AMERICAN UNIVERSITY OF BEIRUT

ON STRUCTURING OFFSHORE HYDROCARBON PRODUCTION SHARING CONTRACTS: LEBANON'S CASE

by
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AN ABSTRACT OF THE THESIS OF

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The high interest in Lebanese offshore hydrocarbon potentials is increasing, especially after the discoveries in Israel and Cyprus, the countries that share the same geological underwater basin with Lebanon. In this report, we propose to structure (i.e. define parameters) and perform an economic feasibility analysis for the potential investment contracts and auctions on Lebanese potential offshore hydrocarbon assets. These contracts and auctions involve the Lebanese government and oil companies looking to invest in Lebanon. We take the Lebanese government perspective. Our objective is to assist the government in formulating and managing the bidding process for hydrocarbon assets. Results include an extensive benchmark study of offshore (production-sharing) contracts and auctions. Based on this study and on country profiling, we propose plausible ranges for the parameters of potential Lebanese production-sharing contracts. We also perform a sensitivity analysis to identify the critical contract parameters that has the highest effect on the government share. An empirical and mathematical analysis of the suitable mechanism to be adopted by the government in auctioning hydrocarbon reserves is also developed. Proposed future work includes a more thorough work on auctioning and its mechanism.

Keywords: Oil and Gas,Contracts**,** Auctions,Offshore**,** Production Sharing, Lebanon

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CHAPTER 1

INTRODUCTION

The Syrian arc area, known as the Levantine basin, encompasses both onshore and offshore Israel, Syria and Cyprus. Potential reservoir beds include many layers throughout the sedimentary sequence down to the crystalline basement (Gill, 1992). Therefore, after the discovery of gas in Israel offshore coasts in Tamar and Dallit (Bar-Eli, 2009) and the Cypriot and the Syrian launchings of oil exploration bids in 2007 (European Weekly, 2007), the interest in the Lebanese offshore area rose. The Lebanese government showed a big interest in this subject since 2000, and hired international oil and survey companies to do 2-D and 3-D seismic surveys. The Petroleum Geo-Services insists that the data acquired was comprehensive and that there is enough evidence to allow the companies to drill (Executive, 2009).

Out of the 122, 378 BCFG[1] estimated to be found in the Levant Basin, 81,437 BCFG are estimated to be in the Levant Sub-Salt Reservoirs Assessment Unit (USGS, 2010). And from Figure 1.1, it can be seen that the Levant Su-Salt Reservoir Assessment Unit highly matches the territorial map of Lebanon. Therefore, it seems that Lebanon's offshore potentials are of high value.

[1] BCFG = billion of cubic feet of gas

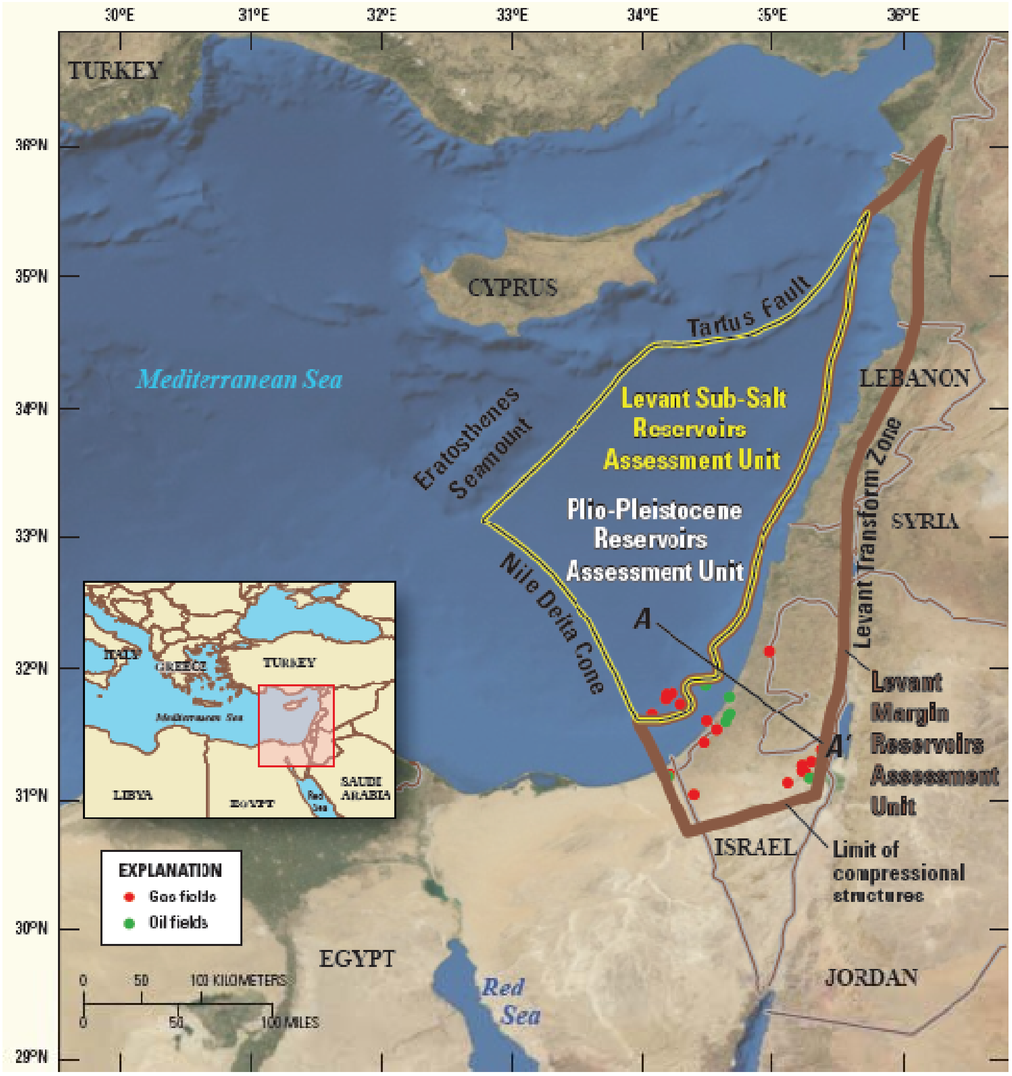

Fig 1.1: Location of potential hydrocarbon reserves in the Eastern Mediterranean (adapted from USGS, 2010)

Accordingly, the Lebanese government approved on a petroleum policy and law (LHL, 2010). However, there is still a lack of managerial and regulatory studies allowing the implementation of the law and policy. Consequently, there is a strong need for further scientific research in support of policy- and law-makers entrusted with the management and exploitation of Lebanon's hydrocarbon resources. This research proposes a comprehensive study which will assist the Lebanese government in structuring production sharing contracts (PSCs), as stipulated in the draft petroleum law and policy. We will also discuss potential optimal auctioning procedure for certain PSC parameters.

Our study has three major research directions: benchmarking, contract model building and analysis, and auction model building and analysis. In the following, we summarize our work on these directions and present the plan for the rest of the thesis.

1.1 The Benchmarking Approach

This is based on a rigorous benchmarking study of different production sharing contracts in various countries with specific focus on neighboring countries and countries with a similar profile to Lebanon; analyzing the pros and cons of the various contract structures from the Lebanese perspective; and recommending a strategy for the Lebanese government. We collect production-sharing agreements (PSA) data used in countries that have offshore oil and gas and employ PSAs such as Norway, Egypt, Libya, Syria, Angola, Indonesia, Nigeria, Cyprus, Iraq and so forth and study the variables of each of these agreements. This is achieved, through literature and Internet searches of publicized PSAs and through searches in specialized databases such as Herold Petroleum Research (Herold, 2009) and Barrows Company (Barrows, 2009). A profile for each of these countries is built based on the political and economic risk level and the status of hydrocarbon reserves (proved or unproved hydrocarbon reserves). This profile is used for categorizing countries and finding the countries with the closest profile to Lebanon. Then a contract structure for Lebanon hydrocarbon assets is recommended. That is, we hypothesize ranges for the value of the parameters of the Lebanese PSC.

1.2 The Contract Model Building and Analysis

This includes analyzing the composition of a PSC, highlighting the role and effect of each element in the contract on risks, rewards, and incentives for both the government and the investing oil company. In particular, we build a mathematical financial feasibility model to relate the various contract parameters to the take[2] of the government and the take of the contractor; then, perform a sensitivity analysis to show how changes in one or more contract parameters or uncertainties (like royalty) influence both takes. This allows identifying the critical parameters of a potential Lebanese PSC, that is, the parameters that the Lebanese government should focus on while negotiating with oil companies.

1.3 Model and Analysis of The Auctioning Process

This covers

i) Studying auction theory literature in oil & gas industry

ii) Building a mathematical model to analyze companies' bids and allow for learning (about the hydrocarbon reserve level) from sequential rounds of bidding

iii) Using this model to experiment with various auctioning strategies

iv) Recommending an auctioning process that will maximize the benefit for the Lebanese government.

[2] The take is the sum of the after-tax profit of a business plus depreciation and other noncash charges; the net income of the government or the contractor. It is a widely used measure in the oil industry.

1.4 Thesis Plan

The rest of this thesis is as follows. In Chapter 2, a review of the literature is provided detailing the types of hydrocarbon contracts and the methods of contracting and auctioning. Chapter 3 describes the data collected on offshore PSCs (49 contracts from 31 countries) and provides descriptive statistics; in chapter 3, we also build a mathematical model for the government take from a PSC. In addition, chapter 3 also presents country profiling. Chapter 4 studies the case of Lebanon with suggestions for quantitative values for PSC parameters and how they fit the Lebanese hydrocarbon law. A mathematical model for auctioning is presented in chapter 5. Finally, the conclusions and suggestions for future work are stated in chapter 6.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

Some countries are unable to extract the resources at a reasonable cost if they lack the technical know-how, expertise or capital to do so. As a result, they rely on international (typically foreign) oil companies to explore and develop these resources. With a second party involved, managing resource becomes more complicated, due to the conflicting interests between foreign oil companies and local governments. The foreign oil company needs to recover its costs and would like to keep as much revenue as possible. The government, on the other hand, wants to maximize its profit as much as possible while making sure there is a remaining interest of the foreign company to continue operating in the country. AsSunley et al. (2002) put it, “both want to maximize rewards and shift as much risk as possible to the other party”. This divergence in objectives yields the need of legislative arrangements that allocate the costs and benefits over a project’s lifetime; which are included in hydrocarbon investment contracts between the government and the oil companies.

The remainder of this chapter presents background information on the types of hydrocarbon contracts with a focus on hydrocarbon Production Sharing Contracts (PSCs) and related on auctions. Section 2.1 provides background information on the process of production in oil and gas industry. In section 2.2, we review the different types of contracts used in gas and oil industry. The hydrocarbon PSA, its parameters and its use are detailed in section 2.2. In section 2.3, we discuss the auctions used in oil and gas industry.

2.1 Hydrocarbon Production Process

The primary method of exploring for hydrocarbon deposits is the seismic studies. Seismic studies reveal the rock structures that might trap hydrocarbon fluids (Thomson, 2010) and geologists interpret the data to map the traps that could contain hydrocarbon potentials. Positive information from seismic studies for a likely presence of hydrocarbons makes the government work on approving a petroleum policy. At that point, the government divides its resources' surfaces (offshore or onshore) into blocks so that it would be able to call for an auction on its reserves blocks. Very few countries replace the auctioning process by a direct negotiation with foreign oil companies, but most of the governments prefer competitive bidding as their method of contracting. Bidding covers some of the hydrocarbon contract's parameters used in this specific country. In general, hydrocarbon contracts involve two phases: the exploration phase and the exploitation phase. During the exploration phase, the oil company granted the lease continues to do seismic studies, in addition to drilling some exploration wells in order to increase the probability of commercial discoveries. It is important to note that even after proving commerciality, it is still likely possible that the hydrocarbon potentials will not be of use for the country. Upon commerciality announcement based on the exploration period, the exploitation phase begins. Based on the type of hydrocarbon found- crude oil or natural gas- the oil company starts to establish its equipments to start extraction and production. The government may participate in one or both, the exploration and the production phases. Hydrocarbon production process is a long period that would extend to 50 years or more (years of seismic studies, up to 10 or more years of exploration phase and up to 30 or more years of production phase), but when proven commercial, it is of high return to the government and the whole country.

2.2 Types of Hydrocarbon Contracts

Four contract types are used throughout the world for oil and gas industry: concessions, service contracts, joint ventures and production sharing agreements (PSA).

Concession contracts, known as the royalty and tax system (R/T system), are not more than a simple combination of royalty and taxes. The government grants a foreign company the license to extract its resources (oil or gas), which becomes the company's property (to sell, transport or refine) once extracted. The company pays the government taxes and royalties for the hydrocarbon. The foreign company bears all the risks and takes all the reward; the government's reward is a function of production and prices (Bindemann, 1999).

Service contracts are contractsfor nationalized industry model where the state makes all of the decisions, and takes all of the revenue (Johnston, 2003). The foreign oil company is paid a cash fee for performing the service of producing mineral resources. The government bears all the risk.

In joint ventures, both the foreign oil company and the government, participate actively in the operation of the hydrocarbon field and acquire ownership of aspecified part of production (Bindemann, 1999). The government and the company do not only share profit, they also share development and operating costs. With joint ventures, the government and the foreign company share risk and reward. PSA is detailed in the next section.

2.3. Production-Sharing Agreement

The production sharing agreement (PSA) is the most popular system for both local governments and the oil corporations (Muttitt, 2005). In theory, the state has ultimate control over the hydrocarbon resources, while a foreign company or consortium of companies do the exploration and production under a contract. In practice, however, the state's hands are tied by restrictions in the contract. This agreement gives the government political and the company commercial satisfaction. While the government may be seen to be running the show, the company actually has high control (AAAUG, 2006). The PSA provides a share of reward to the government and a share to the foreign company; the exploration and production risk is commonly totally tolerated by the company. In a PSA, the foreign company provides the capital investment, first in exploration, then drilling and the construction of infrastructure. Once oil or gas is produced, the foreign company may have to pay royalty charged on gross production to the government. The foreign oil company can recover some of its costs at a pre-specified percentage of production, the so-called 'cost recovery'. Once costs have been recovered, the remaining 'profit share'- 'profit oil' for oil resources and 'profit gas' for gas resources- is divided between state and company in agreed proportions. The company is taxed on its profit share amount. Sometimes the state also participates as a commercial partner in the contract; in this case, the state provides its percentage share of capital investment, and directly receives the same percentage share of cost recovery and profit share. The company's share of the profit is then subdivided according to the production sharing terms. Table 2.1 shows all the parameters of a PSA along with their definitions and Figure 2.1 (adapted from Bindemann, 2009) shows the PSA typical parameters using quantitative values from a PSA signed in Zambia in 2005.

Table 2.1: Parameters of a PSA and their definitions (adapted from Bindemann, 1999)

Variables under a PSA	Definition
Royalty	usage-based payments made by one party (the licensee) to another (the licensor) for ongoing use of an asset
Cost Recovery	A pre-specified percentage of production that will be paid back to oil company (OC) as a recovery
Profit Shate	the share of the OC and the government from the remainder of production
Signature Bonus	A one-off payment on signing a contract
Production Bonus	Payments due when production reaches a certain level.
Discovery Bonus	A one-off fee required after commercial discovery is declared and after the government approves the OC's plan
Tax	An agreed percentage of the profit that the government gets from the OC
Acreage	The size of the area
Relinquishment	The percentage of the contract area that has to be covered by exploration at the end of the first exploration period
Export and Import duties	Oil companies pay no export duties. Import duties may be charged on goods such as foodstuffs that are available in the host country
Work obligation	The OC's commitments with regard to seismic, drilling, information dissemination, financial obligations, employment of local workforce, etc
Participation	The option for the government to participate in the venture
Arbitration	International arbitration should be provided when conflict arises. OC asks for compensation clauses and government asks for penalty clauses
Domestic Market Obligation (DMO)	A percentage of the FOC's production share at a heavily discounted price or at the international market price.
Exploration period	The duration of the exploration
Production period	The duration of the production

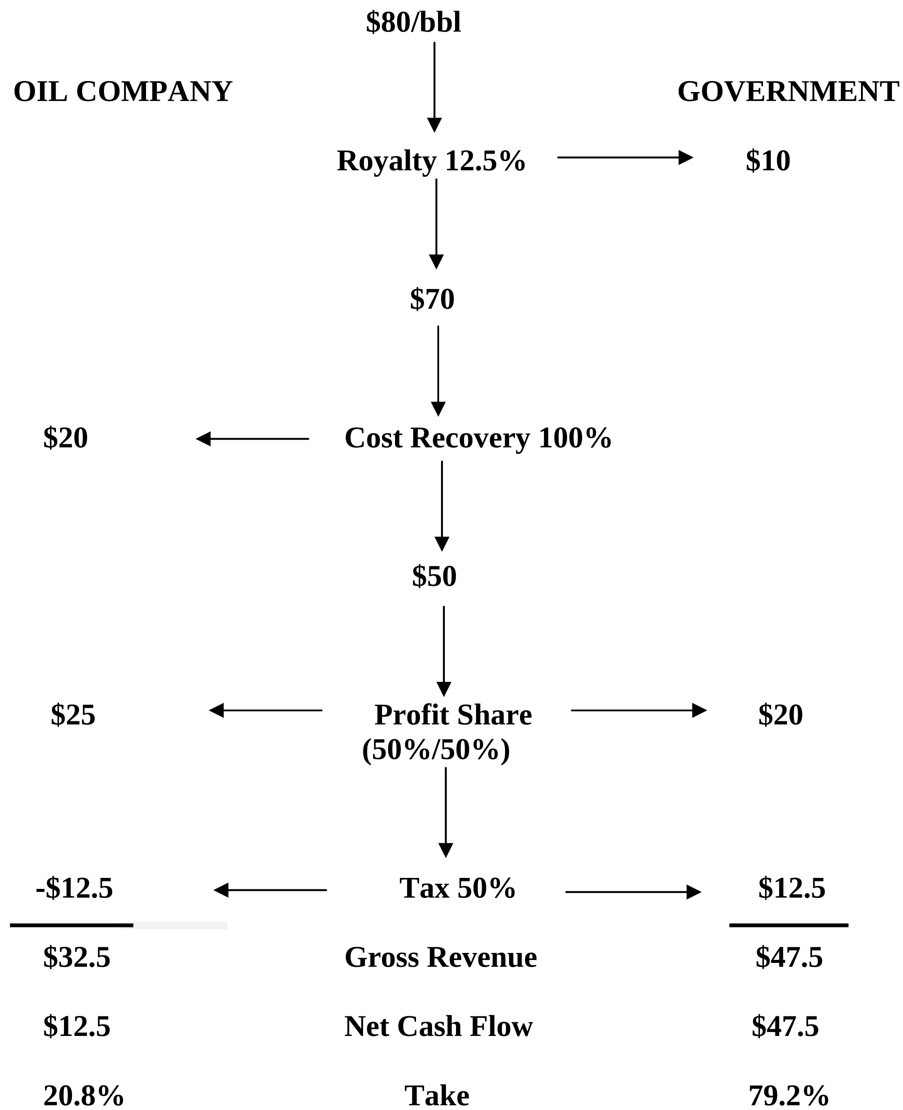

Fig 2.1: PSA structure (adapted from Bindemann, 1999)

In Figure 2.1, upon production of one barrel of oil sold at $80, the government gets a royalty of 12.5%, which leaves $70. Then, a 100% cost oil is applied, i.e. the company recovers all of the cost paid to produce this one barrel. The remaining, $50, is split between the government and the contractor based on a profit oil split; the government receives $25, so does the company. The profit share of the company is then subject to tax; for 50% tax, the company pays the government $12.5. The gross revenue of the company is composed from the cost oil, the company profit share subtracting the amount paid as taxes, whereas the government's gross revenue is composed from the addition of the royalty, profit share and the tax. In the example of Figure 2.1, the company's revenue is 20 + 25 – 12.5 = $32.5 while the government's revenue is 10 + 25 + 12.5 = $47.5. The government net cash flow is equal to its gross revenue ($47.5 in Figure 2.1), whereas the company net cash flow includes its gross revenue subtracting the cost recovery amount ($12.5 in Figure 2.1). Finally, the take of the government (company) is its net cash flow over the total cash flow. For the Zambia 2005 contract in Figure 2.1, the government take is 47.5/ (12.5 + 47.5) = 79.2% and the company's take is 12.5/ (12.5 + 47.5) = 20.8%.

