Ion Motion In Rotating Liquid Helium II

By

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ABSTRACT

According to the theory of Onsager and Feynman, rotating liquid helium II should contain an array of vortex lines arranged parallel to the axis of rotation and approximately evenly distributed throughout the liquid.

Ions produced in liquid helium by ionization with alpha-particles have been exploited as microscopic probe particles to investigate the existence and properties of vortex lines in rotating liquid helium II.

The results obtained are qualitatively in agreement with those obtained in similar experiments. This is true namely in the dependence of the ionic current on temperature, voltage and angular velocity.

Moreover, measurements of ionic current for different axial as well as radial electric fields is believed to provide further evidence in favour of the Onsager - Feynman vortex line model.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Plate Page

SECTION (1)

INTRODUCTION

I. Liquid Helium II and the Two-Fluid Model.

Helium differs from other substances in that it is liquid even down to absolute zero. Classicaly at absolute zero all motion stops, but quantum mechanically this is not so. In fact the most mobile substance known is one at absolute zero.

At 2.18^{o}K, the lambda-point, liquid helium undergoes a transition which may be demonstrated in the following way. Liquid at the boiling point can be cooled by reducing the pressure above its surface with a vacuum pump. During this pumping the high temperature phase liquid helium I, boils vigorously and is clouded by a mass of small bubbles, but suddenly, as soon as the transition temperature is reached, the boiling stops abruptly and the low temperature phase, liquid helium II, appears as a transparent liquid which does not boil anymore. One of the unusual properties of liquid helium II is a very high thermal conductivity and this is the cause of the absence of boiling.

One of the most useful ideas in interpreting the behaviour of the liquid is the two fluid model. One might picture the helium as a background fluid in which excitations move. At absolute zero one has a perfect ideal fluid which may flow frictionlessly with potential flow. If heated, the heat energy excites the liquid. This it does by creating within it excitations of some sort. These excitations can make their way from one place to another, collide with the walls and with each other, and give to helium some properties associated with the so-called normal fluid component, such as viscosity. These excitations are believed to be of two kinds namely, phonons and rotons.

II. Feynman Theory.

The basic available quantum mechanical treatment of liquid helium is that presented by Feynman.[1] The wave function for states representing uniform motion of the superfluid he gives as:

$$\psi = \left[\exp i\vec{k}\cdot(\Sigma_i R_i)\right] \Phi \quad . \; . \; . \; . \; . \; . \; . \qquad (1)$$

where Φ is the ground state wave function. Whereas for the pure superfluid helium flowing he gives the equations:

$$\psi_{flow} = \left\{\exp i \, (\Sigma_i \; S \; (Ri)\right\} \Phi \quad . \; . \; . \; . \; . \qquad (2)$$

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[1] R.P. Feynman, "Application of Quantum Mechanics to Liquid Helium," Progress in Low Temperature Physics, ed. C.G. Gorter, (New York: Interscience Publishers Inc., 1955), I, 17-53.

where the local superfluid velocity is given by

$$V = \hbar m^{-1} \vec{\nabla} S \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

because $\hat{P} = (\hbar/i)\nabla$, $\hat{P}\Psi = P\Psi$ eigenvalue equation,

therefore $P = mv = \hbar \nabla S$, which gives equation (3),

and h is Planck's constant while m is the atomic mass of helium. Equation (3) implies that the motion is irrotational, that is $\nabla \times V_s = 0$. In the case of a circuit enclosing a hole the circulation must be an integral multiple n of a quantized unit $2\pi\hbar/m$, so that Ψ_{flow} will be single valued. This could be expressed as:

$$\Delta S = \oint \vec{\nabla} S \cdot d\vec{r} = n . 2\pi$$

or $\oint \vec{\nabla} s \cdot d\vec{r} = 2\pi\hbar m^{-1}.n = 2\pi n. \; 1.5 \times 10^{-4} \; cm^2/sec.$

If the superfluid must flow irrotationally, at first sight, it cannot lose energy, unless it is moving very rapidly because any loss of energy is a result of the entire fluid changing its velocity.

The only way that gross slowing down can occur for lower velocities is for small parts of the fluid to stop or slow down without the entire fluid having to slow down at once. That is energy loss must be accompanied by flow which is not irrotational; that is, flow which involves local circulation. To understand this effect we must add a new element to our picture of phonons and rotons. These are the quantized vortex lines suggested by Onsager[2].

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[2]L. Onsanger, Nuovo Cimento , vi, Supp. 2, (1949), 249.

III. Quantization of Circulation.

A number of authors (H. London 1946, Onsager 1949, F. London 1954, Landau and Lifshitz 1955) have suggested that macroscopic rotation of the superfluid could be achieved despite the condition $\mathrm{Curl}\ V_s = 0$ if a few small regions of concentrated vorticity (either vortex sheets or vortex lines) were permitted. The quantization condition: $\oint \vec{v}_s \cdot d\vec{r} = nh/m$

resulting from $\Delta S = \oint \vec{\nabla} S \cdot d\vec{r} = 2\pi n$

$$\oint \vec{\nabla} S \cdot d\vec{r} = \frac{m}{\hbar} \oint \vec{v} \cdot d\vec{r} = 2\pi n \qquad \therefore \oint \vec{v}_s \cdot d\vec{r} = \frac{nh}{m} \qquad (4)$$

was also suggested by some of these authors.

Feynman (1955), in the article cited previously[3], gave a satisfactory theoretical argument for the validity of equation (4) in liquid helium, and a fairly detailed quantitative exploration of the possible observable consequencies of quantized circulation:

IV. Quantized Vortex Lines.

The argument which Feyman uses to show the existence of quantized vortex lines, is that it is the state of lowest energy for liquid helium in a rotating container. He showed that the energy of the liquid can always be reduced if more vortices form. Thus this is the lowest existing state but there is a limit to which the energy could be reduced because there is a limit to which the number of these vortices could be increased. Due to the quantization of the vortex strength,

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[3] R.P. Feynman, Op. Cit.

the smallest vortex has circulation $2\pi\hbar m^{-1}$. The lowest energy results if a large number of minimum strength vortex lines form throughout the fluid at nearly uniform density. The lines are all parallel to the axis of rotation. Since the curl of the velocity is the circulation per unit area, and the curl is 2W, there will be $2mW/2\pi\hbar = 2.1 \times 10^3$ W lines/cm^2 with W, the speed of rotation, in radians per second. This result is obtained as follows:

$$2\omega = \frac{\iint \vec{\nabla} \times \vec{v} \cdot d\vec{A}}{A} = \frac{\oint \vec{v} \cdot d\vec{r}}{A} = \frac{nh}{m} \quad \therefore \quad n = \frac{2\omega m}{h}$$

|H?

However, Feynman mentions that it is not self-evident that there is no state of appreciably lower energy. He also points out the fact that Onsager arrived at the same results working independently. Nevertheless, the vortex line model offers an explanation of the various observed properties of rotating liquid helium II among which is the motion of changes on the study of which this work has been based.

V. Experimental Evidence in Favour of the Vortex Line Model.

We have seen above how Onsager (1949) and Feynman (1955) have proposed that in uniformly rotating helium II, the superfluid contains a regular array of quantized vortex lines parallel to the axis of rotation, thus immitating classical solid-body rotation very closely. Experiments on the attenuation of second sound in rotating helium (Hall and Vinen)[4] provide considerable

[4]H.E. Hall and W.F. Vinen, "The Rotation of Liquid Helium II", Proc. Roy. Soc., A238 (1956), 204-214.

support for this model. Hall and Vinen investigated the propagation of second sound in uniformly rotating resonators filled with liquid helium II. They found that the velocity did not change by more than 0.1%. However they observed an extra attenuation due to rotation which, except near the λ -point, is proportional to ω, independent of second sound amplitude, and independent of the frequency in the range 1.5 to 4.5 KC/sec.

The results were interpreted in terms of a mutual friction force due to the collision of the excitations constituting the normal fluid with quantized vortex lines, the density of which is proportional to the angular velocity.[5]

VI. Pellam's Interpretation of Second Sound Experiments.

Pellam (1962) published an article[6] in which he explains the attenuation of second sound in rotating liquid helium II without explicitly assuming the existence of quantized vortex lines. He summarizes his main assumptions as follows:

> The present approach ignores the role of any vortex scheme in explaining absorption, but relies instead upon the over-all Feynman concept of rotating

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[5] Ibid., P.215-238.

[6] J.R. Pellam, "Theory of Second Sound Absorption in Rotating Helium," Physical Review Letters, VII (October, 1962) 281-283.

liquid helium. Losses are attributed to interactions between the second sound waves and the local irrotational regions of superfluid implicit to that region.

The treatment requires one assumption regarding the nature of rotating liquid helium, the existence (following Feynman) of local regions or domains within which the condition $\vec{\nabla} \times \vec{V}_s = 0$ holds. Shape (nearly) and size of these domains remain unimportant, as does also the nature of the "Flaws" or vorticity regions separating them.[7]

Thus Pellam explains the results of second sound attenuation without giving any argument for or against vortex lines or any of their suggested properties, except that certain "flaws" must exist. This shows that the attenuation of second sound could be explained simply by assuming certain irrotational regions whose nature and properties have to be studied by other experiments. We believe studies of ion motion in helium II will help provide such information. Our work is based mainly on such an experiment performed by Careri, McGormick and Scaramuzzi.[8]

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[7] Ibid., P. 281.

[8] G. Careri, W.D. McGormick and F. Scaramuzzi, "Ions in Rotating Liquid Helium II," Physical Rev. Letters, I (April, 1962), 61-63.

VII. Careri's Experiment.

The observations, results and conclusions of this experiment are best summarized by the following quotation from Careri's paper:

> We have measured the voltage-current characteristics for a beam of thermal helium ions passing through rotating helium II. It has been found that the rotating helium is strongly anisotropic only to the passage of the negative ions. When the current was parallel to the axis of rotation its value for a given applied voltage was independent of whether or not the helium was rotating. When the current was perpendicular to the rotational axis, however, it was strongly attenuated during rotation, the decrease depending on the rotational velocity, the applied voltage and the temperature.[9]

As an explanation of the above results, Careri writes:

> The only possible explanation for the observed effects caused by rotation seems to lie in the establishment of "traps" in the rotating helium, that is, a certain number of preferred positions in which the ions, would be trapped and withheld from the general drift motion. The meantime for which an ion is trapped must at least be of the order

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[9] Ibid., P. 61.

of the time taken for an ion to drift from one electrode to the other, a very long time on a molecular scale.[10]

These traps are identified with vortex lines in the Feynman - Onsager theory of rotation of helium II.

Careri's work, however, was incomplete in the sense that it left unanswered a number of questions which might be answered by a more thorough investigation of the same problem. Some of these questions are:

1. Dependence of trapped ion density on W and T.
2. Possibility of drift of trapped ions along the vortex lines.
3. Time constants of the attenuation of negative ion currents effect.

Our plan being to repeat, verify, and extend Careri's experiment to answer question he left unanswered, an apparatus was set up which realizes experimental conditions similar to those under which Careri performed his experiment. The next section presents a brief description of the apparatus and instruments used in this work.

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[10] Ibid., P. 62.

SECTION (2)

DESCRIPTION OF THE APPARATUS AND INSTRUMENTS

I. The Rotating Bucket.

The rotating bucket used is a silver plated brass cylinder having the cross-section and dimensions shown in the diagram in "figure 1". The figure also shows two other electrodes at the top and bottom of the cylinder but separated from the cylindric surface by an insulator. The central axis is the collecting electrode which is insulated from all others and connected to a Micro-micro ammeter by a coaxial wire.

Thus the bucket is in effect an electrical diode, having a P_o^{210} alpha source deposited on the central part of the cylindric surface. This layer produces a thin highly ionized layer in the liquid helium. This layer acts as the ion source, and a potential difference between the electrodes draws out ions of the appropriate sign.

Contacts to the three electrodes are maintained by means of springs, while the central collecting electrode is connected to the Kietheley Micro-micro ammeter. A circuit diagram of this ammeter is shown in "figure 2".[9] The ammeter is capable of measuring currents down to 10^{-13} amperes.

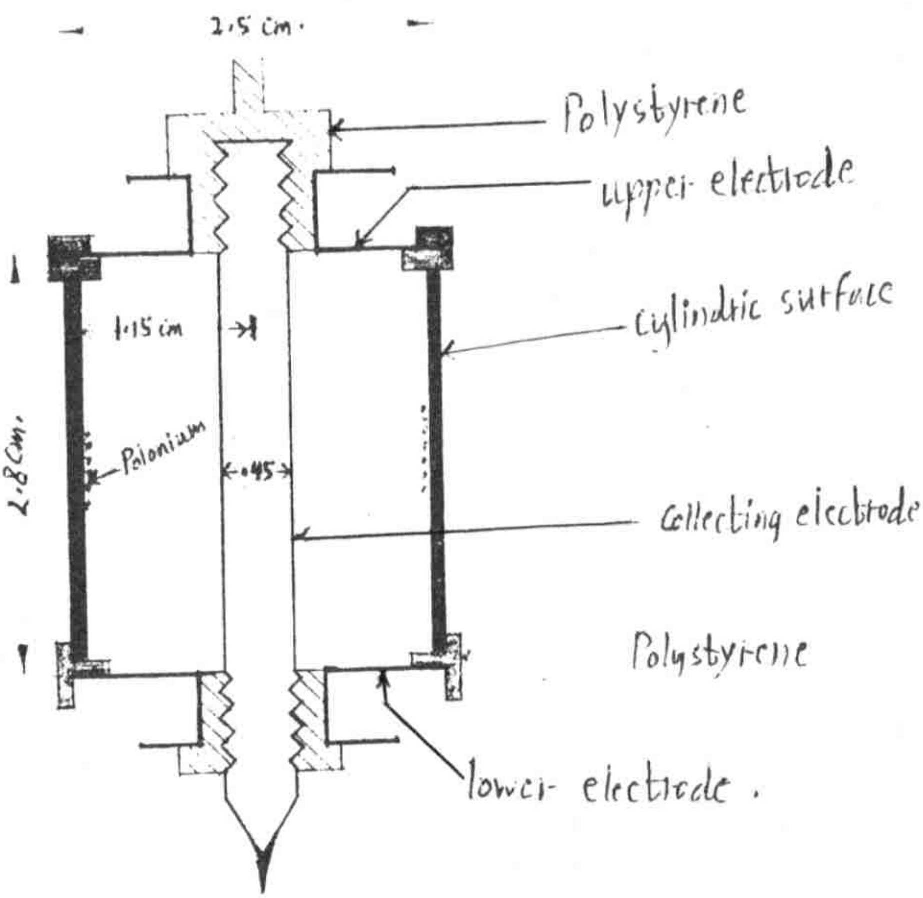


Fig. 1. -- Diagram of the rotating bucket.

