THE DETERMINATION OF SOME OF THE MAGNETIC PROPERTIES
OF A FEW COMMON KINDS OF IRON AND STONE.

by

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The material in this thesis is intended to be mainly a report of a series of experiments, performed on a number of samples of iron and steel with the purpose of obtaining their most important magnetic characteristics.

I have undertaken this series of experiments both for the great interest I have always had in the magnetic phenomena in physics, a field which, for the investigator gives the thrill of an adventure in a domain where many of the facts have not as yet crystallized into one rigid theory; and also on account of the fact that the characteristics I am after might be of help to local designers, restrained to the use of the few kinds of iron and steel available to them.

I would like to mention here a word about the importance of magnetic analysis, a branch of physical investigation which has been progressing rapidly in recent years and is proving itself to be an increasingly useful tool in the hands of the physicist, chemist and engineer, who are not only using it in their theoretical investigations, but also as a means to find the composition of alloys, in the detection of impurities in metals and also in the location of flaws in structures. I have attempted to follow in my experiments the general processes of this analysis.

I feel compelled to acknowledge here my indebtedness to Prof. P. Antippe, who suggested this line of work to me and later was of great help in clearing points in the theory and giving advice in the technique of experiments.

I also wish to express my gratitude to Prof. J. Chevrier who provided workshop facilities in the Refinery of Tripoli.
INTRODUCTION

As was already mentioned the magnetic properties are of great importance for electrical engineers; the necessary data for design could be found in handbooks and catalogues, but comparing different tables and graphs great diversity exists. The cause of it lies in the fact that most of the data are obtained from the factories which keep the manufacturing process a secret and that the small percentage of different impurities is not mentioned in the chemical composition. There is also another point; before the war nobody in this country would trouble about making, for instance, a permanent magnet of cast iron, therefore the concerning data were not necessary. It was much more economical to import products of big factories which are working in masses and with the tendency of using special alloys. When the import became difficult it is of some use to know the magnetic data for the common materials available on the local market. Therefore, samples of iron and steel that are to be found here and for which the magnetic properties have not been exactly determined, form the object of investigation in this thesis.

The samples studied were:
1. Sheet steel
2. Shaft steel
3. Structural rod
4. Soft iron
5. Cast iron

Next step after the choice of the samples was to give them the proper shape for the needs of the experiments. The main work was then to determine the following for each sample:

a) B-H curve
b) Hysteresis curve
c) Permeability
d) Retentivity
e) Coercivity
f) Preisach's equation
g) Hysteresis losses.

To arrive at this determination the methods to be described later on were used.

Having thus obtained the necessary data for each sample, its corresponding curves were drawn and constants calculated. The summary of all the results were then put down in tabular form.
An approximate determination of the magnetic properties of materials is relatively simple. There are many methods available, however, and many kinds of magnetic materials. The method to use should be determined by the purpose for which the tests are being made, the kind of material to be tested, the amount of testing to be done, and the speed and accuracy desired. No one method at present available is most suitable for all purposes. And the progress in magnetic measurements lies rather in perfecting methods already known than in developing fundamentally new ones.

As i.c.e. tests are concerned there are two quantities to be measured namely: the effective magnetising force and the resulting induction in the material.

The methods of measuring magnetising force are as follows:

1) Long solenoid
2) Solenoid with ellipsoid sample
3) Concentric air coil
4) Magnetic potential coil
5) Completely closed ferromagnetic circuit with magnetising coil.
6) Compensation methods
7) Magnetron
8) Mercury helix

The magnetic induction measurements may be divided into the following classes:

\[\text{For details see ref. No. 30, page 206.}
\]
\[\text{** Ref. No. 30, page 204.}\]
1) Magnetometric
2) Rotating ellipsoid
3) Magnetic balance
4) Traction
5) Deflecting coil
6) Rotating coil
7) Biemuth spiral
8) Polarised light
9) Ballistic or fluxmeter
10) Volt-second meter.

It will be not out of place, before coming to the detailed
description of the actually used methods, to give a short summary
of some i.e. methods and peripherals which are in common use for
the ferro-magnetic materials in medium field values.

The magnetometric method is the classical one used in early
magnetic work. A long sample is magnetised longitudinally, magnetic
poles are generated, which act on a compass needle placed at a known
distance. The induction may be calculated from the deflection of
the needle and its distance from the sample.

