Linear Decision Rules for Control of Reactive Power by Distributed Photovoltaic Generators

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***Abstract*—The revised IEEE 1547 Standard together with the grid codes from several European countries require the contribution of photovoltaic inverters to the reactive power and voltage control of distribution networks. This paper presents a decentralized approach for controlling reactive power from a photovoltaic (PV) inverter through a linear decision rule that is in terms of the PV generated real power, conforming to standard characteristic curves in the German grid code. The linear decision rules are computed using an affinely adjustable robust counterpart of an optimal power flow type formulation, with the PV real power generation specified in an uncertainty interval. The robust solution, which is computed via convex quadratic programming, guarantees that both the voltage deviations and power loss will remain within their optimized bounds for all realizations of PV real power sampled from the uncertainty set. The proposed linear decision rules are compared with a local control method that seeks the same type of multiobjective optimization. Monte Carlo simulation results on meshed distribution networks with up to 830 nodes show the superiority of the proposed approach.**

***Index Terms*—Decentralized control, distributed power generation, load flow control, optimization methods, reactive power control, uncertainty, voltage control.**

Nomenclature

A. Notation

$\underline{a}$	A complex-valued quantity, indicated by an underline.
a^{re}, a^{im}	Real and imaginary parts of $\underline{a}$.
$\lvert\underline{a}\rvert$	Magnitude of $\underline{a}$.
A_{ij}	Element in row i and column j of matrix A.
$A(i,:)$	Row i of matrix A.
$z^{\min}, z^{\max}$	Minimum and maximum values of a real-valued quantity z.

B. Parameters

α	Branch reactance to resistance ratio.
Δp_j	Maximum deviation of the jth PV real power generation forecast.
D_g	Diagonal matrix of branch conductances.
k	Half the number of polygon vertices in the polyhedral approximation.
K	Tradeoff parameter.
m	Number of branches.
M	Node branch incidence matrix.
n	Number of nodes.
p	Vector of PV real power generation.
p_{0j}	Forecasted jth PV real power generation.
P_{Di}, Q_{Di}	Real and reactive power demand at node i.
P_{Gi}	Real power generation at node i.
P_i	Real power injection at node i.
V_s	Voltage magnitude at the slack node.
$\underline{Y}$	Nodal equations coefficient matrix.
ϕ	Angle employed in the polyhedral approximation.
Λ	Budget of uncertainty.

C. Variables & Dependent Quantities

β_j	Slope of the jth linear decision rule.
ΔV_i	Voltage magnitude deviation at node i.
Q_{Gi}	Reactive power generation at node i.
Q_i	Reactive power injection at node i.
P_{Loss}	Network real power loss.
q	Vector of reactive power from the PV inverters.
q_{0j}	q_j-intercept of the jth linear decision rule.
$\underline{S}_i$	Complex power injection at node i.
$\underline{V}, \underline{I}$	Nodal voltage and injection current complex vectors.
$\underline{V}_i, \underline{I}_i$	Nodal voltage and injection current complex values at node i.

I. Introduction

PHOTOVOLTAICS are contributing an increased share of power generation driven by both falling prices and environmental concerns. Photovoltaic (PV) generation is commonly connected to distribution networks and poses specific technical challenges in its operation; most importantly, PV generation is intermittent and can rapidly vary with cloud movement. Unlike transmission systems, the high resistance to reactance ratio commonly characterizing distribution networks renders the voltage magnitude more susceptible to variations in real power injection; this is particularly evident in operating instances with high photovoltaic generation and reverse power flows. Voltage control in distribution networks has been an active research problem, aiming to provide voltage regulation by the control of reactive power flow. Until recently, the IEEE 1547 Standard [1] has prohibited reactive power support by distributed generation; however, amendments to this standard [2] together with grid codes issued by Germany and Italy all stipulate the adoption

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of reactive power control strategies under which distributed generation can provide ancillary services for voltage support [3]. This paper proposes a decentralized technique for controlling the inverter reactive power as a function of the real PV power generation; the reactive power is computed locally via a linear decision rule, which is obtained from an affinely adjustable robust optimization framework [4], [5].

A. Related Work

The techniques for reactive power flow control can be categorized based on communication requirements as centralized, distributed, and decentralized with a purely local option [6]. Centralized methods require global system information that is telemetered to the distribution network central controller; this information includes a network model and distribution network load estimation results. The controller computes the set-points based on an optimization technique, and sends the set-points back to the control devices for implementation. From a mathematical perspective, centralized techniques employ an optimal power flow (OPF) solution method for calculating control set-points that either aim to be valid over several operational scenarios [7], [8] or dictate the control of a minimum number of inverters [9]. OPF-type formulations can account for time dependency in controller set-points via receding horizon control [10], and also consider the different time scales at which the controllers can be actuated [11], [12]; for instance, capacitor banks can practically be switched at a slower timescale as compared to PV inverters. More involved OPF techniques account for uncertain active power injections [13] and forecast errors [14].

Contrary to centralized techniques, distributed methods make use of calculations that are distributed at different nodes in the network and require limited communication. For instance, [15] presents a distributed version of the optimal inverter dispatch framework [9] based on the alternating direction method of multipliers (ADMM); the ADMM was also employed for inverter dispatch in radial networks that are modeled by the linear DistFlow equations [16]. An alternative distributed approach that seeks inverter OPF solutions employs limited message passing between PV systems and the utility [17]; the distributed methods [15]–[17] all require a convex network model.

