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TRAFFIC MANAGEMENT CONSIDERING CONGESTION-BASED FLOW REDUCTIONS

by
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Abstract of the Thesis of

Julien Joseph Semaan for Master of Engineering Management
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The cell transmission model (CTM) is widely used in studying optimal traffic management strategies. Since the CTM does not properly model the effect of congestion on traffic flows, a generalized linear model accounting for congestion-based flow reductions is considered. Therefore, the different linearized frameworks will be studied and discussed with their respective properties, as well as the traffic management tools (TMTs) used by these linear programs to obtain optimal solutions. Out of these five TMTs, three rely on traffic holding which is the result of linearizing the CTM model. Because traffic holding is an undesired phenomenon, we discuss a heuristic that eliminates traffic holding on ordinary links, where traffic holding is most unrealistic. Finally, through two numerical examples, we find that strategies under the (realistic) setting of congestion-based flow reduction (produced using the generalized CTM) can differ greatly from those produced using the CTM framework that predominates in the literature, and that the heuristic proposed is an effective tool that yields realistic optimal solutions.

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Abbreviations

EAF	Earliest Arrival Flow
CTM	Cell Transmission Model
DTA	Dynamic Traffic Assignments
LP	Linear Programming
NCT	Network Clearance Time
SO-DTA	System Optimal-Dynamic Traffic Assignment
TMT	Traffic Management Tools
TST	Total System Time

Chapter 1

Introduction

The cell transmission model by Daganzo [10, 9] is a very effective tool to study traffic flows through a certain network over time. The model helps in analyzing macroscopic traffic streams and in visualizing the relation between traffic density and the characteristics of the network. Linearizing the CTM by Ziliaskopoulos [36] certified the use of the model in producing system optimal traffic flows, that is, flows that minimize the sum of time that each vehicle spends in the system before reaching the sink cell. This framework which solves for system optimal traffic flows became widely used in dynamic traffic assignments (DTA) (e.g. Waller and Ziliaskopoulos [31]) and in planning for evacuation strategies (e.g. Bish, Sherali, and Hobeika [4] and Bish and Sherali [3]).

To solve for system optimal-dynamic traffic assignment (SO-DTA) solutions, a controllable and realistic linear model should be used. The linearized model by Ziliaskopoulos [36] considers five traffic management tools to obtain an optimum solution. All five of these tools, however, have various levels of control, difficulty in implementation, and can be used in a manner that does not "make sense," as we discuss in this paper. Two of the tools are implicit in the original CTM [10, 9], and the other three tools are based on traffic holding, which is the result of the linearization of the CTM for the LP framework. Linearizing the CTM required the relaxation of some non-linear equations accounting for less complexity in the problem formulation. This relaxation, however, caused an unrealistic phenomenon known as traffic holding, whereby traffic flow is held at an upstream cell even though there is enough capacity in the downstream cell to advance the traffic forward. Solving this issue has gained wide interest in the literature; however, most of the proposed approaches consider the original CTM, which does not account for congestion-based flow reductions.

Traffic flow usually increases with density (i.e., the free-flow state) until a critical point is reached, after which flow starts to decrease (i.e., congested state), reaching a value of zero at jam density [21]. This reduction in flow due to congestion is an important feature of traffic systems and has been observed in various empirical studies [1, 12, 6, 8]. The original CTM model is capable of displaying this effect of congestion on traffic, where a backward shockwave at the inflow of a congested cell is demonstrated by a reduction in traffic flow. The original CTM, however, does not consider any effect on traffic discharge from a congested cell. A reduction in traffic outflow, especially at bottlenecks, is critical in modeling realistic optimum traffic solutions. A reduction could have severe effects on evacuation strategies knowing that the various bottlenecks in a roadway network are one of the most important network features for determining the performance of the system. This reduction in traffic discharge is due to a delay in a vehicular acceleration when the traffic is at a congested state and is discussed in several researches [1, 12, 6, 8]. Bish, Chamberlayne, and Rakha [2] studied congestion-based traffic flow reductions and proposed a generalized CTM that better models the impact of congestion on traffic discharge at bottlenecks.

We make the following contributions in this paper: 1) we define five traffic management tools used by the LP framework that considers the generalized CTM; 2) we provide structural properties of the optimal solutions from the LP framework that uses the generalized CTM; 3) we use these tools to analyze the strategy from the LP framework with and without congestion-based flow reductions and show that the strategies under congestion-based flow reductions are often better structured; 4) we show that eliminating the unrealistic traffic holding management tools is more difficult under the congestion-based flow reduction setting and provide a heuristic that eliminates traffic holding at ordinary links; and 5) we provide a numerical analysis on a large network from the literature.

The remainder of the paper is structured as follows: Section 2 discusses the literature of dynamic traffic assignment and presents various solutions proposed in the literature to solve traffic holding. Section 3 reviews the CTM proposed by Daganzo [10, 9], as well as a generalization presented in Bish, Chamberlayne, and Rakha [2], and discusses the Linear Programming implementation proposed by Ziliaskopoulos [36]. Section 4 describes the traffic management tools that are used by these models to obtain optimal solutions. Section 5 provides some important structural properties of the optimal solutions. In Section 6, we propose a heuristic that eliminates the unrealistic traffic management tool, traffic holding, at ordinary links. Section 8 illustrates the previous sections using two numerical examples. Finally, section 9 presents the research conclusions.

Chapter 2

Literature Review

With the increase in urban growth and its relative impact on traffic congestion, traffic management models have become crucial to forecast realistic traffic demands. They offer the ability to capture traffic realism and its dependency on human behavior. Dynamic traffic assignment approaches have become an effective tool in modeling traffic flow and have substantially evolved, especially after the work of Merchant and Nemhauser [22, 23], who proposed a mathematical nonlinear discrete-time model for single destination DTA problems. Several methodological DTA approaches have been used in the past years and can be divided into four categories: optimal control, mathematical programming, variational inequality, and simulation-based [26].

The reliability and effectiveness of the DTA depend on two factors, the travel choice principle and the traffic-flow component [29]. The travel choice principle considers the traveler's choice, which is dependent on the travel time required by each path, and the traveler's departure time and destination. On the other hand, the traffic-flow component describes the performance of the network under consideration with respect to time and is represented as a series of constraints. Being reliant on time, traffic patterns vary across different time frames along the network, and what makes DTA diverse from static traffic assignments is its ability to deal with time-varying flows [26]. DTA has been widely used in many areas of transportation systems and problems including advanced traveller information systems (ATIS), advanced traffic management system's (ATMS), telecommunication and computer science [14], and evacuation planning.

Planning for evacuation strategies is one main purpose for utilizing and studying DTA models, especially the Cell Transmission model. The effectiveness of the DTA models in solving for realistic strategies that could be implemented when natural or man-caused disasters occur has been widely examined in the literature. Yan, Liu, and Song [32] argue that models usually focus on minimizing the total time needed to evacuate all vehicles from the network without taking into consideration social fairness. That is, people in areas with the highest risk might be required to sacrifice their evacuation priority in response to improving the system's efficiency. Therefore, Yan, Liu, and Song [32] proposed an extended cell transmission model with a weight function that considers a risk evaluation index as a variable and an emphasis degree of managers on the social fairness principle as a coefficient. On the other hand, [15] consider human behavior in an emergency evacuation by presenting an extended CTM that includes parameters accounting for three characteristics of human behavior: the inertial effect, the unadventurous effect, and panic psychology. Studying the effect of the parameters on the model, an emergency evacuation scenario of a crowd in a supermarket was implemented and it was determined that the parameters considered had a great influence on the evacuation strategy.

Therefore, since it was first suggested by Daganzo [10, 9], the CTM has been widely studied and examined in the literature, with various modifications to the model being proposed. Liu et al. [17] argue that the CTM does not capture the effect of moving bottlenecks, that is, buses travelling in the network. Because moving bottlenecks have great implications on the strategies suggested by the model, Liu et al. [17] provide an analytical formulation which accounts for a mixed traffic system that includes busses and cars. Modifying the CTM, free-flow speed differentials between buses and cars, as well as, capacity reduction caused by buses were incorporated in the CTM resulting in more realistic traffic strategies. Furthermore, Carey [5] proposes a CTM that accounts for varying free-flow speeds. Carey [5] explains that the cell's length in the CTM network is determined by considering the distance covered during one time interval at a specific free-flow speed, and that this length is held constant over time. Carey [5] suggests, however, that the free-flow speed varies with time, depending on factors such as time of day, traffic type or traffic lane, and speed limits that vary over time or space. He proposes that, at cells with a free-flow speed less than that used to determine the length of the cell, traffic should move forward at their free-flow speeds and not at the speed determined by the CTM solution.

Another issue with the CTM is traffic holding. Traffic holding in system optimal dynamic traffic assignments (SO-DTA) has gained wide interest in literature over the past years, as researchers have propositioned various approaches to solve that undesirable phenomenon. Linearizing the Cell Transmission Model, Ziliaskopoulos [36] justifies traffic holding as a tool applied by the LP to optimize the flow and discusses the use of a penalty to eliminate traffic holding. The suggested penalty is based on subtracting a weighted expression from the objective function. Lin and Wang [16] also penalize traffic holding by formulating a lexicographic objective function, which stimulates the advancement of flow. This approach by Lin and Wang [16] will be further elaborated, as it will be used in the proposed heuristic. Similarly, Zhu and Ukkusuri [35] applied a cell-dependent penalty on the occupancy of the cells, with the penalty decreasing as the network moves from the source cell to the sink cell. Such formulation provides a linear approach to remove the traffic holding by giving an incentive for the LP to push traffic forward towards the cell with the lower penalty.

On the other hand, Zheng and Chiu [34] formulated an algorithm that solves for the earliest arrival flow on a time expanded network. The algorithm proposed eliminates traffic holding on ordinary and merging links for single-destination networks. Mixed-integer programs have also been widely used by researchers [25, 33, 20, 2]; however, the use of binary variables was proven to be highly inefficient, especially for large networks, because of the high number of binary variables existing in the formulation.

Chapter 3

The Modeling Framework

In this section, we review the cell transmission model by Daganzo [10, 9], including a generalization (see, Bish, Chamberlayne, and Rakha [2]) that allows a more realistic response to congestion, that is, a reduction in flow discharge from a congested cell. We then review a linear programming implementation of the CTM (Ziliaskopoulos, 2000), modified for the generalized CTM.

