S T R U C T U R A L

T H E S I S

D E S I G N O F A R E I N F O R C E D

C O N C R E T E A R C H B R I D G E

By

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D E S I G N of a R E I N F O R C E D

C O N C R E T E A R C H B R I D G E (Zabadany-Bloudan)

by Vartkes Yacoubian , C.E.

May I946

P R E F A C E

In this design work , we have chosen a reinforced concrete arch bridge with dimensions and specifications that meet the requirements of the system applied in Syria and Lebanon .

However we have made ample use of the American system of analyzing the arch and have tried to obtain the most economical design with due consideration to good appearance by combining the French and the American systems of design .

The bridge designed will span a lovely wooded valley on the Zabadany - Bloudan highway .

Both topographical conditions and architectural and economical considerations made the designer choose an open-spandrel type of arch bridge with two big rised parabolic arch ribs .

The author is under especial obligation to Prof. R. Osborn , Head of the Engineering Department of the American University of Beirut , for his valuable help , without which this thesis work could not have been prepared .

Vartkes Yacoubian , C. E.

C O N T E N T S

I - Advantages of the reinforced concrete arch

If all the loads on an arch were fixed loads , it would be possible in any case to construct an arch ring so that the resultant pressure at all sections would be at the centre of gravity of the section . That arch ring would be parabolic . The compressive stress at any section would then be uniformly distributed over the section , and the arch would be proportioned only for this uniform compression . The line of pressure would lie at the axis of the arch throughout . If however , the arch ring is not made to fit the line of pressure , or if part of the load is a live load , then the resultant pressure will not in general coincide with the axis of the arch . There will exist both bending and direct compression .

In ordinary masonry and plain concrete arches , tensile stresses are not permissible . The arch ring therefore must be designed so that the line of pressure will not pass outside the middle third . In reinforced concrete arches this limitation does not exist . The arch ring is a beam , and if properly reinforced may carry heavy bending moments involving tensile stresses in the steel .

Theoretically the gain in economy by the use of steel in a concrete arch in not great . If the pressure line does not depart from the middle third , the steel reinforces only in compression and in this respect is not as economical as concrete. If the line of pressure deviates farther from the centre , resulting in tensile stresses on the section , the conditions are such that those stresses must be provided for by the use of the steel at very low working stresses .

Practically the value of reinforcement is very considerable . It renders an arch a much more secure and reliable structure , it greatly aids in preventing cracks due to any slight settlement , and by furnishing a form of construction of greater reliability makes possible the use of working stresses in the concrete considerably higher than are usual in plain concrete . The possible increase in average working stress counts greatly toward economy .

When a structure is desired which readily lends itself to artistic treatment , the arch has many advantages over any other type of structure . Its graceful curves blend into the landscape , and being a deck structure , a clear unobstructed view of the surrounding scenery may be secured from the roadway .

S P E C I F I C A T I O N S

Design of a " Reinforced concrete arch bridge "

Clear span of I00'-0"

Roadway including sidewalks of 30'-0"

Materials :

Concrete - 2500 psi @ 28 days

Steel - structural grade

Specifications for concrete :

Report of Joint Committee on Standard Specifications for concrete and reinforced concrete , I924 .

Temperature changes :

Fall of 60 ° F

Rise of 20 ° F

Linear coefficient of expansion of 0.000006

Two inches of bituminous macadam wearing course .

2" of crown for roadway .

Surcharges :

French system of loading .

Two systems shall be considered simultaneously .

a) Uniform live-load of

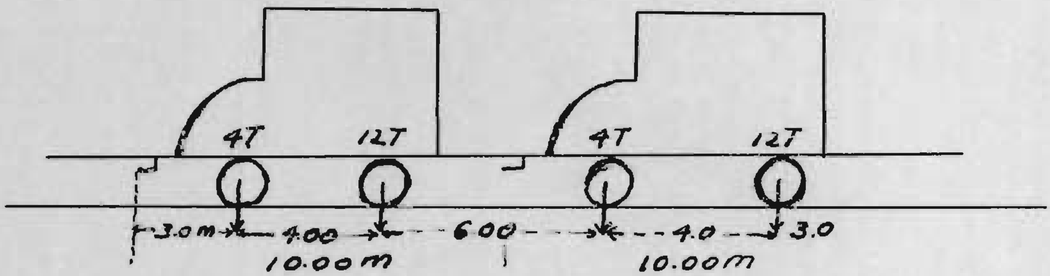
p = (820 - 4 L) kgs/M2 with a minimum of 500 kgs/M2

The sidewalks shall carry a uniform live load of

400 kgs/M2

b) System of two 2-axled trucks having the following characteristics :

Total load	I6 Tons
Rear axle load	I2 Tons
Front axle load	4 Tons
Length of truck	I0 meters
Width of truck	2.5 m
Distance between axles	4 meters
Distance c.to c. of wheels	I.7 m
Width of a wheel rim	0.3 m



As many of those trucks as possible will be assumed travelling side by side and in the same direction on the roadway .

Impact :

Impact I = I + + = I + 0.4 /(I+0.2L) + 0.6 /(I+4P/S)

in which

L is the loaded span

P is the total dead load

S is the live load

D E S I G N   O F   F L O O R   S Y S T E M

Design of :

Roadway slab

Stringer " A "

Sidewalk slab

Stringer " B "

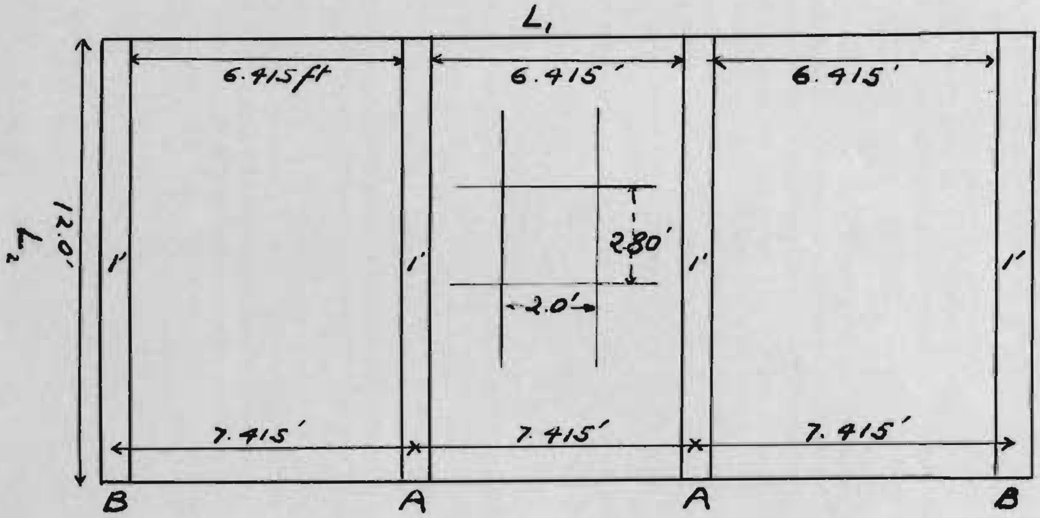
Floor beam

Columns

DESIGN OF ROADWAY SLAB

A ribbed system of roadway slab will be used . Although the spacing of floor beams is not uniform all through the bridge span , a spacing of I2 ft. will be assumed for design purposes .

Design sketch :



Width of tyres W = 30 cms = about I ft.

We have one way design (reinforcement along LI)

Dispersion width along main reinforcement - according to French formulas :

$$\frac{7.415}{3} = 2.47 \text{ ft.}$$

$$2.47 + 0.33 = 2.80 \text{ ft.}$$

The 0.33 is the contact width (about IO cms) .

Dispersion perpendicular to main reinforcement :

$$\alpha + \beta + W = I + I = 2.0 \text{ ft.}$$

The French impact formula is :

$$I = I + \frac{0.4}{1+0.2L} + \frac{0.6}{1+4\frac{P}{S}} = \text{ Metric system}$$

in which :

I = coef. of impact

L = loaded span

P = dead load

S = live load

Converted to ft-lbs system we get :

$$I = I + \frac{0.40}{1+0.061L} + \frac{0.6}{1+4\frac{D}{S}} = \text{ ft-lbs} .$$

The concentrated load of a wheel at the center of a one ft strip of slab is :

$$\frac{12 \times 2000}{2 \times 2.80} = 4280 \text{ lbs} .$$

The live load moment is then :

$$0.8\left(\frac{4280}{2} \times \frac{6.415}{2} \times 12\right) = 65800 \text{ in.lbs}$$

$$-0.8\left(\frac{4280}{2} \times 0.5\right) \times 12 = 10300 \text{ in.lbs} \qquad = 55500 \text{ in.lbs}$$

Dead load : 8.0 inches of concrete slab + 2 inches of bituminous wearing surface

$$= \frac{8}{12} \times 150 + \frac{2}{12} \times 140 = 123 \text{ \#/ft}^2 , \; T.D.L = 123 \times 6.415 = 790\text{\#}$$

D.L. moment =

$$0.8\left(\frac{1}{8} \times 790 \times 6.415 \times 12\right) = 6050 \text{ in.lbs}$$

Impact coef = $1 + \dfrac{0.40}{1+0.061 \times 6.4} + \dfrac{0.60}{1+4\frac{790}{4280}} = 1.64$

L.L. + D.L. + I = $55500 \times 1.64 + 6050 = 98000$ in.lbs

Using : $f_c' = 2500$ psi , f = I8000 psi , n= I2

f = I000 psi

K = I73.3 , p = 0.0III

$M = Kbd^2$

$$d = \sqrt{\frac{98000}{173.3 \times 12}} = 6.85 \text{ inches}$$

Have total thickness 8.0 inches

Main reinforcement = 0.0III x I2 x 6.85 = 0.9I5 in^2

Use 3 - 5/8"ϕ per ft. = 0.9I in^2

3 - 3/8"ϕ per ft. = 0.33I in the other direction .

Shearing stress :

Considering a wheel at the support we have :

$$\frac{6\times 2000}{2.8\times 12\times 6.85} = \quad 52.5 \text{ psi}$$

Allowable according to Joint Code :

$0.03\, f'_c = 0.03\times 2500 = 75$ psi O.K.