Decentralized methods operate based on a control law that takes local measurements as inputs; the control law can be computed centrally and communicated to the local controllers in a periodic manner, or it can be hardwired at the time of installation thus giving rise to purely local control. A common purely local control law requiring no communication adjusts the reactive power as a function of the voltage at the point of connection [18]. $Q(V)$ control [18] is commensurate with the updated IEEE 1547 Standard [2], but its non-incremental control algorithm implementation has restrictive convergence conditions that make it insufficient to maintain voltages in an acceptable range [19]–[21]. Refs. [19]–[22] propose alternative local incremental control schemes with voltage measurements and less restrictive convergence conditions for radial networks; the incremental control algorithm gradually adjusts the reactive power at each time taking into account the previous reactive power value. The German grid code proposes $Q(P)$ control, where reactive power is adjusted as a function of real power measurements [23]. Another class of decentralized methods seeks the multi-objective optimization of a composite function involving both power loss and voltage regulation [24]–[26]; it requires the simple communication of a tradeoff parameter, which is used to tune the emphasis of the local control law on one of the objectives. An alternative solution that makes use of the same objective function in [26] is based on a switching law dictating the choice between the objectives in real time [27]; this solution requires global system information and a power flow to produce a flag that is communicated to all participating inverters.

B. Technical Contribution

This paper builds on the decentralized multi-objective formulation in [24]–[26], but proposes the use of linear decision rules [4], [5] to compute the inverter reactive power contribution as a function of the produced real power. The approach explicitly treats the PV real power generation as an uncertain variable in a predefined uncertainty set, and proposes an affinely adjustable robust optimization problem for calculating the decision rules. The affinely adjustable robust optimization framework makes use of a mathematically rigorous approximation of the power flow equations [28] and the polyhedral norm representation of the magnitude of complex numbers [29]; the result is a convex quadratic OPF-type program that minimizes the multi-objective function of power loss and voltage regulation under the interval uncertainty of PV real power generation. Once the uncertainty is revealed and the PV real power generation is known, the linear decision rules give the reactive power for each inverter such that the power loss and voltage deviations will not exceed the optimized upper bounds from the quadratic program; other interval-based optimization techniques do not allow for adjustable variables that adapt to the uncertainty [30]. As in previous work [9], [13], [15]–[27], the switched capacitor banks and transformer taps are assumed constant during the PV inverter reactive power optimization; this is justified by the fact that classical controllers operate at a much slower pace in comparison with PV inverters that are adjusted in real time [11], [12]. The affinely adjustable robust optimization problem is solved to update the decision rules either based on a fixed time step of 15 to 30 minutes, or whenever there is a major change in the control settings of network configuration switches, capacitor banks, or transformer taps. The proposed approach is simple to implement and is compatible with the German grid code local control characteristic curves [23].

C. Paper Structure

The rest of this paper is organized as follows. Section II introduces the problem formulation using the network model in [28], and derives the expressions for the power loss and voltage deviations as a function of the real and reactive power injections; the problem of reactive power control is reduced to minimizing a quadratic objective function subject to conic constraints, that are further approximated by linear inequality constraints using the polyhedral norm. Section III presents the affinely adjustable robust optimization formulation for computing the linear decision rules under the interval uncertainty of PV real power generation; a budget of uncertainty is considered for limiting the conservativeness of the solution. For comparison purposes, the local control formulae of a competing multi-objective solution method [24]–[26] are summarized in Section IV. Section V includes a numerical comparison, via Monte-Carlo simulation, between the proposed linear decision rules, the local control approach of [24]–[26], and the centralized optimization solution; testing is reported on 33- and 830-node meshed distribution networks with varying levels of PV integration. The paper is concluded in Section VI. The appendix of the paper describes the fundamentals of the polyhedral approximation.

II. Control of Reactive Power by PV Inverters

Consider a distribution network having m branches and n nodes, with the first one being the slack power injection node. The network nodal admittance matrix $\underline{Y}$ accounts for transformers with off-nominal tap positions and switched capacitor banks. Ref. [28] gives a linear approximated relation between the nodal voltages and the complex injection powers (1), in which the first row of the $\underline{Y}$ matrix fixes the voltage at node 1 to the slack voltage magnitude. The complex power injections (4) account for the real/reactive load demand together with any contribution from PV inverters (5):

$$\underline{Y}\,\underline{V} = \underline{S}^* \tag{1}$$

$$\underline{Y} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \underline{Y}_{21} & \cdots & \underline{Y}_{2j} & \cdots & \underline{Y}_{2n} \\ \vdots & & \vdots & & \vdots \\ \underline{Y}_{i1} & \cdots & \underline{Y}_{ij} & \cdots & \underline{Y}_{in} \\ \vdots & & \vdots & & \vdots \\ \underline{Y}_{n1} & \cdots & \underline{Y}_{nj} & \cdots & \underline{Y}_{nn} \end{bmatrix} \tag{2}$$

$$\underline{V} = \begin{bmatrix} \underline{V}_1 \\ \vdots \\ \underline{V}_j \\ \vdots \\ \underline{V}_n \end{bmatrix}; \quad \underline{S} = \begin{bmatrix} V_{sl} \\ \underline{S}_2 \\ \vdots \\ \underline{S}_i \\ \vdots \\ \underline{S}_n \end{bmatrix} \tag{3}$$

$$\underline{S}_i = P_i + \mathrm{j}Q_i, \quad i = 2, \cdots, n \tag{4}$$

$$P_i = P_{Gi} - P_{Di}$$
$$Q_i = Q_{Gi} - Q_{Di}, \quad i = 2, \cdots, n \tag{5}$$

