A M E R I C A N U N I V E R S I T Y O F B E I R U T

Department of Civil Engineering

T H E S I S

on

REINFORCED CONCRETE SPANDRELL FILLED ARCH BRIDGE DESIGN

presented by

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A.U.B. May 1945.

FOREWORD

The analysis of arches is the most common problem encountered in reinforced concrete construction, specially in bridge design. As an arch may be so constructed so that the line of its axis coincides more or less with the rsultant pressure lines of the loads it has to carry, the internal stress at any section in the arch ring will be wholly compressive and uniformly distributed over that section. And as the concrete is the most economical building material to resist compressive stresses it follows clearly that concrete arches are far more economical than any other type of structure. But ideal conditions of loading are never attained so that some bending will always occur in the arch ring, and the need of steel reinforcement to take care of the tensile stresses which might occur becomes imperative. Furthermore the shrinkage and the plastic flow of the concrete complicates very much the problem of design so that an elaborate knowledge of the methods leading to the determination of the critical stresses is necessary in order to be able to design properly any reinforced concrete arch.

In this very mountainous country that is Lebanon, the construction of bridges is the most important factor of communication and it is in the hope of being a bridge designer that I intended to present a thesis upon this aspect of my future professional work.

I wish to gratefully acknowledge the help that Professor Issa Hilu, my supervisor in this work, has given to me.May he find here my warmest thanks and best regards.

May 1945. E. N. Klat.

NOTATION

a -inclination of arch axis at any point;

A -area of cross-section;

d -thickness of arch ring;

E -modulus of elasticity;

g -ratio of dead loads at springing and crown: ws/wc ;

h -rise of highest point of arch axis ;

H -thrust perpendicular to section;

I -moment of inertia ;

I^1 -moment of inertia at crown;

l - span;

m -bending moment at any point in the cantilever due to the external loads, these moments are negative;

M -bending moment;

n -ratio of modulae of elasticity of steel to concrete;

P -any load on the arch;

q -ratio ds/I to ds_1/I_1;

s -length of arch axis;

t -change of temperature;

V -shear;

W -live load;

w -dead load weight per sq. ft. ;

x,y -coordinates of arch axis;

y_1 -vertical ordinate referred to an X-axis through the elastic center

yo -distance from crown to elastic cebtre;

N.B. The subscripts c and s relate respectively to crown and springing

C O N T E N T S

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REINFORCED CONCRETE SPANDRELL FILLED ARCH BRIDGE DESIGN

1.-General Method of Procedure.-

The method of analysis studied here is based on the elastic theory, and it is mainly an algebraic method although certain simple graphical aids are used advantageously. It consists in the determination of the thrust, bending moment and shear at any section.

As an analysis supposes an already made design, we will make a preliminary design by the aid of the approximate or empirical rules laid by Mr. Cochrane resulting in the fair determination of the critical stresses at the crown and at the springing. Then using the ordinary arch formulas we will be able to investigate the dimensions that have been assumed in the tentative design.

2.-Choice of the Data.-

We will take the type and dimensions which will fit most of the requirements in the country: a spandrell earth filled double lane arch bridge, to carry across 30 yards , the maximum loads of the 18-tons trucks. As an 18 tons truck occupies an area of 10 ft. by 30 ft. this will give an equivalent uniform load of $W = 36000/300$ or 120 lbs/ft². And as it is an earth filled arch we do not need adding for impact.

So in all the calculations which follow it is assumed :

the span as : $2 \times 48 = 96$ ft.

the rise as : 16 ft.

and the live load as 120 lbs/ft².

we will also assume :

w of earth = 120 lbs/ft^3

w of concrete = 150 "

fc = 700 lbs/"^{2}

fs = 16000 "

n = 15

Ec = 2,000,000 lbs/in^2

3.-Tentative Design Dimensions.-

By comparison with existing arches of similar span and loading conditions we will take an arch ring having a crown thickness of dc = 12 in. and a springing thickness of ds = 24 in. From fig. 1 it is computed that for the dimensions assumed wc = 370 lbs, ws = 2455 lbs and g = ws/wc = 6.63 .

4.-Dead Load Thrust.-

A closely approximate formula for dead load thrust is :

$$Hc = (1/8\ wc + 0.015(ws - wc))1^2/h$$

In our case Hc = (370/8 + .015(2455 -370))96^2/16 = 44700 lbs.

Assuming the inclination of the arch axis to be 45° at the springing the thrust there would be : Hc/cos45° = Hs = 44700.$\sqrt{2}$ = 63000 lbs.

5.-Moments and Thrusts from Live Load, Temperature, and Rib Shortening

After Cochrane the live load moment for an arch having values of h/l = .167 and ds/dc = 2.0 is Mc = 0.0065Wl^2 and the live load thrust is Hc = 0.48Wl^2. Also the thrust due to temperature changes is Tc = 29.utEI$_1$/h^2 , the moment due to Tc is Mc =.20Tc.h , and the thrust due to Rib-shortening is Hr = .93.H.Tc/Ac.u.t.E where u is the coefficient of expansion of the concrete , it will be taken here as 0.000006. And we assume $t = 40^\circ$

Ref. fig. 1 the moment of inertia at the crown is

$I_1 = 0.1052$ so that $T_c = \frac{29 \times 0.000006 \times 40 \times 2000000 \times 144 \times 0.1052}{16^2}$

$= 29 \times 270 \times 0.1052 = \underline{820 \text{ lbs}}$

$M_c = .20 \times 820 \times 16 = \underline{2600 \text{ ft.lbs.}}$

The dead load thrust is $H_c = .48 \times 120 \times 96 = \underline{5500 \text{ lbs}}$

At the crown the positive moment is the most dangerous stress so that we need to consider the sign of the thrusts accompanying positive moments. Thus the total dead load, live load and temperature thrust is : $44700 + 5500 - 820 = \underline{49400 \text{ lbs}}$

The rib-shortening thrust can now be computed; noting that Ac = 1.153 (Ref fig.1)$H_r = .93 \times 49400 \times 820 / 1,153 \times 69000 = .58 \times 820 = \underline{480 \text{ lb}}$

It is worth to remark that the rib-shortening effects are directly proportional to temperature effects, this proportion being in our case .58 . So while the temperature moment is 2600 ftlbs the rib-shortening moment is therefore : $.58 \times 2600 = \underline{1510 \text{ ftlbs}}$.

Live load positive moment at the crown : $.0065 \times 120 \times 96^2 = \underline{7200 \text{ ft}}$

The total positive moment is:

$7200 + 2600 + 1510 = \underline{\mathbf{11300 \text{ ft-lbs}}}$

The total Thrust is:

$49400 - 480 = \underline{\mathbf{48900 \text{ lbs}}}$

Similarily for the springing section, live load positive moment $= .036 \times 120 \times 96^2 = \underline{39900 \text{ ft-lbs}}$; live load thrust $= .62 \times 120 \times 96^2 = \underline{7150 \text{ lbs}}$; temperature crown thrust for positive moment = 820 and $M_s = 820 \times 16 - 2600 = \underline{10500 \text{ ft-lbs}}$;the rib-shortening moment is $.58 \times 10500 = \underline{6100 \text{ ft-lbs}}$; the thrust due to temperature is : $T_c \cos a = 820 \times 0.712 = \underline{580 \text{ lbs}}$; and the thrust due to rib-shortening is :

580 x .58 = 340 lbs .

The total positive moment at the springing is :

$$Ms = 39900 + 10500 - 6100 = \mathbf{44300\ Ft\text{-}lbs}$$

And the total corresponding thrust at the springing is :

$$Hs = 63000 + 7150 + 580 - 340 = \mathbf{70400\ lbs}$$

For these critical stresses at the crown and at the springing where the depths of the sections are respectively 12 and 24 inches and where the steel reinforcement consistsof two one inch round bars placed at 1.5 in.from the intrados and the extrados, the maximum fibre stresses are respectively 670 lbs/in^2 and 670 lbs/in^2 both compressive. Some tensile stresses also occur at these sections but they are very negligible since they range around 2o lbs/in^2.

6.-Conclusion.

The stresses being very safe we can consider this preliminary design as satisfactory and we can start investigating thouroughly in order to find out the exact stresses.

7.-General Formulas.-

From mechanics it is derived that at the crown the moment is :

$$Mc = \frac{-\sum_A^c (m_R + m_L)q}{2\sum_A^c q} - Hc \times yo$$

the thrust is: $$Hc = \frac{-\sum_A^c (m_R + m_L)y_1 q + \frac{u\ t\ l\ E}{ds_1/I_1}}{2(\sum_A^c y_1^2 q + I_1 \sum_A^c COSa/A)}$$

the shear is $$Vc = \frac{\sum_A^c (m_R - m_L)x.q}{2\sum_A^c x^2 q}$$

In all these expressions m_R and m_L are the moments at correspon-ding points respectively at the right and at the left of the crown. These moments are negative and the summations are for one half arch. yo is the distance between the crown and the elastic centre, its value is : $yo = \sum yq / \sum q$. (See fig. 2 and 3.)

We note that all terms of the denominators are functions of the dimensions of the arch only. The effects of applied loads is included in the terms containing m, and that of temperature in the term containing the coefficient u. The effect of rib-shortening is in the term containing cos a , and its relative importance can readily be observed in any particular case.

