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BY

ADNAN WADI' BITAR - 1951

DESIGN OF A REINFORCED CONCRETE ARCH BRIDGE ON NAHR-AL-DAMUR

BY

ADNAN WADI' BITAR

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JRO

I am particularly indebted to Professor J.R. Osborn, Head of the Engineering Department of the American University of Beirut, who attributed greatly to the making up of this thesis by his important ideas and helpful suggestions. I also thank him for he reviewed and checked the material of my complete work.

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C O N T E N T S

INTRODUCTION

This bridge is to cross a wide but deep valley and is to serve for highway traffic. It is made up of a single arch with a theoretical span of 234 ft. and a theoretical height of 38 ft., thus having a rise-span ratio of $\frac{1}{6.2}$ which is rather economical. I have chosen for it the filled spandral type with fixed ends for reasons that are mentioned in the written part of the thesis.

The procedure of design is what can be called - The Standard Method - which is followed by most designers and is outlined as follows:

1. Assume the dimensions of the arch by the help of some approximate formulas, or by comparison with a previous design.
2. Calculate the extreme fiber stresses due to proper combination of moments and thrusts using approximate formulas with coefficients got out of prepared tables.
3. If these calculated stresses are far from the allowable, repeat steps (1) and (2) until the selected arch dimensions give satisfactory stresses. I made two trials.
4. Calculate either analytically or graphically the exact line of pressure for dead load and adopt this line for the arch axis. I did it analytically.

5. Compute the bending moments and thrusts by the exact method and find the exact stresses in the arch sections. When these stresses are either two small or two large proper change in the dimensions of the arch or in the reinforcement should be made.

In my thesis I did not include the derivations of the arch formulas used, for these are found in any text on arch design; and to put them down means that I should copy them from one source or the other. However, I did study every derivation and have understood them thouroughly.

My prime source of information is the book "Concrete, Plain and Reinforced" - Volume II, by Taylor, Thompson, and Smulsky. Also, I have consulted many other books listed in the bibliography. In addition I am greatly indebted to Prof. J.R. Osborn who directed my work by his useful advices and suggestions.

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N O T A T I O N

APPROXIMATE METHOD OF DESIGN

Let Y = Ordinate for any point on arch axis, center of coordinates at crown;

Y_c = vertical distance of elastic center from crown;

$n = \frac{I}{I_s \cos \phi_s}$ = ratio of moments of inertia at crown and of vertical projection at springing;

x = Abscissa for any point on arch axis, center of coordinates at crown;

l = span length of arch axis, ft.;

r = rise of arch axis, ft;

q_c = unit dead load at crown, lb. per lin. ft. of arch.

q_s = unit dead load at springing, lb. per lin. ft. of arch;

q_x = unit dead load at any point lb. per lin ft. of arch;

$m = \frac{q_s}{q_c}$ = ratio of dead loads;

I = moment of inertia at crown;

I_s = moment of inertia at springing;

I_x = moment of inertia at any point x;

ϕ_x = angle of inclination of tangent to arch axis at point x ;

ϕ_s = angle of inclination of tangent to arch axis at springing;

h_c = depth of section at crown;

h_x = depth of section at intermediate point;

h_s = depth of section at springing;

EXACT ANALYSIS OF ARCH

Let Y = abscissa of arch axis referred to left support as center of coordinates;

X_s and Y_s = location of elastic center with reference to left support;

ds = length of a division of the arch;

l = span of arch;

Ax = area of average section in each division of the arch;

I_x = moment of inertia of average section in each division of arch;

I = moment of inertia of section at crown;

x, y =, coordinates of the center of the division of the arch referred to axes through elastic center;

ϕ_x = angle with vertical of any normal section of the arch;

M_s = static bending moment of loads considering arch as cantilevered at right support;

H = horizontal thrust at both supports for vertical loads;

M_A = bending moment at left support;

VA = vertical reaction at left support;

M = auxiliary bending moment;

M_x = bending moment at any section of the arch;

N_x = Normal thrust at any section the arch.

CHAPTER ONE

ARCHES - GENERAL INFORMATION

An arch bridge is subjected mostly to compressive stresses, and is therefore mostly built of concrete, since this material is especially adapted to resisting compression. In arches of stone or brick, it is essential that the line of pressure for any possible loading should pass within the middle third of each joint of the arch ring, in order to avoid a tendency for any joint to to open. In arches of concrete, however, steel reinforcement may be introduced, and thus, the structure becomes capable of withstanding tension, which means that a considerable variation of the line of pressure may be permitted without endangering the bridge. Reinforced with steel, somewhat larger section may be used since the arch can carry a large bending moment with tension in one face.

When man first discovered the arch, he could build it with only small spans, and it continued to be so until the development of concrete and reinforced concrete which increased the length of the spans considerably. In 1928, a bridge was built in France with an arch span of 558 ft. The most economical rise-span ratio for most archs of all types is generally considered to be between 1/3 and 1/6 . Arches with rise-span ratios of 1/10 and less can be and were built; but are for most of the times uneconomical.

ADVANTAGES OF ARCH CONSTRUCTION

1. PERMANENCY: Reinforced concrete gains strength with time. Thus, if an arch bridge is built on firm unyielding foundations, and serves to carry only those loads it was designed for, it will last eternally.

2. Absence of vibration and noise which is somehow annoying in the case of steel bridges. Both noise and vibration are absorbed by the large mass of the bridge.

3. Aesthetic appearance: Arches may be made graceful and pleasing to a degree unrivalled by any other bridge.

4. Low maintenance cost if any.

5. COST: It has been generally found that with normal conditions of foundations, arch bridges with spans longer than 50 ft., are comparatively cheaper than those of concrete girder bridges or steel bridges.

SELECTION OF TYPE OF BRIDGE

For the bridge I am going to design, I have chosen the filled spandrel type with fixed end points. In this type, the space between the extrados of the arch and the roadway is filled with earth. This fill, after it is properly tamped and rolled, supports the roadway. The fill is retained on both sides by spandrel walls. These walls act as retaining walls and may be built either of masonry, or as reinforced concrete cantilevers, or they may consist of thin slabs tied together by reinforced concrete cross-walls.

The reasons for my choice of the filled spandrel type are outlined as follows:

First: The bridge is rather a shallow one with a rise-span ratio of about 1/7 ; thus the increase in the weight of the superstructure due to the filling materials is not excessive.

Second: If I had chosen the open spandrel type, I would have saved on the thickness of the arch ring; but on the other hand, the open spandrel type of construction requires relatively larger amount of framework, which item is much more expensive than either concrete or steel in the Lebanon.

Third: Considering the architectural or aesthetic point of view, the filled spandrel type appears more graceful and gives a heavy and massive appearance, which characteristic is often desired and is more expressive of the strength and greatness of the bridge.

ARCH ACTION : An arch is a curved beam. It is governed by the same laws of mechanics as the ordinary beam, and its stresses are computed on the same assumptions as any other reinforced concrete structure. It simply differs from a beam in that its cross-sections are subjected to bending moments, normal thrusts, and shears, while those of an ordinary beam are subjected to bending moments and shears only.

This normal thrust on the cross-sections of the arch, is caused by the horizontal thrusts at the supports which are the result of the prevention of the arch ring from straightening up under the vertical loads. It produces compression stresses at all sections of the arch, and thus, it neutralizes some of the tensile stresses produced by other loads on the arch.

The horizontal thrusts, beside producing the normal thrust mentioned, also produce negative bending moments at all sections of the arch which in turn neutralize some of the existing positive bending moments. Hence, we will have our arch greatly subjected to compression and slightly to tension, and since concrete is especially good to resisting compression, we can have, with little reinforcement, very large bridges with comparatively small sections.

DEFLECTION OF ARCH UNDER DIFFERENT LOADINGS

A beam when subjected to vertical loads deflects downwards although its length except at the supports, since they are fixed against downward movement. An arch, however, deflects irregularly depending upon the way it is loaded. It might deflect downwards throughout; or part of it might deflect downwards, while the other part deflects upward.

Figure 1, which is an exact copy of figure 191 on page 463 of the book: "Concrete Plain and Reinforced", by Taylor, Thompson, and Smulsky, serves best to illustrate the deflection of an arch under different types of loading.

USE OF REINFORCEMENT IN CONCRETE ARCHES

An arch may be built either of plain concrete or of reinforced concrete. For the former case to be possible, the compression in the arch sections due to dead loads, should be large

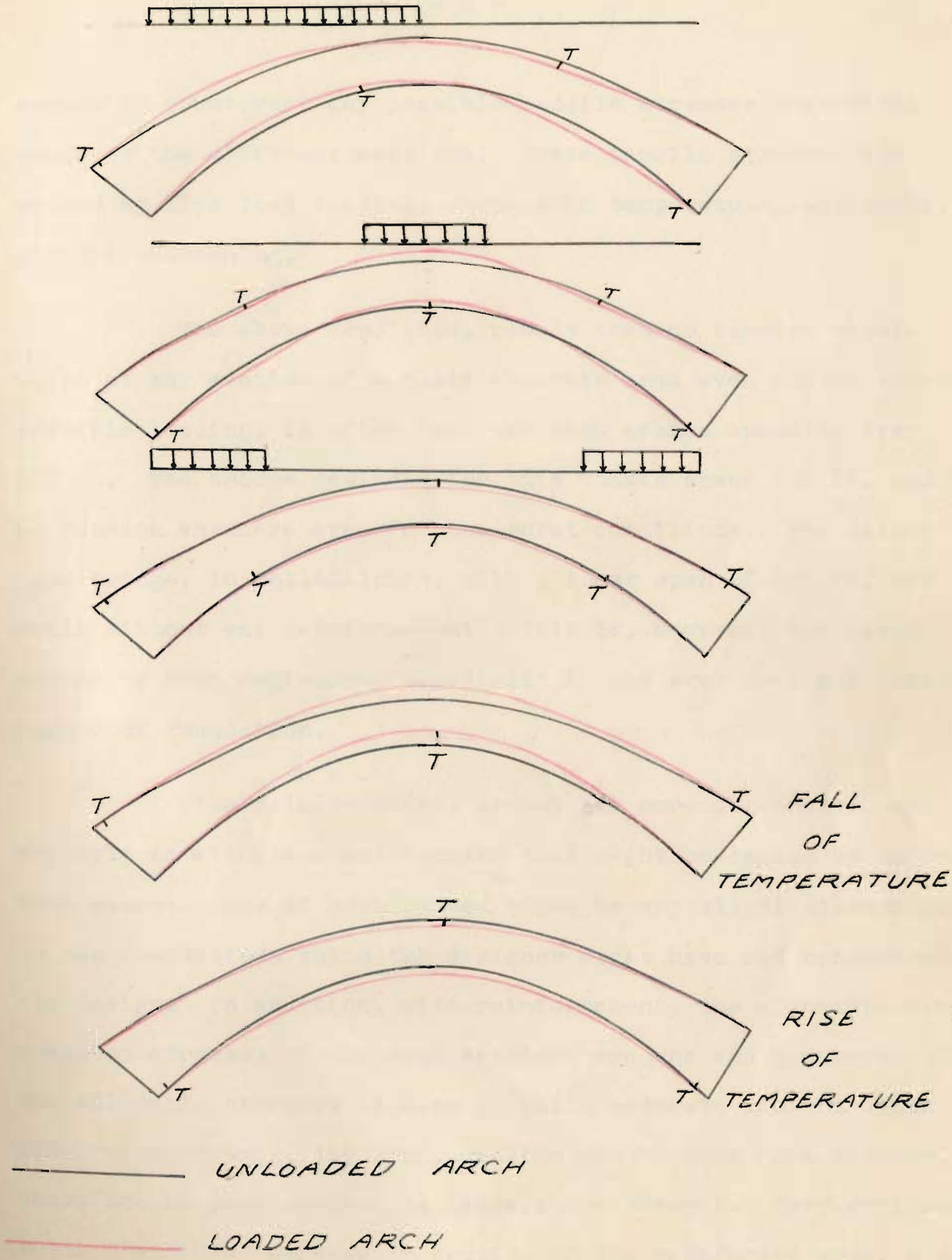


FIG. 1 — DEFLECTION OF ARCH UNDER DIFFERENT TYPES OF LOADING.

enough to counteract any possible tensile stresses that might occur at the different sections. These tensile stresses are caused by live load loading, changes in temperature, shrinkage, and rib shortening.

