AMERICAN UNIVERSITY OF BEIRUT

IDENTIFYING BUSINESS CYCLE ASYMMETRIES BY NONLINEAR DYNAMICS

by
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I thank the Archangels Michael, Gabriel and Rafael.
From my heart I say: Salve Regina!

ABSTRACT

OF THE THESIS OF

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Evidence suggests that macroeconomic variables do not behave in the same manner during different phases of a country business cycle. The aim of the present work is to find potential mathematical explanation for asymmetric behavior of the business cycle by making use of nonlinear dynamic economics. Nonlinear models, even if they are difficult to conceive, help better explaining asymmetries. I examine autoregressive models, Markov Switching autoregressive models, asymmetric time series, models with learning asymmetries, and endogenous business cycle models. Focusing on the mathematical structure of business cycles, I endogenously incorporate asymmetries in a mathematical model using difference and differential equations.

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ABBREVIATIONS

SA: Seasonally Adjusted

NSA: Not Seasonally Adjusted

CTETE: Charlijo True Economic Trend Evolution

y_t: Real GDP series

Yt : Nominal GDP

MS: Nominal Money Supply

C: Consumption

I: Investments

NX: Net Exports

G: Government Spendings

TLRAS: True Long Run Aggregate Supply curve

CHAPTER 1

INTRODUCTION

1.1. Business Cycles

We first begin by defining to the reader the economic meaning of business cycles. Business cycles are alternating sequences of expansions and recessions in the behavior of economic variables for a given economy or country. Cycles can have a specific periodicity but this is not the case in general. The NBER is the research unit in the United States that determines when US expansions and recessions begin and end. Most of the time, the number of months, or more technically the duration, that follow the beginning of a recession is not the same across different business cycles of the US. The same could be said about US expansions. The most important variables measured in a business cycle are GDP or output Y and unemployment U. Many other economic variables are also used. When they move in the same direction as Y, variables are called procyclical. Examples are Consumption C, Investment I, Inventories, Net exports X-M, velocity of money V, money supply MS, interest rates i, as mentioned in the stylized facts of Altug and Labadie (2008). Variables moving in the opposite direction during the same cycle are referred to as countercyclical.

The time of each phase of a given cycle is referred to as the duration. The maximum deviation from trend of the variable Y during a given phase is referred to as the amplitude of the given phase. Technically, we denote by Nr_i and Nex_i the number of months of the recession phase and the expansion phase of a business cycle i of a given time series Y. We denote by a_{r_i} and a_{ex_i} the amplitude of the recession and the expansion of a business cycle i of a given time series Y.

1.2. Asymmetries in Business Cycles

Business cycle asymmetries are perceived when the different phases of the business cycle behave differently with respect to time: it could be that the duration of a complete cycle changes with time, or that one phase shows characteristics that are inherently different than the opposite phase.

As we shall see in chapter 3 below, business cycles do not, in general, show a specific periodicity that pertains to every economy. In other words, we can claim to identify asymmetries in a business cycle if we cannot finds fixed $N1\ and\ N2 \in \mathbb{R}$ such that the time series of y_t in the United States economy experiences a recession for $N1$number of months and an expansion for a $N2$ number of months, for each cycle i. We also have asymmetries when $N1 \neq N2$ or if the amplitude of a phase is not the same as that of the opposite phase or is not constant over time for each phase.

Mathematically, we can define asymmetries in the following way: Business cycle asymmetries are present in the time series of Y when $\nexists\ N1\ and\ N2\ \in \mathbb{R}$ such that the time series of Y possess $N_{r_i} = N_{r_j} = N1$ and $N_{ex_i} = N_{ex_j} = N2\ \forall\ i,j$ cycles Y. We also have asymmetries when $Nr\ i \neq Nex\ i.$ A third case of asymmetry occurs if $a_{r_i} \neq a_{ex_i}$ for a given i, or if $\nexists\ a1\ and\ a2\ \in \mathbb{R}$ such that $a_{r_i} = a_{r_j} = a1$ and $a_{ex_i} = a_{ex_j} = a2\ \forall\ i,j$ cycles in the time series of Y.

1.3. Nonlinear Dynamics

As the Real Business Cycle theory of Kydland and Prescott (1982) would suggest, business cycles are the consequence of shocks to technology A_t. These shocks propagate to the rest of the Economy through intertemporal substitution in consumption C_t, leisure, and lags in investment I_t. However, New Keynesians criticizes the fundamentals of Real Business Cycle theory, the assumption of perfect price flexibility. Namely, prices can adjust rapidly to clear markets. Rotemberg and Woodford (1994) argue that the propagation mechanism comes from imperfect competitions and markups but do not differ much in there are analysis from classical economists in that the business cycle comes solely from shocks to the economy. The debate between the two schools of though is only with respect to the mechanism that propagates the exogenous shocks.

In economics, there are two types of models used to represent or simulate any economic activity: linear models and nonlinear models. We will make use of nonlinear dynamics in our macroeconomic settings by using nonlinear difference and differential equations.

Is qualified as nonlinear any model that cannot be written in the form

$$f(x) = a * x + b, with\ a, b\ \in \mathbb{C}.$$

In other words, a nonlinear system is one with the property that the change in output $f(x)$ is not proportional to the change in the input x.

Nonlinear dynamics are important to economists for many reasons. One of them is the various and complex dynamic outcomes that could be modeled. The traditional linear models can only capture a limited number of possible dynamic phenomena, which are basically convergence to an equilibrium point, steady oscillations, and unbounded divergence.

Nonlinear dynamic can capture both regular and irregular behavior, asymmetric cycles, and erratic behavior. Their ability to generate aperiodic time paths could make any statistician test insignificant (Finkenstädt, 1995).

Mastering nonlinear dynamics requires a good knowledge of the mathematical definitions of time paths or trajectories, attractors, basin of attraction, transient, iterative processes, phase curve, fixed points, aperiodic motions, dissipative systems, limit cycles, spaces of dimension n: $n \in \mathbb{R} \wedge n \notin \mathbb{N}$, and sensitivity to initial conditions to name a few. This should all be detailed and treated with scrutiny in future research, but in the present work, I will try to use nonlinear dynamics and apply them in the study of business cycle asymmetries while being as clear as possible to familiarize readers of different backgrounds with the subject.

In difference equations, an *equilibrium point* makes y_t stationary over time. In other words, an equilibrium point satisfies $y_t = y_{t+1}$.With differential equations, we say the equilibrium point y_e is *stable* if $\frac{\mathrm{dy}}{\mathrm{dt}}$ is negative for $y < y_e$ and positive for $y > y_e$. An equilibrium point is said to be *unstable* if it is not stable. (Gandolfo, 1980).

Suppose V is a one-dimensional function parametrized by an m-dimensional vector α: $V = V(x, \alpha), x \in \mathbb{R}, \alpha \in \mathbb{R}^m$. Then:

$$V(x, \alpha) = x_0 + \alpha_1 x_1 + \cdots + \alpha_n x_n, n \leq m.$$

is the Taylor expansion of V. V is a *structurally stable* function if the number and the character of the extrema of the function do not change when some of the α_i change (Gabisch and Lorenz, 1987).

Readers in the field should note that nonlinear dynamics, while offering a broader range of models for economists, require more difficult and advanced analysis then linear models do.

The remainder of the work is described as follows: in chapter 2, I give a survey of the literature around the importance of nonlinear models and how economists used to deal with business cycle asymmetries and I report their findings. In chapter 3, I examine the US macroeconomic aggregates and out of these observations I build a new theory in Chapter 4. I test the theory in chapter 5 and analyze its implications on many economic models and policies. Finally, I conclude with chapter 6.

CHAPTER 2

A REVIEW ON BUSINESS CYCLE ASYMMETRIES

For most economists, business cycle fluctuations are not periodic. Business cycle expansions characteristics do not match symmetrically recession phases. This is known as business cycle asymmetries. Görtz and Tsoukalas (2013) observed this phenomenon for many economies. In my research, I examine these asymmetries by making use of nonlinear dynamic models.

There are many tools and numerous methods to analyze business cycle asymmetries.

Some researchers looked at the learning behavior and expectations formations of economic agents to explain the observed asymmetries in the different phases of the business cycles.

Van Nieuwerburgh and Veldkamp (2006) build an exhaustive model where household face unlimited liability to try to show that one cause of business cycle asymmetries could be explained by learning processes and adjustment mechanism that differ during expansions and recessions. In other words, expectations of individuals would adapt to different phases of the business cycle. This adaptation, slow or quick, is what makes the behavior of the business cycle different during expansions and recessions.

Studying asymmetries in business cycles, Neftci (1984) defines asymmetric behavior, where gradual upward movements are followed by sudden and sharp drops. He opposes the use of consumption series and prefers examining series related to the production side. He further advices to use trend free variables so that one does not need to estimate and eliminate a trend component to get the empirical results. He also argues that studying asymmetric series using nonlinear time series model would be too difficult. Further, he remarks that "The first implication of the asymmetry from the point of view of economic theory is that models should be built that can explain this common phenomenon endogenously". This is indeed what my research will attempt to achieve.

Many economists do not see linear economic models helpful in dealing with business cycle asymmetries. The simplicity and easiness of computations of linear macroeconomic models is nonetheless associated with numerous costs.

Serrano et al. (2012) argue that while macroeconomic dynamic analysis is rigorous when it deals with linear macro models, it could lead to erroneous and false results if the real, inherent dynamics of the true model are nonlinear. They insist that one needs to examine his economic model on its complete parametric space to be able to form the right conclusions concerning the right policy to implement.

Robinson (1977) says that using linearity in economic time series models provides only an approximation to reality. He advocates to see the response of a discrete, observable, stochastic process $\{Yn\}$ to a sequence of zero-mean, orthogonal random variables, $\{Xn\}$, coming from some unknown, nonlinear mechanism. One way to identify such nonlinear mechanism is to look at irregular, transient behavior of a variable. In his estimation of a nonlinear moving average model, he introduces an interaction term between the random variables and estimates

$$Yn = Xn + \alpha Xn{-}1 + \beta XnXn{-}1, \text{ n} = 1,2, \ldots$$

where β captures the nonlinearity effect on Yn.

According to Finkenstädt (1995), from the practical point of view, a statistician should be aware that applying the common linear statistical apparatus to a time series that is subject to a nonlinear generating mechanism can indeed lead to mistaken conclusions.

Speight (1997), in his study of 16 OECD economies, recalls that macroeconomic variables behave differently in expansions and recessions: as asymmetry appears, expansions are more gradual and recessions very sharp and volatile. This is in accordance to the observation of Neftci (1984). Speight stresses on the necessity to develop and use nonlinear dynamical models to attempt to explain the perceived business cycle asymmetries. He also states clearly that linear models are not perceived to be of any help in this field. Further, he enumerates some methods applied to measure asymmetries: for example, applying the theory of Markov chains to stationary transforms of business cycle indicators, with very little evidence of asymmetries found. Another example would be applying tests for generic nonlinearities in macroeconomic aggregates, and specific nonlinear alternatives to linear autoregressive processes. These tests work well but cannot distinguish between the different types of asymmetries. Finally, the Sichel (1993) tests of different types of asymmetries, which the Speight uses in his own study to the monthly industrial production indices of 16 OECD economies and the OECD aggregate. These tests are able to find asymmetries where the latter is defined in terms of degrees of skewness in stationary series. Asymmetries are characterized them by two types, steepness and deepness. Speight applies the Sichel tests and uses the Hodrick-Prescott filter method of detrending, since these tests are to be used with stationary series. He concludes that his study provides evidence against the use of econometric and linear models with symmetric noise.

Wecker (1981) exposes a property of asymmetric time series, the delay. He states that most of the economic times series models, like the AR model, the MA model and the mixed model, do not reflect asymmetric delay property. These models transform the random shocks sequence $\{u_t\}$ by a linear rule filter to explain the observed sequence $\{x_t\}$. Wecker uses an asymmetric time series model because it treats the random shocks sequence $\{u_t\}$ differently depending on the sign of each u_t . The model applies two distinct filters to $\{u_t\}$, one when u_t > 0 and one when u_t < 0. If the two filters are identical, then we revert back to a symmetric model, but if they differ, $\{x_t\}$ displays more complex behavior and its dynamics depends on the sign of u_t . The author uses a very simple asymmetric model: the asymmetric moving average process of order one, or univariate asMA model, which is given by

$$x_t = u_t + \beta^+ u_{t-1}^+ + \beta^- u_{t-1}^-$$

$where\ \{\{u_t\}\ is\ an\ i.\,i.\,d.\ sequence\ of\ random\ shocks,$

u_{t-1}^+ = max$\{ut$−1, 0}, positive random shock,

u_{t-1}^- = min$\{ut$−1, 0}, negative random shock,

β +and β − differentiate between the two filter.