The best PSA contract is one having the best combination of the parameters in table 2.1. In order to determine a good combination of these PSA parameters, the effect and importance of each of these parameters in a PSA should be well recognized; particularly, their contribution to the country's national objectives.A country's objectives, regarding mineral development, can be classified into three categories: sovereignty, economic growth, and quality of life (Bindemann, 1999).

For example, when the concern of the government is to receive a guaranteed cash flow regardless of the profitability of the project, signature bonus and royalty should be high. On the other hand, governments seeking high potential profitability should require high profit share and tax.

In addition, countries that import oil and gas have a specific interest in minimizing the import cost (Muttitt, 2006), thus they aim to satisfy the domestic demand for oil and gas by imposing a domestic market obligation (DMO) on the company.

The work obligation (seismic surveys, drilling commitments, employment of local workforce, etc.) and the signature bonus dominate the risk side of the contract since they are done before establishing the commerciality of the project; thus these two parameters should take into consideration the availability of information and the geological complexity of the area.

Systems with flexible terms are becoming standard to allow both, the government and company, reach their objectives discussed above. To create a flexible fiscal system, sliding scales are used. The usual approach is an incremental sliding scale based upon average daily production but there are many variations of calculating payments using sliding scales based on cumulative production, water depth, oil prices or R-factors (ratio of revenues to expenses). The setting of rates and the design of the scale is based on the available information and the expected size of the discovery. Profit share split, royalty, cost recovery limits, taxes and production bonuses are the contracts' terms which may possibly be subject to sliding scales. Some contracts tie more than one variable to a sliding scale. An example of a production-based sliding scale is a Syrianmodel contract shown in Table 2.2, signed in 2005 (Johnston, 2003).

Table 2.2: Sliding scale profit share in Syria

Daily production(thousand barrels)	Government profit share	Contractor profit share
Up to 50	79%	21%
50 – 100	80%	20%
100 – 200	82.5%	17.5%
> 200	87.5%	12.5%

We conclude this section by discussing the advantages and disadvantages of PSAs. Production-sharing agreements provide the government with profit shares without direct investment and are well enacted into law. On the other hand, some disadvantages can be found for this type of agreements. First, in a PSA the government will have less knowledge about its hydrocarbon potential than the oil company (OC). Second, PSA grants OCs a strong say in environmental and other standards, since the agreement terminates only in case of material breach of contract. Third, PSAs are contracts into law and there is little possibility of developing coherent and comprehensive legal system. Finally, PSA grants positive legal discrimination for OCs. Investors in other sectors will ask for similar special treatment (Radon, 2008). However, the risk being totally on the foreign company, most governments forgo the inconvenient of the PSA.

2.4 Methods of Contracting and Auctioning

Contracts are developed through one of two methods of contracting: bilateral negotiation and competitive bidding (Bindemann, 1999). Bilateral negotiation is a private negotiation between the company and the government. The foreign company approaches a country's government in order to obtain a contract for hydrocarbon exploration and production in exchange for a royalty, profit share split and many other parameters specified in the contract signed between the government and the company. On the other hand, in an auctioning process, the contract is awarded to a qualified bidder on the basis of either an open auction or a sealed one. Auctions are transparent methods of contracting since they may diminish corruption and encourage competition. The auctioning process tends to assign each block to the most qualified and serious developer, since companies with the highest estimates of value for the blocks are more likely to bid higher than the others, and hence, tend to win the blocks. Whereas, private negotiations lack transparency and are susceptible to favoritism and corruption; this would weaken the competition factor found in auctions (Cramton, 2007). The types of auctions applied in oil and gas industry are detailed in Section 2.4.1.

2.4.1 Types of Auctions

In the oil and gas industry, auctions can be either open or sealed auctions. Among the open there are: English auction, simultaneous clock auction and Dutch descending auctions.

In an English auction (also called simultaneous ascending auction)all the items are up for auction at the same time; this is the most widely used auction. Bidders keep on increasing the value of their bids until no bidder is willing to bid further. Then, the auction ends and each bidder wins the items on which he has the high bid. Sometimes the seller has a minimum sale price in advance, the ‘reserve price’. If the final bid does not reach that price the item remains unsold.

In a simultaneous clock auction, the items on bid have a price specified by the auctioneer, bidders respond to this price by expressing the blocks and quantities desired at that price. If more than one bidder expresses their willingness to pay the price specified to a block, then the auctioneer raises the price of this block. The bidders again have to express their desired blocks at the new prices set. The auction ends when demand equals supply, i.e. when one bidder is willing to pay for an item at the set price (Cramton, 2007).

At the Dutch descending auction also known as an open descending price auction, the auctioneer begins with a high asking price which is lowered until some participant is willing to accept the auctioneer's price (Cramton, 2007). This participant will be the winner.

The sealed bid is another form of auction but in a concealed fashion. The submitted bids are compared and the winner is the participant with the highest bid. The winner pays the highest bid value to the seller in the case of the first-price sealed bid, and pays the second highest bid value in the case of the second-price sealed bid

(Vickrey auction). With sealed bidding, bidders can submit one bid only without any opportunity of adjustments according to other bidders.

Sealed bidding may be less subject to collusion[3] than open bidding. On the other hand, open auctions reveal more information about each bidder's valuation and hence, bidders may be able to update their bidding based on other's valuation. In addition, open auctions raises auction revenue since bidders tend to bid more aggressively.

In general, governments prefer competitive bidding, whereas companies find direct negotiation more advantageous (Johnston, 2003). Competitive bidding tends to raise the revenue of the government from auctioning and from the contracting as a whole. On the other hand and from the point of view of the company, direct negotiation is a flexible way to deal with governments and try to get to a better deal for the contract. For production-sharing contracts on hydrocarbons, the auction is mainly on royalties, profit oil shares, work program or signature bonus.

[3] Collusion is an agreement between companies not to bid against each others

CHAPTER 3

DATA COLLECTION AND ANALYSIS

In this chapter, we present the dataset and some general analysis. In section 3.1, we detail the dataset available and present sliding and fixed scale statistics and some descriptive statistics on the fixed scale data. In section 3.2, we build an economic feasibility model of a production-sharing contract for the takes of the government and of the contractor function of other PSA parameters. Section 3.3 presents the parameters used for profiling contracts and countries and the list of contracts profiled using each of the parameters.

3.1 The Dataset

A thorough literature review of publicized PSAs (e.g. Bindemann, 1999 and Johnston, 2003), and through searches in specialized databases such as Herold Petroleum Research (Herold, 2009) and Barrows Company (Barrows, 2009), we were able to collect, 49 offshore hydrocarbon production-sharing contracts and law terms signed by 31 different countries during the period 1962 to 2007 (a contract adapted from Johnston (2003) is available in Appendix A as a sample of the dataset). The countries and their correspondent available hydrocarbon contracts are detailed in Table 3.1

Table 3.1: Countries and the corresponding PSAs

Country	Oil contract/law available by year	Sources	Label
Algeria	2005	Devon energy, 2007	Alg05
Angola	1979-1991	Bindemann, 1999	Ang79
Angola	Mid 1990s	Johnston, 2003	Ang90s
Azerbaijan	AIOC PSC I, 20-Sep-94	Johnston, 2003	Azer94
China	1990	Johnston, 2003	Chi90
Colombia (ICP)	Ecopetrol – Halliburton, 1998	Johnston, 2003	Col98
Colombia	Association contract post,1994	Johnston, 2003	Col94
Congo	Hydrocarbon Law,1994	Johnston, 2003	Con94
Cote d'Ivoire	Block CI-11 Pluspetrol, 27-Jun-95	Johnston, 2003	Cot95
Cyprus	Mines regulation Law,1997	Barrows, 2009	Cyp97
Cyprus	Forest Oil Contract, 1962	Barrows, 2009	Cyp62
Ecuador	7th round, 1995	Johnston, 2003	Ecua95
Equatorial Guinea	United Meridian/Conoco, 1992	Johnston, 2003	Gui92
Guatemala	1997	Johnston, 2003	Guat97
India	Late 1980s	Johnston, 2003	Indi80
India	Marubeni, ONGC Ravva, 28-Oct-94	Johnston, 2003	Indi94
India	Bidding Announcement, 1994	Bindemann, 1999	Indi94
India	Model contract, 1995	Bindemann, 1999	Indi95
Indonesia	Offshore Northwest Java, 18-Aug-66	Johnston, 2003	Indo66
Indonesia	Southeast Sumatra, 6-Sep-68	Bindemann, 1999	Indo68
Indonesia	Standard, Pre 1984	Johnston, 2003	Indo84
Indonesia	2nd generation, 1976	Bindemann, 1999	Indo76
Indonesia	3rd generation, 1988	Johnston, 2003	Indo88
Iraq	Oil law in Iraq, 15-Feb-07	Council of ministers oil and energy committee, 2007	Ira07
Occupied Palestine	Oil regulation, 2005	Barrows, 2009	OccP05
Libya	Model contract, 1990	Johnston, 2003	Lib90
Malaysia	1994	Johnston, 2003	Mal94
Malaysia	Deepwater terms, 1994	Johnston, 2003	Mal94
Malaysia	Model contract, 1997	Johnston, 2003	Mal97
Nigeria	Shell and Elf, 1994	Johnston, 2003	Nig94
Oman	Conquest, 1989	Johnston, 2003	Oma89
Pakistan	1994	Johnston, 2003	Pak94

Table 3.1: Countries and the corresponding PSAs (continuous)

Country	Oil contract/law available by year	Sources	Label
Peru	License contracts, 1993 law/Dec 1994	Johnston, 2003	Per94
Peru (R/T)	Murphy oil contract, 1995	Johnston, 2003	Per95
Peru	1971	Bindemann, 1999	Per71
Peru	After 1978	Bindemann, 1999	Per78
Philippines (risk service)	Early 1990s	Johnston, 2003	Phi90
Trinidad & Tobago	BHP/ Elf, 29-Feb-96	Johnston, 2003	Trin96
Qatar	Contract model, 1994	Johnston, 2003	Qat94
Russia	Sakhalin II-MMMMS Consortium, 23-Jun-94	Johnston, 2003	Rus94
Syria	SPC & 3companies, 30-Jan-97	Barrows, 2009	Syr97Jan
Syria	Mol Palmyra East agreement, 19-Feb-97	Barrows, 2009	Syr97Feb
Syria	Tel abyad agreement, 23-Jun-92	Johnston, 2003	Syr92
Syria	Model contract, 23-Jun-92	Barrows, 2009	Syr92
Timor Gap – Zoca	License round, 1991/1992	Johnston, 2003	Tim92
Turkmenistan	Monument, 7-Aug-96	Johnston, 2003	Tur96
Venezuela (risk service)	Strategic associations Round 3, 18-Jun-05	Johnston, 2003	Ven05
Yemen	2005	Johnston, 2003	Yem05
Zambia	8-Jun-05	Johnston, 2003	Zam05

Among the 49 contracts of the 31 different countries available in the database, 44 are PSA contracts, 2 are Royalty/Tax system contracts (R/T), 1 is an Incremental Production Contract (ICP) and 2 are Risk Service contracts.

In the following, we present basic descriptive statistics on the elements of the PSA contract. Specifically, in section 3.1.1, we collect sliding and fixed scale statistics. Then, in section 3.1.2, we focus on the fixed scale data and collect the mean, the mode, the median, the quartiles and the standard deviation for each fixed scale parameter. In section 3.1.3, we segregate gas contracts from oil contracts within the dataset in order to analyze the effect of the type of hydrocarbon on the contract itself and its parameters.

3.1.1 Sliding and Fixed Scale Statistics

Classifying the parameters of the 46 PSAs in the dataset into sliding scale based parameters and fixed ones was done. All sliding scale contracts in the database impose a progressively smaller percentile of profit share for the contractor as production rate increases. Table 3.2 presents the statistics of the sliding scale based parameters found in the 46 PSAs of the dataset.

Table 3.2: Sliding scale parameters in percentile

Contracts' parameter	Total number available	Number of sliding scale parameters	Number of fixed scale parameters
Royalty	41	10	31
Profit Share	41	35	6
Cost Recovery	34	7	26
Signature Bonus	24	0	24
Production Bonus	28	16	12
Tax	37	2	35
DMO	19	0	19

From Table 3.2, we see that in the majority of contracts (35 out of 41) , the profit share split is sliding scale, since governments search to increase their take from their natural resources upon commerciality of production. Both royalty and profit share are received upon production; hence, with sliding scale profit share, governments use fixed royalties to build an attractive contract for companies. This can be seen in Table 3.2 with only 10 out of 41 contracts having sliding scale royalty. Table 3.2 also shows that 7 out of 34 have sliding scale cost recovery, this is due to the fact that cost recovery is, in general, function of costs paid not function of the gross production. The signature bonus is received upon signing the contract, which makes having no sliding scale signature bonus very reasonable. On the other hand, the production bonus received upon production is sliding scale in 16 out of 28 contracts to allow the government to capitalize further on commercial discoveries. Surprisingly, taxes are sliding scale only in 2 out of 39 contracts.

3.1.2 Analysis on PSA Parameters

Our analysis here involves examining one PSA parameter at a time. Profit share and production bonus are the PSA parameters that are mostly sliding scale (Table 3.2). Even though, information about the type of production bonus parameter is available, but we lack quantitative values for the volume ranges and their respective bonus. On the other hand, the fixed scale production bonus has a $0 value in all contracts. Therefore, no analysis is done on this parameter.

For the sliding scale profit share parameter, we adjusted its volume ranges, collected statistics on each range as shown in Table 3.3. Table 3.3 shows the volume ranges and their respective descriptive statistics.

Table 3.3: Profit oil volume ranges and their descriptive analysis

Volume Ranges (thousand barrels per day)	**Mean**	**Standard Deviation**	**Mode**	**Median**
0 – 10	60%	0.168	50%	60%
10 – 20	62%	0.166	60%	63%
20 – 30	64%	0.164	75%	69%
30 – 40	66%	0.164	70%	70%
40 – 50	66%	0.16	70%	70%
50 – 60	70%	0.15	80%	73%
60 – 70	70%	0.146	80%	73%
70 – 80	70%	0.146	80%	73%
80 – 90	71%	0.143	80%	75%
90 – 100	71%	0.143	80%	75%
> 100	72%	0.15	74%	72%

Figure 3.1 shows the box plot of the government profit share for fixed and sliding scale contracts. Sliding scale profit share shows a fixed value for all the ranges of production, for example the Indonesian contract in 1966 having a fixed profit share of 35%. Based on Table 3.3 and figure 3.1, it can be seen that the higher the

production volume is, the higher the government profit share gets. We cannot compare sliding scale parameter to fixed scale one; but for both cases, the government profit share is higher in countries with proven reserves such as Angola. On the other hand, an instable and risky country status pushes the government to lower its profit share such as Indonesia in 1966.