8.-Temperature Stresses.-

These may be had by placing m_L AND m_R equal to zero.

Thus $V_t = 0$

$$M_t = H_t \, yo$$

$$H_t = \frac{u \, t \, \dfrac{1 \; E}{ds^1/I_1}}{2 \sum_{A}^{C} y_1{}^2 q + I_1 \sum_{A}^{C} \cos a \, /A}$$

9.-General Observations.-

The method of analysis we are going to follow is simple in theory, and easily followed in the numerical work. The loads and their points of application will be considered apart from the division of the arch ring into ds sections, as the two things are in no wise related.

In wiew of the uncertainty regarding the allowance to be made for shrinkage, plastic flow, and temperature stresses in arches, no great refinement in the calculations may be warranted. However when the formulas involve differences between two quantities, a sufficient number of significant figures must be used so that these differences will have the desired precision.

10.-Method of Procedure.-

We will use the influence-line method by which the values of moment and thrust at the crown and other critical sections are determined for a unit load placed at successive intervals along the arch and sufficiently near together to give thedesired accuracy. Having the influence lines all other stresses due to any type of loading will be easily computed.

11.-Coordinates of the Arch Axis.-

For spandrel filled arches the equation of the arch axis derived by Mr. Cochrane is :

$$y = z^2 h \frac{1 + .1(g - 1)z^3}{1 + .1(g - 1)}$$

The origin being at the crown and $z = 2x/l$

From the tentative design we have $g = 6.63$ (Ref. Art. 3) and $.1(g - 1) = .563$ so that our equation becomes :

$$y = 16z^2 \frac{1 + .563z^3}{1 + .563}$$

$$= 10.23z^2(1 + .563z^3)$$

In order to draw properly the arch axis the coordinates will be computed for 15 different points. Table A contains such computations, at the same time some properties of the arch axis as its length and its inclination are also calculated.

A diagram of the arch axis and of its inclination is drawn on fig. 4 so that the coordinates and the inclination at any point may be read instanteneously.

12.-Thickness of the Arch Ring.-

For ratio of springing to crown thickness of 2.00 Cochrane recommends variations in thickness of the arch rib from crown to springing as :

s	: 0.00	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.00
t	: 1.000:	1.035:	1.048	1.085	1.168	1.311	1.547	1.837	2.000

where s is the proportionate distance along the arch axis from crown and t is the relative thickness. These values are plotted on the diagram of fig. 4 and the curve of the variation of the thickness is drawn in order to give the thickness at any point.

13.-Properties of the Arch Ring.-

In all the calculations which follow, each half of the arch ring has been divided into 13 equal ds sections. From Table A the half length is 52.00 ft. Each ds section has therefore a length of 52/13 = 4.00 ft.

Table B on page 9 gives the calculations of the quantities I, A, q, and cos a /A. The values of I are calculated for the entire section although the bending moment may be such that some tension occurs While this tensile stress in the concrete is ignored the moment of inertia of the entire cross-section will best represent the facts as regards deformations.

Table C on page 10 gives the various needed functions of x, y, y_1, and q. The values of x and y are scaled from the diagram of fig.4 .

T A B L E   A

PROPERTIES OF THE ARCH AXIS

$y = 10.23\ z^2\ (1 + .563\ z^3)\ ;\ ds^2 = dx^2 + dy^2\ ;\ \cos a = dx/ds$

x	26z	26^3z^3	$1 + .563z^3$	26^2z^2	y	dy	dy^2	dx^2	ds^2	ds	cos a
0.000	0	0	1.000	0	0.00						
						0.015	0.000	3.408	3.408	1.847	1.000
1.846	1	1	1.000	1	0.015						
						0.121	0.015	13.63	13.646	3.695	1.000
5.538	3	27	1.001	9	0.136						
						0.244	0.060	13.63	13.691	3.700	0.998
9.231	5	125	1.004	25	0.380						
						0.371	0.138	13.63	13.769	3.706	0.995
12.923	7	343	1.011	49	0.751						
						0.504	0.254	13.63	13.885	3.725	0.990
16.616	9	729	1.023	81	1.255						
						0.656	0.430	13.63	14.061	3.753	0.984
20.308	11	1331	1.043	121	1.911						
						0.829	0.687	13.63	14.318	3.782	0.976
24.000	13	2197	1.071	169	2.740						
						1.03	1.06	13.63	14.69	3.835	0.963
27.692	15	3375	1.108	225	3.77						
						1.29	1.66	13.63	15.29	3.913	0.944
31.384	17	4913	1.158	289	5.06						
						1.60	2.56	13.63	16.19	4.02	0.919
35.077	19	6859	1.220	361	6.66						
						1.99	3.96	13.63	17.59	4.19	0.881
38.769	21	9261	1.297	441	8.65						
						2.47	6.10	13.63	19.73	4.44	0.832
42.461	23	12167	1.390	529	11.12						
						3.08	9.49	13.63	23.12	4.81	0.767
46.153	25	15625	1.501	625	14.20						
						1.80	3.24	3.41	6.65	2.58	0.715
48.000	26	17576	1.563	676	16.00						

Half Total length of the arch axis ....52.00 ft.

TABLE B

PROPERTIES OF THE ARCH RING

Values of I, q, A, and cosa /A

$n = 15$; $A_s = 2 \times .785/144 = .0109 ft^2$; $I_s = (n - 1)A_s.d_1^2 = .153 d_1^2$;

$I_c = d^3/12$; $A = d + (n - 1)A_s = d + .153$; $ds = 4.00$ ft.

Section	d	$2d_1$	d_1^2	Is	Ic	I	A	q	cos a	cosa/A
1	1.004	.754	.142	.0217	.0844	.1061	1.157	1.000	1.000	.865
2	1.011	.761	.145	.0222	.0861	.1083	1.164	.979	.998	.857
3	1.019	.769	.148	.0226	.0882	.1108	1.172	.957	.996	.849
4	1.027	.777	.151	.0231	.0903	.1136	1.180	.934	.991	.840
5	1.034	.784	.154	.0236	.0922	.1158	1.187	.916	.986	.831
6	1.042	.792	.157	.0240	.0945	.1185	1.195	.895	.977	.817
7	1.058	.808	.163	.0249	.0987	.1236	1.211	.859	.964	.796
8	1.102	.852	.182	.0278	.1115	.1393	1.255	.761	.945	.753
9	1.173	.923	.213	.0326	.1343	.1669	1.326	.635	.918	.693
10	1.278	1.028	.264	.0404	.1735	.2139	1.431	.496	.883	.617
11	1.440	1.190	.354	.0541	.2490	.3031	1.593	.350	.836	.525
12	1.635	1.385	.479	.0732	.3640	.4372	1.788	.243	.782	.438
13	1.872	1.622	.657	.1005	.5450	.6455	2.025	.164	.724	.358

$\sum q = 9.189$

$\sum \cos a / A = 9.239$

T A B L E C

PROPERTIES OF THE ARCH RING

Values of x, y, y_1, q, and Products.

Section:	q	y	yq	y_1	x	xq	x^2q	y_1q	y_1^2q	xy_1q
1	1.000	0.010	0.010	-2.91	2.00	2.00	4	-2.91	8.47	-5.8
2	0.979	.150	.147	-2.77	6.00	5.88	35	-2.71	7.51	-16.3
3	.957	.42	.402	-2.50	9.99	9.56	95	-2.39	5.98	-23.9
4	.934	.86	.804	-2.06	13.97	13.03	182	-1.93	3.96	-26.9
5	.916	1.46	1.338	-1.46	17.91	16.42	294	-1.34	1.95	-24.0
6	.895	2.22	1.988	-0.70	21.83	19.55	427	-0.63	0.44	-13.7
7	.859	3.19	2.741	+0.27	25.68	22.07	566	+0.23	0.06	+ 6.2
8	.761	4.35	3.313	+1.43	29.50	22.47	663	+1.09	1.56	+32.2
9	.635	5.82	3.699	+2.90	33.23	21.13	702	+1.84	5.34	+61.2
10	.496	7.58	3.760	+4.66	36.84	18.30	674	+2.31	10.78	+85.2
11	.350	9.57	3.350	+6.65	40.34	14.12	569	+2.33	15.48	+93.9
12	.243	11.96	2.906	+9.04	43.55	10.59	461	+2.20	19.85	+95.6
13	.164	14.58	2.388	+11.66	46.59	7.64	356	+1.91	22.26	+89.0
Summations	9.189		26.846			182.76	5028	-11.91	103.64	-110.6
								+11.91		+463.3
								0.00		+352.7

$$y_0 = \Sigma\, yq / \Sigma\, q = \frac{26.846}{9.189}$$

$$= \underline{2.92 \text{ ft.}}$$

14.-Formulas for Crown Stresses.-

Fig. 5 shows an arch rib with a unit load placed at any point D on the left, at a distance e from the crown C. Between A and D the moment is : $m_L = -(x - e)$; for points between D and C, $m_L = 0$; and for the right half $m_R = 0$. Substituting these values in the formulas of article 7 and replacing the denominators of these formulas by their respective values computed in Tables B and C we have:

$$Hc = \frac{\sum_A^D xy_1q - e\sum_A^D y_1q}{2(103.64 + .106 \times 9.239)}$$

$$= \frac{\sum_A^D xy_1q - e\sum_A^D y_1q}{209.24}$$

$$Mc = \frac{\sum_A^D xq - e\sum_A^D q}{18.38} - 2.92\ Hc$$

$$Vc = \frac{\sum_A^D x^2q - \sum_A^D xq}{10056}$$

In these formulas the summations are taken from the left end of the arch rib at A to the load point D. These summations are computed in Table D from Table C .