The nodal equations can be written in terms of the real and imaginary bus admittance values (7)–(8), the real and imaginary components of nodal voltages (9), and the real and reactive power injections (10):

$$\begin{bmatrix} Y^{re} & -Y^{im} \\ -Y^{im} & -Y^{re} \end{bmatrix} \begin{bmatrix} V^{re} \\ V^{im} \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix} \tag{6}$$

$$Y^{re} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ Y_{21}^{re} & \cdots & Y_{2j}^{re} & \cdots & Y_{2n}^{re} \\ \vdots & & \vdots & & \vdots \\ Y_{i1}^{re} & \cdots & Y_{ij}^{re} & \cdots & Y_{in}^{re} \\ \vdots & & \vdots & & \vdots \\ Y_{n1}^{re} & \cdots & Y_{nj}^{re} & \cdots & Y_{nn}^{re} \end{bmatrix} \tag{7}$$

$$Y^{im} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ Y_{21}^{im} & \cdots & Y_{2j}^{im} & \cdots & Y_{2n}^{im} \\ \vdots & & \vdots & & \vdots \\ Y_{i1}^{im} & \cdots & Y_{ij}^{im} & \cdots & Y_{in}^{im} \\ \vdots & & \vdots & & \vdots \\ Y_{n1}^{im} & \cdots & Y_{nj}^{im} & \cdots & Y_{nn}^{im} \end{bmatrix} \tag{8}$$

$$V^{re} = \begin{bmatrix} V_1^{re} \\ \vdots \\ V_j^{re} \\ \vdots \\ V_n^{re} \end{bmatrix}; \quad V^{im} = \begin{bmatrix} V_1^{im} \\ \vdots \\ V_j^{im} \\ \vdots \\ V_n^{im} \end{bmatrix} \tag{9}$$

$$P = \begin{bmatrix} V_{sl} \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_n \end{bmatrix}; \quad Q = \begin{bmatrix} 0 \\ Q_2 \\ \vdots \\ Q_i \\ \vdots \\ Q_n \end{bmatrix} \tag{10}$$

The objective of PV inverter reactive power control is to minimize both the network power loss and the deviations of the voltage magnitudes from the nominal value [25], [26]:

$$\min K P_{\text{Loss}} + (1 - K) \sum_{i=1}^{n} \Delta V_i \tag{11}$$

where K is a parameter that controls the tradeoff between the two objectives, i.e. $K = 1$ gives priority to loss minimization whereas $K = 0$ places emphasis on minimizing voltage deviations. The real and reactive power passing through the inverter are limited by its capacity:

$$\sqrt{P_{Gi}^2 + Q_{Gi}^2} \leq S_{Gi}^{\max} \tag{12}$$

where it is understood that $P_{Gi} = Q_{Gi} = 0$ at each node i that does not have PV installation. In an optimization setting, the load demands are practically obtained using distribution network load estimation and the PV real power levels are obtained from a forecasting tool.

The use of the weighted objective function for handling multiple objectives (11) is employed in practical distribution management system optimization functions [31], [32]. From an application perspective, [33] states that the weighted sum method is more practical as compared to other multi-objective programming methods thanks to its straightforwardness and performance relative to the software implementation complexity. The weighted sum method is also more suited to convex problems [34], which is the case of the formulation discussed herein. Alternative techniques that have been reported may prove to be useful in power distribution operation, such as the adaptive weighted-sum method [34] and the improved $\varepsilon-$ constraint method [35].

Sections II-A and II-B show that both objectives in (11) can be reformulated using conic constraints. The conic constraints are then approximated by a polyhedral formulation, which reduces the reactive power control problem to a quadratic program with linear inequality constraints; the polyhedral approximation technique is summarized in the appendix.

A. Minimum Loss

The real and imaginary components of the nodal voltages can be obtained from (6):

$$\begin{bmatrix} V^{re} \\ V^{im} \end{bmatrix} = Y_A^{-1} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} Z_A^{re} \\ Z_A^{im} \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} \tag{13}$$

where

$$Y_A = \begin{bmatrix} Y^{re} & -Y^{im} \\ -Y^{im} & -Y^{re} \end{bmatrix},\ Y_A^{-1} = \begin{bmatrix} Z_A^{re} \\ Z_A^{im} \end{bmatrix} \tag{14}$$

The inverse of Y_A in (14) is partitioned into two sub-matrices (Z_A^{re} and Z_A^{im}) that correspond to the real and imaginary components of $\underline{V}$ (V^{re} and V^{im}). Let M denote the node-branch incidence matrix, and D_g denote a diagonal matrix of branch conductances. The network real power loss is:

$$\begin{aligned} P_{\text{Loss}} &= (MV^{re})^T D_g (MV^{re}) + (MV^{im})^T D_g (MV^{im}) \\ &= (V^{re})^T L_A (V^{re}) + \left(V^{im}\right)^T L_A \left(V^{im}\right) \end{aligned} \tag{15}$$

where

$$\begin{aligned} L_A &= M^T D_g M \\ &= \left(D_g^{0.5} M\right)^T \left(D_g^{0.5} M\right) \\ &= L_{Ag}^T L_{Ag} \end{aligned} \tag{16}$$

By defining $L_{Ag} = D_g^{0.5} M$, the network real power loss can be equivalently written in terms of a vector P_A:

$$P_{\text{Loss}} = \begin{bmatrix} V^{re} \\ V^{im} \end{bmatrix}^T \begin{bmatrix} L_A & 0 \\ 0 & L_A \end{bmatrix} \begin{bmatrix} V^{re} \\ V^{im} \end{bmatrix} = P_A^T P_A \tag{17}$$

$$P_A = \begin{bmatrix} L_{Ag} & 0 \\ 0 & L_{Ag} \end{bmatrix} \begin{bmatrix} V^{re} \\ V^{im} \end{bmatrix} \tag{18}$$

Using (13), P_A can be related to P and Q :

$$P_A = \begin{bmatrix} L_{Ag} Z_A^{re} \\ L_{Ag} Z_A^{im} \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} D^{re} \\ D^{im} \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} \tag{19}$$

where

$$D^{re} = L_{Ag} Z_A^{re}, \quad D^{im} = L_{Ag} Z_A^{im} \tag{20}$$

Partition P_A into two vectors:

$$P_A = \begin{bmatrix} \mu^{re} \\ \mu^{im} \end{bmatrix} \tag{21}$$

$$\mu^{re} = D^{re} \begin{bmatrix} P \\ Q \end{bmatrix}, \quad \mu^{im} = D^{im} \begin{bmatrix} P \\ Q \end{bmatrix} \tag{22}$$