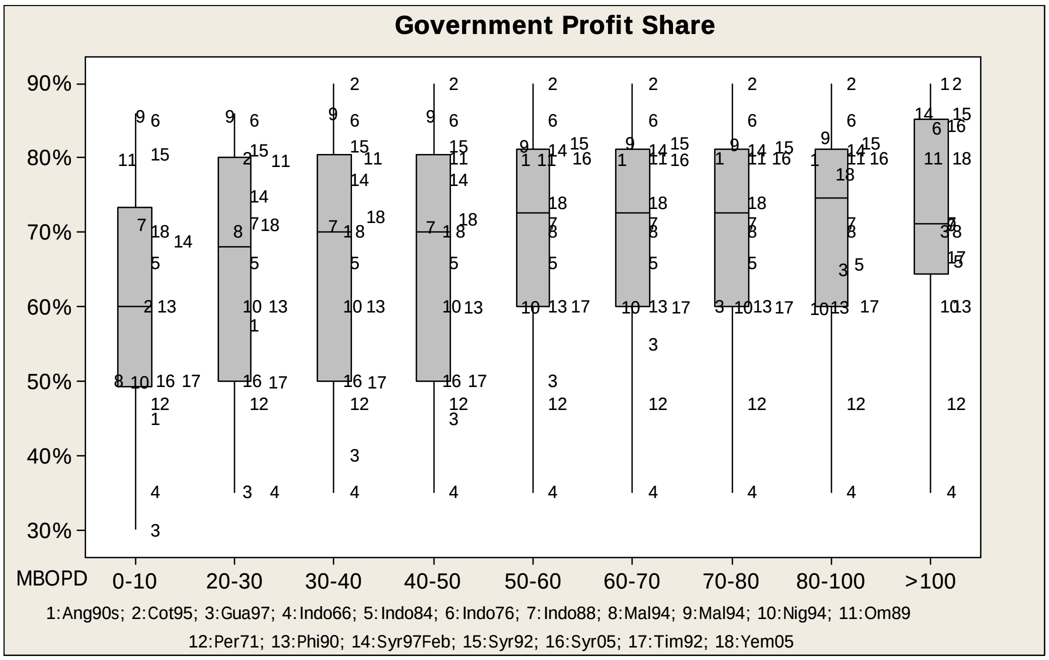

Fig 3.1: Distribution of sliding scale profit share

As for the fixed scale parameters, we looked at its: box plot, mean, median, quartiles, mode and standard deviation. We start with the royalty in a contract; its box plot is shown in Figure 3.2. Figure 3.2 shows that the royalty ranges between 0% and 20% and that its the mode is a 0% royalty value. Q1 is the bottom of the box, it is the first quartile and the top of the box is the third quartile (Q3). For the royalty, Q1 = 0 which means that more than 25% of the data values are equal to 0; and Q3 = 12.5 which means that 75% of the data values are less than or equal to 12.5. The inter-quartile range is equal to 12.5 having values from 0 to 12.5. Several countries with unproven reserves at the time of PSA signature have 0% royalty (e.g. Angola, Ecuador) to make their PSAs attractive; whereas countries like Colombia and Guatemala, which have proven reserves, have a high royalty, around 18%.

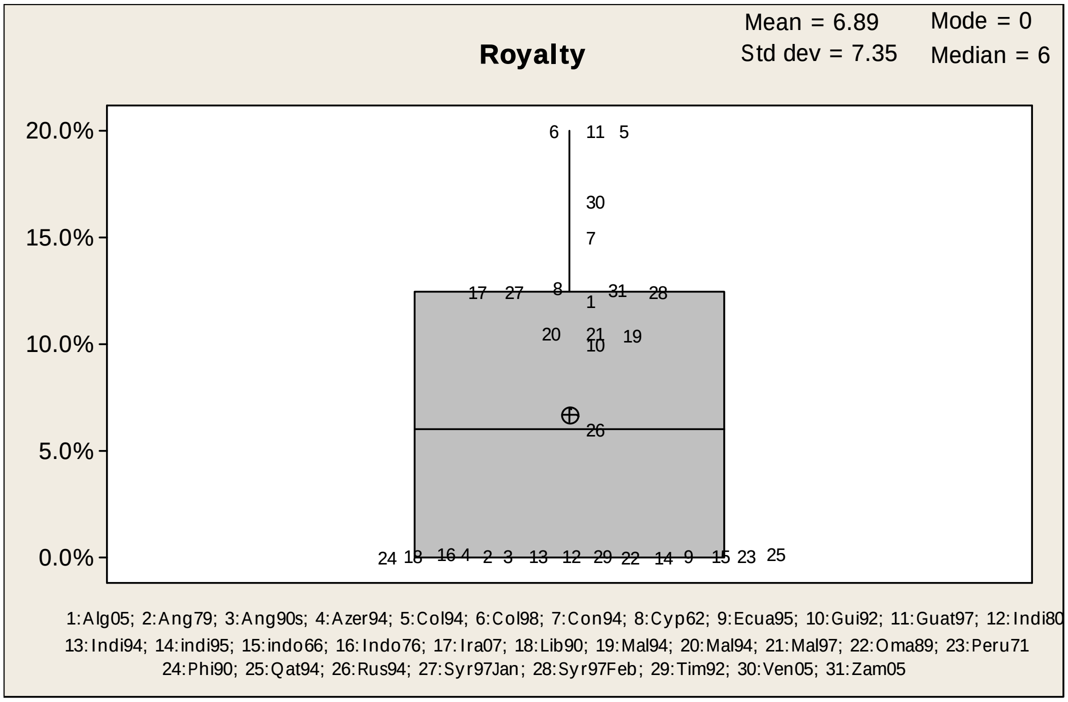

Fig 3.2: Distribution and descriptive statistics of royalty[4]

The box plot of the parameter “cost recovery” is represented in the box plot of Figure 3.3. From Figure 3.3, it can be shown that cost recovery ranges between 0% and 100%; whereas the inter-quartile range is equal to 61.25%, with values ranging between 38.75% and 100%. For the cost recovery, Q1 = 38.75% which means that 25% of the data values are less than or equal to this value; and Q3 = 100% which means that more than 75% of the data values are equal to 100%. Lowest cost rcovery are found in Peru (0%) since the Peruvian reserves are proven and been attractive to oil companies for years. On the other hand, several countries have 100% cost recovery because of the need of incentives to attrat oil companies. For example,

[4] For labeling, refer to Table 3.1

Russia in 1994 was facing major political and economic problems, high cost recovery was an incentive to attract companies despite of its risky situation.

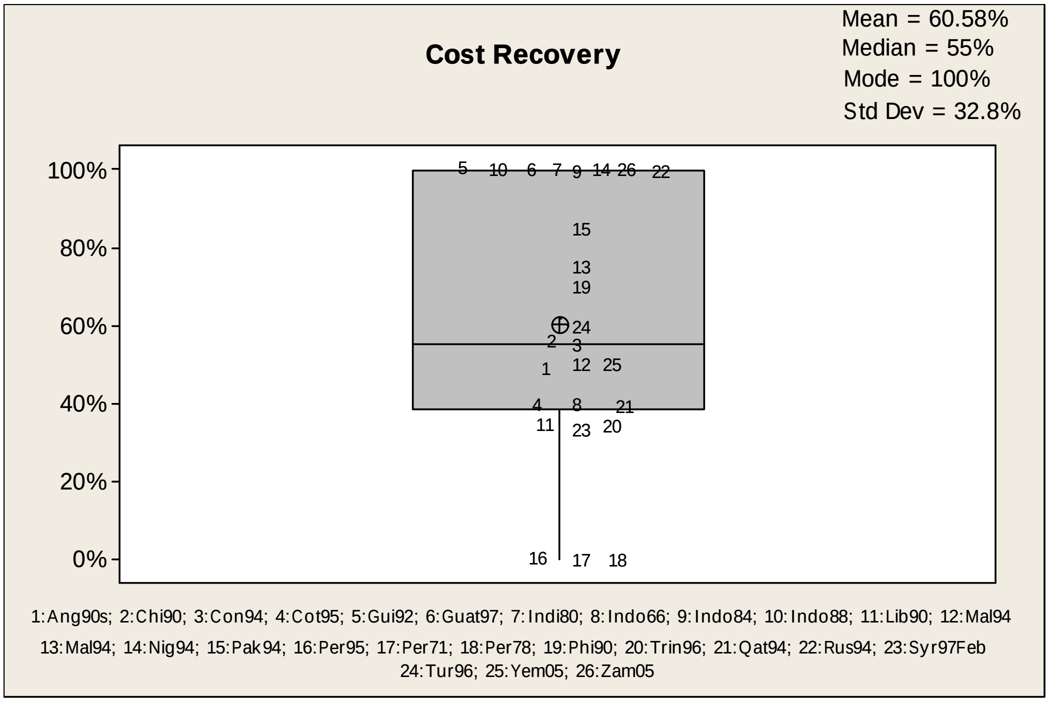

Fig 3.3: Distribution and descriptive statistics for cost recovery

The box plot of the signature bonus parameter is shown in Figure 3.4. Figure 3.4 shows that the signature bonus has a value of $0 for most contracts except for Nigeria and Azerbaijan which are outliers. Implying signature bonus in both Nigeria and Azerbaijan is due to their proven and commercial reserves. In addition to the fact that these countries are in need of cash and search for a quick cash flow from their hydrocarbon resources.

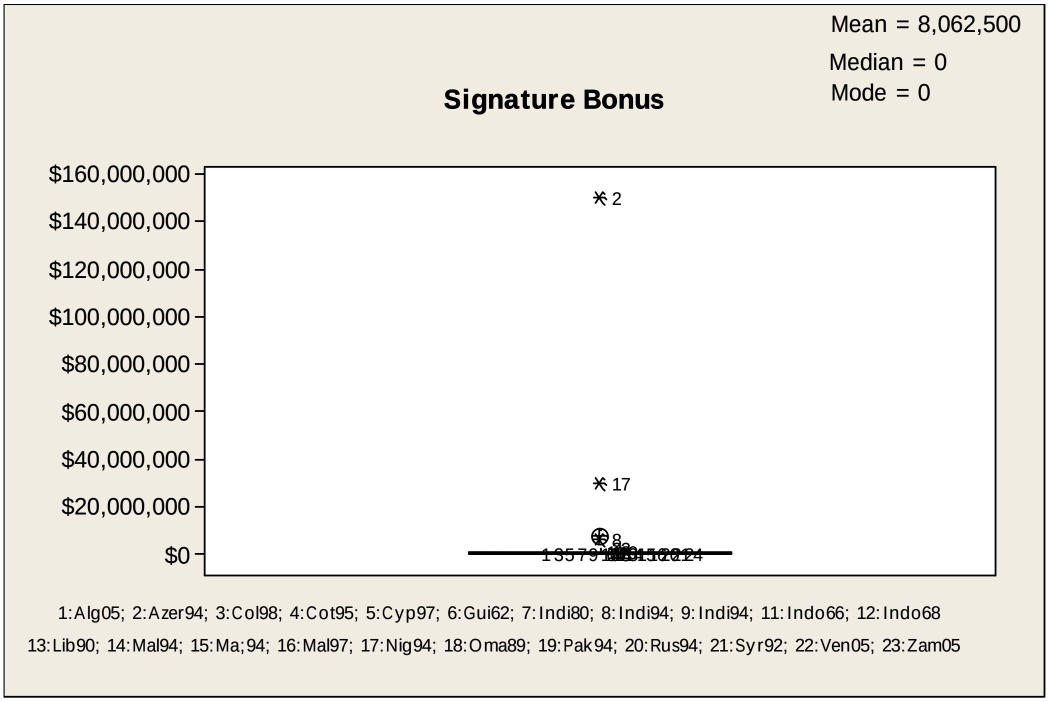

Fig 3.4: Distribution and descriptive statistics of signature bonus

Figure 3.5 shows the box plot of tax. It shows that tax can vary between 0% and 67.7%, whereas the inter-quartile range is equal to 20%, with values between 30% and 50%. Unattractive countries for investment, i.e. high risk countries are forced to lower the values of their contracts' parameters. For example, when Indonesia was a high risk country in 1966, it signed the Northwest Java contract with 0% tax. Whereas, when Indonesia got its stability back in the 1980s, it modeled a model contract with a tax rate of 56%.

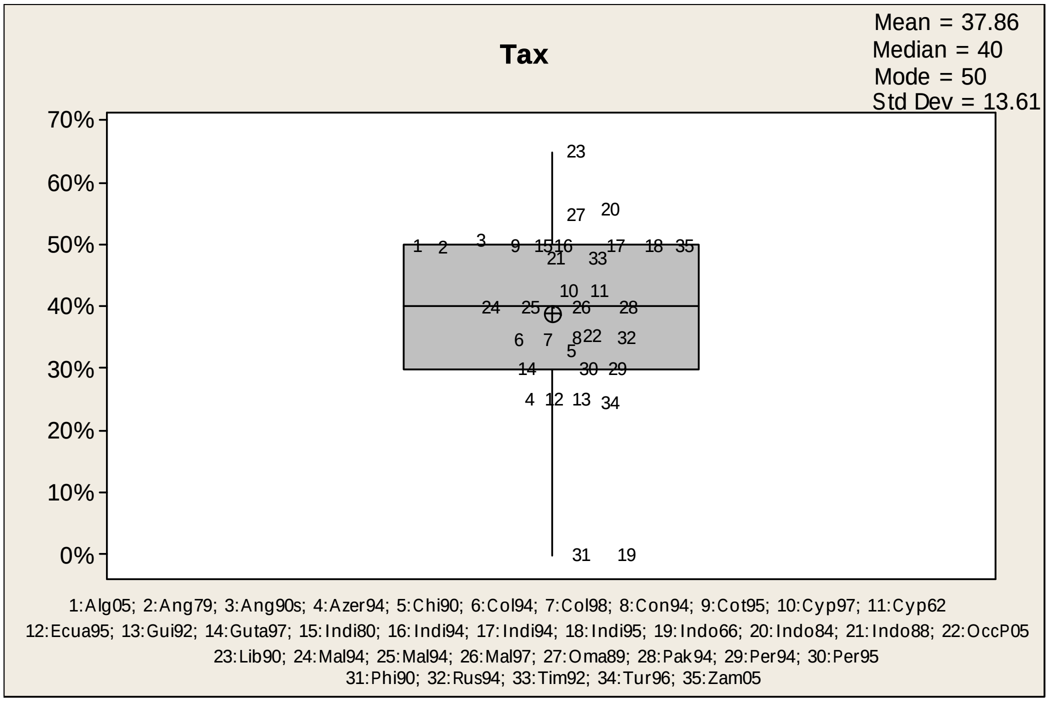

Fig 3.5: Distribution and descriptive statistics of tax

The box plot of the exploration period is shown in Figure 3.6. Figure 3.6 shows that the exploration period is equal to a 2 years period as a minimum and get to a 10 years period as a maximum with a mode of 6 exploration years. Whereas the inter-quartile range for the exploration period is equal to 2 years; with values between 5 and 7 years. Countries working on proving their reserves (e.g. Congo, Philippines) have high exploration period of 10 years. The lowest exploration period is for 4 years, found in Colombia where reserves are already proven.

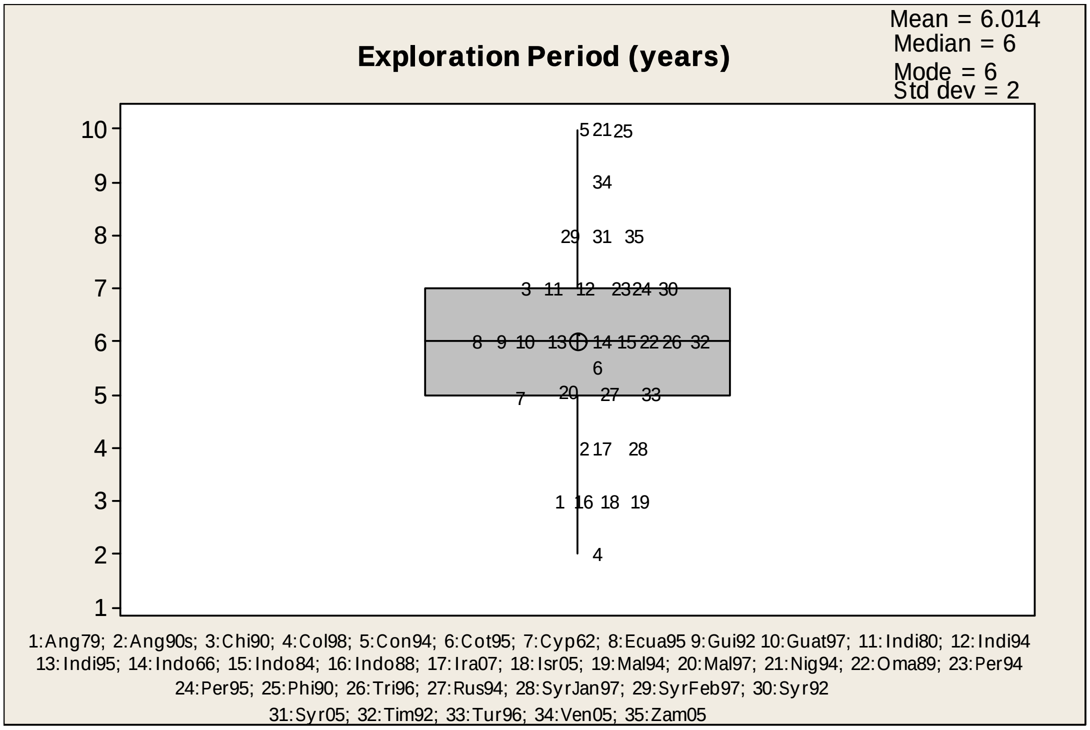

Fig 3.6: Distribution and descriptive statistics of exploration period

The box plot of the production period is shown in the box plot in Figure 3.7. Figure 3.7 shows that the production period can vary from a 10 years period to a 35 years period with a mode of 20 production years period. Its inter-quartile range is equal to 5 years with values between 20 and 25. Peru 1995 had the lowest production period of 10 years, because Peru in 1995 had proven high commercial hydrocarbon reserves, hence, it is of no need to put incentives to attract companies using a long production period. On the other hand, several countries (e.g. Philippines) worked on attracting oil companies with high production period of 30 years.

