

AMERICAN UNIVERSITY OF BEIRUT

OPTIMIZATION OF RESOURCES IN A TRANSSHIPMENT  
CONTAINER TERMINAL: STRATEGIC AND  
OPERATIONAL PERSPECTIVES

by  
NABIL HUSSEIN NEHME

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for the degree of Doctor of Philosophy  
to the Department of Civil and Environmental Engineering  
of the Faculty of Engineering and Architecture  
at the American University of Beirut

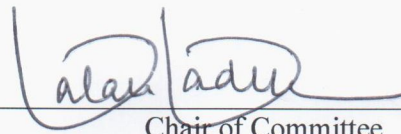
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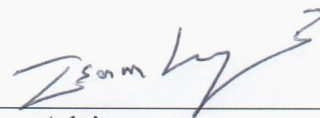
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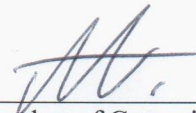
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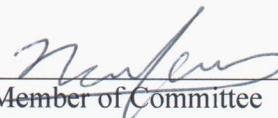
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## ACKNOWLEDGMENTS

*In the name of Allah, the Entirely Merciful, the Especially Merciful.  
He said, "Indeed, with me you will never be able to have patience". (Holy Quran, 18:67)*

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# AN ABSTRACT OF THE DISSERTATION OF

Nabil Hussein Nehme for Doctor of Philosophy  
Major: Civil Engineering

Title: Optimization of Resources in a Transshipment Container Terminal:  
Strategic and Operational Perspectives

With the robust growth in world container throughput during the last decade, every port operator is motivated to expand its business to attract more customers. In addition, to sustain its market position, a port should further develop its competitive edge to stay ahead of its competitor ports.

Challenges arise when a port authority or port operator need to decide how to improve its competitive position by investing more resources in its infrastructure and enhancing the services provided to customers since the scope of such improvements may be diversified. In deciding on resource allocation, most of port authorities and port operators do not explicitly identify and address criteria used by customers in port selection and do not consider the reaction of other competitor ports toward their expansion and investments. Furthermore, the resource allocation process in the port does not typically consider feedback between the strategic and operational levels.

This dissertation is concerned with the optimization of resources in a transshipment container terminal from both perspectives strategic and operational. We first analyze resource allocation strategies used by port authorities and container terminal operators to attract carriers from a strategic perspective. Then, we analyze the resource (mainly quay and yard cranes) allocation inside the port from an operational perspective during a transshipment process. A single ship berthing is considered during unloading of containers and an approach for optimizing the integrated allocation of quay cranes and yard cranes during such operations is proposed. An extension entails a tactical level approach that considers a multi-ship berthing scenario. Finally, we develop a proposed initial “link” between the operational and strategic levels.

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## LIST OF ABBREVIATIONS

AGV	Automated Guided Vehicles
AIMMS	Advanced Interactive Multidimensional Modeling System
AHP	Analytical Hierarchy Process
AR	Average Revenue
BCTC	Beirut Container Terminal Consortium
CPU	Central Processing Unit
CTO	Container Terminal Operator
DSS	Decision Support System
GB	Gigabyte
HCC	Handling Cost of Containers
ILP	Integer Linear Program
MPP	Marginal Physical Product
MR	Marginal Revenue
PIHCC	Percentage Improvement of Handling Cost of Containers
PISCC	Percentage Improvement of Storage Cost of Containers
QC	Quay Crane
RAM	Random Access Memory
SCC	Storage Cost of Containers
TEU	Twenty-foot Equivalent Units
YC	Yard Crane
UNCTAD	United Nations Conference for Trade and Development

*To*

*My Beloved Family*

# CHAPTER 1

## INTRODUCTION

### 1.1 Maritime Trade and Containerization Trends

The expansion of international trade was a crucial factor in the growth of the maritime transport during the last few decades. The concept of globalization in addition to potential efficiencies through consolidation has increased the investments of shipping companies in container vessels (and to some extent in container terminals) and has shifted the shipments of containers from direct freight flows into transshipment flows, in order to sustain their current market shares and to secure new markets (Nehme and Awad, 2010; Van de Voorde, 2005).

With the growth of the world container port throughput by 12.1% from 2006 to reach 487 million Twenty-foot Equivalent Units (TEU) in 2007, according to United Nations Conference for Trade and Development (UNCTAD), operators of ports all around the world sought to sustain their container traffic growth by optimizing the use of their resources (UNCTAD, 2008). Even with the worldwide financial crisis that started in summer 2008, the container port throughput increased by 4.5% to reach 509 million TEU in 2008 (UNCTAD, 2008), but decreased to 466 million TEU in 2009 (UNCTAD, 2009). In 2010, container port throughput accomplished an unexpected recovery with an increase of 12.9% to reach 526 million TEU and in 2011 container port throughput increased to 580 million TEU (UNCTAD, 2011). Figure 1.1 represents the world container port throughput from year 2006 to year 2011, while figure 1.2 represents the annual growth in world container port throughput during the last 6 years. The annual growth rate between 2006 and 2011 was 5.95 %.



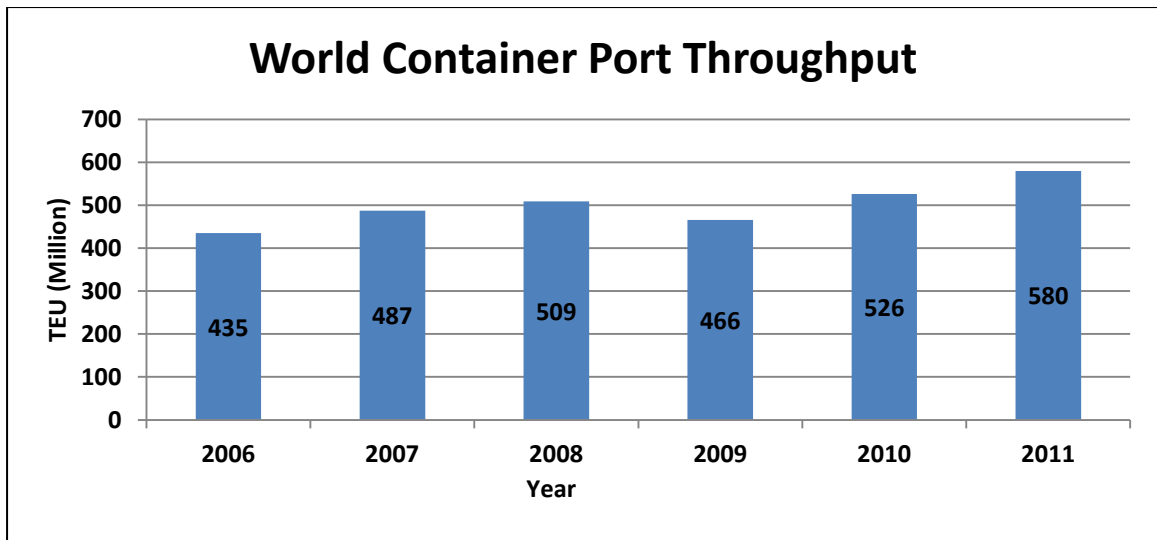


Figure 1. 1: World Container Port Throughput

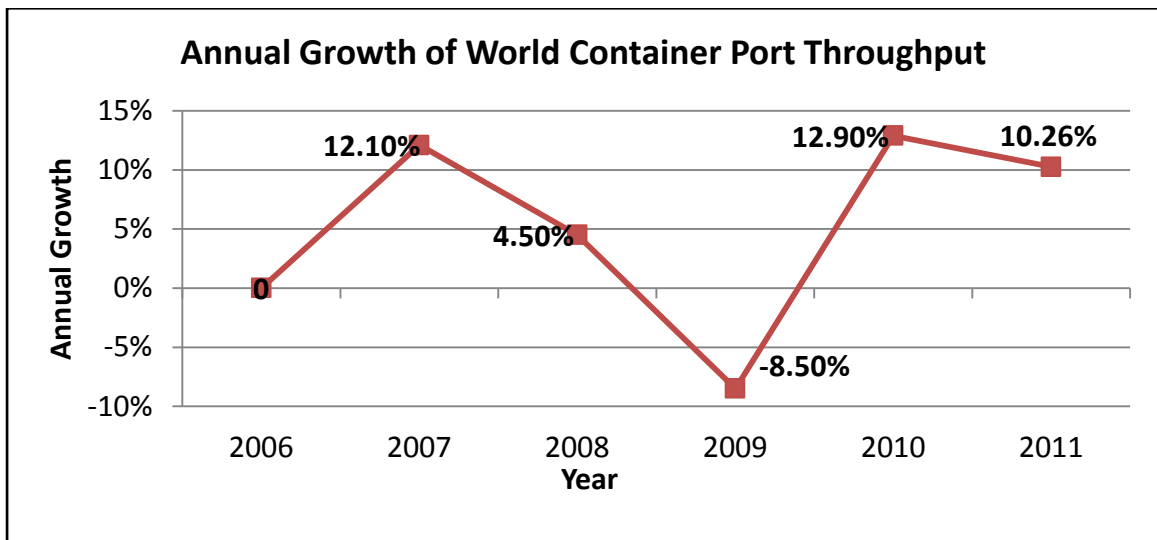


Figure 1. 2: Annual Growth of World Container Port Throughput

This chapter is structured as follows. In the second section, container terminal operations are illustrated. In the third section, the problem statement is presented. In the fourth section, feedback between strategic and operational levels of resource allocation at a container terminal is presented. In the fifth section, the scope of this research is discussed. In the sixth section, the structure of the dissertation is presented.

## 1.2 Container Terminal Operations

A typical container terminal operates as illustrated in Figure 1.3, proposed by Meisel (2011). Normally a container terminal is divided into two main areas: the quay side and the yard side.

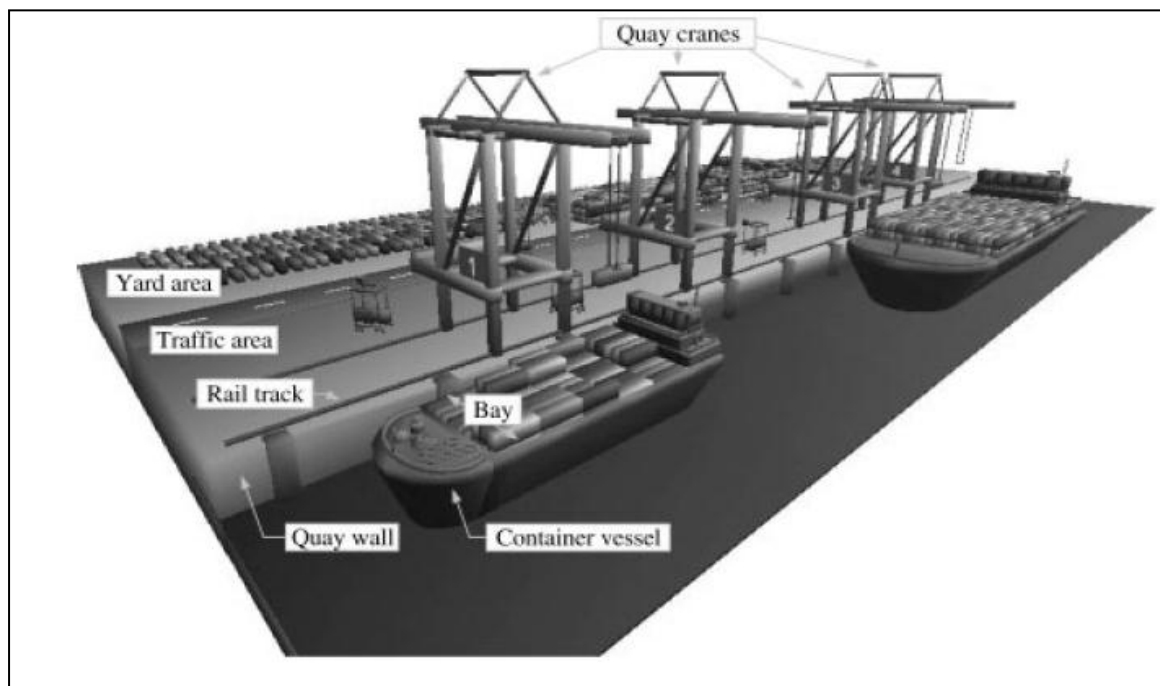


Figure 1. 3: Typical Container Terminal (Meisel, 2011)

Containers are divided into three categories: imported, exported, and transshipped. One critical port operation that involves significant use of resources is the transshipment of containers, which represented 27% (131 million TEU) of the total world container throughput in 2007 (UNCTAD, 2008). Stahlbok and Vob (2008) defined transshipment for maritime containers as “the transfer or change from one conveyance to another with a temporarily limited storage on the container yard”. In 1990, the average loading or unloading of each container was 14 times per year; in 2010, the average was estimated to be 19 times per year (UNCTAD, 2011), which reflects the growth of transshipment for maritime containers.

The loading and unloading of containers to and from vessels is processed at the quay side. When the vessel arrives at the quay side, the assigned quay cranes (QC) are used to unload both the transshipped and the imported containers onto internal trucks for movement to assigned container blocks in the yard. The specified yard cranes (YC) store the transshipped containers in the storage area, and then move the imported containers into the customer specific trucks that transport them out from the container terminal. As for the exported and the transshipped containers, they are discharged from the yard side to the quay cranes via internal trucks to be loaded into outgoing vessels (Choo, 2006). Figure 1.4, extracted from Li et al. (2009), illustrates the loading and unloading process of containers from the quay side to yard side in a container terminal.

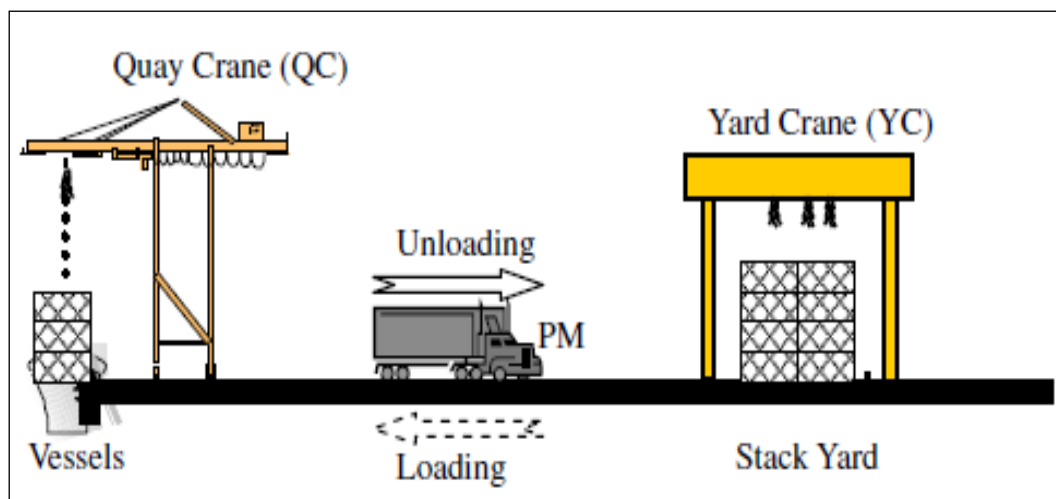


Figure 1. 4: Loading and Unloading Process of Containers (Li et al., 2009)

### 1.3 Problem Statement

With the above mentioned figures demonstrating a robust growth in world container throughput, every port operator is motivated to expand its business to attract more customers. In addition, to sustain its market position, a port should further develop its competitive edge to stay ahead of its competitor ports (Chang et al., 2008).

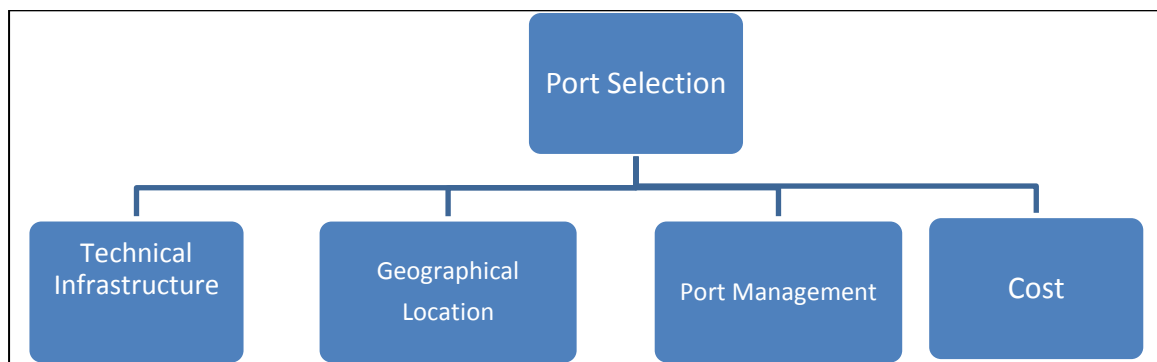
Challenges arise when the port authority or port operator need to decide how to improve its competitive position by investing more resources in its infrastructure and enhancing the services provided to customers since the scope of such improvements may be diversified. However, in deciding on resource allocation, most of port authorities and port operators (i) do not explicitly identify and address criteria used by customers in port selection and (ii) do not consider the reaction of other competitor ports toward their expansion and investments. Furthermore, the resource allocation process in the port does not typically consider feedback between the strategic and operation levels. In the next section, the feedback between the strategic and operational levels is illustrated.

#### **1.4 Feedback between Strategic Level and Operational Level**

From the strategic perspective, the main objective of the port authority or the port operator, referred to as the Container Terminal Operator (CTO), is to attract more carriers and larger volumes. From the carrier's perspective, according to Lirn et al. (2004), there exist four main criteria for port selection which mainly apply to transshipment activities. Figure 1.5 illustrates the four criteria used globally for port selection.

The first criterion is the "port physical and technical infrastructure". This criterion includes depth of the port, available number of berths, degree of integration, equipment, and terminal capacity. The second criterion is the "port geographical location" which includes proximity to import and export areas, and main navigation routes. The third criterion is "port management and administration" which includes management and administration efficiency, vessel turn-around time and port security

and safety. The fourth criterion is the “carriers’ terminal cost” which includes handling cost and storage costs of containers.



**Figure 1. 5: The Four Criteria for Port’s Selection**

From the port authority’s perspective, for the first criterion, investing in technical infrastructure requires availability of significant resources and time, which may be in short supply. As for the second criterion, the change of port location is a complex and difficult decision. Therefore, the port authority may tend to invest in the remaining criteria which are “port management” and “terminal cost”, at least in the short to medium term.

To improve “port management” performance and reduce “terminal cost”, the port authority needs to optimize the resources used during daily operations and in particular transshipment activities.

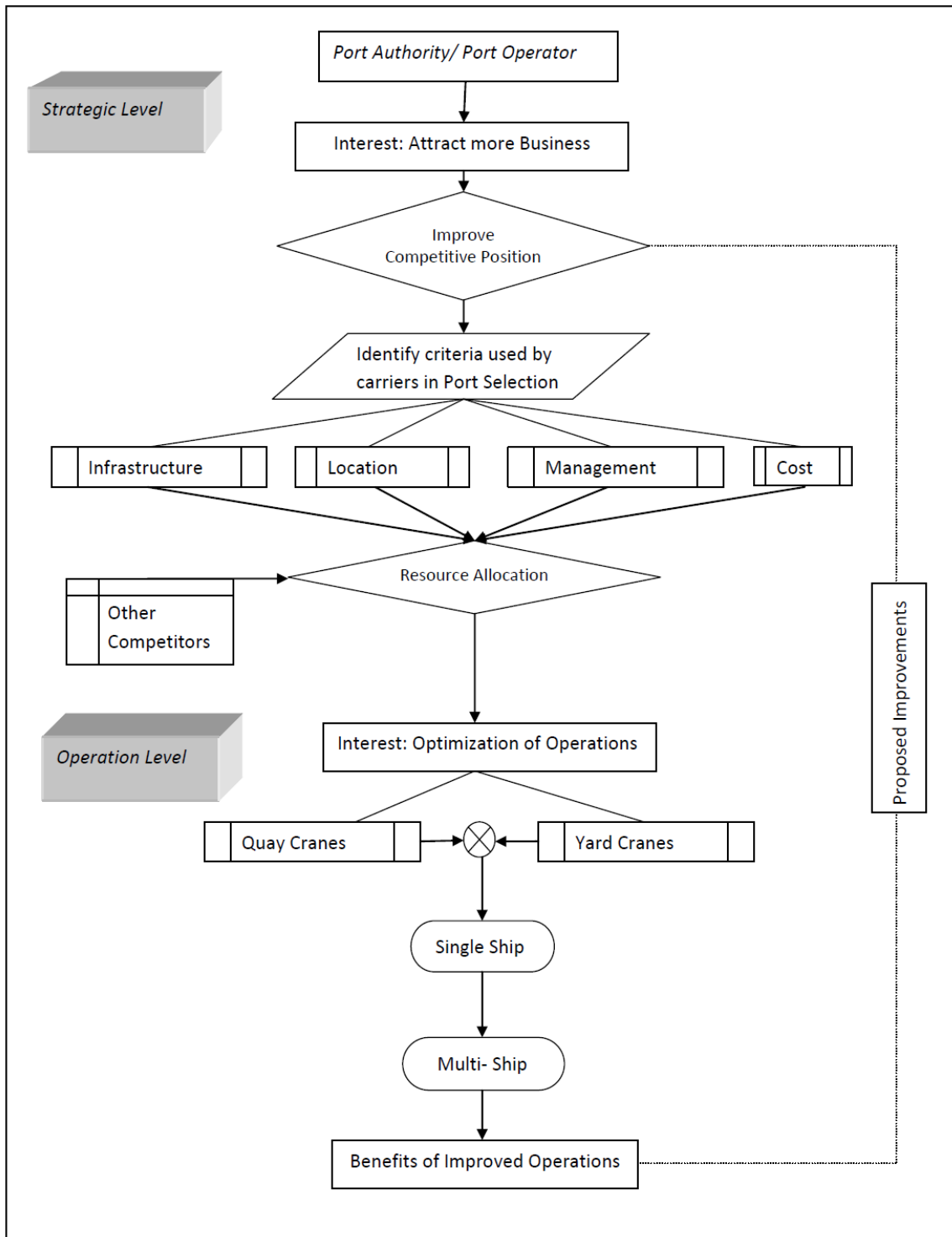
## **1.5 Scope of Research**

This dissertation is divided into three main parts. The first part analyzes resource allocation strategies used by port authorities and CTOs to attract carriers from a strategic perspective. The second part analyzes the resource (mainly quay and yard cranes) allocation inside the port from an operational perspective during a

transshipment process. A single ship berthing is considered during unloading and loading of containers and an approach for optimizing the integrated allocation of quay cranes and yard cranes during such operations is proposed. An extension entails a tactical level approach that considers a multi-ship berthing scenario. The third part develops a proposed initial “link” between the operational and strategic levels. Figure 1.6 illustrates the dissertation framework.

## **1.6 Dissertation Structure**

The remainder of this dissertation is structured as follows. Chapter 2 presents a brief review of relevant literature related to port selection and competition from strategic perspective, and resource allocation at the operation level by considering quay crane scheduling, yard crane scheduling and the integration between the quay side and yard side. Chapter 3 investigates the port selection process by carriers and the investments made by the port authority or port operator to attract more volume based on a game theory approach. Chapter 4 proposes an approach for optimizing the integration of quay side and yard side resources during the transshipment process of containers for a single ship berthing in a container terminal. Chapter 5 explores the extension of the single ship approach to the transshipment process of containers for a multi ship berthing scenario in a container terminal. Chapter 6 proposes an initial approach for linking the operational and strategic levels in the transshipment container terminal. Chapter 7 concludes the dissertation and proposes potential future research.



**Figure 1. 6: Dissertation Framework**

## CHAPTER 2

### LITERATURE REVIEW

This chapter provides a brief overview of the literature that is most relevant to the problems of interest in this dissertation. This chapter is divided into two sections. The first section discusses research related to port selection and competition. The second section investigates research related to resources allocation, mainly cranes, at the operational level.

#### **2.1 Port Selection and Competition: The Strategic Level**

Every port operator is motivated to expand its business to attract more customers. In addition, to sustain its market position, a port should further develop its competitive edge to stay ahead of its competitor ports (Chang et al., 2008). Challenges arise when the port authority or port operator need to decide how to improve its competitive position by investing more resources in its infrastructure and enhancing the services provided to customers since the scope of such improvements may be diversified. However, in deciding on resource allocation, most of port authorities and port operators (i) do not explicitly identify and address criteria used by customers in port selection and (ii) do not consider the reaction of other competitor ports toward their expansion and investments.

##### **2.1.1 Port Selection**

Several studies evaluated criteria in the carriers' port selection decisions, mainly for transshipment operations (Baird, 2000; Branch, 1986; Brooks, 2000;



Browne et al., 1989; Fleming and Baird, 1999; Frankel, 2001; Hayuth, 1995; Lam and Dai, 2012; Murphy et al., 1989; Porcari, 1999; Slack, 1985; Song and Yeo, 2004; Thomas, 1998; Ugboma et al., 2006; Villalon, 1998). There exist more than 44 port selection criteria in the literature. A comprehensive review of criteria used in the literature is presented in figures 2.1 and 2.2, which are adopted and updated from Lirn et al. (2003).

Lirn et al. (2003) studied the transshipment port selection decision by analyzing data collected from a field survey performed in Taiwan. The authors presented six major criteria for port selection from carrier's perspective in the transshipment port selection in Taiwan: (1) water depth of port, marshalling yard, (4) basic cargo volume, (5) geographical advantage, and (6) port efficiency and cost of container- handling for carriers.

Song and Yeo (2004) determined that the port competitiveness in China is based on the four main criteria: (1) Cargo Volume (number of containers), (2) Port Facility, (3) Port Location, and (4) Service Level. Using an empirical analysis of input from 70 professional experts in port management and operations, shippers, terminal operators, academics and researchers in the field, the authors conducted a pair comparing survey to determine that port location is the most powerful criterion in port selection in China with a weight of 45.2 %, followed by (2) local infrastructure and port transportation network, (3) area of the container yard and of port facility criteria with a weight of 19.8 %, then cargo volume with a weight of 17.8 %, and service level with a weight of 17.4 %.

		STUDY														
		Baird	Branch	Brooks	Browne et al.	Fleming & Baird	Frankel	Hayuth	Lam & Dai	Murphy et al.	Porcari	Slack	Song & Yeo	Thomas	Ugboma et al.	Villalon
#	Criteria	2000	1986	2000	1989	1999	2001	1995	2012	1989	1999	1985	2004	1998	2006	1998
1	Available number of berths													X		
2	Back-up space on terminal							X								
3	Congestion								X			X				
4	Cargo volume									X			X			X
5	Cargo-generating effect		X					X							X	
6	Other modes competitiveness		X		X					X						
7	Container cargo proportion				X					X						X
8	Containerised cargo proportion													X		
9	Degree of Integration						X	X							X	
10	Depth of port								X							X
11	Geographical advantage		X	X		X			X	X	X		X		X	X
12	Free time											X				
13	Frequency of feeder service			X	X		X								X	
14	Frequent port of call				X		X	X								
15	Infrastructure		X					X	X	X	X			X	X	X
16	Inland freight rates		X							X		X				
17	labour problems	X														
18	Loading/discharging rate		X	X	X									X		
19	Low cost			X	X				X	X					X	X
20	Major container centre															X
21	Numbers of sailing	X			X							X				
22	Operation							X								
23	Related business operation		X				X				X					
24	Port accessibility				X	X		X						X		

Figure 2. 1: Port Selection Criteria - 1 to 24 (adopted and updated from Lirn et al. (2003))

#	Criteria	STUDY													
		Baird 2000	Branch 1986	Brooks 2000	Browne et al. 1989	Fleming & Baird 1999	Frankel 2001	Hayuth 1995	Lam & Dai 2012	Murphy et al. 1989	Porcari 1999	Slack 1985	Song & Yeo 2004	Thomas 1998	Ugboma et al. 2006
25	Port Working hours			X	X				X				X		
26	Port berthing time length				X										
27	Port charges			X				X			X			X	
28	Port equipment		X	X						X	X	X			
29	Port security								X		X				
30	Port service coverage			X											
31	Port tradition and organisation					X									
32	Port productivity		X		X	X		X						X	
33	Proximity to alternative loading center											X			
34	Quality of customs handling											X			
35	Service considerations											X	X		
36	Size of hinterland							X							
37	Size of port terminal								X		X				
38	State aides and influence on cost														X
39	Superstructure			X				X	X	X			X		
40	Intermodal link network						X	X		X		X			X
41	Transportation and port user cost				X			X							
42	Time on the route	X		X											
43	Transist time			X	X	X									
44	Trade inertia					X									

Figure 2. 2: Port Selection Criteria - 25 to 44 (adopted and updated from Lirn et al.(2003)

Ugboma et al. (2006) used an analytic hierarchy approach (AHP) to study the port selection decisions in Nigeria by using empirical data. Criteria used in this study were: (1) Port Efficiency, (2) Adequate Infrastructure, (3) Frequency of Ship Visits, (4) Quick Responses to Port Users' Needs, (5) Location, (6) Port Charges, and (7) Ports Reputation to Cargo Damage. This study revealed that the port efficiency is the most influential factor in port selection process in Nigeria.