The above condition, namely that no tension should exist at any section of a plain concrete arch even for the worst possible loading, is often realized with arches spanning over 200 ft. The bridge designed for this thesis spans 234 ft. and has no tension anywhere even for the worst conditions. The Walnut Lane Bridge, in Philadelphia, with a clear span of 233 ft. was built without any reinforcement. This is, however, not recommended by most engineers, especially if the arch does not rest on hard rock foundation.

With reinforcement, arches are more dependable, and are able to withstand any tension that might be caused by unforeseen causes. One of such causes might be any slight disarrangement in the foundations which the designer might have not considered in his design. In addition, with reinforcement, the allowable compression stresses on the arch sections are one and a quarter times the allowable stresses in case of plain concrete arches. Thus, with reinforcement, the arch, section can be made more slender, and, therefore be less subject to temperature changes. Many engineers think that this decrease in section of the reinforced arch, more than counterbalances the cost of reinforcement, and hence, that reinforced concrete arches can be built more cheaply.

The most common type of reinforcement and the one universally used, is to put two layers of bars (one near the intrados and the other near the extrados) running longitudinally along the length of the arch rib, together with cross-bars which tie the main bars, distribute the load laterally, and also prevent longitudinal cracks. Also in order tp prevent bucling of the longitudinal bars and to keep them in place bar hoops are used running around the top and bottom bars. The longitudinal reinforcement is placed near both upper and lower surfaces; for a considerable portion of the arch ring is subject to both positive and negative moments.

The amount of longitudinal reinforcement usually ranges from 1/2 - 1 % of the cross-section of the arch. The exact amount depends upon the loading and the form of the arch selected, and must be finally tested by computations.

It is very important that the reinforcement skeleton stays in place while the concrete is poured, and that all the reinforcing bars remain straight. Also the two layers of the reinforcing bars should remain at 2" from the surface of both the intrados and the extrados. If nearer there will be danger from rusting; if farther, the reinforcement will be less effective in resisting bending moments and in preventing cracks.

CHAPTER TWO

DESIGN OF ARCH BY USE OF APPROXIMATE METHOD

DATA AND SPECIFICATIONS:

The bridge to be designed is of the fixed ends type with filled spandrals, and is to serve as a highway bridge for which:

theoretical span, $l = 234$ ft.

theoretical rise, $r = 38$ ft.

$l/r = 6.17$

rise-span ratio $r/l = 0.162$

The roadway is designed to carry a uniform live load of:

$W = (820 - 41)$ Kgs./m^2. which in the present case reduces to : $W = 110$ lb. per sq.ft.

No impact is considered due to the massiveness of the bridge and its type of construction. Also, the investigation for the concentrated type of loading is eliminated for the same reason.

The allowable stresses in the concrete are as follows:

Concentric compression, $f_c = 475$ P.S.I.

Direct stress and bending, small eccentricity, $f_c = 575$ P.S.I.

large eccentricity, $f_c = 650$ P.S.I.

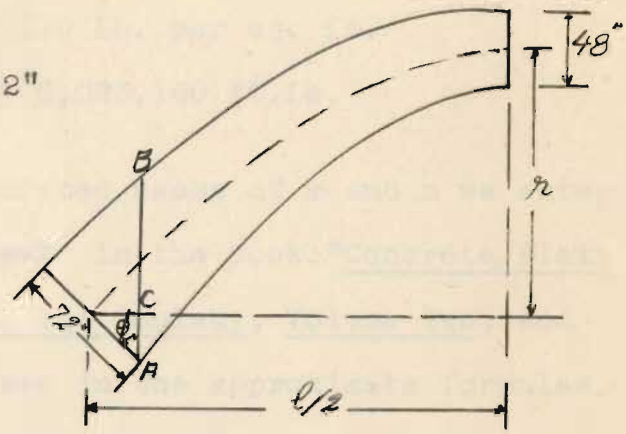
Maximum tension, $f_t = 120$ P.S.I.

$E = 2{,}000{,}000$ P.S.I. , $d = 0.0000055$, $\therefore dE = 11$

Depth of fill at crown equals 1 ft. Unit weight of fill = 100 lb. per cu.ft. Roadway on top of fill weighs 75 lb. per sq.ft. Roadway is assumed to be level longitudinally.

SOLUTION:

Assume: $h_c = 48"$; $h_s = 72"$



$$\tan \phi = \frac{4 \frac{r}{l}}{1+3 \frac{r}{l}} \cdot (1+7.5 \frac{r}{l})$$

$$\tan \phi = \frac{4x0.162}{1+3x0.162} \cdot (1+7x0.162)$$

$\tan \phi = 0.922$

$\phi = 47^{o}$ approx.

$\sin \phi = 0.7314$

$\cos \phi = 0.6820$

$$\therefore AB = \frac{72}{0.682} = 105.5" = 8.80 \text{ ft.}$$

$AC = 36x0.682 = 24.6"$

$BC = 105.5 - 24.6 = 80.9" = 6.73$ ft.

From the above figure we can proceed to get the unit dead loads q_c and q_s at the crown and the springing respectively.

$q_c = 4x150 + 100 + 75 = 775$ lb. per ft.^{2}

$q_s = 8.8x150 + (38+2+1-6.73)x100+75$

$= 4822$ lb. per ft.^{2}

$$m = \frac{q_s}{q_c} = \frac{4822}{775} = 6.35$$

$$n = \frac{I}{I_s \cos\phi_s} = \frac{I}{\cos\phi_s} = 1.36 \text{ as got graphically.}$$

$$n = \frac{48x48x48}{72x72x72} x1.36 = 0.403$$

Use $n = 0.40$

Therefore the constants to be used in the approximate method are:

$m = 6.35$; $n = 0.4$; $w = 110$ Lb. per sq. ft.

$wl = 25740$ Lb. ; $wl^2 = 6,023,160$ ft.Lb.

With the previously calculated vaues of m and n we enter into the several useful diagrams drawn in the book:"Concrete, Plain and Reinforced, by Thompson, Taylor, and Smulsky, Volume Two, and get the various coefficents to be used in the approximate formulas. Thus all the diagrams I refer to, hereafter, are to be found in the specified pages of the above mentioned text.

Let us now calculate the maximum bending moments due to the different loadings and factors affecting the bridge.

LIVE LOAD AT SPRINGING: Find constants from Diagrams 30 and 31 pp. 676 and 677.

$$+M_s = C_{(+s)} wl^2 = 0.0303 x 6,023,160 = +182500 \text{ ft.Lb.}$$

$$H_s = -C_{(hs)} wl \frac{l}{r} = 0.1037 x 25,740 x 6.17 = -16470 \text{ Lb.}$$

$$-M_s = -C_{(-s)} wl^2 = -0.0154 x 6,023,160 = -92760 \text{ ft. Lb.}$$

$$H_s = -C_{(-hs)} wl \frac{l}{r} = -0.0316 x 25740 x 6.17 = -5020 \text{ Lb.}$$

LIVE LOAD AT CROWN: Find constants from Diagrams 26 and 27, pp. 672 and 673.

$$+M_c = C_{(+c)} wl^2 = 0.0071 x 6023160 = +42765 \text{ ft. Lb.}$$

$$H = -C_{(+hc)} wl \frac{l}{r} = 0.0811 x 25740 x 6.17 = -12880 \text{ Lb.}$$

$$-M_c = -C_{(-c)} wl^2 = -0.00302 x 6023160 = -18190 \text{ ft. Lb.}$$

$$H = -C_{(-hc)} wl \frac{l}{r} = -0.0542 x 25740 x 6.17 = -8610 \text{ Lb.}$$

EFFECT OF CHANGE OF TEMPERATURE AND SHRINKAGE:

"C_h is obtained from Diagram 24 p.471"

$$H_t = \frac{dEI(\pm t)}{r^2} \cdot \frac{1}{C_h} \quad ; \qquad dE = 11$$

$$H_t = \frac{11x48^3x(\pm t)}{(12x38)^2x0.0432} = -135.5 \pm t$$

For rise of temperature, $t=25^\circ$ F

$\therefore H_t = 25x-135.5 = -3390$ Lb.

For fall of temperature, $t = -40^\circ$ F

$\therefore H_t = -40x-135.5 = +5420$ Lb.

From these calculated values of H_t, let us find the bending moments that they cause at the crown and at the springing. For this we need to know the value of Y_c, the vertical distance of elastic center from crown.

$Y_c = C_o r$; "Find C_o in Diagram 23 p.670"

$Y_c = 0.21x38 = 8.00$ft.

$r-Y_c = 38-8 = 30.00$ ft.

BENDING MOMENTS

SPRINGING:

RISE, $M_s = 3390x30 = +101700$ ft.Lb.

FALL, $M_s = 5420x30 = -162600$ ft.Lb.

CROWN:

RISE, $M_c = 3390x8 = -27120$ ft.Lb.

FALL, $M_c = 5420x8 = +43360$ ft.Lb.

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EFFECT OF DEAD LOAD:

$H_d = C_d q_c \frac{l^2}{r}$; "C_d from Diagram 25 p.671"

$H_d = 0.21x775x6.17x234 = -234,980$ Lb.

EFFECT OF RIB SHORTENING:

$$H_s = \frac{I}{A_{av} x r^2} \times \frac{1}{C_h} x H_d$$

From Diagram 24 p.671, $C_h = 0.0432$

Assume $A_{av} = (48+72)x\frac{1}{2} = 720$ sq. in.

$$\therefore H_s = -\frac{48^3 x 234980}{720x(12x38)^2 x0.0432} = +4017 \text{ Lb.}$$

The bending moments due to rib shortening are:

SPRINGING:

$M_s = - H_s\ (r-Y_c)$

$M_s = - 4017x30 = 120510$ ft.Lb.

CROWN:

$M_c = H_s\ Y_c$

$= 4017x8 = +32136$ ft. Lb.

Having computed the different thrusts and bending moments of the different loadings and effects, let us tabulate the calculated values and combine them so as to get the maximum possible stresses that might occur at the springing and crown sections of the arch.

SUMMARY OF BENDING MOMENTS AND THRUSTS

TYPE OF LOADING	POSITIVE BENDING MOMENTS		NEGATIVE BENDING MOMENTS	
	HORIZONTAL THRUSTS	BENDING MOMENTS	HORIZONTAL THRUSTS	BENDING MOMENTS
	SPRINGING			
	POUNDS	FOOT-Lb.	POUNDS	FOOT - Lb.
DEAD LOAD	-234980	-------	- 234980	---------
RIB SHORTENING	-------	-------	+ 4017	-120510
LIVE LOAD	- 16470	182500	- 5020	- 92760
EFFECT OF TEMPERATURE AND SHRINKAGE	- 3390	101700	+ 5420	-162600
TOTAL	-254840	+284200	-230563	-375870
	CROWN			
DEAD LOAD	-239980	-------	-234980	--------
RIB SHORTENING	+4017	32136	-------	--------
LIVE LOAD	-12880	42765	- 8610	- 18190
EFFECT OF TEMPERATURE AND SHRINKAGE	+ 5420	+ 43360	- 3390	- 27120
TOTAL	-238423	+118261	-246980	- 45310

Using the maximum bending moments and thrusts given in the previous table, let us proceed to find the required dimensions of the arch sections.

SPRINGIN:

M_s = -375870 ft.Lb.

M_s = 4510440 in.Lb.

H = -230563 Lb.

$$N = \frac{H}{Cos\phi_s} = 230563x1.36 = 313560 \text{ Lb.}$$

$$E = \frac{M}{N} = \frac{375870}{313560} = 14.40 \text{ inches.}$$

$$\frac{bfc}{N} = \frac{12x650}{313560} = 0.025$$

From Diagram 7, p.654 the corresponding h_s = 72 in.

$$\frac{bf_t}{N} = \frac{12x120}{313560} = 0.0046$$

From Diagram 8, p.655 the corresponding h_s = 55 in.

CROWN:

M_c = 118260 ft.Lb.

M_c = 1419120 in.Lb.