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DESIGN of STRINGER "A" - Longitudinal intermediate beam

The span is equal to the floor beam spacing = I2 ft.

Distribution coefficient of load = $\frac{S}{4.5} = \frac{7.415}{4.5} = 1.645$

Dead load from slab =

$7.415 \times 123 = 914$ lbs

Dead load of beam = I50 "

Total I064 Lbs

Dead load moment = $\frac{1}{10}\times 1064\times 12^2\times 12 = 153500$ in. Lbs

Live load moment =

$1.645\times 0.8\left(\frac{12000}{2}\times 6\times 12\right) = 568{,}000$ in. Lbs

Impact = $1+\frac{0.4}{1+0.061\times 12}+\frac{0.6}{1+4\frac{1064\times 12}{12000}} = 1.64$

D.L. + L.L. + Impact moment =

$1.64\times 568000+153500 = 1{,}083{,}000$ in. Lbs

K = I73.3 , P = 0.0III

We have a T-beam :

$v = 0.06 f'_c = 0.06 \times 2500 = 150$ psi

$b'd = \frac{V}{vj} = \frac{12000\times 1.64+1064\times 6}{150\times 0.87} = 182\square''$

Let us consider moment :

$b = \frac{1}{4}L = \frac{1}{4}\, 12 = 3.0$ ft.

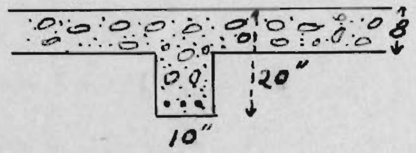
$M_c = \frac{1}{2} f_c\, b\, t\, (d - \frac{1}{2}t)$

$d = \frac{1083000\times 2}{6\times 1000\times 3\times 8}+\frac{1}{2}\times 8 = 11.5''$

Shear governs : $b' = 10''$

Have d = I8.2 inches

Have h = 20.0 inches



Use 6 - 7/8" in 2 rows (3.6I in^2)

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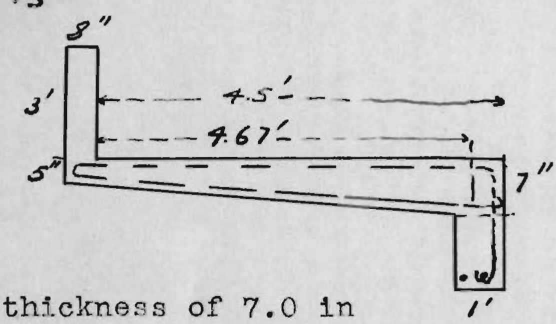
DESIGN of Sidewalk Slab (Cantilever type)

Live load carried by sidewalk - according to French specification

= 400 Kgs/m^2 = 82 lbs/ft^2

Impact : $1 + \frac{0.4}{1+0.06 l \times L} + \frac{0.6}{1+4\, P/S}$



Dead load : assume average thickness of 7.0 in

Uniform D.L. : $\frac{7}{12} \times 4.67 \times 150 = 410$ lbs

Concentrated load :

$1 \times 1 \times 150 = 150$

Impact : $1 + \frac{0.40}{1+0.06\times 4.67} + \frac{0.6}{1+4\,\frac{560}{9\times 82}} = 1.40$

Live load and impact moment = $(82 \times 1.40) \times 4.5 \times \frac{4.5}{2} \times 12 = 13950$ in.lbs

Dead load moment = $(410 \times \frac{4.67}{2} + 150 \times 4.37)12$ 19500 in.lbs

Total = 33450 in.lbs

K = I73.3 , p = 0.0III

M = Kbd^2

d = $\sqrt{\frac{33450}{173.3 \times 12}}$ = 4.I in

Have a thickness of 8.0 inches at the support and 5.0 inches at the end .

$A_s = \frac{33450}{18000 \times 0.87 \times 5} = 0.43$ □"

Use 3 - 5/8" per ft. (= 0.9I in^2) .

Every other two bars is cut short at 2 ft from outside stringer .

Bend down every other bar .

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DESIGN of Stringer "B" - Exterior longitudinal beam .

Span = I2.0 fts.

Loads carried by the beam :

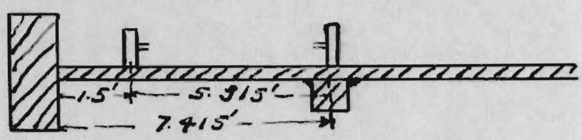
Dead load , live load and impact from sidewalk

$= \frac{6}{12} \times 150 \times 4.67 + 150 + 82 \times 1.40 \times 4.5 = 1015$ lbs

Dead load from roadway $= 123 \times \frac{7.415}{2} = 455$

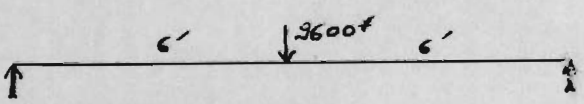
Total : $1015 + 455 = 1470$ lbs/ft

Fraction of a wheel load supported by beam "B"



Fraction = $\frac{5.915}{7.415} = 0.8$

Live load = $12000 \times 0.8 = 9600$ #



Dead load moment = $0.8(\frac{1}{8} 1470 \times 12^2 \times 12) = 253,000$ in. lbs

Live load moment = $0.8(\frac{9600}{2} \times 6 \times 12) = 276,000$ in. lbs

Impact coef. (same as for interior longitudinal beam)

= I.64

L.L. + D.L. + Impact moment =

$276000 \times 1.64 + 253000 = 706,000$ in. lbs

The outside stringer will be designed as a rectangular beam (not a T-beam) .

$v = 0.06\ f'_c = 0.06 \times 2500 = 150$ psi

$bd = V/vj$

$V = 9600 \times 1.64 + 1470 \times 6 = 24600$ #

$bd = \frac{24600}{150 \times 0.87} = 190 \text{ in}^2$

K = I73.3 , p = 0.0III

$M = Kbd^2$

Let b = I0"

$d = \sqrt{\frac{706000}{173.3 \times 10}}$ = 20.3 inches

Moment governs .

$A_s = \frac{706000}{18000 \times 0.87 \times 20.3}$ = 2.22 in^2

Use 4 - 7/8" in 2 rows (= $2.4I \text{ in}^2$)

Have total depth of 22 inches

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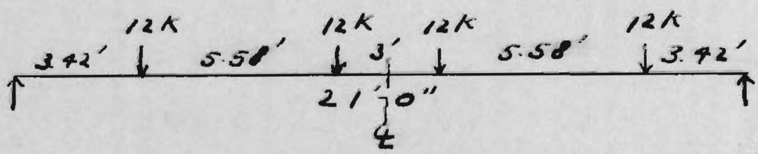
DESIGN of FLOOR BEAMS - Transverse beams

The span of the floor beams is 2I'-0"

The floor beams will be bracketed at the ends .



Dead load per foot of stringer = I064 lbs/ft.

Therefore P = I064 × I2 = I2700 lbs .

Assume section of beam below slab I2" by I8"

Dead load per ft = I50 × I.5 = 225 lbs/ft

Dead load reaction = $12700 + 225 \times \frac{21}{2} = 15075 \text{ lbs}$

Dead load moment at mid-span =

$0.8(15075 \times 10.5 - 225 \times \frac{10.5^2}{2} - 12700 \times 3.65) = 79000\ '\#$

Live load :

Distance between c.to c. of wheels = I.70 m

= I.70 × 3.28 = 5.58 fts

Impact = $1 + \frac{0.4}{1 + 0.061 \times 21} + \frac{0.6}{1 + 4 \times \frac{2 \times 15075}{4 \times 12000}}$ = I.34

Center live load moment = 24000×10.5 - 12000×7.08 - 12000×1.5 = = 148,000 '#

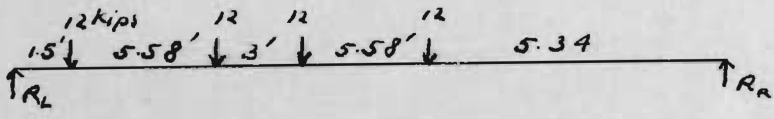
D.L. + L.L. + Impact moment =

$148000 \times 1.34 + 79000 = 276,000$ '#

Maximum shear :

Dead load = I5075 lbs

Live load:



$$R_1 = \frac{12000}{21} (19.5 + 13.92 + 10.92 + 5.34) 1.34 = 38000 \text{ lbs}$$

Total maximum shear = 38000 + I5075 = <u>53075 lbs</u>

Section required as determined by shear :

$v = 0.09 f_c' = 0.09 \times 2500 = 225$ psi

$b'd = V/vj = 53075/225 \times 0.87 = 270$ in^2

Section required as determined by moment :

The floor beams and the slab above are a single monolithic .

We have therefore a T-beam .

$b = \frac{1}{4} L = \frac{1}{4} 21 = 5.25$ fts = 63 inches .

$M = \frac{1}{2} f_c b t (d - \frac{1}{2} t)$ approximately

$$d = \frac{276000 \times 2 \times 12}{63 \times 1000 \times 8} + \frac{1}{2} 8 = 17.15''$$

Shear governs in that case .

Using b' = I3 in , we have d = 20.7 in

As a safety against shear have haunches at the supports .

d at the supports will then be 3I.0 inches

$$A_s = \frac{276000 \times 12}{18000 \times 0.87 \times 17.15} = 12.0 \text{ in}^2 \text{ (too much)}$$

Increase d to 24.0 inches .

A_s = I2 × I7.I5/24 = 8.50 in^2

Use 6 - I"φ = 6.0 in^2

3 - I"φ = 2.35 in^2

8.35 in^2

Have the reinforcements in 3 rows .