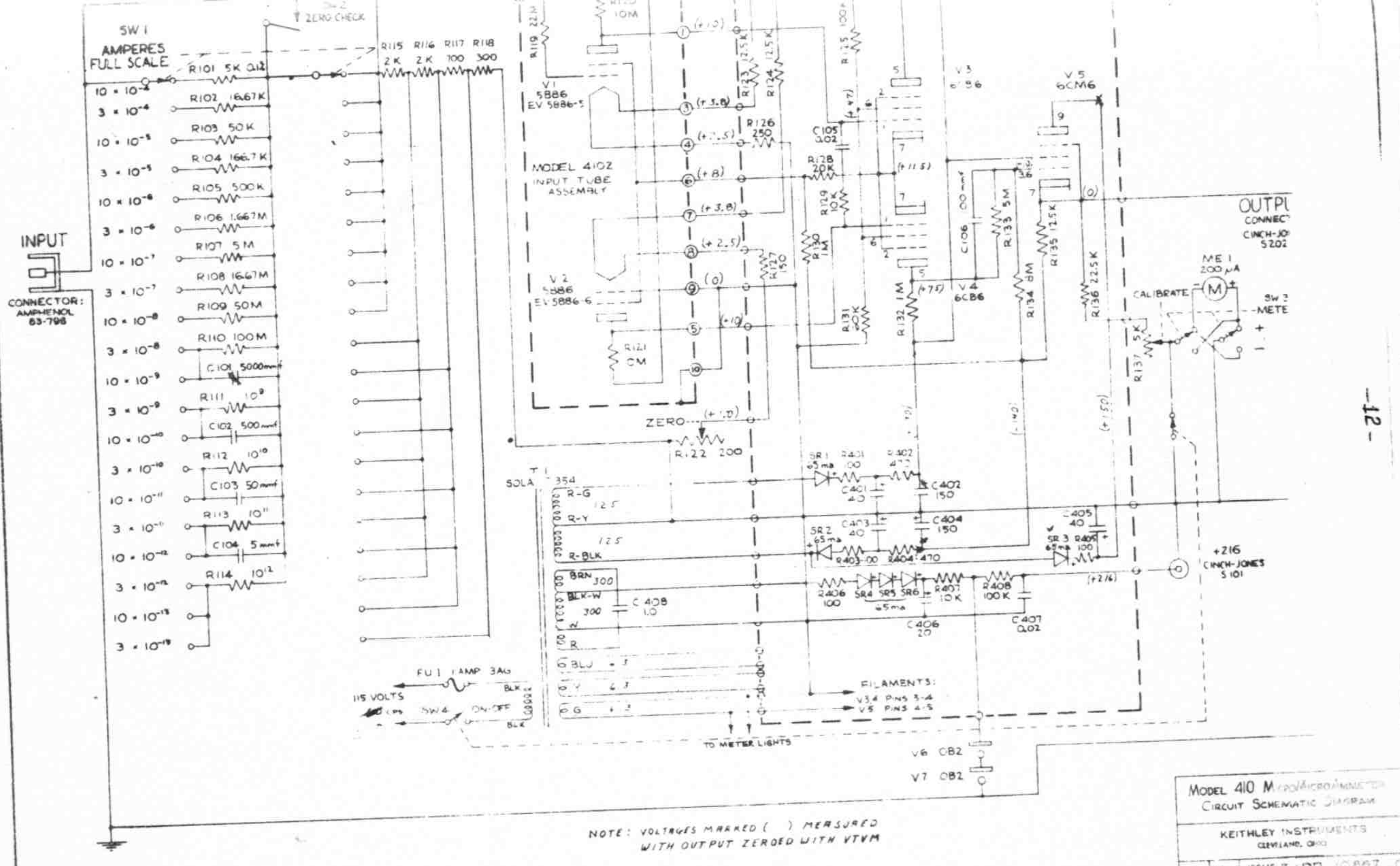


Fig. 2 -- Circuit Diagram of Ammeter

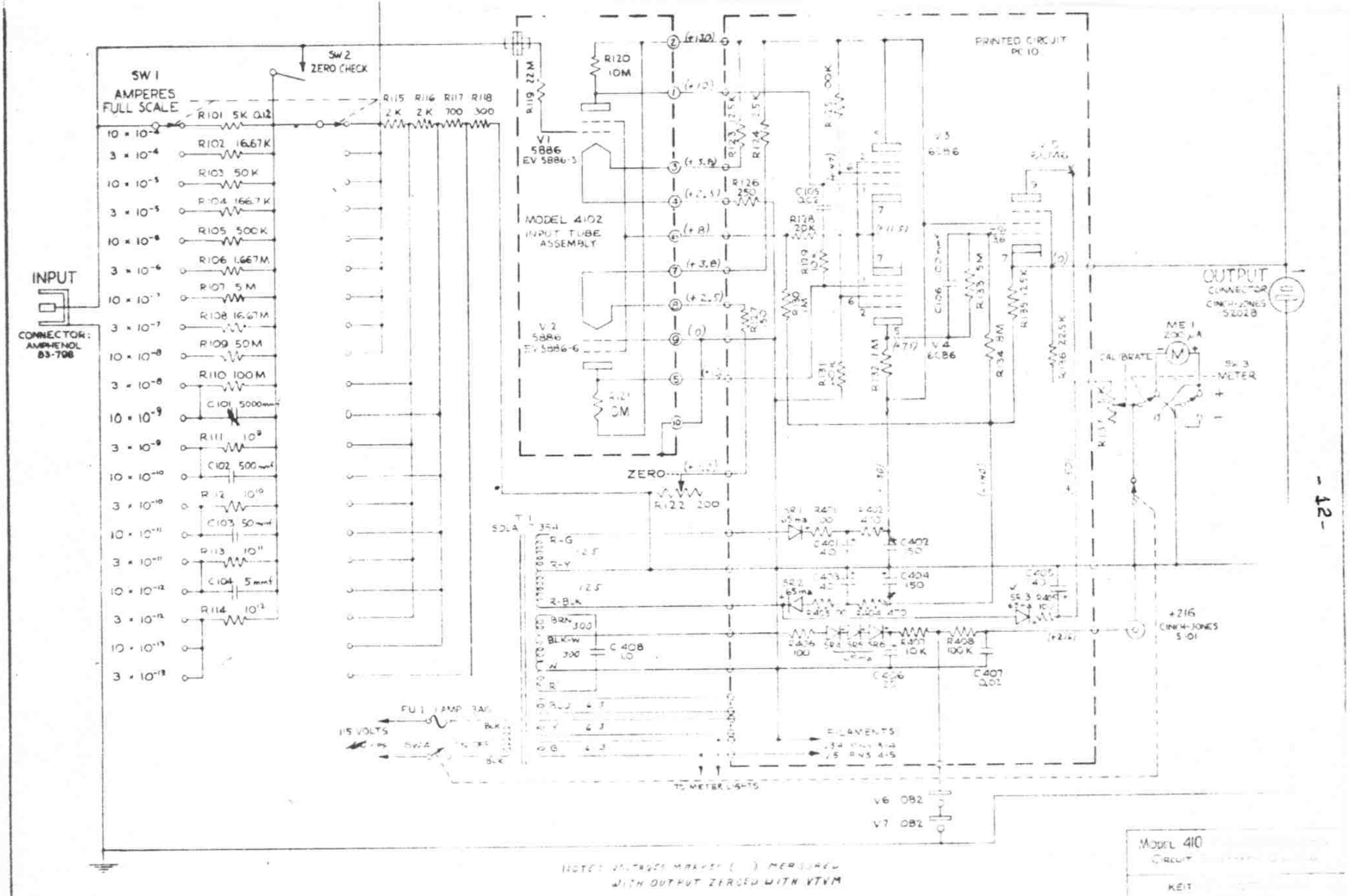


Fig. 2. -- Circuit Diagram of Ammeter

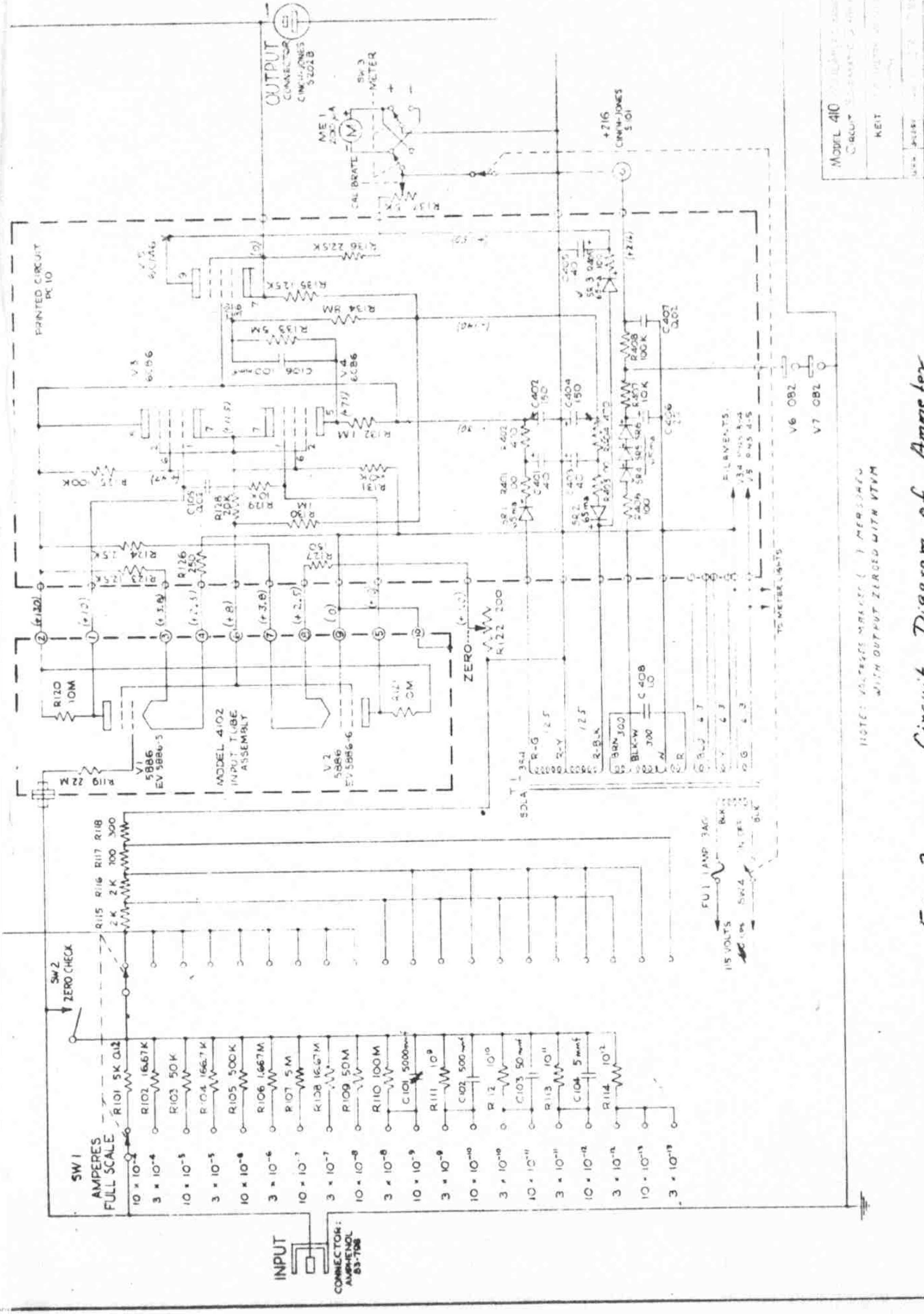


Fig. 2. — Circuit Diagram of Ammeter

The output of the Ammeter is connected to a recorder.

II. The Liquid Helium Dewar.

The rotating cylinder described above is to be filled with liquid helium. This is done by having the cylinder immersed in liquid helium in a dewar having the cross-section shown in "figure 3." The dewar is about one meter long and has an inner diameter of about 12 cm.

III. Suspension.

The cylinder, whose central axis rests on a small platform which hangs down the dewar on two stainless steel tubes, is mounted to, but insulated from, the shaft of a synchronous motor.

The motor and the two stainless steel tubes supporting the platform on which the cylinder rotates hang down from the inside surface of the metal cover of the dewar which also has an opening for the helium transfer tube. In the bottom of the inner dewar are two resistors, one of which acts as a thermometer and the other as a heater. The resistors, springs, and central electrode are electrically connected to terminals at the top of the dewar cover by means of wires through the inside of the supporting tubing. The photograph shown in "plate I" clarifies what is described in this section.