H = N = 238423 Lb.

$$E = \frac{M}{N} = \frac{1419120}{238432} = 5.97 \text{ inches}$$

$$\frac{bf_c}{N} = \frac{12x550}{238423} = 0.028$$

From Diagram 7, p.654 the corresponding h_c = 48 in.

$$\frac{bf_t}{N} = \frac{12x120}{238423} = 0.006$$

From Diagram 8, p.655 the corresponding h_c = 27 in.

From the previous figures, the following dimensions of the arch sections are adopted.

Springing, h_s = 72 inches

Crown, h_c = 50 inches.

CHAPTER THREE

EXACT ANALYSIS OF ARCH

The adopted dimensions are:

$$h_s = 72 \text{ in.}$$

$$h_c = 50 \text{ in.}$$

Theoretical span = 234 ft.

Theoretical height = 38 ft.

Rise-Span ratio = 0.162

$$q_c = \frac{50}{12} \times 150 + 100 + 75 = 800 \text{ Lb.}$$

$$q_s = 8.8 \times 150 + (38 + \frac{25}{12} + 1 - 6.73) \times 100 + 75 = 4830 \text{ Lb.}$$

$$m = \frac{q_s}{q_c} = \frac{4830}{800} = 6.04$$

$$n = \frac{I}{I_s \text{Cos}\phi_s} \; ; \qquad \frac{1}{\text{Cos}\phi_s} = 1.425$$

$$n = \frac{50 \times 50 \times 50}{72 \times 72 \times 72} \times 1.425 = 0.476$$

First let us determine the line of pressure for dead load, which will be accepted as the axis of the arch. For this purpose, let us lay out a preliminary arch to scale whose axis is plotted from a formula given by Victor A. Cochraine. The formula is :

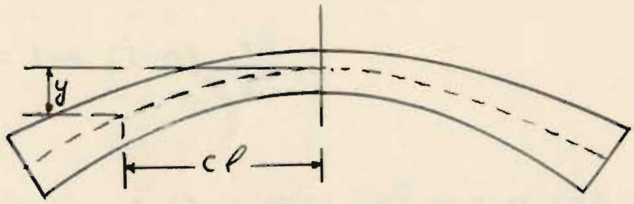
$$y = \frac{4rl}{1+3r} \cdot (c^2 + 24c^5 r), \text{ in which}$$

r = rise-span ratio

l = span of arch axis in ft.

y = ordinate of arch axis, the abscissa for which is cl, the origin of the coordinate axes being at the crown.

The values of cl = x and the corresponding values of y are tabulated below:



c	cl	c^5	$24c^5r$	c^2	c^2+24c^5r	y
0.1	23.40	0.00001	0.000039	0.0100	0.10039	1.024
0.25	58.50	0.00098	0.00381	0.0625	0.06631	6.766
0.30	70.20	0.00243	0.00945	0.0900	0.09945	10.147
0.35	81.90	0.00525	0.02041	0.1225	0.14291	14.582
0.40	93.60	0.01024	0.0398	0.1600	0.1998	20.387
0.45	105.30	0.01845	0.07173	0.2025	0.27423	27.989
0.50	117.00	0.03125	0.1215	0.2500	0.3715	38.000

The above tabulated values of y are calculated from the previously given formula:

$$y = \frac{4rl}{1+3r} \cdot (c^2+24c^5r)$$

$$y = \frac{4x0.162x234}{1+3x0.162} \cdot (c^3+24c^5r)$$

$$y = 102.04\ (c^2+24c^5r)$$

The data from the preceding table enables us to plot the preliminary arch axis. To find the dimensions at the intermediate points, let us use the parabolic variation of the moment of inertia. This variation is expressed in the following formula:

$$\frac{I}{I_x \cos\phi_x} = 1-4\ (1-n)\ \frac{x^2}{l^2}$$

or $$\frac{I}{I_x \cos\phi_x} = 1 - 4\ (1-0.476)\ \frac{x^2}{l^2} = 1-2.096\ \frac{x^2}{l^2}$$

where x is measured from the crown.

$$h_x = \frac{1}{\sqrt{\cos\phi_x (1-2.096\ \frac{x^2}{l^2})}}$$

All the terms in this formula are known except Cos ϕ_x . This is obtained graphically from the arch axis whose ordinates have already been determined. The computations for h_x are given in table one in the following page.

Now we have the coordinates of the preliminary arch axis and the thicknesses at the various sections. Hence, I shall plot the arch axis, the intrados, the extrados, the earthfill, and the roadway. From this plotted diagram, the dead loads acting at the various arch sections are computed and shown in tabular form as shown in table 2.

Knowing the dead loads and their points of application, the line of pressure, which will be the final arch axis of the bridge can be determined analytically. The method of solution follows directly from the requirement that for fixed loads, there should be no bending moment at any point in the arch. For this to be possible, the positive static bending moments due to the loads

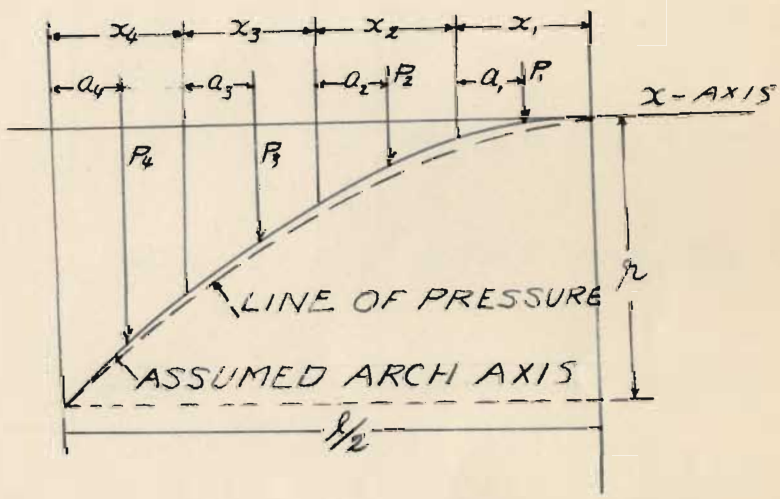
TABLE I.– DIMENSIONS OF ARCH AXIS AT INTERMEDIATE POINTS

DIVISION	$\frac{x}{l}$	$\frac{x^2}{l^2}$	$2.096\ \frac{x^2}{l^2}$	$1-2.096\ \frac{x^2}{l^2}$	$\frac{1}{\cos\phi_x}$	$\frac{1}{\cos\phi_x(1-2.096\frac{x^2}{l^2})}$	$\frac{1}{\sqrt{\cos\phi_x(1-2.096\frac{x^2}{l^2})}}$	h_x INCH
1	$\frac{1}{40}$	0.00062	0.00131	0.999	1	1.001	1	50.00
2	$\frac{3}{40}$	0.0056	0.01180	0.9980	1.005	1.017	1.010	50.50
3	$\frac{1}{8}$	0.0156	0.03276	0.9670	1.010	1.044	1.020	51.00
4	$\frac{7}{40}$	0.0306	0.0640	0.9360	1.015	1.090	1.030	51.50
5	$\frac{9}{40}$	0.0506	0.1060	0.8940	1.025	1.160	1.058	52.90
6	$\frac{11}{40}$	0.0756	0.1590	0.8410	1.040	1.260	1.075	53.75
7	$\frac{13}{40}$	0.1056	0.2220	0.7780	1.065	1.390	1.118	55.90
8	$\frac{3}{8}$	0.1406	0.2950	0.7050	1.120	1.630	1.178	58.90
9	$\frac{17}{40}$	0.1810	0.3790	0.6210	1.230	2.010	1.260	63.00
10	$\frac{19}{40}$	0.225	0.4740	0.5260	1.350	2.710	1.390	69.50

TABLE 2. — DEAD LOAD PER DIVISION OF ARCH

DIVISION	FILL + ROADWAY	ARCH	ARCH + FILL	DEAD LOAD PER DIVISION
	Lb. PER Sq. FOOT	Lb. PER Sq. FOOT	Lb PER Sq. FOOT	POUNDS
1	175.00	4.33 × 150 = 650	825.00	9650
2	235.00	4.35 × 150 = 655	890.00	10 400
3	330.00	4.40 × 150 = 660	990.00	11 600
4	460.00	4.45 × 150 = 670	1130.00	13 200
5	700.00	4.50 × 150 = 675	1375.00	16 100
6	1000.00	4.60 × 150 = 690	1690.00	19 800
7	1375.00	4.85 × 150 = 725	2100.00	24 600
8	1850.00	5.33 × 150 = 800	2650.00	31 000
9	2475.00	6.25 × 150 = 940	3415.00	39 950
10	3295.00	7.60 × 150 = 1140	4435.00	51 900

must be balanced by the negative bending moment due to the horizontal thrust. The bending moment of the horizontal thrust is equal to its magnitude times the vertical distance of the point at the arch axis from the springing, say Hy. By equating the static bending moment to Hy, the value of y, the arch axis ordinate at any point, is readily found. The horizontal thrust is found by dividing the static bending moment at the crown by the rise. The calculations for the values of y are shown in table 3. For better understanding of the notation used, the near-by figure is drawn.



The tabulated values of y'_n in table 3 are the distances measured from the crown for the arch axis at the end of each division.

Now let us find the elastic center for the new arch axis and for the cross-sections of arch worked out in table 1. The following figure and the formula given below it are enough to explain everything about the procedure and the method of solution. The computation is worked out as shown in table 4.

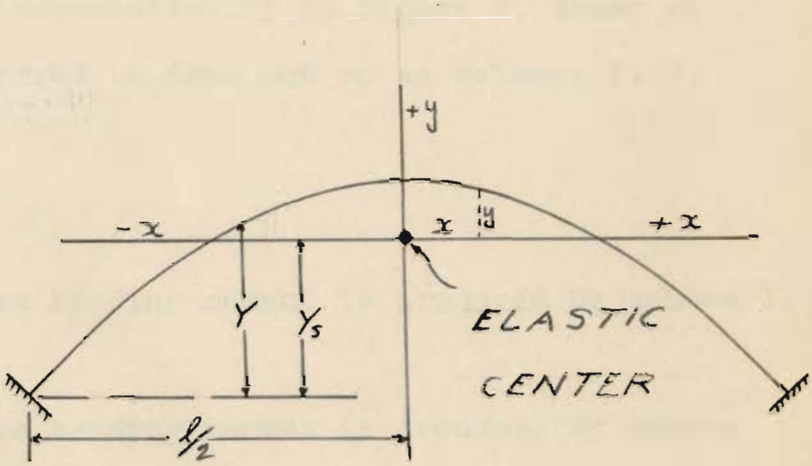
TABLE 3.- LINE OF PRESSURE FOR DEAD LOAD

POINT	LOAD P_n	DISTANCE OF LOAD a_n	$P_n a_n$	LENGTH OF DIVISION	$(P_1+P_2+\cdots+P_n)x_n$	M_{n-1}	M_n (4) + (6) + (7)	$y'_m = \frac{M_n}{H}$ (8) ÷ H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
I	9650	5.85	56 450	11.70	— — — — —	— — —	56 450	0.235′
II	10 400	5.85	60 840	11.70	9650 × 11.7 = 112 900	56450	230 190	0.960′
III	11 600	5.85	67860	11.70	20050 × 11.7 = 234 600	230 190	532 650	2.220′
IV	13200	5.85	77200	11.70	31650 × 11.7 = 370 300	532 650	980 150	4.085′
V	16100	5.85	94200	11.70	44850 × 11.7 = 524 700	980 150	599 050	6.667′
VI	19800	5.85	115 800	11.70	60950 × 11.7 = 713 100	1 599 050	2,427,950	10.120′
VII	24600	5.85	143 900	11.70	80750 × 11.7 = 944 800	2 427 950	3,516,650	14.650′
VIII	31000	5.85	181 350	11.70	105 350 × 11.7 = 1,232,600	3 516 650	4,930,600	20.495′
IX	39950	5.85	233 700	11.70	136 350 × 11.7 = 1,595,300	4 930 600	6,759,600	28.130′
X	51900	5.695	295 550	11.70	176 300 × 11.7 = 2,062,700	6 759 600	9,117,850	38.00′