Have total depth h of 26.0 inches

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DESIGN of floor beams for web reinforcement and Bond

Maximum shear = 53075 lbs

Spacing of bent up bars :

$$S = A_v f_v \; j \; d \; / \; 0.7 \; V'$$

$V' = V - V_c$ = 53075 - 0.03×2500×I3×0.87×24 = 32675 lbs

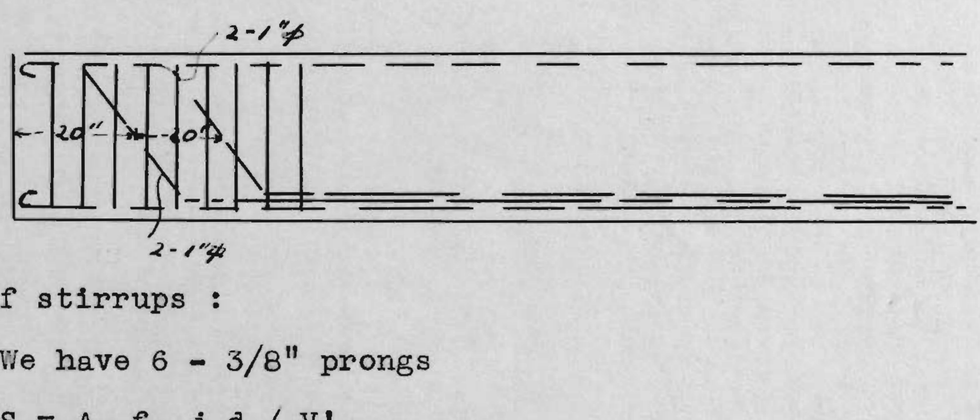
2 - I"φ will be bent first .

$$S = \frac{2 \times 18000 \times 0.87 \times 24}{0.7 \times 32675} = 32 \text{ inches}$$

Maximum spacing =

45d / + I0 = 45 24 / 45 I0 = I9.5 inches

20 in spacing will be used .



Spacing of stirrups :

We have 6 - 3/8" prongs

$$S = A_v \; f_v \; j \; d \; / \; V'$$

$$\frac{6 \times 0.11 \times 18000 \times .87 \times 24}{32675} = 7.5\text{" spacing .}$$

Maximum spacing 0.45 d = 0.45 24 = I0.8 inches

Have spacing of 7.5 in at the supports and I0.0 in at midspan .

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Bond : according to Joint Code $u = 0.05\ f_c' = 0.05 \times 2500 = 125$ psi

$= V / u\ j\ d = 53075 / 125 \times 0.87 \times 24 = 20.2$ in

4 - I"ϕ and I - I"ϕ are required for bond , the remaining bars can be bent up .

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D E S I G N of C O L U M N S

Loads on Columns

Dead load concentration at column (3)

Roadway slab	: II × I0.5 × I0/I2 × I50	= I4450 lbs
Inside stringer	: 8 × I2/I44 × I50 × II	= II00 "
Outside stringer	: 22 × I0/I44 × I50 × II	= 2520 "
Parapet wall	: I50 × II	= I650 "
Floor beam	: I3 × I6/I44 × I0.5 × I50	= 2280 "
Sidewalk slab	: 6.5 × I50/I2 × 4.5 × II	= 4000 "
Sidewalk fill	: 8/I2 × I00 × 3.5 × II	= 2560 "
		28560 lbs

Dead load concentration at column (I)

All items as before except I0.5 ft slab instead of II.0 fts

Total on col. I — 27607 lbs

Dead load concentration at column (2)

All items as before except I0.75 ft slab instead of I2 ft

Total on column (2) — 28080 lbs

Dead load concentration on column (4)

Total : — 295I3 lbs

Dead load concentration on column (5)

Total : — 30466 lbs

DESIGN of Columns :

Columns (I) & (2) will be grouped and have approximately same section .

Load (L.L. + D.L.) = 38000 + 82x4.5xII + 28080 = 70080 lbs.

Length = 5.0 fts.

We have here a short column .

f_c according to Joint Code = 0.225 f_c'

= 0.225x2500 = 562 psi

A value of 550 psi will be used .

Then we have :

$$P = f_c \; A + (n-I) \; A_s'$$

The most economical A_s' is 0.05 °/°

A = 70800 / 550xI.05 = <u>I2I in^2</u>

Have a section of <u>I3.0 by I3.0 inches</u>

A_s = I2I / 200 = 0.605 in^2

Use <u>4 - 5/8"⌀</u> = I.23 in^2

Column (2) : Section <u>I3.5 by I3.5</u> inches

Use same amount of reinforcement .

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Column (3)

Load (L.L. + D.L.) = 42000 + 28560 = 70560 lbs

Length = I3.0 fts

We have here a short column .

70560 = 550 (A + 0.05 A)

A = I22 in^2

Have a section of <u>I4.0 by I4.0</u> inches .

Use 4 - 5/8" ϕ = I.23 in^2

Have I/4" ϕ tie rods 8" c. to c.

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Column (4)

Load = 42000 + 295I3 = 7I5I3 lbs

Length = 22.0 fts

R = d/ I2 = 0.288 d

Assume d = I.25 ft

R = 0.288 x I.25 = 0.36

h/R = 22/0.36 = 6I

We have here a long column .

Design load = P / I.33 - h/I20R

The above formula is given by the joint Code .

Design load = 7I5I3 /(I.33 - 0.505) = 87000 lbs

Therefore

87000 = 550 A + II A / 200

A = I50 in^2

Use section of I5.0 by I5.0 inches = 225 in^2

Use 5 - 5/8" ϕ = 2.46 in^2

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Column (5)

The load on column (5) will be sustained by the retaining wall .

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P R E L I M I N A R Y DESIGN OF THE ARCH RIB

by " COCHRANE's METHOD "

Analysis of the arch ring by Cochrane's method

Design of the arch ring

DESIGN of the ARCH RIB after "COCHRANE"

As already stated , an approximate analysis will first be made by Cochrane's method .

Mr Cochrane's formula for the open spandrel arches is as follows :

$$y = z^2 h \frac{I + I/6 (g - I) z^2}{I + I/6 (g - I)}$$

g in this formula is w_s / w_c

w_s is the unit length load at the springing

w_c is the load per ft at the crown .

In an open spandrel arch , g may be taken as unity . in that case the above formula is reduced to :

$$y = z^2 h \quad (\text{parabola})$$

A big rise - (a clear rise of 35 fts) has been chosen for both topographical and architectural reasons .

Sustituting in the above formula h = 35 , we get the following intrados ordinate table .

z	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
y	0.35	I.4	3.I3	5.6	8.75	I2.6	I7.I	22.4	28.3	35

$$w_c = w_s = I/9 (I8075 + I3 \times I3 \times I50/I44)$$
$$+ 2.25 \times 2 \times I50 = 2400 \text{ lbs/ft}$$

Dead load crown thrust :

A closely approximate formula for the crown thrust is

$$H_c = I/8 \, w_c + 0.0I5 (w_s - w_c) L^2/h$$

This formula reduces to the following simplified form in our case :

$$H_c = I/8\ w_c\ L^2/h$$

$$= I/8 \times 2400 \times I00^2/35 = \underline{85500 \text{ lbs}} .$$

Live load and temperature stresses :

For $d_s/d_c = 2$, $h/l = 35/I00$, Cochrane gives the following crown live load and temperature coefficients .

Live load

$$+ M_c = 0.0063$$

$$T_c = 0.20$$

Temperature

$$M_c = 25$$

$$T_c = I8$$

The uniform live load according to the French system of loading is :

$$p = (820 - 4L) \text{ Kgs/M2}$$

L is the loaded span .

Our span of I00 fts is equal to 30.0 meters .

$$p = (820 - 4 \times 30) = 700 \text{ kgs/M2}$$

Reduced to the foot-lbs system ,

$$p = 700 \times 2.2 / 3.28 \times 3.28 = I44 \text{ lbs/ft}^2$$

Cochrane's equations for crown stresses are :

$$M_c = \text{coef.} \times P\ L^2$$

$$T_c = \text{coef.} \times P\ L$$

P in that case is : I44 x 2I/2 = I5I0 lbs/ft

Then

$$M_c = 0.0063 \times I5I0 \times I00^2 = 95{,}000 \text{ ft.lbs}$$

$$T_c = 0.20 \times I5I0 \times I00 = 3I000 \text{ lbs}$$

Cochrane's temperature equations are :

$$T_c = \text{coef} \times \frac{t\ E\ I_I}{h^2}$$

$$M_c = - \text{coef} \times T_c \, h / 100$$

Substituting we get :

$$T_c = - 18 \times 60 \times 0.000006/35^2 \times 2000000 \times 144 \times 2.17$$
$$= - 3330 \text{ lbs}$$

$$M_c = + 25 \times 3330/100 \times 35 = 29,000 \text{ ft.lbs}$$

<u>Resultant crown stresses :</u>

M_c = 95000 + 29000 = <u>124,000 ft.lbs</u>

T_c = 85500 + 31000 - 3330 = <u>113,170 lbs</u>

Determination of crown section :

About 2 °/° of reinforcement will be used . This amount will be equally divided between the top and bottom faces of the section .

A trial section of 24 x 27 inches will be assumed . The member (arch ring) to be designed is subjected to direct compression and to bending .

The eccentricity e is equal to : M/T

$e = 124000 / 113170 = 1.095$

$p = 0.01$, $n = 12$, $pn = 0.12$

$h/e = 2 / 1.095 = 1.83$

$d' / h = 2 / 24 = 0.083$

Then using the diagrams given in Turneaure , we get

$C = 6.6$, $k = 0.5$

$f_c = C M / bh^2 = 6.6 \times 124000 / 27 \times 24 \times 24 = 630$ psi

$$f_s = nf_c \frac{1 - d'/h)}{k} - 1 = 12 \times 630 \frac{1 - 0.083}{0.5} - 1 = 6300 \text{ psi}$$

Although both the concrete and the steel are found to be understressed , the final stresses found by the " Elastic

Theory " are very satisfactory .

Dead load Springing thrust :

$$85500 / 0.707 = I20500 \text{ lbs}$$

Live load and temperature stresses :

For $d_s / d_c = 2$, $h / l = 35 / I00 = 0.35$,

Cochrane gives the following live load and temperature coef.