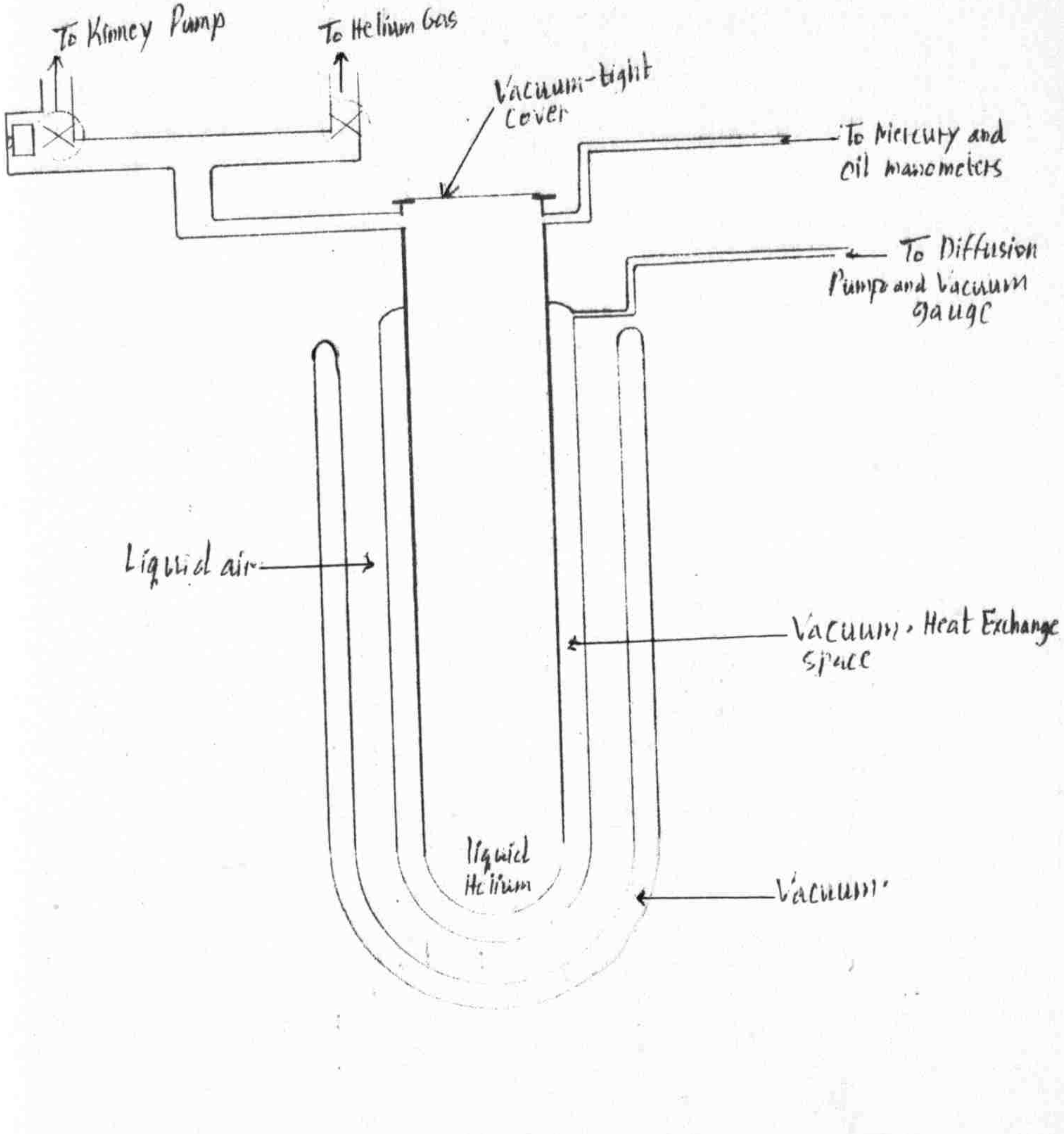
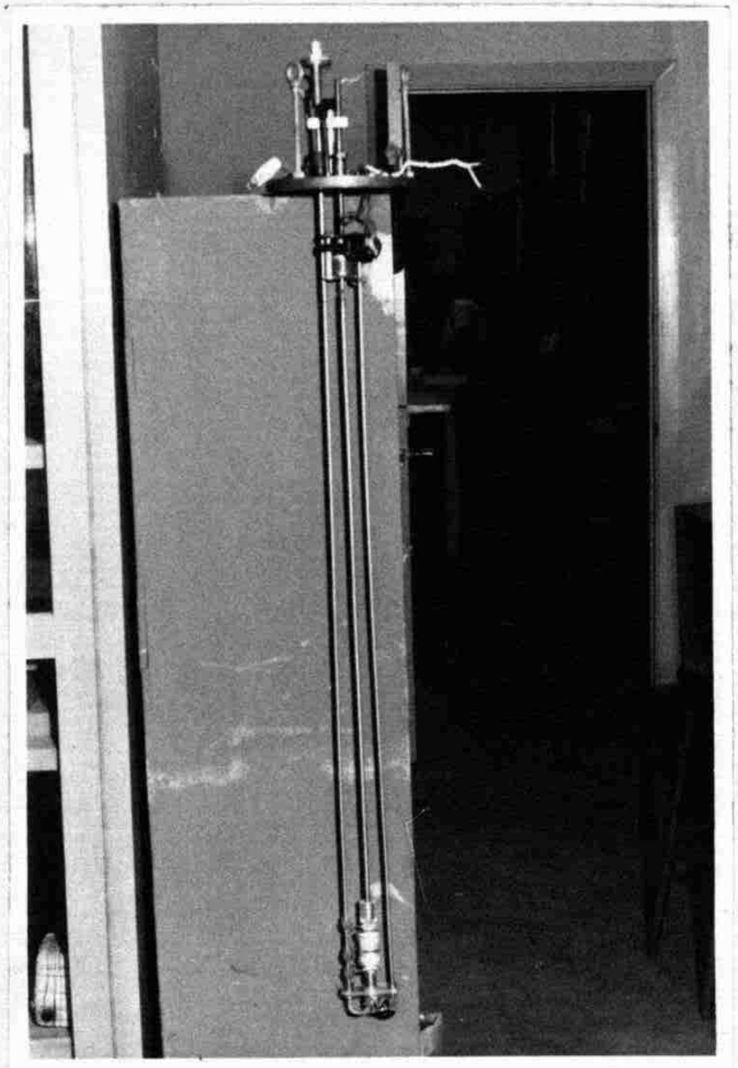


Fig. 3. --Cross-section of Liquid Helium Dewar.

PLATE I.

Dewar Cover and Suspended Parts.



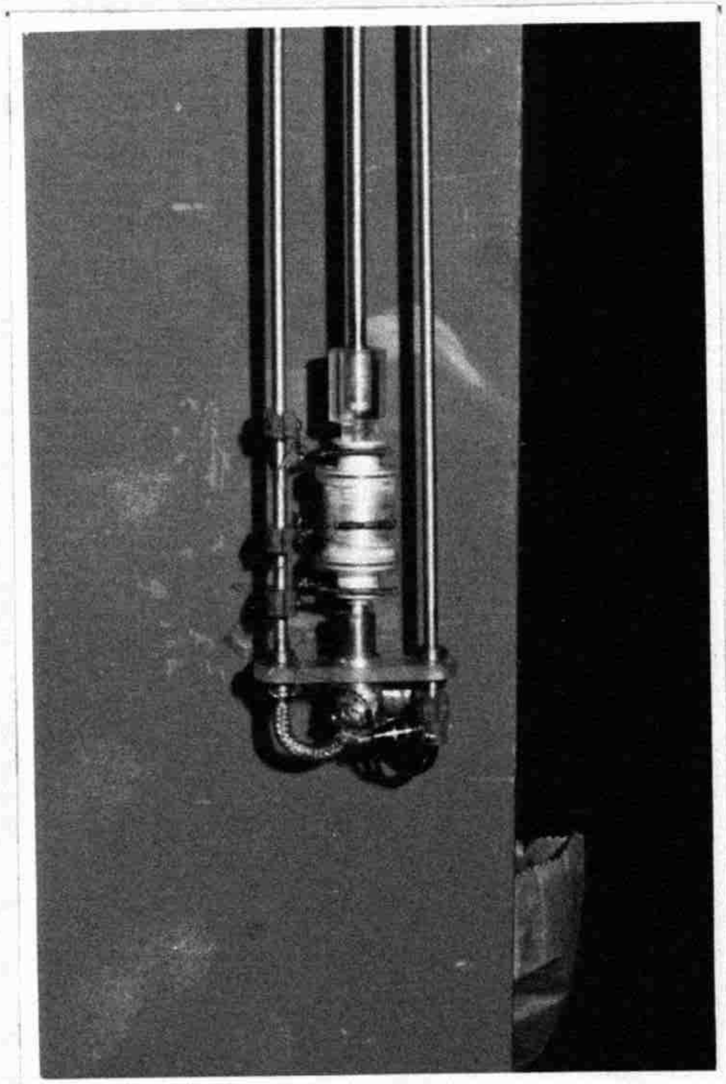
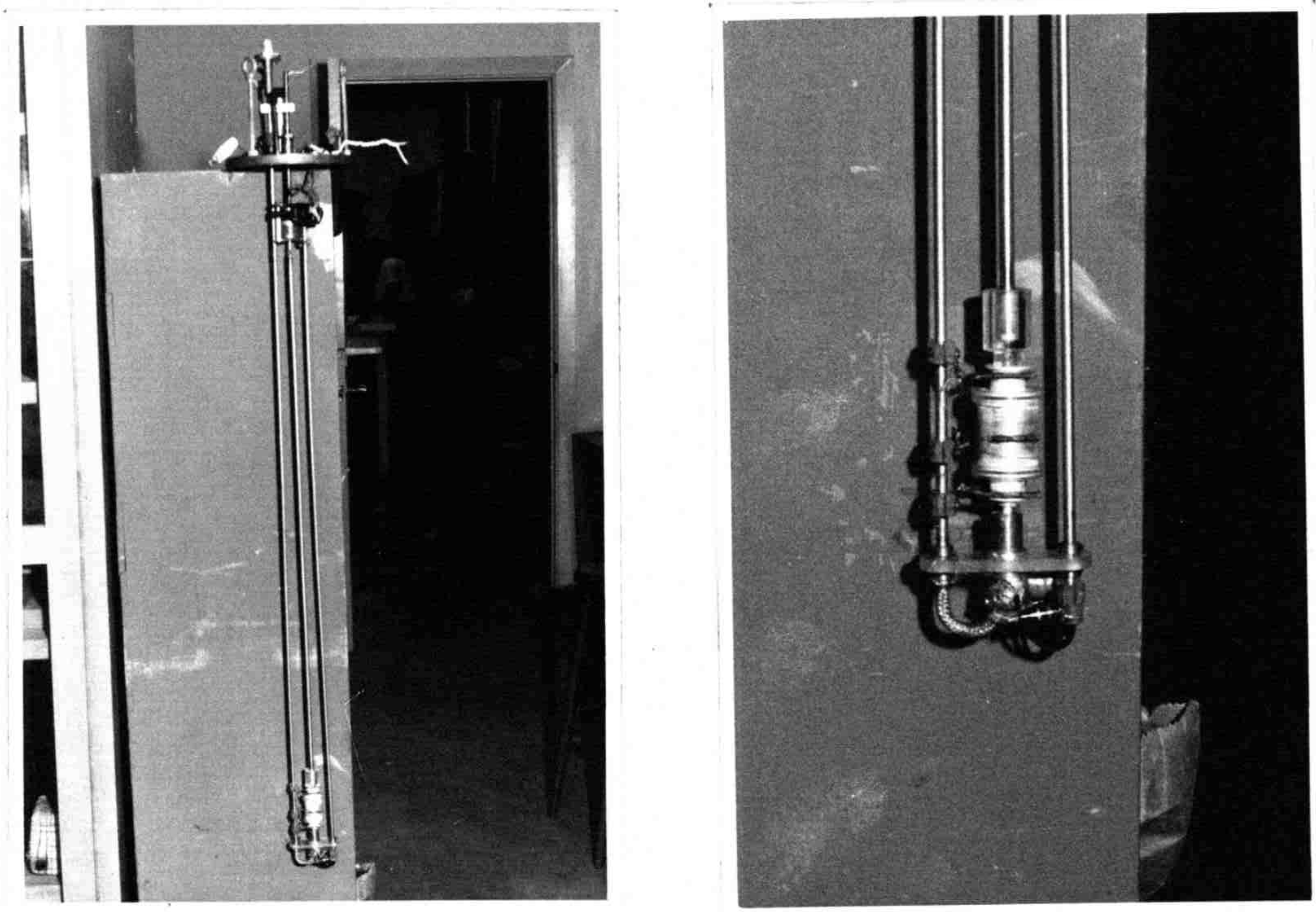


PLATE I.

Dewar Cover and Suspended Parts.



IV. Rotation.

The motor used for rotation is of 5 watts power and makes 25 rpm when run by the line voltage of frequency 50 cycles/sec. and 110 volts rms.

To be able to vary the speed of rotation we need a source capable of giving a voltage of about 100 volts rms at various frequencies and which can provide a current large enough to turn the 5 watt motor whose D.C. resistance is 650 ohms. This has been achieved by means of an audio frequency source whose output is fed into a power amplifier whose output satisfies the required conditions. The main characteristic of the amplifier used is that it has a nearly constant output over the range from 20 to 200 cycles per second. Thus with this arrangement it is possible to vary the speed of rotation of the cylinder between 10 and 100 rpm.

V. Thermometer and Temperature Regulator.

A block diagram of the apparatus used here is shown in "figure 4." The figure is self-explanatory.

The system is used to observe any change in temperature at the bottom of the dewar by its effect on the resistance of the thermometer resistor. This is seen by changes in the signal, from that resistor and the resistance bridge, which is amplified and displayed on the oscilloscope.

