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Percentage rent contracts between co-stores

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ABSTRACT

High rental rates and space scarcity in popular retail areas are encouraging retailers to share available space, creating co-stores. Co-stores with complementary products enjoy a boost in foot traffic, thus higher sales. When the space is owned by one, a common practice for retailers is to engage in a revenue sharing agreement. We study the optimal design of a percentage rent contract between two co-stores, each having his/her own random sales, dependent on effort levels of both retailers. Since the effort level exerted by a retailer is often unverifiable, it results in a double moral hazard problem. We show that a linear percentage rent, which is composed of a sales share and a based rent, achieves the second-best effort levels. The landlord may also find it optimal to set a fixed rent by setting the sales share to zero. With Cobb-Douglas sales and power disutility of effort functions, we obtain closed-form expressions for the optimal contract. We show that the landlord charges a high sales share for a tenant with low demand externality and a high sales scale. The landlord is better off renting out space to a small but high revenue generating retailer, rather than one generating larger traffic, but with lower sales scale. When the landlord's externality potential is low, but he is successful in generating sales, optimal sales share is low, but the base rent is high. The percentage rent can increase the landlord' profit by up to 13% compared to the conventional fixed rent.

1. Introduction

Relative to upstream supply chain, retailing is more sensitive to changes in economic cycles as the consumer spending generally constitutes the largest part of an economy. Moreover, retailers nowadays are looking for ways to cope with the diminishing interest of consumers in brick-and-mortar shopping, as opposed to online shopping. Telegraph.co.uk. (2015) reports that high street sales realized a four percent fall in their March 2015 sales compared to a year ago, which is the largest monthly fall since 2011. The article also notes that although consumers are spending more due to having more disposable income with falling fuel and food prices, it was not materialized amongst high street retailers. The retail industry is continuously searching for best practices to balance its incomes and expenses. Rent, one of the largest operational expenses for retailers, is closely related to the changing demand pattern (D'Arcy, Tsolacos, & McGough, 1997). Ibanez and Pennington-Cross. (2013) argue that rent in metropolitan areas with less available space for construction is both high and very sensitive to demand shifts. High traffic on-street retail locations, where people work, live, and shop, are limited in supply. Thus, their rent is more prone to economic shocks. As a result, retailers in those locations try their best to squeeze the most out of each square foot they are utilizing. According to Useem (2007), Apple aims to locate in close proximity to its customers; Thus, it invests more than its competitors in real estate, enjoying, on the other hand, higher sales per square foot.

To counteract the lingering economic conditions, which are taking their toll on investments in real estate, a larger number of struggling retailers "are teaming up" to keep in close touch with their customers (Porter & Helm, 2008). One way is to co-locate (Sen, Shin, & Sudhir, 2011) or open a store-within-a-store (Jerath & Zhang., 2010), whereby a retailer rents out some of the available floor space to another retailer. The partnering retailers may be either totally independent, or complementary in nature, which boosts the foot traffic and sales for both parties. Co-stores or stores within stores can help save significant construction costs while providing some degree of flexibility (Misonzhnik, 2012b). Edward M. Schmults, the CEO of FAO Schwarz, the oldest toy store in the U.S., explains that "customers aren't going to want to drive to five different places looking for products", while announcing the company's decision to locate their products into 685 Macy's stores. Sephora, a French chain of cosmetic stores, started opening stores inside JCPenney department stores in 2006, and currently

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has branches in more than 375 JCPenney stores across the U.S. (Sephora.com., 2013). According to Sorescu, Frambach, Singh, Rangaswamy, and Bridges (2011), the store-within-a-store concept creates value for the retailers by increasing customer efficiency as customers have access to products easier. In the Sephora-JCPenney partnership, while Sephora gets access to JCPenney's customers through a smaller than its regular branch space, JCPenney attracts beauty product customers without offering its own, thus providing a larger product assortment.

Recently, more partnerships among retailers to co-locate have been emerging. For example, 350 Sony shops were to open inside Best-Buy, the giant U.S. retailer, by the end of 2014 (Time.com. & McCracken, 2014). The concept was introduced at Best Buy a few years ago with both Samsung and Microsoft Windows. Currently, the giant retailer has opened 1400 Samsung and 600 Windows store-within-a-store in the U.S., offering the tech brands to buy spaces in its stores and make themselves visible at a popular destination for U.S. shoppers (NYTimes & Chen, 2013). Similarly, Starbucks has entered a number of licensing agreements to locate in areas where it did not have the ability to locate its outlets. For example, Starbucks has made arrangements to sell an exclusive coffee blend only in Nordstrom stores, operate coffee bars in Barnes and Noble bookstores, and offer coffee service at some Wells Fargo Bank locations in California (Thompson & Strickland, 2003).

Our study is motivated by a real example of a specific branch of a well-known restaurant chain, renting out some of its space to a well-known coffee house chain, at a popular street location. In this context, the restaurant serves as the landlord as he (landlord) has initially reserved the location for his own use. The coffee house needs a relatively smaller floor space as she (tenant) mainly serves her customers on a to-go basis. Due to lack of space availability for the coffee house, aggravated by prohibitively high rent within the target region, the restaurant rents out part of the available space to the coffee house, resulting in a co-store arrangement. In such an arrangement, both retailers agree on a revenue-sharing contract, particularly a percentage rent contract, which entails a rent payment by the tenant to the landlord which is a function of the tenant's revenue. Thus, while the coffee house reduces its high rent to a sales ratio dependent risk, the landlord benefits from the potential high revenue gains of the coffee house. In our example, the retailers' offerings are complementary in nature. Thus, the sales efforts exerted by either party will boost the sales for both parties.

In this paper, we introduce a linear percentage rent contract, whereby the tenant pays the landlord a rent comprised of a fixed rent portion plus a linear percentage of the tenant's revenue. Our investigation of a linear contractual agreement is motivated by the fact that revenue sharing agreements are often based on simple linear rules (Bhattacharya & Lafontaine, 1995). We study the problem as a single season model, with each retailer facing a random demand, function of both the tenant's and the landlord's efforts. The demand functions of the tenant and landlord have different sales scales, which are defined as the average sales per square foot, and are indicators for the sales potential or success of each retailer. We also incorporate into our model demand externalities between retailers (Sen et al., 2011). Demand externalities are defined in Corbett, DeCroix, and Ha (2005) as the benefit that affects a party who did not choose to incur that benefit. For example, higher foot traffic generated by one of the retailers creates higher sales for the other. In addition, each retailer decides on his/her sales effort level individually, thus incurring some sales effort cost.

The resulting model is a double moral hazard problem. Moral hazard is a special case of information asymmetry, in which one party has private information about his/her actions and intentions, thus having an incentive to act inappropriately from the perspective of the party with less information (Corbett et al., 2005; Holmstrom, 1979). The double moral hazard arises when the decisions of two parties in a system are unverifiable, thus not contractible directly. Here, we regard the sales efforts of the retailers as unobservable and unverifiable. As a result, the rent contract is based on the observable output (sales) of the tenant, which is a function of the sales efforts of both parties. When the parties are risk neutral, the first-best solution is unattainable, as shown by several earlier studies on double moral hazard problems (Bhattacharya, Gupta, & Hasija, 2014; Romano, 1994). We show that under general sales functions, a linear percentage rent contract, consisting of a sales share (a percentage of the tenant's sales) and a fixed base rent, results in second-best effort levels.

To the best of our knowledge, this paper is the first study on co-stores investigating the rent contract design problem between these two stores. Moreover, we contribute to the double moral hazard literature by considering two output functions, one for each retailer. Contrary to the previous research studies which have considered only one output function, (the tenant's), this paper considers a situation in double moral hazard in which the output (sales) function of the landlord is of direct relevance, and thus needs to be explicitly accounted for. While our result on the existence of a linear percentage rent contract with second-best effort levels is similar to that in the double moral hazard literature with one output function, the other results that we obtain differentiate our paper from the literature, resulting in completely different practical implications. Specifically, we show that the second-best sales share parameter changes with each party's sales scale, contrary to (Bhattacharya & Lafontaine, 1995) who show that the share parameter is independent of the sales scale of the single output function. This substantial result implies that designing a contract depends on the size and power of the contracting parties. From the perspective of the literature on co-stores, none of the related studies considers rent contract design. Thus, our paper is the first one in the co-storing literature to offer an extensive analysis of the contract design problem between a landlord and a tenant, offering a new research perspective and methodology to analyze co-storing arrangements.

The paper is organized as follows. The related literature is reviewed in Section 2. Section 3 introduces the landlord's rent contract design problem with general sales and cost functions. In Section 4, the sales functions of the tenant and landlord are defined to be of the Cobb-Douglas form. For the particular sales and cost of effort functions, closed-form expressions for the second-best effort levels and contract terms are obtained. In Section 5, we investigate the sensitivity of the second-best solution with respect to the sales scale parameters, externality effects, and scale and elasticity of the cost of effort functions. The performance of the linear percentage rent contract is compared to the first-best solution and the fixed-rent contract in Section 6. We present our conclusions and future research directions in Section 7. All proofs are in the supporting appendix intended for e-publication.

2. Literature review

Our research is mainly based on contract theory with information asymmetry. For a recent survey on contract theory and its applications, we refer the reader to Gibbons (2005) and van Ackere (1993). In our problem, information asymmetry is in the form of moral hazard, where the efforts of the contract parties are not verifiable after the contract is made and thus not contractible. As a result, the contract is based on some other observable/verifiable output of the system. We develop our model using the principal-agent theory subject to double moral hazard, where decisions of both parties in the system are unverifiable.

The moral hazard framework has been applied under different contexts in various disciplines. Among the earliest applications are sharecropping (Stiglitz, 1974) and franchise contracts

(Lal, 1990; Mathewson & Winter, 1985). Crama, De Reyck, and Degraeve (2008); Crama, Reyck, and Degraeve (2013) analyze licensing contracts under incomplete information. The agency theory is used to investigate the sales force diversification and the incentive payment strategies of firms to encourage sales force effort (Caldieraro & Coughlan, 2009; Dai & Jerath, 2016; Rao, 1990). In the finance literature, the principal-agent theory is used to design the compensation contracts for the board of directors (Armstrong, Larcker, & Su, 2010).

The use of the moral hazard framework in supply chain management is more recent. Porteus and Whang (1991) analyze incentive plans that can induce marketing and manufacturing managers to act in a way to maximize the profit of the firm owner. Romano (1994) considers incentives for resale price maintenance between a wholesaler and a retailer. Saouma (2008) analyzes a manufacturer's decision to outsource an assembly task to his supplier that produces the component, where the supplier's effort is unobservable. Krishnan, Kapuscinski, and Butz (2004) consider risk-neutral manufacturer and retailer, and analyze the interaction of incentive conflicts and contractual remedies when the manufacturer uses a return policy or markdown allowances. Iyer, Schwarz, and Zenios (2005) consider the contract problem of a buyer who delegates the product specification and production to the supplier, but retains the flexibility to commit the amount of limited resource to help the supplier, whose resource allocation and capability are unobservable to the buyer.

