On the thermodynamic treatment of diffusion-like economic commodity flows

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Abstract: The flow of a commodity is modelled as a quasi-thermodynamic irreversible process. The flow is treated not as a mass quantity, but more like a transient energy quantity. A commodity is thus considered an artefact that depends on whether it resides or is stored within the system or is in transit across the system boundary to the surroundings. This allows the use of the fundamental laws of thermodynamics and brings further insight into the process of commodity diffusion. The methods of irreversible thermodynamics are also considered and a coupling relationship between commodity price and quality is derived.

Keywords: entropy; exergy; price; production system; quality; thermodynamics; uncoupled flow.

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1 Introduction

The study of thermal systems has had a long history, originally motivated by the age-old desire to produce motive power from burning flames. Thermodynamics has evolved over the years to become the study of energy and entropy. The 'efficiency' of the conversion of energy from an available source (e.g. fire) to a useful form (a rotating shaft for example) has been a primary goal and the concept of entropy has had a major impact on its understanding.

In more recent years, thermodynamics has been used to model systems and processes that behave as though they were thermal systems, at least in some respects. This poses no concern as long as the models used conform to observed physical behaviour and the investigation leads to some insight. In reality, 'copying' natural flow is a least-resistance path that may be chosen when making economic decisions. In this context, the concept of conservation of energy and the concept of exergy destruction (entropy generation) have been found to be useful in providing tools for the analysis of many other systems undergoing certain processes. The last two concepts, better known as the first and second laws of thermodynamics, provide the basis for quantitative and qualitative assessment of comparative systems.

Complete disciplines now exist which merge cost and thermodynamics, the most notable being the field of thermo-economics (Bejan, 1996) and later the associated concepts of extended exergy analysis (Sciubba, 2001). In fact, the thermodynamic concept of exergy has been discussed in relation to the environments, economics and ecosystems (Dincer, 2002; Ertesvag, 2001; Rosen, 2002; Wall and Gong, 2001).

Production systems may be viewed in a way that makes some aspects of their behaviour similar to those systems that can be described by the laws of physics. In management science/operational research these laws may be used when analysing or designing such systems. Information theory, an off-shoot of the concept of entropy and stemming from the statistical mechanics viewpoint, may be used to measure the disorder in a system via Shannon's (1948) entropy function (e.g. Karp and Ronen, 1992; Ronen and Karp, 1994). On the other hand, few have applied the classical thermodynamic approach. For example, Whewell (1997) applied the laws thermodynamic in the analysis of logistics in supply chains. Recently, a new field: 'Econophysics' has also emerged that attempts to study complex economic systems using the methods of mathematics and physics (e.g. Mantegna and Stanley, 2000).

Salamon and Nitzan (1981) were among the first to present the idea of applying the ideas emerging from thermodynamic optimisation to economic systems. Grubbström and Hultman (1989) presented economic models for exergy and heat storage. Using a thermodynamic approach and treating thermal energy as a commodity, their results appeared to be distinct from those using traditional inventory theoretic models. Chen (1999) presented a comprehensive qualitative study that drew parallels between a socio-economic system and a thermodynamic system, and devised appropriate managerially orientated redefinitions of the first and second laws of thermodynamics.

The direct application of the methods of classical thermodynamics for a production-inventory system was detailed by Jaber et al. (2004). The model viewed a commodity flow from a system as an 'energy-like' quantity and using a combination of thermodynamics and inventory models presented some guiding observations. In Jaber et al. (2004) it was stated that 'Price' determines the flow of a commodity. The minimum demand can be viewed as that equal to the human metabolic rate in appropriate units (that which is needed to simply stay alive). On the other hand, demand increases in an undetermined manner that may be linked to affluence. Thus, it is seen that demand can be presented as a 'potential' or 'driving force' in the thermodynamic sense. Since affluence is measured in monetary terms, it becomes possible to translate the potential into a difference in money (price). It was also pointed out that another factor that determines the flow of a commodity from a system is the product 'quality' although the work did not expand much on this. Treating the price-driven flow and the quality-driven flows as uncoupled parallel flows allows some simple observations to be made. But, on the other hand, the two flows are also seen to be coupled and thus dependent. Using the methods of irreversible thermodynamics (Kondepudi and Prigogine, 1998) allows a physical statement to be made on the effect of improving commodity quality on its price. Thus, a 'natural' tendency is presented which may be utilised in economic decisions.

