When the students decide on the task they want to work on and the situation contexts, they influence the discourse of their learning process, take responsibility for it, experience engagement, and find meaningfulness in mathematical problem-solving.

Ju & Kwon (2007) employed a sociocultural perspective to interpret their findings. Building on Cobb's et al. (1989) theory that the belief system is a sociocultural construct and that the classroom culture is viewed as a fundamental factor in forming student beliefs, Ju & Kwon believes that it is necessary to provide an educational experience in which the classroom cultural norms support the emergence of positive students' beliefs about mathematics. The authors stated that when students worked collaboratively on problems that borrowed their life experience in a mathematical context, they worked in small groups to analyze a question, develop solutions, and collectively construct arguments to justify their reasoning. This alternative learning environment characterized by a culture in which the student is the primary agent led to a transformation in their beliefs about mathematics and mathematical problem-solving and how they feel about it.

The sociocultural theory was also utilized by Perrenet & Taconis (2009), where they used the term enculturation to interpret their findings. The authors used Wenger's (1999) definition of enculturation as a product of participation in authentic activities combined with the idea of enculturation into mathematics the way Schoenfeld (1992) used the term. They believe that the school culture, in general, instills faulty mathematical beliefs in students. Perrenet & Taconis (2009) stated that when the culture was changed through an alternative learning environment of authentic problem-solving, where the problems characterize the local mathematical culture, a considerable shift in their views and emotions was clear.

Theoretical Interpretation of the Meta-synthesis Finding on Student's Meaning of a Problem and Problem-Solving

The themes that emerged in our meta-synthesis on the way students negotiate their emotions towards problem-solving show a significant shift in students' emotional disposition when they experience alternative environments across all grade levels and educational systems that we have encountered. Learning environments characterized by collaborative work and authentic problem-solving created a shift in the students' emotional disposition from what McLeod (1992) calls negative to positive emotions towards mathematical problem-solving. DiMartino & Zan (2007) elaborated on emotions by stating that a "positive" emotion is what we perceive as a pleasurable feeling, such as the pleasure experienced while solving a problem. In contrast, anxiety while facing a problem is perceived as a negative emotion.

The shift in students 'emotional disposition that we have noticed in the findings of our meta-synthesis, across all grade levels and different educational systems that we have encountered, may be attributed to the experience that the students had in a more social-constructivist compatible learning environment.

When students collaborated on problem-solving tasks, worked on authentic problems of interest to them, and could construct their knowledge instead of receiving it, they displayed feelings of enjoyment and satisfaction in problem-solving. When students experienced learning and problem-solving as a social activity instead of an individual one, they experienced no anxiety and frustration due to the emotional support provided by the group members. Their social interactions created feelings of satisfaction due to the ability to reach solutions and the feeling of ownership due to the ability to construct knowledge. According to McLeod (1992), These momentary positive experiences that the students went through changed their emotional disposition towards problem-solving over time and repetitive exposure to positive emotions. In a way to interpret the process through which this shift occurred, we would like to resort to the social-constructivist theory, and specific, to McLeod (1992), Hannula (2006), and Ernest (1991).

When the students experienced an alternative learning environment of collaborative work, real-life problem solving, and authentic problem-solving in a new social context, they experienced a wide range of momentary positive emotions. According to Hannula (2006), emotions and cognition are highly related. He calls it "emotional cognition." Hannula (2006) defines *emotional cognition* as the subjective knowledge of one's emotional state. It involves the awareness of one's emotions at different times in different situations and the subjective knowledge of their expected typical emotional reactions in a different situation. Students are aware of the different emotions they experience in mathematics classes while problem-solving. The student's emotional cognition made them also aware of the different emotions they experienced in an alternative learning environment. This awareness represented to them new knowledge, the objective knowledge that they had to negotiate. (Ernest, 1991).

When students experienced these new emotions in an alternative learning environment, they cognitively negotiated it, became cognitively aware of it, and consciously developed a new positive range of emotions towards their problem-solving tasks that they were able to express and justify during their interviews. A shift in their emotional disposition towards problem-solving was noticed because of the repetitive momentarily emotional experiences that they have gone through. This new emotional disposition resembles a new subjective knowledge that they have reformulated and accepted due to their negotiation process. (Ernest, 1991).