Live load $+ M_s = 0.028$

$T_s = 0.290$

Temperature same as before

$$M_s = 0.028 \times I5I0 \times I00^2 = 423{,}000 \text{ ft.lbs}$$

$$T_s = 0.29 \times I5I0 \times I00 = 43{,}800 \text{ lbs}$$

The temperature crown thrust being 3330 lbs for positive moment the springing thrust is :

$$T_s = 3330 \times 0.707 = 2350 \text{ lbs}$$

And the springing temperature moment is :

$$M_s = 3330 \times 35 - 29000 = 87{,}000 \text{ ft.lbs} .$$

Resultant springing stresses :

$$M_s = 423000 + 87000 = 5I0{,}000 \text{ ft.lbs}$$

$$T_s = 43800 + 2350 + I20500 = I66{,}600 \text{ lbs}$$

Investigation of Springing section :

2% of reinforcement equally divided between the top and bottom faces will be used .

The overall section to be investigated is 48 x 27 inches The member is subjected to direct compression and to bending .

$$e = M / T = 5I0000 / I66650 = 3.06$$

$$h / e = 4 / 3.06 = I.30$$

Then

$$C = 6.I \quad , \quad k = 0.45$$

$$f_c = 6.I \times 5I0000 \times I2 \ / \ 27 \times 48 \times 48 = \underline{600 \text{ psi}}$$

$$f_s = I2 \times 600 \quad \frac{I - 0.0625}{0.45} - I \; = \; \underline{8300 \text{ psi}}$$

The section assumed is O.K. as will further be proved by the " Elastic Theory "

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A N A L Y S I S O F T H E R I B by the ELASTIC THEORY

GENERAL FORMULAS for Stress Analysis

by the "ELASTIC THEORY"

The method of analysis of the arch presented here is based on the "Elastic Theory" and is of general application to arches of variable moment of inertia and loaded in any manner . It is mainly an algebraic method , although , if desired ,certain simple graphical aids may be used advantageously . It necessarily assumes that a preliminary design has been made by the aid of approximate or empirical rules or by reference to the proportions of existing arches . This arch is then exactly analyzed and the results used in correcting the design .

In this design work , a preliminary approximate analysis has been made by Cochrane's method .

The method of procedure in the analysis of an arch , will be to determine , first , the thrust , bending moment and shear at the crown . These being known , the values of similar quantities for any other section can readily be determined .

The three equations necessary to determine the unknown values , M_c , H_c and V_c are established by applying to each half of the arch , the laws pertaining to the deflection of curved beams and the condition that the deflection of the point C on one side must correspond exactly with the deflection of this point on the other .

GENERAL NOTATION

H_c = thrust at the crown

V_c = shear at the crown

M_c = bending moment at the crown

T,V and M_x = thrust , V and B.M at any section ds = length of a division of the arch ring measured along the arch axis

ds_I = length of division at the crown

N = number of divisions in one half of the arch

I = moment of inertia of any section = $I_c + n\,I_s$

A = area of cross-section = $A_c + nA_s$

I I = moment of inertia at crown

P = any load on the arch

x , y = coordinates of any point on the arch axis referred to the crown as origin , and all to be considered as positive in sign

y_I = vertical ordinate of any point on the axis referred to an X-axis through the elastic center

y_o = distance from crown to elastic center

m = b.m. at any point in the cantilever due to the external loads P , these moments are negative

= inclination of the arch axis at any point

= radius of curvature of arch axis at any point

f = average compressive fiber stress in the arch at any section = T / A

= coefficient of expansion

t = change of temperature

q = ratio of ds/I for any small division ds of the arch to the quantity ds/I_I AT the crown

l = span length

s = length of arch axis

h = rise

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The equations for the crown stresses given below will not be derived .

Simplified equations for crown stresses H_c , V_c , and M_c for symmetrical arches (transfer of X - axis to the elastic centre) .

$$V_c = \frac{\sum_A^c (m_r - m_l) \, x \, q}{2 \sum_A^c x^2 q}$$

$$M_c = - \frac{\sum_A^c (m_r + m_l) \, q}{2 \sum_A^c q} - H_c \, y_o$$

$$H_c = \frac{- \sum_A^c (m_r + m_l) \, y_I \, q + \omega t \, l \, E \, / \, ds_I / I_I}{2 \sum_A^c y_I^2 \, q + I_I \sum_A^c \cos\alpha / A}$$

T H E   I N V E S T I G A T I O N   O F   T H E   A R C H   R I B

Methods of procedure :

In the application of the forgoing formulas to the analysis of the arch , the general " Influence method " has been used .

By the influence-line method the values of moment and thrust at the crown and other critical sections are determined for a unit load placed at successive intervals along the arch and sufficiently near together to give the desired accuracy . From these values the effect of dead load and any arrangement of live load can be calculated and the maximum stresses determined .

Plotted influence lines have been used .

Dimensions of the arch to be investigated :

The span (clear) of the arch is IOO' - 0"

The rise is 35' - 0"

The arch rib has a crown section of 24 x 27 inches and a springing section of 48 x 27 inches .

I % of reinforcement on the top and I % of reinforcement on the bottom faces is used at any section .

Properties of the arch rib :

In the calculations which follow , each half of the arch rib has been divided into IO equal ds sections . Each ds section has a length of 6.49 fts.

Tables " A " and " B " give the properties of the arch rib .

Formulas for crown stresses :

On substituting values of the several summations given in tables "A" and "B" in the general formulas for crown stresses given above and neglecting the temperature terms which will be evaluated separately , we have :

$$V_c = \frac{\sum_A^c (m_r - m_l) \times q}{2 \times 2796} = \frac{\sum_A^c (m_r - m_l) \times q}{5592} \qquad (I)$$

$$M_c = - \frac{\sum_A^c (m_r + m_l) q}{2 \times 4.966} - 7.49 H_c \qquad (2)$$

$$H_c = \frac{\sum_A^c (m_r + m_l) y_I q}{2 (35";6I + 2.3I6 \times I.I29)}$$

$$H_c = \frac{\sum_A^c (m_r + m_l) yI q}{7I2.5} \qquad (3)$$

TABLE A.- PROPERTIES OF THE ARCH RING

Values of I, 9, A, and $\frac{\text{Cos}\,\alpha}{A}$ p = 1% (reinforcement)

End of Section (1)	Radial Depth d(ft) (2)	d, (3)	d,2 (4)	Is= $2(n-1)A_sd_1^2$ (5)	Ic= $\frac{1}{12}bd^3$ (6)	I= (5)+(6) (7)	A = b d + (8)	Aver I (9)	Q = I,/I (10)	Aver A (11)	α (12)	Cos α (13)	$\frac{\text{Cos}\,\alpha}{A}$ (14)	d s Center (15)
Crown	2.00	0.825	0.680	0.674	1.500	2.174	5.49	2.316	1.000					
								2.316	1.000	5.605	7°00'	0.992	0.177	1
1	2.08	0.865	0748	0.7[illegible]8	1.685	2.457	5.72							
								2.598	0.892	5.840	13° 30'	0.972	0.166	2
2	2.17	0.910	0.828	0.820	1.930	2.740	5.96							
								3.125	0.741	6.180	22°30'	0.924	0.149	3
3	2.33	0.990	0.980	1.130	2.380	3.510	6.40							
								3.810	0.608	6.565	30°00'	0.866	0.132	4
4	2.45	1.050	1.105	1.340	2.770	4.110	6.73							
								4.645	0.500	6.990	35°30'	0.814	0.116	5
5	2.64	1.145	1.315	1.720	3.460	5.180	7.25							
								5.980	0.386	7.585	41°00'	0.755	0.099	6
6	2.88	1.265	1.600	2.280	4.500	6.780	7.92							
								7.880	0.294	8.285	44°00'	0.719	0.087	7
7	3.15	1.400	1.960	3.080	5.880	8.940	8.65							
								10.13	0.228	9.000	47°30'	0.676	0.075	8
8	3.40	1.525	2.320	3.900	7.420	11.320	9.35							
								13.00	0.178	9.750	49°00'	0.656	0.067	9
9	3.70	1.675	2.805	5.130	9.550	14.680	10.15							
								16.66	0.139	10.55	49°30'	0.649	0.061	10
Springing	4.00	1.825	3.330	6.600	12.050	18.650	10.95							
								Σq =	4.966			$\Sigma \frac{\cos\alpha}{A}$ =	1.129	

TABLE B- PROPERTIES OF THE ARCH RING
Values of x, y, q and products

Section	q (1)	y (2)	yq (3)	y_1 (4)	x (5)	xq (6)	x^2q (7)	y_1q (8)	y_1^2q (9)	xy_1q (10)	Σq (11)	Σxq (12)	Σx^2q (13)	Σy_1q (14)	Σxy_1q (15)	Section
	Values of functions at centre of each section										Summation from springing to given section = Σ_A^0					
1	1.000	0.18	0.180	- 7.31	3.25	3.35	10.80	- 7.31	53.30	- 23.80	4.966	97.39	2796.0	- 0.12	537.7	1
2	0.892	1.25	1.110	- 6.24	9.62	8.57	82.50	- 5.56	34.70	- 53.50	3.966	94.04	2785.5	7.19	561.5	2
3	0.741	3.27	2.430	- 4.22	15.66	11.60	181.50	- 3.13	13.25	- 49.00	3.074	85.47	2703.0	12.75	615.0	3
4	0.608	6.07	3.695	- 1.42	21.42	13.0[illegible]	279.50	- 0.86	[illegible]	- 19.40	2.333	73.87	2591.5	15.88	654.0	4
5	0.500	9.55	4.775	2.06	26.80	13.40	359.00	1.03	2.12	27.60	1.725	60.85	2342.0	16.74	682.4	5
6	0.386	13.55	5.240	6.06	31.95	12.32	394.00	2.34	14.18	74.50	1.225	47.45	1883.0	15.71	654.8	6
7	0.294	17.85	5.260	10.36	36.65	10.78	395.00	3.06	31.50	112.00	0.839	35.18	1489.0	13.38	580.3	7
8	0.228	22.55	5.140	15.06	41.15	9.38	386.00	3.44	51.80	141.80	0.545	24.35	1094.00	10.33	468.3	8
9	0.178	27.40	4.870	19.51	45.35	8.07	366.00	3.48	69.00	157.50	0.317	14.97	708.00	6.89	326.5	9
10	0.139	32.40	4.515	24.51	49.60	6.90	342.00	3.41	83.50	169.00	0.139	6.90	342.00	3.41	169.0	10
Σ_1^{10}	4.966		37.205			97.3[illegible]	3796.0	- 0.1[illegible]	358.61	537.70						

$y_0 = \frac{\Sigma yq}{\Sigma q} = \frac{37.205}{4.966} = 7.49$

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May 1946

Scale 1cm = 4ft.

