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Re-ordering policies for inventory systems with recyclable items and stochastic demand – Outsourcing vs. in-house recycling ☆

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ABSTRACT

Investing in recoverable items is an increasing trend in a variety of manufacturing industries. Such industries seek to balance their supply chain costs while reducing solid waste (non-biodegradable). Our work develops mathematical models of inventory systems that rely on newly manufactured and recoverable items to satisfy market demand. Specifically, we consider continuous (r, Q) re-ordering policies for single-item inventory systems with stochastic demand and recycling. The first model assumes outsourcing the recovery of used items to a supplier, where returns (collected used items) arrive in random quantities with every order. The second model assumes performing the recovery process in-house; i.e., at the manufacturer's facility. The proposed mathematical framework considers an infinite time horizon where demand and the amount recovered are stochastic. This work focuses on developing environmentally responsible inventory policies/models that could help in greening supply chains. It also presents a numerical study to compare the proposed models and quantify the trade-off cost between the two; i.e., should the recovery process be in-house or outsourced.

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1. Introduction and motivation

The world population is projected to reach 8.6, 9.8, and 11.2 billion by 2030, 2050, and 2100, respectively [8]. The increase in the world's population, and, subsequently, the consumption behavior and the uncontrolled waste generation with the hazards it brings to the environment are well-documented in the literature [5,12,17,51].

Supply chain systems have become a prime source of pollution, which calls for seriously re-evaluating such manufacturing processes and other culprit activities. The challenge that such manufacturers face is how to reduce the quantities that go into landfill sites and control harmful emissions while continuing to satisfy increases in market demand. Furthermore, the earth has finite resources, and their consumption rates are alarming and unsustainable [5]. For example, global plastic production was about 335 million tons in 2017, a 4% increase from 2016 [35]. Dumping plastic fragments (e.g., from packaging material) into seas and oceans severely damage marine life and habitats (e.g., [5,57]). It takes more than two years for high-density plastics to degrade in briny waters and about 12 months for low-density plastics [49], and much longer (more than 50 years) in landfill sites [2], where many toxic chemicals ooze into soil and water tables [5]. This mismatch between the time to degrade and the rate of disposing plastics creates a new problem due to the limited number of landfill sites and the environmental hazards that a traditional landfill has on its surroundings.

Over the last few decades, governments, industries, and communities have been trying hard to reduce the amount of solid waste that goes to landfill sites [53]. The implementation of recycling, remanufacturing, and reusing has been growing at different levels, from households to large companies. According to the United States Environmental Protection Agency, the recycling rate improved from 6% of municipal solid waste in 1960 to slightly above 25% in 2015. The reported recycling and composting rates are 29.9% for plastic bottles and jars, 33.2% for glass containers, 39.8% for selected consumer electronics, and 54.9% for aluminum beer and soda cans [14]. Over the past years, several companies have implemented collection, recycling, and reuse programs to reduce solid waste and extract value from used items. Apart from government legislation to protect the environment that created this advance, companies found economic opportunities in recov-

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ery programs [29]. Although our discussion relates to the recycling of plastics, mathematical models for recycling systems can include glass and metals [36]. Furthermore, mathematical models for reverse logistics have a wide range of applications that include the steel and iron industry [56], the paper industry [34], the electronics industry [25], among other industries. Due to the complexity of the resulting reverse logistics system, companies, in many cases, resort to outsourcing the process to third-party providers [18].

Classical inventory management does not address recycling and, subsequently, the environmental consequences of an ordering policy. This limitation motivates developing inventory systems that are environmentally responsible and also cost-effective [5]. Models that result from this allow returning assets for recovery (reuse, recycle, etc.) and later reselling (e.g., [3,28]). In this regard, we develop stochastic models to capture the uncertainty in the recovery process and quantify its effects on system performance. The sources of uncertainty are demand over lead-time and the number of returned items in each cycle. To solve these problems, we develop a computationally efficient iterative solution approach that finds the optimal solution. We refer to King et al. [23] for a clear differentiation of end-of-life product recovery strategies, i.e., repairing, reconditioning, remanufacturing, or recycling.

The remainder of the paper is organized as follows. A review of the inventory systems literature with reverse logistics is presented in Section 2. In Section 3, we develop mathematical models and solution procedures for two inventory systems. The first considers remanufacturing in-house (Section 3.1), while the second outsources its remanufacturing activities to a third party (Section 3.2). Numerical examples are presented in Section 4 to illustrate the behavior of the developed models, where we also conduct a sensitivity analysis to compare the performance of the two systems for different parameter-values and demonstrate the efficiency of the solution procedures. Finally, Section 5 concludes the paper.

2. Literature review

Now we present a review of inventory systems with product recovery through reuse, remanufacturing, or recycling. This section has two subsections, 2.1 and 2.2. The first considers the related deterministic models, and the second extends the review to account for the stochastic models. We conclude the section with a table summarizing the literature and highlights the contribution of this work.

2.1. Deterministic models

Inventory management models with reverse flows can be traced back to Schrady [40], who developed a deterministic EOQ-based model for a single-item inventory system with no backorders. The model assumed that products could be returned for repair, with those unrepairable scrapped. Schrady [40] noted that an item designated reparable, i.e., not consumable, is presumably more economical to repair than to dispose of or replace. Nahmias and Rivera [30] extended the model of Schrady for finite repair rate and limited storage. The resulting model also considered the interaction between procurement and repair. Mabini et al. [27] modified Schrady's model to allow for backorders.

With the rise of environmental issues in the 1990s, this research line took a new turn. Richter [38], who rejuvenated this research line, developed an EOQ model with product collection and disposal. Richter [38] considered a system of two shops. The first shop stocked newly produced and recovered items, while the second shop stocked collected/returned/used items. The model in Richter [38] assumed that non-recoverable items are disposed of at a cost and solved for the optimal disposal rate. Teunter [46] studied a deterministic EOQ model with recoverable items with different holding costs. He, who used the terms repaired, refurbished, and remanufactured) interchangeably, accounted for the disposal of used items by categorizing the stock as a manufacturing batch or a recovery batch and obtained a simple EOQ for each stock category over an infinite-time planning horizon. Koh et al. [24] considered joint EPQ and EOQ models with stationary demand, where items are either newly purchased or recovered. Dobos and Richter [9] examined a production and recycling system with a predetermined production-inventory policy and assumed that recovered items are as good as new. Their results showed that a bang-bang (recycle or produce all) and not a mixed strategy is optimal. However, despite what their results showed, Dobos and Richter [9] concluded that such pure strategies are probably not technologically feasible, where relying solely on recycled items entails buying back all sold and used items. In a follow-up paper, Dobos and Richter [10] considered the quality of collected items. They showed that for such an assumption, a mixed strategy of production and recycling is optimal. Singh and Saxena [44], in a similar model, allowed for shortages and backordering. The authors assumed time-dependent rates and investigated coordinating the manufacturing and remanufacturing processes. Other studies, but not limited to, include Hui Oh and Hwang [22] and Matar et al. [28]. The latter being the closest in scope to this paper, where the authors discussed production/recycling/reuse of plastic bottles that are either sold to produce low-grade plastics or disposed of in landfills. Matar et al. [28] put forth two novel ideas, which are (i) using biodegradable plastics to minimize the environmental impact of disposing of bottles and (ii) the rehabilitation of landfill sites. Tighazoui et al. [48] considered a manufacturing/remanufacturing system with two parallel manufacturing and remanufacturing machines. The authors considered several decision variables that include the optimal capacities of the manufacturing and remanufacturing stocks, purchasing warehouses, transport, and the optimal percentage of end-of-life returned products. Most recently, and lastly, Chan et al. [7] considered a reverse logistics system where a vendor does remanufacturing and production and has multiple buyers.

2.2. Stochastic models

Stochastic inventory models include Alinovi et al. [3], who formulated a stochastic EOQ-based inventory control model for a system of manufacturing and remanufacturing activities. The authors utilized the Monte-Carlo simulation to estimate the optimal return policy while accounting for the uncertainty in demand, returned quantity, and return delay. Fleischmann et al. [15] proposed a basic inventory control model with stochastic returns. They adopted Poisson distributions to model the number demanded and returned. Shi et al. [42] formulated a mathematical model to maximize the overall profit by optimizing the production and recycling processes, subject to uncertain demand and return rates. The authors adopted a Lagrangian relaxation and a sub-gradient heuristic. Hsueh [20] investigated time-dependent inventory-control policies in a manufacturing/ remanufacturing system with normally distributed demand and return processes. He obtained closed-form solutions for the optimal production lot size, reorder point, and safety stock for time points.

Benedito and Corominas [4] integrated Markov decision processes with reverse logistics models to obtain the optimal manufacturing policy. The authors assumed that the quantity returned to be stochastic and dependent on sales. The system developed by Benedito and Corominas [4] considers a company that collects, recovers, and sells the product. Serrato et al. [41] considered a Markov decision model where reverse logistics activities could be in-house or outsourced. The authors based their model on a reward function that accounted for the capacity and operating costs. They concluded that as the return fraction increased, the outsourcing threshold was more likely to be crossed, and thus internal reverse logistics would become more favorable [41]. Teunter [47] considered an inventory system with stochastic demand and return rates and a discounted cost with no lead-time. Teunter [47] resorted to simulation to test the best possible values of the decision variables relating to the economic and manufacturing order quantity systems.