The CTM utilizes a discrete time-expanded network of cells and links (C, L) to represent the roadway system of interest. Cells can be either source cells (set S_o), sink cells (set S_e), or roadway cells (set R). In this paper, we study a single commodity flow problem, and thus if there are multiple sinks, the model determines the flow sent to each sink. Links represent allowable movements between cells. When a roadway cell has two incoming links, these links are merge links (L_m); likewise, when a roadway cell has two outgoing links, these links are diverge links (L_d). We use the convention of calling a roadway cell having incoming merge links a merge cell, and a roadway cell having outgoing diverge links a diverge cell. All other links are ordinary links (L_o); an ordinary link is the only outgoing and incoming link for two adjacent cells. Each source cell has only one outgoing link, while each sink cell has only one incoming link.

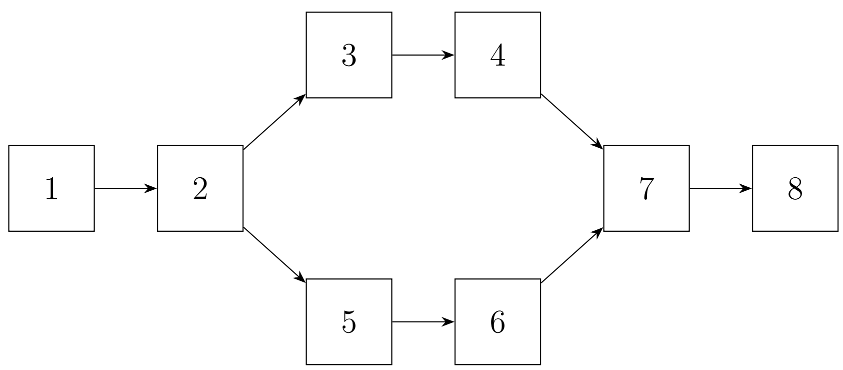


Figure 3.1: A small network example to illustrate the network components used in the CTM

Figure 3.1 illustrates these network components; cell 1 is a source cell, cell 8 is a sink cell, and cells 2-7 are roadway cells; likewise, links (4,7) and (6,7) are merge links, links (2,3) and (2,5) are diverge links, and links (1,2), (3,4), (5,6) and (7,8) are ordinary links. The planning horizon is divided into T time intervals of length τ, and a roadway cell represents a section of a roadway of length ℓ such that vehicles traveling at free-flow speed (u_f) traverse the section in one time interval; that is $\ell = u_f \times \tau$. Additional notation follows:

Decision Variables:

x_i^t : number of vehicles in cell i at the beginning of time interval t, $\forall i \in C, t = 1, ..., T$

y_{ij}^t : number of vehicles flowing from cell i to cell j during time interval t, $\forall (i, j) \in L, t = 1, ..., T$

Parameters:

N_i : maximum number of vehicles that cell i can hold, which is related to the concept of jam density, $\forall i \in R$

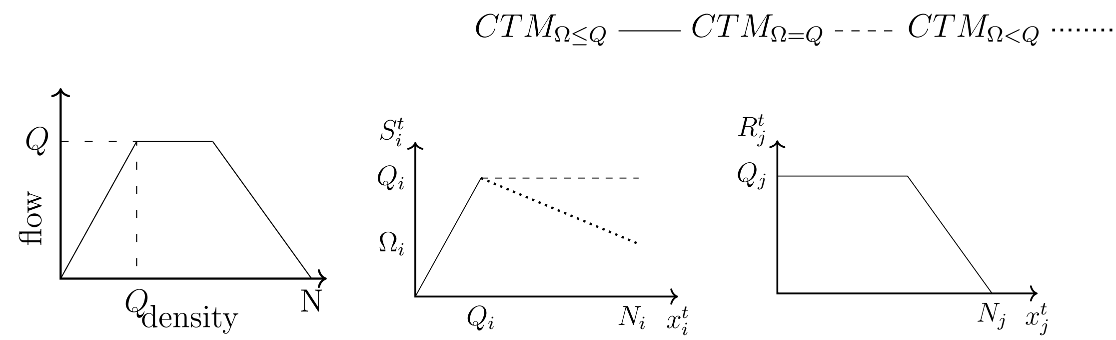
Q_i : maximum attainable flow into or out of cell i, $\forall i \in R$

Ω_i : maximum allowable flow out of cell i when ($x_i^t = N_i$), $\forall i \in R$

δ_i : traffic flow parameter for cell i, $\forall i \in R$

d_i^t : number of vehicles flowing out of source cell i during time interval t, $\forall i \in S_o, t = 1, ..., T$

The parameters N_i, Q_i, Ω_i, and δ_i will be considered to be independent of time; however, it is easy to include time-varying parameters. These parameters have the following relationships for any roadway cell i: $0 < \delta_i \leq 1$, $Q_i \leq N_i$, $0 < \Omega_i \leq Q_i$.



(a) (b) (c)

Figure 3.2: (a) a trapezoidal flow-density diagram and the CTM diagram for (b) S_i^t and (c) R_j^t for $CTM_{\Omega=Q}$ and $CTM_{\Omega<Q}$.

Figure 3.2 shows that as the density of cell i increases, the flow into cell i increases as well (free-flow state, where $x_i^t \leq Q_i$), until the critical point is reached ($x_i^t = Q_i$), after which cell i becomes at the congested state. At the congested state ($x_i^t > Q_i$), the original CTM considers the flow out of cell i to be equal to Q_i; however, under the generalized CTM, congestion-based flow reductions are considered where the flow out of cell i becomes equal to $Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i)$. This relationship is enforced over a link using S_i^t, the maximum flow from the sending or upstream cell, as defined in Eq. (3.1). We note that if $Q_i = \Omega_i$, Eq. (3.1) simplifies to $S_i^t = \min\{x_i^t, Q_i\}$, which is the expression used in the original CTM (see Daganzo [10, 9]. Henceforth, we refer to the original CTM as $CTM_{\Omega=Q}$, the generalized CTM as $CTM_{\Omega\leq Q}$, and the CTM with congestion-based flow reductions as $CTM_{\Omega<Q}$. On the other hand, the flow into the receiving cell j is limited by Q_j when cell j is in the free-flow state ($x_j^t \leq Q_j$) and by $\delta_j(N_j - x_j^t)$ when cell j is in the congested state ($x_j^t > Q_j$). This relationship is represented by Eq. (3.2), where R_j^t is the maximum flow into the receiving or downstream cell .

$$S_i^t = \min\{x_i^t, Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i)\} \tag{3.1}$$
$$R_j^t = \min\{Q_j, \delta_j(N_j - x_j^t)\} \tag{3.2}$$

When cell i is in the congested state ($x_i > Q_i$), $S_i^t = Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i)$ which is equal to Q_i when $x_i^t = Q_i$ (critical point), and decreases to Ω_i when cell i reaches jam density ($x_i^t = N_i$). Likewise, if cell j is in the free-flow state, the flow into j should only be limited by $\min\{S_i^t,\ Q_j\}$, that is, in the free-flow state $Q_j \leq \delta_j(N_j - x_j^t)$, else traffic cannot flow freely (another restriction on the relationship between $Q_i,\ N_i, and\ \delta_i$).

Knowing that the flow out of the upstream cell cannot exceed the flow into the downstream cell, the following equations are used, accounting for the link type. The flow over an ordinary link (i, j) during time interval t is determined by Eq. (3.3). The flow over two merge links (i, k) and (j, k) must adhere to Eq. (3.4); when $S_i^t + S_j^t > R_k^t$, Eq. (3.4) allows the model to determine merge priorities for the flows on these links. The flow over two diverge links (i, j) and (i, k) must adhere to Eq. (3.5).

$$\begin{aligned} y_{ij}^t &= \min\{S_i^t, R_j^t\} \\ &= \min\{x_i^t, Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i), Q_j, \delta_j(N_j - x_j^t)\} \end{aligned} \tag{3.3}$$
$$y_{ik}^t + y_{jk}^t = \min\{S_i^t + S_j^t,\ R_k^t\} \tag{3.4}$$
$$y_{ij}^t + y_{ik}^t \leq \min\{S_i^t,\ R_j^t + R_k^t\} \wedge (y_{ij}^t = R_j^t \vee y_{ik}^t = R_k^t \vee y_{ij}^t + y_{ik}^t = S_i^t) \tag{3.5}$$

The original CTM was linearized by Ziliaskopoulos [36], allowing the usage of the CTM in finding optimal traffic flows. The LP formulated minimizes the total time that vehicles are in the system (i.e., before they reach a sink node) for single commodity flows (i.e., for networks having a single sink or multiple sinks where flows are not specified for a particular sink). The following notations for the LP will be used throughout the paper, where $LP_{\Omega=Q}$, $LP_{\Omega\leq Q}$, and $LP_{\Omega<Q}$ refer to $CTM_{\Omega=Q}$, $CTM_{\Omega\leq Q}$, and $CTM_{\Omega<Q}$ respectively. The linearization by Ziliaskopoulos [36] will be expanded, as follows, to accommodate for the generalized $CTM_{\Omega\leq Q}$:

$$\text{Minmize} \sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t \tag{3.6}$$

subject to:

$$x_j^t = x_j^{t-1} + \sum_{i:(i,j)\in L} y_{ij}^{t-1} - \sum_{k:(j,k)\in L} y_{jk}^{t-1}, \quad \forall j \in C/S_e,\, t = 2, ..., T \tag{3.7}$$

$$\sum_{j:(i,j)\in L} y_{ij}^t \leq x_i^t, \quad \forall i \in C,\, t = 1, ..., T \tag{3.8}$$

$$\sum_{j:(i,j)\in L} y_{ij}^t \leq Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i), \quad \forall i \in R,\, t = 1, ..., T \tag{3.9}$$

$$\sum_{i:(i,j)\in L} y_{ij}^t \leq Q_j, \quad \forall j \in R,\, t = 1, ..., T \tag{3.10}$$

$$\sum_{j:(j,i)\in L} y_{ij}^t \leq \delta_j(N_j - x_j^t), \quad \forall i \in R,\, t = 1, ..., T \tag{3.11}$$

The objective function, Eq. 3.6, minimizes the total system time (TST), that is, the sum of the total time each vehicle spends in the network before reaching a sink cell. Constraint 3.7 is the flow conservation constraint. In this constraint, x_i^1 is the total demand at source cell $i \in S_o$ or the initial number of vehicles on roadway $i \in R$. We note that constraint 3.7 can be easily modified to account for a time variant demand (d_i^t) at the source cell; however, accounting for this time variant parameter, an additional constraint should be added for each source cell $i \in S_o$, to ensure that flow is equivalent to Q_j, where $j : (i, j) \in L$ (source cells have only one outgoing link). This additional constraint eliminates any possible staging of vehicles (see Bish and Sherali [3] and Bish, Chamberlayne, and Rakha [2]). On the other hand, constraints 3.8 and 3.9 are the limits for the flow out of the upstream cell i as specified by Eq. (3.1), while constraints 3.10 and 3.11 are the limits for the flow into the downstream cell j as specified by Eq. (3.2). Finally, the logical non-negativity constraints on the x and y variables are required.