In the case the needle is placed equidistant from the two ends
of the sample (the two-pole method) the corresponding induction is:

\[ B = \frac{4\pi}{3} \frac{H_t g \theta}{\sin \theta} \]

where: 
- \( S \) - cross-sectional area of the test rod
- \( l \) - distance between poles
- \( H \) - horizontal component of the earth's field
- \( \theta \) - angle of deflection

when the needle is adjacent to one end of the sample it is

* Ref. No. 9, page 475.
called single-pole method and under this condition

\[ 3 \approx \frac{4N_i^2}{3} \left( \frac{1}{1 - \left( \frac{3}{s_i} \right)^2} \right) H + \theta \]

\( s \) and \( s_i \) are the distances from the needle to the pole. The
magnetometric method is very simple in principle, but being extreme-
ly susceptible to outside influences is not used for commercial
purposes. For scientific investigations it has the advantage of
absolute method, capable of giving results from the dimensions and
constants of the apparatus.

The deflecting coil method belongs to the electrodynamical group
and is best illustrated by Koepel's permameter. In this instrument
the induction is measured by means of a suspended coil in a narrow
cylindrical gap in the yoke. When the current in the coil is kept
constant the deflection of the coil is proportional to the flux in
the magnetic circuit, which consists of a semicircular yoke and the
sample. This method is disqualified for research work due to many
correction factors which depend on the nature and size of the specimen,
but is very suitable for routine testing and with standard samples
and with care in taking measurements and in applying corrections may
give results accurate to about 5%.

The traction method is based on the fact that the magnetic pull
between two pieces of magnetized material is proportional to the square
of the induction, as expressed by Maxwell's formula

\[ T = \frac{A B^2}{8\pi} \]

where

\( T \) - pulling force
\( A \) - area
\( B \) - induction

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**Ref. No. 9, page 475.**
**Ref. No. 9, page 480; No. 30, page 217; No. 1, page 231.**
**Ref. No. 1, page 266.**
A few years ago the tractive force method was extensively used and many types of permeameters were based on this principle (Thompson and De Bois). At present, however, the ballistic (or induced current) method has the preference as the most convenient and accurate one, and all the commercial permeameters are making use of it. In America the best known and since long time adopted as standard instruments for obtaining the i.e. magnetic characteristics of materials are: Burrows permeameter and May simplex permeameter. A very good description of both permeameters is to be found in Spooner Chapter XVII.

* For ballistic method see page 7.
Taking into consideration that the obtained values are for commercial purposes and because the laboratory equipment is very limited, I have chosen the ballistic method. Using the same parameter, for the sake of comparison of results, I have used the traction method for one of the samples.

I. Ballistic Method.

A. Using circular sample.

Diagram No. I
Referring to the above diagram we note that in this method which is used for the determination of a series of values of the magnetizing force $H$, and the induction $B$ for a given sample, there are in the set up two main circuits: One serving to control and measure $H$, through the relation $H = \frac{0.444N}{L}$ where: $H$ is field strength in gilberts per centimeter

$N$ - number of turns

$I$ - current in amperes

$L$ - mean length of magnetic path in centimeters;

and the other to determine the corresponding values of $B$. The $H$, as thus determined, assumes uniformity of winding and uniformity of specimen both in regard to cross-sectional area and to its magnetic properties in different parts. With regard to $B$, the galvanometer has to be calibrated; that is we must find what torque is produced by a known magnetic force.

Procedure for Testing.

a) Normal induction.

The demagnetization is first carried out. The ammeter ($A$) is now set corresponding to the desired value of $H$, the reversing switch ($SW_1$) is operated a few times (about 20) to put the sample in a cyclic condition. An observation of the deflection of the galvanometer is now made. A further set of reversals is made and the observation taken again. If the value obtained is the same in both observations it may be assumed that the demagnetization is satisfactory. If, however, the second observation gives different

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** All the apparatus in the second circuit must remain unchanged, unless the calibration is made again.
values to B the demagnetization was imperfect and has to be performed once more. The current is increased and the procedure repeated until the desired number of points on the curve have been obtained. h) Hysteresis Loop.

