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Reliability of AASHTO LRFD parameters in multilane reinforced concrete slab bridges

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Abstract. Empirical expressions for estimating live load bending moments are typically specified in AASHTO bridge codes. This paper will evaluate the reliability levels that are inherent in the simplified empirical equations in the AASHTO LRFD to design concrete slab bridges. Typical one-span, multilane, straight bridges, with various span lengths are modeled using finite element analysis (FEA) and subjected to AASHTO live loads. FEA results are compared with LRFD moments to quantify biases that might result from the simplifying assumptions in AASHTO. The reliability index β for bridge cases using AASHTO procedures and FEA results were quantified. The results of this analysis showed that current live load factors in AASHTO LRFD procedures were below the required levels of reliability index (β < 3.5) for one-lane bridges, and those indices were higher for multilane bridges, yet did not reach the required level, thus proposing to increase the AASHTO live load factors.

Keywords: Concrete slab bridges, finite-element analysis, load-carrying capacity, reliability analysis

1. Introduction

The design of highway bridges in the United States conforms to the American Association of State Highway and Transportation Officials (AASHTO) Standard Specifications for Highway Bridges (AAS HTO Specs 2002) or AASHTO Load and Resistance Factor Design (LRFD) Bridge Design Specifications (AASHTO LRFD 2012). The analysis and design of any highway bridge must consider live loads such as AASHTO HS20 (truck or lane) or HL93 (combination of truck or tandem, and lane loading). To analyze and design reinforced concrete slab bridges, AASHTO specifies a distribution width for live loading that simplifies the two-way bending problem into a beam or one-way bending problem. Empirical expressions for estimating the wheel load distribution and live-load bending moment are typically specified in highway bridge codes such as the AASHTO standards. These equations do not take into account the many factors that govern the actual live load such as the transverse position of a truck or tandem on a specific lane, leading to either over-estimation or under-estimation of the live-load longitudinal bending moment. In a previous study using finite element analysis (FEA) by Mabsout et al. (1997 & 2004), it was shown that by alternating the position of the truck loads transversely, the resulting bending moments tend to increase as the applied live loads come closer to the transverse edge of a bridge.

The objective of this paper is to assess the AASHTO LRFD code provisions used in calculating the moment due to live loads, first by comparing with the resulting maximum moments obtained using finite element analysis (FEA) to quantify biases

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that might result from the simplifying assumptions adopted in AASHTO, which is achieved using a reliability approach.

Reliability analysis is an effective tool for developing and assessing new and existing design parameters in the bridge codes. AASHTO LRFD was calibrated to create new load and resistance factors to reach a pre-selected safety target based on a reliability analysis using the basic design Equation 1 (Nowak 1999):

$$\sum \gamma i \, Xi < \phi Rn \tag{1}$$

where γi represents a set of load factors that are greater than one and that are applied to the different load effects Xi, while ϕ represents a resistance factor that is generally less than one and that is multiplied by the nominal resistance Rn.

A preliminary study was conducted by Mahmoud et al. (2017) to evaluate the reliability levels in one- and two-lane concrete slab bridges. In the first step of the analysis conducted in this study, a finite element analysis was performed to evaluate numerically the maximum bending moments of single span, one- and two-lane bridges, with different span lengths and various slab thicknesses subjected to AASHTO LRFD live loads. Next, the bending moments were calculated using the simplified AASHTO LRFD provisions. The ratio of the FEA moments to the LRFD moments (αLL) was then quantified for the bridge cases analyzed. The second step involved defining the statistical characteristics of the different load effects and resistance as per Nowak (1995). This was followed by a reliability analysis that is aimed at quantifying the reliability levels that are inherent in the traditional LRFD design methodology as per the load and resistance factors that are recommended by AASHTO LRFD. The quantification of the reliability level was accomplished using Monte Carlo simulations whereby the reliability index of the bridge design was evaluated for the different bridges analyzed. The reliability analysis was then repeated while correcting the nominal LRFD live load moments to account for the more representative moments that were obtained from the finite element analysis. The final step involved proposing modifications to the live load factors of the AASHTO LRFD equation to achieve a target reliability index of 3.5 for all the concrete slab bridges analyzed in this study.

The current study extends the previous works by Mahmoud et al. (2017) and Mabsout et al. (2004) to include single and multiple lanes (1 to 4 lanes). The comprehensive results of this research will provide structural engineers with more consistent provisions to design or evaluate the load-carrying capacity of concrete slab bridges.