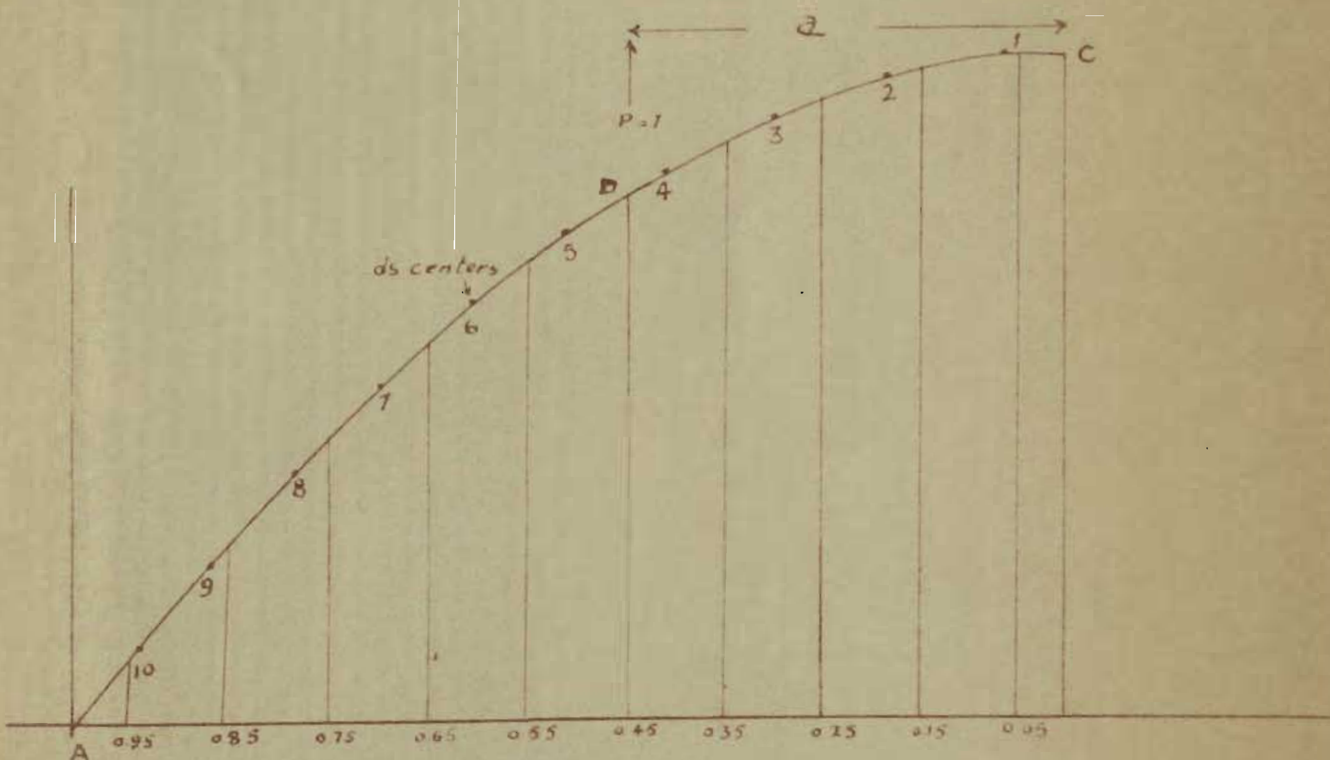
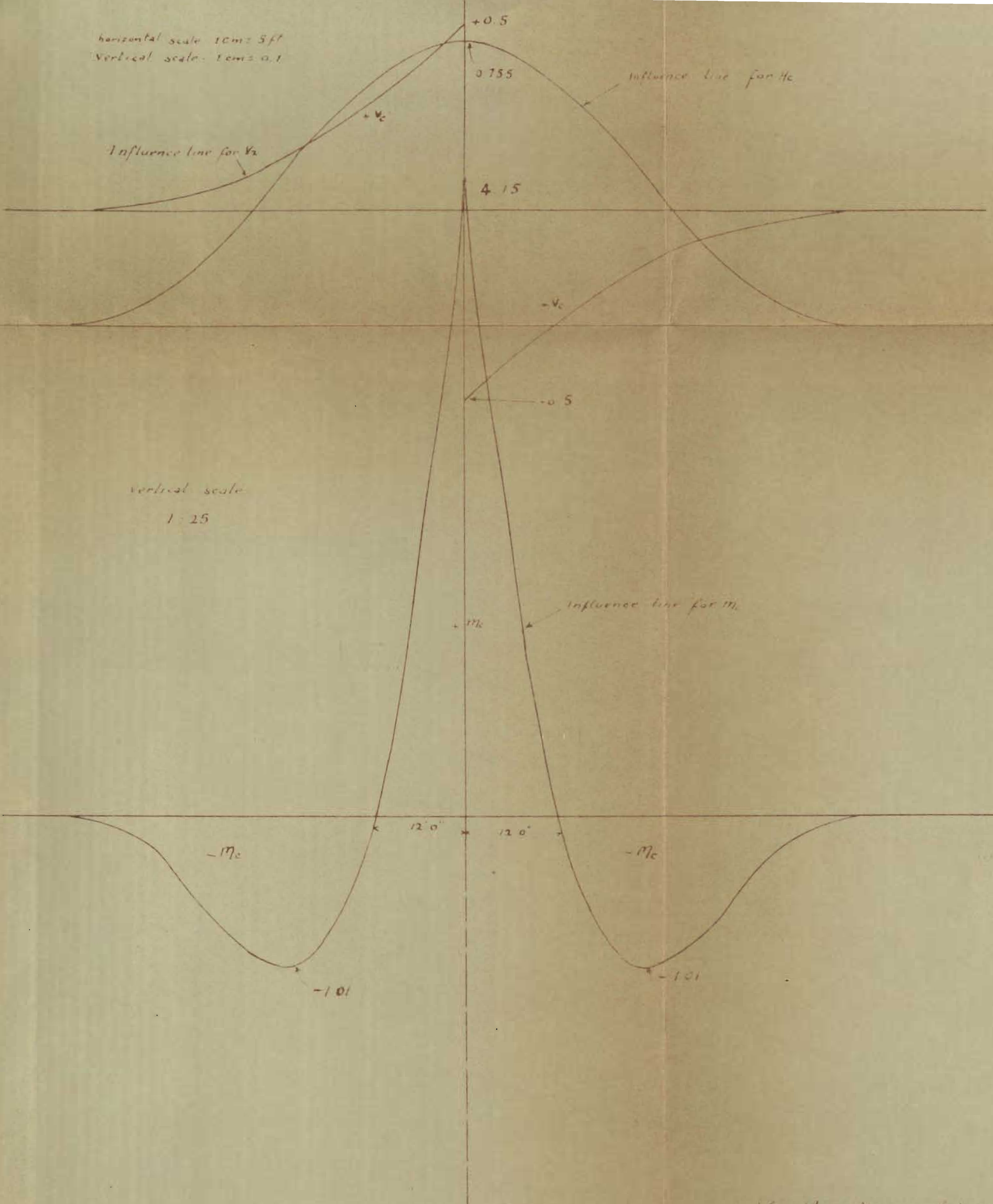


TABLE C

Calculation of H_c, M_c and V_c for unit loads at Various points

Influence-line values for crown sections

Load Points: a/l/2 (1)	a (2)	Sections included in $\sum_A^0$ (3)	$\sum_A^0 x y_1 q$ (4)	$a\sum_A^0 y_1 q$ (5)	$H_c = \frac{4-5}{712.5}$ (6)	$\sum_A^0 x q$ (7)	$a\sum_A^0 q$ (8)	$\frac{(7)^2 - (8)}{9.932}$ (9)	7.49 H_c (10)	M_c (9) - (10) (11)	$\sum_A^0 x^2 q$ (12)	$a\sum_A^0 x q$ (13)	$V = \frac{(12^0 - (13))}{5592}$ (14)	Load Point
Crown	0	1-10	537.7	0	0.755	97.29	0	9.80	5.65	4.15	2796.0	0	0.500	Crown
0.05	2.630	1-10	537.5	-0.330	0.754	97.29	13.10	8.48	5.64	2.84	2796.0	255.0	0.455	0.05
0.15	7.590	2-10	561.5	56.7	0.706	94.04	31.30	6.32	5.29	1.03	27.85.5	744.0	0.373	0.15
0.25	13.17	3-10	615.0	167.5	0.628	85.47	40.50	4.52	4.71	- 0.19	2703.0	1125.0	0.282	0.25
0.35	18.44	4-10	664.0	293.0	0.521	73.87	43.20	3.09	3.90	- 0.81	2521.5	1361.0	0.208	0.35
0.45	23.71	5-10	683.4	397.0	0.401	60.85	41.00	2.00	3.01	- 1.01	2242.0	1442.0	0.143	0.45
0.55	28.98	6-10	654.8	455.0	0.280	47.45	35.50	1.205	2.10	- 0.895	1883.0	1371.0	0.091	0.55
0.65	34.25	7-10	580.3	455.0	0.175	35.13	28.70	0.645	1.315	- 0.670	1489.0	1202.0	0.051	0.65
0.75	39.52	8-10	468.3	409.0	0.083	24.35	21.50	0.287	0.62	- 0.333	1094.0	963.0	0.023	0.75
0.85	44.79	9-10	326.5	508.0	0.026	14.97	14.20	0.075	0.195	- 0.120	708.0	671.0	0.007	0.85
0.95	50.06													0.95