The magnetizing current is set for \( H_{\text{max}} \), the reversing switch operated a few times. Now the current is decreased by varying the resistance \( R_s \) and the galvanometer reading taken. The deflection is proportional to \( A-B = H_{\text{max}} - 3 \). For the points on the negative side of \( H \) one should be very careful with the direction of the current, unless a special double-throw switch is used. Each point is obtained with reference to the tip and in order to maintain the continuity of the loop it is better to return to the tip by going around the loop instead of following a minor loop by going directly back to the tip. If the minor loop was introduced, before obtaining the next point on the loop, several reversals should be made. In our case the first circuit is supplied by d.c., from a 36 volt battery. The magnitude of the current is adjusted by rheostat and measured by a Weston -M 866 No. 1545 Ammeter. The magnetizing coil consists of a primary of \( N_1 = 172 \) turns, uniformly wound all around the circular sample.

For the purpose of demagnetizing the sample after each set of readings there was introduced an a.c. (initial current \( I = 1 \) ampere, frequency = 50 cycles) controlled in magnitude through a resistance of 1400 ohms. In the case of sheet steel the a.c. due to its high frequency did not produce the complete demagnetization of the sample which had to be demagnetized by d.c. The second circuit for measuring B, consisted of a secondary coil (\( N_2 = 200 \) turns, wound on a segment of the sample) a Gibbert magnetic standard (number of turns \( n = 100 \), flux \( \phi = 33600 \) lines) and a slow
reflecting ballistic galvanometer.

The advantages of this method are as follows:

1) Absence of eddy effect
2) Freedom from the disturbance of external magnetic fields
3) Absence of joints which eliminates leakage flux

time creating very nearly an ideal magnetic circuit for the needs of the experiment. On the other hand the difficulty of the method is in the proper preparation of the sample which has to have a ring form and in winding of the primary coil which should be evenly distributed over the entire ring.

B. BALLISTIC METHOD USING THE VIBROMETER.

1) Reading total induction
2) Reading incremental induction

Diagram No. 2.
F - Permeameter, shown on diagram No. 2.

$R_1$ - Variable resistance regulated by steps

$N_1$ - Primary coil = 190 turns

$N_2$ - A search coil with a spring attached = 128 turns

Diagram No. 3.

This method is mainly different from the previous one in its use of the permeameter which sets up for straight bar samples the same closed magnetic path that was present in a circular sample. As is evident, the advantage of the permeameter is to make possible that the experiment be carried on a straight bar sample, an advantage gained at the expense of the accuracy which is now impaired due to the introduction of end effect and joints in the magnetic path. All the same, the accuracy in this case is still within the limits of commercial needs.

The procedure of testing is nearly the same, only that the total induction is measured. In this case $B$ is calculated from the current induced in the secondary coil which is suddenly removed from the field. To do this a spring is so attached that when the test rod is detached this coil is thrown out of the circuit, and the galvanometer reading taken.
In a variation of the method just described the process is
rendered less time-consuming through a procedure known as the step-
by-step reading of the induction which consists in increasing the
current flowing in the primary by steps and taking the reading of
the galvanometer after each step, a reading which represents the
total change in magnetic flux threading through the search coil,
thus not only rendering unnecessary the putting of the sample in
the cyclic condition, but also eliminating the need of repeating
the search coil each time.

II. FRACTION METHOD

Diagram No. 4

This method again uses the permeameter for the same purpose
as in the two foregoing methods. It differs from these, however,
in that the induction $B$ is now measured by the force required to
detach the sample from the yoke, as we notice on the diagram 4
the spring balance takes place of the secondary circuit. The num-
ber of lines broken is equal to $B - H$ and therefore the force is
expressed by: $F = \frac{(B-H) A}{8\pi}$ where $F$ is force in dynes, $A$ - area
of the bar; from this equation $B$ (in gausses) = $\sqrt{\frac{16\pi F (\text{in} \text{gausses})}{(1 \text{r} \text{cm})} + H}$

*Compare with the formula on page 5.*
In the field of electro-magnetism the shape, physical dimensions, previous electromagnetic history, as well as thermal and mechanical treatment to which the material under investigations has been subjected, are important factors that greatly influence the electromagnetic characteristics and constants. That is why I will here give the definite specifications describing the samples to be used.

For the sample of sheet steel in the experiment using method I A the sample had to be in a circular form for which purpose a strip of a material 370 cm. long, 2.1 cm. wide, 0.0645 cm. thick, after being thoroughly cleaned, was wound in a circular coil of 12 layers with the inner diameter 9.3 cm. and the outer diameter 11.1 cm."