Although somewhat controversial and preliminary, the application of physical observation into economic behaviour can be useful. While it may not be apparent in the very short run, many feel that 'ultimately, the effects of physical behaviour will be a key, if not dominant, effect on economic behaviour.' In particular, key aspects of the second law are likely to affect such behaviour. Cengel's (2002) study of societal/political size using thermodynamics provides an interesting treatment of this idea.

In this paper, after a brief review and some clarifications or pertinent background material, the work of Jaber et al. (2004) that connects economic inventory models to thermodynamic principles is extended and modified. The thermodynamic-based methodology is extended to a system with more than one state property and involving entropy generation due to more than one flow. Both independent partial commodity flow and coupled price-quality flow are discussed.

2 Thermodynamics of commodity flow systems

2.1 The system and its state

The system to be studied is a 'production system'. Production systems are involved in purchasing and selling one or more commodities with the task of incurring revenue. In this paper however, we consider the system to be dealing with one and only one kind of commodity.

Similarly to a thermal system, a commodity-production system is an entity that is distinguished from its surroundings. The surroundings are taken to include the market and the supplying system. The system is considered to be at a given 'state' which describes its internal condition. Such a state could be described by a certain number of characteristics; one, for example, could be the price (P) the system ascribes to a certain commodity (or collection of commodities) it produces. Thus, in parallel to thermodynamic jargon, P is taken to be a state property. Another state property is taken to be the commodity quality, herein denoted by g (as a reference to 'goodness').

In order to perform our analyses, the system under study must be defined carefully. By setting the characteristics of the system (e.g. its properties), the internal conditions and thus the state of the system are fully described. The state of the system is defined by two independent intensive properties; by neglecting all internal and external factors that could be encountered we assume that the system depends only on the commodity it deals with. The state of the system is then dependent on the state of the commodity, i.e. the commodity's that has a price P and a quality (goodness) g.

The commodity price P is actually the price (in \$) of one unit of the commodity, while goodness (quality) has units of 'degrees S^*'. The latter unit represents a scale of quality (which can be set arbitrarily, say from 0 to 100) and has been chosen as a logical parallel to the 'absolute temperature scale'. Thus, we are dealing with a closed system, whose state is defined by two intensive properties, the commodity unit price P and the commodity quality/goodness g.

2.2 The surroundings

The system is separated from its surroundings by a real or imaginary surface, the boundary. Surroundings are different from immediate surroundings. By definition, surroundings are everything outside the system boundaries while the immediate surroundings refer to the portion of the surroundings that is affected by the process that may be taking place (Cengel and Boles, 2002). In this study, we are more interested in the immediate surroundings than the surroundings. Thus when specifying the system, it is also necessary to specify the immediate surroundings.

As a start and for the sake of simplicity the immediate surrounding and the surroundings (i.e. the environment) are treated as one and include both the market and the supplying system.

The state of the surroundings is referred to as the equilibrium state and is described by the equilibrium price P_0 and the equilibrium quality (goodness) g_0. This state would be a general representation of the state of the competing market (other economic systems that deal with the same type of commodity), which would be the state of one dominant competitor, or the 'average state' of all the 'important' and influential competitors.

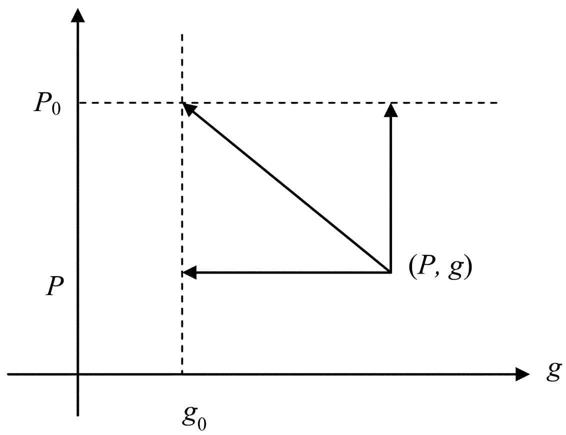
2.3 Commodity flow and the first law

After a system and its state are described, information on the interacting flows is required. Here, the flows are those of one or more commodities in system-surrounding or production unit/market transactions.

