PERFORMANCE OF FOURTH AND FIFTH CLASS LEVEL STUDENTS
ON FOUR INTERPRETATIONS OF A RATIONAL NUMBER

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By

Fida H. Atallah

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PERFORMANCE OF FOURTH AND FIFTH CLASS LEVEL STUDENTS 
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By 
Fida H. Atallah

Approved: 
Dr. Murad Jurdak, Advisor

Dr. George Za'our, Member of Committee

Dr. Yakub Namek, Member of Committee

Date of Thesis Presentation: June, 1986
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Date
I would like to dedicate my thesis to my father Hani and my mother Farida in appreciation of their support and encouragement.
ACKNOWLEDGEMENT

I would like to thank the principals and directors of Saint Mary Orthodox School and Rawdah High School for their help and cooperation in offering their schools for conducting the study. Also, I would like to thank the teachers who helped to validate and conduct this study.
ABSTRACT

Statement of the Problem

The purpose of the study was to investigate student knowledge of the measure, quotient, ratio, and operator interpretations of a rational number in relation to class level, age group, learning opportunities, and kinds of fraction (unit or common).

Procedure

The subjects for this study were: (a) 129 students from Rawda High School and (b) 95 students from Saint Mary Orthodox School. Each student completed a performance test, developed by the researcher, in about 40 minutes. The statistical analysis was performed using Z-test and two-way analysis of variance.

Results

1. Significant differences for class level 4 were found among the different interpretations, with the exception of the quotient by operator comparison, suggesting the following sequence in descending order of facility:
   1. measure interpretation
   2. ratio interpretation
   3. quotient and operator interpretations.
For class level 5, significant differences were found among the different interpretations suggesting the following sequence in descending order of facility:

1. measure interpretation
2. ratio interpretation
3. operator interpretation
4. quotient interpretation.

II. Significant differences for age groups 10, 11, and 12 were found among the different interpretations, with the exception of the quotient by operator comparison, suggesting the following sequence in descending order of facility:

1. measure interpretation
2. ratio interpretation
3. quotient and operator interpretations.

III. A significant difference between class levels 3 and 4, in favour of the former, was found for the operator interpretation. No significant differences were found for the other interpretations.

IV. Significant differences between students with higher learning opportunities and students with lower learning opportunities, in favour of the latter, were found on the measure, ratio, and operator interpretations for class levels 4 and 5, and age groups 11 and 12. For age group 10, a significant difference between both groups was found. The
difference in performance was in favour of the group with lower learning opportunities for the measure interpretation. For the ratio and operator interpretations, the difference found was in favour of the group with higher learning opportunities. No significant differences were found between the two groups for the quotient interpretation.

V. No significant interaction was found between class levels and learning opportunities for any of the four interpretations.

VI. No significant differences were found between age groups 10, 11, and 12 for any of the four interpretations.

VII. A significant interaction was found between age groups and learning opportunities for the ratio interpretation, while no such result was found for the other interpretations.

VIII. A significant difference was found between unit and common fraction mean scores, in favour of the former, for class level 4. No such difference was found for class level 5.

Conclusion

Within the various limitations of the study, it was found that for all class levels and age groups, the measure interpretation was the easiest followed by the ratio.
interpretation. Class level was not found to be related to student performance except in the case of the operator interpretation. Age level was not found to be related to student performance. A confounding of the learning opportunities variable and the school did not allow isolating the effect of the learning opportunities variable hence suggesting the need for further research on this particular issue with a modification of this variable. Finally, unit fractions were observed to be easier to handle than common fractions for fifth elementary students.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>x</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xii</td>
</tr>
<tr>
<td>List of Figure</td>
<td>xiii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Definitions</td>
<td>3</td>
</tr>
<tr>
<td>Variables</td>
<td>6</td>
</tr>
<tr>
<td>Purpose</td>
<td>7</td>
</tr>
<tr>
<td>Hypotheses</td>
<td>8</td>
</tr>
<tr>
<td>Significance</td>
<td>9</td>
</tr>
<tr>
<td>Limitations</td>
<td>10</td>
</tr>
<tr>
<td>II. REVIEW OF LITERATURE</td>
<td>13</td>
</tr>
<tr>
<td>General Review</td>
<td>13</td>
</tr>
<tr>
<td>Review of Research Studies</td>
<td>19</td>
</tr>
<tr>
<td>III. THE PROCEDURE</td>
<td>27</td>
</tr>
<tr>
<td>Population and Sample</td>
<td>27</td>
</tr>
<tr>
<td>Study Design</td>
<td>29</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>35</td>
</tr>
<tr>
<td>IV. RESULTS</td>
<td>38</td>
</tr>
<tr>
<td>Null Hypothesis One</td>
<td>41</td>
</tr>
<tr>
<td>Null Hypothesis Two</td>
<td>42</td>
</tr>
<tr>
<td>Null Hypothesis Three</td>
<td>44</td>
</tr>
<tr>
<td>Null Hypothesis Four</td>
<td>44</td>
</tr>
<tr>
<td>Null Hypothesis Five</td>
<td>48</td>
</tr>
<tr>
<td>Null Hypothesis Six</td>
<td>48</td>
</tr>
<tr>
<td>Null Hypothesis Seven</td>
<td>48</td>
</tr>
<tr>
<td>Null Hypothesis Eight</td>
<td>49</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>---</td>
</tr>
<tr>
<td>V. DISCUSSION</td>
<td>52</td>
</tr>
<tr>
<td>Facility of the Four Interpretations of a Rational Number</td>
<td>52</td>
</tr>
<tr>
<td>Relationship of Class Level and Performance on Each Interpretation</td>
<td>54</td>
</tr>
<tr>
<td>Relationship of Age Group and Performance on Each Interpretation</td>
<td>56</td>
</tr>
<tr>
<td>Relationship of Learning Opportunities and Performance on Each Interpretation</td>
<td>56</td>
</tr>
<tr>
<td>Interaction of Learning Opportunities with the Other Variables</td>
<td>57</td>
</tr>
<tr>
<td>Performance on Items Dealing with Unit and Common Fractions</td>
<td>58</td>
</tr>
<tr>
<td>Recommendations</td>
<td>58</td>
</tr>
</tbody>
</table>

<p>| References | 69 |
| Appendix A | 61 |
| Appendix B | 66 |</p>
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Sample Composition Relative to Class Level</td>
<td>28</td>
</tr>
<tr>
<td>2.</td>
<td>Sample Composition Relative to Age Group</td>
<td>28</td>
</tr>
<tr>
<td>3.</td>
<td>Text-book Survey Relative to the Four Interpretations</td>
<td>34</td>
</tr>
<tr>
<td>4.</td>
<td>The Means and Standard Deviations for Class Levels 4 and 5 Relative to the Four Interpretations of a Rational Number</td>
<td>39</td>
</tr>
<tr>
<td>5.</td>
<td>The Means and Standard Deviations for Age Groups 10, 11 and 12 Relative to the Four Interpretations of a Rational Number</td>
<td>40</td>
</tr>
<tr>
<td>6.</td>
<td>The t-values for Class Levels 4 and 5 (Comparisons of Interpretations)</td>
<td>42</td>
</tr>
<tr>
<td>7.</td>
<td>The t-values for Age Groups 10, 11 and 12 (Comparisons of Interpretations)</td>
<td>43</td>
</tr>
<tr>
<td>8.</td>
<td>Analysis of Variance of the Four Interpretations by Class Level (CL) and Learning Opportunities (LO)</td>
<td>45</td>
</tr>
<tr>
<td>9.</td>
<td>Analysis of Variance of the Four Interpretations by Age Group (AG) and Learning Opportunities (LO)</td>
<td>46</td>
</tr>
<tr>
<td>10.</td>
<td>The Means and Standard Deviations for Class Levels 4 and 5 Relative to the Two Kinds of Fractions</td>
<td>51</td>
</tr>
<tr>
<td>11.</td>
<td>The t-values for Class Levels 4 and 5 (Comparison of Unit and Common Fractions)</td>
<td>51</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1.</td>
<td>Interaction between age group and learning opportunities for the ratio interpretation</td>
<td>50</td>
</tr>
</tbody>
</table>
CHAPTER I

The Problem

Introduction

The field of mathematics education witnessed many changes during the last three decades. The introduction of the so called "new" mathematics was accompanied by a restructuring of the school mathematics curriculum which came to be characterized by the emphasis on mathematical structures and unifying concepts.