Due to the strain to which the material at the edges is subjected, errors in the magnetic characteristics are produced. To reduce these errors the sample was wound of a broader strip. Thus increasing the width of the strip at the expense of the length also had the additional advantage of reducing the difference of the inner and the outer radii of the coil, and tending towards a more equal allocation of number of turns per unit of

* All the measurements of diameters done by micrometer screw, " length done by vernier.
length, between the inner and outer sides. The ratio of radial thickness to outside diameter \( \frac{t}{D} \) comes out to be \( \frac{1}{12.2} \), which is within the allowed limits of \( \frac{1}{10} \) to \( \frac{1}{8} \). For the experiment on steel, in which the permeameter was used a hollow tube was formed from the rectangular piece of material, of dimensions 19 x 4.75 cm, thickness 0.3 cm. The outer diameter came out to be 18.8 cm, thus fitting perfectly into the bore of the instrument. For the remaining 5 kinds of materials the samples were prepared in cylindrical form, 19 cm. in length; 1.6 cm. in diameter: 5- for cast iron; 2- soft iron; 2- steel, 1-structural rail. The purpose of having more than one sample of the same material was naturally to enable comparison of results. We observe that the ratio of length to diameter \( \frac{L}{D} \) has the value 12 which is large enough to render inappreciable the end effect.* All the cylindrical samples had to be given a clean polished surface offering minimum frictional resistance to the motion of the sample within the bore of the permeameter. All the samples used with the permeameter rail to be given each uniform diameter as to leave the minimum clearance between the surface of the material and the yoke, and tend to reduce the leakage flux. Having thus shaped all of the above mentioned samples, it was then necessary to demagnetize them, using the following process. An alternating current of 110 volts, \( f = 50 \) cycles, initial \( I = 0.2 \) amp. was made to flow in many turns around the sample and was then gradually decreased in magnitude to zero, leaving no detectable magnetism in them.**

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* Ref. No. 9, page 229.
** Ref. No. 50, page 61.
*** Ref. No. 2, page 44.
Magnetic flux threading through some materials gives rise to phenomena which are the result of the behaviour of the individual atomic nuclei, electrons, molecules or possibly some other elementary entity of which the material is composed. It is the behaviour of this, as yet unknown entity, which differentiates one magnetic material from another. To fully describe the magnetic properties of such materials it would be necessary to define their: 1) Normal magnetization curve which gives the relation between the normal induction $B$ and the magnetizing force $H$. A curve whose shape exhibits a striking difference in the case of ferromagnetic materials as compared with that of diamagnetic. For any given body the steepness of this curve indicates the magnetisability of the substance. This curve also shows the value of $B$ at which saturation is attained. From the straight part of the $B$-$H$ curve Faseline's equation can be determined and used in many of the mathematical discussions dealing with the magnetic properties of the material. 2) Permeability $\mu = \frac{B}{H}$, a ratio that varies with $H$, for any given value of which it gives a measure of the ease with which the elementary magnetic entities are orientated. We may mention the initial permeability $\mu_0$. The maximum permeability $\mu_{\text{max}}$ which is given by the slope of the tangent drawn through the origin. The incremental permeability $\Delta \mu = \frac{\Delta B}{\Delta H}$ for any position on a magnetization curve, or hysteresis loop. Special case of $\mu_{\text{max}}$ is the reversible permeability $\mu_r$ which is for very small $\Delta H$. 3) Hysteresis loop, many of whose characteristics are of use in classifying magnetic materials. The intercept of the $B$ axis gives the measure of the retentivity ($Br$) which is the induction of the
material when the magnetizing force has been reduced to zero. The intercept of the H-axis measures the coercive force \( H_c \) which is the demagnetizing force necessary to bring the induction in the material to zero. When \( H \) is reduced to zero from the point \( H_c \) the induction goes to the point \( c \) (diagram 6). \( OC \) is often called the elastic induction.

Hysteresis loss is proportional to the area of the hysteresis loop* and expressed by Steinmetz in the form:

\[
W_h = \frac{\pi}{4} B_n H_n \]

Where \( W_h \) = hyst. loss in ergs, per cycle, per c.c.

\( B_n \) = maximum of induction.

\( H = \) Coefficient depending on the material.

\( X = \) Exponent, the mean value of which equals 1.6

Characteristics that are less often met with include \( B_n H_n \); \( B_n H_n \); where the factors represent values that would give the maximum product; \( B_n \) = coefficient of retentivity and \( B_x \) = the specific retentivity.