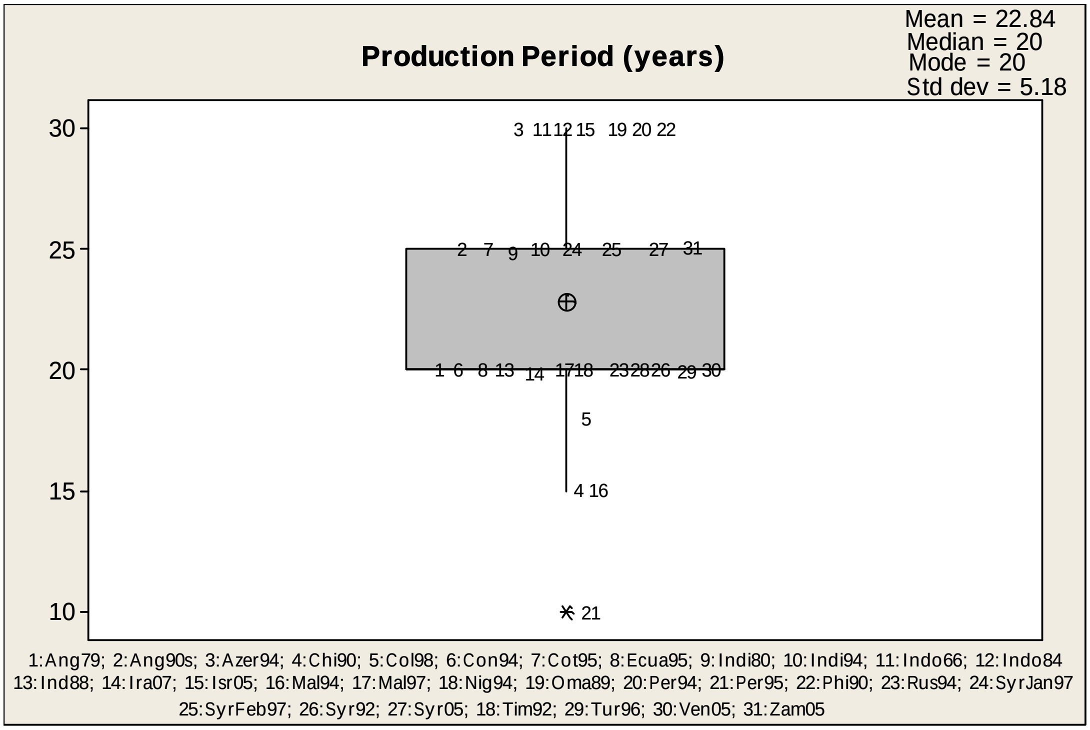

Fig 3.7: Distribution and descriptive statistics of production period

3.1.3 Gas/Oil Analysis

Hydrocarbon contracts can either be oil-only contracts, gas-only contracts or both oil and gas contracts. In our data set, all contracts are either oil-only or oil-and-gas; no gas-only contracts were found. Our data set shows that the main difference between oil-only PSAs and oil-and-gas PSAs lies in the profit share split (fixed or sliding scale) and in the production based sliding scale parameters such as production bonus or royalty. The difference between the two types of PSAs may be due to the following reasons. First, this divergence may be related to the difference in costs for each type of hydrocarbons; expenditures on gas exploration and production are generally higher than oil exploration and production. Second, one barrel of crude oil currently is being sold for $86.88 (Bloomberg, 5-11-2010) whereas an equivalent amount[5]of natural gas is sold at around $23.16(Bloomberg, 5-11-2010).

In our dataset of hydrocarbon contracts and laws, 33 out of 49 are oil-only contracts/laws and 16 out of 49 are oil-and-gas contracts/laws. In an oil-and-gas contract, all parameters have the same value for both types of hydrocarbons, except for profit share split where there are two profit shares, a profit oil and a profit gas. With fixed profit share parameter, the profit oil split share of the government is higher than its profit gas split, e.g. in the third generation Indonesian oil and gas law, the government profit oil is 71.1574% whereas the government profit gas is 42.3077%. In order to be able to compare production based sliding scale profit oil split and profit gas split, we first convert the volume ranges to the same unit; 1 MBOPD = 6 MCFD. The divergence between profit oil and profit gas lies within the ranges of volumes used and/or within the share itself. For the same volume ranges, the government profit gas share increases with volume but in a slower manner than the government profit oil share, therefore, for the same range of volume, the government profit gas share is smaller than the government profit oil share. For the same percentiles of profit share, the volume ranges used for oil production are narrower than those used for gas production; hence, the government profit share increases for a relatively small oil production volume but it needs a bigger gas production volume to increase. Table 3.4 provides examples of sliding scale profit share for more clarification.

[5] 1 barrel of crude oil = 6 cubic feet of natural gas (Rigzone, 2010)

Table 3.4: Examples of government profit oil and gas shares

Contract	Government profit oil share		Government profit gas share		Findings
	MBOPD	Share	MCFD	Share	
Trinidad & Tobago- 1996	0 – 10 10 – 25 25 – 50 50 – 75 > 75	60% 65 70 75 80	0 – 60 60 – 150 150 – 300 300 – 450 > 450	50% 50 55 60 65	Same ranges of volume production (1 MBOPD = 6 MCFD) but slower increase in government gas share
Qatar- 1994	0 – 15 15 – 30 30 – 45 45 – 60 > 60	55% 60 65 70 75	0 – 130 131 – 260 261 – 390 391 – 520 > 520	55% 60 65 70 75	Same percentiles for oil and gas profit share, but wider gas volume production. Hence, slower increase in government gas share

Accordingly, whether with a fixed or a sliding scale profit share, the government profit gas share is smaller or equal to the profit oil share. This may be due to the high exploration and production cost of gas compared to that of oil, as well as to the relatively lower price of gas versus oil.

3.2 A Model Contract

In order to understand the effect of each of the contracts' parameter on the take and therefore the profit of the government and the contractor, a model contract was built and analyzed.

The model contract is afinancial model of a production sharing agreement implemented using Excel spreadsheet. The model contract links the production of hydrocarbon and the take of the government (contractor). It is adapted from Bidemann (1999) and Johnston (2003)

The model is as follows.

$$Government\ take = (government\ NCF)/\ (government\ NCF + OC\ NCF)\ \times 100,$$

$$Oil\ Company\ (OC)\ take = 100 - Government\ take,$$

where,

$$Government\ Net\ Cash\ flow\ (NCF) = royalty + government\ share + bonus + Tax,$$

$$OC\ NetCash\ flow\ NCF = Net\ revenue - Capital\ costs - Operating\ costs$$

$$- government\ Share - Bonus - Tax,$$

$$Net\ revenue = gross\ revenue - royalty$$

where,

$$Gross\ revenue = production \times hydrocarbon\ price,$$

$$Total\ profit = Net\ revenues - cost\ recovery$$

where,

$$Cost\ recovery = Cost\ recovery\ (\%) \times operating\ costs$$

$$Tax = Tax\ (\%) \times Taxable\ Income$$

and,

$$Taxable\ Income = Net\ revenues - Operating\ costs - Depreciation^{6} - Government\ share - Signature\ bonus - TLCF^{7}$$

During the exploration period, there is no gas production. Therefore, the gross revenue royalty, net revenue, operating cost, depreciation, cost recovery and profit shares of the government and the contractor are null. At the beginning of the exploration period, the contractor has to pay the signature bonus, and throughout this period, he has to pay some capital costs to perform the exploration obligations agreed in the PSA. At the end of the first year of exploration, the cash flow of the government is positive due to the signature bonus received, and the cash flow of the contractor is negative because of the signature bonus and the capital costs. At the end

[6] The depreciation of capital costs is a 5-year straight line depreciation
[7] TLCF = Tax Loss Carry Forward

of the second year of exploration, the government has zero cash flow since there was no production or payments from the contractor, whereas the cash flow of the contractor is still negative because of the capital costs. The take of the government and the contractor will be the same during the first and the second exploration period. When production begins, the gross revenue, royalty, net revenue, operating cost, depreciation, cost recovery and profit shares of the government and contractor will no longer be equal to zero. Their values are based on the model above.

3.3 Country Profiling

There are some global factors affecting decision making in any business; the global factors influencing business strategy are: political, social, economical, geological and technological (Bized, 2010). The political determinants of economy wide investment are used to form an index of ownership security. When introduced in empirical models of natural resource use, this index has a significant and quantitatively important effect on the use of petroleum (Bohn and Deacon, 2000). Zanoyan (2004) mentions that the geological preferences based on proven reserves and water depth, and the political and economical status of the host country are the major factors influencing an investment decision taken by an oil company. Accordingly, we chose the *political and economic risk* and the *reserves status* to constitute the elements of the profile built for each contract. The water depth factor was not taken into consideration due to the lack of water depth information for the contracts within our dataset. This profile is used for categorizing countries and finding the countries with the closest profile to Lebanon.

3.3.1 Political and Economic Risk

In the dataset, each contract or law corresponds to a specific country and was signed in a particular period. Hence, each contract was subject to different political and economic threats even if it is for one country. The political and economic risk factor can be low, moderate or high. We determine this by looking at the historical political and economic condition of the country at the specific date of the contract (or law) Table 3.5 presents the PSAs in the dataset with their corresponding date and the political and economical risk factor of the country at that date. (Details about the reason behind each categorization can be found in details in Appendix B)

Table 3.5: Political and Economic risk factor by contract

Country	**Oil contract/law available by year**	**Political and Economic Risk factor**
Algeria	2005	High
Angola	1979-1991	High
Angola	Mid 1990s	High
Azerbaijan	AIOC PSC I, 20-Sep-94	High
China	1990	Moderate
Colombia	Association contract post,1994	High
Congo	Hydrocarbon Law,1994	High
Cote d'Ivoire	Block CI-11 Pluspetrol, 27-Jun-95	Low
Cyprus	Mines regulation Law,1997	High
Cyprus	Forest Oil Contract, 1962	High
Ecuador	7th round, 1995	High
Equatorial Guinea	United Meridian/Conoco, 1992	High
Guatemala	1997	High
India	Late 1980s	Moderate
India	Marubeni, ONGC Ravva, 28-Oct-94	Moderate
India	Bidding Announcement, 1994	Moderate
India	Model contract, 1995	Moderate
Indonesia	Offshore Northwest Java, 18-Aug-66	High
Indonesia	Southeast Sumatra, 6-Sep-68	High

Table 3.5: Political and Economic risk factor by contract (continuous)

Country	**Oil contract/law available by year**	**Political and Economic Risk factor**
Indonesia	Standard, Pre 1984	Moderate
Indonesia	2nd generation, 1976	Moderate
Indonesia	3rd generation, 1988	Moderate
Iraq	Oil law in Iraq, 15-Feb-07	High
Occupied Palestine	Oil regulation, 2005	High
Libya	Model contract, 1990	Moderate
Malaysia	1994	Moderate
Malaysia	Deepwater terms, 1994	Moderate
Malaysia	Model contract, 1997	High
Nigeria	Shell and Elf, 1994	High
Oman	Conquest, 1989	Moderate
Pakistan	1994	High
Peru	License contracts, 1993 law/Dec 1994	High
Peru	1971	High
Peru	After 1978	High
Trinidad & Tobago	BHP/ Elf, 29-Feb-96	High
Qatar	Contract model, 1994	Moderate
Russia	Sakhalin II-MMMMS Consortium, 23-Jun-94	High
Syria	SPC & 3companies, 30-Jan-97	High
Syria	Mol Palmyra East agreement, 19-Feb-97	High
Syria	Tel abyad agreement, 23-Jun-92	High
Syria	Model contract, 23-Jun-92	High
Timor Gap – Zoca	License round, 1991/1992	High
Turkmenistan	Monument, 7-Aug-96	High
Yemen	2005	High
Zambia	8-Jun-05	High

3.3.2 The Condition of Hydrocarbon Reserves

Oil reserves are the estimated quantities of crude oil that are claimed to be recoverable under existing economic and operating conditions (U.S. Energy Information Agency, 2007). All reserve estimates involve uncertainty depending on the amount of data available and the interpretation of those data. They can be divided into two principal classifications: proven and unproven reserves. Proved reserves are those reserves claimed to have a reasonable certainty (at least 90% confidence) of being recoverable under existing economic and political conditions with the existing technology. Reserves are classified as unproven if technical, contractual, or regulatory uncertainties preclude such reserves being classified as proved (Society of Petroleum Engineers, 2005). Table 3.6 shows the PSAs in the dataset with their corresponding date and the condition of the hydrocarbon reserves in the country at that date. The major sources behind the information of the condition of hydrocarbon reserves are the U.S. Energy information administration (EIA, 2007) and Index Mundi (2007), the home of the Internet's most complete country profiles on oil and gas

Table 3.6: Condition of hydrocarbon reserves by contract

Country	**Oil contract/law available by year**	**Condition of Hydrocarbon reserves**
Algeria	2005	Proven
Angola	1979-1991	Proven
Angola	Mid 1990s	Proven
Azerbaijan	AIOC PSC I, 20-Sep-94	Proven
China	1990	Proven
Colombia	Association contract post,1994	Proven
Congo	Hydrocarbon Law,1994	Proven
Cote d’Ivoire	Block CI-11 Pluspetrol, 27-Jun-95	Proven
Cyprus	Mines regulation Law,1997	Unproven
Cyprus	Forest Oil Contract, 1962	Unproven
Ecuador	7th round, 1995	Proven
Equatorial Guinea	United Meridian/Conoco, 1992	Proven
Guatemala	1997	Proven
India	Late 1980s	Proven
India	Marubeni, ONGC Ravva, 28-Oct-94	Proven
India	Bidding Announcement, 1994	Proven
India	Model contract, 1995	Proven
Indonesia	Offshore Northwest Java, 18-Aug-66	Proven

Table 3.6: Condition of hydrocarbon reserves by contract (continuous)

Country	Oil contract/law available by year	Condition of Hydrocarbon reserves
Indonesia	Southeast Sumatra, 6-Sep-68	Proven
Indonesia	Standard, Pre 1984	Proven
Indonesia	2nd generation, 1976	Proven
Indonesia	3rd generation, 1988	Proven
Iraq	Oil law in Iraq, 15-Feb-07	Proven
Occupied Palestine	Oil regulation, 2005	Unproven
Libya	Model contract, 1990	Proven
Malaysia	1994	Proven
Malaysia	Deepwater terms, 1994	Proven
Malaysia	Model contract, 1997	Proven
Nigeria	Shell and Elf, 1994	Proven
Oman	Conquest, 1989	Proven
Pakistan	1994	Proven
Peru	License contracts, 1993 law/Dec 1994	Proven
Peru	1971	Proven
Peru	After 1978	Proven
Trinidad & Tobago	BHP/ Elf, 29-Feb-96	Proven
Qatar	Contract model, 1994	Proven
Russia	Sakhalin II-MMMMS Consortium, 23-Jun-94	Proven
Syria	SPC & 3companies, 30-Jan-97	Proven
Syria	Mol Palmyra East agreement, 19-Feb-97	Proven
Syria	Tel abyad agreement, 23-Jun-92	Proven
Syria	Model contract, 23-Jun-92	Proven
Timor Gap – Zoca	License round, 1991/1992	Unproven
Turkmenistan	Monument, 7-Aug-96	Proven
Yemen	2005	Proven
Zambia	8-Jun-05	Unproven

After collecting the data, profiling the contracts and analyzing their parameters, Lebanon will be profiled and studied in detail throughout Chapter 4 in order to get to some suggestions for Lebanon’s PSA.

CHAPTER 4

LEBANON'S CASE

In this chapter, we use the dataset described in Chapter 3 and perform further analysis in order to suggest likely ranges for the Lebanese PSA parameters. In section 4.1, Lebanon is profiled based on the economic and political risk level and its hydrocarbon reserves' status. Then, in section 4.2 and Section 4.3, we define the type of contract and the design of auction respectively to be used for Lebanon's hydrocarbon potentials. Descriptive statistics for the countries comparable to Lebanon is collected in Section 4.4 followed by sensitivity analysis on PSA parameters in Section 4.5 with ranges extracted from the contracts of countries similar to Lebanon. Section 4.6 includes comparison and analysis of the different results. Suggestions of likely ranges are stated in section 4.7. Finally, a brief summary on how our findings fit and supplement Lebanese hydrocarbon law is presented in section 4.8.

4.1 Lebanon's Profile

The constant threat of instability and regional violence, the large budget deficit and the high government debt at around 160% of GDP make it very difficult for Lebanon's economy to gain momentum. This increases the likelihood that fundamental weaknesses in Lebanon's economy will cause adverse developments for an insurer, which makes of Lebanon a high level economical risk country (A.M. Best Company, 2009). Lebanon is classified as a high political and economic risk country. The interest in the Lebanese hydrocarbons arose since the 1950's. Some Lebanese oil and gas exploration began in the late 1960's and early to mid-1970's with the drilling of several wells across the country (onshore). Then, exploration came to a halt when Lebanon's civil war began in 1975 (Executive, 2009). No drilling has been made in offshore Lebanon to try to verify the condition of natural gas reserves in the Lebanese sea. Therefore, to date, Lebanon has no proved hydrocarbon reserves.

4.2 The Right Contract to Be Used

Paliashvili(1998) argues that it is desirable for the investor to invest under a PSA contract type, because it gives the investor a greater degree of independence from the constantly changing tax system and a stability of the legal relations between the state and himself in the time-period of validity of the agreement. The weaknesses in political and legal institutions and the extensive corruption can be defeated through the usage of PSAs since they provide broad options to international legal and judicial systems, and immunity from changes in host country law. This would help moderate the investor’s risk and increase the benefits of the host country from investment. In addition, PSAs is beneficial to governments lacking expertise and capital to develop their resources.

Since Lebanon is a country with high political risk level, high governmental complications, high political and economic corruption and a lack in expertise and capital, PSA is the type of hydrocarbon contract Lebanon should use in order to attract investors.