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May 1946

TABLE D

Calculation of M1/4 and T 1/4

Influence-line values for quarter Point

Loads A to 1/4 Pt and on Right half

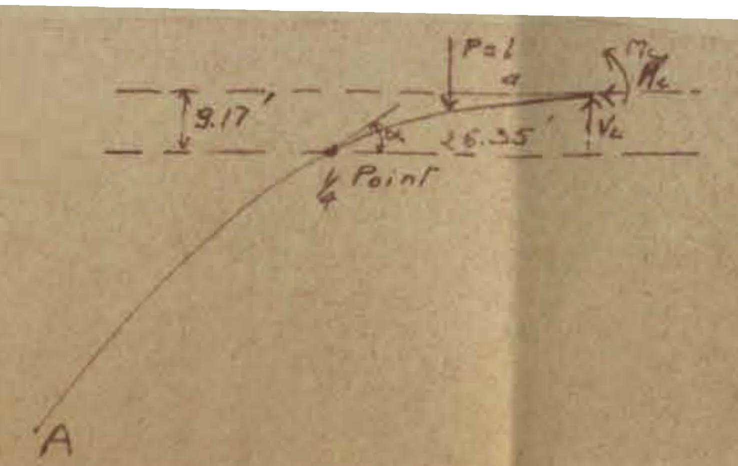
$M\ 1/4 = M_c + 9.17\ H_c + 26.35\ V_c$

$T\ 1/4 = H_c \cos\alpha - V_c \sin\alpha$

Loads 1/4 Pt to C

$M\ 1/4 = M_c + 9.17 H_c + 26.35\ V_c - (26.35 - a)$

$T\ 1/4 = H_c \cos\alpha + (1 - V_c) \sin\alpha$



	Load Point	M_c	$9.17\ H_c$	V_c	$26.35\ V_c$	$-(26.35-a)$	M 1/4	$H_c \cos\alpha$	$(1-V_c)$	$(1-V_c)\sin\alpha$	$V_c \sin\alpha$	T 1/4	Load Point
A	0.95	--	--	--	--	--	--	--	--	--	--	--	0.95
	0.85	-0.120	0.239	0.007	0.184	--	0.303	0.02I	--	--	0.004	0.0I7	0.85
	0.75	-0.333	0.762	0.023	0.6I5	--	1.044	0.067	--	--	0.0I3	0.054	0.75
	0.65	-0.670	1.605	0.05I	1.345	--	2.280	0.I42	--	--	0.029	0.II3	0.65
	0.55	-0.895	2.570	0.09I	2.400	--	4.075	0.228	--	--	0.053	0.I75	0.55
Left1/4 Pt	0.50	-0.950	3.I20	0.II7	3.082	--	5.252	0.277	0.883	0.5I2	0.068	0.209) 0.789 (	0.50 1/4 P
	0.45	-1.01	3.680	0.143	3.770	- 2.64	3.83	0.326	0.857	0.496	--	0.822	0.45
	0.35	-0.8I	4.785	0.208	5.480	- 7.9I	1.545	0.424	0.792	0.458	--	0.882	0.35
	0.25	-0.I9	5.760	0.282	7.430	-13.18	- 0.18	0.5II	0.7I8	0.4I6	--	0.927	0.25
	0.I5	1.03	6.480	0.373	9.840	-18.45	- 1.10	0.575	0.627	0.363	--	0.938	0.I5
	0.05	2.84	6.920	0.455	12.000	-23.72	- 1.96	0.6I2	0.545	0.3I6	--	0.928	0.05
C	Crown	4.I5	6.930	0.500	13.I80	-26.35	- 2.09	0.6I4	0.500	0.290	--	0.904	Crown
	0.05	2.84	6.920	- 0.455	- 12.000	--	- 2.24	0.6I2	--	--	- 0.264	0.876	0.05
	0.I5	1.03	6.480	- 0.373	- 9.840	--	- 2.33	0.575	--	--	- 0.2I6	0.79I	0.I5
	0.25	- 0.I9	5.760	- 0.282	- 7.430	--	- 1.86	0.5II	--	--	- 0.I63	0.674	0.25
	0.35	- 0.8I	4.785	- 0.208	- 5.480	--	- 1.505	0.424	--	--	- 0.I2I	0.545	0.35
Right	0.45	- 1.01	3.680	- 0.I43	- 3.770	--	- 1.100	0.326	--	--	- 0.083	0.409	0.45
	0.55	- 0.895	2.570	- 0.09I	- 2.400	--	- 0.725	0.228	--	--	- 0.053	0.28I	0.55
	0.65	- 0.670	1.605	- 0.05I	- 1.345	--	- 0.4I0	0.I42	--	--	- 0.029	0.I7I	0.65
	0.75	- 0.333	0.762	- 0.023	- 0.6I5	--	- 0.I86	0.067	--	--	- 0.0I3	0.080	0.75
	0.85	- 0.I20	0.239	- 0.007	- 0.184	--	- 0.065	0.02I	--	--	- 0.004	0.025	0.85
B	0.95	--	--	--	--	--	--	--	--	--	--	--	0.95

TABLE E

CALCULATION of M_s and T_s

Influence line values for left Springing

Loads on left:

$M_s = M_c + 36H_c + 52.7V_c - (52.7 - a)$

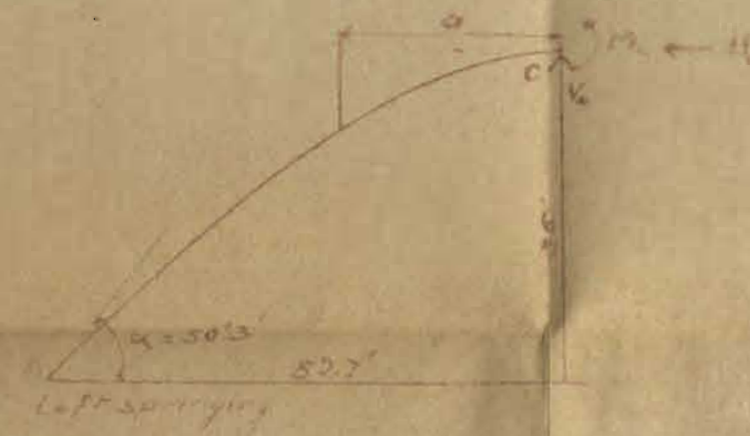
$T_s = H_c \cos\alpha + (1 - V_c)\sin\alpha$

