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ORIGINAL ARTICLE

Re-ordering policies for inventory systems with a fluctuating economic environment – Using economic descriptors to model the demand process

Wassim Dbouk[a], Hussein Tarhini[b] and Walid Nasr[a]

[a]Olayan School of Business, American University of Beirut, Beirut, Lebanon; [b]Department of Industrial Engineering and Management, American University of Beirut, Beirut, Lebanon

ABSTRACT

Customer demand is highly dependent on external environmental factors that are mainly economic in nature. We build on the financial literature to identify two key economic factors that impact the demand process, which are the gross domestic product (GDP) and the economic policy uncertainty (EPU). We accordingly categorize the economic environment into states that jointly capture different realizations of the GDP and EPU. We resort to data from the technology, automotive and oil industries to further validate the state categorizations by illustrating the relationship/dependency between the economic states and the demand process. We utilize a regularization technique to capture the fluctuation in the economic environment by a continuous time Markov Chain. We observe that the demand rates are dependent on the economic environment and accounting for this dependency improves the performance of the inventory system, especially in the technology industry. We numerically investigate the impact of the fluctuations on inventory control systems with continuous (s, S) inventory control policies. Algorithmic approaches, based on matrix representations of the system, are presented to compute the inventory performance measures. Our numerical study shows that savings are highest for the technology industry where the demand process is highly dependent on fluctuations in the economic environment.

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Stochastic demand; economic policy uncertainty; gross domestic product; correlated demand; MMPP; inventory systems; ordering policies

1. Introduction

Over the recent years, the nature and sources of macroeconomic risks have been unpredictable and constantly changing. Companies competing in the global marketplace need to account for known and unknown risks that could lead to significant disruptions in customer demand. Known sources of risk are due to events that have previously occurred and could possibly strike again (such as financial crises, or trade wars). Companies also need to account for unknown and new sources of risk, such as the current COVID-19 pandemic. Operating in such an uncertain environment renders decision-making a challenging task for companies, especially in the context of supply chain systems. Accurate forecasts of customer demand becomes unattainable and firms would face one of two scenarios: producing “too many” or “too few,” where both scenarios comprise several pitfalls (Cachon & Terwiesch, 2020). Leduc and Liu (2016) illustrate that uncertainty raises unemployment levels, lowers inflation, and increases the aggregate demand shock. Similarly, and in order to assess the impacts of uncertainty, both Bloom (2009) and Lee et al. (2010), show the negative effects of uncertainty on demand and output.

Accordingly, economists need to formulate models that capture the uncertainty in the events (or shocks) that affect economic activity and consequently customer demand. Such models have been serving as a basis for improving decision-making for businesses (Hansen & Sargent, 2019). Capturing the extent of inter-dependency between the economic environment and customer behaviour can be challenging, especially in an unstable and dynamic environment. We resort in this article to the gross domestic product (GDP) growth, as well as to the economic policy uncertainty (EPU), as potential shock factors that affect the level of companies’ sales, and their inventory management policies. The EPU index has been recently developed by Baker et al. (2016) and it uses a frequency-based approach. As such, it employs keywords to search through several newspaper articles for indications of uncertainty, using a time series and cross-sectional bases. The EPU index also encompasses several measures of uncertainty, including monetary policy, fiscal policy, taxation, government spending, healthcare, national security, entitlement programs, regulation, financial regulation, trade policy, sovereign debt, and currency crises.

The effect of EPU, and of its uncertainty components, on sales and inventory management is addressed in the literature. For instance, Bloom et al. (2007) present a model in which uncertainty

CONTACT Walid Nasr wn12@aub.edu.lb Olayan School of Business, American University of Beirut, Beirut 1107 2020, Lebanon.

reduces a firms' irreversible investment, in response to shocks that target sales levels due to uncertainty. Furthermore, the tax uncertainty component impacts inventory management and the production process as it affects the forecasts of inventory needs at year-end (Keng, 2018). In addition, factors related to government spending (Baxter & King, 1993), trade restrictions (Boutchkova et al., 2012; Kuttner, 1991), government regulations (Colombo et al., 2015), financial stability (Barrell et al., 2008; Chami et al., 2017) among other components of EPU hold a direct effect on sales and inventory management. Consequently, as the EPU index captures uncertainties related to these factors, it would serve as an adequate proxy for macroeconomic shocks, which we can incorporate in our model. Macroeconomic shocks constitute another important variable that should be taken into consideration when modelling sales and inventory systems. It impacts overall short-term investment decision-making related to inventory investments (Carpenter et al., 1994; Kashyap et al., 1994). A study in (2020) highlights the main sources of uncertainty that have contributed to the evolution of the EPU. These sources include the COVID-19 pandemic, economic crises (such as the 2008 Recession and the dot-com bubble), among other sources of high uncertainty. The COVID-19 pandemic is an on-going and distinctive example on the impact of uncertainty on the global economy where the levels of uncertainty even exceed those of the 2008–2009 recession (Baker et al., 2020). The pandemic is disrupting global supply chains, Nagurney (2021), and has drastically impacted customer demand across different industries. For example, demand has drastically increased for medical supplies, hand sanitizers, and delivery services, but supply was not able to cope with such a situation. In other industries (such as the automotive industry) consumer spending has decreased and demand has drastically dropped (Ivanov, 2020; Shen & Sun, 2021).

The COVID-19 pandemic has ceaselessly affected the global economy, the aggregate demand for products in the financial markets, as well as overall firm activities due to reduced productivity especially in the transportation, services, and manufacturing industries (Pak et al., 2020). For instance, Jiang et al. (2021) report a fall in the world industrial production index by 4.5 percent in the first quarter of 2020. The direct consequences of the pandemic have led to significant reductions in income, business disruptions and a sharp increase in economic uncertainty levels, with a powerful impact on the volatility of commodity and crude oil markets (Jiang et al., 2021). Hence, modelling the behaviour of the demand process, while accounting for the GDP growth and the EPU index factor, will help mitigate the impact of pandemics on economic prosperity. Current macro-level practices have underestimated the risks of the pandemic and have been mostly reactive in their responses (Pak et al., 2020).

Companies strive to achieve the best financial performance through measuring a variety of factors, including more accurate demand forecasts, better inventory management, and an improved responsive supply chain (Fisher, 1997). These three factors have become important to competitive success and survival due to the increasing requirement to reduce prices and inventories, because of the increased competitiveness in a global economy that can erode competitive advantages (Christopher, 2016; Fisher, 1997). The recent COVID-19 pandemic has demonstrated that significant predictors of product demand include GDP growth and economic uncertainty. As a result, when companies confront demand uncertainty, it is critical to respond swiftly based on these macroeconomic factors, in order to meet consumer demands for shorter lead times and coordinate supply to meet demand peaks and troughs (Sabath, 1998).