Many factors may account for bringing about these changes. One of the major factors is attributed to the technological advancement that characterized the last three decades. Another is the expansion of mathematics knowledge which made it necessary to organize the mathematics curriculum in a manner which makes it possible for the students to acquire the maximum amount of knowledge needed. A third factor relates to the psychological theories that emerged in the 1950's and 1960's and stressed the importance of mathematical structures and global unifying concepts.

One of the most important aims of mathematics education is to provide students with the mathematical knowledge needed for everyday life situations. That is, mathematics education is concerned, among other things, with the social utility aspect of mathematics as it relates to individual needs.
National numbers are considered to be an integral part of the school mathematics curriculum. A large amount of research was done on this topic and it is generally agreed that it is one of the most difficult topics in elementary school mathematics whether for learning or teaching. The difficulty lies basically in the lack of understanding of the conceptual aspect of a rational number. Mostly, instruction on rational numbers stresses the computational rather than the conceptual aspect. Hence, rational numbers are viewed as abstract notions unrelated to real life situations. This contradicts one of the most important goals of mathematics education which is to prepare students to implement their mathematical knowledge in real life situations when the need arises.

To understand rational number concepts in a comprehensive manner, students should be familiar with their different interpretations. Several interpretations of rational numbers have been suggested, four of which are the focus of interest in this study. Those four interpretations are the measure, quotient, ratio, and operator numbers (refer to the section on definitions for details). These four basic interpretations are very important for elementary school instruction because they help children view rational numbers as useful tools in everyday life situations. These interpretations have the mathematical characteristics of rational numbers and at the
same time allow for viewing a rational number from different perspectives.

The research studies done on the rational number interpretations are limited. Most of the studies done dealt primarily with measure number interpretation. This lack of attention to the other rational number interpretations is also reflected in school textbooks. The majority of school mathematics textbooks allocate most of the sections of rational number interpretations to the measure number.

Also, little space is allocated in textbooks to rational number interpretations in general and students are not given enough opportunities to learn about them comprehensively despite their general usefulness.

There is need for further research on rational number interpretations and for investigating the relationship between student knowledge of those interpretations and variables like age, class level, and learning opportunities received by students (refer to the section on definitions for details). These relationships would help to form a clearer idea of what should be done to enhance student knowledge of rational number interpretations.

Definitions

**Terminology**

Throughout the study, rational numbers, fractions,
and fractional numbers will refer to using the definitions given by Dwight et al. (1966) as follows:

A rational number is a number that can be expressed as the product of an integer and the multiplicative inverse of a whole number bigger than zero.

A number that is the product of a whole number and the multiplicative inverse of a whole number except zero is called a fraction or a fractional number. When a fraction is named by a numeral of the form $\frac{a}{b}$ where $a$ and $b$ are named whole numbers and $b$ is bigger than zero, the numeral is called a fractional number.

**Interpretations of a Rational Number**

The four basic subconstructs or interpretations dealt with in this study are the measure, quotient, ratio, and operator numbers.

**Measure number.** A rational number is said to be a measure number if it expresses an attribute of a part-whole relationship where the whole is equally partitioned.

An example of a test item dealing with the measure number interpretation is:

What part of the rectangle is shaded?
Quotient number. A rational number is said to be a quotient number if it expresses an attribute of a quantity being equally divided.

An example of a test item dealing with the quotient number interpretation is:

Three boys want to share two apple pies equally, what part of the pie will each have?

Ratio number. A rational number is said to be a ratio number if it expresses a relation between two attributes determined by the number of times one contains the other integrally or rationally.

An example of a test item dealing with the ratio number interpretation is:

A basket of fruits contains three oranges and two apples. What part of the fruits are oranges?

Operator number. A rational number is said to be an operator number if it expresses an attribute which enlarges or shrinks a given quantity.

An example of a test item dealing with the operator number interpretation is:

A wet 25 cm long thread shrinks by one fifth its length upon drying. How many cm does it shrink?
**Variables**

**Criterion Variable**

The dependent variable throughout the study is the student performance on each of the measure, quotient, ratio, and operator interpretations. It is measured by the score the student gets on the items dealing with a particular interpretation under study. The score is the sum of the item scores related to the particular interpretation.

**Classification Variables**

**Age group (AG).** Students participating in the study belong to one of the age groups 10, 11, or 12. For example, a student would belong to the age group of 10 if that student's age is in the range of 9 years and 7 months to 10 years and 6 months at the time of the study. Similar definitions apply to the other age groups.

**Class level (CL).** Students participating in the study belong to one of the class levels 4 or 5. Those belonging to class level 4 have completed the fourth elementary class thus belonging to the fifth elementary class at the time of the study. Those belonging to class level 5 have completed the fifth elementary class thus belonging to the first intermediate class at the time of the study.

**Learning Opportunities (LO).** Learning opportunities relative to each interpretation are expressed in terms of
the number of exercises related to the particular interpretation under study. The exercises that count are those found in the unit of fractional numbers and that are directly related to the nature of the interpretation. The textbooks concerned in this definition are those that were used to finish the class level requirement of the class of interest, where the review will start with the second elementary textbook up to the class level required.

Students with high learning opportunities (LO 2) are those who encounter the larger number of exercises on a particular interpretation, while those with low learning opportunities (LO 1) are those who encounter the smaller number of exercises on that particular interpretation.

Unit and common fractions. A unit fraction (UF) is a proper fraction with a numerator of 1. A common fraction (CP) is a proper fraction with a numerator larger than 1.

Purpose

The purpose of the study was to investigate student knowledge of the four basic interpretations of a rational number which are the measure, quotient, ratio, and operator numbers.

In particular, this knowledge was examined in relation to a set of hypothesized variables which are the class levels,
age groups, learning opportunities, and kinds of fraction (unit or common).

Hypotheses

Based on the purpose of the study, the following null hypotheses were developed:

Null Hypothesis One

There are no significant differences in student performance on the measure, quotient, ratio, and operator interpretations for each class level.

Null Hypothesis Two

There are no significant differences in student performance on the measure, quotient, ratio, and operator interpretations for each age group.

Null Hypothesis Three

There are no significant differences in student performance on each of the measure, quotient, ratio, and operator interpretations among different class levels.

Null Hypothesis Four

There are no significant differences in student performance on each of the measure, quotient, ratio, and operator interpretations between students who have high learning opportunities and those who have low learning opportunities.
Null Hypothesis Five

There is no significant interaction between class level and learning opportunities on student performance for each of the measure, quotient, ratio, and operator interpretations.

Null Hypothesis Six

There are no significant differences in student performance on each of the measure, quotient, ratio, and operator interpretations among different age groups.

Null Hypothesis Seven

There is no significant interaction between age group and learning opportunities on student performance for each of the measure, quotient, ratio, and operator interpretations.

Null Hypothesis Eight

There are no significant differences in student performance between items dealing with unit fractions and items dealing with common fractions on the measure, quotient, ratio, and operator interpretations for each class level.

Significance

Studies done on the different interpretations of a rational number are few and limited in nature. Mostly, they are concerned with one or two interpretations at a time and many of them focus on the measure number interpretation.
The significance of this study lies in its attempt to establish relationships between student knowledge of the four basic interpretations of a rational number and variables that are hypothesized to be important for understanding the nature and development of this knowledge. It might be useful in the process of instruction in the sense that it would encourage teachers to introduce rational numbers through the use of the different interpretations and various real life situations in which they arise, thus giving students the chance to become more familiar with the concept of a rational number and its different usages. It might also be helpful in giving teachers a clearer idea on how students learn fractions and what might be the most meaningful interpretation to start with when introducing the concept of a rational number.

It is hoped that the results of this study will promote future research not only on the four interpretations dealt with here but on other interpretations as well.

Limitations

The study aimed at investigating student knowledge of the four basic interpretations of a rational number through establishing a relationship between the criterion variable and the classification variables (refer to the section on variables for details).
The relationships established are restricted to the particular setting for this study. They cannot be generalized due to many factors. The results of the study can serve as suggestions on which to build future investigations.