4.3 The Right Auctioning Process to be Followed

As mentioned before, hydrocarbon PSCs bidding is based on two parameters: royalty or profit share. The bidding process widely used in the oil and gas industry is the sealed bidding process since it is less subject to corruption and collusion. English auction is a type of auction subject to a high risk of collusion and agreements between oil companies not to bid against each others, which cannot be done when following a sealed process. On the other hand, direct negotiation is a method of contracting highly subject to corruption especially in non-transparent countries and governments such as Lebanon; sealed bidding saves the country from any corrupted actions since all bids and values are submitted closed and envelopes are open either publically or within a group of specialists. Accordingly, a proposed auctioning system for Lebanon's offshore potentials would involve a sealed bidding process on specific blocks based on royalty and/or profit split. Sealed bidding can also be followed by direct negotiations on work program for the same blocks or even on royalty and/or profit split on other blocks. On one hand, by making companies bid on certain blocks, the government makes sure that the company with the highest value for these blocks is the one that would be awarded the license. On the other hand, the combination of sealed bidding and direct negotiation can supply the foreign oil company with some significant flexibility (Johnston, 2003), which makes the country's potentials more attractive to foreign companies.

4.4 Countries Comparable to Lebanon

Lebanon in 2010 is a country with high political and economical risk level and unproven hydrocarbon reserves. Accordingly, countries and contracts are classified in three groups: (i) contracts during a high political and economic risk period regardless of the status of hydrocarbon reserves, (ii) contracts with unproved condition of hydrocarbon reserves regardless of the level of political and economic risk and (iii) contracts during high political and economic risk period and with unproved reserves. For each of the three groups, we present a sliding and fixed scale based analysis as well as descriptive statistics analysis. Section 4.4.3 summarizes the analysis of these three groups of contracts.

4.4.1 Contracts under High Political and Economic Risk

This group contains 31 contracts (28 PSAs, 2 Royalty/Tax contracts and 1 Risk Service contract) signed in 22 different countries through different periods ranging from the 1962 till 2007. Table 4.1 presents the contracts signed during a high risk period, and Table 4.2 presents the number of the fixed scale based parameters found in the 31 high political and economic risk PSAs of the dataset.

Table 4.1: Contracts signed during high risk

Country	Oil contract/law available by year
Algeria	2005
Angola	1979-1991
Angola	Mid 1990s
Azerbaijan	AIOC PSC I, 20-Sep-94
Colombia	Association contract post,1994
Congo	Hydrocarbon Law,1994
Cyprus	Mines regulation Law,1997
Cyprus	Forest Oil Contract, 1962
Ecuador	7th round, 1995
Equatorial Guinea	United Meridian/Conoco, 1992
Guatemala	1997
Indonesia	Offshore Northwest Java, 18-Aug-66
Indonesia	Southeast Sumatra, 6-Sep-68
Iraq	Oil law in Iraq, 15-Feb-07
Occupied Palestine	Oil regulation, 2005
Malaysia	Model contract, 1997
Nigeria	Shell and Elf, 1994
Pakistan	1994
Peru	License contracts, 1993 law/Dec 1994
Peru	1971
Peru	After 1978
Trinidad & Tobago	BHP/ Elf, 29-Feb-96
Russia	Sakhalin II-MMMMS Consortium, 23-Jun-94
Syria	SPC & 3companies, 30-Jan-97
Syria	Mol Palmyra East agreement, 19-Feb-97
Syria	Tel abyad agreement, 23-Jun-92
Syria	Model contract, 23-Jun-92
Timor Gap – Zoca	License round, 1991/1992
Turkmenistan	Monument, 7-Aug-96
Yemen	2005
Zambia	8-Jun-05

Table 4.2: Fixed and sliding scale parameters in high risk countries

Contracts' element	Number of fixed scale based parameters	Number of sliding scale based parameters
Royalty	19	7
Profit Share	2	21
Cost Recovery	17	5
Signature Bonus	13	0
Production Bonus	4	11
Tax	20	0

The statistical analysis involves the box plots of the PSAs' parameters in Figures 4.1 though 4.6. Then, Table 4.4 presents a review of the mean, ranges, mode and standard deviation of each parameter.

Starting with the government profit share, it has two fixed values equal to 65% and 50%. Thus, this element does not need a frequency distribution; its analysis was done based on the based scale used for quantifying the sliding scale element. Within the sliding scale profit share elements, 64% are production based sliding scale and the remaining 36% are R-factor based sliding scale. Contracts signed during a high political and economic risk level with a production based sliding scale profit oil share were subject to further analysis. Table 4.3 shows adjusted[8] volume ranges along with their respective descriptive analysis. Based on Table 4.3, we later give a rough suggestion for a production based sliding scale profit oil split for Lebanon's PSA. The set of sliding scale profit gas is small to collect its statistics; this is why we will focus on profit oil split.

Table 4.3: Profit oil volume ranges and their statistics for high risk countries

Volume Ranges (thousand barrels per day)	Mean	Standard Deviation	Mode	Median
0 – 10	51%	0.166	50%	50%
10 – 20	52.8%	0.173	50%	50%
20 – 30	55.41%	0.172	35%	52.5%
30 – 40	58%	0.171	50%	55%
40 – 50	59%	0.164	50%	55%
50 – 60	66%	0.176	80%	70%
60 – 70	67%	0.17	80%	70%
70 – 80	67%	0.166	60%	70%
80 – 90	68%	0.164	80%	73%
90 – 100	69%	0.164	80%	75%
> 100	72.2%	0.186	66.67%	77%

[8] We adjusted the ranges by splitting all the available ranges into a width equal to the smallest range available

Figure 4.1 shows the box plot of the government profit share during high risk. We have both sliding (e.g. Guatemala 1997) and fixed scale (e.g. Indonesia 1966) proft share The proportionality of the profit share to the production volume is clearly seen from Figure 4.1.

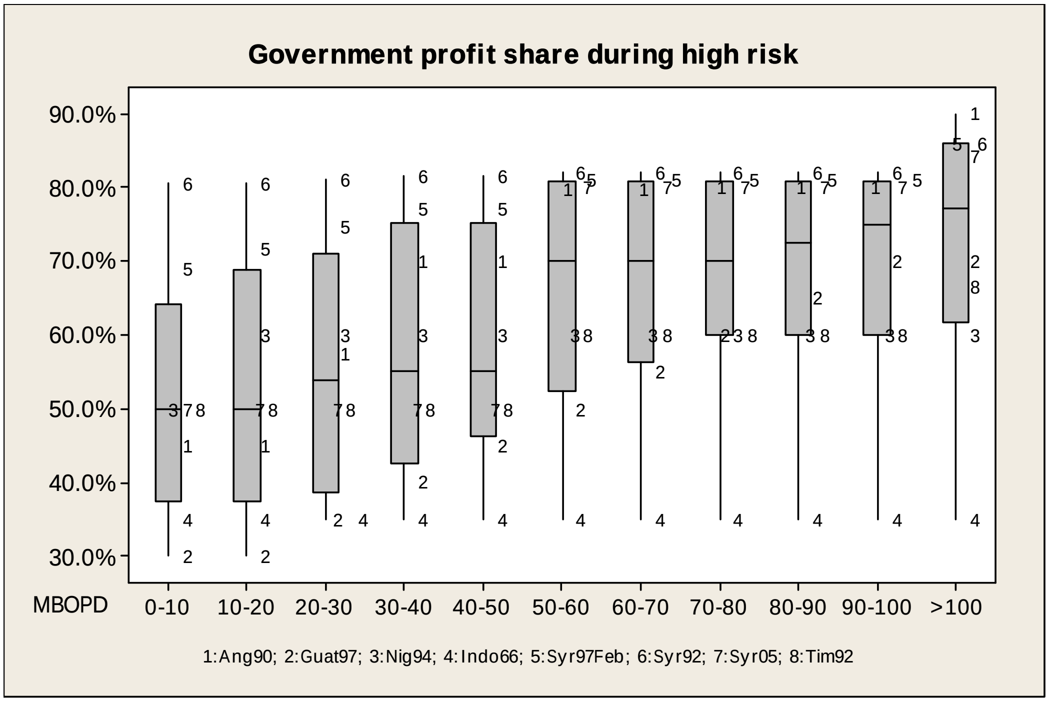

Fig 4.1: Government profit share during high risk

Quantitative values and ranges for sliding scale production bonus is not available in our dataset. And, the production bonus has a $0 value for all of the four fixed bonuses, hence no descriptive statistics analysis was collected for it.

Figure 4.2 shows the royalty of high risk countries ranging between 0% and 20%. Q1 = 0 and Q3 = 12.5 which means that more than 25% of the data values are equal to 0 and 75% of the data are less than or equal to 12.5. The status of the hydrocarbon reserves is the major factor in setting the royalty; countries with proven reserves increase their royalty regardless of the political and economic status of the country.

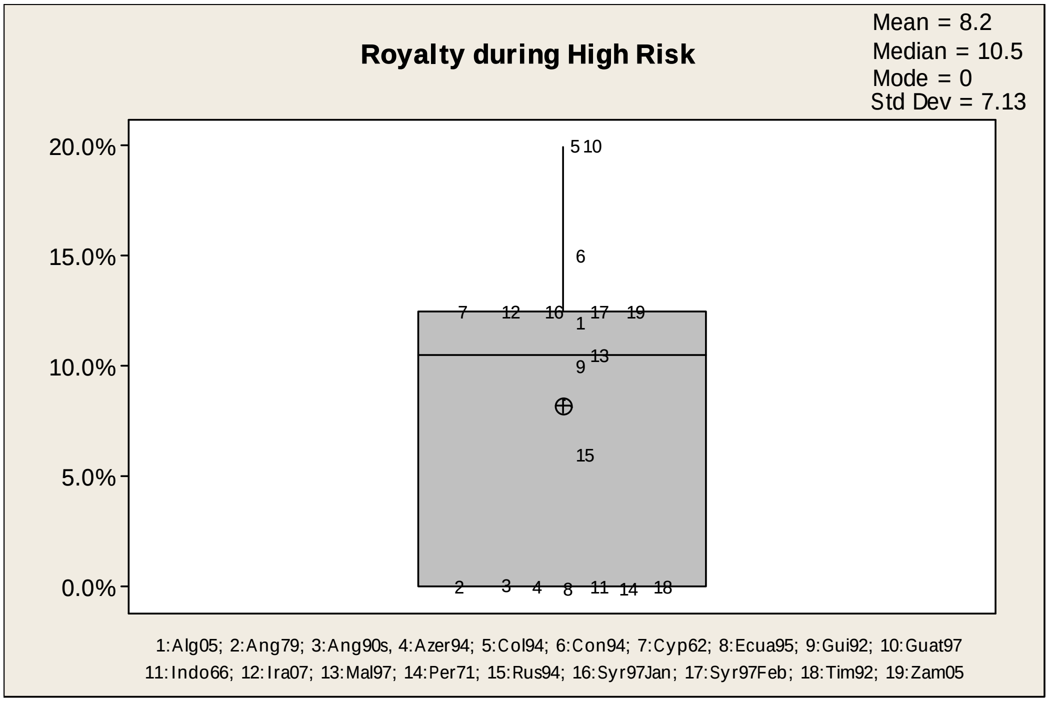

Fig 4.2: Royalty during high risk

Figure 4.3 shows the box plot of the cost recovery parameter during high risk periods. Its mode is equal to 100% and Q3 in Figure 4.3 is also equal to 100%, hence, high political and economic risk level in a country drives it to put incentives in its PSA to attract foreign companies.

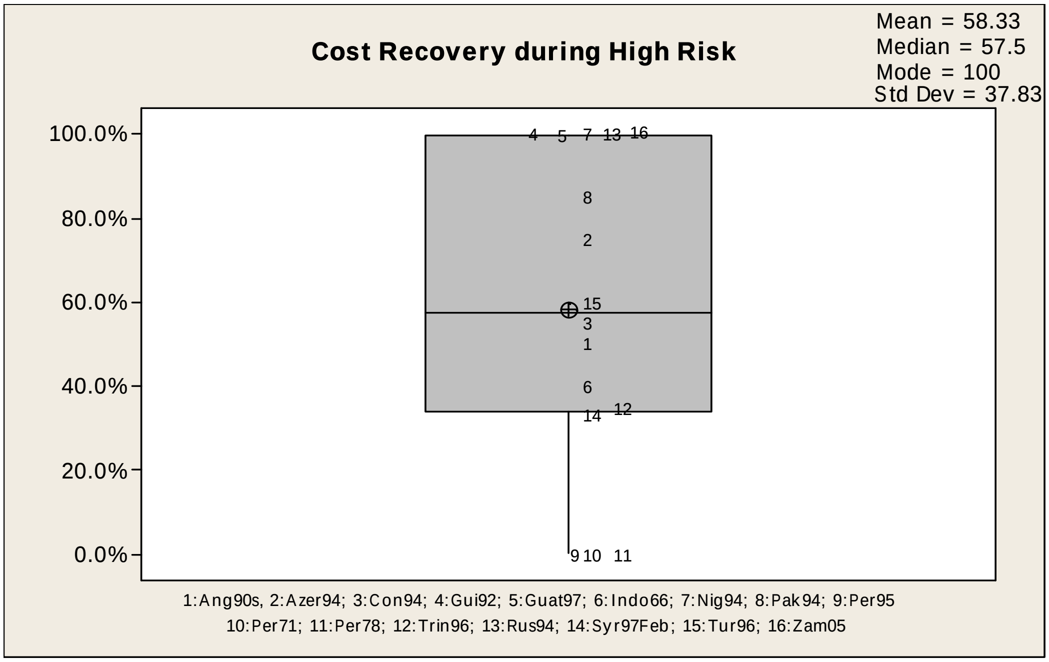

Fig 4.3: Cost recovery during high risk

Figure 4.4 shows the box plot of the signature bonus for contracts signed during high risk periods. It has two outliers- Azerbaijan 1994 and Nigeria 1994- of very high values and all other bonuses are approximatly null. Coutries living high risk periods and unattractivness of investments dismiss the signature bonus parameter.

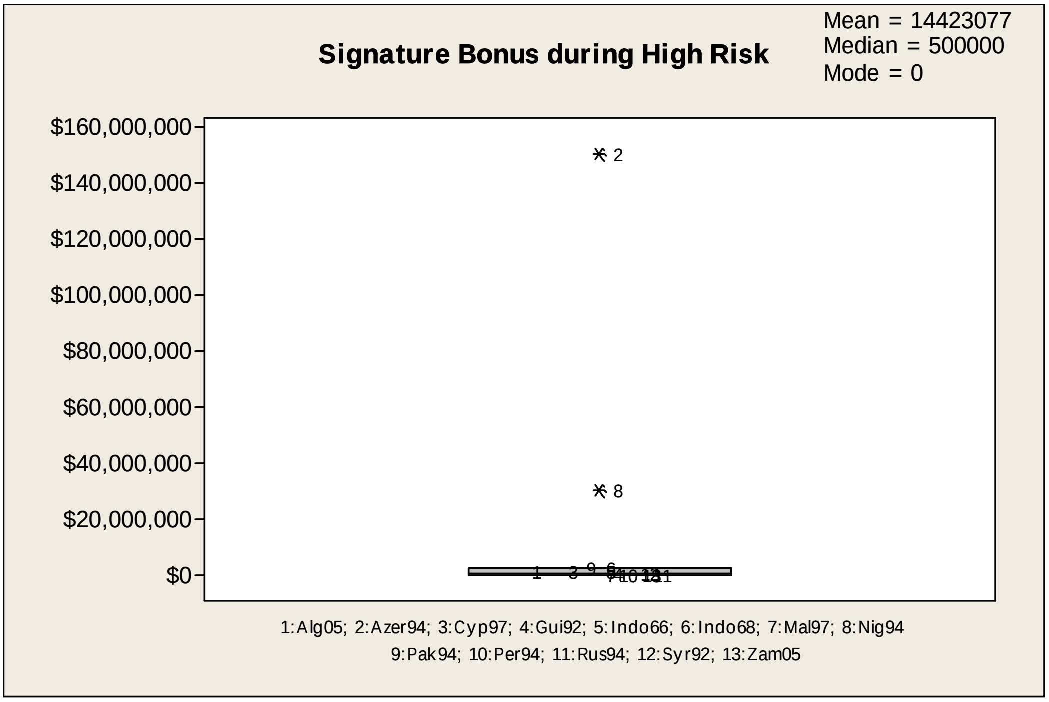

Fig 4.4: Signature bonus during high risk

The box plot of tax during high political and economic risk periods is shown in Figure 4.5. More than 25% of contracts signed during high risk have tax equal to 26.25% (Q1 = 26.25) and 75% of these contracts have tax less than or equal to 41.87% (Q3 = 41.87). In general, countries rely on their own fiscal system to tax hydrocarbon resources regardless of any political or economic status.

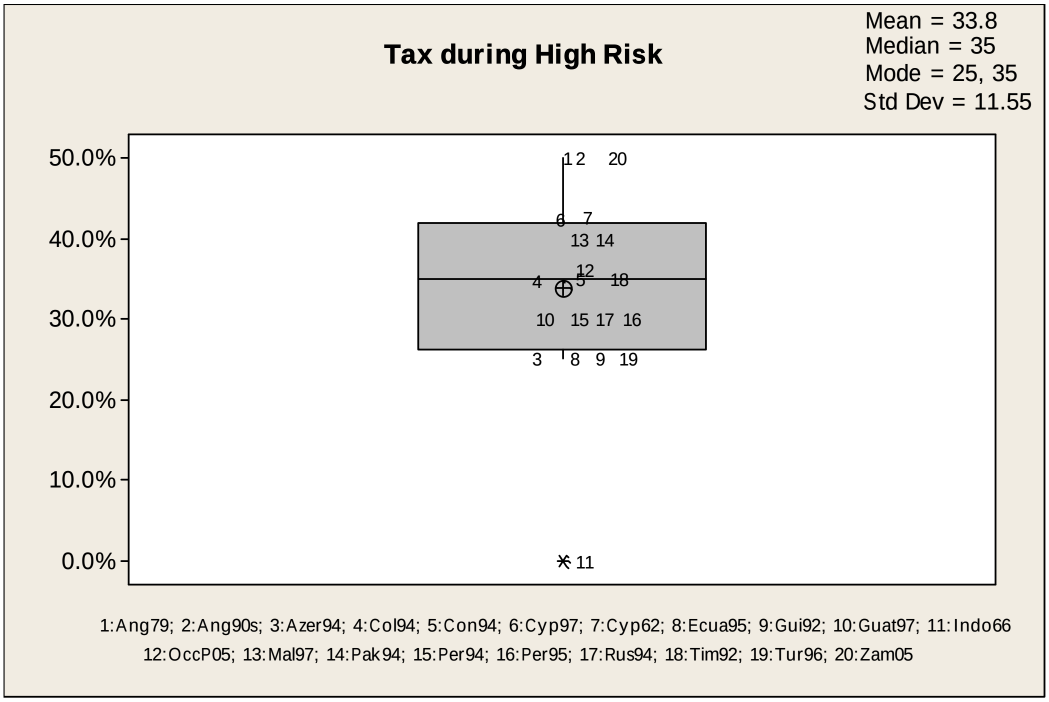

Fig 4.5: Tax during high risk

The exploration period box plot is show in Figure 4.6. It show that 75% of high risky countries have exploration period less than or equal to 10 years (Q3 = 10). The exploration period is in general related to the status of the reserves more than it is to the risk level of the country. But, we can still consider that high risky countries increase their exploration period for the sake of attracting oil companies.