TABLE 4. – FINDING ELASTIC CENTER Y_s

POINT	Y	X	$\frac{I}{I_x \cos\phi_x}$	$\frac{Ids}{I_x}$	$Y\frac{Ids}{I_x}$	y	$y\frac{Ids}{I_x}$	x
0	38.00	117.00	— — —	— — —	— — —	8.74	— — —	0.00
1	37.87	111.15	0.999	11.69	442.70	8.61	100.65	± 5.85
2	37.40	99.45	0.988	11.56	432.34	8.14	94.10	± 17.55
3	36.41	87.75	0.967	11.31	411.80	7.15	80.87	± 29.25
4	34.84	76.05	0.936	10.95	381.50	5.58	61.10	± 40.95
5	32.62	64.35	0.894	10.46	341.20	3.36	35.14	± 52.65
6	29.61	52.65	0.841	9.84	291.36	0.35	3.44	± 64.35
7	25.62	40.95	0.778	9.10	233.14	– 3.64	– 33.12	± 76.05
8	20.43	29.25	0.705	8.25	168.55	– 8.83	– 72.85	± 87.75
9	13.69	17.55	0.621	7.26	99.39	– 15.57	– 113.04	± 99.45
10	3.93	5.70	0.526	6.15	24.17	– 25.33	– 155.78	± 111.30
A	— — —	— — —	— — —	— — —	— — —	– 29.26	— — —	± 117.00
				96.57	2826.15		+ 375.30 – 374.79 CHECK	

In table 4, the values of $\frac{Ids}{I_x}$ are obtained by multiplying $\frac{I}{I_x \cos \phi_x}$ from table 1, by the horizontal length of the division d_x for $\frac{dx}{\cos \phi_x} = ds.$



$$Y_s = \frac{\Sigma_0^{\ell/2} Y \frac{Ids}{I_x}}{\Sigma_0^{\ell/2} \frac{Ids}{I_x}}$$

From table 4,

$$Y_s = \frac{2826.15}{96.57} = 29.26 \text{ ft.}$$

$$y = Y - 29.26 \ ; \quad x = X - 117.00$$

Having reached this stage in the design, I shall investigate the maximum possible moments and thrusts that might act on the different cross-sections of the arch. It is known by experience that only three critical cross-sections need be examined. These are : at the springing, at the quarter point, and at the crown. To determine the maximum thrusts and moments, separate influence lines should be drawn for each case in order to determine the critical positions of the live loads for the different cases. Several engineers have made extensive studies on the influence lines for arch bridges of different spans, and have come out with the following loadings as producing maximum stresses at the three critical cross-sections of the arch rib.

These loadings are shown diagramatically in figure 2, drawn on the next page and are referred to from now on as schemes 1, 2, -----etc.

AT LEFT SPRINGING:

Maximum positive bending moment is produced by scheme 1, fig. 2.

Maximum negative bending moment is produced by scheme 2, fig. 2.

AT LEFT QUARTER POINT:

Maximum positive bending moment is produced by loading scheme 2, fig. 2.

Maximum negative bending moment is produced by loading scheme 1, fig. 2.

AT CROWN:

Maximum positive bending moment is produced by loading scheme 3, fig. 2.

Maximum negative bending moment is produced by loading scheme 4, fig. 2.

In order to find H, V_A, and M for each of the above conditions, it is only necessary to find the values for a load extending over the whole span of the arch and for a load ex- tending ~~over the whole span of the arch and for a load~~ extending over 5/8 of the span of the arch. Then by proper combinations we can find the values of H, V_A, and M for the other schemes.

The formulas for H_A, V_A, and M as given in the different books on arch design are as follows:

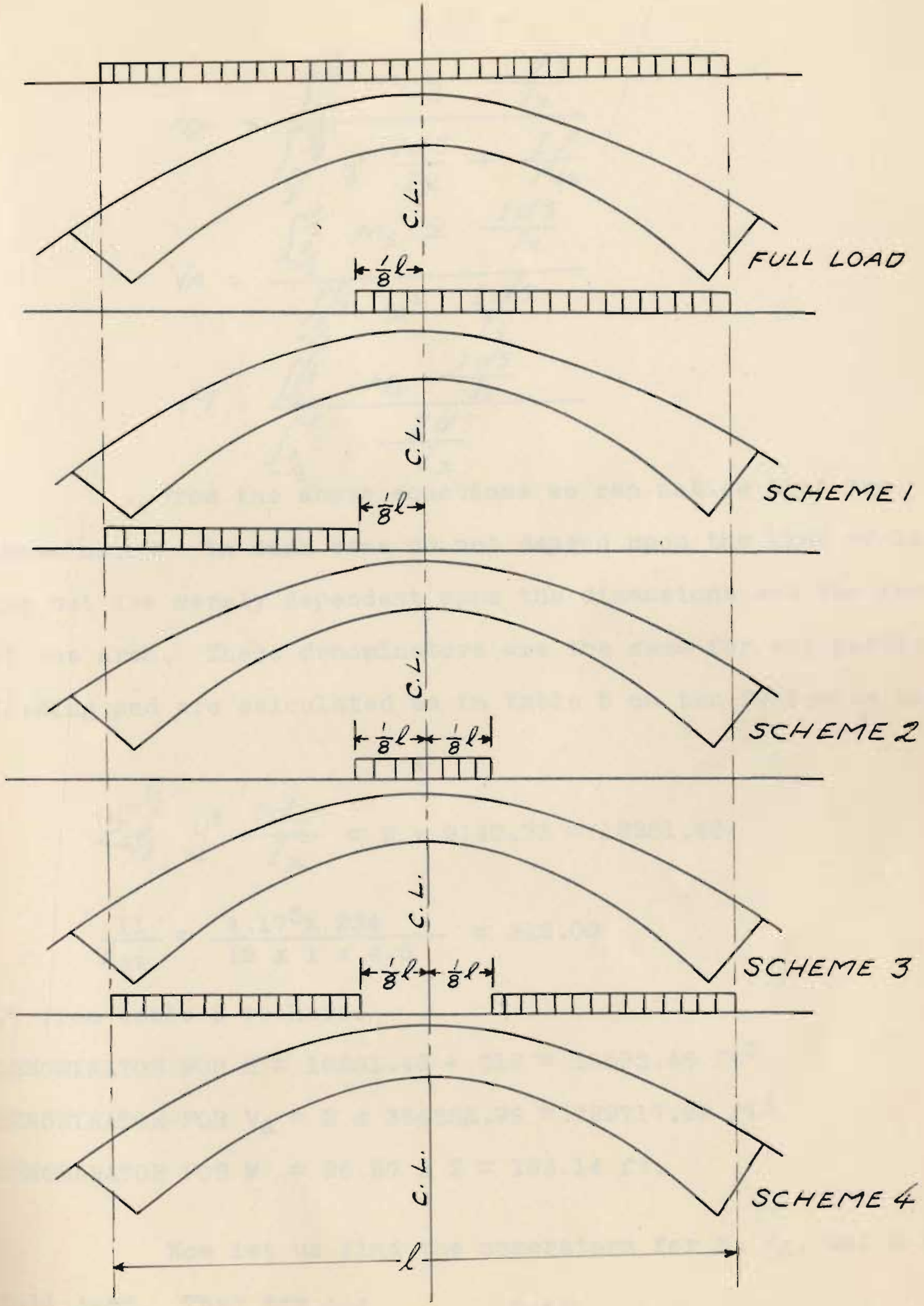


FIG. 2.— POSITION OF LOADING FOR MAXIMUM BENDING MOMENTS

$$H_A = \frac{\int_0^{\frac{l}{2}} M_s\, y\, \frac{I\,ds}{I_x}}{\int_{-\frac{l}{2}}^{\frac{l}{2}} y^2 \frac{I\,ds}{I_x} + \frac{I\,l}{A_{av.}}}$$

$$V_A = \frac{\int_{-\frac{l}{2}}^{\frac{l}{2}} M_s\, x\, \frac{I\,ds}{I_x}}{\int_{-\frac{l}{2}}^{\frac{l}{2}} x^2 \frac{I\,ds}{I_x}}$$

$$M = \frac{\int_{-\frac{l}{2}}^{\frac{l}{2}} M_s \frac{I\,ds}{I_x}}{\int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{I\,ds}{I_x}}$$

From the above equations we can notice that the denominators in each case do not depend upon the kind of loading but are merely dependent upon the dimensions and the form of the arch. These denominators are the same for any particular loading and are calculated as in table 5 on the following page.

$$\Sigma_{-\frac{l}{2}}^{\frac{l}{2}} y^2 \frac{I\,ds}{I_x} = 2 \times 9140.73 = 18281.46$$

$$\frac{Il}{A_{av}} = \frac{4.17^3 \times 234}{12 \times 1 \times 4.5} = 312.00$$

∴ from table 5 we have:

DENOMINATOR FOR H = 18281.46 + 312 = 18593.46 ft^3

DENOMINATOR FOR V_A = 2 x 364858.99 = 729717.98 ft^3

DENOMINATOR FOR M = 96.57 x 2 = 193.14 ft.

Now let us find the numerators for H, V_A, and M for full load. They are :

for H, NUMERATOR = $\Sigma_{-\frac{l}{2}}^{\frac{l}{2}} M_s\, y\, \frac{I\,ds}{I_x}$

but $M_s = -\frac{w}{8}(1+2x)^2$ for any point to the right half of the arch.

and $M_s = -\frac{w}{8}(1-2x)^2$ for any point to the left half of the arch.

The sum of the moments for (+x) and (-x) is

TABLE 5.- DENOMINATORS FOR H, V_A, AND M

SECTION	y	$\frac{Ids}{I_x}$	$y\frac{Ids}{I_x}$	$y^2\frac{Ids}{I_x}$	x	x^2	$x^2\frac{Ids}{I_x}$
1	8.61	11.69	100.65	866.59	±5.85	34.22	400.03
2	8.14	11.56	94.10	765.97	±17.55	308.00	3560.48
3	7.15	11.31	80.87	578.22	±29.25	765.56	8658.48
4	5.58	10.95	61.10	340.93	±40.95	1676.90	18354.39
5	3.36	10.46	35.14	118.07	±52.65	2772.02	28995.33
6	0.35	9.84	3.44	1.20	±64.35	4140.92	40746.65
7	-3.64	9.10	-33.12	120.56	±76.05	5783.60	52630.76
8	-8.83	8.25	-72.85	643.26	±87.75	7700.00	63525.00
9	-15.57	7.26	-113.04	1760.03	±99.45	9890.30	71803.58
10	-25.33	6.15	-155.78	3945.90	±111.30	12378.69	76184.29
		96.57		9140.73			364,858.99

$$-\frac{w}{8}(1+2x)^2 - \frac{w}{8}(1-2x)^2 = -w(\frac{1^2}{4} + x^2)$$

Substituting these in the above formula we get:

$$N \text{ for } H = -w \sum_0^{\ell/2} x^2 y \frac{Ids}{I_x}$$

Similarly,

$$N \text{ for } M = \sum_{-\ell/2}^{\ell/2} M_s \frac{Ids}{I_x}$$

$$= w(\sum_0^{\ell/2} x^2 \frac{Ids}{I_x} + \frac{1^2}{4} \sum_0^{\ell/2} \frac{Ids}{I_x})$$

$V_A = 1/2\ wl = 117\ W$ Lb.

The values of the numerators of H and M are calculated in table 6 as follows:

TABLE 6. - NUMERATORS FOR FULL LOADING

SECTION	x	y	$x^2 \frac{Ids}{I_x}$	$x^2 y \frac{Ids}{I_x}$
1	5.85	8.61	400.03	3444.26
2	17.55	8.14	3560.48	2898.23
3	29.25	7.15	8658.48	61908.13
4	40.95	5.58	18354.39	102417.50
5	52.65	3.36	28995.33	97424.31
6	64.35	0.35	40746.65	14261.33
7	76.05	-3.64	52630.76	-191575.96
8	87.75	-8.83	63525.00	-560925.75
9	99.45	-15.57	71803.58	-1,117981.74
10	111.30	-25.33	76184.29	-1,929,748.06
			364858.99	-3,800231.51
				+ 282353.76
				-3,517877,75

NUMERATOR FOR H = - 3,517,877.75 WLb. ft^3

NUMERATOR FOR M = 364,858.99 + 117^2 x 96.57

= 1,686,805.72 WLb. ft^2

$\therefore$ H FOR FULL LOADING = $\frac{N}{D} = \frac{3{,}517{,}877.75 \text{ W } ft^3}{18593.46 \ ft^3}$

= 189.20 WLb

$\therefore$ M FOR FULL LOADING = $\frac{N}{D} = \frac{1{,}686{,}805.72 \text{ WLbft}^2}{193.14 \text{ ft.}}$

= 8728.33 W ft.Lb.

