



# Synthesis of Interplant Water Networks Using Principal Pipes. Part 1: Network Representation

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## Abstract

This paper presents a novel representation and formulation for the synthesis of interplant water networks with the goal of achieving low-cost and simple water network designs in industrial clusters or eco-industrial parks. Building upon a spatial representation of the problem, we introduce a new interplant connectivity representation that channels water flows collectively through sets of principal pipes. These principal pipes are represented as shared pipelines that can have different functions, i.e., water collection, water distribution, or water collection and distribution. Different principal pipe categories may be structured by selecting a combination of different initial/final/feeding/receiving entities which are allowed to be connected, to form a principal pipe. Moreover, several design aspects can also be explored within a principal pipe category such as (1) the selection of a prescribed water quality, (2) pipe routing options, (3) pipe flow directions, and (4) pipe sharing scenarios. Part 1 of the paper presents the backbone of the methodology that entails the assembly of different principal pipe categories, and also demonstrates how different design aspects may be accounted for in the design process.

**Keywords** Process integration · Network synthesis · Eco-industrial parks · Optimization

## Introduction

The design of water networks with reduced freshwater utilization and wastewater discharge often involves the application of water integration methods. Numerous methods have been proposed, which either rely on graphical techniques (Wang and Smith 1994; Hallale 2002; El-Halwagi et al. 2003; Manan et al. 2004; Prakash and Shenoy 2005), algebraic (Almutlaq et al. 2005; Foo et al. 2006), or mathematical optimization (Huang et al. 1999; Karuppiyah and Grossmann 2006; Kheireddine et al. 2011). The latter methods are capable of addressing larger and more complex systems. Water integration methods have initially focused on applications within a single plant and extended

later to address clusters of plants and eco-industrial parks through interplant water network (IPWN) design. One of the earliest research contributions in water integration across plants is the work by Olesen and Polley (1996) which applies pinch analysis insights based on graphical water integration techniques. Due to the complexity of interplant systems, many methods have subsequently been developed using mathematical programming techniques (Chew et al. 2008, 2009; Chew and Foo 2009; Lim and Park 2009; Lovelady and El-Halwagi 2009; Rubio-Castro et al. 2010; Aviso et al. 2010a, b; Boix et al. 2012; Montastruc et al. 2013). Chew et al. (2008) propose an interplant water integration (IPWI) approach which involves a centralized hub topology for collecting and redistributing water. Chew and Foo (2009) formulate a linear programming model for automated targeting of interplant water networks, primarily derived from pinch analysis techniques. Chew et al. (2009) propose a method for IPWI using game theory techniques that assess various company/plant interactions. Lim and Park (2009) develop interfactory and intrafactory water network designs based on environmental and economic performance factors. Lovelady and El-Halwagi (2009) propose a source-interceptor-sink representation for water network design in eco-industrial parks (EIPs). Rubio-Castro et al. (2010) propose a superstructure for handling wastewater reuse among

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different industries, along with an optimal selection for treatment units that satisfy process and environmental regulations for waste discharges. Aviso et al. (2010a, b) tackled interplant water and wastewater reuse in EIPs using fuzzy mathematical programming. Boix et al. (2012) developed a mixed-integer linear program (MILP) to design interplant industrial water networks, using a multiobjective optimization strategy. Montastruc et al. (2013) investigate the flexibility of water network designs in EIPs, based on the model provided by Boix et al. (2012). All the above contributions demonstrate significant interplant water network design improvements, but none of the methods tackle further the simplification strategies of interplant pipe network design.

Many water integration methods propose ways to retrofit water flows within a plant. It was found that introducing intermediate flow lines with a fixed quality would help manage and operate resulting water networks. Wang and Smith (1998) were the first to incorporate in-plant ‘water mains’ with a fixed water quality, using pinch analysis techniques. Feng and Seider (2001) also presented a water network methodology for single contaminant systems, using internal mains. Wang et al. (2003) integrated principal pipe options using the ‘water-saving factor’ concept. Other extensions were developed for multicontaminant systems (Cao et al. 2004; Su et al. 2012). Ma et al. (2007) introduced a rule-based design methodology to account for internal mains. Liu et al. (2009) presented a methodology using concentration potential information for water main placement, which sets a performing order for processes. Their approach was later utilized by Zhao et al. (2013, 2014).

In terms of interplant network connectivity, many works to date assume a network design representation with full direct connectivity, i.e., a separate pipe connection exists between each water source and water sink of the different plants considered (standard connectivity). The resulting networks tend to be complex and costly, with many parallel pipes across plants, each carrying different water qualities. Chew et al. (2008) state a number of motivations for investigating simplified connectivity in interplant water network designs, such as enhanced network controllability. Moreover, since geographical distances between different plants are normally larger than within plants, reduced piping costs may be achieved. To address such issues, a few works have recently emerged with alternative representations to yield simplified interplant connectivity as compared to the standard connectivity.

Alnouri et al. (2014a, 2015) propose a spatial representation for the industrial cluster to capture important geographical information for network design such as locations of plants, treatment units, and service corridors. The representation enables the routing of pipes to connect various relevant source and sink points in the industrial park or cluster and the identification of parallel pipe lines in service corridors. Building upon this spatial representation, Alnouri et al. (2014b) propose an approach for pipeline merging in IPWN design. The approach

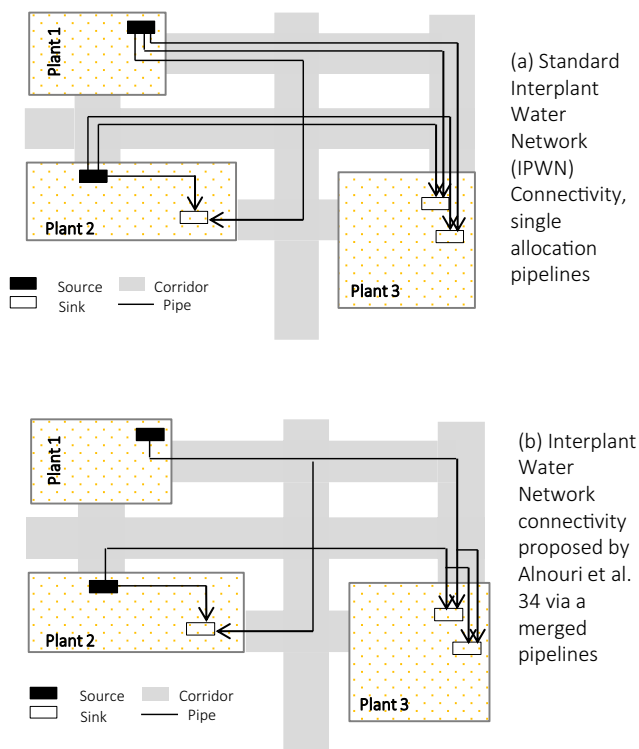
yields networks with reduced piping complexity and cost by avoiding parallel connections without sacrificing the water targets attainable with standard IPWN connectivity. This is because heterogeneity of water qualities is retained throughout the merged pipe networks. Pipeline lengths and sizes together with their placement in the corridors of the industrial park or cluster are determined in optimization.

Unlike for the case of water integration in a single facility, IPWNs typically establish connections across different stakeholders, such as plant owners, a utility to manage interplant water exchanges, and an administration in charge of regulating material and energy exchanges in the cluster. While the benefits of merged pipe connections from the approach by Alnouri et al. (2014b) are appealing in terms of reduced piping costs, further reductions in complexity can be desirable from an implementation standpoint to end up with simpler pipe networks that allow clearer management of water qualities. This work proposes a novel representation to synthesize such simplified networks by organizing interplant connections through principal pipes. In the next section, we review the existing IPWN interconnections proposed to date. The proposed representation of various principal pipe categories is then presented, followed by the detailed mathematical formulation.

## Background

In order to be able to contrast our proposed approach, we start by conceptually illustrating existing interplant connectivity representations in a small cluster of four plants with few water sources and sinks (Fig. 1). Sources and sinks are connected across the plants of the cluster through service corridors. The illustrations in Fig. 1 are for very simple networks considering direct reuse across two sources and three sinks with the purpose of contrasting the different proposed connectivity. Real cases can be expected to be considerably more complex as they may involve additional networks associated with freshwater supplies, wastewater discharges, and network connections associated with central or decentral water treatment.

In the standard connectivity (Chew et al. 2008, 2009; Chew and Foo 2009; Lim and Park 2009; Lovelady and El-Halwagi 2009; Rubio-Castro et al. 2010; Aviso et al. 2010a, b; Boix et al. 2012; Montastruc et al. 2013), a separate pipeline is used for each source to sink water allocation (Fig. 1a). The network is characterized by many parallel pipelines in the service corridors, with some carrying water of identical quality and others carrying water of different qualities. Water may flow in opposite directions through different pipes in the same corridor. Plant sinks may receive mixed quality water flows from different originating plants, which may cause operational problems during plant shutdowns. Complex multilateral stakeholder agreements would be required to assure the



**Fig. 1** a, b A comparison between the different proposed interplant connectivity representations

quality and flow of water received at sinks. Conceptually, the networks are characterized by large numbers of pipes and heterogeneous water qualities throughout the individual source to sink connections.

In the literature, interplant water networks have long not been reported in terms of their spatial layout as shown in Fig. 1. Only very recent work has considered the spatial layout of the industrial cluster (Alnouri et al. 2014a, 2015). This provides a basis for work into simplifying interplant connectivity with respect to the many pipe connections. A first attempt has been our recent contributions of a systematic methodology for simplifying interplant networks through merging strategies for pipe connections that are routed through service corridors (Alnouri et al. 2014b). The resulting networks have significantly less interconnecting pipes and are of lower cost as water is collectively transported. Merged pipe segments are identified through forward and backward branching strategies without mixing of water qualities that would result in increased water use over the standard connectivity (Fig. 1b). Despite the cost savings from merging, the sinks may receive mixed quality water flows from different originating plants as in the case of the standard connectivity, which may cause operational problems during plant shutdowns. Complex multilateral stakeholder agreements would still be required to assure the quality and flow of water received at sinks. Conceptually, the representation of interconnectivities through merged pipelines reduces network complexity and cost, but, in its current form, does not ease the management of water qualities in the network.

Despite of the progress made so far, there is currently no representation that is capable of exploiting the benefits of cost savings through collective water transport in merged connections while handling the issues associated with heterogeneous water qualities. While many water streams of different qualities exist in a plant, it must be emphasized that the sharing of potentially unassured qualities of water across different facilities and stakeholders presents a major obstacle in interplant water integration. Therefore, it is not advisable to consider very large numbers of segregated water qualities as the resulting system can become difficult to manage from a legal, regulatory, as well as an operational point of view. While all controls rest with one stakeholder in the case of in-plant integration, interplant integration is more difficult since different stakeholders are allowed to exchange water, and this often requires quality assurance. This work proposes a new representation as a first attempt to address this problem in IPWN design.

## Proposed Representation of Principal Pipes

Similar to the philosophy applied in the pipe merging approach (Alnouri et al. 2014b), principal pipes collectively transport water and avoid parallel lines that would increase cost. For instance, many water distribution systems, e.g., networks designed for domestic potable water supplies, are commonly designed using principal pipe structures. Similar to water distribution networks, water collection systems, e.g., networks designed for domestic wastewater collection, also employ principal pipes that collectively transport water from various water sources to treatment facilities available.

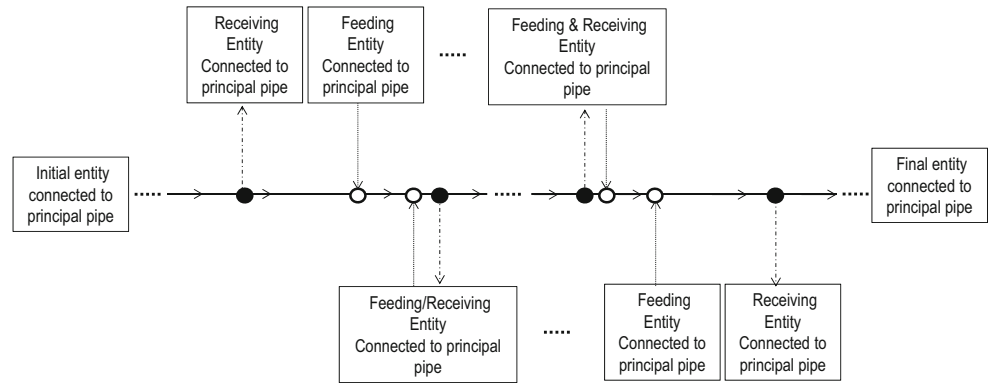
### General Steps for Structuring a Principal Pipe Category

Generally speaking, a principal pipe consists of feeding channels into the pipe, receiving channels from the pipe, as well as in-between pipe connecting segments, as illustrated in Fig. 2. Every receiving channel creates an intermediate splitting point along the pipe, and every water feeding channel creates an intermediate mixing point along the pipe. A special case for principal pipe designs may involve circulating flow, in which the first and last pipe segments must be connected. However, this work does not consider circulating flows in principal pipes. The following section details the steps required to generate a principal pipe category, based on the general representation illustrated in Fig. 2.

#### Step 1: Defining the Initial/Final Entity Connected to a Principal Pipe

The first step considered when defining a principal pipe is to identify which entity is the first pipe segment and last pipe

**Fig. 2** General principal pipe representation



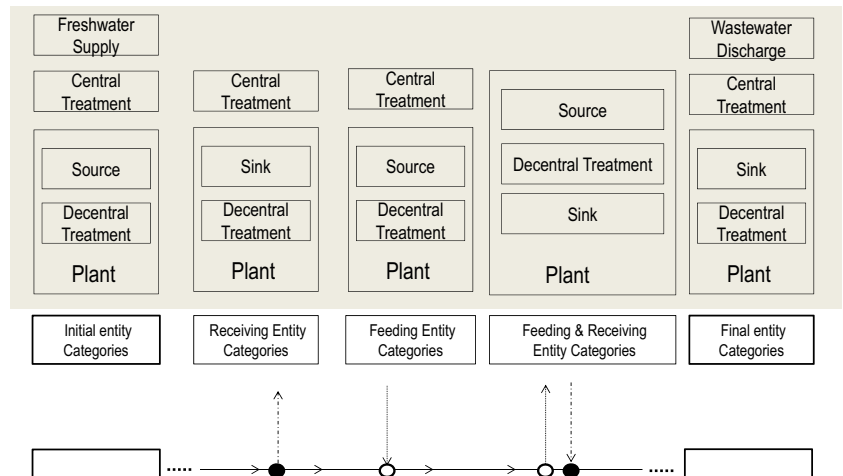
segment connected to. An ‘entity’ may be one of the following: (1) a plant, (2) a centralized treatment, (3) a freshwater supply source, and (4) a wastewater discharge point. It should be noted that a plant entity may refer to a water source type, a decentralized treatment type, a water sink type, or a combination of those types. The first pipe segment of any principal pipe may either be connected to a plant (by means of a source or decentralized treatment within the plant), a centralized treatment unit, or a freshwater supply point. The last pipe segment of any principal pipe may either be connected to a plant (by means of a sink, or a decentralized treatment within the plant), a centralized treatment unit, or a wastewater discharge point. It should be noted that concentrate ‘brine’ discharge streams may result from a number of water treatment options. As a result, concentrate discharge points are located differently than standard wastewater points, due to the disparity between the qualities of any concentrate streams resulting from treatment, compared to standard wastewater before treatment. Hence, a separate brine discharge entity has been considered as an additional final entity option which can accommodate resulting concentrate ‘brine’ water qualities from treatment (if present). This helps account for any streams that fall into this category, separately. Figure 3 illustrates all the different entities which may be used to assemble various combinations of initial-final pipe segment in principal pipes. It is evident that some combinations of

initial and final entities are highly unlikely (such as freshwater supply-to-wastewater discharge). Hence, an appropriate selection process for initial and final entity classifications per principal pipe category must be utilized to ensure that unreasonable connections are avoided.