However, in the author’s work, asymmetries originate from exogenous, random shocks $\{u_t\}$ to the economy.

As Altuğ (2009) indicates, in a new economics literature, macroeconomic time series are shown to exhibit marked nonlinear behavior. These nonlinearities could be of the form of conditional heteroscedasticity such as ARCH or GARCH effects or in the form of asymmetries.

Ashley and Patterson (1989) developed a test to test for deviation from linear stochastic processes, either in the from nonlinear stochastic dynamics or deterministic chaos. They found strong evidence of nonlinearity industrial production, and argued any reasonable macroeconomic model should display some form of nonlinear dynamics.

Valderrema (2007) examines the statistical behavior of National Income and Product Account aggregates for the US and a set of OECD countries including France, Italy, Japan, Mexico and the United Kingdom, and shows that nonlinearities such as skewness, kurtosis and conditional heteroscedasticity are common for many of these aggregates.

Clements and Krolzig (2003) stress on the fact that if business cycle asymmetries could be identified, then linear models with symmetric errors should be abandoned. They adopt the definition of Sichel (1993) for asymmetric cycles and enumerates some types of asymmetries in business cycles like steepness, deepness, and sharpness. Other types include asymmetric persistence to shocks and business cycle duration dependence. They use MS-AR to generate steepness, deepness and sharpness asymmetries. Further, they build tests that they found able to detect asymmetries in the propagation mechanisms of shocks, or first-moment asymmetries. The asymmetries they examine are exogenous and are reflected in detrended variables. However, dependent variables in business cycles are nonstationary. Hence, to remove nonstationarity in their analysis, they set

$$xt = \Delta yt$$

where yt is the log of output. However, the findings of asymmetries might depend of the method of detrending used, and the authors suggest that this problem requires more work and research.

Chauvet (1998) explores the early work of Burns and Mitchell (1946) and remarks that their study lacks the mathematical depth of a probability model that could generate the time series used in identifying business cycles patterns. He proposes a probability model that enables economic analysis of business cycles in real time:

> "For example, the imminence of a recession starting in a certain month can be detected by the inferred probabilities or by the implied coincident indicator at the same time the macroeconomic variables are signaling the recession." (Chauvet 1998)

He uses a dynamic factor model with regime switching. His paper models the idea of business cycles as the simultaneous movement of economic activity in various sectors by using an unobserved dynamic factor. In addition, the asymmetric nature of expansions and contractions is captured by assuming that the underlying factor switches regimes according to a Markov process. Chauvet's analysis demonstrates clearly to the reader that one needs to abandon the idea of linear models and use many economic variables in a single model in order to account for business cycle fluctuations more accurately.

Gabisch (1984) examines how differential and difference equations can be used to analyze business cycles. He states that "[…] it is worthwhile to distinguish business cycle models according to their formal structure, and not according to their underlying economic causes." In this respect, one should consecrate more time dealing with the mechanism of the business cycle rather than examining how asymmetries appear in the data, what are the effects of some variables and whether theses variables affect the dependent variable in a symmetric way or not. Hence, the motivation of this thesis is to focus on the mathematical structures of the business cycle mechanism and to incorporate asymmetries endogenously in business cycle models.

I find therefore that the literature is almost old, since the major work in the subject of business cycle asymmetries has been done before the year 2000. It seems that economists were discouraged by the difficulty of the subject matter. Moreover, most of the literature concentrates on how to test for business cycle asymmetries and how do these asymmetries materialize in the data. Most of the research on business cycle treat asymmetries as effects of exogenous shocks, and not as endogenous components of business cycles. Few researchers try to identify the economic and mathematical mechanism that generates such asymmetries, as I attempt to do in my present work, almost 25 years later.

CHAPTER 3

EVIDENCE ON ASYMMETRIES

In this chapter we will try to identify business cycle asymmetries on the data available to us by observing different time plots and drawing comparisons. We shall try to analyze the time series of the United States in this work.

Gross Domestic Product, or GDP, is computed with the following formula.

$$Y_t = C_t + I_t + G_t + NX_t$$

The price level P_t and the money supply MS_t are considered nominal variables unless specified real.

Unemployment U_t, real wage $\frac{W_t}{P_t}$, and average labor productivity are considered Labor Market Variables.

When a business cycle is identified based on the (absolute) downturn of the level of output, it is called a classical business cycle. This is how the NBER identifies cycles. When we consider the decline in the series measured as a deviation from its long-run trend, we are talking about growth cycles.

Data is taken from the Federal Reserve Bank of Saint Louis official website, BEA and BLS.

3.1 Data with Trend and other Series

The following figures show the time series evolution of real GDP and its different components using seasonally adjusted at annual rates data. It is clear that these macroeconomic aggregates are following a trend and are hence qualified as nonstationary series.

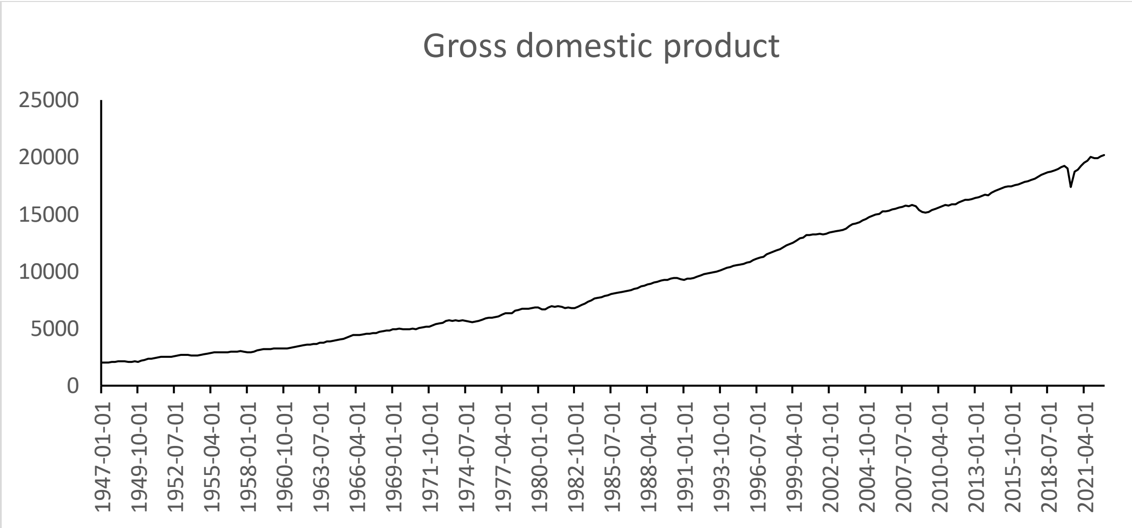

Figure 1. US Real GDP Time Series, SA, in Chained 2012 dollars, Quarterly Rates. Data ranges from 1947 Q1 to 2022 Q4. Source: U.S. Bureau of Economic Analysis

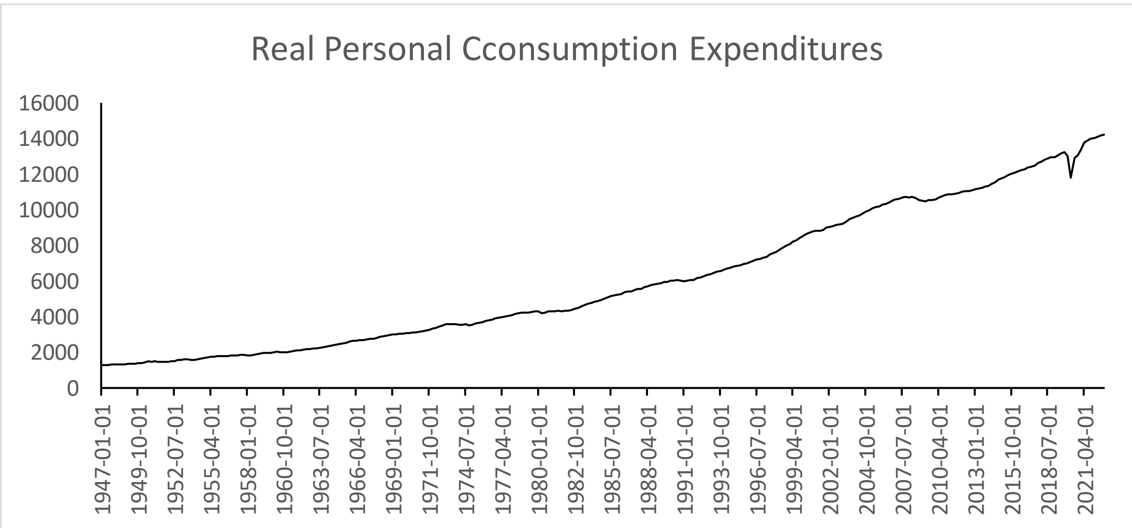

Figure 2. US Consumption Time Series.

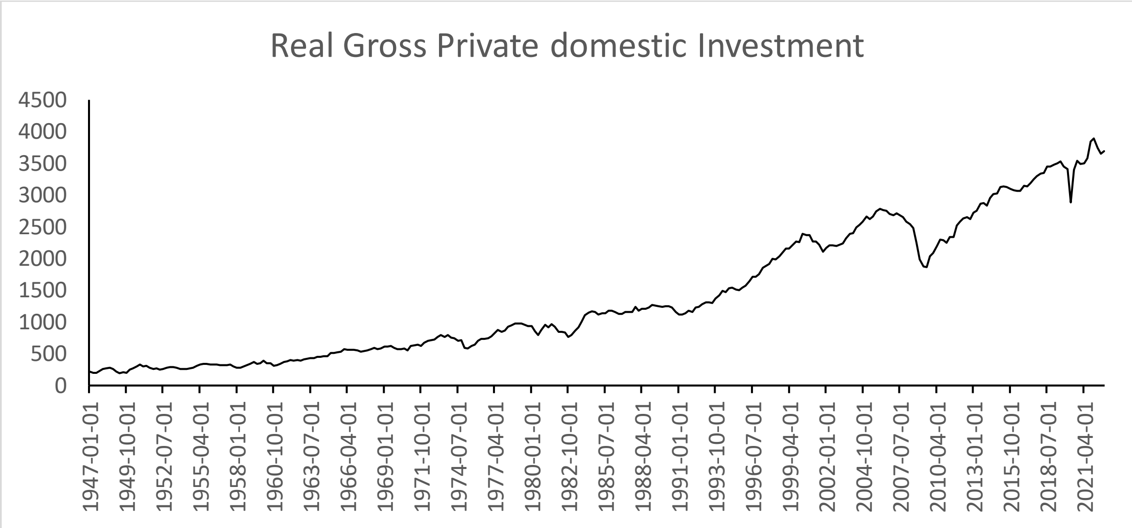

Figure 3. US real Investments Time Series.

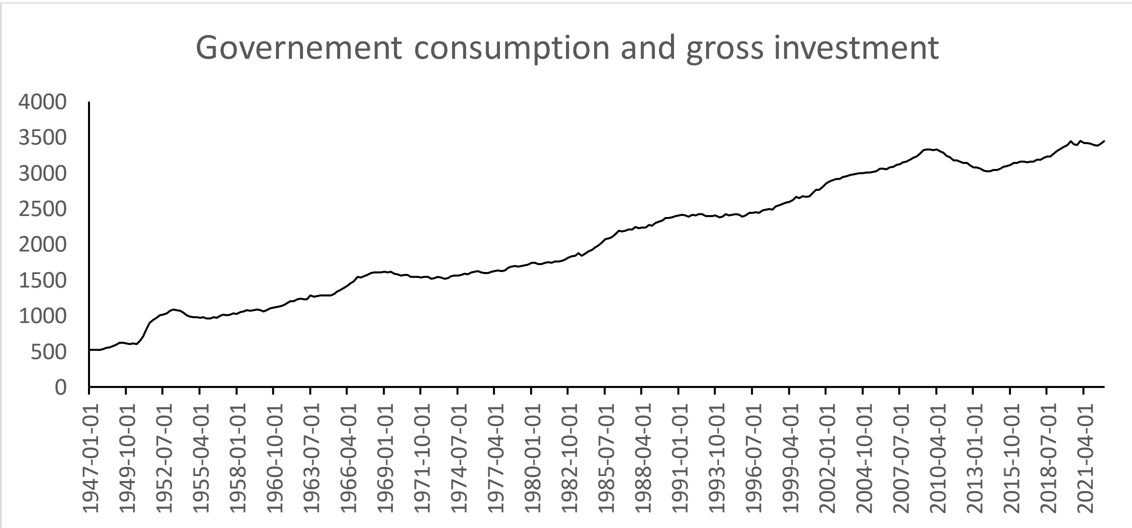

Figure 4. US Government Spendings Time Series.