15.-Influence Lines for Mc, Hc, and Vc.-

These may be obtained by applying a unit load successively at closely spaced intervals along thearch and calculating the resulting stresses by the aid of the formulas of the previous article. We will take the spacing as 1/13 the half span, the first load point being 1/26 x 48 = 1.846 ft. from the crown, and the last load at this same distance of 1.846 ft. from the springing. The positions of the load points with reference to the ds centres from which all the calculations are made are shown on the arch axis of fig. 4 . A strictly precise calculation would involve fractional values of ds in the summations since the load points do not coincide with the ds centres, but the error involved in using whole units in each case is negligible.

Table D contains all the calculations for arriving at the stress values and fig. 6 shows the influence lines drawn from these values. The totals of the positive moment and negative moment areas are calculated numerically by summations of trapezoidal elements areas. The influence line for shear is given to complete the analysis; its area is 9.80 and the maximum live load shear is 120 x 9.80 = 1180 lbs, the unit shear is 1180/144 or 8.1 lbs/in^2 a negligible value. For the springing the shear will not be considered.

As fibre stress is a function of both bending moment and thrust the maximum fibre stress will not in general occur for the particular position of loading giving maximum moment but the increased effect is very small, so that the position for maximum moment can safely be used for fibre stress.

16.-Influence Lines for Moment and Thrust at the Springing.-

From Fig. 7 we have, for a load on the left :

$$Ms = Mc + 16\,Hc + 48\,Vc - (48 - e)$$

$$Hs = Hc \cos a_s + (1 - Vc)\sin a_s$$

For a load on the right :

$$Ms = Mc + 16\,Hc + 48\,Vc$$

$$Hs = Hc \cos a_s - Vc \sin a_s$$

Table E contains the computations and Fig. 8 shows the influence lines with the calculation of the areas.

T A B L E   D

CALCULATION OF Hc, Mc, AND Vc FOR UNIT LOADS.

Influence-line values for Crown Section.

$$H_c = \frac{\sum_A^D xy_1q - e\sum_A^D y_1q}{209.24}\ ; \qquad M_c = \frac{\sum_A^D xq - e\sum_A^D q}{18.38} - 2.92\,H_c$$

$$V_c = \frac{\sum_A^D x^2q - \sum_A^D xq}{10056}$$

e	$\sum_A^D xy_1q$	$\sum_A^D y_1q$	$e\sum_A^D y_1q$	Hc	$\sum_A^D xq$	$\sum_A^D q$	$e\sum_A^D q$	$\frac{\sum xq - e\sum q}{18.38}$	2.92 Hc	Mc	$\sum_A^D x^2q$	$e\sum_A^D xq$	Vc
0.00	352.7	0.00	0.0	1.687	182.76	9.189	0.00	9.94	4.92	+5.02	5028	0	.5000
1.85	352.7	0.0	0.0	1.687	182.76	9.189	16.95	9.01	4.92	+4.09	5028	337	.4660
5.54	358,5	2.9	16.1	1,638	180,76	8.189	45.30	7.36	4.78	+2.58	5024	1000	.4002
9.23	374.8	5.6	51.8	1.544	174.88	7.210	66.60	5.90	4.51	+1.39	4989	1612	.3357
12.92	398.7	8.0	103.5	1.411	165.32	6.253	80,85	4.60	4.12	+0.48	4894	2138	.2739
16.62	425.6	9.9	165.2	1.245	152.29	5.319	88.42	3.48	3.64	-0.16	4712	22532	.2167
20.31	449.6	11.3	228.8	1.057	135.87	4.403	89.50	2.52	3.08	-0.56	4418	2759	.1649
24.00	463.3	11.9	286.0	0.848	116.32	3.508	84.20	1.75	2.48	-0.73	3991	2791	.1193
27.69	457.1	11.7	323.2	0.640	94.25	2.649	73.40	1.13	1.87	-0.74	3425	2610	.0811
31.38	424.9	10.6	332.2	0.444	71.78	1.888	59.20	0.68	1.30	-0.62	2762	2250	.0509
35.08	363.7	8.8	307.2	0.270	50.65	1.253	44.02	0.36	0.79	-0.43	2060	1779	.0279
38.77	278.5	6.4	249.5	0.139	32.35	0.757	29.35	0.16	0.41	-0.25	1386	1253	.0132
42.46	184.6	4.1	174.3	0.049	18.23	0.407	17.28	0.05	0.14	-0.09	817	773	.0044
46.15	89.0	1.9	88.1	0.004	7.64	0.164	7.57	0.004	.0013	-.01	356	353	.0003

TABLE E (Sheet I)

CALCULATION OF Ms AND Hs.

Loads on Left.

$$Ms = Mc + 16\ Hc + 48\ Vc - (48 - e)$$

$$Hs = Hc \cos a_s + (1 - Vc)\sin a_s$$

$$\cos a_s = 0.697\ ;\ \sin a_s = 0.718\ .$$

x	Mc	16 Hc	Vc	48 Vc	-(48-e)	Ms	Hc cosa	1-Vc	(1-Vc) sina	Hs
46.15	-0.01	0.064	.0003	0.01	-1.85	-1.78	0.003	1.000	.718	0.721
42.46	-0.09	0.78	.0044	0.21	-5.54	-4.64	0.034	0.996	.715	0.749
38.77	-0.25	2.22	.0132	0.63	-9.23	-6.63	0.097	.987	.709	0.806
35.08	-0.43	4.32	.0279	1.34	-12.92	-7.69	0.188	.970	.696	0.884
31.38	-0.62	7.10	.0509	2.44	-16.62	-7.70	0.310	.949	.681	0.991
27.69	-0.74	10.24	.0811	3.89	-20.31	-6.92	0.446	.919	.660	1.106
24.00	-0.73	13.58	.1193	5.74	-24.00	-5.41	0.591	.881	.633	1.224
20.31	-0.56	16.90	.1649	7.91	-27.69	-3.44	0.736	.835	.600	1.336
16.62	-0.16	19.91	.2167	10.40	-31.38	-1.23	0.868	.783	.562	1.430
12.92	+0.48	22.60	.2739	13.73	-35.08	+1.13	0.984	.726	.522	1.506
9.23	+1.39	24.72	.3357	16.10	-38.77	+3.44	1.077	.664	.477	1.554
5.54	+2.58	26.20	.4002	19.20	-42.46	+5.52	1.140	.600	.431	1.571
1.85	+4.09	27.00	.4660	22.37	-46.15	+7.31	1.175	.534	.383	1.558
0.00	+5.02	27.00	.5000	24.00	-48.00	+8.02	1.175	.500	.359	1.534

T A B L E E (Sheet II)

CALCULATION OF Ms AND Hs.

Loads on Right.

$$Ms = Mc + 16\ Hc + 48\ Vc$$

$$Hs = Hc \cos a_s - Vc \sin a_s$$

$$\cos a_s = 0.697 \ ; \ \sin a_s = 0.718 \ .$$

x	Mc	16 Hc	Vc	48 Vc	Ms	Hc cos a	Vc sin a	Hs
0.00	+5.02	27.00	.5000	-24.00	+8.02	1.175	-.359	1.534
1.85	+4.09	27.00	.4660	-22.37	+8.72	1.175	-.334	1.509
5.54	+2.58	26.20	.4002	-19.20	+9.58	1.140	-.287	1.427
9.23	+1.39	24.72	.3357	-16.10	+10.01	1.077	-.241	1.318
12.92	+0.48	22.60	.2739	-13.13	+9.95	0.984	-.196	1.180
16.62	-0.16	19.91	.2167	-10.40	+9.35	.868	-.155	1.023
20.31	-.56	16.90	.1649	-7.91	+8.43	.736	-.118	0.854
24.00	-.73	13.58	.1193	-5.74	+7.11	.591	-.086	0.677
27.69	-.74	10.24	.0811	-3.89	+5.61	.446	-.058	0.504
31.38	-.62	7.10	.0509	-2.44	+4.04	.310	-.037	0.347
35.08	-.43	4.32	.0279	-1.34	+2.55	.188	-.020	0.208
38.77	-.25	2.22	.0132	-0.63	+1.34	.097	-.010	0.107
42.46	-.09	0.78	.0044	-0.21	+.48	.034	-.003	0.037
46.15	-.01	.06	.0003	-0.01	+.04	.003	.000	0.003

17.-Dead-Load Stresses.-

The dead-load consists here of the weightsof the earth fill, spandrell wall, roadway slab, and the arch rib.

The roadway slab is formed by 8 in. of concrete. The spandrell wall acts as a retaining wall and must be designed to retain very safely the earth fill and the surcharge of 120 lbs/ft^2. Where the earth fill is more than 4 ft. deep a safe section would be that formed by a wall having 2 ft. for top width and a batter of 1:1 for the interior wall, the front face being vertical. (See Fig. 9). Between the quarter points of the arch the depth of the earth fill is less than 4 ft. we will make the section as 1 ft. for the top width, the batter being 1:1/2.