**Step 2: Identifying Acceptable Feeding Entities/Receiving Entities**

After the initial and final entities in a principal pipe are identified, it is then important to determine (1) which entities are only allowed to supply water into the pipe (feeding entities), (2) which entities are only allowed to receive water from the pipe (receiving entities), and (3) which entities are allowed to both receive and supply water into/from the pipe. Figure 3 shows a classification of the different entities which may be used to assemble the possible feeding/receiving entities for principal pipes. It should be noted that a plant is classified as a receiving entity if the plant is only allowed to accept water from a pipe, into sinks, and decentralized treatment. On the other hand, a plant is defined as a feeding entity if the plant is only allowed to supply water from wastewater sources and decentralized treatment, into the pipe. Moreover, a plant can be both a feeding and a receiving entity, in case the plant is allowed to supply water and receive water simultaneously.

**Fig. 3** Structuring a principal pipe category



### Principal Pipe Category Structuring Examples

The structuring process for any principal pipe category may be achieved according to the steps outlined above. In order to illustrate the principal pipe category structuring process, we have established a number of different principal pipe categories, based on commonly existing examples that are often found useful in interplant water network connectivity. In order to illustrate the logic proposed above, we derive the following five different principal pipe categories, which are described below:

**Freshwater Pipe(s)** This category involves the placement of principal pipes for the purpose of collecting freshwater from freshwater sources and distributing freshwater to sinks across the plants. Often, sources originating from process operations are not allowed to supply water into a fresh principal pipe type (including treated water), so as not to disrupt the prescribed quality of freshwater being supplied, especially if sinks include potential human consumption (e.g., in office buildings). Figure 4a illustrates the fresh principal pipe structuring process in terms of the initial entity, final entity, as well as feeding and receiving entities.

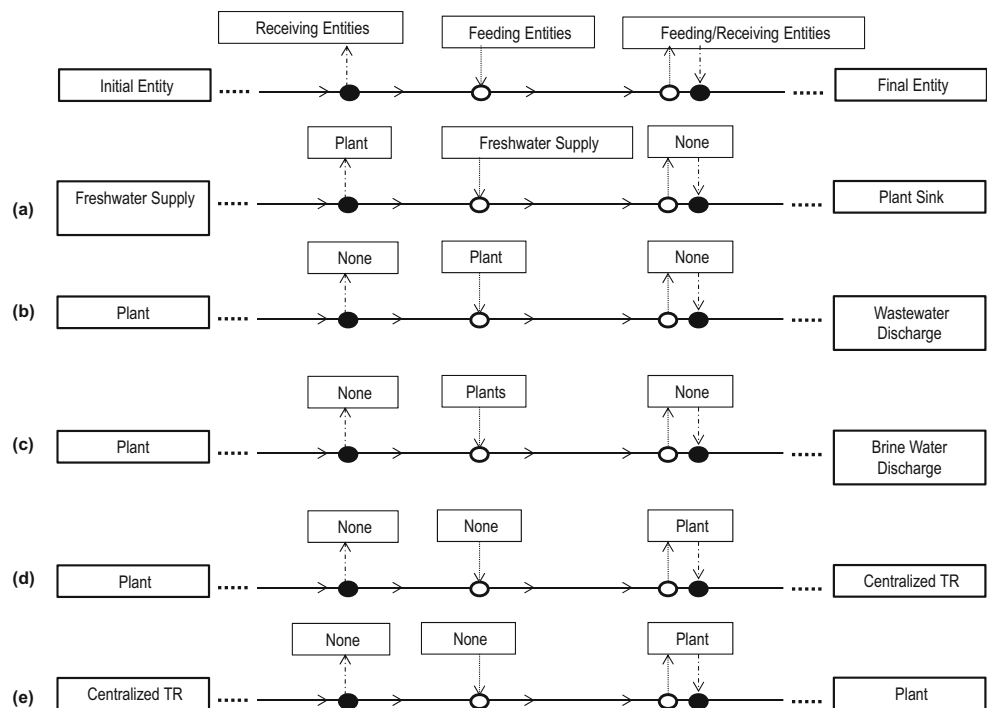
**Wastewater Pipe(s)** This category involves the placement of a shared principal pipe(s) for the purpose of collecting wastewater from onsite wastewater-producing operations, to a common wastewater disposal region. This type of principal pipe is designed to collect process wastewater, mainly with permissible contamination levels that are allowed to be disposed beyond plant boundaries, at designated wastewater disposal region(s).

Moreover, onsite water-consuming facilities are assumed not to be allowed to receive water from a wastewater pipe type, since all unwanted/unused wastewater is collected and sent to disposal in a wastewater pipe type. Figure 4b illustrates wastewater pipe structuring process in terms of the initial entity, final entity, as well as feeding and receiving entities.

**Brine (Reject) Pipe(s)** This category involves the placement of a shared principal pipe(s) for collecting any brine produced from both onsite and offsite wastewater treatment entities, to a common wastewater disposal region. This type of principal pipe is designed to collect wastewater produced from treatment only, mainly in the form of highly saline (brine) waste. Hence, onsite water-consuming entities are not allowed to receive water from a brine pipe type, since all rejected wastewater from central and decentral treatment is collected and sent to disposal, in this category. Figure 4c illustrates the brine pipe structuring process in terms of the initial entity, final entity, as well as feeding and receiving entities.

**Collection-Distribution (Coll-Dist) Pipe(s)** This category involves the placement of a shared principal pipe(s) for the purpose of transporting wastewater from onsite wastewater-producing entities, to centralized treatment facilities. This type of principal pipe is utilized to collectively send wastewater to centralized treatment. Onsite water-consuming facilities are allowed to receive water from a coll-dist pipe in case the water quality being transported before treatment is able to meet sink requirements when being mixed with other water qualities, such as freshwater, at the sink. It should also be noted that

**Fig. 4 a–e** Structuring examples for commonly occurring principal pipe categories



the placement of a coll-dist pipe type is not feasible in industrial areas that do not incorporate centralized treatment facilities into the city infrastructure, because the last pipe element which forms a coll-dist pipe is set to connect to a centralized treatment unit. Hence, coll-dist pipes establish the basic infrastructure for wastewater collection to central treatment among all participating plants. Figure 4b illustrates the coll-dist pipe structuring process in terms of the initial entity, final entity, as well as feeding and receiving entities.

**Distribution-Collection (Dist-Coll) Pipe(s)** This category involves the placement of a shared principal pipe(s) for the purpose of transporting treated water from both onsite and offsite treatment entities, to onsite water-consuming facilities. This type of principal pipe is utilized to distribute treated water (from decentral/central treatment) to all water sinks within an industrial zone. It should also be noted that the placement of a distribution dist-coll pipe is not feasible in industrial areas that do not incorporate centralized treatment facilities into the city infrastructure. This is because the main pipe elements that form a dist-coll pipe are set to emerge from centralized treatment units, and hence, establishes the basic infrastructure for centrally treated water distribution among all participating plants. Figure 4e illustrates the dist-coll pipe structuring process in terms of the initial entity, final entity, as well as feeding and receiving entities.

Hence, the general functions of the principal pipe categories considered in this work include freshwater supply and distribution, untreated or partially treated wastewater collection and distribution (from sources in plants and decentral treatment to further treatment), centrally and/or decentrally treated wastewater collection and distribution (from treatment to sinks across plants, further treatment or discharge), and wastewater collection from sources to discharge. Combining all established principal pipe categories, together with the respective connectivity identified for each, generates a principal pipe interplant superstructure representation, as illustrated in Fig. 5. It should be noted that no connectivity has been allowed between any two different principal pipes of any type, so as to ensure water quality segregation throughout, which enables effective water quality regulation and control within the pipelines. Please note that this is not a spatial representation of the problem. In case more principal pipe categories are required, the same procedure outlined above may be followed.

### General Steps for Obtaining Principal Pipe Options Within a Category

There may be multiple, separate principal pipes in the cluster with the same basic functionality, e.g., multiple freshwater supply lines or multiple distribution lines with different treated water or wastewater qualities, interplant routing, and flow directions. Therefore, several options for principal pipe placement within a category may be applied according to the options that follow.

### Step 1: Set a Water Quality Specification

Water qualities in principal pipes will often vary spatially, especially when streams being supplied into a principal pipe, at various feeding points, are of different qualities. Moreover, water quality considerations in a multistakeholder setting is important due to potential operational variations, e.g., from individual plant shutdowns. Therefore, a water quality specification must be identified for each principal pipe option. This would help set limits on maximum permissible contamination levels for streams that are allowed to be supplied into a pipe, upon collection from a feeding entity. Moreover, this would set a minimum guaranteed standard for the water being transported in a principal pipe, before distribution to a receiving entity. Appropriate water quality constraints must always be introduced for each of the different types of principal pipes, which may be derived from guidelines available for standard types of water quality. Moreover, several heuristics could be applied when prescribing a water quality per pipe, which in turn can be very strict or relaxed, depending on the principal pipe functionality. For instance, a principal pipe designed to distribute freshwater must be associated with a water quality that is able to meet sink quality specifications, or better. Hence, all principal pipes options that are defined in this category would have very tight water quality requirements per pipe. A principal pipe designed to collect wastewater may be associated with discharge limit water quality standard on feeding channels, or alternatively may be assigned a water quality at the discharge limit at the discharge point, standard, where all streams would be assumed to be equally mixed. Moreover, water quality specifications on a pipe connected to a treatment entity must ensure all contamination levels being fed into the principal pipe can be handled by the treatment system.

### Step 2: Set a Routing Option

Routing principal pipes within a cluster of plants must also be established, based on locations/presence of all entities that may be allowed to connect to the principal pipe. In each case, the principal pipe must ensure access to/from all plants. Moreover, all principal pipe pipeline structures must be placed within the prescribed industrial city corridors. The routing must consider access port locations and must be able to cover access to all possible feeding/receiving entities, all the way up the final entity connected to the principal pipe. In this work, two different options are proposed for the routing of a principal pipe while covering the entire city area: (1) a clockwise-like rotation throughout the city and (2) a counterclockwise-like rotation throughout the city. Figures 6 and 7 illustrate examples of the two routing options for each of the principal pipe categories derived in this work.

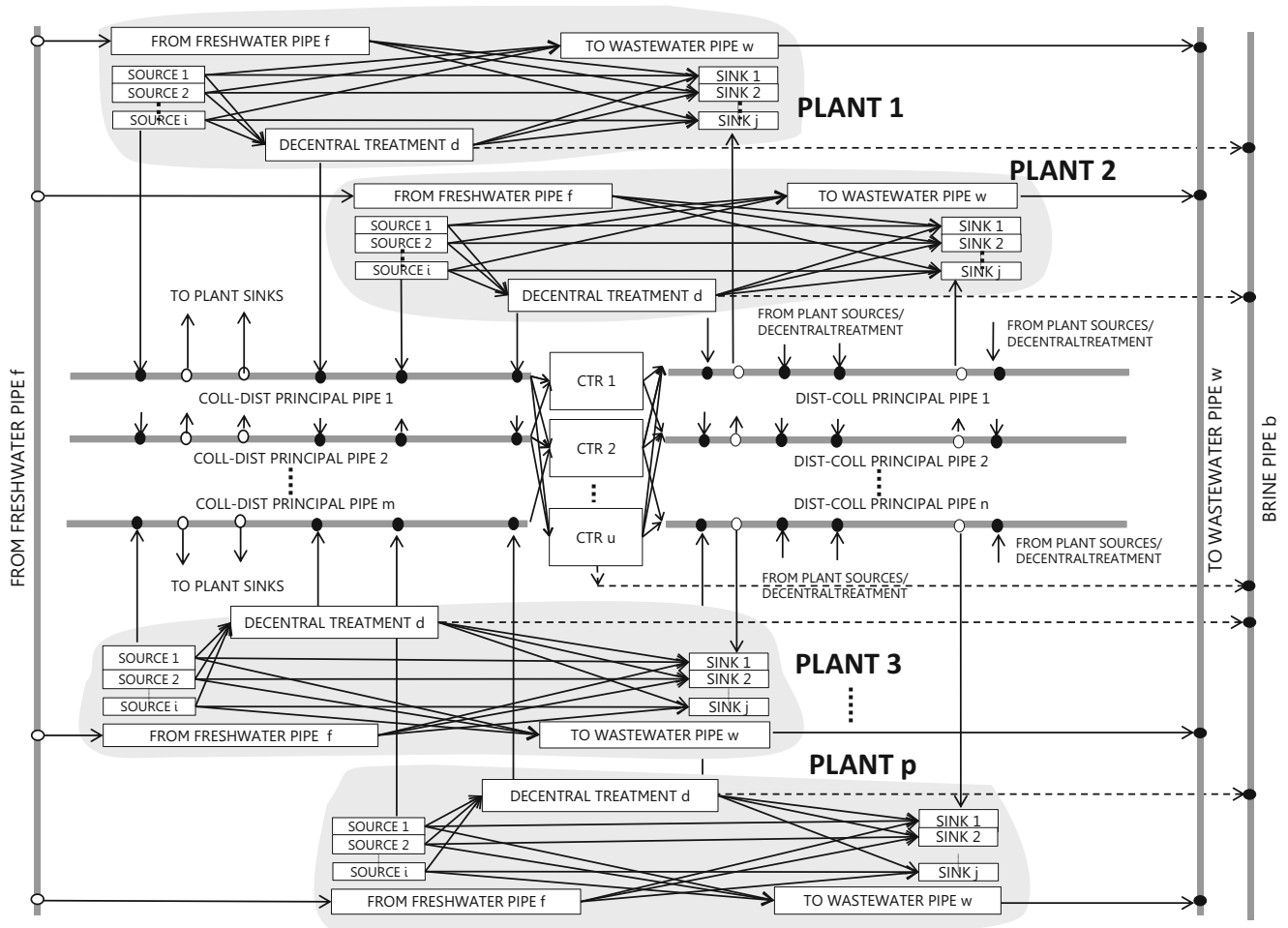


Fig. 5 Principal pipe connectivity for a water reuse/regeneration network, based on established principal pipe categories