While a commodity flow is physically a mass (material) flow, it is taken here to be more akin to an energy flow. In this fashion commodity flow is given a disorganised nature which in turns allows it to be viewed in such a way that it is diminishable due to irreversibilities that arise during the process. Thus, the system is taken to be 'closed' in the thermodynamic sense and only energy may cross its boundary. In general, energy transport across a closed boundary can be either as a disorganised flow (heat) or as an organised flow (work). The more valuable energy flow is obviously the organised work flow. However, using similarity with heat flow automatically gives the flow a 'reduced value' and leaves the idea of 'obtaining work as the final aim' to a later consideration. No other distinction is made between the commodity flows except that part is 'recoverable' and part is not, in accordance with the second law. Thus, any input required (an investment, effort) that may lead to a flow is considered as an affair internal to the system and not needed in a thermodynamic-like treatment.

Although a similarity is drawn between heat and commodity flow, they are dissimilar since heat flows solely due to a temperature difference, while commodity flow depends on at least two variables. Thus, the flow of a commodity is dependent on both price P and quality g. The direction of commodity transfer is always from a lower to a higher price and from higher to lower quality. The total commodity transfer Q is a function of four variables $Q = f(P, g, P_0, g_0)$. Figure 1 shows possible flow paths, including flow at constant price, constant quality and a general flow path.

Figure 1 Commodity flow paths on a $P - g$ diagram



Time comes into play in thermodynamics when the rate of energy transfer is required to be known. In commodity flows between the system and the surroundings the flow occurs at a rate $\dot{Q}$. In general, following the methods of linear irreversible thermodynamics, a flow (*J*) is related to a driving force (a potential) by a linear law

$$J = LX, \tag{1}$$

where *L* is the Onsager coefficient for the flow in question. For heat conduction, for example, *L* is the thermal conductivity and *X* is the gradient of the temperature in a direction. In the case of commodity flow from the system to the surroundings (i.e. a possible sale), the price *P* should be less than its equilibrium price P_0 (the current market price) for a likely flow to occur. Similarly, the quality g of the commodity should be greater than the equilibrium quality g_0 for a likely flow to occur. Phenomenological relations may be hypothesised to exist for each of the price-driven partial commodity flow and the quality-driven partial flow assuming first pure diffusion of each:

$$J_P = \frac{\dot{Q}_{Price}}{A} = -k_P \frac{\partial P}{\partial n} \quad \text{and} \quad J_g = \frac{\dot{Q}_{Quality}}{A} = k_g \frac{\partial g}{\partial n}, \tag{2}$$

where the negative sign for the price-driven flow indicates that the flow is from the system to the surrounding when $P < P_0$ and in the same direction for the quality-driven flow when $g > g_0$. A form of conductivity (k) has been introduced for each flow and a form of 'frontal area' has been defined between the system and its surroundings (immediate) and *J* is thus a flux. While the quality-driven law appears to conform to the usually encountered phenomenological laws of physics such as Fourier's law, Fick's law or Ohms law; the price-driven law seems to have an opposite sense with the partial flows given by:

$$Q_{Price} = -K_P(P - P_0), \tag{3}$$

$$Q_{Quality} = K_g(g - g_0), \tag{4}$$

and

$$Q_{Equilibrium} = f(P_0, g_0). \tag{5}$$

Here, $Q_{Equilibrium}$ represents an equilibrium (base) flow of commodity (or commodities) at $P = P_0$ and $g = g_0$, P is the price of one unit of the commodity sold by the system in dollars per unit, K_P is an 'elasticity' in units per second per (\$/unit) i.e. (unit^2/\$.s), g is the quality (goodness) of the commodity sold by the system in $^\circ S$, and K_g is in units per second per $^\circ S$.