Chapter 4

Traffic Management Tools

For the $LP_{\Omega \leq Q}$ framework , we describe the solution strategy in terms of five traffic management tools that are used to control the traffic flows.

Under the CTM, flows should be governed by Eq. (3.3)-(3.5). These equations form a non-convex feasible solution; thus, they were linearized into constraints 3.8, 3.9, 3.10, and 3.11. This linearization could result in a flow value, over link $(i,\ j)$, which is less than all the limits imposed by those 4 constraints aforementioned instead of setting the flow equal to their minimum. In other words, traffic will be held in the upstream cell i, although there is enough capacity to move that traffic forward into the downstream cell j. This phenomenon is known as traffic holding [36] and is used by the LP as a tool to optimize the traffic flow solution. Given this, we describe the five traffic management tools (TMTs) that can be used by the LP to minimize the objective function.

TMT 1: Prioritizing at merge links. Considering merge links $(i,\ k)$ and $(j,\ k)$, when flows y^t_{ik} and y^t_{jk} are constrained be Eq. (3.4), and R^t_k is the minimizer, the model has the freedom to prioritize flows into cell k.

TMT 2: Routing at diverge links. Considering diverge links $(i,\ j)$ and $(i,\ k)$, when flows y^t_{ij} and y^t_{ik} are constrained be Eq. (3.5), the model has the freedom to route flows into cells j and k.

TMT 3: Traffic holding at merge links. Considering merge links $(i,\ k)$ and $(j,\ k)$, when $y^t_{ik} + y^t_{jk} < \min(S^t_i + S^t_j,\ R^t_k)$, then traffic holding before the merge cell k is used.

TMT 4: Traffic holding at diverge links. Considering diverge links $(i,\ j)$ and $(i,\ k)$, when $y^t_{ij} + y^t_{ik} < \min(S^t_i,\ R^t_j + R^t_k)$, then traffic holding at the diverge cell i is used.

TMT 5: Traffic holding at ordinary links. Considering ordinary link $(i,\ j)$, when $y^t_{ij} < \min(S^t_i,\ R^t_j)$, then traffic holding on the roadway cell i is used.

TMTs 1 and 2 are allowed under Eq. (3.4) and Eq. (3.5), respectively, while the TMTs 3, 4, and 5 are based on traffic holding, and thus, they violate their respective equations in 3.3-3.5. Traffic holding is justified in Ziliaskopoulos [36] as a possible set of traffic controls that could be implemented to optimize flows, while Liu, Lai, and Chang [18] considers traffic holding an unrealistic behavior. Traffic holding was classified into three different TMTs because the type of the cell has important implications; merge and diverge cells are natural locations for traffic controls, and hence traffic holding might be more realistic at these points (for instance, ramp metering is an example of a merge control that might be analogous to traffic holding). On the other hand, traffic holding at ordinary links is more problematic; ordinary links are a modeling construct, and there are probably no analogous traffic controls available at these locations. For that, a heuristic will be proposed to eliminate traffic holding at ordinary links. Moreover, one of the objectives of this paper is to demonstrate how the $LP_{\Omega \leq Q}$ uses these tools to optimally manage traffic flows.

Chapter 5

Important Properties

In this section, we discuss important properties of the CTM and LP modeling framework that will help better understand the structure of optimal solutions.

Before discussing these properties, we define an important network feature, the *bottleneck*. If a link $(i,\ j) \in L$ has $Q_i > Q_j$, then cell j is a bottleneck. Other more complex bottlenecks can be defined, where a merge cell j is considered a bottleneck when $\sum_{i:(i,j)\in L_m} Q_i > Q_j$. Without loss of generality, however, we simplify the discussion of bottlenecks assuming the simple $Q_i > Q_j$ condition. Because Q_i is larger than Q_j, under sufficient demand, cell i will enter the congested state. If a network does not have any bottlenecks, then no congestion will form; and, if a network does have bottlenecks, a bottleneck (or a set of bottlenecks) will determine the performance of the system and will affect how traffic flows are optimally managed.

Under $CTM_{\Omega=Q}$, S_i^t is not reduced when cell i is in the congested state ($x_i^t > Q_i$), even if the number of vehicles in i reaches N_i, the holding capacity of the cell (i.e., the cell's jam density), which is not consistent with the traffic flow theory. Conversely, $CTM_{\Omega<Q}$ reduces the flow below Q_i when cell i is in the congested state, a much more realistic traffic flow behavior. This observation follows directly from Eq. (3.1). This has profound effects on the realism of the traffic flow and on the solution generated. To begin this discussion, Proposition 1 describes how cells transition from the free-flow state to the congested state.

Proposition 1. *Under $CTM_{\Omega\leq Q}$, any roadway cell i in the free-flow state, i.e., $x_i^t \leq Q_i$, will remain in the free-flow state unless the flow out of i is limited to below Q_i by a downstream adjacent cell(s).*

Proof. The flow out of cell i is determined by S_i^t and R_j^t of the adjacent downstream cell(s) j, $j : (i,j) \in L$. Because of the requirement that $Q_i \leq \delta_i(N_i - x_i^t)$ when $x_i^t \leq Q_i$, Eq. (3.2) allows at most Q_i vehicles to enter cell i in interval t. In the free-flow state, by Eq. (3.1), we have $S_i^t = x_i^t$, and thus the flow out of cell i in interval t will be equal to the number of vehicles in cell i at the beginning of interval t (i.e., x_i^t) unless flow is inhibited by the cell(s) j, that is, unless the flow is set by R_j^t as either $y_{ij}^t = Q_j < Q_i$ or $y_{ij}^t = \delta_j(N_j - x_j^t) < Q_i$. ∎

For a link $(i,j) \in L$, R_j^t can inhibit the flow out of cell i if $y_{ij}^t = Q_j < Q_i$, which is based on the structure of the network (e.g., going from a three-lane road to a two-lane road) and is thus a *bottleneck*, or if $y_{ij}^t = \delta_j(N_j - x_j^t) < Q_i$, which is based on congestion (the expression $\delta_j(N_j - x_j^t)$ can only be less than Q_j if cell j is in the congested state). Proposition 2 describes congestion and flow under $CTM_{\Omega=Q}$.

Proposition 2. *Under $CTM_{\Omega=Q}$, congestion on any roadway cell i ($x_i^t > Q_i$) does not reduce the flow out of cell i below Q_i, the maximum attainable flow.*

Proof. S_i^t is the function that limits the flow out of cell i based on the state and characteristics of cell i, see Eq. (3.1). Under $CTM_{\Omega=Q}$, this is simplified to $S_i^t = \min\{x_i^t,\ Q_i\}$, thus when cell i is in the congested state, S_i^t does not limit the flow below Q_i, its maximum value. ∎

Proposition 2 does not state that if cell i is congested, then the flow out of cell i must be Q_i, but it simply proposes that if the flow is less than Q_i, then it is due to the downstream adjacent cell j, $(i,\ j) \in L$, and not congestion in cell i. It is important to note that, when considering congestion and bottlenecks, the flow into cell j, the bottleneck, will not go below Q_j as a result of cell i being in the congested state. Therefore, if cell i is at the highest density allowable or is highly congested (even up to jam density, $x_i^t = N_i$), the bottleneck performance will not be affected, and cell j will remain in the free-flow state. Of course the flow into cell j could be limited below Q_j, if cell j became sufficiently congested, but this would only happen if the flow out of cell j was restricted below x_j^t by a downstream cell k, $k : (j,k) \in L$, because either cell k is also a bottleneck ($Q_j > Q_k$), or cell k is sufficiently congested, such that $x_j^t > \delta_k(N_k - x_k^t)$.

Proposition 2 has some important implications and provides the rational for studying $CTM_{\Omega\leq Q}$: we want to properly penalize congestion in the sense that congestion on cell i can inhibit flow discharge (i.e., S_i^t), which is consistent with the traffic flow theory. Daganzo [10] displays a trapezoidal flow density diagram for the CTM (see Figure 3.2(a)),where congestion (i.e., $x_i^t > Q_i$) first reduces traffic speed and then flow. These congestion effects occur under $CTM_{\Omega=Q}$ due to congestion on the adjacent downstream cell j, not cell i itself. In the case of a bottleneck, however, cell j does not become congested (unless this is caused by a downstream bottleneck), and thus, there is nothing additional to hinder the flow from cell i as congestion builds. By introducing the generalized CTM, however, flow discharge is reduced at the congested cell i. This leads to the question "how does this generalization affect flows at non-bottleneck cells?". We examine this in Proposition 3. Note that, for notational simplicity, we will define $\Omega^* \equiv (Q - \Omega)/(N - Q)$.