One of the factors that limit the possibility of generalizing the results is that the study does not constitute what is known to be a true experimental design. Another factor is that the performance test used, being researcher constructed might not yield scores that describe student performance in a very accurate manner but rather give a general idea about the performance. A third factor is related to one of the classification variables, namely the learning opportunities. This variable is affected by a number of situations. One situation is that each year new students join different classes in a school so there might be a small number of new students in the classes under study that might not have the learning opportunities that are claimed to be theirs. Another situation is that the examples and exercises found in the textbooks are not usually the only sources of information given or practiced by students. The learning opportunities variable is defined in as accurate a manner as possible.

One more source of limitation which affects the interpretation of the statistical analysis is that the repeated administration of the t-test (for testing hypotheses one and two) increases Type I error, thus the probability
of attributing some of the findings to chance increases. Consequently the probability of Type I error was set at .01 so as to reduce Type I error for the hypotheses in toto.
CHAPTER II

Review of Literature

General Review

There seems to be a general agreement about the importance of rational numbers in the school mathematics curriculum. Post (1981) attributed the importance of rational numbers to two factors: social utility and place in future development of advanced mathematics ideas and structures. In another article, Post et al. (1982) emphasized the importance of rational numbers in dealing with real world problems and situations. They continued to say that understanding rational numbers would provide a rich background from which children could develop and expand their mathematical ideas.

Braunfeld and Wolfe (1966) stated that it was generally agreed that, historically, the motivation for inventing and using rational numbers came from real world situations. Nikpa (1981) mentioned in the introductory section of his paper that according to the survey of priorities in the School Mathematics Project done in 1981, four major goals of teaching fractions were related to their use in vocation, consumers' purchase, illustration of basic mathematical
ideas, and provision of solutions to algebraic equations.

Kieren (1980) shared Post's view on the importance of rational numbers despite the current emphasis on metric measures and the widespread availability of calculators because of their practical uses. Kieren also noted the call for earlier as well as greater use of decimals as opposed to common fractions in the mathematics curriculum. Burke et al. (1977) mentioned that the National Advisory Committee on Mathematical Education report written in 1975 emphasized the challenge to the traditional system posed by the new technology. The National Advisory Committee on Mathematical Education report suggested that the elementary school curriculum include early instruction on decimals with much more emphasis, along with de-emphasis and delay of common fraction notation and algorithms.

Although many educators agreed with the suggestions in favor of decimal numbers, nearly all insisted on the important role that rational numbers have in many aspects. Many situations exist where decimal numbers cannot be used to replace rational numbers due to some unique features that are characteristic of rational numbers only.

Rational numbers were noted to be difficult to deal with. Difficulties were encountered both in learning and teaching rational numbers. Kieren (1976) indicated that children laboured over learning fractions to a great extent.
Also, Kieren (1980) asserted that there had been long standing frustration with learning and teaching rational numbers. Nikpa (1981) was of the view that teaching fractions had always been a challenge to teachers at all grade levels. He continued to say that it was still so according to the National Assessment of Educational Progress report issued in 1980.

Carpenter et al. (1975) indicated that the first National Assessment of Educational Progress report written in 1973 stressed the difficulty of the fraction concept both in comprehension and application. In a review of literature for a study done by Novillis (1976), she mentioned that many elementary school students found difficulty in developing a fraction concept. Post et al. (1982) confirmed the general view that students consistently experienced difficulty in dealing with and applying rational number concepts.

The difficulties encountered by students in learning rational numbers were generally attributed to lack of emphasis during instruction on the conceptual development of a rational number in favour of algorithms or operations. Carpenter et al. (1975) stated that the difficulties encountered by students appeared to be attributed to the lack of emphasis on fractions and fraction concepts in upper elementary classes. Post et al. (1982), in reference
to a generalization made by Carpenter and others in 1980, stated that students seemed to be learning mathematics skills at the rote manipulative level and did not understand the underlying concepts. Post et al. (1982) continued to say that Carpenter and others suspected the reason to be mostly that students were usually expected to operate at a higher level of abstraction than what they were capable of and partly that students were expected to operate at the symbolic level too soon and too often.

Kieren (1976) stated that most school curricula treated rational numbers as objects of computation. Hence, children and adolescents missed many of the important interpretations of rational numbers. Post et al. (1982) suggested that one reason for the difficulties encountered in learning fractions might have been that school programs tended to emphasize procedural skills and computational aspects rather than development of important foundational understandings.

Many educators suggested a shift in emphasis during the course of instruction so as to stress the conceptual aspect of development of a rational number as a basic prerequisite to teaching algorithms. What usually happened was that teachers rushed through the introductory level which included the initial basic concepts and the different interpretations of a rational number, the purpose being that students master the computational skills. It was and
still is important to master those skills because of their future need in subsequent education but at the same time, students should be able to deal with rational number concepts and ideas with a reasonable level of familiarity and understanding. They should be able to use them in real life situations when the need arises. They should know when, where, and how to use rational numbers.

Hence, the major goals of instruction in rational numbers should be aimed at a sound understanding of the conceptual development prior to the computational skills. According to Kieren (1976), understanding the basic ideas and concepts related to rational numbers came from having enough experience with their different interpretations. Kieren gave a list of seven interpretations which, admittedly, was not exhaustive. He developed a set of prerequisite skills needed for each interpretation. He continued to say that by failing to take into consideration the unique features of each interpretation, teachers and researchers encountered difficulties that could have been logically anticipated. In another article, Kieren (1980) suggested three levels of knowledge for rational numbers: (a) experimental, (b) symbolic, and (c) axiomatic. The experimental level has four categories (interpretations): (a) measure, (b) quotient, (c) ratio, and (d) operator.
According to Post et al. (1982), Kieren provided in his 1976 article a detailed conceptual analysis of rational numbers highlighting hierarchies of subskills within the various interpretations. Their article gave brief summaries of basic ideas related to some interpretations. They also mentioned a point of view by Dienes, stated in 1967, who suggested that learning was enhanced when children were exposed to a concept in a variety of physical contexts.

Kieren's interpretations were perhaps the most exhaustive analysis done according to Burke et al. (1977). They included a brief summary of each of the interpretations. Other mathematics educators mentioned some interpretations of rational numbers and emphasized the importance of varied student experiences dealing with these interpretations. Sowder (1971) suggested that children should experience various models for fractional numbers. Van Goghen (1960) differentiated between fractions and rate pairs and insisted on the inclusion of both as integral parts of school curriculum. Lawson (1966), in an article dealing with the structure of fractions, recommended that common fractions be introduced as multiples of unit fractions. Pehr (1968) mentioned seven levels with which to view fractions. He went on to introduce the notion of the fraction as an operator with detailed explanation of its role as a stretcher or shrinker. Braunfeld and Wolfe (1966) included in their
article a preliminary view of a program developed for seventh grade low achievers. The program was developed using the operator interpretation to introduce fractions. They introduced fractions through stretcher and shrinker operators.

Ballow (1974) suggested that while working with children towards understanding fraction concepts, they should be introduced in the intuitive sense as representing parts of a whole. Botts (1964) suggested three representations for fractions: (a) numbers, (b) number pairs, and (c) symbols. His textbook review, in which he surveyed currently used ones from 1963 and on, showed that about four textbooks depended on the number representation, at least six textbooks used the number pair representation, and at least five textbooks used the symbol representation. He also found out that none of the books used all three representations together though some used two representations.

Review of Research Studies

An extensive amount of research has been done on rational numbers covering many aspects. Recently, especially during the last decade, the direction of research has shifted more towards the conceptual development of a rational number. Still, there is a limited amount of research studies done on the different interpretations of a rational number.
Gunderson and Gunderson (1957) designed a study to find out what concepts and ideas young children had about fractions. The study was conducted in the form of oral interviews with 22 children. Before the interview sessions, an introductory lesson was taught to the children who were in the second grade. Manipulative materials were used throughout the study. The conclusion they got was that fraction concepts could be taught to children of the second grade and on through using manipulative material.

Novillis (1976) designed a study based on a hierarchy development by Gagne. Novillis hypothesized a hierarchy of selected subconcepts and designed a fraction concept test to investigate if this hierarchy was a supportable sequence of dependent subconcepts. The multiple choice test was given to 274 students in the fourth, fifth, and sixth grades. In discussing the results, she stated that the hierarchy was incomplete. Also, based on the results, the main conclusion reached was that certain stated subconcepts were prerequisite for others. One result was that associating fractions with part-whole and part-group models was prerequisite to association with points on a number line. She also inferred that the instances of the fraction concept that students were exposed to in elementary school were not of sufficient variety to encourage generalization of the fraction concept.
Payne (1976) wrote a paper including summaries of research articles done in Michigan starting 1968 and dealing with fractions. He noted that up till 1973 the research was oriented towards examining various approaches to fraction algorithms and various manipulative material. Starting with 1973, the studies aimed at intensive examination of what children learned while being taught a carefully developed sequence. In the next paragraph, four related studies will be discussed.