The total commodity flow rate $\dot{Q}$ can be defined as is the sum of the partial flows as follows:

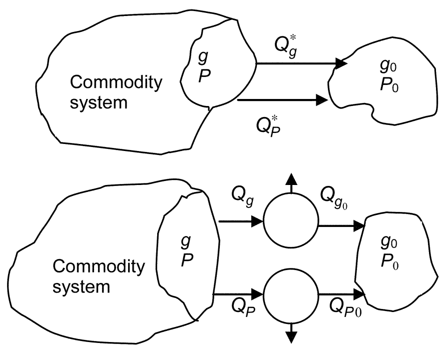
$$\dot{Q} = \dot{Q}_{Price} + \dot{Q}_{Quality} + \dot{Q}_{Equilibrium}. \tag{6}$$

The equilibrium flow is taken to be zero since it is a constant for the problem at hand. Figure 1 shows both price-driven and quality-driven 'partial' flow together with a 'combined' or resultant flow. A combined flow is one where both quality and price drive the commodity flow in a partial manner; giving rise to a resultant flow.

2.4 The second law and entropy

A commodity flow out of the system to the surroundings is assumed to be driven by two forces, quality and price potentials, which are taken for now to be parallel and uncoupled. These flows, which are shown in Figure 2 as Q_g^* and Q_P^*, cannot, however, be fully obtained. Instead, as dictated by the rules of thermodynamics and shown in the lower schematic of Figure 2, Q_g and Q_p are less than the above mentioned undiminished flows.

Figure 2 Undiminished and recoverable commodity flow



In fact, as shown, through appropriate 'economic cycles', only a fraction of Q_g and a fraction of Q_P can be obtained as a useful flow (a beneficial sale). The remainder are what may be termed 'unrecovered' flows which are considered of the same phenomenological nature as the desired flow and are caused by entropy generation (destruction of exergy). That is for example, both Q_{g_0} and $Q_{g,unrecovered}$ are 'outputs' from a 'cyclic engine'. They are only 'distinguishable' through some mechanism yet to be determined. The efficiency of the 'cyclic engines' can be expressed as:

$$\begin{aligned} \eta_g &= \frac{Q_{g_0}}{Q_g} = 1 - \frac{Q_{g,unrecovered}}{Q_g} \\ \eta_P &= \frac{Q_{P_0}}{Q_P} = 1 - \frac{Q_{P,unrecovered}}{Q_P}. \end{aligned} \tag{7}$$

To reverse the above cycles (i.e. to have a 'heat pump-like behaviour) would require an external input which is equal to the unrecovered flows from above. This is for a commodity to flow (be sold) if its price is higher than the equilibrium (current market) price would require an input (an investment/intervention). Similarly, for a commodity of low-quality to be sold would require an external investment. These external investments can be said to be equivalent to raising the quality (a process which carries a cost) in the first case; and lowering the commodity price (also at a cost) in the second case. Thus a firm having a product at a certain price P_0 and a quality g_0 will need to expend effort to decrease its price and increase its quality to P and g respectively.

2.5 Economic quality scale and economic price scale

The thermodynamic temperature scale is based on a reversible engine whose heat input Q_1 and heat rejection Q_2 are related by

$$\frac{Q_1}{Q_2} = \frac{\Phi(T_1)}{\Phi(T_2)}, \tag{8}$$

where any arbitrary function $\Phi(T)$ will satisfy this equation. The simplest temperature scale first proposed by Lord Kelvin involves taking $\Phi(T) = T$ so that:

$$\left(\frac{Q_1}{Q_2}\right)_{reversible} = \frac{T_1}{T_2}. \tag{9}$$

The flow of commodities due to a quality difference follows the same behaviour as heat flow. We can then use the same thermodynamic temperature scale for the 'economic quality scale', where it is noted that both the desired flow and the 'unrecovered' flow are 'rejected' into the environment.