In the context of supply chain systems, fluctuations in the EPU and GDP translate into auto-correlated and state-dependent demand processes. The dependency of the demand process on external, dynamically evolving environments, motivates the use of the Markov Modulated Poisson Process (MMPP) in order to capture the demand process. An MMPP is described by an embedded continuous time Markov chain (embedded-CTMC) with states denoting the "state of the world" or the state of the external environment. In this article, we treat the evolution of the GDP and EPU over time as stochastic processes with discrete states. This results in a stochastic process that describes the joint behaviour of the GDP and EPU. The joint-GDP and EPU process is assumed to follow a CTMC, which is approximated by the embedded-CTCM of the MMPP. The parameters of the resulting embedded-CTMC are fitted to observed data, via a regularization technique presented by (2001).

2. Literature review

The literature on inventory control recognizes the advantages of using the MMPP to capture the behaviour of customer demand. This is especially the case when the demand environment fluctuates between external/uncontrollable "world states" (Avci et al., 2020; Chen et al., 2017; Özkan et al., 2013). Song and Zipkin (1993) show that (s, S) dynamic reordering policies are optimal under MMPP demand. Further extensions on the optimality of (s, S) policies are considered in Sethi and Cheng (1997) to account for non-stationary systems, and in Beyer and Sethi (1997) to include the stationary average-cost problem. The optimality of dynamic (s, S) reordering policies under MMPP demand is thoroughly addressed in the literature for a generalization of classical inventory models. We refer to Beyer et al. (2010) for computational approaches on several classical inventory models. Computational approaches for inventory systems with MMPP demand and (s, S) reordering policies are also considered in 2015, 1997, 1993, 2007, among others. Computational approaches for inventory systems with Markovian demand are also considered in 2022 and 2018. In this work we consider continuous (s, S) reordering policies over a finite time horizon. In such a system, the inventory performance measures are obtained by solving systems of linear equations. Equations for the performance measures are presented in compact matrix notation, which results in a computational framework that is more tractable and easier to replicate.

The work in L. Wang et al. (2020) explores the volatility of copper price due to global economic policy uncertainty and investigates the changes in the demand and supply of refined copper, as a result of this volatility. The authors conclude that the increase in economic policy uncertainty will enhance the long-term volatility of copper. Moreover, the extreme fluctuation in copper price caused by the impact of powerful economic policy uncertainty will weaken the demand confidence of refined copper market and lead to the phenomenon of oversupply. On the contrary, moderate fluctuation in copper price due to the impact of weak economic policy uncertainty will boost the demand confidence of refined copper market and lead to the phenomenon of short supply. The work in Sinaga et al. (2018) shows that economic indicators influence the performance of supply chain systems. Increases in the inflation rate and interest rate decreases the performance of supply chain systems. However, increases in the human development index (HDI) enhances the GDP and promote supply chain performance. Vachon and Mao (2008) investigates the potential link between supply chain characteristics and sustainable development at the country level. They found that supply chain strength has a strong correlation with GDP per capita.

Irvine and Schuh (2005) illustrate that changes in inventory behaviour can account directly for almost half of the total reduction in GDP volatility. However, reduced volatility of sales and lower covariance among the output of major sectors in the economy each account for more than one-fourth of the reduction in GDP volatility. Improved inventory management appears to be associated loosely with lower volatility at the industry level. Khan and Thomas (2007) generalized an equilibrium business cycle model to allow for endogenous (s, S) inventories of an intermediate good. They found that inventory investment co-moves with final sales and GDP.

Recently, and in the context of inventory policy control, several research papers have linked the behaviour of the demand process with a Markovian environment. The work in Barron (2015) considers an EOQ inventory policy and presents a matrix analytic approach that is based on fluid flow techniques to obtain steady state equations for the system cost functions. A state-dependent compound Poisson demand process is considered in Barron and Baron (2020b) where the demand rate is influenced by factors such as service levels, advertisement effectiveness, the age of the on-hand inventory, as well as the quantity of the items in stock. The model in Barron and Baron (2020a) accounts for Poisson demand that is dependent on the state of the inventory. The authors also account for a random lead time and illustrate the importance of reducing the variability in the lead time. We refer to 2005, 2014, 2017 for inventory control models that account for a Markovian environment that captures the dependency between the state of the inventory and the demand rate. In this work we do not account for the age or perishability of the inventory and we assume that the demand process is independent of the quantity and quality of the items in stock. We assume that the Markovian environment is a state of nature that is not influenced by the state of the inventory or the inventory control policy. Another relevant stream of literature accounts for product returns, which many companies allow to remain competitive. Since the return process is stochastic, this significantly effects the uncertainty in the inventory system. We do not account for returns in our model, but we refer the interested reader to the following relevant literature (Barron, 2018; Barron & Dreyfuss, 2021; Huynh et al., 2016).

3. Problem description and article outline

The problem is formulated in two separate but related stages. The objective of the first stage is to capture the dependencies between the economic environment and customer demand via an embedded Continuous-Time Markov Chain (CTMC), with parameters fitted from data. This involves identifying the economic drivers that influence the demand process, which are the gross domestic product (GDP) and the economic policy uncertainty (EPU). We select six companies across three industries (the automotive, gas and oil, and technology industries) and we evaluate the impact of these economic factors on customer demand by categorizing the state of the economy into nine different states denoting different realizations of the GDP and EPU. The state categorizations are presented in Section 4 along with the data pertaining to the transition rates between the states and the demand rates during each state. A main challenge at this stage is fitting the parameters of the CTMC to the economic environment, as governed by the GDP and EPU. This is performed by resorting to regularization fitting techniques. A description of the regularization technique implemented in this work as well as the accuracy of the resulting fits are presented in Section 5.

The second stage involves identifying the state of the resulting inventory system and presenting a compact matrix representation, Section 6. The matrix representation is tailored for inventory systems with MMPP demand with n-states. Such a representation (although tedious to derive) results in a compact and computationally efficient approach to solve for of the performance measures of the inventory system. Heuristics can be efficiently implemented to solve for the re-ordering policies.

The rest of the article is organized as follows. A thorough investigation of the categorization of the states of the environment is presented in Section 4. In Section 5, we fit the parameters of an MMPP to the available data, via a regularization technique, to obtain an MMPP approximation of the demand process. The equations of the performance measures of the resulting inventory systems are presented in section 6. In Section 7, we present heuristics based on the Genetic Algorithm to solve for the re-ordering policy. Numerical examples are presented in Section 8. Finally, a summary discussion, future recommendations and conclusions are presented in Section 9.