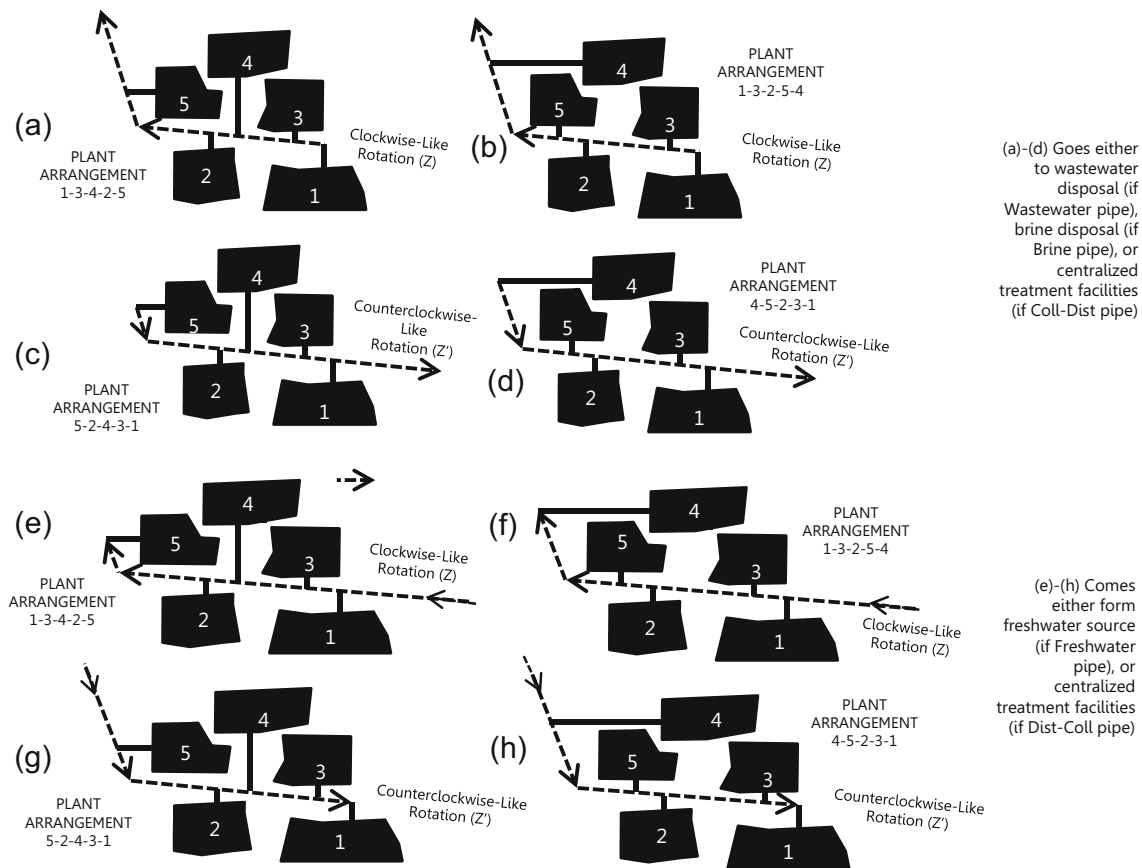
**Step 3: Set a Water Flow Direction for Each Routing Option**

For each routing option, every principal pipe must also be assigned a flow direction, which in turn is dictated by the locations of the initial and final entity connected to principal pipes. The principal pipe flow direction is important for determining any intermediate flow increase/decrease within the various pipe connecting segments, along the pipe. The water flow direction in any pipeline within a water reuse/regeneration network is usually determined according to a prearranged configuration of water sources, sinks, and treatment units, contained by a given industrial city layout. Generally speaking, a water source directs the flow of water away from its location through pipeline arrangements. Conversely, a water sink draws in the flow of water through pipeline structures, toward its location. Moreover, since a treatment unit always operates as an interception point in a water network, all central and decentral treatment units are assigned independent source and sink locations that are used to distinguish treated water flows and wastewater flows, respectively. Hence, source locations within treatment units send the flow of treated water toward onsite water-using

entities, after treatment. On the other hand, sink locations indicated within treatment units are designed to bring in the flow of wastewater toward treatment locations. Since the flow direction of water being transported via any pipeline would need to be from a source location toward a sink location, water flow can always be established by specifying source and sink sites of the respective pipeline assigned to carry out the allocation. Hence, water flow directions within a standard water reuse and regeneration network design problem may be carried out upfront, once the initial and final entities connected to a principal pipe are established. Each of the routing options that cover the entire city area are assigned a flow direction: (1) clockwise-like flow direction and (2) counterclockwise-like flow direction. Figures 6 and 7 illustrate examples of the flow directions attained based on each of the routing options for each of the principal pipe categories derived in this work.

**Step 4: Prescribe a Plant Arrangement for Each Routing Option**

Principal pipe placement also involves the identification of the plant arrangement associated with each flow direction



**Fig. 6** a–g Illustrative demonstration for different principal pipe routing/flow direction/plant arrangements

possibility. Given all access port locations to and from each plant in the city, and given both the clockwise-like and the counterclockwise-like flow direction possibilities based on the different routing options, the respective plant arrangement for each flow direction can be determined. The plant arrangement alongside a principal pipe always sets the order at which the pipe collects/transmits/distributes water, according to the order of access port presence to and from each pipe. Figure 6 demonstrates how the respective plant arrangement can be obtained for each potential flow direction, using allotted corridor and access port locations. The corresponding plant arrangement derived for a clockwise-like flow direction option of a principal pipe is always the reverse of the plant arrangement associated with its counterclockwise-like flow possibility. This applies to any principal pipe category. For instance, Fig. 6a, c, e, g corresponds to the same layout of a five-plant example. In all cases, the corresponding plant arrangements for each clockwise-like (Fig. 6a, e) and counterclockwise-like (Fig. 6c, g) flow direction are derived, and both are shown to yield the exact opposite arrangement. Having altered the access port locations for plants 4 and 5, Fig. 6b, d, f, h corresponds to a slightly altered layout than Fig. 6a, c, e, g, using the same five-plant example. The updated plant arrangement for each clockwise-like (Fig. 6b, f) and counterclockwise-like (Fig. 6d, h) flow direction is derived and was also found to be

reversed, when compared to each other. It is evident from the demonstration provided in Fig. 6 that a slight change in corridor and access port locations, which are assigned to enable water transport to/from every plant within the city, significantly affects the corresponding plant arrangement that is associated with every flow direction.

### Step 5: Establish Selective-Sharing or No-Sharing Scenarios (if Desired)

The proposed methodology is also capable of assigning principal pipes to a selection of plants, or even to a single plant, for cases in which partial pipeline sharing, or no pipeline sharing, is desired. This could be achieved by simply ensuring that the plant arrangement associated with each principal pipes, defined for all flow direction options, only involves the corresponding plants which are assigned to deliver/supply water to/from the pipe defined. A principal pipeline which is defined using a single plant-to-pipe connection corresponds to a no-sharing scenario, while a principal pipeline defined using at least two plant-to-pipe connections corresponds to a selective pipeline-sharing scenario. Figure 7 illustrates examples for both selective pipeline-sharing and no pipeline-sharing options. It should be also noted that if selective-sharing and no-sharing options are to be considered, then all plants should be

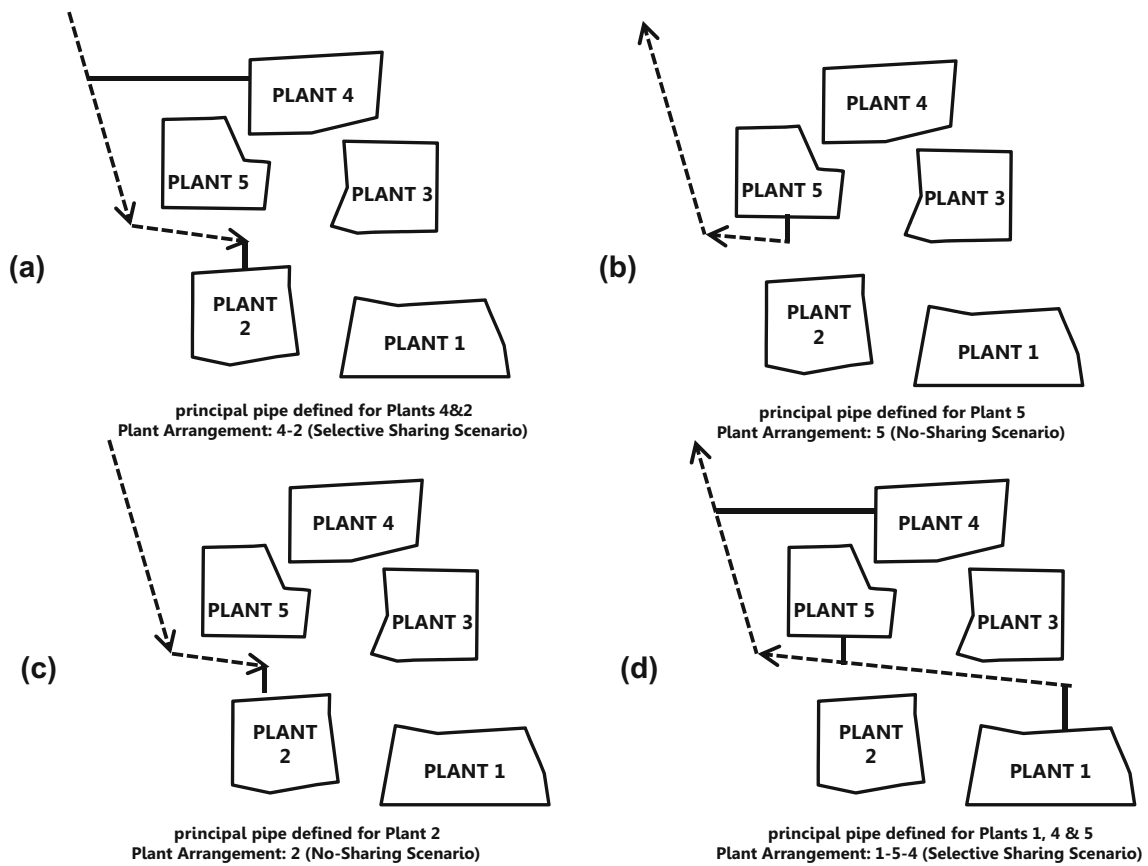


Fig. 7 a–d Example demonstrating selective-sharing and no-sharing scenarios of principal pipelines

connected to at least one defined principal pipe in every category, so as to ensure feasible designs.

### Mathematical Formulation

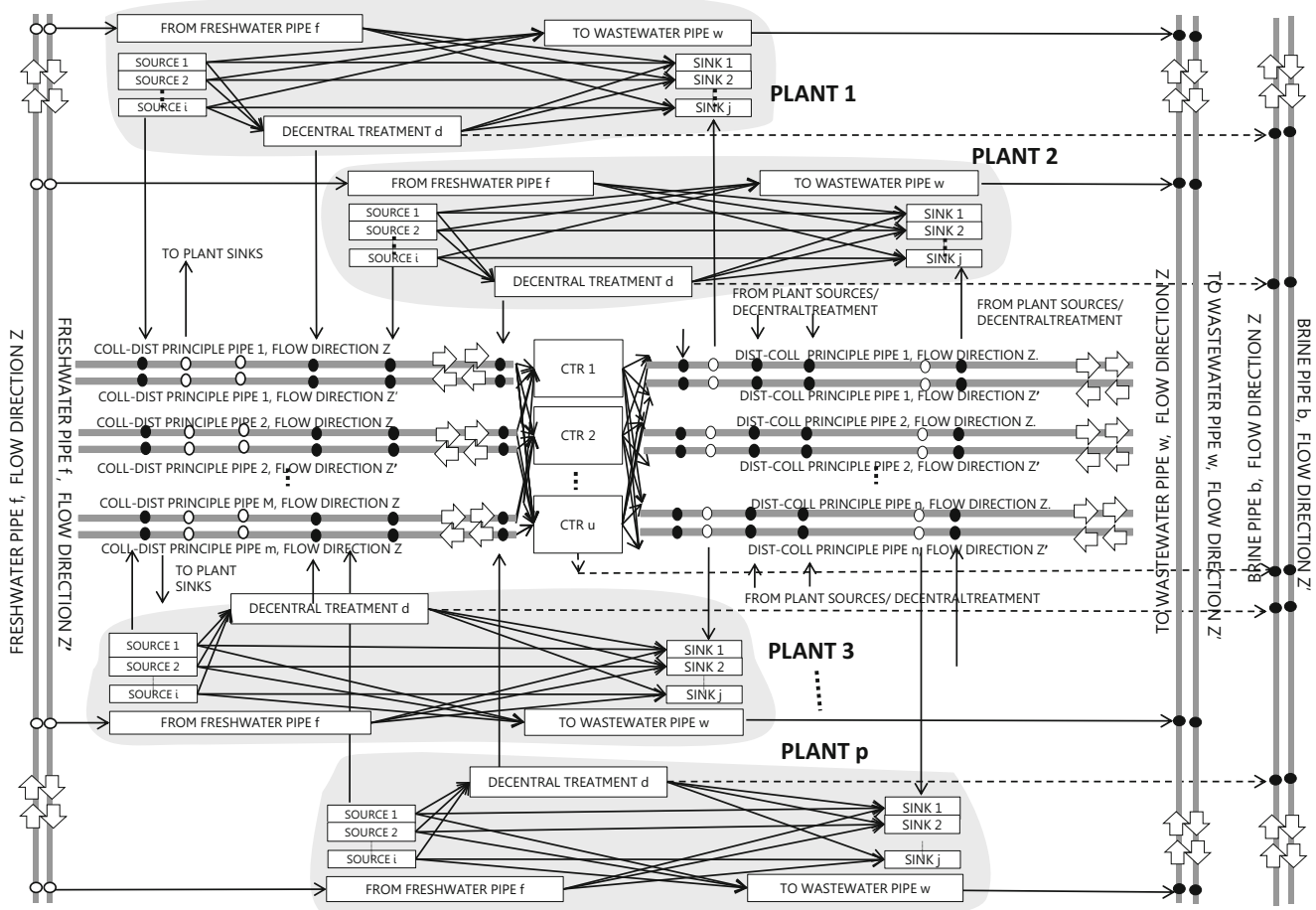
The following problem formulation has been proposed, based on the principal pipe types which have been structured in the previous section. Figure 8 demonstrates an all-inclusive inter-plant superstructure, which accounts for the in-category principal pipe options discussed above. In case more principal pipe categories (and options within category) are to be included in the problem, it would be very straightforward to derive an extended formulation, using the same structure that is described in the following sections. It should also be stressed out that the formulation presented is completely new, and has been carefully designed so as to capture the appropriate IPWN connectivity.

The following sections provide all the details for the mathematical constraints required to formulate the principal pipe connectivity described by the methodology presented above. The formulation presented in this work is very different than previously presented formulations, including Alnouri et al. (2014b). The use of node levels to identify the different branches of a pipe is no longer

adopted. Instead, plant locations are now mainly used as a guideline for identifying the main branches in any generated pipe. Moreover, two different flow directions for each generated pipe have also been accounted for in this work. In doing so, it is realized that a set of mass balances and inequality constraints for each of the sources, sinks, interceptors, and all defined principal pipe categories within the system are required to mathematically formulate this problem, as presented by Eqs. (1)–(70) below.