Collaborative service/production design problems include double moral hazard very often. Corbett and DeCroix (2001) and Corbett et al. (2005) study a shared-savings contract between a supplier and a customer to decrease the consumption of material by the customer with the joint efforts of the two parties. Jain, Hasija, and Popescu (2013) study efficient contracts to align the incentives of the vendor and client in the context of outsourcing equipment repair and restoration. Xue and Field (2008) study the efficiency of the service co-production process in a consulting environment, and determine how work should be allocated between a consultant and client, under various contractual relationships. Kim and Netessine (2013) consider a supplier and a manufacturer engaging in collaborative effort to reduce the uncertainty about the component production cost. Plambeck and Taylor (2006) study a joint production plan of a buyer and a supplier in a dynamic time setting. Balachandran and Radhakrishnan (2005) consider the quality investments of two parties for the incoming material inspection. Rui and Lai (2015) study the inspection mechanisms and deferred payment for reducing the supplier's product adulteration, when the buyer uses deferred payment to manage the supplier's quality. The moral hazard problem is studied in continuous time in Cvitanic, Wan, and Yang (2013).

A common assumption of the double moral hazard literature is that the principal is passive as far as the output is concerned. In other words, both players put effort to form a single output, either for the landlord or the tenant. For instance, in collaborative production/service agreement studies, there is a single common output of the system. Contrary to the literature, each of the principal and tenant in our model provide his/her own output function, which depends on the effort levels of both parties. Thus, our study aims to serve the retailing literature in terms of analyzing an emerging and practical topic of percentage rent contracts, as well as the double moral hazard literature by modeling the output functions of both players. The results we obtain are substantially different from those in the double moral hazard literature with a single output function. For example, while (Bhattacharya & Lafontaine, 1995) show that the optimal share parameter is independent of the sales scale of the single output function, we show that the optimal sales share depends on each parties' sales scale, a result that has significant implications on the practices of contract design between two co-locating retailers.

Although our model focuses on the moral hazard problem, there is also a possible adverse-selection problem in the contract design. While the moral hazard occurs when a party has unobservable behavior and can change the behavior without incurring any costs after a deal is made, adverse selection is the information asymmetry prior to a deal, so that one party has more accurate and different information from the other party. In supply chain management literature, adverse selection often arises between a buyer and supplier, when the quality and/or cost is a private information for the other. Thus, the buyer needs to make a payment plan to the supplier that can reveal the true value of the suppliers quality. Chen and Hu. (2015) investigate the optimal payment plan when both the quality and cost of the supplier are unknown. Lim (2000) studies the payment scheme to a third party logistic company, whose reliability and cost are not known to the buyer. Baiman, Fischer, and Rajan. (2001) investigates the contract between a buyer and supplier where there is an adverse selection problem on the supplier side and a moral hazard problem on the buyer side.

The rent contract design problem is particularly addressed by researchers in several different streams of the literature. Sharecropping, which is a system of agriculture in which a landowner allows a tenant to use the land in return for either cash or a share of the crops, is one of the earliest rent contract problems studied in agricultural economics (Allen & Lueck, 1992; Stiglitz, 1974). In the real estate and urban economics research, commercial real estate rental leasing agreements, arising in shopping malls, are studied. In those studies, the efforts of the tenant stores and the mall owners determine the store sales, but are mostly unverifiable (Benjamin, Boyle, & Sirmans, 1992; Brueckner, 1993; Gould, Pashigian, & Prendergast, 2005; Lee, 1995; Miceli & Sirmans., 1995; Wheaton, 2000). Thus, in all these studies moral hazard problem arises either single sided, from the agent or double, from both the agent and principal's inputs. According to Misonzhnik (2012b), the rent agreements between a landlord and a tenant in the retail industry involve some form of revenue sharing. The origin of percentage rent agreements dates back to the Depression Era, during which many retailers went bankrupt due to high fixed rents. Snively (2000) highlights that percentage rent mechanisms are generally used in prime downtown locations instead of less desirable locations, as location is an important determinant of sales in prime locations. Our purpose in this study is to investigate how the landlord designs a rent contract that includes percentage revenue sharing. Compared to previous work on rent contracts, our study is the first to consider both the tenant and the landlord's efforts and sales separately. As a result of this model, we show that the percentage sales share as a part of the rent changes with each party's sales scale, not only the tenant, which implies that designing a rent contract depends on the size and power of both the landlord and the tenant.

The earlier studies on co-stores do not directly focus on either incentive problems or rent contract design between co-stores. Thus, they are related to our study in terms of the subject, but not the research question or methodology. Using a deterministic and full information model, (Jerath & Zhang., 2010) model a manufacturer's store within a large retailer. They investigate whether the manufacturer should be allowed to have complete autonomy over his retail decisions such as pricing and in-store service, and whether the retailer should manage her own operations after getting the products from the manufacturer. Wang, Bell, and Padmanabhan (2009) analyze the price and service competition between the independent store of a manufacturer and the manufacturer's store within a department store that are closely located. By

using data from a specialty apparel retailer, (Netemeyer, Heilman, & Maxham, 2012) investigate the impact of an in-store boutique on overall performance of the parent stores in which the boutiques are placed. None of these earlier work on co-stores investigate the rent contract design problem between a landlord and a tenant.

3. The general double moral hazard model

We study the rent contract design problem between a landlord and a tenant, each being a retailer having his/her own customer demand. The landlord rents part of his available space to the tenant, so that they become co-stores selling non-substitutable products. The sales effort levels in our model are taken to broadly represent all forms of demand-enhancing and service-providing efforts, including managing the store on a day to day basis, providing service to the customers from the moment a customer walks into the store until the customer leaves, in-store promotions and product placement, hiring, training and supervising employees, staff meetings, developing and maintaining the brand value, promoting and advertising the brand, keeping track of local needs, as well as the overall quality of the shopping experience (Heese & Swaminathan, 2010; Taylor, 2002; Xing & Liu, 2012). Both retailers' effort levels affect their performance. However, the intensity of efforts devoted to such activities is not easily monitored by parties other than the individual providers of the effort (Heese & Kemahlioglu-Ziya, 2016). While some activities such as advertising can be verifiable (He, Prasad, & Sethi, 2009), as the large part of these efforts is unverifiable, each retailer has an incentive to deviate from exerting his/her promised effort level. Given that unverifiable efforts cannot be contractible, we characterize the rent contract design between the two co-locating retailers using the double moral hazard framework.

We model the decision process of the two retailers as a principal-agent model, in which the landlord (principal) first determines the conditions of the percentage revenue sharing contract; then, both retailers simultaneously determine their sales efforts, provided that the tenant receives a fixed reservation profit. Both retailers are assumed to be risk neutral. In our motivational example, we define the landlord as the principal who has initially reserved the location for his own use and the tenant as the agent. In fact, in other situations, the two retailers may have different negotiation power, so they may undergo a rent bargaining game to decide on the rent structure. Given the space scarcity in many retail locations and the significantly higher power of a first-mover (landlord), we model a principal-agent setting in this study, leaving the bargaining model to an extension of the current work.

The landlord's and tenant's effort levels are denoted by l and t, respectively. To capture the aggregate effect of both the landlord's and tenant's effort on sales, we use a composite effort function for the landlord and the tenant, denoted by $W(l, t)$ and $V(l, t)$, respectively. Also, let $X(W(l, t), \epsilon_l)$ and $Y(V(l, t), \epsilon_t)$ be general forms of the landlord's and tenant's random sales revenue, where ϵ_l and ϵ_t are the random terms that capture the uncertainty in the inputs such as demand, with respective density functions $f(x, W(l, t))$ and $g(y, V(l, t))$ and cumulative probabilities $F(., W)$ and $G(., V)$. We assume that the realizations of random terms are unobservable to each other and that effort levels are unverifiable. As a result, contracts based on l and t are not feasible. Any contract has to be based on the observable output level, or sales.

While the retailers' efforts affect each others' sales revenues, each of the retailers bares the cost of his/her own effort. $K(l)$ is the cost of effort for the landlord when he exerts l. Similarly, the cost of effort for the tenant is denoted by $C(t)$. Both the tenant and landlord are risk-neutral, as in Roels, Karmarkar, and Carr (2010) and Corbett et al. (2005).

We set some assumptions on the characteristics of the sales and cost functions, which are quite common in previous studies with double moral hazard (Corbett et al., 2005; Kim & Wang, 1998). The composite effort functions $W(.)$ and $V(.)$ are strictly increasing and jointly concave in (l, t) such that $W_l, W_t, V_l, V_t > 0$ and $W_{ll}, W_{tt}, V_{tt}, V_{ll} \leq 0$. $W_{lt}, V_{tl} \geq 0$, which ensures the complementarity of the effort levels. The condition that the second partial derivatives W_{ll}, W_{tt}, V_{ll}, $V_{tt} \leq 0$ implies that effort levels l and t exhibit diminishing returns. This means that the gains from additional effort become less (though they may still be positive), as the landlord and tenant put in more effort. The other condition that the mixed partial derivatives are non-negative, i.e. W_{lt}, $V_{tl} \geq 0$, implies that effort levels l and t are complementary inputs. This is the case because as one party exerts high effort, more traffic is created, resulting in the other party increasing its effort to provide service to the additional traffic in the store who is interested in the other party's offerings, creating synergies for both parties. The cumulative probabilities $F(., W)$ and $G(., V)$ are decreasing (stochastic dominance condition) and convex, respectively, in W and V. The effort costs $K(l)$ and $C(t)$ are increasing and convex. It also holds that $K'(\infty) = C'(\infty) = \infty$ and $K(0) = C(0) = K'(0) = C'(0) = 0$. All functions are continuous and twice differentiable. All subscripts denote partial derivative.

The assumptions on the sales functions imply the following lemma.

Lemma 1. *The expected sales $E[X]$ and $E[Y]$ are jointly concave and increasing in (l, t).*

The landlord and tenant cannot observe each other's effort, resulting in double moral hazard. Thus, the rent to be paid by the tenant cannot be a function of the tenant's effort, but of the tenant's sales. Let $R(.)$ be the rent paid by the tenant. We study the rent contract design problem in a single period. We model a Stackelberg game, in which the landlord (principal) offers a rent contract to the tenant (agent). The tenant agrees on the terms of the rent given that her expected profit from the contract is not below her outside option utility . If so, both parties choose their respective effort levels within a Nash game setting. If one of the parties sets his/her effort before the other, the sequential game reduces to simultaneous decision making again (Bhattacharya et al., 2014). The net profit of the tenant can be obtained as the tenant's expected sales revenue less the summation of the rent paid to the landlord and the sales effort cost. Similarly, the profit of the landlord consists of the landlord's expected sales plus the rent charged to the tenant, less the landlord's effort cost. The landlord's rent contract design problem is as follows.

$$\pi^L = \max_{R(.)} \quad E[X + R(Y)] - K(l) \tag{1}$$

$$s.t. \quad E[Y - R(Y)] - C(t) \geq \bar{U} \tag{2}$$

$$t = \arg\max_{\bar{t}} \quad E[Y - R(Y)] - C(\bar{t}) \tag{3}$$

$$l = \arg\max_{\bar{l}} \quad E[X + R(Y)] - K(\bar{l}). \tag{4}$$

The landlord designs a rent contract that maximizes his expected profit. In his contract design problem, (2) stands for the participation constraint of the tenant in the rent contract. (3) and (4) represent the effort selection game of the tenant and the landlord.