2. Description of bridge cases

Typical one-span, simply supported, one- to four-lanes, straight reinforced concrete slab bridges are considered. Four span lengths of 24, 36, 46, and 54 ft are assumed, with slab thickness of 18, 21, 24, and 27 in, respectively, to control deflection. Given a standard lane width of 12 ft, the slab widths were assumed to be 14 ft for one lane (including 1 ft offset on each side), 24 ft for two lanes, 36 ft for three lanes, and 48 ft for four lanes.

Live loads are simulated in this analysis as either a combination of AASHTO design truck and design lane loads, or AASHTO design tandem with design lane loads. Lane loads were assumed to be uniform loads centered in each lane with a magnitude of 640 lb-ft/ft. AASHTO design truck load is taken to be 4-kip point load per tire for the front axle while the middle and rear axles is taken to be 16 kips point load per tire for each axle. Design tandem load is assumed as 4-point loads, 12.5 kips each, with a transverse separation distance of 6 ft and a longitudinal separation distance of 4 ft. Transversally, the truck or tandem positions are assumed as either centered in each lane (Centered Load) or located close to the edge of a lane with a 1 ft of separation distance between the edge of the bridge and the first truck while the separation distance for two side by side trucks are taken to be 4 ft (Edge Loading). The AASHTO design truck or design tandem are positioned longitudinally and transversally to produce the maximum bending moments in the bridge slabs. A dynamic load allowance of 33% of the design truck or design tandem is also included with the live load in accordance with AASHTO LRFD Bridge Design Specifications.

Table 1 summarizes the geometry of all the cases analyzed and Fig. 1 shows a typical 36 ft span, two-lane bridge layout with AASHTO design trucks under Edge Loading condition.

3. Finite element analysis and results

The finite element method is used to investigate the effects of live loads on the concrete slab bridges. The bridges are modeled as simply supported slabs, and

Table 1
FEA maximum longitudinal moment vs AASHTO LRFD moment

Number Lanes	Span Length (ft)	Slab Thick (in)	Slab Width (ft)	Governing M_{LL} Source	FEA Mom (Centered) (kip-ft/ft)	FEA Mom (Edge) (kip-ft/ft)	LRFD Moment (kip-ft/ft)	α_{LL} (FEA/ LRFD)
1	24	18	14	Tandem	23.65	24.21	34.95	0.69
	36	21	14	Tandem	38.45	39.10	49.44	0.79
	46	24	14	Truck	54.49	56.91	60.68	0.94
	54	27	14	Truck	69.35	72.91	69.84	1.04
2	24	18	24	Tandem	27.29	29.34	29.97	0.98
	36	21	24	Tandem	44.72	47.07	47.85	0.98
	46	24	24	Truck	63.26	67.86	63.19	1.07
	54	27	24	Truck	80.63	86.52	75.82	1.14
3	24	18	36	Tandem	30.75	35.69	28.13	1.27
	36	21	36	Tandem	44.71	49.18	44.49	1.11
	46	24	36	Truck	63.26	70.44	60.36	1.17
	54	27	36	Truck	85.81	90.05	77.1	1.17
4	24	18	48	Tandem	27.15	31.57	26.74	1.18
	36	21	48	Tandem	44.92	50.76	42.00	1.21
	46	24	48	Truck	66.82	73.00	59.77	1.22
	54	27	48	Truck	85.78	92.56	77.10	1.20