Lirn et al. (2004) extended the work of Lirn et al. (2003) to identify four main criteria embracing twelve sub-criteria for port selection globally defined as follow:

First Criterion: Port Physical and Technical Infrastructure

This criterion includes three sub-criteria (1) basic infrastructure condition such as water access and depth of port, (2) technical structure such available number of berths, port equipment and back –up space on terminal, and (3) intermodal links such as size of port terminal capacity, port accessibility and port service coverage.

Second Criterion: Port Geographical Location

This criterion includes three sub-criteria (1) proximity to import and export areas, (2) proximity to feeder's ports, and (3) proximity to main navigation routes.

Third Criterion: Port Management and Administration

This criterion includes three sub-criteria (1) management and administration efficiency, (2) vessel turn-around time and (3) port security and safety.

Fourth Criterion: Carriers' Terminal Cost

This criterion includes three sub-criteria (1) handling cost of containers, (2) storage costs of containers and (3) terminal ownership exclusive contract policy.

The methodology used by Lirn et al. (2004) consists of an AHP questionnaire survey that was filled by 20 port users and 20 transshipment service providers distributed all over the world in order to determine the extent of impact for each main criterion. The

results of the survey revealed that the carriers' port cost criterion represents 38.12 % of the total decision for transshipment port selection as identified by carriers, while geographical location criterion represents 35.12 %, the physical and technical infrastructure criterion represents 16.38 %, and the port management and administration criterion represents 10.38 %.

In addition, the AHP survey conducted by Lirn et al. (2004) revealed that the top five transshipment port selection sub-criteria are: (1) handling cost of container with 24.27 % weight, (2) proximity to main navigation routes with 15.12 % weight, (3) proximity to feeder ports with 10.26 % weight, (4) proximity to import and export area with 9.75 % weight, and (5) basic infrastructure condition such as water access with 8.51 % weight. Lam and Dai (2012) concluded from their own literature review that the most common criteria in port selection are: geographical location, port charges, port infrastructure, vessel calls, container traffic, and water depth. The authors developed a decision support system (DSS) for port selection using AHP technique and advanced interactive multidimensional modeling system (AIMMS) interface. The DSS is coded on VB net and Visual Basic. Microsoft software products such as Access and Excel are used to maintain the input data.

### ***2.1.2 Modeling Port Competition Using Game Theory***

As for the port competition, from a game theory approach, Anderson et al. (2008) proposed a Nash Equilibrium of a Bertrand pricing game to analyze competition between two ports based on expansion strategy i.e, invest or do not invest, in order to attract more cargo. The study considered competition between the ports of Busan and Shanghai in Asia. Pricing was based on a combination of perfect competition and oligopolistic imperfect competition. However, the value added

services of being a hub and the strategic governmental interference were both ignored in this model. The study revealed that it is advisable to develop strategies for market segments that can be defended from competitors. It was recommended that development efforts should concentrate on markets that generate higher difference in value between competitor hub ports, especially in a country with very low labour costs.

Saeed and Larsen (2010) presented a Bertrand game of competition between four container terminals in two ports via a two stage model where the first stage was to decide on the level of coalition between three container terminals in the same port and the second stage was the competition between the two ports based on the first stage results. The authors determined the net effect on the profits of all players for all possible scenarios. The study revealed that the highest benefit for all players is the grand coalition, when all players are members of the coalition.

Imai et al. (2006) analyzed the competition between container terminals using a non-zero, two-player game approach. The authors considered two service strategies: the first strategy was the hub-and-spoke network, applied to Asia-Europe route, which was modeled via minimizing the travel time from origin to destination (the minimum location) for the container mega-ship. The second strategy considered a multi-port-calling network of ordinary ships and was modeled as a classic traveling salesman problem, applied to Asia-North America route. This study revealed that the container mega-ship service was competitive in all scenarios for the European trade, while it was feasible for the North American trade only when the feeder cost and freight rate were low. In addition, this study proved the relationship between ship size selection and freight rate.

In this research, we contribute to the literature by formulating a game theory framework for port's strategic investment to attract more carriers based on the four main criteria identified by Lirn et al. (2004). We formulate a mathematical model to attract carriers taking into consideration the existence of competition among ports. This contribution is vital to understand the strategic investments in ports.

## **2.2 Resource Allocation at the Operational Level**

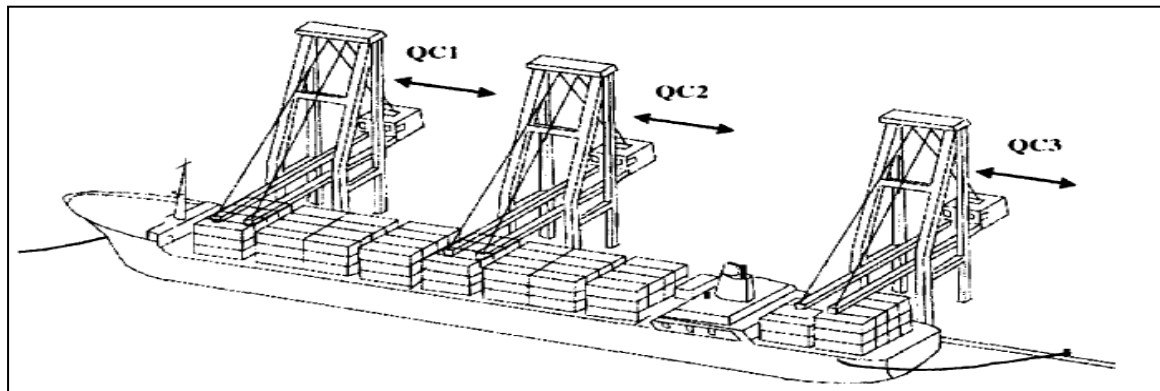
In this section, research related to quay cranes (QC) scheduling is first highlighted, then research related to yard cranes (YC) scheduling is highlighted. This part is concluded with research related to integration between QC and YC.

Limited research efforts in the field of port operations address the particular needs of transshipment hubs; most focus on import and export activities at general container terminals. Additionally, most of the literature tackles separate problems at a terminal, while the need is crucial for holistic approaches and integrated optimization of operations in different terminal areas (Stahlbok and Vob, 2008). Research on integrated optimization for control of terminal operations, such as the one in this research, is vital for improving terminal performance.

### **2.2.1 QC Scheduling**

Several researchers discussed quay crane (QC) scheduling. Daganzo (1989) studied the QC scheduling problem by developing an algorithm to schedule cranes with the objective of minimizing the aggregate cost of delay of the served ships. Peterkofsky and Daganzo (1990) developed an algorithm, using the branch and bound approach, to schedule cranes to individual bays of vessels in a specific time segment.

Figure 2.3, extracted from Choo (2006) illustrates the operation process of QC.



**Figure 2. 3: Operation Process of QC (Choo, 2006)**

Murty et al. (2005) optimized the allocation of internal trucks to QC in order to minimize the number of internal trucks and to maximize their utilization in Hong Kong International Terminals. Kim and Park (2004) formulated a mixed integer programming model to schedule QC taking into consideration various operational constraints such as the sequences of tasks, the earliest available time for QC, and the physical location of quay cranes. The authors used the branch and bound algorithm to reach optimality. Lee et al. (2008) proposed a genetic algorithm to determine a handling sequence of holds for a QC scheduling problem with non-interference constraints involving a single container vessel. The authors proved that the problem is NP-complete. Giallombardo et al. (2010) modeled the Berth Allocation Problem by considering two decisions variables: the allocation of vessels inside the container terminal and the QC assignment. The objective function aimed to optimize the resources utilization of the QC while minimizing the cost of transshipment flows between ships.

Meisel (2011) presented a new approach for the quay crane scheduling problem by restricting the availability of cranes for a ship to certain time windows during loading and unloading of containers. This restriction attempts to enhance the service of high priority ships while serving low priority ships.



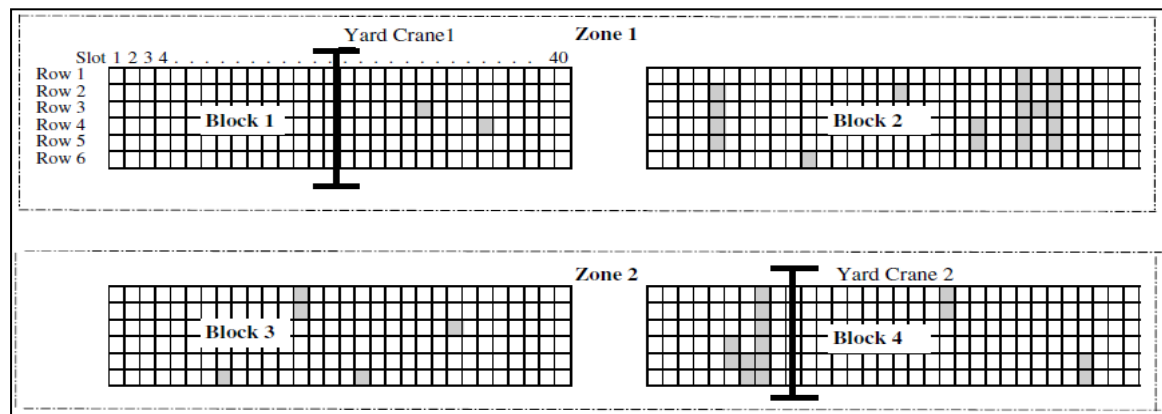
Choo et al. (2010) studied quay crane sequencing in the quay side during discharging and loading of containers. The authors formulated a mixed integer programming model to minimize berth allocation time of ships taking into consideration the clearance between quay cranes. The authors presented a new algorithm for the single ship problem, more realistic than the one presented by Daganzo (1989), by considering QC sequencing. The authors assumed that a QC can only operate at a single bay per specific time, and that the quay cranes are limited by clearance constraints and cannot cross each other's path. In the model the number of quay cranes allocated (fixed for the entire process duration), the number of bays, and the number of containers (per flag) are assumed to be known a priori. In addition, the authors considered the quay cranes to be identical in work rate and neglected the QC gantrying time and the delay in trucks delivering containers. All of these studies did not consider coordination with yard cranes as we do in this research.

### **2.2.2 YC Scheduling**

As for the yard crane (YC) deployment problem, Ng and Mak (2005) analyzed this problem by assigning YC to arriving trucks to minimize waiting times using the branch and bound algorithm. A numerical example was used to reveal the efficiency of the algorithm. Figure 2.4, extracted from Ng and Mak (2005), represents a top view of the storage area in a container terminal where the movement of yard cranes is illustrated.

Li et al. (2009) developed a model for YC scheduling by taking into account realistic operational constraints such as inter-crane interference, fixed YC separation distances and simultaneous storage and retrieval, with a multi-criteria objective of minimizing

retrieval and storage delays. The computational results for this model required an extensive amount



**Figure 2. 4: Top View of Storage Area (Ng and Mak, 2005)**

of time to reach a good solution, especially when retrieval moves are involved, and the time to reach a solution increased exponentially as the number of jobs increased. In order to decrease the time required to reach the solution, the authors assumed that the job finish time of each move is placed inside a certain range around its target time. Kozan and Preston (2006) presented an integrating algorithm that determines both the optimal locations and the corresponding handling schedule for the YC. In order to solve large problems, three algorithms were used (1) a genetic algorithm, (2) a tabu search algorithm and a (3) tabu search/genetic algorithm hybrid.

Lin (2000) studied the problem of scheduling of YC considering multiple time periods with the objectives of balancing the workload and minimizing the unfinished work of YC at the end of each time period. Lee et al. (2006) investigated the yard storage problem in a transshipment hub. They formulated a mixed integer programming model to minimize the number of YC to deploy and to determine the storage locations of unloaded containers. Cordeau et al. (2007) solved the resource allocation problem inside the container terminal by minimizing the intensity of traffic service in the yard. The objective was to minimize the handling operations inside the yard area. Travel

distance was the decision criterion. A branch and bound algorithm was applied to solve real data applied to Gioia Tauro Maritime Container Terminal. Cao et al. (2010) considered the integration between yard trucks and yard cranes in studying the yard crane deployment. The authors formulated a mathematical model to minimize the makespan (total time required) of loading all outbound containers in a finite time horizon. The authors proposed heuristic algorithms based on Benders' decomposition to reach a near optimal solution. Chen and Langevin (2012) formulated a mixed integer programming model for scheduling multiple yard cranes during loading operations. The objective of the model is to minimize the makespan of all the loading operations. Interference between yard cranes, movement of yard cranes among container blocks, and sequencing of yard cranes within each block are considered. A genetic algorithm and a tabu search algorithm were used to reach near optimal solutions. The authors deduced that tabu search surpassed the genetic algorithm in the efficiency of reaching a solution for large sized problems. These papers did not consider coordination with the quay side as we do in this research.

### ***2.2.3 Integration between QC and YC***

Few papers considered integrated quay crane and yard crane decisions. Gambardella et al. (2001) solved the problem of allocating and scheduling quay crane (QC) and yard crane (YC) utilizing a network model. Gambardella et al.'s work is different from ours in that (i) resource allocation and scheduling are handled separately through different models and (ii) the sequence of containers loading and unloading is assumed to be given. Lau and Zhao (2008) considered the problem of scheduling automated guided vehicles (AGV), QC and automated YC with the objective of minimizing the total travel and delay costs. Lau and Zhao's work is

different from ours in that (i) they assumed that the number of used quay cranes and automated yard cranes is given, and (ii) the tasks assigned to each QC are predetermined, in contrast to our model where we determine the number of quay cranes and yard cranes at every time period.

## CHAPTER 3

### STRATEGIC LEVEL: PORT SELECTION

In this chapter, the optimal investment strategy by the port authority or container terminal operator (CTO) is investigated based on a game theory approach that takes into account a number of port selection criteria by carriers. The methodology suggested in this study is to develop a utility function that represents carrier preference for a specific port, and then maximize the utility subject to financial, physical, and location constraints. This chapter is structured as follows. In the first section the formulated mathematical model is presented along with utility functions and constraints. In the second section the game type process is illustrated. In the third section sensitivity analysis for the weight used is conducted via five different scenarios. In the fourth section resource allocation analysis is presented via five selective cases. In the fifth section insights are drawn from the port management perspective.

#### **3.1 Mathematical Model**

Lirn et al. (2004) identified four main criteria for port selection globally defined as follow:

##### First Criterion: Port Physical and Technical Infrastructure

This criterion includes depth of the port, available number of berths, degree of Integration, equipments, and terminal capacity.

##### Second Criterion: Port Geographical Location

This criterion includes proximity to import and export areas, feeder's ports, and main navigation routes.

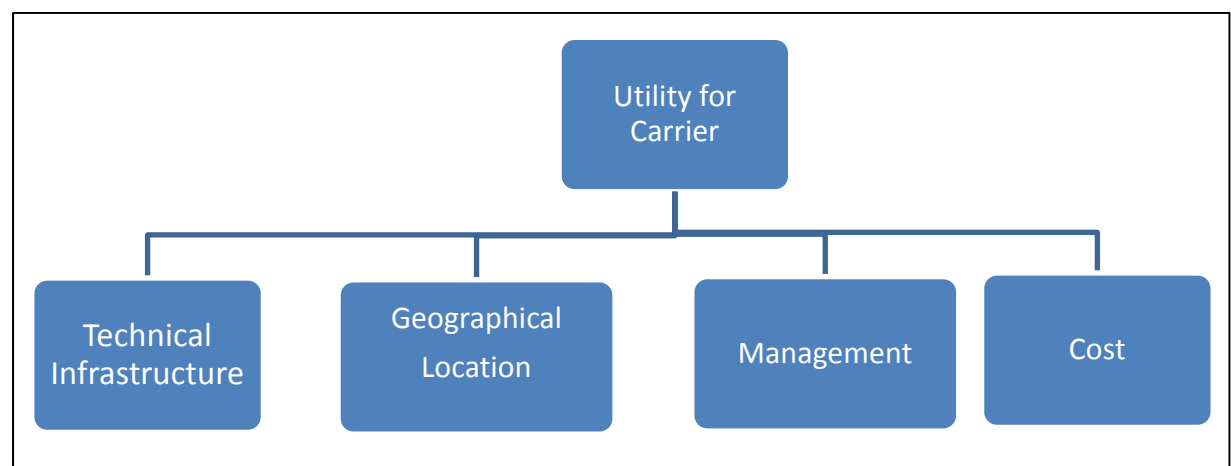
### Third Criterion: Port Management and Administration

This criterion includes management and administration efficiency, vessel turn-around time and port security and safety.

### Fourth Criterion: Carriers' Terminal Cost

This criterion includes handling cost of containers, storage costs of containers and terminal ownership exclusive contract policy.

Based on the above global criteria we develop a utility function for port selection from the carrier's perspective. Figure 3.1 summarizes the suggested criteria for carrier to choose a port.



**Figure 3. 1: Criteria used for port selection according to carrier's utility**

In the next sub-sections, we first define the utility functions used for port selection from both revenue and cost perspectives. Then, in the second sub-section, we identify the constraints that bound the port investments.

#### ***3.1.1 Utility Functions for Port Selection***

We define the attractiveness for a carrier to select a specific port  $i$  to be:

$$u_i = w_1a_i + w_2b_i + w_3c_i + w_4d_i \quad \text{Equation (3.1)}$$

Where:

- $w_1$  is the weight for the technical infrastructure criterion
- $a_i$  is the score on Likert scale ( from 1 to 5, 1 = lowest scale and 5 = highest scale) for port  $i$  with respect to the technical infrastructure criterion
- $w_2$  is the weight for geographical location criterion
- $b_i$  is the score on Likert scale ( from 1 to 5, 1 = lowest scale and 5 = highest scale) for port  $i$  with respect to the port geographical criterion
- $w_3$  is the weight for the port management and administration criterion
- $c_i$  is the score on Likert scale ( from 1 to 5, 1 = lowest scale and 5 = highest scale) for port  $i$  with respect to the port management and administration criterion
- $w_4$  is the weight for the carrier's terminal cost criterion
- $d_i$  is the score on Likert scale ( from 1 to 5, 1 = lowest scale and 5 = highest scale) for port  $i$  with respect to the carrier's terminal cost criterion

The scores of  $a_i$ ,  $b_i$ ,  $c_i$  and  $d_i$  are determined on Likert scale via questionnaires sent to carriers to evaluate the performance of ports and container terminals for the four mentioned criteria, similar to the methodology used by Lirn et al. (2004).

Since the summation of weights should be equal to 1, then

$$w_1 + w_2 + w_3 + w_4 = 1 \quad \text{Equation (3.2)}$$

In this research, the value of  $w_1$ ,  $w_2$ ,  $w_3$ , and  $w_4$  are determined from the study conducted by Lirn et al. (2004). Therefore, we consider the following weights:

- $w_1 = 16.38 \%$
- $w_2 = 35.12 \%$
- $w_3 = 10.38 \%$

- $w_4 = 38.12 \%$

We define the utility of a carrier, assuming a rational behavior, to choose a specific port  $i$  among a set of ports  $N$  as:

$$V = \max_{i \in N} u_i \quad \text{Equation (3.3)}$$

Equations (3.1) and (3.3) are easily solved if the competition to attract carriers among ports is ignored. However, this competition exists. Every port  $i$  is aware of the existence of other ports and seeks to increase its utility function by investing in the port. The investment in ports is translated according to Equation (3.1) in increasing the value of  $a$ ,  $b$ ,  $c$  and  $d$  (base or current situation) to  $a + \Delta a$ ,  $b + \Delta b$ ,  $c + \Delta c$ , and  $d + \Delta d$  (after investment). Thus, Equation (3.1) becomes

$$u_i = w_1(a + \Delta a) + w_2(b + \Delta b) + w_3(c + \Delta c) + w_4(d + \Delta d) \quad \text{Equation (3.4)}$$

The new attributes  $a + \Delta a$ ,  $b + \Delta b$ ,  $c + \Delta c$ , and  $d + \Delta d$  have enhancement boundaries such that

Upper Bound Condition: the maximum scale achieved on Likert scale is 5, the port has limitation on the level of enhancement.

$$a + \Delta a, b + \Delta b, c + \Delta c, d + \Delta d \leq 5 \quad \text{for every port } i \quad \text{Equation (3.5)}$$

Lower Bound Condition: After any investment, if any, the base or current situation of a specific port should not be more than the “after investment situation”.

$$\Delta a, \Delta b, \Delta c, \Delta d \geq 0 \quad \text{for every port } i \quad \text{Equation (3.6)}$$

The port is selected by the carrier according to Equation (3.3).

### 3.1.1.1 Payoff for Port

Every variation in the criteria due to investment leads to additional cost

$C(\Delta a, \Delta b, \Delta c, \Delta d)$  for the port. As such the payoff function for a specific port  $i$  is

$$\pi_i = P_i \cdot I_i - C(\Delta a, \Delta b, \Delta c, \Delta d) \quad \text{Equation (3.7)}$$



Where

- $P_i$  is the revenue generated by a port  $i$  when the carrier choose the specific port  $i$ .

The revenue generated by port is affected by (1) the port operation costs, (2) the size of the carrier and (3) the agreement made with the carrier. The utility function of the revenue is defined in the next sub-section.

- $I_i$  is a binary variable such that  $I_i = \begin{cases} 1 & \text{if port } i \text{ is selected by the carrier} \\ 0 & \text{otherwise} \end{cases}$

Since we are considering that the carrier will select only one port for agreement, then

$$\sum_{i \in I} I_i = 1 \quad \text{Equation (3.8)}$$

- $C(\Delta a_i, \Delta b_i, \Delta c_i, \Delta d_i)$  is the cost function occurred due to port investments in the four criteria to compete with other ports to attract the carrier.

### 3.1.1.2 Revenue function

In developing the revenue function generated from investing in the port, we assume that the function shall have diseconomies of scale based on diminishing marginal productivity. These assumptions are reasonable due to the large scale investment required for port development which is likely to entail some losses in efficiency when increasing the inputs as discussed by “Adam Smith” in analyzing the production of pins (Snyder and Nicholson, 2008). These assumptions are supported by Haralambides (2002) for applications related to ports. The author discussed the case where the port is faced with a situation where the demand for its services is higher than its handling capacity which leads to over utilization of port capacity, more accidents in cargo handling, imposing of surcharges on shippers by carriers and more

demurrages claims. In this case, the port incurs diseconomies of scale and has to allocate its scarce resources according to carriers' willingness to pay (Haralambides, 2002).

The initial revenue function for a port  $i$  is defined as follow:

$$P_i = a_i(1 - e^{-a_i}) + b_i(1 - e^{-b_i}) + c_i(1 - e^{-c_i}) + d_i(1 - e^{-d_i}) \quad \text{Equation (3.9)}$$

This revenue function is the current payoff function before any investment in any criterion in the four mentioned criteria.

**LEMMA 1: The initial revenue function has diseconomies of scale**

**Proof:**

$$\text{Consider } P(a) = a(1 - e^{-a})$$

$$\text{Marginal Revenue: } MR = \frac{\partial P(a)}{\partial a} = \frac{\partial a(1 - e^{-a})}{\partial a} = 1 - e^{-a} + ae^{-a}$$

$$\text{Average Revenue: } AR = \frac{P(a)}{a} = \frac{a(1 - e^{-a})}{a} = 1 - e^{-a}$$

$$\frac{AR}{MR} = \frac{1 - e^{-a}}{1 - e^{-a} + ae^{-a}} = \frac{1}{ae^{-a}} < 1, \text{ thus we have diseconomies of scale for product } a$$

Same applies for products  $b$ ,  $c$ , and  $d$  by considering  $P(b)$ ,  $P(c)$ , and  $P(d)$ . Since the initial revenue function is a summation of  $P(a)$ ,  $P(b)$ ,  $P(c)$ , and  $P(d)$ , therefore the function has diseconomies of scales for products  $a$ ,  $b$ ,  $c$ , and  $d$ . ■

The revenue function for a port  $i$  after investment is:

$$P_i = (a_i + \Delta a_i)(1 - e^{-(a_i + \Delta a_i)}) + (b_i + \Delta b_i)(1 - e^{-(b_i + \Delta b_i)}) + (c_i + \Delta c_i)(1 - e^{-(c_i + \Delta c_i)}) + (d_i + \Delta d_i)(1 - e^{-(d_i + \Delta d_i)}) \quad \text{Equation (3.10)}$$

**LEMMA 2: The revenue function after investment has a diminishing marginal productivity**

**Proof:**

$$\text{Consider } P(a) = (a + \Delta a)(1 - e^{-(a + \Delta a)})$$

*Marginal Physical Product (MPP) or Marginal Revenue (MR) for product a*

*(Nicholson, 1998) is:*

$$MPP_a = \frac{\partial P(a)}{\partial \Delta a} = \frac{\partial (a + \Delta a)(1 - e^{-(a + \Delta a)})}{\partial \Delta a} = a e^{-(a + \Delta a)} + 1 - e^{-(a + \Delta a)} + \Delta a e^{-(a + \Delta a)}$$

$$\frac{\partial P(a)}{\partial \Delta a} = 1 + e^{-(a + \Delta a)}(a + \Delta a - 1)$$

*Now considering the second derivative:*

$$\frac{\partial MPP_a}{\partial \Delta a} = \frac{\partial (1 + e^{-(a + \Delta a)}(a + \Delta a - 1))}{\partial \Delta a} = e^{-(a + \Delta a)} - e^{-(a + \Delta a)}(a + \Delta a - 1)$$

$$\frac{\partial MPP_a}{\partial \Delta a} = e^{-(a + \Delta a)}(2 - (a + \Delta a))$$

$\frac{\partial MPP_a}{\partial \Delta a}$  is  $< 0$  if  $(a + \Delta a) > 2$ , or the minimum value for  $a$  is 1 (Likert scale) and the

minimum investment unit is 1 unit. Therefore  $P(a)$  has a diminishing marginal

productivity (Nicholson, 1998). Same applies for products  $b$ ,  $c$ , and  $d$  by considering

$P(b)$ ,  $P(c)$ , and  $P(d)$ . ■

### 3.1.1.3 Cost Function

The cost function is defined as follow:

$$C(\Delta a_i, \Delta b_i, \Delta c_i, \Delta d_i) = a_i * \Delta a_i^{\Delta a_i + b_i} + b_i * \Delta b_i^{\Delta b_i} + c_i * \Delta c_i^{\Delta c_i} + d_i * \Delta d_i^{\Delta d_i}$$

Equation (3.11)

In the absence of any increase (investment) in a specific attribute, the term of the attribute becomes zero. For instance if  $\Delta a$  and  $\Delta b$  are zero equation Equation (3.11)

becomes

$$C(\Delta a, \Delta b, \Delta c, \Delta d) = c * \Delta c^{\Delta c} + d * \Delta d^{\Delta d}$$

**LEMMA 3: The cost function has an increasing return to scale property**

**Proof:**

Consider  $F(m\Delta a, m\Delta b, m\Delta c, m\Delta d)$ , where  $m$  is input constant.

$$F(m\Delta a, m\Delta b, m\Delta c, m\Delta d) = a * m\Delta a^{m\Delta a} + b * m\Delta b^{m\Delta b} + c * m\Delta c^{m\Delta c} + d * m\Delta d^{m\Delta d}$$

In this case  $F(m\Delta a, m\Delta b, m\Delta c, m\Delta d) > m F(\Delta a, \Delta b, \Delta c, \Delta d)$ . Therefore, the cost function has an increasing return to scale property (Nicholson, 1998). ■

### 3.1.2 Constraints

In this sub-section, we discuss the financial, physical and location constraints that limit the investment in the port.

#### 3.1.2.1 Budget Constraint

There is a budget constraint that restricts the “investment” for every port  $i$  based on fund availability such that:

$$\Delta a_i + \Delta b_i + \Delta c_i + \Delta d_i \leq budget_i \quad \text{Equation (3.12)}$$

It is vital to differentiate between the budget constraint (Equation 3.12) and the cost function (Equation 3.11). The term “budget” in this context is related to the level of possible improvement in the port, while the “cost” term in this context is related to the expenditures or fees related to such improvement.

#### 3.1.2.2 Capacity Constraint

Every port  $i$  has a capacity constraint due to physical infrastructure capacity of the port

$$\Delta a_i \leq capacity_i \quad \text{Equation (3.13)}$$

### 3.1.2.3 Location Constraint

Unless the port operator can shift location of the port, no investment could be done for the attribute of “Port Geographical Location”, thus

$$\Delta b_i = 0 \quad \text{Equation (3.14)}$$

### 3.1.2.4 Manpower Constraint

The enhancement of (i) management efficiency, (ii) vessel turn-around and (iii) port security and safety is restricted by the availability of manpower resources.

$$\Delta c_i \leq \text{manpower}_i \quad \text{Equation (3.15)}$$

### 3.1.2.5 Price Constraint

Every port  $i$  has its own price policy that could not be altered drastically, especially regarding handling cost and storage cost of containers.

$$\Delta d_i \leq \text{price}_i \quad \text{Equation (3.16)}$$

## **3.2 Game Theory Process**

In this section, the game theory process is illustrated. First, the game type suggested in this research is presented. Then, a numerical example is presented to illustrate the game process.