D being the vertical depth of the arch ring, and F being the depth of the earth fill, it is calculated in Fig. 9 that :

$$F = y + 1.5 - D/2$$

and assuming the dead-load to be uniformly distributed over the width of the arch, the concentrated load assumed to act at the middle of each division 3.692 ft long is also calculated to be :

$$\text{for } F \angle\ 4 \qquad P = 2F^2 + 419F + 60 \text{ lbs.}$$

$$\text{for } F \setminus\ 4 \qquad P = 4F^2 + 459F + 60 \text{ lbs.}$$

Table F contains all the calculations for the determination of these dead-load concentrations. The stresses for each concentration will be the products of the value of the load by the ordinates of the influence lines. The summation will give the total stresses, as in Table G .

18.-Live-Load Stresses.-

The live-load assumed is 120 lbs/ft^2 and the moments and thrusts for this uniform loading can be obtained by multiplying the influence-line area by 120.Such areas for maximum positive and negative moments,together with the corresponding area for simultaneous thrust, are shown on the influence-lines of Figs. 6 and 8. For positive moment the influence-lines areas for moment and thrust are 31.6 and 24.4 respectively. For negative moment the values are 13.3 and 16.1 . So the live-load stresses are: (at the springing the areas are 349.7 ,56.7 and 167.6 , 34.25)

for positive moment

$M_c = 31.6 \times 120 \times 2 =$ 7590 ftlbs.

$H_c = 24.4 \times 120 \times 2 =$ 5850 lbs

$M_s = 349.7 \times 120 =$ 42000 ftlbs.

$H_s = 56.7 \times 120 =$ 6800 lbs

for negative moment

$M_c = 13.3 \times 120 \times 2 =$ -3190 ftlbs.

$H_c = 16.1 \times 120 \times 2 =$ 3860 lbs

$M_s = 167.6 \times 120 =$ -20100 ftlbs

$H_s = 34.25 \times 120 =$ 4100 lbs

19.-Temperature Stresses.-

As considerable uncertainty exists regarding the exact nature of temperature changes in an arch and their relation to external temperature conditions, and as the effect of a permanent change is small hence only the seasonal changes need be considered, we will consider the direct effect upon stresses of a variation of temperature of 40°F. Such a high variation will represent the worst condition which may occur in the Syria and the Lebanon.

Taking the value of the coefficient of expansion of the concrete as $u = 0.000006$ and considering the formulas of article 8, the thrust at the crown due to temperature is :

T A B L E   F

DEAD - LOAD CONCENTRATIONS,

(Weight of an element 3.692 ft long)

$D = d/\cos a$; $F = y + 1.5 - D/2$; $P = 2F^2 + 419F + 60$ lbs for $F \angle 4$ ft.

and $P = 4F^2 + 459F + 60$ lbs for $F \searrow 4$ ft.

x	ring thickness d	cosa	D	D+ slab	P_1	D/2	1.5-D/2	y	F	$2F^2$	419F	P	$P + P_1$
1.85	1.00	1.00	1.00	1.67	924	0.50	1.00	0.02	1.02	2	427	489	1413
5.54	1.01	.998	1.01	1.68	929	0.50	1.00	.14	1.14	2	477	539	1468
9.23	1.01	.996	1.02	1.69	935	0.51	.99	.38	1.37	4	574	638	1573
12.92	1.02	.993	1.03	1.70	941	.51	.99	.75	1.74	6	728	794	1735
16.62	1.03	.987	1.04	1.71	946	.52	.98	1.26	2.24	10	938	1008	1954
20.31	1.04	.981	1.06	1.73	956	.53	.97	1.91	2.87	16	1200	1276	2232
24.00	1.05	.970	1.08	1.75	968	.54	.96	2.74	3.70	28	1550	1638	2606
										$4F^2$	459F		
27.69	1.08	.954	1.13	1.80	996	.57	.93	3.77	4.70	88	2160	2308	3304
31.38	1.14	.933	1.22	1.89	1045	.61	.89	5.06	5.95	142	2730	2932	3977
35.08	1.22	.902	1.35	2.02	1117	.68	.82	6.66	7.48	224	3440	3724	4841
38.77	1.36	.859	1.58	2.25	1243	.79	.71	8.65	9.36	350	4300	4710	5953
42.46	1.56	.802	1.94	2.61	1442	.97	.53	11.12	11.65	544	5120	5724	7166
46.15	1.84	.733	2.51	3.18	1759	1.26	.24	14.20	14.44	See	Fig 9	7466	9225

Weight of one half of the arch......47447 lbs

TABLE G

DEAD - LOAD STRESSES AT CROWN AND SPRINGING.

Load	Infl.Mc	Mc	Infl.Hc	Hc	Infl.Ms	Ms	Infl.Hs	Hs
9230	-0.01	-92	0.004	37	-1.78	-16410	0.721	6650
7170	- .09	-645	.049	351	-4.64	-33300	.749	5360
5950	- .25	-1485	.139	825	-6.63	-39450	.806	4790
4840	- .43	-2086	.270	1305	-7.69	-37300	.884	4280
3980	- .62	-2470	.444	1760	-7.70	-30700	.991	3940
3300	- .74	-2450	.640	2110	-6.92	-22850	1.106	3630
2610	- .73	-1910	.848	2210	-5.41	-14140	1.224	3180
2230	-. 56	-1250	1.057	2355	-3.44	-7700	1.336	2980
1950	-. 16	-312	1.245	2430	-1.23	-2400	1.430	2780
1740	+ .48	+830	1.411	2450	+1.13	+1960	1.506	2720
1570	+1.39	+2180	1.544	2420	+3.44	+5400	1.554	2440
1470	+2.58	+3940	1.638	2400	+5.52	+8100	1.571	2310
1410	+4.09	+5750	1.687	2370	+7.31	+10300	1.558	2190
1410					+8.72	+12350	1.509	2120
1470					+9.58	+14050	1.427	2090
1570					+10.01	+15700	1.318	2060
1740					+9.95	+17300	1.180	2050
1950					+9.35	+18200	1.023	1990
2230					+8.43	+18800	.854	1900
2610					+7.11	+18500	.677	1760
3300					+5.61	+18450	.504	1660
3980					+4.04	+16000	.347	1380
4840					+2.55	+12340	.208	1005
5950					+1.34	+7960	.107	635
7170					+0.48	+3440	.037	255
9230					+0.04	+370	.003	25

+Mc = +12700 23023

-Mc = -12700

Mc = 0

Total Hc at Crown = 2 x 23023 = 46,050 Lbs

Mc = 0

-Ms = -204250

+Ms = +199220

Ms = - 5030 Ftlbs.

Hs = 66190 Lbs.

$$Hc_t = \frac{0.000006 \times 40 \; \frac{96 \times 2000000 \times 144}{4.00 \;/\; 0.1061}}{209.24}$$

or $Hc_t = 840$ Lbs.

$Mc_t = 2.92 \times 840$ or $Mc_t = 2460$ ftlbs.

A change of temperature in the arch produces moments of opposite signs at the crown and at the springing, therefore the moment at the springing has a value of :

$Ms_t = -2460 + 840 \times 16$ or $Ms_t = 10980$ Ftlbs.

The thrust at the springing is $Hs_t = Hc_t \cos a = 840 \times 0.697$

or $Hs_t = 590$ Lbs.

We must note that for a fall of temperature, a positive moment is produced at the crown and a negative moment at the springing. For a rise of temperature, opposite effects are produced.

20.-Total Moments and Thrusts.-

In Table H are performed all the calculations giving the total stresses at the crown and the springing, due to dead-load, live-load and temperature.In making the several combinations, the temperature conditions are so chosen as to place the arch under the worst conditions that may arrive. As it may be seen in table H the most critical stresses are :

$Mc = 10050$ Ftlbs.

$Hc = 51060$ Lbs.

$Ms = 47950$ Ftlbs.

$Hs = 73580$ Lbs.

These values are a little higher than those fpund in the tentative design exept for Mc which is a little bit less.

T A B L E H

COMBINED MOMENTS AND THRUSTS

Dead-Load, Live-Load, and Temperature

	CROWN				SPRINGING			
	Positive Moment		Negative Moment		Positive Moment		Negative Moment	
	M	H	M	H	M	H	M	H
Live-Load	+ 7590	5850	- 3190	3860	+ 42000	6800	- 20100	4100
Dead-Load	0000	46050	0000	46050	- 5030	66190	- 5030	66190
Temperature	+ 2460	- 840	- 2460	+ 840	+ 10980	+ 590	- 10980	- 590
T O T A L	+ 10050	-51060	- 5650	50750	+ 47950	73580	- 36110	69700

- -

21.-Critical Fibre Stresses in the Arch Rib.-

From TURNEAURE AND MAURER's diagrams for bending and direct stresses for rectangular sections, for the crwn stresses Mc = 10050 ftlbs and Hc = 51060 lbs we get :

fc = 630 lbs/in².

fs = 750 "

fs' = 8200 "

at the springing where the stresses are Ms = 47950 ftlbs and Hs = 73580 lb

fc = 715 lbs/in².

fs = 4070 "

fs' = 9650 "

The allowable stresses on concrete and steel are assumed to be respectively 700 and 16000 lbs/in^2. For our arch the steel stresses are very safe while under the worst conditions the concrete works at 715 lbs/in^2. or approximately at the allowable stress value. Our design is therefore safe and considering that the excidingly unfavorable conditions to which we designed our arch practically do not occur we can state that the design is even more than safe.