Then P_{Loss} can be written as a sum of squares:

$$\begin{aligned} P_{\text{Loss}} &= (\mu^{re})^T \mu^{re} + \left(\mu^{im}\right)^T \mu^{im} \\ &= \sum_{i=1}^{m} (\mu_i^{re})^2 + \left(\mu_i^{im}\right)^2 \end{aligned} \tag{23}$$

$$\mu_i^{re} = D^{re}(i,:) \begin{bmatrix} P \\ Q \end{bmatrix}, \quad \mu_i^{im} = D^{im}(i,:) \begin{bmatrix} P \\ Q \end{bmatrix} \tag{24}$$

The loss minimization objective can be therefore expressed as a quadratic objective function (25) subject to a set of conic constraints (26):

$$\min P_{\text{Loss}} = \sum_{i=1}^{m} t_i^2 \tag{25}$$

$$t_i \geq \sqrt{\left(\mu_i^{re}\right)^2 + \left(\mu_i^{im}\right)^2},\ i = 1, \cdots, m \tag{26}$$

B. Minimum Voltage Deviation

The minimum voltage deviation objective can be written as a linear objective function (27) subject to a set of conic constraints (28):

$$\min \sum_{i=1}^{n} \Delta V_i \tag{27}$$

$$\Delta V_i \geq \sqrt{\left(V_i^{re} - V_{sl}\right)^2 + \left(V_i^{im}\right)^2},\ i = 1, \cdots, n \tag{28}$$

where

$$V_i^{re} = Z_A^{re}(i,:) \begin{bmatrix} P \\ Q \end{bmatrix}, \quad V_i^{im} = Z_A^{im}(i,:) \begin{bmatrix} P \\ Q \end{bmatrix} \tag{29}$$

C. Polyhedral Formulation

The conic constraints (12), (26), and (28) can be interpreted as magnitudes of complex valued functions; for instance, (12) is equivalent to:

$$S_{Gi}^{\max} \geq |P_{Gi} + \mathtt{j} Q_{Gi}| \tag{30}$$

As shown in the appendix, (30) can be approximated by a number of linear inequalities on the real and imaginary parts of this argument by making use of the polyhedral norm:

$$\begin{aligned} -S_{Gi}^{\max} \leq\ & \cos(\ell\phi) P_{Gi} + \sin(\ell\phi) Q_{Gi} \leq S_{Gi}^{\max} \\ \phi = \frac{\pi}{k}, \quad & \ell = 1, \cdots, k \end{aligned} \tag{31}$$

Eq. (26) can be also approximated by k double-sided linear inequalities:

$$-t_i \le \cos(\ell\phi)\,\mu_i^{re} + \sin(\ell\phi)\,\mu_i^{im} \le t_i$$
$$\phi = \frac{\pi}{k}, \quad \ell = 1, \cdots, k \tag{32}$$

which in view of (24) could be directly expressed in terms of the real and reactive power injection vectors:

$$-t_i \le \left[\cos(\ell\phi)\,D^{re}(i,:) + \sin(\ell\phi)\,D^{im}(i,:)\right]\begin{bmatrix} P \\ Q \end{bmatrix} \le t_i$$
$$\phi = \frac{\pi}{k}, \quad \ell = 1, \cdots, k \tag{33}$$

Similarly, (28) can be approximated by:

$$\Delta V_i \ge \left[\cos(\ell\phi)\,Z_A^{re}(i,:) + \sin(\ell\phi)\,Z_A^{im}(i,:)\right]\begin{bmatrix} P \\ Q \end{bmatrix} - \cos(\ell\phi)\,V_{sl}$$
$$\Delta V_i \ge -\left[\cos(\ell\phi)\,Z_A^{re}(i,:) + \sin(\ell\phi)\,Z_A^{im}(i,:)\right]\begin{bmatrix} P \\ Q \end{bmatrix} + \cos(\ell\phi)\,V_{sl}$$
$$\phi = \frac{\pi}{k}, \quad \ell = 1, \cdots, k \tag{34}$$

Therefore, the inverter reactive power control problem can be written as a quadratic program (QP):

$$\min K \sum_{i=1}^{m} t_i^2 + (1-K) \sum_{i=1}^{n} \Delta V_i \tag{35}$$

subject to (31), (33), and (34); it is understood that the real and reactive power injections $[P, Q]^T$ in the constraints are substituted by (5).

III. Linear Decision Rules by Affinely Adjustable Robust Optimization

The reactive power control problem is a quadratic program, having the canonical from of a quadratic objective function (36) and linear inequality constraints (37):

$$\min_{q,u} \frac{1}{2} \sum_j a_j u_j^2 + \sum_j c_j u_j \tag{36}$$

subject to

$$\sum_j A_{ij} u_j + \sum_j B_{ij} q_j \le \sum_j C_{ij} p_j + b_i, \ \forall i \tag{37}$$

where the vector u (with elements u_j) includes the variables in the objective function (35), p denotes the vector of PV real power, and q the vector of reactive power generation from the PV inverters; the constant coefficients $(a_j, A_{ij}, b_j, B_{ij}, c_j, C_{ij})$ are obtained by identification with (31), (33), (34), (35) and (5). With a forecast of the PV real power generation p_j, the reactive power generation q_j could be computed by solving the QP (36)–(37). However, the PV real power generation is highly uncertain, and can be considered to belong to an interval uncertainty set:

$$p_j \in \left[0, p_j^{\max}\right] \tag{38}$$

The PV inverters respond to changes in the real power generation by adjusting the reactive power according to the following linear decision rule:

$$q_j = q_{0j} + \beta_j p_j \tag{39}$$

where q_{0j} and β_j are constants; their optimal values could be obtained using an affinely adjustable robust counterpart (AARC) of (36)–(37). The AARC formulation seeks computing q_{0j} and β_j to minimize (36) and such that for every value of p_j in the interval uncertainty set, there exists a value of q_j (calculated according to (39)) for which the constraints (37) remain satisfied; the feasibility of the solution under interval uncertainty can be ensured by requiring [4], [5]:

$$\sum_j A_{ij} u_j + \sum_j B_{ij} q_{0j} + \max_{p \in [0, p^{\max}]} \sum_j (B_{ij}\beta_j - C_{ij})\, p_j \le b_i, \ \forall i \tag{40}$$