Note that the amount of space to be rented by the landlord to the tenant determines the actual rent, as is the practice in the real estate industry. In this paper, we investigate the terms of the rent contract under an ex-ante preset floor space agreement. That is, we assume that the landlord is utilizing a part of the total floor space to meet his needs, and is renting out the remaining floor space. The amount of space rented is implicitly considered in the model

by the sales function parameters, such as the tenant's sales scale, with higher tenant's sales scale partly due to larger rented floor space.

3.1. Coordinated effort levels

When both stores are managed by the same parent company, the first-best effort levels of both stores can be obtained as follows.

$$\pi^C = \max_{l,t} \; E[X] + E[Y] - K(l) - C(t) \tag{5}$$

The first-order conditions for (5) are

$$W_l \int_0^\infty x\, f_W(x, W(l,t))dx + V_l \int_0^\infty y\, g_V(y, V(l,t))dy = K'(l) \tag{6}$$

$$W_t \int_0^\infty x\, f_W(x, W(l,t))dx + V_t \int_0^\infty y\, g_V(y, V(l,t))dy = C'(t). \tag{7}$$

From the concavity of $E(X)$ and $E(Y)$ in (l, t) and convexity of $K(.)$ and $C(.)$, there exists a unique solution to these first-order conditions.

3.2. A linear percentage rent contract

Percentage rent contracts are quite common in the retail industry. This is mainly due to the effect of these contracts in improving the tenant's performance, while being easy to implement. Let the rent be a linear function of the tenant's sales, as follows

$$R(Y) = aY + M,$$

where a is the sales share ratio and M is the base (fixed) rent to be paid by the tenant to the landlord. Next, we show that the landlord can obtain a second-best solution for the effort levels, by offering the tenant a linear percentage-rent contract.

Theorem 1. *(i) There always exists a linear rent contract $R^{**}(Y) = a^{**}Y + M^{**}$, which results in a second-best effort pair (l^{**}, t^{**}) where $l^{**}, t^{**} \in (0, \infty)$. (ii) These second-best effort levels do not achieve the coordinated solution.*

Several previous studies indicate that when both parties are risk neutral, there is no contract that can coordinate the principal and agent to exert system-optimal efforts (Bhattacharya et al., 2014; Bhattacharya & Lafontaine, 1995; Kim & Wang, 1998; Roels et al., 2010; Romano, 1994). Furthermore, a linear contract that is composed of a fixed base rent and a payment that is a linear function of the tenant's sales revenue achieves the second-best solution for the landlord's expected profit. Note that here, we define the rent as a percentage of the tenant's gross income, not the profit. This is considered in Wheaton (2000), as a common practice in the real estate industry. In the context of our motivating example, we expect the restaurant and coffee shop owners to be risk neutral in general, so the linear percentage rent contract is one potential contract that is very simple to use in practice to achieve a second-best solution.

Theorem 2. *In the optimal contract, $R^{**}(Y) = a^{**}Y + M^{**}$, (i) the optimal sales share a^{**} cannot be 1, (ii) the optimal base rent M^{**} cannot be 0.*

The result on the non-optimality of the tenant sharing her entire revenue with the landlord $(a = 1)$ is quite expected. A more interesting result is that the second-best sales share can be equal to zero, i.e., $a = 0$, implying that the coffee shop owner may not share any portion of the sales with the restaurant owner, paying only a fixed fee as the rent, resulting in a fixed-fee contract. This will be more explained in the numerical analyses section. In previous double moral hazard studies with a single joint output function, it is shown that the optimal sales share cannot be zero, which excludes the optimality of fixed fee contracts (Bhattacharya & Lafontaine, 1995; Roels et al., 2010).

Moreover, we show in the numerical analyses in Sections 5 and 6 that the second-best rent contract may involve a percentage and/or fixed payment by the landlord to the tenant $(a < 0, M < 0)$ (i.e. a reverse payment). In the case of a single output function, the principal may offer the agent a fixed payment to raise the agent's profit to her outside option level $\bar{U}$, giving the agent an incentive to accept the contract (see Roels et al., 2010). A reverse percentage payment is not common for double moral hazard models with a single output function. However, in our model, the landlord might decide to offer the tenant an incentive as a function of the tenant's sales, which would encourage the tenant to exert a higher effort level, resulting in a further increase in the landlord's sales.

4. A particular sales model

To derive further insights on the rent contract design problem for co-stores, we specify particular sales and cost of effort functions that satisfy the model assumptions defined in Section 3. The double-sided moral hazard model between the landlord and the tenant has appealing solutions when the sales functions are of the Cobb–Douglas form, $W = Cl^\gamma t^\alpha$ and $V = Dl^\gamma t^\alpha$, respectively for the landlord and the tenant. Here, C and D denote the sales scale parameters, and γ and α represent the elasticities of the landlord's and retailer's efforts, respectively, where $\gamma, \alpha > 0$ and $0 < \gamma + \alpha \leq 1$. The Cobb-Douglas function used hereafter is similar to those used in Bhattacharya and Lafontaine (1995); Corbett et al. (2005); Roels et al. (2010), and Kim and Netessine (2013), to characterize the joint sales function of the two collaborating parties.

Here α can be interpreted as the extent of the demand externality generated by the tenant's effort. Similarly, γ denotes the externality created by the landlord's effort. For instance, in a shopping mall context, an anchor store is a major shopping store that attracts a much larger portion of customers than other tenants, because it is renting a much larger space than other stores. It generates demand for the other stores as well by attracting a higher customer traffic to the mall (Bernstein & Winter, 2008). With high sales effort levels, an anchor store can have a high externality effect. On the other hand, the scales of the landlord's and tenant's sales, respectively, C and D, can be interpreted as the level of sales, such as the average sales per square foot. Generally, the average sales per unit area is a good indicator of the store success in terms of operations.

The landlord's and tenant's cost of effort functions have the following power form $K(l) = kl^m/m$ and $C(t) = ct^n/n$, respectively, with $m, n > 1$. Here k and c are called the cost of effort parameters and m and n are the cost elasticities, as in Bhattacharya and Lafontaine (1995). Moreover, we assume the following power distribution functions $f(x, W) = (1/W)e^{-x/W}$ and $g(y, V) = (1/V)e^{-y/V}$. Then the expected sales become

$$\mathrm{E}(X) = W = Cl^\gamma t^\alpha,$$
$$\mathrm{E}(Y) = V = Dl^\gamma t^\alpha,$$

Note that according to these particular sales functions, if one of the parties exerts zero effort, it makes the sales of both parties equal to zero, which may not be valid in practice in our co-store setting. However, according to Theorem 1, none of the players would put zero effort at optimality. Thus, a solution leading to zero effort(s), resulting in zero sales for the system, is not valid in our model.

With these particular sales functions, the second-best effort levels can be obtained by simultaneously solving (3) and (4). The resulting effort level for the landlord as a function of the sales share a is as follows

$$l^{**} = \left[\frac{(aD+C)\gamma}{k}\left(\frac{(1-a)\alpha D}{c}\right)^{\frac{\alpha}{n-\alpha}}\right]^{\frac{n-\alpha}{mn-\gamma n-\alpha m}}. \tag{8}$$

The second-best effort level of the tenant can be obtained as a function of a and l^{**}.

$$\begin{aligned} t^{**} &= \left[\frac{(1-a)\alpha D(l^{**})^{\gamma}}{c}\right]^{\frac{1}{n-\alpha}} \\ &= \left[\left(\frac{(aD+C)\gamma}{k}\right)^{\gamma}\left(\frac{(1-a)\alpha D}{c}\right)^{m-\gamma}\right]^{\frac{1}{mn-\gamma n-\alpha m}}. \end{aligned} \tag{9}$$

Theorem 3. *With the Cobb–Douglas sales and power cost of effort functions, there exists a unique second-best sales share a^{**} such that*

$$a^{**} = \frac{\sqrt{\gamma(n-\alpha)} - C/D\sqrt{\alpha(m-\gamma)}}{\sqrt{\alpha(m-\gamma)} + \sqrt{\gamma(n-\alpha)}}. \tag{10}$$

According to (10), it always holds that $-C/D < a^{**} < 1$. The optimal sales share determined by the landlord is independent of the cost of effort parameters c and k. Moreover, when the expected sales are equal, the optimal sales share is also independent of the sales scale parameters C and D. Unlike (Bhattacharya & Lafontaine, 1995), these results indicate that the contract terms between the two parties are non-uniform, but depend on the specific characteristics of the retailers, as in the restaurant and coffee shop examples, such as average sales per square foot of each retailer, their effort elasticities and cost of effort parameters. Accordingly, the restaurant owner will use different contract terms with different coffee shop chains. Next, the relationship between the landlord's and tenant's effort levels and the sales share is detailed by Theorem 4.

Theorem 4. *(i) The second-best effort levels of the landlord and the tenant are both unimodal in the sales share a, i.e., they first increase in a and then decrease.*
(ii) Let a^{0l} and a^{0t} be, respectively, the sales shares at which the landlord and the tenant exert their highest effort levels. They satisfy

$$a^{0t} \le a^{**} \le a^{0l}.$$

Theorem 4 states that both the landlord's and tenant's effort levels are not monotone in the sales share a. While the tenant prefers a low a, even negative if possible, the effort level of the landlord will be significantly low for a very low a. As a increases, the landlord exerts more effort, leading the tenant to put higher effort in response. So, both l^{**} and t^{**} first increase in a. As a increases further, t^{**} reaches its highest attainable value a^{0t}, beyond which the tenant puts less effort due to a lower marginal return from effort. If the sales share is very high ($>a^{0l}$), then the tenant's second-best effort decreases significantly, which leads the landlord to respond with a lowered effort level as well. The sales share that optimizes the landlord's profit is between these two values a^{0t} and a^{0l}. Theorem 4 is illustrated for a sample problem instance in Fig. 1.