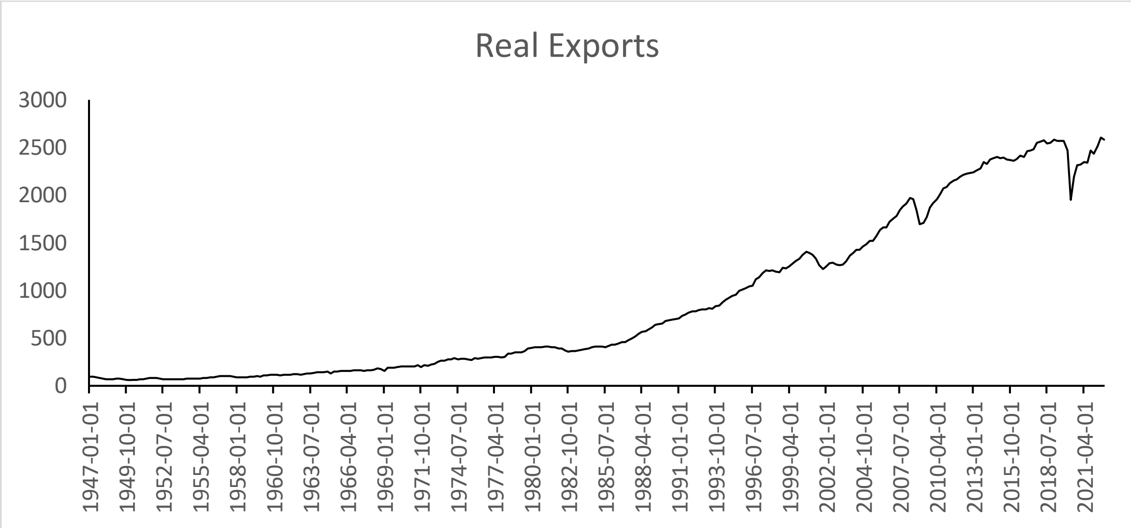

Figure 5. US Real Exports

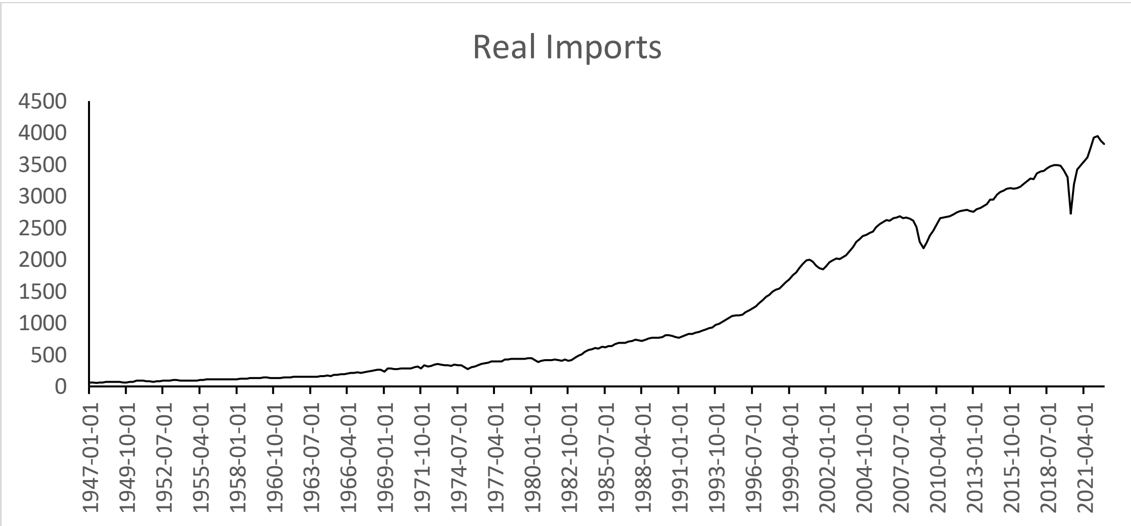

Figure 6. US Real Imports

We now take a look at the time series that represents the evolution of monetary aggregates growth rates, inflation rates, unemployment rates, and GDP growth rates.

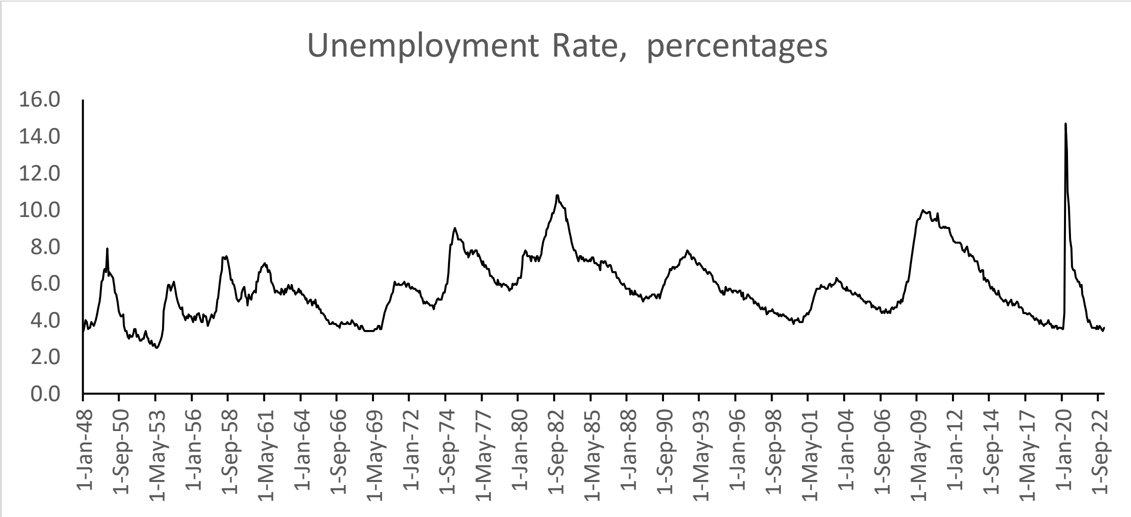

Figure 7. U.S. Unemployment Rate in percent, Monthly. Data ranges from January 1948 to February 2023. Source: U.S. Bureau of Labor Statistics.

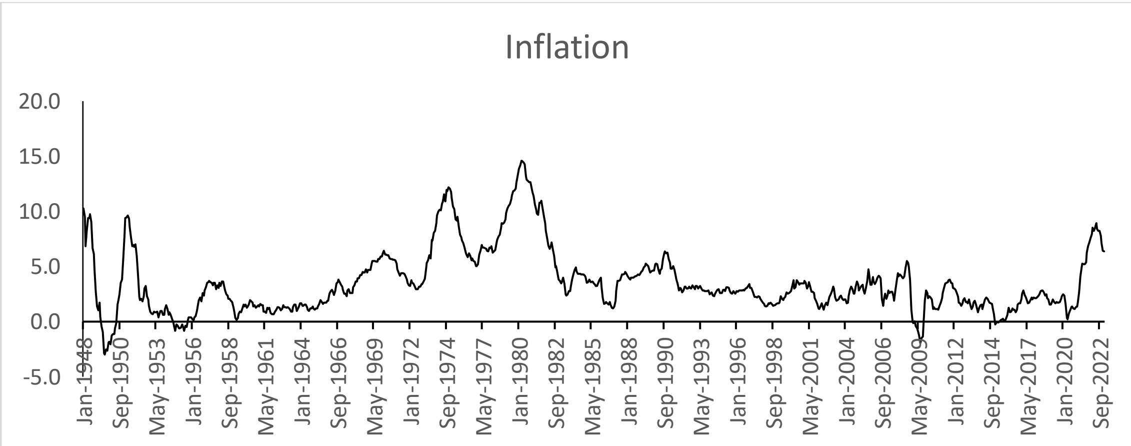

Figure 8. US Consumer Price Index for All Urban Consumers, CPI-U, in Percentages, Monthly, NSA. Data Ranges from January 1948 to January 2023. Source: U.S. Bureau of Labor Statistics.

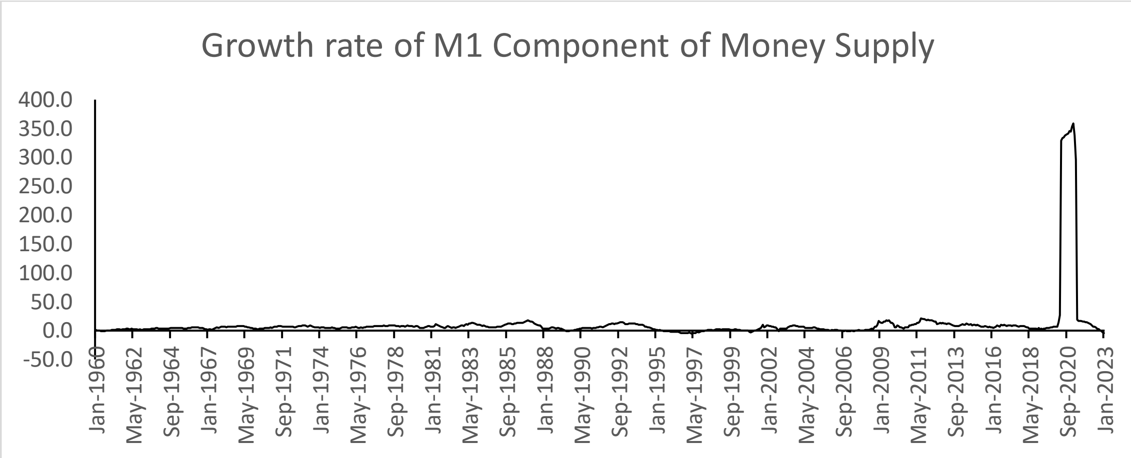

Figure 9. US Growth rate of Money Supply, Percentage Change from a Year Ago, Monthly. Data ranges from January 1960 to January 2023. Source: Federal Reserve Economic Data, Federal reserve Bank of Saint Louis.

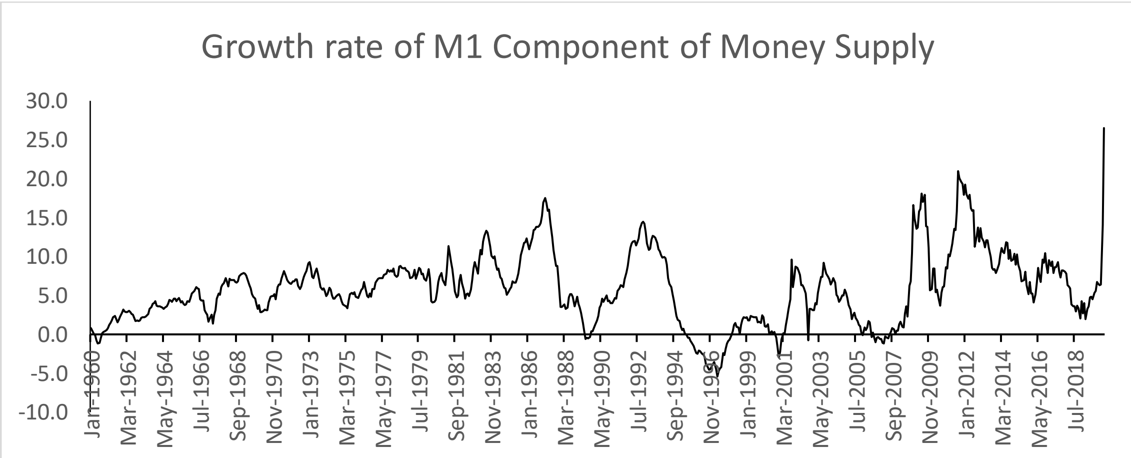

Figure 10. US Growth rate of Money Supply, Percentage Change from a Year Ago, Monthly. Data ranges from January 1960 to April 2020. Source: Federal Reserve Economic Data, Federal reserve Bank of Saint Louis.