The maximization term in (40) can be equivalently expressed using:

$$\max_{p \in [0, p^{\max}]} \sum_j (B_{ij}\beta_j - C_{ij})\, p_j = \sum_j \max_{p_j \in \left[0, p_j^{\max}\right]} (B_{ij}\beta_j - C_{ij})\, p_j = \sum_j \max\left(0, (B_{ij}\beta_j - C_{ij})\, p_j^{\max}\right) \tag{41}$$

Combining (36), (40) and (41) gives the following AARC formulation:

$$\min_{\beta,\theta,q_0,u} \frac{1}{2} \sum_j a_j u_j^2 + \sum_j c_j u_j \tag{42}$$

subject to

$$\sum_j A_{ij} u_j + \sum_j B_{ij} q_{0j} + \sum_j \theta_{ij} p_j^{\max} \le b_i, \ \forall i \tag{43}$$
$$\theta_{ij} \ge 0, \quad \theta_{ij} \ge B_{ij}\beta_j - C_{ij}, \quad \forall i, \forall j \tag{44}$$

The above AARC could have been alternatively derived by using the dual of the inner maximization problem in (40) [36], [37]. In the context of reactive power control under uncertain real power generation, the AARC formulation (42)–(44) computes the coefficients of the linear decision rule (39) for each PV inverter; the decision rule allows local control of reactive power so that for all realizations of real power (38) the inverter reactive power remains within its maximum capacity, and the power loss and voltage deviations do not exceed their minimum computed upper bounds.

A. Budget of Uncertainty

The uncertainty set (38) allows each of the uncertain PV power generation levels to vary between their minimum and maximum generation levels, leading to conservative solutions. To overcome the conservativeness issue, it is common to employ a budget of uncertainty Λ [38], [39] that limits the deviations of the uncertain parameters from their nominal values:

$$U = \{p : p_j = p_{0j} + \Delta p_j z_j,\ \forall j,\ z \in Z\} \tag{45}$$

$$Z = \left\{ z : |z_j| \le 1,\ \forall j,\ \sum_j |z_j| \le \Lambda \right\} \tag{46}$$

where p_{0j} is the forecasted PV real power generation and Δp_j is the maximum magnitude of the deviation from the forecast, for instance: $p_{0j} = \Delta p_j = \frac{p_j^{\max}}{2}$. With the budget of uncertainty, the AARC constraint (40) is expressed as follows:

$$\sum_j A_{ij} u_j + \sum_j B_{ij} q_{0j} + \max_{p \in U} \sum_j (B_{ij}\beta_j - C_{ij})\, p_j \le b_i,\ \forall i \tag{47}$$

The inner maximization problem in (47) can be equivalently written as:

$$\max \sum_j (B_{ij}\beta_j - C_{ij}) \left(p_{0j} + \Delta p_j \left(z_j^+ - z_j^-\right)\right) \tag{48}$$

subject to

$$0 \le z_j^+ \le 1,\ 0 \le z_j^- \le 1,\ z_j^+ + z_j^- \le 1,\ \forall j \tag{49}$$

$$\sum_j \left(z_j^+ + z_j^-\right) \le \Lambda \tag{50}$$

Taking the dual of (48)–(50) gives the AARC formulation with an uncertainty budget [36]–[38]:

$$\min_{\beta, \theta^+, \theta^-, \theta^1, \theta^\Lambda, q_0, u} \frac{1}{2} \sum_j a_j u_j^2 + \sum_j c_j u_j \tag{51}$$

subject to

$$\sum_j A_{ij} u_j + \sum_j B_{ij} q_{0j} + \sum_j (B_{ij}\beta_j - C_{ij})\, p_{0j} + \sum_j \left(\theta_{ij}^+ + \theta_{ij}^- + \theta_{ij}^1\right) + \Lambda \theta_i^\Lambda \le b_i, \quad \forall i \tag{52}$$

$$\theta_{ij}^+ + \theta_{ij}^1 + \theta_i^\Lambda \ge (B_{ij}\beta_j - C_{ij})\, \Delta p_j, \quad \forall i, \forall j \tag{53}$$

$$\theta_{ij}^- + \theta_{ij}^1 + \theta_i^\Lambda \ge (C_{ij} - B_{ij}\beta_j)\, \Delta p_j, \quad \forall i, \forall j \tag{54}$$

$$\theta_{ij}^+ \ge 0,\ \theta_{ij}^- \ge 0,\ \theta_{ij}^1 \ge 0,\ \forall i, \forall j, \quad \theta_i^\Lambda \ge 0,\ \forall i \tag{55}$$

IV. Other Rules for Local Control

In contrast to the local control rule (39) that is derived via robust optimization, other rules for minimizing power loss and voltage deviations by the local control of inverters have been proposed in [24]–[26]. These rules employ the measured reactive power consumption Q_{Di} at the bus where the inverter is connected and the PV real power generation P_{Gi}. The scheme requires introducing a helper function, Constr, to enforce the inverter reactive power limit (12):

$$\mathrm{Constr}(x, x^{\max}) = \begin{cases} x, & |x| \le x^{\max} \\ \left({}^{x}/_{|x|}\right) x^{\max}, & \text{otherwise} \end{cases} \tag{56}$$