### Total and Component Balances

The total flow rate for any given source  $i$  in plant  $p$  ( $W_{p,i}^{Source}$ ) must equal the summation of flows from source  $i$ , plant  $p$  that are directly passed onto all sinks  $j$  in the same plant  $p$  ( $F_{p,i,j}$ ), plus the summation of flows from source  $i$ , plant  $p$  into all decentral treatment units  $d$  in the same plant  $p$  ( $F_{p,i,d,p}$ ), plus the summation of flows from source  $i$ , plant  $p$  to all coll-dist principal pipes  $m$  across all directions  $z$  and  $z'$  ( $F_{p,i,m,z}$  and  $F_{p,i,m,z'}$ ), plus the summation of flows from source  $i$ , plant  $p$  to all dist-coll principal pipes  $n$  across all directions  $z$  and  $z'$  ( $F_{p,i,n,z}$  and  $F_{p,i,n,z'}$ ), plus the summation of flows from source  $i$ , plant  $p$  to all wastewater pipes  $w$  across all directions  $z$  and  $z'$  ( $F_{p,i,w,z}$  and  $F_{p,i,w,z'}$ ), as per Eq. (1) below:



**Fig. 8** Generalized principal pipe connectivity in a water reuse/regeneration network revisited with principal pipeline flow direction possibilities incorporated

$$\begin{aligned}
 W_{p,i}^{\text{Source}} &= \sum_{p \in P} \sum_{j \in \text{SN}_p} F_{p,i,j,p} + \sum_{d \in \text{DTR}_p} \sum_{p \in P} F_{p,i,d,p} \\
 &+ \sum_{m \in \text{CPIPES}} (F_{p,i,m,z} + F_{p,i,m,z'}) \\
 &+ \sum_{n \in \text{DPIPES}} (F_{p,i,n,z} + F_{p,i,n,z'}) \\
 &+ \sum_{\substack{w \in \text{WPIPES} \\ \forall p \in P; \forall i \in \text{SU}_p}} (F_{p,i,w,z} + F_{p,i,w,z'})
 \end{aligned} \tag{1}$$

The total flow rate into any decentralized treatment unit  $d$  defined across a given plant  $p$ , received from all sources  $i$  that are available within the same plant  $p$ , ( $F_{i,p,d,p}$ ), multiplied by a water recovery factor dictated by the treatment technology ( $\xi_{d,p}$ ), must equal the summation of flow rates passed onto all coll-dist principal pipes  $m$  across all corresponding directions  $z$  and  $z'$  ( $F_{d,p,m,z}$  and  $F_{d,p,m,z'}$ ), plus the summation of flow rates passed onto all dist-coll principal pipes  $n$  across all corresponding flow directions  $z$  and  $z'$  ( $F_{d,p,n,z}$  and  $F_{d,p,n,z'}$ ), plus the summation of flow rates passed onto all sinks  $j$  within

the same plant  $p$  ( $F_{d,p,j,p}$ ), as shown by Eq. (2) below. This equation corresponds to the total treated water balance for a decentralized treatment unit.

$$\begin{aligned}
 \xi_{d,p} \sum_{p \in P} \sum_{i \in \text{SU}_p} F_{i,p,d,p} \\
 = \sum_{m \in \text{CPIPES}} (F_{d,p,m,z} + F_{d,p,m,z'}) \\
 + \sum_{n \in \text{DPIPES}} (F_{d,p,n,z} + F_{d,p,n,z'}) \\
 + \sum_{p \in P} \sum_{j \in \text{SN}_p} F_{d,p,j,p} \quad \forall p \in P; \forall d \in \text{DTR}_p
 \end{aligned} \tag{2}$$

The total flow rate losses in any decentralized treatment unit  $d$  defined across a given plant  $p$  ( $F_{d,p}^{\text{loss}}$ ) must equal the total flow rate into any decentralized treatment unit  $d$  defined across a given plant  $p$ , received from all sources  $i$  that are available within the same plant  $p$ , ( $F_{i,p,d,p}$ ), multiplied by a water loss factor dictated by the treatment technology ( $\eta_{d,p}$ ), according to Eq. (3) below. This equation corresponds to the total water loss balance for a decentral treatment unit.

$$F_{d,p}^{loss} = \eta_{d,p} \sum_{p \in P} \sum_{i \in SU_p} F_{i,p,d,p} \quad \forall p \in P; \forall d \in DTR_p \quad (3)$$

The total flow rate into any decentralized treatment unit  $d$  defined across a given plant  $p$ , received from all sources  $i$  that are available within the same plant  $p$ , ( $F_{i,p,d,p}$ ), multiplied by the corresponding untreated water factor ( $1 - \xi_{d,p} - \eta_{d,p}$ ), for which both water recovery ( $\xi_{d,p}$ ) and water losses ( $\eta_{d,p}$ ) are dictated by the treatment technology, must equal the summation of flow rates into all brine pipes  $b$  across all directions  $z$  and  $z'$  ( $F_{d,p,b,z}$  and  $F_{d,p,b,z'}$ ) as shown in Eq. (4) below. This equation corresponds to the total untreated water balance for a decentral treatment unit.

$$\begin{aligned} & (1 - \xi_{d,p} - \eta_{d,p}) \sum_{p \in P} \sum_{i \in SU_p} F_{i,p,d,p} \\ &= \sum_{b \in BPIPES} (F_{d,p,b,z} + F_{d,p,b,z'}) \end{aligned} \quad (4)$$

$$\forall p \in P; \forall d \in DTR_p$$

The inlet concentration of a component  $c$ , into a decentral treatment unit  $d$ , in plant  $p$  ( $x_{d,p,c}^{Inlet}$ ) can be obtained from Eq. (5) below, which corresponds to the product of the given concentration levels of component  $c$  associated with water source  $i$ , plant  $p$ , ( $y_{p,i,c}^{Source}$ ), each multiplied by the corresponding stream flow rate term from source  $i$ , plant  $p$  into all decentral treatment units  $d$  in the same plant  $p$  ( $F_{p,i,d,p}$ ), divided by the summation of all flow rate terms, as per Eq. (5) below:

$$x_{d,p,c}^{Inlet} = \frac{\sum_{p \in P} \sum_{i \in SU_p} y_{p,i,c}^{Source} F_{p,i,d,p}}{\sum_{p \in P} \sum_{i \in SU_p} F_{p,i,d,p}} \quad (5)$$

$$\forall p \in P; \forall d \in DTR_p; \forall c \in C$$

The treated water stream concentration from a decentral treatment unit  $d$ , in plant  $p$  can be calculated from the inlet concentration into the system ( $x_{d,p,c}^{Inlet}$ ) coupled with the respective contamination removal level of component  $c$  in the system ( $1 - \psi_{d,p,c}$ ), dictated by the decentral treatment technology per component in the system, all divided by water recovery ( $\xi_{d,p}$ ) as shown in Eq. (6) below:

$$x_{d,p,c}^{Treated} = x_{d,p,c}^{Inlet} (1 - \psi_{d,p,c}) / \xi_{d,p} \quad (6)$$

$$\forall p \in P; \forall d \in DTR_p; \forall c \in C$$

The untreated water stream concentration from the decentral treatment unit  $d$ , in plant  $p$  can be calculated from

the inlet concentration into the system ( $x_{d,p,c}^{Inlet}$ ) coupled with the respective contamination of component  $c$  removed from the system ( $\psi_{d,p,c}$ ), dictated by the decentral treatment technology per component in the system, all divided by the corresponding untreated water factor ( $1 - \xi_{d,p} - \eta_{d,p}$ ), composed of both water recovery ( $\xi_{d,p}$ ) and water losses ( $\eta_{d,p}$ ), as shown in Eq. (7) below:

$$x_{d,p,c}^{Untreated} = x_{d,p,c}^{Inlet} (\psi_{d,p,c}) / (1 - \xi_{d,p} - \eta_{d,p}) \quad (7)$$

$$\forall p \in P; \forall d \in DTR_p; \forall c \in C$$

The total flow rate in a coll-dist principal pipe  $m$  from flow directions  $z$  and  $z'$  that are passed onto all central treatment units  $u$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ) must equal the summation of treated flow rates received from all decentralized treatment units  $d$  across all plants  $p$ , from coll-dist principal pipe  $m$  with flow direction  $z$  from flow directions  $z$  and  $z'$  ( $F_{d,p,m,z}$  and  $F_{d,p,m,z'}$ ) plus flow rates received from all sources  $i$ , across all plants  $p$  from coll-dist principal pipe  $m$  from flow directions  $z$  and  $z'$  ( $F_{p,i,m,z}$  and  $F_{p,i,m,z'}$ ), minus the summation of flow rates passed on to all sinks  $j$ , across all plants  $p$  from coll-dist principal pipe  $m$  from flow directions  $z$  and  $z'$  ( $F_{m,z,j,p}$  and  $F_{m,z',j,p}$ ), according to Eq. (8) below:

$$\begin{aligned} & \sum_{c \in CTR} (F_{m,z,u} + F_{m,z',u}) \\ &= \sum_{d \in DTR_p} \sum_{p \in P} (F_{d,p,m,z} + F_{d,p,m,z'}) \\ &+ \sum_{p \in P} \sum_{i \in SU_p} (F_{p,i,m,z} + F_{p,i,m,z'}) \\ &- \sum_{p \in P} \sum_{i \in SU_p} (F_{m,z,j,p} + F_{m,z',j,p}) \quad \forall m \in CPIPES \end{aligned} \quad (8)$$

Equations (9)–(12) ensure that the water quality in each coll-dist principal pipe  $m$ , with possible flow directions  $z$  and  $z'$ , must at least meet the defined water quality, ( $z_{m,c}^{Pipe}$ ), in each of the streams providing water into the pipe, for each contaminant present in the system.

$$\sum_{d \in DTR_p} \sum_{p \in P} z_{m,c}^{Pipe} (F_{d,p,m,z}) \geq \sum_{d \in DTR_p} \sum_{p \in P} x_{d,p,c}^{Treated} (F_{d,p,m,z}) \quad (9)$$

$$\forall m \in CPIPES; \forall c \in C$$

$$\sum_{d \in DTR_p} \sum_{p \in P} z_{m,c}^{Pipe} (F_{d,p,m,z'}) \geq \sum_{d \in DTR_p} \sum_{p \in P} x_{d,p,c}^{Treated} (F_{d,p,m,z'}) \quad (10)$$

$$\forall m \in CPIPES; \forall c \in C$$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{m,c}^{\text{Pipe}}(F_{p,i,m,z}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} y_{p,i,c}^{\text{Source}}(F_{p,i,m,z}) \quad (11)$$

$$\forall m \in \text{CPIPES}; \forall c \in C$$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{m,c}^{\text{Mains}}(F_{p,i,m,z'}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} y_{p,i,c}^{\text{Source}}(F_{p,i,m,z'}) \quad (12)$$

$$\forall m \in \text{CPIPES}; \forall c \in C$$

The total flow rate from coll-dist principal pipe  $m$  across directions  $z$  and  $z'$  into a centralized treatment unit  $u$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ) multiplied by a treated water recovery factor dictated by the treatment technology ( $\xi_u$ ) must equal the summation of treated flow rates from centralized treatment unit  $u$  passed onto all dist-coll principal pipe  $n$  across directions  $z$  and  $z'$  ( $F_{u,n,z}$  and  $F_{u,n,z'}$ ), according to Eq. (13) below. This equation corresponds to the total treated water balance for a central treatment unit.

$$\begin{aligned} \xi_u \sum_{m \in \text{CPIPES}} (F_{m,z,u} + F_{m,z',u}) \\ = \sum_{n \in \text{DPIPES}} (F_{u,n,z} + F_{u,n,z'}) \\ \forall u \in \text{CTR} \end{aligned} \quad (13)$$

The total flow rate losses in any centralized treatment unit  $u$  ( $F_u^{\text{loss}}$ ) must equal the total flow rate into any centralized treatment  $u$ , received from all coll-dist principal pipe  $m$  across directions  $z$  and  $z'$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ), multiplied by a water loss factor dictated by the treatment technology ( $\eta_u$ ), according to Eq. (14) below. This equation corresponds to the total water loss balance for a central treatment unit.

$$F_u^{\text{loss}} = \eta_u \sum_{m \in \text{CPIPES}} (F_{m,z,u} + F_{m,z',u}) \quad \forall u \in \text{CTR} \quad (14)$$

The total flow rate into any centralized treatment unit  $u$ , received from all coll-dist principal pipe  $m$  across directions  $z$  and  $z'$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ), multiplied by the corresponding untreated water factor ( $1 - \xi_u - \eta_u$ ), for which both water recovery ( $\xi_u$ ) and water losses ( $\eta_u$ ) are dictated by the treatment technology, must equal the summation of flow rates into all brine principal pipe  $b$  across all directions  $z$  and  $z'$  ( $F_{u,b,z}$  and  $F_{u,b,z'}$ ) as shown in Eq. (15) below. This equation corresponds to the total untreated water balance for a central treatment unit.

$$\begin{aligned} (1 - \xi_u - \eta_u) \sum_{m \in \text{CPIPES}} (F_{m,z,u} + F_{m,z',u}) \\ = \sum_{b \in \text{BPIPES}} (F_{u,b,z} + F_{u,b,z'}) \\ \forall u \in \text{CTR} \end{aligned} \quad (15)$$

The inlet concentration of a component  $c$ , into a central treatment unit  $u$  ( $x_{u,c}^{\text{Inlet}}$ ), can be obtained from Eq. (16) below, which corresponds to the product of the given concentration levels of component  $c$  associated with received from all coll-dist principal pipe  $m$  across directions  $z$  and  $z'$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ), multiplied by the corresponding stream flow rate term from all coll-dist principal pipe  $m$  across directions  $z$  and  $z'$  ( $F_{m,z,u}$  and  $F_{m,z',u}$ ), divided by the summation of the total flow rate terms, according to Eq. (16) below:

$$x_{u,c}^{\text{Inlet}} = \frac{\sum_{m \in \text{CPIPES}} z_{m,c}^{\text{Pipe}} (F_{m,z,u} + F_{m,z',u})}{\sum_{m \in \text{CPIPES}} (F_{m,z,u} + F_{m,z',u})} \quad (16)$$

$$\forall u \in \text{CTR}; \forall c \in C$$

The treated water stream concentration from the central treatment unit  $u$  can be calculated from the inlet concentration into the system ( $x_{u,c}^{\text{Inlet}}$ ) coupled with the respective contamination removal level of component  $c$  in the system ( $1 - \psi_{u,c}$ ), dictated by the central treatment technology per component in the system, all divided by water recovery ( $\xi_u$ ) as shown in Eq. (17) below:

$$x_{u,c}^{\text{Treated}} = x_{u,c}^{\text{Inlet}} (1 - \psi_{u,c}) / \xi_u \quad (17)$$

$$\forall u \in \text{CTR}; \forall c \in C$$

The untreated water stream concentration from the central treatment unit  $u$  can be calculated from the inlet concentration into the system ( $x_{u,c}^{\text{Inlet}}$ ) coupled with the respective contamination of component  $c$  removed from the system ( $\psi_{u,c}$ ), dictated by the central treatment technology per component in the system, all divided by the corresponding untreated water factor ( $1 - \xi_u - \eta_u$ ), composed of both water recovery ( $\xi_u$ ), and water losses ( $\eta_u$ ), as shown in Eq. (18) below:

$$x_{u,c}^{\text{Untreated}} = x_{u,c}^{\text{Inlet}} (\psi_{u,c}) / (1 - \xi_u - \eta_u) \quad (18)$$

$$\forall u \in \text{CTR}; \forall c \in C$$

The total flow rate from all centralized treatment units  $u$  into dist-coll principal pipe  $n$  across flow direction  $z$  and  $z'$  ( $F_{u,n,z}$  and  $F_{u,n,z'}$ ), plus the summation of flow rates from all decentralized treatment units  $d$  across plant  $p$  into dist-coll principal pipe  $n$  with flow direction  $z$  and  $z'$  ( $F_{d,p,n,z}$  and  $F_{d,p,n,z'}$ ), plus the summation of flow rates from all sources  $i$ , across all plants  $p$  into dist-coll principal pipe  $n$  with flow direction  $z$  and  $z'$  ( $F_{p,i,n,z}$  and  $F_{p,i,n,z'}$ ), must equal the total flow rate from dist-coll principal pipe  $n$  with flow directions  $z$

and  $z'$  into all sinks  $j$  across plants  $p$  ( $F_{n,z,j,p}$  and  $F_{n,z',j,p}$ ), shown by Eq. (19) below:

$$\begin{aligned} & \sum_{u \in \text{CTR}} (F_{u,n,z} + F_{u,n,z'}) + \sum_{p \in P} \sum_{d \in \text{DTR}_p} (F_{d,p,n,z} + F_{d,p,n,z'}) \\ & + \sum_{p \in P} \sum_{i \in \text{SU}_p} (F_{p,i,n,z} + F_{p,i,n,z'}) \\ & = \sum_{p \in P} \sum_{j \in \text{SN}_p} (F_{n,z,j,p} + F_{n,z',j,p}) \quad \forall n \in \text{DPIPES} \end{aligned} \quad (19)$$

Equations (20)–(25) ensure that the water quality in each coll-dist principal pipe  $m$ , with possible flow directions  $z$  and  $z'$ , must at least meet the defined water quality ( $z_{m,c}^{\text{Mains}}$ ) in each of the streams providing water into the pipe.