4) Sily current losses are resulting from the change of magnetic flux and are proportional to the square of the induced currents and may be expressed by the formula

\[
W_o = \frac{2}{3} n^2 f^2 B^2
\]

Where \( W_o \) is the sily current loss per unit volume, \( \frac{2}{3} \) a constant depending on the material; \( n \) = form factor; \( f \) = frequency; \( B \) = induction. But those losses, although very important, were not determined for the materials under investigation and that on

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* Ref. No. 9, page 450.
** \* = 31, page 23.
*** \* = 7, page 423.
account of insinuate laboratory facilities.

5) A line of investigation, which could not be carried out due to laboratory limitations, in the study of the magnetic properties of the material after it had been subjected to thermal treatment or at a given temperature.

**Sheet steel**

To determine the above mentioned characteristics for sheet steel using method I A, the data shown on diagram No. 7 were obtained.

Dimensions of the sample on diagram No. 6

- the mean diameter \( D = 10.2 \) cm.
- cross section of material \( A = 12 \times 0.0645 \times 2.1 = 1.665 \) cm²

\[
\begin{align*}
B & = \frac{Q}{A} = \frac{53900}{1.665} = 32417.8 \\
J & = 6.745 J
\end{align*}
\]

At

\[
\begin{align*}
B & = \frac{Q}{A} = \frac{53900}{1.665} = 32417.8 \\
\text{St} & = \frac{1.665}{\text{St}} \\
\text{max} & = 378 \text{ for the magnetizing force}
\end{align*}
\]

H = 12.4 gilb / cm.

Remanence \( B_r = 5680 \) gauss

Coercive force \( B_c = 6.7 \) gilb / cm.

Froehlich's equation \( B = \frac{a}{b + H} \)

\[
\begin{align*}
a & = 13000 \\
b & = 19.6
\end{align*}
\]

Hysteresis loss \( W_h = 20000 \) ergs / cycle / cm

Steinmetz' coefficient \( \eta = 0.0118 \)

Diagram No. 8 shows the data for sheet steel using method I Bc.

Outer diameter \( d_0 = 1.56 \) cm
Sheet steel

Diagram No 7
Sheet steel tube sample

Diagram No. 8.
inner diameter $d_1 = 0.96$ cm
length $l = 19$ cm
Mean area $A = 1.14$ cm$^2$

$H = \frac{0.47}{1.14} \cdot \frac{180}{0.3} = 22.6$ J

$B = \frac{1}{\mu_0} = \frac{33300 \cdot 10^6 \cdot 4}{1.19 \cdot 125 \cdot 30} = 22550 \frac{A}{\text{m}}$

$M_{\text{max}} = 435$ for $H = 16.3$ gilb / cm
$B_r = 3500$ gauss
$B_0 = 4.6$ gilb / cm.
$B_{al} = 773$ gauss
Persi's equation $B = \frac{14360 \cdot H}{20 + H}$

Magnet. loss $W = 22800$ ergs / cycle / cm.
Strixes' coeff. $\eta = 0.03526$

**Structural Rod, Sample No. 1**

Dimensions: $d = 18.52$ mm, $l = 19$ cm.

$A = 1.36$ cm$^2$

$H = 28.6$ J

$B = 13320 \frac{A}{\text{m}}$

From diagram No. 9 the following is deduced

$M_{\text{max}} = 487$ for $H = 9$ gilb / cm.

$B_r = 4050$ gauss.

$B_0 = 5.2$ gilb / cm.

$B_{al} = 745$ gauss

* The Mean area was obtained by dipping the tube in water and calculated from the water displaced.
Diagram No. 9.

Structural rod sample № 1.
\[ a = 19000 \]
\[ b = 25 \]
\[ W_a = 28500 \text{ ergs / cycle / cc} \]
\[ \eta = 0.00697 \]

**Soft Iron**

Sample No. 2  \( d = 18.83 \text{ mm} \)
\[ A = 1.94 \text{ cm}^2 \]

Constants obtained from diagram No. 10

\[ J_{\text{max}} = 478 \quad \text{for} \quad H = 4.1 \text{ gilb / cm} \]
\[ B_r = 3600 \text{ gauss} \]
\[ H_0 = 5.1 \text{ gilb / cm} \]
\[ a = 13750 \]
\[ b = 32.5 \]
\[ W_a = 31600 \text{ ergs / cycle / cc} \]
\[ \eta = 0.00685 \]