Proposition 3. *Consider a roadway segment, initially in the free-flow state, represented by n identical ordinary cells ($Q_i = Q$, $N_i = N$, $\Omega_i = \Omega < Q$, and $\delta_i = \delta$, $i = 1, ..., n$). For this roadway segment, under $CTM_{\Omega\leq Q}$, the flow on the n-1 links connecting these cells is governed by $y_{ij}^t = \min\{S_i^t,\ R_j^t\} = \min\{x_i^t,\ Q,\ \delta(N - x_j^t)\}$, that is, the term $Q - (x_i^t - Q)\Omega^*$ is redundant in Eq. (3.3).*

Proof. Consider an arbitrary link (i, j). The expression $Q - (x_i^t - Q)\Omega^*$ cannot restrict flows on link (i, j) while in the free-flow state. Suppose that t is the first time interval where $Q - (x_i^t - Q)\Omega^*$ limits the flow over link (i, j), yielding the following:

$$y_{ij}^t = Q - (x_i^t - Q)\Omega^* < \delta(N - x_j^t) \tag{5.1}$$

This can only occur if $x_i^t > Q$ (i.e., cell i is in the congested state), which, by Proposition 1, implies that $y_{ij}^t = \delta(N - x_j^{t-1})$, that is, in the previous time interval $t-1$, the congestion in cell j was sufficient to limit the flow over (i, j) and force cell i to become congested such that Eq. (5.1) holds. Substituting $y_{ij}^t = \delta(N - x_j^{t-1})$ into Eq. (3.7) for cell j yields $x_j^t = x_j^{t-1} + \delta(N - x_j^{t-1}) - y_{jk}^{t-1}$ or:

$$y_{jk}^{t-1} = (1-\delta)x_j^{t-1} + \delta N - x_j^t \tag{5.2}$$

Also by Eq. (3.7), we have $x_j^{t-1} = x_j^t - y_{ij}^{t-1} + y_{jk}^{t-1}$; substituting $x_j^t - y_{ij}^{t-1} + y_{jk}^{t-1}$ in Eq. (5.2) yields $y_{jk}^{t-1} = (1-\delta)x_j^t - (1-\delta)y_{ij}^{t-1} + (1-\delta)y_{jk}^{t-1} + \delta N - x_j^t = \delta(N - x_j^t) - (1-\delta)y_{ij}^{t-1} + (1-\delta)y_{jk}^{t-1}$, and then, the rearranging yields $\delta y_{jk}^{t-1} = \delta(N - x_j^t) - (1-\delta)y_{ij}^{t-1}$ or $\delta(N - x_j^t) = \delta y_{jk}^{t-1} + (1-\delta)y_{ij}^{t-1}$ and thus by Eq. (5.1) we get:

$$y_{ij}^t < \delta y_{jk}^{t-1} + (1-\delta)y_{ij}^{t-1} \tag{5.3}$$

Using $y_{ij}^{t-1} = \delta(N - x_j^{t-1})$ and $y_{hi}^{t-1} \leq \delta(N - x_i^{t-1})$ (this holds by Eq. (3.3)) in Eq. (3.7) yields $x_i^t \leq x_i^{t-1} + \delta(N - x_i^{t-1}) - \delta(N - x_j^{t-1})$ or $x_i^t \leq (1-\delta)x_i^{t-1} + \delta x_j^{t-1}$, which we substitute into Eq. (5.1) to get $y_{ij}^t \geq Q - ((1-\delta)x_i^{t-1} + \delta x_j^{t-1} - Q)\Omega^*$, which when combined with Eq. (5.3) yields:

$$(Q - (1-\delta)x_i^{t-1} + \delta x_j^{t-1} - Q)\Omega^* < \delta y_{jk}^{t-1} + (1-\delta)y_{ij}^{t-1} \tag{5.4}$$

Once again, by Eq. (3.3), we have $y_{pq}^t \leq Q - (x_p^t - Q)\Omega^* \; \forall (p,q) \in L$; replacing the y-variables in the RHS of Eq. (5.4) with this expression yields $\delta y_{jk}^{t-1} + (1-\delta)y_{ij}^{t-1} \leq \delta(Q - (x_j^{t-1} - Q)\Omega^*) + (1-\delta)(Q - (x_i^{t-1} - Q)\Omega^*)$, which simplifies to $\delta y_{jk}^{t-1} + (1-\delta)y_{ij}^{t-1} \leq Q + \Omega^* Q - \delta\Omega^* x_j^{t-1} - (1-\delta)\Omega^* x_i^{t-1}$. The LHS of Eq. (5.4) also simplifies to $Q + \Omega^* Q - \delta\Omega^* x_j^{t-1} - (1-\delta)\Omega^* x_i^{t-1}$;thus, we have for Eq. (5.4) LHS=RHS, which contradicts the strict inequality of Eq. (5.4) and completes the proof. ∎

This proposition states that flows on cells that are not before a bottleneck behave the same under $CTM_{\Omega<Q}$ and $CTM_{\Omega=Q}$. We note that at bottlenecks, the generalized constraint also gives a trapezoidal flow-density curve, as the density in cell i (the cell before the bottleneck) must increase until the flow becomes governed by $Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i) < Q_j$ after which flow is reduced, and thus there is a level of congestion for which there is no reduction in flow. While Proposition 3 states that the CTM generalization only impacts traffic flows at bottlenecks, this is not the case in the LP (remember there is no traffic holding under CTM; this only occurs for the LP). The traffic management tools using traffic holding, TMTs 3, 4, and 5, can cause congestion that reduces flow under $CTM_{\Omega<Q}$, unlike $CTM_{\Omega=Q}$. The lack of flow reduction at a bottleneck under $CTM_{\Omega=Q}$ naturally leads to Proposition 4.

Proposition 4. *For $LP_{\Omega=Q}$ there exists an optimal solution that does not have traffic holding.*

Proof. See Bish and Sherali [3];Nie [24];Shen, Nie, and Zhang [27]. ∎

Empirically we observe that the optimal solutions to $LP_{\Omega=Q}$ (found using commercial solver), almost always have traffic holding, at least in the interesting case where a bottleneck exists. By Proposition 4 this is just one of the possible alternate optimal solutions. Proposition 4 is intuitive, traffic holding can help improve the solution when traffic congestion reduces the flow rate, because it can be used to mitigate congestion at the bottlenecks; but, when congestion has no negative effect on flows, traffic holding does not improve the solution. Conversely, Bish, Chamberlayne, and Rakha [2] provides examples where traffic holding is required for an optimal solution to $LP_{\Omega<Q}$. Despite this, we will demonstrate that solutions to $LP_{\Omega\leq Q}$ use traffic holding in an undesirable and unrealistic manner. To deploy traffic holding as a tool, we would like to generate solutions that only include traffic holding that improves the system's performance and further uses traffic holding in a sensible manner. Zheng and Chiu [34] developed an algorithm that generates a solution with no traffic holding for single commodity problems under $CTM_{\Omega=Q}$, but this algorithm will not work under $CTM_{\Omega<Q}$ because an earliest arrival flow (EAF) solution does not necessarily exist under $CTM_{\Omega<Q}$ as we later show. Lin and Wang [16] suggested the use of the following lexicographic objective function to remove traffic holding:

$$Minimize \sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t + \epsilon \sum_{t=1}^{T} \sum_{(i,j)\in L} t y_{ij}^t \tag{5.5}$$

where ϵ is a number small enough to optimize the two expressions in preemptive order; Lin and Wang [16] did not elaborate on the appropriate ϵ-value. The secondary objective, $\sum_{t=1}^{T}\sum_{(i,j)\in L} t y_{ij}^t$, which we denote as f_2, rewards advancement of the flow and penalizes traffic holding. In Proposition 6, we show how to derive an appropriate ϵ-value; but, first, we consider the following proposition to help with this endeavour.

Proposition 5. *The objective function min $\sum_{t=1}^{T}\sum_{i\in C/S_e} x_i^t$ is equivalent to min $\sum_{t=1}^{T}\sum_{i:(i,j)\in S_e} t y_{ij}^t$.*

Proof. Consider a unit of flow (i.e., a vehicle) that leaves the system (i.e., enters a sink cell) in interval t. The contribution to $\sum_{t=1}^{T}\sum_{i\in C/S_e} x_i^t$ for this unit of flow is t (note that all the demand is in the system in the beginning of the first time interval), which is also its contribution to $\sum_{t=1}^{T}\sum_{i:(i,j)\in S_e} t y_{ij}^t$. As this is true for every unit of flow, the two objective functions are equivalent. ∎

Proposition 5 assumes that the time horizon T is large enough to allow every vehicle in the system to reach a sink cell, and we note that the constraint $\sum_{t=1}^{T}\sum_{i:(i,j)\in S_e} t y_{ij}^t = \sum_{t=1}^{T}\sum_{i\in C/S_e} x_i^t$ must be added to force the vehicles from the system. This proposition still holds when the demand at the source cells is a time-dependent parameter; in that, the solutions to the two objectives would be identical, but with a constant difference between the values of the objective functions.

Proposition 6. *The objective function (5.5) will minimize $\sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t$ and $\sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t$ in a preemptive order if $\epsilon < 1/(max \sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t)$.*

Proof. Consider two objective functions, f_1 and f_2, to be minimized over a bounded region for which an optimal solution exists; Sherali [28] shows that min $f_1 + \epsilon f_2$ will be minimized in a preemptive order if

$$\epsilon < min(\frac{1}{f_{1max} - f_{1min}}, \frac{1}{f_{2max} - f_{2min}})$$

Setting $f_1 = \sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t$ and $f_2 = \sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t$ and observing that these functions are bounded and always have non-negative objective function values, we have

$$min(\frac{1}{f_{1max}}, \frac{1}{f_{2max}}) \leq min(\frac{1}{f_{1max} - f_{1min}}, \frac{1}{f_{2max} - f_{2min}})$$

Using Proposition 5, we have

$$\sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t \Leftrightarrow \sum_{t=1}^{T} \sum_{i:(i,j) \in S_e} t y_{ij}^t < \sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t$$

Thus, setting $\epsilon < 1/(max \sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t)$ will minimize $\{f_1, f_2\}$ in preemptive order. ∎

Proposition 7. *Objective function (5.5), with an ϵ that ensures a lexicographic ordering of the two objectives, produces a solution to the $LP_{\Omega \leq Q}$ that only has traffic holding that is required to optimize objective function (3.6).*

Proof. Based on the lexicographic property (see Proposition 6), objective function (5.5) will always give the optimal solution to (3.6). Objective function (5.5) will then try to minimize the secondary objective $f_2 = \sum_{t=1}^{T} \sum_{(i,j) \in L} t y_{ij}^t$. This expression provides a time based penalty for movement on the links; if we consider a solution where all demand reaches the sinks on or before time interval T, then every unit of flow has a given path from its source cell to a sink cell, and thus a set number of link traversals. To minimize f_2, flow will be advanced as much as possible on its given path considering the network configuration and the primary objective, and thus traffic holding will only exist if it reduces the primary objective function. ∎

Thus, objective function (5.5) eliminates traffic holding for $LP_{\Omega = Q}$, but does not necessarily do so for $LP_{\Omega < Q}$, as we see in the numerical examples in Section 8. Under $LP_{\Omega = Q}$, Lo [20], Shen, Nie, and Zhang [27], and Zheng and Chiu [34] provide methods to eliminate traffic holding; but, the methods used either do not work under $\Omega < Q$ or are computationally too expensive. For $LP_{\Omega < Q}$, Bish, Chamberlayne, and

Rakha [2] propose a mixed binary program (thus adding binary variables to the LP; we denote this formulation as $BP_{\Omega\leq Q}$) to eliminate TMT 5, traffic holding on ordinary cells; we use this model in the numerical section.