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{u,n,z}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} x_{u,c}^{\text{Treated}} (F_{u,n,z}) \quad (20)$$

$\forall n \in \text{DMAINS}; \forall c \in C$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{u,n,z'}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} x_{u,c}^{\text{Treated}} (F_{u,n,z'}) \quad (21)$$

$\forall n \in \text{DPIPES}; \forall c \in C$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{d,p,n,z}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} x_{d,p,c}^{\text{Treated}} (F_{d,p,n,z}) \quad (22)$$

$\forall n \in \text{DPIPES}; \forall c \in C$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{d,p,n,z'}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} x_{d,p,c}^{\text{Treated}} (F_{d,p,n,z'}) \quad (23)$$

$\forall n \in \text{DPIPES}; \forall c \in C$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{p,i,n,z}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} y_{p,i,c}^{\text{Source}} (F_{p,i,n,z}) \quad (24)$$

$\forall n \in \text{DPIPES}; \forall c \in C$

$$\sum_{d \in \text{DTR}_p} \sum_{p \in P} z_{n,c}^{\text{Pipe}} (F_{p,i,n,z'}) \geq \sum_{d \in \text{DTR}_p} \sum_{p \in P} y_{p,i,c}^{\text{Source}} (F_{p,i,n,z'}) \quad (25)$$

$\forall n \in \text{DPIPES}; \forall c \in C$

The total flow rate for any given sink  $j$  in plant  $p$  ( $G_{j,p}^{\text{Sink}}$ ) must equal the summation of flows from all water sources  $i$  in plant  $p$  to water sinks  $j$  in the same plant  $p$  ( $F_{p,i,j,p}$ ), plus all flows from all coll-dist principal pipe  $m$  across flow directions  $z$  and  $z'$  into sink  $j$ , plant  $p$  ( $F_{m,z,j,p}$  and  $F_{m,z',j,p}$ ), plus all flows from all dist-coll principal pipes  $n$  across flow directions

$z$  and  $z'$  into sink  $j$ , plant  $p$  ( $F_{n,z,j,p}$  and  $F_{n,z',j,p}$ ), plus the summation of flows from all freshwater pipe  $f$  across flow directions  $z$  and  $z'$  into sink  $j$ , plant  $p$  ( $F_{f,z,j,p}$  and  $F_{f,z',j,p}$ ), according to Eq. (26) below:

$$\begin{aligned} G_{j,p}^{\text{Sink}} &= \sum_{p \in P} \sum_{i \in \text{SU}_p} F_{p,i,j,p} + \sum_{m \in \text{CPIPES}} (F_{m,z,j,p} + F_{m,z',j,p}) \\ &+ \sum_{n \in \text{DPIPES}} (F_{n,z,j,p} + F_{n,z',j,p}) \\ &+ \sum_{f \in \text{FPIPES}} (F_{f,z,j,p} + F_{f,z',j,p}) \quad \forall p \in P; \quad \forall j \in \text{SN}_p \end{aligned} \quad (26)$$

Similarly, Eq. (27) defines the individual component inequalities which are based on the total sink balance given by Eq. (26), by combining it with the respective concentrations of all individual streams that represent the total balance.

$$\begin{aligned} G_{j,p}^{\text{Sink}} z_{j,p,c}^{\text{Sink}} &\geq \sum_{p \in P} \sum_{i \in \text{SU}_p} y_{p,i,c}^{\text{Source}} F_{p,i,j,p} \\ &+ \sum_{m \in \text{CPIPES}} z_{m,c}^{\text{Mains}} (F_{m,z,j,p} + F_{m,z',j,p}) \\ &+ \sum_{n \in \text{DPIPES}} z_{n,c}^{\text{Mains}} (F_{n,z,j,p} + F_{n,z',j,p}) \\ &+ \sum_{f \in \text{FPIPES}} z_{f,c}^{\text{Fresh}} (F_{f,z,j,p} + F_{f,z',j,p}) \end{aligned} \quad (27)$$

$\forall p \in P; \quad \forall j \in \text{SN}_p; \forall c \in C$

The total water flow rate used from freshwater pipe  $f$  ( $F_f$ ) must equal the summation of flows from freshwater pipe  $f$  from both directions  $z$  and  $z'$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$  and  $F_{f,z',j,p}$ ), according to Eq. (28) below:

$$F_f = \sum_{p \in P} \sum_{j \in \text{SN}_p} (F_{f,z,j,p} + F_{f,z',j,p}) \quad \forall f \in \text{FPIPES} \quad (28)$$

The total wastewater flow rate into wastewater pipe  $w$  ( $F_w$ ) must equal the summation of flows from all sources  $i$  across all plants  $p$  into wastewater pipe  $w$  from both directions  $z$  and  $z'$  ( $F_{p,i,w,z}$  and  $F_{p,i,w,z'}$ ), according to Eq. (29) below:

$$F_w = \sum_{p \in P} \sum_{j \in \text{SN}_p} (F_{p,i,w,z} + F_{p,i,w,z'}) \quad \forall w \in \text{WPIPES} \quad (29)$$

Equation (30) defines the individual component constraints imposed on any waste stream sent to a wastewater pipe  $w$ , based on the total waste balance given by Eq. (29). The summation of all flow rate terms combined with the respective concentrations of all individual streams that represent the total

balance must not exceed the maximum allowable limit represented by  $(F_{wz,w,c}^{Waste})$ :

$$F_{wz,w,c}^{Waste} \geq \sum_{p \in P} \sum_{j \in SN_p} (F_{p,i,w,z} + F_{p,i,w,z'}) y_{p,i,c}^{Source} \quad (30)$$

$\forall w \in WPIPES; \forall c \in C$

The total wastewater flow rate into brine principal pipe  $b$  ( $F_b$ ) must equal the summation of flows from all decentral treatment units  $d$  across all plants  $p$  into brine principal pipe  $b$  from both directions  $z$  and  $z'$  ( $F_{d,p,b,z}$  and  $F_{d,p,b,z'}$ ), plus the summation of flows from all central treatment units  $u$  from both directions  $z$  and  $z'$  ( $F_{u,b,z}$  and  $F_{u,b,z'}$ ), according to Eq. (31) below:

$$F_b = \sum_{p \in P} \sum_{d \in DTR_p} (F_{d,p,b,z} + F_{d,p,b,z'}) + \sum_{u \in CTR} (F_{u,b,z} + F_{u,b,z'}) \quad (31)$$

$\forall b \in BPIPES \forall c \in C$

Similar to Eq. (31), Eq. (32) defines the individual component constraints imposed on any waste stream sent to a brine principal pipe  $w$ , based on the total waste balance given by Eq. (30). The summation of all flow rate terms combined with the respective concentrations of all individual streams that represent the total balance must not exceed the maximum allowable limit represented by  $(F_{bz,b,c}^{Brine})$ :

$$F_{bz,b,c}^{Brine} \geq \sum_{p \in P} \sum_{d \in DTR_p} (F_{d,p,b,z} + F_{d,p,b,z'}) x_{d,p,c}^{Untreated} + \sum_{u \in CTR} (F_{u,b,z} + F_{u,b,z'}) x_{u,c}^{Untreated} \quad (32)$$

$\forall b \in BPIPES; \forall c \in C$

### Flow Balances for Principle Pipe Segments

Equations (33)–(70) provided in the following section below describe the individual water balance flows of each of the principal pipe categories, within the respective pipeline segments that are used to assemble each pipe. The equations described below utilize clockwise flow directions ( $z$ ) as an illustration, so as to keep this section as concise as possible. However, counterclockwise ( $z'$ ) flow balances for each of the principal pipe categories must also be derived for the problem in a similar manner to Eqs. (33)–(70). This can simply be done by replacing all

plant arrangement sets with the ones associated with counterclockwise flow directions, as well as by replacing all flow variables with the ones that correspond to counterclockwise flow, in Eqs. (33)–(70). Hence, an additional set of equations that resemble Eqs. (33)–(70), but correspond to the counterclockwise flow options, must also be incorporated in the overall problem formulation.

### Freshwater Pipes

The summation of flows from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$ ), must equal the summation of freshwater passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{f,z,j,A}$ ), plus the summation of freshwater passed onto all sinks in the second plant that lies in the  $z$  flow direction ( $F_{f,z,j,B}$ ), plus the summation of freshwater passed onto all sinks in the third plant that lies in the  $z$  flow direction ( $F_{f,z,j,C}$ ), plus all the remaining summations up to the  $N^{\text{th}}$  existing plant ( $F_{f,z,j,N}$ ).

$$\sum_{p \in P} \sum_{j \in SN_p} F_{f,z,j,p} = \sum_{j \in SN_A} F_{f,z,j,A} + \sum_{j \in SN_B} F_{f,z,j,B} + \sum_{j \in SN_C} F_{f,z,j,C} + \dots + \sum_{j \in SN_{N-1}} F_{f,z,j,(N-1)} + \sum_{j \in SN_N} F_{f,z,j,N} \quad (33)$$

$\forall f \in FPIPES; A, B, C..N \in P_{f,z} [A, B, C..N \in P \leftrightarrow A, B, C..N \in P_{f,z}]$

The summation of the flow in freshwater pipe  $f$  with direction  $z$  (corresponding to segment before the first plant, according to the flow arrangement  $z$  ( $F_{f,z,A}^{SEG}$ )), must equal the summation of flows from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$ ).

$$F_{f,z,A}^{SEG} = \sum_{p \in P} \sum_{j \in SN_p} F_{f,z,j,p} \quad (34)$$

$\forall f \in FPIPES; A \in P_{f,z} [A \in P \leftrightarrow A \in P_{f,z}]$

The summation of remaining flow in freshwater pipe  $f$  with direction  $z$  (corresponding to the segment between the first and second plant) that is not passed onto any sinks in the first plant according to the  $z$  flow direction ( $F_{f,z,B}^{SEG}$ ) must equal the summation of flows from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$ ), minus the summation of freshwater passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{f,z,j,A}$ ).

$$F_{f,z,B}^{SEG} = \sum_{p \in P} \sum_{j \in SN_p} F_{f,z,j,p} - \sum_{j \in SN_A} F_{f,z,j,A} \quad \forall f \in FPIPES; A \in P_{f,z} [A \in P \leftrightarrow A \in P_{f,z}] \quad (35)$$

The summation of remaining flow in freshwater pipe  $f$  with direction  $z$  (corresponding to the segment between the second and third plant) that is not passed onto any sinks in the first and second plants according to the  $z$  flow direction ( $F_{f,z,C}^{SEG}$ ) must equal the summation of flows from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$ ), minus the summation of freshwater passed onto all sinks in the first and second plants in the  $z$  flow direction ( $F_{f,z,j,A}$  and  $F_{f,z,j,B}$ ), respectively.

$$F_{f,z,C}^{SEG} = \sum_{p \in P} \sum_{j \in SN_p} F_{f,z,j,p} - \sum_{j \in SN_A} F_{f,z,j,A} - \sum_{j \in SN_B} F_{f,z,j,B} \quad (36)$$

$\forall f \in FPIPES; A, B \in P_{f,z} [A, B \in P \leftrightarrow A, B \in P_{f,z}]$

The summation of remaining flow in freshwater pipe  $f$  with direction  $z$  (corresponding to the segment between the  $(N-1)^{th}$  and  $N^{th}$  plant) that passed onto  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{f,z,N-1}^{SEG}$ ) must equal the summation of flows from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{f,z,j,p}$ ), minus the summation of freshwater passed onto all sinks in the first, second, third all up to the  $(N-1)^{th}$  plant in the  $z$  flow direction ( $F_{f,z,j,A}$  and  $F_{f,z,j,B}$  and  $F_{f,z,j,C}$  and  $F_{f,z,j,(N-1)}$ ), respectively.

$$F_{f,z,N}^{SEG} = \sum_{p \in P} \sum_{j \in SN_p} F_{f,z,j,p} - \sum_{j \in SN_A} F_{f,z,j,A} - \sum_{j \in SN_B} F_{f,z,j,B} - \sum_{j \in SN_C} F_{f,z,j,C} - \dots - \sum_{j \in SN_{N-1}} F_{f,z,j,(N-1)}$$

$\forall f \in FPIPES; A, B, C \dots N-1 \in P_{f,z} [A, B, C \dots N-1 \in P \leftrightarrow A, B, C \dots N-1 \in P_{f,z}]$

(37)

The summation of freshwater passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{f,z,j,A}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{f,z,j,p}$ )

$$\sum_{j \in SN_A} F_{f,z,j,A} = \sum_{j \in SN_p} F_{f,z,j,p} \quad \forall f \in FPIPES; \forall (A = p); \quad (38)$$

$p \in P [p \in P \leftrightarrow p \in P_{f,z}]; A \in P_{f,z} [A \in P \leftrightarrow A \in P_{f,z}]$

Similarly, the summation of freshwater passed onto all sinks in the second plant, third plant, and any  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{f,z,j,N}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from freshwater pipe  $f$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{f,z,j,p}$ )

$$\sum_{j \in SN_N} F_{f,z,j,N} = \sum_{j \in SN_p} F_{f,z,j,p} \quad \forall f \in FPIPES; \forall (N = p); \quad (39)$$

$p \in P [p \in P \leftrightarrow p \in P_{f,z}]; N \in P_{f,z} [N \in P \leftrightarrow N \in P_{f,z}]$

### Coll-Dist Pipe(s)

The summation of flow in coll-dist principal pipe  $m$  with direction  $z$ , that is passed across for treatment to all centralized treatment units  $u$  ( $F_{m,z,u}$ ), must equal the summation of collected water from all sources within plants  $A$  up until  $N$ , and decentralized treatment units within plants  $A$  up until  $N$ , minus the summation of collected water passed onto all sinks within plants  $A$  up until  $N$ .

$$\sum_{u \in CTR} F_{m,z,u} = \left( \sum_{i \in SU_A} F_{A,i,m,z} + \sum_{i \in SU_B} F_{B,i,m,z} + \sum_{i \in SU_C} F_{C,i,m,z} + \dots + \sum_{i \in SU_{N-1}} F_{N-1,i,m,z} + \sum_{i \in SU_N} F_{N,i,m,z} \right) + \left( \sum_{d \in DTR_A} F_{d,A,m,z} + \sum_{d \in DTR_B} F_{d,B,m,z} + \sum_{d \in DTR_C} F_{d,C,m,z} + \dots + \sum_{d \in DTR_{N-1}} F_{d,N-1,m,z} + \sum_{d \in DTR_N} F_{d,N,m,z} \right) - \left( \sum_{j \in SN_A} F_{m,z,j,A} + \sum_{j \in SN_B} F_{m,z,j,B} + \sum_{j \in SN_C} F_{m,z,j,C} + \dots + \sum_{j \in SN_{N-1}} F_{m,z,j,(N-1)} + \sum_{j \in SN_N} F_{m,z,j,N} \right)$$

$\forall m \in CPIPES; A, B, C \dots N \in P_{m,z} [A, B, C \dots N \in P \leftrightarrow A, B, C \dots N \in P_{m,z}]$

The summation of remaining flow in coll-dist principal pipe  $m$  with direction  $z$  (corresponding to the segment between the first and second plant) that is not passed onto any sinks in the first plant according to the  $z$  flow direction ( $F_{m,z,A}^{SEG}$ ) must equal the summation of water received from all sources  $i$  in the

first plant that lies in the  $z$  flow direction ( $F_{A,i,m,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the first plant that lies in the  $z$  flow direction ( $F_{d,A,m,z}$ ), minus the summation of collected water passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{m,z,j,A}$ ).