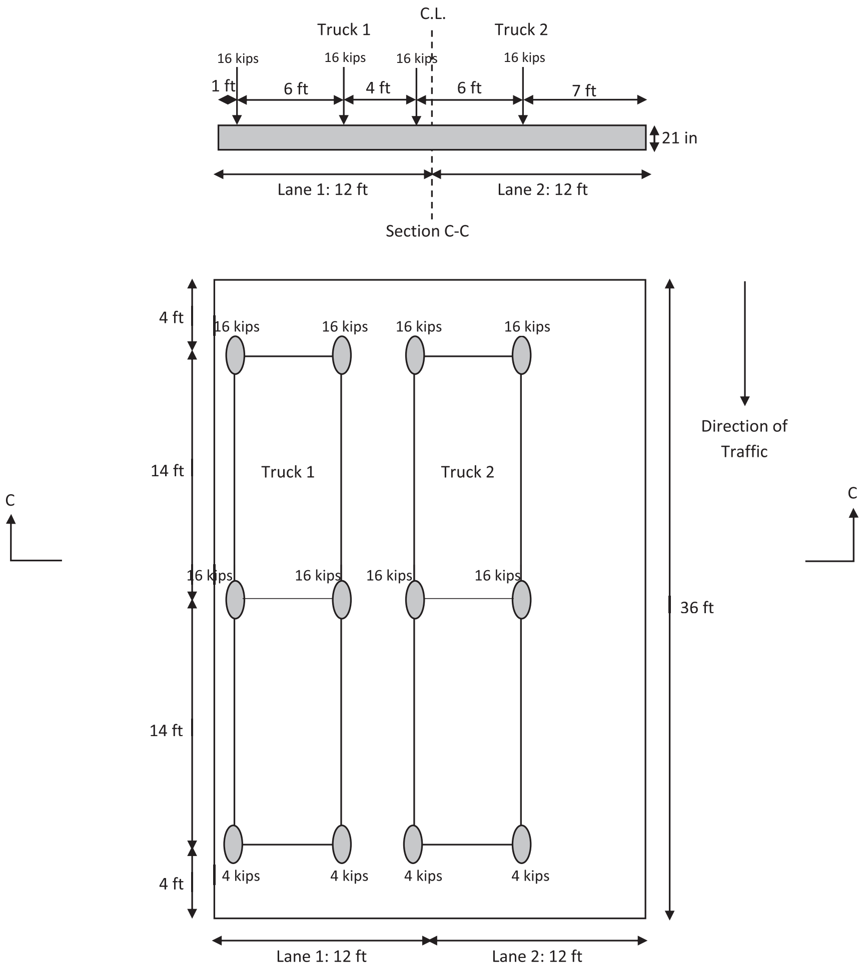

Fig. 1. Typical cross-section and plan of a 36 ft span, two-lane bridge under Edge Loading condition.

the computer program SAP2000 (version 19) is used to discretize the bridge into four-node shell elements with six degrees of freedom at each node. Following a sensitivity analysis, the size of each shell element is taken to be 1 ft x 1 ft (Mabsout et al. 2004). AASHTO design truck and tandem loads are modeled as point loads applied at the nodes. Figures 2 and 3 show the finite element discretization of a typical 36 ft, two-lane bridge under Centered (a) and Edge (b) Loading conditions, for AASHTO design truck and tandem configurations, respectively.

The FEA longitudinal bending moments per unit foot along the critical cross-section for all two-lane bridges with the variable span lengths considered are plotted in Fig. 4, under AASHTO design truck Centered and Edge Loadings. The results are reported in terms of the maximum moment in the slab at the critical section, defined as the first peak value after the left edge peak moment. It is assumed that the maximum peak moment at the edge is resisted by an edge beam, and is not the subject of study in this current paper. Results show that the maximum moments that were calculated for the Edge Loading condition are generally larger than the moments from Centered Loading for all bridge cases considered.

Table 1 presents a summary of results for all the bridge cases analyzed. As indicated in the table, the FEA maximum longitudinal moments with the critical Edge Loading are obtained from a combination of tandem and lane loads govern in short spans (24 and 36 ft) while the moments from AASHTO design truck and lane loads govern in the longer spans (46 and 54 ft).

4. Design using AASHTO LRFD

The AASHTO LRFD live load design bending moments are obtained from the simplified method by dividing the moment due to one lane live load by

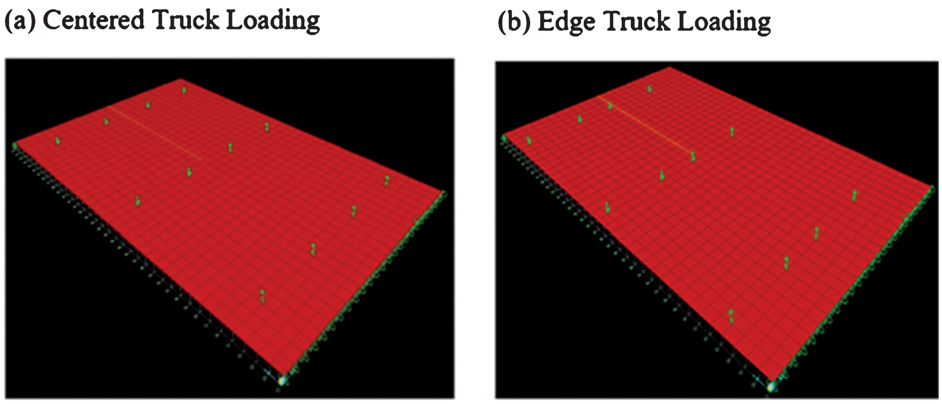

Fig. 2. Typical finite element discretization of a 36 ft span, two-lane bridge, under AASHTO design truck configuration for Centered (a) and Edge (b) Loading conditions.