### **3.2.1 Game Type**

The proposed methodology suggested in this part of the thesis is modeling the port selection process using game theory approach. The game type suggested is a

first-price sealed-bid auction. The set of actions of each player is the set of possible bids (Osborne, 2009).

In this study the term player refers to port authority or port operator in charge of the potential investment in the four criteria: (1) port physical and technical Infrastructure, (2) port geographical location, (3) port management and administration, and (4) carriers' terminal cost. The bid term refers to investment made. The objective of the player is to maximize the utility function from carrier's perspective to "win" the bid. The assumptions made here are:

- Every player has perfect and complete information about other players. In this case, each player knows the set of available choices for him and for the other players, the payoff functions of each possible choice made or strategy pursued by him or by the other players, and is aware that other players have complete information about him (Osborne, 2009).
- Factor of Time and number of rounds: 1 round only (Sealed-Bid).

We illustrate the game theory process via a numerical illustration.

### ***3.2.2 Numerical Illustration***

Consider 2 different ports as follows.

#### **3.2.2.1 Port 1 Characteristics**

Port 1 has scored: 3 over 5 for the first criterion "Port Physical and Technical Infrastructure"  $a_1 = 3$ , 4 over 5 for the second criterion "Port Geographical Location"  $b_1 = 4$ , 3 over 5 for the third criterion "Port Management and Administration"  $c_1 = 3$ , and 3 over 5 for the fourth criterion "Carriers' Terminal Cost"  $d_1 = 3$ . The current

attractiveness of Port 1 to a specific carrier is  $u_1 = w_1a_1 + w_2b_1 + w_3c_1 + w_4d_1 = 16.38$   
 $\% * 3 + 35.12 \% * 4 + 10.38 \% * 3 + 38.12 \% * 3 = 3.3512$ .

The constraints for Port 1 are as follows:

- Budget Constraint: Port 1 has a fund availability of 5 units

$$\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 5$$

- Capacity Constraint: Port 1 can enhance its technical infrastructure by 2 units

$$\Delta a_1 \leq 2$$

- Location Constraint: Port 1 is restricted by the geographical location; the location in this case cannot be enhanced.  $\Delta b_1 = 0$ .

- Manpower Constraint: Port 1 can enhance its management and administration by investing 2 units in its manpower

$$\Delta c_1 \leq 2$$

- Price Constraint: Port 1 enhance the fourth criterion by reducing its price to carrier by 1 unit

$$\Delta d_1 \leq 1$$

In this case Port 1 has 18 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 1$
7.  $\Delta a = 1, \Delta b = 0, \Delta c = 0, \Delta d = 0$
8.  $\Delta a = 1, \Delta b = 0, \Delta c = 0, \Delta d = 1$
9.  $\Delta a = 1, \Delta b = 0, \Delta c = 1, \Delta d = 0$

10.  $\Delta a = 1, \Delta b = 0, \Delta c = 1, \Delta d = 1$
11.  $\Delta a = 1, \Delta b = 0, \Delta c = 2, \Delta d = 0$
12.  $\Delta a = 1, \Delta b = 0, \Delta c = 2, \Delta d = 1$
13.  $\Delta a = 2, \Delta b = 0, \Delta c = 0, \Delta d = 0$
14.  $\Delta a = 2, \Delta b = 0, \Delta c = 0, \Delta d = 1$
15.  $\Delta a = 2, \Delta b = 0, \Delta c = 1, \Delta d = 0$
16.  $\Delta a = 2, \Delta b = 0, \Delta c = 1, \Delta d = 1$
17.  $\Delta a = 2, \Delta b = 0, \Delta c = 2, \Delta d = 0$
18.  $\Delta a = 2, \Delta b = 0, \Delta c = 2, \Delta d = 1$

Table 3.1 represents all the possible investments in the first five columns, the attractiveness utility of each port for the carrier in column 6, revenue generated in case of attracting the carrier in column 7, cost of investment in column 8, and payoff generated (revenue – cost) in column 9 for Port 1.

**Table 3. 1: Possible Investment Scenario for Port 1**

	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>u1</b>	<b>Revenue</b>	<b>Cost</b>	<b>Payoff</b>
<b>Investment</b>	<b>3</b>	<b>4</b>	<b>3</b>	<b>3</b>	<b>3.35</b>	<b>12.48</b>	<b>0</b>	<b>12.48</b>
<b>(0,0,0,0)</b>	0	0	0	0	3.35	12.48	0	12.48
<b>(0,0,0,1)</b>	0	0	0	1	3.73	13.55	3	10.55
<b>(0,0,1,0)</b>	0	0	1	0	3.46	13.55	3	10.55
<b>(0,0,1,1)</b>	0	0	1	1	3.84	14.63	6	8.63
<b>(0,0,2,0)</b>	0	0	2	0	3.56	14.59	12	2.59
<b>(0,0,2,1)</b>	0	0	2	1	3.94	15.67	15	0.67
<b>(1,0,0,0)</b>	1	0	0	0	3.52	13.55	3	10.55
<b>(1,0,0,1)</b>	1	0	0	1	3.90	14.63	6	8.63
<b>(1,0,1,0)</b>	1	0	1	0	3.62	14.63	6	8.63
<b>(1,0,1,1)</b>	1	0	1	1	4.00	15.71	9	6.71
<b>(1,0,2,0)</b>	1	0	2	0	3.72	15.67	15	0.67
<b>(1,0,2,1)</b>	1	0	2	1	4.10	16.75	18	-1.25
<b>(2,0,0,0)</b>	2	0	0	0	3.68	14.59	12	2.59
<b>(2,0,0,1)</b>	2	0	0	1	4.06	15.67	15	0.67
<b>(2,0,1,0)</b>	2	0	1	0	3.78	15.67	15	0.67
<b>(2,0,1,1)</b>	2	0	1	1	4.16	16.75	18	-1.25
<b>(2,0,2,0)</b>	2	0	2	0	3.89	16.71	24	-7.29
<b>(2,0,2,1)</b>	2	0	2	1	4.27	17.79	27	-9.21



### 3.2.2.2 Port 2 Characteristics

Port 2 has scored: 2 over 5 for the first criterion “Port Physical and Technical Infrastructure”  $a_2 = 2$ , 3 over 5 for the second criterion “Port Geographical Location”  $b_2 = 3$ , 3 over 5 for the third criterion “Port Management and Administration”  $c_2 = 3$ , and 5 over 5 for the fourth criterion “Carriers’ Terminal Cost”  $d_2 = 5$ . The current attractiveness of Port 2 to a specific carrier is  $u_2 = w_1a_2 + w_2b_2 + w_3c_2 + w_4d_2 = 16.38\% * 2 + 35.12\% * 3 + 10.38\% * 3 + 38.12\% * 5 = 3.5986$ .

The constraints for Port 2 are as follows:

- Budget Constraint: Port 2 has a fund availability of 5 units

$$\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 3$$

- Capacity Constraint: Port 2 can enhance its technical infrastructure by 2 units

$$\Delta a_2 \leq 2$$

- Location Constraint: Port 2 is restricted by the geographical location; the location in this case cannot be enhanced.

$$\Delta b_2 = 0$$

- Manpower Constraint: Port 2 can enhance its management and administration by investing 2 units in its manpower

$$\Delta c_1 \leq 2$$

- Price Constraint: Port 1 enhance the fourth criterion by reducing its price to carrier by 1 unit  $\Delta d_1 \leq 1$ . Or since already Port 2 scored 5 over 5 on this criterion, any reduction of price (enhancing the the score of  $d_1$ ) is redundant.

$$\text{Thus, } \Delta d_2 = 0$$

In this case Port 2 has 8 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$

3.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
4.  $\Delta a = 1, \Delta b = 0, \Delta c = 0, \Delta d = 0$
5.  $\Delta a = 1, \Delta b = 0, \Delta c = 1, \Delta d = 0$
6.  $\Delta a = 1, \Delta b = 0, \Delta c = 2, \Delta d = 0$
7.  $\Delta a = 2, \Delta b = 0, \Delta c = 0, \Delta d = 0$
8.  $\Delta a = 2, \Delta b = 0, \Delta c = 1, \Delta d = 0$

Table 3.2 represents all the possible investments in the first five columns, the attractiveness utility of each port for the carrier in column 6, revenue generated in case of attracting the carrier in column 7, cost of investment in column 8, and payoff generated (revenue – cost) in column 9 for Port 2.

**Table 3. 2: Possible Investment Scenario for Port 2**

	a	b	c	d	u2	Revenue	Cost	Payoff
<b>Investment</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>5</b>	<b>3.60</b>	<b>12.40</b>	<b>0</b>	<b>12.40</b>
<b>(0,0,0,0)</b>	0	0	0	0	3.60	12.40	0	12.40
<b>(0,0,1,0)</b>	0	0	1	0	3.70	13.47	3	10.47
<b>(0,0,2,0)</b>	0	0	2	0	3.81	14.51	12	2.51
<b>(1,0,0,0)</b>	1	0	0	0	3.76	13.52	2	11.52
<b>(1,0,1,0)</b>	1	0	1	0	3.87	14.59	5	9.59
<b>(1,0,2,0)</b>	1	0	2	0	3.97	15.63	14	1.63
<b>(2,0,0,0)</b>	2	0	0	0	3.93	14.59	8	6.59
<b>(2,0,1,0)</b>	2	0	1	0	4.03	15.67	11	4.67

### 3.2.2.3 Game Process

In this case, we have 18 possible investment options for Port1 and 8 possible investment options for Port 2. In order to understand the reaction of players toward every possible investment, we need to evaluate each possible investment option of player 1 (Port 1) with all possible investment option of player 2 (Port 2) and vice versa. Hence, we need to evaluate  $18 * 8 = 144$  possible scenarios.

After presenting the available investment options in tables 3.1 and 3.2, in table 3.3 we tabulate the attractiveness utility of each port for the carrier by investment option.

Table 3.4 tabulates which port will win the bid for potential investment option, based on the highest attractiveness utility of carrier as per Equation (3.3). From this table, we can deduce that for investment options (1, 0, 2, 1), (2, 0, 0, 1), (2, 0, 1, 1) and (2, 0, 2, 1) Port 1 has a strictly dominant solution; Port 2 has no dominant solution. All the other investment options are weakly dominant solution for both Port 1 and Port 2. Further details about the definition of strictly dominant solution and weakly dominant solution are available in the last section of this chapter. The dominant solutions for Port 1 (1, 0, 2, 1), (2, 0, 0, 1), (2, 0, 1, 1) and (2, 0, 2, 1) reveal that Port 1 will attract the carrier, regardless of the reaction of Port 2, in case of applying any of the mentioned investment strategy. However, to determine the payoff for every investment we tabulate in table 3.5 the payoff for Port 1 and Port 2 for every potential investment option as per Equation (3.7).

From table 3.5, we notice that the payoff of investment strategy (1, 0, 2, 1) for Port 1 will generate a loss of 1.25, the investment strategy (2, 0, 0, 1) will generate a very minimal payoff of 0.67, the investment strategy (2, 0, 1, 1) will generate a loss of 1.25, and the investment strategy (2, 0, 2, 1) for Port 1 will generate a loss of 9.21. The above reveals that even if an investment strategy will attract the carrier, the port authority should make sure that the cost of investment will not exceed the revenue generated from the business of the new carrier; otherwise the port's payoff will not be profitable.

Tables 3.4 and 3.5 are tools to assist the port authority to make decision regarding the best investment strategy to use taking into consideration the other competitor reaction toward this investment.

Table 3. 3: Attractiveness Utility for Carrier

		PORT 2															
		(0,0,0,0)		(0,0,1,0)		(0,0,2,0)		(1,0,0,0)		(1,0,1,0)		(1,0,2,0)		(2,0,0,0)		(2,0,1,0)	
PORT 1	(0,0,0,0)	3.35	3.60	3.35	3.70	3.35	3.81	3.35	3.76	3.35	3.87	3.35	3.97	3.35	3.93	3.35	4.03
	(0,0,0,1)	3.73	3.60	3.73	3.70	3.73	3.81	3.73	3.76	3.73	3.87	3.73	3.97	3.73	3.93	3.73	4.03
	(0,0,1,0)	3.46	3.60	3.46	3.70	3.46	3.81	3.46	3.76	3.46	3.87	3.46	3.97	3.46	3.93	3.46	4.03
	(0,0,1,1)	3.84	3.60	3.84	3.70	3.84	3.81	3.84	3.76	3.84	3.87	3.84	3.97	3.84	3.93	3.84	4.03
	(0,0,2,0)	3.56	3.60	3.56	3.70	3.56	3.81	3.56	3.76	3.56	3.87	3.56	3.97	3.56	3.93	3.56	4.03
	(0,0,2,1)	3.94	3.60	3.94	3.70	3.94	3.81	3.94	3.76	3.94	3.87	3.94	3.97	3.94	3.93	3.94	4.03
	(1,0,0,0)	3.52	3.60	3.52	3.70	3.52	3.81	3.52	3.76	3.52	3.87	3.52	3.97	3.52	3.93	3.52	4.03
	(1,0,0,1)	3.90	3.60	3.90	3.70	3.90	3.81	3.90	3.76	3.90	3.87	3.90	3.97	3.90	3.93	3.90	4.03
	(1,0,1,0)	3.62	3.60	3.62	3.70	3.62	3.81	3.62	3.76	3.62	3.87	3.62	3.97	3.62	3.93	3.62	4.03
	(1,0,1,1)	4.00	3.60	4.00	3.70	4.00	3.81	4.00	3.76	4.00	3.87	4.00	3.97	4.00	3.93	4.00	4.03
	(1,0,2,0)	3.72	3.60	3.72	3.70	3.72	3.81	3.72	3.76	3.72	3.87	3.72	3.97	3.72	3.93	3.72	4.03
	(1,0,2,1)	4.10	3.60	4.10	3.70	4.10	3.81	4.10	3.76	4.10	3.87	4.10	3.97	4.10	3.93	4.10	4.03
	(2,0,0,0)	3.68	3.60	3.68	3.70	3.68	3.81	3.68	3.76	3.68	3.87	3.68	3.97	3.68	3.93	3.68	4.03
	(2,0,0,1)	4.06	3.60	4.06	3.70	4.06	3.81	4.06	3.76	4.06	3.87	4.06	3.97	4.06	3.93	4.06	4.03
	(2,0,1,0)	3.78	3.60	3.78	3.70	3.78	3.81	3.78	3.76	3.78	3.87	3.78	3.97	3.78	3.93	3.78	4.03
	(2,0,1,1)	4.16	3.60	4.16	3.70	4.16	3.81	4.16	3.76	4.16	3.87	4.16	3.97	4.16	3.93	4.16	4.03
(2,0,2,0)	3.89	3.60	3.89	3.70	3.89	3.81	3.89	3.76	3.89	3.87	3.89	3.97	3.89	3.93	3.89	4.03	
(2,0,2,1)	4.27	3.60	4.27	3.70	4.27	3.81	4.27	3.76	4.27	3.87	4.27	3.97	4.27	3.93	4.27	4.03	

**Table 3. 4: Bidding Award for Every Potential Investment Option**

		PORT 2							
		(0,0,0,0)	(0,0,1,0)	(0,0,2,0)	(1,0,0,0)	(1,0,1,0)	(1,0,2,0)	(2,0,0,0)	(2,0,1,0)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,1,1)	Port 1	Port 1	Port 1	Port 1	Port 2	Port 2	Port 2	Port 2
	(0,0,2,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,2,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 2	Port 1	Port 2
	(1,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(1,0,0,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 2	Port 2	Port 2
	(1,0,1,0)	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(1,0,1,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 2
	(1,0,2,0)	Port 1	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(1,0,2,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1
	(2,0,0,0)	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(2,0,0,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1
	(2,0,1,0)	Port 1	Port 1	Port 2	Port 1	Port 2	Port 2	Port 2	Port 2
	(2,0,1,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1
	(2,0,2,0)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 2	Port 2	Port 2
(2,0,2,1)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1	

Table 3. 5: Payoff for Every Potential Investment Option

		PORT 2															
		(0,0,0,0)	(0,0,1,0)	(0,0,2,0)	(1,0,0,0)	(1,0,1,0)	(1,0,2,0)	(2,0,0,0)	(2,0,1,0)								
PORT 1	(0,0,0,0)	0.00	12.40	0.00	10.47	0.00	2.51	0.00	11.52	0.00	9.59	0.00	9.59	0.00	1.63	0.00	6.59
	(0,0,0,1)	10.55	0.00	10.55	-3.00	-3.00	2.51	-3.00	11.52	-3.00	9.59	-3.00	9.59	-3.00	1.63	-3.00	6.59
	(0,0,1,0)	-3.00	12.40	-3.00	10.47	-3.00	2.51	-3.00	11.52	-3.00	9.59	-3.00	9.59	-3.00	1.63	-3.00	6.59
	(0,0,1,1)	8.63	0.00	8.63	-3.00	8.63	-12.00	8.63	-2.00	-6.00	9.59	-6.00	9.59	-6.00	1.63	-6.00	6.59
	(0,0,2,0)	-12.00	12.40	-12.00	10.47	-12.00	2.51	-12.00	11.52	-12.00	9.59	-12.00	9.59	-12.00	1.63	-12.00	6.59
	(0,0,2,1)	0.67	0.00	0.67	-3.00	0.67	-12.00	0.67	-2.00	0.67	-5.00	-15.00	9.59	0.67	-14.00	-15.00	6.59
	(1,0,0,0)	-3.00	12.40	-3.00	10.47	-3.00	2.51	-3.00	11.52	-3.00	9.59	-3.00	9.59	-3.00	1.63	-3.00	6.59
	(1,0,0,1)	8.63	0.00	8.63	-3.00	8.63	-12.00	8.63	-2.00	8.63	-5.00	-6.00	9.59	-6.00	1.63	-6.00	6.59
	(1,0,1,0)	8.63	0.00	-6.00	10.47	-6.00	2.51	-6.00	11.52	-6.00	9.59	-6.00	9.59	-6.00	1.63	-6.00	6.59
	(1,0,1,1)	6.71	0.00	6.71	-3.00	6.71	-12.00	6.71	-2.00	6.71	-5.00	6.71	-5.00	6.71	-14.00	-9.00	6.59
	(1,0,2,0)	0.67	0.00	0.67	-3.00	-15.00	2.51	-15.00	11.52	-15.00	9.59	-15.00	9.59	-15.00	1.63	-15.00	6.59
	(1,0,2,1)	-1.25	0.00	-1.25	-3.00	-1.25	-12.00	-1.25	-2.00	-1.25	-5.00	-1.25	-5.00	-1.25	-14.00	-1.25	-8.00
	(2,0,0,0)	2.59	0.00	-12.00	10.47	-12.00	2.51	-12.00	11.52	-12.00	9.59	-12.00	9.59	-12.00	1.63	-12.00	6.59
	(2,0,0,1)	0.67	0.00	0.67	-3.00	0.67	-12.00	0.67	-2.00	0.67	-5.00	0.67	-5.00	0.67	-14.00	0.67	-8.00
	(2,0,1,0)	0.67	0.00	0.67	-3.00	-15.00	2.51	0.67	-2.00	-15.00	9.59	-15.00	9.59	-15.00	1.63	-15.00	6.59
	(2,0,1,1)	-1.25	0.00	-1.25	-3.00	-1.25	-12.00	-1.25	-2.00	-1.25	-5.00	-1.25	-5.00	-1.25	-14.00	-1.25	-8.00
	(2,0,2,0)	-7.29	0.00	-7.29	-3.00	-7.29	-12.00	-7.29	-2.00	-7.29	-5.00	-24.00	9.59	-24.00	1.63	-24.00	6.59
(2,0,2,1)	-9.21	0.00	-9.21	-3.00	-9.21	-12.00	-9.21	-2.00	-9.21	-5.00	-9.21	-5.00	-9.21	-14.00	-9.21	-8.00	

### 3.3 Sensitivity Analysis for Weights

In this section, we investigate the impact of weights used for the four criteria defined above based on a sensitivity analysis of the four weights.

We consider 5 scenarios in our analysis. Scenario 1 is “base case”, where we use the weights for the four criteria as determined by Lirn et al. (2004). In Scenario 2, we consider that the carrier has equal preferences for the four criteria. In Scenario 3, we consider that the carrier is seeking for the best geographical port location. In Scenario 4, we consider that the carrier is cost reduction seeker. In Scenario 5, we consider that the carrier is seeking more efficient port management.

#### 3.3.1 Base Scenario

In this scenario considered as a base case scenario, we assume that the weights assigned for each of the four criteria defined above are identical to Lirn et al. (2004), such as  $w_1 = 16.38\%$ ,  $w_2 = 35.12\%$ ,  $w_3 = 10.38\%$ , and  $w_4 = 38.12\%$ .

Consider 2 different ports as follow.

##### 3.3.1.1 Port 1 Characteristics

Port 1 has scored: 3 over 5 for the first criterion “Port Physical and Technical Infrastructure”  $a_1 = 3$ , 4 over 5 for the second criterion “Port Geographical Location”  $b_1 = 4$ , 2 over 5 for the third criterion “Port Management and Administration”  $c_1 = 2$ , and 3 over 5 for the fourth criterion “Carriers’ Terminal Cost”  $d_1 = 3$ . The current attractiveness of Port 1 to a specific carrier is  $u_1 = w_1a_1 + w_2b_1 + w_3c_1 + w_4d_1 = 16.38\% * 3 + 35.12\% * 4 + 10.38\% * 2 + 38.12\% * 3 = 2.75$

For Port 1, in this case, we assume that we have only 1 unit to invest, and the port location cannot be altered such as:

- Budget Constraint:  $\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 1$
- Capacity Constraint:  $\Delta a_1 \leq 1$
- Location Constraint:  $\Delta b_1 = 0$
- Manpower Constraint:  $\Delta c_1 \leq 1$
- Price Constraint:  $\Delta d_1 \leq 1$

In this case Port 1 has 4 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 1, \Delta b = 0, \Delta c = 0, \Delta d = 0$

Table 3.6 represents all the possible investments in the first five columns, the attractiveness utility of each port for the carrier in column 6, revenue generated in case of attracting the carrier in column 7, cost of investment in column 8, and payoff generated (revenue – cost) in column 9 for Port 1.

**Table 3. 6: Base Case - Port 1 Options**

	a	b	c	d	u1	Revenue	Cost	Payoff
<b>Port 1</b>	<b>3</b>	<b>2</b>	<b>4</b>	<b>3</b>	<b>2.75</b>	11.36	0	11.36
<b>(0,0,0,0)</b>	0	0	0	0	2.75	11.36	0	11.36
<b>(0,0,0,1)</b>	0	0	0	1	3.13	12.43	3	9.43
<b>(0,0,1,0)</b>	0	0	1	0	2.86	12.40	4	8.40
<b>(1,0,0,0)</b>	1	0	0	0	2.92	12.43	3	9.43

### 3.3.1.2 Port 2 Characteristics

Port 2 has scored: 3 over 5 for the first criterion “Port Physical and Technical Infrastructure”  $a_2 = 3$ , 3 over 5 for the second criterion “Port Geographical Location”  $b_2 = 3$ , 3 over 5 for the third criterion “Port Management and Administration”  $c_2 = 3$ ,



and 3 over 5 for the fourth criterion “ Carriers’ Terminal Cost”  $d_1 = 3$ . The current attractiveness of Port 2 to a specific carrier is  $u_2 = w_1a_2 + w_2b_2 + w_3c_2 + w_4d_2 = 16.38\% * 3 + 35.12\% * 3 + 10.38\% * 3 + 38.12\% * 3 = 3$ .

The constraints for Port 2 are the same as for Port.

- Budget Constraint:  $\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 1$
- Capacity Constraint:  $\Delta a_2 \leq 1$
- Location Constraint:  $\Delta b_2 = 0$
- Manpower Constraint:  $\Delta c_2 \leq 1$
- Price Constraint:  $\Delta d_2 \leq 1$

In this case Port 2 has 4 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 1, \Delta b = 0, \Delta c = 0, \Delta d = 0$

Table 3.7 represents all the possible investments in the first five columns, the attractiveness utility of each port for the carrier in column 6, revenue generated in case of attracting the carrier in column 7, cost of investment in column 8, and payoff generated (revenue – cost) in column 9 for Port 2.

**Table 3. 7: Base Case - Port 2 Options**

	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>u2</b>	<b>Revenue</b>	<b>Cost</b>	<b>Payoff</b>
<b>Port 2</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3.00</b>	11.40	0	11.40
<b>(0,0,0,0)</b>	0	0	0	0	3.00	11.40	0	11.40
<b>(0,0,0,1)</b>	0	0	0	1	3.38	12.48	3	9.48
<b>(0,0,1,0)</b>	0	0	1	0	3.10	12.48	3	9.48
<b>(1,0,0,0)</b>	1	0	0	0	3.16	12.48	3	9.48

### 3.3.1.3 Game Process

In this case we have 4 possible investment options for Port1 and 4 possible investment options for Port 2. We need to evaluate  $4 * 4 = 16$  possible scenarios.

Table 3.8 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.9 tabulates which port will win the bid for potential investment option.

Table 3.10 tabulates the payoff for each scenario.

From table 3.9, Port 1 is able to “win” the bid in case the investment strategy of (0, 0, 0, 1) is applied and Port 2 avoids investment strategy of (0, 0, 0, 1) or (1, 0, 0, 0).

Otherwise, Port 2 has 2 dominant investment strategies: (0, 0, 0, 1) and (1, 0, 0, 0).

From table 3.10, Port 2 will tend to use the investment strategy (0, 0, 0, 1) since it generates higher payoff than (0, 0, 0, 1).

It might be argued that Port 1 will tend to avoid any investment due to the dominant investment strategy of Port 2. In this case Port 1 would select investment strategy (0, 0, 0, 0), if Port 2 has a motive to minimize cost, i.e. avoid capital investment, then he will use strategy (0, 0, 0, 0).

**Table 3. 8: Base Case - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	2.75	3.00	2.75	3.38	2.75	3.10	2.75	3.16
	(0,0,0,1)	3.13	3.00	3.13	3.38	3.13	3.10	3.13	3.16
	(0,0,1,0)	2.86	3.00	2.86	3.38	2.86	3.10	2.86	3.16
	(1,0,0,0)	2.92	3.00	2.92	3.38	2.92	3.10	2.92	3.16

**Table 3. 9: Base Case - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 2
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2
	(1,0,0,0)	Port 2	Port 2	Port 2	Port 2

**Table 3. 10: Base Case - Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	0.00	11.40	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.43	0.00	-3.00	9.48	9.43	-3.00	-3.00	9.48
	(0,0,1,0)	-4.00	11.40	-4.00	9.48	-4.00	9.48	-4.00	9.48
	(1,0,0,0)	-3.00	11.40	-3.00	9.48	-3.00	9.48	-3.00	9.48

### 3.3.2 Scenario 2: Equal Preferences

In this scenario, we consider that the carrier has equal weight for each of the four criteria such that  $w_1 = 25\%$ ,  $w_2 = 25\%$ ,  $w_3 = 25\%$ , and  $w_4 = 25\%$ .

Table 3.11 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.12 tabulates which port will win the bid for different potential investment options. Table 3.13 tabulates the payoff for each scenario. In this case, when all weights are equal then all investment strategies will result in the same payoff per port.

**Table 3. 11: Scenario 2- Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.00	3.00	3.00	3.25	3.00	3.25	3.00	3.25
	(0,0,0,1)	3.25	3.00	3.25	3.25	3.25	3.25	3.25	3.25
	(0,0,1,0)	3.25	3.00	3.25	3.25	3.25	3.25	3.25	3.25
	(1,0,0,0)	3.25	3.00	3.25	3.25	3.25	3.25	3.25	3.25

**Table 3. 12: Scenario 2- Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port1/Port2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port1/Port2	Port1/Port2	Port1/Port2
	(0,0,1,0)	Port 1	Port1/Port2	Port1/Port2	Port1/Port2
	(1,0,0,0)	Port 1	Port1/Port2	Port1/Port2	Port1/Port2

**Table 3. 13: Scenario 2- Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,1,0)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	0.00	0.00	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.43	0.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00
	(0,0,1,0)	8.40	0.00	-4.00	-3.00	-4.00	-3.00	-4.00	-3.00
	(1,0,0,0)	9.43	0.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00

### 3.3.3 Scenario 3: Geographical Location

In this scenario, we consider that the carrier has the port geographical location as the main objective such as  $w_1 = 10\%$ ,  $w_2 = 50\%$ ,  $w_3 = 10\%$ , and  $w_4 = 30\%$ .

Table 3.14 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.15 tabulates which port will win the bid for potential investment option. Table 3.16 tabulates the payoff for each scenario.

In this scenario, Port 2 which has the highest initial score on “Port Geographical Location” is in a dominant position regardless of the investment strategy used by Port 1, since the port geographical location cannot be altered.