$$F_{m,z,A}^{SEG} = \sum_{i \in SU_A} F_{A,i,m,z} + \sum_{d \in DTR_A} F_{d,A,m,z} - \sum_{j \in SN_A} F_{m,z,j,A} \quad (41)$$

$$\forall m \in CPIPEs; A \in P_{m,z} [A \in P \leftrightarrow A \in P_{m,z}]$$

The summation of remaining flow in coll-dist principal pipe  $m$  with direction  $z$  (corresponding to the segment between the second and third plant) that is not passed onto any sinks in the first and second plants in the  $z$  flow direction ( $F_{m,z,B}^{SEG}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{m,z,A}^{SEG}$ ), plus the summation of water received from all sources  $i$  in the second plant that lies in the  $z$  flow direction ( $F_{B,i,m,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the second plant that lies in the  $z$  flow direction ( $F_{d,B,m,z}$ ), minus the summation of collected water passed onto all sinks in the second plant that lies in the  $z$  flow direction ( $F_{m,z,j,B}$ ).

$$F_{m,z,B}^{SEG} = F_{m,z,A}^{SEG} + \sum_{i \in SU_B} F_{B,i,m,z} + \sum_{d \in DTR_B} F_{d,B,m,z} - \sum_{j \in SN_B} F_{m,z,j,B}$$

$$\forall m \in CPIPEs; A, B \in P_{m,z} [A, B \in P \leftrightarrow A, B \in P_{m,z}] \quad (42)$$

The summation of remaining flow in coll-dist principal pipe  $m$  with direction  $z$  (corresponding to the segment between the third and fourth plant) that is not passed onto any sinks in the first, second, and third plants in the  $z$  flow direction ( $F_{m,z,C}^{SEG}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{m,z,A}^{SEG}$ ), plus the summation of flow in the segment between the second and

third plant ( $F_{m,z,B}^{SEG}$ ), plus the summation of water received from all sources  $i$  in the third plant that lies in the  $z$  flow direction ( $F_{C,i,m,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the third plant that lies in the  $z$  flow direction ( $F_{d,C,m,z}$ ), minus the summation of collected water passed onto all sinks in the third plant that lies in the  $z$  flow direction ( $F_{m,z,j,C}$ ).

$$F_{m,z,C}^{SEG} = F_{m,z,A}^{SEG} + F_{m,z,B}^{SEG} + \sum_{i \in SU_C} F_{C,i,m,z} + \sum_{d \in DTR_C} F_{d,C,m,z} - \sum_{j \in SN_C} F_{m,z,j,C}$$

$$\forall m \in CPIPEs; A, B, C \in P_{m,z} [A, B, C \in P \leftrightarrow A, B, C \in P_{m,z}] \quad (43)$$

The summation of remaining flow in coll-dist principal pipe  $m$  with direction  $z$  (corresponding to the segment between the  $(N-1)^{th}$  and  $N^{th}$  plant) that is passed onto  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{m,z,N-1}^{SEG}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{m,z,A}^{SEG}$ ), plus the summation of flow in the segment between the second and third plant ( $F_{m,z,B}^{SEG}$ ), plus the summation of flow in the segment between the third and fourth plant ( $F_{m,z,C}^{SEG}$ ), all up until the segment corresponding to  $(N-2)^{th}$  plant, plus the summation of water received from all sources  $i$  in the  $(N-1)^{th}$  plant that lies in the  $z$  flow direction ( $F_{N-1,i,m,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the  $(N-1)^{th}$  plant that lies in the  $z$  flow direction ( $F_{d,N-1,m,z}$ ), minus the summation of collected water passed onto all sinks in the  $(N-1)^{th}$  plant that lies in the  $z$  flow direction ( $F_{m,z,j,N-1}$ ).

$$F_{m,z,(N-1)}^{SEG} = F_{m,z,A}^{SEG} + F_{m,z,B}^{SEG} + F_{m,z,C}^{SEG} + \dots + F_{m,z,N-2}^{SEG} + \sum_{i \in SU_{N-1}} F_{N-1,i,m,z} + \sum_{d \in DTR_{N-1}} F_{d,N-1,m,z} - \sum_{j \in SN_{N-1}} F_{m,z,j,N-1} \quad (44)$$

$$\forall m \in CPIPEs; A, B, C, \dots, N-1 \in P_{m,z} [A, B, C, \dots, N-1 \in P \leftrightarrow A, B, C, \dots, N-1 \in P_{m,z}]$$

The summation of remaining flow in coll-dist principal pipe  $m$  after the  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{m,z,N}^{SEG}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{m,z,A}^{SEG}$ ), plus the summation of flow in the segment between the second and third plant ( $F_{m,z,B}^{SEG}$ ), plus the summation of flow in the segment between the third and fourth plant ( $F_{m,z,C}^{SEG}$ ), all up until the segment corresponding to  $(N-1)^{th}$  plant, plus the summation of water received from all sources  $i$  in the  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{N,i,m,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{d,N,m,z}$ ), minus the summation of collected water passed onto all sinks in the  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{m,z,j,N}$ ), which also equals the total

flow in the coll-dist principal pipe  $m$  with direction  $z$ , that is passed across for treatment to all centralized treatment units  $u$  ( $F_{m,z,u}$ )

$$F_{m,z,N}^{SEG} = F_{m,z,A}^{SEG} + F_{m,z,B}^{SEG} + F_{m,z,C}^{SEG} + \dots + F_{m,z,N-1}^{SEG} + \sum_{i \in SU_{N-1}} F_{N,i,m,z} \quad (45)$$

$$+ \sum_{d \in DTR_{N-1}} F_{d,N,m,z} - \sum_{j \in SN_{N-1}} F_{m,z,j,N} = \sum_{u \in CTR} F_{m,z,u}$$

$$\forall m \in CPIPEs; N \in P_{m,z} [N \in P \leftrightarrow N \in P_{m,z}]$$

The summation of flow rate terms received from or passed onto the first plant in the  $z$  flow direction ( $F_{m,z,j,A}$ ,  $F_{A,i,m,z}$ ,  $F_{d,A,m,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from coll-dist principal pipe  $m$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{m,z,j,p}$ ,  $F_{p,i,m,z}$ ,  $F_{d,p,m,z}$ )

$$\begin{aligned}
 \sum_{j \in \text{SN}_A} F_{m,z,j,A} &= \sum_{j \in \text{SN}_p} F_{m,z,j,p}; \sum_{i \in \text{SU}_A} F_{A,i,m,z} \\
 &= \sum_{i \in \text{SU}_p} F_{p,i,m,z}; \sum_{d \in \text{DTR}_A} F_{d,A,m,z} = \sum_{d \in \text{DTR}_p} F_{d,p,m,z} \quad (46) \\
 \forall m \in \text{CPIPES}; \forall (A = p); p \in P [p \in P \leftrightarrow p \in P_{m,z}]; \\
 &A \in P_{m,z} [A \in P \leftrightarrow A \in P_{m,z}]
 \end{aligned}$$

Similarly, the summation of flow rate terms received from or passed onto the second plant, third plant, and any  $N^{\text{th}}$  plant in the  $z$  flow direction ( $F_{n,z,j,N}$ ,  $F_{N,i,n,z}$ ,  $F_{d,N,n,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from coll-dist principal pipe  $m$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{m,z,j,p}$ ,  $F_{p,i,m,z}$ ,  $F_{d,p,m,z}$ )

$$\begin{aligned}
 \sum_{j \in \text{SN}_N} F_{m,z,j,C} &= \sum_{j \in \text{SN}_p} F_{m,z,j,p}; \sum_{i \in \text{SU}_N} F_{C,i,m,z} \\
 &= \sum_{i \in \text{SU}_p} F_{p,i,m,z}; \sum_{d \in \text{DTR}_N} F_{d,C,m,z} = \sum_{d \in \text{DTR}_p} F_{d,p,m,z} \quad (47) \\
 \forall m \in \text{CPIPES}; \forall (N = p); p \in P [p \in P \leftrightarrow p \in P_{m,z}]; \\
 &N \in P_{m,z} [N \in P \leftrightarrow N \in P_{m,z}]
 \end{aligned}$$

### Dist-Coll Pipe(s)

The summation of flows from dist-coll principal pipe  $n$  with direction  $z$  into all sinks  $j$ , across all plants  $p$  ( $F_{n,z,j,p}$ ) must equal the summation of distributed water passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{n,z,j,A}$ ), plus the summation of distributed water passed onto all sinks in the second plant that lies in the  $z$  flow direction ( $F_{n,z,j,B}$ ), plus the summation of distributed water passed onto all sinks in the third plant that lies in the  $z$  flow direction ( $F_{n,z,j,C}$ ), plus all the remaining summations up to the  $(N - 1)^{\text{th}}$  existing plant ( $F_{n,z,j,N-1}$ ).

$$\begin{aligned}
 \sum_{p \in P} \sum_{j \in \text{SN}_p} F_{n,z,j,p} &= \sum_{j \in \text{SN}_A} F_{n,z,j,A} + \sum_{j \in \text{SN}_B} F_{n,z,j,B} \\
 &+ \sum_{j \in \text{SN}_C} F_{n,z,j,C} + \dots + \sum_{j \in \text{SN}_{N-1}} F_{n,z,j,(N-1)} \quad (48) \\
 \forall n \in \text{DPIPES}; A, B, C, \dots, N \in P_{n,z} [A, B, C, \dots, N \in P \leftrightarrow A, B, C, \dots, N \in P_{n,z}]
 \end{aligned}$$

The summation of flow in dist-coll principal pipe  $n$  with direction  $z$  (corresponding to the segment before the first plant) must equal the summation of flow from centralized treatment units  $u$ , that is passed onto dist-coll principal pipe  $n$  with direction  $z$  ( $F_{u,n,z}$ )

$$F_{n,z,A}^{\text{SEG}} = \sum_{u \in \text{CTR}} F_{u,n,z} \quad \forall n \in \text{DPIPES}; A \in P_{n,z} [A \in P \leftrightarrow A \in P_{n,z}] \quad (49)$$

The summation of remaining flow in dist-coll principal pipe  $n$  with direction  $z$  (corresponding to the segment between the first and second plant) that is not passed onto any sinks in the first plant along the  $z$  flow direction) ( $F_{n,z,A}^{\text{SEG}}$ )

must equal the summation of the initial flow in the distributed principal pipe  $f$  with direction  $z$  into all sinks  $j$ , which corresponds to the treated water flow passed onto the main ( $F_{u,n,z}$ ), plus the summation of water received from all sources  $i$  in the first plant that lies in the  $z$  flow direction ( $F_{A,i,n,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the first plant that lies in the  $z$  flow direction ( $F_{d,A,n,z}$ ), minus the summation of distributed water passed onto all sinks in the first plant that lies in the  $z$  flow direction ( $F_{n,z,j,A}$ ).

$$\begin{aligned}
 F_{n,z,B}^{\text{SEG}} &= \sum_{u \in \text{CTR}} F_{u,n,z} + \sum_{i \in \text{SU}_A} F_{A,i,n,z} + \sum_{d \in \text{DTR}_A} F_{d,A,n,z} - \sum_{j \in \text{SN}_A} F_{n,z,j,A} \\
 &\quad \forall n \in \text{DPIPES}; A \in P_{n,z} [A \in P \leftrightarrow A \in P_{n,z}] \quad (50)
 \end{aligned}$$

The summation of remaining flow in dist-coll principal pipe  $n$  with direction  $z$  (corresponding to the segment between the second and third plant) that is not passed onto any sinks in the first and second plants in the  $z$  flow direction ( $F_{n,z,B}^{\text{SEG}}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{n,z,A}^{\text{SEG}}$ ), plus the summation of water received from all sources  $i$  in the second plant that lies in the  $z$  flow direction ( $F_{B,i,n,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the second plant that lies in the  $z$  flow direction ( $F_{d,B,n,z}$ ), minus the summation of distributed water passed onto all sinks in the second plant that lies in the  $z$  flow direction ( $F_{n,z,j,B}$ ).

$$\begin{aligned}
 F_{n,z,C}^{\text{SEG}} &= F_{n,z,A}^{\text{SEG}} + \sum_{i \in \text{SU}_B} F_{B,i,n,z} + \sum_{d \in \text{DTR}_B} F_{d,B,n,z} - \sum_{j \in \text{SN}_B} F_{n,z,j,B} \\
 &\quad \forall n \in \text{DPIPES}; A, B \in P_{n,z} [A, B \in P \leftrightarrow A, B \in P_{n,z}] \quad (51)
 \end{aligned}$$

The summation of remaining flow in dist-coll principal pipe  $n$  with direction  $z$  (corresponding to the segment between the  $(N - 1)^{\text{th}}$  and  $N^{\text{th}}$  plant) that is passed onto  $N^{\text{th}}$  (last) plant in the  $z$  flow direction ( $F_{n,z,N-1}^{\text{SEG}}$ ) must equal the summation of flow in the segment between the first and second plant ( $F_{n,z,A}^{\text{SEG}}$ ), plus the summation of flow in the segment between the second and third plant ( $F_{n,z,B}^{\text{SEG}}$ ), plus the summation of flow in the segment between the third and fourth plant ( $F_{n,z,C}^{\text{SEG}}$ ), all up until the segment corresponding to  $(N - 2)^{\text{th}}$  plant, plus the summation of water received from all sources  $i$  in the  $(N - 1)^{\text{th}}$  plant that lies in the  $z$  flow direction ( $F_{N-1,i,n,z}$ ), plus the summation of water received from all decentral treatment units  $d$  in the  $(N - 1)^{\text{th}}$  plant that lies in the  $z$  flow direction ( $F_{d,N-1,n,z}$ ), minus the summation of distributed water passed onto all sinks in the  $(N - 1)^{\text{th}}$  plant that lies in the  $z$  flow direction ( $F_{n,z,j,N-1}$ ).

$$\begin{aligned}
 F_{n,z,N}^{SEG} &= F_{n,z,A}^{SEG} + F_{n,z,B}^{SEG} + F_{n,z,C}^{SEG} + \dots \\
 &+ F_{n,z,N-2}^{SEG} + \sum_{i \in SU_{N-1}} F_{N-1,i,n,z} + \sum_{d \in DTR_{N-1}} F_{d,N-1,n,z} \\
 &- \sum_{j \in SN_{N-1}} F_{n,z,j,N-1} \\
 \forall n \in DPIPES; A, B, C..N-1 \in P_{n,z} [A, B, C..N-1 \in P \leftrightarrow A, B, C..N-1 \in P_{n,z}]
 \end{aligned} \tag{52}$$

The summation of flow rate terms received from or passed onto the first plant in the  $z$  flow direction ( $F_{n,z,j,A}, F_{A,i,n,z}, F_{d,A,n,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from dist-coll principal pipe  $n$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{n,z,j,p}, F_{p,i,n,z}, F_{d,p,n,z}$ )

$$\begin{aligned}
 \sum_{j \in SN_A} F_{n,z,j,A} &= \sum_{j \in SN_p} F_{n,z,j,p}; \sum_{i \in SU_A} F_{A,i,n,z} = \sum_{i \in SU_p} F_{p,i,n,z}; \\
 \sum_{d \in DTR_A} F_{d,A,n,z} &= \sum_{d \in DTR_p} F_{d,p,n,z} \\
 \forall n \in DPIPES; \forall (A = p); p \in P [p \in P \leftrightarrow p \in P_{n,z}]; A \in P_{n,z} [A \in P \leftrightarrow A \in P_{n,z}]
 \end{aligned} \tag{53}$$