4. Categorizing the state of the environment

For our analysis we use the period 1985 to 2017 as sample years (since data from Standard and Poor's Compustat is significantly abundant starting from 1985). In order to compute the real GDP growth, we use the quarterly real GDP data from the U.S. Bureau of Economic Analysis (A191RX series), then we compute the quarterly growth as $g = (GDP_t - GDP_{t-1})/GDP_{t-1}$. Then, we define periods of high growth, normal, and low growth based on the respective tertile bucket in which the growth falls. On the other hand, monthly EPU data for the US is obtained from Baker et al. (2016)[1]. Similar to real GDP growth we classify our periods of low, normal, and high EPU periods based on the respective tertile in which they fall.

Let n_e and n_g be the number of possible states for the EPU and GDP, respectively. The state of the economy is represented by the joint EPU and GDP state realizations, (i, j) for $i = 1, \ldots, n_e, j = 1, \ldots, n_g$. This results in a total of $N = n_e \times n_g$ states. In this work we define three states for the EPU, $n_e = 3$, where 1, 2 and 3 represent high, normal and low EPU value periods respectively. Similarly, we define three states for the GDP, $n_g = 3$, where 1, 2, and 3 represent high, normal and low growth periods respectively. This results in nine possible allocations of EPU and GDP, $N = 9$.

For each month, we assign the monthly GDP growth state value to be that of the quarter within which the month falls. Based on these 9 possible states we classify each month in our sample from January 1985 to December 2017. Then, we compute in Table 1 the number of transitions from a state to another based on the historical change in states, observed in our 396 months sample[2]; whereby "Sum" refers to the total number of transitions from a given state to another.

We select companies that are dependent on the economic states by resorting to Compustat to compute the sales of all companies provided by the database. Then, we select companies that have high variation in sales across the 9 states. We standardize the level of sales of each company by dividing the sales of the company across all sample years by its minimum level and then select firms with the maximum level of standardized sales. Our results confirm that firms with the highest variation in sales in the sample are also those with the maximum standardized sales.

Tables 2–4 present the characteristics of six chosen firms which are: Apple and HP from the technology sector, Exxon Mobil and Royal Dutch from the oil and gas sector, and Daimler and

Table 1. One month state transitions – Observed from data.

State	1	2	3	4	5	6	7	8	9	Sum
1	25	5	2	3	0	0	0	0	0	35
2	4	29	5	1	3	0	0	0	0	42
3	5	3	33	0	0	3	0	0	1	45
4	1	0	0	25	1	3	11	1	0	42
5	0	4	0	5	32	6	0	3	0	50
6	0	1	5	2	6	29	1	0	3	47
7	0	0	0	5	2	0	36	6	6	55
8	0	0	0	1	7	0	4	24	2	38
9	0	0	0	0	0	6	3	4	28	41

Table 2. Demand by state (millions of dollars).

Company/State	1	2	3	4	5	6	7	8	9
Apple	27585	15766	4437	14713	17198	5592	14219	10977	7823
Exxon Mobil	65945	60036	49438	71060	44634	34980	55949	51834	43288
HP	17489	17094	15316	16583	10223	11504	18495	14705	7383
Royal Dutch	66856	58368	47912	71944	49720	34731	58995	52630	42337
Daimler	43496	37940	40454	36176	44218	42974	38590	36896	38702
Toyota	54877	51277	50479	52065	48044	66802	57807	48544	37312

Table 3. Demand by state – Normalized.

Company/State	1	2	3	4	5	6	7	8	9
Apple	6.22	3.55	1.00	3.32	3.88	1.26	3.20	2.47	1.76
Exxon Mobil	1.89	1.72	1.41	2.03	1.28	1.00	1.60	1.48	1.24
HP	2.37	2.32	2.07	2.25	1.38	1.56	2.51	1.99	1.00
Royal Dutch	1.92	1.68	1.38	2.07	1.43	1.00	1.70	1.52	1.22
Daimler	1.20	1.05	1.12	1.00	1.22	1.19	1.07	1.02	1.07
Toyota	1.47	1.37	1.35	1.40	1.29	1.79	1.55	1.30	1.00

Table 4. Demand fluctuation by company.

Company/State	cv	max
Apple	0.54	6.22
Exxon Mobil	0.22	2.03
HP	0.26	2.51
Royal Dutch	0.22	2.07
Daimler	0.07	1.22
Toyota	0.15	1.79

Toyota from the automotive sector. Table 2 shows the average sales, in millions of dollars, generated by the six selected companies in each of the nine economic states. We assume that the dollar value of the demand is equal to the sales. This is based on the assumption that the unfulfilled demand for the selected companies is negligible. Table 3 shows the normalized average demand (based on minimum demand), that each of the six selected companies has generated, in each of the nine respective economic states. Table 4 shows the variation in the normalized demand; whereby "cv" refers to the coefficient of variation in the average sales, under each economic state, for each of the six selected companies; and "max" refers to the maximum difference in normalized sales between the best and worst economic states, for each of the six selected companies.

In times of low economic uncertainty and high living standards, consumers tend to increase their consumption of commodities such as gasoline, cars, latest mobiles, and smart electronics. Such is contrary to times of economic recession and high uncertainty, where people's purchasing power decreases. Consequently, production levels and the consumption of oil and gas drop as a result of a drop in demand. Similarly, the demand for used cars overtakes the demand for new cars in the automotive industry. Similarly for the technology sector (Apple, HP, etc.), demand for the latest technologies decreases, as older devices become the favoured choice. Table 2 shows that, in general, the level of sales deteriorates when the economic state deteriorates, which implies that demand for a firm's products depends on economic conditions. These results are reinforced in Table 3 where we report these companies' normalized sales. Table 4, on the other hand, shows the variation in the standardized sales and the maximum difference in standardized sales between the best economic state and worst economic state. Our results indicate that the selected technology firms are more volatile, in terms of sales, than firms selected from the oil and gas sector, which are in turn more volatile than those selected from the automotive industry.

5. Capturing the economic environment and demand process by an MMPP

An n-state MMPP is defined by an infinitesimal generator $\underset{(n\times n)}{\boldsymbol{\sigma}}$ and an arrival rate vector $\underset{(n\times 1)}{\boldsymbol{\lambda}}$. The generator $\boldsymbol{\sigma}$ describes the behaviour of an embedded-CTMC (ECTMC). The vector $\boldsymbol{\lambda}$ denotes the arrival rates of the MMPP conditioned on the state of the ECTMC. Accordingly, λ_i is the arrival rate conditioned on the ECTMC being in state i, for $i = 1, \ldots, n$. We refer to Fischer and Meier-Hellstern (1993) for a thorough description of the MMPP notation and characteristics.