Sample No. 3  \( d = 18.86 \text{ mm} \)
\[ A = 1.946 \text{ cm}^2 \]

Diagram No. 11a.

\[ J_{\text{max}} = 505 \quad \text{for} \quad H = 18.06 \text{ gilb / cm} \]
\[ B_r = 4000 \text{ gauss} \]
\[ H_0 = 5.1 \text{ gilb / cm} \]
\[ a = 23075 \]
\[ b = 30 \]
\[ W_a = 20850 \text{ ergs / cycle / cc} \]
\[ \eta = 0.00445 \]
Soft iron sample No. 2.
Diagram No. 41a

Soft iron
sample No. 3.

[incremental reading of induction]
For comparison from diagram No. 11b we have:

\[ \frac{V}{V_{\text{max}}} = \frac{500}{4200} \quad \text{for} \quad \frac{H}{H_{\text{max}}} = \frac{15}{15} \text{ gilb/cm.} \]

\[ B_{r} = 2000 \text{ gausses} \]

\[ H_{0} = 4.0 \text{ gilb/cm.} \]

\[ a = 28.28 \]

\[ b = 31 \]

The traction method, as shown on diagram No. 11c, gives very different results due to the difficulty of taking readings:

1) motion of weight

2) effect of any, even momentary, change of the current.

For these reasons instead of theoretical accuracy of measuring the force (it should be possible to detect the difference of force = 6 gr. which corresponds to \( \Delta B = 200 \) lines of force) it was possible only to distinguish force of 60 gr., equivalent to

\[ 48 = \frac{1}{3} 738 \text{ gausses.} \]

Using the galvanometer one can read as accurate as \( 1 = 0.06 \text{ cm.} \) therefore \( \Delta B = 75 \text{ gausses.} \)

The traction method was not used for any other sample as being inconvenient and giving not truthful results.

**Shaft Steel**

Sample No. 4 Diagram No. 12

\[ i = 15.95 \text{ cm} \quad A = 1.986 \text{ cm}^2 \]

\[ \frac{V}{V_{\text{max}}} = \frac{370}{4200} \quad \text{for} \quad \frac{H}{H_{\text{max}}} = \frac{21.7}{21.7} \text{ gilb/cm.} \]

\[ B_{r} = 4000 \text{ gausses} \]

\[ H_{0} = 6.4 \text{ gilb/cm.} \]

\[ B_{el} = 890 \text{ gausses} \]
Soft iron
sample №3.
Total reading of induction.
Soft iron
sample No. 3.
raction method.

Diagram No. 11c.

- normal induction
- decreasing current
- increasing

$H = 123 \text{ Gm}/\text{cm}^3$

$J = 100$

$B = 2000$
Shaft steel
sample №4.
\[ a = 17960 \]
\[ b = 26.9 \]
\[ W_n = 32200 \text{ ergs / cycle / cc} \]
\[ \eta = 0.0073 \]

Sample No. 5  Diagram No. 13
\[ a = 15.66 \text{ mm} \quad A = 1.98 \text{ cm}^2 \]
\[ \beta_{max} = 638 \quad \text{for } H = 6.0 \text{ gilb / cm} \]
\[ B_x = 3270 \text{ gauss} \]
\[ H_0 = 5.12 \text{ gilb / cm} \]
\[ B_{\text{al}} = 396 \text{ gauss} \]
\[ a = 15800 \]
\[ b = 28.2 \]
\[ W_n = 28400 \text{ ergs / cycle / cc} \]
\[ \eta = 0.00474 \]
Cast Iron

From diagrams XI, 14, 16, 17, 19, the following characteristics were obtained.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>8-10</th>
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<tbody>
<tr>
<td>Diameter d in mm</td>
<td>16.48</td>
<td>16.95</td>
<td>17.06</td>
<td>15.96</td>
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<td>Cross section A in cm²</td>
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<td>1.965</td>
<td>1.965</td>
<td>1.966</td>
<td>1.966</td>
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<td>Max. permissible J max</td>
<td>170</td>
<td>160</td>
<td>160</td>
<td>135</td>
<td>130</td>
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<td>14.6</td>
<td>15.6</td>
<td>11.6</td>
<td>16.6</td>
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<td>3350</td>
<td>3300</td>
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<td>3400</td>
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<tr>
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<td>11.3</td>
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<td>11.5</td>
</tr>
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<td>p³ in g/cm³</td>
<td>832</td>
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<td>840</td>
<td>1170</td>
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<td></td>
<td>14300</td>
<td>14000</td>
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<td></td>
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<td>67.78</td>
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<td>Syntaxis loss</td>
<td>19080</td>
<td>22278</td>
<td>27080</td>
<td>24300</td>
<td>25000</td>
</tr>
<tr>
<td>Stress' σ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress' σ</td>
<td>0.0107</td>
<td>0.0124</td>
<td>0.0148</td>
<td>0.0158</td>
<td>0.0182</td>
</tr>
</tbody>
</table>