Under second-best effort levels (8) and (9) and sales share in (10), the base rent can be obtained as follows.

$$\begin{aligned} M^{**} &= (1-a^{**})D(l^{**})^{\gamma}(t^{**})^{\alpha} - c(t^{**})^{n}/n - \bar{\mathcal{U}} \\ &= c\left(\frac{1}{\alpha}-\frac{1}{n}\right) \\ &\quad\times\left[\left(\frac{(a^{**}D+C)\gamma}{k}\right)^{\gamma n}\left(\frac{(1-a^{**})\alpha D}{c}\right)^{mn-\gamma n}\right]^{\frac{1}{mn-\gamma n-\alpha m}} - \bar{\mathcal{U}}. \end{aligned} \tag{11}$$

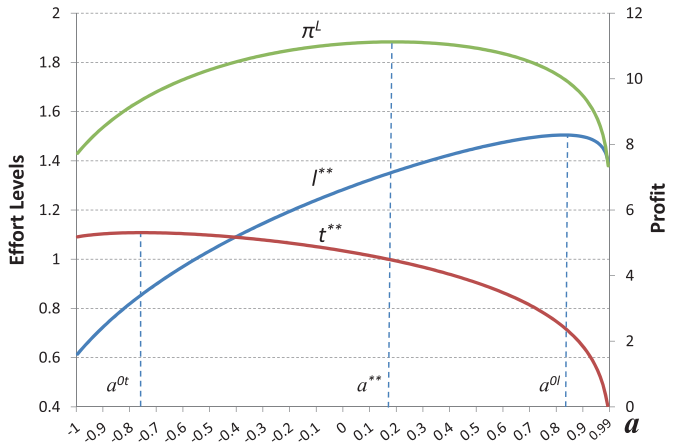

Fig. 1. Second-best sales share $a^{**} = 0.18$ versus $a^{0t} = -0.76$ and $a^{0l} = 0.82$ for a sample problem with $\gamma = 0.6$, $\alpha = 0.4$, $C = 6$, $D = 5$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{\mathcal{U}} = 0$.

Note that when the outside option utility $\bar{\mathcal{U}}$ is equal to zero, it always holds that M^{**} is positive. Even when the net rent paid by the tenant is negative due to a negative sales share a, there is always a positive base rent that is paid by the tenant.

5. System characteristics within different environments

In this section, we first analyze the sensitivity of the first-best effort levels of the coordinated system. This is done in Section 5.1. Then, we analytically and numerically study in Section 5.2 the effects of the system parameters on the second-best effort levels and contract terms, when a linear percentage rent contract is used.

5.1. Characteristics of the coordinated system

When both parties are jointly managed by the same company under a single objective (5), the first-best effort levels can be obtained for Cobb-Douglas sales functions and power cost of effort functions, as defined in Section 4, by simultaneously solving (6) and (7). We obtain the following closed form expressions

$$l^{*} = \left[\left(\frac{\gamma}{k}\right)^{n-\alpha}(C+D)^{n}\left(\frac{\alpha}{c}\right)^{\alpha}\right]^{\frac{1}{mn-\gamma n-\alpha m}}, \tag{12}$$

$$\begin{aligned} t^{*} &= \left[\frac{\alpha(C+D)(l^{*})^{\gamma}}{c}\right]^{\frac{1}{n-\alpha}} \\ &= \left[\left(\frac{\gamma}{k}\right)^{\gamma}(C+D)^{m}\left(\frac{\alpha}{c}\right)^{m-\gamma}\right]^{\frac{1}{mn-\gamma n-\alpha m}}. \end{aligned} \tag{13}$$

The resulting first-best total system profit, π^{C}, can be obtained by replacing (12) and (13) in (5).

$$\pi^{C} = \left[\left(\frac{\gamma}{k}\right)^{\gamma n}(C+D)^{mn}\left(\frac{\alpha}{c}\right)^{\alpha m}\right]^{\frac{1}{mn-\gamma n-\alpha m}}\left(1-\frac{\gamma}{m}-\frac{\alpha}{n}\right). \tag{14}$$

The characteristics of the first-best effort levels with respect to the system parameters are summarized in Theorems 5 and 6.

Theorem 5. *The first-best effort levels l^{*} and t^{*} under coordination increase as C and D increase; decrease as k and c increase; increase if $l^{*} < 1$, decrease otherwise as m increases; increase if $t^{*} < 1$, decrease otherwise as n increases.*

In a coordinated system, the co-stores react to changes in sales scales C and D, cost of efforts k and c, or cost elasticities m and n in a similar way. When either the restaurant's or the coffee shop's

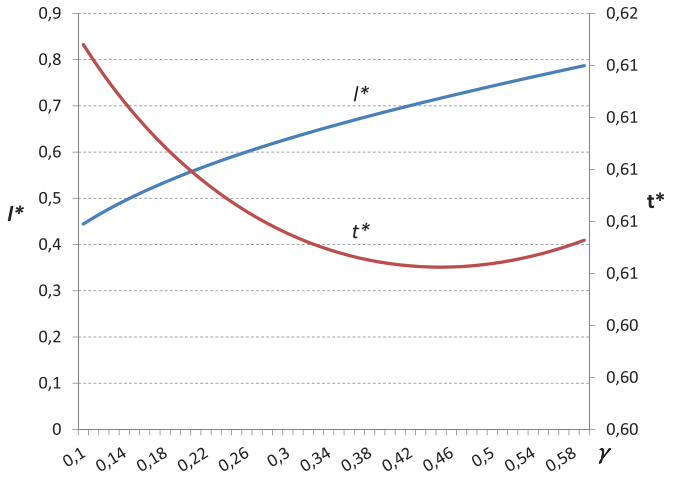

Fig. 2. First-best effort levels vs. γ for a sample problem with $\alpha = 0.1$, $C = D = 1$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{\mathcal{U}} = 0$.

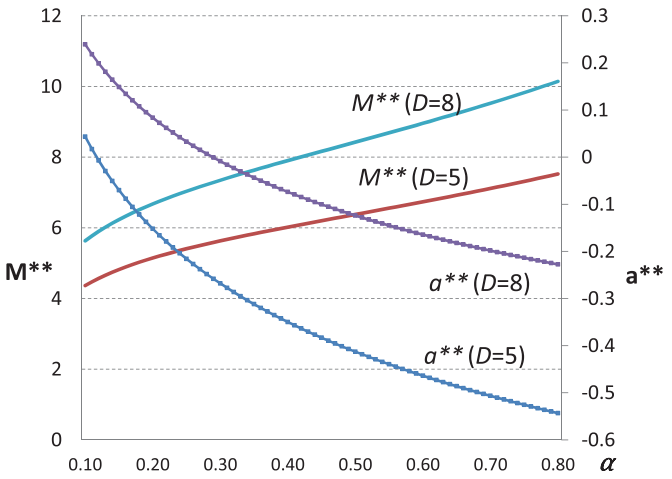

Fig. 3. Second-best contract parameters vs. α for a sample problem with $\gamma = 0.1$, $C = 6$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{\mathcal{U}} = 0$.

sales scale increases, both parties exert more effort. When the effort becomes more costly, i.e. having a higher cost of effort for either of the parties, both exert less effort.

Theorem 6. *i. l^* is either increasing-decreasing or always increasing with γ.*
ii. t^* is either decreasing-increasing or always decreasing with γ.
iii. t^* is either increasing-decreasing or always increasing with α.
iv. l^* is either decreasing-increasing or always decreasing with α.

On the other hand, the landlord's and tenant's effort levels move in opposite directions for any change in the elasticity of effort for any of the two parties, as stated in Theorem 6. The result is illustrated in Fig. 2 for a sample problem, where the elasticity of the landlord's effort γ increases from 0.1 to 0.6. As γ increases, it is optimal for the landlord to exert more effort. The tenant will rely on the landlord's effort and exert less effort, so that the system-wide effort costs are balanced out. For instance, when the coffee shop engages in a rent agreement with a larger restaurant (in terms of store space), who can generate higher customer traffic, the coffee shop may choose to exert less effort. However, at some point when the restaurant's externality effect is very high compared to the coffee shop's, the latter should also contribute to the system by exerting more effort.

5.2. Characteristics of the linear percentage rent contract

Next, we analyze how the second-best linear percentage rent contract terms and effort levels change with the landlord's and tenant's characteristics. Note that the landlord selects the contract terms such that the tenant's individual rationality constraint is always binding. If the tenant's reservation utility $\bar{\mathcal{U}}$ is set to zero, without loss of generality, the tenant's profit will be zero at optimality. Then the landlord's profit becomes the total system's profit.

We first study analytically the sensitivity of the optimal sales share to the system parameters. Note that from (10), a^{**} is insensitive to the cost of effort parameters c and k.

Theorem 7. *The second-best sales share a^{**} of the second-best linear rent contract increases as D, n, and γ increase; decreases as C, m, and α increase.*

To interpret the sensitivity of a^{**}, we need to consider the changes in the equilibrium effort levels as well as in the second-best base rent.

Theorem 8. *i. The second-best effort levels l^{**} and t^{**} increase as C and D increase; decrease as k and c increase.*
ii. The second-best base rent M^{**} increases as C and D increase; decrease as k and c increase.

The behaviors of l^{**}, t^{**}, and M^{**} with respect to γ, α, m, and n are not easily tractable analytically, thus will be illustrated numerically in what follows. For instance, M^{**} can be decreasing-increasing or increasing-decreasing in m and n.

5.2.1. Effect of the tenant's sales scale (D) and externalities (α)

Particularly, the sensitivity results with respect to C, D, α and γ are worth discussing. According to Theorem 7, a tenant with a higher demand externality α, will be awarded by the landlord with a lower sales share, as an incentive for the tenant to exert a higher effort level. Fig. 3 shows that a lower a is also accompanied with a higher base rent M, to compensate for the decrease in the marginal return of the landlord's effort. This result is consistent with the shopping mall contracting literature, where it was shown that anchor stores with high externalities are offered lower sales share and a higher base rent when compared to stores with lower externalities (Gould et al., 2005). On the other hand, when the tenant has a higher sales scale D, both the sales share and the base rent become higher. In fact, it is interesting to observe that the tenant's externality effect α and the sales scale D have opposite effects on the second-best sales share a^{**}.

To further elaborate on the last observation, we analyze real shopping mall tenant information obtained from the Dollars and Cents of Shopping Centers-2004 report (ULI, 2004), which is published by The Urban Land Institute (ULI), and includes real data on more than 20,000 tenants in 947 U.S. shopping centers. While our co-store model is quite different then a shopping mall setting, we can still benefit from the similarities in terms of the tenant's and the landlord's characteristics: The sales scale parameters C and D in our model correspond to the median sales per square foot in the Dollars and Cents of Shopping Centers (2004) report; The externality effect measure by γ and α in our model correspond to the Gross Leasable Area (GLA). Unfortunately, to the best of our knowledge there is no published data on co-store rent contracts where both parties are independent retailers. Thus, we use the published data on shopping mall rent contracts, to empirically support our study.