The objective function becomes more interesting under $LP_{\Omega<Q}$. Zheng and Chiu [34] show that an Earliest Arrival Flow (EAF) solution exists under $LP_{\Omega=Q}$. An EAF solution maximizes $\sum_{t=1}^{t'}\sum_{(j:s)\in L} y_{js}^{t}$, $\forall t' = 1, ..., T$, and it is equivalent to minmizing objective function (3.6). Likewise, Jarvis and Ratliff [13] show that without congestion-based flow reductions, EAF solutions exist, and these solutions also minimize the network clearance time (NCT), the time interval when the last flow enters a sink cell (note, an EAF solution in a multi-commodity may not exist Fleischer [11]). The NCT is an important objective for evacuation problems. For $LP_{\Omega<Q}$, an EAF solution does not necessarily exist, and thus to minimize NCT, a different objective function is required. We can formulate this objective function as follows, including additional constraints.

$$\text{Minimize} \sum_{t=1}^{T} E^{t} \tag{5.6}$$

$$\epsilon^{t} \geq 1 - \frac{\sum_{i\in S_e} x_i^t}{\sum_{i\in S_o} x_i^1} \tag{5.7}$$

$$\epsilon^{t} \in \{0,1\}, t = 1, ..., T \tag{5.8}$$

Chapter 6

Solution Heuristic

In this section we provide a heuristic for determining traffic management strategies under $LP_{\Omega\leq Q}$ considering restrictions on the allowable TMTs. If we allow all five TMTs, then the solution can be obtained directly from the $LP_{\Omega\leq Q}$, and by using the objective function (5.5), we can eliminate all traffic holding (i.e., TMTs 3, 4 and 5) under $CTM_{\Omega=Q}$ and limit unnecessary traffic holding under $CTM_{\Omega<Q}$. Eliminating all traffic holding under $CTM_{\Omega<Q}$, however, is more difficult, as this requires the addition of nonlinear constraints to the LP or the addition of binary variables (i.e., $BP_{\Omega\leq Q}$, Bish, Chamberlayne, and Rakha [2]). As mentioned in section 4, the traffic holding TMTs (i.e., TMTs 3, 4, and 5) violate the CTM equations (3.3)-(3.5), and thus they might not be desirable tools to use. Adding to that, merge and diverge cells can be natural locations for traffic controls, such as ramp metering at merge cells or routing at diverge; thus, traffic holding might be realistic at these cells, unlike traffic holding at ordinary cells that are an artifact of the modeling framework and do not necessarily align with any controllable section of roadway. For that reason, the proposed heuristic will only focus on eliminating traffic holding on ordinary links (i.e., TMT 5).

The heuristic proposes a new objective function, which replaces objective function (3.6) and accommodates for the secondary objective suggested by Lin and Wang [16]. This alternation requires some additional constraints, yielding the following $LP_{Heuristic}$:

$$\text{Maximize} \sum_{t=1}^{T} \sum_{i\in C_{BM}} x_i^t - \epsilon \sum_{t=1}^{T} \sum_{(i,j)\in L} t y_{ij}^t \tag{6.1}$$

subject to:

$$x_j^t = x_j^{t-1} + \sum_{i:(i,j)\in L} y_{ij}^{t-1} - \sum_{k:(j,k)\in L} y_{jk}^{t-1}, \quad \forall j \in C/S_e,\ t = 2, ..., T \tag{6.2}$$

$$\sum_{j:(i,j)\in L} y_{ij}^t \leq x_i^t, \quad \forall i \in C,\ t = 1, ..., T \tag{6.3}$$

$$\sum_{j:(i,j)\in L} y_{ij}^t \leq Q_i - (x_i^t - Q_i)(Q_i - \Omega_i)/(N_i - Q_i), \quad \forall i \in R,\ t = 1, ..., T \tag{6.4}$$

$$\sum_{i:(i,j)\in L} y_{ij}^t \leq Q_j, \quad \forall j \in R, \, t = 1, ..., T \tag{6.5}$$

$$\sum_{j:(j,i)\in L} y_{ij}^t \leq \delta_j (N_j - x_j^t), \quad \forall i \in R, \, t = 1, ..., T \tag{6.6}$$

$$x_i^1 = 0, \quad \forall i \in C/S_o \tag{6.7}$$

$$\sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t \geq TST_m, \tag{6.8}$$

$$x_i^{NCT_n} = 0, \quad \forall i \in C \tag{6.9}$$

As mentioned in section (4), traffic holding is used by the LP as a tool to yield optimal solutions by usually preventing traffic from building up at the bottlenecks. The primary objective function in Eq. (6.1) targets that issue by forcing the traffic at cells from which merge links branch to be maximized. The set of these cells will be noted as C_{BM} and will include all cells that are directly before merge cells, as a result, traffic will be forced to build up at the bottleneck, and thus, reduce traffic holding. The secondary objective function has the same impact as discussed previously, which is, rewarding the advancement of flow and penalizing traffic holding. The two objective functions will be optimized in a preemptive order when ϵ has a value less than that discussed in proposition (6). On the other hand, constraints (6.2)-(6.6) are the same constraints (3.7)-(3.11) used in the generalized model (i.e., $LP_{\Omega \leq Q}$). Constraint (6.6) ensures that all cells, except source cells, initially (i.e., $t = 1$) do not hold any traffic. Constraint (6.8) ensures that $TST_{Heuristic}$ is greater or equal to the TST_m set by the algorithm, with index m representing the number of the iteration (i.e., m=0,1,2,3,4,...). Constraint (6.9) forces the flow to end at an NCT_n assigned by the algorithm, with index n representing the number of the iteration (i.e., n=0,1,2,3,4,...). Finally, the logical non-negativity constraints on the x and y variables are required. In order to initiate the heuristic, the initial conditions are set by defining TST_0 and NCT_0 as the solution that is obtained by solving $LP_{\Omega \leq Q}$. That is, first, $LP_{\Omega \leq Q}$ is solved, and the optimal solution obtained (i.e., TST_0 and NCT_0) will be set as initial conditions in $LP_{Heuristic}$.

In order to solve for a feasible solution that has no traffic holding at ordinary links, the following steps shall be followed:

Algorithm 1:

Initialization: Set UB=∞ and LB= 0;
Solve $LP_{\Omega=Q}$ and set optimal TST and NCT values obtained as TST_0 and NCT_0;
Solve $LP_{Heuristic}$ by setting TST_0 in (6.8) and NCT_0 in (6.9) ;
Check for traffic holding at ordinary links;
if *Traffic holding at ordinary links does not exist (i.e., $y_{ij}^t = min(S_i^t, R_j^t)$)* **then**
    Stop heuristic;
    NCT_0 and TST_0 are a feasible solution;
else
    Set NCT_0 as LB and iterate by increasing NCT in (6.9) by increments of 1, while keeping TST_0 in (6.8) fixed;
    Solve $LP_{Heuristic}$ at each iteration and check whether traffic holding at ordinary links still exists or not;
    while *Traffic holding at ordinary links exists (i.e., $y_{ij}^t < min(S_i^t, R_j^t)$)* **do**
        $NCT_{n+1} \leftarrow NCT_n + 1$;
        if *Traffic holding at ordinary links is eliminated* **then**
            Set NCT_n value reached by iteration as UB;
            Check whether there exists a feasible solution at NCT_{n-1} with a better TST value by iterating through TST at NCT_{n-1}:
            Solve $LP_{Heuristic}$ at NCT_{n-1} and get $TST_{n-1,heuristic}$ ($TST_{n-1,heuristic}$ is the TST value obtained from solving the heuristic using NCT_{n-1});
            Alter constraint (6.8) into $\sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t \leq TST_{n-1,heuristic}$;
            Solve $LP_{Heuristic}$ while fixing NCT_{n-1} in (6.9) and iterating through TST in (6.8);
            while *Traffic holding at ordinary links exists (i.e., $y_{ij}^t < min(S_i^t, R_j^t)$)* **do**
                $TST_{m+1} \leftarrow TST_m - 1$;
                $TST_0 \leq TST_m \leq TST_{n-1,heuristic}$;
                if *Traffic holding at ordinary links is eliminated at a certain TST_m for that fixed NCT_{n-1}* **then**
                    Repeat same process at $NCT_{n-2}, NCT_{n-3}, ...$;
                    Stop when traffic holding is not eliminated for any TST value between TST_0 and $TST_{n-1,heuristic}$;
                else
                    NCT_n gives the best feasible solution with no traffic holding at ordinary links;
                    Set $TST_{n,heuristic}$ as UB;
                    Alter constraint (6.8) into $\sum_{t=1}^{T} \sum_{i \in C/S_e} x_i^t \leq TST_{n,heuristic}$ and iterate by $TST_{m+1} \leftarrow TST_m - 1$;
                    Stop at a TST value where traffic holding still exists and set this TST as LB
                end
            end
        end
    end
end

The heuristic proposed follows a grid search that results in a feasible solution to $LP_{\Omega \leq Q}$. The heuristic focuses on iterating between upper bound and lower bound values for NCT and TST until a feasible solution that eliminates traffic holding at ordinary links is reached.

Chapter 7

Revised Cell-Transmission Formulation

Knowing that an efficient optimization problem is especially important, particularly with large-scale networks, Liu, Lai, and Chang [19] proposed a revised cell-transmission formulation that allows simplification of the network as seen in Figure 7.1 and thus, reduces the run-time required to solve the LP. Accordingly, the LP constraints are modified to account for the simplification of the network (see Liu, Lai, and Chang [19]); the revised LP will be denoted as $LP_{revised}$.

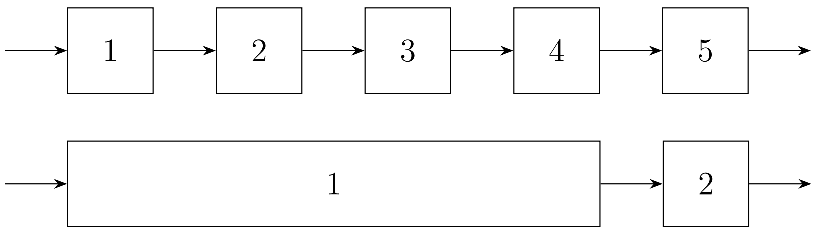


Figure 7.1: Revised LP network

For δ-values of 1, the $LP_{revised}$ and $LP_{\Omega\leq Q}$ frameworks are equivalent. This can be justified by proposition 1 which states that the flow in a certain cell can only be limited by the downstream cell. As a result, for every stream of ordinary cells, the formulation can be simplified by using two cells only and thus increasing the tractability of the model.