Similarly, the summation of flow rate terms received from or passed onto the second plant, third plant, and any  $N^{\text{th}}$  plant in the  $z$  flow direction ( $F_{n,z,j,N}, F_{N,i,n,z}, F_{d,N,n,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) from dist-coll principal pipe  $n$  with direction  $z$  into all sinks  $j$ , in the same plant  $p$  ( $F_{n,z,j,p}, F_{p,i,n,z}, F_{d,p,n,z}$ )

$$\begin{aligned}
 \sum_{j \in SN_N} F_{n,z,j,C} &= \sum_{j \in SN_p} F_{n,z,j,p}; \sum_{i \in SU_N} F_{C,i,n,z} = \sum_{i \in SU_p} F_{p,i,n,z}; \\
 \sum_{d \in DTR_N} F_{d,C,n,z} &= \sum_{d \in DTR_p} F_{d,p,n,z} \\
 \forall n \in DPIPES; \forall (N = p); p \in P [p \in P \leftrightarrow p \in P_{n,z}]; N \in P_{n,z} [N \in P \leftrightarrow N \in P_{n,z}]
 \end{aligned} \tag{54}$$

### Wastewater Pipes

The summation of flows to wastewater pipe  $w$  with direction  $z$  received from all sources  $i$ , across all plants  $p$  ( $F_{p,i,w,z}$ ), must equal the summation of wastewater received from all sources in the first plant that lies in the  $z$  flow direction ( $F_{A,i,w,z}$ ), plus the summation of wastewater received from all sources in the second plant that lies in the  $z$  flow direction ( $F_{B,i,w,z}$ ), plus the summation of wastewater received from all sources in the third plant that lies in the  $z$  flow direction ( $F_{C,i,w,z}$ ), plus all the remaining summations up to the  $N^{\text{th}}$  existing plant ( $F_{N,i,w,z}$ ).

$$\begin{aligned}
 \sum_{p \in P} \sum_{i \in SU_p} F_{p,i,w,z} &= \sum_{i \in SU_A} F_{A,i,w,z} + \sum_{i \in SU_B} F_{B,i,w,z} \\
 &+ \sum_{i \in SU_C} F_{C,i,w,z} + \dots + \sum_{i \in SU_{N-1}} F_{N-1,i,w,z} + \sum_{i \in SU_N} F_{N,i,w,z} \\
 \forall w \in WPIPES; A, B, C..N \in P_{w,z} [A, B, C..N \in P \leftrightarrow A, B, C..N \in P_{w,z}]
 \end{aligned} \tag{55}$$

The summation of the flow portion in wastewater pipe  $w$  with direction  $z$  corresponding to the segment between the first and second plant that is received from all sources  $i$  in the first plant according to the  $z$  flow direction ( $F_{w,z,A}^{SEG}$ ) must equal the summation of wastewater received from all sources in the first plant that lies in the  $z$  flow direction ( $F_{A,i,w,z}$ )

$$F_{w,z,A}^{SEG} = \sum_{i \in SU_A} F_{A,i,w,z} \forall w \in WPIPES; A \in P_{w,z} [A \in P \leftrightarrow A \in P_{w,z}] \tag{56}$$

The summation of the flow portion in the wastewater pipe  $w$  with direction  $z$  corresponding to the segment between the second and third plant that is received from all sources  $i$  in the first plant according to the  $z$  flow direction ( $F_{w,z,B}^{SEG}$ ) must equal the summation of wastewater received from all sources in the first and second plants that lie in the  $z$  flow direction ( $F_{A,i,w,z}$  and  $F_{B,i,w,z}$ ), respectively.

$$\begin{aligned}
 F_{w,z,B}^{SEG} &= \sum_{i \in SU_A} F_{A,i,w,z} + \sum_{i \in SU_B} F_{B,i,w,z} \\
 \forall w \in WPIPES; A, B \in P_{w,z} [A, B \in P \leftrightarrow A, B \in P_{w,z}]
 \end{aligned} \tag{57}$$

The summation of the flow portion in wastewater pipe  $w$  with direction  $z$  corresponding to the segment between the third and fourth plant that is received from all sources  $i$  in the first plant according to the  $z$  flow direction ( $F_{w,z,C}^{SEG}$ ) must equal the summation of wastewater received from all sources in the first, second, and third plants that lie in the  $z$  flow direction ( $F_{A,i,w,z}$  and  $F_{B,i,w,z}$  and  $F_{C,i,w,z}$ ), respectively.

$$\begin{aligned}
 F_{w,z,C}^{SEG} &= \sum_{i \in SU_A} F_{A,i,w,z} + \sum_{i \in SU_B} F_{B,i,w,z} + \sum_{i \in SU_C} F_{C,i,w,z} \\
 \forall w \in WPIPES; A, B, C \in P_{w,z} [A, B, C \in P \leftrightarrow A, B, C \in P_{w,z}]
 \end{aligned} \tag{58}$$

The summation of the flow portion in wastewater pipe  $w$  with direction  $z$  corresponding to the segment between the  $(N-1)^{\text{th}}$  and  $N^{\text{th}}$  plant that is received from all sources  $i$  in the  $N^{\text{th}}$  (last) plant in the  $z$  flow direction ( $F_{w,z,N-1}^{SEG}$ ) must equal the summation of flows received from all sources in the first, second, third all up to the  $(N-1)^{\text{th}}$  plant in the  $z$  flow direction ( $F_{A,i,w,z}$  and  $F_{B,i,w,z}$  and  $F_{C,i,w,z}$  and  $F_{N-1,i,w,z}$ ), respectively.

$$\begin{aligned}
 F_{w,z,N-1}^{SEG} &= \sum_{i \in SU_A} F_{A,i,w,z} + \sum_{i \in SU_B} F_{B,i,w,z} + \sum_{i \in SU_C} F_{C,i,w,z} \\
 &+ \dots + \sum_{i \in SU_{N-1}} F_{N-1,i,w,z} \\
 \forall w \in WMAINS; A, B, C..N-1 \in P_{w,z} [A, B, C..N-1 \in P \leftrightarrow A, B, C..N-1 \in P_{w,z}]
 \end{aligned}
 \tag{59}$$

The summation of the flow portion in wastewater pipe  $w$  with direction  $z$ , after the  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{w,z,N}^{SEG}$ ), must be equal to the summation of flows to wastewater pipe  $w$  with direction  $z$  received from all sources  $i$ , across all plants  $p$  ( $F_{p,i,w,z}$ ),

$$F_{w,z,N}^{SEG} = \sum_{p \in P} \sum_{i \in SU_p} F_{p,i,w,z} \forall w \in WPIPIPES; \forall N \in P_{w,z} [N \in P \leftrightarrow N \in P_{w,z}]
 \tag{60}$$

The summation of wastewater received from all sources in the first plant that lies in the  $z$  flow direction ( $F_{A,i,w,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) to wastewater pipe  $w$  with direction  $z$  from all sources  $i$ , in the same plant  $p$  ( $F_{p,i,w,z}$ )

$$\begin{aligned}
 \sum_{i \in SU_A} F_{A,i,w,z} &= \sum_{j \in SU_p} F_{p,i,w,z} \forall w \in WPIPIPES; \forall (A = p); \\
 p \in P [p \in P \leftrightarrow p \in P_{w,z}]; A \in P_{w,z} [A \in P \leftrightarrow A \in P_{w,z}]
 \end{aligned}
 \tag{61}$$

Similarly, the summation of wastewater received from all sources in the second plant, third plant, and any  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{N,i,w,z}$ ) must equal the summation of all flow rate terms (defined for the total and component balances) to wastewater pipe  $w$  with direction  $z$  from all sources  $i$ , in the same plant  $p$  ( $F_{p,i,w,z}$ )

$$\begin{aligned}
 \sum_{i \in SU_N} F_{N,i,w,z} &= \sum_{i \in SU_p} F_{p,i,w,z} \forall w \in WPIPIPES; \forall (N = p); \\
 p \in P [p \in P \leftrightarrow p \in P_{w,z}]; N \in P_{w,z} [N \in P \leftrightarrow N \in P_{w,z}]
 \end{aligned}
 \tag{62}$$

### Brine Principal Pipes

The summation of brine water received from all decentral treatment units  $d$ , across all plants  $p$  ( $F_{d,p,b,z}$ ), must equal the summation of wastewater received from all decentral treatment units in the first plant that lies in the  $z$  flow direction ( $F_{d,A,b,z}$ ), plus the summation of wastewater received from all decentral treatment units in the second plant that lies in the  $z$  flow direction ( $F_{d,B,b,z}$ ), plus the summation of wastewater received from all decentral treatment units in the third plant that lies in the  $z$  flow direction ( $F_{d,C,b,z}$ ), plus all the remaining summations up to the  $N^{th}$  existing plant ( $F_{d,N,b,z}$ ).

$$\begin{aligned}
 \sum_{p \in P} \sum_{d \in DTR_p} F_{d,p,b,z} &= \sum_{d \in DTR_A} F_{d,A,b,z} + \sum_{d \in DTR_B} F_{d,B,b,z} \\
 &+ \sum_{d \in DTR_C} F_{d,C,b,z} + \dots + \sum_{d \in DTR_{N-1}} F_{d,N-1,b,z} + \sum_{d \in DTR_N} F_{d,N,b,z} \\
 \forall b \in BPIPIPES; \forall p \in P [p \in P \leftrightarrow p \in P_{b,z}];
 \end{aligned}
 \tag{63}$$

$$A, B, C..N \in P_{b,z} [A, B, C..N \in P \leftrightarrow A, B, C..N \in P_{b,z}]$$

The summation of the flow portion in the brine principal pipe  $b$  with direction  $z$  corresponding to the segment between the first and second plant that is received from all decentral treatment units  $d$  in the first plant according to the  $z$  flow direction ( $F_{b,z,A}^{SEG}$ ) must equal the summation of brine water received from all decentral treatment units  $d$  in the first plant that lies in the  $z$  flow direction ( $F_{d,A,b,z}$ ), added to flows from all central treatment units  $u$  ( $F_{u,b,z}$ )

$$\begin{aligned}
 F_{b,z,A}^{SEG} &= \sum_{u \in CTR} F_{u,b,z} + \sum_{d \in DTR_A} F_{d,A,b,z} \\
 \forall b \in BPIPIPES; A \in P_{b,z} [A \in P \leftrightarrow A \in P_{b,z}]
 \end{aligned}
 \tag{64}$$

The summation of the flow portion in wastewater pipe  $w$  with direction  $z$  corresponding to the segment between the second and third plant that is received from all decentral treatment units  $d$  in the first plant according to the  $z$  flow direction ( $F_{b,z,B}^{SEG}$ ) must equal the summation of brine water received from all decentral treatment units  $d$  in the first and second plants that lie in the  $z$  flow direction ( $F_{d,A,b,z}$  and  $F_{d,B,b,z}$ ), respectively, all added to flows from all central treatment units  $u$  ( $F_{u,b,z}$ )

$$\begin{aligned}
 F_{b,z,B}^{SEG} &= \sum_{u \in CTR} F_{u,b,z} + \sum_{d \in DTR_A} F_{d,A,b,z} + \sum_{d \in DTR_B} F_{d,B,b,z} \\
 \forall b \in BPIPIPES; A, B \in P_{b,z} [A, B \in P \leftrightarrow A, B \in P_{b,z}]
 \end{aligned}
 \tag{65}$$

The summation of the flow portion in the brine principal pipe  $b$  with direction  $z$  corresponding to the segment between the third and fourth plant that is received from all decentral treatment units  $d$  in the first plant according to the  $z$  flow direction ( $F_{b,z,C}^{SEG}$ ) must equal the summation of brine water received from all decentral treatment units  $d$  in the first, second, and third plants that lie in the  $z$  flow direction ( $F_{d,A,b,z}$  and  $F_{d,B,b,z}$  and  $F_{d,C,b,z}$ ), respectively, all added to flows from all central treatment units  $u$  ( $F_{u,b,z}$ )

$$\begin{aligned}
 F_{b,z,C}^{SEG} &= \sum_{u \in CTR} F_{u,b,z} + \sum_{d \in DTR_A} F_{d,A,b,z} + \sum_{d \in DTR_B} F_{d,B,b,z} \\
 &+ \sum_{d \in DTR_C} F_{d,C,b,z} \\
 \forall b \in BPIPIPES; A, B, C \in P_{b,z} [A, B, C \in P \leftrightarrow A, B, C \in P_{b,z}]
 \end{aligned}
 \tag{66}$$

The summation of the flow portion in the brine principal pipe  $b$  with direction  $z$  corresponding to the segment between

the  $(N - 1)^{th}$  and  $N^{th}$  plant that is received from all decentral treatment units  $d$  in the  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{b,z,N-1}^{SEG}$ ) must equal the summation of flows received from all decentral treatment units  $d$  in the first, second, third, and all up to the  $(N - 1)^{th}$  plant in the  $z$  flow direction ( $F_{d,A,b,z}$  and  $F_{d,B,b,z}$  and  $F_{d,C,b,z}$  and  $F_{d,N-1,b,z}$ ), respectively, all added to flows from all central treatment units  $u$  ( $F_{u,b,z}$ ).

$$F_{b,z,N-1}^{SEG} = \sum_{u \in CTR} F_{u,b,z} + \sum_{d \in DTR_A} F_{d,A,b,z} + \sum_{d \in DTR_B} F_{d,B,b,z} + \sum_{d \in DTR_C} F_{d,C,b,z} + \dots + \sum_{d \in DTR_{N-1}} F_{d,N-1,b,z}$$

$\forall b \in BPIPES; A, B, C..N-1 \in P_{b,z} [A, B, C..N-1 \in P \leftrightarrow A, B, C..N-1 \in P_{b,z}]$  (67)

The summation of the flow portion in the brine principal pipe  $b$  with direction  $z$ , after the  $N^{th}$  (last) plant in the  $z$  flow direction ( $F_{b,z,N}^{SEG}$ ), must be equal to the summation of flows to wastewater pipe  $w$  with direction  $z$  received from all decentral treatment units  $d$ , across all plants  $p$  ( $F_{p,i,w,z}$ ),

$$F_{b,z,N}^{SEG} = \sum_{u \in CTR} F_{u,b,z} + \sum_{p \in P} \sum_{d \in DTR_p} F_{d,p,b,z}$$

$\forall b \in BPIPES; \forall N \in P_{f,z} [N \in P \leftrightarrow N \in P_{m,z}]$  (68)

The summation of brine water received from all decentral treatment units in the first plant that lies in the  $z$  flow direction ( $F_{d,A,b,z}$ ) must equal the summation of all flow rate terms received from all decentral treatment units in the same plant  $p$  ( $F_{d,p,b,z}$ ), which are defined for the total and component balances of brine principal pipe  $b$  with direction  $z$

$$\sum_{d \in DTR_A} F_{d,A,b,z} = \sum_{d \in DTR_p} F_{d,p,b,z} \forall b \in BPIPES; \forall (A = p); p \in P [p \in P \leftrightarrow p \in P_{b,z}]; A \in P_{b,z} [A \in P \leftrightarrow A \in P_{b,z}]$$
 (69)

Similarly, the summation of brine water received from all decentral treatment units in the second plant, third plant, and any  $N^{th}$  plant that lies in the  $z$  flow direction ( $F_{d,N,b,z}$ ) must equal the summation of all flow rate terms received from all decentral treatment units in the same plant  $p$  ( $F_{d,p,b,z}$ ) which are defined for the total and component balances of brine principal pipe  $b$  with direction  $z$

$$\sum_{d \in DTR_N} F_{d,N,b,z} = \sum_{d \in DTR_p} F_{d,p,b,z} \forall b \in BMAINS; \forall (N = p); p \in P [p \in P \leftrightarrow p \in P_{b,z}]; N \in P_{b,z} [N \in P \leftrightarrow N \in P_{b,z}]$$
 (70)

### Conclusions

This paper presents a novel methodology that entails the design of interplant water network systems with pipeline connectivity, handling, and management strategies, using

different types of principal pipes. Each of the principal pipe (or water mains) categories is classified according to the type of water that is allowed to be transported. Moreover, depending on the water classification involved (freshwater, wastewater, or treated water), each principal pipe was allowed to either receive water, distribute water, or conduct both actions of receiving and distributing water simultaneously. This type of principal pipe classification was found to be particularly important since the purpose of the pipe within a water network can vary significantly depending on the type of water involved. Hence, the design of interplant water networks with principal pipe options was conducted in a more structured and systematic method that relies on categorizing the different principal pipe options involved. A total of five different principal pipe categories have been identified in this work, and each category has been associated with a different set of allowed connectivity within the network. Moreover, principal pipe categories that were allowed to conduct both actions of receiving and distributing water simultaneously were associated with a minimum standard water quality, so as to ensure a guaranteed minimum of the quality of water being transported within the principal pipe. The proposed method also accounts for two different but complementary flow direction options that may be associated with each of the principal pipe options. The benefits of the proposed approach have all been quantified and highlighted using several case studies, which are presented in part 2 of this manuscript.