$V_A = 1/2 \text{ Wl} = 117.00$ W Lb

Having found H, V_A AND M for a load extending all over the span, let us find the values of the same terms for a load extending over 5/8 of the span, i.e. as in scheme 1 of fig.2. Let the terms for this loading be H_1, V_{A1}, and M_1 . Since the denominators are the same for all loadings we need only find the numerators.

NUMERATOR FOR H_1 $= \sum_{-\frac{l}{8}}^{\frac{1}{2}l} M_s \, y \, \frac{Ids}{I_x}$

NUMERATOR FOR V_{A1} $= \sum_{-\frac{l}{8}}^{\frac{1}{2}l} M_s \, x \, \frac{Ids}{I_x}$

NUMERATOR FOR M_1 $= \sum_{-\frac{l}{8}}^{\frac{1}{2}l} M_s \, \frac{Ids}{I_x}$

But $M_s = -\frac{1}{2} w(\frac{1}{8} l + x)^2$

Substituting this value of M_s in the above equations we get,

NUMERATOR FOR H_1 $= - W \sum_{-\frac{l}{8}}^{\frac{l}{2}} \frac{1}{2}(\frac{1}{8} l + x)^2 \, y \times \frac{Ids}{I_x}$

NUMERATOR FOR VA_1 $= - W \sum_{-\frac{l}{8}}^{\frac{l}{2}} \frac{1}{2}(\frac{1}{8} l + x)^2 \, x \times \frac{Ids}{I_x}$

NUMERATOR FOR M_1 $= - W \sum_{-\frac{l}{8}}^{\frac{l}{2}} \frac{1}{2}(\frac{1}{8} l + x)^2 \times \frac{Ids}{I_x}$

These values are computed as shown in table 7 - on the next page:

From table 7 -

NUMERATOR FOR H_1 = - 2,535,471.54 W Lb - ft^3

NUMERATOR FOR V_{A1} = 376,208.44 W Lb - ft^2

NUMERATOR FOR M_1 = 28,054,271.61 W Lb - ft^3

Let H, V, and M for the different loading schemes shown in fig. 2 be designated accordingly with the following notation:

Let M = auxiliary moment for full load;

M_1 = auxiliary moment for left springing, loading scheme 1;

M_2 = auxiliary moment for left springing, loading scheme 2;

M_{2R} = auxiliary moment for right springing, loading scheme 2;

M_3 = auxiliary moment for left springing, loading scheme 3;

M_4 = auxiliary moment for left springing, loading scheme 4;

V_A = VERTICAL reaction at left springing, for full load;

V_{A1} = vertical reaction at left springing, for loading scheme 1;

V_{A2} = vertical reaction at left springing, for loading scheme 2;

V_{B2} = vertical reaction at right springing, for loading scheme 2;

V_{A3} = vertical reaction at left springing, for loading scheme 3;

V_{A4} = vertical reaction at left springing, for loading scheme 4;

H = horizontal thrust for full load;

H_1 = horizontal thrust for loading scheme 1 ;

H_2 = horizontal thrust for loading scheme 2;

H_3 = horizontal thrust for loading scheme 4;

H_4 = horizontal thrust for loading scheme 4;

W = uniformly distributed load per unit of length;

l = span of arch.

TABLE 7.- NUMERATORS FOR LOADING SCHEME ONE

SECTION	x	$\frac{\ell}{8}+x$	$\left(\frac{\ell}{8}+x\right)^2$	$\frac{1}{2}\left(\frac{\ell}{8}+x\right)^2$	$\frac{Ids}{I_x}$	$\frac{Ids}{I_x}\cdot\frac{1}{2}\left(\frac{\ell}{8}+x\right)^2$	$x\frac{Ids}{I_x}\cdot\frac{1}{2}\cdot\left(\frac{\ell}{8}+x\right)^2$	y	$y\frac{Ids}{I_x}\cdot\frac{1}{2}\left(\frac{\ell}{8}+x\right)^2$
3′	-26.325	2.925	8.55	4.275	11.45	48.95	-1288.61	7.7	376.91
2′	-17.55	11.70	136.89	68.445	11.56	791.22	-13885.91	8.14	6440.53
1′	-5.85	23.40	547.56	273.78	11.69	3200.49	-18722.86	8.61	27844.22
1	5.85	35.10	1232.01	616.00	11.69	7201.04	42126.08	8.61	62000.95
2	17.55	46.80	2190.24	1095.00	11.56	12659.59	222175.80	8.14	103049.06
3	29.25	58.50	3422.25	1711.13	11.31	19352.82	566069.98	7.15	138372.66
4	40.95	70.20	4928.04	2464.02	10.95	26981.02	1104872.77	5.58	150554.09
5	52.65	81.90	6707.61	3353.80	10.46	35080.80	1847004.12	3.36	117871.49
6	64.35	93.60	8760.96	4380.48	9.84	43103.92	2773737.25	0.35	15086.37
7	76.05	105.30	11088.09	5544.05	9.10	50450.81	3836784.10	-3.64	-183640.95
8	87.75	117.00	13689.00	6844.50	8.25	56467.12	4954989.78	-8.83	-498604.67
9	99.45	128.70	16563.69	8281.85	7.26	60126.19	5979549.60	-15.57	-936164.78
10	111.30	140.55	19754.30	9877.15	6.15	60744.47	6760859.51	-25.33	-1,538657.42
						367,208.44	28,088,168.99		-3,157,067.82
							- 33897.38		621,596.28
							28,054,271.61		-2535471.54

LOADING SCHEME I :

From tables 5 and 7 we have,

$$H = \frac{-2,535,471.54 \text{ WLb. ft}^3}{18593.46 \text{ ft}^3} = 136.38 \text{ W Lb.}$$

$$M_1 = \frac{376,208.44 \text{ W Lb.ft}^2}{193.14 \text{ ft.}} = 1947.85 \text{ w ft.Lb.}$$

$$V_{A1} = \frac{28,054,271.61 \text{ W Lb ft}^3}{729,717.98 \text{ ft}^3} = 38.44 \text{ W Lb.}$$

LOADING SCHEME 2 :

$$H_2 = H - H_1 = (189.20 - 136.38)W = 52.82 \text{ W Lb.}$$

$$M_2 = M - M_1 = (8728.335 - 1947.85)W = 6780.335 \text{ W ft.Lb.}$$

$$V_{A2} = \frac{wl}{2} - V_{A1} = (117 - 38.44)W = 78.56 \text{ W Lb.}$$

LOADING SCHEME 3 : Scheme 3 may be obtained by subtracting from scheme 1 the reversed loading of scheme 2. If equations are written and solved they give :

$$H_3 = H_1 - H_2 = (136.38 - 52.82)W = 83.56 \text{ W Lb.}$$

$$M_3 = 2M_1 - M + \frac{15}{128} w\, l^2$$

$$M_3 = 3895.70 \text{ W} - 8728.335 \text{ W} + 6416.72 \text{ W}$$

$$M_3 = 1584.085 \text{ W ft.Lb.}$$

$$V_{A3} = \frac{1}{8} wl = 29.25 \text{ W Lb.}$$

LOADING SCHEME 4 :

$$H_4 = H_1 - H_3 = (189.20 - 83.56)W = 105.64 \text{ W Lb.}$$

$$M_4 = M - M_3 = (8728.335 - 1584.085) \text{ W} = 7144.25 \text{ W Ft.Lb.}$$

$$V_{A4} = \frac{3}{8} wl = 87.75 \text{ W Lb.}$$

Let us now tabulate the undeterminate values of H, M, and V_A of all the schemes in the following table :

STATISTICALLY INDETERMINATE VALUES FOR ALL SCHEMES OF LOADING

TYPE OF LOADING	H IN Lb.	V_A IN Lb.	M IN ft.Lb.
FULL LOAD	189.20w=20812.00	117.00w=12870.00	8728.335w=960116.70
SCHEME 1	136.38w=15001.80	38.44w=4228.40	1947.85w =214263.50
SCHEME 2	52.82w= 5810.20	78.56w=8641.60	6780.335w=745.836.80
SCHEME 3	83.56w= 9191.60	29.25w=3217.50	1584.085w=174249.30
SCHEME 4	105.64w=11620.40	87.75w=9652.50	7144.25w =785867.50

w = 110 Lb. per sq. ft.

All values for a width of one ft. of arch.

BENDING MOMENTS FOR LIVE LOADS:

Using the values given in the preceeding table, the bending moments at the springing, the crown, and the quarter points are found as follows:

SCHEME 1

BENDING MOMENT AT SPRINGING

$M_A = M' - V_A' \frac{1}{2} = H' Y_s$

$= 214263.50 - 4228.40x117 - 15001.80 \times 29.25$

$= + 158,493.30$ ft.Lb.

BENDING MOMENT AT QUARTER POINT

$M_{1/4} = M' - V_A' \frac{1}{4} - H' y_{1/4}$

$= 214263.50 - 4228.40 \times 58.50 - 15001.80 \times 2.073$

$= - 64196.70$ ft. Lb.

SCHEME 2:

BENDING MOMENT AT SPRINGING

BENDING MOMENT AT SPRINGING

$M_A = M_2 - V_{A2} \frac{1}{2} - H_2 Y_s$

$= 745836.80 - 8641.60 \times 117 - 5810.20 \times 29.26$

$= - 95223.90$ ft. Lb.

BENDING MOMENT AT QUARTER POINT

$M_{1/4} = M_2 - V_{A2} \frac{L}{4} - H_2 y_{1/4} - \frac{1}{2} w \frac{l^2}{16}$

$= 745836.80 - 8641.60 \times 58.50 - 5810.20 \times 2.073 - 55 \times 58.5^2$

$= + 40035.00$ ft. Lb.

SCHEME 3 :

BENDING MOMENT AT CROWN

$M_c = M_3 - H_3 (r - y_s) - \frac{1}{2} w \frac{l^2}{64}$

$= 174249.3 - 9191.60 (38 - 29.26) - 55 \times 29.25$

$= + 46858.90$ ft.Lb.

SCHEME 4 :

BENDING MOMENT AT CROWN

$M_c = M_4 - H_4 (r - y_s) - \frac{3}{8} wl \times \frac{5}{16} l$

$= 785867.5 - 11620.4 (38 - 29.26) - \frac{3 \times 110 \times 234 \times 5 \times 234}{8 \times 16}$

$= - 21534.00$ ft. Lb.

EFFECT OF RIB SHORTENING:

$$H_s = \frac{\sum_{-\ell/2}^{\ell/2} \frac{Ids}{A_x}}{\sum_{-\ell/2}^{\ell/2} y^2 \frac{Ids}{I_x} + \sum_{-\ell/2}^{\ell/2} \frac{Ids}{A_x}} \times H_d$$

The numerator may be replaced by $\frac{Il}{A_{av}}$ which is equal to 312 as found previously. The denominator is got from table 5 and is equal to: $9140.73 \times 2 + 312 = 18593.46$; $H_d = 239940$ Lb.

$\therefore H_s = \frac{312.00}{18593.46} \times 239940 = 4026.20$ Lb.