In the period from 1973 to 1975 a series of related studies were conducted on learning initial fraction concepts and on algorithm learning. With an initial goal of having more pupils master the content being taught, pilot studies were done with small groups of students before using the instructional sequences with larger groups in more controlled experimental situations. The pilot studies were done by Payne and six other investigators after designing an initial sequence on fractions. The studies were done on third and fourth grade students. After the third pilot study, it was agreed to delete the set model because of its difficulty and the conflict it created with the measure models. The investigators found out that difficulties persisted in identifying fractional parts of sets and number lines. Muangnapo (cited in Payne, 1976) used this sequence in a study done in 1975. He found out that major difficulties
persisted with the number line. He also noted that the region model was easier than the number line one. Williams (cited in Payne, 1976) used Maangnspoon's sequence with some modifications for her study on students in second, fourth, and sixth grades. She found out that by second grade, fractions related to concrete materials could be taught quite successfully. Galloway (cited in Payne, 1976) conducted a study in 1975 for grades one through five. She found out that all age levels had difficulties with the number line model. She also confirmed that most students from age 8 and on could master the initial fraction concepts.

The initial fraction concept sequence included as spatial representations the region and the number line models. It was generally agreed that more attention should be given to the number line model because of the major difficulties that students found in it.

A working paper by Burke et al. (1977) included a review of important research articles and dissertations on rational numbers. The following four paragraphs include some significant conclusions of related studies.

In 1946, Sebold (cited in Burke et al., 1977) found out that in general, unit fractions were easier to handle than proper ones in the third and fourth grades. In a study done in 1971 comparing a set treatment and an area treatment for teaching basic concepts of rational numbers,
Sension (cited in Burke et al., 1977) found no significant difference between both treatments. In a study aiming at investigating children's understanding of three interpretations (part of whole, part of set, operation of division) of certain unit fractions prior to formal instruction, Campbell (cited in Burke et al., 1977) found out that part of a whole is understood better by 5, 6, and 7 year olds than either of the other two interpretations.

In 1974, Carpenter and others published a draft of a report from the National Committee for Teachers of Mathematics project for Interpretative Reports on National Assessment. The general conclusion regarding the 9 year old students showed absence of knowledge rather than erroneous work. The overall conclusion indicated that concepts of fractional numbers were poorly understood by the majority of students.

In a book written by Piaget, Inhelder, and Szeminska in 1960, and in a part dealing with the subdivision of areas and the concept of fractions, a study was reported dealing with tracing spontaneous development of the fraction concept in children. The most important finding was that children acquired the concept at several stages. Before the age of four or four and a half, children were not able to divide a whole into halves. Between the ages of four and five, children could dichotomize a small regularly shaped whole. Between the ages of six and seven they could divide a
whole into thirds and after that they could extend to fifths, sixths and so on. The authors viewed the concept of a fraction as the synthesis of part-whole and part-part notions.

Carpenter et al. (1973) wrote an article on the results and implications of the first National Assessment of Educational Progress. In this assessment, the number of exercises on fractions was very small. The assessment was given to 9 and 13 year old students. Concerning the results of the 9 year old, one third of the subjects answered correctly when asked what fractional part of a region was shaded. The data indicated that about 2 out of 3 9 year old students did not have enough knowledge of fractions to translate a pictorial representation of a fraction into a numerical form. The errors suggested lack of knowledge rather than misunderstanding. Data from other fraction exercises supported these observations. As for the 13 year old students, 65% could name simple fractional parts and 20% could name complex ones.

Carpenter et al. (1980) reported on the results of the second National Assessment of Educational Progress. The focus in the article was on the results of the 13 and 17 year olds. About 72% could identify fractional parts of sets. About 50% could identify division problems. In general, about one third of the thirteen year old subjects didn’t seem to master the fundamental fraction skills needed for learning operations. About two thirds, learned most of the very elementary skills.
Post (1981) reported on the results and implications from national assessment. The National Assessment of Educational Progress allocated a substantial portion of the second mathematics assessment to fractions due to their importance in any well-balanced school program. The items on fractions included region, set, and number line models. About 20% of the 9 year old and 82% of the 13 year old students were able to respond correctly to the questions dealing with naming part of a shaded region. About 62% of the 9 year olds could shade three fourths of a simple region. As for the set model 72% of the 13 year olds and 12% of the 9 year olds could name part of a set that is colored. About 58% of the 13 year old students could mark the point one half on the number line. This age group, as the author concluded, did not seem to have a firm grasp on fractions as they related to the number line measurement. In general, results indicated less proficiency with the set model than the region one.

In general, fraction concepts were found to be difficult to handle at all age levels. Some studies indicated that these concepts could be taught successfully to students of the second grade using manipulative material.

A major conclusion reached in many of the studies mentioned in this section was that the part-whole (region) model was easier to handle than the part-group (set) model.
Another was that the number line model was more difficult to handle than the part-group and part-whole models. In general, the number line model was found to be difficult at nearly all age levels. Another major conclusion dealing with rational number interpretations was that the division operation model was less understood by students than the part-whole and part-group models. Also, unit fractions were observed to be easier to work with than proper fractions.
CHAPTER III

Procedure

Population and Sample

The population for the study consisted of two groups of students. The first group consisted of students who had finished the fourth elementary class and belonged to class level 4 at the time of the study. The second group consisted of students who had finished the fifth elementary class and belonged to class level 5 at the time of the study. Both groups came from private schools which implemented mathematics instruction in English. The schools were in Beirut District.

The sample consisted of 129 students from Rawda High School and 95 students from Saint Mary Orthodox School, after deleting the cases who did not belong to any of the specified age groups (see section on definitions) and the cases of missing data. Class level 4 had 109 students and class level 5 had 115 students. (See Tables 1 and 2 for sample composition.)

The study was conducted in the beginning of this school year between November and December, 1985.
Table 1
Sample Composition Relative to Class Level

<table>
<thead>
<tr>
<th>School</th>
<th>Class Level 4 (CL 4)</th>
<th>Class Level 5 (CL 5)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sawda High School</td>
<td>67</td>
<td>66</td>
<td>129</td>
</tr>
<tr>
<td>Saint Mary Orthodox School</td>
<td>46</td>
<td>49</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td>109</td>
<td>115</td>
<td>224</td>
</tr>
</tbody>
</table>

Table 2
Sample Composition Relative to Age Group

<table>
<thead>
<tr>
<th>School</th>
<th>Age Group 10 (AG 10)</th>
<th>Age Group 11 (AG 11)</th>
<th>Age Group 12 (AG 12)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sawda High School</td>
<td>40</td>
<td>55</td>
<td>34</td>
<td>129</td>
</tr>
<tr>
<td>Saint Mary Orthodox</td>
<td>24</td>
<td>36</td>
<td>35</td>
<td>95</td>
</tr>
<tr>
<td>School</td>
<td>64</td>
<td>91</td>
<td>69</td>
<td>224</td>
</tr>
</tbody>
</table>
Study Design

The Performance Test

A performance test was developed to measure student knowledge of the four basic interpretations of a rational number. The test was revised and modified more than once based on the results of two pilot studies that were administered.

Initial test development. After developing basic definitions of the four interpretations of a rational number, four sets of questions were constructed each belonging to one of the four interpretations. In constructing the questions, care was taken to include as much as possible real life situations where these interpretations were needed. Some sample items were selected from each set of questions and arranged to form the first performance test which included 18 items. There were five items on each of the measure and quotient interpretations and four items on each of the ratio and operator interpretations.

First Pilot Study. The performance test was administered to a sample of 19 students in the fifth elementary class in Has Beirut School. An analysis of variance was done together with an index of difficulty to select the items which are reasonably difficult and at the same time clear and precise.
The second performance test. After the first pilot study, the definitions were reviewed and put in operational terms and many of the items were deleted or modified according to the analysis done. The items that remained, together with new ones were arranged to form the second performance test. There were 30 items in this test, constructed in such a way that each situation had two items corresponding to it, one using unit fractions and one using common ones. A copy of the test together with the newly developed definitions of the measure, quotient, ratio, and operator interpretations were given to a fellow teacher with an educational background in mathematics, so as to ensure that the items are representative of the interpretations (content validity). The items belonging to the ratio interpretation were labelled correctly. Six out of eight items on the operator interpretation were labelled correctly. The measure interpretation items were all labelled as belonging to the quotient interpretation.

