= T H E S I S =

Refinement of nomographic method as applied to problems of practical astronomy.

by

Salah Ud-Dine KHARBUTLI ,

A . U . B .

----- 1940 - 1941 -----

REFERENCES.

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II.- A First Course in Nomography.........by S.Brodetsky Printed in I920.

III.- Graphical & Mechanical Computation....by Joseph Lipka Copyright I9I8.

IV.- Traité de Nomographie................Maurice d'Ocagne I92I.

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N O M O G R A P H Y

It was d'Ocagne who applied the name Nomography to the method of representing graphically on a plane mathmatical formula or relations that contain two or more variables.

In recent years the utility and convenience of nomography have been increasingly realised,and the subject has gained in importance and recognition,particularly in engineering practice. It is,in the main,a product of French mahmatical genius. Articles have appeared in one or two English journals, and excellent accounts of the subject in English are to be found in Hezlett's Nomography (Royal Military Institution,Woolwich) and Lipka's Graphical and Mechanical Computation (Wiley,New-York).

But the reader who is interested in the subject cannot do better than read d'Ocagne's excellent Traité de Nomographie, (G.Villars,Paris,I899). I will show in the following pages, the essential steps in nomography.

OBJECT of NOMOGRAPHY

We are all familier with graph-drawing as means of solving equations. But ordinary graphical methods are often inconvenient,because as separate graph is required for pratically each equation we have to solve. Thus to solve the quadratic equation

$$X^2 + X-2 = 0$$

We need to draw the graph $Y = X^2 + X$, and find where it is

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cut by the line $Y=2$. To solve de equation $X^2 + 2X - 2 = 0$, we most draw the graph $Y = X^2 + 2X$, and similarly for other values of the coefficient of X.

<u>The object of Nomography is to enable us to solve all equations of a given type by means of one diagram.</u>

Thus all quadratic equations of the type $X^2 + aX + b = 0$ can be solved by means of one graph which can be drawn once for all. In the same way all cubic equations of, say, the form $X^3 + aX + b = 0$ can be solved graphically by means of one diagram. Such a diagram is called à NOMOGRAM.

It is also the object of Nomography to enable us to find the value of a complicated expression graphically. And we will try to give examples about what we are speaking, after expressing the essential steps for this method.

<u>METHOD of NOMOGRAPHY</u>

The method of Nomography is as follows. Suppose that a number X is determined when two numbers, a, b, are given; e.g. Let $X = ab$ or a/b or $3a^2/b^3$, or let X be given by $X^2+aX+b=0$ or $a \sin X + b \cos X + I = 0$;
then two straight lines a, b, are graduated and a curve (which we shall, when necessary, call the X curve) is also graduated in such way that, if a straight line is drawn joining the graduation a, on scale a, to the graduation b, on scale b, the line cuts

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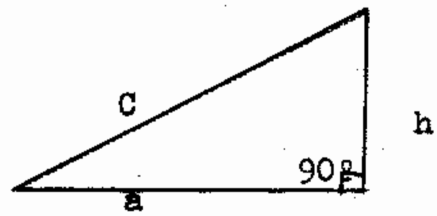
the X curve at the graduation X, where X is the desired result.

The coming examples will give us a clear idea about what we said before.

In order to reduce the inclined distance to the corresponding horizontal distance a correction must be subtracted, and we can see that in the following figure which is a right triangle.



$C^2 - a^2 = h^2$

$(C + a)(c - a) = h^2$

assuming $c = a$ and applying it to the first parenthesis only we get :

$2\,C\,(c - a) = h^2$ approximatly

$c - a = \frac{h^2}{2c}$ ----dº------

----dº------

Similarly

$c - a = \frac{h^2}{2a}$ ----dº------

It is evident that the smaller h is in comparison with the other two sides the more exact will be the results obtained by this formula.

Now from the above formula we get $a = c - \frac{h^2}{2c}$ which is the horizental distance we see that the value of a is equal to the length of the tape after subtracting the amount $\frac{h^2}{2c}$ from it.

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It is required now to draw the nomogram which gives the correction $-\frac{h^2}{2c}$.

If we make a table by considering c to be varied from I metre to I5 metres h from I metre to I5 metres we get the following table :

c	h	$-\frac{h^2}{2c}$	(a) approximate value	(A) real value	error A - a
I	.I	-.005	.995	.99495	5/I00000
2	.2	-.0I0	I.990	I.98990	I0/I00000
3	.3	-.0I5	2.985	2.98485	I5/I00000
4	.4	-.020	3.980	3.97980	20/I00000
5	.5	-.025	4.975	4.97475	25/I00000
6	.6	-.30	5.970	5.96970	30/I00000
7	.7	-.35	6.965	6.96465	35/I00000
8	.8	-.40	7.960	7.95960	40/I00000
9	.9	-.45	8.955	8.95455	45/I00000
I0	I.0	-.50	9.950	9.94950	50/I00000
II	I.I	-.55	I0.945	I0.94445	55/I00000
I2	I.2	-.60	II.940	II.93940	60/I00000
I3	I.3	-.65	I2.935	I2.93435	65/I00000
I4	I.4	-.70	I3.930	I3.92930	70/I00000
I5	I.5	-.75	I4.925	I4.92425	75/I00000

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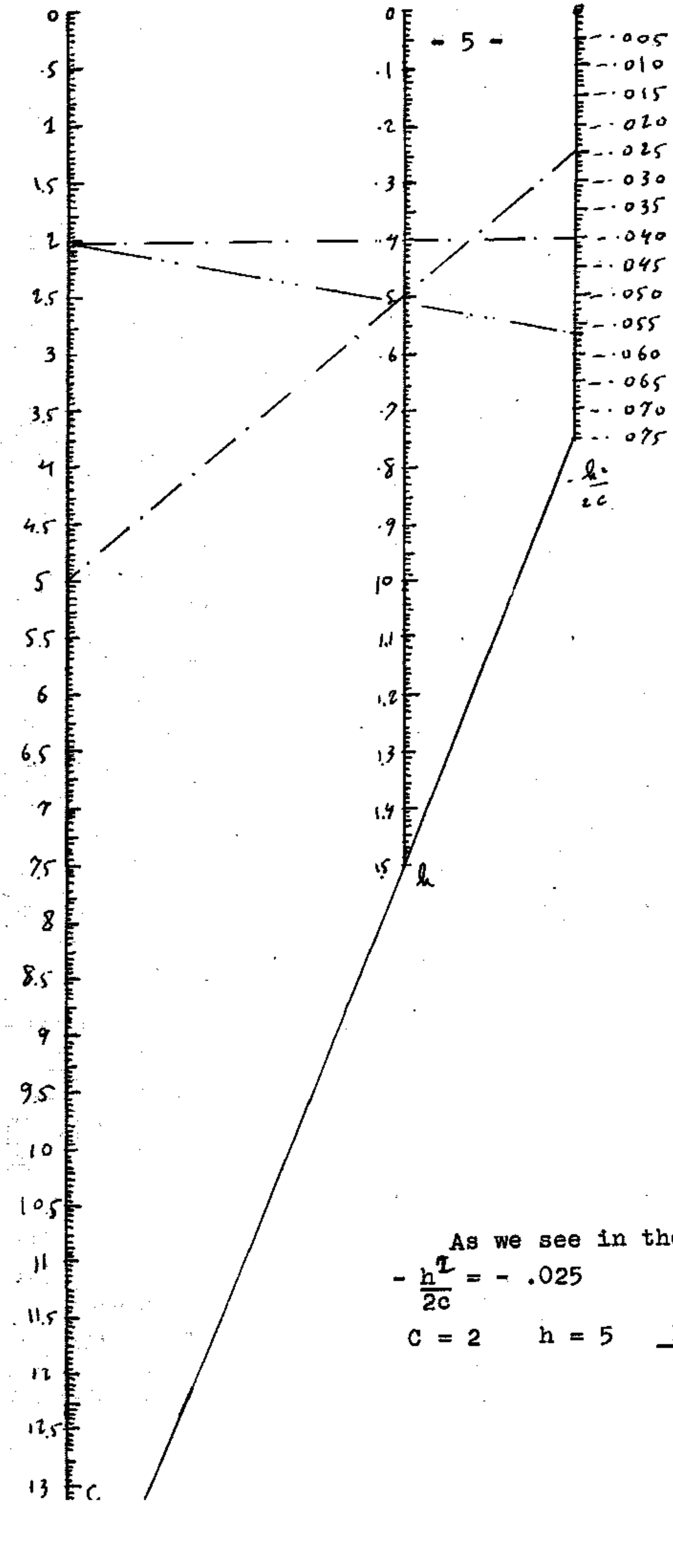


As we see in the figure when c=5 and h=5,

$-\frac{h^2}{2c} = -.025$

$C = 2 \quad h = 5 \quad \frac{h^2}{2c} = 0.06$

FUNDAMENTAL PRINCIPLE :

The fundamental principle involved in the construction of nomography consists in the representation of an equation connecting three vriables for instance, f (u,v w) = 0,by means of three scales along three curves (or straight lines) in such a manner that a straight line cuts the three scales in values u,v,and w satisfying the equation. (1)

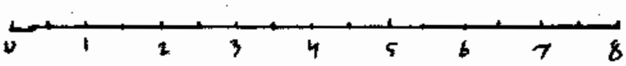
The transversal is called an index line.

We shall now make a study of some of the equations which can be represented in this way,and of the nature and relations of the saoles representing the variables involved.

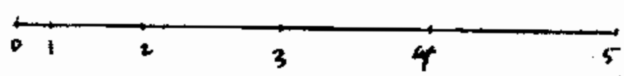
Before studying the construction of the nomograms I found it interesting to mention few words about the scales and the representation of a function by a scale.

I. Definition of a scale.- A graphical scale is a curve or axis on which are marked a series of points corresponding in order to a set of numbers arranged in order of magnitude.

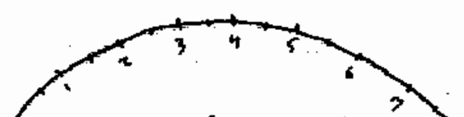
If the distance between successive points are equal,the scale is said to be uniform. If the distance between successive points are unequal,the scale is said to be non-uniform. (2)



A uniform scale



A non-uniform scale



A uniform scale

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(1) Lipka page 44 (2) Lipka page (44 - 45)

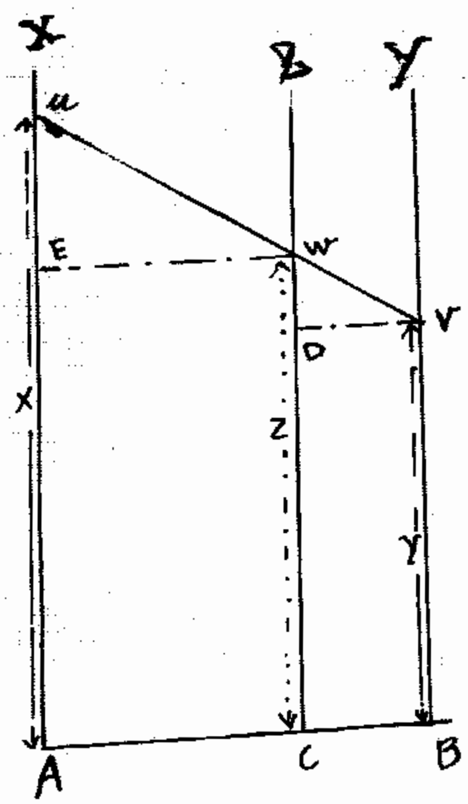
<u>Representation of a function by a scale</u> .- Consider the function u^2 of a variable u from the table .