Our work considers a continuous (r, Q) inventory-control system with stochastic demand and product return. We present a computationally efficient iterative approach instead of simulation to find the optimal cost per unit of time. This work builds on an approach proposed in Silver et al. [43] to calculate the reorder point and order quantity for fast-moving items. Silver et al. [43] developed a procedure to find the optimal solution by iterating between two values, the order quantity and the reorder point. We implement a similar approach in our model to compute the ordering policy by integrating the remanufacturing/recycling process. The work in Silver et al. [43] does not account for recycling or reverse logistics. Our model extends the work in Silver et al. [43] by jointly accounting for the stochastic demand process and the stochastic return process. The return process is either performed in-house or outsourced to a supplier. We present a closed-form expression of the optimal solution for the solution pair r and Q and an algorithm that iterates between both values until convergence. The convex nature of the developed cost functions ensures the convergence to a simultaneous solution of r and Q. For further details concerning convexity when the solution converges, we refer to Hadley and Whitin [19].

Table 1 presents a concise summary of the relevant literature and lists the inventory models with uncertainty in demand or recovery processes with their assumptions and solution procedures. Our work considers the (r, Q) policy over an infinite time horizon, and we show that the cost per unit time is convex (for outsourcing and in-house recycling models). The convexity of both models allows us to develop an iterative algorithm that solves for the optimal solution.

3. Model formulation and assumptions

We consider a continuous review inventory system with recoverable (e.g., recyclable) items over an infinite time horizon, where a manufacturer procures new items (raw material) from an external supplier. Each mathematical model calculates the total cost, which is the performance measure. We develop two (r, Q) continuous reordering models, where r is the reordering level and Q is the order quantity. The first model outsources recycling activities to a supplier (Model 1). The second does recovery in-house (Model 2).

The models assume demand over lead-time and the number of recovered items to be random variables. The latter is related to the demand process through parameter θ, which denotes the proportion of items returned from the market for recovery.

We assume that recovered and newly manufactured items comparable, meaning they sell for the same price and, subsequently, both have equal holding costs [46]. Both models assume that there are never two or more outstanding orders. The stockout cost of both inventory systems is p (in \$/unit). We summarize the notations used in the mathematical models:

- **Monetary parameters**

c_Q: purchase cost of newly manufactured item (\$ /unit)
c_{R_T} : purchase cost of a returned item (\$ /unit)
h: holding cost (\$ /unit / unit-time)
p: stockout cost (\$ /unit)
K: ordering cost (\$ / production cycle)

- **System parameters**

d: demand rate (unit /unit-time)
d_e: effective demand rate (unit / unit-time)
r_e: recovery rate (unit / unit-time)
L: lead-time (unit-time)
T: cycle time - time to consume inventory (unit-time) –random variable
γ: proportion of demand collected, $0 \leq \gamma \leq 1$
θ: proportion of collected demand that is recoverable, $0 \leq \theta \leq 1$
r: reorder point (units) – decision variable
Q: order quantity (units) – decision variable
D_L: demand over lead-time (unit) – random variable
R_T: number of returned items during a cycle (unit) – random variable
$n(r)$: expected shortage per cycle (unit)
$cv(R_T)$: coefficient of variability of recovered items during a cycle
$cv(D_L)$: coefficient of variability of demand during lead time
$cv(R_T)$: coefficient of variability of recovered items during lead time

Next, we describe Models 1 and 2 and present the corresponding cost equations.

3.1. Model 1

The recycling system of Model 1 is described in Fig. 1, and the behavior of the inventory system is illustrated in Fig. 2. The supplier needs exactly L units of time to deliver an order of size Q and a random number of recovered items R_T at the beginning of each cycle/period. The supplier is responsible for collecting used items from the market, disposing of the unrecoverable ones, recovering those that are, and delivering the lot to the manufacturer.

We assume that the inventory system of Fig. 2 follows a renewal process, where the time between replenishments, T, represents a renewal cycle. The demand process is a stochastic renewal process with X_i denoting the time between the $(i-1)^{st}$ and the i^{th} arrival epochs. Therefore, E[X_i] is the expected inter-arrival time of a demand item, and the demand rate, d, is expressed as follows,

$$d = \frac{1}{\mathrm{E}[X_i]}$$

where the manufacturer consumes inventory at a rate of d items per unit of time. Furthermore, the duration of the inventory renewal cycle, as denoted by T, is expressed as follows,

$$T = \sum_{i=1}^{Q+R_T} X_i,$$

where R_T is the number of recovered items received in a cycle of duration T, and Q as defined earlier. Since $X_1, \ldots, X_{Q+R_T}$ is a sequence of independent and identically distributed observations, then Wald's equation [39] is used to calculate the expected value of the cycle time,

$$E[T] = \frac{Q + E[R_T]}{d} \; .$$

The portion of items that the supplier collects from consumers over T is γ. The portion, θ, of those items is recovered. Therefore, the generation rate of recoverable items is $r_e = \gamma\theta d$ items per unit time. Consequently, the expected number of recovered items in a cycle of length T is expressed as follows,

$$E[R_T] = r_e \; E[T] = \gamma \; \theta \; d \; E[T] = \; \gamma \; \theta \; (Q + E[R_T]) \; \Rightarrow E[R_T] = \frac{\gamma \; \theta}{1 - \gamma \; \theta} \; Q.$$

Table 1
Literature summary table.

Authors	Stochastic demand	Stochastic returns	In-house	Supplier	Decision variables	Solution procedure
Schrady [40]	-	-	X	-	(r, Q)	Closed-form solution
Nahmiasj and Rivera [30]	-	-	X	-	(r, Q)	Closed-form solution
Mabini et al. [27]	-	-	X	-	(r, Q)	Closed-form solution
Richter [38]	-	-	-	X	(r, Q)	Closed-form solution
Teunter [46]	-	-	X	-	Manufacturing and recovery batch size	Closed-form solution
Koh et al. [24]	-	-	X	-	Quantity of newly produced items, inventory level of recoverable items, number of orders	Search heuristic
Dobos and Richter [9]	-	-	X	-	Marginal use and buyback rates, number and size of recycling lots, number and size of production lots	Closed-form solution
Dobos and Richter [10]	-	-	X	-	Marginal use and buyback rates, number and size of recycling lots, number and size of production lots	Closed-form solution
Singh and Saxena [44]	-	-	X	-	Acceptable returned quantity for used items, maximum inventory level from production and remanufacturing	Closed-form solution
Hui Oh and Hwang [22]	-	-	X	-	Number of production setups, number of raw material orders, production lot size, order size of raw material, cycle time	Closed-form solution
Matar et al. [28]	-	-	X	-	Cycle time	Closed-form solution
Teunter [47]	X	X	X	-	Economic order quantity for manufacturing and remanufacturing	Near Optimal Equations verified via simulation
Hsueh [20]	X	X	X	-	Number of production activities and safety stock for every stage of the product life cycle (finite time horizon)	Closed-form solution for every life every stage
Benedito and Corominas [4]	X	X	X	-	Number of products to be manufactured	MOLP adapted to the approximated Markov model
Silver et al. [43]	X	-	X	-	(r,Q)	Optimal Solution obtained via iterative algorithmic approach
Alinovi et al. [3]	X	X	X	-	Size of the manufacturing purchasing order	Simulation
Fleischmann et al. [15]	X	X	X	-	(s,Q)	Optimal control policy
Shi et al. [42]	X	X	X	-	Stocking, manufacturing and remanufacturing quantities	Lagrangian based Heuristic
This Work	X	X	X	X	(r,Q)	Optimal Solution obtained via iterative algorithmic approach

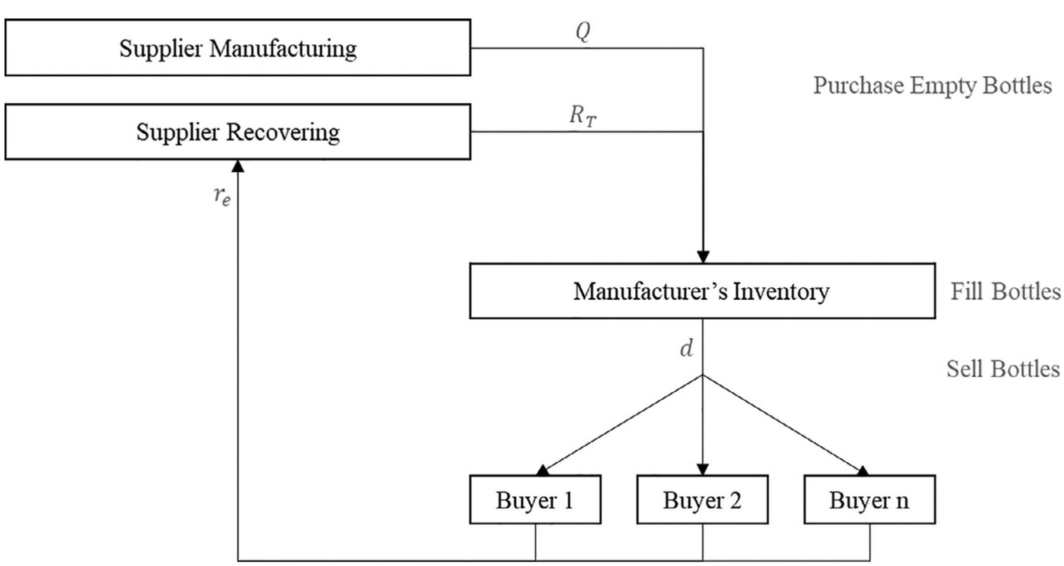

Fig. 1. Model 1 process flow.