It was clear that the measure and quotient interpretation definitions needed modification. Also, before administering the test for the second pilot study, it was split to two forms due to time limits imposed by the length of the class periods.

Second Pilot Study. The modified performance test was administered to two groups of students. One of the groups
had 13 students of the fourth elementary class in Ras Beirut School. The other group had 17 students of the fifth elementary class in Ras Beirut School. Each group was given both forms. An item analysis was done to select the items to be included in the final form of the performance test.

The final form of the test. Based on the results of the item analysis, some items were deleted due to their relative difficulty. Also, the items dealing with the quotient interpretation concerning unit fractions were deleted because they reflected a case of measure rather than quotient interpretation according to the last modified definitions (refer to section on definitions for details).

The final form of the test (See Appendix A for this form) included 16 items selected from the previous test with slight language modifications. There were six items on the measure interpretation, two items on the quotient interpretation, four items on the ratio interpretation and four items on the operator interpretation. Each item dealing with a common fraction, except in the case of the quotient number, has a corresponding item dealing with a unit fraction.

Reliability of the performance test. The test reliability was established through the use of the Kuder-Richardson reliability coefficient. Using the Kuder-
Richardson formula 21, the reliability coefficient was 0.64 indicating a moderate reliability.

Validity of the performance test. The content validity was established through having two fellow teachers label each item as belonging to one of the four interpretations of a rational number. The teacher having an educational background in mathematics labelled them all correctly while the teacher having an educational background in civil engineering labelled nearly two-thirds the items correctly.

Scoring of the performance test. The items of the test were fill-in were subjects were expected to provide the answers. Each item had either a correct answer (scored 1) or a wrong answer (scored 0). Any missing answer was considered as wrong.

Implementation. The time for reading instructions and administering the test was 50 minutes. Two instruction sheets accompanied the test (See Appendix B for both forms). The first was for the teachers whose classes were participating in the study. This sheet gave a general idea about the purposes of the study together with test administration directions. The second sheet was for the students participating as subjects. This included guidelines for filling in the responses.

Data Collection

Prior to the test administration time, a day or two
before, students were asked to check their birth dates (day, month, year). These dates were recorded by the students upon test administration.

As for the learning opportunities, the textbook review was done and the results were recorded in Table 3. An examination of Table 3 indicates that one of the schools (LO 2) was superior in learning opportunities to the other school (LO 1) relative to the measure, ratio, and operator interpretations. As for the quotient interpretation, no relevant exercises were found in the reviewed text books of both schools.

The data related to student performances was obtained from the individual scores on the performance test. Each student had six different scores, each of which reflected the number of correct items on a particular topic. The scores were the following:

1. The Measure Score (MS)
2. The Quotient Score (QS)
3. The Ratio Score (RS)
4. The Operator Score (OS)
5. The Unit Fraction Score (UF)
6. The Common Fraction Score (CF)
Table 2

Textbook Review Relative to the Four Interpretations

<table>
<thead>
<tr>
<th>Learning Opportunity</th>
<th>Measure</th>
<th>Quotient</th>
<th>Ratio</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (LO 1)</td>
<td>168</td>
<td>0</td>
<td>33</td>
<td>4</td>
</tr>
<tr>
<td>High (LO 2)</td>
<td>195</td>
<td>0</td>
<td>114</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: Each number represents the total number of exercises found for a particular interpretation in the textbooks reviewed for classes second up to fifth elementary.
Data Analysis

Null Hypothesis One

The first null hypothesis was tested using the \( t \)-test for correlated data. For each class level, six \( t \)-tests were performed to test the following comparisons:

1. Measure Score with Quotient Score
2. Measure Score with Ratio Score
3. Measure Score with Operator Score
4. Quotient Score with Ratio Score
5. Quotient Score with Operator Score
6. Ratio Score with Operator Score

Null Hypothesis Two

The second null hypothesis was tested using the \( t \)-test for correlated data. For each class level, six \( t \)-tests were performed to test the following comparisons:

1. Measure Score with Quotient Score
2. Measure Score with Ratio Score
3. Measure Score with Operator Score
4. Quotient Score with Ratio Score
5. Quotient Score with Operator Score
6. Ratio Score with Operator Score

Null Hypotheses Three, Four, and Five

The third, fourth, and fifth hypotheses were tested using an analysis of variance method. Four 2 x 2 analyses of variance were performed, one for each of the four interpretations.
The classification variables used here were the class level (CL), which served as the independent variable, and the learning opportunities (LO) which served as the moderator variable.

Null hypotheses three and four were tested by examining individual effects of the independent variable (CL) and moderator variable (LO) respectively. Null hypothesis five was tested by examining interaction effects (CL x LO).

**Null Hypotheses Six, Four, and Seven**

The sixth, fourth, and seventh hypotheses were tested using an analysis of variance method. Four 3 x 2 analyses of variance were performed, one for each of the four interpretations.

The classification variables used here were the age group (AG), which served as the independent variable, and the learning opportunities (LO) which served as the moderator variable.

Null hypotheses six and four were tested by examining individual effects of the independent variable (AG) and moderator variable (LO) respectively. Null hypothesis seven was tested by examining interaction effects (AG x LO).
Null Hypothesis Eight

The eighth null hypothesis was tested using the t-test for correlated data. For each class level one t-test was performed to examine the significance of the difference between the means of unit fraction scores and common fraction scores.
CHAPTER IV

Results

The first, second, and eighth hypotheses were tested using the t-test for correlated data. All the t-values are found in Tables 6, 7, and 11.

The third, fourth, fifth, sixth, and seventh hypotheses were tested using the analysis of variance method. All related statistics are found in Tables 8 and 9.

The means and standard deviations of class levels 4 and 5; and age groups 10, 11, and 12 (for each school and for both schools) relative to each of the four interpretations of a rational number are found in Tables 4 and 5. The means and standard deviations of unit and common fractions are found in Table 10.

One important point to be noted is that the repeated administration of the t-test would increase Type I error, thus increasing the probability of rejecting the null hypotheses. For this reason, a high level of confidence \( p < .01 \) was chosen for rejecting null hypotheses one and two for which the t-test was repeatedly used. A level of confidence \( p < .05 \) was chosen for rejecting the other null hypotheses.
Table 4
The Means and Standard Deviations for Class Levels 4 and 5 Relative to the Four Interpretations of a Rational Number

<table>
<thead>
<tr>
<th>Class Level</th>
<th>NS</th>
<th>QS</th>
<th>RS</th>
<th>OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO 1</td>
<td>4.48(1.69)</td>
<td>0.38(0.55)</td>
<td>2.37(1.20)</td>
<td>0.30(0.64)</td>
</tr>
<tr>
<td>LO 2</td>
<td>3.61(2.05)</td>
<td>0.35(0.53)</td>
<td>1.87(1.26)</td>
<td>0.20(0.4)</td>
</tr>
<tr>
<td>Both</td>
<td>4.11(1.89)</td>
<td>0.37(0.54)</td>
<td>2.16(1.24)</td>
<td>0.26(0.55)</td>
</tr>
<tr>
<td>CL 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO 1</td>
<td>4.89(1.34)</td>
<td>0.45(0.59)</td>
<td>2.62(1.16)</td>
<td>0.88(1.09)</td>
</tr>
<tr>
<td>LO 2</td>
<td>3.45(2.26)</td>
<td>0.37(0.64)</td>
<td>1.69(1.52)</td>
<td>0.51(0.79)</td>
</tr>
<tr>
<td>Both</td>
<td>4.28(1.92)</td>
<td>0.42(0.61)</td>
<td>2.23(1.40)</td>
<td>0.71(0.99)</td>
</tr>
</tbody>
</table>

Note: 1. LO 1 represents the group with lower learning opportunities.