**Table 3. 14: Scenario 3- Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	2.60	3.00	2.60	3.30	2.60	3.10	2.60	3.10
	(0,0,0,1)	2.90	3.00	2.90	3.30	2.90	3.10	2.90	3.10
	(0,0,1,0)	2.70	3.00	2.70	3.30	2.70	3.10	2.70	3.10
	(1,0,0,0)	2.70	3.00	2.70	3.30	2.70	3.10	2.70	3.10

**Table 3. 15: Scenario 3- Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 2	Port 2	Port 2	Port 2
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2
	(1,0,0,0)	Port 2	Port 2	Port 2	Port 2

**Table 3. 16: Scenario 3- Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	0.00	11.40	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	-3.00	11.40	-3.00	9.48	-3.00	9.48	-3.00	9.48
	(0,0,1,0)	-4.00	11.40	-4.00	9.48	-4.00	9.48	-4.00	9.48
	(1,0,0,0)	-3.00	11.40	-3.00	9.48	-3.00	9.48	-3.00	9.48

### 3.3.4 Scenario 4: Cost Reduction

In this scenario, we consider that the carrier is cost reduction seeker such that  $w_1 = 10\%$ ,  $w_2 = 30\%$ ,  $w_3 = 10\%$ , and  $w_4 = 50\%$ .

Table 3.17 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.18 tabulates which port will win the bid for potential investment option. Table 3.19 tabulates the payoff for each scenario.

In this scenario, Port 2 has a dominant investment strategy (0, 0, 0, 1).

**Table 3. 17: Scenario 4- Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	2.80	3.00	2.80	3.50	2.80	3.10	2.80	3.10
	(0,0,0,1)	3.30	3.00	3.30	3.50	3.30	3.10	3.30	3.10
	(0,0,1,0)	2.90	3.00	2.90	3.50	2.90	3.10	2.90	3.10
	(1,0,0,0)	2.90	3.00	2.90	3.50	2.90	3.10	2.90	3.10

**Table 3. 18: Scenario 4- Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 1
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2
	(1,0,0,0)	Port 2	Port 2	Port 2	Port 2

**Table 3. 19: Scenario 4- Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	0.00	11.40	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.43	0.00	-3.00	9.48	9.43	-3.00	9.43	-3.00
	(0,0,1,0)	-4.00	11.40	-4.00	9.48	-4.00	9.48	-4.00	9.48
	(1,0,0,0)	-3.00	11.40	-3.00	9.48	-3.00	9.48	-3.00	9.48

### 3.3.5 Scenario 5: Port Efficiency

In this scenario, we consider that the carrier has the port efficiency as main objective such as  $w_1 = 10\%$ ,  $w_2 = 20\%$ ,  $w_3 = 40\%$ , and  $w_4 = 30\%$ . The reason behind taking the main criterion  $w_3 = 40\%$  and not  $w_3 = 50\%$  as we did for the above scenarios is the initial value of  $w_3 = 10.38\%$  in the base scenario which renders a  $w_3$  value of  $50\%$  rather unreasonable.

Table 3.20 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.21 tabulates which port will win the bid for potential investment option. Table 3.22 tabulates the payoff for each scenario.

In this scenario, Port 1 instead of Port 2 has 2 dominant investment strategies (0, 0, 0, 1) and (0, 0, 1, 0) with a payoff of 9.43 and 8.4 respectively.

**Table 3. 20: Scenario 5- Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.20	3.00	3.20	3.30	3.20	3.40	3.20	3.10
	(0,0,0,1)	3.50	3.00	3.50	3.30	3.50	3.40	3.50	3.10
	(0,0,1,0)	3.60	3.00	3.60	3.30	3.60	3.40	3.60	3.10
	(1,0,0,0)	3.30	3.00	3.30	3.30	3.30	3.40	3.30	3.10

**Table 3. 21: Scenario 5- Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 1	Port 2	Port 2	Port 1
	(0,0,0,1)	Port 1	Port 1	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 1	Port 1	Port 1
	(1,0,0,0)	Port 1	Port1/Port2	Port 2	Port 1

**Table 3. 22: Scenario 5- Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	11.36	0.00	0.00	9.48	0.00	9.48	11.36	-3.00
	(0,0,0,1)	9.43	0.00	9.43	-3.00	9.43	-3.00	9.43	-3.00
	(0,0,1,0)	8.40	0.00	8.40	-3.00	8.40	-3.00	8.40	-3.00
	(1,0,0,0)	9.43	0.00	-3.00	-3.00	-3.00	9.48	9.43	-3.00

Table 3.23 summarizes the sensitivity analysis of weight for the above 5 scenarios.

**Table 3. 23: Summary of Sensitivity Analysis for Weights**

Scenario	Weights	Optimal Investment Strategy
Base Case	$w_1 = 16.38 \%$ , $w_2 = 35.12 \%$ , $w_3 = 10.38 \%$ , $w_4 = 38.12 \%$	Port 2 has 2 dominant solutions (0, 0, 1, 0) and (1, 0, 0, 0)
Scenario 2: Equal Preferences	$w_1 = 25 \%$ , $w_2 = 25 \%$ , $w_3 = 25 \%$ , $w_4 = 25 \%$	No Dominant Solution
Scenario 3: Geographical Location	$w_1 = 10 \%$ , $w_2 = 50 \%$ , $w_3 = 10 \%$ , $w_4 = 30 \%$ .	Port 2 has dominant solutions regardless of the strategy used
Scenario 4: Cost Reduction	$w_1 = 10 \%$ , $w_2 = 30 \%$ , $w_3 = 10 \%$ , $w_4 = 50 \%$	Port 2 has a unique dominant strategy (0, 0, 1, 0)
Scenario 5: Port Efficiency	$w_1 = 10 \%$ , $w_2 = 20 \%$ , $w_3 = 40 \%$ , $w_4 = 30 \%$	Port 1 has 2 dominant solution (0, 0, 0, 1) and (0, 0, 1, 0)

### 3.4 Resource Allocation Analysis

In this section, numerical examples are conducted to analyze resource allocation at the strategic level in port investments. Five cases are presented in this section. The objective of this section is to study the impact of resource allocation on

the investment strategy used by ports. Our study focuses on analyzing the impact of the third criterion “Port Management and Administration” and the fourth criterion “Carriers’ Terminal Cost”. The reason behind our interest in those two criteria is their short term impact. While (i) investing in “Port Physical and Technical Infrastructure” requires long term commitment and implementation, and (ii) enhancing the “Port Geographical Location” is not always practical and possible investing in (iii) “Port Management and Administration” and (iv) “Carriers’ Terminal Cost” is feasible to implement in the short to medium term.

#### **3.4.1 Base Case**

The base case considered in this section is the same base case presented in Section 3.3. In this case, Port 1 and Port 2 have only 1 unit of resource to allocate. The allocation is allowable in all criteria except in the criterion of “Port Geographical Location” since the location is considered to be fixed. Tables 3.8, 3.9, and 3.10 represent the matrix for attractiveness utility, port selection by carrier and payoff to ports, respectively.

As mentioned before in Section 3.3.1.3, Port 1 is able to “win” the bid in case the investment strategy of (0, 0, 0, 1) is applied and Port 2 avoids investment strategy of (0, 0, 0, 1) or (1, 0, 0, 0). Otherwise, Port 2 has 2 dominant investment strategies: (0, 0, 0, 1) and (1, 0, 0, 0).

#### **3.4.2 Case 2: Two Resources to Allocate**

In this case, compared to the base case, two unit resources are available to invest in both ports.

However, due to our interest in the third and fourth criteria as explained above, no resource is allocated in the first and second criteria.

For Port 1, we have only 2 units to invest such as:

- Budget Constraint:  $\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 2$
- Capacity Constraint:  $\Delta a_1 = 0$
- Location Constraint:  $\Delta b_1 = 0$
- Manpower Constraint: if  $c_1 = 4$ , then only 1 unit can be invested  $\Delta c_1 \leq 1$

In this case Port 1 has 5 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 2$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$

The constraints for Port 2 are the same as for Port.

- Budget Constraint:  $\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 2$
- Capacity Constraint:  $\Delta a_2 = 0$
- Location Constraint:  $\Delta b_2 = 0$

In this case Port 2 has 6 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 2$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$



Table 3.24 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.25 tabulates which port will win the bid for potential investment option. Table 3.26 tabulates the payoff for each scenario.

**Table 3. 24: Case 2 - Attractiveness Utility**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,0,2)		(0,0,2,0)		(0,0,1,1)	
PORT 1	(0,0,0,0)	2.75	3.00	2.75	3.38	2.75	3.10	2.75	3.76	2.75	3.21	2.75	3.49
	(0,0,0,1)	3.13	3.00	3.13	3.38	3.13	3.10	3.13	3.76	3.13	3.21	3.13	3.49
	(0,0,1,0)	2.86	3.00	2.86	3.38	2.86	3.10	2.86	3.76	2.86	3.21	2.86	3.49
	(0,0,0,2)	3.52	3.00	3.52	3.38	3.52	3.10	3.52	3.76	3.52	3.21	3.52	3.49
	(0,0,1,1)	3.24	3.00	3.24	3.38	3.24	3.10	3.24	3.76	3.24	3.21	3.24	3.49

**Table 3. 25: Case 2 - Port Selection**

		PORT 2						
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(0,0,0,2)	(0,0,2,0)	(0,0,1,1)	
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 2	Port 2	Port 2	
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	
	(0,0,0,2)	Port 1	Port 1	Port 1	Port 2	Port 1	Port 1	
	(0,0,1,1)	Port 1	Port 2	Port 1	Port 2	Port 1	Port 2	

**Table 3. 26: Case 2 - Payoff**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,0,2)		(0,0,2,0)		(0,0,1,1)	
PORT 1	(0,0,0,0)	0.00	11.40	0.00	9.48	0.00	9.48	0.00	1.52	0.00	1.52	0.00	7.55
	(0,0,0,1)	9.43	0.00	-3.00	9.48	9.43	-3.00	-3.00	1.52	-3.00	1.52	-3.00	7.55
	(0,0,1,0)	-4.00	11.40	-4.00	9.48	-4.00	9.48	-4.00	1.52	-4.00	1.52	-4.00	7.55
	(0,0,0,2)	1.47	0.00	1.47	-3.00	1.47	-3.00	-12.00	1.52	1.47	-12.00	1.47	-6.00
	(0,0,1,1)	6.47	0.00	-7.00	9.48	6.47	-3.00	-7.00	1.52	6.47	-12.00	-7.00	7.55

In this case, Port 2 has a dominant solution strategy (0, 0, 0, 2) with a low payoff of 1.52. Port 2 may have a higher payoff of 9.48 if the investment strategy (0, 0, 0, 1) is applied given that Port 1 avoids using the investment strategy (0, 0, 0, 2).

### 3.4.3 Case 3: Three Resources to Allocate

In this case, compared to the base case, three resources are available to invest in both ports.

For Port 1, we have only 3 units to invest such as:

- Budget Constraint:  $\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 3$
- Capacity Constraint:  $\Delta a_1 = 0$
- Location Constraint:  $\Delta b_1 = 0$
- Manpower Constraint:  $\Delta c_1 \leq 1$
- Price Constraint:  $\Delta d_1 \leq 2$

In this case Port 1 has 6 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 2$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 2$

The constraints for Port 2 are the same as for Port.

- Budget Constraint:  $\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 3$
- Capacity Constraint:  $\Delta a_2 = 0$
- Location Constraint:  $\Delta b_2 = 0$
- Manpower Constraint:  $\Delta c_2 \leq 2$
- Price Constraint:  $\Delta d_2 \leq 2$

In this case Port 2 has 8 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 2$

5.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
7.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 2$
8.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 1$

Table 3.27 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.28 tabulates which port will win the bid for potential investment option. Table 3.29 tabulates the payoff for each scenario.

**Table 3. 27: Case 3- Attractiveness Utility**

		PORT 2															
		(0,0,0)		(0,0,1)		(0,1,0)		(0,0,2)		(0,2,0)		(0,1,1)		(0,1,2)		(0,2,1)	
PORT 1	(0,0,0,0)	2.75	3.00	2.75	3.38	2.75	3.10	2.75	3.76	2.75	3.21	2.75	3.49	2.75	3.87	2.75	3.59
	(0,0,0,1)	3.13	3.00	3.13	3.38	3.13	3.10	3.13	3.76	3.13	3.21	3.13	3.49	3.13	3.87	3.13	3.59
	(0,0,1,0)	2.86	3.00	2.86	3.38	2.86	3.10	2.86	3.76	2.86	3.21	2.86	3.49	2.86	3.87	2.86	3.59
	(0,0,0,2)	3.52	3.00	3.52	3.38	3.52	3.10	3.52	3.76	3.52	3.21	3.52	3.49	3.52	3.87	3.52	3.59
	(0,0,1,1)	3.24	3.00	3.24	3.38	3.24	3.10	3.24	3.76	3.24	3.21	3.24	3.49	3.24	3.87	3.24	3.59
	(0,0,1,2)	3.62	3.00	3.62	3.38	3.62	3.10	3.62	3.76	3.62	3.21	3.62	3.49	3.62	3.87	3.62	3.59

**Table 3. 28: Case 3- Port Selection**

		PORT 2							
		(0,0,0,0)	(0,0,0,1)	(0,1,1,0)	(0,0,0,2)	(0,2,2,0)	(0,0,1,1)	(0,0,1,2)	(0,0,2,1)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,2)	Port 1	Port 1	Port 1	Port 2	Port 1	Port 1	Port 2	Port 2
	(0,0,1,1)	Port 1	Port 2	Port 1	Port 2	Port 1	Port 2	Port 2	Port 2
	(0,0,1,2)	Port 1	Port 1	Port 1	Port 2	Port 1	Port 1	Port 2	Port 1

**Table 3. 29: Case 3 - Payoff**

		PORT 2															
		(0,0,0,0)	(0,0,0,1)	(0,1,1,0)	(0,0,0,2)	(0,2,2,0)	(0,0,1,1)	(0,0,1,2)	(0,0,2,1)								
PORT 1	(0,0,0,0)	0.00	0.00	0.00	9.48	0.00	9.48	0.00	1.52	0.00	1.52	0.00	7.55	0.00	-0.41	0.00	-0.41
	(0,0,0,1)	9.43	11.40	-3.00	9.48	9.43	-3.00	-3.00	1.52	-3.00	1.52	-3.00	7.55	-3.00	-0.41	-3.00	-0.41
	(0,0,1,0)	-4.00	0.00	-4.00	9.48	-4.00	9.48	-4.00	1.52	-4.00	1.52	-4.00	7.55	-4.00	-0.41	-4.00	-0.41
	(0,0,0,2)	1.47	0.00	1.47	-3.00	1.47	-3.00	-12.00	1.52	1.47	-12.00	1.47	-6.00	-12.00	-0.41	-12.00	-0.41
	(0,0,1,1)	6.47	0.00	-7.00	9.48	6.47	-3.00	-7.00	1.52	6.47	-12.00	-7.00	7.55	-7.00	-0.41	-7.00	-0.41
	(0,0,1,2)	-1.49	0.00	-1.49	-3.00	-1.49	-3.00	-16.00	1.52	-1.49	-12.00	-1.49	-6.00	-16.00	-0.41	-1.49	-12.00

In this case, in addition to the dominant investment strategy (0, 0, 0, 2) in case 3, Port 2 has another dominant investment strategy (0, 0, 1, 2). However, the second

dominant investment strategy (0, 0, 1, 2) generates a negative payoff (-0.41) for Port 2, which is not profitable. As for the first dominant investment strategy (0, 0, 0, 2), the generated payoff for Port 2 is considered low (1.52).

#### **3.4.4 Case 4: Three Resources to Allocate with Restriction on Fourth Criterion (Price) for Both Ports**

In this case, compared to Case 3, we restrict the investment in the fourth criterion “Carriers’ Terminal Cost” (price/fee charged to carrier) to 1 unit only. This restriction is legitimate since the port authority or port operator may have a tight margin of fees reduction to carriers.

For Port 1, constraints are as follow:

- Budget Constraint:  $\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 3$
- Capacity Constraint:  $\Delta a_1 = 0$
- Location Constraint:  $\Delta b_1 = 0$
- Manpower Constraint: if  $c_1 = 4$ , then only 1 unit can be invested  $\Delta c_1 \leq 1$
- Price Constraint:  $\Delta d_1 \leq 1$

In this case Port 1 has 4 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$

The constraints for Port 2 are the same as for Port 1.

- Budget Constraint:  $\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 3$
- Capacity Constraint:  $\Delta a_2 = 0$

- Location Constraint:  $\Delta b_2 = 0$
- Manpower Constraint:  $\Delta c_2 \leq 3, c_2 = 3$ , then only 2 unit can be invested  
 $\Delta c_2 \leq 2$
- Price Constraint:  $\Delta d_2 \leq 1$

In this case Port 2 has 6 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 1$

Table 3.30 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.31 tabulates which port will win the bid for potential investment option. Table 3.32 tabulates the payoff for each scenario.

**Table 3. 30: Case 4 – Attractiveness Utility**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,2,0)		(0,0,1,1)		(0,0,2,1)	
PORT 1	(0,0,0,0)	2.75	2.84	2.75	3.22	2.75	2.94	2.75	3.04	2.75	3.32	2.75	3.43
	(0,0,0,1)	3.13	2.84	3.13	3.22	3.13	2.94	3.13	3.04	3.13	3.32	3.13	3.43
	(0,0,1,0)	2.86	2.84	2.86	3.22	2.86	2.94	2.86	3.04	2.86	3.32	2.86	3.43
	(0,0,1,1)	3.24	2.84	3.24	3.22	3.24	2.94	3.24	3.04	3.24	3.32	3.24	3.43

**Table 3. 31: Case 4 – Port Selection**

		PORT 2					
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(0,0,2,0)	(0,0,1,1)	(0,0,2,1)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 1	Port 2	Port 2
	(0,0,1,0)	Port 1	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,1,1)	Port 1	Port 1	Port 1	Port 1	Port 2	Port 2

**Table 3. 32: Case 4 – Payoff**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,2,0)		(0,0,1,1)		(0,0,2,1)	
PORT 1	(0,0,0,0)	0.00	0.00	0.00	8.36	0.00	8.36	0.00	0.40	0.00	6.43	0.00	-1.53
	(0,0,0,1)	9.43	0.00	-3.00	8.36	9.43	-3.00	9.43	-12.00	-3.00	6.43	-3.00	-1.53
	(0,0,1,0)	8.40	0.00	-4.00	8.36	-4.00	8.36	-4.00	0.40	-4.00	6.43	-4.00	-1.53
	(0,0,1,1)	6.47	0.00	6.47	-3.00	6.47	-3.00	6.47	-12.00	-7.00	6.43	-7.00	-1.53

In this case, Port 2 has 2 dominant investment strategies (0, 0, 1, 1) and (0, 0, 2, 1).

The first dominant investment strategy generates a payoff of (6.43), while the second dominant investment strategy generates a negative payoff (-1.53).

### 3.4.5 Case 5: Three Resources to Allocate with Restriction on the Fourth

#### Criterion (Price) for One Port Only

In this case, compared to Case 3 and Case 4, we restrict the investment in the fourth criterion “Carriers’ Terminal Cost” to 1 unit only for Port 2 only, while Port 1 is allowed to have 2 units on this criterion. This restriction is legitimate since a specific port authority or port operator can go for “price war” to attract a specific carrier.

For Port 1, constraints are as follow:

- Budget Constraint:  $\Delta a_1 + \Delta b_1 + \Delta c_1 + \Delta d_1 \leq 3$
- Capacity Constraint:  $\Delta a_1 = 0$
- Location Constraint:  $\Delta b_1 = 0$
- Manpower Constraint: if  $c_1 = 4$ , then only 1 unit can be invested  $\Delta c_1 \leq 1$
- Price Constraint:  $\Delta d_1 \leq 2$

In this case Port 1 has 6 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$

2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 2$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 1$

Constraints for Port 2 are as follow.

- Budget Constraint:  $\Delta a_2 + \Delta b_2 + \Delta c_2 + \Delta d_2 \leq 3$
- Capacity Constraint:  $\Delta a_2 = 0$
- Location Constraint:  $\Delta b_2 = 0$
- Manpower Constraint: if  $c_2 = 3$ , then only 2 unit can be invested  $\Delta c_2 \leq 2$
- Price Constraint:  $\Delta d_2 \leq 1$

In this case Port 2 has 6 options to consider for investment.

1. Do nothing :  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 0$
2.  $\Delta a = 0, \Delta b = 0, \Delta c = 0, \Delta d = 1$
3.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 0$
4.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 0$
5.  $\Delta a = 0, \Delta b = 0, \Delta c = 1, \Delta d = 1$
6.  $\Delta a = 0, \Delta b = 0, \Delta c = 2, \Delta d = 1$

Table 3.33 tabulates the attractiveness utility of each port for the carrier by investment option. Table 3.34 tabulates which port will win the bid for potential investment option. Table 3.35 tabulates the payoff for each scenario.

**Table 3. 33: Case 5 – Attractiveness Utility**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,2,0)		(0,0,1,1)		(0,0,2,1)	
PORT 1	(0,0,0,0)	2.75	3.00	2.75	3.38	2.75	3.10	2.75	3.21	2.75	3.49	2.75	3.59
	(0,0,0,1)	3.13	3.00	3.13	3.38	3.13	3.10	3.13	3.21	3.13	3.49	3.13	3.59
	(0,0,1,0)	2.86	3.00	2.86	3.38	2.86	3.10	2.86	3.21	2.86	3.49	2.86	3.59
	(0,0,0,2)	3.52	3.00	3.52	3.38	3.52	3.10	3.52	3.21	3.52	3.49	3.52	3.59
	(0,0,1,1)	3.24	3.00	3.24	3.38	3.24	3.10	3.24	3.21	3.24	3.49	3.24	3.59
	(0,0,1,2)	3.62	3.00	3.62	3.38	3.62	3.10	3.62	3.21	3.62	3.49	3.62	3.59

**Table 3. 34: Case 5 – Port Selection**

		PORT 2					
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(0,0,2,0)	(0,0,1,1)	(0,0,2,1)
PORT 1	(0,0,0,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 2	Port 1	Port 2	Port 2	Port 2
	(0,0,1,0)	Port 2	Port 2	Port 2	Port 2	Port 2	Port 2
	(0,0,0,2)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 2
	(0,0,1,1)	Port 1	Port 2	Port 1	Port 1	Port 2	Port 2
	(0,0,1,2)	Port 1	Port 1	Port 1	Port 1	Port 1	Port 1

**Table 3. 35: Case 5 – Payoff**

		PORT 2											
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(0,0,2,0)		(0,0,1,1)		(0,0,2,1)	
PORT 1	(0,0,0,0)	0.00	0.00	0.00	9.48	0.00	9.48	0.00	1.52	0.00	7.55	0.00	-0.41
	(0,0,0,1)	9.43	11.40	-3.00	9.48	9.43	-3.00	-3.00	1.52	-3.00	7.55	-3.00	-0.41
	(0,0,1,0)	-4.00	0.00	-4.00	9.48	-4.00	9.48	-4.00	1.52	-4.00	7.55	-4.00	-0.41
	(0,0,0,2)	1.47	0.00	1.47	-3.00	1.47	-3.00	1.47	-12.00	1.47	-6.00	-12.00	-0.41
	(0,0,1,1)	6.47	0.00	-7.00	9.48	6.47	-3.00	6.47	-12.00	-7.00	7.55	-7.00	-0.41
	(0,0,1,2)	-1.49	0.00	-1.49	-3.00	-1.49	-3.00	-1.49	-12.00	-1.49	-6.00	-1.49	-15.00

In this case, Port 2 loses the advantage of having a dominant investment strategy. Port 1 has now a dominant investment strategy (0, 0, 1, 2), but this investment strategy generates a negative payoff (-1.49) for Port 1, so it is not advisable to apply this strategy. However, if we investigate the investment strategy (0, 0, 0, 2), it is a dominant investment strategy for Port 1 except for the investment strategy (0, 0, 2, 1) by Port 2. Nevertheless, the investment strategy (0, 0, 2, 1) generates a negative payoff (-0.41) for Port 2. Thus, applying the investment strategy (0, 0, 0, 2) is beneficial for Port 1 unless Port 2 has a “kamikaze” behavior, and not rational behavior, to eliminate the competitor regardless of the negative payoff.



### 3.5 Insights and Discussion

The presented approach in this chapter provides a managerial tool for port authorities around the world to allocate their investments in the optimal manner with the objective of better positioning their port to attract carriers.

Using this game theory approach, a port authority is able to identify its target, which is the carrier, and its other competitors. The port authority, based on the utility function developed in the first section of this chapter, is able to assess its weaknesses and strengths, in addition to weaknesses and strengths of other port competitors. In this model, the reaction of other players in the market is presented taking into consideration financial, physical, and location constraints. The port authority shall consider the existence of competition in any future investment strategy to enhance its ability to attract carriers, which is reflected in maximizing its utility function to attract carriers and minimizing losses in case of inability to attract carriers from competitors. In the below, some managerial insights related to the investment strategy policy are drawn based on the above sections.

#### 3.5.1 Strictly Dominant Bidder

**LEMMA 4:** *A port  $i$  is considered to be a strictly dominant bidder if and only if for all  $i'$  in set of port  $N$  excluding  $i$*

$$w_1 a_i + w_2 b_i + w_3 c_i + w_4 d_i > w_1 (a_{i'} + \max \Delta a_{i'}) + w_2 (b_{i'} + \max \Delta b_{i'}) + w_3 (c_{i'} + \max \Delta c_{i'}) + w_4 (d_{i'} + \max \Delta d_{i'}) .$$

*Proof*

Since the carrier according to Equation (3.3) will choose  $\max_{i \in N} u_i$  therefore any investment made by port  $i'$  will not exceed the utility of port  $i$ . ■

This Lemma implies that in case a strictly dominant bidder exists, ports should not make any investments in their ports since the dominant port will attract the carrier regardless of any investment made. In this case, other port authorities and port operators will minimize their investment cost due to inability to attract carriers.

### 3.5.2 Weakly Dominant Bidder

**LEMMA 5:** A port  $i$  is considered to be weakly dominant bidder if and only if for all  $i'$  in set of port  $N$  excluding  $i$

$$w_1 a_i + w_2 b_i + w_3 c_i + w_4 d_i > w_1 a_{i'} + w_2 b_{i'} + w_3 c_{i'} + w_4 d_{i'}$$

and there exists a feasible set of  $\Delta a_{i'}$ ,  $\Delta b_{i'}$ ,  $\Delta c_{i'}$ , and  $\Delta d_{i'}$  such that

$$w_1 a_i + w_2 b_i + w_3 c_i + w_4 d_i < w_1 (a_{i'} + \Delta a_{i'}) + w_2 (b_{i'} + \Delta b_{i'}) + w_3 (c_{i'} + \Delta c_{i'}) + w_4 (d_{i'} + \Delta d_{i'}) .$$

*Proof*

Same as LEMMA 4. ■

This Lemma implies that in the case of a weakly dominant bidder, ports should invest in their facilities to keep attracting carriers. Otherwise, carriers are lost to other competitors.

### 3.5.3 Profitable Investment Strategy

**LEMMA 6:** A feasible set of  $\Delta a_i$ ,  $\Delta b_i$ ,  $\Delta c_i$ , and  $\Delta d_i$  for port  $i$  is considered to be a profitable investment strategy for port  $i$  if and only if

$$(\mathbf{a}_i + \Delta \mathbf{a}_i)(1 - e^{-(\mathbf{a}_i + \Delta \mathbf{a}_i)}) + (\mathbf{b}_i + \Delta \mathbf{b}_i)(1 - e^{-(\mathbf{b}_i + \Delta \mathbf{b}_i)}) + (\mathbf{c}_i + \Delta \mathbf{c}_i)(1 - e^{-(\mathbf{c}_i + \Delta \mathbf{c}_i)}) + (\mathbf{d}_i + \Delta \mathbf{d}_i)(1 - e^{-(\mathbf{d}_i + \Delta \mathbf{d}_i)}) > \mathbf{a}_i * \Delta \mathbf{a}_i^{\Delta \mathbf{a}_i} + \mathbf{b}_i * \Delta \mathbf{b}_i^{\Delta \mathbf{b}_i} + \mathbf{c}_i * \Delta \mathbf{c}_i^{\Delta \mathbf{c}_i} + \mathbf{d}_i * \Delta \mathbf{d}_i^{\Delta \mathbf{d}_i}$$

This Lemma implies that the existence of a feasible investment set is not sufficient to be adopted if the revenue generated from this set is not enough to cover the cost of implementing this investment strategy. Otherwise, the port authority will not generate any profit from this investment strategy.