For loss minimization, the reactive power produced by the inverter attempts to balance the reactive power consumption at this node; it is set to [24]:

$$Q_{Gi}^{\mathrm{Loss}} = \mathrm{Constr}\left(Q_{Di}, \sqrt{(S_{Gi}^{\max})^2 - P_{Gi}^2}\right) \tag{57}$$

For minimizing voltage deviations, [25], [26] suggest minimizing the absolute value of the combined power flow $rP_j + xQ_j$, where the ratio of line reactance to line resistance is assumed to be nearly constant ${}^{x}/_{r} = \alpha$. The real power demand P_{Di} at the node is also required to give the reactive power generation:

$$Q_{Gi}^{\Delta V} = \mathrm{Constr}\left(Q_{Dj} + \frac{P_{Dj} - P_{Gj}}{\alpha}, \sqrt{(S_{Gi}^{\max})^2 - P_{Gi}^2}\right) \tag{58}$$

A rule that balances between the objectives of loss minimization and voltage regulation could be obtained by combining (57) and (58) through the Constr function [25], [26]:

$$Q_{Gi} = \mathrm{Constr}\left(K Q_{Gi}^{\mathrm{Loss}} + (1-K)\, Q_{Gi}^{\Delta V}, \sqrt{(S_{Gi}^{\max})^2 - P_{Gi}^2}\right) \tag{59}$$

where K is a parameter that needs to be communicated to the controllers for adjusting the tradeoff between the two objectives.

V. Numerical Results

The AARC formulation for computing the local decision rules was programmed in MATLAB, and the QPs were solved using CPLEX [40] with default termination criteria. The simulations were carried out on a MacBook Pro having 2.9 GHz dual-core Intel Core i5 processor with a memory of 8 GB 2133 MHz. The method was tested on two weakly meshed networks, a 33-node network [12] and an 830-node network [41]. Each network was investigated under three levels of installed photovoltaic generation along the lines described in [25]: low (L) at around 20% of the real load, medium (M) at around 50%, and high (H) where there is over-generation of real power leading to reversal of power flow into the transmission grid; this gives a total of six network instances: 33_L, 33_M, 33_H, 830_L, 830_M, and 830_H. Additionally, two levels of inverter capacity were considered as in [25]: the 33-node network had an inverter capacity to installed photovoltaic generation ratio of 1.1, and the 830-node network had high capacity inverters with a ratio of 2.2; [27], [42] discuss the importance of oversizing inverters to increase the controller bandwidth with larger percentages of renewable power supplied to the grid. Each of these test instances has an optimized set of linear decision rules for reactive power control (39), computed from the AARC formulation; the decision rules can be computed for different values of the tradeoff

TABLE I
LOCAL CONTROL & CS FOR LOSS MINIMIZATION ($K = 1$) – PERCENTAGE IMPROVEMENT RELATIVE TO ZERO REACTIVE POWER GENERATION

Net.	max $\lvert\Delta V\rvert_\infty$			max P_{Loss}			average P_{Loss}		
	AARC	(59)	CS	AARC	(59)	CS	AARC	(59)	CS
33_L	9	3	9	18	7	18	11	8	18
33_M	15	3	15	26	7	26	22	10	33
33_H	18	3	18	29	7	29	40	12	49
830_L	8	3	8	27	19	27	29	22	31
830_M	15	3	15	31	19	32	38	25	41
830_H	19	3	19	32	19	34	48	30	53

TABLE II
LOCAL CONTROL & CS FOR VOLTAGE REGULATION ($K = 0$) – PERCENTAGE IMPROVEMENT RELATIVE TO ZERO REACTIVE POWER GENERATION

Net.	max $\lvert\Delta V\rvert_\infty$			max P_{Loss}			average P_{Loss}		
	AARC	(59)	CS	AARC	(59)	CS	AARC	(59)	CS
33_L	9	8	9	18	16	18	16	11	18
33_M	15	9	15	26	18	26	30	–1	33
33_H	19	–6	19	27	–44	27	44	–63	47
830_L	8	4	8	27	23	27	31	25	31
830_M	15	4	15	31	24	31	39	27	40
830_H	19	4	19	32	24	32	51	22	52

TABLE III
LOCAL CONTROL & CS FOR THE COMPROMISE OBJECTIVE ($K = 0.5$) – PERCENTAGE IMPROVEMENT RELATIVE TO ZERO REACTIVE POWER GENERATION

Net.	max $\lvert\Delta V\rvert_\infty$			max P_{Loss}			average P_{Loss}		
	AARC	(59)	CS	AARC	(59)	CS	AARC	(59)	CS
33_L	9	5	9	18	12	18	11	10	18
33_M	15	6	15	26	13	26	22	5	33
33_H	19	6	19	28	13	29	41	–19	49
830_L	8	3	8	27	22	27	29	24	31
830_M	15	3	15	31	22	32	38	26	41
830_H	19	3	19	32	22	33	49	27	52

parameter K in (35). A 32-vertex polygon ($k = 16$) is employed in the polyhedral approximation.

The linear decision rules from AARC are compared against:

1) The local formulae reported in [24]–[26] and summarized in Section IV.
2) A fully centralized optimization solution that solves the quadratic program (36)–(37) for each instance of renewable power production levels; the centralized solution (CS) assumes that any change in PV real power generation is telemetered to the distribution control center, and the optimization solution is sent back for implementation at the inverter. The centralized optimization solution is equivalent to the wait-and-see decisions in robust optimization with full-recourse [5], and in this context Ref. [36] refers to the use of the fully centralized solution as the folding horizon approach.