2. LO 2 represents the group with higher learning opportunities.

3. The values within the parentheses represent the standard deviations.

4. The values not within the parentheses represent the mean scores.

5. NS represents the score on the measure interpretation

QS represents the score on the quotient interpretation

RS represents the score on the ratio interpretation

OS represents the score on the operator interpretation
Table 5
The Means and Standard Deviations for Age Groups 10, 11, and 12 Relative to the Four Interpretations of a Rational Number

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Interpretaions</th>
<th>NS</th>
<th>μS</th>
<th>RS</th>
<th>DS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO 1</td>
<td></td>
<td>4.45(1.69)</td>
<td>0.38(0.59)</td>
<td>1.65(1.17)</td>
<td>0.35(0.70)</td>
</tr>
<tr>
<td>LO 2</td>
<td></td>
<td>4.04(2.22)</td>
<td>0.54(0.66)</td>
<td>2.21(1.10)</td>
<td>0.38(0.58)</td>
</tr>
<tr>
<td>Both</td>
<td></td>
<td>4.30(1.90)</td>
<td>0.44(0.61)</td>
<td>2.14(1.13)</td>
<td>0.36(0.65)</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO 1</td>
<td></td>
<td>4.90(1.44)</td>
<td>0.58(0.6)</td>
<td>2.84(1.13)</td>
<td>0.62(1.01)</td>
</tr>
<tr>
<td>LO 2</td>
<td></td>
<td>3.58(1.99)</td>
<td>0.28(0.43)</td>
<td>1.86(1.50)</td>
<td>0.39(0.75)</td>
</tr>
<tr>
<td>Both</td>
<td></td>
<td>4.37(1.79)</td>
<td>0.43(0.56)</td>
<td>2.45(1.37)</td>
<td>0.53(0.91)</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO 1</td>
<td></td>
<td>4.65(1.47)</td>
<td>0.29(0.46)</td>
<td>2.41(1.16)</td>
<td>0.82(1.03)</td>
</tr>
<tr>
<td>LO 2</td>
<td></td>
<td>3.11(2.25)</td>
<td>0.31(0.63)</td>
<td>1.40(1.40)</td>
<td>0.31(0.63)</td>
</tr>
<tr>
<td>Both</td>
<td></td>
<td>3.87(2.04)</td>
<td>0.30(0.55)</td>
<td>1.90(1.37)</td>
<td>0.57(0.88)</td>
</tr>
</tbody>
</table>

Note: 1. LO 1 represents the group with lower learning opportunities.
2. LO 2 represents the group with higher learning opportunities.
3. The values within the parantheses represent the standard deviation scores.
4. The values not within the parantheses represent the mean scores.
Wall Hypothesis One

This hypothesis focused on the mean differences among the four interpretations of a rational number for each class level. The results of the $t$-test for correlated data are presented in Table 6.

For class level 4, there were significant differences between the measure and quotient interpretations, measure and ratio interpretations, measure and operator interpretations, ratio and quotient interpretations, ratio and operator interpretations (df = 216, $p < .01$). There was no significant difference between the operator and quotient interpretations. An examination of CL4 of Table 4 suggests the following sequence in descending order of facility:

1. Measure interpretation
2. Ratio interpretation
3. Quotient and operator interpretations.

For class level 5, there were significant differences between the measure and quotient interpretations, measure and ratio interpretations, measure and operator interpretations, ratio and quotient interpretations, ratio and operator interpretations, operator and quotient interpretations (df = 228, $p < .01$). An examination of CL5 of Table 5 suggests the following sequence in descending order of facility:

1. Measure interpretation
2. Ratio interpretation
3. Operator interpretation
4. Quotient interpretation
Table 6
The t-values for Class Levels 4 and 5 (Comparisons of interpretations)

<table>
<thead>
<tr>
<th>Class Level</th>
<th>MS-QS</th>
<th>MS-RS</th>
<th>MS-QS</th>
<th>RS-QS</th>
<th>OS-QS</th>
<th>RS-OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>21.93*</td>
<td>12.93*</td>
<td>21.13*</td>
<td>17.90*</td>
<td>-1.83</td>
<td>19.00*</td>
</tr>
<tr>
<td>5</td>
<td>23.05*</td>
<td>13.76*</td>
<td>17.73*</td>
<td>15.08*</td>
<td>2.90*</td>
<td>8.94*</td>
</tr>
</tbody>
</table>

* $p < .01$

Note: MS represents the score on the measure interpretation
QS represents the score on the quotient interpretation
RS represents the score on the ratio interpretation
OS represents the score on the operator interpretation

Null Hypothesis Two

The hypothesis focused on the mean differences among the four interpretations of a rational number for each age group. The results of the t-test for correlated data are presented in Table 7.

For age groups 10, 11, and 12, there were significant differences between the measure and quotient interpretations, measure and ratio interpretations, measure and operator.
interpretations, ratio and quotient interpretations, ratio
and operator interpretations (df = 126, df = 180, df = 136
for age groups 10, 11, and 12 respectively, * p < .01).
There was no significant difference between the operator
and quotient interpretations for any of the three age groups.
An examination of Table 5 suggests the following sequence
in descending order of facility:
1. Measure interpretation
2. Ratio interpretation
3. Quotient and operator interpretations.

Table 7
The t-values for Age Groups 10, 11 and 12 (Comparisons of
Interpretations)

<table>
<thead>
<tr>
<th>Age Group</th>
<th>MS-QS</th>
<th>MS-RS</th>
<th>MS-OS</th>
<th>RS-QS</th>
<th>QS-OS</th>
<th>RS-OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>17.34*</td>
<td>10.81*</td>
<td>16.83*</td>
<td>12.14*</td>
<td>-0.89</td>
<td>12.71*</td>
</tr>
<tr>
<td>11</td>
<td>21.94*</td>
<td>13.33*</td>
<td>17.91*</td>
<td>15.43*</td>
<td>1.00</td>
<td>11.29*</td>
</tr>
<tr>
<td>12</td>
<td>15.35*</td>
<td>9.85*</td>
<td>12.80*</td>
<td>7.86*</td>
<td>1.93</td>
<td>7.82*</td>
</tr>
</tbody>
</table>

* p < .01

Note: MS represents the score on the measure interpretation
QS represents the score on the quotient interpretation
RS represents the score on the ratio interpretation
OS represents the score on the operator interpretation
Null Hypothesis Three

This hypothesis focused on the mean differences between class levels 4 and 5 for each of the four interpretations of a rational number. The results of the analyses of variance are found in Table 8.

A significant difference was found between the sample means of class levels 4 and 5, \( F(1,220) = 15.63, p < .01 \) for the operator interpretation. No significant differences were found for the other three interpretations.

An examination of Table 4 indicates that class level 5 group had a higher mean score on the items dealing with the operator interpretation than class level 4 group.

Null Hypothesis Four

This hypothesis focused on the mean differences between the performances of students with higher learning opportunities (LO 2 group) and students with lower learning opportunities (LO 1 group), relative to each of the four interpretations of a rational number. An examination of Tables 8 and 9, in which the results of the CL by LO and AG by LO analyses of variance are presented, suggests a consistency of results.

For the measure interpretation, a significant difference was found between the sample means of LO 1 and LO 2 groups using the CL by LO analysis of variance, \( F(1,220) = 20.33, p < .01 \) and the AG by LO analysis of variance, \( F(1,218) = 18.03, p < .01 \).
Table 8
Analysis of Variance of the Four Interpretations by Class Level (CL) and Learning Opportunities (LO)

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure Score (MS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class Level (CL)</td>
<td>1</td>
<td>1.00</td>
<td>0.31</td>
</tr>
<tr>
<td>Learning Opportunities (LO)</td>
<td>1</td>
<td>66.50</td>
<td>20.33**</td>
</tr>
<tr>
<td>CL x LO</td>
<td>1</td>
<td>3.50</td>
<td>1.07</td>
</tr>
<tr>
<td>error</td>
<td>220</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Quotient Score (QS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>0.50</td>
<td>1.52</td>
</tr>
<tr>
<td>CL x LO</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>error</td>
<td>220</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Ratio Score (RS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>25.50</td>
<td>15.64**</td>
</tr>
<tr>
<td>CL x LO</td>
<td>1</td>
<td>2.00</td>
<td>1.23</td>
</tr>
<tr>
<td>error</td>
<td>220</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td>Operator Score (OS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>1</td>
<td>10.00</td>
<td>15.63**</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>3.00</td>
<td>4.69*</td>
</tr>
<tr>
<td>CL x LO</td>
<td>1</td>
<td>0.50</td>
<td>0.78</td>
</tr>
<tr>
<td>error</td>
<td>220</td>
<td>0.64</td>
<td></td>
</tr>
</tbody>
</table>