### 3.5.4 Strictly Dominant Investment Strategy

**LEMMA 7: A feasible set of  $\Delta \mathbf{a}_i$ ,  $\Delta \mathbf{b}_i$ ,  $\Delta \mathbf{c}_i$ , and  $\Delta \mathbf{d}_i$  for port  $i$  is considered to be a strictly dominant investment strategy for port  $i$  if and only if**

$$(\mathbf{a}_i + \Delta \mathbf{a}_i)(1 - e^{-(\mathbf{a}_i + \Delta \mathbf{a}_i)}) + (\mathbf{b}_i + \Delta \mathbf{b}_i)(1 - e^{-(\mathbf{b}_i + \Delta \mathbf{b}_i)}) + (\mathbf{c}_i + \Delta \mathbf{c}_i)(1 - e^{-(\mathbf{c}_i + \Delta \mathbf{c}_i)}) + (\mathbf{d}_i + \Delta \mathbf{d}_i)(1 - e^{-(\mathbf{d}_i + \Delta \mathbf{d}_i)}) > \mathbf{a}_i * \Delta \mathbf{a}_i^{\Delta \mathbf{a}_i} + \mathbf{b}_i * \Delta \mathbf{b}_i^{\Delta \mathbf{b}_i} + \mathbf{c}_i * \Delta \mathbf{c}_i^{\Delta \mathbf{c}_i} + \mathbf{d}_i * \Delta \mathbf{d}_i^{\Delta \mathbf{d}_i}$$

**and for all  $i'$  in set of port  $N$  excluding  $i$**

$$\begin{aligned} & w_1(\mathbf{a}_i + \Delta \mathbf{a}_i) + w_2(\mathbf{b}_i + \Delta \mathbf{b}_i) + w_3(\mathbf{c}_i + \Delta \mathbf{c}_i) + w_4(\mathbf{d}_i + \Delta \mathbf{d}_i) \\ & > w_1(\mathbf{a}_{i'} + \max \Delta \mathbf{a}_{i'}) + w_2(\mathbf{b}_{i'} + \max \Delta \mathbf{b}_{i'}) + w_3(\mathbf{c}_{i'} + \max \Delta \mathbf{c}_{i'}) + w_4(\mathbf{d}_{i'} + \max \Delta \mathbf{d}_{i'}) \end{aligned}$$

This Lemma implies that if a specific port has a strictly dominant investment strategy, which is a profitable investment strategy that guarantees the winning of the bid, this strategy should be adopted by the specific port, and the competitors' ports should avoid making any kind of investment in their facilities in order to minimize their investment cost.

## CHAPTER 4

### OPERATIONAL LEVEL: SINGLE SHIP SCENARIO

In this chapter, the integration between the quay sides and yard sides for a single berthing ship with transshipment containers is investigated. Figure 4.1, extracted from Ng and Mk (2005), illustrates typical container flow in the container terminal for a single berthing ship. The methodology suggested is to formulate a mathematical model to mimic, as much as possible, the transshipment process in a container terminal in order to optimize the number of quay crane (QC) and yard crane (YC) used. Only the process of unloading containers is considered in this research. The number of containers unloaded by crane used, bay location, and storage location is determined for each time period. The authors plan to address the container loading process in future research. Interested reader can refer to Stahlbock and Vob (2008) for the difference between the container unloading and loading problems. Major constraints related to transshipment operation are taken into consideration such as: crane capacities on both quay and yard sides, time constraints, and spatial constraints on both the discharge and storage areas. The formulated model is first tested on small scale size problems. Then large “industry-size” problems are considered. The primary objective is to determine the assignment of QCs to bays and of YCs to sub-blocks over time in a way that minimizes the total number of utilized cranes (Kaysi et al., 2012).

This chapter is structured as follows. In the first section the mathematical model is presented along with the objective function and the constraints. In the second section the complexity of the model is highlighted. In the third section numerical

examples are presented for both small scale size problems and large scale size problems. In the fourth section insights are drawn.

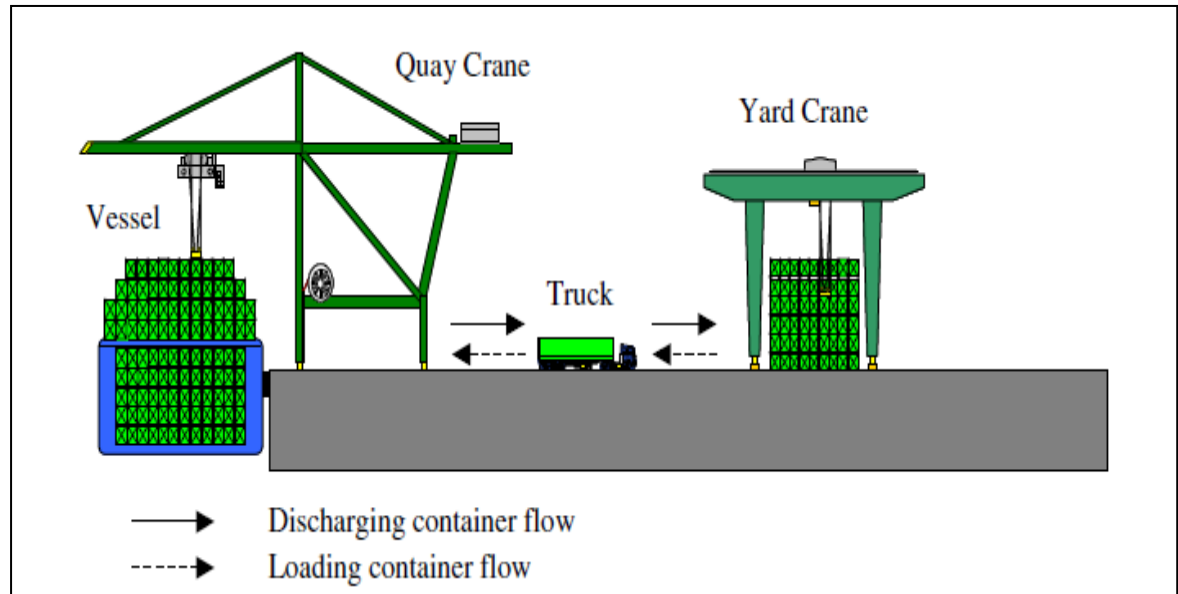


Figure 4. 1: Typical Transshipment Container Flow for Single Ship (Ng and Mak, 2005)

#### 4.1 Mathematical Model

Consider a single berthing ship with transshipment containers. Containers are stored in different locations of the ship called bays. The number of bays is  $B$ . A container is classified into one of different flags (types) according to attributes such as weight class, size (20" or 40" length), and next port of destination. The number of flags is  $F$ . The number of containers from flag  $f$ , in bay  $b$  is  $N_{bf}, f=1, \dots, F, b=1, \dots, B$ . A container is unloaded from the ship via quay cranes and placed on an internal truck. We assume that the number of internal trucks is large enough to handle the workload, which is justified since the cost of internal trucks is low relative to the cost of quay cranes and yard cranes.

Each bay can accommodate only one quay crane (QC) per time period. A QC can handle a maximum of  $C_Q$  containers per time period. The truck moves the container into a designated yard storage location, which we refer to as a sub-block. The number of sub-blocks is  $S$ . Once the truck reaches the yard, the container is stored at the appropriate sub-block via a yard crane (YC). A YC can handle a maximum of  $C_Y$  containers per time period.

We assume that enough YC capacity is available during the unloading time period to handle all the containers to be unloaded from the ship. Based on this, the problem formulation ensures that enough YC capacity is available during each time period to handle the containers unloaded from the ship during that same time period. This assumption is justified as it reduces congestion at the yard caused by loaded internal trucks. This assumption also implies that the crane capacities at both quay and yard side are coordinated.

The sub-blocks where a container may be stored are pre-determined based on the flag of the container. Specifically, we define binary variables  $I_{fs}$ , where  $I_{fs} = 1$  if flag  $f$  can be stored in sub-block  $s$ ,  $f=1, \dots, F$ ,  $s=1, \dots, S$  and  $I_{fs} = 0$  otherwise. The time horizon,  $T$ , is finite and divided into equal periods (e.g. hours) that we denote by  $t = 1, 2, \dots, T$ . The numbers of quay cranes and yard cranes available over a time period  $t$  ( $QC_t$  and  $YC_t$ ) are assumed to be known a priori. In practice, the port operator determines  $QC_t$  and  $YC_t$  based on the number of ships berthing in the port at a given day and the type of activity involved (e.g., import, export, transshipment). Due to physical restrictions, a QC can serve at most one bay per time period. The number of YC that can serve a sub-block in a time period is also limited; a maximum number of  $K_{st}$  yard cranes can serve sub-block  $s$  at time  $t$ . Finally, the yard storage space is limited; a maximum of  $C_{fs}$  containers of flag  $f$  can be stored in sub-block  $s$ .

### 4.1.1 Objective Function

The objective of the model is to determine the number of quay cranes and yard cranes needed at every time period from a set of available cranes. The number of available cranes is provided by the Container Terminal Operator (CTO). Since the model assumes a single ship, the CTO allocates a specific number of cranes to serve this ship, and this number varies with time. Our objective is to determine the assignment of quay cranes to bays and of yard cranes to sub-blocks over time in a way that minimizes the total number of utilized cranes. Let  $y_{bt}$  be a binary variable which indicates whether a QC is assigned to bay  $b$  in time period  $t$ , i.e.,  $y_{bt} = 1$  if a crane is assigned to bay  $b$  at time  $t$ , and  $y_{bt} = 0$ , otherwise. Define also  $z_{st}$  as the number of yard cranes assigned to sub-block in time period  $t$ . Our decision variables are  $y_{bt}$  and  $z_{st}$ . We also define the number of containers of flag  $f$  which are unloaded from bay  $b$  to sub-block  $s$  in time period  $t$ ,  $x_{bfts}$ , as auxiliary decision variables. Then, our problem is formulated with the following integer linear program (ILP).

$$\text{Minimize } w \sum_{b=1}^B \sum_{t=1}^T y_{bt} + (1-w) \sum_{s=1}^S \sum_{t=1}^T z_{st} \quad \text{Equation (4.1)}$$

The objective in Equation (4.1) is to minimize the number of cranes used in the quay side and the yard side during the transshipment process. We assign a weight  $w$ , which varies between 0 and 1, for representing the importance of the yard crane (YC) with respect to quay crane (QC). For example, we could represent the ratio of the operating cost of the QC per time period over the sum of the operating costs of both QC and YC.

### 4.1.2 Constraints

Several constraints are applied for the single ship model such as crane capacities on both quay and yard sides, time constraints, and spatial constraints on both the discharge and storage areas.

In every time period, the total number of QC at all bays will not exceed the number of available QC, which is represented in Equation (4.2).

$$\sum_{b=1}^B y_{bt} \leq QC_t, \forall t \quad \text{Equation (4.2)}$$

The number of containers unloaded will not exceed the capacity of the QC at every time period for every bay, which is represented in Equation (4.3).

$$\frac{\sum_{f=1}^F \sum_{s=1}^S x_{bfts}}{C_Q} \leq y_{bt}, \forall b, \forall t \quad \text{Equation (4.3)}$$

All transshipment containers are unloaded from the ship over the time horizon of length  $T$ , which is represented in Equation (4.4).

$$\sum_{t=1}^T \sum_{s=1}^S x_{bfts} = N_{bf}, \forall b, \forall f \quad \text{Equation (4.4)}$$

In every time period, the total number of YC used at all sub-blocks will not exceed the number of available YC, which represented in Equation (4.5).

$$\sum_{s=1}^S z_{st} \leq YC_t, \forall t \quad \text{Equation (4.5)}$$

The number of containers discharged will not exceed the capacity of the YC available in every time period and for every sub-block, which represented in Equation (4.6).

This constraint is crucial as it captures our assumption that unloaded containers do not wait at the yard side, which requires coordination of quay and yard cranes capacities.



$$\frac{\sum_{f=1}^F \sum_{b=1}^B x_{bfts}}{C_Y} \leq z_{st}, \forall s, \forall t \quad \text{Equation (4.6)}$$

The storage of containers having a flag type  $f$  is restricted to a sub-block  $s$  such that  $I_{fs} = 1$ , which is represented in Equation (4.7).

$$x_{bfts} \leq I_{fs} M, \forall b, \forall f, \forall t, \forall s \quad \text{Equation (4.7)}$$

The constant  $M$  is a large enough number which could be set equal to the total number of containers on the vessel.

The number of YC assigned to sub-block  $s$  is limited to the maximum allowable number for every sub-block  $s$ , which is represented in Equation (4.8).

$$z_{st} \leq K_{st}, \forall s, \forall t \quad \text{Equation (4.8)}$$

Finally, the number of containers discharged in every sub-block should fit within the spatial area allowed for every flag type, which is represented in Equation (4.9).

$$\sum_{t=1}^T \sum_{b=1}^B x_{bfts} \leq C_{fs}, \forall f, \forall s \quad \text{Equation (4.9)}$$

Figure 4.2 summarizes the single ship model.

## Single Ship Model

$$\text{Minimize } w \sum_{b=1}^B \sum_{t=1}^T y_{bt} + (1-w) \sum_{s=1}^S \sum_{t=1}^T z_{st} \quad \text{Equation (4.1)}$$

Subject to

$$\sum_{b=1}^B y_{bt} \leq QC_t, \forall t \quad \text{Equation (4.2)}$$

$$\frac{\sum_{f=1}^F \sum_{s=1}^S x_{bfts}}{C_Q} \leq y_{bt}, \forall b, \forall f \quad \text{Equation (4.3)}$$

$$\sum_{t=1}^T \sum_{s=1}^S x_{bfts} = N_{bf}, \forall b, \forall f \quad \text{Equation (4.4)}$$

$$\sum_{s=1}^S z_{st} \leq YC_t, \forall t \quad \text{Equation (4.5)}$$

$$\frac{\sum_{f=1}^F \sum_{b=1}^B x_{bfts}}{C_Y} \leq z_{st}, \forall s, \forall t \quad \text{Equation (4.6)}$$

$$x_{bfts} \leq I_{fs} M, \forall b, \forall f, \forall t, \forall s \quad \text{Equation (4.7)}$$

$$z_{st} \leq K_{st}, \forall s, \forall t \quad \text{Equation (4.8)}$$

$$\sum_{t=1}^T \sum_{b=1}^B x_{bfts} \leq C_{fs}, \forall f, \forall s \quad \text{Equation (4.9)}$$

$$x_{bfts}, z_{st} \text{ Integer, } y_{bt} \text{ binary, } \forall b, \forall f, \forall t, \forall s$$

Figure 4. 2: Single Ship Model Summary

## 4.2 Computational Complexity

In this section, we discuss computational complexity, i.e., the difficulty of solving the integer linear programming (ILP) problem in Section 4.1. We first show that this problem is NP-hard by proving that it reduces to the multi-dimensional knapsack problem in a special case. Then, we discuss the number of integer decision variables needed in the solution.

Consider a special case of the problem in Section 4.1 with a ship having one bay, a yard having one sub-block, and no restriction on the number of available cranes and storage capacity. That is,  $B = 1$  (1 bay),  $F = 1$  (1 flag),  $S = 1$  (1 sub-block),  $QC_t$ ,  $YC_t$ ,  $K_{st}$ , and  $C_{fs}$  are very large. In this special case, denote the total number of containers by  $N$ , the number of quay and yard cranes utilized in period  $t$  by  $y_t$  and  $z_t$ , and the number of unloaded containers in period  $t$  by  $x_t$ . The model in Section 4.1 simplifies to:

$$\text{Minimize } w \sum_{t=1}^T y_t + (1-w) \sum_{t=1}^T z_t \quad \text{Equation (4.10)}$$

Subject to

$$\frac{x_t}{C_Q} \leq y_t, \forall t \quad \text{Equation (4.11)}$$

$$\sum_{t=1}^T x_t = N \quad \text{Equation (4.12)}$$

$$\frac{x_t}{C_Y} \leq z_t, \forall t \quad \text{Equation (4.13)}$$

$x_t, z_t$  integer,  $y_t$ , binary.

Replacing the value of  $x_t$  from the second constraint,  $x_t = N - \sum_{i \in Y \setminus \{t\}} x_i$ , where

$Y = \{1, 2, \dots, T\}$  is the set of time periods, in the first and third constraints, the

problem becomes equivalent to the following model:

$$\text{Minimize } w \sum_{i \in Y} y_i + (1-w) \sum_{i \in Y} z_i \quad \text{Equation (4.14)}$$

Subject to

$$C_Q y_t + \sum_{i \in Y \setminus \{t\}} x_i \geq N, \quad \forall t \quad \text{Equation (4.15)}$$

$$\sum_{i \in Y} x_i = N \quad \text{Equation (4.16)}$$

$$C_Q z_t + \sum_{i \in Y \setminus \{t\}} x_i \geq N, \quad \forall t \quad \text{Equation (4.17)}$$

$x_t, z_t$  integer,  $y_t$ , binary.

This model is equivalent to a multi-dimensional knapsack problem with decision variables,  $x_i, y_i$ , and  $z_i$ . Since the knapsack problem is NP hard as discussed by Wolsey (1998), we conclude that our transshipment ILP model is NP hard problem.

The high complexity of the problem is also obvious from the large number of binary and integer decision variables for realistic instances. The number of decision variables  $x_{bfts}, y_{bt}$ , and  $z_{st}$  are respectively  $B \times F \times T \times S, B \times T$  and  $S \times T$ , leading to a total number of variables of  $B \times F \times T \times S + B \times T + S \times T$ . For example, in a problem with 10 bays ( $B = 10$ ), 10 flags ( $F = 10$ ), 24 time periods ( $T = 24$ ) and 10 sub-blocks ( $S = 10$ ), the total number of integer decision variables is 24,480 which implies that the problem is not likely to be solved within a reasonable time with any available integer programming solver.

### 4.3 Numerical Results

In this section, we present numerical results. The results were obtained by coding the integer program model of Section 4.1 on AMPL compiler and then solving it using the GUROBI solver version 3.0.1 on Intel Core i5 Central Processing Unit (CPU) with 4 Gigabyte (GB) random access memory (RAM) computer. The AMPL code is available in Appendix A.

First, we consider a small instance for our base example. This is intended to be a “proof of concept” example which facilitates drawing useful insights. Then we present selective large scale problems to validate our insights, by showing that they apply to realistic settings.

#### 4.3.1 Small Scale Problems

Consider a problem instance with three time intervals ( $T = 3$ ), two bays ( $B = 2$ ), two flags ( $F = 2$ ), and two sub-blocks ( $S = 2$ ). Consider the following data as a base case:

- The quay crane (QC) capacity is  $C_Q = 20$  containers/unit time.
- The yard crane (YC) capacity is  $C_Y = 15$  containers/unit time.
- The cost of QC is equal to the cost of YC; therefore  $w = 0.5$ .
- The number of QC available at every time period is 3, i.e.,  $\mathbf{QC} = (3 \ 3 \ 3)$ .
- The number of YC available at every time period is 3, i.e.,  $\mathbf{YC} = (3 \ 3 \ 3)$ .
- The number of containers of flag  $f$  to be unloaded from vessel bay  $b$ ,  $N_{bf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$\mathbf{N} = \begin{pmatrix} 10 & 10 \\ 10 & 10 \end{pmatrix}.$$

- The allocation of flag type  $f$  to the sub-block  $s$  is such that  $I_{fs}=1$  for all  $f$  and  $s$ , i.e.,

$$\mathbf{I} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

- The maximum allowed YC per sub-block  $s$  at time period  $t$   $K_{st}$  is given by the following matrix:

$$\mathbf{K} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix}.$$

- The sub-block capacity for all flags is  $C_{fs} = 40$ , i.e.,

$$\mathbf{C} = \begin{pmatrix} 40 & 40 \\ 40 & 40 \end{pmatrix}.$$

The optimal solution to this problem obtained from AMPL compiler is as follows. In terms of the quay crane (QC),  $y_{11} = 1$ ,  $y_{21} = 1$ , and  $y_{bt} = 0$  otherwise. For the yard crane (YC),  $z_{11} = 1$ ,  $z_{21} = 2$ , and  $z_{st} = 0$  otherwise. For the schedule of containers unloaded,  $x_{1112} = 10$ ,  $x_{1212} = 10$ ,  $x_{2111} = 10$ ,  $x_{2212} = 10$ , and  $x_{bfts} = 0$  otherwise. That is, 2 quay cranes are used at full-capacity in time period 1 to unload 20 containers from each of the two bays. The containers unloaded from bay 1 are all stored in sub-block 2 and those unloaded from bay 2 are split between the two sub-blocks. This requires that 2 yard cranes (working at full-capacity) be assigned to sub-block 2 to handle 30 containers and 1 yard crane be assigned to sub-block 1 to handle the remaining 10 containers.

Table 4.1 presents the solution for several variations of the base case above. The “Change” column shows the variation from the base case while keeping other parameters at their base values. To present the solution in a compact form, table 4.1

reports the total number of QCs and YCs utilized in every time period ,  $\mathbf{y} = (y_1 \ y_2 \ y_3)$

and  $\mathbf{z} = (z_1 \ z_2 \ z_3)$ , where  $y_t = \sum_{b=1}^B y_{bt}$  and  $z_t = \sum_{s=1}^S z_{st}$  .

Details for the solution of each case are available in Appendix B – Small Scale Problems for Single Ship.

#### 4.3.2 Large Scale Problems

We apply our model to selected large scale problems space. Consider a base large scale problem with twelve time intervals ( $T = 12$ ), eight bays ( $B = 8$ ), eight flags ( $F = 8$ ), and four sub-blocks ( $S = 4$ ) and 1,280 containers to handle. Consider the following data as the base case:

- The QC capacity is  $C_Q = 20$  containers/unit time.
- The YC capacity is  $C_Y = 15$  containers/unit time.
- The cost of QC is equal to the cost of YC; therefore  $w = 0.5$ .
- The number of QC available at every time period is 10,  
i.e.,  $\mathbf{QC} = (10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10)$ .
- The number of YC available at every time period is 20,  
i.e.,  $\mathbf{YC} = (20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20)$ .
- The number of containers of flag  $f$  to be unloaded from vessel bay  $b$ ,  $N_{bf}$ , is 20 for all  $b$  and  $f$ , i.e.,

$$N = \begin{pmatrix} 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \end{pmatrix} .$$

**Table 4. 1: Summary of Small Scale Problems**

<b>Case</b>	<b>Change from Base Case</b>	<b>Description</b>	<b>y</b>	<b>z</b>
1	None	Base case	(2 0 0)	(3 0 0)
2	<b>YC</b> = (2 1 1)	Decrease number of yard cranes	(2 1 0)	(2 1 0)
3	<b>YC</b> = (1 1 1)	Decrease number of yard cranes	(2 1 1)	(1 1 1)
4	<b>QC</b> = (1 1 1)	Decrease number of quay cranes	(1 1 0)	(2 2 0)
5	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Restrict flag allocation	(1 1 0)	(2 2 0)
6	$\mathbf{I} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$	Use only 1 sub-block for all flags	(2 1 0)	(2 1 0)
7	$\mathbf{C} = \begin{pmatrix} 10 & 10 \\ 10 & 10 \end{pmatrix}$	Decrease sub-block capacity	(1 1 0)	(2 2 0)
8	$\mathbf{C} = \begin{pmatrix} 0 & 20 \\ 0 & 20 \end{pmatrix}$	Use 1 sub-block with less capacity	(2 1 0)	(2 1 0)
9	$\mathbf{I} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ , $w = 0.9$ , $K_{22} = 2$	Use 1 sub-block, quay cranes are more costly than yard cranes, and increase $K_{22}$ to allow more yard cranes on sub- block 2	(1 1 0)	(2 2 0)
10	$\mathbf{I} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ , $w = 0.1$ , $K_{22} = 2$	Use 1 sub-block, yard cranes are more costly than quay cranes, and increase $K_{22}$ to allow more yard cranes on sub- block 2	(1 2 0)	(1 2 0)



- The allocation of flag type  $f$  to the sub-block  $s$  is such that  $I_{fs}=1$  for all  $f$  and  $s$ , i.e.,

$$\mathbf{I} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

- The maximum allowed YC per sub-block  $s$  at time period  $t$   $K_{st}$  is five at every time period, i.e.,

$$K = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}.$$

- The sub-block capacity for all flags is  $C_{fs} = 500$ , i.e.,

$$C = \begin{pmatrix} 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \end{pmatrix}.$$

Table 4.2 shows the results of applying our model to several variations of the large base case problem. Appendix C contains a brief solution for each case of the large scale size problems. For the large scale size problems, the time required to reach optimal solution can be quite high as shown in table 4.3. However, we present these results here to (i) show that the managerial insights observed for small scale problems

continue to hold for realistic industry-size problems, and (ii) to further illustrate the difficulty of solving the model.

In the next section, we present insights based on the results from Tables 4.1 and 4.2.

Table 4. 2: Summary of Large Scale Problems

Case	Change from Base Case	Description	y	z
1	None	Base case	64	86
2	$YC_t = 5, t = 7, \dots, 12$	Decrease number of yard cranes	67	86
3	$QC_t = 5, t = 1, \dots, 10$	Decrease number of quay cranes	64	89
4	$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$	Restrict flag allocation	64	88
5	$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$	Restrict flag allocation	64	87
6	$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} .$ $K_{It} = 10, t = 1, \dots, 6$	Restrict flag allocation just for sub-block 1, and allow more cranes on yard	67	86

7	$C = \begin{pmatrix} 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \\ 25 & 55 & 40 & 40 \end{pmatrix}.$	Decrease sub-block capacity	64	88
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**Table 4. 3: Time needed to reach optimality for large scale problems**

Case	1	2	3	4	5	6	7
Time (second)	308	14,539	27,185	18,845	14,784	23,260	48,291

#### 4.4 Insights and Analysis

Tables 4.1 and 4.2 reveal the following insights.

***First Insight: The limitation of the available crane capacity over time at either the quay side or the yard side will affect allocation and scheduling of cranes on the other side.***

For example, in case 2 of table 4.1, the number of available yard cranes in period 1 is 2, which, in the optimal solution, are utilized at full-capacity (handle 30 containers). This leads to two quay cranes not operating at full capacity in period 1 at the quay side. Then, in period 2, the remaining 10 containers are handled by one half-utilized crane on each side. Compared to case 1 of table 4.1, the base case, the limitation of available yard crane capacity in case 2 led to utilizing one more quay crane in the optimal solution in order to maintain the coordination between the quay and the yard. A similar effect is observed in case 3 of table 4.1 where further limitation on the yard crane capacity in periods 1 and 2 lead to utilizing 4 quay crane in the optimal solution,

which is twice the number in the base case. Case 4 of table 4.1 also reveals a similar situation, but with the quay crane capacity being binding. In this case, the limitation of quay crane capacity leads to using an additional yard crane relative to the base case. In the large scale problems, this insight continues to hold as illustrated in case 2 and 3 of table 4.2, where decreasing the number of available yard cranes in case 2 led to 3 more quay cranes utilized in the optimal solution. A similar observation is made in case 2 of table 4.2 but with restricting the availability of quay cranes leading to more yard cranes utilization.

***Second Insight: Restricting flag allocation to sub-blocks leads to using more resources not only on the yard side, but also on the quay side.***

For example, in case 5 of table 4.1, containers of Flag 1 can only be stored in sub-block 1 and containers of Flag 2 can only be stored in sub-block 2. In the optimal solution, the number of yard cranes increases to 4 from 3 in the base case. Case 6 of table 4.1, where containers of all flags are to be stored in sub-block 2, shows that restriction of flag allocation may also affect the number of quay cranes used in the quay side as 3 quay cranes are used (up from 2 in the base case). In the large scale problems, this insight is illustrated in cases 4, and 5 of table 4.2, where restricting flags allocation on the yard side led to increase the numbers of quay cranes utilized in the optimal solution. In case 6 of table 4.2, restricting flags allocation and allowing more crane capacities on the yard side led to increase both the number of yard cranes and quay cranes utilized in the optimal solution.

***Third Insight: Restricting storage capacities in sub-blocks leads to using more resources not only on the yard side, but also on the quay side.***

For example, in case 7 of table 4.1 we limit sub-block capacity for each flag to 10 containers, which leads to utilizing an additional yard crane compared to the base

case. In addition, in case 8 of table 4.1, we force all the flags to be stored in sub-block 2, which leads to utilizing an additional quay crane relative to the base case. In the large scale problems, this insight is illustrated in case 7 of table 4.2 where decreasing sub-block capacity led to using two more yard cranes.

***Fourth Insight: The weight assigned to yard crane and quay crane in the objective function is critical and will affect the level of resources used at both yard and quay side.***

In case 9 of table 4.1, we consider the same parameters as in case 6 but we change the weight  $w$  of the quay crane to 0.9 instead of 0.5, which means that quay crane is more important (higher cost) than the yard crane. We also change the maximum number of yard crane allowed for sub-block 2 to 2 instead of 1. In the optimal solution, the number of assigned quay crane is 1 in each of periods 1 and 2, with 2 yard cranes in each of these two periods. Next, consider case 10 of table 4.1, which is the same as case 9 but now the weight of quay crane is 0.1 instead of 0.9. In this case, the yard crane is more important than quay crane. We notice that the optimal solution is different than case 9, with 1 quay crane and 1 yard in period 1, and 2 quay cranes and yard cranes in period 2. That is, when the cost of quay crane is higher (as in case 9), less quay cranes are used in the optimal allocation.

## CHAPTER 5

### OPERATIONAL LEVEL: MULTI SHIP SCENARIO

In this chapter, the integration between the quay and yard sides for multiple berthing ships with transshipment containers is investigated, as illustrated in figure 5.1 (Murty et al., 2005) where the loading and unloading of containers for several ships at the same time are represented.

The methodology suggested is to extend the mathematical model formulated for the single ship scenario in Chapter 4 to take into consideration the allocation of resources among multiple ships berthing at the same time in the container terminal for transshipment operations. Only the process of unloading containers is considered in this research. The number of containers unloaded by crane used, bay location, and storage location is determined for each time period. The authors plan to address the container loading process in future research. Interested reader can refer to Stahlbock and Vob (2008) for the difference between the container unloading and loading problems. Major constraints related to transshipment operation are taken into consideration such as: crane capacities on both quay and yard sides, time constraints, and spatial constraints on both the discharge and storage areas.