$$\left(\frac{Q_{g1}}{Q_{g2}}\right)_{reversible} = \frac{\Phi(g_1)}{\Phi(g_2)} = \frac{g_1}{g_2}. \tag{10}$$

Then,

$$\eta_g = \frac{Q_{g0}}{Q_g} = \frac{g_0}{g} \leq 1.$$

Commodities flow due to a price difference from low to high prices. This is obviously opposite to heat flow. To conform to this observation, it is proposed to take $\Phi(P) = 1/P$ to define the 'economic price scale' as follows:

$$\left(\frac{Q_{P1}}{Q_{P2}}\right)_{reversible} = \frac{P_2}{P_1}. \tag{11}$$

Then,

$$\eta_P = \frac{Q_P}{Q_{P0}} = \frac{P}{P_0} \leq 1.$$

2.6 Entropy

Analogous to the Clausius statement of the second law, it may be hypothesised that

> "It is impossible to have an economic system that operates in a cycle and produces no effect other than the transfer of commodity from a higher-price medium to a lower-price one and from a lower-quality medium to a higher-quality one".

The previous statement does not imply that such a system can not exist. It simply states that an economic system will not operate unless a certain effort is done to force the flow to occur against its natural tendency.

In thermal systems, the entropy associated with a heat transfer from a system at temperature T is defined by $dS_{th} \geq \delta Q/T$. Based on this 'thermodynamic entropy' an '*economic entropy*' is defined assuming that it fulfils the requirement of being a 'thermodynamic property'.

Following the above and the logical relationship between all variables, we postulate that the *economic entropy* is such that:

$$dS_e \geq \frac{P}{g}\delta Q. \tag{12}$$

The above equation is quite complex in that it relates the change in entropy due to a flow of commodity (or commodities) to both P and g simultaneously. Solving this equation is not attempted in this work. Instead, as a first step and in order to simplify the analysis, it is assumed that the partial flows are uncoupled, parallel flows. The total economic entropy is thus obtained by addition

$$dS_e = \partial S_P + \partial S_g, \tag{13}$$

where δS_P is the economic entropy change due to the flow of commodities as a result of a price difference and δS_g is the economic entropy change due to the flow of commodities as a result of a quality difference.

The flow of commodities due to a quality difference follows the same behaviour as regular heat transfer (the flow direction goes from high to lower quality/temperature) and thus generates an economic entropy that has a similar form as the thermal entropy

$$dS_g \geq \frac{\delta Q_g}{g}. \tag{14}$$

The flow of commodities due to a price difference has a different behaviour from heat transfer as the flow follows a higher to a lower path. Thus, it is considered to be equal to

$$dS_P \geq P\delta Q_P. \tag{15}$$

With the two partial flows being considered as uncoupled parallel flows, the total economic entropy is found by addition

$$dS_e \geq (P\delta Q_P) + \left(\frac{\delta Q_g}{g}\right). \tag{16}$$

The equality in Equation (16) applies in the 'reversible/best' limit and thus the total entropy change is seen to be greater or equal to the entropy transfer, due to the two partial flows induced by both a price and a quality change.

3 Quality, price and their relationship

Price and quality are taken to be independent variables in the treatment described so far. In fact, it may be argued that they are not and are proportional to each other in some fashion ($g \propto P$). Generally, a product having a higher quality tends to have a higher price and vice versa. However, this actually depends on how quality is defined and on the object (commodity) in question. Different kinds of quality vs time relationships can be observed for different kinds of products. Figure 3 shows three possible modes of quality vs time behaviour for certain commodities. A combination of the three modes shown in the figure is also possible. For example, an object that obeys the third mode could after a certain amount of time, turn into a collection object and then obey the second mode shown in the figure below. Of course, the timescales are of no concern here as we only seek to demonstrate the variability of price with time. For quality-driven flow occurring at a given time, the equilibrium quality and equilibrium prices are fixed quantities since the times involved are much shorter than those of Figure 4. The relationship between g and P is not well defined and may not be clear. In any event, what is being considered here is the relationship between the partial flows induced by a change in g and a change in P. Thus, while the partial flows have been shown (Figure 2) to be parallel flows, the flows may not be independent and one flow (or a part of it) may be induced by the creation of a driving force that is initiated by another. In this case, a resort to the methods of irreversible thermodynamics is attempted and a logical mapping of commodity diffusion with other diffusion processes is described (Kondepudi and Prigogine, 1998).