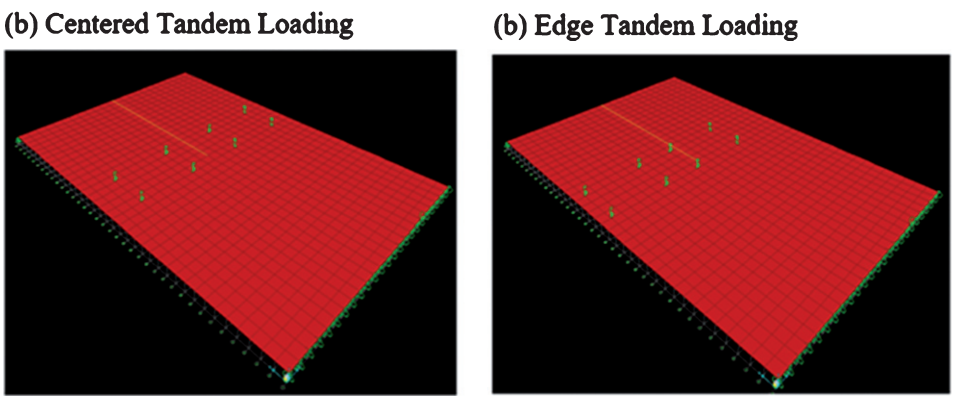

Fig. 3. Typical finite element discretization of a 36 ft span, two-lane bridge, under AASHTO design tandem configuration for Centered (a) and Edge (b) Loading conditions.

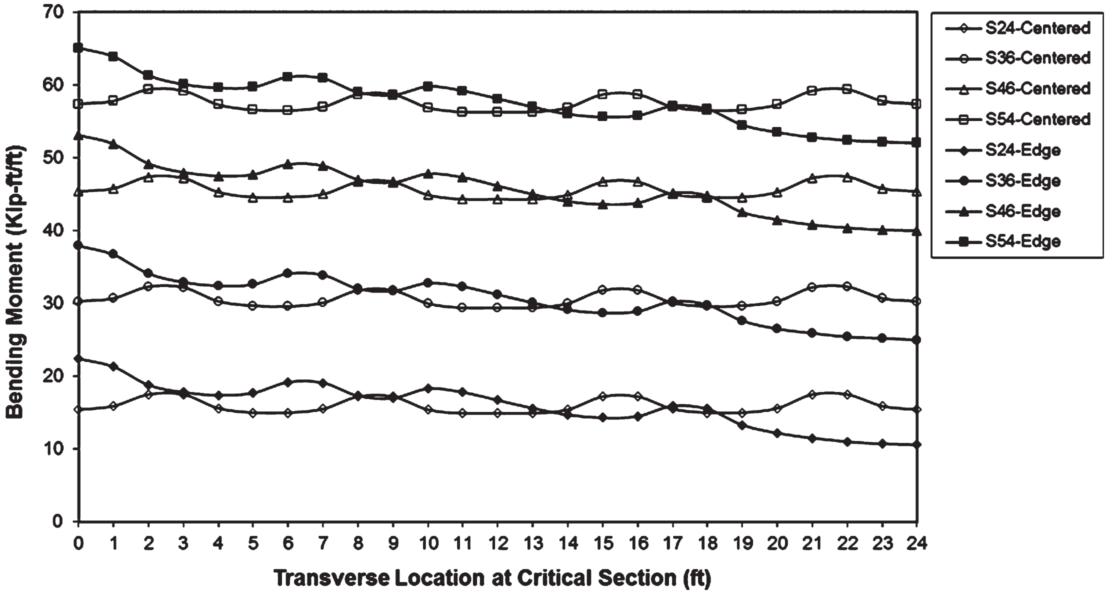

Fig. 4. Typical FEA longitudinal bending moments for two-lane bridges under AASHTO design truck configuration for Centered (a) and Edge (b) Loading conditions.

an equivalent width (AASHTO LRFD, 2012). This moment is divided by the equivalent width (E) for each span, as determined from Equations 2 or 3:

$$E = 10 + 5\sqrt{L_1}W_1/12 \text{ for single-lane bridges} \quad (2)$$

$$E = 84 + 1.44\sqrt{L_1}W_1/12 \text{ for multiple-lane bridges} \quad (3)$$

where:

L_1 = span length in feet, the lesser of the actual span or 60 ft.