Let $\{\mathrm{A}(t) : t \geq 0\}$ be the state of the ECTMC at time t where $\mathrm{A}(t) = 1, \ldots, N$. Let $\underset{(N\times N)}{\mathbf{p}}$ represent the one-month transition probabilities where $p_{i,j}$ is the probability the economy is in state i after one month, conditioned on starting in state j, for $i, j = 1 \ldots, N$. Denote the point estimate of the one-month conditional probabilities by $\hat{\mathbf{p}}$. In this section, we model the behaviour of the economy by a CTMC with nine states, $N = 9$ as defined in Section 4. The one-month probabilities, $\hat{p}_{i,j}$, are calculated from Table 1 and presented in Table 5, for $i, j = 1, \ldots, 9$.

Let $\underset{(N\times N)}{\boldsymbol{\sigma}}$ denote the infinitesimal generator of the CTMC where,

$$\underset{(N\times N)}{\mathbf{p}} = \underset{(N\times N)}{e^{\boldsymbol{\sigma}\tau}}, \quad \text{for } \tau = 1 \text{ month}. \tag{1}$$

Solving for an estimate of the generator, $\boldsymbol{\sigma}$, requires solving for the matrix logarithm of $\hat{\mathbf{p}}$,

$$\underset{(N\times N)}{\mathbf{L}} = ln(\underset{(N\times N)}{\hat{\mathbf{p}}}). \tag{2}$$

Notice that implementing $\mathbf{L}$ as an estimate of $\boldsymbol{\sigma}$ is not possible in the case where the matrix logarithm, $\mathbf{L}$, has negative entries in the non-diagonal entries, $L_{i,j} < 0$ for an $i \neq j$. For such cases, regularization algorithms are proposed, Kreinin and Sidelnikova (2001), to fit a generator matrix to the system state probabilities for a specific time τ for $\tau > 0$. We describe a regularization approach in Algorithm 1 to calculate the estimate of the CTMC generator, $\hat{\boldsymbol{\sigma}}$.

Algorithm 1. Regularization Approach

- ▷ **Step 1:** Set $\mathbf{L} \equiv$ Matrix Logarithm of $\hat{\mathbf{p}}$.
- ▷ **Step 2:** Set $\hat{\sigma}_{i,j} = \max(L_{i,j}, 0)$, for $i, j = 1, \ldots, N$ and $i \neq j$.
- ▷ **Step 3:** Set $S_i = \sum_{j=1, \; j \neq i}^{9} \hat{\sigma}_{i,j}$, for $i = 1, \ldots, N$.
- ▷ **Step 4:** Set $\hat{\sigma}_{i,j} = -L_{i,i} \frac{\hat{\sigma}_{i,j}}{S_i}$, for $i = 1, \ldots, N$ and $i \neq j$.
- ▷ **Step 5:** Set $\hat{\sigma}_{i,i} = L_{i,i}$, for $i = 1, \ldots, N$.

Table 5. One-month transition probabilities – From data.

	1	2	3	4	5	6	7	8	9
1	0.7143	0.1429	0.0571	0.0857	0	0	0	0	0
2	0.0952	0.6905	0.119	0.0238	0.0714	0	0	0	0
3	0.1111	0.0667	0.7333	0	0	0.0667	0	0	0.022
4	0.0238	0	0	0.5952	0.0238	0.0714	0.2619	0.0238	0
5	0	0.08	0	0.1	0.64	0.12	0	0.06	0
6	0	0.0213	0.1064	0.0426	0.1277	0.617	0.0213	0	0.064
7	0	0	0	0.0909	0.0364	0	0.6545	0.1091	0.109
8	0	0	0	0.0263	0.1842	0	0.1053	0.6316	0.053
9	0	0	0	0	0	0.1463	0.0732	0.0976	0.683

Accordingly, the matrix logarithm, $\mathbf{L}$, as calculated from Equation 2 is presented in Table 6 and has negative non-diagonal entries (Step 1 of Algorithm 1). The generator, $\hat{\boldsymbol{\sigma}}$, is calculated by the regularization approach of Algorithm 1 and presented in Table 7. To test the accuracy of the regularization approach, we generate the one-month probabilities after regularization,

$$\hat{\mathbf{p}}^r = e^{\hat{\boldsymbol{\sigma}}}, \tag{3}$$

where $\mathbf{p}^r$ is presented in Table 8. Comparing $\hat{\mathbf{p}}$ and $\hat{\mathbf{p}}^r$ (as presented in Tables 5 and 8, respectively), the average absolute difference is 0.0071 and the maximum absolute difference is 0.0242. This illustrates the accuracy of the regularization approach in calculating a generator, $\hat{\boldsymbol{\sigma}}$ that closely captures the fluctuations in the states of the economy.

Note that the data sets we resorted to in Sections 4 and 5 have been obtained from the financial statements of publicly listed companies in the US market. Specifically, we obtain the quarterly revenues of manufacturing firms from 1985 to 2018 using the Compustat database, accessible through the Wharton Research Data Services (WRDS). We focus on manufacturing firms because they regularly practice inventory management. Our data sample consists of manufacturing firms (SIC codes between 2000 and 3999). On the other hand, we obtain EPU data from the Baker et al. (2016) economic policy uncertainty website. Finally, we obtain real GDP data from the FRED data desk at the Federal Reserve Bank of St. Louis.

6. The continuous MMPP inventory system

The evolution of the EPU and GDP are considered jointly when describing the behaviour of the external economic state. We discretize the state of the economy and model the transition/fluctuation between the economic states via a continuous-time Markov Chain (CTMC). We consider a continuous re-ordering policy where the re-order levels are dynamic and change with the state of the environment. Accordingly, let (s_i, S_i) denote the re-ordering policy when the state of the environment is in state i for $i = 1, \ldots, N$. Let $\{\mathrm{IP}(t) : t \geq 0\}$, be the state of the inventory position at time t. The inventory position is bounded by $\mathrm{IP}(t) \in [s_{min} + 1, S_{max}]$, where $s_{min} = \min(s_i)$ and $S_{max} = \max(S_i)$, for $i = 1\ldots, N$. The range on the inventory position is $y = S_{max} - s_{min}$.