Figure 3 Proof of commodity Clausius relation

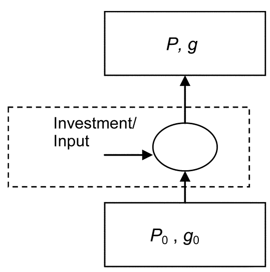
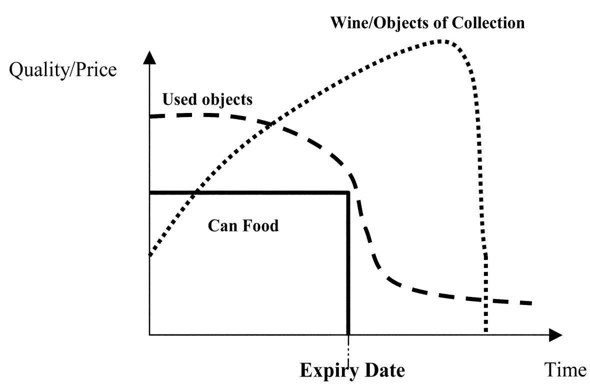


Figure 4 Commodity quality/price behaviour with time



With two driving forces stipulated to be present in the system there are two flows that are induced which are not necessarily functions of unique driving forces. This coupled flow is similar to the flow of heat in a solid due to a temperature difference which, in turn, induces a heat flow that is carried by charge carriers (thermoelectricity). For the commodity two-flow system, the so-called Omsager coupling relations can be written as:

$$\begin{aligned} J_g &= L_{gg}X_g + L_{gP}X_g \\ J_P &= L_{Pg}X_P + L_{PP}X_P, \end{aligned} \tag{17}$$

where X_g and X_P are the driving forces (gradients) due to quality and price changes, and the L's are coupling coefficient to be determined subject to the total entropy generation rate being given by:

$$\frac{dS_{gen}}{dt} = J_g X_g + J_P X_P. \tag{18}$$

The phenomenological relations describing the partial commodity diffusion due to the price potential is given by

$$J_P = k_P \frac{\partial P}{\partial x}, \tag{19}$$

where it has been assumed that there is a direction of flow x and thus a flow-path whose size is not of significance in this work and k_p is a form of commodity-conductivity. Following the logic of the previous section and borrowing from thermoelectric theory (Bejan, 1988), the entropy flow along this path is

$$J_{S,P} = PJ_P = Pk_P \frac{dP}{dx}. \tag{20}$$

The entropy generation rate is given per unit volume is

$$\left(\frac{dS}{dt}\right)_{gen} = J_P \frac{dP}{dx}. \tag{21}$$

In a similar fashion, the partial commodity diffusion due to a quality potential is given by

$$J_g = -k_g \frac{\partial g}{\partial x}, \tag{22}$$

where k_g is a quality-conductivity coefficient. The entropy flow in this case is

$$J_{S,g} = \frac{J_g}{g} = -k_g \frac{1}{g}\frac{dg}{dx}. \tag{23}$$

On a per unit volume basis

$$\left(\frac{dS}{dt}\right)_{gen} = -J_g \frac{1}{g^2}\frac{dg}{dx}. \tag{24}$$

The total entropy generated is

$$\left(\frac{dS}{dt}\right)_{gen;Tot} = -J_g \frac{1}{g^2}\frac{dg}{dx} + J_P \frac{dP}{dx}. \tag{25}$$

The Onsager coupling equations (Equation (23)) can now be written in a form that satisfies Equation (25):

$$J_g = -L_{gg}\frac{1}{g^2}\frac{dg}{dx} + L_{gP}\frac{dP}{dx},$$
$$J_P = -L_{Pg}\frac{1}{g^2}\frac{dg}{dx} + L_{PP}\frac{dP}{dx}. \tag{26}$$