$u =$	0	I	2	3	4	5	6	7	8	9	IO
$u^2 =$	0	I	4	9	I6	25	36	49	64	8I	IOO

and on an axis OX lay off from the origin O, lengths equal to $X = 0.04\ U^2$ inches ; mark at the points or strokes indicating the end of each segment the corresponding value of u. Thus,a stroke marked u is at a distance of $0,04\ u^2$ inches [1] from the origin. The length 0.04 inches is chosen arbitrarily in this case to represent the unit segment used in laying off the value of u^2 on the axis. This unit segment is called the <u>scale modulus</u> : usually we give it the letter(m),in our coming work. So X = m (u) is our equation of the above scale.

NOMOGRAMS FOR ADDITION AND SUBTRACTION

(I) Equation of the form $f_1(u) + f_2(v) = f_3(w)$



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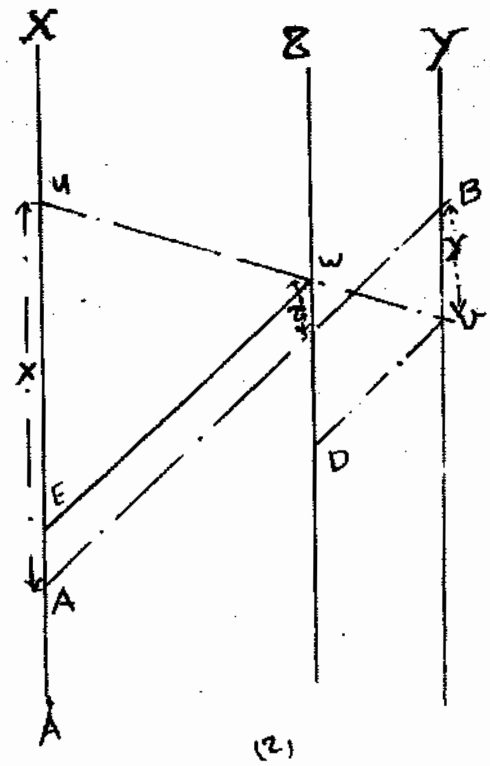

(2)

1) Lipka page (4)

Let A X, by, C Z, be three palallel axes with ABC any transversal or base line Fig. { (I) (2) } . Draw any index line cutting the axes in the points u, v, w, respectively, so that A u = X, B O = y, cw = z . Hew are x, y, z related ?

If $\frac{A\ e}{C\ B} = \frac{m_I}{m_2}$ and if through v and w we draw lines parallel to AB, then the triangles u E w and w D v are similar, and $\frac{E\ u}{D\ w} = \frac{E\ w}{D\ v} = \frac{A\ C}{C\ B}$ or $\frac{X-Z}{Z-Y} = \frac{m_I}{m_2}$.

$$m_2\ X + m_I\ Y = (m_I + m_2)\ z \text{ or } \frac{X}{m_I} + \frac{Y}{m_2} = \frac{Z}{\frac{m_I\ m_2}{m_I + m_2}}$$

Now if A X, B Y, C Z carry the scales $X = m_I\ f_,(u)$, $y = m_2\ f_2(v)$, $Z = \frac{m_I\ m_2}{m_I + m_2}\ f_3\ (w)$, respectively, the last equation becomes $f_I\ (u) + f_2\ (v) = f_3\ (w)$ and any index line will cut the axes in three points whose corresponding values, u, v, w satisfy this equation.

We also note that for the equation $f_I\ (u) - f_2(v) = f_3(w)$, the scales $X = m_I\ f_I(u)$ and $y = -\ m_2\ f_2(v)$ are constructed in opposite directions as in fig (2). (1)

Hence in order to construct the nomogram of the equation $f_{I(u)} + f_{2(v)} = f_{3(w)}$, we proceed as follows :

I. Draw two parallel lines (X and Y axes) any distance apart, and on these construct the scales $x = m_I f_I(u)$ and $y = m_2 f_2(v)$ where m_I and m_2 are arbitrarily moduli. The graduations of the u and v scales may start at any points on the axis.

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(1) Lipka page 44

2. Draw a third line (z axis) parallel to the X-and Y-axes, such that $\frac{\text{distance from X axis to Z axis}}{\text{distance from Z axis to Y axis}} = \frac{m_I}{m2}$

3. Determine a starting point for the graduations of the w scale. This may be the point C (Z=0) cut out by the line from A (X=0) to B (Y=0). If the range of the varaibles u and v is such that the points A and B do not appear on the scales,a starting point for the w-graduations may nevertheless be found by noting that three values of u,v,w satisfying equation $m_I f_I (u) + m_2 f_2(v) = m_2 f_3 (w)$ must be on a straight line ; thus, assign values to u and v,say U_o and V_o , and compute the corresponding value of w,say w_o from our equation ; mark the point in which the line joining $u = u_o$ and $v = v_o$ cuts the z axis with the value $w = w_o$ and use this last point as a starting point for the w graduations.

4. From the starting point for the w - graduations,construct the scale $z = m_3 f_3 (w) = \frac{m_I m_2}{m_I + m_2} f_3 (w)$.

In laying off the w-scale which the help of the above procedure,the following operation will increase the accuracy of the construction. Assign two or three sets of values to u and v,and compute the corresponding values of w ; let these be $(u_o\ v_o\ w_o),(u_I,v_I,w_I)$ and $(u_2,v_2\ w_2)$. (1)

Draw the index lines $(y_o\ v_o)$ $(u_I\ v_I)$,and $(u_2\ v_2)$,and mark the points in which these lines cut the z-axis with the corresponding values of w,and then we will notice whether the

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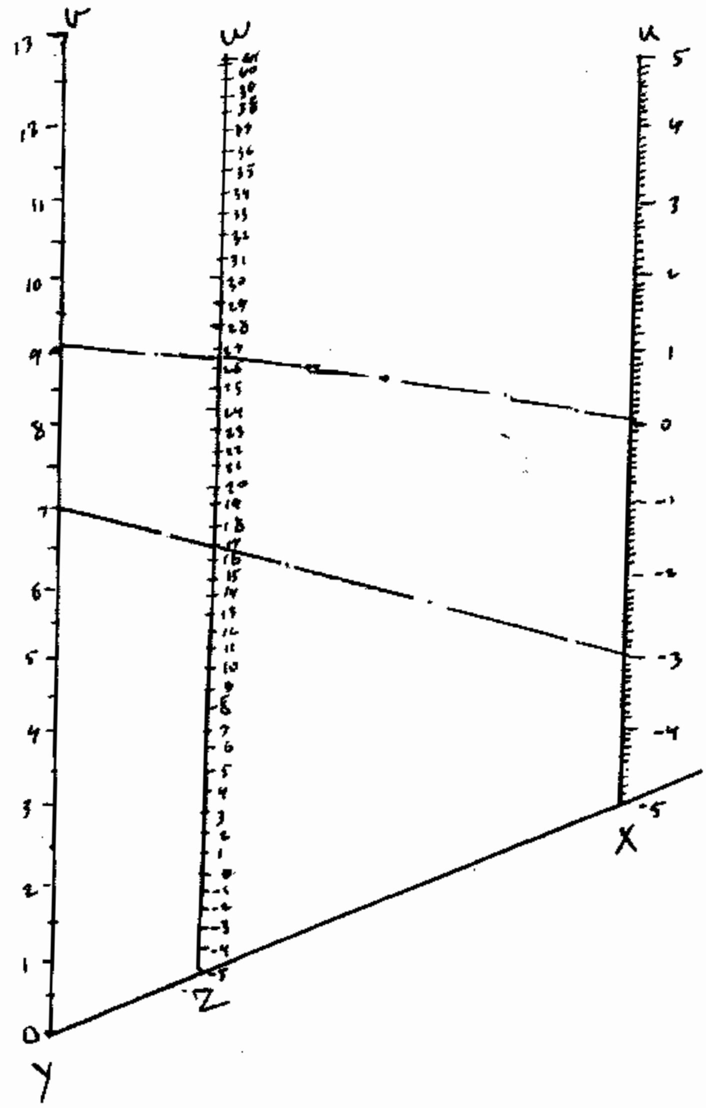
(1) link page 45

points of the scale numbered w_o , w_I , w_2 , pratically coincide with the like-numbered points of the axis. This procedure is especially important when the modulus, m_3 , is quite small.

Using the previous explaination, we can now construct a nomogram for $z = x + 3 \ y$.

Let $m_I = I \quad m_2 = I/3 \quad \therefore \quad m_3 = \dfrac{m_I \, m_2}{m_I + m_2} = I/3/4/3 = I/4$

and $m_I : m_2 = I/I/3 = 3$



The equation of the scale are $x=u \quad y=v \quad z=I/4 \ w$ and we see that when $x=-3 \quad y=7 \quad z=I7$

also $x= o \quad y=9 \quad z=27$

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¹) Brodsky page 26

(II).-Equation of Form $f_I(u) + f_2(v) + f_3(w) + \ldots\ldots = f_4(t)$

Nomogram for equation II : The above equation is merely an extension of equation (I) and the method of finding the nomogram of the former is an extension of the method employed in charting (chart or nomogram) of the latter. (1)

For difiniteness, let us consider the case of four variables and the equation in the form $f_I(u) + f_2(v) + f_3(w) = f_4(t)$ let $f_I(u) + f_2(v) = q$. This equation is in the form (I) and can therefore be charted by means of three parallel scales, but the q scale need not be graduated. We then have $q + f_3(w) = f_4(t)$, which is also in the form (I) and can therefore be charted by means of three parallel scales one of which is the q-scale already constructed. The graduations of the u, v, and w scales may start any where along their axes, but a starting point for the graduations of the (t) scale must be determined by a set of values $u=u_o$, $v=v_o$, $w=w_o$, $t=t_o$ satisfaying equation II ; thus join u_o and v_o by a straight line and mark its point of intersection with the q axis ; join this point with w_o cutting the t scale in a point which must be marked t_o ; this last point is then used as a starting point for constructing the t scale. To read the comp leted nomogram we thus use two index lines, one joining points on the u and v scales, the other joining points on the w and t scales, intersecting the q axis in the same point ,

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(1) Lipka page (55)