6.1. Inventory position

The Markovian representation of the inventory system at time $t \geq 0$ is determined by the state of the inventory position at time t, $\mathrm{IP}(t)$ and the state of the ECTMC, $\mathrm{A}(t)$. Define the steady state probability of the Markovian representation by,

$$P_{(i,j)} = \lim_{t\to\infty} \mathrm{Prob}(\mathrm{IP}(t) = i, \mathrm{A}(t) = j), \quad \text{for} \quad i = s_{min}, \ldots, S_{max} \quad \text{for} \quad j = 1, \ldots, N. \tag{4}$$

Let $\mathbf{Q}_{1(Ny\times Ny)}$ denote the infinitesimal generator of the corresponding CTMC.

$$\underset{(Ny\times Ny)}{\mathbf{Q}_1} = \begin{array}{c} \\ \end{array}\begin{matrix} 1 & 2 & 3 & \cdots & N \end{matrix}$$
$$\underset{(Ny\times Ny)}{\mathbf{Q}_1} = \begin{bmatrix} \underset{(y\times y)}{\mathbf{D}_{1,1}} & \underset{(y\times y)}{\mathbf{D}_{1,2}} & \underset{(y\times y)}{\mathbf{D}_{1,3}} & \cdots & \underset{(y\times y)}{\mathbf{D}_{1,N}} \\ \underset{(y\times y)}{\mathbf{D}_{2,1}} & \underset{(y\times y)}{\mathbf{D}_{2,2}} & \underset{(y\times y)}{\mathbf{D}_{2,3}} & \cdots & \underset{(y\times y)}{\mathbf{D}_{2,N}} \\ \underset{(y\times y)}{\mathbf{D}_{3,1}} & \underset{(y\times y)}{\mathbf{D}_{3,2}} & \underset{(y\times y)}{\mathbf{D}_{3,3}} & \cdots & \underset{(y\times y)}{\mathbf{D}_{3,N}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underset{(y\times y)}{\mathbf{D}_{N,1}} & \underset{(y\times y)}{\mathbf{D}_{N,2}} & \underset{(y\times y)}{\mathbf{D}_{N,3}} & \cdots & \underset{(y\times y)}{\mathbf{D}_{N,N}} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ N \end{matrix}$$

where the non-diagonal entries $\mathbf{D}_{r,c}$ for $r, c = 1, \ldots, N$ and $r \neq c$,

$$\mathbf{D}_{r,c}(i,j) = \begin{cases} \hat{\sigma}_{r,c} & \text{if } i = j \\ \hat{\sigma}_{r,c} & \text{if } j = S_c \text{ and } i \leq s_c, \\ 0 & \text{otherwise} \end{cases},$$

and the diagonal entries $\mathbf{D}_{r,r}$ for $r = 1, \ldots, N$,

$$\mathbf{D}_{r,r}(i,j) = \begin{cases} -\hat{\lambda}_r + \hat{\sigma}_{r,r} & \text{if } i = j \\ \hat{\lambda}_r & \text{if } j = i-1 \text{ and } i \neq s_r + 1 \\ \hat{\lambda}_r & \text{if } i = s_r + 1 \text{ and } j = S_r \\ 0 & \text{if } i \neq j. \end{cases}$$

The steady state probabilities are calculated by solving the linear set of equations,

Table 6. Matrix logarithm from observed data – **L**.

State	1	2	3	4	5	6	7	8	9
1	0.3572	−0.3572	0.2045	0.0641	0.1337	−0.0116	−0.0098	−0.0273	0.001
2	0.3973	0.1246	−0.3973	0.1675	0.0214	0.1129	−0.0205	−0.0022	−0.0055
3	0.3297	0.1509	0.0791	−0.3297	−0.0136	−0.0135	0.1002	0.0006	−0.0008
4	0.5597	0.0392	−0.0061	−0.0112	−0.5597	0.0145	0.1278	0.4318	0.0058
5	0.4886	−0.0088	0.1234	−0.0246	0.1606	−0.4886	0.1899	−0.0439	0.0998
6	0.5268	−0.0146	0.0146	0.1624	0.055	0.211	−0.5268	0.0207	−0.0203
7	0.4751	−0.0029	−0.0011	0.0034	0.1465	0.0371	−0.0311	−0.4751	0.1587
8	0.4934	0.001	−0.0178	0.0057	0.0086	0.2979	−0.0373	0.1649	−0.4934
9	0.406	0.0022	0.0008	−0.0187	−0.0137	−0.0476	0.2377	0.099	0.1462

Table 7. Transition rates after regularization – $\hat{\boldsymbol{\sigma}}$.

State	1	2	3	4	5	6	7	8	9
1	−0.3572	0.18	0.0564	0.1176	0	0	0	0.0009	0.0023
2	0.1161	−0.3973	0.156	0.02	0.1052	0	0	0	0
3	0.1391	0.0729	−0.3297	0	0	0.0924	0.0006	0	0.0249
4	0.0354	0	0	−0.5597	0.0132	0.1155	0.3904	0.0053	0
5	0	0.1051	0	0.1367	−0.4886	0.1617	0	0.085	0
6	0	0.0137	0.1523	0.0515	0.1979	−0.5268	0.0194	0	0.0919
7	0	0	0.0031	0.1365	0.0346	0	−0.4751	0.1478	0.1532
8	0.0009	0	0.0051	0.0078	0.2679	0	0.1483	−0.4934	0.0634
9	0.0018	0.0007	0	0	0	0.1986	0.0827	0.1222	−0.406

Table 8. One-month probabilities from fitted CTMC – $(\hat{\mathbf{p}}^r = e^{\hat{\sigma}})$.

State	1	2	3	4	5	6	7	8	9
1	0.7116	0.1261	0.0505	0.0772	0.0076	0.007	0.015	0.0019	0.0031
2	0.0883	0.6875	0.1123	0.0221	0.0693	0.0114	0.004	0.0032	0.0021
3	0.1026	0.0607	0.7314	0.0081	0.0091	0.0632	0.0028	0.0016	0.0205
4	0.0231	0.0036	0.0069	0.5915	0.0212	0.0709	0.2378	0.0223	0.0226
5	0.0064	0.0697	0.0141	0.0866	0.6351	0.1043	0.0219	0.0542	0.0078
6	0.0089	0.0196	0.1021	0.0402	0.1229	0.6127	0.0222	0.0103	0.0613
7	0.002	0.002	0.0034	0.0854	0.0363	0.0172	0.6498	0.1	0.1038
8	0.0016	0.0094	0.0051	0.0225	0.1688	0.0189	0.0975	0.6274	0.0489
9	0.002	0.0026	0.0105	0.0085	0.0242	0.1274	0.0619	0.0834	0.6795

$$\underset{(1\times Ny)}{\mathbf{P}}\ \underset{(Ny\times Ny)}{\mathbf{Q}_1} = \underset{(Ny\times Ny)}{\mathbf{0}} \quad \text{and} \quad \sum_{i=s_{min}+1}^{S_{max}} \sum_{j=1}^{N} P_{(i,j)} = 1, \tag{5}$$

where $\mathbf{P} = [P_{(s_{min}+1,\,1)}, \ldots, P_{(S_{max},\,\ldots,\,1)}, P_{(s_{min}+1,\,2)}, \ldots, P_{(S_{max},\,\ldots,\,2)}, P_{(s_{min}+1,\,3)}, \ldots \ldots, P_{(S_{max},\,\ldots,\,N)}]$.