In Table 1, a summary of tenant information is given on median sales per square foot, median gross leasable area (GLA) (in square foot), and median rate of percentage rent paid by various tenant

Table 1
Summary tenant information for U.S. super regional and regional shopping centers.

Tenant classification	Super regional centers			Regional centers		
	Median sales per square feet (dollars)	Median GLA in square feet	Median rate of % rent	Median sales per square feet (dollars)	Median GLA in square feet	Median rate of % rent
General merc.	176.18	117,030	2.25	141.85	66,582	2
Food	365.63	1128	7	359.29	1200	6
Food service	459.36	795	8	367.14	1216	6
Clothing acc.	310.12	3502	6	259.09	3999	5
Shoes	283.86	2651	6	224.57	3000	6
Home furnish.	354.35	2658	6	209.25	5287	5
Home app./music	379	2489	5	315.05	2676	5
Hobby	299.84	2732	6	230.75	3537	6
Gifts	268.26	2529	6	215.58	3171	6
Jewelry	899.31	1233	6	720.24	1200	6
Drugs	317.34	10,594	3	–	–	–
Other retail	421.92	815	7	277.09	1500	6
Personal serv.	294.76	1146	6	228.56	1200	6

Source: Urban Land Institute, Dollars and Cents of Shopping Centers (2004).

Table 2
Correlation coefficients obtained from summary tenant information for U.S. super regional and regional shopping centers.

	Super regional centers median rate of % rent	Regional centers median rate of % rent
Median sales per square feet (dollar)	0.34	0.38
Median GLA in square feet	−0.72	−0.94

types, located in either super regional (around 1 million square feet) or regional (around 500,000 square feet) shopping centers. We interpret the sales per square foot as the sales scale of a tenant, which is denoted by D in our model. Also, since anchor stores attract high customer traffic, and thus create high externality for other stores, they generally tend to have the largest GLA in a shopping center. As a result, GLA is interpreted as the externality effect, which is denoted by α.

Using the data reported in Table 1, we obtained the correlation of the percentage rent with the sales per square foot and GLA. According to Table 2, in both super regional and regional malls, the tenant sales scale and percentage rent paid by the tenant are positively correlated.

On the other hand, GLA is negatively correlated with the percentage rent. Thus, consistent with our results, stores creating higher externalities are charged lower sales share to enhance their effort levels. However, as the tenant's sales scale increases, the sales share increases as well. Snively (2000) also states that retailers with a higher profit margin generally pay higher percentage rents. For instance, according to Table 1, while a median general merchandiser pays 2.25% as sales share, a jewelry store, which makes five times more sales per square foot, pays 6%.

The net expected rent paid by the tenant shows a non-monotonic behavior with respect to the tenant's externality factor α, as illustrated in Fig. 4. If the tenant becomes more effective in creating externalities, the rent paid by the tenant first decreases, due to the decrease in the sales share a, despite the increasing base rent M. Thus, the expected profit of the landlord decreases. A tenant with a much higher α puts in significantly higher effort according to Fig. 5, resulting in a higher shared revenue, and increases the landlord's profit. Moreover, when the tenant can create higher externalities, the landlord may rely on the tenant's effort, and exert less effort. Thus, a landlord benefits from a tenant with high externality effect, by receiving a higher rent and incurring a lower cost of effort due to exerting less effort.

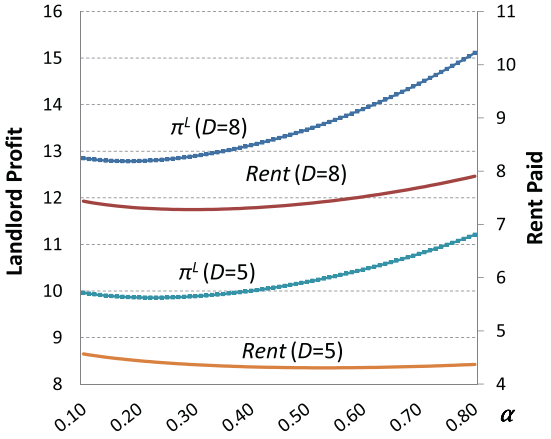

Fig. 4. Landlord's Profit and Rent Paid vs. α.

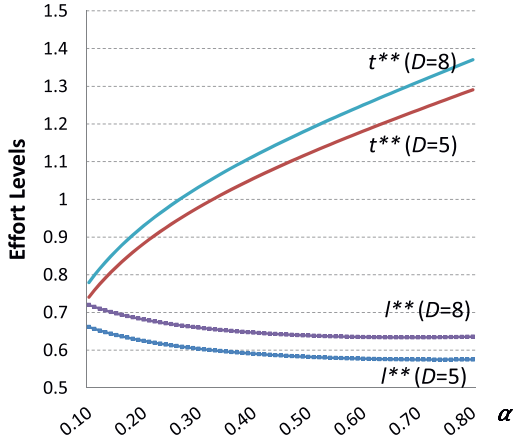

Fig. 5. Second-best effort levels vs. α, for a sample problem with $\gamma = 0.1$, $C = 6$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{U} = 0$.

It can also be observed in Figs. 4 and 5 that as D, the tenant's sales scale, increases, the effort levels, total rent, and landlord's profit increase. For instance, when the tenant is 60% more capable of generating sales for given effort levels (D=8/D=5), both the tenant and landlord will exert more effort, and the landlord increases his profit by around 30%, according to Fig. 4. The change in the tenant's sales scale has a greater effect in boosting the landlord's profit through the increased rent payments. Thus, the landlord is

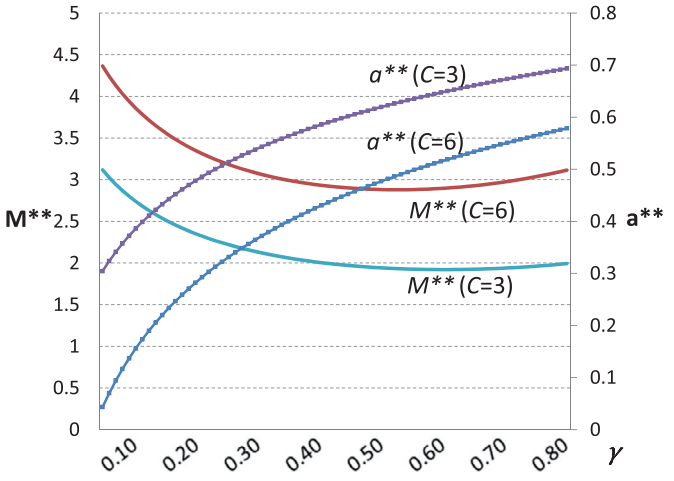

Fig. 6. Second-best contract parameters vs. γ for a sample problem with $\alpha = 0.1$, $D = 5$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{u} = 0$.

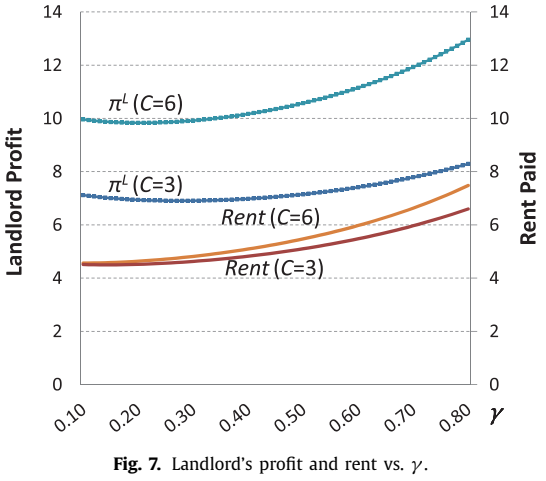

Fig. 7. Landlord's profit and rent vs. γ.

better-off renting out space to a small size but high revenue generating retailer, instead of one generating higher traffic, but with lower sales scale. Some shopping centers in the U.S. have already started programs to fill up their small-size vacant spaces, by renting them out for free to start-up entrepreneur stores, and giving them the chance to become high revenue generators (Misonzhnik, 2012a).

5.2.2. Effect of the landlord's sales scale (C) and externalities (γ)

The landlord's sales characteristics γ and C have opposite effects on the contract terms. As γ increases, the landlord has higher externality effect, and requires a higher sales share. However, as C increases, indicating a landlord with a larger sales scale, the optimal sales share decreases. The result is illustrated in Fig. 6.

The result on the effect of the landlord's externality effect on the sales share is consistent with the shopping mall data. According to Dollars and Cents of Shopping Centers (2004) report, the median rate of the percentage rent increases in the mall size (the total gross leasable area (GLA)), an equivalent to the landlord's externality effect. Table 3 includes the median rate of percentage rent paid by various tenant types in four different types of shopping centers. For instance, the median sales share paid by food restaurants is 7% in superregional, 6% in regional, 1.6% in community, and 1% in neighborhood shopping centers.

The base rent has a convex behavior with respect to the landlord's externality effect. While it is decreasing up to some γ value, it is increasing again for a very high value of γ. This is due to the significant increase in the tenant's revenue with the increasing effort of the landlord, as shown in Fig. 8. A higher externality effect encourages the landlord to put in more effort. Relying on the landlord's high effort level, the tenant exerts less effort. Due to the externality created by one party, both parties will generate higher revenues. As a result, the landlord's profit decreases for a slight increase in the landlord's externality effect. However, as γ increases even further, the landlord is better off, due to both his higher effort level and higher rent, as illustrated by Fig. 7.

On the other hand, as the landlord's sales scale, C, increases, both parties put in more effort and the landlord will make more profit (associated with higher sales and higher rent). The rent composition slightly changes to include a lower sales share and a higher base rent. The opposite effects of C and γ on the rent contract are due to their varying effects on the landlord's effort level. For the same increase separately in C and γ, the landlord's effort increases more with the increase in γ compared to that in C. Thus, a landlord with a high externality effect can increase the sales share irrespective of the tenant's effort level. Moreover, for a given landlord's externality effect, γ, the landlord's effort does not increase significantly with a higher sales scale C. So the landlord might decrease the share to encourage the tenant to exert more effort.

Table 3
Median rate of percentage rent paid in shopping centers of various sizes.

Tenant classification	Median rate of percentage rent			
	Super regional	Regional	Community	Neighborhood
General merchandise	2.25	2	2	3
Food	7	6	1.6	1
Food service	8	6	6	6
Clothing accessories	6	5	5	6
Shoes	6	6	5	5
Home furnishings	6	5	5	5
Home appliances/music	5	5	3	4.47
Hobby/special interest	6	6	5	6
Gifts/specialty	6	6	5	6
Jewelry	6	6	6	6
Drugs	3	–	2	2.25
Other retail	7	6	5	5
Personal services	6	6	6	6
Entertainment/community	12	10	6	–

Source: Urban Land Institute, Dollars and Cents of Shopping Centers (2004).