b) The numbers for σ are of value only for the significant digit.
Diagram No. 13.

Shaft steel
sample No. 5.
Cast iron
sample No. 6.

Diagram No. 14.

H = 12.8 gilbf/sqm.
J = 100
S = 2000 gauss.
Cast iron sample № 7.
Cast iron

sample № 3-10

Diagram № 18.
The accuracy of the results depends on the instruments used and is affected by errors introduced by the method itself and by the external disturbances.

The apparatus consists of an ammeter the accuracy of 2%, the sensitivity of the electric galvanometer is much more than ample; as for the magnetic standard the accuracy of 0.5% was assumed, because there was no means to check the permeability of the field magnet which might be changed due to aging. The permeability used due to its construction is also a source of big and uncertain error; namely

1) Loss in the reluctance of the yoke
2) Leakage flux through air and joints
3) Residual magnetism in the yoke
4) Induced currents in the yoke
5) Skin effect.

All this could be eliminated if the apparatus would be provided with a number of standard samples, thus to determine the constant of the permeability for any kind of material, and for any value of the magnetizing force, because as we know the errors are not constant throughout the whole range of B.

Now let us go over some of the possible disturbances. As concerning the current the voltage of the battery was not very stable, and often two successive readings of B, for the same value of the current were not equal; this is due to small hysteresis loop superimposed on the main one. Another difficulty was in taking the simultaneous readings of the ammeter and galvanometer. The zero of the galvanometer was nearly stable, only affected by

* Ref. No. 20, page 914.
the leakage flux through keys and by the vibrations due to the trans passing by. The experiments were carried out under the assumption that the sample had been completely demagnetized and no strains present, but actually we do not know to what extent the machining operation will alter the magnetic properties. The condition of the surface was supposed to be perfect, but a thin layer of rust possibly present, is not only increasing the reluctance of the joints enormously, but also changing the uniform distribution of flux over the cross-section of the sample. The projecting ends of the sample may not be taken under consideration, according to Sprower, for points below the knee of induction curve they do not give any noticeable effect, for points above the knee the length of the sample projecting might produce a slight positive error in the magnetizing force. The effect of variation of temperature can be omitted, because these fluctuations are rather small. The principal error is also of some importance. All the calculations were made on the slide rule and by the counting machine.

Some of the mentioned errors can be approximately calculated, but a number of them is undetermined. To find the approximate value of the reluctance of the yoke an additional experiment was performed in order to obtain the permeability of the material of the yoke at low flux densities.

For a rough calculation of the reluctance of the yoke, the permeability, thus obtained, is accurate enough. The percentage error produced by the reluctance of the yoke as compared with the reluctance of the sample, will be different at different flux densities and for each kind of material. To give the idea I have

* Ref. No. 36.
The computation for 3 points namely for $B = 4000$ gauss, $B = 6500$ gauss and $B = 10000$ gauss and for one of the samples of maste steel, soft iron, shaft steel and cast iron.

Diagram No. 15.

<table>
<thead>
<tr>
<th>% Error due to the Reluctance of the Yoke</th>
<th>Steel</th>
<th>Soft Iron</th>
<th>Shaft Steel</th>
<th>Cast Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Density $B$ in Gausses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>3.0%</td>
<td>9.1%</td>
<td>6.84%</td>
<td>3.07%</td>
</tr>
<tr>
<td>6500</td>
<td>3.4%</td>
<td>7.68%</td>
<td>4.3%</td>
<td>2.07%</td>
</tr>
<tr>
<td>10000</td>
<td>5.88%</td>
<td>4.96%</td>
<td>5.12%</td>
<td></td>
</tr>
</tbody>
</table>

This error is a systematic one and always negative, that means that part of the magnetizing force is used up to overcome the reluctance of the yoke.
The reluctance of the joints plays an important part too, but even approximately it is impossible to account for it because the condition of the contact between the sample and the yoke is unknown and at that place the distribution of the flux is not uniform.