For each network instance, the local control is reported with emphasis on loss minimization ($K = 1$ in (35) and (59)), voltage regulation ($K = 0$), and a compromise of objectives ($K = 0.5$). The comparison between the decision rules ((39) versus (59)) and the centralized solution (CS) is carried out using a Monte-Carlo simulation with 10,000 trials; each trial involves firstly choosing values of PV real power generation uniformly drawn from the interval uncertainty set (38), secondly computing the reactive power using the decision rules, and thirdly solving for the voltage profile and power loss using the current injection power flow method [43]. In each instance, the improvement is measured as a percentage relative to the case of zero reactive power generation ($Q_{Gj} = 0$):

$$\%\text{ improvement in } x = \frac{x|_{QG_j=0} - x|_{\text{Local Control}}}{x|_{QG_j=0}} \times 100 \quad (60)$$

The percentage improvement is measured over 10,000 trials and reported for the maximum voltage magnitude deviation, the maximum power loss, and the average power loss. Table I shows the results for $K = 1$, Table II for $K = 0$, and Table III for $K = 0.5$. It is evident that the AARC gives better improvement in all cases, particularly those with high photovoltaic integration leading to reverse power flows. In fact, the local control formula (59) can even lead to reduced performance as compared to zero reactive power control when the photovoltaic generation and load are in relatively close balance; this behavior is noted in [26], and reflected in the simulations through the negative values of percentage improvement. The level of the improvement of the local control method (59) is highly dependent on the choice of K that is transmitted to the inverter; for instance, the improvement always turned out to be positive for $K = 1$ (Table I). For the AARC results, the reported improvement in the maximum voltage deviation and power loss is relatively insensitive to the choice of K. The AARC formulation does not explicitly minimize the average real power loss, instead it minimizes an upper bound on power loss and voltage deviations; the average realized real power loss over the 10,000 Monte-Carlo trials turned out to be the least for the results in Table II (for $K = 0$). The AARC results are also consistent with the CS, whose expected value of improvement serves as an upper bound on AARC; in fact, most instances do not show much difference between the improvement levels of AARC and CS. Very similar performance was also observed when the 33- and 830-node networks were simulated using their radial configurations [12], [41].

A. Results With a Budget of Uncertainty

Tables IV and V report results with a budget of uncertainty for the 33_H and 830_H networks; four values for the budget of uncertainty Λ are considered for each simulation instance; they correspond to 100%, 75%, 50%, and 25% of the number of PV generators (the 100% is equivalent to the case without an uncertainty budget). While it is observed that the robust solutions are less conservative (less voltage deviation and max/average power loss) with decreasing uncertainty budget, it is interesting to note that the percentage improvement levels (60) for the three approaches (AARC, (59), CS) remain largely unchanged for the different simulated budgets of uncertainty.

TABLE IV
LOCAL CONTROL & CS WITH A BUDGET OF UNCERTAINTY – PERCENTAGE IMPROVEMENT RELATIVE TO ZERO REACTIVE POWER GENERATION (33_H NETWORK)

K	Λ	max $\lvert\Delta V\rvert_\infty$			max P_{Loss}			average P_{Loss}		
		AARC	(59)	CS	AARC	(59)	CS	AARC	(59)	CS
1	4	18	3	18	29	7	29	40	12	49
1	3	19	4	19	32	8	32	41	13	49
1	2	21	4	21	34	9	35	45	13	50
1	1	20	3	20	34	9	35	45	13	51
0	4	19	–6	19	27	–44	27	44	–63	47
0	3	21	–10	21	31	–57	32	44	–62	47
0	2	22	–15	22	31	–71	31	44	–66	48
0	1	26	–14	26	33	–69	34	48	–67	49
0.5	4	19	6	19	28	13	29	41	–19	49
0.5	3	19	5	19	31	12	32	41	–19	49
0.5	2	22	4	22	34	4	35	45	–20	50
0.5	1	21	5	21	34	9	35	48	–21	51

TABLE V
LOCAL CONTROL & CS WITH A BUDGET OF UNCERTAINTY – PERCENTAGE IMPROVEMENT RELATIVE TO ZERO REACTIVE POWER GENERATION (830_H NETWORK)

K	Λ	max $\lvert\Delta V\rvert_\infty$			max P_{Loss}			average P_{Loss}		
		AARC	(59)	CS	AARC	(59)	CS	AARC	(59)	CS
1	80	19	3	19	32	19	34	48	30	53
1	60	19	3	19	38	22	39	48	30	53
1	40	19	3	19	44	26	45	49	30	53
1	20	20	3	20	48	29	50	51	31	54
0	80	19	4	19	32	24	32	51	22	52
0	60	19	4	19	38	26	38	51	22	52
0	40	19	4	19	44	27	44	51	22	52
0	20	20	4	20	39	25	50	41	23	53
0.5	80	19	3	19	32	22	33	49	27	52
0.5	60	19	3	19	38	24	39	49	27	52
0.5	40	19	3	19	44	27	44	49	27	52
0.5	20	20	4	20	48	27	50	51	28	54

B. Approximation Accuracy and Computational Performance

The percentage improvement values quoted in the above tables were computed using an exact power flow method [43]; the decision variables in both AARC and CS are however calculated starting from a convex quadratic program (36)–(37) that approximates the optimal power flow formulation. The polyhedral formulation that forms the basis of the quadratic program can be used to compute the voltage deviation from the nominal value at each node, which is given by the maximum of the inequality constraint right-hand sides in (34); to quantify the accuracy, this deviation is gauged against the deviation value computed from the exact power flow solution within the Monte-Carlo simulation framework. For each Monte-Carlo trial, two indices are computed: the ℓ_2- norm of the difference in voltage deviations (polyhedral versus exact) normalized by the number of nodes, and the $\ell_\infty-$ norm of the difference in voltage deviations. Tables VI and VII show the maximum and average values of these two indices, as computed from 10,000 Monte-Carlo trials of the 33- and 830-node networks (corresponding to the results in Tables I–III); these results suggest that the polyhedral approximation with a 32-vertex polygon gives an approximation with good accuracy.