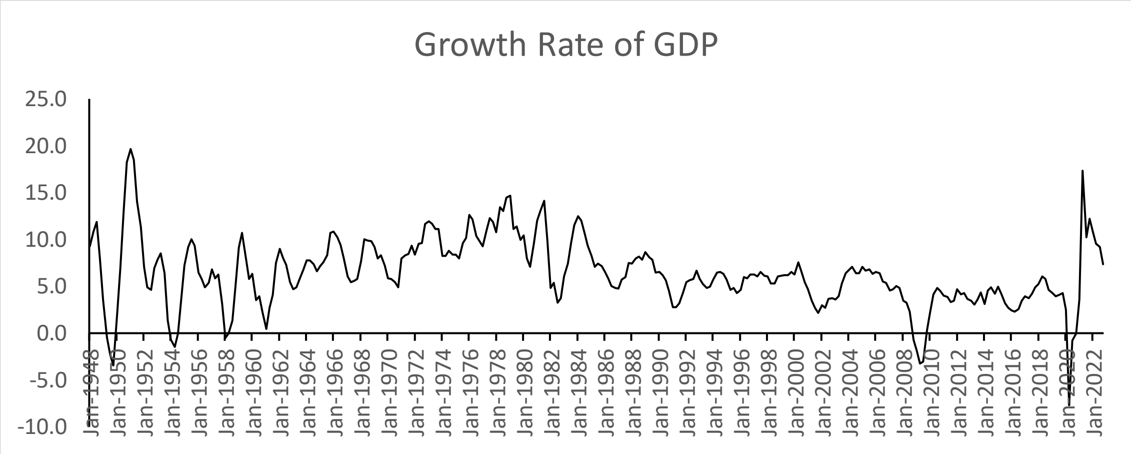

Figure 11. US Growth rate of nominal GDP, Percentage Change from a Year Ago. Quarterly. Data ranges from Q1 1948 to Q4 2022. Source: U.S. Bureau of Economic Analysis.

The Unemployment rate of Figure 7 shows much more cyclicality and is more or less stationary. Moreover, sudden and sharp increases in U_t are clear in most NBER confirmed recessions while decreases in U_t appear more gradual and take more time during expansions.

In the GDP growth rate time series of Figure 11, we see that the growth rate was around 5% for about eight years, between 2011 and 2019. Then in 2020 the growth rate suddenly decreases to -7% and stays in the negative region for only 6 months. Were the use growth cycle symmetric, we would expect the growth rate of GDP to stay in the negative region of -5% for about eight years. But this is not what happened in reality.

We can conclude that cyclicality is not very well observe looking at raw data observations of nominal GDP. However, the growth rate of GDP depicts business cycles more clearly and asymmetries start to appear. Finally, unemployment series show exactly what business cycle asymmetries are about in terms of amplitude and duration of the different phases.

3.2 Removing the Trend Component

Figures 1 to 6 indicate that the macroeconomic variables are following a trend or are growing exponentially with time. In other words, these time series are not stationary. Since a non-stationary time-series can be decomposed into trend and cyclical component, we can write:

$$y_t = g_t + v_t$$

Here g_t is the trend component and v_t is the cyclical component.

The HP filter, worked out by Hodrick and Prescott (1981), deals with time series that tends to infinity in theory but is used to calculate the trend component g_t of time series with finite observations in practice. Now, we remove the trend component of the real GDP time series of figure 1 using the HP filter. For the HP filter, I selected λ to be equal to 1600 since it is the value that is typically chosen for dealing with quarterly data (Canova, 1998). To obtain the cyclical component of GDP, we just perform the following calculation:

$$v_t = g_t - y_t$$

Hence, we can represent the newly obtained time series as follows:

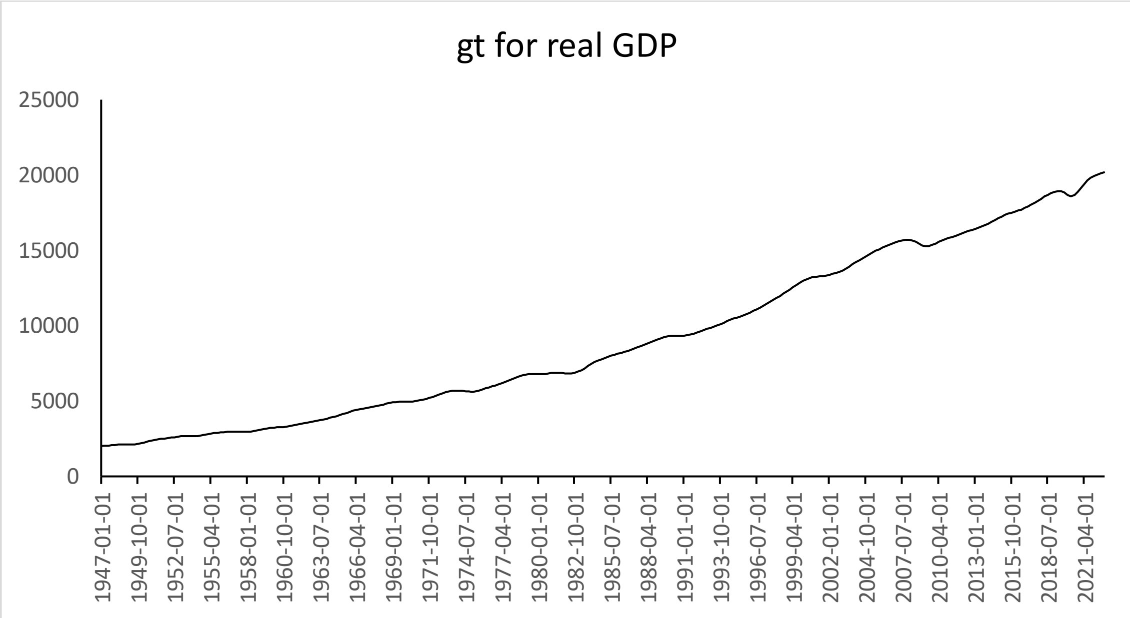

Figure 12. HP Trend Component of real GDP, SA. Quarterly. Data ranges from Q1 1947 to Q4 2022.

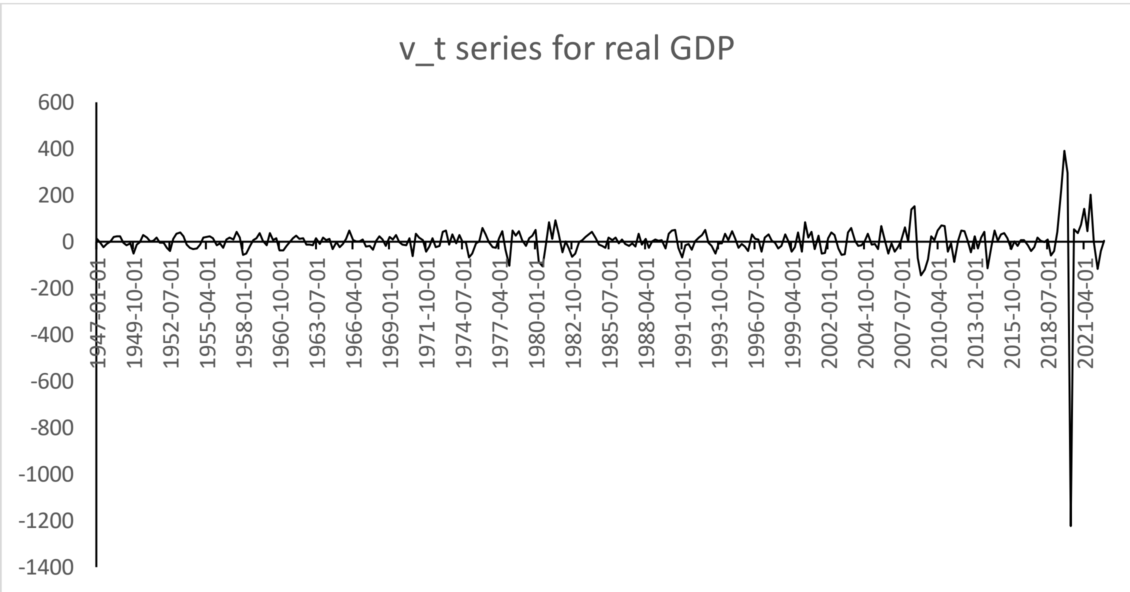

Figure 13. Cyclical component of Real GDP, SA. Quarterly. Data ranges from Q1 1947 to Q4 2022.

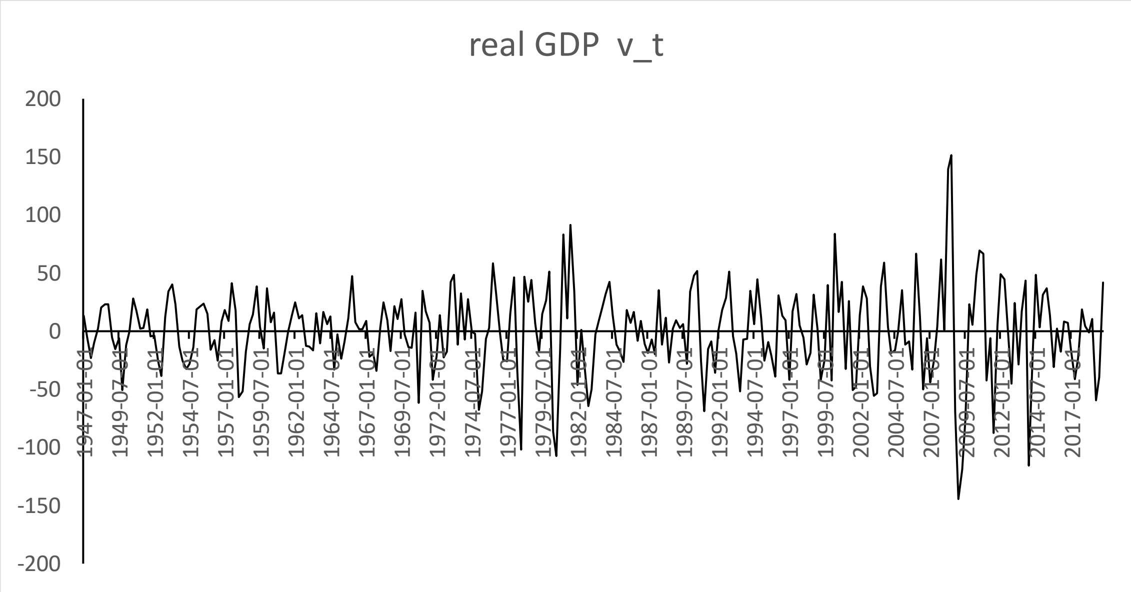

Figure 14. Cyclical component of Real GDP, SA. Quarterly. Data ranges from Q1 1947 to Q2 2019.

After removing the trend component, we see that the cyclical component of GDP is subject to repeated and somehow chaotic oscillations. We can conclude that, in the light of Altug, Ashley and Patterson (1999), there could be some nonlinearities in generating mechanism of real output.

3.3 The Logistic Map

We are given the function:

$$f(x) = ax(1 - x), \text{ let } x \in [0, 1] \qquad (3.3.1)$$

This function is known as the logistic map. It has been thoroughly studied by Lorenz (1963), Li and York (1975), May (1976), and Hoppensteadt and Hyman (1977) to name a few.

Now, let $x_{t+1} = f(x_t) = 4x_t(1 - x_t), a = 4$. We have a nonlinear difference equation. Choosing $x_0 = 0.33$, we show in the following figure the evolution of the $\{x_t\}$ sequence as time evolves.

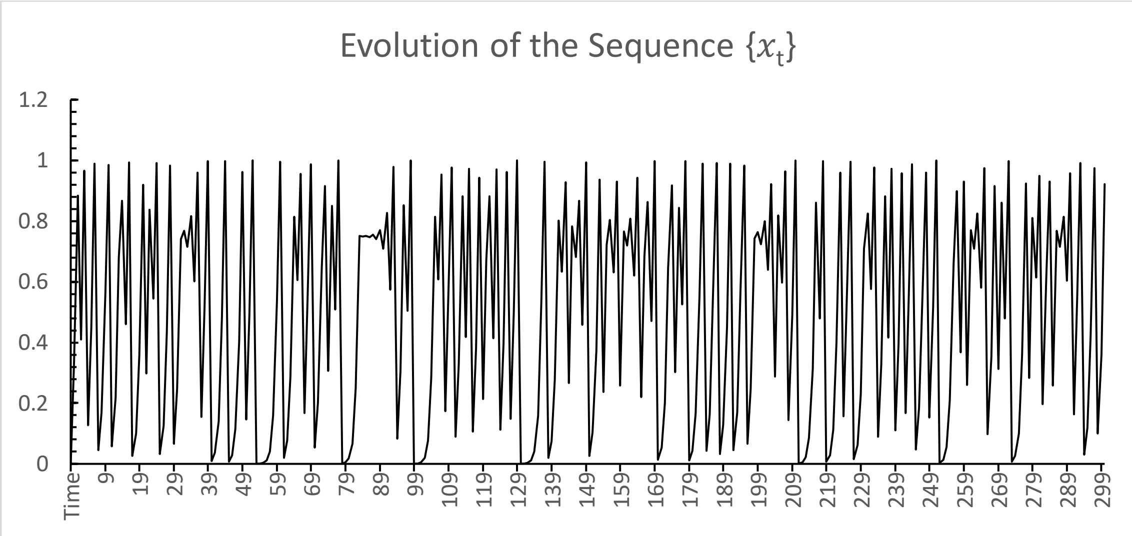

Figure 15. Logistic Map. First 300 iterations starting at $x_0 = 0.33$

Hence, we observe a similarity between Figure 15 and Figure 14. Fluctuations appear to be numerous and unpredictable in the case of the chaotic Logistic Map as well as for the cyclical component of GDP, v_t.