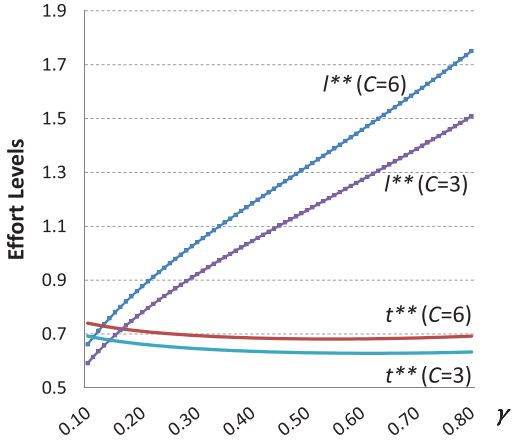

Fig. 8. Second-best effort levels vs. γ, for a sample problem with $\alpha = 0.1$, $D = 5$, $k = c = 2$, $m = 3$, $n = 5$, and $\bar{U} = 0$.

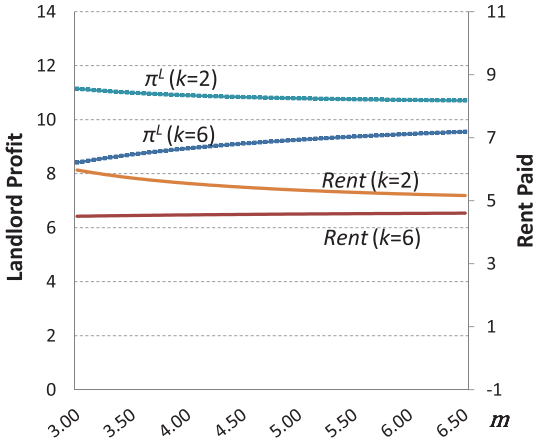

Fig. 9. Landlord's profit and rent vs. m.

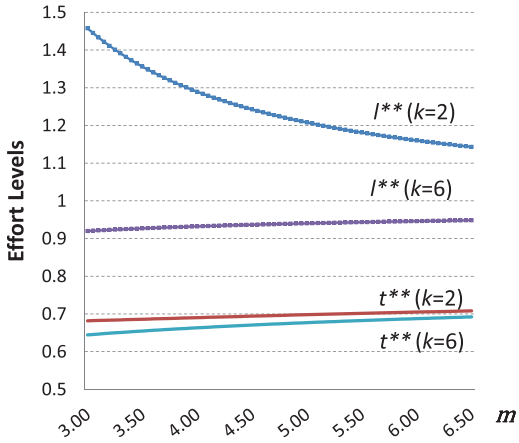

Fig. 10. Second-best effort levels vs. m, for a sample problem with $\alpha = 0.1$, $\gamma = 0.6$, $C = 6$, $D = 5$, $c = 2$, $n = 5$, and $\bar{U} = 0$.

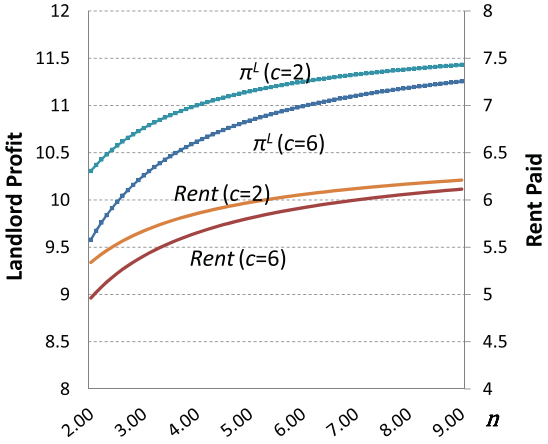

Fig. 11. Landlord's profit and rent vs. n.

Overall, the landlord's second-best sales share is high when the landlord has small sales scale but has high externality effect. In this case, a low base rent accompanies the high sales share. On the contrary, when the landlord has high sales scale and low externality effect, he charges a small sales share and high base rent. It is also interesting to note that, when the landlord's externality effect is small, the net rent received by the landlord is insensitive to his sales scale, as illustrated in Fig. 7.

5.2.3. Effect of the cost of effort parameters

Next, we investigate the effect of the cost of efforts on the second-best rent contract. The effects of k and c on the contract terms and effort levels are shown in Theorems 7 and 8. If a retailer's effort becomes more costly, he/she exerts less effort. Since the second-best sales share is independent of the cost of effort parameters, one party, being affected by the other party's low effort, will also reduce its own effort level. As both parties observe lower sales with their reduced effort levels, the base rent charged by the landlord will decrease as well, resulting in a smaller rent and landlord's profit.

On the other hand, the elasticity of the landlord's cost of effort, m, might have more variable effects on the system. As indicated by Theorem 7, under a higher m, the landlord charges a lower sales share. A reduction in the sales share is offset by a higher base rent. The change in the net rent depends on landlord's effort. If the landlord's effort exceeds 1, then as m increases, the landlord reduces his effort, which reduces the tenant's sales, total rent, and landlord's profit. However, if the landlord's effort is below 1, then as m increases, the landlord increases his effort, resulting in the increase of the rent and profit. From (10), as m increases, a^{**} approaches $-C/D$. Then, l^{**} and t^{**} approach, respectively, 1 and $((C+D)\alpha/c)^{1/(n-\alpha)}$. Figs. 9 and 10 illustrate the effects of m on the landlord's profit and second-best effort levels, respectively.

The elasticity of the tenant's cost of effort, n, also has variable effects on the system. Contrary to m, Theorem 7 indicates that under a higher n, the landlord charges a higher sales share and a lower base rent. The total rent depends on the tenant's effort. In the example illustrated in Figs. 11 and 12, when the tenant's effort is below 1, the tenant increases her effort with n. With the increasing sales share, the landlord also puts in more effort, resulting in higher tenant's sales and total rent, and thus higher landlord's profit. However, if the landlord's effort exceeds 1, with a higher n, the tenant reduces her effort, causing a decrease in the tenant's sales. The total rent paid can either decrease or increase as the sales share increases. From (10), we observe that as n increases, both a^{**} and t^{**} become 1 in the limit.

To summarize, the landlord is definitely better off under higher sales scales C and D, and lower cost of effort parameters k and c. The effects of the cost elasticities may not be clear. The landlord may be better or worse off under higher cost elasticity, depending on whether it leads to higher or lower second-best effort levels. The landlord is better off when he or the tenant enjoys a high externality effect. For moderate levels of externality factors, slight increases are not preferred. For example, when the landlord's externality increases slightly, the tenant relies on the landlord's

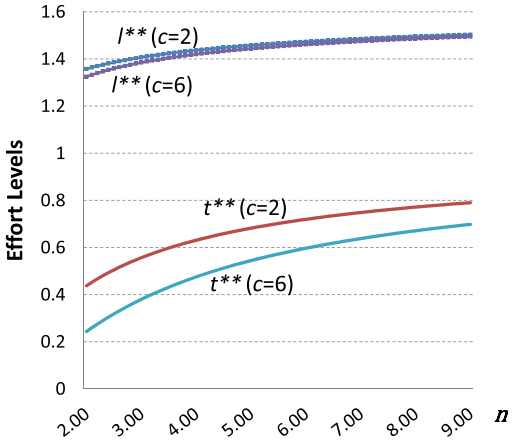

Fig. 12. Second-best effort levels vs. n, for a sample problem with $\alpha = 0.1$, $\gamma = 0.6$, $C = 6$, $D = 5$, $k = 2$, $m = 3$, and $\bar{U} = 0$.

increasing effort and externalities, and will put in less effort, causing lower rent and landlord's profit. Similarly, when the tenant's externality increases slightly and the sales share decreases, the total rent decreases when the tenant's effort does not increase significantly, which results in a reduced landlord's profit.

6. Performance of the linear percentage rent contract

In this section, the second-best effort levels and profits under a linear contract are compared to the first-best, obtained in coordinated and fixed-rent systems.

Theorem 9.

(i) *Under a linear rent contract, both parties exert less effort than the first-best, i.e.* $l^{**} < l^{*}$ and $t^{**} < t^{*}$.
(ii) *The ratios of the first-best and second-best effort levels* (l^{*}/l^{**} *and* t^{*}/t^{**}) *are insensitive to the cost of effort parameters k and c and sales scale parameters C and D.*
(iii) *The ratio of the first-best system profit to the second-best system profit under a linear rent contract is insensitive to the cost of effort parameters k and c and sales scale parameters C and D.*

Theorem 1 demonstrates that a linear percentage rent contract cannot achieve the first-best solution obtained under a coordinated system. Under the linear rent contract, both retailers exert less effort than the first-best levels. The ratios of the first-best and second-best effort levels are shown to be independent of the sales scales of the retailers and the cost of effort parameters.

The retailer effort levels under an independent system, in which retailers are not bound to any rent payment, is also analogous to a fixed-rent system, in which the landlord charges a fixed rent that is enough to wipe off all the profit that a tenant would get over an outside option. Thus, the optimal effort levels under a fixed-rent system are also valid when the tenant rents the space from another landlord or owns it. The retailers' effort setting problem under the optimal fixed rent, M^F, is as follows.

$$\begin{aligned} \pi^F = \max_{M} \;& E[X] - K(l) + M \\ s.t.\;& E[Y] - C(t) - M \geq \bar{U} \\ & t = \arg\max_{\bar{t}} \; E[Y] - C(\bar{t}) \\ & l = \arg\max_{\bar{l}} \; E[X] - K(\bar{l}). \end{aligned} \tag{15}$$

Theorem 10. *Under a fixed-rent contract, neither the landlord nor the tenant can generate more profit in comparison to a linear percentage rent contract.*

The proof is clear. Under a fixed rent, the sales share $a = 0$. Thus, problem (15) is a special case of the retailers' problem with linear percentage rent. Consequently, the landlord cannot achieve a higher profit with the fixed rent over the linear percentage rent case. The landlord determines this fixed rent such that the tenant gets a profit as much as her outside option $\bar{U}$. So under a fixed rent, the tenant gets the same profit as in a percentage rent contract.

For the Cobb–Douglas sales functions defined in Section 4, the resulting optimal effort levels l^F and t^F under the fixed rent contract are as follows.

$$l^F = \left[\left(\frac{\gamma C}{k}\right)^{n-\alpha}\left(\frac{\alpha D}{c}\right)^{\alpha}\right]^{\frac{1}{mn-\gamma n-\alpha m}} \tag{16}$$

$$t^F = \left[\left(\frac{\gamma C}{k}\right)^{\gamma}\left(\frac{\alpha D}{c}\right)^{m-\gamma}\right]^{\frac{1}{mn-\gamma n-\alpha m}}. \tag{17}$$

Theorem 11 explains how the retailers' relative effort levels change with the system parameters under coordinated, linear percentage rent, and fixed-rent systems.