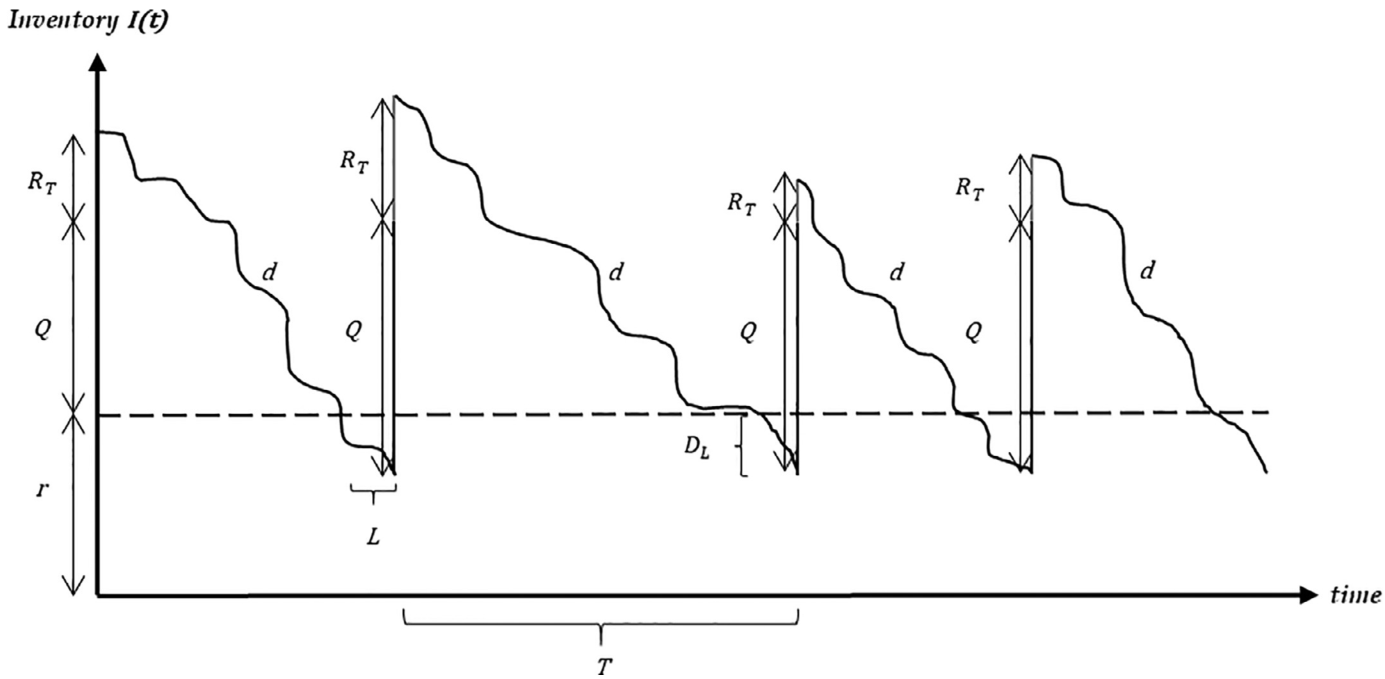

Fig. 2. Behavior of inventory for Model 1 (recovery outsourced).

We consider the demand over lead time, D_L, as a random variable with a coefficient-of-variation, $cv(D_L)$. Accordingly [39],
$E[D_L] = d\ L$, and $Var[D_L] = cv(D_L)^2 E[D_L]^2$.

To capture the variability of the number of recovered items delivered to the supplier during an inventory cycle and over the lead time, we define cv_{R_T} and cv_{R_L} to be the coefficient of variations of the number of recovered items during intervals of duration T and L, respectively. We related cv_{R_T} and cv_{R_L} by the following equation,

$$\mathrm{cv}(R_T)^2 \times E[T] \approx \mathrm{cv}(R_L)^2 \times L \ \Rightarrow \mathrm{cv}(R_T) \approx \mathrm{cv}(R_L)\sqrt{\frac{L\ E[d]}{Q+E[R_T]}}\ . \tag{1}$$

Notice that the approximation in Equation (1) is exact if the demand process is Poisson (time between arrivals follows an exponential distribution). Furthermore, the assumption in Equation (1) is accurate if the inter-arrival epochs are renewal, and the accuracy improves for long lead times where the error is $o(1)$ [32,55]. Accordingly, Equation (1) relates the variability of the number of recovered items during a cycle and over the lead-time. Therefore, the variance of the number of recovered items over the cycle time is $Var[R_T] = \mathrm{cv}(R_T)^2 \times E[R_T]^2$, [39].

We define the safety stock, SS, as the lowest inventory level realized by the system, i.e., the inventory just before the order is received. The safety stock is a random variable and is expressed as $SS = r - D_L$.

3.1.1. Performance Measures/System Costs – Model 1

We now calculate the system cost per cycle (CPC) of Model 1, which is the sum of the purchasing, holding, ordering, and shortage costs. The manufacturer purchases newly manufactured items for c_Q each. The supplier provides the recovered items for a discounted price of c_{R_T} for each. Therefore, the expected purchase cost per cycle is calculated as $PPC_1 =\ c_Q\ Q + c_{R_T}\ E[R_T]$. Recovered items delivered to the manufacturer, including the purchased items, are assumed to be of the same quality and holding cost, h. The expected inventory level in a cycle, EIL_1, is calculated as,

$$EIL_1 = \frac{Q+E[R_T]}{2} +\ \mathrm{E}[SS],$$

which is the average value of the ordered quantity $(Q+E[R_T])/2$ plus the expected safety stock $E[SS]$. Accordingly, the expected holding cost per cycle HPC_1 is,

$$HPC_1 = h\left(\frac{Q+R_T}{2} + SS\right)\sum_{i=1}^{Q+R_T} X_i.$$

Let K be the fixed ordering cost per cycle, and p be the stockout cost per unit incurred by the manufacturer. A shortage is present when the demand over the lead-time is more than the reorder point inventory. Let $f_{D_L}(x)$ be the density function of the number of demanded items over the lead-time. Therefore, the expected shortage per cycle in this model is [43],

$$n(r) = \int_r^\infty (x-r) f_{D_L}(x)dx = \int_r^\infty (x-r) f_{D_L}(x)dx = \sigma_{D_L} \times L\left(z_{\alpha(r)}\right), \tag{2}$$

where $z_{\alpha(r)}$ is the standard normal value satisfying a service level of $\alpha(r)$ and $L(z_{\alpha(r)})$ is the corresponding standard loss function, i.e., the expected number of lost sales as a fraction of the standard deviation σ_{D_L}. The loss function is the expected quantity by which demand exceeds a determined threshold value. This threshold value corresponds to the reorder point, r. If demand exceeds r, a shortage cost per item, p, is incurred.

The expected cost function per unit time is,

$$E[CPUT_1] = \frac{(c_Q\ Q + c_{R_T}\ E[R_T]\ )d}{Q+E[R_T]} +\ h\ \times \frac{d}{Q+E[R_T]} \times\left(\left(\frac{Q}{2} + \mathrm{E}[SS]\right)\times\frac{Q+E[R_T]}{d} + \frac{E\left[{R_T}^2\right]}{2\ d} + Q\times\frac{E[R_T]}{2\ d}\right) + K\times\frac{d}{Q+E[R_T]} + p\times\frac{d}{Q+E[R_T]}\times n(r)$$

is derived and proven to be convex in a detailed derivation in the appendix section.