EFFECT OF TEMPERATURE CHANGES AND SHRINKAGE:

$$H_t = \frac{\alpha E \ell I}{\sum_{-\frac{\ell}{2}}^{\frac{\ell}{2}} y^2 \frac{I ds}{I_x} + \sum_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{I ds}{I_x}} \pm t^\circ$$

It should be noticed that the denominator for H_t is the same as that for H_s. E = 2000000 P.S.I. ; d = 0.0000055; dE = 11 P.S.I.; or dE = 11 x 144 = 1584 Lb per sq. ft.; $I = \frac{1}{12} \times \frac{50^3}{12^3} = 6.05\ ft^4$.

$$\therefore H_t = - \frac{1584 \times 234 \times 6.05}{18593.46} \pm t^o = 120.60 \pm t^o$$

For rise in temperature $t = 25^o$,

$$H_t = - 120.6 \times 25 = - 3015 \text{ Lb.}$$

For fall in temperature plus shrinkage $t = - 40^o$

$$H_t = 120 \times 40 = 4824 \text{ Lb.}$$

The bending moments due to rib shortening and temperature changes (including shrinkage) are calculated in the following table:

BENDING MOMENTS DUE TO TEMPERATURE CHANGES AND RIB SHORTENING.

KIND	H	SPRINGING y = -29.26 ft.	QUARTER POINT y = 2.073	CROWN y = 8.74
	POUNDS	FOOT-Lb.	FOOT-Lb.	FOOT-Lb.
FALL	4824.00	- 140151.20	+ 10000.10	42161.80
RISE	-3015.00	+ 88218.90	- 6250.10	- 26351.10
RIB SHORTENing	+4026.20	- 117806.60	+ 8346.30	- 35189.00

COMBINED BENDING MOMENTS: The following table gives the final bending moments and thrusts for a combination of live load, dead load, rib shortening, and temperature changes. These values are used to compute the maximum stresses in the arch sections.

TYPE OF LOADING	POSITIVE BENDING MOMENTS		NEGATIVE BENDING MOMENTS	
	HORIZONTAL THRUSTS	BENDING MOMENTS	HORIZONTAL THRUSTS	BENDING MOMENTS
		SPRINGING		
	POUNDS	FOOT-Lb.	POUNDS	FOOT-Lb.
DEAD LOAD	-239940	----------	-239940	---------
RIB SHORTENING	--------	----------	- 4026.20	-117806
LIVE LOAD	- 15001.80	+158493	- 5810	- 95224
TEMPERATURE AND SHRINKAGE	- 3015.00	+ 88219	+ 4824	-140151
TOTAL	-257956.8	+246712	-236900	-353181
		QUARTER POINT		
DEAD LOAD	-239940	----------	-239940	---------
RIB SHORTENING	+ 4026.20	+ 8346.00	----------	---------
LIVE LOAD	- 5810.20	+ 40035.00	- 15002.00	- 64196.70
TEMPERATURE AND SHRINKAGE	+ 4824.00	+ 10000.00	- 3015.00	- 6250.10
TOTAL	-236900	+ 58381	-257957	- 70446.8
		CROWN		
DEAD LOAD	-239940	----------	-239940	-----------
RIB SHORTENING	+ 4026.20	+ 35189	---------	-----------
LIVE LOAD	- 9191.60	+ 46859	- 11620.40	- 21534
TEMPERATURE AND SHRINKAGE	+ 4824.00	+ 42162	- 3015.00	- 26351
TOTAL	-240281.80	+124210	-254575.4	- 47885

STRESSES DUE TO FINAL BENDING MOMENTS AND THRUSTS:

In calculating these stresses, certain coefficients have been taken from the proper diagrams in the book "Concrete, Plain and Reinforced", Volume II, by Taylor, Thompson, and Smulsky. Thus all references to pages of diagrams are meant to pages of the book mentioned.

SPRINGING:

$H = -236900$ Lb.

$M_A = 353181 \times 12 = 4238180$ INCH-Lb.

$$N_A = \frac{H}{\cos\phi_x} = 236900 \times 1.425 = 337582 \text{ Lb.}$$

$$E = \frac{M_A}{N_A} = \frac{4238180}{337582} = 12.54''$$

SECTION:

$b = 12''$; $h = 72''$

$d' = 2''$; $d = 70''$

$A_s = 0.01 \times 12 \times 72 = 8.64$ sq.in.

With this eccentricity, there is no tension in the section, and

$$f_c = C_e \frac{N}{bh}$$

$$\frac{2a}{h} = \frac{68}{72} = 0.945 \; ; \quad \frac{e}{h} = \frac{12.54}{72} = 0.174 \; ; \quad p = 0.01$$

From diagrams on pp. 652 and 653, $Ce = 1.64$

$$\therefore f_c = \frac{1.64 \times 237582}{12 \times 72} = 640.80 \text{ P.S.I. (COMPRESSION)}$$

The allowable is 650 P.S.I. and $\therefore$ this result is good.

CROWN: $N = H = -\ 240281.80$ Lb.

$M_c = 124210 \times 12 = 1490520$ INCH-Lb.

$$E = \frac{M_c}{N} = \frac{1490520}{240281.80} = 6.2 \text{ IN.}$$

After proper investigation it was found that no tension exists in this section.

SECTION: $b = 12"$; $h = 50"$; $d' = 2"$

$$\frac{2a}{h} = \frac{46}{50} = 0.92 \quad ; \quad \frac{e}{h} = \frac{6.2}{50} = 0.124$$

$$A_s = 0.01 \times 12 \times 50 = 6.00 \text{ Sq. In.}$$

$$f_c = C_e \frac{N}{bh}$$

From diagram 4 on p.651, $C_e = 1.43$

$$\therefore f_c = \frac{1.43 \times 240281}{12 \times 50} = 573.10 \text{ P.S.I.}$$

The allowable is 575 P.S.I. and therefore this result is excellent.

QUARTER POINT:

$H = -\ 257957$ Lb.

$M_{1/4} = -\ 70446.8 \times 12 = -\ 845,364$ INCH-Lb.

$$N_{1/4} = \frac{H}{\cos\phi_x} = 257957 \times 1.03 = 265696 \text{ Lb.}$$

$$E = \frac{M_{1/4}}{N_{1/4}} = \frac{845364}{265696} = 3.18"$$

SECTION:

$b = 12"$; $h = 53.30"$; $d' = 2"$

$$\frac{2a}{h} = \frac{49.3}{53.3} = 0.905 \quad ; \quad \frac{e}{h} = \frac{3.18}{53.3} = 0.06$$

$$A_s = 0.01 \times 12 \times 12 \times 53.3 = 6.40 \text{ Sq. in.}$$

With this eccentricity there is no tension in the section.

$$f_c = C_e \frac{N}{bh}$$

From diagram 4 on p.651, $C_e = 1.145$

$$\therefore f_c \frac{1.145 \times 265696}{12 \times 53.3} = 476.10 \text{ P.S.I}$$

The allowable is 475 P.S.I. and therefore this result is excellent.

RESULTS

All stresses at all section are satisfactory. The assumed dimensions are good, and the bridge shall be built as it is designed without any necessary changes.

CHAPTER FOUR

DESIGN OF ABUTMENTS

The abutments lie on a rocky bed with a safe bearing power of 8 tons per square foot. They are subjected to the following forces:

(1) Horizontal thrust. This as found previously for the worst loading is - 236900 Lb.

(2) Negative bending moment of - 353182 ft.Lb.

(3) Vertical reaction of arch. This is the greater value as got either from the horizontal thrust,

$V_A = H_A \tan \phi$

or as got due to total dead load of the arch and fill plus full live load on the arch.

FIRST CASE: $V_A = H_A \tan \phi$; but $\frac{1}{\cos\phi} = 1.425$

$\therefore \phi = 44^\circ\ 45'$, and $\tan \phi = 0.9913$

$V_A = 236900 \times 0.9913 = 235000$ Lb.

SECOND CASE: From table 2,

V_A - D.L. = 9650 + 10400 + 11600 + 24600 + 3100 + 39950 + 51900 = 228200 Lb.

V_A - L.L. $\frac{1}{2}$ wl = 117 x 110 = 12870 Lb.

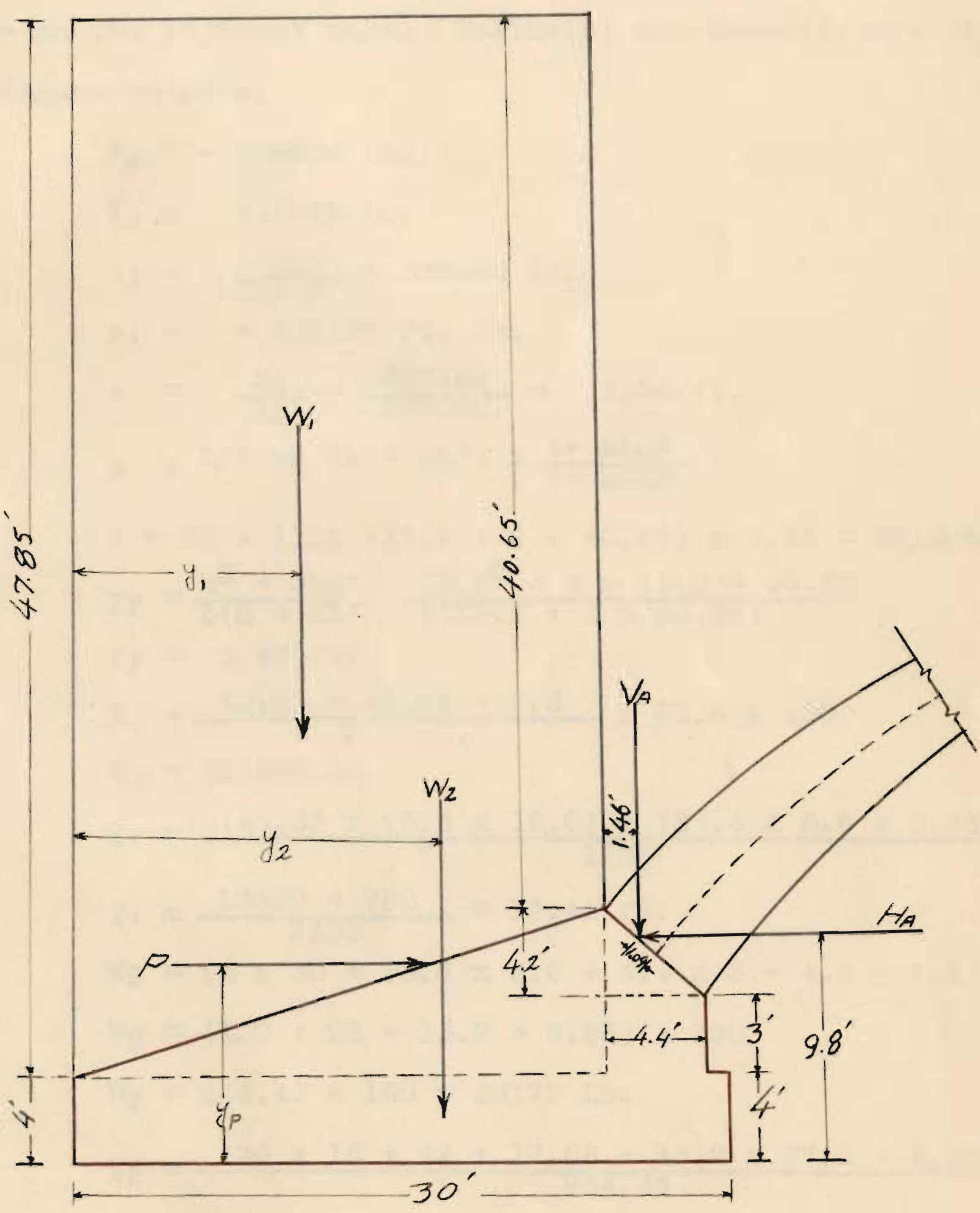
TOTAL V_A = 228200 + 12870 = 241070 Lb.

Use in design V_A = 241070 Lb.
H_A = 236900 Lb.

$\phi = 45^\circ$ approx.

(4) Resultant force due to fill and surcharge directly above the abutment. This force has both a horizontal and a vertical component.

LOAD LINE INCLUDING L. LOAD



ANALYSIS OF ABUTMENT

(5) Weight of the abutment itself.