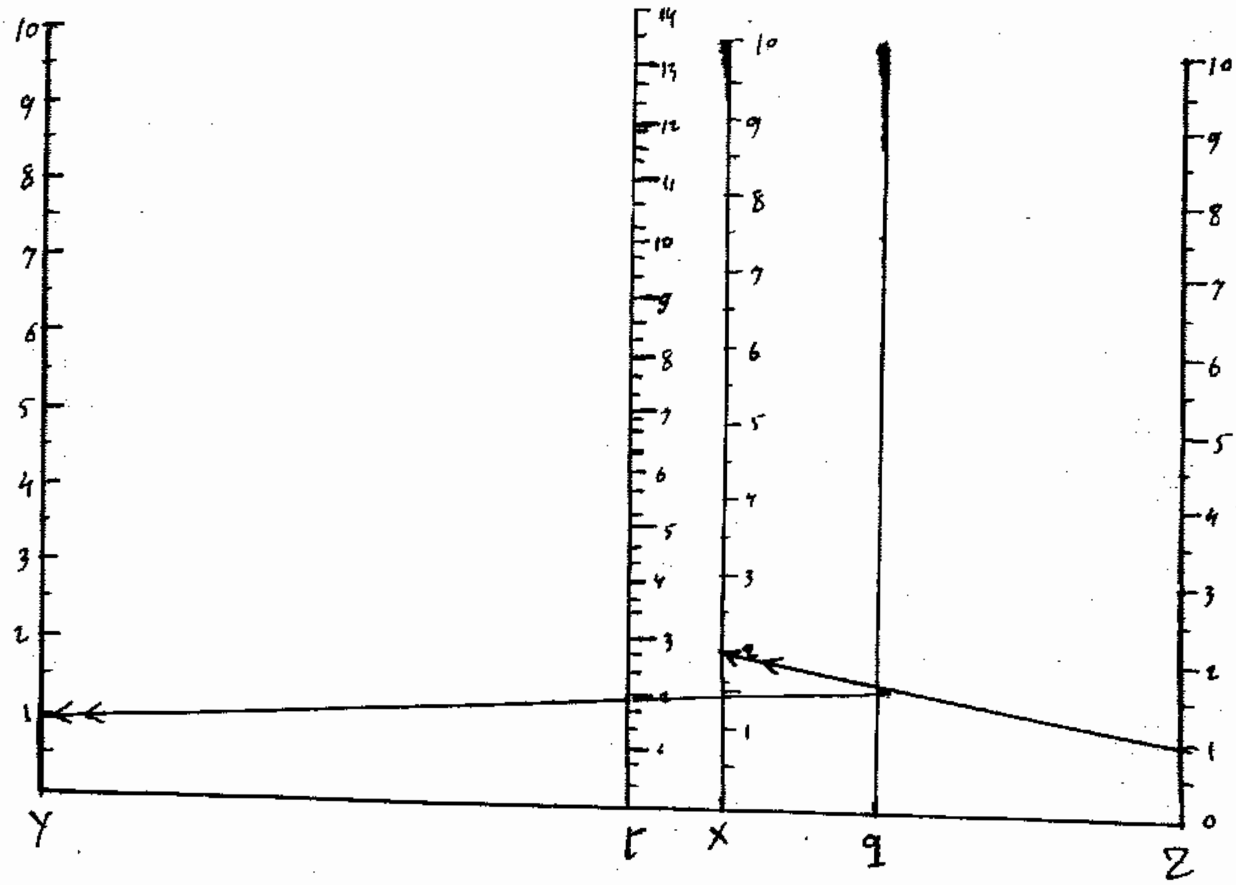
and the following figure will illustrate all that we said :

It is required to construct a nomogram for :

$$Z + 2X + Y = 3\,t$$

Let $Z + 2X = q$. and let $m_1=1$ $m_2=\frac{1}{2}$ therefore $m_3=1/3$.

$q + y = 3\,t$



If the modulus of y is $m_4=1$ then $m_5= \dfrac{m_4\ m_3}{m_4+m_3} = 1/4$

and the ratio $m_3 : m_4=1/3 : 1=1/3$

our q axis is called the reference line, and as we see in the figure when $z=1$ $x=2$ $y=1$ we get $t = 2$ (1)

Now we may say the extension of this method to equations of the form II containing more than four variables is obvious.

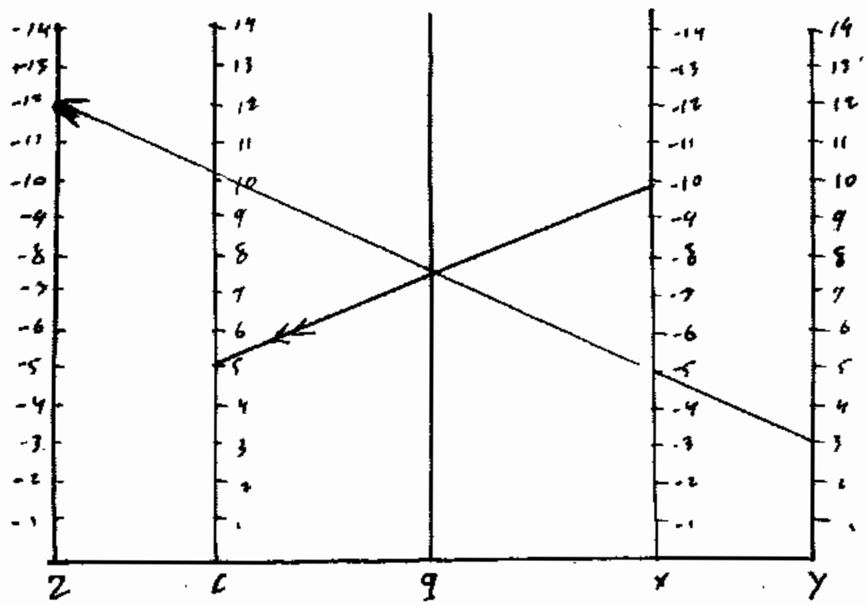
(1) hyrka page (55)

Subtraction .

If we have now more than three variables we proceed like in addition but the graduation of some of the axises must be reserved. The method to be adopted will perhaps be most easily grasped by considering one simple case. (1)

It is required to find the nomogram of $t=x+y-z$

Let $m_3=I$ and $m_z=I$ then $m_q=\frac{1}{2}$ where $q=y-z$ and $\frac{m_y}{m_z}=I$



If we take $m_x = I$ we get $m_t = \frac{m_q\, m_t}{m_x+m_q} = \frac{-I\times\frac{1}{2}}{-I+\frac{1}{2}} = +I$ and

$\frac{m_x}{m_q} = \frac{I}{\frac{1}{2}} = 2$ so the graduation of t out its position in known and if we suppose $y=3$ $z=-I2$ we draw the fisrt index line(q) After that if we give to X the value (-IO),we join this point by u and we get finally the value(5) on the T axis and truely $t=-IO+3-(-I2)=5$.

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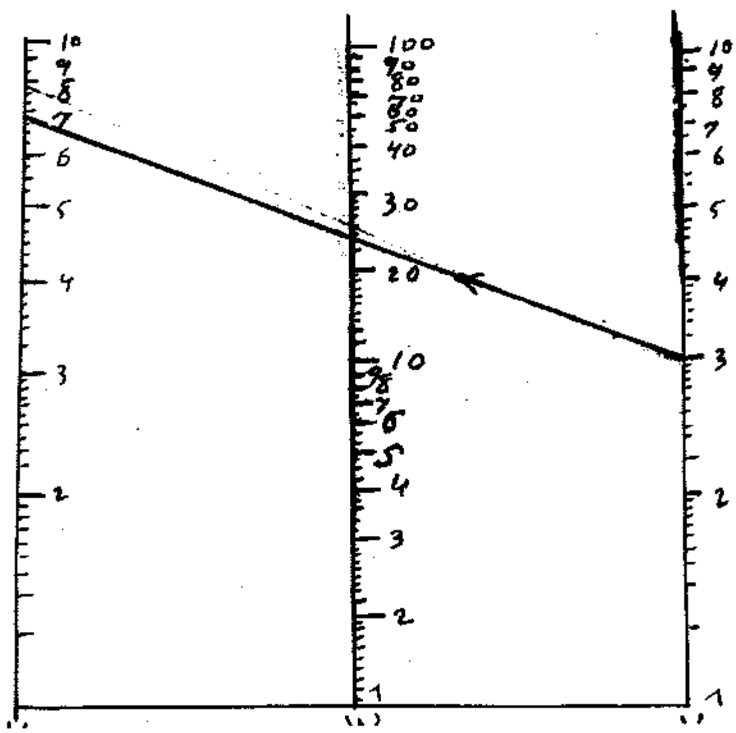
(1) Brodesky page 38

In case of successive subtraction and addition we will have more than one reference line and the work is the same as for four variables.

NOMOGRAM for MULTIPLICATION and DIVISION

The equation u.v=w   If we write this equation as log u + log v=log w, we get an equation of the form (I). Let u and v range from I to (IO) ; then w ranges from (I) to (IOO). Construct IO, in apart, the parallel scales $X = m_I \log u = IO \log u$ and $y = m_2 \log v = IO \log v$. Since $m_I : m_2 = I : I$, the Z axis is midway between the X - and y axes. The line joining u=I and v=I must cut the Z axis in w=I, and using this last point as a starting points, construct the scale $z = \frac{m_I \ m_2}{m_I + m_2} = w = 5 \log w$. The index line in the completed nomogram (in the following figure) gives the reading u=7, v=2I. Since the u and v scales are logarithmic scales ; and after a while I'll mention few words about the logarithmic scale, we may read these scales as ranging from IO^p to IO^{p+I} where p is any integer, with a corresponding change in the position of the decimal point in the value of w. 1)



1) Lipka hal. 41

LOGARITHMIC SCALES

The distance of any graduation from the begining of the scale is proportional to the logarithm of the number of the graduation. Hence the beginning of each scale is marked unity and the end should be marked ten. Since, however, powers of ten do not affect the mantissa, i.e. the decimal fractional part of a logarithm, it is usual to mark the end of each scale unity, we can, in fact, consider the scale to refer to numbers IO^p to IO^{p+I} (as we said before) : where p is a positive or negative whole number. (1)

To construct a logarithmic scale we take a uniformly graduated line, as e.g. on a sheet of squared paper, and mark off the logarithms as given in the tables-This is sufficiently accurate for our purpose.

We fisrt put in the numbers 2,3...9. Then we estimate how many subdivisions to include between I and 2, 2 and 3, and so on we note that.

(I) Graphically it is of little use to deal with execively small intervals ; in fact, the smallest subdivisions must not be much less than I milimetre ; or I/25 inch.

(II) The whole of any one interval I to 2, 2 to 3,..must be subdivided in the same way. (2)

The fundamental principle in the use of a logarithmic scale is that if we add up the geometrical distances corresponding to two numbers we get the geometrical distance for the product.

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(1) Brodsky page 47 (2) Brodsky page 48

EQUATION of FORM.

$$f_I(u).f_2(v).f_3(w).........= f_4(t)$$

This form of equation can be brought immediately into the first form by taking logarithms of both numbers ; $\log f_I(u)+\log f_2(v)+\log f_3(w)........................ = \log f_4(t)$.

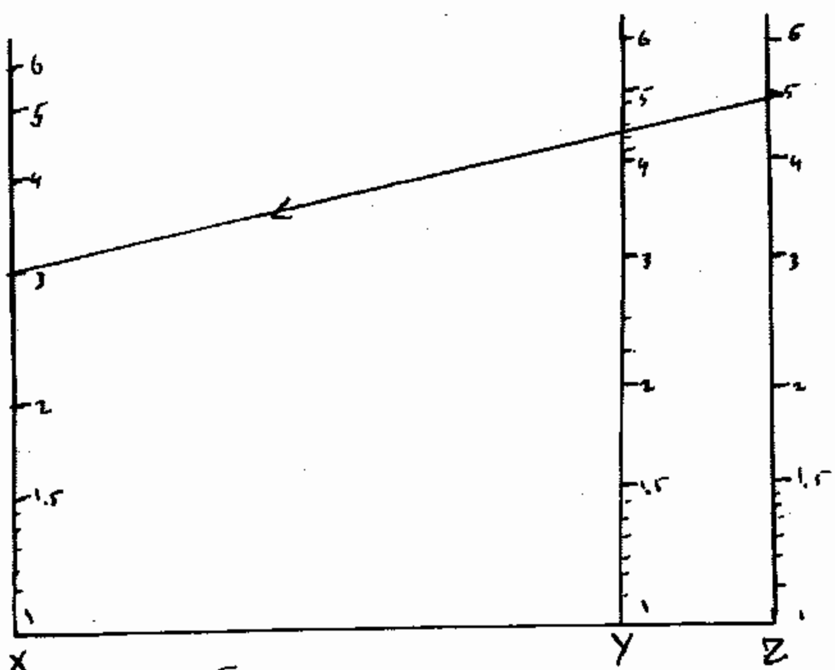
And so we proceed to convert the addition and subtraction into multiplication and division nomograms, by means of converting the uniform scales into corresponding logarithmic scales : If we construct now the nomogram of $y = \sqrt[5]{x.(z)^4}$ we will have a clear idea about what we illustrated before.