The part of the leakage flux through air between the sample and the coil is expressed in the formula:

\[ B_{\text{correct}} = B_{\text{observed}} - K \delta \]

where \( K = \frac{A}{a} = 0.106 \)

\( A = \) cross section of the coil

\( a = \) " " sample.

For \( K = 1000 \) gauss, this correction is less than 0.1%. The residual magnetism of the yoke gives very small change of the galvanometer zero, this was noticeable only for low values of \( B \).

The demagnetizing effect of the edges of the sample depends upon the ratio of the length to the diameter, according to Sporer, the so-called coefficient of shape or demagnetizing coefficient \( N \) for \( \frac{L}{d} = 12 \) was found to be 0.01** and in that case \( K = B_{0} - 0.01 \)*** is not much affected by it. This correction becomes important in the case if materials of high permeability, scaling only with low permeability materials this factor does not influence much the readings and may not be taken into consideration.

It might be of some interest to have the statistical distribution of errors. For this reason, 30 readings of \( B \) were taken for a constant value of the current. This has been done for 3 different values of \( B \) for one of the cast iron samples.

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* Ref. No. 4.
*** Ref. No. 1, page 62.
$\sigma$ - deviation from the mean.
$f$ - frequency

Diagram No. 20.

for $B = 800$ gauss

Diagram No. 21.

for $B = 2000$ gauss
For \( B = 6500 \) gauss.

Diagram No. 22

These diagrams show the accidental error which represents the reproducibility of results.

The percentage error has been calculated for the flux density of 500, 2000 and 6500 gauss.

The percentage error as calculated.

<table>
<thead>
<tr>
<th>Flux Density</th>
<th>from the mean deviation.</th>
<th>from the max positive deviation.</th>
<th>from the max negative deviation.</th>
<th>from the most probable deviation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>7.33 %</td>
<td>4.17,0</td>
<td>-22.4</td>
<td>+4.9</td>
</tr>
<tr>
<td>2000</td>
<td>2.26</td>
<td>4.765</td>
<td>-8.8</td>
<td>+2.12</td>
</tr>
<tr>
<td>6500</td>
<td>0.59</td>
<td>1.96</td>
<td>-1.77</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

* The symmetry of the curve is indicated by the extreme values.
As we can see it is impossible to state the final accuracy of the results, there are many sources of errors, some of them big, others negligible. A table of mean values may be of some use, which I am including below.

<table>
<thead>
<tr>
<th></th>
<th>Sheet steel</th>
<th>Shaft steel</th>
<th>Structural rod</th>
<th>Soft Iron</th>
<th>Cast Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum permeability</td>
<td>408</td>
<td>432</td>
<td>487</td>
<td>492</td>
<td>178</td>
</tr>
<tr>
<td>Hyst. loss in ergs/cycle/deg</td>
<td>21,400</td>
<td>27,300</td>
<td>22,000</td>
<td>21,275</td>
<td>24,135</td>
</tr>
<tr>
<td>Remanence in gauss</td>
<td>4,420</td>
<td>3,600</td>
<td>4,000</td>
<td>3,600</td>
<td>5,266</td>
</tr>
<tr>
<td>Coercive force in gilb/mm</td>
<td>5.56</td>
<td>5.76</td>
<td>8.2</td>
<td>6.1</td>
<td>10.85</td>
</tr>
<tr>
<td>$B \times H_c$</td>
<td>25,030</td>
<td>20,736</td>
<td>20,800</td>
<td>19,350</td>
<td>35,328</td>
</tr>
<tr>
<td>Freundlich's $a$</td>
<td>13,650</td>
<td>18,376</td>
<td>19,000</td>
<td>20,325</td>
<td>13,950</td>
</tr>
<tr>
<td>Const. $b$</td>
<td>19.75</td>
<td>27.56</td>
<td>25</td>
<td>51.25</td>
<td>58.31</td>
</tr>
<tr>
<td>Steinmetz' coeff.</td>
<td>0.00594</td>
<td>0.00809</td>
<td>0.00588</td>
<td>0.00575</td>
<td>0.01111</td>
</tr>
</tbody>
</table>
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