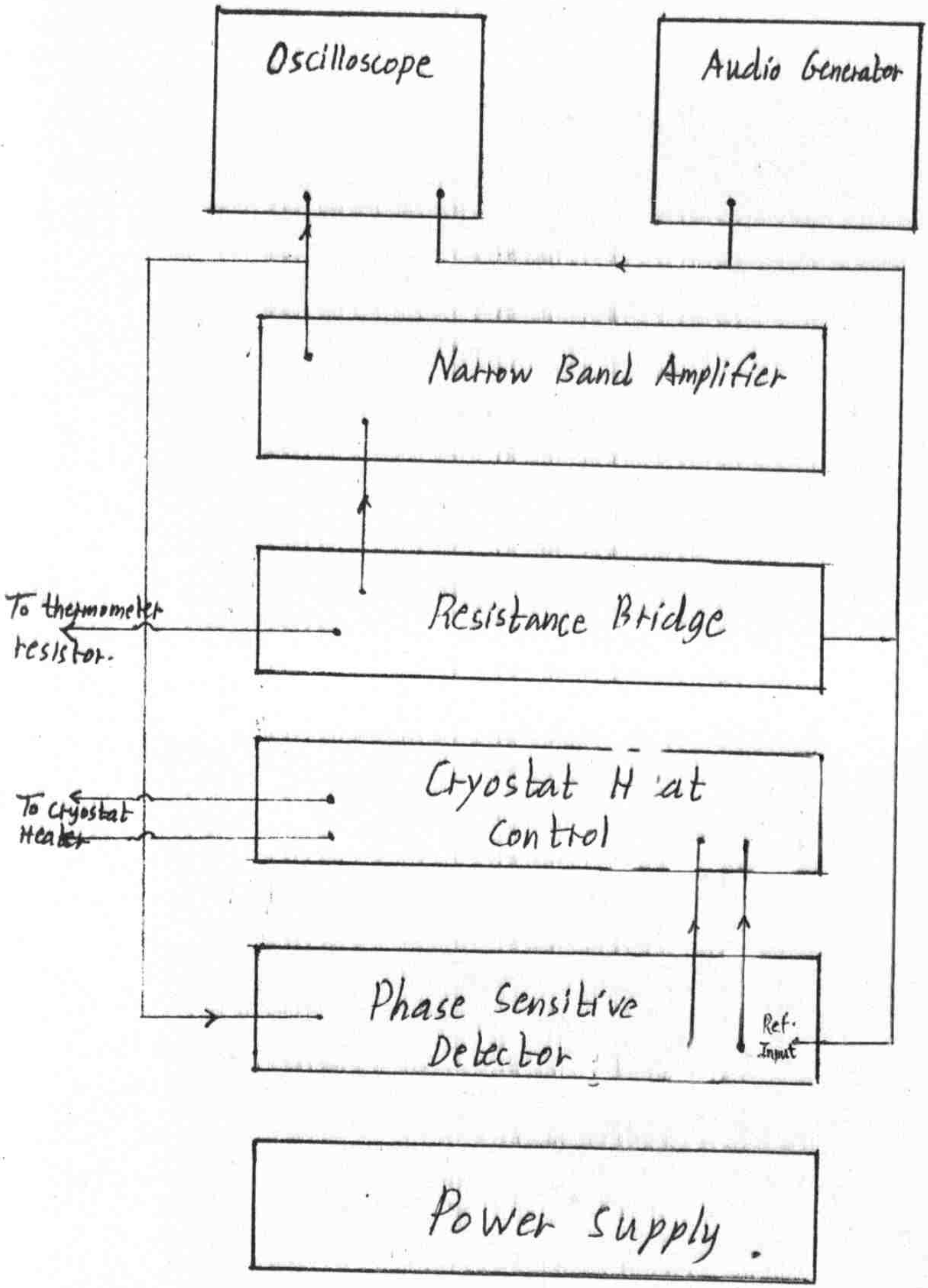


Fig. 4. Block diagram of temperature regulator.

The main purpose for using this, however, is to maintain constant temperature when readings are to be taken at a certain fixed temperature. This is done by automatic variations in the value of the current supplied by the automatic heat control unit according to the signal it receives from the phase sensitive detector.

VI. The Helium Temperature.

Mercury and oil manometers are used to determine the pressure of the helium gas above liquid helium (the vapour pressure), and hence the temperature of the liquid helium in which the cylinder is immersed. This is done by using tables of The "1958 He^4 Scale of Temperatures" published by the National Bureau of Standards, U.S.A.

VII. Source of Variable Voltage.

To vary the voltage applied to the different electrodes the battery, potentiometer, and potential divider connected as shown in "figure 5" are used. Thus the voltage applied to each electrode could be varied between zero and $\pm$ 40 volts.

The photograph shown in "plate II" gives a general view of the apparatus and the different instruments.

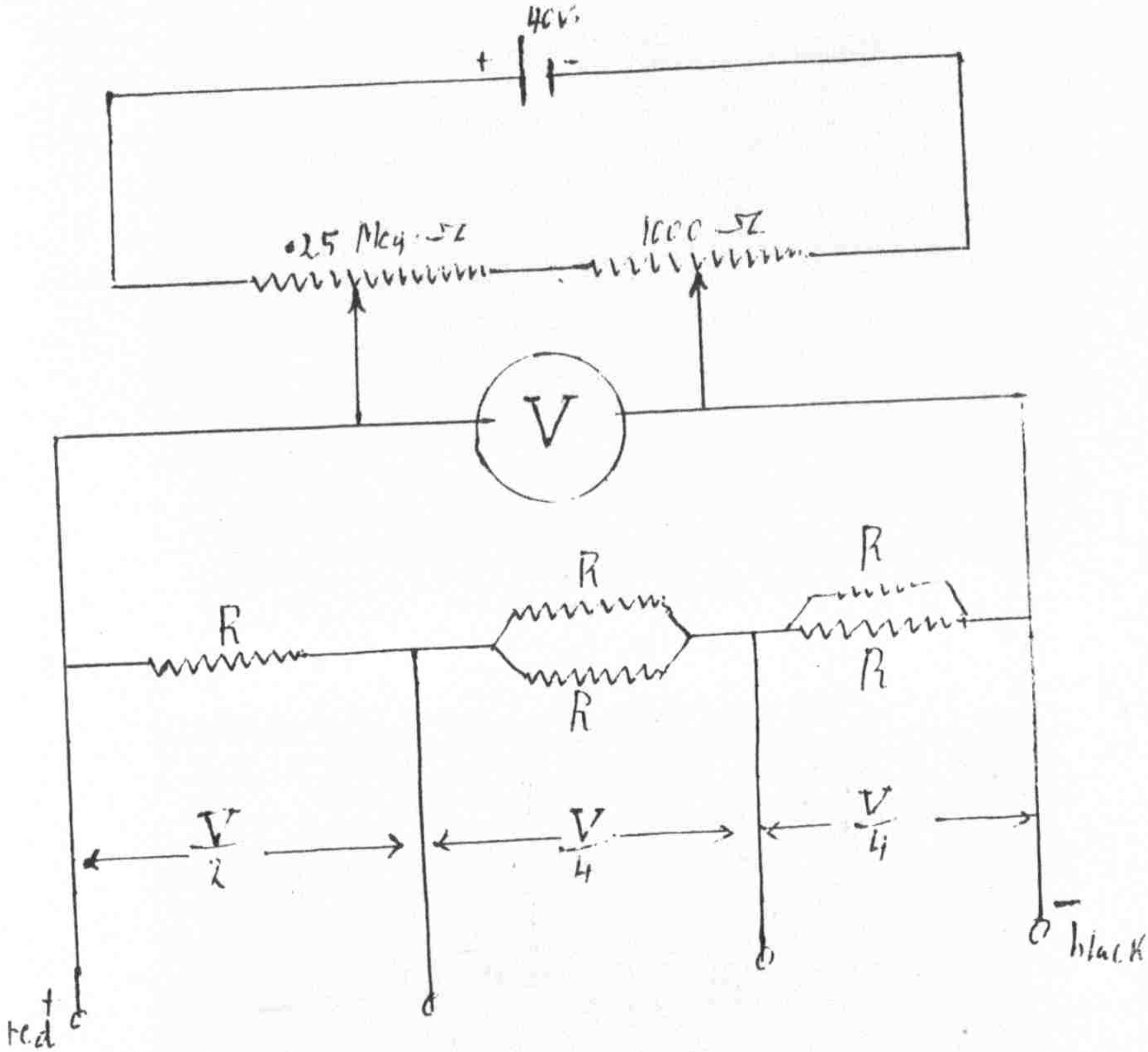
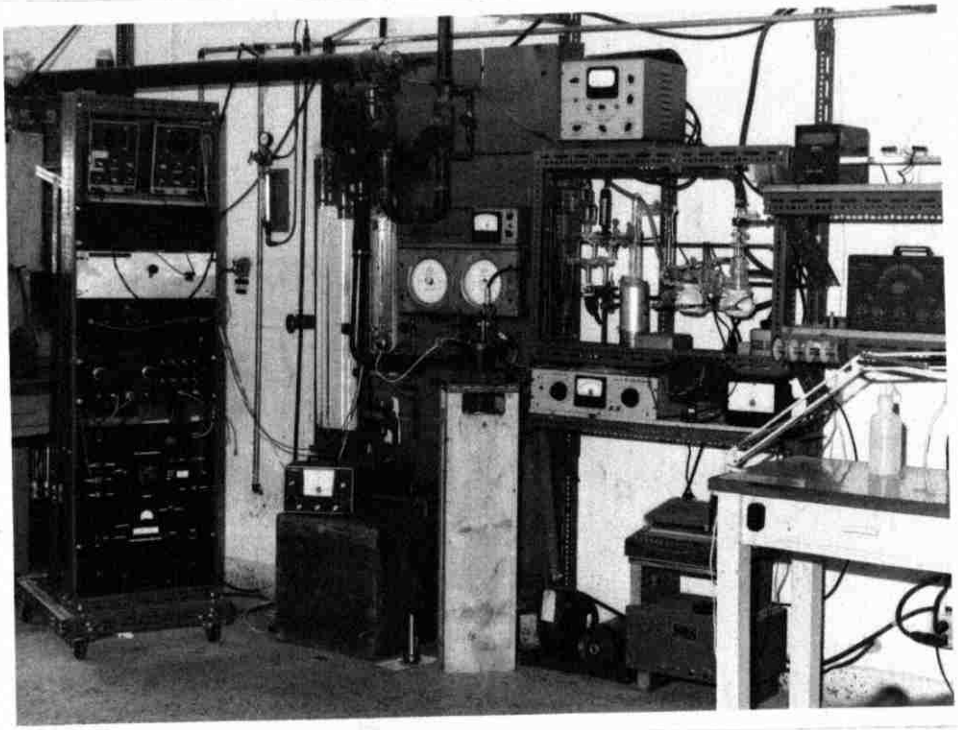


Fig. 5. -- Potentiometer and Potential Divider.

PLATE II

General Photograph of Apparatus



VIII. Experimental Problems.

In using the apparatus described in the previous 7 parts of this section, a number of problems were faced some of which were solved completely while the effects of others were diminshed but not completely eliminated. Most of these problems are related to electrical noise which effected the current readings from the Micro-micro ammeter. A list of the most important of these problems follows:

1. Contacts to the rotating cylinder were originally made through thin wires which proved to be poor contacts. This problem was solved by replacing the thin wires by the springs mentioned above.

2. Unsmooth rotation of the cylinder was another source of noise. Here making a more rigid connection between the shaft of the motor and the stainless steel tube holding the cylinder, together with some fine adjustments, proved to be very helpful.

3. Polonium was put on the inside of the cylinder by dropping few drops of polonium in 1 normal solution of nitric acid from a dropper while the cylinder was rotating on the Lathe. This is the method used in most similar previous experiments.

After the experiment had been in progress several months, we observed a large positive current which persisted in the absence of any applied voltage at liquid helium temperatures. It seems probable that this positive current is a result of the formation of some sort of a non-conducting layer on the cylinder caused by the reaction between the acid and the surface.

After several tests were made on cylinders with different surfaces, it was thought best to find a way for depositing the polonium without the acid in the hope that this might eliminate the possible causes of the large positive current at zero potential. This was done by electroplating the polonium on the cylinder which served as the cathode while a platinum wire was used as the anode. A potential difference of 12 volts was applied for half an hour between these two electrodes which were placed in distilled water containing 6 drops of polonium in 1 normal nitric acid solution. Tests on the cylinder showed that the polonium has been plated and the surface was clean. The large positive current at zero potential was no more observed. However, a small positive current has been observed all through and this zero field current will be taken into account in further calculations.

The value of the current in the electroplated cylinder was,however, less than that in the previous one. This is due mainly to two reasons. First, the intensity of a space charge limited current is pr oportional to the area of the source. The electroplated area is less than that covered previously. Secondly, the period of time during which the polonium solution was kept exceeded the half-life of this radioactive element, and thus it is expected that the intensity of alpha-particles, and hence the current, will become less than they were several months ago.

Yet it was found out that the current was too small to be accounted for by these two factors only. The only possible explanation left is that the electroplating has not been very efficient and only part of the area assumed to be electroplated is actually covered with polonium. This makes it necessary to replace the actual height of that cylindric area by an effective height which will give the actual area covered. This is done in section (5).

4. Another serious source of noise was the Micro-micro Ammeter itself. In this case it was possible to reduce the noise but not to eliminate it. The recordings shown in "figure 6" show the noise level before and after the following modifications in the meter were made.

From the circuit diagram of the Meter shown in "figure 2", it is seen that the Hi-Meg resistor is practically grounded because of the small values of the resistors which join it to ground. A capacitor placed in parallel with the high resistor will provide a path to ground of relatively low resistance for any A.C. pick-up noise.

The Meter already had small capacitors joined to the input resistances of the more sensitive scales. This however did not seem to be enough and it was decided to replace these capacitors by larger ones since that will lower the impedance, although this will lead to an increase in the relaxation time of the needle.

PLATE III

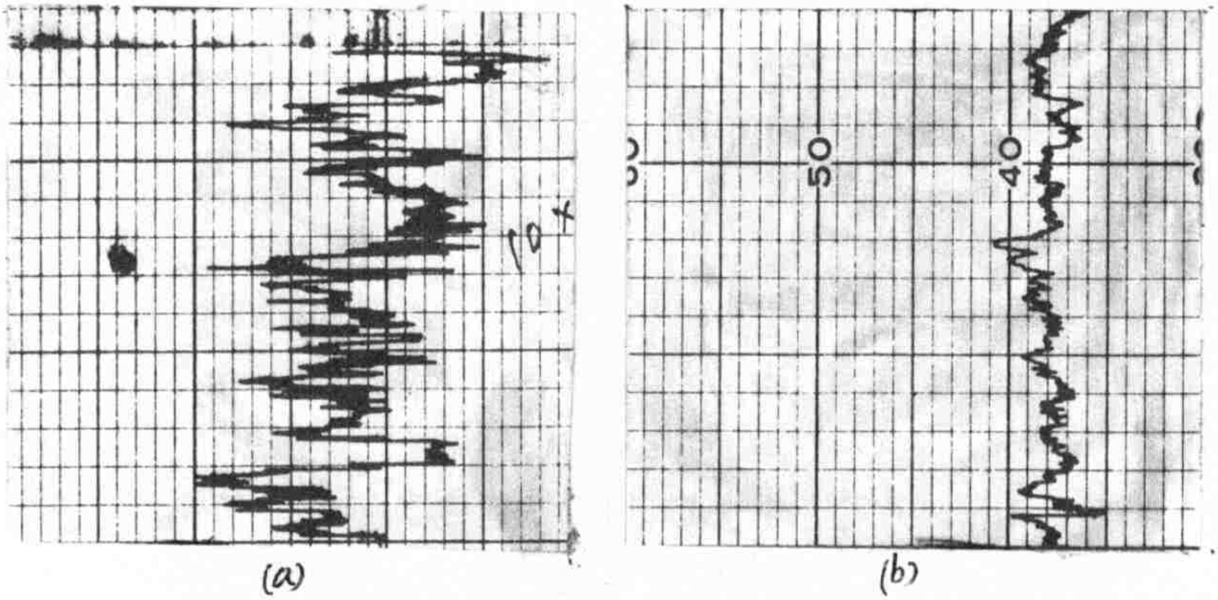


Fig. 6..(a) Recording of noise level on 10 x 10^{-13} scale before modification. Each small unit is .1 x 10^{-13} amps. (b) Noise level on same scale after modification.