The efficiency of the revised LP formulation will be further examined in the following numerical examples.

Chapter 8

Numerical Examples

In this section, two numerical examples will be presented, where two different networks will be solved, using Gurobi solver, under $LP_{\Omega=Q}$, $LP_{\Omega<Q}$, and the heuristic proposed. Analyzing the solution obtained by solving these networks will help in illustrating the performance of the heuristic, the difference in performance under different values of Ω, and the strategies used by the various modeling frameworks. The first example is a simple network, which is easy to solve; however, the second example considers a much more complex network.

Example 1: Consider the network in Figure (8.1), adapted from Bish, Chamberlayne, and Rakha [2]. All roadway cells have Q-values of 30, N-values of 210, and δ-values of 1. Cells S_o1 and S_o2 are source cells, having $x^1_{S_o1} = 750$ and $x^1_{S_o2} = 750$. Cell S_e is the sink cell.

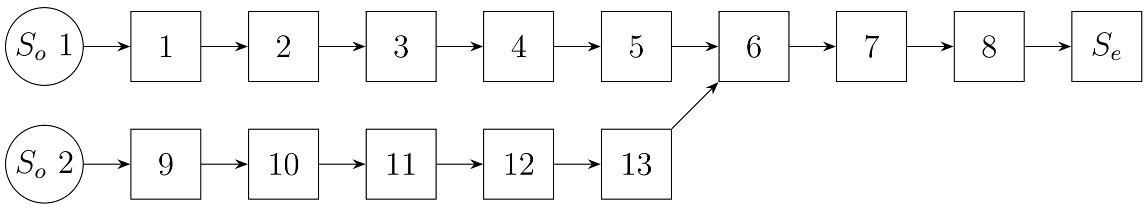


Figure 8.1: Small tree, single merge network

Because the network in this example does not have diverge links, only TMT 1 (prioritizing at merge links), TMT 3 (traffic holding at merge links), and TMT 5 (traffic holding at ordinary links) are applicable. Furthermore, an optimal solution never uses TMT 3 because traffic holding at the merge (holding traffic on cells 5 and 13) would reduce the flow through cell 6, the one and only bottleneck. Using $LP_{\Omega=Q}$ yields an optimal solution that has a total system time (TST) of 48,750 and a network clearance time (NCT) of 59. Analyzing the solution obtained, $LP_{\Omega=Q}$ framework uses TMT 5, although there is an optimal solution that does not require any traffic holding (see proposition (4)). For instance, one solution pushes cell 10 to its maximum capacity, reaching a jam density of 210 vehicles, although there is enough capacity in cell 11 to move that traffic forward, with cell 11 holding only 30 vehicles. Adding to that, at some other time intervals, cell 10 reaches a congested state, with a density of 150 vehicles, while cell 11 holds 0 vehicles. This shows how TMT 5 can be very problematic and unrealistic, especially given that no traffic controls can exist at ordinary links. On the other hand, solving for a solution using $LP_{\Omega=Q}$ but with objective function (5.5) also yields a TST= 48,750 and an NCT= 59. As mentioned previously, however, using objective function 5.5 for $LP_{\Omega=Q}$ eliminates traffic holding. Therefore, TMT 5 is eliminated, and traffic behaves much more realistically in a continuous manner. For example, cell 10 is never congested, remaining in a free-flow state with a density of 30 vehicles (the maximum number in the free-flow state). This is because the downstream cell, cell 11, also does not reach a congested state. Additionally, cells 5 and 13 reach jam density (i.e., $N = 210$) causing the upstream cells 4 and 12 to reach a congested state, as well as a jam density at certain time intervals. Thus, the location of the congestion is more realistic in this solution, where an upstream cell becomes congested when the flow is limited by the downstream cell (see proposition (1)). In both solutions, however, roadway cells reach the maximum density (a veritable traffic jam), and yet there is no reduction in traffic flow discharge. On the other hand, TMT 1 is used, under $LP_{\Omega=Q}$ (3.6) and (5.5), in an erratic manner, where a 100% merge priority is given to cell 5 in some time intervals and to cell 13 in other intervals. For instance, in one of the solutions, cell 5 is given a 100% merge priority for 7 consecutive time intervals, after which cell 13 is given a 100% priority for only one time interval. Interestingly, under $LP_{\Omega=Q}$, any particular TMT 1 strategy is optimal since both cells 5 and 13 can supply the bottleneck at its maximum flow rate. Given this, we would prefer a more sensible strategy, for instance, giving links (5; 6) and (13; 6) each a 50% merge priority while eliminating traffic holding.

Using $LP_{\Omega<Q}$ framework, the optimal solution obtained was the same as that for $LP_{\Omega=Q}$ (i.e., TST= 48,750 and NCT= 59), for all values of Ω; but, $LP_{\Omega<Q}$ had to use TMTs 1 and 5 to obtain an optimal solution for smaller values of Ω. For analysis and illustrative purposes, we will study $\Omega = 0.2Q$ case, which was deemed realistic in Chamberlayne, Rakha, and Bish [7] using simulations studies. Since both LP frameworks yielded the same TST value, traffic holding will be studied by comparing the value of the secondary objective function in Eq. (5.5), that is, $\sum_{t=1}^{T}\sum_{(i,j)\in L} ty_{ij}^{t}$, which will be denoted as z_2. It must be noted that the greater the value of z_2 is, the more traffic holding exists. Table 8.1 presents the TST, z_2, and NCT values obtained for the different LP frameworks used under objective functions (3.6) and (5.5) for Example 1.

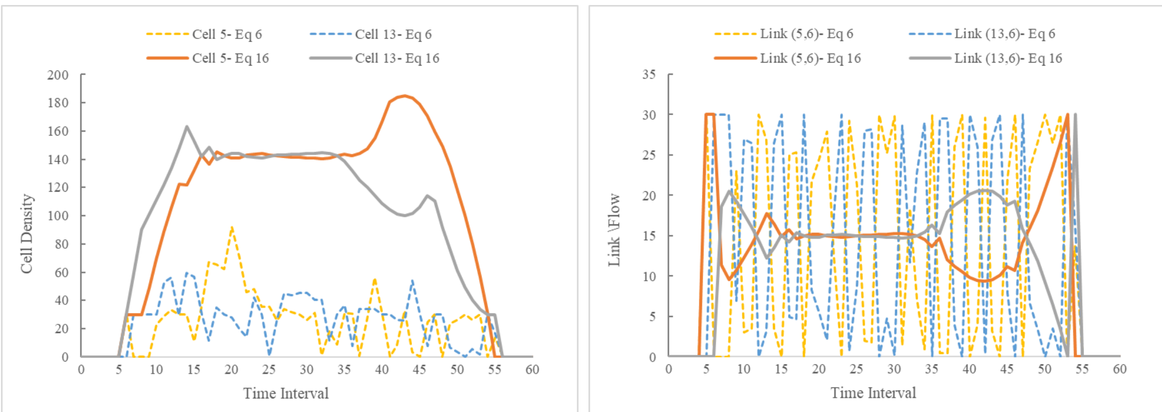
Knowing that objective function (5.5) eliminates traffic holding for $LP_{\Omega=Q}$, $z_2 = 309,600$ represents the value for the solution that has no traffic holding. Therefore, any value greater than that will account for traffic holding. Comparing the results in Table (8.1), we can deduce that $LP_{\Omega=0.2Q}$(3.6) produces the solution with the most traffic holding, because traffic holding is required under that framework to minimize the TST, especially for lower values of Ω. Of course, under $LP_{\Omega=0.2Q}$, traffic holding can also penalize flow through flow reductions, thus TMT 5 must be used carefully. On the other hand, $LP_{\Omega=0.2Q}$(5.5) accounts for less traffic holding, where only traffic holding required to obtain the optimal solution is used. This is done by holding

Table 8.1: TST, z_2, and NCT values for $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$ under objective functions (3.6) and (5.5)

Model	TST	z_2	NCT
$LP_{\Omega=Q}$(3.6)	48,750	364,470	59
$LP_{\Omega=Q}$(5.5)	48,750	309,600	59
$LP_{\Omega=0.2Q}$(3.6)	48,750	378,575	59
$LP_{\Omega=0.2Q}$(5.5)	48,750	314,564	59

traffic before the bottleneck, which is the most critical component of the network. For example, in one of the solutions, cells 5 and 13 never reach jam density (i.e., N=210); instead, cells 4 and 12 reach a highly congested state, with traffic densities of 153 and 187 vehicles respectively, protecting cells 5 and 13 from excess congestion (the maximum number of vehicles in cells 5 and 13 are 185 and 145, respectively). In fact, the strategy balances the congestion in cells 5 and 13 and the flow out of these cells, such that the maximum flow is maintained through the bottleneck.

Analyzing the obtained solutions for $LP_{\Omega=0.2Q}$ framework under objective functions (3.6) and (5.5), cell densities have a much more smoother transitions than that under $LP_{\Omega=Q}$(3.6), where adjacent cells can be at opposite extremes, one at jam density and the other at a free-flow state.