Limitations of the Study

One limitation of the study is that qualitative research studies differ in their design and approach. This study selected qualitative studies, irrespective of their designs and approach. The study used affective constructs as defined by the respective authors without further examination

Implications of the Results to Research and Practice

The power of qualitative meta-synthesis is that it allows a shift from knowledge generation to knowledge application since it is grounded in lived experience. Qualitative meta-synthesis studies are rare in mathematics education and this study could be an encouragement for more qualitative meta-synthesis studies in the field. The findings of this study ought to be of significance to the mathematical community, from stakeholders to educators and curriculum developers who strive to have a body of students with a positive affective field towards mathematics, as it affects the way they approach mathematical problem-solving and ultimately their abilities to be good problem solvers. Altering the learning environment and adapting a collaborative social-constructivist approach in the mathematics classroom, where students can collectively build and share their knowledge with their colleagues, is the suggestion of our study supported by our findings.

Suggestions for Further Research

Since the affective field plays a significant role in the way students approach mathematics, we believe that more qualitative research is needed on students affect towards mathematical problem-solving. More qualitative research is also needed on the impact of alternative learning environments on students affect towards problem-solving once students start learning mathematics formally at the elementary grade level. This grade level represents a steppingstone into the students' learning journey.

There is a need to cross-validate the findings of this study with the findings of meta-analysis of quantitative studies on students' meaning and affect toward problem and problem-solving.

APPENDIX

The following is an example on the analysis process for the first theme.

First step is labeling text related to our research question in an individual article.

Group 1 (kindergarten and grade 1) As was predicted, in answering the first question of the

individual section, the children in Group 1 did not refer to the mathematical problem, but to

everyday life problems. We collected a great amount of drawings and we tried to find categories of problems within these.

Except for some specific singularities, we classified the majority (almost three out of four) of problems produced by the children into three macro-categories: to be sick, to get hurt, to have some physical constraints, or to die in the most extreme cases; something breaking or not working, to lose an object

To lose an object is by far the most frequent category in the reported examples of problems:

almost one out of two (57 of 121) students in Group 1 refer to this kind of problem. It would be

interesting to further study this result from a sociological point of view.

The text in Fig. 5 shows an optimistic characterization of problems shared in Group 1: the

only problem recognized as not being certainly solvable by all children is death. A thorough

analysis of the data collected shows how the majority of the students characterize a problem as something that can be solved and they want to solve, but—at least initially—they are not able to solve. Therefore, despite the shared optimistic view, students in Group 1 declared to be upset, sad, and disheartened when a problem occurs because they are not able to do something they wish to do.

So, there seems to be a differentiation between a personal level (I am not able to do something: so, there is a problem) and a social level (there exists someone who can solve the problem if we ask for his/her help).

As well as asking for help, three other ways of solving problems characterize the answers of students in Group 1: thinking/reasoning, trying in various ways, not giving up.

During the group discussion focusing on whether and how the problems were solvable, we

observed, on the one hand, the children's optimism about the possibility to solve almost all problems; on the other hand, their naturalness in analyzing in depth a solution, criticizing it, and finding multiple solutions to a single problem.

For example, we report on the activity developed in a kindergarten with 25 children (age 5). Cristina, the teacher, introduced the problem proposed by Anna: "a problem is when I

cannot open the door because I cannot reach the doorknob," asking children to propose and

then dramatize their solutions. After an in-depth critical analysis of the pros and cons of the solutions proposed, children had to choose their preferred solution. Seven different solutions

emerged: I jump (seven children, see Fig 6); I call dad/mom (five children); I ring the bell (four children); I take a ladder/a chair and I climb on it (three children, see Fig. 7); I go in the tavern and I get the keys (one child); I wear my heels (one child).

Group 2 (grades 3–5) The data collected offer a very different picture from the one emerging from Group 1. First of all, a problem is no longer characterized as something that students want to solve, but as something that must be solved (Grade 4: a problem is a difficulty and we must resolve it^). The emergence of this normative/prescriptive component seems to be related to the fact that school problems are hetero posed (i.e., posed by others). Zan (2011) developed meaningful reflections about the implications of this fact for children's sense making of the problematic situation described in the mathematical problem texts.