The solution equations are solved iteratively to calculate Q^* and r^*:

$$Q^* = \frac{(1-\theta)\sqrt{2\ h\ d\ (p\ n(r) + K)}}{h},\ \text{ and}$$

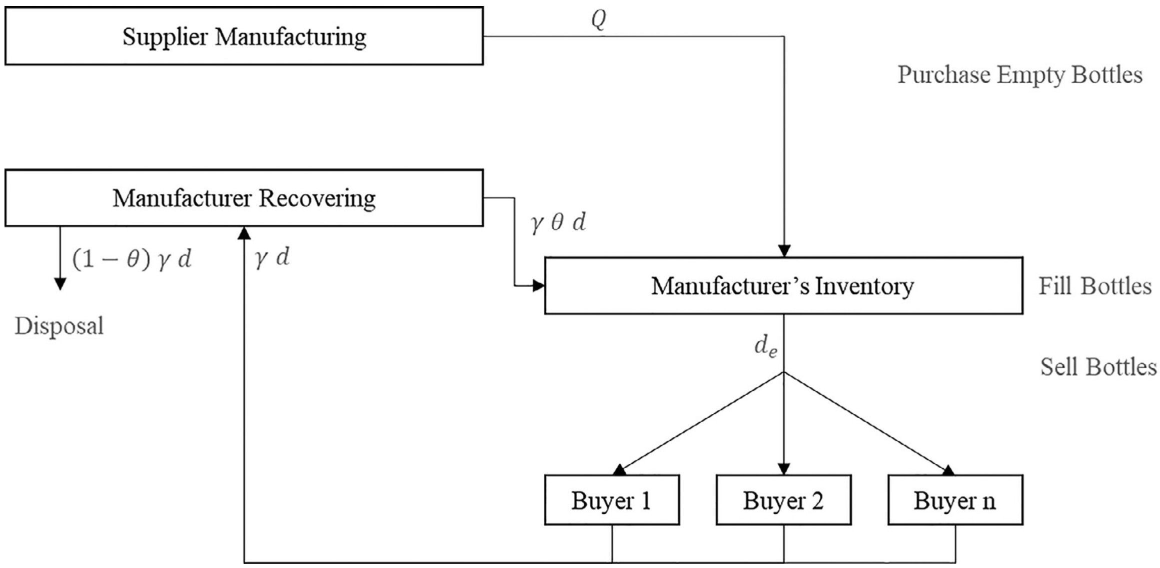

Fig. 3. Model 2 process flow.

$$F(r^*) = 1 - \frac{(Q + E[R_T])\ h}{p\ d}\ .$$

3.1.2. Iterative Procedure – Model 1

Silver et al. [43] discussed the simultaneous determination of Q and r. We describe a similar iterative procedure to find the ordering policy. Let $Q_i$and r_i denote the order size and reorder point for the i^{th} iteration, respectively.

Step 1: Calculate Q_i using the basic EOQ re-ordering quantity, $Q_i = \sqrt{\frac{2Kd}{h}}$.
Step 2: Calculate $F(r_i) = 1 - \frac{h(Q_i + E[R_T])}{p\ d}$.
Step 3: Calculate the relevant $z_{\alpha(r_i)}$ from $F(r_i)$, the inverse of the cumulative distribution of the standard Normal.
Step 4: Calculate r_i using the value $z_{\alpha(r_i)}$ (as calculated in Step 3), corresponding to a safety level $\alpha(r_i)$, $r_i\ = \sigma_{D_L} z_{\alpha(r_i)} + E[D_L]$.
Step 5: Calculate $n(r_i)$ using Equation (2), $n(r_i) = \sigma_{D_L} \times L(z_{\alpha(r_i)})$.
Step 6: Calculate $Q_{i+1} =\ Q^* = \frac{(1-\theta)\sqrt{2\ h\ d\ (p\ n(r_i) + K)}}{h}$.
Step 7: If $|Q_{i+1} - Q_i| \le \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_i). Otherwise, set $Q_i = Q_{i+1}$ and repeat **Step 2** to **Step 4**.
Step 8: If $|r_{i+1} - r_i| \le \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_{i+1}). Otherwise set $r_i = r_{i+1}$ and repeat **Step 5** to **Step 6.**
Step 9: Set $Q_i = Q_{i+1}$, and restart algorithm from **Step 2**.

A figure summarizing the above algorithm is presented in the Appendix.

3.2. Model 2

The system described by Model 2 is presented in Fig. 3, and the inventory behavior is presented in Fig. 4, where a period/cycle is the elapsed time between two successive replenishments. It takes the supplier exactly L units of time to deliver an order to the manufacturer. The model assumes that the recovered items are added to inventory as they arrive throughout the cycle at a rate of r_e items per unit time. The stock of returned items is consumed at a rate of d. Thus, we define the effective demand rate as the difference between the demand rate, d, and the manufacturer's rate of collecting used items, r_e. This results in a lower effective demand rate, $d_e = d - r_e$.

The number of effectively demanded items over the lead time is a random variable, De_L, with an expected value and variance of, [39],

$$E[De_L] = E[D_L] - E[R_L],$$

$$Var[De_L] = cv(D_L)^2 \times E[D_L]^2 + cv(R_L)^2 \times E[R_L]^2,$$

where $cv(D_L)$ and $cv(R_L)$ are the coefficients of variability of the number of demanded items and returned items over the lead time respectively. The cycle time as illustrated in Figure becomes, $T = Q/d_e$. Similar to Model 1, the manufacturer adopts a safety stock policy to decrease the number of random stock outs $n(r)$ to accommodate for the variation over the lead-time, $SS = r - D_{Le}$.

3.2.1. Performance measures/system costs –Model 2

We now calculate the system cost per cycle (CPC) of Model 2 as the sum of purchasing, holding, ordering, and shortage costs. The recycling cost is discussed in detail in section 3.2.2. Like Model 1, the manufacturer purchases newly manufactured items for c_Q each. Therefore, the expected purchase cost per cycle is calculated as follows, $PPC_2 = c_Q\ Q$. Since the quality and price of the recovered items are assumed in the literature to be "as-new" (e.g., [38,46]), we use the same holding cost assumptions adopted in Model 1. Consequently, purchased and recovered items have an equal holding cost, h. The expected inventory level, EIL_2, held per cycle is calculated as follows,

$$EIL_2 = \frac{Q}{2} + E[SS],$$

which is the average value of the ordered quantity $Q/2$ plus the expected safety stock $E[SS]$. Accordingly, the holding cost per cycle HPC_2 is

$$HPC_2 = h\left(\frac{Q}{2} + SS\right)\sum_{i=1}^{Q}(X_i - Z_i).$$

The manufacturer incurs a fixed cost when it places an order. The shortage cost in this model is calculated by a similar expression to Model 1, except it uses the standard deviation of the 'effective' demanded items when calculating the expected number of shortages per cycle,

$$n(r) =\ \sigma_{De_L}\ \times L\left(z_{\alpha(r)}\right). \qquad (3)$$

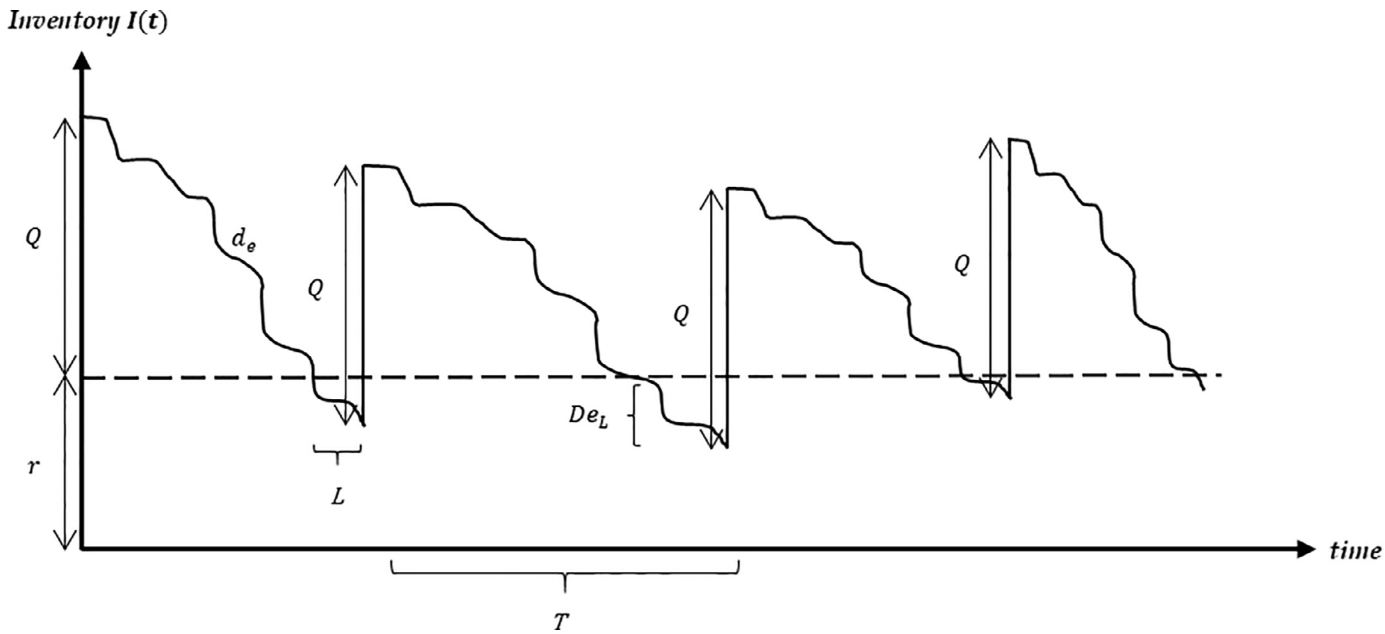

Fig. 4. Behavior of inventory for Model 2 (recover in-house).

3.2.2. In-house recycling cost – Model 2

In addition to the inventory-related costs (holding, ordering, and shortage costs), Model 2 incurs in-house recycling costs. The in-house recovery process consists of three components with three costs. The first sub-process is the collection process, where we assume γ to be the collection rate of items ($\gamma \geq \theta$). Let c_1 be the collection cost of one item ($ / item). The collection cost per unit time is $CC_2 = c_1\ \gamma\ d$. Collected items that do not pass inspection in the second sub-process, which involves screening and testing, are labeled unusable or waste, i.e., sent to disposal. The second sub-process is for dispensing items that do not pass inspection, which involves screening and testing the collected items and labeling them unusable, i.e., not repairable/recoverable.