W_1 = edge-to-edge width of bridge in feet taken to be the lesser of the actual width or 60 ft for multilane loading, or 30 ft for single-lane loading.

Table 1 shows the AASHTO LRFD design moments in comparison with the FEA corresponding moments. The resulting moments calculated from the AASHTO LRFD design live load deviate from FEA, expressed by the ratio $\alpha_{LL(FEA/LRFD)}$, as follows: The moment deviation is about 30% less or 5% more for one-lane bridges (ratio ranging from 0.69 to 1.04); this deviation ratio is about equal or 30% more for multilane bridges (ratios ranging from 0.98 to 1.14 for two lanes, 1.11 to 1.27 for three lanes, and 1.18 to 1.22 for four lanes).

The design of the bridge slab using Equation 1, derived from the reliability-based AASHTO LRFD approach, requires knowledge about the nominal values of the bending moments due to dead load, live load, and impact load. The nominal bending moment due to dead loads includes the effects of the dead load coming from the slab's self-weight (DC) and the weight of the wearing surface above it (DW) (Nowak 1999). To determine the stress due to self-weight of the slab, the thickness of the slab was multiplied by the unit weight of concrete (0.150 kcf as per AASHTO LRFD, Table 3.5.1–1). Similarly, the stress due to the wearing surface was calculated as the product of the thickness (0.25 ft.) and the unit weight of 0.140 kcf. The nominal bending moment due to the components of the dead loads (M_{DC} and M_{DW}) was then determined based on the simply sup-

Table 2
Nominal longitudinal bending moments due to dead loads

Span Length (ft)	Slab Thickness (in)	M_{DC} (kip-ft/ft)	Wear. Surf. Thickness (ft)	M_{DW} (kip-ft/ft)	$M_{(DC+DW)}$ (kip-ft/ft)
24	18	15.66	0.25	2.52	18.18
36	21	41.11	0.25	5.67	46.78
46	24	76.71	0.25	9.25	85.96
54	27	118.92	0.25	12.75	131.67

ported moment equation. Table 2 presents a summary of the nominal bending moments determined based on the different dead load components for all bridge cases considered.

The total nominal maximum live load moment (M_{LL} +M_{IL}) was calculated as the summation of the statical live load moment (M_{LL}), and the dynamic/impact live load moment (M_{IL}) applied on truck or tandem only. AASHTO LRFD defines the ratio of the dynamic load allowance as 33% of the static moment of the truck or tandem components of the statical live load only. To calculate the impact load, the contribution of the truck/tandem load to the statical live load was isolated and multiplied by a factor of 0.33. Table 3 shows the values of the statical, impact, and total nominal live load moments.

Given the nominal dead load and live load moments from Tables 2 and 3, the AASHTO LRFD design Equation 1 can be applied to calculate the nominal moment resistance (Rn) for each bridge such that:

$$Rn = \frac{1.5M_{DW} + 1.25M_{DC} + 1.75\,(M_{LL} + M_{IL})}{0.9} \quad (4)$$

5. Reliability analysis

Monte Carlo simulations were utilized to conduct a reliability analysis for concrete slab bridges that are designed based on the AASHTO LRFD design equation. Failure was defined using the performance function shown in Equation 5:

$$g = R\text{–}[DC + DW + (LL + IL)] \quad (5)$$

where R, DC, DW, and (LL + IL) were assumed to be random variables, representing the nominal moment resistance and the applied moments resulting from dead and live loads. The probability of failure (P_f) was determined from the Monte Carlo simulations by counting the realizations with ($g < 0$), and dividing them by the total number of simulations. The number of simulations was selected based on a sensitivity analysis assuming a 95% confidence level and 10% error for the results to reach a target reliability index of 3.5 based on the probability of failures. Based on this analysis, the target reliability index was reached using 1,000,000 simulations.

The reliability index β, which is a measure of structural safety, was then calculated as:

$$\beta = -\phi^{-1}(P_f) \quad (6)$$

where Φ^{-1} constitutes the inverse of the standard normal cumulative distribution function. The probabilistic models and the statistical parameters (mean and standard deviation) describing the uncertainty in the different design variables are discussed in the following sections.