22.-Shrinkage and Plastic Flow.-

The affect of shrinkage is to increase the stress in the steel and decrease that in concrete. The result on total distortion is a shortening of the arch rib, thus producing bending and direct stresses throughout the arch in a manner similar to a change of temperature. But this last effect is small compared with the first one. Plastic flow also increases the compressive stress of the steel and reduces compression in the concrete. In our case their combined effect is to reduce our concrete stresses and to increase the compressive stresses in the steel. This is a very desirable effect so long as the compressive stress in the steel remains below the allowable value of 16000 lbs/in^2. The maximum steel stress in the arch is 9650 lbs/in^2 compressive. Therefore shrinkage and plastic flow effects on steel must not be more than 6000 lbs/in^2. So in order to remain safe and as shrinkage and plastic flow always occur we better avoid them as much as possible.

Since the major portion of the deformations from shrinkage and plastic flow takes place at early ages, their effects may be greatly reduced by casting the arch in a series of segments with gaps of about one foot in between. These segments should age for approximately one month with careful curing before the small gaps are finally closed. Five segments will be enough for oue arch.

23.-Expansion Joints.-

As the rise and fall of the arch ring due to temperature changes are marked, special consideration must be given to expansion joints, that experience has found to be very necessary. Accordingly our arch will have four expansion joints : two over the abutments and two at the quarter points. These expansion joints will be filled withe soft asphalt.

Also the concrete road slab shall be constructed in squares of six feet sides, and shall be connected with each other through soft asphalt.

24.-Lateral Reinforcement.-

This is to prevent longitudinal cracks due to temperature, shrinkage and unequal loading durꝋng construction. We will use :

2 round bars per foot 1/2 " at the crown;
-do- 5/8" at quarter point;
-do- 3/4" at springing.

25.-Foundations.-

Foundations are beyond the scope of this thesis. It is therefore assumed that adequate foundations exist, they must be very rigid and no deformation should take place.

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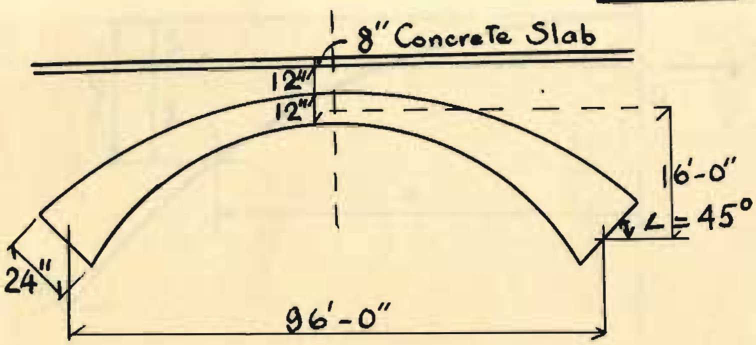
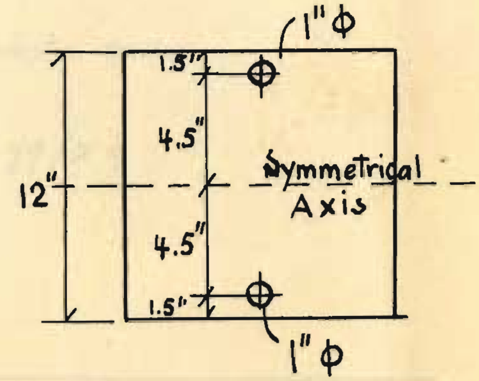
FIG. 1





Cross-Section at Crown

Unit Dead-Load at Crown : w_c

$$\frac{12+8}{12} \times 150 + 120 = 370 \text{ lbs/ft}^2$$

Unit Dead-Load at Springing : w_s

for Concrete: $\frac{24\sqrt{2}+8}{12} \times 150 = 525$

for Earth: $\left(16 + \frac{12+6-\frac{1}{2}24\sqrt{2}}{12}\right)120 = 1930$

$$\text{Total} = 525 + 1930 = 2455 \text{ lbs/ft}^2$$

$$g = \frac{w_s}{w_c} = \frac{2455}{370} = 6.63$$

Moment of Inertia

for Concrete:

$$I = \frac{bd^3}{12} = \frac{1}{12} = 0.0834$$

for Steel

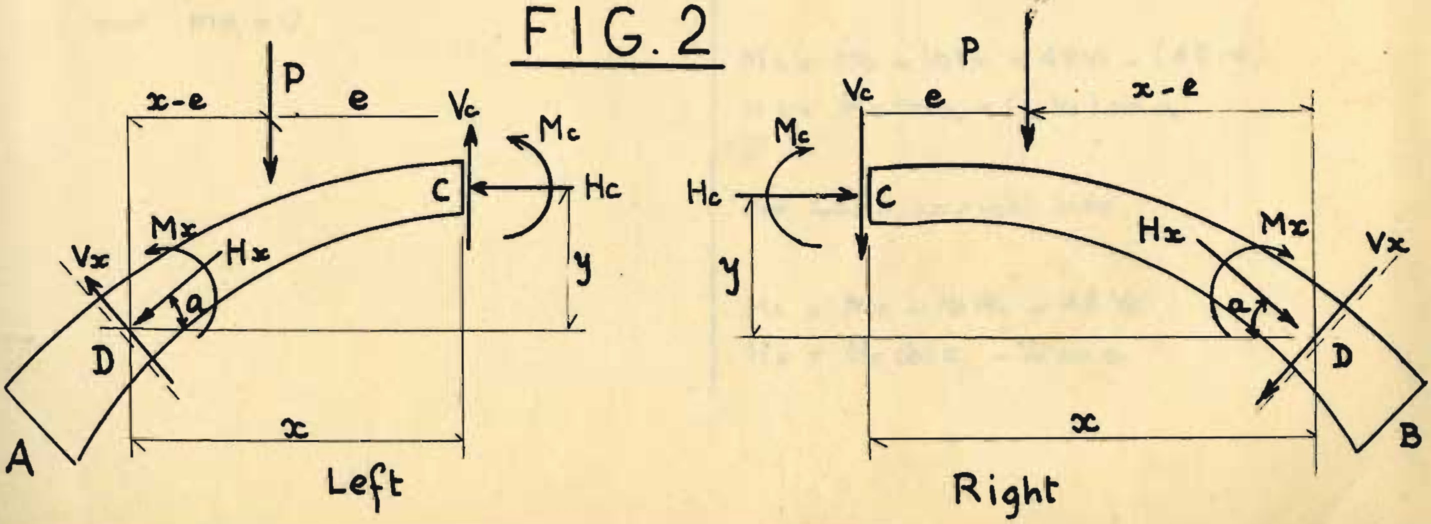
$$I = nA_s h^2 = \frac{2 \times 14 \times 0.785}{144} \times \left(\frac{4.5}{12}\right)^2 = 0.0218$$

$$\text{Total} = 0.1052 \text{ ft}^4$$

Area of the Transformed Section at Crown:

$$A = bd + (n-1)A_s = 1 \times 1 + \frac{14 \times 2 \times 0.785}{144} = 1.153 \text{ ft}^2$$

FIG. 2



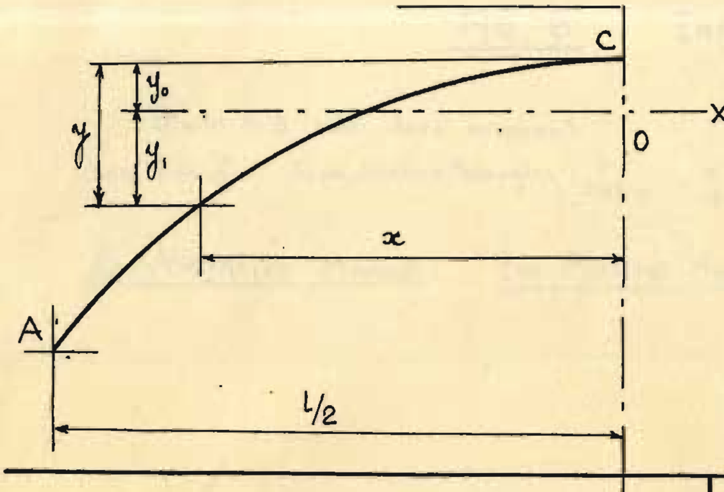
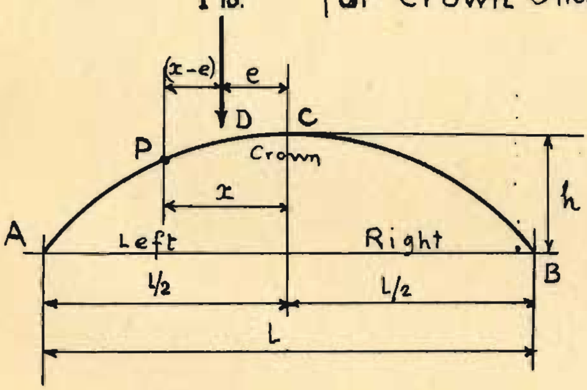