Theorem 11. *When the cost elasticity parameters are equal, i.e.,* $m = n$,

(i) *Under both a linear rent contract and a coordinated system, the relative values of the retailers' best effort levels* (l^{*}/t^{*} *and* l^{**}/t^{**}) *are insensitive to the sales scale parameters C and D. However, under a fixed rent contract, the relative value of efforts* l^F/t^F *is increasing in C and decreasing in D.*
(ii) *Each of* l^{*}/t^{*}, l^{**}/t^{**}, *and* l^F/t^F *is increasing in* γ *and c and decreasing in* α *and k.*

In all systems where the elasticities of the cost of efforts are equal, the landlord exerts more effort compared to the tenant when the landlord's externality effect increases or his cost of effort decreases. The result is reversed when the tenant generates higher externalities or the landlord's effort becomes more costly. The interesting result is that, under coordinated and linear percentage rent contract systems, retailers react similarly to changes in the sales scales, but not in a fixed-rent system. When the rent is fixed, a retailer is relatively insensitive to the increase in the sales scale of his/her co-store. The result justifies the use of a percentage rent contract to align the efforts of co-stores.

Next, we investigate the sensitivity of the improvement in profit obtained as a result of using a linear percentage rent over a fixed-rent contract, and the loss incurred from using the linear percentage contract over the coordinated solution. The percentage increase in the total expected profit from the use of a linear percentage rent contract instead of a fixed-rent is denoted by $\Delta\pi^F$ such that $\Delta\pi^F = (\pi^L - \pi^F)/\pi^F * 100$. The percentage decrease in the total expected profit from the use of a linear percentage rent contract compared to a coordinated solution is denoted by $\Delta\pi^C$ such that $\Delta\pi^C = (\pi^C - \pi^L)/\pi^C * 100$.

Figs. 13 and 14 illustrate the change in $\Delta\pi^C$ and $\Delta\pi^F$, respectively, as the tenant's and landlord's externality effects, α and γ change. As stated in Theorem 9, the percentage profit gap between the linear percentage rent contract and the coordinated solution, $\Delta\pi^C$, is independent of the sales scales of the two stores, C and D. Thus, as the tenant's or the landlord's externality effect changes, the effect on $\Delta\pi^C$ is exactly the same for any value of C for a fixed D, or vice versa. We observe that, as the tenant's or the landlord's externality effect increases, $\Delta\pi^C$ increases as well. The reason is that with increasing externalities, the increase in the effort level of

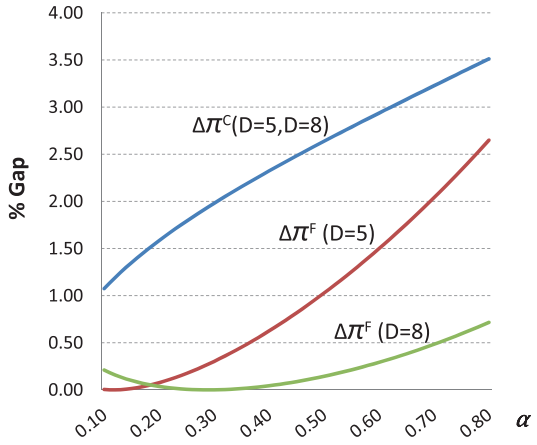

Fig. 13. % change in profit vs. α.

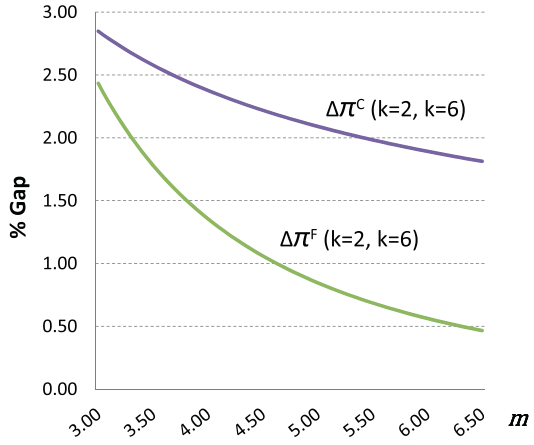

Fig. 15. % change in profit vs. m.

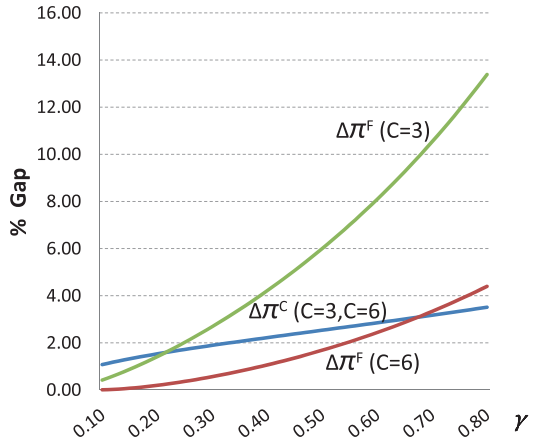

Fig. 14. % change in profit vs. γ, $\alpha = 0.1$, $\gamma = 0.6$, $C = 6$, $D = 5$, $k = c = 2$, $m = 3$, $n = 5$ and $\bar{\mathcal{U}} = 0$.

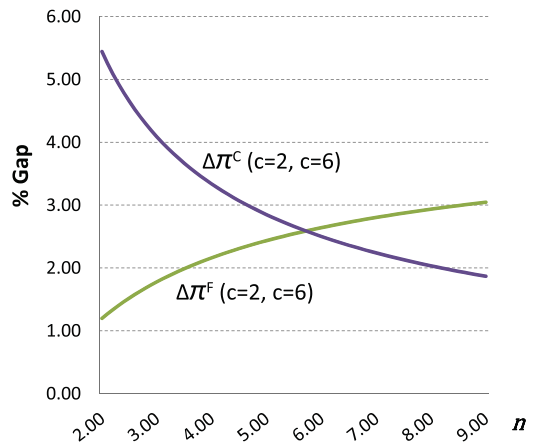

Fig. 16. % change in profit vs. n, $\alpha = 0.1$, $\gamma = 0.6$, $C = 6$, $D = 5$, $k = c = 2$, $m = 3$, $n = 5$ and $\bar{\mathcal{U}} = 0$.

at least one of the parties in a coordinated system is higher than in a linear rent contract system.

The improvement in the landlord's profit, which is equal to the system profit less $\bar{\mathcal{U}}$, over a fixed rent contract, is observed to be unimodal in the second-best sales share a^{**}, which attains the lowest value of zero when $a^{**} = 0$. As a^{**} deviates from zero, $\Delta\pi^F$ increases as well. Thus, there is no monotone behavior of $\Delta\pi^F$ in any of the system parameters α, γ, C, D, m, or n. Its behavior depends solely on how a^{**} changes around zero. In the example illustrated in Fig. 14, the gap can exceed 13%. Moreover, although the analytical proof is omitted here, $\Delta\pi^F$ is also insensitive to the cost of effort parameters k and c, similarly to $\Delta\pi^C$.

We show in Section 5.2 that as the elasticities of the cost of effort for the tenant and landlord increase, the effort levels of both retailers approach constant values. A similar result is also obtained for the coordinated system as stated by Theorem 5. Therefore, as m or n increase, the percentage profit change $\Delta\pi^C$ decreases and approaches a constant number. On the other hand, $\Delta\pi^F$ may decrease or increase depending on whether a^{**} approaches or moves away from zero, respectively, as illustrated in Figs. 15 and 16.

7. Concluding remarks

Being the main source of supply chain revenues, retail operations have been an important part of operations management. In the last decade, and with the fast customers' adoption of online shopping, brick and mortar retailers need to pay special attention to the operational issues specific to physical stores, for the sustainability of their business. The rent of the floor space is an important one of these operational issues. High rental rates and construction costs, limited space availability, and significant externality effects in popular shopping areas, encourage some retailers to share floor space with other retailers, creating co-store arrangements. Co-stores share the rent expense and benefit from mutual sales efforts that lead to higher foot traffic.

In this paper, we study the rent contract design problem between two co-stores, in which the floor space is owned by one of the stores. Each of the co-stores realizes individual sales as a function of the sales efforts of both retailers. While higher effort of one party increases the sales of both, each party incurs his/her own effort cost. Motivated by the common practice, the rent paid by the tenant store to the landlord store is modeled as a linear function of the tenant's gross sales plus a base or fixed fee. The rent is not a function of the tenant's effort, but sales, because effort such as salaries of salespeople, publicity, and promotion are not verifiable, leading to a double moral hazard problem. By analyzing the resulting double moral hazard problem, our objective in this work is to understand the dynamics of the optimal linear percentage rent contract offered by the landlord to the tenant store, and the resulting equilibrium effort levels of the co-stores.

We show that for general sales functions, a linear percentage contract cannot achieve the first-best effort levels, but leads to second-best effort levels. At the second-best solution, the optimal sales share can be zero, but the fixed base rent cannot. Thus, the

tenant would always pay a base rent even if she is exempted from a percentage sales payment.

To further explore the properties of the optimal contract and equilibrium effort levels, we use Cobb-Douglas sales functions and power cost of effort functions. We derive closed-form expressions of the second-best contract parameters and effort levels. We explore the sensitivity of the second-best solution with respect to the problem parameters, such as the tenant's and landlord's sales scales, demand externalities, and cost of efforts. We show that the landlord's contract parameters, when the landlord has low sales scale and high externalities, consists of a high sales share and a low base rent. On the contrary, when the landlord has high sales scale and low externality effect, he charges a low sales share, but high base rent. We also find that when the tenant has a low sales scale and high externalities, the landlord charges a low sales share, but a relatively high base rent. Furthermore, we compare the linear contract to the first-best solution, in which both efforts are observable, and the traditional fixed rent contract, in which the percentage sales share is equal to zero. Although the profit improvement provided by a linear percentage rent contract over the fixed rent is not monotone in any system parameter, the improvement exceeds 13%. The percent gap with the first-best solution is shown to be independent of the sales scales and cost of efforts of the co-stores and it is generally lower with low externality effects.

The main results are also supported by real evidence from data on the rent contracts in US shopping malls, which is the most relevant and accessible data on percentage rents. Our co-store setting is different from a shopping mall system where only tenant stores directly observe customer sales, but not the mall owner. Still, the externality effect and sales scale of a store can be interpreted similarly. The data indicates that the mall owner charges a lower sales share to stores with higher gross leasable area, i.e. with higher foot traffic or externality effect. On the other hand, stores making higher sales per square foot, i.e. with higher sales scale, are charged higher sales share. Moreover, we observe that in larger malls, where the externality effect is higher, the sales share is higher. Our theoretical findings are consistent with these observations on the rent terms currently used in practice.