(a) (b)

Figure 8.2: (a) Cell Densities before the bottleneck (cells 5 and 13) and (b) flows into the bottleneck (links (5,6) and (13,6)) for $LP_{\Omega=0.2Q}$(3.6) and $LP_{\Omega=0.2Q}$(5.5)

Furthermore, examining Figure 8.2(a), which displays the densities in cells 5 and 13 for $LP_{\Omega=0.2Q}$(3.6) and $LP_{\Omega=0.2Q}$(5.5), we can deduce that, under objective function (3.6), cells 5 and 13 do not reach high congestion levels. This can be justified by being the LP framework with the most traffic holding, where traffic is held, during different time intervals, at all the ordinary links upstream of the bottleneck (i.e., cell 6), even at links (S_o1, 1) and (S_o2, 9). On the other hand, by using objective function (5.5), all unnecessary traffic holding is eliminated, and traffic is only held at links close to the bottleneck (i.e., links (3,4), (4,5), (11,12), and (12,13)). Additionally, traffic flow reductions are considered under $LP_{\Omega=0.2Q}$; thus, when a congested state is reached (i.e., $x_i^t \geq Q = 30$), traffic outflow from the congested cell is reduced to below the maximum outflow capacity (i.e., 30 vehicles). For instance, in one of the solutions under $LP_{\Omega=0.2Q}$(5.5), at t=15, cells 5 and 13 are congested with cell densities of 132 and 153 respectively. During that time interval, traffic outflow through links (5,6) and (13,6) is reduced to below 30 vehicles, 16 and 14 vehicles respectively. This justifies why under $LP_{\Omega=0.2Q}$(5.5), a 100% merge priority is rarely given to either cells, as can be seen in Figure 8.2(b). Therefore, under the lexicographic objective function (5.5), TMTs 1 and 5 are used in a more coordinated approach. Interestingly, for Example 1, there is an optimal $LP_{\Omega=0.2Q}$ solution where TMT 1 uses a static 50% merge priority that has the same z_2 value as that found by $LP_{\Omega=0.2Q}$(5.5). Symmetry in the densities of cells that are before the bottleneck is also exhibited under that case. This demonstrates that there are multiple optimal solutions for $LP_{\Omega=0.2Q}$(5.5), in Example 1, and the solution from the solver, with its more intricate plan, might not be the most desirable from an implementation point of view.

Knowing that TMT 5 (i.e., traffic holding at ordinary links) is undesirable, the heuristic proposed in this paper will be used to eliminate that traffic management tool. Since $LP_{\Omega=0.2Q}$(5.5) does not eliminate TMT 5, the solution obtained from the heuristic will be compared to that obtained by the binary program (BP), which also targets TMT 5, presented in Bish, Chamberlayne, and Rakha [2].

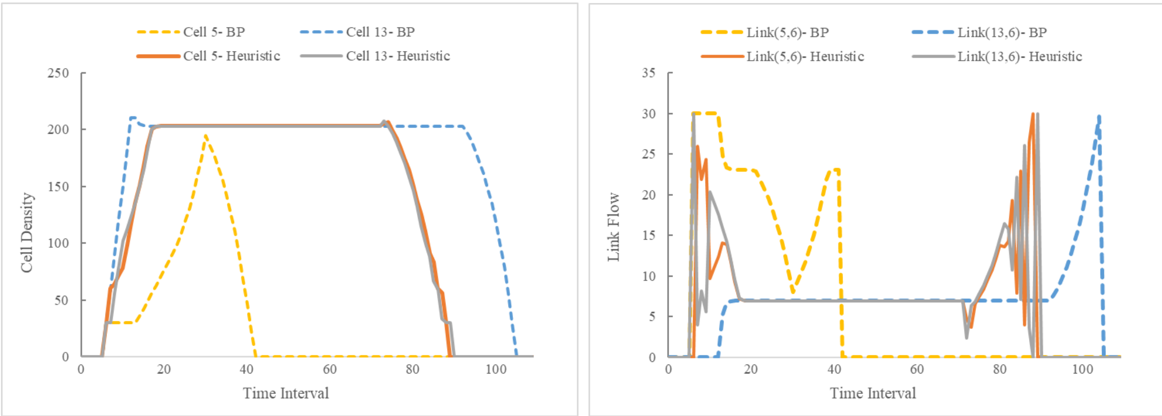
Table 8.2 displays the solutions obtained by $LP_{Heuristic}$ for different Ω cases, as well as the solutions by the BP proposed in Bish, Chamberlayne, and Rakha [2].

Table 8.2: Results from BP and the heuristic, for a range of values for Example 1.

	Ω	$0.8Q$	$0.7Q$	$0.6Q$	$0.5Q$	$0.4Q$	$0.3Q$	$0.2Q$
BP	TST	48,750	48,750	48,750	48,750	50,785	56,723	67,827
	NCT	59	59	59	59	63	81	108
Heuristic	TST	48,750	48,750	48,750	48,750	51,583	59,242	74,887
	NCT	59	59	59	59	63	73	93

Comparing the solutions obtained by solving the network, in Example 1, using the BP and heuristic, it can be deduced that the heuristic does not provide the optimal solution for TST; however, its does yield a better NCT value than that obtained by the BP. Adding to that, the heuristic is more tractable than the BP, which requires much more time to be solved due to the large number of binary variables existing in the formulation. Thus, considering the fact that the network in Example 1 is small and easy to solve, the BP might not be efficient to use for large networks.

Figure 8.3(a) shows the densities of cells 5 and 13 (i.e., cells directly before the bottleneck) and (b) the flow on the merging links, links (5,6) and (13,6). By analyzing the cell densities, it can be deduced that the binary program does not balance the flow priority out of cells 5 and 13, with cell 13 reaching jam density at a



(a) (b)

Figure 8.3: (a) Cell Densities before the bottleneck (cells 5 and 13) and (b) flows into the bottleneck (links (5,6) and (13,6)) for the BP and the Heuristic under $\Omega = 0.2Q$

certain time interval. Thus, the BP minimizes the TST by using a highly asymmetric strategy; in this case, the BP gives precedence to flows on link (5; 6) to move the traffic from source $S_0 1$ out of system quickly, with just enough priority given to flows over link (13, 6) to avoid too much congestion. On the other hand, the heuristic provides a more symmetric solution, with cells 5 and 13 holding approximately equal densities at most intervals, and accounts for a scenario closer to a 50% merge priority. Therefore, the solution of the heuristic is somewhat easier to implement.

Finally, the $LP_{revised}$ framework, suggested by Liu, Lai, and Chang [19], will be examined and compared to the generalized LP framework. The network in Figure 8.1 was simplified as shown in Figure 8.4.

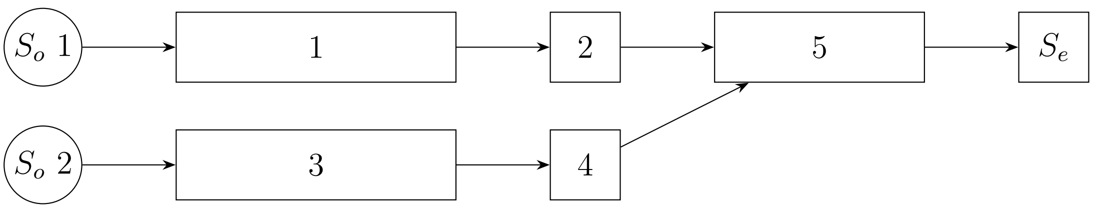


Figure 8.4: Small tree, single merge network

Considering proposition 1, the flow in a certain cell can only be limited by the downstream cell. Therefore, for instance, the flow in cell 2 will be limited by the bottleneck cell 5; the traffic will then propagate backward to cell 1. Thus, due to these aforementioned properties, the revised cell-transmission formulation proposed by Liu, Lai, and Chang [18] is equivalent to the generalized LP formulation. To prove that, $LP_{revised:\Omega=Q}$ and $LP_{revised:\Omega=0.2Q}$ were used to solve the network in figure 8.4. The solution obtained and its efficiency are shown in Table 8.3.

Analyzing the results in Table 8.3, the optimal solution did not change after simplifying the network proving that both models, the generalized and the revised

Table 8.3: TST, NCT, and Run-time values for $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$ using the generalized and revised model

Model	TST	NCT	Run-time (sec)
$LP_{\Omega=Q}$(3.6)	48,750	59	0.761
$LP_{\Omega=0.2Q}$(3.6)	48,750	59	0.985
$LP_{revised:\Omega=Q}$(3.6)	48,750	59	0.309
$LP_{revised:\Omega=0.2Q}$(3.6)	48,750	59	0.339

cell-transmission, are equivalent; however, the revised LP is much more efficient, as can be seen by comparing the run-times of the LP's.

Studying and analyzing the solutions obtained under the different LP frameworks used to solve the simple network of Example 1 aided in identifying some important properties associated with each framework. It is of interest, however, to examine more complex networks, accounting for multiple bottlenecks. Thus, we study next a much larger network, considered from the literature [31], to further study the strategies produced by the LP and the heuristic.

Example 2-a: The network presented in Figure (8.5) has freeway cells (represented by circles), having $Q = 12$ and $N = 36$, and arterial cells (represented by squares), having $Q = 6$ and $N = 18$. All roadway cells have $\delta = 1$. Cells 54, 55, and 56 are source cells, having $x^1_{54} = 500$, $x^1_{55} = 150$, and $x^1_{56} = 150$. Cell 59 is the sink cell.

Unlike Example 1, this network has multiple bottlenecks with several merge and diverge cells. Therefore, all traffic management tools are applicable (i.e., TMTs 1-5). Solving for the optimal solution using $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$ frameworks results in a TST of 35,868 and an NCT of 80. Table 8.4 presents the TST, z_2, and NCT values obtained for the different LP frameworks used under objective functions (3.6) and (5.5) for Example 2-a.

Table 8.4: TST, z_2, and NCT values for $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$ under objective functions (3.6) and (5.5)

Model	TST	z_2	NCT
$LP_{\Omega=Q}$(3.6)	35,868	364,470	80
$LP_{\Omega=Q}$(5.5)	35,868	309,600	80
$LP_{\Omega=0.2Q}$(3.6)	35,868	378,547	80
$LP_{\Omega=0.2Q}$(5.5)	35,868	314,564	80

Analyzing the results presented in Table 8.4, we can deduce that the LP frameworks perform the same in both examples, Examples 1 and 2-a, where under $LP_{\Omega=Q}$(5.5) there is no traffic holding, with $z_2 = 309,600$ representing that state. $LP_{\Omega=Q}$(3.6) uses traffic holding although there is an optimal solution that does not require traffic holding. On the other hand, $LP_{\Omega=0.2Q}$(3.6) had the most traffic holding, where the TMTs are required by this framework to obtain an optimal solution. Using objective function (5.5) under $LP_{\Omega=0.2Q}$ eliminates all unnecessary traffic holding, thus