6.2. The demand process over lead time

Denote the inventory lead time by L and let $\{\mathrm{D}_t(L) : t \geq 0\}$ be the demand count over the time interval $[t, t+L]$. The distribution of the demand count process over lead time is commonly approximated by a Normal distribution in the inventory control literature, Silver et al. (1998). The Normal distribution provides an accurate approximation to the demand count process, especially if demand is a renewal process. In this work, the demand process is dependent on a fluctuating environment and is highly correlated. Furthermore, the demand count process over lead-time, conditioned on the state of the environment starting in state i, will have a different probability distribution if we condition on starting the state of the environment in state j $(i \neq j)$. In other words, $\mathrm{Prob}(\mathrm{D}_t(L) = d|\mathrm{A}(t-L) = i)$ is not necessarily equal to $\mathrm{Prob}(\mathrm{D}_t(L) = d|\mathrm{A}(t-L) = i$. In this section, we present the equations to compute the conditional distribution of the demand over lead-time in a matrix compact form. We also note that this work makes the simplifying assumptions that lead-time, L, is constant and independent of the demand process. We refer to in (Barron, 2019; Barron & Baron, 2020b) for Markovian inventory systems with random lead-time and to P. Wang et al. (2009) for inventory systems with correlated demand and lead-time.

At steady state, the demand over lead time is denoted by,

$$\mathrm{D}(L) = \lim_{t\to\infty} D_t(L). \tag{6}$$

Let $\Gamma_{(d|j)}(t, L) = \mathrm{Prob}(\mathrm{D}_t(L) = d|\mathrm{A}(t-L) = j)$ and $\Gamma_{(d,j)}(t, L) = \mathrm{Prob}(\mathrm{D}_t(L) = d, \mathrm{A}(t-L) = j)$ be the conditional and joint (respectively) probability distributions of the counting process with respect to the ECTMC at time t – L, for $d = 0, \ldots, \infty$ and $j = 1, \ldots, N$. The steady state of the conditional probability is expressed as follows,

$$\begin{aligned} \Gamma_{(d|j)}(L) &= \lim_{t\to\infty} \Gamma_{(d|j)}(t, L) \quad \text{and} \\ \Gamma_{(d,j)}(L) &= \lim_{t\to\infty} \Gamma_{(d,j)}(t, L), \quad \text{and } d = 0, \ldots, \infty \\ &\quad \text{and } j = 1, \ldots, N. \end{aligned} \tag{7}$$

For computational efficiency, we define d_{max} as the maximum demand during lead time. The value for d_{max} is set such that d_{max} is the smallest positive

Table 9. Illustration of the structure of a chromosome for the case of three states.

s_1	s_2	s_3	S_1	S_2	S_3

integer that satisfies $\text{Prob}(D(L) > d_{max}) < \epsilon$, for an appropriate choice of ϵ. The resulting infinitesimal generator of the CTMC of the counting process is augmented with the state of the ECTMC is $\mathbf{Q}_2$ and expressed as follows,

$$\underset{(Nd_{max}\times Nd_{max})}{\mathbf{Q}_2} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & \cdots & d_{max} \end{matrix} \\ \begin{bmatrix} \underset{(N\times N)}{\mathbf{C}_{1,1}} & \underset{(N\times N)}{\mathbf{C}_{1,2}} & \underset{(N\times N)}{\mathbf{0}} & \cdots & \underset{(N\times N)}{\mathbf{0}} \\ \underset{(N\times N)}{\mathbf{0}} & \underset{(N\times N)}{\mathbf{C}_{2,2}} & \underset{(N\times N)}{\mathbf{C}_{2,3}} & \cdots & \underset{(N\times N)}{\mathbf{0}} \\ \underset{(N\times N)}{\mathbf{0}} & \underset{(N\times N)}{\mathbf{0}} & \underset{(N\times N)}{\mathbf{C}_{3,3}} & \cdots & \underset{(N\times N)}{\mathbf{0}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underset{(N\times N)}{\mathbf{0}} & \underset{(N\times N)}{\mathbf{0}} & \underset{(N\times N)}{\mathbf{0}} & \cdots & \underset{(N\times N)}{\mathbf{C}_{d_{max},d_{max}}} \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ d_{max} \end{matrix}$$

where the non-diagonal entries $\mathbf{C}_{r,r+1}$ for $r = 1, \ldots, d_{max} - 1$ and $i, j = 1, \ldots, N$,

$$\mathbf{C}_{r,r+1}(i,j) = \begin{cases} \hat{\lambda}_i & \text{if } i = j, \\ 0 & \text{otherwise} \end{cases},$$

and the diagonal entries $\mathbf{C}_{r,r}$ for $r = 1, \ldots, d_{max}$ and $i, j = 1, \ldots, N$,

$$\mathbf{C}_{r,r}(i,j) = \begin{cases} -\hat{\lambda}_i + \hat{\sigma}_{i,i} & \text{if } i = j, \\ \hat{\sigma}_{i,j} & \text{if } i \neq j. \end{cases}$$

Consider the following homogeneous set of differential equations (HSDE) where the derivative is in respect to τ,

$$\mathbf{\Gamma}'(\tau) = \mathbf{\Gamma}(\tau)\ \mathbf{Q}_2. \tag{8}$$

We obtain the conditional probability matrix $\mathbf{\Gamma}(L)$ by solving the HSDE in (8) from $\tau = 0$ to L, where $\Gamma_{(d|j)} = \sum_{\ell=1}^{N} \mathbf{\Gamma}(j, \ell + N\ (d-1))$ for $d = 1, \ldots, d_{max}$ and $j = 1, \ldots, N$. Note that the HSDE of Equation (8) can be solved by numerical integration over the interval $\tau = 0 \to L$, or by solving the matrix exponential, $\mathbf{e}^{Q_2 L}$.

6.3. System performance measures

In this section, we present the equations to calculate the steady-state expected values for the net-inventory (NI), on-hand inventory (OH), number-in-backorder (B), and the number of orders per unit time (ρ). The expected net-inventory at steady state is calculated as follows,

$$\begin{aligned} \text{E[NI]} &= \text{E[IP]} - \text{E[D}(L)] \\ &= \sum_{j=1}^{N} \sum_{i=s_{min}+1}^{S_{max}} i\ P_{(i,j)} - \sum_{j=1}^{N} \hat{\lambda}_j\ \pi_j\ L, \end{aligned} \tag{9}$$

where π_j is the probability of being in state j of the ECTMC at stationarity, for $j = 1, \ldots, N$. The expected on-hand inventory is calculated as a function of the inventory position distribution and conditional-demand distribution, obtained from Equations (5) and (8), respectively.

$$\text{E[OH]} = \sum_{j=1}^{N} \sum_{i=0}^{S_{max}} \sum_{d=0}^{i-1} (i-d)\ \Gamma_{(d|j)}\ P_{(i,j)}. \tag{10}$$

Note that the derivation of Equation (10) is based on $\text{OH} = \max(\text{NI}, 0)$. The expected number-in-backorder becomes, $\text{E[B]} = \text{E[NI]} - \text{E[OH]}$. The number of orders can be modelled as an increasing (pure-birth) counting process. The numbers of orders per unit time is calculated as the rate the pure-birth process is incremented.

$$\rho = \sum_{j=1}^{N} \hat{\lambda}_j\ P_{(s_j+1,j)} + \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{i=s_{min}}^{s_j} \hat{\sigma}_{k,j}\ P_{(i,k)}. \tag{11}$$

The total cost per unit time is calculated as follows,

$$\phi = h\ \text{E[OH]} + b\ \text{E}[B] + K\ \rho, \tag{12}$$

where h is the holding cost per item per unit time, b is the back-ordering cost per item per unit time, and k is the cost of placing an order.