Let c_2 be the cost of dispensing one unusable item ($/item). The dispensing cost per unit time is $DC_2 = c_2\ (1-\theta)\ \gamma\ d$.

The third sub-process is the recycling process where the cost to recycle one item is c_3. The recycling cost per unit time is, $RC_2 = c_3\ \gamma\ \theta\ d$.

We summarize the notation for the in-house recycling process,

c_1: unit collection cost ($ / item)
c_2: unit cost disposal cost ($ / item)
c_3: cost of recovery one item ($ / item)

The total cost per unit time of the in-house recovery process is denoted by TRC_2 and calculated as follows,

$$TRC_2 = c_1\ \gamma\ d + c_2\ (1-\theta)\ \gamma\ d + c_3\ \gamma\ \theta\ d.$$

The expected cost function per unit time used in this model,

$$E[CPUT_2] = \frac{\left(c_Q\ Q + \frac{h\ Q\ \left(\frac{Q}{2}+E[SS]\right)}{d_e} + K + p\ n(s)\right)\ d_e}{Q} + TRC_2$$

is derived and proven to be convex in a detailed derivation in the appendix section.

The solution equations are solved iteratively to calculate Q^* and r^*:

$$Q^* = \frac{\sqrt{2\ h\ d_e\ (p\ n(r)+K)}}{h},\ \text{and}\ F(r^*) = 1 - \frac{Q\ h}{p\ d_e}\ .$$

3.2.3. Iterative procedure – Model 2

Step 1: Calculate Q_i using the basic EOQ re-ordering quantity, $Q_i = \sqrt{\frac{2Kd_e}{h}}$.
Step 2: Calculate $F(r_i) = 1 - \frac{Q_i\ h}{p\ d_e}$.
Step 3: Calculate the relevant $z_{\alpha(r_i)}$ from $F(r_i)$, the inverse of the cumulative distribution of the standard Normal.
Step 4: Calculate r_i using the value $z_{\alpha(r_i)}$ (as calculated in Step 3), corresponding to a safety level $\alpha(r_i)$; $r_i = \sigma_{De_L} \times z_\alpha + E[De_L]$.
Step 5: Calculate $n(r_i)$ using Equation (3), $n(r_i) = \sigma_{De_L} \times L(z_{\alpha(r)})$.
Step 6: Calculate $Q_{i+1} = Q^* = \frac{\sqrt{2\ h\ d_e\ (p\ n(r_i)+K)}}{h}$.
Step 7: If $|Q_{i+1} - Q_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_i). Otherwise, set $Q_i = Q_{i+1}$ and repeat **Step 2** to **Step 4**.
Step 8: If $|r_{i+1} - r_i| \leq \varepsilon$ then stop the procedure and set the solution as (Q_{i+1}, r_{i+1}). Otherwise set $r_i = r_{i+1}$ and repeat **Step 5** to **Step 6**
Step 9: Set $Q_i = Q_{i+1}$, and restart algorithm from **Step 2**.

A figure summarizing the above algorithm is presented in the Appendix.

3.3. Model comparison

As mentioned in Section 3.2, the in-house recycling process involves collecting the items, re-manufacturing the recyclables, and disposing of the un-recyclable items. Consequently, implementing Model 2 necessitates a cost-efficient in-house recycling process. This is one of the main challenges when considering Model 2 since the collection process can be expensive, rendering in-house recycling (Model 2) cost-inefficient and impractical. The cost of collecting a unit can be as high as purchasing a new PET bottle [37], which decreases the incentive to collect and recycle. The deposit refund system is an approach to reduce the collection cost of recyclables. This approach is adopted in several countries. For example, Norway's high recycling rate is attributed to its efficient deposit return scheme [45]. This idea of charging a deposit has proved to be very successful and resulted in several countries following suit [6,45]. As a condition for in-house recycling (Model 2) to be considered as an alternative to Model 1, then the following should be satisfied, $c_{R_T} > E[\textit{Cost to recycle one collected item inhouse}]$. This condition guarantees that the average cost to recycle an item in-house is cheaper than purchasing a recycled item from a supplier. We expand on this condition as follows,

$$\begin{aligned} &E[\textit{Cost to recycle one collected item inhouse}] \\ &= c_1 + c_2\ P(\textit{item requires disposal})(1-\theta) \\ &+c_3\ P(\textit{item is recoverable}) \ = \ c_1 \ + c_2(1-\theta) \\ &+c_3\theta,\ \Rightarrow c_{R_T} > c_1 \ + c_2\ (1-\theta)\ + c_3\ \theta \end{aligned} \tag{4}$$

The right-hand side value of Equation (4) would correspond to the average cost to recycle one item in-house. This term accounts for the collection cost of an item, c_1, the expected disposal cost of an item, $c_2\ (1-\theta)$, and the expected recovery cost of an item, $c_3\ \theta$.

From an inventory management perspective, it would be easier to manage the variability in the demand process for Model 1. This could be seen when comparing the safety stocks equations for Models 1 and 2. The demand over lead time in Model 2, D_{Le}, is more variable than the demand over lead time in Model 1, D_L, since D_{Le} is affected by the variability in the demand process and the variability in the number of recycled items over lead time. The recycled items in Model 1 are received at the beginning of the cycle. Hence the variability in R_T does not play a role when calculating the safety stock in Model 1.

Accordingly, if the condition in Equation (4) is not satisfied, then Model 1 would provide a better inventory policy. If it is satisfied, then recycling items in-house is cheaper. However, higher inventory might offset the savings. In such a case, we can use the presented mathematical framework to compare different scenarios numerically.

4. Numerical analysis

This section provides numerical examples and sensitivity analysis to illustrate the behavior and the flexibility of our mathematical framework in comparing in-house recycling vs. outsourcing. The base case we consider captures the characteristics of 500ml PET water bottles that weighing around 9.9 grams each. The price of PET plastic is $0.83-$0.85 per pound and $0.58-$66 per pound for recycled ones [26]. Accordingly, we set the price of new and recycled PET water bottles to $c_Q = \$0.018$/bottle and $c_{R_T} = \$0.014$/bottle, respectively. Waste disposal in landfills in the US ranges from $40.92 to $73.03 per ton, with a national average of $55.36 per ton [54]. We calculate the disposal cost of one PET bottle to be $c_2 = 0.0005$ $/bottle, in line with the national average. The recycling cost of plastics is $0.32/Kg, of which 18% is for collection [16]. The collection and recycling cost are $c_1 = \$0.0006$/bottle and $c_3 = \$0.0026$/bottle, respectively. EPA [13] estimates the recycling rate of PET bottles as 26.8%, which is the percentage adopted in our numerical examples where we set $(\gamma \times \theta) = 0.25$, with $\gamma = \theta = 0.5$. The stockout cost is $0.25/bottle, the same as the selling price of 500ml water bottles. The holding cost per month is 2%. We consider a monthly demand rate of 50,000 bottles per month, which is a reasonable assumption for a population of around one million with the company's market is, say, 15%.

Notice that the values of the parameters selected vary over time. For example, a 500ml water bottle weighs today almost half of it is many years ago. The recycling costs also vary depending on location, and new governmental policies could result in new and improved recycling rates. The values of the parameters that we use in the numerical analysis are plausible and in line with what one could realize in reality. We conduct a sensitivity analysis to study the model's behavior to account for variations in some of the parameters, as they are in practice.

Table 2
Input parameters for base case.

Parameter	Value	Unit
d	50,000	bottles/month
γ	0.5	-
θ	0.5	-
L	0.2	months
cv_{D_L}	0.3	-
cv_{R_L}	0.1	-
c_Q	0.018	$/bottle
c_{R_T}	0.014	$/bottle
c_1	0.0006	$/bottle
c_2	0.0005	$/bottle
c_3	0.0026	$/bottle
h	0.00036	$/bottle/month
p	0.25	$/bottle
K	20	$

The base case parameters are summarized in Table 2, where we assume that items (plastic bottles) are recovered via recycling.

The percent savings of Model 2 (in-house recycling) compared to Model 1 (outsourcing the recycling process) is denoted by Δ and is calculated as follows,

$$\Delta = \frac{E[CPUT_1]-E[CPUT_2]}{E[CPUT_1]} \times 100.$$

Define Δ as a measure of the cost-efficiency of choosing to recycle in-house instead of outsourcing the recycling process. The optimal ordering policy is calculated in Table 3, along with the expected cost per unit time.

The cost-efficiency of in-house recycling is $\Delta = 14.43\%$, where the results in Table 3 show that Model 1 (outsourcing) results in a lower order quantity yet a higher reorder point. The adoption of Model 2 instead of Model 1 saves 23% in holding cost, 14% in ordering cost, and 21% in purchase cost. We perform a one-way sensitivity analysis by varying the demand rate, recovered proportion, lead-time, coefficients of variability of the demanded new and recovered items over the lead-time, purchase costs, holding cost, shortage cost, and ordering cost.