This chapter is structured such as follows. In the first section the mathematical model is presented along with the objective function and the constraints. In the second section, numerical examples are presented. In the third section insights are drawn.

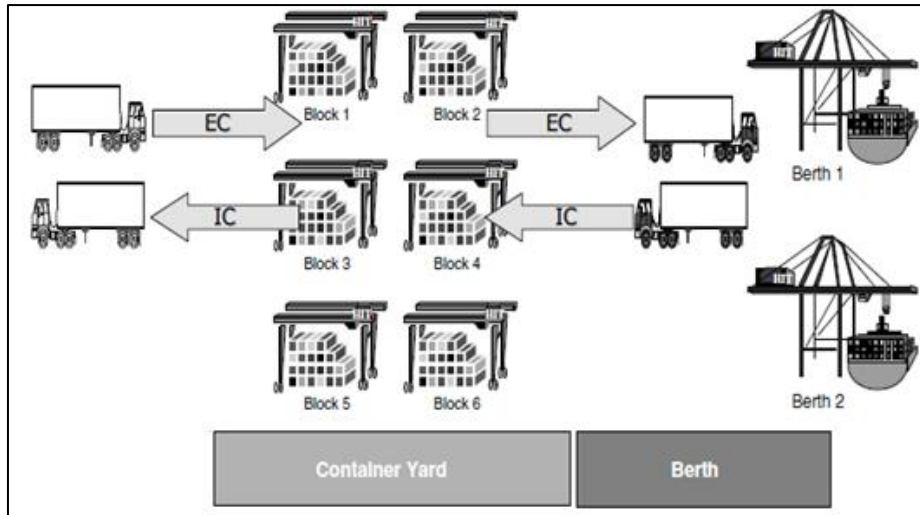


Figure 5. 1: Loading and Unloading of Containers from Multi Ship (Murty et al., 2005)

## 5.1 Mathematical Model

In a manner similar to Chapter 4, we consider  $V$  berthing ships with transshipment containers. Containers are stored in different locations of the ship called bays. The number of bays in ship  $v$  is  $B_v$ . A container is classified into one of different flags (types) according to attributes such as weight class, size (20” or 40” length), and next port of destination. The number of flags is  $F$ . The number of containers from flag  $f$ , in bay  $b$  for ship  $v$  is  $N_{vbf}, f=1, \dots, F, b=1, \dots, B_v$ . A container is unloaded from the ship  $v$  via quay cranes and placed on an internal truck. We assume that the number of internal trucks is large enough to handle the workload, which is justified since the cost of internal trucks is low relative to the cost of quay cranes and yard cranes.

Each bay can accommodate only one quay crane (QC) per time period. A QC can handle a maximum of  $C_Q$  containers per time period. We define  $Q_t$  to be the number of available quay crane at time  $t$  for all ships. The truck moves the container into a designated yard storage location, which we refer to as a sub-block. The number of



sub-blocks is  $S$ . Once the truck reaches the yard, the container is stored at the appropriate sub-block via a yard crane (YC). A YC can handle a maximum of  $C_Y$  containers per time period. We assume that enough YC capacity should be available in every period to handle all the containers unloaded from ships. We define  $YC_t$  to be the total number of available yard cranes available at time  $t$ .

The sub-blocks where a container may be stored are pre-determined based on the flag of the container. Specifically, we define binary variables  $I_{fs}$ , where  $I_{fs} = 1$  if flag  $f$  can be stored in sub-block  $s$ ,  $f=1, \dots, F$ ,  $s=1, \dots, S$  and  $I_{fs} = 0$  otherwise. The time horizon,  $T$ , is finite and divided into equal periods (e.g. hours) that we denote by  $t = 1, 2, \dots, T$ . The numbers of quay cranes and yard cranes available over a time period  $t$  are assumed to be known a priori. The number of YC that can serve a sub-block in a time period is limited due to spatial and maneuver constraints inside the sub-block; a maximum number of  $K_{st}$  yard cranes is allowed to serve sub-block  $s$  at time  $t$ . Finally, the yard storage space is limited; a maximum of  $C_{fs}$  containers of flag  $f$  can be stored in sub-block  $s$ .

In addition to the model of single ship in Chapter 4 we consider that sharing of the same yard crane is allowed between ships at the same time unit in the storage area.

The mathematical model is presented as follows:

### 5.1.1 Indices

In the model the indices are:

- $v$ : ship number,  $v=1, \dots, V$
- $b$ : bay number for ship  $v$ ,  $b=1, \dots, B_v$
- $s$ : sub-block area number,  $s=1, \dots, S$
- $f$ : flag type,  $f=1, \dots, F$

- $t$ : time period,  $t = 1, \dots, T$

### 5.1.2 Parameters

In this model the parameters are:

- $Q_t$ : number of available quay cranes (QC) at time  $t$  for all ships
- $C_Q$ : maximum number of containers a QC can handle in one period of time
- $C_Y$ : maximum number of containers a YC can handle in one period of time
- $YC_t$ : total number of available YC at time  $t$
- $K_{st}$ : number of allowed YC for every sub-block  $s$  at time  $t$
- $N_{vbf}$ : number of containers in bay  $b$  of ship  $v$  and of flag type  $f$
- $I_{fs} = \begin{cases} 1 & \text{if flag } f \text{ is assigned to sub-block } s \\ 0 & \text{otherwise} \end{cases}$
- $C_{fs}$ : number of containers of flag type  $f$  allowed to be discharged in sub-block  $s$

### 5.1.3 Decision Variables

- $y_{vbt}$  = number of QC assigned to ship  $v$ , bay  $b$  at time  $t$ , ( $y_{vbt}$  is binary)
- $z_{st}$  = number of YC assigned to sub-block  $s$  in time period  $t$ , ( $z_{st}$  is integer)
- $x_{vbfts}$  = number of containers of flag type  $f$  which are unloaded from ship  $v$  bay  $b$  to sub block  $s$  in time  $t$ , integer (auxiliary decision variable)

### 5.1.4 Objective Function

The objective of the model is to determine (1) the number of quay cranes needed for every ship at every time period from a set of available cranes and (2) yard cranes needed at every time period from a set of available cranes. The number of

available cranes is provided by the Container Terminal Operator (CTO). Our objective is to determine the assignment of quay cranes to bays and of yard cranes to sub-blocks over time in a way that minimizes the total number of utilized cranes. Let  $y_{vbt}$  be a binary variable which indicates whether a QC is assigned to bay  $b$  in time period  $t$  for ship  $v$ , i.e.,  $y_{1bt} = 1$  if a crane is assigned to bay  $b$  at time  $t$  for ship 1, and  $y_{1bt} = 0$ , otherwise. Define also  $z_{st}$  as the number of yard cranes assigned to sub-block in time period  $t$ . Our decision variables are  $y_{vbt}$  and  $z_{st}$ . We also define the number of containers of flag  $f$  which are unloaded from ship  $v$  from bay  $b$  to sub-block  $s$  in time period  $t$ ,  $x_{vbfst}$ , as auxiliary decision variables. Then, our problem is formulated with the following objective function.

$$\text{Minimize } w \sum_{v=1}^V \sum_{b=1}^{B_v} \sum_{t=1}^T y_{vbt} + (1 - w) \sum_{s=1}^S \sum_{t=1}^T z_{st} \quad \text{Equation (5.1)}$$

The objective in Equation (5.1) is to minimize the number of cranes used in the quay side and the yard side during the transshipment process for berthing ships. We assign a weight  $w$ , which varies between 0 and 1, for representing the relative cost of the yard crane (YC) with respect to quay crane (QC), similar to Chapter 4.

### 5.1.5 Constraints

Several constraints are applied for the multi ship model such as crane capacities on both quay and yard sides, time constraints, and spatial constraints on both the discharge and storage areas.

At every time period, the total number of QC at all bays of all ships will not exceed the number of available QC,

$$\sum_{v=1}^V \sum_{b=1}^{B_v} y_{vbt} \leq Q_t, \forall t, \quad \text{Equation (5.2)}$$

The number of containers unloaded will not exceed the capacity of the QC allocated for every bay of every ship at every time period,

$$\frac{\sum_{f=1}^F \sum_{t=1}^T x_{vbf ts}}{C_Q} \leq y_{vbt} \quad \forall v, \forall b, \forall t, \quad \text{Equation (5.3)}$$

All transshipment containers, for a given ship, are unloaded over the time horizon of length  $T$ ,

$$\sum_{t=1}^T \sum_{s=1}^S x_{vbf ts} = N_{vbf} \quad \forall v, \forall b, \forall f, \quad \text{Equation (5.4)}$$

At every time period, the number of YC used at any sub-block will not exceed the number of allowed YC,  $K_{st}$ ,

$$z_{st} \leq K_{st} \quad \forall s, \forall t, \quad \text{Equation (5.5)}$$

At every time period, the total number of YC used for all sub-blocks will not exceed the number of available YC,

$$\sum_{s=1}^S z_{st} \leq YC_t \quad \forall t, \quad \text{Equation (5.6)}$$

The total number of containers discharged from all ships will not exceed the capacity of the YC available at every time period, for every sub-block,

$$\frac{\sum_{v=1}^V \sum_{f=1}^F \sum_{b=1}^{B_v} x_{vbf ts}}{C_Y} \leq z_{st} \quad \forall s, \forall t, \quad \text{Equation (5.7)}$$

Containers having a flag type  $f$  are restricted to be stored in sub-block  $s$  such that  $I_{fs} = 1$ ,

$$x_{vbf ts} \leq I_{fs} \cdot M \quad \forall v, \forall b, \forall f, \forall t, \forall s, \quad \text{Equation (5.8)}$$

Where  $M$  is a large constant number which could be set equal to the total number of containers on the ship.

The total number of containers discharged in every sub-block for all ships will fit within the spatial area of sub-block  $s$  allocated to every flag type  $f$ ,

$$\sum_{v=1}^V \sum_{b=1}^{B_v} \sum_{t=1}^T x_{vbf ts} \leq C_{fs} \quad \forall f, \forall s, \quad \text{Equation (5.9)}$$

$$x_{vbf ts}, z_{st} \text{ integer, } y_{vbt} \text{ binary } \forall v, \forall b, \forall f, \forall t.$$

Figure 5.2 summarizes the multi ship model.

## Multi Ship Model

$$\text{Minimize } w \sum_{v=1}^V \sum_{b=1}^{B_v} \sum_{t=1}^T y_{vbt} + (1-w) \sum_{s=1}^S \sum_{t=1}^T z_{st} \quad \text{Equation (5.1)}$$

Subject to

$$\sum_{v=1}^V \sum_{b=1}^{B_v} y_{vbt} \leq Q_t, \forall t \quad \text{Equation (5.2)}$$

$$\frac{\sum_{f=1}^F \sum_{t=1}^S x_{vbfst}}{c_Q} \leq y_{vbt} \quad \forall v, \forall b, \forall t \quad \text{Equation (5.3)}$$

$$\sum_{t=1}^T \sum_{s=1}^S x_{vbfst} = N_{vbf} \quad \forall v, \forall b, \forall f \quad \text{Equation (5.4)}$$

$$z_{st} \leq K_{st} \quad \forall s, \forall t \quad \text{Equation (5.5)}$$

$$\sum_{s=1}^S z_{st} \leq YC_t \quad \forall t \quad \text{Equation (5.6)}$$

$$\frac{\sum_{v=1}^V \sum_{f=1}^F \sum_{b=1}^{B_v} x_{vbfst}}{c_Y} \leq z_{st} \quad \forall s, \forall t \quad \text{Equation (5.7)}$$

$$x_{vbfst} \leq I_{fs} \cdot M \quad \forall v, \forall b, \forall f, \forall t, \forall s \quad \text{Equation (5.8)}$$

$$\sum_{v=1}^V \sum_{b=1}^{B_v} \sum_{t=1}^T x_{vbfst} \leq C_{fs} \quad \forall f, \forall s \quad \text{Equation (5.9)}$$

$$x_{vbfst}, z_{st} \text{ integer, } y_{vbt} \text{ binary } \forall v, \forall b, \forall f, \forall t$$

Figure 5. 2: Multi Ship Model Summary

## 5.2 Numerical Results

In this section, we present numerical results. The results were obtained by coding the integer program model of Section 5.1 on AMPL compiler and then solving it using GUROBI solver version 4.5.2 on Intel Core i5 Central Processing Unit (CPU) with 4 Gigabyte (GB) random access memory (RAM) computer. The AMPL code for multi ship model is available in Appendix D.

In this section, we first validate our model by considering 1 ship in the multi ship model. Then, we consider a small instance for our base example. This is intended to be a “proof of concept” example which facilitates drawing useful insights. Then, we conclude this section by solving selective large scale problems to validate our insights, by showing that they apply to realistic settings.

### 5.2.1 Model Validation

In order to validate our multi ship model, we consider the base case of the single ship model with three time intervals ( $T = 3$ ), two bays ( $B = 2$ ), two flags ( $F = 2$ ), and two sub-blocks ( $S = 2$ ). Consider the following data:

- The quay crane (QC) capacity is  $C_Q = 20$  containers/unit time.
- The yard crane (YC) capacity is  $C_Y = 15$  containers/unit time.
- The cost of QC is equal to the cost of YC; therefore  $w = 0.5$ .
- The number of QC available at every time period is 3, i.e.,  $\mathbf{QC} = (3\ 3\ 3)$ .
- The number of YC available at every time period is 3, i.e.,  $\mathbf{YC} = (3\ 3\ 3)$ .
- The number of containers of flag  $f$  to be unloaded from ship 1 bay  $b$ ,  $N_{vbf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$N_1 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}.$$

- The number of containers of flag  $f$  to be unloaded from ship 2 bay  $b$ ,  $N_{bf}$ , is 0 for all  $b$  and  $f$ , i.e.,

$$N_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

- The allocation of flag type  $f$  to the sub-block  $s$  is such that  $I_{fs}=1$  for all  $f$  and  $s$ , i.e.,

$$I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

- The maximum allowed YC per sub-block  $s$  at time period  $t$   $K_{st}$  is given by the following matrix:

$$K = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}.$$

- The sub-block capacity for all flags is  $C_{fs} = 40$ , i.e.,

$$C = \begin{bmatrix} 40 & 40 \\ 40 & 40 \end{bmatrix}$$

The optimal solution to this problem obtained from AMPL compiler is as follows. In terms of the quay crane (QC),  $y_{111} = 1$ ,  $y_{121} = 1$  and  $y_{1bt} = 0$ , otherwise. For the yard crane (YC),  $z_{11} = 2$ ,  $z_{21} = 1$  and  $z_{st} = 0$ , otherwise.

For the schedule of containers unloaded,  $x_{11111} = 10$ ,  $x_{11211} = 10$ ,  $x_{12111} = 10$ ,  $x_{12212} = 10$ , and  $x_{vbfst} = 0$ , otherwise. That is, 2 quay cranes are used at full-capacity in time period 1 to unload 20 containers from each of the two bays. The containers unloaded from bay 1 are all stored in sub-block 1 and those unloaded from bay 2 are split between the two sub-blocks. This requires that 2 yard cranes (working at full-capacity) be assigned to sub-block 1 to handle 30 containers and 1 yard crane be assigned to sub-block 2 to handle the remaining 10 containers. The results of this case match with results of the base case for small scale problems presented in Section 4.3.1.

### 5.2.2 Small Scale Problems

Consider a problem instance with two ships ( $V=2$ ), three time intervals ( $T=3$ ), two bays ( $B=2$ ), two flags ( $F=2$ ), and two sub-blocks ( $S=2$ ). Consider the following data as a base case:

- The quay crane (QC) capacity is  $C_Q = 20$  containers/unit time.
- The yard crane (YC) capacity is  $C_Y = 15$  containers/unit time.
- The cost of QC is four times the cost of YC; therefore  $w = 0.8$ .
- The number of QC available at every time period for all ships is 6, i.e.,  $\mathbf{Q} = (6 \ 6 \ 6)$ .
- The number of YC available at every time period for all ships is 3, i.e.,  $\mathbf{YC} = (3 \ 3 \ 3)$ .
- The number of containers of flag  $f$  to be unloaded from ship 1 bay  $b$ ,  $N_{1bf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$N_1 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}.$$

- The number of containers of flag  $f$  to be unloaded from ship 2 bay  $b$ ,  $N_{2bf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$N_2 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}.$$

- The allocation of flag type  $f$  to the sub-block  $s$  is such that  $I_{fs} = 1$  for all  $f$  and  $s$ , i.e.,

$$I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

- The maximum allowed YC per sub-block  $s$  at time period  $t$   $K_{st}$  is given by the following matrix

$$K = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

- The sub-block capacity for all flags is  $C_{fs} = 80$ , i.e.,



$$C = \begin{bmatrix} 80 & 80 \\ 80 & 80 \end{bmatrix}.$$

The optimal solution to this problem obtained from AMPL compiler is as follows. In terms of the quay crane (QC),  $y_{111} = 1$ ,  $y_{122} = 1$ ,  $y_{211} = 1$ ,  $y_{222} = 1$  and  $y_{1bt} = 0$ , otherwise. For the yard crane (YC),  $z_{11} = 2$ ,  $z_{21} = 1$ ,  $z_{12} = 2$ ,  $z_{22} = 1$ , and  $z_{st} = 0$ , otherwise.

For the schedule of containers unloaded,  $x_{11111} = 10$ ,  $x_{11211} = 10$ ,  $x_{12121} = 10$ ,  $x_{12221} = 10$ ,  $x_{21111} = 10$ ,  $x_{21212} = 10$ ,  $x_{22121} = 10$ ,  $x_{22222} = 10$ , and  $x_{bfts} = 0$ , otherwise.

At time period 1, 1 quay crane for each ship (2 QCs for both ships) is used working at full capacity. The containers unloaded from ship 1, bay 1 of flag 1 (10 containers), the containers unloaded from ship 1, bay 1 of flag 2 (10 containers), and the containers unloaded from ship 2, bay 1 of flag 1 (10 containers) are all stored in sub-block 1, which requires 2 YCs working at full capacity. While the container unloaded from ship 2, bay 1 of flag 2 (10 containers) are stored in sub-block 2, which requires 1 YC working at (2/3) capacity.

At time period 2, 1 quay crane for each ship (2 QCs for both ships) is used working at full capacity. The containers unloaded from ship 1, bay 2 of flag 1 (10 containers), the containers unloaded from ship 1, bay 2 of flag 2 (10 containers), and the containers unloaded from ship 2, bay 2 of flag 1 (10 containers) are all stored in sub-block 1, which requires 2 YCs working at full capacity. While the container unloaded from ship 2, bay 2 of flag 2 (10 containers) are stored in sub-block 2, which requires 1 YC working at (2/3) capacity.

Table 5.1 presents the solution for several variations of the base case above. The “Change” column shows the variation from the base case while keeping other parameters at their base values. To present the solution in a compact form, table 5.1 reports the total number of QCs and YCs utilized in every time period ,  $\mathbf{y}_1 = (y_{11} \ y_{12}$

$y_{13}$ ) for ship 1,  $\mathbf{y}_2 = (y_{21} \ y_{22} \ y_{23})$  for ship 2, and  $\mathbf{z} = (z_1 \ z_2 \ z_3)$ , where  $y_{1t} = \sum_{b=1}^{B_1} y_{1bt}$ ,  
 $y_{2t} = \sum_{b=1}^{B_2} y_{2bt}$  and  $z_t = \sum_{s=1}^S z_{st}$ .

Details for the solution of each case are available in Appendix E (Small Scale Problems – Multi Ship).

**Table 5. 1: Summary of Small Scale Problems - Multi Ship Scenario**

Case	Change from Base Case	Description	$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{z}$
1	None	Base case	(1 1 0)	(1 1 0)	(3 3 0)
2	$\mathbf{YC} = (2 \ 2 \ 2)$	Decrease number of yard cranes	(2 0 1)	(0 1 1)	(2 2 2)
3	$\mathbf{Q}_t = (6 \ 1 \ 1)$	Decrease number of quay cranes	(2 0 0)	(0 1 1)	(3 2 2)
4	$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Restrict flag allocation	(2 0 1)	(0 1 1)	(2 2 2)
5	$I = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	Use only 1 sub-block for all flags	(1 2 0)	(1 0 1)	(2 2 2)
6	$C = \begin{bmatrix} 40 & 0 \\ 40 & 0 \end{bmatrix}$	Decrease sub-block capacity	(1 2 0)	(1 0 1)	(2 2 2)

After investigating the multi ship model's variations compared to the base case of two ship scenarios, we investigate the multi ship model's variations compared to a single ship model developed in Chapter 4.

We consider a ship in isolation as the ship that is served alone without the existence of other ships to be served. Hence, all the resources at the container terminal are available for the service of this ship in isolation. In order to clearly identify the impact of other ships on the service of a single ship, we apply our multi ship model by considering a single ship in isolation. The service of 2 ships in isolation requires double the resources needed to serve a single ship with the same total number of containers and an identical distribution of container flags.

Table 5.2 presents the solution for cases presented in table 5.1 with respect to 2 ships in isolation. The “Change” column shows the variation from the base case while keeping other parameters at their base values. To present the solution in a compact form, table 5.2 reports the total number of QCs and YCs utilized in every time period ,  $\mathbf{y}_1 = (y_{11} \ y_{12} \ y_{13})$  for ship 1,  $\mathbf{y}_2 = (y_{21} \ y_{22} \ y_{23})$  for ship 2, and  $\mathbf{z} = (z_1 \ z_2 \ z_3)$ , where  $y_{1t} = \sum_{b=1}^{B_1} y_{1bt}$ ,  $y_{2t} = \sum_{b=1}^{B_2} y_{2bt}$  and  $z_t = \sum_{s=1}^S z_{st}$  for the multi ship scenario. As for the 2 ships in isolation, the the total number of QCs and YCs utilized in every time period,  $\mathbf{y} = (y_1 \ y_2 \ y_3)$  and  $z_t = \sum_{s=1}^S z_{st}$  are reported where  $y_t = \sum_{b=1}^B y_{bt}$ .

Table 5. 2: Summary of Multi Ship Compared to Single Ship in Isolation

Case	Change from Base Case	Description	2 Ships - Multi Ship Optimization			2 Ships In Isolation		Observations
			$y_1$	$y_2$	$z$	$y$	$z$	
1	None	Base case	(1 1 0)	(1 1 0)	(3 3 0)	2*(2 0 0)	2*(3 0 0)	---
2	$YC = (2 2 2)$	Decrease number of yard cranes	(2 0 1)	(0 1 1)	(2 2 2)	2*(1 1 0)	2*(2 2 0)	Multi ship requires more QC and less YC
3	$Q_t = (6 1 1)$	Decrease number of quay cranes	(2 0 0)	(0 1 1)	(3 2 2)	2*(2 0 0)	2*(3 0 0)	Multi ship requires more YC
4	$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Restrict flag allocation	(2 0 1)	(0 1 1)	(2 2 2)	2*(1 1 0)	2*(2 2 0)	Multi ship requires more QC and less YC
5	$I = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	Use only 1 sub-block for all flags	(1 2 0)	(1 0 1)	(2 2 2)	2*(1 1 0)	2*(2 2 0)	Multi ship requires more QC and less YC
6	$C = \begin{bmatrix} 40 & 0 \\ 40 & 0 \end{bmatrix}$	Use 1 sub – block and decrease sub-block capacity	(1 2 0)	(1 0 1)	(2 2 2)	2*(1 1 0)	2*(2 2 0)	Multi ship requires more QC and less YC

### 5.2.2 Large Scale Problems

We apply our model on selected large scale problems space. Consider a base large scale problem with two ships ( $V=2$ ), eight time intervals ( $T=8$ ), eight bays ( $B=8$ ), four flags ( $F=4$ ), and four sub-blocks ( $S=4$ ). Consider the following data as a base case:

- The quay crane (QC) capacity is  $C_Q = 20$  containers/unit time.
- The yard crane (YC) capacity is  $C_Y = 15$  containers/unit time.
- The cost of QC is four times the cost of YC; therefore  $w = 0.8$ .
- The number of QC available at every time period for all ships is 20, i.e.,

$$Q = (10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10)$$

- The number of YC available at every time period for all ships is 20, i.e.,

$$YC = (20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20)$$

- The number of containers of flag  $f$  to be unloaded from ship 1 bay  $b$ ,  $N_{1bf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$N_1 = \begin{bmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix}.$$

- The number of containers of flag  $f$  to be unloaded from ship 2 bay  $b$ ,  $N_{2bf}$ , is 10 for all  $b$  and  $f$ , i.e.,

$$N_2 = \begin{bmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix}$$

- The allocation of flag type  $f$  to the sub-block  $s$  is such that  $I_{fs}=1$  for all  $f$  and  $s$ ,  
i.e.,

$$I = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- The maximum allowed YC per sub-block  $s$  at time period  $t$   $K_{st}$  is given by the following matrix

$$K = \begin{bmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{bmatrix}$$

- The sub-block capacity for all flags is  $C_{fs} = 500$ , i.e.,

$$C = \begin{bmatrix} 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \\ 500 & 500 & 500 & 500 \end{bmatrix}$$

Table 5.3 shows the results of applying our model on several variations of the large base case problem. The “Change” column shows the variation from the base case while keeping other parameters at their base values. To present the solution in a compact form, table 5.3 reports the total number of QCs and YCs utilized,  $\mathbf{y}_1$

$$= \sum_{t=1}^T \sum_{b=1}^{B_1} y_{1bt}, \mathbf{y}_2 = \sum_{t=1}^T \sum_{b=1}^{B_2} y_{2bt} \text{ and } \mathbf{z} = \sum_{t=1}^T \sum_{s=1}^S z_{st}.$$

Details for the solution of each case are available in Appendix F – Large Scale Problems for Multi Ship.

**Table 5. 3: Summary of Large Scale Problems - Multi Ship Scenario**

Case	Change from Base Case	Description	$y_1$	$y_2$	$z$
1	None	Base case	16	16	43
2	$YC = (10\ 10\ 10\ 10\ 3\ 0\ 0\ 0)$	Restrict yard cranes	18	16	43
3	$Q_t = (5\ 5\ 5\ 5\ 5\ 5\ 5)$	Restrict quay cranes	16	16	44
4	$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Restrict flag allocation	16	16	44
5	$C = \begin{bmatrix} 160 & 0 & 0 & 0 \\ 0 & 160 & 0 & 0 \\ 0 & 0 & 160 & 0 \\ 0 & 0 & 0 & 160 \end{bmatrix}$	Decrease sub-block capacity	16	16	44

As we did for the small scale problems, we apply our multi ship model by considering a single ship in isolation for large scale size problems. Table 5.4 presents the solution for cases presented in table 5.3 with respect to a single ship in isolation. The “Change” column shows the variation from the base case while keeping other parameters at their base values. To present the solution in a compact form, table 5.4 reports the total number of QCs and YCs utilized,  $y_1 = \sum_{t=1}^T \sum_{b=1}^{B_1} y_{1bt}$ ,  $y_2 = \sum_{t=1}^T \sum_{b=1}^{B_2} y_{2bt}$  and  $z = \sum_{t=1}^T \sum_{s=1}^S z_{st}$ .

As for the single ship in isolation, the the total number of QCs and YCs utilized  $y =$

$\sum_{t=1}^T \sum_{b=1}^B y_{1bt}$  and  $z = \sum_{t=1}^T \sum_{s=1}^S z_{st}$  are reported.

Table 5. 4: Summary of Multi Ship Compared to Single Ship in Isolation – Large Scale Size Problems

Case	Change from Base Case	Description	2 Ships - Multi Ship Optimization			2 Ships – In Isolation	
			$y_1$	$y_2$	$z$	$y$	$z$
1	None	Base case	16	16	43	2*16	2*22
2	$YC = (10\ 10\ 10\ 10\ 3\ 0\ 0\ 0)$	Decrease number of yard cranes	18	16	43	2*16	2*22
3	$Q_t = (5\ 5\ 5\ 5\ 5\ 5\ 5\ 5)$	Decrease number of quay cranes	16	16	44	2*16	2*22
4	$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Restrict flag allocation	16	16	44	16	24
5	$C = \begin{bmatrix} 160 & 0 & 0 & 0 \\ 0 & 160 & 0 & 0 \\ 0 & 0 & 160 & 0 \\ 0 & 0 & 0 & 160 \end{bmatrix}$	Decrease sub-block capacity	16	16	44	16	24



### 5.3 Insights and Analysis

In this section, in the first sub-section we confirm the insights deduced in Chapter 4 for the single ship model, and then in the second sub-section we draw insights related to model representing the multiple ships berthing at the same time.