7. Genetic algorithm

Genetic algorithm (GA) is used to find the optimal (s, S) policy for the different states in the model earlier. GA is a random search technique that mimics natural selection processes (Goldberg & Holland, 1988). GA starts with an initial solution, then starts searching for better ones by applying neighbourhood search operators (crossover and mutation). Those operators are then applied to randomly selected solutions from a current set with a given probability, which is proportional to an obtained objective function value. GAs have been successfully implemented to a wide range of combinatorial optimization problems (Gen et al., 1997). The chromosome is represented by a one-dimensional matrix, as shown in Table 9, and is comprised of the decision variables of the optimisation problems. In our case, the order points s_i and the order-up-to points S_i are part of the chromosome, for $i, \ldots, N$.

7.1. Generation of initial population

An initial solution is fed to the GA to reduce the probability of returning a bad local minimum. This is due to the non-convexity of the model at hand. We denote λ_i to be the demand in state i. Since the model is of the continue-review type, the initial solution consists of the optimal solution of the

Table 10. Base case input parameters.

Fixed order cost K	Holding cost h	Backorder cost b	Lead Time τ	Minimum demand d_{min} per month
20	0.0125	0.05	4 months	30

Table 11. Apple's normalized demand rates at states 1–9.

1	2	3	4	5	6	7	8	9
6.22	3.55	1	3.82	3.88	1.26	3.2	2.47	1.76

economic order quantity model (EOQ) with shortages, where a non-negative safety stock is ensured. The initial solution used is as follows,

$$s_i = \lambda_i \tau \text{ and } S_i = \sqrt{\frac{2K\lambda_i}{h}}\sqrt{\frac{p}{p+h}}, i = 1, \ldots, N. \quad (13)$$

7.2. Fitness function

The fitness function of the genetic algorithm is the total cost ϕ of the model that was defined in equation (12). The decision variables are the ordering point s_i and the order-up-to point S_i. In addition, the following constraint has been added to the GA model:

$$s_i \leq S_i \quad \forall i = 1, 2, \ldots, N. \quad (14)$$

The value of the objective function is returned to the GA, and the process, as discussed below, continues until a solution is reached.

7.3. Mutation

Mutation in GA is used to search for better solutions by altering the current pool of solutions. Mutation is additionally aimed at maintaining diversity in the population. Mutation generates new solutions in the neighbourhood of a current solution by introducing a small change in some aspect of the current solution. This ensures that no point in the search space has a zero probability of being examined (Srinivas & Rao, 2010). The resulting chromosomes form the new population are evaluated to check their feasibility and optimality. The algorithm continues from one generation to the next and stops once some criteria are satisfied.

8. Numerical analysis

In this section, a numerical study is performed over the selected companies. The industries that the selected companies belong to are sensitive to changes in economic conditions. However, they're not all equally affected by these conditions because of the different nature of each industry, as well as the various inventory practices of each company. The cost unit is 10^6 $\month. To be able to compare the results with the different sectors, d_{min} is set to a constant value and the base model parameters are given in Table 10.

Apple is considered first. Table 11 shows the normalized demand based on the minimum demand level d_{min} which in the base case is set to 30.

As discussed earlier, the state-to-state transitions are modelled as a continuous time Markov chain. Thus, to obtain Apple's four-month transition rate matrix, we use the infinitesimal matrix with transition rates calculated in Table 7. To obtain the probability of being in state j after t time units, starting in state i, we solve for the exponential matrix P(t)= $e^{\hat{\sigma}t}$.

In this case, the vendor spends a month in each state but the lead time is four months. To obtain the probability that the system is in state j after 4 months, starting from state i, we compute P(4)=$e^{\hat{\sigma}4}$ in Table 12. The vendor has nine possible pairs of re-order points and order-up-to levels (s_i, S_i), for $i = 1...9$. These re-order points depend on the demand levels of every state, as well as on the probability transition matrix. The initial solution fed to the solver considers the demand at every state, without considering the probability transition matrix. The initial solution is tabulated in Table 13.

The initial solution yielded a total cost of 14.9. $s_i, \forall i = 1, \ldots, N$ is set to be equal to the demand during lead time, which yield a zero safety stock on average. $S_i, \forall i = 1, \ldots, N$, is the Economic Order Quantity formula with shortages. The generated optimal solution is presented next in Table 14.

The optimal solution yielded a cost of 9. We can notice from the results that the order points and order-up-to quantities are more evenly spread out, since the system changes its demand-state every month. We can also notice that the highest (s, S) occurs in demand state 1 since it has the highest demand level, while the lowest values occur in state 8 since its demand level is low, and the probability of transitioning to a state with a high demand is low as seen in Table 12. Also, the average (s, S) in the optimal solution is (292, 739) which is significantly lower than that of the initial solution (347, 918) provided since the optimal solution accounts for the transition between states.

Next we perform some sensitivity analysis on Apple's base case. We first consider changing the minimum to 50. The total order-up-to quantity S did increase across the different states, since the demand per month increased (Table 15).

Similarly, when the fixed order cost K is changed to 50, S across the different demand states scaled up

Table 12. Four month transition rate matrix.

0	0.1800	0.0564	0.1176	0	0	0	9.0470e-04	0.0023
0.1161	0	0.1560	0.0200	0.1052	0	0	0	0
0.1391	0.0729	0	0	0	0.0924	5.8413e-04	0	0.0249
0.0354	0	0	0	0.0132	0.1155	0.3904	0.0053	0
0	0.1051	0	0.1367	0	0.1617	0	0.0850	0
0	0.0137	0.1523	0.0515	0.1979	0	0.0194	0	0.0919
0	0	0.0031	0.1365	0.0346	0	0	0.1478	0.1532
8.9943e-04	0	0.0051	0.0078	0.2679	0	0.1483	0	0.0634
0.0018	6.9173e-03	0	0	0	0.1986	0.0827	0.1222	0

Table 13. Initial solution (s, S) at each economic state for Apple.