Table 4 presents the sensitivity analysis results of varying the demand rate, d, over the range [1,000, 1,000,000] in multiples of 10. Model 2 results in better cost savings compared to all the demand values considered. As the demand rate increases, the percent saving of using Model 2 is stable at about 14% (Δ ranges between 14.31% and 14.45% in Table 3). At $d = 1{,}000{,}000$, the savings reached 32% in holding cost, 14% in ordering cost, and 21% in purchase cost.

Next, we conduct a one-way sensitivity analysis on the recycling proportion $(\gamma\ \theta)$. We increase $(\gamma\ \theta)$ from 0.05 to 0.5 by increasing θ and fixing γ. These recycling parameters are dependent on societal recycling behaviors and can improve by implementing governmental regulations. As $(\gamma\ \theta)$ increases, Model 2 becomes more cost-efficient, as Table 5 indicates. The order quantity Q^* decreases as the number of recovered items in both models increases, mainly due to the continuous utilization of recovered items in supplying the demand in Model 2. However, the reorder point r^* does not vary in Model 1 since it is not directly dependent on $(\gamma\ \theta)$. By comparing the total expected cost behavior of Model 1 and Model 2, anyone would also notice that as $(\gamma\ \theta)$ increases, the cost per unit time decreases at a faster rate in Model 2. Looking at the behavior of cost components, one can see that the holding and order costs in Model 1 are insensitive to variations in $(\gamma\ \theta)$. The parameter that drives the decrease in the total expected cost in Model 1 is the purchase cost since the manufacturer acquires more recovered items for a lower unit cost. On the other hand, the holding, order, and purchase costs decrease in Model 2, resulting in 33.68% cost savings. One notices by analyzing the reorder point that de-

Table 3
Base case results.

Example (#)	Model 1			Model 2			Δ %
	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	
1	56,567	18,556	$ 882.75	65,451	15,945	$ 755.35	14.43%

Table 4
Sensitivity analysis - demand.

		Model 1			Model 2			
Ex(#)	d	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
1	1,000	7,922	330	$20.9	9,150	277	$108.57	14.31%
2	10,000	25,142	3,553	$183.13	29,060	3,028	$887.01	14.40%
3	100,000	80,354	37,754	$1,750.01	93,046	32,542	$8,273.11	14.44%
4	1,000,000	262,161	397,318	$17,247.29	305,177	345,413	$14,755.21	14.45%

Table 5
Sensitivity analysis – recycling proportion.

		Model 1			Model 2			
Ex(#)	$\gamma\theta$	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
5	0.05	71,651	18,556	$922.75	73,537	18,032	$917.29	0.59%
6	0.125	65,995	18,556	$907.75	70,615	17,249	$856.6	5.63%
7	0.375	47,139	18,556	$857.75	59,839	14,641	$653.94	23.76%
8	0.5	37,711	18,556	$832.75	53,633	13,328	$552.31	33.68%

Table 6
Sensitivity analysis – lead-time.

		Model 1			Model 2			
Ex(#)	L	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
9	0.4	57,241	37,089	$888.67	66,368	31,863	$758.71	14.62%
10	0.6	57,925	55,600	$894.58	67,298	47,754	$762.07	14.81%
11	0.8	58,617	74,088	$900.49	68,244	63,618	$765.42	15.00%
12	1	59,319	92,553	$906.39	69,205	79,454	$768.76	15.18%

Table 7
Sensitivity analysis - holding cost.

		Model 1			Model 2			
Ex(#)	h	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
13	0.0001	106,697	19,155	$865.84	123,328	16,555	$741.98	14.31%
14	0.001	34,239	18,048	$910.7	39,678	15,426	$776.35	14.75%
15	0.01	11,414	16,755	$1,139.76	13,354	14,094	$928.23	18.56%
16	0.1	4,412	15,022	$2640.81	5,367	12,252	$1,740.58	34.09%

creases in Model 2 are because the manufacturer recovers more items over the inventory cycle.

We conducted a one-way sensitivity on the lead-time by increasing L from 0.2 months (base case) to 1 month in increments of 0.2-month. The results presented in Table 6 indicate that although the total cost per unit time for both models increases with lead-time, Model 2 becomes more efficient. Specifically, the holding and shortage costs in both models increase with longer lead-times. The order cost decreases while the order quantity Q^* and reorder point s^* increase in both models because the manufacturer prefers to hold more inventory to avert a stockout situation. By comparing the reorder point of Model 2 to the reorder point in Model 1, at L of 1 month, one notices that it is lower by 14.15%, which indicates that Model 2 is more resilient to longer lead times.

We also conducted a one-way sensitivity analysis for holding cost h by increasing the base case h from $0.0001/bottle/month to $0.1 in multiples of 10. Table 7 shows that the savings of Model 2 reach 34.09% for higher holding costs. The detailed results showed that as we increase h to $0.1, Model 2 holding costs are less than that of Model 1 by 50%. At that h, the holding cost in Model 2 constitutes 42.71% of the total cost compared to 56.67% in Model 1, which indicates that Model 2 is less sensitive to increases in holding costs. Furthermore, the number of ordered items Q^* and reorder point r^* decrease in both models since the solution equations are inversely proportional to h. We, therefore, expect the manufacturer to respond as suggested to increases in its holding costs.

The results in Table 8 show that $E[CPUT_2]$ is more sensitive to changes in $cv(D_L)$ than $E[CPUT_1]$. As it increases from 0.5 to 3, $E[CPUT_2]$ increases by about 3.7% compared to $E[CPUT_1]$, which increases by 2.2%. The term $cv(D_L)$, the coefficient of variation of the demand counts over lead time, captures the uncertainty/variability in the number of demanded items over the lead-time. As variability increases, the manufacturer is interested in holding inventory to decrease the probability of stockouts. Both models recommend that the manufacturer makes larger orders (higher Q^*) at a higher reorder point r^*. The results of a detailed analysis showed, as expected, that increases in holding and shortage costs negatively affect the total cost. As we further analyze the savings at the holding cost level, we observe that Model 2 provides an 18% saving in holding cost for $cv(D_L) = 0.5$. When $cv(D_L) = 2$, Model 1 outperforms Model 2 as it has lower holding costs by 5%, reaching 13% when $cv(D_L) = 3$. From a managerial perspective, holding more inventory (as proposed by Model 1) is a better solution in a market with a highly variable demand process.

Table 8
Sensitivity analysis - coefficient of variability of the number of demanded items over the lead time.

		Model 1			Model 2			
Ex(#)	$cv(D_L)$	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
17	0.5	57,015	24,248	$884.29	66,057	21,529	$757.58	14.33%
18	1	58,155	38,432	$888.15	67,602	35,458	$763.15	14.07%
19	1.5	59,319	52,553	$891.99	69,189	49,316	$768.71	13.82%
20	2	60,510	66,610	$895.82	70,816	63,101	$774.26	13.57%
21	2.5	61,726	80,603	$899.64	72,485	76,810	$779.8	13.32%
22	3	62,970	94,533	$903.46	74,195	90,442	$785.32	13.08%

Table 9
Sensitivity analysis - order cost.

		Model 1			Model 2			
Ex(#)	K	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
23	10	40,177	18,877	$875	46,522	16,271	$748.65	20.71%
24	30	69,141	18,363	$888.71	79,973	15,749	$760.51	20.66%
25	40	79,740	18,223	$893.75	92,214	15,607	$764.86	20.57%
26	50	89,078	18,113	$898.20	102,998	15,495	$768.70	20.54%
27	60	97,520	18,023	$902.21	112,746	15,403	$772.18	20.51%
28	70	105,282	17,945	$905.91	121,711	15,324	$775.38	20.48%
29	80	112,507	17,875	$909.36	130,054	15,256	$778.36	20.45%

Table 10
Sensitivity analysis - newly manufactured items purchase cost.

		Model 1			Model 2			
Ex(#)	c_Q	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
30	0.002	56,567	18,556	$282.75	65,451	15,945	$155.35	45.06%
31	0.02	56,567	18,556	$957.75	65,451	15,945	$830.35	13.30%
32	0.2	56,567	18,556	$7,707.75	65,451	15,945	$7,580.35	1.65%
33	0.25	56,567	18,556	$9,582.75	65,451	15,945	$9,455.35	1.33%

In Table 9, as we increase the order cost K from 10 dollars to 80 dollars in steps of 10, both models propose that the manufacturer increases the order quantity Q^* and lowers the reorder point r^*, which leads to a higher total cost per unit time in both models. Even though the efficiency of Model 2 decreases slightly with the increase of K, Model 2 remains more efficient than Model 1 in our analysis. The slight change in Δ indicates that both models are influenced equally by the variation of K.

We also performed a one-way sensitivity analysis for the purchase cost of newly manufactured items where we varied c_Q from $0.002/unit to $0.25/unit in Table 10. The results show that $E[CPUT_1]$ and $E[CPUT_2]$ increased to reach $9582.75/month and 9,455.35/month, respectively. Q^* and r^* values are insensitive to changes in c_Q for both models. The difference in expected total costs between Model 1 and 2 decreased from 45.06% to 1.33%. The results suggest that there is a c_Q value for which both models become indifferent. The purchase cost is the only parameter that influences this variation in both models. One could justify the effect of c_Q by analyzing the source of bottles in both models, where, in Model 1, it is the sum of the purchase cost of newly manufactured items and the recovered items provided by the supplier. On the other hand, Model 2 depends on purchasing 'newly' manufactured items and recycling collected items.