#### *5.3.1 Confirmation of Insights from Single Ship Model in case of Multiple Ships*

Tables 5.1 and 5.3 confirm the insights deduced in Chapter 4 for the single ship model scenario:

***The limitation of the available cranes over time at either the quay side or the yard side will affect allocation and scheduling of cranes on the other side.***

In case 2 of table 5.1, decreasing the number of yard cranes to 2 YCs at every time period leads to the need for an additional QC to unload all the required containers. In case 3 of table 5.1, the limitation of quay cranes to 1 QC at time period 2 and time period 3 leads to the need for an additional YC at time period 1 to balance the unloading of containers. A similar observation is made in case 2 of table 5.3, the restriction of yard cranes leads to more usage of quay cranes. In case 3 of table 5.3, the restriction of quay cranes leads to more usage of yard cranes.

***Restricting flag allocation to sub – blocks in a multi ship scenario leads to using more resources on the quay side.***

In case 4 of table 5.1, the restriction of flag allocation leads to an increase of 1 QC because the yard cranes allowed at each sub-block is 2,  $K_{st} = 2$  for all  $s$  and  $t$ . Thus, the quay side has to increase the number of cranes to adjust to this flag restriction. In case 5 of table 5.1, the restriction of flags to a single sub- block has forced the quay side to increase one QC to balance with the yard side, similar to case 4. In the large

scale problems, this insight is illustrated in case 4 of table 5.3 where restricting flags allocation on the yard led to an increase in the required number of yard cranes.

***Restricting storage capacities in sub-blocks in a multi ship scenario leads to using more resources on the quay side.***

In case 6 of table 5.1, we have decreased the sub-block capacity to 40 containers for each flag, which leads to an increase in QC requirements. In the large scale problems, this insight is illustrated in case 5 of table 5.3 where restricting sub-block capacity on the yard led to an increase in the number of yard cranes.

### ***5.3.2 Insights from Multi Ship Model***

Tables 5.2 and 5.4 reveal the following insights regarding the berthing of multiple ships at the same time compared to two single ships in isolation.

***First Insight: Restricting resources at the yard side, in case of several ships berthing at the same time, leads to (i) an increase in quay crane requirements and (ii) a decrease in yard cranes required to serve the berthing ships.***

In case 2 of table 5.2, we have restricted the yard crane availability to 2 YCs at every time period. In the single ship in isolation 2 QCs and 4 YCs are required to serve the ship. Thus, for two single ships in isolation, 4 QCS and 8 YCs are required. However, in integrating the two ships in a multi ship model 5 QCs and 6 YCs are required instead of 4 QCs and 8 YCs.

As for the large scale size problems, in case 2 of table 5.4 the number of cranes required to serve a single ship in isolation is 16 QCs and 22 YCs. Thus, for two single ships in isolation, 32 QCs and 44 YCs are required. However, in integrating the two ships 34 QCs and 43 YCs are required.

***Second Insight: Restricting flags allocation or decreasing sub-block capacities leads to using additional resources on the quay side.***

In case 4 of table 5.2, the flag allocation is restricted so containers of flag type 1 are restricted to be stored in sub-block 1, while containers of flag type 2 are restricted to be stored in sub-block 2. In this case, for the single ship in isolation 2 QCs and 4 YCs are required; thus, for two single ships in isolation 4 QCS and 8 YCs are required. However, in integrating the two ships in a multi ship model 5 QCs and 6 YCs are required instead of 4 QCs and 8 YCs. In case 5 of table 5.2, all containers of different flag types are stored in a single sub-block. In this case, for two single ships in isolation 4 QCS and 8 YCs are required. However, in integrating the two ships in a multi ship model 5 QCs and 6 YCs are required instead of 4 QCs and 8 YCs. In case 6 of table 5.2, sub-block capacities are restricted. In this case, for two single ships in isolation 4 QCS and 8 YCs are required. However, in integrating the two ships in a multi ship model 5 QCs and 6 YCs are required instead of 4 QCs and 8 YCs.

As for the large scale size problems, in case 4 of table 5.4 where flag allocation is restricted, the number of cranes required for two single ships in isolation is 32 QCs and 48 YCs. However, in integrating the two ships 32 QCs and 44 YCs are required. The same applies for case 5 of table 5.4 where we restrict the sub-block capacity; the number of cranes required to serve two single ships in isolation is 32 QCs and 48 YCs. However, in integrating the two ships 32 QCs and 44 YCs are required.

***Third Insight: The limitation in quay cranes, in case of several ships berthing at the same time, leads to an increase in the number of required yard cranes; however, the number of quay cranes required remains constant.***

In case 3 of table 5.2, we have limited the quay crane availability to 1 QC at time period 2 and time period 3. In this case, for the single ship in isolation 2 QCs and 3 YCs are required to serve the ship; thus, for two single ships in isolation 4 QCs and 6 YCs are required. However, in integrating the two ships in a multi ship model 4 QCs and 7 YCs are required instead of 4 QCs and 6 YCs. As for the large scale problems, case 3 of table 5.4 confirms that the number of quay cranes remains constant.

# CHAPTER 6

## LINK BETWEEN

### THE OPERATIONAL AND STRATEGIC LEVELS

In this chapter, the link between the operational and strategic levels in a transshipment container terminal is investigated. The methodology suggested in this study is to extend the mathematical model formulated in Chapter 3 via embedding the outcome from the optimization of quay side and yard side resources during the transshipment process of containers in a container terminal, as presented in Chapter 4 and Chapter 5, into the utility functions formulated in Chapter 3.

The objective of this chapter is to propose an “initial” approach for linking the operational and strategic levels in the transshipment container terminal. This chapter is structured as follows. In the first section the initial approach to link the optimization of resources at the operational level with the port selection process at the strategic level is discussed. In the second section numerical examples are presented and discussed.

#### **6.1 Extended Mathematical Model**

The integration of quay side and yard side resources during the transshipment process of containers, at the operational level, leads to optimization of resources used in handling and storing containers at the container terminal, as illustrated in Chapter 4 and Chapter 5. Thus, two factors are affected at this level: (1) handling cost of containers, and (2) storage cost of containers.

From Chapter 3, the four criteria recognized by Lirn et al. (2004) for port selection are: (1) Port Physical and Technical Infrastructure, (2) Port Geographical Location, (3) Port Management and Administration and (4) Carriers' Terminal Cost.

Lirn et al. (2004) identified the handling cost of containers (HCC) and the storage cost of containers (SCC) as falling under the fourth criterion "Carriers' Terminal Cost".

Lirn et al. (2004) estimated the weight of HCC and SCC to be 24.27 % and 6.53 % respectively for global port selection from carriers' perspectives. The mentioned weights are the results of evaluation by 18 global carriers (Lirn et al., 2004, Table 6).

In this section, we amend the utility function for port selection at the strategic level, the revenue function, and the cost function based on the optimization of resources used at the quay side and yard side at the operational level.

### 6.1.1 Utility Function for Port Selection

The attractiveness for a carrier to select a specific port  $i$  defined in Chapter 3 (Equation 3.1) is amended, after optimization of resources used at the quay side and yard side, to be:

$$u_i = w_1a_i + w_2b_i + w_3c_i + w_4 \left[ d_i + \left( \frac{w_{HCC}}{w_4} \right) * PIHCC + \left( \frac{w_{SCC}}{w_4} \right) * PISCC \right]$$

(Equation 6.1)

Where:

- $w_{HCC}$  is the weight of handling cost of containers
- $PIHCC$  is the percentage improvement of handling cost of containers after optimization at the operational level and equal to  $1 - \frac{HCC \text{ after optimization}}{HCC \text{ before optimization}}$

- $w_{SCC}$  is the weight of storage cost of containers
- $PISCC$  is the percentage improvement of storage cost of container after optimization at the operational level  $1 - \frac{SCC \text{ after optimization}}{SCC \text{ before optimization}}$

All other terms remain the same as defined in Chapter 3.

As we have defined the enhancement boundaries for investments in Chapter 3 in Equation 3.5 and Equation 3.6, we define the upper and lower bounds for the improvements to be

$$d + \left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC \leq 5 \text{ for every port } i \quad \text{Equation (6.2)}$$

$$\left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC \geq 0 \text{ for every port } i \quad \text{Equation (6.3)}$$

### 6.1.2 Revenue Function

The revenue function for a port  $i$  after optimization of resources at quay side and yard side, based on Equation 3.9, is amended as follows:

$$P_i = a_i(1 - e^{-a_i}) + b_i(1 - e^{-b_i}) + c_i(1 - e^{-c_i}) + \left(d_i + \left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC\right) \left(1 - e^{-\left[d_i + \left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC\right]}\right) \quad \text{Equation (6.4)}$$

The revenue function for a port  $i$  after optimization, based on Equation 3.10, is amended to be:

$$P_i = (a_i + \Delta a_i)(1 - e^{-(a_i + \Delta a_i)}) + (b_i + \Delta b_i)(1 - e^{-(b_i + \Delta b_i)}) + (c_i + \Delta c_i)(1 - e^{-(c_i + \Delta c_i)}) + \left(d_i + \left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC + \Delta d_i\right) \left(1 - e^{-\left(d_i + \left(\frac{w_{HCC}}{w_4}\right) * PIHCC + \left(\frac{w_{SCC}}{w_4}\right) * PISCC + \Delta d_i\right)}\right)$$

$$\text{Equation (6.5)}$$

### 6.1.3 Cost Function

The cost function after optimization, based on Equation 3.11, is amended as follow:

$$C(\Delta a_i, \Delta b_i, \Delta c_i, \Delta d_i) = a_i * \Delta a_i^{\Delta a_i + b_i} * \Delta b_i^{\Delta b_i} + c_i * \Delta c_i^{\Delta c_i} + \left( d_i + \left( \frac{w_{HCC}}{w_4} \right) * PIHCC + \left( \frac{w_{SCC}}{w_4} \right) * PISCC \right) * \Delta d_i^{\Delta d_i}$$

Equation (6.6)

## 6.2 Numerical Examples

In this section, we present numerical examples to illustrate the link between the operational level and the strategic level in a container terminal.

We consider 5 scenarios in this section. In scenario 1, which is the base case, we have 2 identical ports with the following attributes:  $a=3$ ,  $b=3$ ,  $c=3$  and  $d=3$ , and only 1 resource unit is available for investment. In the following scenarios, we assume that Port 1 only has optimized the resources used at the quay side and yard side at the operational level.

In scenario 2 we consider that the optimization of resources leads to an improvement of 5 % only in both the HCC and SCC. In scenario 3 we consider that the optimization of resources leads to an improvement of 25 % in both the HCC and SCC. In scenario 4 we consider that the optimization of resources leads to an improvement of 50 % in both the HCC and SCC. In scenario 5 we consider that the optimization of resources leads to an improvement of 100 % in both the HCC and SCC, in other terms the optimization leads to eliminating the HCC and SCC.



### 6.2.1 Scenario 1 – Base Case Scenario

Table 6.1 tabulates the attractiveness utility of each port for the carrier by investment option. Table 6.2 tabulates which port will win the bid for potential investment option. Table 6.3 tabulates the payoff for each scenario.

**Table 6. 1:Base Case - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.00	3.00	3.00	3.38	3.00	3.10	3.00	3.16
	(0,0,0,1)	3.38	3.00	3.38	3.38	3.38	3.10	3.38	3.16
	(0,0,1,0)	3.10	3.00	3.10	3.38	3.10	3.10	3.10	3.16
	(1,0,0,0)	3.16	3.00	3.16	3.38	3.16	3.10	3.16	3.16

**Table 6. 2: Base Case - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port1/Port2	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port1/Port2	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 2	Port1/Port2	Port 2
	(1,0,0,0)	Port 1	Port 2	Port 1	Port1/Port2

**Table 6. 3: Base Case – Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	0.00	0.00	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.48	0.00	-3.00	-3.00	9.48	-3.00	9.48	-3.00
	(0,0,1,0)	9.48	0.00	-3.00	9.48	-3.00	-3.00	-3.00	9.48
	(1,0,0,0)	9.48	0.00	-3.00	9.48	9.48	-3.00	-3.00	-3.00

### 6.2.2 Scenario 2- 5 % Improvement

Table 6.4 tabulates the attractiveness utility of each port for the carrier by investment option. Table 6.5 tabulates which port will win the bid for potential investment option. Table 6.6 tabulates the payoff for each scenario.

**Table 6. 4: 5 % Improvement - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.02	3.00	3.02	3.38	3.02	3.10	3.02	3.16
	(0,0,0,1)	3.40	3.00	3.40	3.38	3.40	3.10	3.40	3.16
	(0,0,1,0)	3.12	3.00	3.12	3.38	3.12	3.10	3.12	3.16
	(1,0,0,0)	3.18	3.00	3.18	3.38	3.18	3.10	3.18	3.16

**Table 6. 5: 5 % Improvement - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 1	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 1	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 2	Port 1	Port 2
	(1,0,0,0)	Port 1	Port 2	Port 1	Port 1

**Table 6. 6: 5 % Improvement – Payoff**

		PORT 2							
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)				
PORT 1	(0,0,0,0)	11.45	0.00	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.48	0.00	9.48	-3.00	9.48	-3.00	9.48	-3.00
	(0,0,1,0)	9.52	0.00	-3.00	9.48	9.52	-3.00	-3.00	9.48
	(1,0,0,0)	9.52	0.00	-3.00	9.48	9.52	-3.00	9.52	-3.00

**6.2.3 Scenario 3 - 25 % Improvement**

Table 6.7 tabulates the attractiveness utility of each port for the carrier by investment option. Table 6.8 tabulates which port will win the bid for potential investment option. Table 6.9 tabulates the payoff for each scenario.

**Table 6. 7: 25 % Improvement - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)				
PORT 1	(0,0,0,0)	3.08	3.00	3.08	3.38	3.08	3.10	3.08	3.16
	(0,0,0,1)	3.46	3.00	3.46	3.38	3.46	3.10	3.46	3.16
	(0,0,1,0)	3.18	3.00	3.18	3.38	3.18	3.10	3.18	3.16
	(1,0,0,0)	3.24	3.00	3.24	3.38	3.24	3.10	3.24	3.16

**Table 6. 8: 25 % Improvement - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 1	Port 2	Port 2	Port 2
	(0,0,0,1)	Port 1	Port 1	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 2	Port 1	Port 1
	(1,0,0,0)	Port 1	Port 2	Port 1	Port 1

**Table 6. 9: 25 % Improvement – Payoff**

		PORT 2							
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)				
PORT 1	(0,0,0,0)	11.62	0.00	0.00	9.48	0.00	9.48	0.00	9.48
	(0,0,0,1)	9.49	0.00	9.49	-3.00	9.49	-3.00	9.49	-3.00
	(0,0,1,0)	9.70	0.00	-3.00	9.48	9.70	-3.00	9.70	-3.00
	(1,0,0,0)	9.70	0.00	-3.00	9.48	9.70	-3.00	9.70	-3.00

### 6.2.4 Scenario 4 - 50 % Improvement

Table 6.10 tabulates the attractiveness utility of each port for the carrier by investment option. Table 6.11 tabulates which port will win the bid for potential investment option. Table 6.12 tabulates the payoff for each scenario.

**Table 6. 10: 50 % Improvement - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.15	3.00	3.15	3.38	3.15	3.10	3.15	3.16
	(0,0,0,1)	3.54	3.00	3.54	3.38	3.54	3.10	3.54	3.16
	(0,0,1,0)	3.26	3.00	3.26	3.38	3.26	3.10	3.26	3.16
	(1,0,0,0)	3.32	3.00	3.32	3.38	3.32	3.10	3.32	3.16

**Table 6. 11: 50 % Improvement - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 1	Port 2	Port 1	Port 2
	(0,0,0,1)	Port 1	Port 1	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 2	Port 1	Port 1
	(1,0,0,0)	Port 1	Port 2	Port 1	Port 1

**Table 6. 12: 50 % Improvement – Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	11.84	0.00	0.00	9.48	11.84	-3.00	0.00	9.48
	(0,0,0,1)	9.50	0.00	9.50	-3.00	9.50	-3.00	9.50	-3.00
	(0,0,1,0)	9.92	0.00	-3.00	9.48	9.92	-3.00	9.92	-3.00
	(1,0,0,0)	9.92	0.00	-3.00	9.48	9.92	-3.00	9.92	-3.00

### 6.2.5 Scenario 5 - 100 % Improvement

Table 6.13 tabulates the attractiveness utility of each port for the carrier by investment option. Table 6.14 tabulates which port will win the bid for potential investment option. Table 6.15 tabulates the payoff for each scenario.

**Table 6. 13: 100 % Improvement - Attractiveness Utility**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	3.31	3.00	3.31	3.38	3.31	3.10	3.31	3.16
	(0,0,0,1)	3.69	3.00	3.69	3.38	3.69	3.10	3.69	3.16
	(0,0,1,0)	3.41	3.00	3.41	3.38	3.41	3.10	3.41	3.16
	(1,0,0,0)	3.47	3.00	3.47	3.38	3.47	3.10	3.47	3.16

**Table 6. 14: 100 % Improvement - Port Selection**

		PORT 2			
		(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(1,0,0,0)
PORT 1	(0,0,0,0)	Port 1	Port 2	Port 1	Port 1
	(0,0,0,1)	Port 1	Port 1	Port 1	Port 1
	(0,0,1,0)	Port 1	Port 1	Port 1	Port 1
	(1,0,0,0)	Port 1	Port 1	Port 1	Port 1

**Table 6. 15: 100 % Improvement – Payoff**

		PORT 2							
		(0,0,0,0)		(0,0,0,1)		(0,0,1,0)		(1,0,0,0)	
PORT 1	(0,0,0,0)	12.28	0.00	0.00	9.48	12.28	-3.00	12.28	-3.00
	(0,0,0,1)	9.51	0.00	9.51	-3.00	9.51	-3.00	9.51	-3.00
	(0,0,1,0)	10.35	0.00	10.35	-3.00	10.35	-3.00	10.35	-3.00
	(1,0,0,0)	10.35	0.00	10.35	-3.00	10.35	-3.00	10.35	-3.00

### 6.2.6 Discussion

The presented approach in this chapter provides a link between the operational and strategic levels in the transshipment container terminal. The numerical examples reveal the high impact of resource optimization at the operational level on the resource investment at the strategic level. In the base scenario (Scenario 1), table 6.1 and table 6.2 indicate that no dominant strategy is available for any of the two ports. In Scenario 2 the 5 % improvement at the operational level for Port 1 has increased the initial attractiveness utility for Port 1 to 3.02 at the strategic level, which led to a dominant investment strategy of (0, 0, 0, 1) compared to the base scenario. Compared to Scenario 2, the 25 % improvement at the operational level in Scenario 3 has increased the initial attractiveness utility for Port 1 to 3.08 at the strategic level, which led to more winning investment strategies for Port 1. Compared to Scenario 2 and Scenario 3, the 50 % improvement at the operational level in Scenario 4 has increased the initial attractiveness utility for Port 1 to 3.15 at the strategic level. In this case, any investment strategy used by Port 1, given that Port 2 avoids investment strategy (0, 0, 0, 1), leads to Port 1 “winning” the bid. In

Scenario 5 any investment strategy used by Port 1 is a strictly dominant investment strategy and Port 2 is advised to avoid applying any investment in its facilities.

Even though two scenarios of an improvement of 50 % (Scenario 4) and an improvement of 100 % (Scenario 5) are analyzed in this section, we feel that in reality the integration between quay side and yard side at the operational level will generate a maximum improvement of 25 % for medium and large size container terminal operators and a maximum improvement of 50 % for small container terminal operators.

This “initial” link provides a managerial tool for port authorities to enhance their strategic investment strategies via optimizing their resource allocation at the operational level.

## CHAPTER 7

### SUMMARY AND FUTURE RESEARCH

#### 7.1 Summary

In this dissertation, we study the optimization of resources in a transshipment container terminal from both strategic and operational perspectives. In addition, an “initial” approach is presented to link the improvements made at the operational level to the strategic level investment decision in port.

##### 7.1.1 *Strategic Level*

In the first part of this dissertation, the resource allocation strategies used by port authorities and container terminal operators (CTOs) to attract carriers from a strategic perspective are analyzed based on a game theory approach that takes into account a number of port selection criteria by carriers. Utility functions are developed to represent carrier’s preference for a specific port subject to financial, physical, and location constraints. Four criteria, based on Lirn et al. (2004), are selected in the attractiveness utility to select a specific port by a carrier: (1) port physical and technical infrastructure, (2) port geographical location, (3) port management and administration and (4) carriers’ terminal cost. Then, a revenue function is developed, reflecting a diminishing marginal productivity, and a cost function is developed having an increasing return to scale property. Budget, capacity, location, manpower and price constraints are formulated to limit investments inside ports. Then an auction game with two players having complete and perfect information is illustrated.

In order to assess the weights used for the four mentioned criteria, a sensitivity analysis is conducted for five different scenarios. In the first scenario a base case is formulated according to weights determined by Lirn et al. (2004). In the second scenario an equal preference for each of the four criteria is assumed. In the third scenario the port geographical location is considered to be the most important criterion for the carrier. In the fourth scenario the carrier is assumed to be cost reduction seeker. In the fifth scenario the port efficiency is considered to be the primary criterion for the carrier. In the base scenario, Port 2 has 2 dominant investment strategies while in the second scenario no port has a dominant investment strategy due to the equality of preferences. In the third scenario, Port 2 has 2 dominant solutions regardless of the strategy used by the other port. In the fourth scenario, Port 2 has a unique dominant investment strategy. In the fifth scenario, Port 1 has 2 dominant investment strategies. From the above scenarios, we conclude that the weights assigned to each criterion affect the optimal investment strategy used by ports based on the initial attractiveness utility of each port for the carrier and the payoff generated from each investment strategy.

A sensitivity analysis for resource allocation is presented. Five cases are illustrated in this analysis. In the first case one resource unit is available for investment in both ports, while in the second case two resource units are available to invest in both ports. In the third case three resource units are available to invest in both ports. In the fourth case three resource units are available to invest, with a restriction on the level of investment related to the fourth criterion “carriers’ terminal cost” for both ports. In the fifth case three resource units are available to invest, with a restriction on the level of investment related to the fourth criterion “carriers’ terminal cost” for one port only. In the base case, Port 2 has 2 dominant investment strategies, while in the second case

only 1 dominant investment strategy is available to Port 2 with a low payoff. In the third case, a new dominant investment strategy is generated for Port 2; however, the new dominant strategy has a negative payoff (not profitable). In the fourth case, Port 2 has 2 dominant investment strategies, one of them having a negative payoff. In the fifth case, Port 2 loses the advantage of having a dominant strategy and Port 1 has a dominant investment strategy with a negative payoff.

This part is concluded with managerial tools and insights for port authorities and port operators around the world to advise them about the optimal manner in allocating their resources at the strategic level based on the above scenarios and cases. Notions of strictly dominant bidder, weakly dominant bidder, profitable investment strategy and strictly dominant investment strategy are defined.

### ***7.1.2 Operational Level***

The second part of this dissertation analyzes the resource (mainly quay and yard cranes) allocation inside the port from an operational perspective during the container transshipment process. The integration between the quay side and yard side for a single berthing ship with transshipment containers is investigated. This dissertation presents a novel optimization model for transshipment operations that effectively coordinates the workloads of the quay and the yard sides.

A mathematical model is formulated to optimize the number of quay cranes (QCs) and yard cranes (YCs) used during the container transshipment process in a container terminal. The optimization approach assists in determining the number of containers loaded at each time period according to crane used, quay location and storage location. Crane capacities on both quay and yard sides, time constraints, and spatial



constraints on both the discharge and storage areas are taken into consideration. The formulated model is an integer linear programming problem and we prove it to be NP-hard problem since it is reduced to the multi-dimensional knapsack problem in a special case. The mathematical model is first tested on small scale size problems. Then large “industry-size” problems from the Beirut Container Terminal are considered. The model output gives the complete location assignments and the schedules of quay cranes and yard cranes.

All numerical examples are solved using GUROBI solver on AMPL compiler.

For the single ship model, four insights are drawn as follow:

- First Insight: The limitation of the available crane capacity over time at either the quay side or the yard side will affect allocation and scheduling of cranes on the other side.
- Second Insight: Restricting flag allocation to sub-blocks leads to using more resources not only on the yard side, but also on the quay side.
- Third Insight: Restricting storage capacities in sub-blocks leads to using more resources not only on the yard side, but also on the quay side.
- Fourth Insight: The weight assigned to utilization of yard cranes and quay cranes in the objective function is critical and will affect the number of resources used at both yard and quay side.

After considering a single ship berthing scenario, an extension that considers a multi-ship berthing scenario was introduced. The integration between the quay and yard sides for multiple berthing ships with transshipment containers is investigated, where the loading and unloading of containers for several vessels at the same time is represented. In the multi ship model, at the quay side, a QC can serve at most one bay per time period for a specific vessel, and the number of QCs to serve a specific vessel

is limited. At the yard side, sharing the same yard crane between vessels in the same sub-block at the same time unit is allowed.

We validate that the insights drawn for the single ship model are also applied for the multi ship model, and then we conduct numerical examples to deduce the following new insights:

- First Insight: Restricting resources at the yard side, in case of several ships berthing at the same time, leads to (i) an increase of quay cranes and (ii) a decrease of yard cranes required to serve the berthing ships.
- Second Insight: Restricting flag allocation or decreasing sub-block capacities leads to using additional resources on the quay side.
- Third Insight: The limitation in quay cranes, in case of several ships berthing at the same time, leads to an increase of yard crane requirements; however, the number of quay cranes required remains constant.

### ***7.1.3 Link between Operational Level and Strategic Level***

The link between the operational and strategic levels in the transshipment container terminal is investigated. By considering the handling cost and storage cost of containers at the operational level, an initial feedback to the attractiveness utility of ports to carriers is drawn at the strategic level. The utility function for port selection, the revenue function, and the cost function at the strategic level are amended to embed the optimization of resources used at the quay side and yard side at the operational level. Five scenarios, based on 2 ports, are presented to highlight the impact of resource optimization at the operational level on the resource investment at

the strategic level. In each scenario a certain percentage of improvement at the operational level is considered for Port 1 only.

In the base scenario (Scenario 1) the optimization at the operational level is not considered at the strategic level; in this case no dominant strategy is available for any of the two ports. In Scenario 2 a 5 % improvement at the operational level for Port 1 is considered, which led to a dominant investment strategy compared to the base scenario. Compared to Scenario 2, the 25 % improvement at the operational level in Scenario 3 for Port 1 led to more “winning” investment strategies for Port 1.

Compared to Scenario 2 and Scenario 3, the 50 % improvement at the operational level in Scenario 4 led to Port 1 “winning” the bid in the majority of the potential investment strategies. In Scenario 5, where a 100 % improvement at the operational level is considered, any investment strategy used by Port 1 is a strictly dominant investment strategy and Port 2 is advised to avoid applying any investment in its facilities.

## **7.2 Future Research**

### ***7.2.1 Strategic Level***

At the strategic level, our main target in the near future is to apply the formulated model to ports in the Middle East and Arab Gulf countries.

The formulated model assumes complete and perfect information among players.

Considering incomplete and imperfect information among players is a possible interest for future research. The quality of information in the incomplete and imperfect situations can be assessed in terms of cost paid to enhance the quality of

information about competitors. The work performed by Yassine et al. (2012) regarding the optimal information exchange can be integrated to estimate the cost of information in mitigating risks in port investment.

Coalition among different terminal operators is another potential research to minimize investment cost and avoid competition in attracting carriers.

### ***7.2.2 Operational Level***

At the operational level, the numerical results on small scale problems and large scale problems indicate that coordination of the quay and yard sides implies that scheduling and resource allocation at one side may be affected by the physical constraints on the other side. However, due to the complexity of the problem the time needed to reach optimal solutions can be quite high. We are currently experimenting with two heuristic approaches to solve the integration of quay side with the yard side. In the first approach, cranes are assigned to the quay side in a feasible way which provides a sufficient number of cranes to unload the ship over allowable time, and then the number of required yard cranes is determined. In the second approach, the idle time of cranes is minimized using greedy assignments. Given the difficulty of solving large problems, the main objective of the future research will be to develop solution methodologies for large “industry-size” problems.

We are currently investigating structural properties of the optimal solution that may aid in developing efficient solution algorithms and heuristics. Once these efficient solution methodologies are developed, they will be tested on realistic problem instances from our industry partner, Beirut port.

In addition, considering the effect of the waiting and delay cost of internal trucks on the transshipment process is a worthwhile direction for further research. The same holds for the queuing that may occur in dealing with berthing priorities amongst vessels.

Finally, a model that integrates transshipment with other concurrent activities at the port such as import and export is also an interesting venue for future research.

### ***7.2.3 Link between Operational Level and Strategic Level***

The proposed approach in this dissertation to link the operational level and the strategic level in optimization of resources in a transshipment container terminal is preliminary. Further investigation regarding the structure of the suggested model shall be pursued in the near future. Detailed sensitivity analysis for the extended model is required. In addition, the impact of sharing resources between players at the strategic level is of possible interest. Finally, integrating other criteria than handling cost and storage cost of containers is also interesting venue for future research.