For pure quality-induced flow

$$J_g = -\frac{L_{gg}}{g^2}\frac{dg}{dx} = -k_g\frac{dg}{dx}, \tag{27}$$

while for pure price-induced flow

$$J_g = L_{pp}\frac{dp}{dx} = k_p\frac{dp}{dx}. \tag{28}$$

Now, assuming only quality-driven flow by setting $J_P = 0$ in Equation (26) gives a form of quality-power relating the gradient in price to that of quality as follows:

$$\varepsilon = \frac{dp}{dg} = \frac{L_{pg}}{g^2 L_{pp}}. \tag{29}$$

This shows that even with no commodity flowing directly due to a price-change, there will still exist a potential gradient in price due to the quality change.

Manipulating the phenomenological equation allows the writing of a commodity flow with a single driving gradient and a coupled linear coefficient

$$J_g = -\left(\frac{L_{gg}L_{pp} - L_{gp}^2}{L_{pp}g^2}\right)\frac{dg}{dx}. \tag{30}$$

For the coupled flow case, appropriate manipulation leads to the following relationship

$$J_g = -k_g\frac{dg}{dx} + \varepsilon g^2 J_p. \tag{31}$$

This expresses the fact that, as a result of a quality gradient, not only does a commodity flow due to that gradient, but in addition there is a partial flow due to an arising price gradient. This equation is similar to the modified Fourier equation for thermoelectricity, which includes the so-called Peltier heat flow. The use of Equation (31) requires appropriate experimental determination of the quality-power factor ϵ. In general, it is shown that commodity price and quality are coupled in a 'natural' way. Advantage could be taken of this coupling when economic sale policies are to be made although ϵ needs experimental determination.

4 Conclusions

The behaviour of a commodity flow from a system to given market surroundings can be modelled using concepts borrowed from the field of thermodynamics. Treating the flow of a commodity as a 'disorganised' energy-type flow to/from a closed system allows entropy generation to be evaluated. A commodity flows partially as a result of driving forces emerging from at least price and quality changes. The partial commodity flows due to price changes and due to quality changes can be modelled as either separate additive parallel flows or as coupled flows that result from the induction of flow from the other. In this way, it is shown that, in the two-property flow system, a flow due to a quality increase may give rise naturally to a flow due to a price decrease.

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Nomenclature

P	Price (in \$) of a unit of commodity
g	Quality (goodness) of a unit commodity
P_0	Equilibrium price of a unit of commodity
g_0	Equilibrium quality (goodness) of a unit of commodity
Q	Total commodity transfer
J	Flow
L	Onsager coefficient for flow J
X	Gradient of the temperature in a direction
$Q_{Equilibrium}$	Equilibrium (base) flow of commodity (or commodities) at price P_0 and quality g_0
Q_{Price}	(Q_P) flow of commodity (or commodities) at price P and quality g_0
$Q_{Quality}$	(Q_g) flow of commodity (or commodities) at price P_0 and quality g
K_P	Elasticity associated with price
K_g	Elasticity associated with quality/goodness
$\dot{Q}$	Commodity flow rate
Q_P	Recoverable part of undiminished flow Q_P^*
Q_g	Recoverable part of undiminished flow Q_g^*
$Q_{g,unrecovered}$	Unrecovered flow due to quality/goodness
$Q_{P,unrecovered}$	Unrecovered flow due to price
Q_1	Heat input
Q_2	Heat rejection
S_P	Economic entropy generated by the flow of commodities due to a price difference
S_g	Economic entropy generated by the flow of commodities due to a quality difference.
dE_C	A change in a property of the system (akin to the total energy of the system)
J_P	Partial commodity diffusion due to a price potential

J_g	Partial commodity diffusion due to a quality potential
$J_{S,P}$	Entropy flow resulting from a partial commodity diffusion due to a price potential
$J_{S,g}$	Entropy flow resulting from a partial commodity diffusion due to a quality potential
k_p	Price-conductivity coefficient
k_g	Quality-conductivity coefficient
ϵ	Quality-power factor