The equation can be written $\quad \operatorname{Log} y=\frac{I}{5}\log x + \frac{4}{5}.\log z$ or

$5 \log y= \log X + 4 \log 2$

Let M_x=IO $\quad m_2= \frac{IO}{4} \quad m_q=2$ and $m_x : m_2=4$. The equation of our scales are $X = IO \log x \qquad Y = IO \log y \quad Z=IO \log z$

A starting point for the Z scale is found by noting that when x=I and y=I then z=I, and then by using the same units on the three scales, we get by drawing our index line that when X=3 y=5 z=4.5 ١)



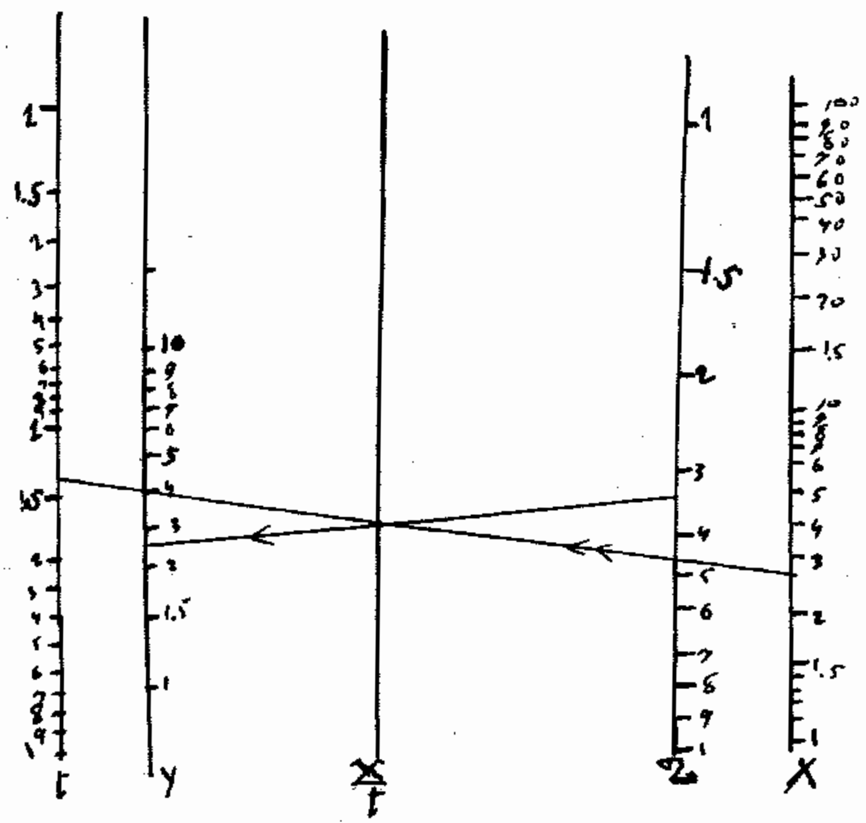
Finally if we construct the nomogram of the expression $y=675X\,Z^2/t$ we will grasp the nomographic representation of both division, and multiplication in the same time.

by taking the logarithm of both numbers we get

$\text{Log } y = \text{Log } 675 + \text{Log } X + 2 \text{ Log } Z - \text{Log } t.$

let $m_y = I0 \quad m_x = I0 \quad m_z = \frac{I0}{2} \quad m_t = I0$.

Here the graduation of t axis must be reversed. We have to notice the starting point of the y scale which is found by putting X = I . Z = I and t = I we get log y = log 675 = 2.83 so we make the y scale have its zero point .I7 to the point 3 because 3 - 2 . 83 = . I7



NOMOGRAMS WITH TRIGONOMETRICAL FUNCTIONS .

We have seen till now nomograms that do not concern us too much,and I mentioned them in order to get a simple idea about this branch of mathematic (nomography).

I found it necessary also to write few lines about the general theory of nomograms with two parallel scales,and what do we mean by parallel coordinates,before having our trigonometrical function ?

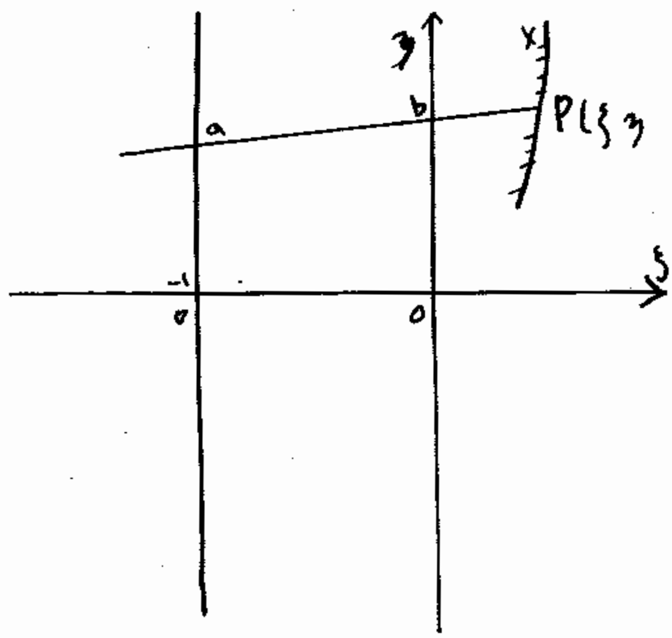
Distances a,b cut off on parallel uniform scales are called parallel coordinates.

NOMOGRAMS WITH PARALLEL COORDINATES

We shall proceed to show how to find the equation of the X scale in a nomogram with parallel coordinates,and how to graduate it.

Choose rectangular axes O ξ , o n,so that o n is the b scale and O is the zero of the b scale. Let the a scale be at a distance of ~~one~~ ξ unit to the left (negative side) of the b scale.

Consider any point P whose coordinates are (ξ n). We shall find what relation there must be between the lengths a,b cut of on the a,b scale for all lines that pass through P.



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Take the line through P given by a,b the equation of this line is $Y = b + (b-a)\,X$,

Where (X,Y) are coordinates of any point on it. Since P is on this line this equation also hold for (ξ,n) the coordinates of P itself. This is true for all lines through P. Hence the a,b intercepts for all lines through P satisfy the single equation $\quad n = b + (b-a)\xi \quad$ i.e.

$$\xi/n \;\; a - \frac{\xi + I}{n}\, b + I = 0$$

Let then the equation,given to be solved,be written

$$A\,(x)\; a + B\,(x)\; b + I = 0$$

so that A (x) and B (x) are known functions of x. If there is to be a difinite graduation x at P it follows that the two equations $\frac{\xi}{n}\, a - \frac{\xi + I}{n}\, b + I = 0$ and $A\,(x)\; a + B\,(x)\; b + I = 0$ are both true for any number of straight lines through P. This means that the equations are really the same,so,that we must have '

$$A\,(x) = \frac{\xi}{n}\;, \quad B\,(x) = -\,\frac{\xi + I}{n} \quad \text{i.e.}$$

$$\xi = \frac{A\,(x)}{A(x)+B(x)} \qquad n = -\,\frac{I}{A\,(x) + B(x)}$$

If we eliminate x we obtain an equation between ξ,n telling us what the relation between the coordinates of P must be if P is to be graduation in the nomogram,i.e. a point on the x curve. This relation defines the x curve,which can be plotted. The graduation of the curve is affected by finding from the above equations what is the value x for any point (ξ ,n).

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In practice, it is often simpler to consider x as a parameter in terms of which ξ, n are given, this parameter being in fact the graduation.

TRIGONOMETRICAL FUNCTIONS

Threr being no restriction on the form of A (x), B (x) in the method mentioned before ; we can use this method for the construction of nomograms for transcendental equations ; as e.g. equations involving trigonometrical functions.

Nomograms for $a \tan X + b \sec X + I = 0$ 1)

In order to decide on the best positions of the a, b scales let us put the a scale on the line $\xi = a$, the b scale on the line $\xi = B$, in the previous method.

The equation of the line joining the a graduation to the b graduation is

$$n = \frac{b-a}{B-a}(\xi - a) + a. \text{ i.e. } \frac{\xi - B}{B-a}\frac{a}{n} - \frac{\xi - a}{B-a}\frac{b}{n} + I = 0$$

To make this agree with

$$a \tan x + b \sec x + I = 0$$

we make $\dfrac{\xi - B}{(B-a)\,n} = \tan X$, $\dfrac{\xi - a}{(B-a)\,n} = -\sec X$

we at once get, since $\sec^2 x = I + \tan^2 x$, the equation

$(\xi - a)^2 - (\xi - B)^2 = (B-a)^2 n^2$

i.e $2\xi = (B - a) n^2 + (B + a)$ to get the simple result, we therefore use $a + B = 0$

This gives $2\xi = 2 B n^2$

Hence we choose $B = I$ $a = - I$

and the x curve is the parabola $\xi = n^2$

The graduations are seen to be given by $x = \dfrac{I - \xi}{I + \xi}$

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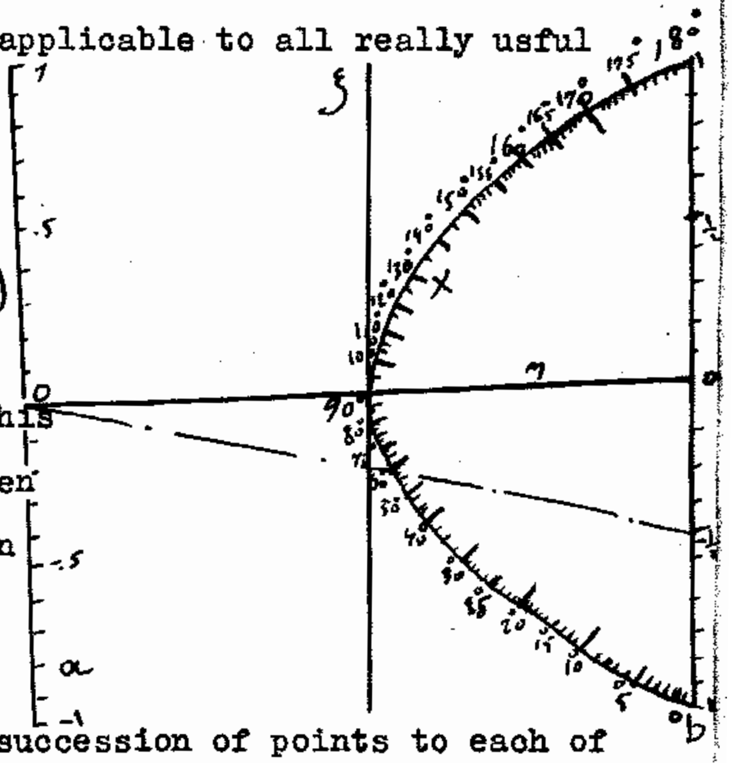
1) Brodetsky page 100

The coming figure will be applicable to all really usful values of a,b



We see that when a = o

b = $-\frac{1}{2}$ = - . 5,X =(60°)

<u>The practical and automatic construction of this nomogram.</u> (1)

Suppose we wish to construct this nomogram for values of a between I and -I and values of b between I and -I also our object is to find the X curve and its graduations - in other,we want a succession of points to each of which is assigned a certain graduation X.

We draw the two scales a,and b with some convenient units. Since the ranges are equal,we use equal units. The distance apart is chosen so as to make the figure as square as possible.Choose some value of a,say a=o.Then,if we join this point on the a scale to any point on the b scale,we have somewhere on this line the graduation X given by b sec X + I = 0 cos X = - b

Thus to get X points for

0° I0° 20° 30° 40° 50° 60° 70° 80° 90°

we join the point a = 0 to the following points respectively on the b scale

-I -.985 -.939 -.867 -.767 -.643 -.5 -.343 -.I74 0

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(1) Brodesky page 122

for the values larger than 90º we simply change the signs of the previous values.