FIG. 3

O is the elastic centre

$$y_0 = \Sigma yq / \Sigma q$$

FIG. 5 Values of m_L and m_R for Crown Stresses



For points between A and D

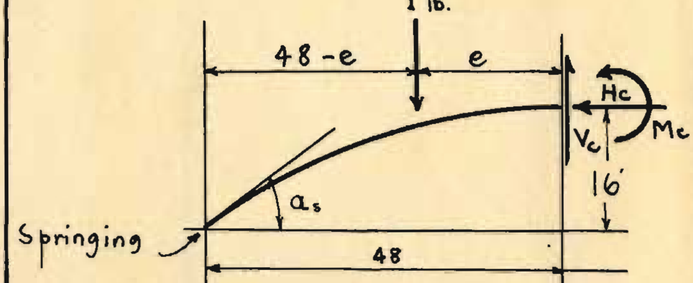
$$m_L = -(x-e)$$

For points between D and C

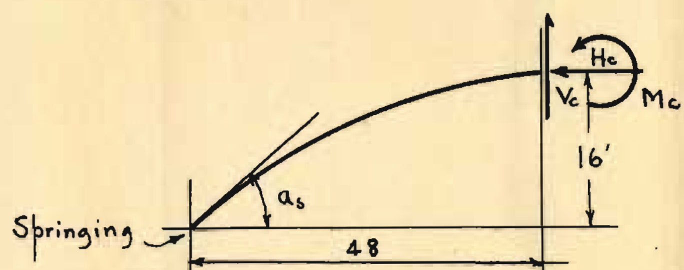
$$m_L = 0$$

and $m_R = 0$

FIG. 7 Values of Springing Stresses



Load on Left Side



Load on Right Side

For Load on left Side

$$M_s = M_c + 16H_c + 48V_c - (48-e)$$

$$H_s = H_c \cos a_s + (1-V_c) \sin a_s$$

For Load on right Side

$$M_s = M_c + 16H_c + 48V_c$$

$$H_s = H_c \cos a_s - V_c \sin a_s$$

FIG. 4 _ Properties of the Arch Axis

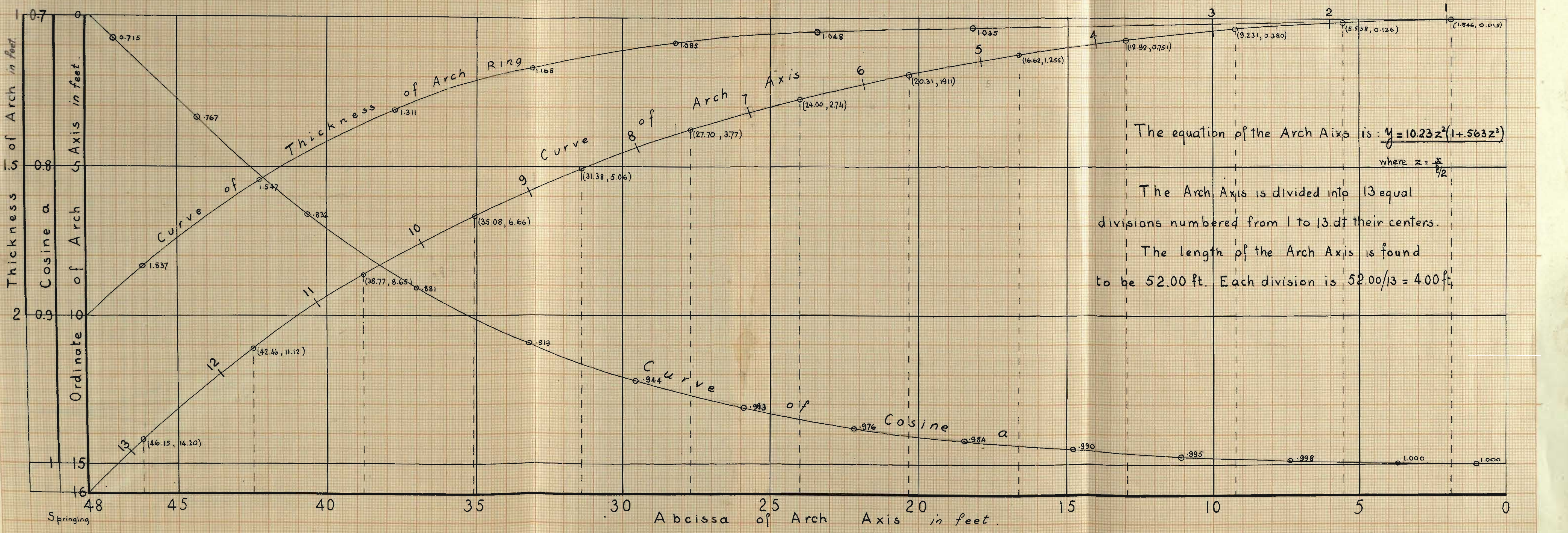


FIG. 6 Influence Lines Areas at Crown

Axis 0-0 cuts zero moment.

Calculation of r. From Similar triangles : $\frac{r}{3.69-r} = \frac{0.48}{0.16}$; $r = 2.77$; $3.69 - 2.77 = 0.92$

for Negative Moment

− 0.01
− 0.09
− 0.25
− 0.43
− 0.62
− 0.74
− 0.73
− 0.56
Σ = − 3.43

Area = $3.43 \times 3.692 = 12.67$

+ tr. OAB = $.92 \times 0.16 \times \frac{1}{2}\left(\frac{1.85+0.92}{0.92}\right)^2 = .66$

Total Area = −13.3

for Positive Moment

+ 4.09
+ 2.58
+ 1.39
+ 0.48
Σ = + 8.54

Area = $8.54 \times 3.692 = 31.5$

+ tr. OCD = $\left(\frac{.92}{2.77}\right)^2 \times \frac{2.77 \times 0.48}{2} = .1$

Total Area: + 31.6

Thrust for Positive Mom.

1.687
1.638
1.544
1.411
Σ = 6.280

Area = $6.280 \times 3.692 = 23.2$

+ $1.3 \times .92$ = 1.2

Total Area = 24.4

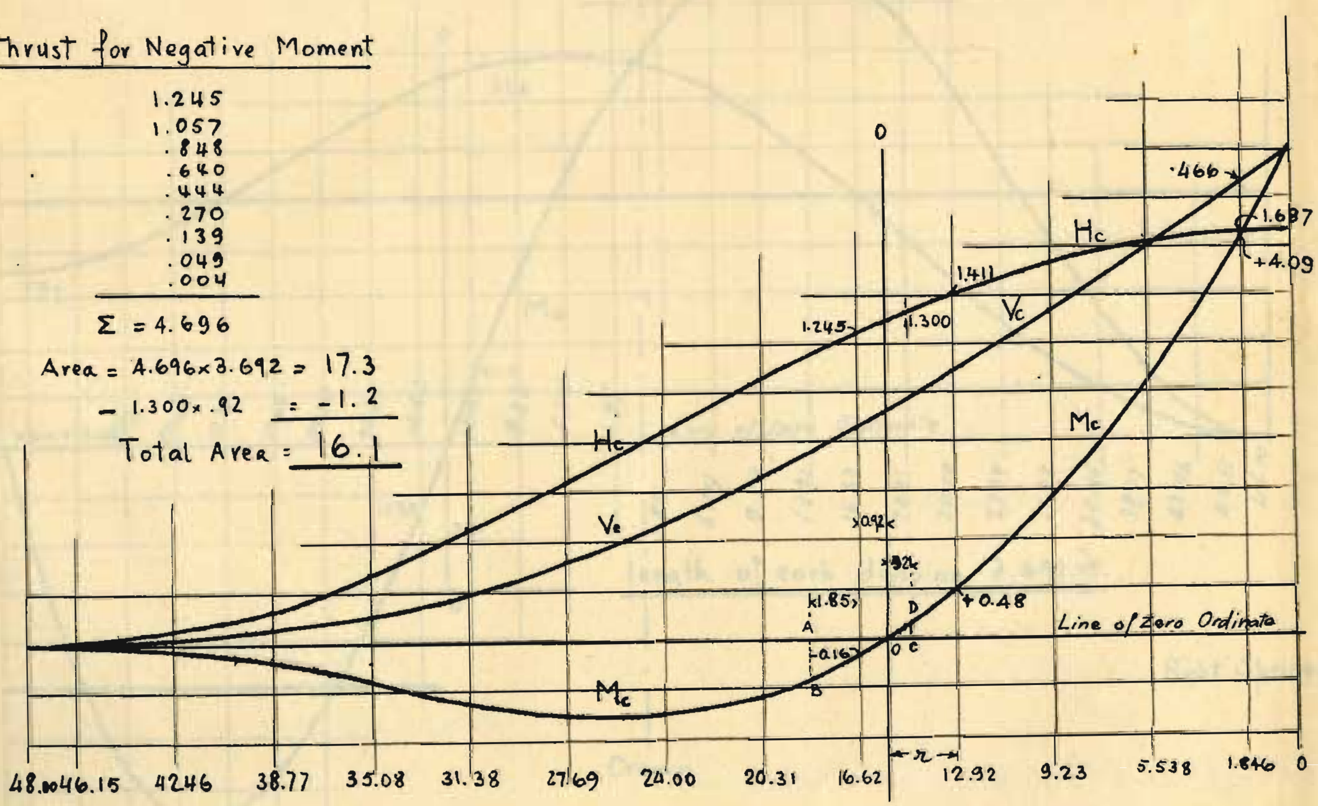
Thrust for Negative Moment

1.245
1.057
.848
.640
.444
.270
.139
.049
.004
Σ = 4.696

Area = $4.696 \times 3.692 = 17.3$

− $1.300 \times .92$ = −1.2

Total Area = 16.1



length of each division: 3.692 ft.