As stated by Bhattacharya and Lafontaine (1995), a linear contract implements the second-best effort levels, but it is not the unique contract to give a second-best solution. For instance, a percentage rent is sometimes not payable unless the tenant's sales exceed a threshold level within the given time period. The tenant pays a base rent as usual, but in addition, pays a sales share of the amount of sales in excess of a threshold level. Our study is devoted to understanding the characteristics of an easy-to-implement linear percentage rent contract. Analyzing piecewise linear contracts is a future research work worthy of investigation.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2016.08.063.

References

Allen, D., & Lueck, D. (1992). Contract choice in modern agriculture: Cash rent versus cropshare. *The Journal of Law & Economics, 35*(2), 397–426.

Armstrong, C. S., Larcker, D. F., & Su, C.-L. (2010). Endogenous selection and moral hazard in compensation contracts. *Operations Research, 58*(4), 1090–1106.

Baiman, S., Fischer, P. E., & Rajan. , M. V. (2001). Performance measurement and design in supply chains. *Management Science, 47*(1), 173–188.

Balachandran, K. R., & Radhakrishnan, S. (2005). Quality implications of warranties in a supply chain. *Management Science, 51*(8), 1266–1277.

Benjamin, J. D., Boyle, G. W., & Sirmans, C. F. (1992). Price discrimination in shopping center leases. *Journal of Urban Economics, 32*, 299–317.

Bernstein, S., & Winter, E. (2008). Throwing a party: Contracting with type dependent externalities. *Working paper*. Stanford University.

Bhattacharya, S., Gupta, A., & Hasija, S. (2014). Joint product improvement by client and customer support center: The role of gain-share contracts in coordination. *Information Systems Research, 25*(1), 137–151.

Bhattacharya, S., & Lafontaine, F. (1995). Double-sided moral hazard and the nature of share contracts. *RAND Journal of Economics, 26*(4), 761–781.

Brueckner, J. K. (1993). Inter-store externalities and space allocation in shopping centers. *Journal of Real Estate Finance and Economics, 7*, 5–16.

Caldieraro, F., & Coughlan, A. T. (2009). Optimal sales force diversification and group incentive payments. *Marketing Science, 28*(6), 1009–1026.

Chen, B. X. (2013) NY Times. [Available at http://bits.blogs.nytimes.com/2013/04/04/samsung-will-open-mini-stores-inside-1400-best-buys/?_php=true&_type=blogs&_r=0.]. Accessed 01.07.16.

Chen, J., & Hu, Q. (2015). Optimal payment scheme when the supplier's quality level and cost are unknown. *European Journal of Operational Research, 245*, 731–742.

Corbett, C. J., & DeCroix, G. A. (2001). Shared savings contracts for indirect materials in supply chains: Channel profits and environmental impacts. *Management Science, 47*(7), 881–893.

Corbett, C. J., DeCroix, G. A., & Ha, A. J. (2005). Optimal shared savings contracts in supply chains: Linear contracts and double moral hazard. *European Journal of Operational Research, 163*, 653–667.

Crama, P., De Reyck, B., & Degraeve, Z. (2008). Milestone payments or royalties? Contract design for R&D licensing. *Operations Research, 56*(6), 1539–1552.

Crama, P., Reyck, B., & Degraeve, Z. (2013). Step by step. the benefits of stage-based R&D licensing contracts. *European Journal of Operational Research, 224*(3), 572–582.

Cvitanic, J., Wan, X., & Yang, H. (2013). Dynamics of contracts design with screening. *Management Science, 50*(5), 1229–1244.

Dai, T., & Jerath, K. (2016). Technical note- impact of inventory on quota-bonus contracts with rent sharing. *Operations Research, 64*(1), 94–98.

D'Arcy, E., Tsolacos, S., & McGough, T. (1997). An empirical investigation of retail rents in five european cities. *Journal of Property Valuation and Investment, 15*(4), 308–322.

Gibbons, R. (2005). Incentives between firms (and within). *Management Science, 51*(1), 2–17.

Gould, E. D., Pashigian, B. P., & Prendergast, C. J. (2005). Contracts, externalities, and incentives in shopping malls. *The Review of Economics and Statistics, 87*(3), 411–422.

He, X., Prasad, A., & Sethi, S. P. (2009). Cooperative advertising and pricing in a dynamic stochastic supply chain: Feedback stackelberg strategies. *Production and Operations Management, 18*(1), 78–94.

Heese, H. S., & Kemahlioglu-Ziya, E. (2016). Don't ask, don't tell: Sharing revenues with a dishonest retailer. *European Journal of Operational Research, 248*(2), 580–592.

Heese, H. S., & Swaminathan, J. M. (2010). Inventory and sales effort management under unobservable lost sales. *European Journal of Operational Research, 207*, 1263–1268.

Holmstrom, B. (1979). Moral hazard and observability. *The Bell Journal of Economics, 10*(1), 74–91.

Ibanez, M. R., & Pennington-Cross. , A. (2013). Commercial property rent dynamics in u.s. metropolitan areas: An examination of office, industrial, flex and retail space. *The Journal of Real Estate Finance and Economics, 46*(2), 232–259.

Iyer, A. V., Schwarz, L. B., & Zenios, S. A. (2005). A principal-agent model for product specification and production. *Management Science, 51*(1), 106–119.

Jain, N., Hasija, S., & Popescu, D. G. (2013). Optimal contracts for outsourcing of repair and restoration services. *Operations Research, 61*(6), 1295–1311.

Jerath, K., & Zhang, Z. J. (2010). Store within a store. *Journal of Marketing Research, 47*(4), 748–763.

Kim, S.-H., & Netessine, S. (2013). Collaborative cost reduction and component procurement under information asymmetry. *Management Science, 59*(1), 189–206.

Kim, S. K., & Wang, S. (1998). Linear contracts and the double moral hazard. *Journal of Economic Theory, 82*, 342–378.

Krishnan, H., Kapuscinski, R., & Butz, D. A. (2004). Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Science, 50*(1), 48–63.

Lal, R. (1990). Improving channel coordination through franchising. *Marketing Science, 9*(4), 299–318.

Lee, K. (1995). Optimal retail lease contracts: The principal-agent approach. *Regional Science and Urban Economics, 25*, 727–738.

Lim, W. S. (2000). A lemons market? An incentive scheme to induce truth-telling in third party logistics providers. *European Journal of Operational Research, 125*, 519–525.

Mathewson, G. F., & Winter, R. A. (1985). The economics of franchsie contracts. *The Journal of Law and Economics, 28*(3), 503–526.

Miceli, T. J., & Sirmans, C. F. (1995). Contracting with spatial externalities and agency problems: The case of shopping center leases. *Regional Science and Urban Economics, 25*, 355–372.

Misonzhnik, E. (2012a). Tenant incubators take off as landlords try to fill small shop space. National Real Estate Investor. [Available at http://nreionline.com/corporate-real-estate/tenant-incubators-take-landlords-try-fill-small-shop-space.].

Misonzhnik, E. (2012b). Store-within-a-store becomes expansion model dujour. National Real Estate Investor. [Available at http://nreionline.com/corporate-real-estate/store-within-store-becomes-expansion-model-dujour.].

Netemeyer, R. G., Heilman, C. M., & Maxham, J. G., III (2012). The impact of a new retail brand in-store boutique and its perceived fit with the parent retail brand on store performance and customer spending. *Journal of Retailing, 88*(4), 462–475.
Plambeck, E. L., & Taylor, T. A. (2006). Partnership in a dynamic production system with unobservable actions and noncontractible output. *Management Science, 52*(10), 1509–1527.
Porter, J., & Helm, B. (2008). Doing whatever gets them in the door. In *Businessweek (4090)* (pp. 60–61).
Porteus, E. L., & Whang, S. (1991). On manufacturing/marketing incentives. *Management Science, 37*(9), 1166–1181.
Rao, R. C. (1990). Compensating heterogeneous salesforces: Some explicit solutions. *Marketing Science, 9*(4), 319–341.
Roels, G., Karmarkar, U. S., & Carr, S. (2010). Contracting for collaborative services. *Management Science, 56*(5), 849–863.
Romano, R. E. (1994). Double moral hazard and resale price maintenance. *RAND Journal of Economics, 25*, 455–466.
Rui, H., & Lai, G. (2015). Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management, 24*(6), 934–946.
Saouma, R. (2008). Optimal second-stage outsourcing. *Management Science, 54*(6), 1147–1159.
Sen, B., Shin, J., & Sudhir, K. (2011). Demand externalities from co-location: Evidence from a natural experiment. *Working paper, Mimeo*. Yale University.
Sephora.com. (2013). Sephora Inside Jcp(accessed online on February 8, 2013) [Available at http://www.sephora.com/sephora-inside-jcpenney].
Snively, S. W. (2000). Killpercentagerent.com: Internet sales and shopping center leases. *Probate & Property Magazine*. July/Aug, http://www.americanbar.org/publications/probate_property_magazine_home/probate_2000_index/probate_july_aug_2000_index/rppt_publications_magazine_2000_ja00snively.html
Sorescu, A., Frambach, R. T., Singh, J., Rangaswamy, A., & Bridges, C. (2011). Innovation in retail business models. *Journal of Retailing, 87*(Supplement 1), S3–S16.
Stiglitz, J. E. (1974). Incentives and risk sharing in sharecropping. *The Review of Economic Studies, 41*(2), 219–255.
Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management Science, 48*(8), 992–1007.
Telegraph.co.uk. (2015). High street sales suffer sharpest fall in more than three years in march. [Available at http://www.telegraph.co.uk/finance/newsbysector/retailandconsumer/11513883/] High-street-sales-suffer-sharpest-fall-in-more-than-three-years-in-March.html].
Thompson, A. A., & Strickland, A. J. (2003). *Strategic management* (13th). McGraw-Hill.
Time.com., & McCracken, H. (2014). [Available at http://time.com/83648/sony-best-buy].
ULI (2004). *Dollars and cents of shopping centers: 2004*. Washington, D.C.: ULI- the Urban Land Institute.
Useem, J. (2007). Apple: America's best retailer. Fortune (March 8, 2007) (accessed online on February 8, 2013) [Available at http://money.cnn.com/magazines/fortune/fortune_archive/2007/03/19/8402321/index.htm].
van Ackere, A. (1993). The principal/agent paradigm: Its relevance to various functional fields. *European Journal of Operational Research, 70*, 83–103.
Wang, Y., Bell, D. R., & Padmanabhan, V. (2009). Manufacturer-owned retail stores. *Marketing Letters, 20*(2), 107–124.
Wheaton, W. C. (2000). Percentage rent in retail leasing: The alignment of landlord–tenant interests. *Real Estate Economics, 28*(2), 185–204.
Xing, D., & Liu, T. (2012). Sales effort free riding and coordination with price match and channel rebate. *European Journal of Operational Research, 219*(2), 264–271.
Xue, M., & Field, J. M. (2008). Service coproduction with information stickiness and incomplete contracts: Implications for consulting services design. *Production and Operations Management, 17*(3), 357–372.