Again, take some definite value of b say b=0 . Then if the point b = 0 is joined to any point a on the a scale, this join passes through the graduation X on the X curve, if a tan X=-I

cot X = - a (1)

Hence to get the X points for

0º IOº 20º 30º 40º 50º 60º 70º 80º 90º we join the point b = 0 to the following points, respectively, on the a scale

∞ - 5.68 -2.75 -I.74 -I.I92 -.643 -.578 -.364 -.I77 0

for the values X larger than the above values, we get them by reversing the signs only.

The intersections of the pairs of lines for each value of X gives the points on the X curve for these values of X we will be obliged sometimes to choose another value for b or for a . (2)

We now join the points thus obtained by as smooth a curve as possible, and insert additional graduations, either by repeating one or other of these processes, or by free-hand interpolation, taking note of the way in which the graduations already obtained suggest these subdivisions. This way is called the practical and automatic construction of nomograms .

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(1) Brodsky page 123 (2) Brodsky page (124)

MEASUREMENT of TIME .

The measurement of intervals of time is made to depend upon the period of the earth's rotation on its axis,which is known to be uniform. The most natural unit of time for ordinary purposes is the solar day,on the time corresponding to one rotation of the earth with respect to the sun's direction. On account of the motion of the earth around the sun once a year the direction of this reference line is continually changing with reference to the directions of fixed stars,and the lengh of the solar day is not the true time of one rotation of the earth. In some kinds of astronomical work it is more convenient to employ a unit time of one rotation,namely sidereal time (or Star time). (1)

Since most of our work will be on the subject of time, I want to mention breively the different times and then,I'll construct the nomogram which gives us the correction that must headded or subtracted from the mean solar time in order to get the sidereal time and vice versa.

Sidereal Time. The sidereal time at a given meridian at any specific instant is equal to the hour angle of the vernal equinox measured from the upper half of that meridian.(2) It is therefore a measure of the angle through which the earth has rotated since the equinox was on the meridian,and shows at once the position on the sphere at this instant with respect ......

(1) Hosmer page 40 (2) Hosmer page 41

to the observer's meridian.

Solar time. The solar time at any instant is equal to the hour angle of the sun's center plus 180º or 12 hours ; in other words it is the hour angle counted from the lower transit.

It is the angle through which the earth has rotated, with respect to the sun's direction, since midnight, and measures the time interval that has elapsed.

Since the earth revolves around the sun in an elliptical orbit in accordance with the law of gravitation, the apparent angular notion of the sun is not uniform, and the days are therefore of different length at different seasons.

Under modern conditions, which demand accurate measure of time by the use of cloks and chronometers, and an invariable unit of time is essential. The time ordinarily employed is that kept by a fictitious point called the "mean sun", which is imagined to move at a uniform rate along the equator, its rate of motion being such that it makes one revolution around the earth in the same time as the actual sun, that is, in one year. The time indicated by the position of the mean sun is called mean solar time. The time indicated by the position of the real sun is called apparent solar time. Mean time cannot, of course, be observed directly, but must be derived by computation. (1) Equation of time.

The difference between mean time and apparent time at any instant is called the equation of time and depends upon how

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(1) Hosmer page (42)

much the real sun*is ahead of or behind its average position.

Conversion of Mean solar and sidereal intervals of Time.

It has already been stated that on acount of the earth's orbital motion the sun has an apparent eastward motion among the stars of nearly (1º per day). This eastward motion of the sun makes the intervals between the sun's transits greater by nearby 4^m than interval between the transits of the equinox,that is the solar day is nearly 4^m longer than the sidereal day. (1)

Since each kind of day is subdivided into hours,minutes, and seconds,all of these units in solar time will be proportionally larger than the corresponding units of sidereal time. If two cloks,are regulated to mean solar time and the other sidereal time,were started at the same instant,both reading 0^h,the sidereal clok would immediatly begin to gain on the solar clok,the gain being exactly proportional to the time elapsed,that is,about 10^s per hour,or more nearly 3^m56^s per day.

This fact enables us to establish the exact relation between the two time units. It is known that the tropical year (equinox to equinox) contains 365,2422 mean solar days.Since the number of sidereal days is one greater (2)

We have............366,2422 sidereal days = 365,2422 solar days

a 1 " day = 0.99726957 " day

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(1) Hosmer page 54 (2) Hosmer page 55

and I solar day = I.0027379I sidereal days.

These two relations may be written : (1)

24^h sidereal time = (24^h + 3^m55^s,909) solar time

24^h mean solar time=(24^h + 3^m56^s,555) sidereal time

Also we can put the same relation into more convenient form for computation by expressing the difference in time as a correction to be applied to any interval of time to change it from one unit to the other. If I_m is a mean solar interval and I_s the corresponding number of sidereal units

then $I_s = I_m + 0.0027379I \times I_m$

and $I_m = I_s - 0.00273043 \times I_s$

These give + 9^s,8565 and - 9^s,8296 as the corresponding corrections for one hour of solar and sidereal time respectively. (2)

Now if we use the two previous relations to have two tables, one for converting sidereal into mean solar time, and the other for converting mean solar into sidereal time, we can then construct their nomograms.

If we call e the correction that should be subtracted from the sidereal time in order to get the mean time we will have

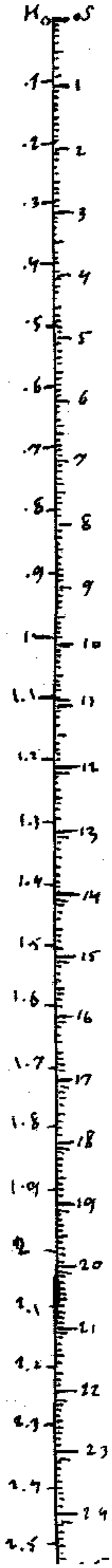
Mean time = Sidereal time-$\bar{e}$

and the small following table will help us too much for our nomographic method

Sidereal hours	Correction	Sidereal minut.	Correction	Sider. Sec.	Correct.
I	$^m0\ 9.^s83$	I	$0.^s164$	I	$0.^s003$
2	0I9.66	2	0.328	2	0.005

(1) Hosmer page 55 (2) Hosmer page (56)

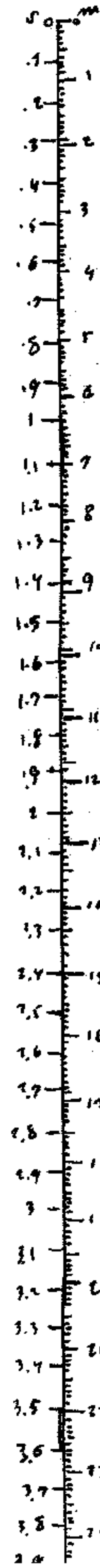
Hours



Correction in second

For instance the correction for on hour in the left scale is equal to $9^{s}.8$. For I5 minutes is equal to $2^{s}.9$ and so on

On the right scale we have the correction for one minute is equale to,I6^{s} for 25 minutes we have to subtract 4 seconds and so on



From the previous small table we can construct three nomograms, one representing the hours, the second the minutes, and the third the seconds and for all of them we can use stationary scales. By the way it is interresting to explaine what do we mean by stationary scales.

If we have two varaibles u and v of the form $v=f(u)$ we may represent the relation connecting them grafically by constructing the two scales $x = m\,v$ and $x = m\,f\,(u)$ on opposite sides of the same axis or on adjacent or parallel axes with the same modulus and from the same origine.

Standard time.

From the definition of mean solar time we see that at any given instant the solar times at two places will differ by an amount equal to their difference in longitude expressed in hours, minutes and seconds. Before 1883 it was customary in all the countries for each large city or town to use the mean solar time of a meridian passing through that place, and for the smaller towns in that vicinity to adopt the same time.

Before railroad travel become extensive this change of time from one place to another caused no great difficulty, but with the increased amount of railroad and telegraph these frequent and irregular changes of time became so inconvenient and confusing that in 1883 a uniform system of time was adopted. (1)

The country is devided into times belts, each one theorically

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(1) Hosmer from the chapter of time (50)

15º wide. Wherever the change of time occurs the amount of the change is always exactly one hour.

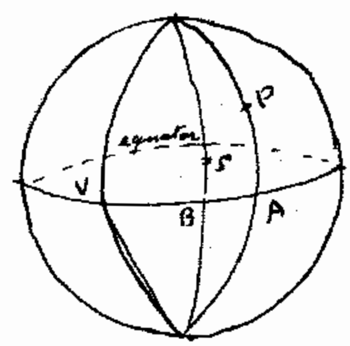
The minutes and seconds of all clocks are the same as those of the Greenwich clock. When it is noon at Greenwich it is 2^h PM at Beirut { 3^h PM at Bagdad}.

Standard time is now in use in the principal contries of the world ; in most cases the systems of standard time are based on the meridian of Greenwich. (1)

To change from local to standard time or the contrary.

The change from local to standard time, or the contrary, is made by expressing the difference in longitude between the given meridian and the standard meridian in units of time and adding or subtracting this correction, remembering that the farther west a place is the earlier it is in the day at that given instant of time.

Relation between Sidereal time, Right Ascention, and Hour angle of any point at a given instant.



In the above figure the hour angle of equinox, or local sidereal time, at the meridian of P, is the arc AV. The hour angle of the star S at the meridian of P is the arc A B .

(1) Hosmer page 52

The right ascension of the star S is the arc V B. It is evident from the figure that

A V = V B + A B   or

S   =   a   + t

where S = the sidereal time at P, a = the right ascension and t = the hour angle of the star. This relation is a general one and will be found to hold true for all position, except that it will be necessary to add 24^h to the actual sidereal time when the sum of a and t exceed 24^h. For instance, if the hour angle is $I0^h$ and the right ascenssion is 20^h the sum is 30^h, so that the actual sidereal time is 6^h. When the sidereal time and the right ascension are given and the hour angle is required we must first add 24^h (if necessary) to the sidereal time ($24^h + 6^h = 30^h$) before subtracting the 20^h right ascension, to obtain the hour angle $I0^h$. If however, it is prefered to compute the hour angle in a direct manner the result is the same. When the right ascension is 20^h the angle from V westward to the point must be $24^h - 20 = 4^h$. This 4^h added to the 6^h sidereal time give $I0^h$ for the hour angle as before. (1)

STAR ON THE MERIDIAN

When a star is on any meridian the hour angle of the star at that meridian becomes 0^h. The sidereal time at the place then becomes numerically equal to the right ascension of the star. This of great practical importance because one of the

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(1) Hosmer page (53)

best methods of determining the time is by observing transits of stars over the plane of the meridian. The sidereal time thus becomes known at once when a star of known right ascension is on the meridian. (1)

Our method of determining time is by transit of a northern star over vertical line through Polaris . By this method the hour angle of the observed star is not zero but it is few minutes only, and our aim is to compute this hour angle by constructing nomograms.

DETERMINATION OF TIME BY TRANSIT OF A STAR OVER VERTICAL LINE THROUGH POLARIS .

We will not use in our observation ~~xxx~~ a telescope or any other astronomical instrument, but all what we are in need of is a thread attached to it a weight of about one kilograme.