## APPENDIX A

### SINGLE SHIP – AMPL CODE

```
# Define sets#

set B;           # set of bays
set T;           # set of Time
set S;           # set of sub-blocks
set F;           # set of flags

# Define parameters #
param W >=0 , <= 1;           # Weight of QC and YC Cranes
param QC {T} >=0;             # Number of QC allowed at each time period
param YC {T} >=0;             # Number of YC allowed at each time period
param CQ >=0;                  # Capacity of QC [containers move/hour]
param CY >=0;                  # Capacity of YC [containers move/hour]
param I{F,S} >=0;             # Allowed Area for containers of flag f into sub
block s
param N{B,F} >=0;             # Number of containers from bay b for flag f
param K{S,T} >=0;             # Number of YC allowed at each time t for sub
block s
param C{F,S} >=0;             # Number of Containers allowed for each f in
sub block s

# Define variables #

var x{b in B,f in F, t in T, s in S} integer >= 0;
# Number of Container unloaded from each bay, flag, time to subblock

var y {b in B, t in T} binary ;
# Number of bay used at time t

var z {s in S,t in T} integer;
# Number of RTG used in time t for sub block s

# Write the objective function #

minimize Total_Y_Z: (W*(sum {b in B,t in T} y[b,t])) + ((1-W)*(sum { s in S , t in
T} z[s,t]));
# sum

# Specify Constraints #
```

#constraint 2 - Number of available STS at each time period t

subject to equation2 { t in T}:  $\sum \{b \text{ in } B\} y[b,t] \leq QC[t];$

# constraint 3 -number of containers to be unloaded from QUay Yard at each time period t for each bay b and flag f to be transported to sub block s

subject to equation3 { b in B, t in T}:  $(\sum \{f \text{ in } F, s \text{ in } S\} x[b,f,t,s])/CQ \leq y[b,t];$

# constraint 4- Number of Containers unload  $x[b,f,t,s]$  should be equal to number of containers  $N[b,f]$

subject to equation4 { b in B, f in F}:  $\sum \{t \text{ in } T, s \text{ in } S\} x[b,f,t,s] = N[b,f];$

# constraint 5- Number of RTG available at each time t

subject to equation5 { t in T}:  $\sum \{s \text{ in } S\} z[s,t] \leq YC[t];$

# constraint 6 - Number of containers to be stored in the Storage Yard at each time period t per sub block s

subject to equation6 { s in S, t in T}:  $(\sum \{f \text{ in } F, b \text{ in } B\} x[b,f,t,s])/CY \leq z[s,t];$

# constraint 7 - Allowed Area in the Storage Yard where the containers could be stored based on their flag f

subject to equation7 { b in B, f in F, t in T, s in S}:  $x[b,f,t,s] \leq 1000 * I[f,s];$

# constraint 8 - Allowed RTG available to be used at every sub block s at time t

subject to equation8 { s in S, t in T}:  $z[s,t] \leq K[s,t];$

# constraint 9 - the number of containers discharged in every sub block will not exceed the spatial area allowed for every flag type of containers

subject to equation9 { s in S, f in F}:  $\sum \{b \text{ in } B, t \text{ in } T\} x[b,f,t,s] \leq C[f,s];$

## APPENDIX B

### SINGLE SHIP - SMALL SCALE PROBLEMS

#### *Base Case Solution*

##### Quay Cranes

y :=  
1 1 1  
1 2 0  
1 3 0  
2 1 1  
2 2 0  
2 3 0

##### Yard Cranes

z :=  
1 1 1  
1 2 0  
1 3 0  
2 1 2  
2 2 0  
2 3 0

##### Containers Unloading

x [1,1,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0

[1,2,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0

[2,1,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[2,2,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0



## ***Case 2 Solution***

### Quay Cranes

y :=  
1 1 0  
1 2 1  
1 3 1  
2 1 0  
2 2 1  
2 3 0

### Yard Cranes

z :=  
1 1 0  
1 2 1  
1 3 0  
2 1 0  
2 2 1  
2 3 1

### Containers Unloading

x [1,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10

[1,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 5 5  
3 0 0

[2,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0

[2,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0

### **Case 3 Solution**

#### Quay Cranes

```
y :=  
1 1 1  
1 2 0  
1 3 1  
2 1 1  
2 2 1  
2 3 0
```

#### Yard Cranes

```
z :=  
1 1 1  
1 2 1  
1 3 0  
2 1 0  
2 2 0  
2 3 1
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 5 0  
2 0 0  
3 0 5
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10
```

```
[2,1,*,*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

### **Case 4 Solution**

#### Quay Cranes

```
y :=  
1 1 1  
1 2 0  
1 3 0  
2 1 0  
2 2 0  
2 3 1
```

#### Yard Cranes

```
z :=  
1 1 1  
1 2 0  
1 3 2  
2 1 1  
2 2 0  
2 3 0
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 5 5  
2 0 0  
3 0 0
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[2,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0
```

### **Case 5 Solution**

#### Quay Cranes

```
y :=  
1 1 0  
1 2 1  
1 3 0  
2 1 0  
2 2 0  
2 3 1
```

#### Yard Cranes

```
z :=  
1 1 0  
1 2 1  
1 3 1  
2 1 0  
2 2 1  
2 3 1
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

```
[2,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10
```

### **Case 6 Solution**

#### Quay Cranes

```
y :=  
1 1 1  
1 2 0  
1 3 0  
2 1 1  
2 2 1  
2 3 0
```

#### Yard Cranes

```
z :=  
1 1 0  
1 2 0  
1 3 0  
2 1 2  
2 2 1  
2 3 0
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[2,1,*,*]  
: 1 2 :=  
1 0 5  
2 0 5  
3 0 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

### **Case 7 Solution**

#### Quay Cranes

```
y :=  
1 1 1  
1 2 0  
1 3 0  
2 1 0  
2 2 1  
2 3 0
```

#### Yard Cranes

```
z :=  
1 1 0  
1 2 2  
1 3 0  
2 1 2  
2 2 0  
2 3 0
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[2,1,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

**Case 8 Solution**

Quay Cranes

y :=  
1 1 1  
1 2 0  
1 3 0  
2 1 1  
2 2 1  
2 3 0

Yard Cranes

z :=  
1 1 0  
1 2 0  
1 3 0  
2 1 2  
2 2 1  
2 3 0

Containers Unloading

x [1,1,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0

[1,2,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0

[2,1,\*,\*]  
: 1 2 :=  
1 0 5  
2 0 5  
3 0 0

[2,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0

**Case 9 Solution**

Quay Cranes

y :=

```
1 1 1
1 2 0
1 3 0
2 1 0
2 2 1
2 3 0
```

### Yard Cranes

```
z :=
1 1 0
1 2 0
1 3 0
2 1 2
2 2 2
2 3 0
```

### Containers Unloading

```
x [1,1,*,*]
: 1 2 :=
1 0 10
2 0 0
3 0 0
```

```
[1,2,*,*]
: 1 2 :=
1 0 10
2 0 0
3 0 0
```

```
[2,1,*,*]
: 1 2 :=
1 0 0
2 0 10
3 0 0
```

```
[2,2,*,*]
: 1 2 :=
1 0 0
2 0 10
3 0 0
```



### **Case 10 Solution**

#### Quay Cranes

```
y :=  
1 1 1  
1 2 1  
1 3 0  
2 1 0  
2 2 1  
2 3 0
```

#### Yard Cranes

```
z :=  
1 1 0  
1 2 0  
1 3 0  
2 1 1  
2 2 2  
2 3 0
```

#### Containers Unloading

```
x [1,1,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[1,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

```
[2,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

```
[2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

## APPENDIX C

### SINGLE SHIP – LARGE SCALE PROBLEMS

#### *Large Base Case Solution*

Quay Cranes

y[\*,\*] (tr):

	1	2	3	4	5	6	7	8	:=
1	1	1	0	0	1	0	0	0	1
2	0	0	0	1	0	1	0	0	1
3	1	1	0	0	1	1	1	1	
4	0	0	1	1	0	0	1	0	
5	1	1	1	1	1	1	1	1	
6	1	1	0	1	1	1	1	0	
7	0	1	1	0	1	0	0	0	
8	1	1	1	0	1	1	1	0	
9	1	1	1	1	1	1	1	1	
10	1	0	1	1	0	1	1	1	
11	1	1	1	1	0	1	0	1	
12	0	1	0	1	1	1	1	1	

Yard Cranes

z[\*,\*] (tr)

	1	2	3	4	:=
1	0	0	0	4	
2	4	0	0	0	
3	4	3	1	0	
4	4	0	0	0	
5	3	4	0	4	
6	0	0	4	4	
7	0	0	4	0	
8	3	0	5	0	
9	4	0	4	3	
10	0	3	0	5	
11	0	0	3	5	
12	0	0	5	3	

## Large Case 2 Solution

### Quay Cranes

y[\*,\*](tr):

```
1 2 3 4 5 6 7 8 :=
1 1 1 1 1 1 1 1 1
2 1 1 1 1 1 1 1 1
3 1 1 0 1 1 1 1 1
4 1 1 1 1 1 1 1 1
5 1 0 1 1 1 1 1 1
6 1 1 1 1 1 1 1 1
7 0 1 0 1 0 0 0 1
8 0 1 1 0 1 0 0 1
9 1 0 0 0 0 1 1 0
10 0 0 0 1 1 1 0 0
11 1 1 0 1 0 0 0 1
12 1 1 1 0 0 0 1 0
```

### Yard Cranes

z[\*,\*](tr)

```
: 1 2 3 4 :=
1 2 0 5 3
2 1 2 4 3
3 5 0 4 1
4 5 3 0 2
5 1 3 0 5
6 4 2 0 4
7 0 2 0 2
8 4 0 1 0
9 0 4 0 0
10 1 0 3 0
11 0 0 1 4
12 0 0 1 4
;
```

### *Large Case 3 Solution*

Quay Cranes

```
y[*,*](tr)
: 1 2 3 4 5 6 7 8 :=
1 1 0 1 1 0 1 1 0
2 1 1 0 0 1 1 1 0
3 1 0 0 0 1 0 0 1
4 0 1 1 1 1 0 0 1
5 0 1 1 1 0 1 1 0
6 1 0 1 1 0 1 0 1
7 0 1 1 1 1 1 0 0
8 1 0 1 0 0 1 1 1
9 1 1 0 1 0 0 1 1
10 0 1 0 0 1 1 1 1
11 1 1 1 1 1 1 1 1
12 1 1 1 1 1 1 1 1
```

Yard Cranes

```
z[*,*](tr)
: 1 2 3 4 :=
1 3 4 0 0
2 4 0 3 0
3 0 0 0 4
4 0 3 4 0
5 0 0 4 3
6 2 0 0 5
7 3 0 0 4
8 4 0 3 0
9 0 5 0 2
10 3 2 2 0
11 5 5 1 0
12 4 0 5 2
;
```

### *Large Case 4 Solution*

#### Quay Cranes

```
y[*,*](tr)
: 1 2 3 4 5 6 7 8 :=
1 1 1 1 1 1 1 1 1
2 1 1 1 1 1 1 1 1
3 0 0 0 1 1 0 1 0
4 0 0 0 1 1 1 1 1
5 0 0 0 1 0 1 0 1
6 0 0 1 1 1 0 0 0
7 1 1 1 0 0 1 1 1
8 1 1 1 1 0 0 0 1
9 1 1 1 0 1 1 1 0
10 1 1 1 0 0 1 0 0
11 1 1 1 0 1 0 1 1
12 1 1 0 1 1 1 1 1
```

#### Yard Cranes

```
z[*,*](tr)
: 1 2 3 4 :=
1 0 4 3 4
2 3 3 5 0
3 0 0 0 4
4 4 0 3 0
5 0 0 0 4
6 0 4 0 0
7 5 0 3 0
8 4 0 3 0
9 0 0 5 3
10 1 3 0 2
11 0 3 0 5
12 5 5 0 0
;
```

### *Large Case 5 Solution*

#### Quay Cranes

```
y[*,*](tr)
: 1 2 3 4 5 6 7 8 :=
1 0 0 1 1 1 1 0 1
2 0 1 1 0 1 0 0 0
3 1 1 0 1 1 0 1 1
4 0 1 1 1 0 1 1 1
5 1 1 0 1 0 1 0 1
6 0 1 1 0 1 1 1 0
7 1 0 1 0 1 1 1 0
8 1 0 0 1 1 1 1 1
9 1 1 0 1 1 0 1 1
10 1 0 1 1 0 1 1 1
11 1 1 1 0 0 0 0 0
12 1 1 1 1 1 1 1 1
```

#### Yard Cranes

```
z[*,*](tr)
: 1 2 3 4 :=
1 5 2 0 0
2 2 0 0 2
3 5 2 1 0
4 4 0 3 1
5 2 4 0 1
6 4 0 3 0
7 3 1 0 3
8 3 0 1 4
9 4 0 2 2
10 3 0 1 4
11 4 0 0 0
12 4 2 0 5
;
```

### *Large Case 6 Solution*

#### Quay Cranes

```
y[*,*](tr)
: 1 2 3 4 5 6 7 8 :=
1 1 0 1 1 1 1 1 1
2 1 1 1 1 1 1 1 1
3 1 0 1 1 1 1 1 1
4 1 1 1 1 1 1 1 1
5 1 0 1 1 1 1 1 1
6 1 1 1 1 0 1 1 1
7 0 1 0 1 1 0 0 1
8 0 0 1 0 1 1 0 0
9 0 1 0 1 1 0 1 0
10 1 1 0 0 0 1 0 1
11 0 1 1 1 0 0 1 0
12 1 1 0 0 1 1 0 0
```

#### Yard Cranes

```
z[*,*](tr)
: 1 2 3 4 :=
1 9 0 0 0
2 10 0 0 0
3 9 0 0 0
4 10 0 0 0
5 9 0 0 0
6 10 0 0 0
7 5 0 0 0
8 4 0 0 0
9 5 0 0 0
10 5 0 0 0
11 5 0 0 0
12 5 0 0 0
;
```

### *Large Case 7 Solution*

#### Quay Cranes

```
y[*,*](tr)
: 1 2 3 4 5 6 7 8 :=
1 0 1 1 1 1 1 1 1
2 0 1 1 1 1 1 1 0
3 0 1 0 0 1 0 0 0
4 1 0 1 1 0 0 1 1
5 1 1 0 0 1 1 1 1
6 1 0 1 1 1 1 1 0
7 0 0 1 0 1 1 1 1
8 1 1 0 1 0 0 0 0
9 1 0 1 0 0 0 0 1
10 1 1 1 1 0 1 1 1
11 1 1 0 1 1 1 0 1
12 1 1 1 1 1 1 1 1
```

#### Yard Cranes

```
z[*,*](tr)
: 1 2 3 4 :=
1 2 2 5 1
2 3 4 0 1
3 0 1 1 1
4 0 2 3 2
5 2 1 5 0
6 0 3 2 3
7 0 2 2 3
8 2 0 0 2
9 1 2 0 1
10 1 5 0 4
11 1 4 1 2
12 2 4 3 2
;
```



## APPENDIX D

### MULTI SHIP – AMPL CODE

# Define sets#

```
set V;           # set of ships
set B;           # set of bays
set T;           # set of Time
set S;           # set of sub-blocks
set F;           # set of flags
```

# Define parameters #

```
param W >=0 , <= 1;           # Weight of QC and YC Cranes
param Q {T} >=0;               # Number of QC allowed at each time period
param YC {T} >=0;              # Number of YC allowed at each time period
param CQ >=0;                  # Capacity of QC [containers move/hour]
param CY >=0;                  # Capacity of YC [containers move/hour]
param I{F,S} >=0;              # Allowed Area for containers of flag f into sub block s
#param N{V,B,F} >=0;           # Number of containers of ship v from bay b for flag f
param N1{B,F} >=0;             # SHIP 1
param N2{B,F} >=0;             # SHIP 2
param K{S,T} >=0;              # Number of YC allowed at each time t for sub block s
param C{F,S} >=0;              # Number of Containers allowed for each f in sub block
s
```

# Define variables #

```
var x {v in V, b in B, f in F, t in T, s in S} integer >= 0;
# Number of Container unloaded from ship v each bay, flag, time to subblock
```

```
var y {v in V, b in B, t in T} binary ;
# Number of bay used at time t for ship v
```

```
var z {s in S, t in T} integer >= 0;
# Number of RTG used in time t for sub block s for ship v
```

# Write the objective function #

```
minimize Total_Y_Z: (W*(sum {v in V, b in B, t in T} y[v,b,t])) + ((1-W)*(sum { s in
S , t in T} z[s,t])); # sum
```

# Specify Constraints #

#constraint 2

subject to equation2 { t in T}:  $\sum \{v \text{ in } V, b \text{ in } B\} y[v,b,t] \leq Q[t];$

# constraint 3

subject to equation3 { v in V,b in B, t in T}:  $(\sum \{f \text{ in } F,s \text{ in } S\} x[v,b,f,t,s])/CQ \leq y[v,b,t];$

# constraint 4

subject to equation41 { b in B, f in F}:  $\sum \{t \text{ in } T, s \text{ in } S\} x[1,b,f,t,s] = N1[b,f];$

subject to equation42 { b in B, f in F}:  $\sum \{t \text{ in } T, s \text{ in } S\} x[2,b,f,t,s] = N2[b,f];$

# constraint 5

subject to equation5 { s in S, t in T}:  $z[s,t] \leq K[s,t];$

# constraint 6

subject to equation6 { t in T}:  $\sum \{s \text{ in } S\} z[s,t] \leq YC[t];$

# constraint 7

subject to equation8 { s in S, t in T}:  $(\sum \{v \text{ in } V, f \text{ in } F, b \text{ in } B\} x[v,b,f,t,s])/CY \leq z[s,t];$

# constraint 8

subject to equation8 { v in V, b in B, f in F, t in T, s in S}:  $x[v,b,f,t,s] \leq 5000*I[f,s];$

# constraint 9

subject to equation9 { s in S, f in F}:  $\sum \{v \text{ in } V,b \text{ in } B,t \text{ in } T\} x[v,b,f,t,s] \leq C[f,s];$

## APPENDIX E

### MULTI SHIP – SMALL SCALE PROBLEMS

#### *Base Case Solution*

##### Quay Cranes

y :=  
1 1 1 1  
1 1 2 0  
1 1 3 0  
1 2 1 0  
1 2 2 1  
1 2 3 0  
2 1 1 1  
2 1 2 0  
2 1 3 0  
2 2 1 0  
2 2 2 1  
2 2 3 0

##### Yard Cranes

z :=  
1 1 1  
1 2 1  
1 3 1  
2 1 2  
2 2 2  
2 3 0

##### Containers Unloading

x [1,1,1,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[1,1,2,\*,\*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0

[1,2,1,\*,\*]  
: 1 2 :=

```
1 0 0
2 0 10
3 0 0
```

```
[1,2,2,*,*]
: 1 2 :=
1 0 0
2 10 0
3 0 0
```

```
[2,1,1,*,*]
: 1 2 :=
1 0 10
2 0 0
3 0 0
```

```
[2,1,2,*,*]
: 1 2 :=
1 0 10
2 0 0
3 0 0
```

```
[2,2,1,*,*]
: 1 2 :=
1 0 0
2 0 10
3 0 0
```

```
[2,2,2,*,*]
: 1 2 :=
1 0 0
2 0 10
3 0 0
;
```

### ***Case 2 Solution***

Yard Cranes

```
y :=
1 1 1 1
1 1 2 0
1 1 3 0
1 2 1 1
1 2 2 0
1 2 3 1
2 1 1 0
2 1 2 0
2 1 3 1
2 2 1 0
2 2 2 1
```

2 2 3 0

### Quay Cranes

z :=  
1 1 2  
1 2 0  
1 3 0  
2 1 0  
2 2 2  
2 3 2

### Containers Unloading

x [1,1,1,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[1,1,2,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[1,2,1,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[1,2,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10

[2,1,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10

[2,1,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0

3 0 10

```
[2,2,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0
```

```
[2,2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0  
;
```

### *Case 3 Solution* Quay Cranes

```
y :=  
1 1 1 0  
1 1 2 1  
1 1 3 0  
1 2 1 0  
1 2 2 0  
1 2 3 1  
2 1 1 1  
2 1 2 0  
2 1 3 0  
2 2 1 1  
2 2 2 0  
2 2 3 0
```

### Yard Cranes

```
z :=  
1 1 1  
1 2 2  
1 3 0  
2 1 2  
2 2 0  
2 3 2
```

### Containers Unloading

```
x [1,1,1,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[1,1,2,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[1,2,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10
```

```
[1,2,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 0 10
```

```
[2,1,1,*,*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0
```

```
[2,1,2,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[2,2,1,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0
```

```
[2,2,2,*,*]  
: 1 2 :=  
1 0 10  
2 0 0  
3 0 0  
;
```

**Case 4 Solution**  
Quay Cranes

```
y :=  
1 1 1 0  
1 1 2 1  
1 1 3 0
```

1 2 1 1  
1 2 2 1  
1 2 3 0  
2 1 1 1  
2 1 2 0  
2 1 3 0  
2 2 1 0  
2 2 2 0  
2 2 3 1

### Yard Cranes

z :=  
1 1 1  
1 2 1  
1 3 1  
2 1 1  
2 2 1  
2 3 1

### Containers Unloading

x [1,1,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0

[1,1,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 10  
3 0 0

[1,2,1,\*,\*]  
: 1 2 :=  
1 5 0  
2 5 0  
3 0 0

[1,2,2,\*,\*]  
: 1 2 :=  
1 0 5  
2 0 5  
3 0 0

[2,1,1,\*,\*]  
: 1 2 :=  
1 10 0



```
2 0 0
3 0 0
```

```
[2,1,2,*,*]
: 1 2 :=
1 0 10
2 0 0
3 0 0
```

```
[2,2,1,*,*]
: 1 2 :=
1 0 0
2 0 0
3 10 0
```

```
[2,2,2,*,*]
: 1 2 :=
1 0 0
2 0 0
3 0 10
;
```

**Case 5 Solution**  
Quay Cranes

```
y :=
1 1 1 1
1 1 2 1
1 1 3 0
1 2 1 0
1 2 2 1
1 2 3 0
2 1 1 0
2 1 2 0
2 1 3 1
2 2 1 1
2 2 2 0
2 2 3 0
```

Yard Cranes

```
z :=
1 1 2
1 2 2
1 3 2
2 1 0
2 2 0
2 3 0
```

Containers Unloading

```
x [1,1,1,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[1,1,2,*,*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0
```

```
[1,2,1,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[1,2,2,*,*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0
```

```
[2,1,1,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0
```

```
[2,1,2,*,*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0
```

```
[2,2,1,*,*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0
```

```
[2,2,2,*,*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0  
;
```

### **Case 6 Solution**

#### Quay Cranes

y :=  
1 1 1 1  
1 1 2 1  
1 1 3 0  
1 2 1 0  
1 2 2 1  
1 2 3 0  
2 1 1 0  
2 1 2 0  
2 1 3 1  
2 2 1 1  
2 2 2 0  
2 2 3 0

#### Yard Cranes

z :=  
1 1 2  
1 2 2  
1 3 2  
2 1 0  
2 2 0  
2 3 0

#### Containers Unloading

x [1,1,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0

[1,1,2,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[1,2,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 10 0  
3 0 0

[1,2,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 10 0

3 0 0

[2,1,1,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0

[2,1,2,\*,\*]  
: 1 2 :=  
1 0 0  
2 0 0  
3 10 0

[2,2,1,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0

[2,2,2,\*,\*]  
: 1 2 :=  
1 10 0  
2 0 0  
3 0 0  
;

## APPENDIX F

### MULTI SHIP – LARGE SCALE PROBLEMS

#### *Base Case Solution*

Quay Cranes

```
Y[1,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 0 0 1 1 0 0 0 0  
2 0 1 1 0 0 0 0 0  
3 0 1 0 0 0 0 0 1  
4 1 1 0 0 0 0 0 0  
5 1 0 0 0 0 0 0 1  
6 1 1 0 0 0 0 0 0  
7 1 1 0 0 0 0 0 0  
8 1 0 0 1 0 0 0 0
```

```
[2,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 1 0 0 0 1 0 0 0  
2 0 1 0 0 0 0 1 0  
3 1 1 0 0 0 0 0 0  
4 1 0 0 0 0 0 1 0  
5 0 0 0 0 1 0 0 1  
6 1 0 1 0 0 0 0 0  
7 0 1 0 0 1 0 0 0  
8 0 1 0 0 0 0 1 0  
;
```

Yard Cranes

```
Z[*,*](TR)  
: 1 2 3 4 :=  
1 4 1 5 2  
2 1 4 4 3  
3 0 2 0 2  
4 3 0 0 0  
5 2 2 0 0  
6 0 0 0 0  
7 3 0 0 1  
8 3 0 1 0  
;
```

## Case 2 Solution

### Yard Cranes

```
y [1,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 0 0 1 1 0 0 0 0  
2 1 1 1 1 0 0 0 0  
3 1 0 0 1 0 0 0 0  
4 0 0 1 0 1 0 0 0  
5 1 1 0 0 0 0 0 0  
6 1 0 1 0 0 0 0 0  
7 0 1 0 0 1 0 0 0  
8 0 1 0 1 0 0 0 0
```

```
[2,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 1 1 0 0 0 0 0 0  
2 1 0 0 1 0 0 0 0  
3 0 1 1 0 0 0 0 0  
4 1 1 0 0 0 0 0 0  
5 0 1 1 0 0 0 0 0  
6 1 0 0 1 0 0 0 0  
7 0 0 1 1 0 0 0 0  
8 0 0 1 1 0 0 0 0  
;
```

### Quay Cranes

```
z [*,*] (tr)  
: 1 2 3 4 :=  
1 1 4 1 4  
2 5 1 1 3  
3 1 3 2 4  
4 4 1 2 3  
5 0 2 0 1  
6 0 0 0 0  
7 0 0 0 0  
8 0 0 0 0  
;
```

### Case 3 Solution

#### Quay Cranes

```
y [1,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 0 0 1 0 0 1 0 0  
2 0 0 1 0 1 0 0 0  
3 0 1 0 0 0 0 1 0  
4 0 1 0 0 0 0 0 1  
5 0 1 1 0 0 0 0 0  
6 0 0 0 1 0 0 0 1  
7 0 0 1 1 0 0 0 0  
8 0 0 1 0 0 1 0 0
```

```
[2,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 0 0 0 0 1 0 0 1  
2 1 0 0 0 1 0 0 0  
3 0 0 0 1 0 1 0 0  
4 1 0 0 1 0 0 0 0  
5 0 1 0 0 0 0 1 0  
6 0 0 0 1 0 0 1 0  
7 0 1 0 0 0 0 0 1  
8 1 0 0 0 0 0 0 1  
;
```

#### Yard Cranes

```
z [*,*] (tr)  
: 1 2 3 4 :=  
1 2 1 1 0  
2 4 1 1 1  
3 2 4 1 0  
4 1 4 0 2  
5 1 0 2 1  
6 2 2 0 0  
7 4 0 0 0  
8 3 1 3 0  
;
```

### Case 4 Solution

#### Quay Cranes

```
y [1,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 0 1 0 0 0 0 0 1  
2 1 0 0 0 0 0 0 1  
3 0 0 0 0 0 0 1 1  
4 1 0 0 0 0 0 0 1  
5 1 0 0 0 0 0 1 0  
6 0 0 1 0 0 0 0 1  
7 1 0 0 0 1 0 0 0  
8 0 0 0 0 0 1 0 1
```

```
[2,*,*]  
: 1 2 3 4 5 6 7 8 :=  
1 1 0 0 0 0 0 0 1  
2 1 0 0 0 0 0 0 1  
3 0 1 0 0 1 0 0 0  
4 1 0 0 1 0 0 0 0  
5 0 0 1 1 0 0 0 0  
6 1 0 0 0 1 0 0 0  
7 0 0 1 1 0 0 0 0  
8 0 1 0 0 0 0 1 0  
;
```

#### Yard Cranes

```
z [*,*] (tr)  
: 1 2 3 4 :=  
1 3 3 3 2  
2 0 2 0 2  
3 0 2 0 2  
4 2 0 2 0  
5 1 1 1 1  
6 0 1 1 0  
7 1 0 2 1  
8 4 2 2 3  
;
```



### Case 5 Solution

#### Quay Cranes

y [1,\*,\*]

```
: 1 2 3 4 5 6 7 8 :=  
1 0 1 0 0 0 0 0 1  
2 1 0 0 0 0 0 0 1  
3 0 0 0 0 0 0 1 1  
4 1 0 0 0 0 0 0 1  
5 1 0 0 0 0 0 1 0  
6 0 0 1 0 0 0 0 1  
7 1 0 0 0 1 0 0 0  
8 0 0 0 0 0 1 0 1
```

[2,\*,\*]

```
: 1 2 3 4 5 6 7 8 :=  
1 1 0 0 0 0 0 0 1  
2 1 0 0 0 0 0 0 1  
3 0 1 0 0 1 0 0 0  
4 1 0 0 1 0 0 0 0  
5 0 0 1 1 0 0 0 0  
6 1 0 0 0 1 0 0 0  
7 0 0 1 1 0 0 0 0  
8 0 1 0 0 0 0 1 0  
;
```

#### Yard Cranes

z [\*,\*] (tr)

```
: 1 2 3 4 :=  
1 3 3 3 2  
2 0 2 0 2  
3 0 2 0 2  
4 2 0 2 0  
5 1 1 1 1  
6 0 1 1 0  
7 1 0 2 1  
8 4 2 2 3  
;
```

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