CHAPTER 4

THEORY

We can now ask the following question: what do the Logistic Map examined in Chapter 3, Differential Equations and the Exponential Growth have in common? We shall get the answer to this strange question by the end of this chapter.

Suppose we have the first order homogeneous linear ordinary differential equation:

$$\frac{d}{dt}N(t) = r * N(t) \qquad (4.1.1)$$

where r is the rate of increase in the variable of interest $N(t)$, whose behavior is depicted by $(4.1.1)$. The family of solutions is $N(t) = a * \exp(r * t)$, with a being a parameter that depends on initial conditions. Here the variable $N(t)$ grows exponentially at the constant rate r.

Suppose now that r is not constant but varies with the level of the variable of interest. Hence, (4.1.1) can be written into $\frac{d}{dt}N(t) = r(N(t)) * N(t)$. Further, letting

$$r\big(N(t)\big) = A - b * N(t) \qquad (4.1.2)$$

also implies that that r varies with time but through the variable of interest N(t). In this manner, there is a negative linear relationship between the rate of growth r and the variable of interest $N(t)$. If $N(t)$ increases as time increases, then r would also be negatively related to t. We now have:

$$\frac{d}{dt}N(t) = N(t) * \big(A - b * N(t)\big) \qquad (4.1.3)$$

The write hand side of $(4.1.3)$ is again a generalization of the logistic map in $(3.3.1)$. Here we use it in a nonlinear autonomous first order differential equation instead of assigning the value of consecutive terms of a sequence $\{x_t\}$ to it.

It is worth mentioning to the interested reader that $(4.1.3)$ is also a special type a more general Bernoulli Differential Equations:

$\frac{d}{dt}N(t) + Z(t) * N(t) = W(t) * N(t)^m$, with $m \in \mathbb{R} - \{0; 1\}$.

Also, $(4.1.3)$ is a special case of differential equations of the form

$$\frac{d}{dt}N(t) = \frac{k}{a} * \big(a - N(t)\big) * N(t)$$

that are used to model sigmoid functions for positive input t.

We are now interested in the solution of $(4.1.3)$. Using integral calculus, we obtain:

$$N(t) = {}^{A}/_{b} * \left(1 + \frac{{}^{A}/_{b} - N(0)}{N(0)} * e^{-A*t}\right)^{-1} \qquad (4.1.4)$$

Analytically, we understand $N(0)$ to be the initial value taken by the variable $N(t)$ at the first period $t = 0$. When I apply (4.1.3) in the next chapter, I obtain the same solution form for all macroeconomic aggregates and I call the solution with the Yt time series the *Charlijo True Economic Trend Evolution Equation* or $CTETE\ Equation$

Going back to (4.1.3), we want to analyze the behavior of the solution function. Since $r = A - b * N$ then there is a non-zero value of N for which r will be 0 and ultimately dN/dt will be 0, so that a stable equilibrium value of $N(t)$ will be attained where $\frac{d}{dt}N(t)$ is negative for N smaller than this particular value and positive for greater N.

In fact, r could be 0 is N=0 but this is an unstable equilibrium. At this particular non-zero value, $N(t)$ will not grow anymore as time elapses. This value of N(t) is the maximum number that the function $N(t)$ can attain and corresponds to the limit of the function itself. In other words, $N(t)$ converges to its maximum value as t goes to infinity. This is visible in (4.1.3) where the $\lim_{t\to+\infty} N(t) = \frac{A}{b}$.

Now, we are interested in the equation of the derivative $\frac{d}{dt}N(t)$. Differentiating (4.1.4), we obtain:

$$\frac{d}{dt}N(t) = (-A^2 N(0)(bN(0) - A)\exp(At)) * (bN(0)(e^{A*t} - 1) + \mathrm{A})^{-2} \qquad (4.1.5)$$

Now, using (4.1.3) we find that $\frac{d}{dt}N(t)$ attains its maximum at $N = \frac{A}{2b}$. This value of $N(t)$ corresponds to

$$t = \frac{\ln\left(\frac{A}{b} - N(0)\right) - \ln\big(N(0)\big)}{A} \qquad (4.1.6)$$

This also implies that the greatest growth rate of N(t) is equal to $A^2/4b$ at that time.

CHAPTER 5

QUANTITATIVE ANALYSIS

In this chapter, we will apply the theory studied in chapter 4 to the US macroeconomic aggregates time series and derive conclusions about the newly proposed hypothesis on the dynamical evolution of GDP. Many implications on economics theory and practices follow. US y_t aggregates are in billions of chained 2012 dollars. Yt, C, I NX, and G are in billions of current dollars. Variables in capital letters are nominal variables unless preceded with the term "real". Small case variables, like y_t, represent real variables. We use the M2 definition for Money Supply and the subsequent Velocity. One period if defined as one quarter in all the time series used in this research and in all subsequent computations. Data is retrieved from the official websites of the Federal Reserve Bank of St Louis, BEA, and BLS.

5.1 Applying the theory to US real GDP

In this section, we examine the relationship between the rate of growth r and the variable of interest. We can model r by setting $r_{y_t} = A - b * y_t$.

In order to estimate r, we compute it as follows. If the dynamics of the macroeconomic variable of interest, say $y(t)$, follow equation (4.1.1), we can calculate the rate of growth r by setting $r = y(t)^{-1} * \frac{d}{dt}y(t)$. Since our measurements are discrete and one period is a quarter, we find $\frac{d}{dt}y(t)$ by setting $\frac{d}{dt}y(t) \approx \frac{\Delta yt}{\Delta t}$. We choose to write $r_{Yt,i,t} = A - b * yt$ when $\Delta t = i$ quarters for each measurement of r. We now regress the $r_{yt,i,t}$ variable on yt according to $(4.1.2)$. Hence, for US real GDP series computed with chained 2012 dollars, yt, and setting $i = 10$ we obtain the following equation of r:

$$r_{y_t,10} = 0.010676 - 3.364635 \times 10^{-7}\, y_t$$

With the above findings, and (4.1.4), we set $t = 0$ at Q1 1947 and we can write the equation of the dynamics of US real GDP as follows:

$$y(t) = 31730 * \left(1 + \frac{31730 - 2034.45}{2034.45} * e^{-0.010676*t}\right)^{-1}.$$

I call the above equation the Charlijo True Economic Trend Evolution equation, or CTETE equation of US real GDP. Further, when time extends to infinity, $y(t)$ converges to $\lim_{t\to+\infty} y(t) = 31{,}730$ billions of chained 2012 US dollars.

Below is the graph of the trend from Q1 1947 to Q4 2080 and the actual available time series $y(t)$.

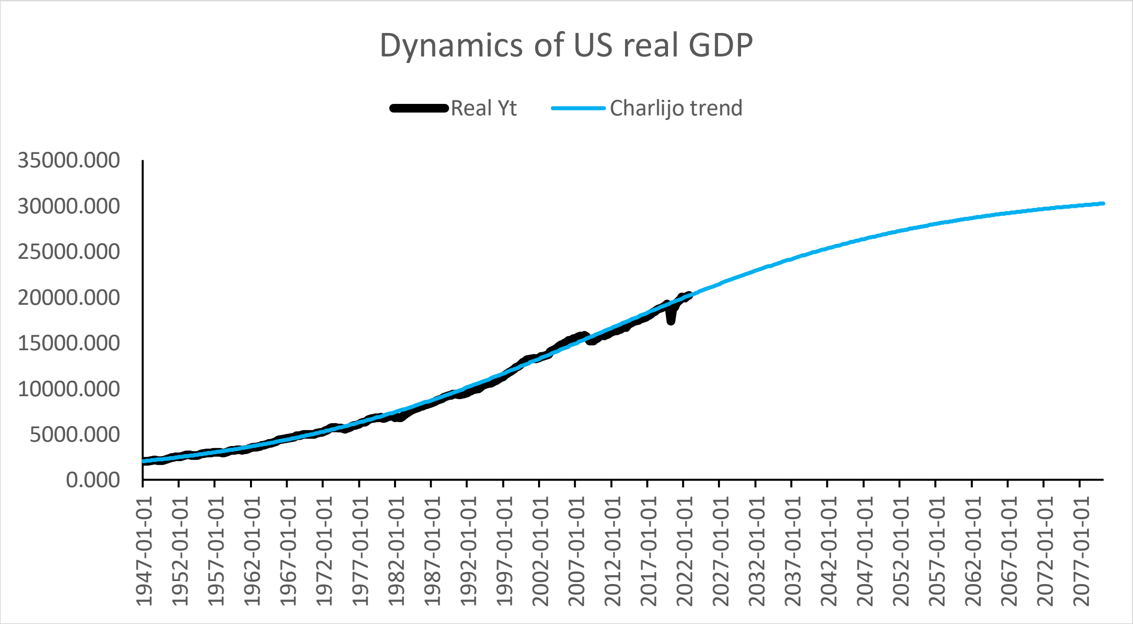

Figure 16. US real GDP time series in billions of 2012 chained dollars in black and its corresponding CTETE equation time series in blue.

To evaluate the results of my work, I regress the following equation:

$$y_t = \beta * \tilde{y}_t + e_t$$

Again, y_t is US real GDP and $\tilde{y}_t$ represents the CTETE time series. I use all the data available from Q1 1947 to Q4 2022. The theory suggests that β should be 1.

The regression results show that β equals 0.9869, with t-stat equal to 605 and R^2 equal to 0.999. Hence, we can write the following equation relating the actual observed time series y_t to its corresponding trend component $\tilde{y}_t$:

$$y_t = \tilde{y}_t + cj_{y,t}$$

Where $cj_{y,t}$ represents the deviation from trend.

Now I repeat the same steps for $\dot{y}_t$ and its corresponding CTETE trend. The trend is in blue and the time series in black.

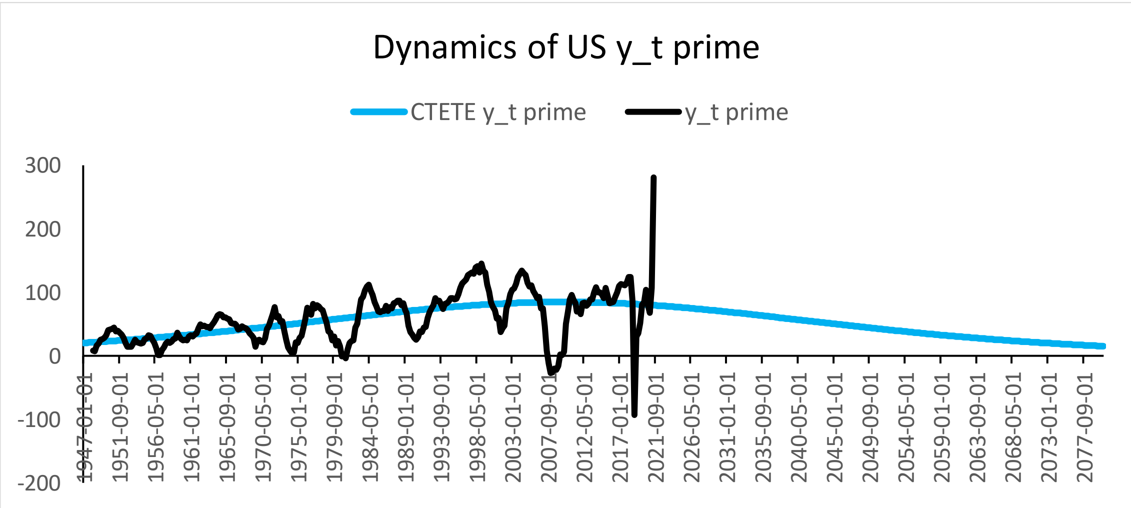

Figure 17. US y_t prime and its corresponding CTETE trend in blue. From Q1 1947 to Q4 2080.