FIG. 8 Influence Lines Areas at Springing

Same procedure as in FIG. 6 ; $r = \frac{1.13(3.69-r)}{1.23}$; $r = 1.77$; $3.692 - r = 1.92$

Area for Neg. Mom.	For Pos. Mom.	Thrust for Neg. Mom.	Thrust for Pos. Mom.
	+ 1.13	.721	1.506
- 1.78	3.44	.749	1.554
- 4.64	5.52	.806	1.571
- 6.63	7.31	.884	1.558
- 7.69	8.72	.991	1.509
- 7.70	9.58	1.106	1.427
- 6.92	10.01	1.224	1.318
- 5.41	9.95	1.336	1.180
- 3.44	9.35	1.430	1.023
- 1.23	8.43	Σ = 9.247	.854
Σ = - 45.44	7.11		.677
	5.61		.504
	4.04		.347
	2.55		.208
	1.34		.107
	.48		.037
	.04		.003
	Σ = 94.61		Σ = 15.383

Area: 45.44 × 3.692 = 167.6
+ 0.0
Total Area = **167.6**

Area = 9.247 × 3.692 = 34.15
+ .7 × 1.47 = .1
Total Area = **34.25**

Area = 15.383 × 3.692 = **56.7**

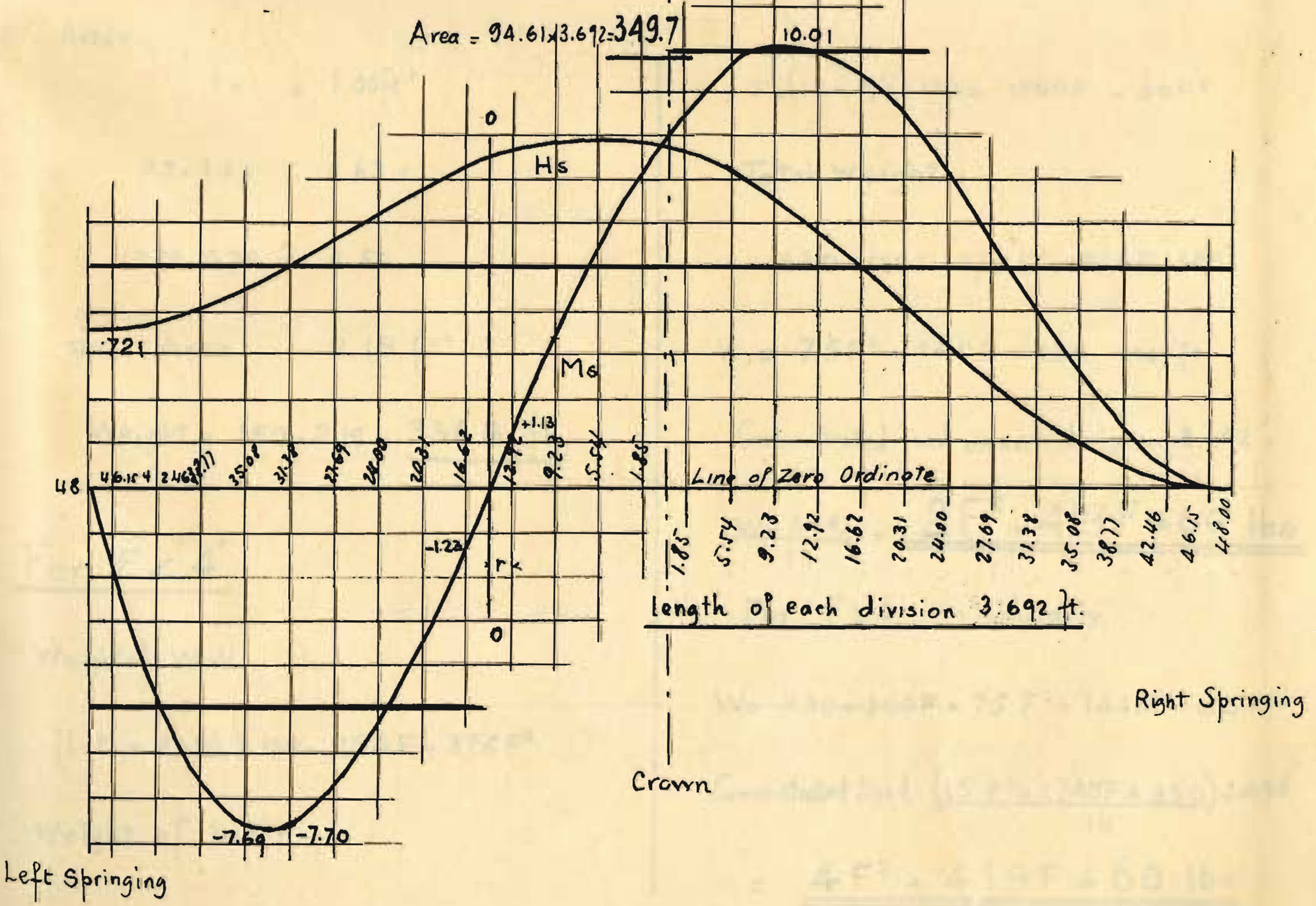
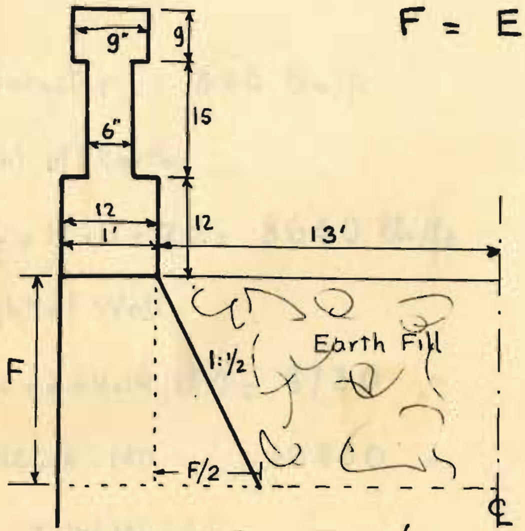
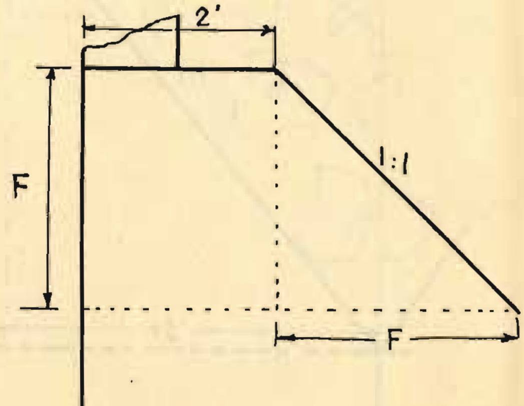


FIG. 9 Spandrel Wall Dead Weight (Sheet I)

F = Earth Fill Depth



Section for F∠4'



Section for F > 4'

Weight of Baluster:

Areas

$1 \times 1 = 1.00 ft^2$

$0.5 \times 1.25 = 0.63$ "

$0.75 \times 0.75 = 0.56$

Total Area $2.19 ft^2$

Weight $= 150 \times 2.19 =$ 330 lbs/ft.

For F∠4'

Weight of Wall :

$$\left(1 \times F + \frac{F \times F/2}{2}\right) \times 150 = 150F + 37.5F^2$$

Weight of Earth :

$$= F \times (13 - F/4)\,120 = 1560F - 30F^2$$

Total Weight :

$$330 + 150F + 37.5F^2 + 1560F - 30F^2$$

$$W = 7.5F^2 + 1710F + 330 \quad lbs/ft$$

Concentrated load on each division 3.692':

$$\frac{W \times 3.692}{14} = 2F^2 + 419F + 60 \text{ lbs}$$

For F > 4' Similarly

$$W = 330 + 300F + 75F^2 + 1440F - 60F^2$$

Concentrated Load : $\left(\frac{15F^2 + 1740F + 330}{14}\right) 3.692$

$$= 4F^2 + 459F + 60 \text{ lbs}$$

P.T.O.