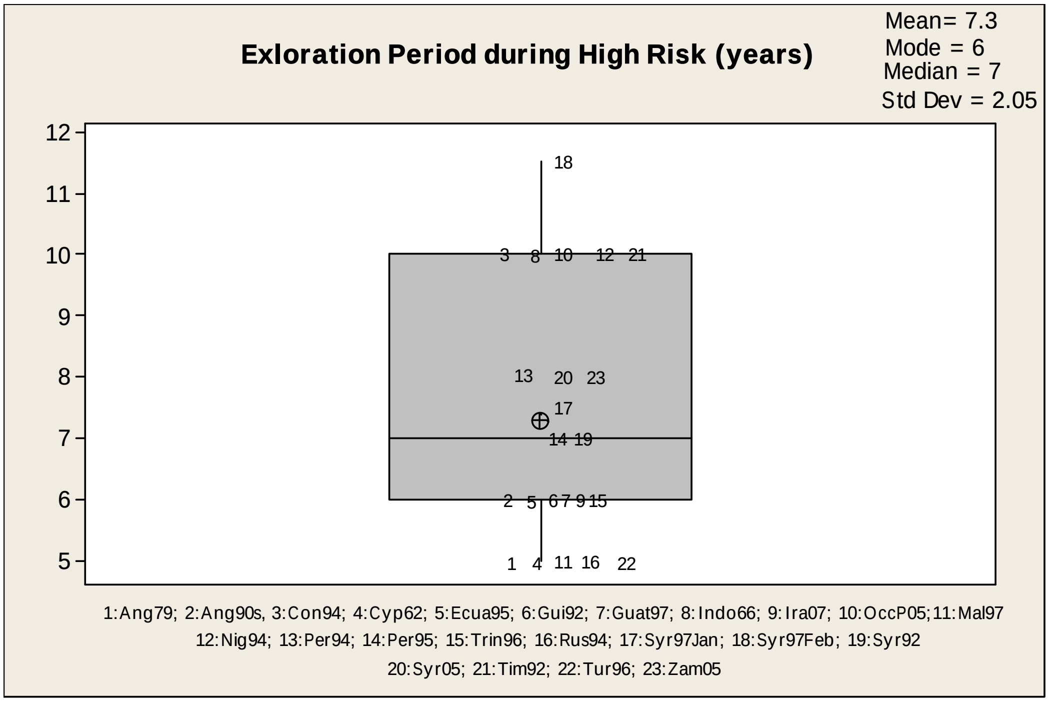

Fig 4.6: Exploration period during high risk

The box plot of the production period for contracts in high risk countries is available in Figure 4.7. Extensive production periods are considered incentives for oil companies to invest in hydrocarbon resources, since they would be able to benefit from these resources for the longest period of time.

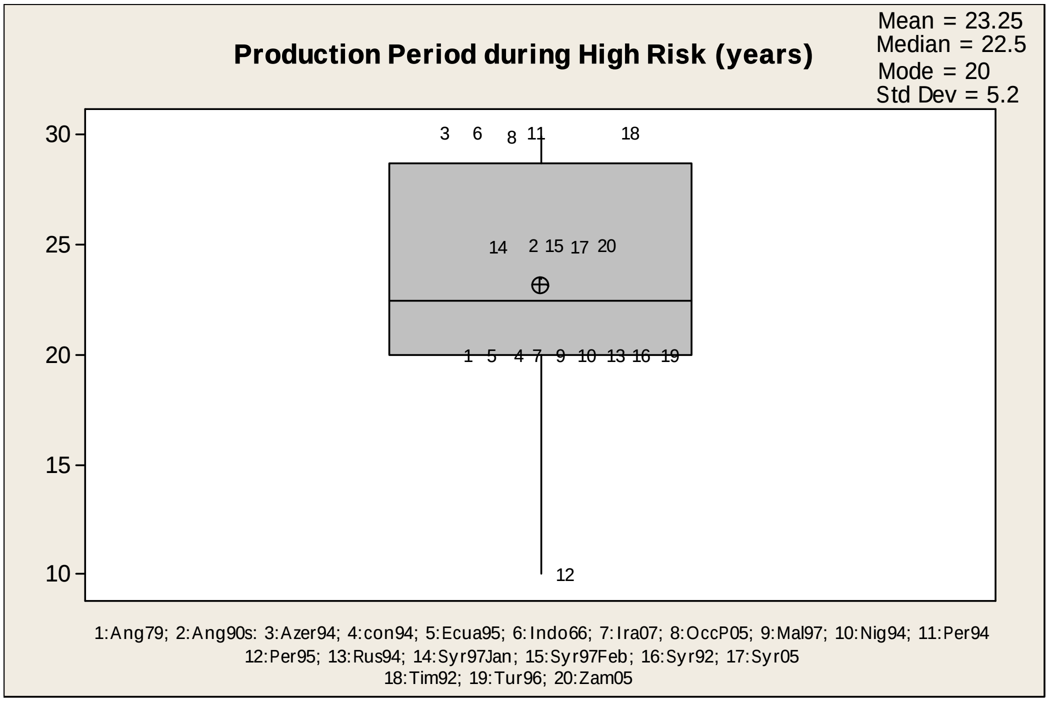

Fig 4.7: Production period during high risk

Table 4.4 presents a summary of statistics for the contract parameters in high risk countries.

Table 4.4: Statistics for the contract's parameters in high risk countries

Parameter	**Ranges**	**Mean**	**Mode**	**Standard deviation**
Royalty	0 – 20%	8.21%	0%	7.13%
Cost Recovery	0 – 100%	58.3%	100%	37.83%
Signature Bonus	0 – $150M	$ 14423077	$ 0	$ 41543575
Tax	0 – 50%	33.8%	25%, 35%	11.55%
Exploration period	5 – 11.5 years	7.3 years	6 years	2.05 years
Production period	20 – 30 years	23.25 years	20 years	5.2 years

4.4.2 Contracts Signed in Countries with Unproven Reserves

The second group of contracts "contracts signed on unproven reserves" contains five PSAs signed in 4 different countries through different periods ranging from the 1962 till 2005. Table 4.5 shows the contracts signed while reserves were still unproven yet and Table 4.6 presents the number of the fixed and sliding scale based parameters found in the five unproven reserves PSAs of the dataset.

Table 4.5: Contracts on unproven reserves

Country	Contract
Cyprus	Mines regulation Law, 1997
Cyprus	Forest Oil Contract, 1962
Occupies Palestine	Oil regulation, 2005
Timor Gap – Zoca	License round, 1991/1992
Zambia	8-Jun-2005

Table 4.6: Fixed and sliding scale parameters in unproven reserves contracts

Contracts' parameter	Number of fixed scale based parameter	Number of sliding scale based parameter
Royalty	3	1
Profit Share	0	2
Cost Recovery	2	0
Signature Bonus	2	0
Production Bonus	2	0
Tax	5	0

Statistical analysis of the parameters of the unproved reserves PSAs are not given here because of the small number of PSAs (5 PSAs). The descriptive statistics of the PSAs' parameters are shown in Table 4.7.

Table 4.7: Descriptive analysis of PSA parameters for unproven reserves blocks

Parameter	**Ranges**	**Mean**	**Mode**	**Standard deviation**
Royalty	0 – 12.5%	8.33%	12.5%	7.22%
Cost Recovery	80 – 100%	90%	-	14.14%
Tax	35 – 50%	41%	35% & 42.5%	6.3%
Exploration period	5 – 10 years	8.25 years	10 years	2.36 years
Production period	25 – 30 years	28.33 years	30 years	2.89 years

The fixed based scale signature and production bonus in the PSAs having unproven hydrocarbon reserves do not appear in Table 4.7 since they are null in the five unproven reserves contracts. Sliding scale profit share analysis is not available because of the small size of the dataset; we have one numerical production based profit share (Timor Cap - Zoca 1991/1992) and one R-factor based profit share (Zambia 2005).

4.4.3 Contracts Signed On Unproven Reserves under High Political and Economic Risk

The third group of contracts "contracts signed on unproven reserves during high political and economic risk period" contains five PSAs signed in four different countries - Cyprus, Occupied Palestine, Timor Gap – Zoca and Zambia- through different periods ranging from the 1962 till 2005. This group has the same contracts as the 'unproven reserves' group (Table 4.5). Then, the descriptive statistics of the corresponding PSAs' parameters are shown in Table 4.7.

4.5 Comparison and Analysis

Having categorized countries with similar profile to Lebanon, a comparison between the three groups and the whole dataset is provided next.

First, it can be seen that contracts with high political and economic risk level have a smaller tax percentile but a higher exploration and production periods when compared to the whole dataset. Countries trying to sign hydrocarbon contracts in a high political and economical risk level period tend to offer incentives to attract foreign oil companies such as minimizing tax and increasing the exploration and production periods, thus increasing the company's profit over a longer time period.

Second, comparing the values in Table 4.7 with the whole dataset, it can be shown that countries with unproven reserves limit their royalty (to 12.5%), increase the minimum the cost recovery, require no signature and production bonuses, decrease tax and increase both the exploration and the production periods. These results are consistent with the fact that countries with unproven reserves are less attractive to foreign companies than countries with high political and economic risk level. Countries with unproven reserves seek a fast development of local petroleum industry, hence, they seek fast flow of money; this is why the mode of the royalty for countries and contracts with unproven reserves is 12.5% and not 0% (the case of high political and economic risk countries and the whole dataset).

Third, countries with unproven reserves, in our sample, were living a period of high political and economic risk. This is why, the 'unproven reserves' group and the 'high risk and unproven reserves' group perfectly matched.

4.6 Sensitivity Analysis for Lebanon's Profile

After building the general PSA model in section 3.2, the royalty, tax and government profit share, in addition to the cost recovery, are simulated by taking one range of values for each of the four elements and changing the value of one parameter at a time in the Excel spreadsheet, to recognize the relevance of each of these parameters on the take of the government and that of the contractor. The ranges used for each parameter are based on the statistical findings for the high risk, unproven reserves group available in Table 4.7. Building this model starts with a base case (equal to the mean of each parameter in high risk and unproven reserves countries, Table 4.7); having the following PSA parameters: royalty = 8%; cost recovery = 90% of the cost; government profit share = 62% (mean of profit oil mean from Table 4.3); tax = 41%; signature bonus = \$0; exploration period = 8 years; production period = 28 years. Then, using the mathematical expression from Section 3.2 that links the production of gas to the take of the government/contractor, sensitivity analysis is performed by changing each PSA parameter along some ranges[9] while holding other parameters at their base values. The chart relating the array of takes to the parameter's set of numbers are plotted and found below, with a linear equation trend.

Figure 4.8 presents the take of the government and the take of the contractor in function of the changing profit share of the government for a range between 28% ($62.31 - 2\times17$) and 96% ($62.31 + 2\times17$). In Figure 4.8, the slope of the line relating the government profit to the government take is positive (0.571), hence, the government profit share is proportional to the government's take; when increasing the government profit share, the government take would increase. Whereas, it is inversely proportional to the contractor's take.

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[9] $Ranges\ used: parameter's\ mean\ \mp\ 2 \times parameter's\ standard\ deviation$

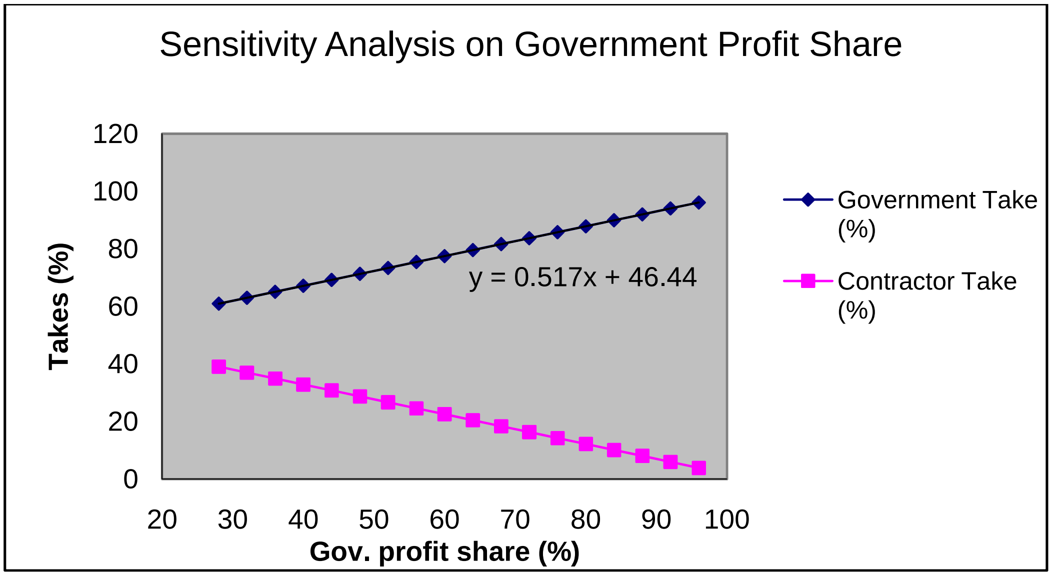

Fig 4.8: Sensitivity analysis on the government 'profit share'

Figure 4.9 represents the take of the government and the take of the contractor with respect to the royalty change over a range between 0% ($8.33 - 2\times7.22 < 0$) and 23% ($8.33 + 2\times7.22$). In Figure 4.9, the slope of the line relating the royalty to the take of the government is positive (0.327), hence, the royalty is proportional to the government's take and inversely proportional to the contractor's take.

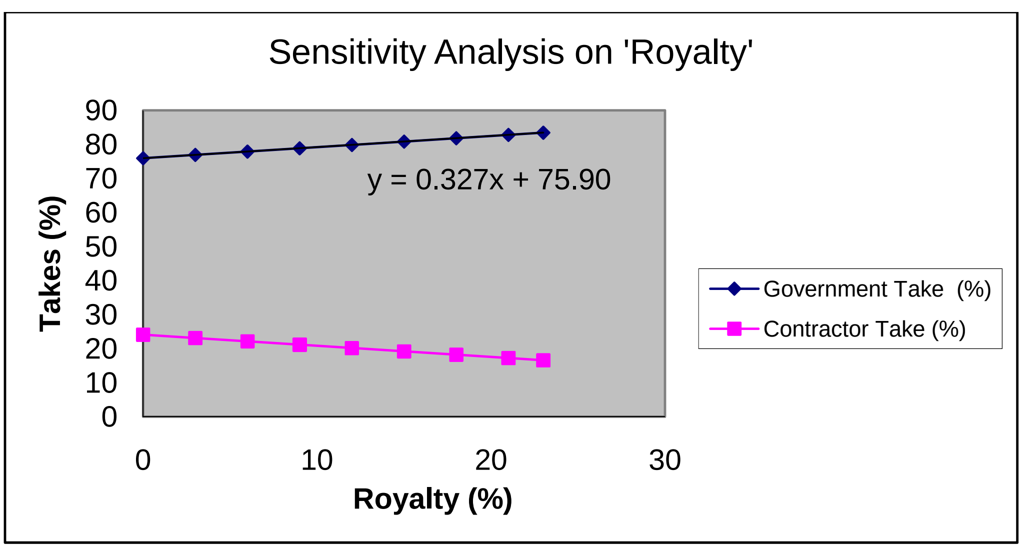

Fig 4.9: Sensitivity analysis on 'royalty'

Figure 4.10 represents the take of the government and the take of the contractor in function of the changing tax between 26% (40.6 − 2×7.18) and 55% (40.6 + 2×7.18). In Figure 4.10, the slope of the line relating the tax to the take of the government is positive (0.32), hence, tax is proportional to the government's take and inversely proportional to the contractor's take.