The second difference is the absolute predominance (156 answers out of 163) of mathematical problems in the collected examples of problems. The analysis of these examples allowed us to reconstruct a shared idea about what a mathematical problem is (and how to solve it) for these children a problem is a text, with a familiar context and a final question. The question has a unique numerical answer, and it is necessary to combine all the numerical data present in the text with a single operation in order to get it. Moreover, except for extremely rare cases, combining all and only the data provided in the text.

The emerging structure associated with a mathematical problem is rigid and stereotypical:

the collected examples are pretty much shaped by an interchangeable frame, depending on the arithmetic operation involved. For example, considering problems that refer to the operation of subtraction, we found interchangeable texts all referring to the ‘take away’ meaning of subtraction (Grade 3: Paolo had 19 cookies; he ate 7. How many cookies are left?).

In particular, the context appears to be considered as completely irrelevant—an alibi using Gerofsky’s (1996) words—just like its trueness or realism. Almost all the examples collected refer to a real-life context (apart from 12 examples of geometrical problems), but these problems are often immersed within rather unrealistic—or sometimes insignificant—situations (Grade 5: A couple has 135 children, the three ninths of them are males. How many of them are female?).

The analysis of the group discussions about how to solve a problem also shows significant differences compared to the opinions collected from Group 1.

For the majority of the children in Group 2, a mathematical problem can always be solved, and it has only one solution, which should be reached in a single correct way.

In particular, 26 children describe explicitly a shared standard procedure to solve a problem: highlighting data and finding keywords in the text, therefore, finding the right arithmetical operation (tracing a drawing/diagram appears not very related to finding the solution, but it is something to do to please the teacher).

In this view, numerical data and arithmetic operations are considered necessary in order to solve mathematical problems.

Analyzing the answers of Group 2 to the questions: What do not you like about problems?

Why? It emerges that what students in grades 3–5 dislike is their inability to solve the problems and, therefore, the difficulty of a problem is considered to be a negative aspect.

The answers collected show a worrisome evolution from grade 3, where the evaluation is on being in a particular condition (state) of difficulty (when I am not able to solve a problem I feel bad^), to grade 5, where frequently students make reference to a priori evaluation (trait) of difficulty (I am not able to solve problems; this is why when the teacher gives them to us, I would like to go home^).

This evolution seems to have an impact on the emotional disposition towards mathematical problems: in grades 4–5, the answers (and only for these grades) include statements expressing a definitive idea about the perceived competence in solving mathematical problems (Grade 4: I feel so dumb^; Grade 5: "I will never be able to do this! When I have to solve a problem, I hold my head in my hands and I despair".

The spread and the consequences of a mechanized approach to problems appear in simple problems where no operations are required. Within step 3 of the present study, we used the following problem from the 2009 Italian National Standardized Assessment for grade 5:

Maria, Renata and Fabio measure the length of their classroom through their step. Maria counts 26 steps, Renata counts 30 and Fabio 28. "Who has the longest step?" Almost 50% of

the grade 5 students of the national sample chose the option 'Renata', and we obtained a similar percentage in our sample of students from grades 3 to 5 (43% of them answered 'Renata'). We proposed a similar problem at the kindergarten level (simply using smaller numbers in the text) and it was correctly solved by the children through an empirical approach:

they used steps to calculate the length of their classroom noticing that the smaller children,

with shorter steps, counted more steps.

The results obtained show a worrisome evolution between kindergarten and the end of the primary school of the students' attitude towards problems, in terms of the three components of attitude: vision, emotional disposition, and perceived competence.

This is true throughout the primary school trajectory: the autobiographical essays of these

students show that the percentage of those who express negative emotions towards mathematical problems increase from grade 1 to grade 5 and, at the same time, there is a decrease in average perceived competence in solving mathematical problems.

But this is also true in the transition from kindergarten to the more structured (and directly linked to mathematics) primary school level. This phenomenon seems to highlight the negative effect of the exposition to the word problems in primary school.

Students in kindergarten and in the first year of primary school hold an idea of problems that appears to be very promising, not fixed to a stereotypical model.

Also, their idea about how a problem can be solved (their idea of problem solving) appears to be quite advanced: they do not describe prescriptive and reproductive actions. Moreover, in many cases, they accept and search for a multiplicity of solutions for the same problem, referring to actions such as collaborating with others, thinking/reasoning, trying in various ways, not giving up.