(Sheet II) FIG. 9 (Contnd)

For F = 14.44 (Ref Table F for x = 46.15)

for Baluster : 330 lbs/ft

weight of Earth:

$\frac{1}{2} \times 12 \times 12 \times 120 = 8640$ lbs/ft

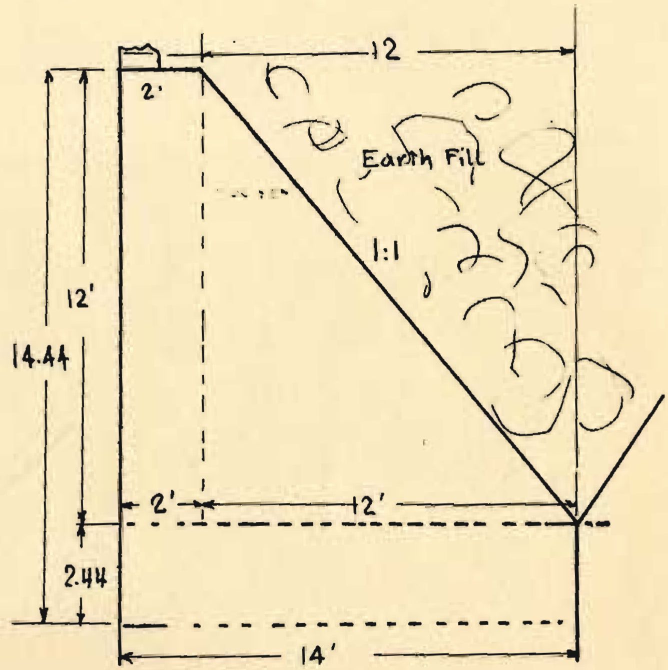
Weight of Wall:

$(2 \times 12 + 2.44 \times 14)150 = 8730$ "

$+ \frac{1}{2}\ 12 \times 12 \times 150 = 10800$ "

Total Weight

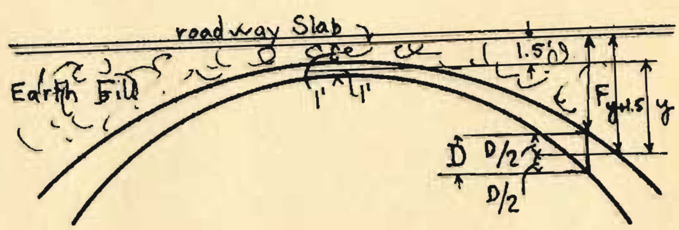
= 28500 lbs/ft



Concentrated Load:

$\frac{28500 \times 3.692}{14} = 7466$ lbs.

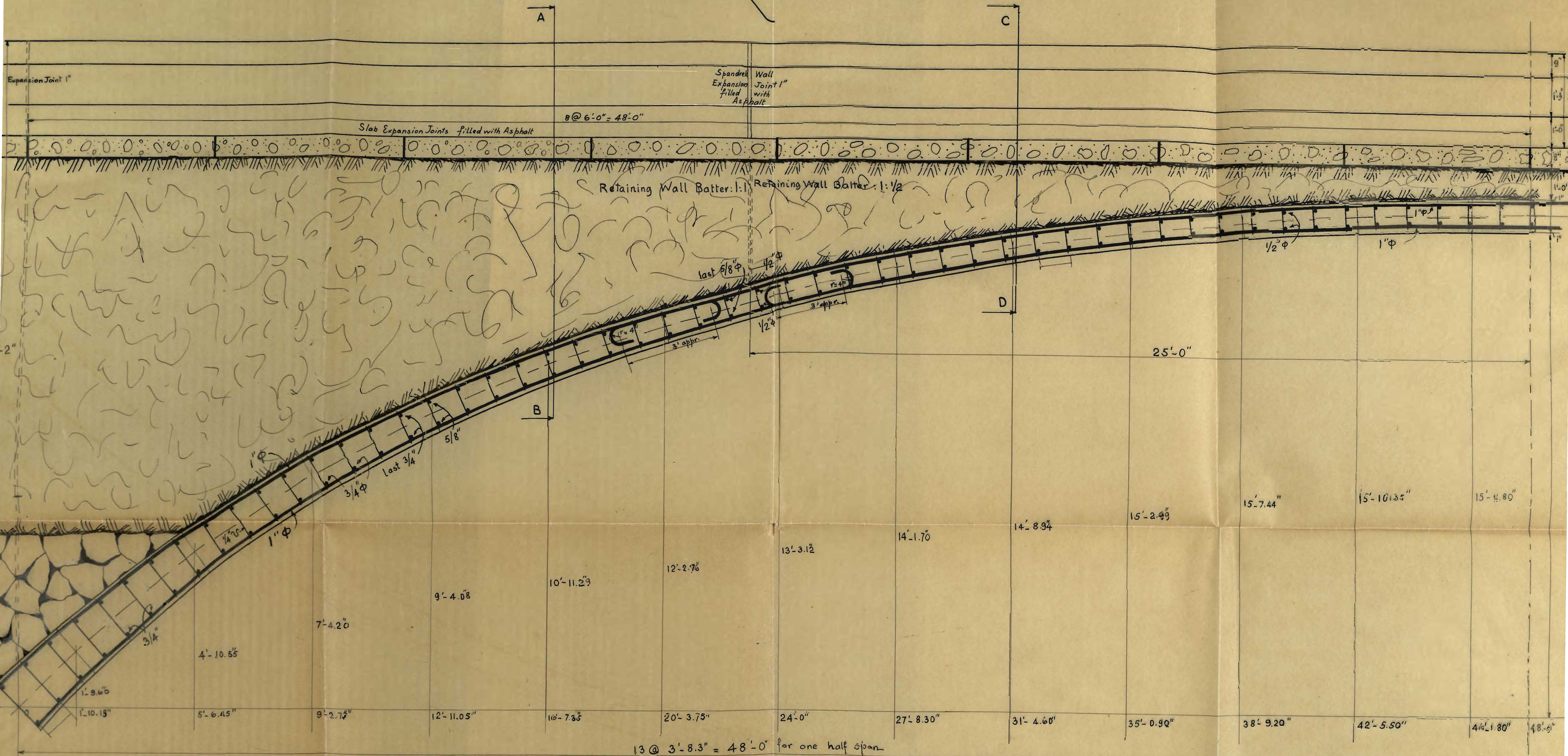
Value of F



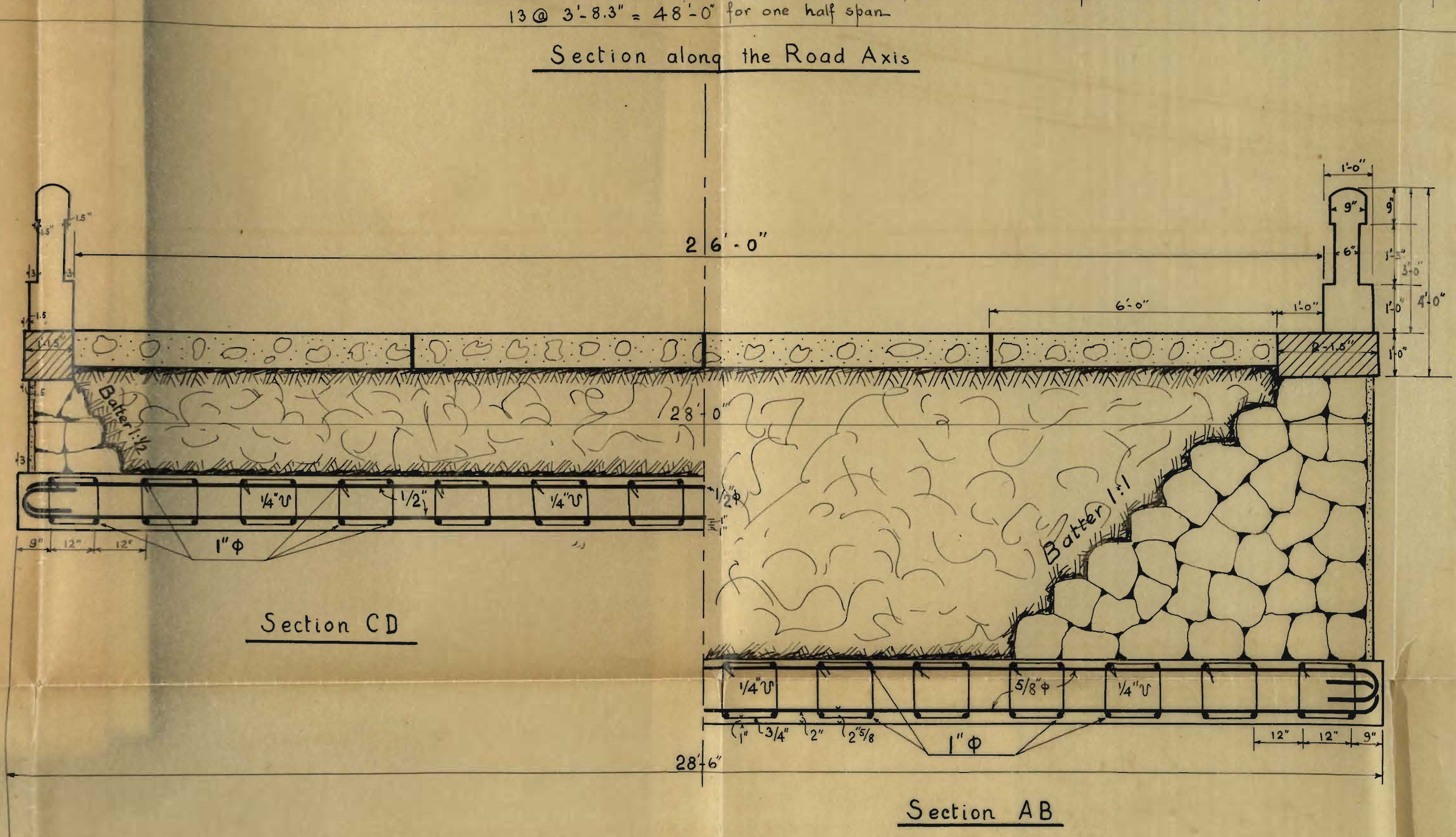
D being the vertical depth it is clear that

$F = y + 1.5 - D/2$

PLAN OF AN ARCH BRIDGE



Section along the Road Axis



THESIS

PRESENTED BY

EMILE NEMEH KLAT, B.A.

A. U. B. MAY 28, 1945