5.1. Statistical load and capacity models

The statistical parameters (bias λ and coefficient of variation V) for the bending moments due to slab self-weight (DC) and wearing surface (DW) were adopted

Table 3
AASHTO LRFD nominal longitudinal bending moments due to live loads with impact

Number of Lanes	Span Length (ft)	Slab Width (ft)	Governing M_{LL} Source	M_{LL} LRFD LRFD (kip-ft/ft)	M_{IL} LRFD LRFD (kip-ft/ft)	$M_{(LL+IL)}$ LRFD (kip-ft/ft)
1	24	14	Tandem	34.95	9.84	44.79
	36	14	Tandem	49.44	13.13	62.57
	46	14	Truck	60.68	18.93	79.61
	54	14	Truck	69.84	18.77	88.61
2	24	24	Tandem	29.97	7.69	37.66
	36	24	Tandem	47.85	10.23	58.08
	46	24	Truck	63.19	14.71	77.90
	54	24	Truck	75.82	14.57	90.39
3	24	36	Tandem	28.13	7.92	36.05
	36	36	Tandem	44.49	12.71	57.20
	46	36	Truck	60.36	18.17	78.53
	54	36	Truck	77.1	18.77	95.87
4	24	48	Tandem	26.74	7.53	34.27
	36	48	Tandem	42.00	11.16	53.16
	46	48	Truck	59.77	18.00	77.77
	54	48	Truck	77.10	19.22	96.32

Table 4
Live load statistical parameters and analysis

Number of	Span Length (ft)	λ_{LL}	V_{LL}	Mean of $M_{(LL+IL)}$ (kip-ft/ft)	SD of $M_{(LL+IL)}$
1	24	1.38	0.12	61.80	9.77
	36	1.39	0.12	86.97	13.36
	46	1.37	0.12	109.01	18.14
	54	1.36	0.12	120.5	18.85
2	24	1.16	0.12	43.68	7.43
	36	1.19	0.12	69.11	10.66
	46	1.19	0.12	92.70	14.83
	54	1.18	0.12	106.65	15.85
3	24	1.16	0.12	41.81	7.45
	36	1.19	0.12	68.07	11.99
	46	1.19	0.12	93.46	16.90
	54	1.18	0.12	113.12	18.56
4	24	1.16	0.12	39.75	7.08
	36	1.19	0.12	63.26	10.76
	46	1.19	0.12	92.54	16.74
	54	1.18	0.12	113.66	18.86

from Nowak (1995) as $\lambda_{DC} = 1.05$ and $V_{DC} = 0.1$, and $\lambda_{DW} = 1.0$ and $V_{DW} = 0.25$, respectively. The bias factor is defined as the ratio of the mean of a given parameter to the nominal value of that parameter. As a result, the mean values for DC and DW for all bridges considered can be defined from λ_{DC} and λ_{DW} together with the nominal moment values shown in Table 2. The bias factors λ_{LL} for the statical live load moments M_{LL} were presented by Nowak (1999) and are dependent on the number of lanes and span lengths, while the coefficient of variation V_{LL} has been found to be constant with a value of 0.12. Table 4 shows the bias factors and coefficient of variation of the live load moment for each span and number lanes. The mean and standard deviation (SD) of the nominal live load $M_{(LL+IL)}$ are determined according to Equations 7 and 8 by combining the statistics of the statical live load and the impact load. In Equation 8, the values of 0.12 and 0.8 represent the coefficients of variation of the statical live load and the dynamic impact load, respectively:

$$Mean\ of\ M_{(LL+IL)} = \lambda_{LL} x\, M_{(LL+IL)nominal} \quad (7)$$

$$SD\ of\ M_{(LL+IL)} = \sqrt{(0.12\lambda_{LL} M_{LL})^2 + (0.8 M_{IL})^2} \quad (8)$$

The resulting mean and standard deviations of the total live load for the cases analyzed in this study are presented in Table 4 together with the corresponding estimates of the coefficient of variation of the nominal moment $M_{(LL+IL)}$.

Finally, the statistical parameters for the moment capacity for reinforced concrete slab bridges were adopted from Kulicki et al. (2007) based on a bias factor λR of 1.14 and a VR of 0.13. In the reliability analysis, the moments due to slab weight, wearing surface, and total live load were assumed to be normally distributed as per the recommendations of Kulicki et al. (2007). Along the same lines, the moment capacity was taken to be lognormal distributed. It is worth noting that the assumption of the normal distribution for the total live load is based on the study of Kulicki et al. (2007). The live load (being an extreme 75 year load) could have also been represented by a Gumble distribution. However, the fact that the live load is a summation of two components (lane load+impact) that are not correlated makes the total live load approach the normal distribution based on the central limit theorem. It is expected that the results of the reliability analysis will not be affected in any significant manner by the adopted distribution for the total live load.