Next, we conduct a sensitivity analysis concerning the purchase cost of recovered items c_{R_T} in Table 11. As c_{R_T} increases from $0.001/unit to $0.15/unit, the efficiency of Model 2 increases from -4.87% to 70.75%, suggesting that in-house recycling is recommended when the supplier of recovered items charges a high unit price. This happens when the supplier's collection and recycling process are expensively prohibiting it from offering the manufacturer recovered items at a competitive price.

The results in Table 11 show that the number of recovered items over the lead-time is insensitive to the coefficient-of-variation, $cv(R_L)$. As we increase $cv(R_L)$ from 0.1 to 0.5 in steps of

Table 11
Sensitivity analysis - recovered items purchase cost.

		Model 1			Model 2			
(#)	c_{R_T}	Q^*	r^*	$E[CPUT_1]$	Q^*	r^*	$E[CPUT_2]$	Δ
34	0.001	56,567	18,556	$720.25	65,451	15,945	$755.35	-4.87%
35	0.01	56,567	18,556	$832.75	65,451	15,945	$755.35	9.29%
36	0.1	56,567	18,556	$1,957.75	65,451	15,945	$755.35	61.42%
37	0.15	56,567	18,556	$2,582.75	65,451	15,945	$755.35	70.75%

0.1, the solution given by Model 1 remains the same, and thus the total cost is not affected. This result is mainly due to the outsourcing assumption in Model 1, where it decreases the effect of variability on the manufacturer's inventory and incurs it on the suppliers. Furthermore, another justification is that the base case adopts a short lead-time, which decreases the effect of $cv(R_L)$ on the models. The results show a slight increase (by about 0.5%) in the total cost per unit time in Model 2. Varying the shortage cost from $0.15/unit to $0.35/unit in steps of 0.05 produced similar results. The results showed a decrease in the shortage cost component of the total cost per unit time, where both models recommend using a higher reorder point r^* to reduce the cost of probable stockouts. The numerical examples quantify the improvement of in-house recycling over outsourcing by Δ. We summarize the findings of our numerical results as they related to Δ.

1- Fast/Slow moving items: The cost-efficiency of recycling in-house, as calculated by Δ, remained almost unchanged as the demand rate increased from 1,000 (Δ = 14.31%) to 1,000,000 (Δ = 14.45%). This finding shows that the cost efficiency of in-house recycling is insensitive to increases in the demand rate. Applications of recycling models can include fast-moving items, e.g., PET bottles, or items with relatively lower demand, such as home appliances and electronics (Engeland et al., 2020; [7]). The results show that as demand increases by increasing, for example, the market share, the change in cost-effectiveness is negligible. From a managerial perspective, this implies that an increase in market volume does not motivate a change in recycling policy.

2- High recycling proportion: The numerical results of Table 5 show that for high recycling proportions, in-house recycling is more financially lucrative (cost efficiency improved from 0.59% to 33.68% when the proportion recovered increased from 0.05 to 0.5). A deposit refund system, shown as a practical and effective approach, increases the recycling proportion. This system proved to be successful in several countries [6,45]. Accordingly, countries with efficient and high recycling proportions would be highly motivated to consider in-house recycling over outsourcing the recycling process.

3- Long lead-time: When the system experiences longer lead times by the supplier, our results illustrate that the cost-efficiency of in-house recycling improves (Δ increased from 14.43% to 15.18% as the lead-time increased from 0.4 to 1-time unit). Long and variable lead times are characteristics of reverse logistics inventory systems [11]. The mathematical framework presented in this paper quantifies the sensitivity of the cost-efficiency to changes in the lead time.

4- High holding cost: Higher holding costs also favor in-house recycling (Δ increased from 14.31% to 34.09% as h increased from 0.0001 to 0.1). Real-world applications of recyclable items vary from low-cost items such as water bottles to more expensive items, e.g., electronics and appliances [53]. From a managerial perspective, this further motivates in-house recycling for expensive items due to high opportunity costs.

5- Demand variability over lead-time: High variations in demand reduce the cost-efficiency of in-house recycling (Δ decreased from 14.33% to 13.08% as coefficient-of-variation over lead-time increased from 0.5 to 3). Variable and uncertain demand is a challenge faced by supply chain managers [21], where sources of uncertainty can be due to the economic environment, competition, customer behavior, among other sources [31]. The mathematical framework allows a manager to quantify the impact of demand variability on the cost-effectiveness of the recycling process.

To evaluate the cost benefits of introducing recycling to our model, we compare our model to the case where the recycling proportion is set to 0, $(\gamma\ \theta) = 0$. This results in an order quantity Q^* of 75,422 bottles and a reorder point r^* of 18,556 bottles. The total expected cost based on the previous solution is $932.75/month, which is greater by 5% and 15%, compared to the total costs of Model 1 and Model 2, respectively. Although the environmental benefits of recycling are well-documented, the analyses in this paper reflect the added value of the proposed models in terms of cost savings.

Several models found in the literature do not consider a reorder point, $r = 0$, and only account for batch ordering/producing when the inventory level is zero (see, e.g., [3,10,46], among others). We numerically examine the performance of our models for a reorder point of 0 by running the base case when $r = 0$. This results in an order quantity $Q^* = 624{,}375$ bottles and a total expected cost of $1,148.62/month, which corresponds to a 23% increase in cost.

5. Conclusion and future work

The well-established environmental benefits of product recovery (e.g., recycling, repair, remanufacturing) motivated this work, especially for industries seeking a balance between environmentally friendly processes and managing an economically efficient supply chain system. Furthermore, the effects of uncertainty on inventory systems and, subsequently, policies in a wide range of industries are well established in practice and thoroughly investigated in the inventory-control literature. Calculating the reordering levels to manage and offset uncertainty effects becomes a key challenge for supply chain managers. Accordingly, this work addressed the operational side of a stochastic inventory system of recyclable items by providing economically optimal reordering policies. The first model investigates outsourcing the recycling process to an external supplier, while the second model considers in-house recycling. Both models assume demand over lead-time, and the number of recovered items is a cycle as random variables and have a continuous reordering policy with deterministic lead times.

Although this work is motivated by the plastic-bottle industry, extensions to include other recyclable items are also possible [1,33]. This work provides a managerial tool for practitioners who wish to make outsourcing decisions and setting inventory reordering levels. Numerical analyses in this paper illustrated the performance behavior for both systems (in-house and outsourcing manufacturing activities) for a range of input parameters. We adopted percent savings as a measure to compare the results from both models, which serves to quantify the monetary advantages/disadvantages of recycling in-house vs. outsourcing. The numerical results illustrate that in-house recycling becomes significantly more favorable when the proportion of recovered items is high. The numerical study also shows that in-house recycling is more profitable than outsourcing for high holding costs and long lead times. However, its profitability decreases with an increase in demand variability over long lead times.

A limitation of this work is that it did not investigate the operational complexity or feasibility of recycling in-house. Future work would address this limitation and account for investing machinery, workers, transportation, among other operational activities of in-house manufacturing. Another extension worth exploring is analyzing the stochastic nature of demand and recovery in practice (repair/refurbish/remanufacture/recycle). To validate what probability distribution functions demand and return rates necessitates collecting and analyzing data from various recovery programs. Conducting this experiment will help in better positioning the models of this paper or those viewed as extensions. Another limitation of this work is that it does not consider the carbon emissions resulting from the remanufacturing process. We refer the reader to Turki et al. [52] and Turki and Rezg [50] for reverse logistics systems that account for carbon emissions.

Declaration of Competing Interest

None.

CRediT authorship contribution statement

Bassam K. Hallak: Conceptualization, Methodology, Writing - original draft, Software. **Walid W. Nasr:** Supervision, Conceptualization, Methodology, Writing - original draft. **Mohamad Y. Jaber:** Conceptualization, Writing - review & editing.