Again, to evaluate the significance of the work, I run the same form of regression and I find that β equals 0.9945, with t-stat equal to 32.5 and R^2 equal to 0.783. Indeed, results are again very significant.

5.2 Business Cycles Identification

We can compare the deviation series v_t derived from the HP filter with the $cj_{y,t}$ deviation series of the new proposed dynamics in section 5.1. In figure 18, we plot deviation series in black and the NBER-defined recession indicator in orange. The indicator works as follows: for recession quarters, the indicator shows +1 and 0 otherwise. Hence, we see that each NBER-proclaimed recession period corresponds very well to periods of decrease in the deviation series $cj_{y,t}$. This is in contrast with the v_t series of the HP filter where it appears that there is no correlation between officially NBER-approved recessions and the v_t series itself as it is shown in the below figure.

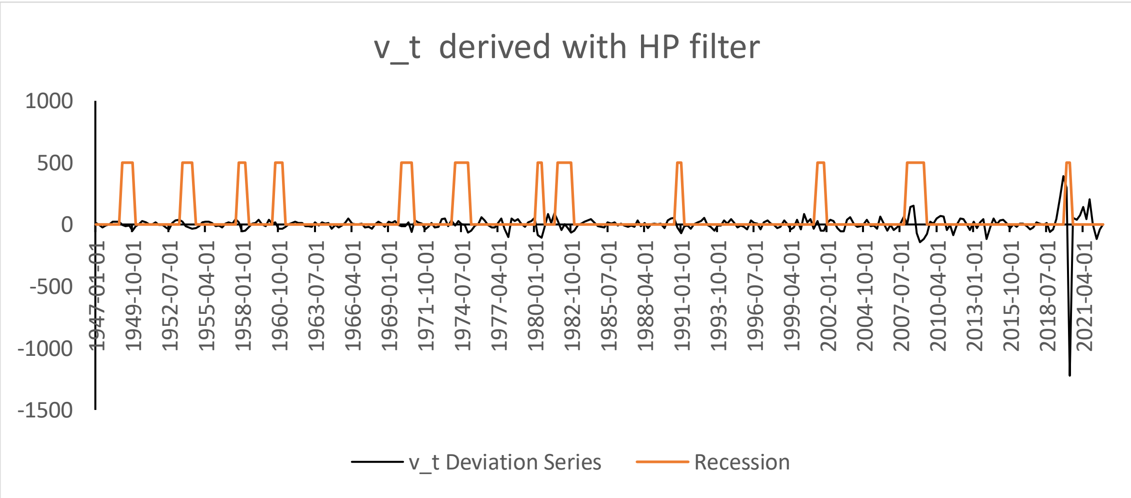

Figure 18. Deviation series with HP filter. No particular relationship between v_t and recession periods is visible.

It is worth noting that the NBER looks at absolute downturns and thus examines classical cycles and not growth cycles. But my work's growth cycles are confirming the NBER classical cycles.

Looking at the below figure, it is clear that any deviation from the Charlijo Trend leads to inverse phases where the business cycle adjusts itself and comes back to the True Long Run Trend. Cycles appear clearer when CTETE is applied to the variable of concern.

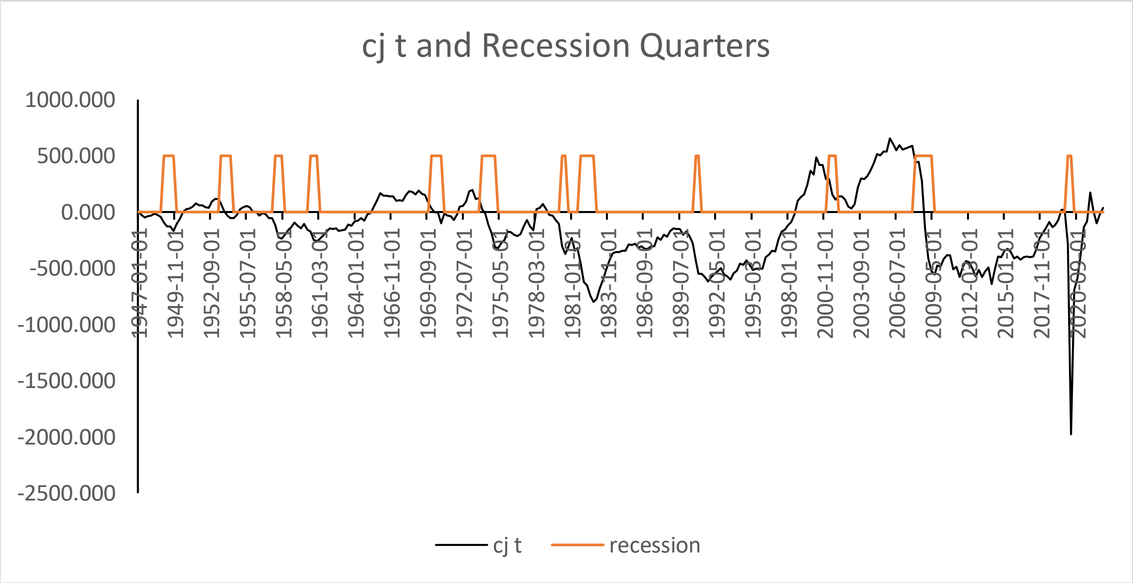

Figure 19. $cj_{y,t}$ deviations series of the real GDP series with chained 2012 prices

5.3 Money Supply and The Velocity of Money

We apply the nonlinear differential equation dynamics exposed in chapter 4 to MS and nominal GDP time series. The results are simply stated:

$$r_{Yt,10} = 0.01869 - 5.0216 \times 10^{-7}\, Yt$$

$$r_{MS,10} = 0.01856 - 4.9942 \times 10^{-7}\, MS_t$$

Proceeding in the same way as earlier, we find the limits of $Y(t)$ and MS(t) equal $\lim_{t\to+\infty} Y(t) = \$\,37{,}218.5$ Billion and $\lim_{t\to+\infty} MS(t) = \$\,37{,}167.4$ Billion. So as time evolves, the two series get closer to each other and converge to the same value.

Subsequently, we plot the evolution of Velocity of money as a function of time and we observe that velocity, the ratio of Yt to MS, approaches the value of 1, where in theory, it should converge to. This is indeed what the theory exposed above suggests: in the Charlijo True Long Run, real GDP attains a maximum at which capacity is fully used, productivity is at its maximum and Money Supply is also at its maximum value. Moreover, MS and Yt have the same maximum or TLR value.

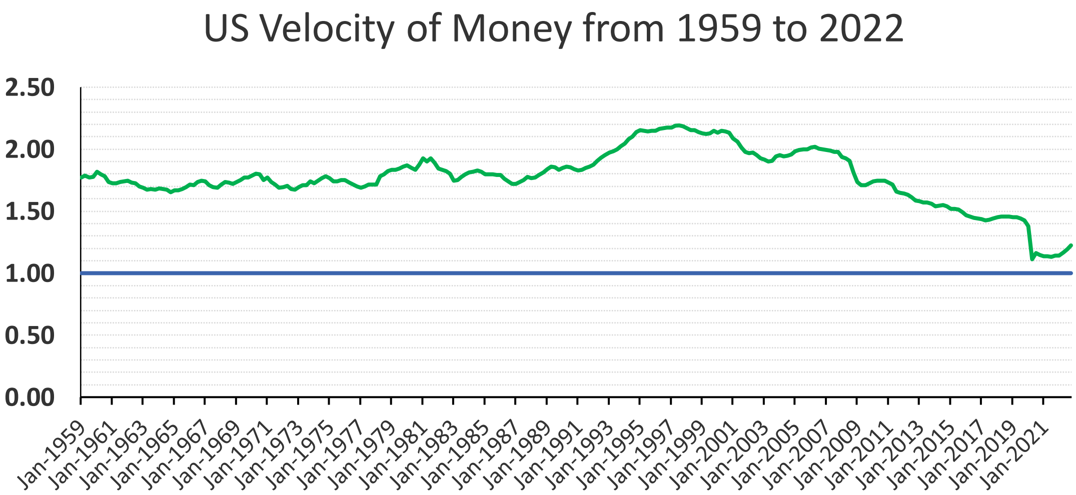

Figure 20. US Velocity of Money is decreasing with time and approaching 1.

In the True Long Run stage, Real money supply attains a maximum. This maximum corresponds to the highest level of real MS above which the cost in real terms to increase real MS is higher than the value represented by the newly created real MS. This implies that the limit of V in the Charlijo True Long Run tends to the ratio of the latter two variables' limits. Since Yt and MS converge, V should converge, from calculus theory.

I argue that Money supply cannot decrease with time. This is evident and explained in the following way: Because of continuous learning and experiments during the production process, new techniques and innovations are being developed and used so that eventually households and businesses are able to buy more and invest more to get higher rates of return. Further, as a result of the need for more transactions, as Keynesians would suggest, more money will be demanded from businesses and consumers and thus interest rates would rise, impeding the economy's evolution. Even if velocity increases with lower MS available, time does not change and there would always be impediments to the production process since the production is not instantaneous. Hence, agents would go to a new money other than that being controlled by the central bank because the newly available amount of money supplied will not suffice to produce as much as the economy required before any decrease in MS. So, any decrease would represent enormous and impeding costs for the country and its production. Hence, MS should not be decreasing with time.

Moreover, V should converge to 1 since V decreases with MS, and Yt and MS converge to the same TLR value. In fact, V cannot go below 1 since this would have two implications: the first is that the value of MS would be higher than the value of nominal GDP. But then, in real terms, the value of GDP would decrease to reflect the real value relative to the real MS available until they are equal. The second implication is that with $V < 1$, there would be some created amount of money that is not used at all, in the economy. This idle amount would be equivalent to 0, which would bring V back to 1.

Velocity cannot increase with time since it would require money to change hands faster, but this cannot continue forever since agents exchange goods and services in the economy and the production process is not instantaneous. Moreover, the period, one quarter, is a fixed amount of time in which velocity itself is calculated. If production was instantaneous, then we can imagine GDP to increase infinitely since every new period, production would take half the time it took in the previous period.

Hence, we have established that MS and V are monotone sequences that are bounded from above and below respectively. This means that MS and V are converging sequences and their product is a limit in $\mathbb{R}$. Hence, real and nominal GDP would converge to a maximum value, in theory. Moreover, the maximum TLR value of MS(t) should equal the TLR value of Y(t).

5.4 New Perspective for Business Cycle Asymmetries

The new discovery proposes that any economy has three phases during its evolution: The starting phase, the second or Transitory phase and the True Long Run phase.

In the starting phase, the country produces what is evident and easy to produce using part of the potential of the available factors of production. Growth is not significant in this stage as the production process is just repetitive from period to period, but agents participating in the production process notice and learn from their experience and start developing new techniques and innovations to be used in production, though with little to no success.

In the Transitory phase, the right and efficient new methods of production start to be implemented and productivity starts rising as efficiency and capacity utilization increase. Agents start to make use of more and more of the potential of available factors of production that are present in the economy. GDP growth starts rising. In the same time, new discoveries, experiments and techniques continue to be developed and enhanced as agents learn more and more about the potential of the available factors of production. Asymmetries could be portrayed as such: since GDP growth is positive in this stage, any positive deviation from trend is represented by gradual increases. Any subsequent decreases in GDP growth should be steep and short because the trend in GDP growth is in general positive at this stage. Recessions are steep because there is a need in the economy to compensate for the amount and time of positive deviations by reflecting them in the opposite direction. However, as GDP growth is positive in this phase, recessions do not last much as the actual GDP series needs to adjust and continue its evolution process. In Figure 19, showing the cj_t deviation component of real GDP, it is clear that the deviations from trend appear more important in the period post 1970. This could indicate that the US has entered its Second Phase since the 1970s.

Finally, in the True Long Run phase, the economy starts experiencing the effect of diminishing marginal productivity and heads towards utilization of the full potential of its available factors of production. Agents have learned to make the most out of what is available for them to produce. In this phase, asymmetries should decrease and more symmetries should take place between expansions and recessions since GDP growth is more or less equal to 0. This gives enough time for recessions to last as much as expansions. Since the starting phase is somehow similar but not identical to the TLR phase, business cycles in the starting phase should also reflect more symmetry than in the Transitory phase.

Business cycles are now defined as deviations from the True Evolution Dynamics of the Economy, or True Trend that differs across the three different phases.