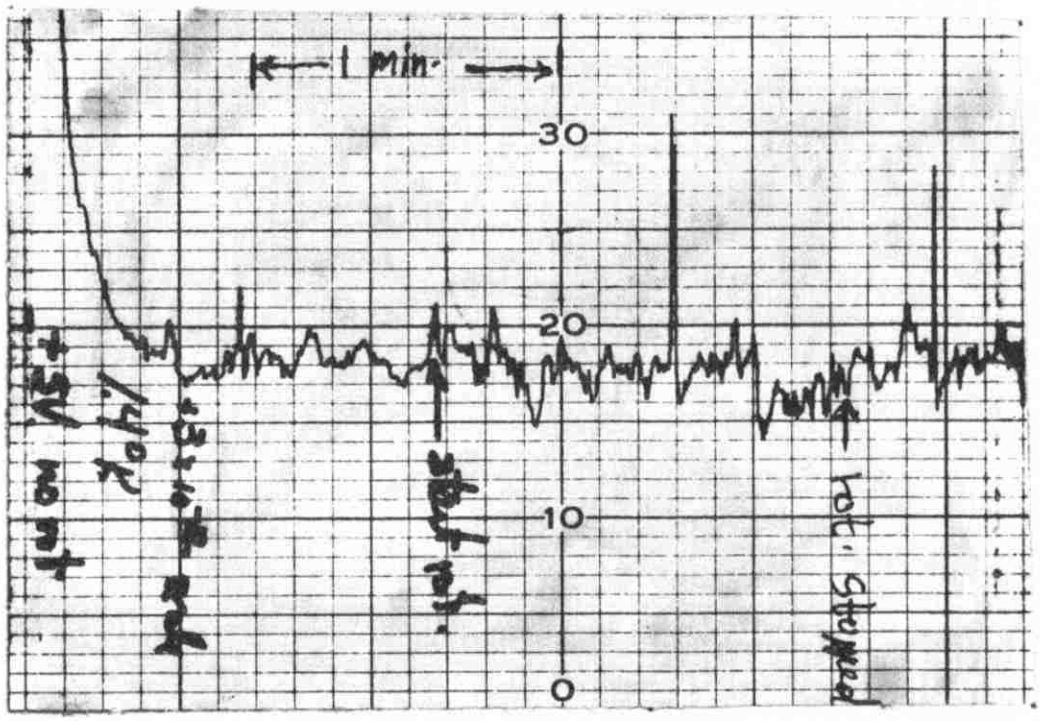


Fig. 7.. Recording of positive current for W = 0 & 25 rpm.

The following changes in the capacitors were made and can be followed on the circuit diagram in "figure 2."

a. The 500 uuf capacitor was moved from the 10×10^{-10} scale to the 10×10^{-11} scale.

b. The 50 uuf capacitor was moved from the 10×10^{-11} scale to the 10×10^{-12} scale.

c. On the input resistance of the 10×10^{-13} scale it was decided to put a 10 uuf capacitor which, of course, should have very high leakage resistance. For this a piece of Amphenol RG 58/u wire 10.6 cm long was used.

The effect of these changes on the noise level has already been pointed out in the recordings of "figure 6."

It was suspected that the remaining noise might be partly due to the power supply not being regulated. Thus the wires leading to the power supply were disconnected and an external regulated power supply was used. The noise level did not change, and we were not able to eliminate this remaining noise.

Information recently obtained from the manufacturer (Kietheley Inc.) indicates that this noise results from the input tubes, and hence can be reduced only by reducing the input capacitance.

SECTION (3)

EXPERIMENTAL RESULTS

Using the apparatus as described and improved in the previous section, a total of seven helium runs have been made with another five unsuccessful ones. The data collected and observations made during these runs may be summarized as follows:

1. Positive Current: It has been observed at various temperatures, voltages and speeds of rotation, that a positive ion current is not affected by rotation. A typical recording of such a result is shown in "figure 7" of Plate III.

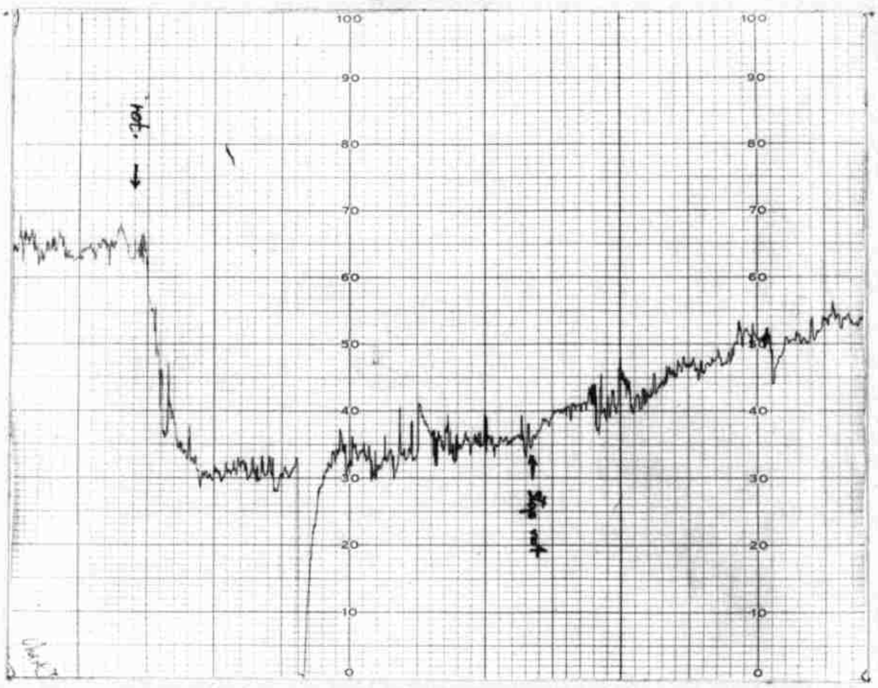
2. Negative Current: It was observed that a negative current perpendicular to the axis of rotation was attenuated during rotation. Typical recordings of this effect are shown in Plate IV. The recordings show the attenuation due to rotation at 1.4°K and 1.195°K and the relaxation times on starting and stopping rotation.

The dependence of this attenuation on the voltage and speed of rotation is given in part (4).

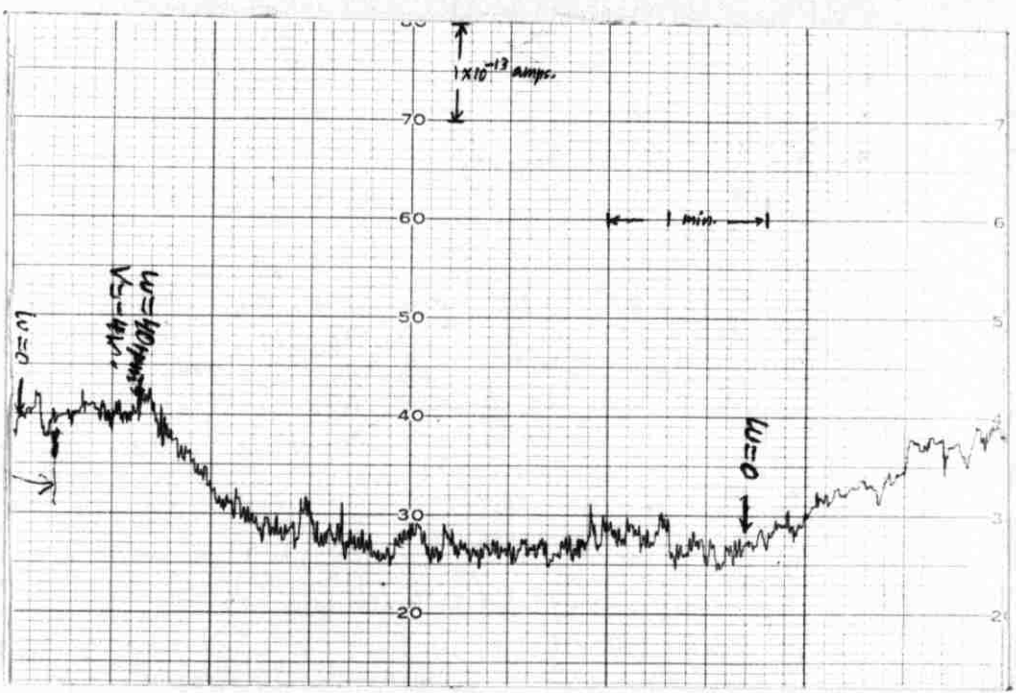
All other parts of this section refer to negative currents only. Also voltages mentioned in parts except (6) refer to the voltage applied on cylindric surface while other two electrodes are left at zero potential.

PLATE IV

Attenuation of Negative Current



$T = 1.4^{o}K$, $V_1 = 2V_2 = 2V_3 = -5v.$ 3×10^{-12} amps. scale, W = 25.



$T = 1.195^{o}K$, V = - 4v., 10×10^{-13} amps. scale, W = 40 rpm.

3. For low applied voltages and for the cylindrical container used the current is found to be proportional to the ionic mobility and to the square of the applied voltage, as expected from the theory of space charge limited currents, It has been attempted to determine the region of validity of $I \propto \mu V^2$ for different temperatures. The graphs of "Plates V and VI" show the curves obtained for I versus V^2 at T = 1.187°K and T = 1.29°K. As mentioned previously it is seen in the curves for I versus V^2 that there is a positive current at zero field. This fact is to be taken into consideration by defining an effective voltage to be used in calculations.

4. As stated in part (2), the negative current is attenuated when the cylinder rotates. To study the dependence of this attenuation on W and V, several sets of values for I versus V at constant W and T have been obtained and used in plotting the curves shown in "Plate V" and "Plate VI".

5. The effect of applying various voltages to the upper and lower electrodes has been studied to a small extent. The results obtained at 3 different temperatures and for various voltage combinations and speeds of rotation are given in tables 1 to 3.

It is observed from these tables that the attenuation is maximum when the three electrodes are at the same negative potential, while it is least when the upper and lower electrodes are at zero potential; and has an intermediate value when one of them only is at zero potential.

PLATE V

Graph of I Versus V^2

T = 1.187°K.