Loads on Right: $M_s = M_c + 36H_c + 52.7V_c$

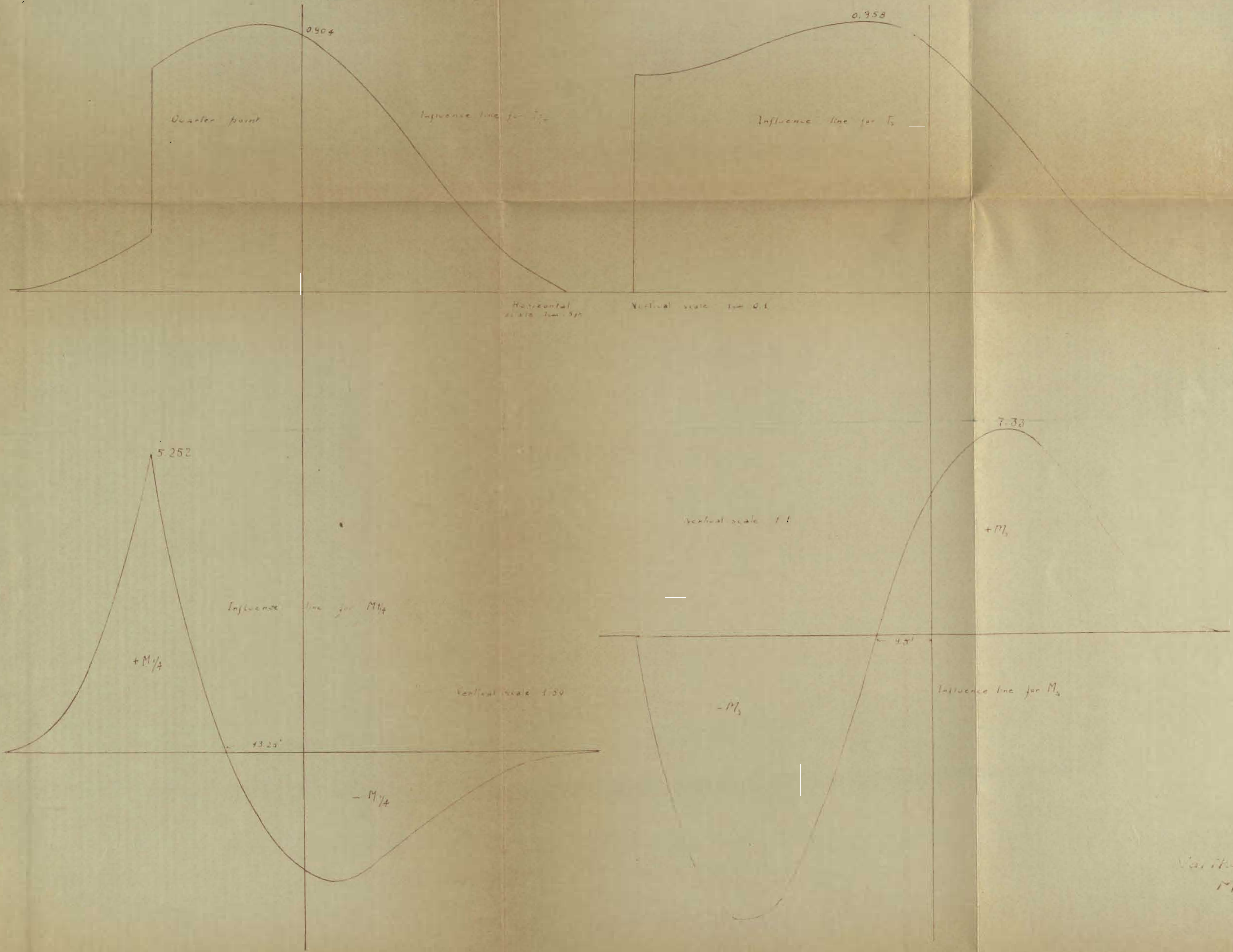
$T_s = H_c \cos\alpha - V_c \sin\alpha$

$\cos\alpha = \frac{5.2}{8.1} = 0.642$

$\sin\alpha = \frac{6.25}{8.1} = 0.772$



Load Point	M_c	36 H_c	V_c	52.7V_c	-(52.7-a)	M_s	H_c cos∝	(1- V_c)	(I -V_c)	V_c sin	T_s	Load Point
0.95	--	--	--	—	—	—	—	--	—	--	--	0.95
0.85	- 0.120	0.935	0.007	0.368	- 7.91	- 6.727	0.017	0.993	0.766		0.783	0.85
0.75	- 0.333	2.980	0.023	1.210	-13.18	- 9.323	0.053	0.977	0.754		0.807	0.75
0.65	- 0.670	6.30	0.051	2.680	-18.45	-10.14	0.112	0.949	0.732		0.844	0.65
0.55	- 0.895	10.10	0.09I	4.80	-23.72	- 9.715	0.180	0.909	0.70I		0.88I	0.55
0.45	- 1.010	14.45	0.143	7.55	-28.99	- 8.00	0.258	0.857	0.662		0.920	0.45
0.35	- 0.8I	18.80	0.208	10.95	-34.26	- 5.32	0.334	0.792	0.6I2		0.946	0.35
0.25	- 0.19	22.62	0.282	14.90	-39.53	- 2.20	0.404	0.7I8	0.554		0.958	0.25
0.15	1.03	25.50	0.373	19.75	-44.80	1.45	0.454	0.627	0.484		0.938	0.15
0.05	2.84	27.18	0.455	23.90	-50.07	3.85	0.484	0.545	0.43I		0.905	0.05
Crown	4.15	27.20	0.500	26.35	-52.70	5.00	0.485	0.500	0.386	0.386	0.871	Crown
0.05	2.84	27.18	-0.455	-23.90		6.12	0.484			-0.35I	0.835	0.05
0.15	1.03	25.50	-0.373	-19.75		6.78	0.454			-0.288	0.743	0.15
0.25	- 0.19	22.62	-0.282	-14.90		7.35	0.404			-0.218	0.622	0.25
0.35	- 0.81	18.80	-0.208	-10.95		7.04	0.334			-0.161	0.495	0.35
0.45	- 1.010	14.45	-0.143	- 7.55		5.89	0.258			-0.110	0.368	0.45
0.55	- 0.895	10.10	-0.091	- 4.80		4.405	0.180			-0.070	0.250	0.55
0.65	- 0.670	6.30	-0.051	- 2.68		2.95	0.112			-0.039	0.151	0.65
0.75	- 0.333	2.98	-0.023	- 1.21		1.437	0.053			-0.018	0.071	0.75
0.85	- 0.120	0.935	-0.007	- 0.368		0.447	0.017			-0.005	0.022	0.85
0.95	—	--	—	—	—	—	—	—	—	—	--	0.95


0.904
Quarter point
0.958
Influence line for
Horizontal
Vertical scale
5.252
Influence line for M1/4
+M1/4
13.25'
-M1/4
7.33
+M5
4.5'
-M5
Influence line for M5

T A B L E   F

Dead load concentrations - Distance from Crown

Section	Loads	Dist.	Column	Loads	Distance
I	4380 #	3.2	I	27607 #	5.25
2	4600	9.65	2	28080	I6.00
3	4960	I5.7	3	28560	27.00
4	5I80	2I.4	4	295I3	38.50
5	5400	26.7			
6	5840	3I.85			
7	6370	36.70			
8	6800	4I.I0			
9	7450	45.30			
I0	8I00	49.55			

TABLE G

DEAD LOAD STRESSES AT CROWN, QUARTER POINT AND SPRINGING

INFLUENCE LINE METHOD

Load Point	Load		CROWN: Influence line ordinate for M_c	M_c	Influence line ordinate for T_c	T_c	QUARTER POINT: Influence line ordinate for M 1/4	M 1/4	Influence line ordinate for T 1/4	T 1/4	LEFT SPRINGING: Influence line ordinate for M_s	M_s	Influence line ordinate for T_s	Ts
50.5	C_5		- 0.03		+ 0.005		0.07			0.005	- 1.35		0.77	
49.55	P_{10}	8100	- 0.03	-243	0.010	81	0.10	810	81	0.01	- 2.7	- 21900	0.775	6380
45.30	P_9	7450	- 0.10	-745	0.025	186	0.32	2380	149	0.03	- 5.9	-43800	0.78	5800
41.10	P_8	6800	- 0.24	-1632	0.065	442	0.75	5090	306	0.045	- 8.0	-54200	0.80	5440
38.5	C_4	29513	- 0.38	-10800	0.09	2610	1.24	36500	2060	0.07	- 8.35	-246000	0.815	24000
36.70	P_7	6370	- 0.51	-3250	0.12	765	1.70	10800	540	0.085	- 0.75	-55000	0.825	5250
31.85	P_6	5840	- 0.76	-4430	0.215	1255	2.92	17000	784	0.135	- 09.0	-52800	0.855	5000
27.0	C_3	28560	- 0.95	-27100	0.30	8540	4.85	137000	5400	0.19	- 0.35	-236000	0.885	25200
26.7	P_5-	5400	- 0.96	-4960	0.465	2405	2.50	12900	4450	0.86	- 7.05	-36500	0.925	4780
21.4	P_4	5180	- 0.96	-4960	0.465	2405	2.50	12900	4450	0.86	- 7.05	-36500	0.925	4780
16.0	C_2	28080	- 0.56	-15700	0.53	14850	0.70	19600	25500	0.91	- 4.1	-115000	0.95	26720
15.7	P_3	4960	- 0.55	-2730	0.56	2780	0.60	2980	4540	0.915	- 3.9	-19300	0.955	4740
9.65	P_2	4600	+ 0.62	+2850	0.62	2850	- 0.80	-3680	4320	0.94	0	0000	0.955	4390
5.25	C_1	27607	+ 2.06	+57000	0.705	19400	- 1.60	-44300	25700	0.935	+ 2.5	+63800	0.93	25200
3.20	P_1	4380	+ 2.84	+12450	0.73	5190	- 1.85	-8100	408	0.93	+ 3.6	+15750	0.91	3980
3.20	P_1	4380		-4460		61054	- 2.23	-9800	3780	0.865	+ 5.90	+26950	0.82	3580
5.25	C_1	27607					- 2.28	-63000	22800	0.83	+ 6.75	+188000	0.79	21600
9.65	P_2	4600					- 2.15	-9900	3450	0.75	+ 6.95	+32000	0.685	3140
15.7	P_3	4960					- 1.70	-8450	3080	0.62	+ 7.35	+36650	0.565	2800
16.0	C_2	28080					- 1.76	-49500	17000	0.605	+ 7.35	+207000	0.55	15500
21.4	P_4	5180					- 1.35	-7070	2480	0.48	+ 6.9	+37100	0.42	2170
26.7	P_5	5400					- 0.95	-5180	1770	0.33	+ 5.6	+30500	0.305	1650
27.0	C_3	28560					- 0.89	-25400	8850	0.31	+ 5.3	+152000	0.29	8250
31.85	P_6	5840					- 0.55	-3210	1250	0.215	+ 3.45	+20150	0.195	1140
36.70	P_7	6370					- 0.32	-2040	820	0.13	+ 2.65	+17300	0.12	760
38.5	C_4	29848					- 0.65	-5800	2951	0.10	+ 2.25	+66600	0.095	2800
41.10	P_8	6800					- 0.13	-885	475	0.07	+ 1.45	+9800	0.06	410
45.30	P_9	7450					- 0.05	-372	224	0.03	+ 0.5	3720	0.025	180
49.55	P_{10}	8100					- 0.02	-162	81	0.01	+ 0.1	810	0.005	40
50.5	C_5						- 0.005			0.005	+ 0.05		0.002	
				-8920		122108		25470	144500			-14900		215660

T A B L E   H

Live load Moments and Thrusts

Influence line Method

Concentrated load per column : 42,000 lbs

Uniform load per column : 26,000 lbs

Front axle load = 1/3 rear axle load

Section	Maximum Moment		Thrusts corresponding to Max M.	
	Sum of influence line ordinates	Moment(ft. lbs)	Sum of influence line ordinates	Thrust lbs.
Crown	+ 2.76 (Con)	+ 116,000	+ 0.975	41,000
	- 3.68 (Uni)	- 95,700	+ 2.02	52,500
Quarter point	+ 5.5 (C)	+ 231,000	0.81	34,000
	- 6.65 (U)	- 173,000	2.78	72,400
Springing	+22.95 (U)	+ 596,000	2.66	69,200
	-23.9 (U)	- 620,000	3.42	89,000

Shear influence line ordinate for Maximum Moment at crown = 0.26

Shear = 0.26 x 38000 = 7870

T A B L E I

Combined Moments and Thrusts

D. L. , L. L. and Temperature

Moments in ft. lbs, Thrusts in lbs.

			Dead load	Live load	T. Cond.	T. Effect	Total
CROWN	Positive moment	$+M_c$	-8920	+ 116,000	Fall	+ 52,800	+160800
		T_c	122100	41,000		- 7,000	156100
	Negative moment	$-M_c$	- 8920	- 95,700	Rise	- 17,400	-122020
		T_c	122100	52,500		+ 1,300	176930
Quarter Point	Positive moment	$T_{\frac{1}{4}}$	144300	34,000	Rise	+ 1,900	180200
		$+M_{\frac{1}{4}}$	25470	+ 231,000		+ 4,000	+260470
	Negative moment	$-M_{\frac{1}{4}}$	25470	- 173,000	Fall	- 11,600	-159130
		$T_{\frac{1}{4}}$	144300	72,400		- 5,700	211000
Springing	Positive moment	$+M_s$	- 14800	+ 596,000	Rise	+ 56,600	+847700
		T_s	215660	+ 69,200		+ 1,500	286360
	Negative moment	$-M_s$	- 14900	- 620,000	Fall	-199,500	-834400
		T_s	215660	89,000		- 4,500	300160

Moment under column (I) :

$+ M = 160800 + 156100 \times 0.35 + 7870 \times 5.25 - 4380 \times (5.25 - 2.84) =$

$= 245,600$ ft. lbs.