We highlight how these ways of solving a problem contrast with the typical Italian setting for

problem solving sessions (but also with the setting of international standardized assessments):

students have little time to solve a problem (so it is difficult to think much, or to try in various

ways) and cooperation is not encouraged, to say the least.

The common vision of problems developed at the end of primary school appears to be the exact opposite. This is another qualitative indicator of the deterioration of students' attitude towards

The fact that the vision of problems is much broader, open, and productive in children who have not yet met the mathematical problem is an aspect to reflect upon: in a certain sense, it seems that educational experience—and, in particular, the exposition to mathematical problems in primary school—has a negative effect on students' vision of problems, but also on their self-perception and emotional disposition towards mathematical problems (i.e., on students' attitude towards mathematical problems.

Second step

After labeling relevant text in all the articles being analyzed, all codes are written using constant comparison. Codes that have a common concept are highlighted in a similar way.

Status-Quo Learning Environment.

- **Students noted that the Problem differed from other problems that they had encountered in their mathematics classes.**
- **Initially she thought it had nothing to do with mathematics.**

I realized with a lot of these problems it took me a lot longer than 5 to 10 minutes to solve them.

- **Formal mathematics has little or nothing to do with real thinking or problem - solving.**
- **In a problem that calls for discovery, formal mathematics will not be evoked**
- *Mathematics problems are always solved in less than ten minutes, if they are solved at all.*
- *mathematics problems have quick solutions or they cannot be solved.*
- *Students are well conditioned* to view problems as solvable in a short time frame.
- **I think the way the grammar is set up makes the mathematics confusing. If they just came right out and said it, I think it would be much easier than putting it in a whole paragraph form.**
- they did not take our non-routine problems as mathematics, since they were not calculable (H3-8, S3-3), did not involve numbers (Y3-6), and involved decision-making (H3-7).
- **This question looks like logical reasoning more than mathematics.**
- Logical reasoning involves words more, and for math, all are numbers

- Mathematics should involve calculation, drawing figures [is] more like cartoon drawing.

.

- **Most of them found a distinction between the two notions when they compared these non-routine problems with the types of problems they met day-to-day.**
- Most [routine] problems in textbooks involve calculations and these [non-routine] problems involve thinking. (Y3-3)

- Some even suggested that those that needed explanations were not mathematics, "as they do not involve numbers" (B3-4; T3-4, L3-4).
- *not only is finding the answer stressed in most cases, but also obtaining the answer in a few minutes*.
- ***each mathematics [problem] has only one solution.***
- ***There is definitely one answer, very definite. One won't be asked of possibilities" (W7-9).***
- ***on meeting these non-routine problems, which might have more than one answer, they "didn't proceed after getting an answer [one of the answers]" (K6-7).***
- **To some of them, these open-ended problems did not look like mathematics very much: "These problems involve real life situations [rather than mathematics], not asking for a definite answer**" (W9-3).
- **Some felt that these non-routine problems were more challenging while others "don't like these problems as they do not follow a fixed rule [for solution]" (P7-3).**
- ***For routine problems, you can find a method after reading through the question. I can't [even] find a way to tackle them for these problems. (W9-4)***
- **I feel these problems are like composition rather than mathematics. (M6-8)**
- **These problems require writing, like "arts subjects."**

Alternative Learning Environments.

- **The name really doesn't matter. That's neither here nor there. I mean, just knowing how to do it, that's the important part, that's what we learned. And that's being able to do it, being able to teach it to somebody else, to explain it, to use it for what you need to use it for. That's what really matters, not being able to know the name of it, or how to draw it up, or anything like that.**
- **"all [of] the problems that have been given to us, I feel like, somehow, one is related to each other.**
- **Ankur rejected the idea of just memorizing names or labels of concepts without knowing "why."**
- **I understand a lot better the whole concept behind each problem.**
- **Brian mentioned getting to "the full meaning of a problem" through an in-depth coverage of concepts in class.**

- **I really have gained a lot of knowledge and learned how to look into things deeper than just surface things like, “Why is it like this?” Now, I start thinking like that.**
- **They were motivated to do mathematics if they were allowed to discover and express their ideas, work together and use manipulative objects whenever needed.**

- **the students in this study did not exhibit the paradoxical positions of viewing mathematics as being about mathematical thinking while, at the same time, holding firmly to the expectation that mathematics problems should be solved relatively quickly.**
- **the students in this study suggested explicitly that problem solving is a long discovery process. Finally, the students in this study exhibited epistemologically sophisticated views about learning.**