Another point to look for is that, according to my research, and contrary to the beliefs of Classicals and Keynesians, business cycles do not originate from arbitrary, exogenous shocks as much as they come from ill-guided policies or decisions made by households, firms, banks, governments and central banks. I think that the 2008 financial crisis is only one example of the many recessions that support this view. The theory exposed in this work argues that any exogeneous shock can only contribute to a temporal rise in GDP when GDP is above the true dynamic trend represented by equation (4.1.3). Moreover, the economy cannot be simply subject to a permanent positive technology shock and have the GDP series increase infinitely at an increasing or constant rate, since, as the model suggests, any gain from any exogenous shock will be countered by losses from any other factor of production, be it human or capital, so that the true dynamic growth process of GDP is not altered.

Hence, in this research's perspective, business cycles are abnormal deviations from the dynamic trend that are triggered by agents in the economy, be they the government, central bank or labor, and are followed by the economy's self-correcting mechanism that brings the macroeconomic aggregate series back to its true trend.

The self-correcting mechanism that Mishkin (2019) talks about generates the business cycle mechanism in a way that ensures that any deviations from the Charlijo True Economic Trend Evolution are followed by movements in the opposite direction to bring the time-series back to its CTETE equation.

5.5 Implications on Monetary Policy

Keynes argues that monetary policy and fiscal policy should be used with the purpose of smoothing the business cycle. But in fact, this research argues that it is the bad monetary or fiscal policy decisions that make business cycles asymmetric, and hence hurtful for the economy. Monetary policy should neither be interventionist nor achieve price stability prematurely because in the True Long Run, prices will stabilize.

In the Transitory phase, monetary policy targets should be centered on real interest rates, considering that MS is increasing until we arrive near the True Long Run phase where MS should begin to stabilize. Before the True Long Run phase of the evolution of the economy, central banks cannot use real interest rates as the monetary policy instrument with the consent to fix MS. It is also possible to assume MS fixed during the starting phase of the economy since velocity is also constant and MS and nominal GDP do not change much.

But obviously major economies in the world are now in their second phase and some are heading toward the True Long Run.

5.6 Implications on Mc Callum Theory of GDP growth

Mc Callum (1989) argues that the rate of growth of nominal GDP is the sum of the rate of growth of real GDP and the inflation rate $\dot{Y}_{t,N} = \dot{Y}_{t,R} + \pi_t$. In order to stabilize the growth rate of real GDP, the monetary policy should focus on stabilizing the nominal GDP. Further, to minimize inflation, the central bank should target the growth rate of real GDP and set the nominal GDP growth to be equal to real GDP growth.

My analysis shows indeed that when the economy enters its True Long Run phase, nominal GDP and Real GDP should converge to their respective maximum values and the Price Level series starts stagnating around its maximum. This indicates and confirms the view of McCallum that achieving the same growth rate in nominal and real GDP will lead to stable price levels.

5.7 Implications on AS – AD analysis

In AS – AD analysis, the equilibrium points determine the equilibrium price level and output level. In the Short Run, the intersection between the curves AS and AD is the short run equilibrium point. In the same analysis, the intersection between AS, AD and LRAS is the Long Run equilibrium point that determines the potential output and the corresponding equilibrium price level.

Before my research, output Y was assumed to be growing infinitely over time. This means that the LRAS curve was supposed in general to shift to the right as time evolves.

My work shows that there exists a True Long Run Aggregate Supply curve TLRAS that depicts the optimal output level an economy can achieve (See Figure 21 below). This output level is constant and cannot change since it represents a maximum that is not related to technological progress or any exogenous factors but is derived from the fundamentals of the economy and its factors of production specifications.

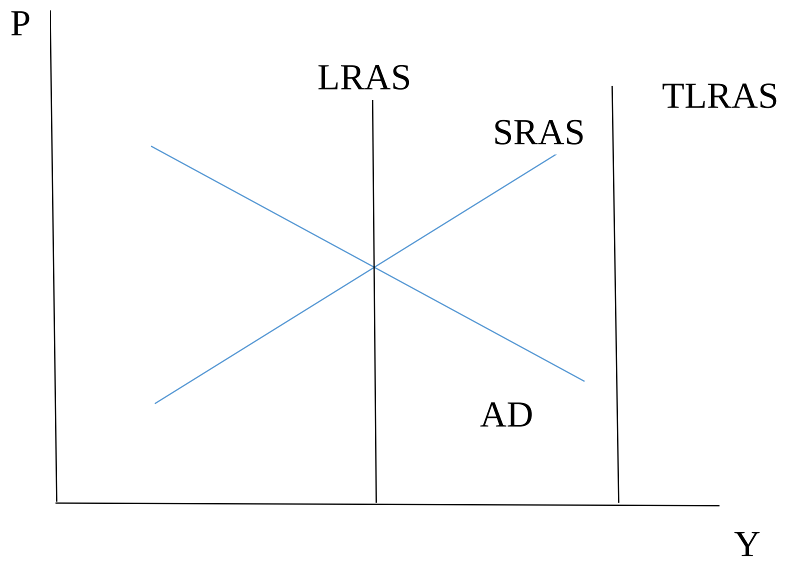

Figure 21. The TLRAS curves added to AS – AD analysis

If a permanent negative supply shock occurs, LRAS would shift to the left. Now, either another innovation or discovery let LRAS shifts again to the right and continue its move towards TLRAS or TLRAS shifts also to the left by the same amount of the LRAS shift. As an illustration, consider a country that loses suddenly a territory without receiving money in exchange, which otherwise would count as an exportation. The TLRAS will shift down by the amount of TLR production value that was coming from the old resource.

If, however, a permanent positive supply shock occurs, LRAS will shift to the right. Now, either the Dutch disease effect will bring back LRAS to the left and LRAS continue its progressive move towards TLRAS or TLRAS shifts also to the right by the same amount of the shift in LRAS. For illustration, a country acquires a new land. This resource is acquired for free, otherwise it would count as an investment. The TLRAS will shift up by the amount of TLR production value that originates from the new resource found.

5.8 Implications on the Solow Growth Model

Robert Solow (1956) argues that in the Long Run or steady state, GDP, consumption, investments and capital should grow at constant rates together with labor and technological growth.

The model and analysis present in this research confirm that in fact C, I, Y as well as NX and G, which Solow do not take onto account, grow together over time. However, the growth rate is not constant as Solow suggests and these series experience a decreasing rate of growth over time. We have shown indeed, in theory and in practice, that the rate of growth is not constant but varies with time and level of the series itself: it decreases as the variable itself grows. Practically, this has been verified from the high t-statistics and the very low p-value of the numerous regressions done in this work. The decrease in the rate of growth is very significant from the point of view of the econometrician.

In fact, Solow's model implies that the macroeconomic aggregates should tend to infinity as time evolves because a function with constant and positive growth rate diverges to infinity. Solow's model cannot be true in a finite period of time since resources are finite in a finite period of time and the production process is not instantaneous. For example, even if agents decide to plant 100 apple seeds in a land that can only contain 50 apple trees, it is more realistic to assume that the land will return less than 50 apple trees in due time because some trees would have been so clause to each other that the process of development of these trees would be altered and not work properly.

Other assumptions of the Solow Model could also be debatable. The Labor Market for example is assumed to always be at full employment. However, many Keynesians, as Robert Mundell for example, argues that wages are sticky and therefore create labor shortages and unemployment possibilities. Finally, the dynamics of my model show that TLR value of employed labor agents would be about 246 million of people employed.

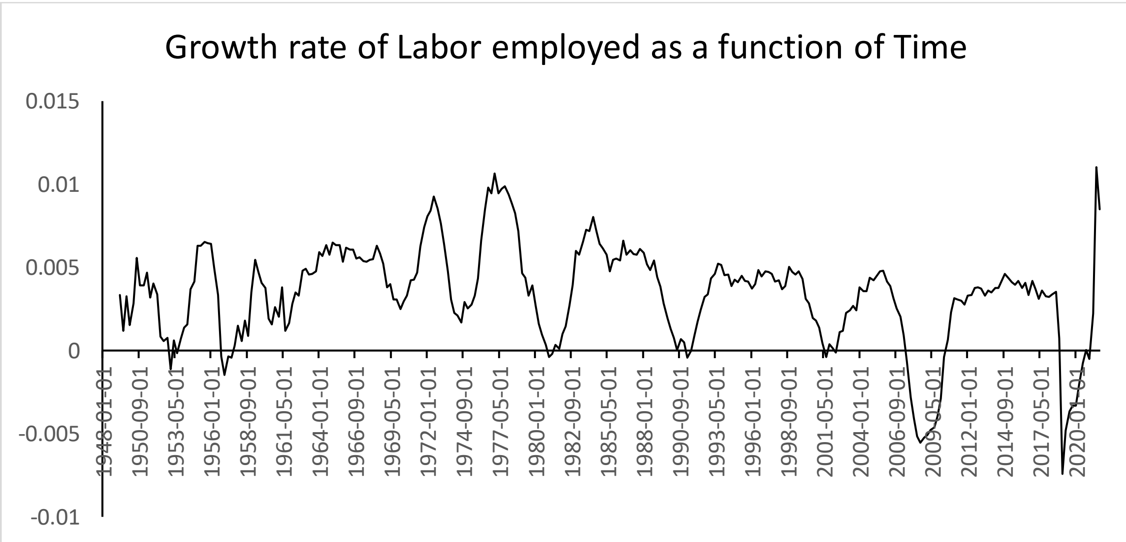

Figure 22. This plot contradicts Solow's assumption about constant growth rate of Labor for the US economy. Q2 1949 to Q4 2021. Author's computation of the growth rate using section 5.1

5.9 Implications on the Analysis of Financial Crises

The financial crises in developed economies start by credit booms and asset prices booms (Mishkin, 2019). This will lead to abnormal increases in macroeconomic aggregates and hence in Yt series. Yt will exceed $\tilde{Y}_t$ by a considerable amount. This in turn will make the economy overheat and eventually a crisis will follow where the economy will get back to $\tilde{Y}_t$ and could even reach lower levels. The adjustment process can contain periods where Yt will be below $\tilde{Y}_t$ before Yt converges back to $\tilde{Y}_t$ as is the case in the cobweb model of Kaldor (1934).

Starting in Q2 of 2003, we see y_t overtaking $\tilde{y}_t$ and the distance separating the two series was increasing until 2008 Q1 approximately. This period corresponds to the credit boom and asset boom and expansionary monetary policy of the Fed that started in 2003. In 2008, the economy moved to a severe recession and the model I examined shows indeed that y_t crossed the trend $\tilde{y}_t$ and by a considerable amount. This led the recession to last for a relative long time before the economy completely recovered by 2018 where y_t converged back to $\tilde{y}_t$.

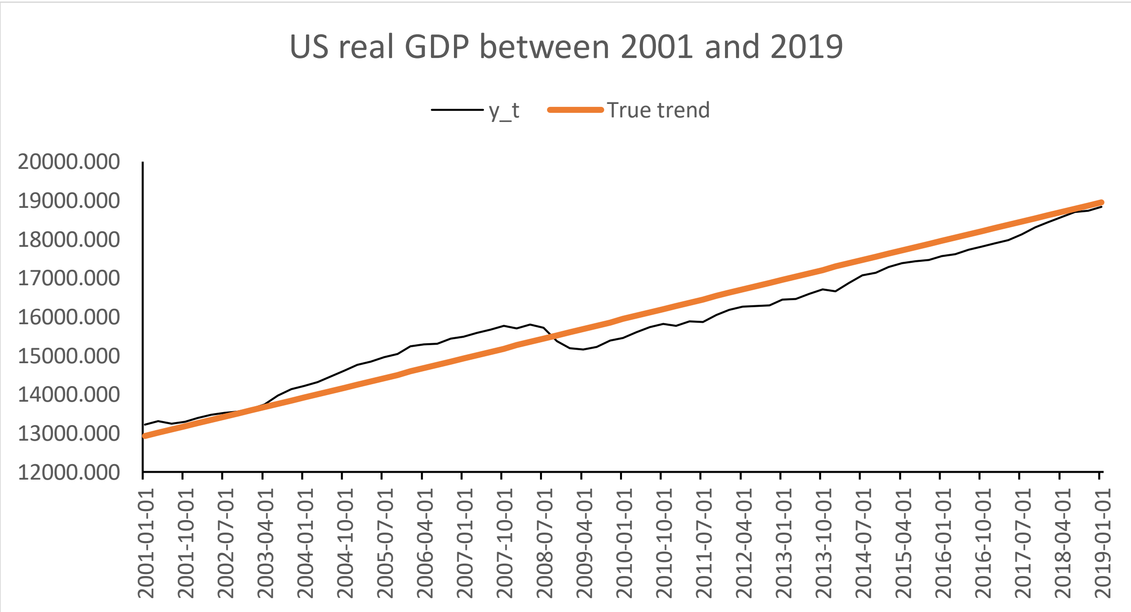

Figure 23. US real GDP and its corresponding CTETE trend. Positive deviations from CTETE trend overheat the economy and corresponds to Assets and Credits booms that precede Financial Crisis in developed economies.