SOLUTION:

Assume the dimensions of the abutment as shown in the figure on the adjacent page. Following the nomenclature of this same figure we have,

$$H_A = -\ 236900 \text{ Lb.}$$

$$V_A = 241070 \text{ Lb.}$$

$$R_A = \frac{H_A}{\cos \phi} = 338000 \text{ Lb.}$$

$$M_A = -\ 353182 \text{ ft. Lb.}$$

$$e = \frac{M_A}{R_A} = \frac{353182}{338000} = 1.04 \text{ ft.}$$

$$P = 1/2\ wh\ (h + 2h') \times \frac{1- \sin\phi}{1+ \sin\phi}$$

$$P = 50 \times 11.2\ (11.2 + 2 \times 40.65) \times 0.33 = 8310 \text{ Lb.}$$

$$y_P = \frac{h^2 + 3hh'}{3(h + 2h')}\quad \frac{11.2^2 + 3 \times 11.2 + 40.65}{3(11.2 + 2 \times 40.65)}$$

$$Y_P = 5.40 \text{ ft.}$$

$$W_1 = \frac{40.65 + 40.65 + 7.2}{2} \times 25.6 \times 100$$

$$W_1 = 113500 \text{ Lb.}$$

$$Y_1 = \frac{(40.65 \times 25.6 \times 12.8) + (25.6 \times 3.6 \times 8.53)}{1135}$$

$$Y_1 = \frac{13320 + 780}{1135} = 12.41 \text{ ft.}$$

$$W_2 = (4 \times 30 + 25.6 \times 3.6 + 4.4 \times 3 + 4.4 \times 2.1) \times 150$$

$$W_2 = (120 + 92 + 13.2 + 9.25) \times 150$$

$$W_2 = 234.45 \times 150 = 35170 \text{ Lb.}$$

$$Y_2 = \frac{120 \times 15 + 92 \times 17.06 + 13.2 \times 27.8 + 9.25 \times 27.07}{234.45}$$

$$Y_2 = \frac{3981}{234.45} = 17.00 \text{ ft.}$$

Now we know all the forces acting on the abutment and their points of application. Let us find the resultant of all of them "R" and the distance of its point of application $\bar{x}$ from the tow of the abutment A. This is done by summing up moments about A.

$\bar{x}$ = (113500 x 12.41 + 8310 x 5.4 + 35170 x 17 + 241070 x 27.06 - 236900 x 9.8) ÷ (113500 + 35170 + 241070)

$$\bar{x} = \frac{6253020}{389740} = 16.10 \text{ ft.}$$

<u>CRUSHING:</u>

$$P = \frac{P}{A} \pm \frac{Mc}{I}$$

$$P = \frac{389740}{30} \quad \frac{389740 \times 1.1 \times 15 \times 12}{30 \times 30 \times 30}$$

$$P = 12991 \pm 2859$$

∴ MAX. PRESSURE = 15850 Lb. per sq. ft.

∴ MIN. PRESSURE = 10132 Lb. per sq. ft.

These stresses are very satisfactory, the allowable being 8 tons per sq. fit.

<u>OVERTURNING</u>

$$F.S. = \frac{\text{Stabilizing moments}}{\text{Overturning moments}}$$

$$F.S. = \frac{8574640}{2321620} = 3.7 \quad \text{which is very safe.}$$

<u>SLIDING:</u>

$$F.S. = \frac{UN}{\Sigma H} = \frac{389740 \times 0.6}{236900 - 8310} = 1.02$$

The F.S., to be satsifactory, should be at least 1.5. In the present investigation, however, the abutment was considered as entirely exposed. The true fact is that the abutment should enter into the solid rock a minimum of 6 inches, and is then covered with earth for the remaining part of its depth. Thus if all this be considered and foreseen, we can easily say that the abutment is safe against sliding.

<u>RESULTS:</u>

The assumed dimensions are good and are adopted without any change.

CHAPTER FIVE

DESIGN OF SPANDREL WALLS

The spandrel walls vary considerably in height depending upon their location on the arch rib. At the springing the wall is 36 ft. high, and since the ordinary cantilever type is exceedingly uneconomical at such big heights, I shall build a certain length of the walls to be of the counterfort type. In this type, the main wall is designed as a slab spanning horizontally and continuously over counterforts. The loading from the earth thrust with surcharge, will be uniform at any given depth. The counterfort is designed to take the cantilever moment from the earth thrust for any full bay, the wall slab assisting it in the nature of a flange to a T - beam section. Ample horizontal links should be used to tie the slab to the counterfort, and the main tension steel, as much as requires to be carried to the top of the counterfort, should be well anchored there.

The length of the walls to be built of the counterfort type, is 30 ft. measured from every springing point along a horizontal axis towards the center. The spacing of the counterforts shall be 10 ft. center to center and their thickness 24 inches for the deeper sections and 18 inches for the shallower ones as indicated in the design to follow. Thus there shall be four walls on each side of one half of the arch. These four walls have heights of 36,30,23, and 18 ft respectively including backfill and surcharge reduced to backfill - the unit volume weight being 100 Lb. per cubic ft.

The remaining 87 ft. length of the spandral walls on each half of the arch, shall be built of the ordinary cantilever type. Taking the left half of the arch and with center of co-ordinates at the left springing, there shall be:

A wall 18' high from x = 30' to x = 40' ;

A wall 14' high from x = 40' to x = 60' ;

A wall 8' high from x = 60' to x = 80' ;

A wall 5' high from x = 80' to x = 117';

The allowable unit stresses are as follows:

f_s = tensil stress = 16000 P.S.I. ; f_c = 650 P.S.I.

U = bond stress = 100 P.S.I. ;

V = Shearing stress = 40 P.S.I. ;

N = 15 ; K = 108

<u>VERTICAL SLAB :</u> The vertical slab is designed as a simple slab supported by the counterforts. The thickness of the slab required is governed by the pressure on the top of the base slab which, in this case, is the arch ring.

(1) Taking a horizontal strip one ft. high and 36 ft. from the top of the wall, the pressure per linear foot is :

$$P = wx \frac{1 - \sin\phi}{1 + \sin\phi} \quad ; \quad (\phi = 33^\circ 42')$$

$$P = 100 \times 36 \times 0.286 = 1030 \text{ Lb. per lin.ft.}$$

The max. bending moment of the strip assuming the length as the clear distance between counterforts, is

$$M = \frac{1}{8} wl^2 = 1/8 \times 1030 \times 8.5^2 \times 12 = 112000 \text{ in.Lb.}$$

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{112000}{108 \times 12}} = 9.3''$$

V = 1/2 x 1030 x 8.5 = 4375 Lb.

$$d = \frac{V}{vjb} = \frac{4375}{40 \times 0.88 \times 12} = 10.2''$$

Use effective d = 11.00"

plus 2" insulation, t = 13.00"

$$A_s = \frac{M}{f_s jd} = \frac{112000}{16000 \times 0.88 \times 11} = 0.72 \text{ in}^2 \text{ per ft.}$$

5/8" ∅ bars at 12 ÷ $\frac{0.72}{0.3068}$ = 5.1, say 5" center to center are selected.

$$U = \frac{V}{\Sigma_o jd} = \frac{4375}{1.964 \times 12/5 \times 0.88 \times 11} = 91 \text{ P.S.I.}$$

The allowable is 100 P.S.I.

(2) Taking a horizontal strip 1 ft. high, and 30 ft. below top of wall we have,

P = 100 x 30 x 0.286 = 858 Lb. per lin.ft.

$M = 1/8 \times 858 \times 8.5^2 \times 12 = 93000$ in. Lb.

$$A_s = \frac{93000}{16000 \times 0.88 \times 11} = 0.60 \text{ sq. in. per ft.}$$

5/8"∅bars at 12 ÷ $\frac{0.60}{0.3068}$ = 6.1", say 6" center to center are selected.

(3) Taking a horizontal strip 1 ft. high and 24 ft. felow top of wall we have,

P = 100 x 24 x 0.286 = 686 Lb. per lin.ft.

$M = 1/8 \times 686 \times 8.5^2 \times 12 = 74400$ in. lb.

$$A_s = \frac{74400}{16000 \times 0.88 \times 11} = 0.48 \text{ sq. in. per ft.}$$

1/2" ∅ bars at 12 ÷ $\frac{0.48}{0.1967}$ = 4.9" say 4.5" center to center are selected.

(4) Taking a horizontal strip 1 ft. high and 18 ft. below top of wall we have,

$$P = 100 \times 18 \times 0.286 = 513 \text{ Lb. per lin. ft.}$$

$$M = 1/8 \times 513 \times 8.5^2 \times 12 = 55800 \text{ in.lb.}$$

$$A_s = \frac{55800}{16000 \times 0.88 \times 11} = 0.36 \text{ sq. in. per ft.}$$

1/2" ∅ bars at 12 ÷ $\frac{0.36}{0.1964}$ = 6.5" say 6" center to center are selected.

(5) Taking a horizontal strip 1 ft. high and 12 ft below top of wall we have,

$$P = 100 \times 12 \times 0.286 = 343 \text{ lb. per lin. ft.}$$

$$M = 1/8 \times 343 \times 8.5^2 \times 12 = 37100 \text{ in.Lb.}$$

$$A_s = \frac{37100}{16000 \times 0.88 \times 11} = 0.25 \text{ sq. in per ft.}$$

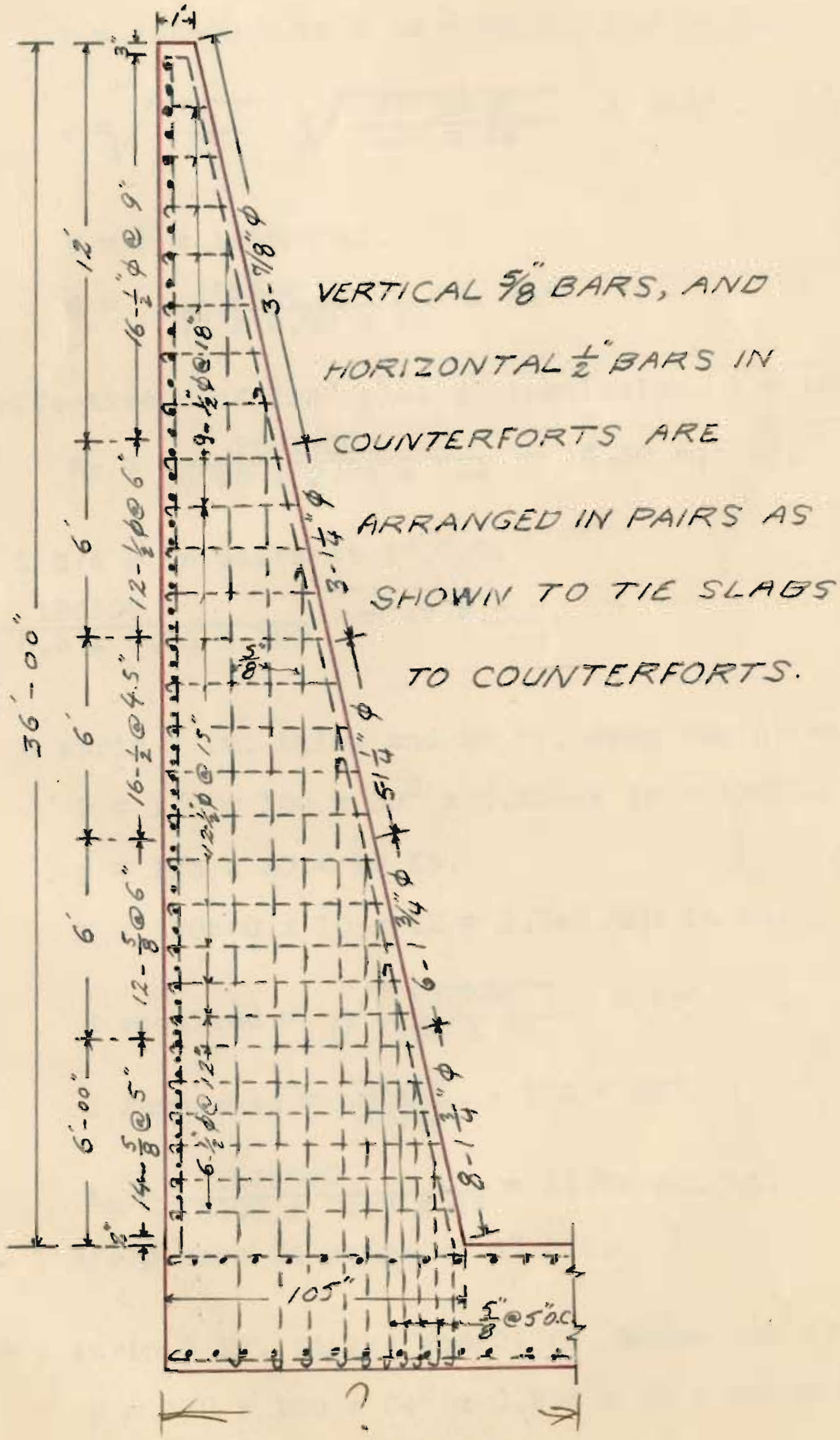
1/2" ∅ bars at 12 ÷ $\frac{0.25}{0.1946}$ = 9.3" say 9" center to center are selected. These are continued to the top of the wall.