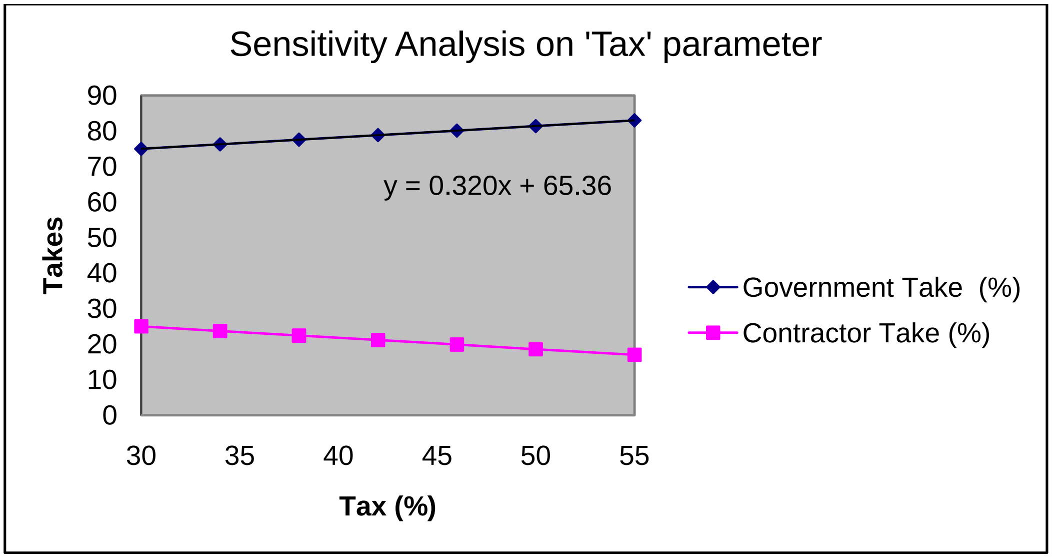

Fig 4.10: Sensitivity analysis on 'tax'

Figure 4.11 illustrates the take of the government and the take of the contractor in function of the changing cost recovery. The range used is between 62% (90 − 2×14.14) and 100% (90 + 2×14.14 > 100). In Figure 4.11, the slope of the line relating the cost recovery to the take of the government is negative (-0.188), hence, the cost recovery is inversely proportional to the government's take; when increasing the cost recovery percentile, the government take will decrease, whereas it is proportional to the contractor's take; with an increasing cost recovery, the contractor is increasing his take with a slope of 0.188.

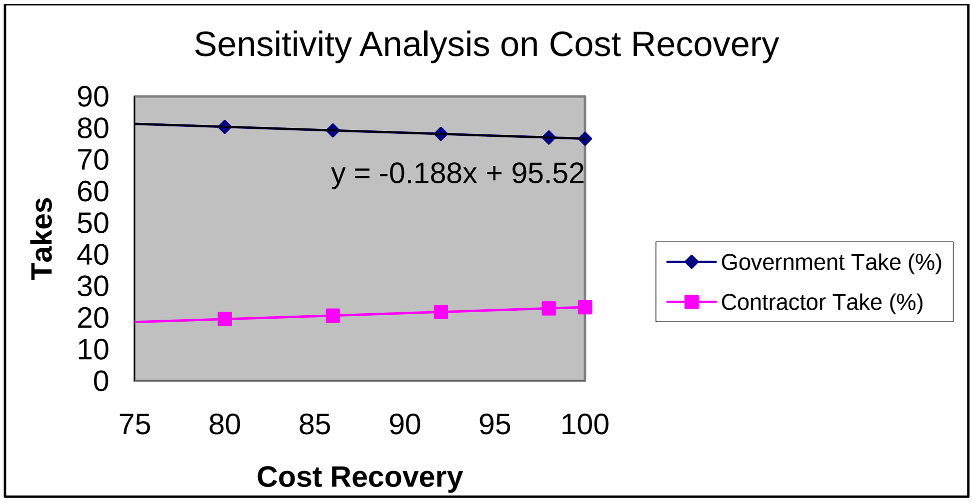

Fig 4.11: Sensitivity analysis on 'cost recovery'

The ranges used for each parameter and the slope of the linear trend relating each parameter to the take of the government are shown in Table 4.8

Table 4.8: Results of the sensitivity analysis

Parameters under simulation	**Ranges**	**Slope**
Cost Recovery	62 – 100%	– 0.188
Royalty	0 – 23%	0.327
Tax	26 – 55%	0.320
Government Profit Oil	28 – 96%	0.517

Table 4.8 indicates that increasing the government's profit share has the highest effect on the take of the government, followed by royalty, tax and cost recovery. Therefore, when negotiating a PSA contract, the government can be strict on setting profit share, conservative about royalty, tax and somewhat flexible about cost recovery.

4.7 Suggestions

Since Lebanon is a high risk with unproven reserves country, our suggestions are based in the statistics for high risk with unproven reserves countries (Table 4.7). But, since the data available for this group is relatively small, we also use the analysis on the group of high risk countries (Table 4.4). Then, based on the findings of section 4.4, the suggested ranges and values for a Lebanese model PSA are shown in Table 4.9.

Table 4.9 Suggested ranges and values for a Lebanese hydrocarbon PSA

Variables	**Ranges and Values**
Royalty	0 – 12.5%
Profit Share	Available in details in Table 4.3[10]
Cost Recovery	80 – 100%
Tax	35 – 50% / based on the Lebanese fiscal system
Signature Bonus	$ 0
Production Bonus	$ 0
Exploration period	5 – 10 years
Production period	25 – 30 years

[10] We use the high risk country analysis because data for high risk and unproven reserves is not available

4.8 The Lebanese Hydrocarbon Law

On the 24th of August 2010, the Lebanese Council of Ministers approved on the Lebanese offshore hydrocarbon law – law number 132 (LHL, 2010).

Upon this law, all hydrocarbons found within the Lebanese borders, the offshore territorial waters, the continental shelf and the exclusive zone of the state of Lebanon are property of the state and no petroleum operation shall be conducted in Lebanon by any entity, unless such entity has entered into a petroleum contract and has been granted a license. Potential contracts signed between the Lebanese government and the foreign oil companies will be a PSA. The law also stipulates that the council of Ministers sets the policy that must be followed as regards the Lebanese hydrocarbon resources and the Minister of Energy and Water signs the exploration and production licenses in accordance with the provisions of the law. The Minister will have to ensure the implementation of the licensing strategy, to work on promoting the hydrocarbon potentials, to monitor and to supervise the activities linked with the hydrocarbon resources. According to this law, an agency called “organization department of the petroleum sector” linked to the Minister of Energy and Water will be delegated some functions such as establishing and organizing the conditions for bids, helping the Minister in short listing the best bidders and negotiating with them, in addition to supervising and monitoring the execution of the petroleum contracts (Article 10, LHL, 2010). The LebanesePSA includes two phases: an exploration phase for a period of 10 years maximum and a production phase for 30 years maximum (Article 13/2, LHL, 2010). The contract shall be awarded to oil companies based on a private negotiation between the Minister and the oil companies (each block in the Lebanese offshore will be explored and exploited by at least 3 oil companies (Article

1, LHL, 2010)). Each license holder has to pay taxes upon the Lebanese fiscal system (Article 19/6, LHL, 2010).

The law includes several articles that specify the potential PSA parameters. Table 4.10 summarizes the PSA parameters instituted in the Lebanese law.

Table 4.10: Parameters of contracts as found in the Lebanese hydrocarbon law

Parameters/conditions found the Lebanese hydrocarbon contract	**Article within the law**(LHL, 2010)
Exploration obligations	19/7/d
Participation of the Lebanese government	19/7/e
Rental fees per sqkm	41/2
Fixed royalty	43
No signature bonus	
No production bonus	
Fixed cost recovery	44
Sliding scale profit share	44
Tax paid according to the Lebanese fiscal terms	19/6

Accordingly, comparing Table 4.10 to our suggestions in Table 4.9, it can be seen that our study supply and complement the Lebanese law with the needed quantitative values for the contract’s parameters.

CHAPTER 5

AUCTIONS

Even though the Lebanese law (Section 4.8) mentions direct negotiations with the contractors, we believe that sealed bidding is the right Lebanese method of contracting having the profit share as the parameter subject to the auctioning; we propose this parameter for bidding because profit share was identified as the most critical PSA parameter in Section 4.6; in addition to its popularity in auctioning process. Sealed envelope bidding is preferred for Lebanon since it provides more transparency and can generate better revenue for the government than direct negotiations. It can be followed by direct negotiations with potential bidders on other parameters of the contract. The two key factors in sealed auction are the optimal minimum number of bidders and the reserve price. In order to be able to quantify these values, we develop a mathematical modelthat maximizes the benefit of the government from the sealed auctions. Most models in the literature deal with this problem from the company perspective. Models that take the government's perspective are rare

Section 5.1 includes the key factors in sealed bidding process, followed by the mathematical model in Section 5.2. Section 5.3 focuses on understanding both mathematical models by setting a numerical example.

5.1 Key Factors in Sealed Bidding

One of the important key factors in auctioning is the level of competition i.e. the number of bidders (Cramton, 2007). Therefore, the design of the auction should take into consideration the level of competition and the number of bidders that would participate in this auction. High revenues from open auctions are accomplished because of the aggressive bidding of the auctioneers when competition is high, which is not the case with a low level of competition. Therefore, countries expecting high competition on their hydrocarbon reserves choose to go for open auctions; whereas countries with low competition expectation would rather choose sealed-bid auctions. Figure 5.1 plots data generated through simulations to illustrate the relationship between the number of bidders and the revenue of the government (Villa-Boas, 2006). It can be seen from Figure 5.1 that with an increasing number of bidders, the expected revenue from auctions increases, getting to a stability number called the “optimal minimum number of bidders”. Hence, the design of an auction should involve a cautious estimate of the number of bidders.

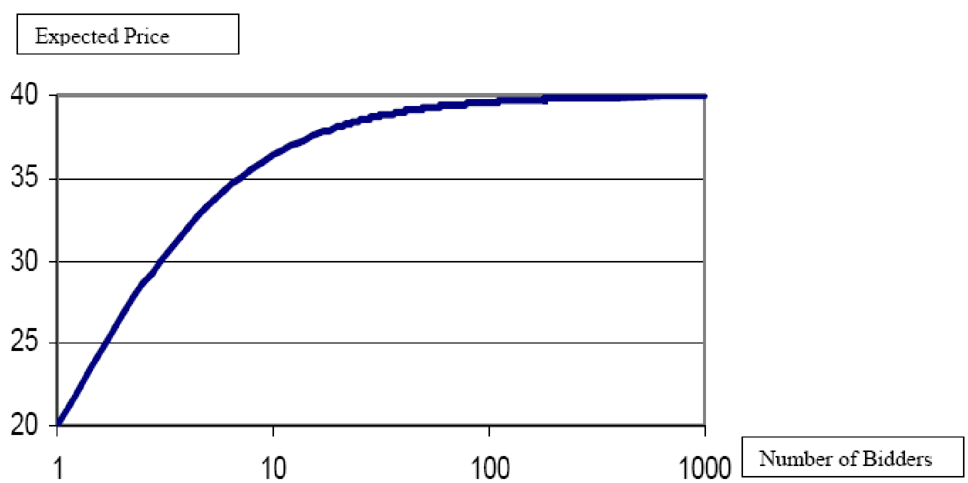

Fig 5.1: Effect of the number of bidders on the expected price (Villas-Boas, 2006)

The different auction formats – open and sealed- acquire the same expected revenue for the seller; this result is known as the 'Revenue Equivalence Proposition"

(Riley, 1989). Riley and Samuelson (1981) have shown that the mechanism which maximizes the seller's expected gain should involve choosing the reserve priceoptimally. Therefore, another key factor in auctioning design is the reserve price. Mathematical models for both auctioning key factors – minimum optimal number of bidders and optimal reserve price- are developed later in the following section.

5.2 Mathematical Model

Our reserve price model heavily relies on a model by Riley andSamuelson (1981). Table 5.1 defines the notations for this model.

Table 5.1: Notations used in the models

n	Total number of bidders
b_i	Bid of bidder i
v_i	Valuation of bidder i for the object in auction; continuous random variable
$F^{'}(v)$	Probability density function of the valuation
$F\ (v)$	Cumulative density function with a support upon the range [v*, $\overline{\text{v}}$]
$P\ (v_{i)}$	Expected payment of bidder i having a valuation v_i
v_0	Valuation of the government
$b_0$or $v*$	Reserve price
π	Government expected revenue from one bidder
Π	Government expected revenue from the auction

The payments of bidders are the main source for the revenue of the government from auctions; hence, a closed mathematical form for the payment of each bidder is developed next.

Assuming that all bidders' valuations belong to the same cumulative density function$F\ (v)$, then, the expected gain of bidder $i\ =\ v_i \times\ F^{n-1}(v_i) -\ P(v_i)$

where, v_i is the valuation of bidder i

$F^{n-1}(v_i)$is the probability that bidder i wins. $\prod_{j\,=\,1, j\neq\, i}^{n} P\{V_j \,\leq\, v_i\}$

$P\ (v_i\ )$ is the expected payment of bidder i

The reserve price is chosen in a way that makes the expected bidder gain equal to zero. That is, at $v\ *$, $v * \times\ F^{n-1}(v *) -\ P(v *) =\ 0$

$$P(v *) = v * \times\ F^{n-1}\ (v *)(1)$$

$$\text{For} v_1 > v *, v_1 \times \frac{\partial F^{n-1}\,(v_1)}{\partial v_1} = P^{'}(v_1) \qquad (2)$$

The expected payment for bidder i would be equal to his payment at $v_i = v *$ in addition to the payment of bidder i with$v_i > v *$. Then,

$$P(v_i) = P(v_i = v *) +\ P(v_i > v *)$$

$$P(v_i) =\ P(v *) +\ \int_{v*}^{v_i} P^{'}(x)dx$$

Substituting $P(v *)$ using equation (1) and $P^{'}(x)$ by its equation (2), we have

$$P(v_i) =\ v * \times\ F^{n-1}(v *) +\ \int_{v*}^{v_i} x\partial F^{n-1}(x)dx$$

Applying the integration by part (full mathematics are available in Appendix C), we get

$$P(v_i) =\ v_i \times\ F^{n-1}(v_i) -\ \int_{v*}^{v_i} F^{n-1}(x)\ dx \qquad (3)$$

After finding a closed form for the expected payment of one bidder, the government revenue from bidder i will be easily generated since it is equal to the expectation of the expected payment $P\ (v_i)$

Then, $\pi(v_i)$=$\int_{v*}^{\bar{v}} P(v_i) \times F^{'}(v_i)dv_i$; substituting equation (3) of the payment yields

$$\pi\ (v_i) = \int_{v*}^{\bar{v}} (v \times F^{'}(v) +\ F(v) -\ 1) \times\ F^{n-1}(v)dv \qquad (4)$$

After finding the expected revenue from one bidder, we find the expected revenue of the government from the whole auction. The expected government gain from an auction can have two cases; the first case of the gain is when there is no acceptance of biddings, and the second case is when of the government's gain comes from bidders.

The case called 'no bidding acceptance' is when all the bidders' valuations are less than the reserve price$v*$, in this case the government cancels the auction and the situation is considered as a 'no bid submission'. Therefore, with no submission, the gain of the government is equal to the valuation of the government multiplied by the probability that all bidders' valuation is less than$v*$. Whereas, the gain of the government from n bidders is the total expected payments from the n bidders; hence using equation (4) gives

$$\Pi(v) = v_0 \times F^n(v*) + n \times \int_{v*}^{\bar{v}} (v \times F'(v) + F(v) - 1) \times F^{n-1}(v) dv \qquad (5)$$

5.2.1 Optimal Reserve Price

In order to find the optimal reserve price$v*$, we use the first order optimality conditions, which implies that $\frac{\partial \Pi(v)}{\partial v*} = 0$

$$v* = v_0 - \frac{F(v*)}{F'(v*)} + \frac{1}{F'(v*)} \qquad (6)$$

All the detailed mathematical steps are found in Appendix C. Equation (6) proves that the reserve price should always be bigger than the valuation of the government (Riley and Samuelson, 1981).

In order to prove whether the reserve price obtained from the above model is a local maximum, the concavity of equation (6) is proven in Appendix C. Therefore, the mathematical model for the optimal reserve price gives a unique optimal value for the reserve price. A numerical example is detailed in section 5.3.

5.2.2 Optimal Minimum Number of Bidders

To find the optimal number of bidders, we use the first-order optimality conditions, i.e. $\frac{\partial \Pi\ (v)}{\partial n} = 0$

All the mathematical steps are detailed in Appendix C. And, the optimal minimum number of bidders is obtained from the following equation:

$$v_0 F^n(v *)\, ln[F(v *)] +$$

$$\int_{v*}^{\bar{v}} [(v \times F'(v) + F(v) - 1) F^{n-1}(v)] \times [\, 1 - n(n-1)F^{-1}] dv\ = 0 \quad (7)$$

The second derivative of equation (7) is found in order to check on the concavity of the equation and the uniqueness of the solution;

$$\frac{\partial \Pi^2(v)}{\partial^2 n} =$$

$$v_0 F^n(v *)\, ln^2\, F(v *) + \int_{v*}^{\bar{v}} (vF'(v) + \ F(v) - \ 1) \times \ F^{n-1}(v)\, ln\, F(v *)\, dv \quad (8)$$

Equation (8) cannot be known whether it is positive or negative. This is why we took several examples of distributions; and for all theexamples,we find that the minimum optimal number of bidders was unique. Hence, it seems that equation (7) has one unique solution.

Looking at both equations above (6) and (7), it can be seen that the reserve price is independent of the number of bidders whereas the number of bidders is highly dependent on the reserve price. In addition to this, the two key factors are inversely proportional: as the reserve price increases, the minimum number of bidders required gets smaller. With anincreasing the reserve price, the government’s revenue from auction automatically gets higher since the government accepts no bids lower than the reserve price. Therefore, the need for competition decreases, i.e. the number of bidders decreases.

5.3 Numerical Analysis

In order to understand each of the above mathematical models, we will illustrate an example by taking some assumptions:

- The valuations are uniformly distributed over the unit interval [0,1], then $F(v) = v$ and $F^{'}(v) = 1$
- The government has a 0 valuation for the block on bid, i.e. $v_0 = 0$

Replacing these assumptions in equation (6), we obtain $v* = -v + 1$. v and $v*$ belong to the unit uniform distribution and $v* \geq 0$, then for $0 \leq v \leq 1$, $v* = {}^{1}/_{2}$. Substituting the reserve price by its value in equation (7), we get the optimal minimum number of bidders: $n = 3$ bidders (mathematical details are available in Appendix C). We replace $v*$ and $n = 3$ by their values in equation (5) and get the government expected revenue from the auction equal to 0.53.

CHAPTER 6

CONCLUSION

The main objective of this study is to assist the Lebanese government and other governments in structuring hydrocarbon contracts for their offshore potentials. Such assistance needs anin depth study of the contracts and auctions in oil and gas industry.