Finally, we should note that both the equation of $\frac{dY_t}{dt}$ as derived from the solution equation (4.1.3) and the equation (4.1.6) indicate that at between Q4 2009 and Q1 2010 the US economy should experience the fastest growth in its real GDP before noticing a decrease in the growth rate for all subsequent periods. This period corresponds to the maximum value attainable by $\frac{dY_t}{dt}$ during the whole economic evolution of the economy. In fact, during this period, the most severe recession of the US happened because the unfortunately the US economy was growing faster than the maximum calculated growth of its evolution.

CHAPTER 6

CONCLUSION

Business cycles are the economic phenomena that drive a society into its different states, namely expansions and recessions. However, the different phases of the cycle do not look exactly the same. This is indeed what the data shows. Asymmetries in the business cycles are characterized by phases that differ in length and amplitude.

We attempted to study the business cycle and its asymmetries in the United States of America from as early as the 1940s to the present. We first surveyed some methods and techniques used by several scholars to deal with this particular subject. Then, we tried to use difference and differential equations in nonlinear dynamics settings in order to understand and analyze business cycle asymmetries. We found that real GDP as well as other real and nominal macroeconomic aggregates follow a very specific evolution mechanism. In my research, I found that Business Cycles are nothing but deviations from the true evolution mechanism of theses macroeconomic variables. We attempted to find this evolution mechanism and discovered, by building a new theory and using mathematical analysis and nonlinear dynamics, that all macroeconomic aggregates will eventually converge to some ceiling values that we call True Long Run values. Further, any deviation from the true evolution mechanism in any macroeconomic variable will lead to a phase of the business cycle where this previous deviation from the true trend needs to be eliminated. This is a result of the balance in any economic and real-life environment that must be satisfied.

In our theory, we were also able to explain and confirm the findings of previous scholars that recessions tend to be shorter and steeper than expansions because the true trend of macroeconomic variables is always in continuous increase in absolute terms, leaving little time for recessions to last. We project that, as we approach the True Long Run phase of evolution of macroeconomic aggregates, we should notice recessions that are relatively longer and less steep until we have perfect symmetry between expansions and recessions.

As Canova (1998) points out, many observed facts from business cycles could not reflect well the behavior and properties of business cycles in general since the different detrending methods imply different data manipulation. This produces different outputs and leads to different interpretations and analysis. I argue in my research that the best detrending method is to choose CTETE equation as it is the trend that the macroeconomic aggregates follow.

Economists always tried to understand the velocity of money function that seems more difficult to interpret and model than other macroeconomic variables. Keynesians argue that V is a function of the expected nominal interest rates and Monetarists say that V is constant and predictable. However, the dynamics proposed here show that in fact V starts constant than decreases until it reaches 1, where it will be constant again. I argue in my work that the role of economists is to know in which phase of the CTETE the economy is currently at.

We have defined the "true long run" in an economy for the first time in the history of economics. I intend, by this research, to initiate a new school of thought whose existence builds upon Keynesians and Monetarists and complement them. In this new school, recessions and expansions come from perturbations of GDP and its deviation from its CTETE trend. Once a crisis hits, the economy adjusts by following again the CTETE Equation.

In order to perform this research, I had to search and learn the different and heterogeneous areas of mathematics, physics, biology and economics. Because all the tools that lead me to my conclusions are from these disciplines. I think every research should incorporate a little bit of material from all the fields of science. After all, the great philosophers of ancient times were all at once mathematicians, physicians, physicists, astronomers, etc.

In future work, it is possible to include many countries in a single analysis and try to identify if the findings made earlier would be compatible with the new analysis in the larger and broader dataset. We can also examine other macroeconomic variables like capacity utilization and level of inventories. Moreover, I also plan to make use of higher order ordinary differential equations as well as start using partial differential equations.

Finally, in future work, I plan to analyze the derivatives of the solution function and study the behavior of Money Demand functions in relations to these derivatives. I also plan to define and analyze mathematically the start and end of the 3 different stages of the evolution of an economy as explained in section 5.4 as well as the transition between each stage. Moreover, I will apply my theory to other countries and use it to try to prevent financial crisis or any type of crisis to occur, especially for the countries which might be in the transitory stage with maximum rate of growth. I will also study the Cobb-Douglas production function in light with the theory proposed in this research and apply my theory to Labor employed L, capital used K and technology advancement A components, where all variable should be expressed in a CTETE equation form.

The actual data realizations of any economy show its underlying fundamentals that govern the evolution mechanism followed by the economy. This conclusion is taken from the results and implications of the theory I advance in this research.

APPENDIX

A.1 Solving the Nonlinear Differential Equation $(4.1.3)$

Here we will solve equation (4.1.3) in detailed steps. We start from the beginning.

$$\frac{d}{dt}N(t) = N(t) * \left(A - b * N(t)\right)$$

$$\frac{\frac{d}{dt}N(t)}{N(t) * \left(A - b * N(t)\right)} = 1$$

Partial Fraction Decomposition yields:

$$\frac{\frac{d}{dt}N(t)}{N(t) * \left(A - b * N(t)\right)} = 1$$

$$\frac{dN(t)}{A * N(t)} + \frac{b * dN(t)}{A * \left(A - b * N(t)\right)} = 1 * dt$$

Integrating both sides:

$$\int \frac{dN(t)}{N(t)} + \int \frac{b * dN(t)}{\left(A - b * N(t)\right)} = \int A * dt$$

$$ln|N(t)| - ln|A - b * N(t)| = At + C$$

Where C is a constant. Using the properties of ln (x):

$$\ln\left(\frac{|N(t)|}{|A - b * N(t)|}\right) = At + C$$

Using the exponential function transformation on both sides:

$$\frac{|N(t)|}{|A - b * N(t)|} = D * \exp(At)\,, D = \exp(C)$$

At $t = 0$, $D = \frac{|N(0)|}{|A - b*N(0)|}$. Hence:

$$N(t) = {}^{A}\!/_{b} * \left(1 + \frac{{}^{A}\!/_{b} - N(0)}{N(0)} * e^{-A*t}\right)^{-1}$$

And the nonlinear differential equation is solved.

BIBLIOGRAPHY

Altuğ, S. G. (2009). Matching the model to the data. *Business cycles: Fact, fallacy and fantasy* (). World Scientific Publishing Company.

Altuğ, S., Ashley, R. A., & Patterson, D. M. (1999). are technology shocks nonlinear? *Macroeconomic Dynamics, 3*(4), 506-533. https://doi.org/10.1017/S1365100599013036

Altug, S., & Labadie, P. (2008). *Asset pricing for dynamic economies*. Cambridge University Press. https://doi.org/10.1017/CBO9780511753909

Ashley, R. and Patterson, D. (1989). Linear versus Nonlinear Macroeconomics: A Statistical Test. *International Economic Review 30*, 685-704.

BEA, US Bureau of Economic Analysis; [online] Available at: https://www.bea.gov/. [Accessed March 16, 2023].

BLS, US Bureau of Labor Statistics; [online] Available at: https://www.bls.gov/. [Accessed March 16, 2023].

Canova, F. (1998). Detrending and business cycle facts. *Journal of Monetary Economics, 41*(3), 475-512. https://doi.org/10.1016/S0304-3932(98)00006-3

Clements, M.P. &Krolzig, H-M. (2003). Business cycle asymmetries: Characterization and testing based on markov-switching autoregressions. *Journal of Business & Economic Statistics, 21*(1), 196-211. DOI: 10.1198/073500102288618892

Desmos Graphing Calculator. 2023. *Desmos Graphing Calculator*. [online] Available at: https://www.desmos.com/calculator?create_account. [Accessed March 16, 2023].

Finkenstädt, B. (1995). Lecture notes in economics and mathematical systems 426.

Nonlinear dynamics in economics (). Springer Berlin / Heidelberg.

FRED, Federal Reserve Bank of St. Louis; [online] Available at: https://fred.stlouisfed.org. [Accessed March 16, 2023].

Gabish, G. (1984). Nonlinear models of business cycle theory, in: Hammer G. and Pallaschke D.(Eds.), *Selected topics in operations research and mathematical economics: Proceedings of the 8th symposium on operations research, held at the University of Karlsruhe, West Germany August 22–25, 1983*. Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-45567-4

Gabisch, G., & Lorenz, H. (1987). *Business cycle theory: A survey of methods and concepts*. Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-662-011782

Gandolfo, G. (1980). *Economic dynamics, methods and models* (2nd., rev ed.). North-Holland.

Görtz, C., & Tsoukalas, J. D. (2013). Learning, capital-embodied technology and aggregate fluctuations. *Review of Economic Dynamics*, *16*(4), 708-723. https://doi.org/10.1016/j.red.2012.04.003

Hodrick, R. J., & Prescott, E. (1981). Post-war U.S. business cycles: An empirical investigation. (). St. Louis: Federal Reserve Bank of St Louis.

Hoppensteadt, F. C., & Hyman, J. M. (1977). Periodic solutions of a logistic difference equation. *SIAM Journal on Applied Mathematics*, *32*(1), 73-81. https://doi.org/10.1137/0132005

Kaldor, N. (1934). A classificatory note on the determinateness of equilibrium. *The Review of Economic Studies*, *1*(2), 122-136. https://doi.org/10.2307/2967618

Kydland, F. & Pescott, E. (1982). Time-to-Build and Aggregate Fluctuations.

Econometrica, 50(6), 1345-1370.

Li, T., & Yorke, J. A. (1975). Period three implies chaos. *The American Mathematical Monthly, 82*(10), 985. https://doi.org/10.2307/2318254

Lorenz, E.N. (1963). Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences 20*(2),130-141. http://dx.doi.org/10.1175/1520-0469(1963)020<0130:dnf>2.0.co;2

May, R. M. (1976). Simple mathematical models with very complicated dynamics. *Nature (London), 261*(5560), 459-467. https://doi.org/10.1038/261459a0

McCallum, B. T. (1989). *Targets, indicators, and instruments of monetary policy.* (). Cambridge: National Bureau of Economic Research, Inc. https://doi.org/10.3386/w3047

Mishkin, F. S. (2019). *The economics of money, banking and financial markets* (Twelfth, Global ed.). Pearson.

Neftci, S. N. (1984). Are economic time series asymmetric over the business cycle? *The Journal of Political Economy, 92*(2), 307-328. https://doi.org/10.1086/261226

Robinson, P. M. (1977). The estimation of a nonlinear moving average model. *Stochastic Processes and their Applications, 5*(1), 81-90. https://doi.org/10.1016/0304- 4149(77)90052-7

Rotemberg, J. J., & Woodford, M. (1995). Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets. In T. F. Cooley (Ed.), *Frontiers of Business Cycle Research* (pp. 243–293). Princeton University Press. https://doi.org/10.2307/j.ctv14163jx.15

Serrano, S., Barrio, R., Dena, A., & Rodríguez, M. (2012). Crisis curves in nonlinear business cycles. *Communications in Nonlinear Science & Numerical*

Simulation, 17(2), 788-794. https://doi.org/10.1016/j.cnsns.2011.06.022

Sichel, D. E. (1993). Business cycle asymmetry: A deeper look. *Economic Inquiry, 31*(2), 224- 236. https://doi.org/10.1111/j.1465-7295.1993.tb00879.x

Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics, 70*(1), 65-94. https://doi.org/10.2307/1884513

Speight, A. E. H. (1997). Testing for business cycle asymmetry: Some international evidence. *Applied Economics Letters, 4*(10), 603-606. https://doi.org/10.1080/758533282

Stock, J. (2003). The Econometric Analysis of Business Cycles. *Medium Econometrisch Toepassingen, 11*(1), 23-26.

Valderrama, D. (2007). Statistical Nonlinearities in the Business Cycle: A Challenge for the Canonical RBC Model. *Journal of Economic Dynamics and Control, 31*(9), 2957-2983.

Van Nieuwerburgh, S., & Veldkamp, L. (2006). Learning asymmetries in real business cycles. *Journal of Monetary Economics, 53*(4), 753-772. https://doi.org/10.1016/j.jmoneco.2005.02.003

Wecker, W. E. (1981). Asymmetric time series. *Journal of the American Statistical Association, 76*(373), 16-21. https://doi.org/10.1080/01621459.1981.10477595