The simulation results in Tables I–III required solving 18 optimization problems to obtain 18 sets of linear decision rules (three problems corresponding to $K = 1, 0, 0.5$ for each of the six network instances); in particular, 12 of these problems are quadratic programs and 6 are linear programs (corresponding to $K = 0$). Table VIII shows the CPLEX computational time, and reveals that the worst-case time is less than one minute for the largest network. With the uncertainty budget accounted for, the maximum time is around two minutes. This is acceptable given the local nature of the decision rules that are expected to operate over an uncertainty range of photovoltaic power generation.

TABLE VI
VOLTAGE DEVIATION ERROR VALUES (PU) FOR THE 33-NODE NETWORK

Net.	K	ℓ_2- norm		$\ell_\infty-$ norm	
		max	average	max	average
33_L	1	2.0E-04	1.9E-04	1.8E-03	1.7E-03
33_M	1	1.7E-04	1.5E-04	1.5E-03	1.3E-03
33_H	1	1.5E-04	9.0E-05	1.3E-03	8.1E-04
33_L	0	1.9E-04	1.8E-04	1.7E-03	1.7E-03
33_M	0	1.6E-04	1.3E-04	1.4E-03	1.2E-03
33_H	0	1.5E-04	8.2E-05	1.3E-03	7.4E-04
33_L	0.5	2.0E-04	1.9E-04	1.8E-03	1.7E-03
33_M	0.5	1.7E-04	1.5E-04	1.5E-03	1.3E-03
33_H	0.5	1.5E-04	8.8E-05	1.3E-03	8.0E-04

TABLE VII
VOLTAGE DEVIATION ERROR VALUES (PU) FOR THE 830-NODE NETWORK

Net.	K	ℓ_2- norm		$\ell_\infty-$ norm	
		max	average	max	average
830_L	1	3.2E-05	3.1E-05	1.9E-03	1.8E-03
830_M	1	2.5E-05	2.2E-05	1.5E-03	1.4E-03
830_H	1	1.8E-05	1.4E-05	1.0E-03	9.4E-04
830_L	0	3.1E-05	3.0E-05	1.8E-03	1.8E-03
830_M	0	2.5E-05	2.1E-05	1.5E-03	1.3E-03
830_H	0	1.7E-05	1.1E-05	9.4E-04	8.1E-04
830_L	0.5	3.2E-05	3.1E-05	1.9E-03	1.8E-03
830_M	0.5	2.4E-05	2.1E-05	1.5E-03	1.4E-03
830_H	0.5	1.7E-05	1.2E-05	9.5E-04	8.5E-04

TABLE VIII
COMPUTATIONAL TIME (S) FOR THE AARC FORMULATION (THE ASTERISK SIGN DENOTES SIMULATION WITH AN UNCERTAINTY BUDGET)

Net.	$K = 1$	$K = 0$	$K = 0.5$
33_L	0.1	0.2	0.2
33_M	0.2	0.2	0.2
33_H	0.2	0.2	0.2
33_H*	0.2	0.3	0.5
830_L	26.8	27.0	46.9
830_M	26.2	25.7	48.8
830_H	23.1	25.7	42.2
830_H*	81.6	84.6	133.0

VI. Conclusion

This paper presents an affinely adjustable robust optimization approach for computing linear decision rules that dictate the amount of reactive power supplied or absorbed by photovoltaic inverters. The rules are computed via quadratic programming such that for each value of generated real power from an interval uncertainty set, the reactive power is automatically adjusted to maintain the power loss and voltage deviation within optimized upper bounds. The AARC formulation is based on a new polyhedral formulation of the reactive power control problem. The numerical results reveal that the AARC formulation gives better performance as compared to the case of zero reactive power generation, and that the level of improvement of the worst-case performance values is relatively insensitive to the tradeoff parameter. The results also suggest that the performance of the proposed linear decision rules is superior to that of a competing multi-objective local control method, particularly at high levels of photovoltaic integration.

Appendix

Let $\underline{\vartheta} = \vartheta^{re} + \mathrm{j}\vartheta^{im}$ denote a complex number in the complex plane. Ref. [29] shows that the magnitude of $\underline{\vartheta}$ satisfies:

$$|\underline{\vartheta}| \geq \wp_k\left(\underline{\vartheta}\right) \geq \cos\left(\frac{\phi}{2}\right)|\underline{\vartheta}| \tag{61}$$

where

$$\wp_k\left(\underline{\vartheta}\right) = \max_{\ell=1,2,\cdots,k}\left|\vartheta^{re}\cos\left(\ell\phi\right) + \vartheta^{im}\sin\left(\ell\phi\right)\right|, \quad \phi = \frac{\pi}{k} \tag{62}$$

Therefore, the magnitude $|\underline{\vartheta}|$ can be approximated by the polyhedral norm $\wp_k\left(\underline{\vartheta}\right)$ with a relative accuracy of $1 - \cos\left(^{\pi}/_{2k}\right)$. For example, the 32-vertex polygon ($k = 16$) approximates $|\underline{\vartheta}|$ with a maximum error of 0.48%. Let $\underline{F}\left(w\right) = F^{re}\left(w\right) + \mathrm{j}F^{im}\left(w\right)$ denote a complex-valued function whose variables are in the vector w, and consider the optimization problem:

$$\min_{t,w}\left\{t \middle| t \geq |\underline{F}\left(w\right)|\right\} \tag{63}$$

Given (61), the optimization problem (63) can be approximated using:

$$\min_{t,w}\left\{t \middle| t \geq \wp_k\left(\underline{F}\left(w\right)\right)\right\} \tag{64}$$

By using the definition of the polyhedral norm (62), problem (64) can be written as follows:

$$\min_{t,w}\left\{\begin{array}{c} t\middle| -t \leq \cos\left(\ell\phi\right)F^{re}\left(w\right) + \sin\left(\ell\phi\right)F^{im}\left(w\right) \leq t \\ \ell = 1, 2, \cdots, k, \quad \phi = \dfrac{\pi}{k}\end{array}\right\} \tag{65}$$

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