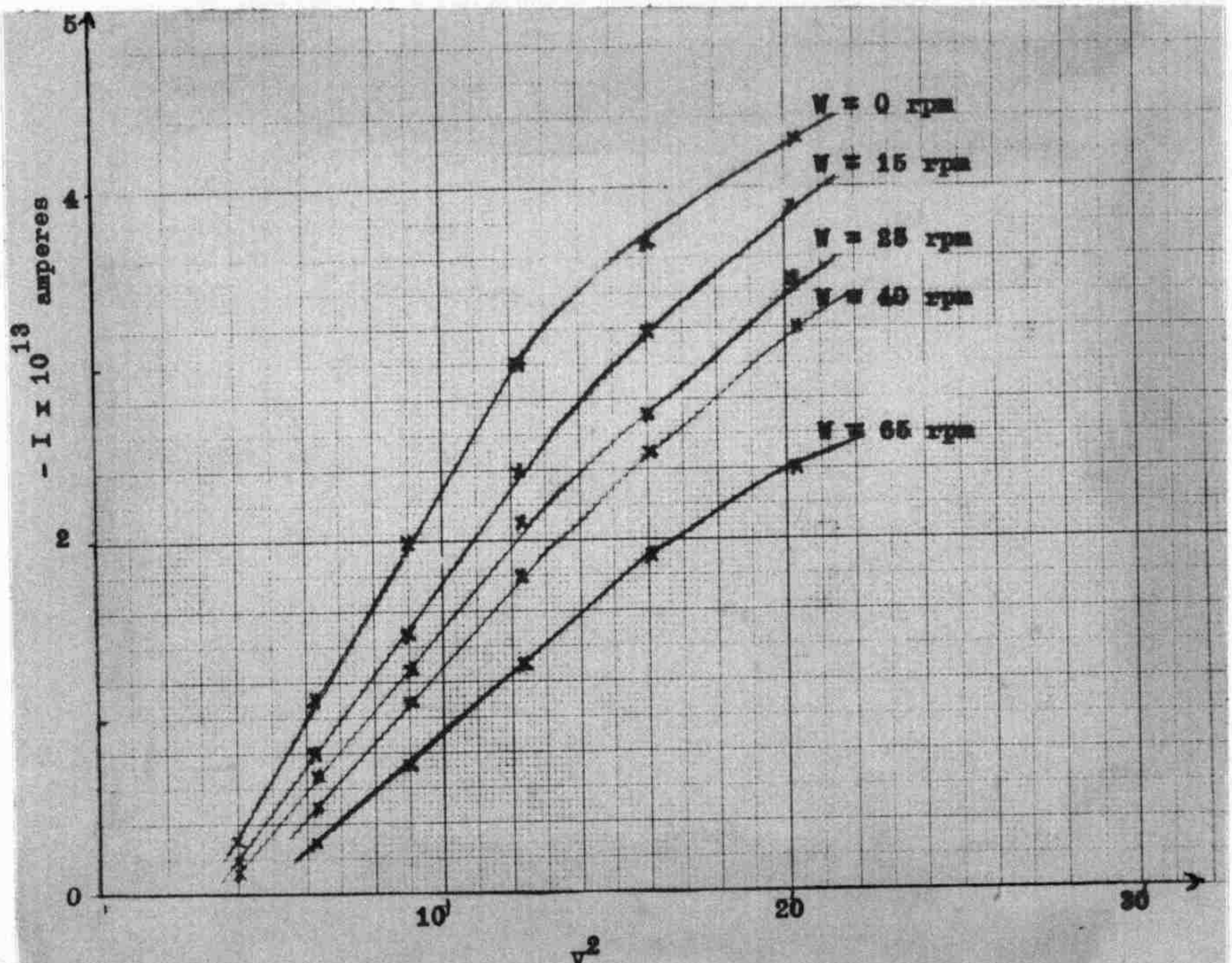


PLATE VI

Graph of I Versus V^2

T = 1.29°K

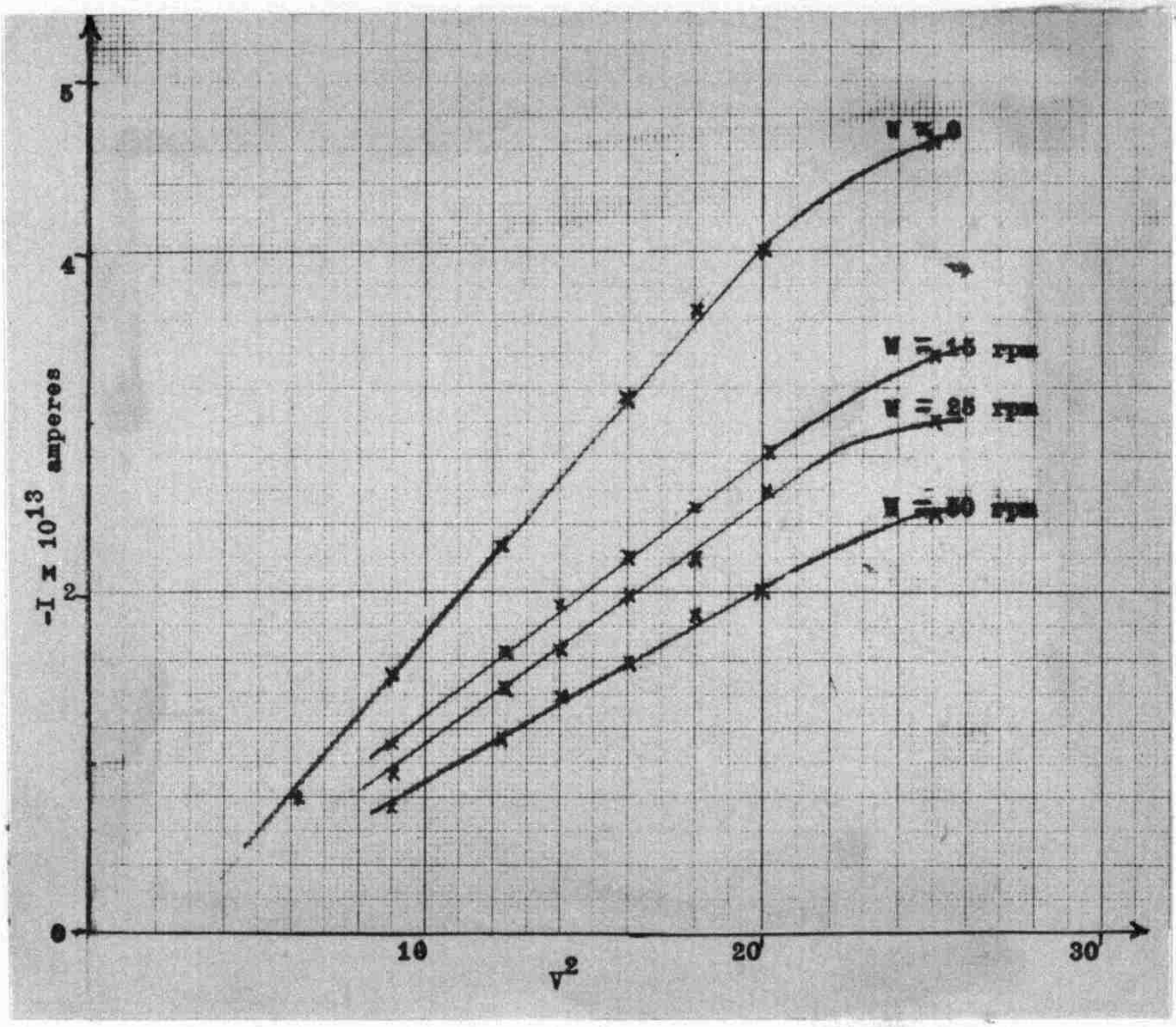


TABLE 1

CURRENT VARIATIONS WITH AXIAL FIELDS FOR DIFFERENT SPEEDS OF ROTATION AT T = 1.195°K.

Wrpm	V_1^a	V_2^b	V_3^c	$-I \times 10^{13}$ amps.
0	-3.5	0	0	2.65
25	-3.5	0	0	1.7
50	-3.5	0	0	1.3
0	-3.5	0	-3.5	2.1
25	-3.5	0	-3.5	1.2
50	-3.5	0	-3.5	0.75
0	-3.5	-3.5	-3.5	1.8
25	-3.5	-3.5	-3.5	0.5
50	-3.5	-3.5	-3.5	0.35

TABLE 2
T = 1.29°K.

Wrpm	V_1	V_2	V_3	$-I \times 10^{13}$ amps.
0	-3.5	0	0	2.55
25	-3.5	0	0	1.9
50	-3.5	0	0	1.4
0	-3.5	0	-3.5	2.0
25	-3.5	0	-3.5	1.0
50	-3.5	0	-3.5	0.6
0	-3.5	-3.5	-3.5	1.85
25	-3.5	-3.5	-3.5	0.4
50	-3.5	-3.5	-3.5	0.25

TABLE 3
T = 1.443°K.

Wrpm	V_1	V_2	V_3	$-I\times10^{13}$ amps.
0	-4	0	0	2.3
25	-4	0	0	1.9
50	-4	0	0	1.3
0	-4	0	-4	2.0
25	-4	0	-4	1.3
50	-4	0	-4	0.75
0	-4	-4	-4	1.9
25	-4	-4	-4	0.6
50	-4	-4	-4	0.35

[a] V_1 : Voltage applied to cylinder
[b] V_2 : " " " lower Elect.
[c] V_3 : " " " upper "

The attentuation in the first and last cases is observed to exceed the sum of the attenuations produced by rotation when upper and lower electrodes are grounded plus the attenuation produced when one or both electrodes are at a negative potential but the cylinder is not rotating.

6. It was originally planned to study the relaxation times of the current when rotation is started or stopped. Recordings of these effects can be seen in plate IV given above. However, this study could not be continued due to the great increase in the response time of the Micro-micro ammeter, brought about by the modifications made in it to reduce the noise level.

The results obtained before that modification was made show that more than one equilibrium process sets in at the same time. It was found that on starting rotation the current needs a longer time to reach equilibrium than when rotation is stopped. The two periods are of the order of 3 minutes and half a minute respectively.

An analysis of the above results as well as a theoretical explanation of some of the effects observed and recorded in this section is given in section 5.

SECTION (4)

THEORY.

I. Ion Motion in Liquid Helium II

We now present a theoretical explanation of the motion of ions in rotating liquid helium II based on the assumption that an array of vortex lines is formed in rotating liquid helium, and that these vortex lines behave like holes in which the ions are trapped. This is intended to explain the attenuation in the current when the cylinder is set into rotation.

II. The Linear Region

The simplest situation to interpret is that where the electric field E is kept sufficiently small so that the ion never acquires an energy appreciably in excess of its equilibrium thermal energy in the fluid. The ion then resembles most closely an ideal probe which disturbs the medium minimally. Under these circumstances u, the drift velocity is proportional to E, and the mobility $\mu = u/E$ of the ion is independent of E.[11] This is what is meant

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[11] F. Reif and L. Meyer, "Study of Superfluidity in Liquid Helium by Ion Motion," Physical Review, Vol. 119 (August, 1960), 1164-1174.

by the linear region.

To express this condition in terms of physical constants we can say that it is the region in which E is kept small enough so that the energy imparted to an ion by the applied field E in a mean free path l is small compared to its thermal energy, that is,

$$eEl/(\tfrac{3}{2}\ KT) \ll 1.$$

We shall confine our treatment of ion motion to cases in which this condition is satisfied.

III. Solution of Poisson's Equation With Fixed Space Charges in Cylindrical Geometry and Assuming Cylindric Symmetry.

To interpret our results, we need to relate the measured space charge limited current to the electrode voltage V, ion mobility μ and density of **trapped** ions ρ_2 .

The radial component of poission's equation in cylindrical coordinates is,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{1}{\varepsilon_o}\left(\rho_2 + \rho_1(r)\right) \quad \ldots\ldots\ldots \quad (5)$$

V is in volts, ρ_2 stands for fixed charge density, ρ_1 (r) density of charge drifting with speed μE.

In the linear region considered, the conservation and mobility equations give

$$\frac{I}{2\pi hr} = j(r) = -\rho_1(r)\,\mu\,\frac{dV}{dr}$$

$$\text{or } \rho_1 = \frac{-I}{2\pi h\, \rho_1(r)\, \mu\, dV/dr}$$

where h is the height of the ion source or polonium covered surface, and I is the measured total current resulting from a source of height h on the cylindrical surface of radius r_o.

Substituting this in Poisson's equation we get:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dV}{dr}\right) + \frac{\rho_2}{\varepsilon_o} - \frac{I}{\varepsilon_o \mu} \frac{1}{2\pi h r \frac{dV}{dr}} = 0 \quad \ldots\ldots \quad (7)$$

$$\therefore \frac{dV}{dr} \frac{d}{dr} \left(r \frac{dV}{dr}\right) + \frac{\rho_2}{\varepsilon_o} r \frac{dV}{dr} - \frac{I}{\varepsilon_o \mu 2\pi h} = 0 \quad ,$$

Now multiply by $\frac{\varepsilon_o}{\rho_2^2 r_o^2}$ and write $\frac{d}{dr} = \frac{1}{r_o} \frac{d}{d(r/r_o)}$.
Also make the following substitutions:

$$x = r/r_o, \quad y = -\frac{d\varphi}{dx}, \quad \varphi = \frac{-\varepsilon_o}{\rho_2 r_o^2} V$$

$$\ldots\ldots \quad (8)$$

$$Z = \frac{-\varepsilon_o I}{2\pi h \mu \rho_2^2 r_o^2}$$

These substitutions reduce our equation to

$$y \frac{d}{dx} (xy) + xy + z = 0 \quad \ldots\ldots\ldots\ldots \quad (9)$$

For the solution of this equation we refer to a paper by Ghizetti and Gross[12] who solve equation (9) with $y = \frac{-d\varphi}{dx}$ and for $0 \angle x \leq 1$, with the initial conditions:

$$y = \varphi = 0 \qquad \text{for } x = 1.$$

$x = 1$ at the surface of the outer cylinder, that is $r = r_o$; hence $y = \varphi = 0$ corresponds to an electric field of zero at the source, and if $\varphi = 0$, then from equation (8), we see that ρ_2 should be infinite. Thus all these boundary conditions at $x = 1$ for which the equation (9) is solved correspond to space charge limit case.

The solution they get is of the form:

$$\varphi_{x\ min} = F_2(\rho)\, Z^{1/2} + \frac{1}{3} F_3(\rho) + \frac{1}{36} F_4(\rho)\, Z^{-\frac{1}{2}} + \ldots \qquad (10)$$

where $\rho = \sqrt{1 - x^2_{min}}$ and the various F functions are given in the Appendix where it is also shown that the third term is negligible. This gives for the voltage V the following expression (only first two terms of the series are considered in what follows) :

$$V = \frac{-r_o^2 \rho_2 F_2(\rho)\sqrt{Z}}{\varepsilon_o} - \frac{r_o^2 \rho_2}{3\varepsilon_o} F_3(\rho) \ \ldots . \qquad (11)$$

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[12] A. Ghizzetti and W. Gross, Quaderno No. 2, Instututo Nazionale Per le Applicazioni del Calcolo, Consiglio Nazionale delle Ricerche, Roma (June 2, 1962).

But $Z = \dfrac{-\varepsilon_o I}{2\pi h\mu\rho_2^{\ 2}\ r_o^{\ 2}}$ $\therefore \sqrt{Z} = \dfrac{\sqrt{\varepsilon_o}\ \sqrt{I}}{|\rho_2|\ r_o \sqrt{-2\pi h\mu}}$

and $F_2(\rho) = 1.37$

$\frac{1}{3} F_3(\rho) = 0.37$

$$\therefore\ V = \frac{(1.37)\ r_o \sqrt{I}}{\sqrt{2\pi h\varepsilon_o}\ \sqrt{-\mu}} - \frac{0.37\ r_o^2 \rho_2}{\varepsilon_o} \quad \ldots\ldots \quad (12)$$

in which the MKS system of units is to be used and in our case:

$r_o = 1.15 \times 10^{-2}$m.

$\varepsilon_o = 0.88 \times 10^{-11}$

and μ is negative for negative ions and should be expressed in m^2 / volt-sec.

If there are no trapped charges we have:

$$V = \frac{(1.37)\ r_o \sqrt{I}}{\sqrt{2\pi h\varepsilon_o}\ \sqrt{-\mu}} \quad \ldots\ldots\ldots \quad (13)$$

where it should be kept in mind that both equations (12) and (13) apply only in the linear region because this has been a basic assumption in the derivation.

This is the main theory in the light of which the results given in section (3) will be analyzed in the next section.

SECTION (5)

ANALYSIS OF RESULTS.

In the experimental results of section (3),we have presented 6 points which summarize the main results and observations. We shall now present a theoretical explanation and analysis, in as far as it is possible, for each of the points mentioned in section (3).

1. Positive Current: That positive ions are not affected by rotation agrees with Careri's observation.[13] As to the reason for this, it is believed to be due to the different structure of the positive ion in liquid helium. The exact structures of both positive and negative ions is not well known. A recent paper on this subject[14] reaches the conclusion that:

a. The positive ion is a solid cluster of He atoms polarized around a positive charge, which may change site while still remaining inside the cluster.

b. The negative ion is a cloud of charge in a cage, self-trapped by the shell of polarized atoms.

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[13]Careri, Op. Cit.

[14]G. Careri, Fasoli and Gacta, "Experimental Behaviour of Ionic Structures in Liquid Helium II," Nuovo Cimento, XV (No. 5, 1960), 774.