- **Kira: When I first saw it I thought it was a joke. I did not take time to go through it step by step to discover whetheror not it was possible to solve it. After I realized that it was possible [for the problem to be solved] I took my time andsolved it step by step.**

- **(i.e. 81%) seemed to have recognized by the end of the intervention that a problem that appears to be unsolvable can actually be solvable and within their capabilities.**
- over 75% of the students seeme,d to have realized that effective problem solving requires perseverance.
- Aleara noted that her engagement with the Problem changed her “whole outlook on how to solve problems” and learned that one does not “need specific numbers, or even variables, when solving a problem.”
- Beth said that the Problem“gave [her] a new sense of looking at word problems” in terms of what kind of information is necessary to have in a problem or how this information can be presented.
- **Nadia noted that, prior to her experience with the Problem, she would glance at a mathematics problemand draw a quick conclusion. The Problem challenged this approach by teaching her “to not assume something when [she] just read[s] through [a problem for] the first time, but to carefully go back and examine every piece of info.”**
- **Sophie expressed a similar idea when she said that the Problem “taught her to look at the problems that are presented[to her] in more than just the given way, to look more deeply into them and do not think that they are impossible.”**
- Erin noted that her engagement with the Problem showed her “that in math, as in life, things aren’t always the way they seem” there by teaching her the importance of “always keep[ing] an open mind and attempt[ing] to look at problems in more than one way.”
- *Evans noted that the Problem taught him the importance of perseverance in mathematical problem-solving.*

- **Kimberly noted that the Problem was fun because it was not a typical mathematics problem, explaining that it was presented like a story and did not follow the typical format in which the givens are offered first and a question follows. She added that the Problem was challenging and, although initially she thought it had nothing to do with mathematics, after some thinking she realized it was indeed a mathematical problem.**
- *I realize the more you think about it and the more you understand what they're saying, it's easier to solve the problem and it's not always going to take 5-10 minutes.*

- **"7 of these students informed me that their teachers did give them impossible problems to work on. They claimed that it was all right because the problems made them think hard and that one learns by working on such problems." (p.19)**

- **the mathematics problems I met at school came up within the context of a certain technique (such as a chapter with only problems about integration) or they were solved using standard recipes.**
- **For these kinds of problems the approach to be followed was so clear that the first try in almost all cases directly led to the solution.**
- At university it quickly became clear that problems often consist of several sub-problems and one has to choose from several available techniques, some of which get hopelessly stuck.

- However, now it has appeared that, in mathematics, a lot of things have to be investigated and discovered.

- Every new mathematical discovery gives way to more and deeper investigations. Mathematics is a discipline that does not stand still; new knowledge is always added.
- ***Problems at secondary school were standard and, almost always, there was a unique and easy method for solving a problem.***

- **At university, problems are non standard; precisely reading the question is important.**
- ***At school, precise answers were possible. For problems encountered at university, precise answers are often not possible.***

Third step

Codes that had common meaning were grouped into categories.

1. The way students negotiate the problem’s definition in relation to mathematics versus to a real-world situation.
2. The way students negotiate the characteristics of a problem as being of a stereotypical nature or a specific structure.
3. The way students negotiate the problem as being posed by a teacher or a textbook versus a real-life challenge without a mediator.
4. The way students negotiate the problem’s definition as a text or a sense making activity.
5. The way students negotiate the characteristics of a problem as being of a stereotypical nature versus a thinking activity without specific structure.

Fourth step: The theme emerged based on categories

Theme Related to Students’ Negotiation of the Meaning of a Problem across Learning Environments

Our analysis of journal articles that dealt with students' view of a problem suggests that when students experience alternative learning environments characterized by individual/collaborative work, as contrasted to a status quo whole-class learning environment, they negotiate their meaning of a problem. The students display a shift in the way they view a problem. In status quo learning environments students view a problem as a closed, narrow-focused narrative posed by a textbook or a teacher in a familiar context and asking for a numerical answer; whereas in alternative learning environments, students start to view the problem as an open and broad-focused narrative that may be non-routine, in a non-familiar context, that may require thinking and analyzing the data to resolve the issue posed in the narrative.

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