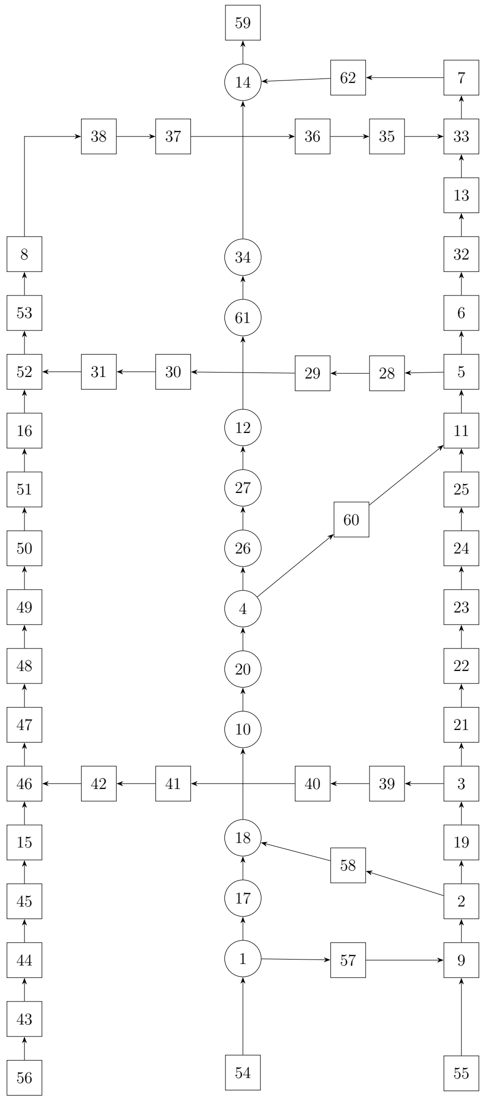


Figure 8.5: Large complex network with multiple bottlenecks

accounting for a smaller z2 value. Adding to that, all LP frameworks resulted in an NCT of 80; thus, these solutions are EAF solutions. For LP solutions in general, we make the following observation:

Observation 1 *An $LP_{\Omega<Q}$ solution will produce an EAF solution by using the traffic holding TMTs 3-5, provided there are sufficient cells to hold traffic.*

This is the case for most networks, but a small network can have insufficient space to hold traffic without incurring a traffic holding penalty that reduces flow through the network's bottleneck.

In order to eliminate TMT5, the heuristic was utilized, and the solution obtained was compared to the shortest path (SP) solution described in Tarhini and Bish [30]. The BP, proposed by Bish, Chamberlayne, and Rakha [2], was not used in Example 2 because it was highly inefficient due to the large number of binary variables existing in the formulation (the BP was not able to solve a single case within 2 hours).

As mentioned previously, there are many possible optimal solutions under $LP_{\Omega=Q}$

that do not have traffic holding. For this network, these solutions include the shortest path (SP) solution. This optimal solution uses only the shortest path from each source cell to the sink cell and also includes rules on merge priorities. Specifically, in this example, the cell with a lower Q-value has 100% merge priority (remember, this does not mean that the other cell does not send flow into the merge cell, but it can only utilize the remaining capacity). In fact, Tarhini and Bish [30] demonstrate that, for the network in Figure 8.5, this solution is optimal for any demand realization and thus is the optimal policy when demand is uncertain. Here we do not study demand uncertainty, but the SP solution is useful because the strategy is simple, and thus can be easily adapted to the $\Omega < Q$ setting. For comparison purposes, however, since this SP solution does not use TMTs 3-5, the heuristic will be applied on the shortest path network, which would only eliminate TMT 5. Table 8.5 presents the TST and NCT values for the heuristic and the shortest path problem at different Ω values for Example 2-a.

Table 8.5: Results from the heuristic and shortest path, for a range of values for Example 2-a

	Ω	$0.8Q$	$0.7Q$	$0.6Q$	$0.5Q$	$0.4Q$	$0.3Q$	$0.2Q$
Heuristic	TST	35,868	35,868	35,868	35,868	35,868	35,868	35,868
	NCT	80	80	80	80	80	80	80
SP	TST	35,868	35,868	35,868	35,868	37,409	37,511	43,735
	NCT	80	80	80	80	84	85	102

Analyzing Table 8.5, the heuristic was able to eliminate traffic holding at ordinary links at the optimal solution at all Ω-values, unlike under the shortest path problem, where a feasible solution was obtained and the TST increased with the decrease in the Ω-value. Having a z_2 value of 431,977 at $\Omega = 0.2Q$, the heuristic was capable of resulting in the optimal solution but with a higher z-value compared to $z_2 = 309,600$ obtained under $LP_{\Omega=Q}$5.5 (refer back to Table 8.4). This is because of the fact that the heuristic targets only traffic holding at ordinary links.

Having a large network with a small number of vehicles at the source cells allowed the heuristic to diverge traffic into different paths, facilitating the goal of the heuristic in eliminating traffic holding at ordinary links. On the other hand, forcing traffic flow into only three separate paths (i.e., shortest path from each source cell to the sink cell) constrained the performance of the heuristic and resulted in larger TST at smaller Ω values. Therefore, in order to further examine the performance of the heuristic, Example 2-b will consider the network in Figure 8.5 but with a larger number of vehicles at the source cells.

Example 2-b: The network presented in Figure (8.5) will also be considered with freeway cells (represented by circles) having $Q = 12$ and $N = 36$ and arterial cells (represented by squares) having $Q = 6$ and $N = 18$. All roadway cells have $\delta = 1$. Cells 54, 55, and 56 are source cells, having $x^1_{54} = 600$, $x^1_{55} = 600$, and $x^1_{56} = 600$.

Solving the linear problem using $LP_{\Omega \leq Q}$ framework gave an optimal solution of

$TST = 155,700$ and $NCT = 164$.

Table 8.6 presents the TST and NCT values for the heuristic and the shortest path problem at different Ω values for Example 2-b.

Table 8.6: Results from the heuristic and shortest path, for a range of values for Example 2-b

	Ω	$0.8Q$	$0.7Q$	$0.6Q$	$0.5Q$	$0.4Q$	$0.3Q$	$0.2Q$
Heuristic	TST	155,700	155,700	155,700	155,700	156,919	183,402	196,241
	NCT	164	164	164	164	178	182	195
SP	TST	155,700	155,700	155,700	155,700	178,862	198,850	262,105
	NCT	164	164	164	164	187	216	288

As discussed previously, increasing the initial number of vehicles at the source cells restrained the heuristic from eliminating TMT5 at the optimal solution for lower Ω-values. The feasible solution obtained by performing the heuristic on the network in Figure 8.5, however, was still superior to the solution obtained from applying the heuristic on the shortest path, as can be analyzed from Table 8.6.

The $LP_{revised}$ model by Liu, Lai, and Chang [18] will be tested on the network of example 2-b. Simplifying the network and modifying the LP formulation accordingly, $LP_{revised:\Omega=Q}$ and $LP_{revised:\Omega=0.2Q}$ will be solved and compared to $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$. Table 8.7 presents TST, NCT, and the run-times required by the generalized and revised LP models.

Table 8.7: TST, NCT, and Run-time values for $LP_{\Omega=Q}$ and $LP_{\Omega=0.2Q}$ using the generalized and revised model for example 2-b

Model	TST	NCT	Run-time (sec)
$LP_{\Omega=Q}$(3.6)	155,700	164	51.281
$LP_{\Omega=0.2Q}$(3.6)	155,700	164	52.439
$LP_{revised:\Omega=Q}$(3.6)	155,700	164	11.723
$LP_{revised:\Omega=0.2Q}$(3.6)	155,700	164	13.889

The results in Table 8.7 show that both, revised and generalized, LP models are equivalent due to proposition 1 as explained in example 1. On the other hand, comparing the run-times of both models, it can be deduced that $LP_{revised}$ is much more efficient compared to the generalized. Therefore, applying the revised model on the heuristic proposed in this paper will greatly reduce the run-time required by the heuristic to find a feasible solution.

Chapter 9

Conclusion

The generalized CTM, $CTM_{\Omega \leq Q}$, follows a trapezoidal flow-density relationship; thus, this framework is capable of capturing the effect of traffic congestion on upstream cells by accurately modeling queue spill backs. Moreover, under $CTM_{\Omega \leq Q}$, a cell only becomes congested when the downstream cell is also congested. Under $CTM_{\Omega = Q}$, however, this only works well at non-bottleneck locations, because at a bottleneck the downstream cell is never congested and thus, the flow reduction mechanism of $CTM_{\Omega = Q}$ does not work properly, where flow through the bottleneck is not adversely affected by the congestion. On the other hand, under the generalized CTM, $CTM_{\Omega \leq Q}$, traffic flow reductions through the bottleneck are enforced when congestion builds before a bottleneck. This reduction in flow is one of the defining characteristics of traffic flows. There is a large body of literature that uses the CTM embedded in a linear program (LP) framework (first proposed in Ziliaskopoulos [36]), which also suffers from the same lack of flow reduction in response to congestion before a bottleneck.

In this paper, we explore the $LP_{\Omega \leq Q}$ framework, which produces an optimal solution, that is a set of traffic flows. To better understand these flows, we illustrate five traffic management tools that describe any solution. These tools are prioritizing at merge links, routing at diverge links, traffic holding at merge links, traffic holding at diverge links, and traffic holding at ordinary links. The traffic management tools have various levels of realism and can be used in more or less sensible ways. Only the first two tools are allowed under CTM, while the traffic holding tools are produced because of the linearization of the CTM. We show that for $LP_{\Omega = Q}$ there are many optimal solutions; all a solution must do to be optimal is to ensure sufficient demand at the network's bottlenecks so that flows through the bottlenecks are maintained at there maximum level. Because of this, solutions often use the traffic management tools in undesirable ways. While this is still a problem under $LP_{\Omega \leq Q}$, it is less so, as the number of optimal solutions is reduced, and the solution must ensure sufficient demand at the bottlenecks, as before, but also must control congestion before the network's bottlenecks in order to maintain maximum flows, if possible. Because of the issues with how the LP framework uses the traffic management tools and their varying realism, we explore how to reduce or eliminate the use of these tools. This can increase the complexity of the problem, requiring alternative objective functions and/or the inclusion of binary variables. Because adding binary variables quickly reduces the tractability of the framework, we provide a heuristic for developing solutions. The heuristic proposed targets traffic holding at ordinary links, as it is the most unrealistic tool since no traffic controls can exist at these links. Analyzing the heuristic by studying networks from the literature, we deduced that the heuristic is efficient and effective to use and that it yields feasible solutions accounting for realistic traffic flow scenarios.

Future research into further enhancing the CTM's realism is of interest. Finding solutions to the linearized framework with more controllable traffic management tools can be examined. Analyses on the implementation of the CTM on signalized intersections can also be targeted in order to improve the model under multi-class signalized control and to enhance the model's ability in simulating better queue formation and dissipation. Finally, with the emergence of automated vehicles, it is of great importance to analyze and integrate the effect of such vehicles in traffic management models.

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