We start by the part of contracts by reviewing the literature on the type of hydrocarbon contracts used; production-sharing agreements are the focus of this study. Then, a collection of offshore hydrocarbon PSAs from all over the world was put in our dataset, studied and analyzed. Descriptive statistics on PSAs parameters is collected and sensitivity analysis was conducted. Royalty, cost recovery, tax, signature bonus, and exploration and production periods are fixed scale parameters; whereas the profit split and the production bonus are sliding scale parameters in most contracts. Our financial feasibility analysis concludes that the government's profit share split is the most critical parameter on its take; hence, it is the parameter that should be greatly taken into consideration.

In order to be able to compare the data available to Lebanon's case, we profile contracts and countries on the basis of 'political and economic risk level' and 'condition of hydrocarbon reserves'. And then, contracts in countries similar to Lebanon's profile were analyzed and quantitative suggestions of ranges for Lebanon's hydrocarbon PSA parameters were given.

The types of auctions used in oil and gas industry were reviewed; a sealed bidding process on certain blocks for the royalty and/or profit share split followed by a direct negotiation on work program is the auctioning design recommended for

Lebanondue to the transparency of sealed bidding, the critical effect of the profit share parameter on the government’s take and the interest of the Lebanese government in the “work program” parameter of the contract. The design of a sealed bid is optimized based on optimizing two key factors: the reserve price and the number of bidder. A mathematical model was built to achieve the best government’s revenue from auctions.

This study is useful for the Lebanese government since it yields a production-sharing contract with the best combination of parameters for the case of Lebanon. Having these quantitative values for all the parameters will ensure the government better revenue from its offshore potentials and will enhance the bargaining position of the Lebanese government. In addition, the auctioning procedure suggested by this study will give the optimal minimum number of bidders, the optimal reserve price and therefore, the best revenue for the Lebanese government.

Not only Lebanon will benefit from this study; other governments, working on developing or enhancing the terms of their offshore PSAs, can use our data, analysis methodology and framework.

Finally, this study will be beneficial for oil companies too since it will help them understand how governments structure their contracts and auctions, hence, they will have better insights on the needs of the governments.

Future work will include (i) collecting more data on PSAs and auctions in order to refine the benchmarking study, (ii) getting some data on the valuation distributions and (iii) refining the auction model by performing sensitivity analysis with realistic valuation distribution.

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APPENDIX A

Sample Hydrocarbon Contract

SYRIA

PSC Unocal 23 June, 1992

Area	Block III
Duration	**Exploration** 3+2+1 (I believe) **Production** 20 years typically + 10 yr extension
Relinquishment	25% + 25% of original area each time
Exploration Obligations	
Bonuses	**Signature** $2.5 MM **Production** $1.5, 2, 4, 4, & $8 MM at 25, 50, 100, 150, & 200 MBOPD Not recoverable
Training	$50,000/year (recoverable)
Royalty	**MBOPD** **Rate** 0–50 14% 50–100 15% > 100 16%
Cost Recovery Ceiling "net of royalty"	25% up to 50,000 BOPD 20% over 50,000 BOPD Excess cost oil goes directly to National Oil Corp. - SPC.
Depreciation	Exploration capital and operating costs expensed Development Costs 5 years SLD
Profit Oil Split	**BOPD** **Contractor Share** Up to 25,000 19.5% 25,001–50,000 18.5 50,001–100,000 18.0 100,001–200,000 15.0 > 200,000 12.0
Taxation	In lieu "Paid by the State"
Ringfencing	Yes
Gvt. Participation	None

Government Take				Effective Royalty Rate	Lifting	Savings Index	Data Quality
Downside	Mid-range	Upside	Margin				
100%	91%	86%	88%	63%	36%	19¢	Good

APPENDIX B

Justification of Countries' Political and Economic Profiling

Table B-1: Justifications for political and economic risk level by contract

Country	Political and Economic Risk factor	Justification
Algeria (2005)	High	
Angola (1979-1991)	High	Intense civil war from 1975 until 2002 (Wikipedia, 2010)
Angola (mid 1990s)	High	Intense civil war from 1975 until 2002 (Wikipedia, 2010)
Azerbaijan(20-Sep-94)	High	In 1994, Azerbaijan was subject to several coups. GDP fell by 22%; hyper-inflation and the monthly retail price increases exceeded 50% (Bayulgen, 1998)
China (1990)	Moderate	China was experiencing political and social stability, economic growth and successes in foreign relations (Shambaugh, 1991)
Colombia (1994)	High	Daily crimes, thieves (Sturner, 1998)
Congo (1994)	High	Subject to several attempts along with the need for the presence of international troops (Bureau of African affairs, 2010)
Cote d'Ivoire (27-Jun-95)	Low	Ivory Coast grew rich and stable since 1993 (Alley, Valley and Watkins, 2009)
Cyprus (1997)	High	Turkish and Greek debate on it (Fouskas, 2002)
Cyprus (1962)	High	Getting out of independence at 1960, Cyprus was not stable at that time. In 1963, a Turkish invasion took place (Fouskas, 2002)
Ecuador (1995)	High	Economic crisis
Equatorial Guinea (1992)	High	Corruption and very slow growth rate (Bureau of African Affairs, 2010)
Guatemala (1997)	High	The civil war ended in 1996 and the situation was not stable yet (Gutierrez, 1999)
India (late 1980s)	Moderate	Political conflicts and collapsing of the government (Maps of India, 2010)

Table B-1: Justifications for political and economic risk level by contract (continuous)

Country	Political and Economic Risk factor	Justification
India (28-Oct-94)	Moderate	Economic liberalization, Indian opening to global trade and investment (Maps of India, 2010)
India (1994)	Moderate	Economic liberalization, Indian opening to global trade and investment (Maps of India, 2010)
India (1995)	Moderate	Economic liberalization, Indian opening to global trade and investment (Maps of India, 2010)
Indonesia (18-Aug-66)	High	In 1965, a violent anti-communist remove took place with a deterioration of the economy till 1968 (Indo.com, 2010)
Indonesia (6-Sep-68)	High	In 1965, a violent anti-communist remove took place with a deterioration of the economy till 1968 (Indo.com, 2010)
Indonesia (pre 1984)	Moderate	In the 1970s, Indonesian economy was sustained mainly due to oil export (Indo.com, 2010)
Indonesia (1976)	Moderate	In the 1970s, Indonesian economy was sustained mainly due to oil export (Indo.com, 2010)
Indonesia (1988)	Moderate	In the 1970s, Indonesian economy was sustained mainly due to oil export (Indo.com, 2010)
Iraq (15-Feb-07)	High	American war
Occupied Palestine (2005)	High	Palestinian conflict
Libya (1990)	Moderate	Wealthiest countries along with the highest GDP per person in Africa. High economic stability (Wikipedia, 2010)
Malaysia (1994)	Moderate	High economic growth and stability (Wikipedia, 2010)
Malaysia (1994)	Moderate	High economic growth and stability (Wikipedia, 2010)

Table B-1: Justifications for political and economic risk level by contract (continuous)

Country	**Political and Economic Risk factor**	**Justification**
Malaysia (1997)	High	Shaken by the Asian financial crisis and political unrest(Wikipedia, 2010)
Nigeria (1994)	High	Inhomogeneous ethnic groups yielding continuous conflict, poverty and corrupted political system (Oritsejafor, 2000)
Oman (1989)	Moderate	Stability, economic growth and building good foreign relations (Day, 2003)
Pakistan (1994)	High	Economic and political difficulties, in addition to the effect of Afghani problems to its status (Bhatia, 2009)
Peru (1993/ Dec 1994)	High	Reforms permitted an economic growth since 1993, but the overall status of the country was still unstable and unattractive yet (Wikipedia, 2010)
Peru (1971)	High	Subject to the War of the Pacific (Wikipedia, 2010)
Peru (after 1978)	High	Subject to the War of the Pacific (Wikipedia, 2010)
Trinidad & Tobago (29-Feb-96)	High	Political and economic instability, presence of violence and corruption (Worldwide Governance Indicators, 2009)
Qatar (1994)	Moderate	Emphasis on economic issues more than political, which made the country in a small political insecurity (Day, 2003)
Russia (23-Jun-94)	High	Early 1990s, economic crisis struck all post-Soviet countries with a great effect on it GDP (Wikipedia, 2010)
Syria (30-Jan-97)	High	Conflict with Israel in addition to low rates of investments and attractiveness to foreign interest (Wikipedia, 2010)

Table B-1: Justifications for political and economic risk level by contract (continuous)

Country	Political and Economic Risk factor	Justification
Syria (19-Feb-97)	High	Conflict with Israel in addition to low rates of investments and attractiveness to foreign interest (Wikipedia, 2010)
Syria (23-Jun-92)	High	Instability in political situation regarding Israel and the just ending war in Lebanon
Syria (23-Jun-92,model contract)	High	Instability in political situation regarding Israel and the just ending war in Lebanon
Timor Gap – Zoca (1991/1992)	High	Conflict between Australia and Indonesia on the ownership of the country in front of the International Court of Justice (Catry, 2004)
Turkmenistan (7-Aug-96)	High	Launching of an economic plan but instability was the main title of the period (Columbus, 1999)
Zambia (8-Jun-05)	High	Political and economic instability (Wikipedia, 2010)

APPENDIX C

Mathematical Details for Auctioning Model

Bidder i payment derivation

$$P(v_i) = v* \times F^{n-1}(v*) + \int_{v*}^{v_i} x \partial F^{n-1}(x) dx$$

Doing an integration by part; let $u = x$; $\frac{\partial u}{\partial x} = 1$ and $\frac{\partial w}{\partial x} = \partial F^{n-1}(x)$; $w = F^{n-1}(x)$

$$P(v_1) = v* \times F^{n-1}(v*) + [uw]_{v*}^{v_1} - \int_{v*}^{v_1} w \times \frac{\partial u}{\partial x} dx$$

$$P(v_1) = v* \times F^{n-1}(v*) + [x \times F^{n-1}(x)]_{v*}^{v_1} - \int_{v*}^{v_1} F^{n-1}(x)\, dx$$

$$P(v_1) = v* \times F^{n-1}(v*) + v_1 \times F^{n-1}(v_1) - v* \times F^{n-1}(v*) - \int_{v*}^{v_1} F^{n-1}(x)\, dx$$

$$P(v_1) = v_1 \times F^{n-1}(v_1) - \int_{v*}^{v_1} F^{n-1}(x)\, dx$$

Reserve price derivation

In order to find the optimal reserve price$v*$, we derive$\Pi(v)$, the expected revenue of the government from the auction in function of$v*$ and set it to 0

$$\frac{\partial \Pi(v)}{\partial v*} = 0$$

$$n \times v_0 \times F^{'}(v*) \times F^{n-1}(v*) - n \times [v* \times F^{'}(v*) + F(v*) - 1] \times F^{n-1}(v)$$
$$= 0$$

$$n \times F^{n-1}(v*) \times [v_0 \times F^{'}(v*) - v* F^{'}(v*) - F(v*) + 1] = 0$$

$$v_0 \times F^{'}(v*) - F(v*) + 1 = v* \times F^{'}(v*)$$

Optimal reserve price: $v* = v_0 - \frac{F(v*)}{F'(v*)} + \frac{1}{F'(v*)}$

Concavity of reserve price equation

To prove concavity, the second derivative of the reserve price equation is found.

$$\frac{\partial \Pi^2\ (v)}{\partial^2\ (v*)} = v_0 \times F''(v*) - F'(v*) + 0 - F'(v*) - v* \times F''(v*)$$

$= v_0 \times F''(v*) - 2F'(v*) - v* \times F''(v*)$

$=(v_0 - v*) \times F''(v*) - 2F'(v*)$

$F'(v*) > 0$,which implies that $-2F'(v*) < 0$. The reserve price should be bigger than the valuation of the government (Riley and Samuelson, 1981), then $(v_0 - v*) < 0$. $F''(v*) > 0$, therefore $(v_0 - v*) \times F''(v*) < 0$; resulting in $(v_0 - v*) \times F''(v*) - 2F'(v*) < 0$. Hence, the reserve price equation is concave resulting in a unique optimal value for the reserve price.

Optimal minimum number of bidders' derivation

$$\frac{\partial \Pi(v)}{\partial n} = 0$$

$$v_0 \times \frac{\partial F^n(v*)}{\partial n} + \int_{v*}^{\bar{v}} (v \times F'(v) + F(v) - 1) \times F^{n-1}(v)dv - n \times \frac{\partial}{\partial n} \int_{v*}^{\bar{v}} (v \times F'(v) + F(v) - 1) \times F^{n-1}(v)dv = 0$$

$\frac{\partial F^n(v*)}{\partial n}$; let $y = F^n(v*)$

$\ln y = n \times \ln F(v*)$; derive both sides with respect to n

$$\frac{1}{y} \times \frac{\partial y}{\partial n} = \ln F(v*)$$

$$\frac{\partial y}{\partial n} = y \times \ln F(v*)$$

$$\frac{\partial F^n(v*)}{\partial n} = F^n(v*) \times \ln F(v*)$$

$\frac{\partial}{\partial n}\int_{v*}^{\bar{v}}(v \times F'(v) + F(v) - 1) \times F^{n-1}(v)dv$ can be solved using Leibniz rule.

Leibniz Rule: $\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z}dx + f(b(z),z) \times \frac{\partial b}{\partial z} - f(a(z),z) \times \frac{\partial a}{\partial z}$

$$\frac{\partial}{\partial n}\int_{v*}^{\bar{v}}(v \times F'(v) + F(v) - 1) \times F^{n-1}(v)dv$$

$$= \int_{v*}^{\bar{v}}(n-1) \times (v \times F'(v) + F(v) - 1) \times F^{n-2}dv + 0 - 0$$

$$\int_{v*}^{\bar{v}}(n-1) \times (v \times F'(v) + F(v) - 1) \times F^{n-2}dv$$

$\frac{\partial \Pi(v)}{\partial n} = v_0 \times F^n(v*) \times ln\, F(v*) + \int_{v*}^{\bar{v}}(v \times F'(v) + F(v) - 1) \times F^{n-1}(v)dv -$

$n \times \int_{v*}^{\bar{v}}(n-1) \times (v \times F'(v) + F(v) - 1) \times F^{n-2}dv = 0$

And, the optimal minimum number of bidders is obtained from the following equation:

$v_0 \times F^n(v*) \times ln\, F(v*) + \int_{v*}^{\bar{v}}[(v \times F'(v) + F(v) - 1) \times F^{n-1}(v)] \times$

$[1 - n \times (n-1) \times F^{-1}]dv = 0.$

Numerical model details

For a valuation uniform unit distribution, the reserve price is $v* = {}^{1}/_{2}$ and

substituting the reserve price value in equation (7) yields:

$$0 + \int_{1/2}^{1}[(v + v - 1) \times v^{n-1}] \times \left[1 - \frac{n(n-1)}{v}\right]dv = 0$$

$$\int_{1/2}^{1}[(2v-1) \times v^{n-2}] \times [v - n(n-1)]dv = 0$$

$$\int_{1/2}^{1} [\, 2\, v^{n-1} - v^{n-2}] \times [v - n(n-1)\,] dv = 0$$

$$\int_{1/2}^{1} [\, 2v^{n} - 2n(n-1)v^{n-1} - v^{n-1} + n(n-1)v^{n-2}]\, dv = 0$$

$$\frac{2v^{n+1}}{n+1} - \frac{2n(n-1)v^{n}}{n} - \frac{v^{n}}{n} + \frac{n(n-1)v^{n-1}}{n-1}]_{1/2}^{1} = 0$$

$$\left[\frac{2}{n+1} - 2(n-1) - \frac{1}{n} + n\right] - \left[\frac{(1/2)^{n}}{n+1} - 2(n-1)(1/2)^{n} - \frac{(1/2)^{n}}{n} + 2n\left(\frac{1}{2}\right)^{n}\right] = 0$$

(B)

Figure C-1 plots equation (B). The plot crosses the x axis in one point which shows that this equation has a unique zero solution at the point of 3. This implies that for this numerical example, the minimum optimal number of bidders is 3 bidders.

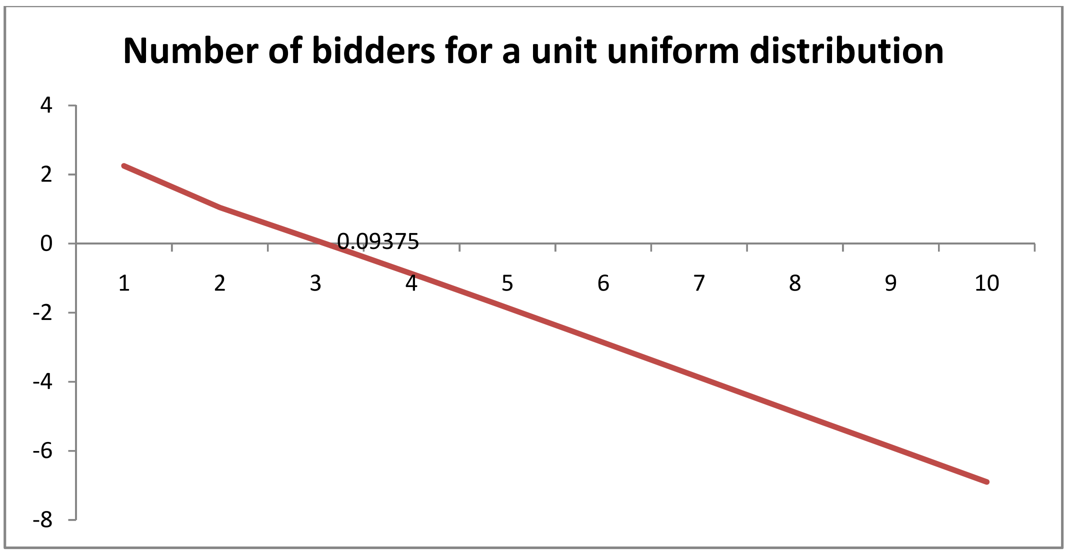

Fig C-1: Number of bidders' plot for the uniform unit distribution