* p < .05  ** p < .01
Table 9
Analysis of Variance of the Four Interpretations by Age Group (AG) and Learning Opportunities (LO)

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measure Score (MS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class Level (CL)</td>
<td>2</td>
<td>2.92</td>
<td>0.89</td>
</tr>
<tr>
<td>Learning Opportunities (LO)</td>
<td>1</td>
<td>59.33</td>
<td>18.03**</td>
</tr>
<tr>
<td>CL X LO</td>
<td>2</td>
<td>0.22</td>
<td>0.07</td>
</tr>
<tr>
<td>Error</td>
<td>218</td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td><strong>Quotient Score (QS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>2</td>
<td>0.67</td>
<td>2.03</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>0.33</td>
<td>1.00</td>
</tr>
<tr>
<td>CL X LO</td>
<td>2</td>
<td>0.34</td>
<td>1.03</td>
</tr>
<tr>
<td>Error</td>
<td>218</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td><strong>Ratio Score (RS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>2</td>
<td>4.34</td>
<td>2.78</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>11.33</td>
<td>7.26**</td>
</tr>
<tr>
<td>CL X LO</td>
<td>2</td>
<td>13.33</td>
<td>8.54**</td>
</tr>
<tr>
<td>Error</td>
<td>218</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td><strong>Operator Score (OS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>2</td>
<td>0.84</td>
<td>1.24</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>1.00</td>
<td>4.41*</td>
</tr>
<tr>
<td>CL X LO</td>
<td>2</td>
<td>0.84</td>
<td>1.24</td>
</tr>
<tr>
<td>Error</td>
<td>218</td>
<td>0.68</td>
<td></td>
</tr>
</tbody>
</table>

* p < .05  ** p < .01
For the quotient interpretation, there was no significant difference between the sample means of LO 1 and LO 2 groups using both the CL x LO and the AG x LO analyses of variance.

For the ratio interpretation, a significant difference was found between the sample means of LO 1 and LO 2 groups using the CL x LO analysis of variance, $F(1, 220) = 15.64$, $p < .01$, and the AG x LO analysis of variance, $F(1, 218) = 7.26$, $p < .01$.

For the operator interpretation, a significant difference was found between the sample means of the LO 1 and LO 2 groups using the CL by LO analysis of variance, $F(1, 220) = 4.69$, $p < .05$, and the AG by LO analysis of variance, $F(1, 218) = 4.41$, $p < .05$.

An examination of Table 4 indicates that for class levels 4 and 5, the mean scores of the LO 1 group were higher than those of the LO 2 group for the measure, ratio, and operator interpretations.

An examination of Table 5 indicates that for age group 10, the mean score of the LO 1 group was higher than that of the LO 2 group for the measure interpretation. As for the ratio, and operator interpretations, the mean scores of the LO 2 group were higher than those of the LO 1 group.

For age groups 11 and 12, the mean scores of the LO 1 group were higher than those of the LO 2 group for the measure, ratio, and operator interpretations.
Null Hypothesis Five

This hypothesis focused on the interaction between class level and learning opportunities as this relates to student performance on each of the four interpretations of a rational number. The results of the analysis of variance are found in Table 8.

The F-ratio was not significant for any of the four interpretations indicating no interaction. In other words, the different class levels did not affect the two groups LO 1 and LO 2 differentially. The superiority of the LO 1 group over the LO 2 group remained nearly constant from one class level to the other for the four interpretations.

Null Hypothesis Six

This hypothesis focused on the mean differences between age groups 10, 11, and 12 for each of the four interpretations of a rational number. The results of the analyses of variance are found in Table 9.

No significant differences were found between the sample means of the three age groups for any of the four interpretations.

Null Hypothesis Seven

This hypothesis focused on the interaction between age group and learning opportunities as this relates to student performances on each of the four interpretations of
a rational number. The results of the analysis of variance are found in Table 9.

The F-ratio was significant only in the case of the ratio interpretation, \( F(2,218) = 9.54, p < .01 \), indicating a significant interaction between age group and learning opportunities.

An examination of Figure 1 suggests that for the age group 10 the L0 2 group is superior to L0 1, while for age groups 11 and 12 the L0 1 group is superior to L0 2.

**Null Hypothesis Right**

This hypothesis focused on the mean difference between unit fraction scores and common fraction scores for each class level. The results of the t-test for correlated data are presented in Table 11.

There was no significant difference in the mean scores of unit and common fraction items for class level 5. While, there was a significant difference between the two mean scores for class level 4 (df = 216, \( p < .02 \)). This difference was in favour of unit fraction scores (See Table 10.)
Figure 1. Interaction between age group and learning opportunities for the ratio interpretation.
Table 10
The Means and Standard Deviations for Class Levels 4 and 5 Relative to the Two Kinds of Fractions

<table>
<thead>
<tr>
<th>Class Level</th>
<th>Kinds of Fractions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit Fraction (UF)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.61 (1.61)</td>
<td>3.28 (1.89)</td>
</tr>
<tr>
<td>5</td>
<td>3.99 (1.66)</td>
<td>3.74 (1.96)</td>
</tr>
</tbody>
</table>

Note: The values within the parentheses represent the standard deviation scores.
The value not within the parentheses represent the mean scores.

Table 11
The t-values for class Levels 4 and 5 (Comparison of Unit and Common Fractions)

<table>
<thead>
<tr>
<th>Class Level</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UF - CF</td>
</tr>
<tr>
<td>4</td>
<td>2.36*</td>
</tr>
<tr>
<td>5</td>
<td>1.30</td>
</tr>
</tbody>
</table>

* p < .02
CHAPTER V

Discussion

This study was designed to investigate student knowledge of the measure, quotient, ratio and operator interpretations of a rational number through its possible relationship to four hypothesized variables: class level, age group, learning opportunities, and kinds of fraction (unit and common).

The study was not designed to establish a cause-effect relationship. Other than the limitations mentioned in Chapter I, the sample used is not a large one. In addition, the sample comes from schools with relatively high academic standard. Due to those factors combined, the investigator recognizes the difficulty of generalizing from the findings of the study. However, the following interpretations appear to be plausible.

Facility of the Four Interpretations of a Rational Number

With the exception of class level 5 for which the operator interpretation was easier than the quotient interpretation, the following sequence, in descending order of facility, is observed for all class levels and age groups:

1. measure interpretation
2. ratio interpretation
3. quotient and operator interpretations.

52
This conclusion is partly in agreement with the results found by Post (1981) and Carpenter et al. (1980). Post (1981) noted that the results of national assessment showed in general less proficiency with the set model (ratio) than the region model (measure). Carpenter et al. (1980), in reporting on the results of the second National Assessment of Educational Progress mentioned that about 72% of the subjects could identify fractional parts of sets (ratio), while about 50% could identify division problems (quotient). Campbell (cited in Burke et al., 1977) found out that part of whole (measure) is understood better than part of set (ratio) and operation of division (quotient).

On the other hand, the results of this study are not in agreement with the results of Sension (cited in Burke et al., 1977) who found no significant difference between an area treatment (measure) and a set treatment (ratio).

The sequence of facility observed in this study can be interpreted in terms of the learning opportunities that the students have. The overall effects of the learning opportunities variable correlates positively and highly with the sequence of facility. Specifically, the measure interpretation which has the highest learning opportunities was found to be first in the sequence of facility. The ratio interpretation which has the second highest learning opportunities was second in the sequence of facility.
Although the operator interpretation has very few learning opportunities and the quotient interpretation has none, both came third in the sequence of facility (except in the case of class level 5 where the operator interpretation was third in the sequence of facility and the quotient interpretation was fourth).

The differences in the difficulty level of the four interpretations of a rational number and the observation that such differences can be accounted for by the learning opportunities do not provide evidence for the existence of transfer of learning from one interpretation to another. Thus, knowing one interpretation of a rational number is no guarantee for knowing other interpretations. Consequently, unless specifically taught, most probably an interpretation will not be learned.

Relationship of Class Level and Performance on Each Interpretation

There are no significant differences between class levels 4 and 5 on the measure, quotient, and ratio interpretations while there is a significant difference in favour of class level 5 on the operator interpretation.