$- M = - 122020 + 176930 \times 0.35 - 7870 \times 5.25 - 4380 \times (5.25 - 2.84) =$

$= - 112,120$ ft. lbs.

T A B L E J

Maximum fiber Stresses .

Dead load , Live load and Temperature

Sect		Moment	Thrust	e	h	h/e	p	pn	d'/h	C	f_c (psi)	f_s (psi)
Crown	+M	I60300	I56I00.	I.025	2.00	I.95	0.0I	0.I2	.083	6.3	[illegible]00	5I50
	-M	I22020	I76930	0.69I	2.00	2.88	0.0I	0.I2	.083	6.8	[illegible]12	2800
C-I	+M	245600	I52000	I.6I	2.I5	I.33	0.0I	0.I2	.083	5.6	[illegible]I5	9I50
	-M	II2I20	I7I000	0.66	2.I5	3.26	0.0I	0.I2	.063	7.0	[illegible]60	2250
Q.P.	+M	260470	I80200	I.38	2.50	I.8I	0.0I	0.I2	.066	6.I	[illegible]85	7060
	-M	I59I30	2II000	0.755	2.50	3.30	0.0I	0.I2	.066	6.5	[illegible]25	2850
Sp.	+M	647700	286360	2.25	4.00	I.77	0.0I	0.I2	.063	5.9	[illegible]	6700
	-M	834400	300I60	2.76	4.00	I.45	0.0I	0.I2	.063	5.6	[illegible]00	9450

CALCULATION of Stresses by the Influence - Line Method .

Influence lines have been drawn for V_C , H_C and M_C ; $T_{\frac{1}{4}}$, $M_{\frac{1}{4}}$; and T_s and M_s . Those are plotted from Tables "C" , "D" and "E".

From those influence lines , dead load and live load stresses have been calculated and tabulated in tables Gand H .

The arch ring has been divided into IO sections , and the dead load of each computed . The dead load of the roadway system is concentrated at the centers of the columns .

The French system of loading has been used . Both the uniform loading and the concentrated loading systems have been considered simultaneously and the one giving bigger effect used for design .

The concentrated load per colomn due to the concentrated loading system is

38,000 + 4000 = 42,000 lbs .

The above 38,000 is the maximum floor beam reaction , while the 4,000 is the sidewalk liveload .

On the other hand , the concentrated load per column due to the uniform system of loading is 26,000 lbs .

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TEMPERATURE STRESSES

It is sometimes desirable to have separate expressions for crown stresses due to temperature changes .

If V_t , M_t , and Ht represent respectively the crown shear , moment and thrust due to a rise in temperature , we have :

$$V_t = 0$$

$$M_t = H_t\ y_o$$

$$Ht = \frac{\omega t \frac{\ell E}{ds/I_1}}{2\left[\sum_A^c y_1^2 g + I_1 \sum_A^c \frac{\cos\alpha}{A}\right]}$$

- - - - - - - - - - - - - - - - - -

Temperature range :

Fall of 60° F

Rise of 20° F

Stresses due to a 60° f fall :

Substituting in the above formulas we get for a coef. of linear expansion per degree Fahrenheit of 0.000006 :

H_c = 0.000006 x 60 x I00 x 2000000 x I44 / 2.805 x 7I2.5

= - 7000 lbs (Tension)

M_c = + 7.49 H_c = + 7.49 x 7000 = + 52,500 ft.lbs

Quarter point values :

$M_{\frac{1}{4}}$ = 52500 - 7000 x 9.I7 = - II600 ft.lbs

$T_{\frac{1}{4}}$ = - 7000 x cos = -7000 x 0.8I4 = - 5700 lbs (Comp

Springing values :

M_s = + 52500 - 7000 x 36 = - I99500 ft.lbs

T_s = 7000 x 0.642 = - 4500 lbs (Tension)

Stresses due to 20° F Rise :

H_c = II6.8 x 20 = 2330 lbs (Compression)

M_c = - 7.49 x 2330 = - I7400 ft.lbs

$T_{\frac{1}{4}}$ = 2330 x 0.8I4 = I900 lbs (Compression)

$M_{\frac{1}{4}}$ = - I7400 + 2330 x 9.I7 = 4000 ft.lbs

T_s = 2330 x 0.642 = I500 lbs (Compression)

M_s = - I7400 + 2330 x 36 = 66600 ft.lbs

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As seen in table " J " , the arch rib as designed is on the safe side . Minor modifications at the crown of the rib and in the columns may be had without any danger of failure .

These modifications will be made essentially for architectural reasons .

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D E S I G N of A B U T M E N T

The foundation soil is hard rock . An allowable bearing capacity of I0 Tons / ft^2 has been taken for the design of the footings . Although the bearing capacity assumed above seems to be too low , the designer wanted to be on the safe side .

The thrust and moment acting on the footing is equal to the respective springing values .

+ M = 647700 ft.lbs T = 286300 lbs

- M = 834400 ft.lbs T = 300I60 lbs

Weight of earth = 5 x 4 x 35 x I00 = 70,000 lbs

Resultant thrust - found graphically to be 350,000 lbs

Resultant moment = 834000 - 70000 x 4 = 554,000 ft.lbs

Then the following equation can be written :

$$I0 \times 2200 = P/A + Mc/I$$

$$= 350000/4d + 554 \times 6 / 4d^2$$

From which we get

d = 8.5 fts

Have d = 9.0 fts

There is no need to calculate for the reinforcement .

The reinforcements used are shown on the detail drawing .

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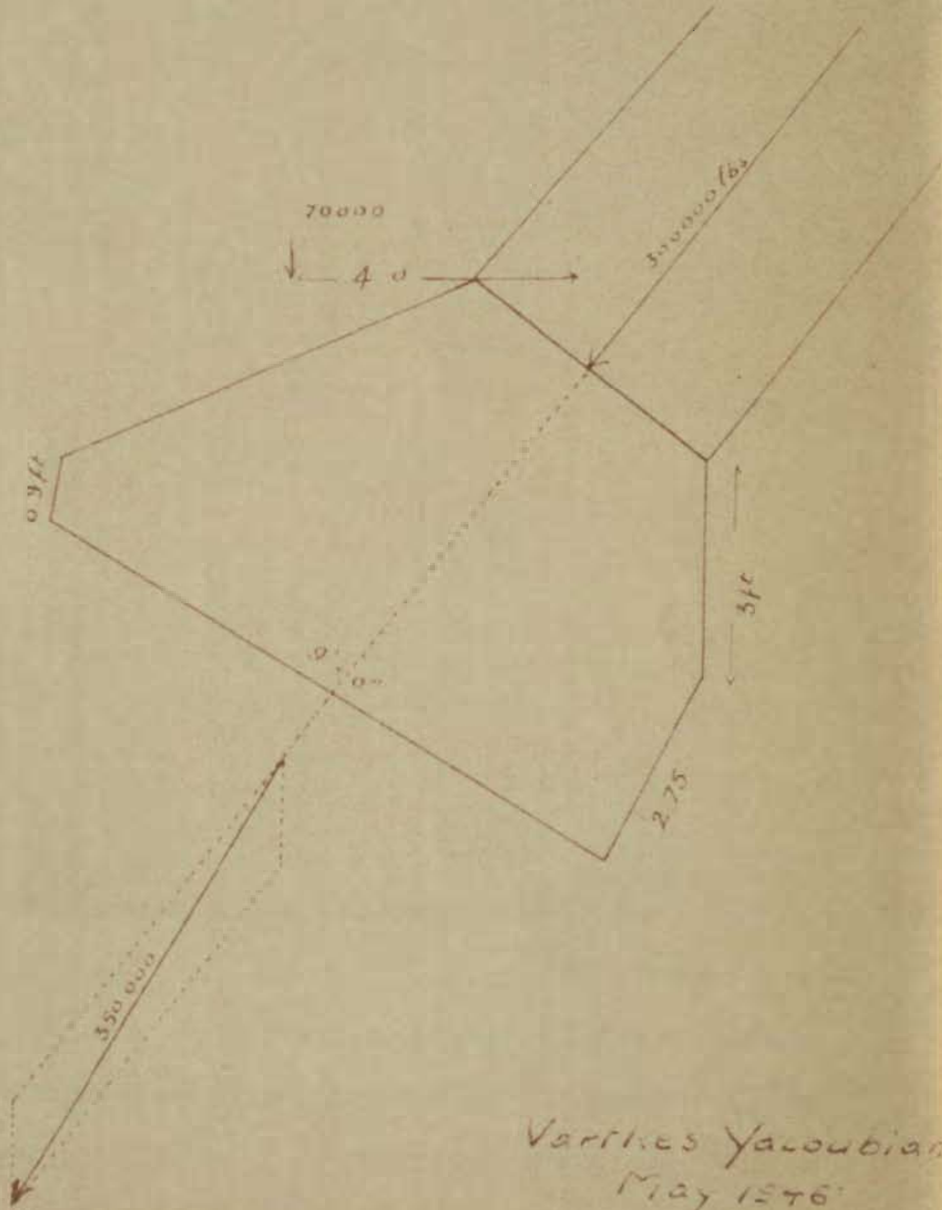
Footing of arch rib

Scale 1cm = 1ft



Vartkes Yacoubian
May 1946

D E S I G N of Retaining and wing W A L L S

The soil backing the retaining and wing walls is hard rock . The angle of friction (natural slope) of rock is 90° , so that there is no need to consider earth pressure provided proper drainage has been prepared . Minimum requirements have been used .

The dimensions and reinforcements adopted are shown in the detail drawings .

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A R C H D E T A I L S

Expansion joints are provided in the floor system near the abutments and the quarter points . Those expansion joints are to be filled with bitumen .

The public utilities (electric wirings , pipings etc) are located under the sidewalks .

Drainage holes are amply provided in the retaining and wing walls .