After having this we suspend the thread with the attached weight from a branch of tree or from something similar to that. The thread which takes the direction of a plumb line and Polaris determine a vertical plane. By this plane we can begin our observation. (2)

In making observation by this method the line of sight of the observer is set in the vertical plane through Polaris, and the time of transit of some northern star (called the time-star) across this plane is observed immediately ; (2) the correction

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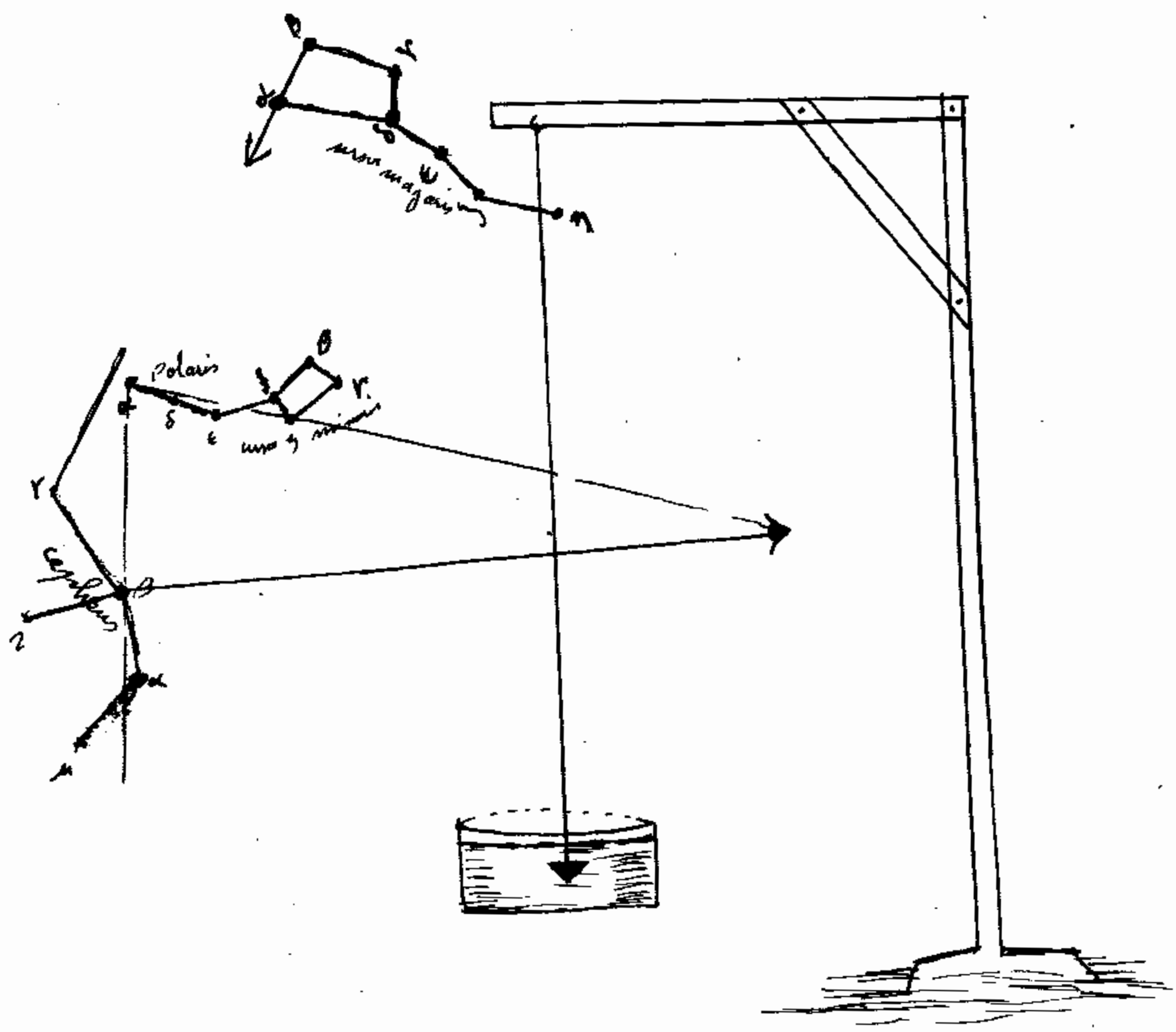
(1) Hosmer page (54) (2) Hosmer page 140

for reducing the star's right ascension to the true sidereal time of the observation is then computed and added to the right ascension. The advantages of this method are that the direction of the meridian does not have to be established before time of observation can begun,and the errors due to the instrument is reduced to a zero. (1)

If Polaris is near its elongation then the azimuth of the sight line will be a maximum.



(1) Hosmer page 140

In order to deduce an expression for the difference in time between the meridian transit and the observed transit let a and a_o the right ascension of the stars S and so the sidereal times of transit over the thread, t and to the hour angles of the stars, the subscripts referring to Polaris. Then we can write the following relation : (1)

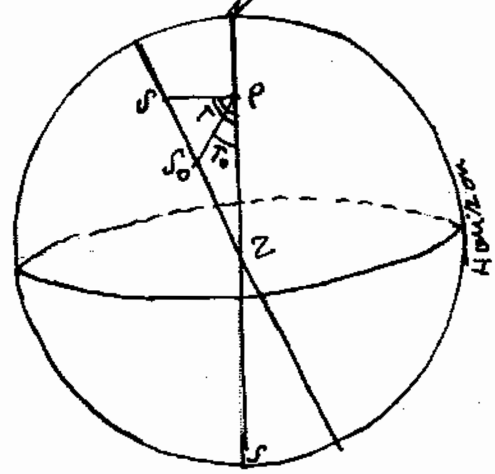
$$t = S - a$$

$$t_o = S_o - a_o$$

but since we observed the two stars in the same time we have $S = S_o$, so if we subtract the two relation we get :

$$t - t_o = a_o - a$$

Now if we draw the figure of the celestial sphere and the horizon we can get another relation which helps us to compute the hour angle t of the time star.



Let so be the position of Polaris when S it is observed; P the celestial north pole ; Z, the zenith of the observer ; and S the time star in the position in which it is observed.

Let also P_o be the polar distance of Polaris ; ζ and ζ_o the zenith distances of the two stars ; and δ and δ_o their ~~altitudes~~ declinations : then in the triangle P S_o S (2)

$$\frac{\sin S}{\sin P_o} = \frac{\sin (t-t_o)}{\sin S S_o} = \frac{\sin (t-t_o)}{\sin (\zeta-\zeta_o)}$$

(1) Hosmer page 141 (2) Hosmer page 141

from which we get

$$\sin S = \sin P_0 \sin (t-t_0) \csc (\zeta-\zeta_0) \qquad (I)$$

In triangle P z s ,

$$\frac{\sin S}{\sin(90-\varphi)} = \frac{\sin t}{\sin \zeta}, \text{ or } \sin S = \cos \varphi \sin t \csc \zeta \; (2)$$

Where φ is the latitude of the place.

equating equation number (I) and equation number (2) we get

$$\sin t \cos \varphi \csc \zeta = \sin P_0 \sin (t-t_0) \csc (\zeta - \zeta_0)$$

Since t and P_0 are small the angle may be substitued for their sines , and

$$t = P_0 \sec \varphi \sin (t - t_0) \csc (\zeta - \zeta_0) \sin \zeta$$

But since we do not know ζ and ζ_0 the factor $\sin \zeta$ and csc $(\zeta - \zeta_0)$ may be replaced by $\sin (\delta - \varphi)$ and $\csc (\delta - \delta_0)$ because when a star is on the meridian its altitude is equal to

$$\varphi + 9^0 - \delta \quad \therefore \zeta = 9^0 - (90 + \varphi + \delta) = \delta - \varphi$$

and $\zeta_0 = \delta_0 - \varphi$ the difference $\zeta - \zeta_0 = \delta - \delta_0$

then equation number (3) becomes

$$t = P_0 \sec \varphi \sin (t-t_0) \csc (\delta - \delta_0) \sin (\delta - \varphi) \text{ or}$$

$$t = P_0 \sec \varphi \sin (a_0 - a) \csc (\delta - \delta_0) \sin (\delta - \varphi) \quad (4) \text{ because } a_0 - a = t - t_0$$

In this method the latitude is supposed to be known.

Now if we compute t from (4) in second of the time, and if we add it to the right ascension of the time star, we obtain the local sidereal time of the observation of this star. This sidereal time may then be converted into local civil time and then into standard time and the wach correction obtained.

Example

Observation of B Cepheus over vertical line through Polaris ; Latitude 33° 54' 22'' N (Beirut)
Longitude = 35° 28' 10'' east date April 3, 1941

a =	21^h	27^m	$54^s.30$	Observed time = $8^h\ 9^m\ 50^s$
a_0 =	1^h	42^m	00,00	time by the tower clock
δ =	70°	18'	$05^s.60$	of A.U.B.
δ_0 =	88°	58'	00	

Our formula is

$$t = 4\, P_0 \sec\varphi \ \sin(a_0 - a)\ \sin(\delta - \varphi) \csc(\delta - \delta_0)$$

the factor 4 has been introduced in our example in order to reduce minutes of angle to seconds of time.

$$t - t_0 = a_0 - a = -(19^h 45^m 54^s.3) = -296° \ 28' \ 12''.65$$

$$\delta - \varphi = 70° \ 18' \ 0^s5.6 - 33° \ 54' \ 22'' = 36° \ 23' \ 43''.6°$$

$$\delta - \delta_0 = 70 \ 18' \ 05^s.6 - 88° \ 58' = -18° \ 39' \ 54''.4°$$

$$P_0 = 90° - 88° \ 58' = 0 \ 62'$$

Log 4 = .60206

Log P_0 = 1.79239

Log sin $(a_0 - a)$ = 9.95191

Log sin $(\delta - \varphi)$ = 9.77331

Log csc $(\delta - \delta_0)$ = .49481

Log sec 4 = . 08095

Log t = 22.69543 − 20
= 2.69543

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$$t = -495^s.9$$
$$= - \quad 8^m\ 15^s.9$$

The true sidereal time may now be found by subtracting 8^m $15^s.9$ from the right ascension of B Cepheus, the result being as follows :

a	=	21^h	27^m	$54^s.3$
t	= -		8^m	$15^s.9$
Local sidereal time	=	21^h	15^m	$38^s.4$
Longitude of Beirut	=	2^h	21^m	$52^s.60$
Greenwich Sidereal time	=	18^h	53^m	$45^s.74$
The right ascension of the sun + 12^h	=	12^h	43^m	$43^s.38$
		6^h	10^m	$2^s.36$

We must subtract $60^s.62$ from the above result in order to reduce it into mean solar time

	6^h	10^m	$2^s.36$
			$60^s.62$
Greenwich civil time	6^h	9^m	$1^s.74$
	2		
Beirut Standard time	8^h	9^m	$1^s.74$

The difference between this and the wach time ($8^h 9^m 50^s$)(which is the tower clock of A.U.B.) shows that the ~~xxxx~~ tower clock was <u>$48^s.26$</u> fast .

Now it is required to find the value of the hour angle (t) by constructing a nomogram, and this is the essential part in our work.

Let our variables ranges as follows :

$a - a_0$	From	60°	to	70°
$\delta - \varphi$	"	30°	to	40°
$\delta - \delta_0$	"	- 20°	to	-10°

and we consider 4 and P_0 to be constant, so our formula will take the following form

$\text{Log } t = \log c + \log \sin(a_0 - a) + \log \sin(\delta - \varphi) + \log \csc$ $(\delta - \delta_0)$ we first construct a nomogram for $\log \sin(a_0 - a) + \log \sin(\delta - \varphi) = q$. The scale are

$$X = m_I \log \sin(a_0 - a) \qquad y = m_2 \log \sin(\delta - \varphi)$$

$$z = m_3 q$$

Now $\log \sin(a_0 - a)$ varies from $\log \sin 60° = 9.93753$ to $\log 70°$ $= 9.97299$, a range of .03546 ; if we choos $m_I = 282.28$ the equation of the $\sin(a_0 - a)$ scale will be $X = 282.28 \log \sin(a_0 - a)$ and the scale will be about I0 centimetres. Again $\log \sin(\delta - \varphi)$ varies from $\log \sin 30° = 9.69897$ to $\log \sin 40° = 9.80807$, a range of 0.I09I0 ; if we choose $m_2 = 9I.66$ the equation of the $\sin(\delta - \varphi)$ scale will be about I0 centimetres also.