The reinforcement of the slab in the vertical direction shall consist of 3/8" ∅ bars at 10 inches intervals.

COUNTERFORTS:

The moment in the counterforts is due to the pressure on the vertical slab over a length of wall equal to the distance center to center of counterforts.

$$P = 1/2 \; wh^2 \frac{1-\sin\phi}{1+\sin\phi} \times 1$$



DETAILS OF COUNTERFORT WALL

$P = 1/2 \times 100 \times 36^2 \times 0.286 \times 10 = 185500$ Lb.

$y = 1/3 \times 36 = 12$ ft.

$M = 185500 \times 12 \times 12 = 26,712,000$ in.Lb.

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{26712000}{108 \times 24}} = 102''$$

$V = P = 185500$ Lb.

$$d = \frac{185500}{40 \times 0.88 \times 24} = 68''$$

Use an effective d of 102" plus 3" insulation, t = 105"

$$A_s = \frac{26812000}{16000 \times 0.88 \times 102} = 18.60 \text{ sq. in.}$$

Use 8 - 1 3/4 Ø in pairs at 5" O.C.

$$U = \frac{185500}{8 \times 5.5 \times 0.88 \times 102} = 47 \text{ P.S.I.}$$

(2) For a strip 2 ft. thick and 30 ft. from top of wall we have,

$P = 1/2 \times 100 \times 30^2 \times 0.286 \times 10 = 128500$ Lb.

$y = 1/3 \times 30 = 10$ ft.

$M = 128500 \times 10 \times 12 = 1,540,000$ in Lb.

$$d \text{ required} = \sqrt{\frac{1540000}{108 \times 24}} = 77''$$

$$d \text{ furnished} = \frac{34.25}{40.25} \times 102 = 86''$$

$$A_s = \frac{1540000}{16000 \times 0.88 \times 86} = 12.75 \text{ sq. in.}$$

Use 6 - 1 3/4 Ø bars in pairs at 6" O.C.

(3) For a strip 2 ft. thick, and 24 ft. below top of wall we have,

$P = 1/2 \times 100 \times 24^2 \times 0.286 \times 10 = 82500$ Lb.

$y = 1/3 \times 24 = 8$ ft.

$$M = 82500 \times 8 \times 12 = 7{,}930{,}000 \text{ in.Lb.}$$

$$d \text{ required} = \sqrt{\frac{7930000}{108 \times 24}} = 55''$$

$$d \text{ supplied} = \frac{28.25}{40.25} \times 102 = 71''$$

$$A_s = \frac{7930000}{16000 \times 0.88 \times 71} = 7.9 \text{ sq. in.}$$

Use 5 - 1 1/4" Ø bars at 6" O.C.

(4) For a strip 18" thick and 18 ft. below top of wall we have,

$$P = 1/2 \times 100 \times 18^2 \times 0.286 \times 10 = 46250 \text{ Lb.}$$

$$y = 1/3 \times 18 = 6 \text{ ft.}$$

$$M = 46250 \times 6 \times 12 = 3{,}320{,}000 \text{ in.Lb.}$$

$$d \text{ required} = \sqrt{\frac{3320000}{108 \times 18}} = 41.5''$$

$$d \text{ furnished} = \frac{22.25}{40.25} \times 102 = 57''$$

$$A_s = \frac{3320000}{16000 \times 0.88 \times 57} = 4.1 \text{ sq. IN.}$$

Use 3 - 1 1/4" Ø bars at 4.5" O.C.

(5) For a strip 18" and 12 ft. below top of wall,

$$P = 1/2 \times 100 \times 12^2 \times 0.286 \times 10 = 20600 \text{ Lb.}$$

$$y = 1/3 \times 12 = 4 \text{ ft.}$$

$$M = 20600 \times 4 \times 12 = 990{,}000 \text{ ft. Lb.}$$

$$d \text{ required} = \sqrt{\frac{990000}{108 \times 18}} = 22.6''$$

$$d \text{ furnished} = \frac{18.25}{40.25} \times 102 = 45 \text{ INS.}$$

$$A_s = \frac{990000}{16000 \times 0.88 \times 45} = 1.75 \text{ sq. in.}$$

Use 3 - 7/8" ∅ bars at 4.5" O.C.

Two of these are continued to top of wall beyond the section which is at 6 ft. below top of wall.

DESIGN OF CANTILEVER WALLS:

The following is the design of an 18 ft. high cantilever wall, with investigations at sections which are at 12, 8 and 5 ft. below top of wall. The design as a whole enables us t us to build the total length of the spandrel walls to be of the cantilever type.

(1) For a strip 18 ft. below the top of the wall,

$$p = 1/2\ wh^2\ \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$P = 1/2 \times 100 \times 18^2 \times 0.286 = 4625 \text{ Lb.}$$

$$y = 1/3\ h = 6 \text{ ft.}$$

$$M = 4625 \times 6 \times 12 = 332000 \text{ in. Lb.}$$

$$d = \sqrt{\frac{332000}{108 \times 12}} = 16"$$

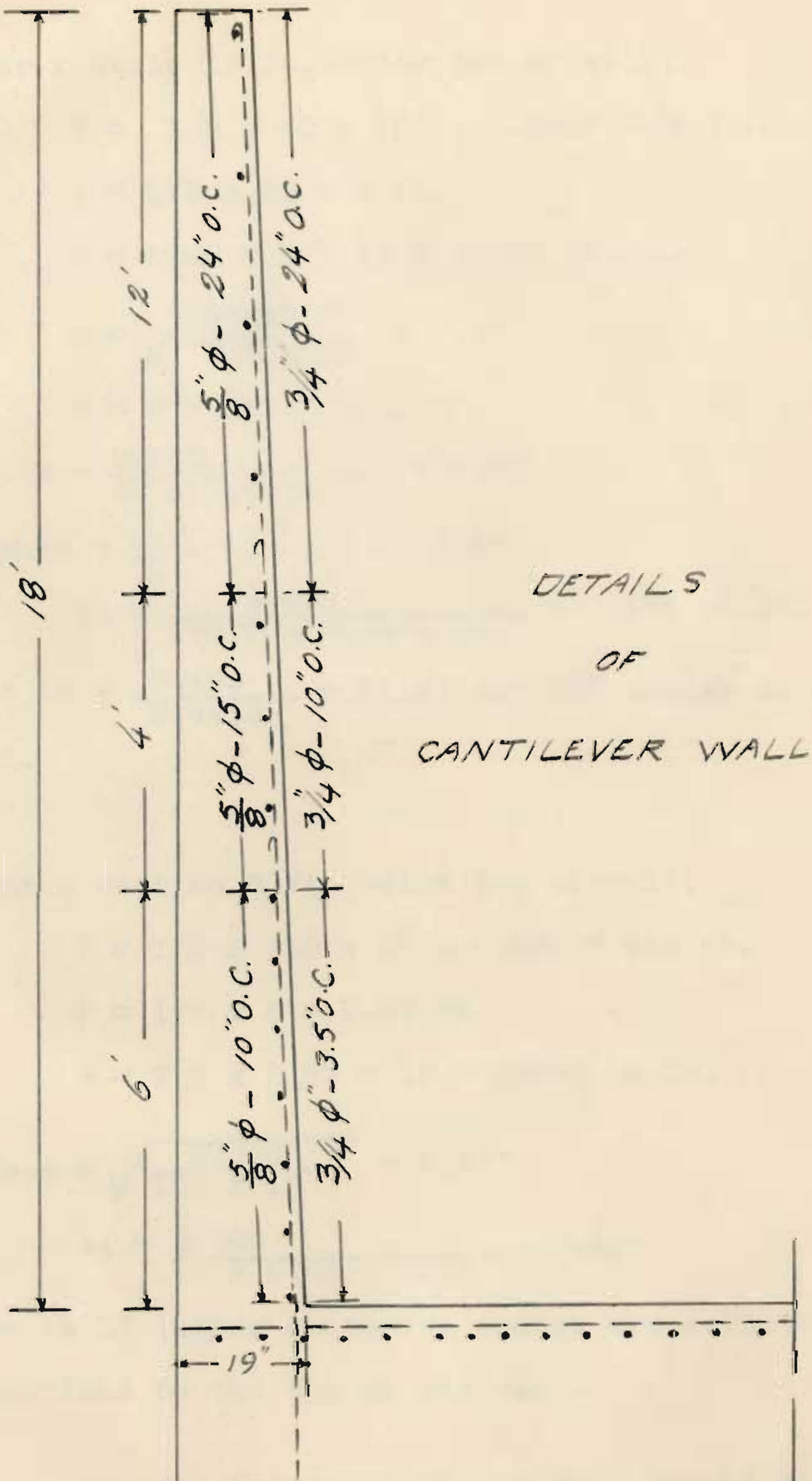
$$V = P = 4625 \text{ Lb.}$$

$$d = \frac{V}{vjb} = \frac{4625}{40 \times 0.88 \times 12} = 11"$$

Use an effective d of 16" plus 3" insulation, t = 19"

$$A_s = \frac{332000}{16000 \times 0.88 \times 16} = 1.48 \text{ sq. in.}$$

3/4" ∅ bars at $12 \div \frac{1.48}{0.4418} = 3.6$ say 3.5" center to center are selected.



DETAILS OF CANTILEVER WALL

(2) For a strip 12 ft. below top of wall,

$$P = 1/2 \times 50 \times 12^2 \times 0.86 = 2060 \text{ Lb.}$$

$$y = 1/3 \times 12 = 4 \text{ ft.}$$

$$M = 2060 \times 4 \times 12 = 99000 \text{ in.Lb.}$$

$$d = \sqrt{\frac{99000}{108 \times 12}} = 8.7"$$

$$V = P = 2060 \text{ Lb.}$$

$$d = \frac{20600}{40 \times 0.88 \times 12} = 4.8"$$

d furnished = 19 - 7/3 - 3 = 13.7"

$$As = \frac{99000}{16000 \times 0.88 \times 13.7} = 0.465 \text{ sq.in.}$$

3/4 ∅ at 12 $\div \frac{0.465}{0.4418} = 11.3"$ say 10" center to center are selected.

(3) For a section 8 ft. below top of wall,

$$P = 1/2 \times 100 \times 8^2 \times 0.286 = 918 \text{ Lb.}$$

$$y = 1/3 \times 8 = 2.67 \text{ ft.}$$

$$M = 918 \times 2.67 \times 12 = 29300 \text{ in.Lb.}$$

$$d \text{ required} = \sqrt{\frac{29300}{108 \times 12}} = 2.17"$$

$$A_s = \frac{29300}{16 \times 0.88 \times 12.1} = 0.172"$$

3/4 bars at 18 inches center to center are selected. These bars shall continue to the top of the wall.

The reinforcement of the wall in the horizontal section shall consist of 3/8" ∅ at spacings as shown in the details drawing of the wall.

B I B L I O G R A P H Y

PRIMARY SOURCES:

1. "Concrete, Plain and Reinforced", by Taylor, Thompson, and Smulski. (vol. I & II)

2. "Design of Concrete Structures", by Urquhart and O'Rourke.

SECONDARY SOURCES:

1. "Reinforced Concrete Construction", by Hool, (vol. II & III)

2. "Reinforced Concrete Bridge Design", by Chettoe and Adams

3. "Concrete Engineers Handbook", by Hool and Johnson.

4. "Reinforced Concrete Design" , by Sutherland and Clifford.