5.2. *Results from reliability analysis*

The first set of reliability analyses were conducted to assess the reliability levels that are inherent in concrete slab bridges that are designed in accordance with the current LRFD design equation which is based on a live load factor of 1.75. The results of this set of analyses are presented in Fig. 5, indicating that the reliability index Beta (β) ranges from 2.6 to 3.0 for the cases involving single-lane bridges and is slightly below 3.5 for the cases involving two-lane

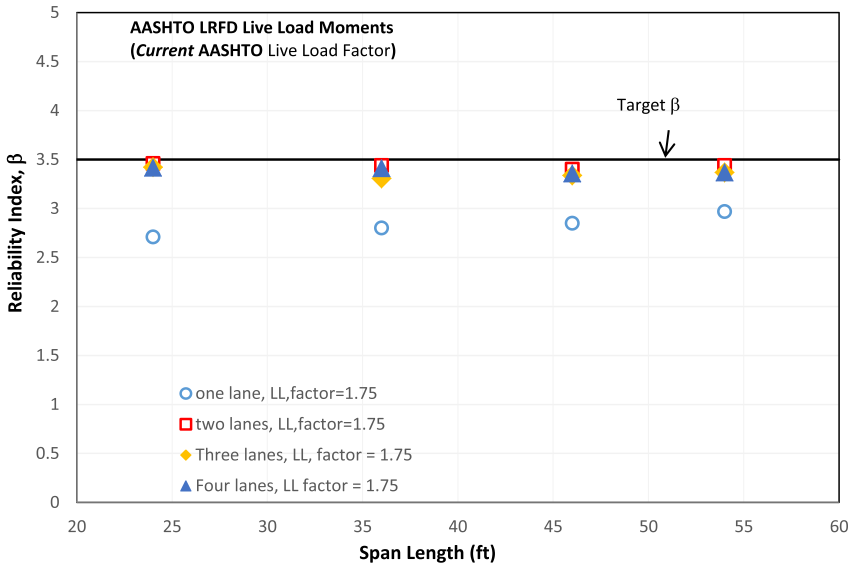

Fig. 5. Reliability indices for concrete slab bridges per simplified AASHTO LRFD moments for AASHTO live load factors.

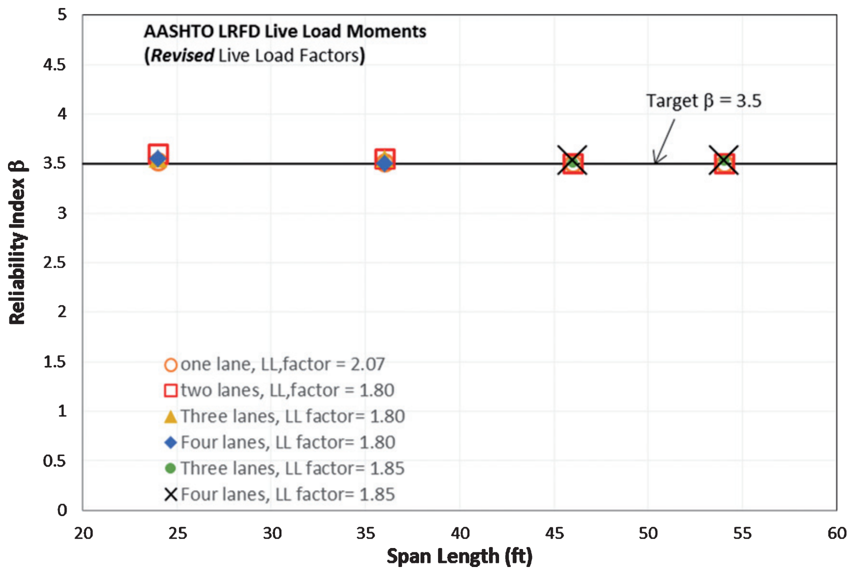

Fig. 6. Reliability indices for concrete slab bridges per simplified AASHTO LRFD moments for revised live load factors.

slab bridges. The results for the three-lane bridges show that the reliability index is between 3.31 and 3.42 for short spans and between 3.34 and 3.37 for longer spans. For the four-lane case, the results show that the reliability index is between 3.36 and 3.42. Results of the single lane concrete bridges reflect reliability levels that fall short of the target reliability index of 3.5 that was set by AASHTO LRFD. On the other hand, the results of the two-lane, three-lane, and four-lane bridges are closer to the target reliability level.