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### Compliance with ethical standards

**Conflict of interest** The authors declare that they have no conflict of interest.

### Nomenclature

**Subscripts:** ;  $p$ , Plant/process;  $i$ , Water source;  $j$ , Water sink;  $c$ , Contaminant/component;  $f$ , Freshwater pipe;  $w$ , Wastewater pipe;  $b$ , Brine principal pipe;  $d$ , Decentral treatment interceptor unit;  $u$ , Central treatment interceptor unit;  $m$ , Principal pipe that collects water from plants to central treatment;  $n$ , Principal pipe that distributes water from central treatment to plants;  $q$ , Quality standard associated with coll-dist principal pipe;  $r$ , Quality standard associated with dist-coll principal pipe;  $z$ , Flow direction (indeterminate);  $z'$ , Complementary (indeterminate) flow direction;  $A$ , First plant located along the direction of flow of a principal pipe;  $B$ , Second plant located along the direction of flow of a principal pipe;  $C$ , Third plant located along the direction of flow of a principal pipe;  $N$ ,  $N^{th}$  plant located along the direction of flow of a principal pipe; **Sets:** ;  $P$ , Set of plants/processes in industrial city;  $C$ , Set of contaminants/pollutants;  $SU_p$ , Set of water sources in plant  $p$ ;  $SU_A$ , Set of water sources in plant  $A$ , which corresponds to the first plant according to an arrangement for a direction of flow defined;  $SU_B$ , Set of water sources

in plant  $B$ , which corresponds to the second plant according to an arrangement for a direction of flow defined;  $SU_C$ , Set of water sources in plant  $C$ , which corresponds to the third plant according to an arrangement for a direction of flow defined;  $SU_{N-1}$ , Set of water sources in plant  $N - 1$ , which corresponds to the  $(N - 1)^{th}$  plant according to an arrangement for a direction of flow defined;  $SU_N$ , Set of water sources in plant  $N$ , which corresponds to the very last plant according to an arrangement for a direction of flow defined;  $SN_p$ , Set of water sinks in plant  $p$ ;  $SN_A$ , Set of water sinks in plant  $A$ , first plant according to an arrangement for a direction of flow defined;  $SN_B$ , Set of water sinks in plant  $B$ , second plant according to an arrangement for a direction of flow defined;  $SN_C$ , Set of water sinks in plant  $C$ , third plant according to an arrangement for a direction of flow defined;  $SN_{N-1}$ , Set of water sinks in plant  $N - 1$ , which corresponds to the  $(N - 1)^{th}$  plant according to a defined arrangement for a direction of flow defined;  $SN_N$ , Set of water sinks in plant  $N$ , which corresponds to the very last plant according to a defined arrangement for a direction of flow defined;  $DTR_p$ , Set of decentralized wastewater treatment interceptors in plant  $p$ ;  $DTR_A$ , Set of decentralized wastewater treatment interceptors in plant  $A$ , first plant according to a defined arrangement for a direction of flow defined;  $DTR_B$ , Set of decentralized wastewater treatment interceptors in plant  $B$ , second plant according to a defined arrangement for a direction of flow defined;  $DTR_C$ , Set of decentralized wastewater treatment interceptors in plant  $C$ , third plant according to a defined arrangement for a direction of flow defined;  $DTR_{N-1}$ , Set of decentralized wastewater treatment interceptors in plant  $N - 1$ , which corresponds to the  $(N - 1)^{th}$  plant according to an arrangement for a direction of flow defined;  $DTR_N$ , Set of decentralized wastewater treatment interceptors in plant  $N$ , which corresponds to the very last plant according to an arrangement for a direction of flow defined;  $CTR$ , Set of centralized wastewater treatment interceptors;  $FPIPES$ , Set of freshwater principal pipes;  $WPIPES$ , Set of wastewater principal pipes;  $BPIPES$ , Set of brine principal pipes;  $CPIPES$ , Set of coll-dist principal pipes from plants to central treatment;  $DPIPES$ , Set of dist-coll principal pipes from central treatment to plants;  $P_{f,z}$ , Set of plant orders defined for freshwater pipe  $f$ , with indeterminate flow direction  $z$ , starting from the first plant to receive water from the pipe;  $P_{f,z'}$ , Set of plant orders defined for freshwater pipe  $f$ , with a complementary indeterminate flow direction  $z'$ , starting from the first plant to receive water from the pipe;  $P_{w,z}$ , Set of plant arrangement order defined for wastewater pipe  $w$ , with an indeterminate flow direction  $z$ , starting from the first plant to supply water to the pipe;  $P_{w,z'}$ , Set of plant arrangement order defined for wastewater pipe  $w$ , with a complementary indeterminate flow direction  $z'$ , starting from the first plant to supply water to the pipe;  $P_{m,z}$ , Set of plant arrangement order defined for coll-dist principal pipe  $m$ , with an indeterminate flow direction  $z$ , starting from the first plant to supply water to the pipe;  $P_{m,z'}$ , Set of plant arrangement order defined for coll-dist principal pipe  $m$ , with a complementary indeterminate flow direction  $z'$ , starting from the first plant to supply water to the pipe;  $P_{n,z}$ , Set of plant arrangement order defined for dist-coll principal pipe  $n$ , with an indeterminate flow direction  $z$ , starting from the first plant to receive water from the pipe;  $P_{n,z'}$ , Set of plant arrangement order defined for dist-coll principal pipe  $n$ , with a complementary indeterminate flow direction  $z'$ , starting from the first plant to receive water from the pipe;  $P_{b,z}$ , Set of treatment arrangement (combining both central and decentral) orders defined for brine principal pipe  $b$ , with an indeterminate flow direction  $z$ , starting from the first unit to supply water to the pipe;  $P_{b,z'}$ , Set of treatment arrangement (combining both central and decentral) orders defined for brine principal pipe  $b$ , with a complementary indeterminate flow direction  $z'$ , starting from the first unit to supply water to the pipe; **Parameters:** ;  $W_{p,i}^{Source}$ , Water source flow rate available in plant  $p$ , source  $i$ ;  $G_{j,p}^{Sink}$ , Water sink flow rate required in plant  $p$ , sink  $j$ ;  $x_{d,p,c}^{Inlet}$ , Concentration of component  $c$  into decentral treatment unit  $d$ , plant  $p$ ;  $x_{d,p,c}^{Treated}$ , Concentration of component  $c$  in treated stream out of decentral treatment unit  $d$ , plant  $p$ ;  $x_{d,p,c}^{Untreated}$ , Concentration of component  $c$  in untreated stream out of decentral treatment unit  $d$ , plant  $p$ ;  $x_{u,c}^{Inlet}$ , Concentration of

component  $c$  in treated stream into central treatment unit  $u$ ;  $x_{u,c}^{Treated}$ , Concentration of component  $c$  in untreated stream out of central treatment unit  $u$ ;  $x_{u,c}^{Untreated}$ , Concentration of component  $c$  in untreated stream out of central treatment unit  $u$ ;  $y_{p,i,c}^{Source}$ , Given concentration level, in plant  $p$  source  $i$ , of component  $c$ ;  $z_{f,p,c}^{Fresh}$ , Given concentration level, in freshwater pipe  $f$ , of component  $c$ ;  $z_{j,p,c}^{Sink}$ , Maximum allowable concentration level, in sink  $j$  plant  $p$ , of component  $c$ ;  $z_{m,c}^{Pipe}$ , Specified concentration level of contaminant  $c$  allowed into coll-dist principal pipe  $m$ ;  $z_{n,c}^{Pipe}$ , Specified concentration level of contaminant  $c$  allowed into dist-coll principal pipe  $n$ ;  $z_{w,c}^{Waste}$ , Total maximum concentration level of contaminant  $c$  that can be disposed by wastewater pipe  $w$ ;  $z_{b,c}^{Brine}$ , Total maximum concentration level of contaminant  $c$  that can be disposed by brine pipe  $b$ ;  $F_{d,p}^{loss}$ , Total water loss in decentral treatment unit  $d$  in plant  $p$ ;  $F_u^{loss}$ , Total water loss in central treatment unit  $u$ ;  $a, p, c$ , Removal ratio of component  $c$  by decentral treatment unit  $d$  in plant  $p$ ;  $a, p$ , Treated water recovery by decentral treatment unit  $d$  in plant  $p$ ;  $a, p$ , Water loss factor in decentral treatment unit  $d$  in plant  $p$ ;  $u, c$ , Removal ratio of component  $c$  by central treatment unit  $u$ ;  $u$ , Treated water recovery by central treatment unit  $u$ ;  $u$ , Water loss factor in central treatment unit  $u$ ; **Variables:** ;  $F_{p,i,j,p}$ , Water flow rate from plant  $p$ , source  $i$  into sink  $j$  in the same plant  $p$ ;  $F_{p,i,d,p}$ , Water flow rate from plant  $p$ , source  $i$  into a corresponding decentral treatment unit in the same plant  $p$ ;  $F_{p,i,m,z}$ , Water flow rate from plant  $p$ , source  $i$  to coll-dist principal pipe  $m$  with flow direction  $z$ ;  $F_{p,i,m,z'}$ , Water flow rate from plant  $p$ , source  $i$  to coll-dist principal pipe  $m$  with flow direction  $z'$ ;  $F_{p,i,n,z}$ , Water flow rate from plant  $p$ , source  $i$  to dist-coll principal pipe  $n$  with flow direction  $z$ ;  $F_{p,i,n,z'}$ , Water flow rate from plant  $p$ , source  $i$  to dist-coll principal pipe  $n$  with flow direction  $z'$ ;  $F_{p,i,w,z}$ , Water flow rate from plant  $p$ , source  $i$  to wastewater pipe  $w$  with flow direction  $z$ ;  $F_{p,i,w,z'}$ , Water flow rate from plant  $p$ , source  $i$  to wastewater pipe  $w$  with flow direction  $z'$ ;  $F_{d,p,j,p}$ , Treated water flow rate from decentral treatment unit plant  $p$  to sink  $j$  in the same plant  $p$ ;  $F_{d,p,m,z}$ , Treated water flow rate from decentral treatment unit plant  $p$  to a coll-dist principal pipe  $m$  with flow direction  $z$ ;  $F_{d,p,m,z'}$ , Treated water flow rate from decentral treatment unit plant  $p$  to a coll-dist principal pipe  $m$  with flow direction  $z'$ ;  $F_{d,p,n,z}$ , Treated water flow rate from decentral treatment unit plant  $p$  to a dist-coll principal pipe  $n$  with flow direction  $z$ ;  $F_{d,p,n,z'}$ , Treated water flow rate from decentral treatment unit plant  $p$  to a dist-coll principal pipe  $n$  with flow direction  $z'$ ;  $F_{d,p,b,z}$ , Untreated water flow rate from decentral treatment unit plant  $p$  to a brine principal pipe  $b$  with flow direction  $z$ ;  $F_{d,p,b,z'}$ , Untreated water flow rate from decentral treatment unit plant  $p$  to a brine principal pipe  $b$  with flow direction  $z'$ ;  $F_m, z, u$ , Water flow rate from a coll-dist principal pipe  $m$  with flow direction  $z$  into central treatment unit  $u$ ;  $F_{m,z',u}$ , Water flow rate from a coll-dist Principal pipe  $m$  with flow direction  $z'$  into central treatment unit  $u$ ;  $F_u, n, z$ , Treated water flow rate from central treatment unit  $u$  into dist-coll principal pipe  $n$  with flow direction  $z$ ;  $F_{u,n,z'}$ , Treated water flow rate from central treatment unit  $u$  into dist-coll principal pipe  $n$  with flow direction  $z'$ ;  $F_u, b, z$ , Untreated water flow rate from central treatment unit  $u$  into brine principal pipe  $b$  with flow direction  $z$ ;  $F_{u,b,z'}$ , Untreated water flow rate from central treatment unit  $u$  into brine principal pipe  $b$  with flow direction  $z'$ ;  $F_m, z, j, p$ , Water flow rate from coll-dist principal pipe  $m$  with flow direction  $z$  to source in plant  $p$ ;  $F_{m,z',j,p}$ , Water flow rate from coll-dist principal pipe  $m$  with flow direction  $z'$  to source in plant  $p$ ;  $F_n, z, j, p$ , Water flow rate from a dist-coll principal pipe  $n$  with flow direction  $z$  to sink  $j$  in plant  $p$ ;  $F_{n,z',j,p}$ , Water flow rate from a dist-coll principal pipe  $n$  with flow direction  $z'$  to sink  $j$  in plant  $p$ ;  $F_f, z, j, p$ , Water flow rate from freshwater pipe  $f$  with flow direction  $z$  to sink  $j$  in plant  $p$ ;  $F_{f,z',j,p}$ , Water flow rate from freshwater pipe  $f$  with flow direction  $z'$  to sink  $j$  in plant  $p$ ;  $F_p, i, w, z$ , Water flow rate from source  $i$  in plant  $p$  to wastewater pipe  $w$  with flow direction  $z$ ;  $F_{p,i,w,z'}$ , Water flow rate from source  $i$  in plant  $p$  to wastewater pipe  $w$  with flow direction  $z'$ ;  $F_{f,z,p}^{SEG}$ , Flow rate in the first segment that forms freshwater principal pipe  $f$  with flow direction  $z$  before plant  $p$ ;  $F_{f,z',p}^{SEG}$ , Flow rate in the first segment that forms freshwater principal pipe  $f$  with flow direction  $z'$  before plant  $p$ ;  $F_{m,z,p}^{SEG}$ , Flow rate in the first

segment that forms coll-dist principal pipe  $m$  with flow direction  $z$  after plant  $p$ ;  $F_{m,z,p}^{\text{SEG}}$ , Flow rate in the first segment that forms coll-dist principal pipe  $m$  with flow direction  $z'$  after plant  $p$ ;  $F_{n,z,p}^{\text{SEG}}$ , Flow rate in the first segment that forms dist-coll principal pipe  $n$  with flow direction  $z$  before plant  $p$ ;  $F_{n,z',p}^{\text{SEG}}$ , Flow rate in the first segment that forms dist-coll principal pipe  $n$  with flow direction  $z'$  before plant  $p$ ;  $F_{w,z,p}^{\text{SEG}}$ , Flow rate in the first segment that forms wastewater pipe  $w$  with flow direction  $z$  after plant  $p$ ;  $F_{w,z',p}^{\text{SEG}}$ , Flow rate in the first segment that forms wastewater pipe  $w$  with flow direction  $z'$  after plant  $p$ ;  $F_{b,z,p}^{\text{SEG}}$ , Flow rate in the first segment that forms brine pipe  $b$  with flow direction  $z$  after plant  $p$ ;  $F_{b,z',p}^{\text{SEG}}$ , Flow rate in the first segment that forms brine pipe  $b$  with flow direction  $z'$  after plant  $p$ ;  $F_f$ , Total flow rate in freshwater pipe  $f$ ;  $F_w$ , Total flow rate in wastewater pipe  $w$ ;  $F_b$ , Total flow rate in brine principal pipe  $b$

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