Thus the difference in the behaviour of positive and negative ions in rotating liquid helium is believed to be due to the difference in their structure or, possibly, due to the difference in their effective masses although these have not yet been measured.

2. Negative Current: The observed attenuation of a negative current perpendicular to the axis of rotation, we believe is due to the establishment of "traps" in the rotating helium, that is, a certain number of preferred positions in which the ions would be withheld from the general drift motion. We believe that the results support the Feyhman - Onsager vortex line theory of rotation of superfluid helium which has been summarized above. Further discussion of this is given in part (4) below.

3. Linear Region: Since, as explained above, it is much easier to study the attenuation of the current in the linear region, it was essential to determine that region. The graphs given in "plate V" and "plate VI" for I versus V^2 up to high voltages show the voltage at which the curve deviates from linearity at two different temperatures. This is seen to be true for low values of V from -5 volts and below.

The fact that $I \propto V^2$ for low V could be used to show that $I \propto \mu$. Since in the linear region, $\frac{I}{V^2} \propto \mu$ and $\mu \propto e^{\Delta/KT}$ [15], a plot of $\ln(I/V^2)$ versus (I/T), where all

[15] L. Meyer and F. Reif, "Mobilities of the Ions in Liquid Helium," <u>Physical Review</u>, vol. 110 (April, 1958), 279-280.

points are chosen to fall in the linear region should give a straight line of slope $\frac{\Delta}{K} \approx 8.3°K.$[16] However, since we are considering only two different temperatures, it will suffice to calculate the slope of the line joining these two points without actually plotting it. The following table could be formed.

TABLE 4.

VALUES FOR (I/V^2) FOR TWO DIFFERENT TEMPERATURES.

(I/V^2)	$\ln(I/V^2)$	T °K	$1/T$
2.25	0.81	1.29	0.775
3.30	1.19	1.187	0.842

From the above table one can directly obtain the slope of the line joining the two points which we have considered. The slope turns out to be about 6°K which is comparable with the previously measured value of 8.3°K when we take into consideration the experimental error in the current readings.

4. Attentuation Due to Rotation: The decrease in current can not be attributed to a decrease in the ionic mobility caused by the addition of new scattering centers

[16] Ibid. P. 280.

because recent experimental results mentioned later seem to show that mobility is not affected by rotation. This conclusion is also reached ~~if~~ from the temperature dependence of the mobility.[17] Thus the only explanation seems to lie in the establishment of traps in the rotating helium.

For the case of trapped charges in the region of lihear mobility we have derived the expression:

$$V = \frac{(1.37)\, r_o \sqrt{I}}{\sqrt{2\pi h \epsilon_o}\ \sqrt{-\mu}} - \frac{0.37\ r_o^2 \rho_2}{\epsilon_o}$$

A theoretical plot of the curve representing this function for constant ρ_2 in which I is plotted versus V^2 will give a curve very close to a straight line. Actually ρ_2 changes with V and hence assuming ρ_2 constant does not give the real picture. However, the general shape of the slightly curved straight line is in agreement with the result. obtained by Careri[18] which also agrees qualitatively with the results obtained here and shown in plates V and VI. I say qualitatively, because, both the recordings and the graphs show an attenuation which is much smaller than that found by Careri and shown recorded in his paper.[19]

To apply equation (12), we have to determine $h_{effective}$ as explained previously. This has been done by using the slopes of the straight lines for I versus V^2 for the

[17] Ibid. P. 280.

[18] Careri, McGormick, Scarammuzzi, Op. Cit. P. 62.

[19] Ibid.

non-rotating case as obtained from the graphs in plates V and VI. These slopes are equated to the coefficient of I in equation (12). Using the slopes has the advantage of giving a result which is independent of the zero field current. The calculations gave an average value of $h_{eff.} = 0.17 \times 10^{-2}$ meters.

Recent experimental results[20] suggest that the mobility does not change with rotation. Thus the value for mobility in a stationary container will be used, and this means that all the attenuation in the current is attributed to the trapped ions.

A typical calculation for determining ρ_2 is as follows:

At $T = 1.187°K$, $\mu = -0.96$ cm^2/volt-sec.

$$\sqrt{-\mu} = 0.98 \times 10^{-2} \text{ in MKS units.}$$

And equation (12) gives:

$$\rho_2 = 0.93 \sqrt{I} - 1.8 \times 10^{-7} V_{eff.}$$

where $V^2_{eff.} = V^2 - 3$ in this case.

Similarly, at $T = 1.29°K$ we have:

$$\rho_2 = 1.2 \sqrt{I} - 1.8 \times 10^{-7} V_{eff.}$$

and here $V^2_{eff.} = V^2 - 2.5$.

With the above two formulas, we can calculate ρ_2 for the various speeds of rotation. The number of trapped charges per cm^3, n, is of more interest than the density of trapped charges. Thus n is obtained from ρ_2 using the fact that the charge of an electron is 1.6×10^{-19} coulombs. Table V shows the number of trapped charges for various voltages and speeds

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[20] Experiment performed by Dr. R.L. Douglass.

TABLE 5

VARIATIONS OF THE NUMBER OF CHARGES TRAPPED WITH THE APPLIED VOLTAGE FOR DIFFERENT TEMPERATURES AND SPEEDS OF ROTATION.

$-V_{eff}$ Volts	$n \times 10^{-6}$ electrons/cm^3.						
	T = 1.187°K				T = 1.29°K.		
	W = 15	W=25	W=40	W=65	W=15	W=25	W=50
1	0.34						
1.8	.45	0.56	.79	1.07			
2.45	.58	.77	.89	1.21			
2.55					0.40	0.60	0.85
3.12	.73	.90	1.12	1.46	0.47	0.69	0.92
3.39					0.54	0.75	1.0
3.61	.89	1.14	1.25	1.62			
3.67					0.61	0.78	1.06
3.94					0.68	0.91	1.16

PLATE VII

Graph of Number of Trapped Charges Versus Voltage

at W = 25 rpm

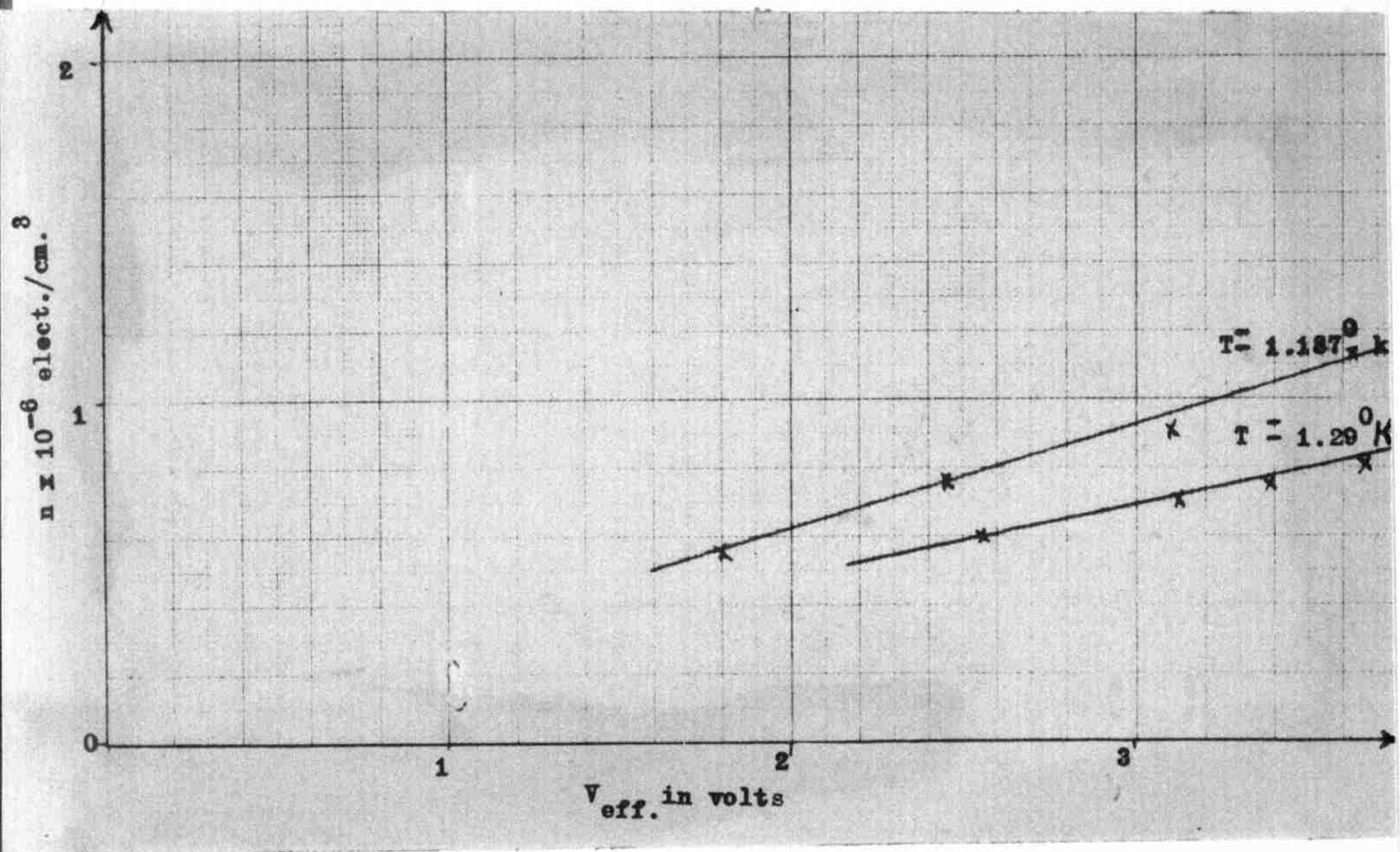


PLATE VIII

Graph of Number of Trapped Charges Versus
Speed of Rotation at V = -3.5V and
Constant Temperature

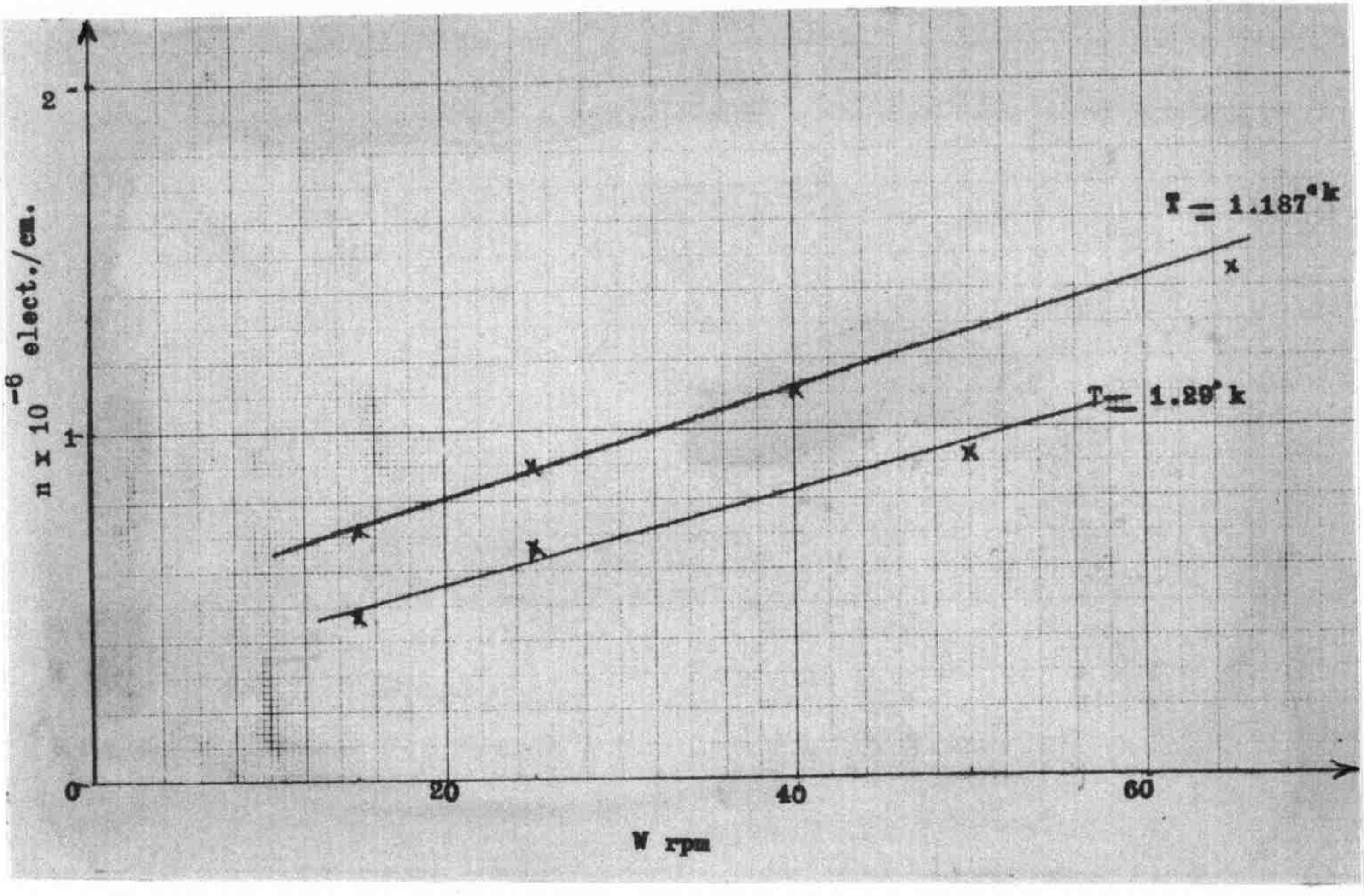
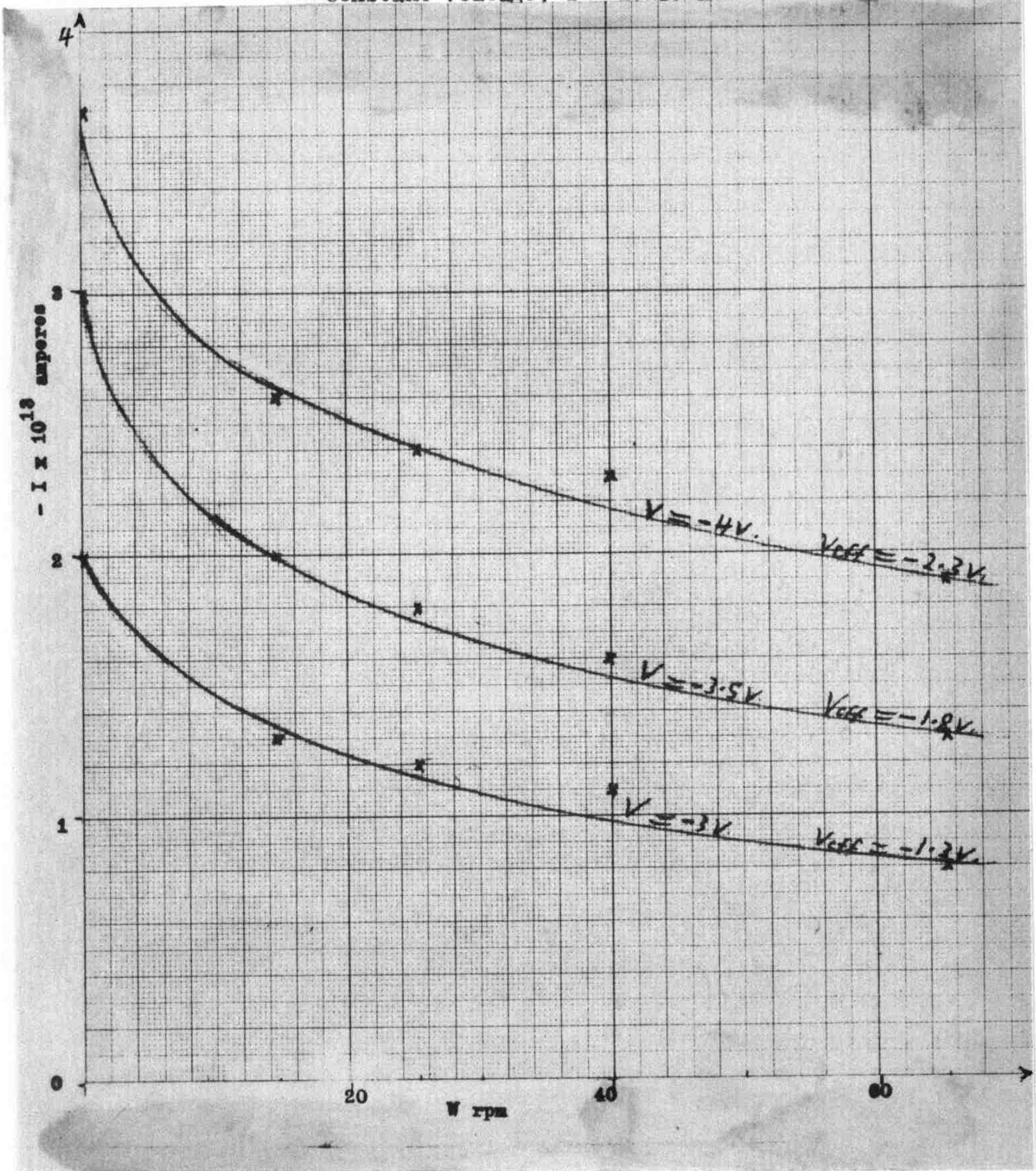