Then $m_3 = \dfrac{m_I \; m_2}{m_I + m_2} = 69.I8$. The equation of our scale are

$$X = 282.28 \log \sin(a_0 - a)$$

$Y = 91.66 \log \sin (69)$

$z = 69.189$

The Z axis must divide the distance between the X and Y axes in the ratio $m_1 : m_2 = 282,28/91.66 = 3.0$

Now it is interesting to explain how we graduate our scale, **the X, Y scales.**

We get from the logarithmic table.

		successive differences ,
Log sin	60º = 9.93753	.00429
Log "	61º = 9.94182	.00411
Log "	62º = 9.94593	.00395
Log "	63º = 9.94988	.00378
Log "	64º = 9.95366	.00362
Log "	65º = 9.95728	.00345
Log "	66º = 9.96073	.00330
Log "	67º = 9.96403	.00314
Log "	68º = 9.96717	.00298
Log "	69º = 9.97015	.00284
Log "	70º = 9.97299	

Now we say for the range .003546 we have 10 centimetres keeping the same proportion for the range .00429 we will have 1.21 centimetres (which is the first successive differences)

range	chosing length	
.03546	10	
.00429	X_1	$X_1 = 1.21$ c

by the same way we get the following points.

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Centimetres.

X_2	=	I.I6
X_3	=	I.II
X_4	=	I.07
X_5	=	I.02
X_6	=	.97
X_7	=	.92
X_8	=	.88
X_9	=	.84
X_{IO}	=	.80
sum	=	9 .98 centimetres

By following the same method we get a graduated scale for log sin $(\delta - \varphi)$ with the following points on it

Log sin 30º $\longrightarrow$ 40º

Centimetres

Y_I	=	I.I9
Y_2	=	I.I3
Y_3	=	I.09
Y_4	=	I.05
Y_5	=	I.02
Y_6	=	.98
Y_7	=	.95
Y_8	=	.9I
Y_9	=	.86
YIO	=	.83
Sum		IO .OI centimetres

log sin IOº $\longrightarrow$ 20º

Centimetres

V_I	=	I.36
V_2	=	I.26
V_3	=	I.I7
V_4	=	I.05
V_5	=	.98
V_6	=	.94
V_7	=	.87
V_8	=	.82
V_9	=	.77
VIO	=	.73
		9 . 95 centi...

Construct the X and the Y axes (I2) cent~~s~~ apart the Z axis must divide this distance in the ratia (3) = $\frac{m_I}{m_2}$ we draw the z axis at a distance of 9 centimetres from the x axis and 3 cent from the Y axis the q scale need not be gratuated.

We now continue the construction by finding the nomogram of

$$q + \log c + \log \csc(\delta-\delta_o) = \log\ t$$

the scalesare

$$z = m_3\ q = 69.189$$

$$v = m_4 \log\csc(\delta-\delta_o)$$

$$w = m_5 \log\ t$$

We use the same q scale as above so that $m_3 = 69.I8$

$\log\csc(\delta - \delta_0)$ varies from $\log\csc IO = -\log\sin IO^\circ = -(9.23967)$ to $-\log\sin 20^\circ = -(9.53405)$ a range of .29438 ; if we choose $m_4 = 33.96$ the equation of the

$\log_{\csc}(\delta-\delta_0)$ will be $V = 33.96 \log\csc(\delta-\delta_0)$ and the scale will be about IO centi. Then $m_5 = \frac{m_3\ m_4}{m_3+m_4} = 22.77$. The equation of our scales are

$$z = 69.I89 \quad q \qquad V = 33.96 \quad \log_{\csc}(\delta-\delta_0)$$

$$w = 22.77 \quad \log \quad t$$

Construct the V axis at a distance of 6 cet.from the z axis. The w axis must ~~be~~ divide the distance between the z axis and the v axis in the retia $m_3/m_4 = 2,03$ and is therefore at a distance 4 cent from the z axis and 2 cent from v axis.

....

We get a starting point for the W scale by making two different computations,

taking some examples in order to be sure of our nomogram .

Let $a_o - a = 65^g$

$$\delta - \varphi = 40^g$$

$$\delta - \delta_o = 15^g$$

by joining the point (65^g) on the X scale and (40^g) on the Y scale,we cut the q axis in a point,and then we join ~~s~~ this point by $(\delta-\delta_o) = 15^g$ on the V scale we get $t = 6\ 71^s$

If we compute the value of t by the formula $t = P_o \sec \varphi \sin(a_o - a) \sin(\delta-\varphi) \csc(\delta-\delta_o)$ we get

$$t = 672.6^s$$

Also for our previous example for time in Beirut we had

$$a_o - a = 63^g \quad 3\bar{1} \quad 4\bar{\bar{7}}.35$$

$$\delta - \varphi = 36^g \quad 2\bar{\bar{3}} \quad 4\bar{\bar{3}}.\ 6$$

$$\delta - \delta_o = 18^g \quad 5\bar{4} \quad 2\bar{\bar{2}}$$

by joining lines between these points like the previous example we get

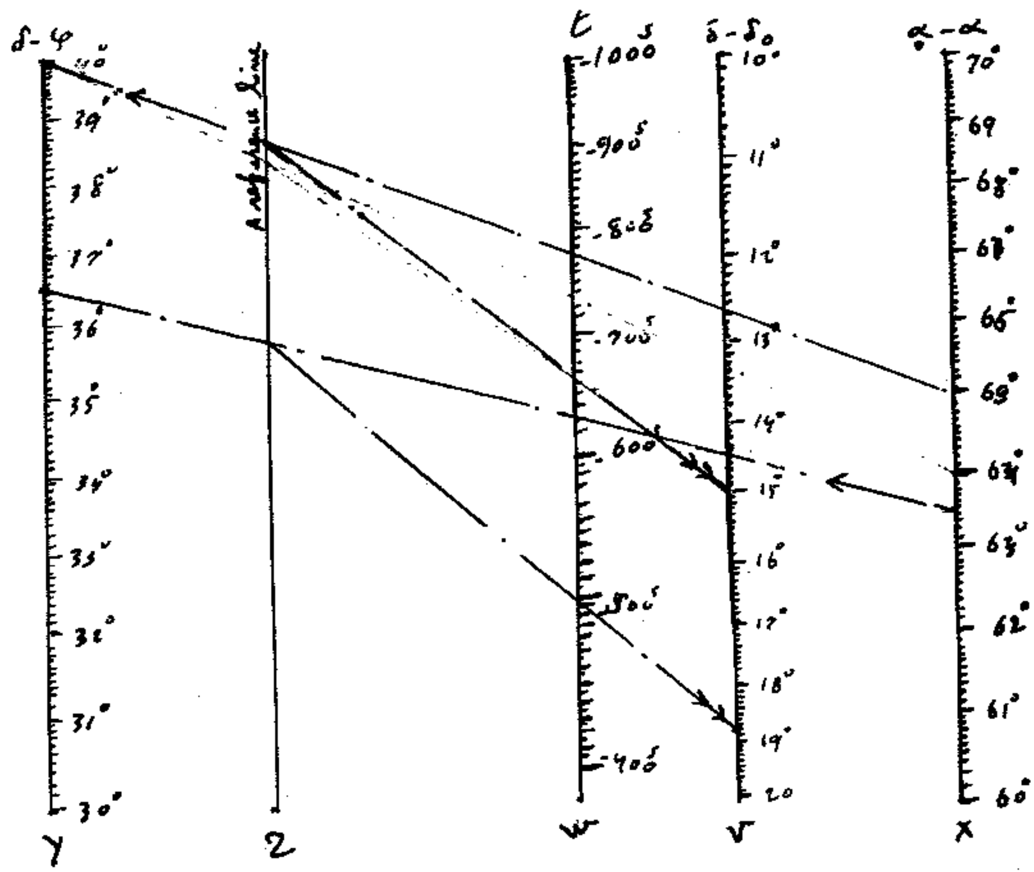
$$t = 495.9^s$$

the correct answer was 495.9^s

......

A NOMOGRAM FOR THE FORMULA

$$t = P_0 \sec\varphi \sin(a_0 - a)\sin(\delta - \varphi)\csc(\delta - \delta_0)$$



Time by upper transit of a northern star.

Observation of a urso majoris over a vertical throught Polaris, Latitude $33^\circ\ 54'\ 22''$ N Longitude $35^\circ\ 28'\ I0''$ on April, 28 I94I.

$$\alpha = II^h\ 00^m\ 08^s.05$$
$$\delta = 62^\circ\ 3'\ 72''.66$$
$$a_o = I^h\ 43^m\ I0^s.39$$
$$\delta_o = 88^\circ\ 59'\ 02''.38$$

Right ascension of mean sun + $I2^h$ = $I4^h\ 22^m\ I7^s.55$

$P_o = 62''$

First we solve this problem by the formula,and then by nomographic method,our formula is

$$t = P_o \sec\varphi \sin\ (a_o - a) \sin(\alpha_o - \alpha)\ \csc\ (\delta - \delta_o)$$

$$a_o - a = I39^\circ\ I4'\ 25''.5$$
$$\delta - \delta_o = 26^\circ\ 54'\ 59''.72$$
$$\delta - \varphi = 29^\circ\ 9'\ 40''.66$$
$$\varphi = 33^\circ\ 54'\ 22''\ N$$

$$\log 4 = .60206$$
$$\log P_o = I.79239$$
$$\log \sec\varphi = .08095$$
$$\log \sin (a_o - a) = 9.8I483$$
$$\log \sin (\delta - \varphi) = 9.68776$$
$$\log \csc (\delta - \delta_o) = .34420$$
$$\log - t = 22.322I9$$
$$= 2.322I9$$
$$t = .209.9^s = -3^m\ 29^s.9$$

......

The true sidereal time is found by subtracting t from a

a =	II^h	oo^m	$08^s.05$	
t = -		3^m	$29^s.9$	
a - t =	IO^h	56^m	$47^s.85$	Local sidereal time

and the Greenwich sidereal time will be

	IO^h	56^m	$38^s.I5$	
Longitude of Beirut. -	2^h	$2I^m$	$52^s.66$	
Greenwich sid. time	8^h	34^m	$45^s.49$	
R.A. Sun + $I2^h$	I4	22	I7.55	
	$I8^h$	$I2^m$	$27^s.94$	or
	6^h	$I2^m$	$27^s.94$	
			$-61^s.06$	correction to reduce

it to means solar time

Greenwich Civil Time	6^h	II^m	$26^s.88$
	2		
	8^h	II^m	$26^s.88$
Watch	8^h	$I4^m$	$4^s.00$
Watch		2^m	$22^s.88$ fast.

Now let us draw a nomogram for the observed star and some other stars near to it. To do that let our variables range as follows :

$a - a_o$	from	35º	to	45º
$\delta - \delta_o$	"	25º	"	35º
$\delta - \varphi$	"	25º	"	35º

....

and by considering φ and P_o to be constant our formula will take the form

$$\text{Log } t = \log C + \log \sin (a_o - a) + \log \sin (\delta - \varphi) + \log \csc (\delta - \delta_o)$$

the first nomogram will be for the two variables

$\text{Log } \sin (a_o - a) + \log \sin (\delta - \varphi) = q$. The scales are

$$X = M_I \log \sin (a_o - a)$$
$$= m_2 \log \sin (\delta - \varphi)$$
$$Z = m_3 q$$

Log sin $(a_o - a)$ in our example varies from log sin (35º) = 9.75859 to log sin (45º) = 9.84949 a range of .09090, if we want the scale to be 20 centimetres long we have to consider m_I to be equal to 220,02 and m_2 = I49.64 because we know that the range times the modulus = length of the scale :

and $m_3 = \dfrac{m_I \, m_2}{m_I + m_2} = 89.06$

and the equation of the scales will be

$$X = 220.02 \log \sin (a_o - a)$$
$$Y = 149.64 \log \sin (\delta - \varphi)$$
$$Z = 89.06 \ q$$

by following the same steps of the previous example we get the following points of the X axis

X_I = 2.34 centimetres	X_3 = 2.I7 cent.	X_5=2.02 cent.
X_2 = 2.25 "	X_4 = 2.09 "	X_6=I.95 "

.....