This result could be explained in terms of the school mathematics programs for the fourth and fifth elementary classes. In the fourth elementary class, students are exposed to a somewhat detailed study of the nature of fractional
numbers mostly through the use of the measure interpretation, and occasionally, the ratio interpretation. As for the quotient interpretation, no learning experiences are offered in that field. In the fifth elementary class, emphasis is shifted towards the computational aspect of fractional numbers. Hence, whatever the students learn in the fourth elementary class is the basic knowledge that the student acquires because the fifth elementary mathematics program does not provide additional enrichment beyond what the students already know.

The significant differences found in favour of class level 5 on the operator interpretation could be explained in terms of the school mathematics program which provides in the fifth elementary class few exercises that deal with the operator interpretation while it rarely provides any in the fourth elementary class.

The absence of growth in the ability to use the different interpretations of the rational number provides evidence that building computational skills of students does not by itself facilitate the learning of untaught interpretations.
Relationship of Age Group and Performance on Each Interpretation

There were no significant differences among the age groups on any of the four interpretations of a rational number. Hence, age is not a variable which affects student performance on any of the measure, quotient, ratio, or operator interpretations. Consequently, there seems to be no developmental growth in understanding and using such interpretations.

Relationship of Learning Opportunities and Performance on Each Interpretation

There were significant differences between the group with higher learning opportunities (LO 2) and the group with lower learning opportunities (LO 1) for the measure, ratio, and operator interpretations. For the quotient interpretation, there was no significant difference between both groups.

Before interpreting this result, one important point should be mentioned. The way the learning opportunities were defined resulted in confounding the learning opportunities variable and the school. In other words, all student classified as belonging to LO 1 were from one school whereas all students classified as belonging to LO 2 were from the other school.

The confounding of the learning opportunity and the school variables was true for all interpretations. This may suggest a plausible explanation for the fact that the group with the lower learning opportunities scored higher in general than
the group with higher learning opportunities. This confounding of variables suggests that the school itself, with all or some of its particular characteristics, might be a variable affecting student performance on each of the measure, ratio, and operator interpretations.

The two groups had no significant differences between them on the quotient interpretation. This result is predictable due to the fact that none of the groups had any learning experiences concerning this interpretation.

**Interaction of Learning Opportunities with the Other Variables**

There was no significant interaction between learning opportunities and class level for any of the four interpretations of a rational number. That is, the superiority of one group to another is constant. In particular, for both class levels 4 and 5, the L0 1 is always superior to the L0 2 group for all four interpretations.

There were no significant interactions between learning opportunities and age groups except for the ratio interpretation. The significant interaction in the ratio interpretation suggests a non-consistency in performance between the two groups where this superiority is in favour of L0 2 for age groups 11 and 12 and in favour of L0 1 for age group 10.
Performances on Items Dealing with Unit and Common Fractions

There were significant differences between unit and common fraction scores only for class level 4. These differences were in favour of unit fraction scores. Sebold (cited in Burke et al., 1977) found out that in general unit fractions were easier to handle than proper ones. This result is in agreement with the findings of the study for class level 4 but not class level 5.

The non-significant difference between unit and common fraction scores could be attributed to the fact that most students who are in class level 5 have already mastered the basic concept of a fraction hence they could manipulate unit and common fractions as well with little difficulty.

Recommendations

Based on the discussion of the study results, the following recommendations are made in relation to curriculum development and teaching:

1. It seems evident that by the end of the elementary stage, students still do not master the four interpretations of a rational number. It is suggested that all four interpretations be included as a basic part of the elementary school curriculum and that teachers are made aware of the existence of those different interpretations and of their
importance in acquiring a better knowledge of the nature of a rational number.

2. It is observed from the discussion that there is no evidence of transfer of learning from one interpretation to another. Also, building computational skills that are underlying concepts for certain interpretations does not by itself facilitate learning these interpretations. Hence, it is important to expose students to a variety of situations including the different interpretations of a rational number.

In view of the results of the study, the following recommendations for further research are made:

1. It is suggested that the study be replicated with a larger and more representative sample of students, with a wider range of age groups and class levels and with a modified learning opportunities variable so as to include several possible factors that affect learning opportunities (e.g. supplementary material provided by teachers, use of manipulative material and audio-visual aids, and teacher mode of instruction).

2. There are possibly many variables which affect student performance on rational number interpretations other than the ones mentioned in this study. It is suggested that the future studies include other variables also (e.g. IQ,
achievement in English language, and skills in problem solving especially division problems).

3. There are many real life situations in which the four interpretations under study can be implemented. It is suggested that the performance test used in this study be modified so as to include as many situations as possible. For example, the measure interpretation has many situations in which it can be used in real life applications like the part-whole region, number line, capacity measure, and others.
Appendix A

The Performance Test

Do not write inside these squares

Name ____________________________
Age
Date month Year

AG CL UF CF

Class ____________________________
School __________________________

MS QS RS OS

1. What part of the pie is missing?

   answer __________

2. One of three children in the family is a girl. What part of the children are girls?

   answer __________

3. A lens makes an object look two thirds its size. A ruler is 9 cm long. What will its length look like through the lens?

   answer __________
4. What part of the square is shaded?

5. Three girls want to share two loaves of bread equally. What part of the loaves will each girl get?

6. A wet cotton thread is 24 cm long. Upon drying, it shrinks by one sixth its length. How many cm does it shrink?

7. One cup of sugar is mixed with two cups of flour. What part of the mixture is sugar?
8. What part of the road from home to school did Samir reach?

9. A lens makes an object look one-fourth its size. A ruler is 8 cm long. What will its length look like through the lens?

10. Four boys want to share three apples equally. What part of the apples will each boy get?

11. What part of the chocolate bar is missing?
12. A wet cotton thread is 30 cm long. Upon drying, it shrinks by two fifths its length. How many cm does it shrink? 

answer

13. Two of five children in the family are boys. What part of the children are boys? 

answer

14. What part of the rectangle is shaded? 

answer

15. Three cups of sugar are mixed with four cups of flour. What part of the mixture is flour? 

answer
16. What part of the road from home to school did Manal reach?
Appendix B
Teacher and Student Instruction Sheets

Teacher Instruction Sheet

Dear Fellow Teacher,

Enclosed with this letter is a set of questions which constitutes a performance test designed to measure student knowledge of four basic interpretations of a rational number (measure, quotient, ratio and operator interpretations). It is a part of a study designed for a master's degree thesis at the American University of Beirut.

Enclosed also is a sheet including definitions of the interpretations with examples. In addition, there is a copy of the set of questions (performance test) together with the instruction sheet that will be distributed to each student along with the test.

This performance test will be given to students of the first elementary and first intermediate classes to measure their knowledge of the previously mentioned interpretations. Its duration is 50 minutes (10 minutes for distributing the test and reading the instructions, 40 minutes for the test itself).

A day or two before the test administration, ask the students to check their birthday dates (date, month, year - for example 12, 6, 1970) from their parents or guardians.

No indication should be given to the students before
hand that the test will be administered. It should come as
a usual class activity not preannounced.

Test Administration Procedure

1. At the beginning of the class period when the test is to
be administered, it can be introduced to the student as a
sort of classwork or class activity.

2. After the test sheets together with the student instruction
sheets are distributed to the students, please, read and explain
the instruction sheet carefully. Next, read all the test
questions and if necessary explain (even in Arabic) without
giving any hints or suggestions as to the answer or its nature.

3. Give the signal for starting after checking that the
students have filled out the left side of the box on the first
page of the test (which includes name, age, class and school).
Make sure that they do not fill in the part on the right.

4. Remember that the time for the test is 40 minutes. It is
preferable to have all students filling out the answers for
each question without leaving any questions unanswered.

5. Please make sure that they understand where to place their
answer and remind them to use either whole numbers of fractions
for answering.

6. Give the signal for stopping and collect the sheets at
the end of the allocated time.
7. Please send the sheets to the School administration after collecting them.

Thank you for your help and cooperation.

Note: In case you would like to know more about the test administration procedure or about any details please call me at the number 351645.

Sincerely yours,

Fida Atallah

Student Instruction Sheet

Dear Student:

Please read the following carefully before answering the set of questions given to you:

1. Write your answer inside the small empty square.
   
   Example:
   
   What is the sum of 12 and 16?
   
   answer 28

2. All your answers should be given as fractions or whole numbers.

3. The time is 40 minutes.