1	2	3	4	5	6	7	8	9
(746,1610)	(426,1079)	(120,466)	(398,1029)	(466,1148)	(151,540)	(384,1004)	(296,840)	(211,671)
Total cost= 14.9								

Table 14. Optimal solution (s,S) at each economic state for Apple.

1	2	3	4	5	6	7	8	9
(313,965)	(280,834)	(293,460)	(312,698)	(238,838)	(288,779)	(316,777)	(300,538)	(307,766)
Total cost= 9								

Table 15. Optimal solution (*s*, *S*) at each economic state when $d_{min} = 50$.

1	2	3	4	5	6	7	8	9
(113,836)	(138,913)	(148,480)	(213,921)	(148,970)	(154,867)	(163,856)	(149,863)	(179,897)
Total cost= 14.4								

Table 16. Optimal solution (*s*, *S*) at each economic state when $K = 50$.

1	2	3	4	5	6	7	8	9
(88, 1504)	(85, 1036)	(64, 929)	(71, 1391)	(81, 1321)	(79, 1166)	(81, 1201)	(55, 1825)	(44, 1296)
Total cost = 10.35								

Table 17. Optimal solution (*s*, *S*) at each economic state when $\tau = 6$.

1	2	3	4	5	6	7	8	9
(668,1620)	(614,1595)	(583,839)	(366,1320)	(714,1577)	(516,1490)	(741,1528)	(609,858)	(605,1272)
Total cost = 12.06								

Table 18. Optimal solution (*s*, *S*) at each economic state when $b = 0.1$.

1	2	3	4	5	6	7	8	9
(746,1566)	(426,1045)	(120,449)	(398,997)	(466,1113)	(151,520)	(384,972)	(296,812)	(211,647)
Total cost= 15.32								

by a factor of 1.6 which is expected since the EOQ formula in equation (13) is proportional to the square root of K. When the lead time τ is increased to 6 months, we can notice that s increased (Table 16) significantly due to the fact that a higher safety stock is needed since the items take a longer time to arrive to the inventory (Table 17).

Table 18 shows the results when $b = 0.1$. The values for S and s increase significantly since the cost of having backorder inventory is significantly higher. Similarly, when h is increased, the optimal S values remain relatively stable, however s decreased and as a result the average inventory level went down, which results in a lower inventory.

The same analysis is conducted for the other sectors. When a sector is highly variable, as in the technology sector, the variability in the demand translates into the variability in the optimal solution as shown in Table 14. Whereas in the automotive industry, Daimler and Toyota, Table 20 shows that less variability exists in the optimal solution for the automotive sector when compared to the technological sector.

Note that in this numerical study, the inventory is expressed in monetary units ($'s) rather than (Table 19) number of items. This can serve as a decision making tool for the purposes of aggregate planning, which is the process of developing, maintaining, and analysing the approximate scope of the operations of a company. Traditional Aggregate planning methodologies require the assumption that demand is known with certainty and does not provide any buffer against unanticipated forecast errors. From a managerial perspective, assigning an inventory policy to each economic state complements other important decisions that include determining the appropriate workforce size, production capacity and inventory policy for the families of product

Table 19. Optimal solution (*s*, *S*) at each economic state when $h = 0.025$.

1	2	3	4	5	6	7	8	9
(144,1085)	(145,923)	(151,838)	(144,728)	(168,933)	(145,1093)	(179,747)	(136,574)	(123,534)
Total Cost= 10.24								

Table 20. Optimal solution (*s*, *S*) at each economic state for Excon, HP, Royal Dutch, Damilier and Toyota.

	1	2	3	4	5	6	7	8	9
Excon	(92,598)	(70,750)	(94,708)	(94,708)	(103,371)	(102,644)	(102,574)	(90,711)	(101,605)
HP	(199,722)	(182,819)	(181,777)	(190,879)	(186,430)	(193,718)	(193,687)	(190,747)	(188,286)
Royal Dutch	(109,676)	(112,621)	(108,298)	(105,617)	(107,625)	(111,612)	(110,508)	(110,595)	(103,529)
Damilier	(109,573)	(106,387)	(107,462)	(104,517)	(119,398)	(103,594)	(104,503)	(107,506)	(107,505)
Toyota	(176,596)	(165,571)	(162,565)	(167,576)	(155,548)	(215,679)	(186,617)	(156,551)	(120,466)

being produced. The output can be desegregated for individual products and a Master Production Schedule (MPS) can be generated for the Material Requirement Plan (MRP) (Table 20) (Nahmias & Olsen, 2015).

9. Conclusion and recommendation for future research

The dependency of customer demand on external and dynamically evolving environments motivates the use of the MMPP to capture the behaviour of the demand process. This also allows us to model and examine the inter-dependencies between the different economic states, where the focus is on the GDP growth and the EPU index factor. Selected firms were assessed in terms of their demand's reliance on the different economic states at hand. The data illustrates that firms with the highest variation in sales levels are also those with the maximum standardized sales. Six firms from three industries are carefully chosen and analysed (two main firms from each industry). The sought-after industries/sectors considered in this work are the oil and gas, technology, and automotive sectors. We select these three sectors since our data showed that firms operating in these sectors are highly dependent on the economic conditions considered. An (*s*, *S*) model with a dynamic re-ordering policy is developed to account for the fluctuations in the economy. Since the model is complex and non-convex, optimizing over the (*s*, *S*) values is performed via a genetic algorithm where an initial solution is suggested so that the optimization yields a good local optimum. A limitation of this work is that the GDP and EPU are the sole economical factors considered. Future work would investigate other economic factors that could influence customer demand.

The existing literature that relates to inventory-control systems with Markovian demand presents relatively complex mathematical models and consequently complex inventory policies. From a managerial perspective, disadvantages of implementing these polices include complexity and applicability. This work simplifies the mathematical complexity by presenting the equations of the inventory system with compact matrix equations. Another main contribution of this work is illustrating the applicability of the model across different industries by using real world data, which according to our research has not been a focus of the related literature. We use real data, based on key economic indicators, to model the economic environment with a CTMC. Since the parameters of the CTMC are fitted from real data, via a regularization technique, this provides a demand planner with a modelling tool to describe the demand process and to quantify the impact of the economic environment on their inventory control policy. Our data also showed that firms operating in the technology, oil and gas, and automotive sectors are not immune to fluctuations in the economic environment. From a managerial perspective, this illustrates that ignoring these fluctuations can lead to costly inventory policies. This would justify the added complexity of adopting Markovian distributions, such as the MMPP, to capture the behaviour of the demand process.

Notes

1. Available for download from: https://www.policyuncertainty.com
2. Our sample consists of 396 months. However, our total number of observations for the transition matrix is 395 observations since the transition from December 1984 to January 1985 is missing due to the exclusion of December 1984 from our sample.

Disclosure statement

No potential conflict of interest was reported by the authors.

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