PLATE IX

Current Variation with Speed of Rotation,

Constant Voltage; T = 1.187°K.



of rotation for the two different temperatures considered.

Data for this table is used in plotting "plate VII" and "plate VIII". Plate VII gives n versus V for W = 25 rpm and constant T, while plate VIII gives n versus W for constant V and T. Furthermore, variations in I with W for constant V and T are shown in plate IX.

The trapped ion densities shown in table 5 are much smaller than those reported by Careri.[21] This difference is largely due to differences in the electric field configuration used in the two experiments, as will be discussed in part (5) of this section. We also note that a recent private communication from Careri states that he feels his attenuation is too large to be accounted for by present vortex line theory, and that he is checking for errors in his work.

We do not have any readings for voltages less than -3 volts which is the main part of the linear region. This is impossible to obtain because of the very small current which is of the same order of magnitude as the noise in our meter.

Inspite of these problems and the experimental error in the current reading, we can make the following conclusions:

(1) The trapped ion density increases with the applied voltage and approaches a saturation value (if we assume zero trapped ion density at W = 0), before the limit of the linear region. This agrees with Careri's result.

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[21] Careri, Op. Cit. P. 63.

(2) The trapped ion density increases with W but there seems to be no sign of saturation in the range investigated. This agrees with the theory given in section (1) where it was found that the number of lines (traps) = 2.1×10^3 W lines/cm^2. Thus the number of traps is expected to increase with W and hence the number of trapped charges is expected to increase also.

(3) The trapped ion density decreases when the temperature increases.

(4) "Plate IX" as well as "plates V and VI" show that the attenuation in the current approaches a saturation value very close to W = 65 rpm.

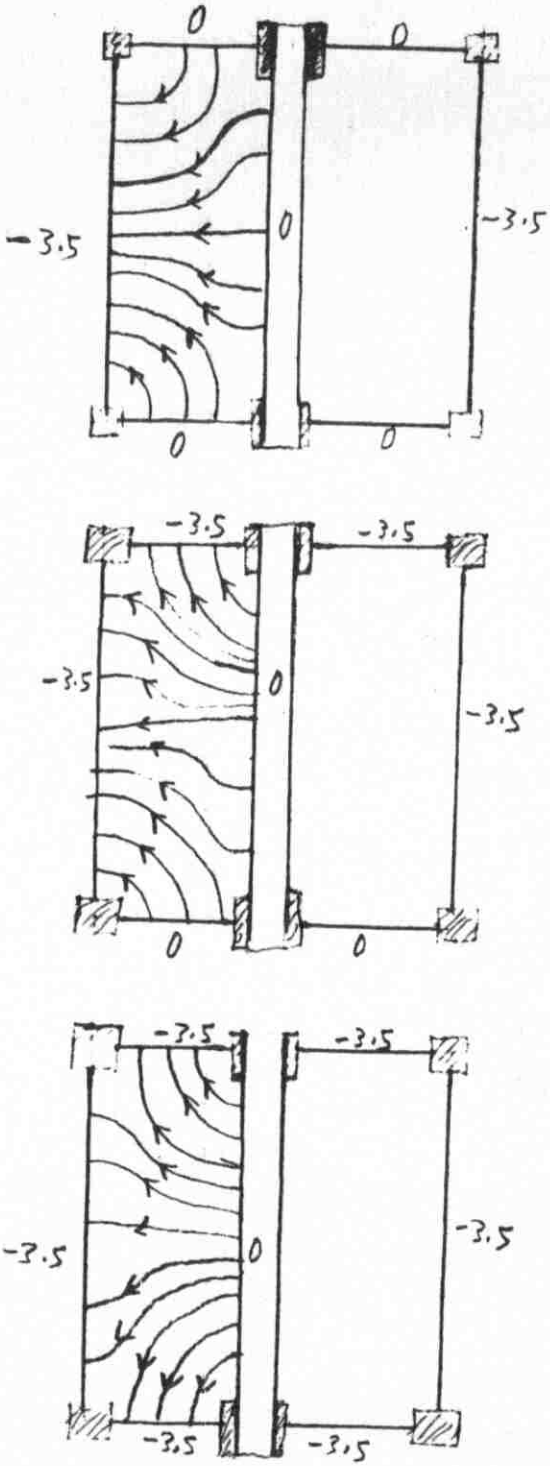
(5) Axial Fields: Three forms of electric fields have been investigated and "figure 9" shows the shape of the electric field in each case. From "Tables 1, 2 and 3" we can obtain the ratios of current with rotation, I_W, to current without rotation, I_o, as shown in "Table6". The results in this table show the following points:

a. In the non-rotating case, the current for a fixed voltage on the cylinder is a maximum when the upper and lower electrodes are grounded.

b. The ratio of I_W/I_o is minimum when the 3 electrodes are at the same potential, and is maximum when the upper and lower are grounded and has an intermediate value in between.

c. The ratios of I_W/I_o remain in the same order but all increase uniformly with temperature and decrease with the speed of rotation.

These conclusions are believed to provide evidence in favour of the Onsager - Feynman vortex line theory and Careri's



(a)

$V_1 = -3.5$ V.

$V_2 = V_3 = 0$

Minimum Trapping

(b)

$V_1 = V_3 = -3.5$ V

$V_2 = 0$

Intermediate Trapping

(C)

$V_1 = V_2 = V_3 = -3.5$ V.

Maximum Trapping.

Fig. 9.-- Electric Fields For Various Applied Voltages.

TABLE 6

RATIO OF THE CURRENT DURING ROTATION TO THAT WHEN STATIONARY FOR VARIOUS AXIAL FIELD COMPONENTS AT TWO DIFFERENT TEMPERATURES

		I_w/I.		
T^oK	Wrpm	a	b	c
1.29	25,	0.716	0.50	0.22
1.29	50,	.550	.30	.13
1.443	25,	.820	.65	.31
1.443	50,	.560	.36	.18

$^{a}V_1 = -3.5$ v, $V_2 = V_3 = 0$

$^{b}V_1 = V_3 = -3.5$ v, $V_2 = 0$.

$^{c}V_1 = V_2 = V_3 = -3.5$ V.

idea of trapped ions in these vortices. The interpretation of these results in the light of this theory are as follows:

Referring to the diagrams of the electric fields in cases (a), (b) and (c), we see that at the central part of the cylinder, where the polonium is deposited and where the main part of the current flows, in each of the 3 cases, the field has an axial as well as a radial part. However, the following differences exist between the various parts:

Case (a): The axial field component acts downward and upward (away from the center) on negative charges, and since the top and bottom electrodes are grounded, negative charges will continue being attracted there. Thus one expects little accumilation of charges at the center in the stationary state and very little trapping in the rotating state. Thus the current should be high in both rotating and stationary cases.

Case (b): The axial field acts only downward on negative charges and they are attracted to the lower electrode which is grounded. So few charges accumilate at the center (but more than in case (a). Thus the current is less than in case (a) when stationary and when rotating because more ions are trapped.

Case (c): The axial field acts toward the center on negative charges and thus tends to accumilate more charges and make the current less than the two previous cases when the bucket is stationary. When rotation starts, the presence of a large number of ions makes it more probable that more will be trapped, hence the current drops tremendously as observed.

If a flushing effect exists (that is trapped ions being moved down or up the vortex lines under the effect of an electric field), the current will be much less in cases (a) and (b) during rotation because their axial fields would drive ions away from the center and toward the upper and lower electrodes. Since our experimental result is contrary to this, it means that either such a flushing effect does not exist or if it exists it is negligible in comparison with the trapping effect. Thus all what is observed in this part is in accordance with the theory of trapped ions in vortex lines.

Careri's bucket has an electric field which corresponds to case (c) here, while we did most of our measurements on the attenuation of current under the conditions of case (a). This offers an explanation for the greater attenuation observed by Careri.

(6) Relaxation Time : For lack of data nothing will be added to what has been said about this part.

SECTION (6)

CONCLUSION

All the results obtained in this work are in accordance with what is expected from the Onsager - Feynman theory of vortex lines in rotating liquid helium II, and with Careri's idea of negative ions being trapped in these vortices.

Although few quantitative conclusions have been reached, the qualitative dependence of the current on various factors have been obtained and can be summarized as follows:

1. The trapped ion density increases with the speed of rotation.

2. The trapped ion density decreases when the temperature increases.

3. The attenuation in the current seems to move toward a saturation value at about 65 rpm.

4. The trapped ion density increases with voltage and approaches a saturation value at a voltage close to the upper limit of the linear region.

The results obtained in the case of small axial fields provide evidence in favour of the theory in general and especially the idea of trapped ions. This effect needs further investigations which would give more quantitative results which it is thought will provide conclusive evidence in favour of the fact that the "flaws" which Pellam speaks about are actually vortex lines.

As for the apparatus used, it has been found to operate quite efficiently except for two exceptions. These are the Micro-micro Ammeter and the Polonium ion source. In future work with this apparatus, a Meter should be used which is less influenced by the capacitance of the input cables so that most of the noise would be eliminated. Moreover, the effect of the noise would be diminished if larger currents are used. This means that a larger area of the cylinder should be more effectively covered with polonium.

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APPENDIX.

A. Evaluation of the constant coefficients in the solution of Poisson's equation for trapped charge ion density, which was given in section (4) of this paper:

Ghizzetti and Gross give for these coefficients the following series:

$$F_{2n}(\rho) = \frac{1}{2} l_n \frac{1+\rho}{1-\rho} - \sum_{i=1}^{n} \frac{\rho^{2i-1}}{2i-1}$$

$$F_{2n+1}(\rho) = -\frac{1}{2} l_n (1-\rho^2) - \sum_{i=1}^{n} \frac{\rho^{2i}}{2i}$$

where $\rho = \sqrt{1-x^2}$ and $x = \frac{r}{r_o}$

For our apparatus: $x = \frac{0.215}{1.150} = 0.19$

$$\therefore \quad F_2(\rho) = \frac{1}{2} l_n \left[\frac{1+\sqrt{1-(0.19)^2}}{1-\sqrt{1-(0.19)^2}} \right] - \sqrt{1-(0.19)^2}$$

$F_2(\rho) = 1.37$

and similarly $\frac{1}{3} f_3(\rho) = 0.37$.

The coefficient of the third term is $\frac{1}{36} f_4(\rho) = 0.03$.

B. Order of magnitude of the third term in the series expansion:

The third term consists of the following:

$$\frac{(0.03)\, r_o^{\,3}\, \rho_2^2 \sqrt{2\pi h} \sqrt{-\mu}}{\sqrt{\epsilon_o^{\,3}} \sqrt{I}}$$

$$r_o = 1.15 \times 10^{-2} m$$

$$2\pi = 6.28$$

$$h = 0.12 \times 10^{-2} m$$

$$\sqrt{-\mu} = 0.7 \times 10^{-2}$$

$$e_o = 0.88 \times 10^{-11}$$

$$\sqrt{I} = 1 \times 10^{-13}$$

$$\rho_2 \sim 1 \times 10^{-7}$$

When substituted these give a number of order 10^{-2} which is negligible in comparison with second term which is of order unity.

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