X_7 = I.88 centimetres X_9 = I.75 centimetres

X_8 = I.8I " X_{IO} = I.69 "

and for the Y scale we have

Y_I = 2.37 centimetres Y_2 = 2.30 Centimetres Y_3 = 2.I8 cent.

Y_4 = 2.09 " Y_5 = 2.00 " Y_6 = I.92 "

Y_7 = I.88 " Y_8 = I.79 " Y_9 = I.7I "

Y_{IO} = I.65 "

the q scale need not be graduated.

We continue the construction by establishing the nomogram of

$q + \log C + \log \csc(\delta - \delta_o) = \log t$

$z = m_3 q$

$V = m_4 \log \csc(\delta - \delta_o)$

$W = m_5 \log t$

If m_4 = I49.64 we have $m_5 = \dfrac{m_3 m_4}{m_3 + m_4} = 55.78$

and on the V scale we have the same points as on the Y scale ; namely we have

V_I = 2.37 centimetres V_2 = 2.30 cent. V_3 = 2.I8 centimet.

V_4 = 2.09 " V_5 = 2.00 " V_6 = I.92 "

V_7 = I.88 " V_8 = I.79 " V_9 = I.7I "

V_{IO} = I.65 "

the distances between the scales will be chosen arbitrarily.

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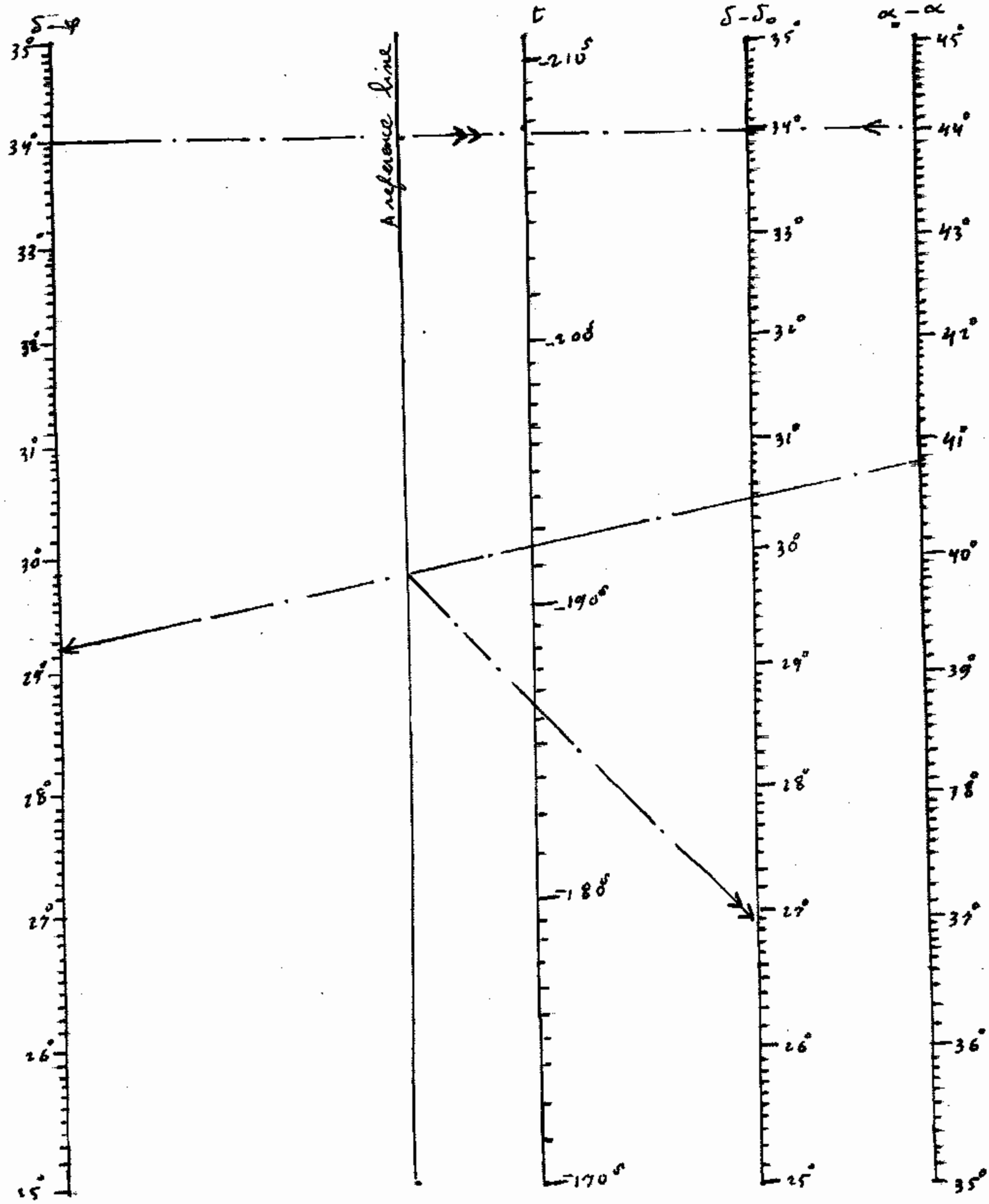


δ-φ
Reference line
t
δ-δ₀
α₀-α
35°
34°
33°
32°
31°
30°
29°
28°
27°
26°
25°
210ˢ
200ˢ
190ˢ
180ˢ
170ˢ
45°
44°
43°
42°
41°
40°
39°
38°
37°
36°
35°

We get the value of **t** in our nomogram to be $- 3^m\ 5^s.5$ and for another example we see that when

$a - a_o = 44°$, $\delta - \varphi = 34°$ and $\delta - \delta_o = 34°$

we get $t = - 207^s = - 3^m\ 27^s$, and if we solve this last example by logarithm we get $- 3^m\ 27^s.7$ the difference between this result and the first got by nomographic method is $.7^s$ which can be neglected.

R E M A R K S

I. Our constant in the formula

$$t = P_o \sec \varphi \sin (a_o - a) \sin (\delta - \varphi) \csc (\delta - \delta_o)$$

depends upon the polar distance of Polaris (P_o) which can be considered to be constant for few years and upon the latitude of the place ; and since this is changing from one place to another we get different nomograms for different places.

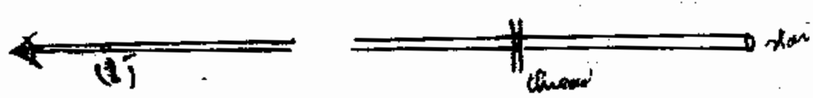
But all the constructed nomograms for the different places will be alike, the t scale only will be shifted up or down according to the value of sec (φ) for Syria and Palestine this value varies between 25° N and 39° N

2. It is good to chose the stars which are not very hear to Polaris, because their apparent motions will be very slow

....

another thing it is interesting to take into consideration it is the thikness of the thread which must be about 2 to 4 milimetres in diameter otherwise the star will take time to cross it and this will introduce an error to the computed hour angle (t).

3. The observer should stand at a distance of 5 meters for a thread 4 milimetres in thikness in order to have an angle about ~~half a degree~~ one minute (the vertex of this angle is the eye's observer and the two limbs of our time stars :



The star must disappear behind the thread when we take the time.

AnOther advantage of this method of determining time

We noticed that we did not use any astronomical instrument in our work ; so for the places in which we do not have observatories (like the arabic countries) or radio we may follow this method to determine the exact standard time of our watch. But we should have at least

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an ephemeries in our hands to know the declinations of the circoompolar stars, their sight ascensions, and the right ascension of the mean sun for the whole year. So amateurs of astronomy only can be familiar with such method.

Wed. May 7/ 194I

S. K H A R B U T L I

The applicability and utility of nomographic method.

For what nomography is used generally ?

As a general conclusion, I can say that it is better to use nomographic method in two cases.

1.- For finding values with few figures through quick work.

2.- For computing the correction of a certain given or known quantity taken by measurements.

I have mentioned at the beginning of this thesis something about the operations of nomographic method, and its fundamental principles.

It remains to discuss this method and to show its practical value and application to engineering practices, and to the problems of functional relations containing many terms of the type.

$$W = Z + U + F(X,Y\ldots)$$

in which Z and U are values easily found or directly known, while $f\ (X,Y\ldots)$ is a function which cannot be found directly but it needs some mathematical operations and we tried to have the value of $F\ (X,Y\ldots)$ by nomography instead of ~~either~~ using other methods.

It is true that we do not use nomographic method in the simple mathematical operations such as

$$X + Y = Z$$

Which gives $3 + 5 = 8$ for it requires less time with simple operation than to construct its nomogram, but we use nomography in the thesis to give us the correction that should be added to the right ascension in order to get the exact time, and it gave us 3 figures after the whole given number (a) the right ascension,in a case of large and more precisly constructed nomographic charts we get a result up to four figures.

Anyhow we use nomographic method for higher mathematical relations like those having trigonometric relations or the similars which cannot be solved by the simple method and we have to use logarithmic tables, or slide rules or some other calculating divices.

Sometimes we can get our requirements approximately or correctly to one decimal place by graphical or analytical methods.

To get results more correct we have to use logarithmic tables or other calculating devices which need a lot of time while we can save the time by using the nomographic method and to come with as accurate results as the previous method.

At any rate nomography is not to be used for solving problems of any kind. Due to the fact that it is best used for corrections, for in this field it saves a good percentage of the engineer's time.

For example he has to find the correction $(-\frac{h^2}{2c})$ found in the problem of geodecy, mentioned in page 3 of the thesis which is $a = c - \frac{h^2}{2c}$.

First this functional relation is of the type

$$W = U + Z + F(X,Y\ldots)$$

where c represents $(U + Z)$ which is directly found, and $(-\frac{h^2}{2c})$ represents the function $f(X,Y)$.

In this example instead of squaring and dividing then subtracting, he can find the correction $(-\frac{h^2}{2c})$ directly by the nomographic method saving his time and being far from making a mistake in calculating his results.

And if he is required to be more accurate and to go through the series of the higher powers as $a = c - (\frac{h^2}{2c} + \frac{h^4}{8c^3})$ and so on and so forth which is more complicated and more distructive for time.

On such occasions it is far more preferable and more profitable to use nomographic method for it gives the requirement directly as we have seen in page 5 of this thesis.

Or in such example as that given in page (36) of the thesis.

Sid time = A + t.

Or

Sid time = A + (P_o sec φ sin (d- φ) sin (A_o-A) csc (d-d_o)) which is also of the type

$W = U + Z + F(X,Y...)$

Where A represents U + Z and t stands for F (X,Y...) which is the correction that should be added to A (the right ascension) to get the sidereal time.

So in this case also we see the supereminence of the nomographic method on all other method for which it is supersede especially when we know that it gives accurate results up to three figures after the whole known number.

For instance sidereal time = A $\pm$ c (c is the correction that should be added to the right ascension in order to get the exact sidereal time, and we get this correction which is accurate to the third figure by using nomographic method.

As a result the nomographic method gave an answer of (497) sec while the long process of logarithm and accurate calculations gave an answer of (495.9) sec. the difference is (1,1) sec. which is not woerth the time spent in calculating that answer.

All this show the superiority of nomographic method to the other methods used for the same purpose, from this point of view concerning time and labor with such a good accuracy of two thousandth.