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Appendix

Computing the Re-ordering Policy – Model 1

The cost per cycle equation is expressed as a function of the holding, ordering, and shortage costs as follows,

$$CPC_1 = c_Q\ Q + c_{R_T}\ E[R_T] + h\left(\frac{Q+R_T}{2} + SS\right)\sum_{i=1}^{Q+R_T} X_i + K + p \times n(r).$$

The expected cost per cycle is then calculated by the following equation,

$$E[CPC_1] = c_Q\ Q + c_{R_T}\ E[R_T] + h \times \left(\left(\frac{Q}{2} + E[SS]\right)\frac{Q+E[R_T]}{d} + \frac{E\left[{R_T}^2\right]}{2d} + Q\ \frac{E[R_T]}{2d}\right) + K + p \times n(r)$$

Since the behavior of the inventory system of Model 1 is a renewal process with a cycle time of duration T, the cost per unit time of Model 1 ($CPUT_1$) is calculated as the expected cost per cycle $E[CPUT_1]$ divided by the expected cycle time, $E[T]$ [39],

$$E[CPUT_1] = \frac{E[CPC_1]}{E[T]}.$$

This results in the following equation,

$$E[CPUT_1] = \frac{(\ c_Q\ Q + c_{R_T}\ E[R_T]\ )d}{Q+E[R_T]} + h \times \frac{d}{Q+E[R_T]} \times \left(\left(\frac{Q}{2} + E[SS]\right) \times \frac{Q+E[R_T]}{d} + \frac{E\left[{R_T}^2\right]}{2\ d} + Q \times \frac{E[R_T]}{2\ d}\right)$$
$$+K \times \frac{d}{Q+E[R_T]} + p \times \frac{d}{Q+E[R_T]} \times n(r)$$

The cost function of can be shown to be convex as a function of the order quantity Q. To prove convexity, the Hessian for $E[CPUT_1]$ is calculated as,

$$H(Q,r)_1 = \begin{bmatrix} \frac{-2(-1+\theta)(p\ n(r)+K)d}{Q^3} & \frac{(-1+\theta)p\ n(r)d}{Q^2} \\ \frac{pd(1-F(r))\left(1+\frac{\theta}{1-\theta}\right)}{\left(Q+Q\frac{\theta}{1-\theta}\right)^2} & -\frac{pd(1-f(r))}{Q+Q\frac{\theta}{1-\theta}} \end{bmatrix}.$$

The resulting Hessian matrix is positive semidefinite since,

$$\frac{p(1-\theta)\left(Q^2\ d\ z_2^2[1-f(r)] + 2p\ n(r)\ d\ z_1^2 + 2\ K\ d\ z_1^2\right)}{Q^3},$$

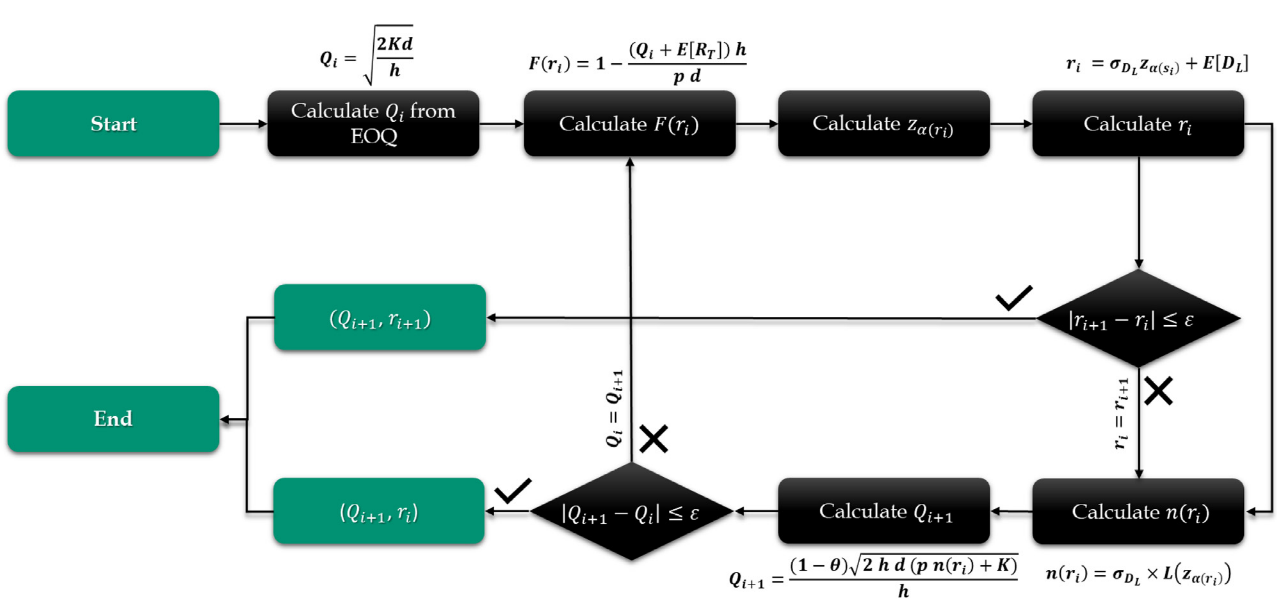
is positive for all positive values of Q and r, and, hence, completes the proof of convexity of $E[CPUT_1]$. Thus, the closed-form solution, i.e., the order quantity Q, is obtained by taking the derivative of the expected total cost per unit time with respect to Q,

$$Q^* = \frac{(1-\theta)\sqrt{2\ h\ d\ (p\ n(r)+K)}}{h}.$$

Furthermore, the equation for the reorder point s is calculated by considering the derivative of $E[CPUT_1]$with respect to r,

$$F(r^*) = 1 - \frac{(Q+E[R_T])\ h}{p\ d}.$$

Model 1 Iterative Procedure



Computing the Re-ordering Policy - Model 2

The cost per cycle equation for Model 2 (CPC_2) is now expressed as follows,

$$CPC_2 = c_Q\ Q + h\left(\frac{Q}{2} + SS\right)\sum_{i=1}^{Q}(X_i - Z_i) + K + p\ n(r)$$

The expected cost per cycle is then calculated as,

$$E[CPC_2] = h\ \times E\left[\left(\frac{Q}{2} + SS\right)\ \sum_{i=1}^{Q}(X_i - Z_i)\right] + K + p\ n(r)$$

$$= h\ \times\left(\left(\frac{Q}{2} + E[SS]\right)\ \frac{Q}{d_e}\right) + K + p\ n(r).$$

Since the inventory system of Model 2 is also a renewal process, the cost per cycle is divided by the cycle time $E[T]$ to obtain the cost per unit time for Model 2, ($CPUT_2$). This results in the following equation for Model 2,

$$E[CPUT_2] = \frac{\left(c_Q\ Q + \frac{h\ Q\ \left(\frac{Q}{2}+E[SS]\right)}{d_e} + K + p\ n(s)\right)\ d_e}{Q} + TRC_2.$$

To prove convexity, the Hessian for $E[CPUT_2]$ is calculated as,

$$H(Q, r)_2 = \begin{bmatrix} \frac{2\ d_e\ (p\ n(r)\ + K)}{Q^3} & \frac{-p(1-F(r))\ \ d_e}{Q^2} \\ \frac{p(1-F(r))\ \ d_e}{Q^2} & \frac{p\ d_e(1-f(r))}{Q} \end{bmatrix}.$$

The resulting Hessian matrix is positive semidefinite since,

$$\frac{p\ Q^2\ \ d_e\ z_2^2\ [1 - f(r)] + 2p\ n(r)\ d_e\ z_1^2 + 2\ K\ d_e\ z_1^2}{Q^3},$$

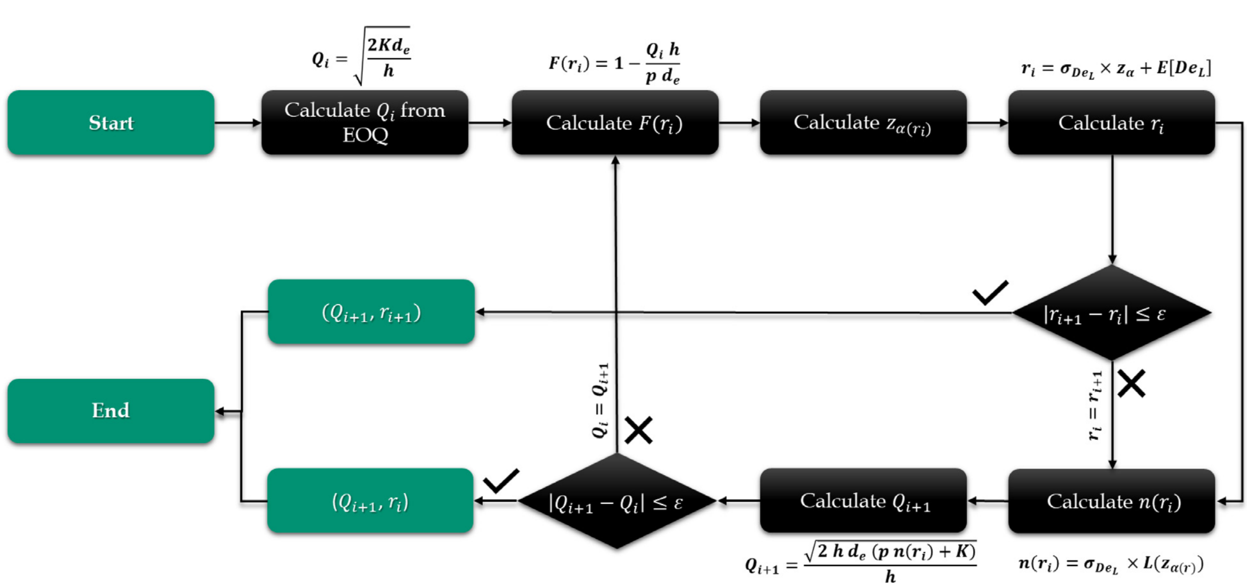
is positive for all positive values of Q and s then $CPUT_2$ is convex. Thus, the equation for the order quantity Q is calculated by taking the derivative of the expected total cost per unit time with respect to Q,

$$Q^* = \frac{\sqrt{2\ h\ d_e\ (p\ n(r) + K)}}{h}\ .$$

The equation for the reorder point s is calculated by deriving the expected total cost per unit time with respect to r,

$$F(r^*) = 1 - \frac{Q\ h}{p\ d_e}$$

Model 2 Iterative procedure



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