Results shown in Fig. 5 point to the need for revising the AASHTO LRFD live load factors if a reliability index as high as 3.5 is to be targeted. This is particularly important for the case involving single-lane bridges. The reliability analysis was therefore repeated assuming different live load factors to identify the factors that would ensure the desired level of reliability in the design. Results indicated that, for single lane bridges, a live load factor that is as high as 2.07 is required to ensure that bridges with all span lengths would achieve a target reliability index of 3.5.

Table 5
Reliability indices for concrete slab bridges per simplified AASHTO LRFD moments for AASHTO live load factors and revised live load factors

Number of Lanes	Span Length (ft)	Beta (LL factor = 1.75)	Beta (LL factor = 2.07)	Beta (LL factor = 1.80)	Beta (LL factor = 1.85)
1	24	2.71	3.52	–	–
	36	2.80	3.51	–	–
	46	2.85	3.50	–	–
	54	2.97	3.51	–	–
2	24	3.46	–	3.60	–
	36	3.44	–	3.55	–
	46	3.40	–	3.50	–
	54	3.44	–	3.50	–
3	24	3.42	–	3.56	–
	36	3.31	–	3.55	–
	46	3.34	–	–	3.52
	54	3.37	–	–	3.53
4	24	3.42	–	3.55	–
	36	3.41	–	3.51	–
	46	3.36	–	–	3.53
	54	3.37	–	–	3.54

For the two-lane bridges, the LRFD load factor needs to be increased slightly from 1.75 to 1.8 to achieve the target reliability level. For the three-lane and four-lane bridges, the reliability indices found when the live load factor was increased to 1.80 showed that for the short spans, the reliability indices reached the target index while for the longer spans the live load factor had to be increased to 1.85 to reach the target index. This paper proposes revising LRFD design equations for the single and multiple lane concrete slab bridges as presented in Equations 9, 10, and 11:

One-lane bridges, all spans:

$$\phi Rn = \left(1.25M_{DC} + 1.5M_{DW} + 2.07M_{(LL+IL)}\right) \tag{9}$$

Two-, three-, and four-lane bridges, short spans:

$$\phi Rn = \left(1.25M_{DC} + 1.5M_{DW} + 1.80M_{(LL+IL)}\right) \tag{10}$$

Two-, three-, and four-lane bridges, long spans:

$$\phi Rn = \left(1.25M_{DC} + 1.5M_{DW} + 1.85M_{(LL+IL)}\right) \tag{11}$$

The resulting reliability levels for the revised cases are presented in Fig. 6. Tables 5 summarizes the reliability indices found for the different cases using the AASHTO LRFD live load factor of 1.75, as well as the reliability indices for the revised AASHTO LRFD live load factors.

6. Summary and conclusions

In this paper, AASHTO LRFD simplified approach used in the design of reinforced concrete slab bridges is assessed using finite element analysis, and the reliability levels that are inherent in the empirical live load equations specified in AASHTO are assessed. Slab bridges with various practical geometries, mainly span lengths and number of lanes, were considered with AASHTO truck or tandem, and lane live loading.

For one- and two-lane bridge cases, the AASHTO LRFD bending moment equations tend to overestimate the live load moments obtained from the finite element analysis for short spans, and slightly underestimate those moments for longer spans. For three- and four-lane bridge cases, AASHTO LRFD tends to underestimate the live load moments obtained from FEA for all spans considered.

The reliability analysis performed in this study is used to check the level of safety for the concrete slab bridges considered that are designed with the AASHTO LRFD provisions. The results of the analysis showed that the reliability indices are slightly lower than the target reliability index of 3.5 for two-, three-, and four-lane concrete slab bridges, while these indices for one-lane bridges are considerably lower than the target reliability index.

To reach a consistent level of safety and ensure the target reliability index of 3.5 for all bridges considered, the results of this study suggest that the live load factor of 1.75 in the AASHTO LRFD load combination is revised by increasing the live load factor to the following concrete slab bridges: 2.07 for one-lane concrete slab bridges with all spans; 1.8 for two-lane bridges with all spans, and for three- and four-lane bridges with short spans; and 1.85 and for three- and four-lane bridges with longer spans. This research provides structural engineers with more consistent reliability index to design or evaluate the load-carrying capacity of concrete slab highway bridges.

Conflict of interest

None to report.

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